Force On a Charged Particle in Magnetic Field

(i) If a particle carrying a positive charge q and moving with velocity $\vec{v}$ enters a magnetic field $\vec{B}$ then it experiences a force $\vec{F}$ given by

$\vec{F} = q(\vec{V} \times \vec{B})$, |F| = qVB sinθ

where θ is the angle between $\vec{v}$ and $\vec{B}$ and direction of F is perpendicular to the plane formed by $\vec{v}$ and $\vec{B}$.

(1) **Zero force :** Force on charged particle will be zero (*i.e.* $F = 0$) if

(i) No field *i.e.* $B = 0 \Rightarrow F = 0$

(ii) Neutral particle *i.e.* $q = 0 \Rightarrow F = 0$

(iii) Rest charge *i.e.* $v = 0 \Rightarrow F = 0$

(iv) Moving charge *i.e.* $\theta = 0^o$ or $\theta = 180^o \Rightarrow F = 0$

(2) **Direction of force :** The force $\vec{F}$ is always perpendicular to both the velocity $\vec{v}$ and the field $\vec{B}$ in accordance with Right Hand Screw Rule,

Direction of force on charged particle in magnetic field can also be find by Fleming's Left Hand Rule (FLHR).

When the first three fingers of the left hand are stretched mutually perpendicular to each other, *First finger* (indicates) $\rightarrow$ Direction of magnetic field

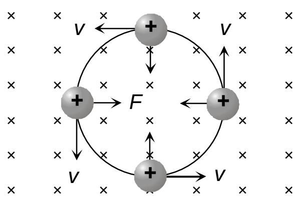
Middle finger $\rightarrow$ Direction of motion of positive charge or direction, Opposite to the motion of negative charge.

Thumb $\rightarrow$ Direction of force

Trajectory of a Charged Particle in a Magnetic Field

(1) **Straight line :** If the direction of a $\vec{v}$ is parallel or antiparallel to $\vec{B}$, $\theta = 0$ or $\theta = 180^o$ and therefore $F = 0$. Hence the trajectory of the particle is a straight line.

(2) **Circular path :** If $\vec{v}$ is perpendicular to $\vec{B}$ *i.e.* $\theta = 90^o$, hence particle will experience a maximum magnetic force $\boldsymbol{F_{max} = qvB}$ which act's in a direction perpendicular to the motion of charged particle. Therefore the trajectory of the particle is a circle.



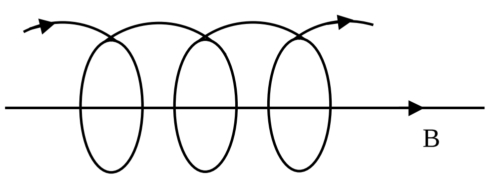
(i) In this case path of charged particle is circular and magnetic force provides the necessary centripetal force *i.e.* $qvB = \frac{mv^2}{r}$ $\Rightarrow$ radius of path

$$\boldsymbol{r = \frac{mv}{qB} = \frac{p}{qB} = \frac{\sqrt{2mK}}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}}}$$

where p = momentum of charged particle and K = kinetic energy of charged particle (gained by charged particle after accelerating through potential difference V) then $p = mv = \sqrt{2mK} = \sqrt{2mqV}$

(ii) If T is the time period of the particle then $T = \frac{2\pi m}{qB}$ (*i.e.*, time period (or frequency) is independent of speed of particle).

(3) **Helical path :** When the charged particle is moving at an angle to the field (other than 0^o, 90^o, or 180^o). Particle describes a path called helix.



(i) The radius of this helical path is $\boldsymbol{r = \frac{m(v\sin\theta)}{qB}}$

(ii) Time period and frequency do not depend on velocity and so they are given by $T = \frac{2\pi\, m}{qB}$ and $\nu = \frac{qB}{2\pi\, m}$

(iii) The *pitch* of the *helix*, (*i.e.*, linear distance travelled in one rotation) will be given by $p = T(v\cos\theta) = 2\pi \frac{m}{qB}(v\cos\theta)$

(iv) If pitch value is p, then number of pitches obtained in length l given as

Number of pitches $= \frac{l}{p}$ and time required $t = \frac{l}{v\cos\theta}$

Lorentz Force

When the moving charged particle is subjected simultaneously to both electric field $\vec{E}$ and magnetic field $\vec{B}$, the moving charged particle will experience electric force $\vec{F_e} = q\vec{E}$ and magnetic force $\vec{F_m} = q(\vec{v} \times \vec{B})$; so the net force on it will be $\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})]$. Which is the famous 'Lorentz-force equation'.

