Video 12

Probability Theory

Picture থেকে output দেয় → Cat <.75> → Cat Final Decision

→ Dog <.25>

Agent → Store knowledge

System → detects the general weather of march.

weather → Sunny, rainy, cold.

Need data of previous years march weather.

50 years → 45 march (sunny/hot)

In general, March → P(sunny/hot) → $\frac{45}{50}$

= 90%

Cat
→ Final
Decision

Machine learning classifier → Naive Bayes classifier
↳ model
↳ based on probability theory.

Terminology of probability

Events/variables → A B C {has domain value}

Event A → (A, ¬A → not A)

Event B → (B, ¬B → not B)

একটা variable এর under এ multiple domain ও থাকতে পারে।

Variable weather → ⟨sunny, rainy, cloudy, snowy⟩
4 Domain Values

প্রত্যেকটা Domain এর সাথে কিছু probabilistic value associated থাকবে, and summation of all probabilistic value will be 1.

Variable weather $\rightarrow \langle$ sunny, rainy, cloudy, snowy $\rangle$

$\langle 0.72, \; .1, \; 0.08, \; 0.1 \rangle$

$\Rightarrow .72 + .1 + .08 + .1 = 1$

$P(\text{sunny}) = .72 \qquad P(\text{raining}) = 0.1$

$P(\text{cloudy}) = .08 \qquad P(\text{snowy}) = .1$

এগুলো represent করে Domain এর probabilistic value.

হওয়ার probability

না হওয়ার probability

$P(A) \qquad P(\neg A)$

$P(B) \qquad P(\neg B)$

representation.

Probability এর value always $\boxed{0 \leq P(A) \leq 1}$

also can't be negative

$P(\neg A) = 1 - P(A)$

$A \to A, \neg A$

$P(A) + P(\neg A) = 1$

$P(\neg A) = 1 - P(A)$

we can write.

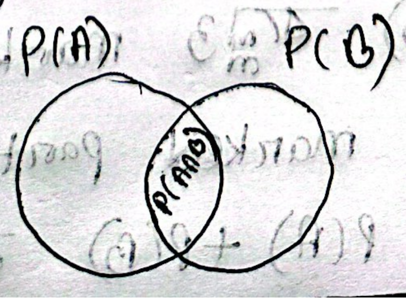
Probability multiple variable combination এও হতে পারে।

A

$P(A \cap B) \leftarrow$ Joint Probability.

$P(A \cap B) \rightarrow$ যেখানে Duita event A and B একসাথে হওয়ার Probability.

Venn Diagram দিয়ে represent করতে পারি।



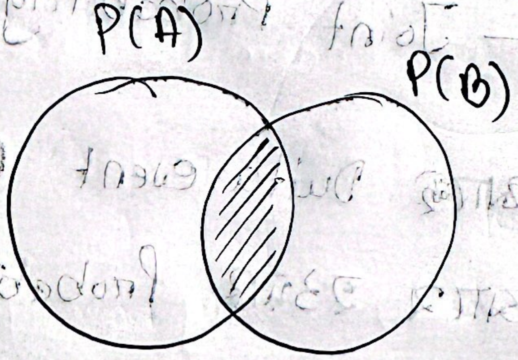
$P(A \cup B) \rightleftarrows P(A \text{ or } B)$

এটির মানে হলো হয় দুটি একসাথে হবে আর না হলে একটি হবে।

$P(A \cap B) \rightleftarrows P(A, B) \rightleftarrows P(A \text{ and } B)$ } Same meaning

formula

$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$ } relation between or/and.



P(A) বললেও marked part আসতেছে

P(B) বললেও marked part আসতেছে।

and marked part হচ্ছে $(A \cap B)$.

As, $P(A) + P(B)$ করলে marked part 2 বার

আমাদের তাই একবার $P(A \cap B)$ substract করেছি।

Another formula

$$P(A \setminus B)$$

$A \to$ covid.

$B \to$ fever.

A given B মানে হচ্ছে B situation already occured. Given that situation, what is the outcome of A.

Situation: আমার already জ্বর আসছে, Now what is Any probability of getting affected by covid.

$$P(\text{covid} \setminus \text{fever}) = \frac{P(\text{covid} \cap \text{fever})}{P(\text{fever})}$$

covid $\to$ A, fever $\to$ B

$$= \frac{P(A \cap B)}{P(B)}$$

Joint Probability

↳ $P(A \cap B) \rightarrow$ Probability of A and B happening at the same time.

