Kinetic Energy

The energy possessed by a body by virtue of its motion is called **kinetic energy**. It is measured by the amount of work which the body can do before coming to rest.

Running water, a released arrow, a bullet fired from a gun, blowing wind, etc. possess kinetic energy.

If a body of mass m is moving with a velocity v, then its kinetic energy = $\frac{1}{2}mv^2$.

A flying bird possesses both K.E. and P.E.

The work done on a body at rest in order that it may acquire a certain velocity is a measure of its kinetic energy.

If the kinetic energy of a body of mass m is E and its momentum is P, then E = $\frac{P^2}{2m}$.

If the momentum of the body increased by ‘n’ times, K.E increase by n^2 times.

If the K.E of the body increases by ‘n’ times, the momentum increases by $\sqrt{n}$ times.

a) If the momentum of the body increases by p Percent, percentage increase in K.E.=$\left(2+\frac{p}{100}\right)$ p%

b) If the momentum of the body decreases by p Percent, decrease in K.E.= $\left(2-\frac{p}{100}\right)$ p%.

If two bodies, one heavier and the other lighter are moving with the same momentum, then the lighter body possesses greater kinetic energy.

If two bodies, one heavier and the other lighter have the same K.E. then the heavier body possesses greater momentum.

Two bodies, one is heavier and the other is lighter are moving with the same momentum. If they are stopped by the same retarding force, then

i) the distance travelled by the lighter body is greater. ($s \propto \frac{1}{m}$)

ii) They will come to rest within the same time interval

Two bodies, one is heavier and the other is lighter are moving with same kinetic energy. If they are stopped by the same retarding force, then

i) The distance travelled by both the bodies are same.

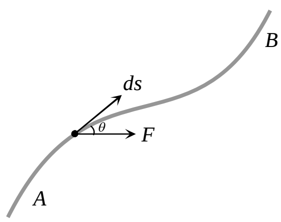
ii) The time taken by the heavier body will be more. ($t \propto \sqrt{m}$)

Two bodies, one is heavier and the other is lighter are moving with same velocity. If they are stopped by the same retarding force, then.

i) The heavier body covers greater distance before coming to rest. ($s \propto m$)
ii) The heavier body takes more time to come to test. ($t \propto m$)

Work Done by a Variable Force

When the magnitude and direction of a force varies with position, the work done by such a force for an infinitesimal displacement is given by $dW = \vec{F}.d\vec{s}$



The total work done in going from A to B as shown in the figure is

$$W = \int_A^B \vec{F}.d\vec{s} = \int_A^B (F\cos\theta)ds$$

In terms of rectangular component $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$$d\vec{s} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$\therefore\ W = \int_A^B (F_x\hat{i} + F_y\hat{j} + F_z\hat{k}).(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

or $W = \int_{x_A}^{x_B} F_x dx + \int_{y_A}^{y_B} F_y dy + \int_{z_A}^{z_B} F_z dz$

Dimension and Units of Work

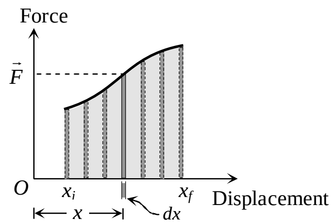
Dimension : As work = Force × displacement

$$[W] = [MLT^{-2}] \times [L] = [ML^2T^{-2}]$$

Units : The units of work are of two types

Work Done Calculation by Force Displacement Graph

Let a body, whose initial position is x_i, is acted upon by a variable force (whose magnitude is changing continuously) and consequently the body acquires its final position x_f.



Let F be the average value of variable force within the interval dx from position x to $(x + dx)$ *i.e.* for small displacement dx. The work done will be the area of the shaded strip of width dx. The work done on the body in displacing it from position x_i to x_f will be equal to the sum of areas of all the such strips

$$dW = \vec{F}\, dx$$

$$\therefore W = \int_{x_i}^{x_f} dW = \int_{x_i}^{x_f} F\, dx$$

$$\therefore W = \int_{x_i}^{x_f} (\text{Area of strip of width } dx)$$

$$\therefore W = \text{Area under curve between } x_i \text{ and } x_f$$

i.e. Area under force-displacement curve with proper algebraic sign represents work done by the force.

Work–Energy theorem :

The work done by the resultant force acting on a body is equal to the change in its kinetic energy.

$$W = Fs;$$

$$W = \frac{1}{2}m(v^2 - u^2) = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

In general, the work done = change in energy.

