Equation of Kinematics (From Graph)

These are the various relations between u, v, a, t and s for the particle moving with uniform acceleration where the notations are used as :

u = Initial velocity of the particle at time t = 0 *sec*

v = Final velocity at time t *sec*

a = Acceleration of the particle

s = Distance travelled in time t *sec*

s_n = Distance travelled by the body in n^{th} *sec*

When particle moves with zero acceleration

(i) It is a unidirectional motion with constant speed.

(ii) Magnitude of displacement is always equal to the distance travelled.

(iii) $v = u$, $s = u\,t$ [As $a = 0$]

When particle moves with constant acceleration

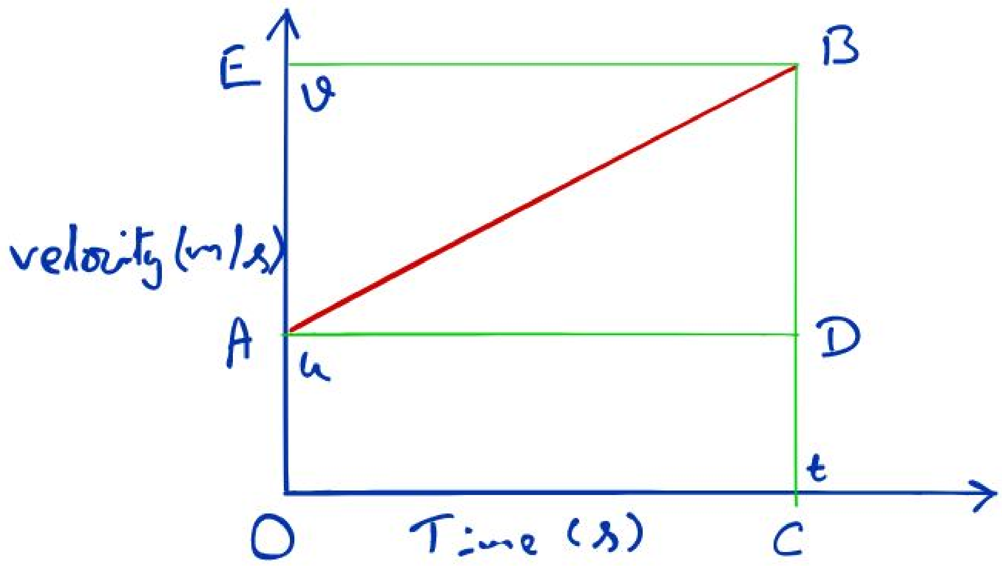
(i) Acceleration is said to be constant when both the magnitude and direction of acceleration remain constant.

(ii) There will be one dimensional motion if initial velocity and acceleration are parallel or anti-parallel to each other.

(iii) Equations of motion (in scalar from)	Equation of motion (in vector from)
$\upsilon = u + at$	$\vec{v} = \vec{u} + \vec{a}t$
$s = ut + \frac{1}{2}at^2$	$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$
$\upsilon^2 = u^2 + 2as$	$\vec{v}.\vec{v} - \vec{u}.\vec{u} = 2\vec{a}.\vec{s}$
$s = \left(\frac{u+v}{2}\right)t$	$\vec{s} = \frac{1}{2}(\vec{u} + \vec{v})t$
$s_n = u + \frac{a}{2}(2n-1)$	$\vec{s}_n = \vec{u} + \frac{\vec{a}}{2}(2n-1)$

Equations of Motion by Graphical Method

Consider the velocity-time graph of an object that moves under uniform acceleration as shown in Fig.



From this graph,the initial velocity of the object is u (at point A) and then it increases to v (at point B) in time t. The velocity changes at a uniform rate a.

The perpendicular lines BC and BE are drawn from point B on the time and the velocity axes respectively, so that the initial velocity is represented by OA, the final velocity is represented by BC and the time interval t is represented by OC. BD = BC – CD, represents the change in velocity in time interval t. Let us draw AD parallel to OC. From the graph, we observe that BC = BD + DC = BD + OA Substituting BC = v and OA = u, we get v = BD + u or BD = v – u

From the velocity-time graph , the acceleration of the object is given by

a = Change in velocity/ time taken = BD /AD = BD /OC

Substituting OC = t, we get a = BD/ t

or BD = at

But BD=v-u

Hence, we get v = u + at

EQUATION FOR POSITION-TIME RELATION

Let us assume that the object travelled a distance s in time t under uniform acceleration a. In Fig. above, the distance travelled by the object can be obtained by the area enclosed within OABC under the velocity-time graph AB. Thus, the distance s travelled by the object is given by s = area of OABC (which is a trapezium) = area of the rectangle OADC + area of the triangle ABD.

$$= OA \times OC + \frac{1}{2}(AD \times BD)$$

Substituting OA = u, OC = AD = t and BD = at, we get

$$s = u \times t + \frac{1}{2}(t \times at)$$

or $$s = ut + \frac{1}{2}at^2$$

EQUATION FOR POSITION–VELOCITY RELATION

From the velocity-time graph shown above, the distance s travelled by the object in time t, is given by the area enclosed within the trapezium OABC under the graph.

That is, s = area of the trapezium OABC

$$s = \left(\frac{u+v}{2}\right)t$$

From the velocity-time relation we get, $t = \dfrac{v-u}{a}$

From the two above equations we get, $$s = \left(\frac{u+v}{2}\right)\frac{v-u}{a}$$

Therefore $$\upsilon^2 - u^2 = 2as$$