$A \rightarrow A, \neg A$
$B \rightarrow B, \neg B$

} $A \cap B$ এর outcome এ 4টা outcome আসতে পারে।

$P(A \cap B)$ $\quad$ $P(\neg A \cap \neg B)$
$P(A \cap \neg B)$ $\quad$ $P(\neg A \cap B)$

add করলে 1 হবে

A and B একসাথে হওয়া।

A হওয়া and B না হওয়া।

Joint Probability distribution.

A না হওয়া and B হওয়া

A and B কোনটিই একসাথে না হওয়া

We can represent it in tabular form.

	A	¬A
B	$P(A \cap B)$	$P(\neg A \cap B)$
¬B	$P(A \cap \neg B)$	$P(\neg A \cap \neg B)$

Joint distribution table.

	A	¬A
B	.75	.15
¬B	.05	.05

Sum = 1.

$$.75 + .15 + .05 + .05 = 1$$

$P(A \cap B) = .75$

$P(\neg A \cap B) = .15$

$P(A \cap \neg B) = .05$

$P(\neg A \cap \neg B) = .05$

alarm $\longrightarrow$ alarm, $\neg$alarm

$\downarrow\uparrow$

buglary $\longrightarrow$ buglary, $\neg$buglary.

	alarm	$\neg$ alarm
buglary	.09	.01
$\neg$buglary	.1	.8

Exam.

$P(\text{alarm} \cap \text{buglary}) = .09$

$P(\neg\text{alarm} \cap \text{buglary}) = .01$

$P(\neg\text{buglary} \cap \text{alarm}) = .1$

$P(\neg\text{alarm} \cap \neg\text{buglary}) = .8$

$P(\text{alarm}) = .09 + .1 = .19$

$P(\neg\text{alarm}) = .01 + .8 = .81$

$p(\text{alanm})$

$= p(\text{alanm} \cap \text{buglany}) + p(\text{alanm} \cap \neg\text{buglany})$

$= .09 + .1$

$= .19$

$P(\text{buglany}) = P(\text{buglany} \cap \text{alanm}) + P(\text{buglany} \cap \neg\text{alanm})$

$= .09 + .01$

$= .1$

$P(\neg\text{buglany}) = P(\neg\text{buglany} \cap \text{alanm}) + P(\neg\text{buglany} \cap \neg\text{alanm})$

$= .1 + .8$

$= .9$

$P(\neg\text{alanm}) = P(\neg\text{alanm} \cap \text{buglony}) + P(\neg\text{alanm} \cap \neg\text{buglony})$

$= .01 + .8$

$= .81$

$P(alarm \mid buglary) \rightarrow$ conditional probability

$$\Rightarrow \frac{P(alarm \cap buglary)}{P(buglary)} = \frac{.09}{.1} = .9$$

এটা বুঝাচ্ছে যে buglary হলে, alarm বাজার probability .9 অথবা 90%.

$$P(\neg alarm \mid \neg buglary) = \frac{P(\neg alarm \cap \neg buglary)}{P(\neg buglary)}.$$

$$= \frac{.8}{.9}$$

$$= .88$$

↳ buglary না হলে alarm না বাজার probability.

Cheated on College Exam?

	Yes	No
Male	.32	.22
Female	.28	.18

$P(\text{Male} \cap \text{Yes}) = .32$ $\qquad$ $P(\text{Yes}) = .32 + .28 = .6$

$P(\text{Male} \cap \text{No}) = .22$ $\qquad$ $P(\text{No}) = .22 + .18 = .4$

$P(\text{Female} \cap \text{Yes}) = .28$ $\qquad$ $P(\text{Male}) = .32 + .22 = .54$

$P(\text{Female} \cap \text{No}) = .18$ $\qquad$ $P(\text{Female}) = .28 + .18 = .46$

$$P(\text{Male} \setminus \text{cheated}) = \frac{P(\text{Male} \cap \text{cheated})}{P(\text{cheated})}$$

$$= \frac{.32}{.6} = .533$$

$P(\text{female} \cap \text{No}) = .18$

$$P(\text{Male} \setminus \text{Yes}) = \frac{P(\text{Male} \cap \text{Yes})}{P(\text{Yes})} = \frac{.32}{.6} = .53$$

if any already did cheat then, ~~propa~~ probability of the person being male is $.53\%$.

	Right handed	Left handed	
Male	.41	.08	.49 = Male
female	.45	.06	.51 = female
	.86	.14	
	"	"	
	Right handed	Left handed.	

(a)

$$P(M \mid R) = \frac{P(M \cap R)}{P(R)} = \frac{.41}{.86} = .477$$

(b)

$$P(R \mid M) = \frac{P(R \cap M)}{P(M)} = \frac{.41}{.49} = .837.$$

(c)

$$P(F \mid L) = \frac{P(F \cap L)}{P(L)} = \frac{.06}{.14} = .429$$

(d)

$$\left.\begin{array}{l} P(F \mid L) = .429 \\ P(F) = .51 \end{array}\right\} \neq$$

So, F and L are not independent.