Stopping distance of a vehicle is directly proportional to the square of its velocity and inversely proportional to the braking force.

If a body is thrown on a horizontal plane and comes to rest after travelling a distance 's', then

μ m g s = ½ mv^2

'μ' coefficient of friction

distance travelled before coming to rest

$$s = \frac{\frac{1}{2}mv^2}{\mu mg} = \frac{\text{Inital K.E.}}{\text{retarding force}}$$

When a body of mass m falls freely from a height, its total energy is mgh.

When it falls through a distance x, its K.E. is mgx and P.E. is mg(h -x).

For a freely falling body or for a body thrown up K.E. at the ground is equal to the P.E. at the maximum height.

Potential Energy

Potential energy is defined only for conservative forces. In the space occupied by conservative forces every point is associated with certain energy which is called the energy of position or potential energy. Potential energy generally are of three types : Elastic potential energy, Electric potential energy and Gravitational potential energy.

(1) **Change in potential energy :** Change in potential energy between any two points is defined in the terms of the work done by the associated conservative force in displacing the particle between these two points without any change in kinetic energy.

$$U_2 - U_1 = -\int_{r_1}^{r_2} \vec{F}.d\vec{r} = -W \qquad \text{...(i)}$$

We can define a unique value of potential energy only by assigning some arbitrary value to a fixed point called the reference point. Whenever and wherever possible, we take the reference point at infinity and assume potential energy to be zero there, *i.e.* if we take $r_1 = \infty$ and $r_2 = r$ then from equation (i)

$$U = -\int_{\infty}^{r} \vec{F}.d\vec{r} = -W$$

In case of conservative force (field) potential energy is equal to negative of work done by conservative force in shifting the body from one position to another position.

This is why, in shifting a particle in a conservative field (say gravitational or electric), if the particle moves opposite to the field, work done by the field will be negative and so change in potential energy will be positive *i.e.* potential energy will increase. When the particle moves in the direction of field, work will be positive and change in potential energy will be negative *i.e.* potential energy will decrease.

(2) **Three dimensional formula for potential energy:** For only conservative fields $\vec{F}$ equals the negative gradient $(-\vec{\nabla})$ of the potential energy.

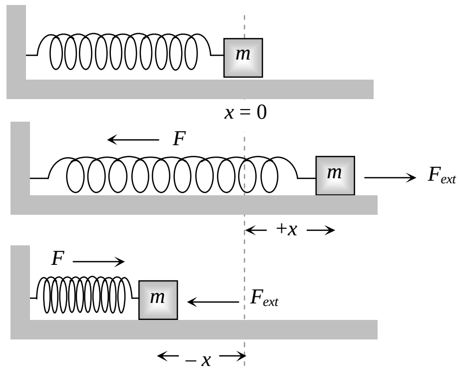
So $\vec{F} = -\vec{\nabla}U$ ($\vec{\nabla}$ read as Del operator or Nabla operator and $\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$)

$$\Rightarrow \qquad \vec{F} = -\left[\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right]$$

Spring Potential Energy

(1) **Restoring force and spring constant :** When a spring is stretched or compressed from its normal position ($x = 0$) by a small distance x, then a restoring force is produced in the spring to bring it to the normal position.

According to Hooke's law this restoring force is proportional to the displacement x and its direction is always opposite to the displacement.



i.e. $\vec{F} \propto -\vec{x}$

or $\qquad \vec{F} = -k\vec{x}$ $\qquad\qquad$ …(i)

where k is called spring constant.

If $x = 1$, $F = k$ (Numerically)

or $\qquad k = F$

Hence spring constant is numerically equal to force required to produce unit displacement (compression or extension) in the spring. If required force is more, then spring is said to be more stiff and vice-versa.

Actually k is a measure of the stiffness/softness of the spring.

Dimension : As $k = \frac{F}{x}$

$\therefore \; [k] = \frac{[F]}{[x]} = \frac{[MLT^{-2}]}{L} = [MT^{-2}]$

Units : S.I. unit *Newton*/*metre*, C.G.S unit *Dyne*/*cm*. Dimension of force constant is similar to surface tension.

(2) **Expression for elastic potential energy :** When a spring is stretched or compressed from its normal position ($x = 0$), work has to be done by external force against restoring force. $\vec{F}_{\text{ext}} = -\vec{F}_{restoring} = k\vec{x}$

Let the spring is further stretched through the distance dx, then work done

$dW = \vec{F}_{ext}.d\vec{x} = F_{ext}.dx\cos 0^o = kx\,dx$ [As $\cos 0^o = 1$]

Therefore total work done to stretch the spring through a distance x from its mean position is given by

$$W = \int_0^x dW = \int_0^x kx\,dx = k\left[\frac{x^2}{2}\right]_0^x = \frac{1}{2}kx^2$$

This work done is stored as the potential energy in the stretched spring.

