Acceleration Due to Gravity

The relation between g and G is given by $g = \frac{GM}{R^2} = \frac{4}{3}\pi R\rho G$ *nhyg*

Variations of g

are due to i) shape of the earth (Pear shaped, more flattened at the S–pole than at the N–pole),
ii)Spin of the earth,
iii)Latitude,
iv) Altitude and
v) Local conditions.

Earth is flat at the poles and some what bulky at the equator. The polar radius is lesser than the equatorial radius by 21 km. Hence g is greater at the polar regions than at the equatorial region.

Because of the spin of the earth, more centrifugal force acts on bodies near the equator. Hence g value is less at the equator.
Variation of g due to rotation of the earth is given by $g_1 = g - R\omega^2\cos^2\lambda$ where λ= latitude angle

Spin of the earth does not affect the value of g at the poles.

If the earth stops spinning, g increases slightly near the equator.

If the earth shrinks without change in its mass, g increases.

The reduction in value of 'g' at the equator is 0.034 ms^{-2} due to the rotation of earth ($\because R\square^2 = 0.034$)

If the earth spins at 17 times the present speed, g becomes zero at the equator.
The angular velocity of rotation of the earth is $7.27\times10^{\square 5}$ $rads^{\square 1}$. The linear velocity of a body at the equator is 0.465 $kms^{\square 1}$.
With the help of isograms, mineral deposits and mineral oils are located.
Isograms are the lines joining the places of equal g on the earth.

As the height from the surface of the earth increases, the value of g decreases.
If g is the acceleration due to gravity on the surface of earth and g_h at a height h above the earth, then $g_h = g(1-\frac{2h}{R})$ approximately or $g_h = \frac{gR^2}{(R+h)^2}$ exactly.
As the depth from the surface of the earth increases, the value of g decreases.
If d is the depth below the surface, then $g_d = g(1-\frac{d}{R})$.

Gravitational Potential Energy

The gravitational potential energy of a body at a point is defined as the amount of work done in bringing the body from infinity to that point against the gravitational force.

$$W = \int_{\infty}^{r} \frac{GMm}{x^2} dx = -GMm\left[\frac{1}{x}\right]_{\infty}^{r}$$

$$W = -\frac{GMm}{r}$$

This work done is stored inside the body as its gravitational potential energy

$$\therefore U = -\frac{GMm}{r}$$

Potential energy is a scalar quantity.

Unit : Joule

Dimension : $[ML^2T^{-2}]$

Gravitational potential energy is always negative in the gravitational field because the force is always attractive in nature.

As the distance r increases, the gravitational potential energy becomes less negative i.e., it increases.

If $r = \infty$ then it becomes zero (maximum)

In case of discrete distribution of masses

Gravitational potential energy

$$U = \sum u_i = -\left[\frac{Gm_1m_2}{r_{12}} + \frac{Gm_2m_3}{r_{23}} + ...\right]$$

If the body of mass m is moved from a point at a distance r_1 to a point at distance $r_2 (r_1 > r_2)$ then change in potential energy $\Delta U = \int_{r_1}^{r_2} \frac{GMm}{x^2} dx = -GMm\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$

or $\Delta U = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

As r_1 is greater than r_2, the change in potential energy of the body will be negative. It means that if a body is brought closer to earth it's potential energy decreases.

Relation between gravitational potential energy and potential $U = -\frac{GMm}{r} = m\left[\frac{-GM}{r}\right]$

$$\therefore = 2\pi \sqrt{\frac{R}{g}}\left(1 + \frac{h}{R}\right)^{3/2}$$

(x) Gravitational potential energy at the centre of earth relative to infinity.

$$U_{centre} = mV_{centre} = m\left(-\frac{3}{2}\frac{GM}{R}\right) = -\frac{3}{2}\frac{GMm}{R}$$

Escape Velocity

The minimum velocity with which a body must be projected up so as to enable it to just overcome the gravitational pull, is known as escape velocity.

The work done to displace a body from the surface of earth ($r = R$) to infinity ($r = \infty$) is

$$g = \frac{4}{3}\pi\rho GR = -GMm\left[\frac{1}{\infty} - \frac{1}{R}\right]$$

$$\Rightarrow \qquad W = \frac{GMm}{R}$$

This work required to project the body so as to escape the gravitational pull is performed on the body by providing an equal amount of kinetic energy to it at the surface of the earth.

If v_e is the required escape velocity, then kinetic energy

which should be given to the body is $\frac{1}{2}mv_e^2$

$$\therefore \therefore \qquad \frac{1}{2}mv_e^2 = \frac{GMm}{R} \Rightarrow v_e = \sqrt{\frac{2GM}{R}}$$

$$\Rightarrow \quad v_e = \sqrt{2gR} \qquad \text{[As } GM = gR^2\text{]}$$

Escape velocity is independent of the mass and direction of projection of the body.