Joint Distribution table ২টির বেশি variable নিয়েও হতে পারে।

	tootache		¬tootache	
	Show on XRay	¬Show on XRay	Show on XRay	¬Show on XRay
Cavity	.108	.012	.072	.008
¬cavity	.016	.064	.144	.576

t = toodache

xi = xRay

c = cavity.

3 variable each 2 outcomes.

$2^3 = 8$ values.

if 4 variable with 2 outcomes.

then, $2^4 = 16$ outcome values.

if 5 variable with 2 outcomes.

then, $2^5 = 32$ outcome values.

$P(t \cap a \cap c) = .108$

$P(\neg t \cap \neg x \cap \neg c) = .008$

$P(x) = .108 + .016 + .072 + .144$

= marginal probability

Question

↳ $P(t \cap c)$

$= .108 + .012$

$= .12.$

↳ $P(\neg c \cap x)$

$= .016 + .144$

$= .16$

↳ $P(\neg x)$

$= .012 + .064 + .008 + .576$

$= .66$

↳ $P(t \mid \neg x) = \dfrac{P(t \cap \neg x)}{P(\neg x)} = \dfrac{.012 + .064}{.66} = \dfrac{.076}{.66}$

$= .115\%$

Video 13

	Toothache		¬ Toothache	
	Xray	¬ Xray	Xray	¬ Xray
Cavity	.108	.012	.072	.008
¬ Cavity	.016	.064	.144	.576

Question

↳ $P(\text{toothache} \mid \text{cavity} \cap \text{Xray})$

$$= \frac{P(\text{toothache} \cap \text{cavity} \cap \text{Xray})}{P(\text{cavity} \cap \text{Xray})}$$

$$= \frac{.108}{.108 + .072}$$

$$= \frac{.108}{.18} = 0.6$$

Independence

Two branch

- → full independence
- → Conditional independence

independence usually 2 টি event এর perspective এ discuss করা হয়।

A   B

A এর outcome doesn't effect the outcome of B → A and B are independent.

Scenario

Fair Toin Toss $P(h) = .5$ $P(T) = .5$

30 বার টানা head পড়লেও 31 বার এ head and tail পড়ার probability same. Doesn't affect. Independent.

So, fair coin tosses are independent.

Formula

$$P(A \cap B) = P(A) \times P(B)$$

$$L.H.S = \cancel{P(H) *} P(H \cap T)$$

$$= .25$$

$$R.H.S = P(A) * P(B)$$

$$= P(H) * P(T)$$

$$= .5 * .5$$

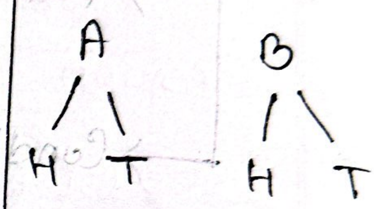
$$= .25$$

$$\therefore L.H.S = R.H.S$$

$$P(A \cap B) = P(A) * P(B)$$

So, coin tosses are independent.

Two coin toss.



Outcomes:

HH → .25
HT → .25
TT → .25
TH → .25

Conditional Independence

Bias Coin Toss {But don't know which side bias}.

Scenario: Coin toss করলাম, 3 বার, 3 বারই head পরলো and আমি জানি coin টা bias. So, 4th time head and tail পরার probability same থাকবে না।

$\boxed{\begin{array}{l} P(H) = .5 \\ P(T) = .5 \end{array}}$ } Prothom 2 বার এর পর probability change হয়ে গেলো।

$P(H) = .75$

$P(T) = .25$

Bias টা condition হিসেবে set করে দিলে তখন এটা conditional independence

We are introducing a particular condition and after this conditional independence is achieved?

Example: 2 টা person একটি place এ যেতে চেয়েছে and আমি জানি storm হলে তাদের বাসায় আসতে late হবে। initially আমাকে বলা হয় নাই যে, storm হচ্ছে নাকি না।

Person 1 late করে আসলো বাসায়। person 1 কে late করতে দেখে আমি assume করলাম, যে good chance যে storm হচ্ছে।

So, Person 2 ও বাসায় late করে আসতে পারে।

Person 1 এর late এ আসা বা না আসা Person 2 এর আসা বা না আসার probability effect করতেছে।

Now, storm হচ্ছে এটা একটা given condition. already আমি জানি যে ~~late 2~~ storm হচ্ছে,

So, তখন both person 1 and 2 এর বাসায় late করে আসার probability বেশি থাকবে।

And, তখন, person 1 late করে আসতিসে নাকি না এটার উপর base করে person 2 এর late করে আসার probability change হবে না।

