CARNOT ENGINE

It consists of the following parts

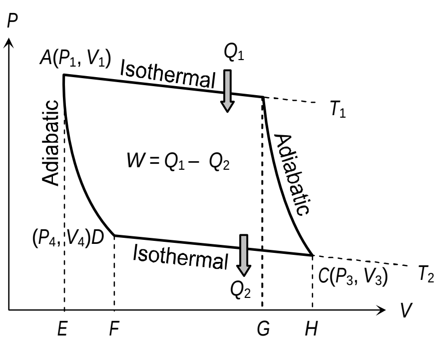
(i) A cylinder with perfectly non-conducting walls and a perfectly conducting base containing a perfect gas as working substance and fitted with a non-conducting frictionless piston

(ii) A source of infinite thermal capacity maintained at constant higher temperature $T_{1.}$

(iii) A sink of infinite thermal capacity maintained at constant lower temperature T_2.

(iv) A perfectly non-conducting stand for the cylinder.

Carnot cycle : As the engine works, the working substance of the engine undergoes a cycle known as Carnot cycle. The Carnot cycle consists of the following four strokes



(i) First stroke (Isothermal expansion) (curve *AB*) :

The cylinder containing ideal gas as working substance allowed to expand slowly at this constant temperature T_1.

Work done = Heat absorbed by the system

$$W_1 = Q_1 = \int_{V_1}^{V_2} P\,dV = RT_1 \log_e\left(\frac{V_2}{V_1}\right) = \text{Area } ABGE$$

(ii) Second stroke (Adiabatic expansion) (curve *BC*) :

The cylinder is then placed on the non conducting stand and the gas is allowed to expand adiabatically till the temperature falls from T_1 to T_2.

$$W_2 = \int_{V_2}^{V_3} P\,dV = \frac{R}{(\gamma-1)}[T_1 - T_2] = \text{Area } BCHG$$

(iii) Third stroke (Isothermal compression) (curve CD) :

The cylinder is placed on the sink and the gas is compressed at constant temperature T_2.

Work done = Heat released by the system

$$W_3 = Q_2 = -\int_{V_3}^{V_4} P\,dV = -RT_2 \log_e \frac{V_4}{V_3}$$

$$= RT_2 \log_e \frac{V_3}{V_4} = \text{Area } CDFH$$

(iv) Fourth stroke (adiabatic compression) (curve DA) :

Finally the cylinder is again placed on non-conducting stand and the compression is continued so that gas returns to its initial stage.

$$W_4 = -\int_{V_4}^{V_1} P\,dV = -\frac{R}{\gamma-1}(T_2 - T_1) = \frac{R}{\gamma-1}(T_1 - T_2) = \text{Area } ADFE$$

Efficiency of Carnot cycle : The efficiency of engine is defined as the ratio of work done to the heat supplied *i.e.* $\eta = \frac{\text{Work done}}{\text{Heat input}} = \frac{W}{Q_1}$

Net work done during the complete cycle

$$W = W_1 + W_2 + (-W_3) + (-W_4) = W_1 - W_3 = \text{Area } ABCD$$

[As $W_2 = W_4$]

$$\therefore \quad \eta = \frac{W}{Q_1} = \frac{W_1 - W_3}{W_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{W_3}{W_1} = 1 - \frac{Q_2}{Q_1}$$

or $\eta = 1 - \frac{RT_2 \log_e (V_3 / V_4)}{RT_1 \log_e (V_2 / V_1)}$

Since points B and C lie on same adiabatic curve

$$\therefore \quad T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1} \text{ or } \frac{T_1}{T_2} = \left(\frac{V_3}{V_2}\right)^{\gamma-1} \qquad \text{.....(i)}$$

Also point D and A lie on the same adiabatic curve

$$\therefore \quad T_1 V_1^{\gamma-1} = T_2 V_4^{\gamma-1} \text{ or } \frac{T_1}{T_2} = \left(\frac{V_4}{V_1}\right)^{\gamma-1} \qquad \text{.....(ii)}$$

From (i) and (ii), $\frac{V_3}{V_2} = \frac{V_4}{V_1}$ or $\frac{V_3}{V_4} = \frac{V_2}{V_1} \Rightarrow \log_e\left(\frac{V_3}{V_4}\right) = \log_e\left(\frac{V_2}{V_1}\right)$

So efficiency of Carnot engine $\eta = 1 - \frac{T_2}{T_1}$

Efficiency of a heat engine depends only on temperatures of source and sink and is independent of all other factors.