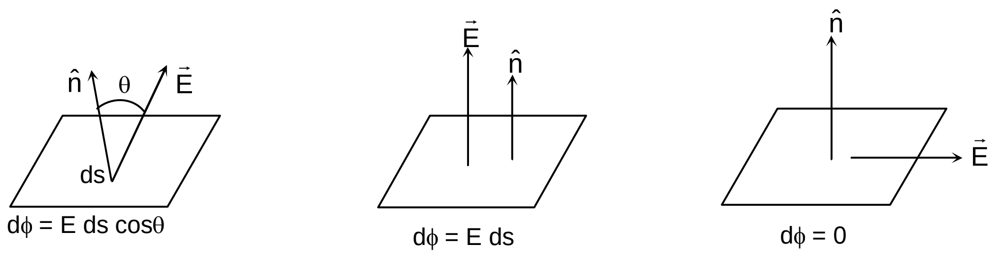
Electric flux and Gauss Law

Electric flux is Proportional to the number of electric field lines through a surface. It is equal to the product of an area element and the perpendicular component of $\vec{E}$, integrated over a surface.

i) The number of electric lines of force crossing a surface normal to the area proportionally gives electric flux



ii) Electric flux through an elementary area ds is defined as the scalar product of area and field.

$d\phi_E. = \vec{E}.d\vec{s} = Eds\cos\theta$

iii) In case of variable electric field or curved area $\phi_E = \int \vec{E}.d\vec{s}$

iv) Flux will be maximum when electric field is normal to the area ($d\phi = Eds$)

v) Flux will be minimum when field is parallel to area ($d\phi = 0$)

vi) For a closed surface, outward flux is positive and inward flux is negative.

Gauss's Law :

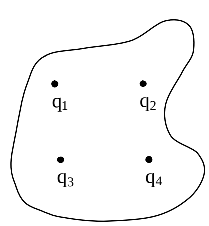
i) The total flux linked with a closed surface is ($1/\in_0$) times the charge enclosed by the closed surface.

$$\oint \vec{E}.d\vec{s} = \frac{1}{\in_0} q$$

The closed surface can be hypothetical and then it is called a Gaussian surface.

If the closed surface enclosed a number of charges q_1 , q_2 .... q_n etc. then

$$\phi = \oint \vec{E}.\vec{ds} = \frac{\Sigma q}{\varepsilon_0} = \frac{(q_1 + q_2 + ...q_n)}{\varepsilon_0}$$



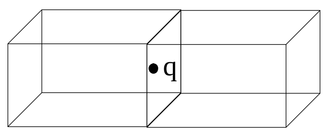
ii) Electric field in $\oint \vec{E}.\vec{dA}$ is complete electric field. It may be partly due to charge with in the surface and partly due to charge outside the surface. However if there is no charge enclosed in the Gaussian surface, then $\oint \vec{E}.\vec{dA} = 0$.

Applications :

1) A point charge q is placed inside a cube of edge 'a'. The flux through each face of the cube is $\frac{q}{6 \epsilon_0}$.

A charge q is placed at the centre of a face of a cube ,then total flux through cube = $\frac{q}{2\varepsilon_0}$

A second cube can be assumed adjacent to the first cube total flux through both cubes = $\frac{q}{\varepsilon_0}$, So flux through each cube = $\frac{q}{2\varepsilon_0}$



Now ,if q is placed at a corner then the flux will be $\frac{q}{8\varepsilon_0}$

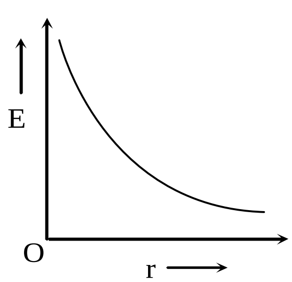
2) In case of infinite line of charge , at a distance 'r'. E = $\frac{1}{4\pi \epsilon_0} \frac{2\lambda}{r} = \frac{\lambda}{2\pi \epsilon_0 r}$.

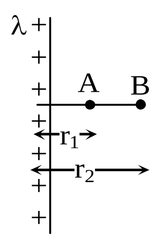
Where λ is the linear charge density.

If wire is positively charged , direction of E will be away from the wire while for a negatively charged wire, direction of $\vec{E}$ will be towards the wire .