Depending on the directions of $\vec{v}, \vec{E}$ and $\vec{B}$ following situations are possible

(i) **When $\vec{v}, \vec{E}$ and $\vec{B}$ all the three are collinear :** In this situation the magnetic force on it will be zero and only electric force will act and so $\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m}$

(ii) The particle will pass through the field following a straight-line path (parallel field) with change in its speed. So in this situation speed, velocity, momentum and kinetic energy all will change without change in direction of motion as shown

(iii) **$\vec{v}, \vec{E}$ and $\vec{B}$ are mutually perpendicular :** In this situation if $\vec{E}$ and $\vec{B}$ are such that $\vec{F} = \vec{F_e} + \vec{F_m} = 0$ *i.e.*, $\vec{a} = (\vec{F}/m) = 0$

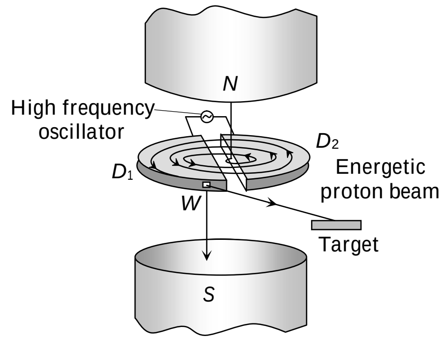
the particle will pass through the field with same velocity, without any deviation in path.

And in this situation, as $F_e = F_m$ *i.e.*, $qE = qvB$ $v = E/B$

This principle is used in 'velocity-selector' to get a charged beam having a specific velocity.

Cyclotron

Cyclotron is a device used to accelerated positively charged particles (like, α-particles, deutrons *etc.*) to acquire enough energy to carry out nuclear disintegration *etc.*



It is based on the fact that the electric field accelerates a charged particle and the magnetic field keeps it revolving in circular orbits of constant frequency.

It consists of two hollow *D*-shaped metallic chambers D_1 and D_2 called dees. The two dees are placed horizontally with a small gap separating them. The dees are connected to the source of high frequency electric field. The dees are enclosed in a metal box containing a gas at a low pressure of the order of 10^{-3} *mm* mercury. The whole apparatus is placed between the two poles of a strong electromagnet *NS* as shown in fig. The magnetic field acts perpendicular to the plane of the dees.

(1) **Cyclotron frequency :** Time taken by ion to describe a semicircular path is given by $t = \frac{\pi r}{v} = \frac{\pi m}{qB}$

If T = time period of oscillating electric field then $T = 2t = \frac{2\pi m}{qB}$ the cyclotron frequency $\nu = \frac{1}{T} = \frac{Bq}{2\pi m}$

(2) **Maximum energy of particle :** Maximum energy gained by the charged particle $E_{\max} = \left(\frac{q^2B^2}{2m}\right)r^2$

where r_0 = maximum radius of the circular path followed by the positive ion.

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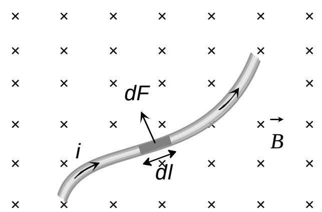
Force On a Current Carrying Conductor In Magnetic Field

In case of current carrying conductor in a magnetic field force experienced by its small length element is $d\vec{F} = id\vec{l} \times \vec{B}$; $id\vec{l}$ = current element $d\vec{F} = i(d\vec{l} \times \vec{B})$



Total magnetic force $\vec{F} = \int d\vec{F} = \int i(d\vec{l} \times \vec{B})$. If magnetic field is uniform *i.e.*,

$\vec{B}$ = constant $\vec{F} = i[\int d\vec{l}] \times \vec{B} = i(\vec{L} \times \vec{B})$

$\int d\vec{l} = \vec{L}' =$ vector sum of all the length elements from initial to final point. Which is in accordance with the law of vector addition is equal to length vector $\vec{L}$ joining initial to final point.

(For a straight conductor $F = Bil\sin\theta$)

Direction of force : The direction of force is always perpendicular to the plane containing $id\vec{l}$ and $\vec{B}$ and is same as that of cross-product of two vectors $(\vec{A} \times \vec{B})$ with $\vec{A} = i\,d\vec{l}$.

The direction of force when current element $id\vec{l}$ and $\vec{B}$ are perpendicular to each other can also be determined by applying either of the following rules

Fleming's left-hand rule : Stretch the fore-finger, central finger and thumb of left hand mutually perpendicular. Then if the fore-finger points in the direction of field $\vec{B}$ and the central in the direction of current *i*, the thumb will point in the direction of force.