Condition → Snow storm is happening. Based on the condition person 1 and person 2 independent.

formula

$$\underset{\text{independence}}{A \rightleftarrows B} \xrightarrow[\text{condition}]{\text{Given}} C$$

L.H.S R.H.S

$$P(A \cap B \mid C) = P(A \mid C) * P(B \mid C)$$

* কোন event independent মানে যে তারা conditional independent এটা তেমন না।

L.H.S = R.H.S

equal হলে conditional independent otherwise not.

$$P(\underbrace{A \cap B}_{A} \mid \underbrace{C}_{B}) = P(A \mid C) * P(B \mid C)$$

same

$$\frac{P(A \cap B \cap C)}{P(C)} = P(A \mid C) * P(B \mid C)$$

same

$$P(A \cap B \cap C) = P(A \mid C) * P(B \mid C) * P(C)$$

Maths of Normal Independence

$P(A \cap B) = P(A) * P(B)$.

if equal then independent.

	Smart		¬Smart	
	Study	¬Study	Study	¬Study
Prepared	.432	.16	.084	.008
¬Prepared	.048	.16	.036	.072

↳ is smart independent of study.

$P(\text{Smart} \cap \text{Study}) = P(\text{smart}) * P(\text{study})$

L.H.S

$= P(\text{smart} \cap \text{study})$

$= P(.432 + .048)$

$= .48$

R.H.S

$P(\text{smart}) * P(\text{study})$.

$= (.432 + .16 + .048 + .16) *$
$(.432 + .048 + .084 + .036)$

$= .8 * .6$

$= .48$

L.H.S = R.H.S

Smart and study are independent.

↳ Is prepared independent of study.

$P(\text{prepared} \cap \text{study}) = P(\text{prepared}) * P(\text{study})$.

L.H.S	R.H.S
$P(\text{prepared} \cap \text{study})$	$= P(\text{prepared}) * P(\text{Study})$
$= P(.432 + .084)$	$= P(.432 + .16 + .084 + .008)$
	$* (.432 + .048 + .084 + .036)$
$= P(.516)$	
$= .516$	$= (.684) * (0.6)$
	$= .4104$.

So, they are not independent.

Math of Conditional Independence

	Smart		¬ smart	
	Study	¬ Study	Study	¬ Study
Prepared	.432	.16	.084	.008
¬ prepared	.048	.16	.036	.072

↳ is smart conditionally independent of prepared, given study.

$P(\text{Smart} \cap \text{Prepared} \setminus \text{Study}) = P(\text{Smart} \setminus \text{Study}) * P(\text{Prepared} \setminus \text{Study})$

L.H.S

$P(\text{Smart} \cap \text{Prepared} \setminus \text{Study})$

$= \dfrac{P(\text{Smart} \cap \text{Prepared} \cap \text{Study})}{P(\text{Study})}$

$= \dfrac{.432}{.432 + .048 + .084 + .036} = \dfrac{.432}{.6}$

$= .72$

R.H.S

$= P(\text{Smart} \setminus \text{Study}) * P(\text{Prepared} \setminus \text{Study})$

$= \dfrac{P(\text{Smart} \cap \text{Study})}{P(\text{Study})} * \dfrac{P(\text{Prepared} \cap \text{Study})}{P(\text{Study})}$

$= \dfrac{.432 + .048}{.6} * \dfrac{.516}{.6}$

$= .8 \times .86$

$= .688$

L.H.S = .72

R.H.S = .688

$\therefore$ L.H.S $\neq$ R.H.S

$\therefore$ smart and prepared are not conditionally independent given study.

↳ Is study conditionally independent of prepared, given smart?

$P(\text{study} \cap \text{prepared} \setminus \text{smart}) = P(\text{study} \setminus \text{smart}) * P(\text{prepared} \setminus \text{smart})$

L.H.S

$= P(\text{study} \cap \text{prepared} \setminus \text{smart})$

$= \dfrac{P(\text{study} \cap \text{prepared} \cap \text{smart})}{P(\text{smart})}$

$= .432/.8$

$= .54$

R.H.S

$P(\text{study} \setminus \text{smart}) * P(\text{prepared} \setminus \text{smart})$

$= \dfrac{P(\text{study} \cap \text{smart})}{P(\text{smart})} * \dfrac{P(\text{prepared} \cap \text{smart})}{P(\text{smart})}$

$= \dfrac{.432 + .048}{.8} * \dfrac{.432 + .16}{.8}$

$= .6 * .74$

$= .44$

$L.H.S = .54$

$R.H.S = .44$

$\therefore L.H.S \neq R.H.S$

So, study and prepared are not conditionally independence given smart.