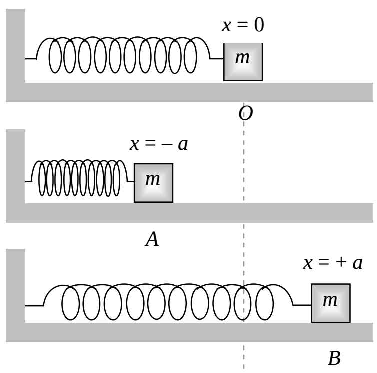
$\therefore$ Elastic potential energy $U = \frac{1}{2}kx^2$

$$U = \frac{1}{2}Fx \qquad \left[\text{As } k = \frac{F}{x}\right]$$

$$U = \frac{F^2}{2k} \qquad \left[\text{As } x = \frac{F}{k}\right]$$

$\therefore$ Elastic potential energy $U = \frac{1}{2}kx^2 = \frac{1}{2}Fx = \frac{F^2}{2k}$

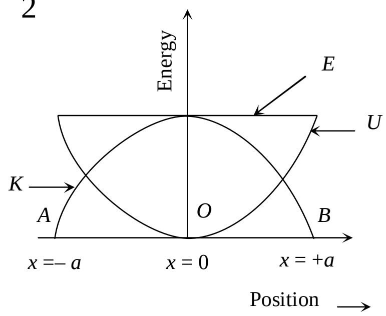
(3) **Energy graph for a spring :** If the mass attached with spring performs simple harmonic motion about its mean position then its potential energy at any position (x) can be given by



$$U = \frac{1}{2}kx^2 \qquad \ldots\text{(i)}$$

So for the extreme position

$$U = \frac{1}{2}ka^2 \qquad [\text{As } x = \pm a \text{ for extreme}]$$



This is maximum potential energy or the total energy of mass.

$\therefore$ Total energy $E = \frac{1}{2}ka^2$ ...(ii)

[Because velocity of mass is zero at extreme position]

$\therefore\ K = \frac{1}{2}mv^2 = 0$]

Now kinetic energy at any position

$K = E - U = \frac{1}{2}k\,a^2 - \frac{1}{2}k\,x^2$

$K = \frac{1}{2}k(a^2 - x^2)$ ...(iii)

From the above formula we can check that

$U_{\max} = \frac{1}{2}ka^2$ [At extreme $x = \pm a$]

and $U_{\min} = 0$ [At mean $x = 0$]

$K_{\max} = \frac{1}{2}ka^2$ [At mean $x = 0$]

and $K_{\min} = 0$ [At extreme $x = \pm a$]

$E = \frac{1}{2}ka^2 =$ constant (at all positions)

It means kinetic energy and potential energy changes parabolically *w.r.t.* position but total energy remain always constant irrespective to position of the mass

Law of conservation of total energy :

The total energy of a system is constant. Energy can neither be created nor destroyed. But it can be converted from one form to the other.

Examples on conversion of energy :

1. Electrical → Heat, Eg. Iron, geyser, over
2. Electrical → Light, Eg. Filament bulb,
   Fluorescent tube
3. Electrical → Sound, Eg. Loud speaker,
   Telephone receiver
4. Electrical → Mechanical. Eg. Fan, Motor
5. Heat → Electrical. Eg : Thermal power plant
6. Heat → Mechanical, Eg. Steam locomotive
7. Mechanical → Electrical. Eg : Dyano (Generator)
8. Sound → Electrical. Eg: Microphone
9. Light → Electrical. Eg: Photoelectric effect
10. Chemical → Electrical. Eg. Primary cell

Power

Power of a body is defined as the rate at which the body can do the work.

Average power $(P_{\text{av.}}) = \dfrac{\Delta W}{\Delta t} = \dfrac{W}{t}$

Instantaneous power $(P_{\text{inst.}}) = \dfrac{dW}{dt} = \dfrac{\vec{F}.d\vec{s}}{dt}$ [As $dW = \vec{F}.d\vec{s}$]

$P_{\text{inst}} = \vec{F}.\vec{v}$ [As $\vec{v} = \dfrac{d\vec{s}}{dt}$]

i.e. power is equal to the scalar product of force with velocity