Right-hand palm rule : Stretch the fingers and thumb of right hand at right angles to each other. Then if the fingers point in the direction of field $\vec{B}$ and thumb in the direction of current *i*, then normal to the palm will point in the direction of force

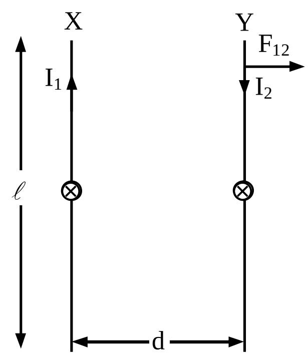
Force Between Two Parallel Current Carrying Conductors

The force on a length l of each of two long, straight, parallel wires carrying currents i_1 and i_2 and separated by a distance a is

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2i_1 i_2}{a} \times l$$

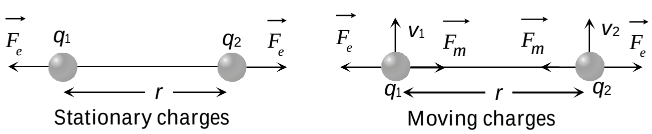
Hence force per unit length $\frac{F}{l} = \frac{\mu_0}{4\pi} \cdot \frac{2i_1 i_2}{a} \left(\frac{N}{m}\right)$ or $\frac{F}{l} = \frac{2i_1 i_2}{a} \left(\frac{dyne}{cm}\right)$

Direction of force : If conductors carries current in same direction, then force between them will be attractive. If conductor carries current in opposite direction, then force between them will be repulsive.



Force Between Two Moving Charges

If two charges q_1 and q_2 are moving with velocities v_1 and v_2 respectively and at any instant the distance between them is r, then



Magnetic force between them is $F_m = \frac{\mu_0}{4\pi} \cdot \frac{q_1 q_2 v_1 v_2}{r^2}$ .... (i)

and Electric force between them is $F_e = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$ .... (ii)

From equation (i) and (ii) $\frac{F_m}{F_e} = \mu_0 \varepsilon_0 v^2$ but $\mu_0 \varepsilon_0 = \frac{1}{c^2}$;

where c is the velocity of light in vacuum. So $\frac{F_m}{F_e} = \left(\frac{v}{c}\right)^2$

As $v < c$ so $F_m < F_e$

Standard Cases For Force on Current Carrying Conductors

***Case* 1 :** When an arbitrary current carrying loop placed in a magnetic field ($\perp$ to the plane of loop), each element of loop experiences a magnetic force due to which loop stretches and open into circular loop and tension developed in it's each part.

***Case* 2 : Equilibrium of a current carrying conductor :** When a finite length current carrying wire is kept parallel to another infinite length current carrying wire, it can suspend freely in air as shown below

In both the situations for equilibrium of XY it's downward weight = upward magnetic force *i.e.* $mg = \frac{\mu_0}{4\pi}.\frac{2i_1 i_2}{h}.l$

***Case* 3 : Current carrying spring :** If current is passed through a spring, then it will contract because current will flow through all the turns in the same direction.

***Case* 4 : Tension less strings :** In the following figure the value and direction of current through the conductor XY so that strings becomes tensionless?

Strings becomes tensionless if weight of conductor XY balanced by magnetic force (F_m).

Current Loop as a Magnetic Dipole

A current carrying circular coil behaves as a bar magnet whose magnetic moment is $M = NiA$; *Where* N = Number of turns in the coil, i = Current through the coil and A = Area of the coil

Magnetic moment of a current carrying coil is a vector and it's direction is given by right hand thumb rule

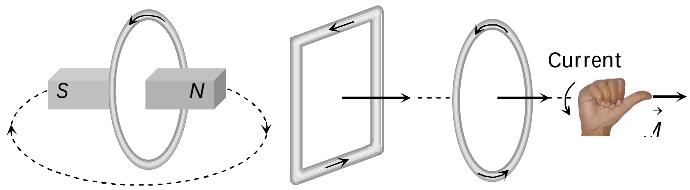


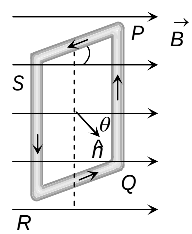
Fig. 21.47

(1) For a given perimeter circular shape have maximum area. Hence maximum magnetic moment.

(2) For a any loop or coil $\vec{B}$ at centre due to current in loop, and $\vec{M}$ are always parallel.

Behaviour of Current Loop in a Magnetic Field

(1) **Torque :** Consider a rectangular current carrying coil *PQRS* having N turns and area A, placed in a uniform field $\vec{B}$, in such a way that the normal $(\hat{n})$ to the coil makes an angle θ with the direction of $\vec{B}$. the coil experiences a torque given by $\boldsymbol{\tau} = \boldsymbol{NBiA\sin\theta}$. Vectorially $\vec{\tau} = \vec{M} \times \vec{B}$



(i) τ is zero when $\theta = 0$, *i.e.*, when the plane of the coil is perpendicular to the field.

(ii) τ is maximum when $\theta = 90^\circ$, *i.e.*, the plane of the coil is parallel to the field $\tau_{\max} = NBiA$

(2) **Workdone :** If coil is rotated through an angle θ from it's equilibrium position then required work. $W = MB(1 - \cos\theta)$. It is maximum when $\theta = 180^\circ \Rightarrow W_{\max} = 2\,MB$

(3) **Potential energy :** $U = -MB\cos\theta \Rightarrow U = -\vec{M}.\vec{B}$