E at point p

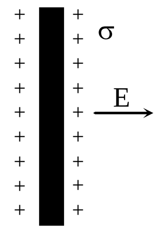
$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r} \qquad \text{or} \qquad E = \frac{\lambda}{2\pi\varepsilon_0 r}$$





3) The intensity of electric field near a plane sheet of charge is $E = \frac{\sigma}{2\varepsilon_o K}$

where σ = surface charge density



$\vec{E}$ is independent of distance of the point from the sheet and also of the area of sheet i.e..

Direction of electric field is perpendicular to the sheet.

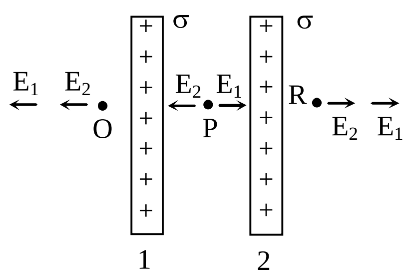
4) **Electric field due to two infinite parallel plates of charge**

Both plates have same type of charge

$$E_O = E_1 + E_2 = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

$$E_P = E_1 + E_2 = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

$$E_R = E_1 + E_2 = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

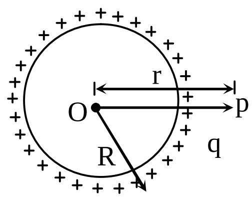


5) The intensity of electric field near a plane charged conductor $E = \frac{\sigma}{\varepsilon_o K}$ in a medium of dielectric constant K. If dielectric medium is air, then $E_{air} = \sigma / \varepsilon_o$.

6) Field between two parallel plates of a condenser is $E = \frac{\sigma}{\epsilon_0}$, where σ is the surface charge density.

7) Charged Conducting sphere (or shell of charge) :

If charge on a conducting sphere of radius R is Q (and σ = surface charge density) as shown in figure then electric field is



(i) **Out side the sphere :** If point P lies outside the sphere

$$E_{out} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r^2} = \frac{\sigma R^2}{\varepsilon_0 r^2} \text{ and } V_{out} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r} = \frac{\sigma R^2}{\varepsilon_0 r}$$

$(Q = \sigma \times A = \sigma \times 4\pi R^2)$

(ii) **At the surface of sphere :** At surface $r = R$

So, $$E_s = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R^2} = \frac{\sigma}{\varepsilon_0} \text{ and } V_s = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R} = \frac{\sigma R}{\varepsilon_0}$$

(iii) **Inside the sphere :** Inside the conducting charge sphere electric field is zero and potential remains constant every where and equals to the potential at the surface.

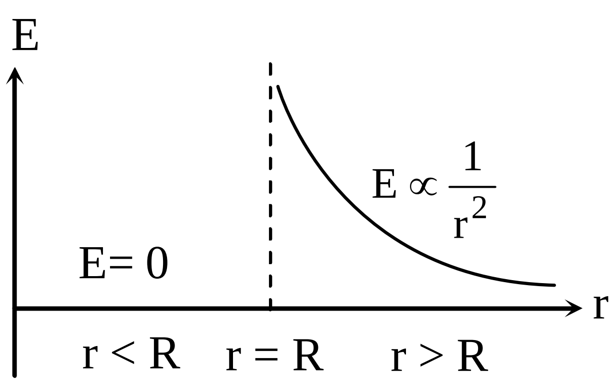
$E_{in} = 0$ and V_{in} = constant $= V_s$

Graph

(iv) Electric field at the surface is always perpendicular to the surface.

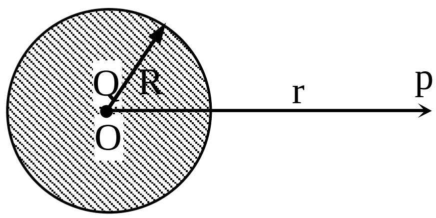
(v) For points, near the surface of the conductor, E = perpendicular to the surface

(vi) Graphically ,



(8) **Uniformly charged non-conducting sphere :**

Suppose charge Q is uniformly distributed in the volume of a non-conducting sphere of radius R as shown below



(i) **Outside the sphere :** If point P lies outside the sphere

$$E_{out} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}$$

If the sphere has uniform volume charge density $\rho = \dfrac{Q}{\frac{4}{3}\pi R^3}$

then $E_{out} = \dfrac{\rho R^3}{3\varepsilon_0 r^2}$

(ii) **At the surface of sphere :** At surface $r = R$

$$E_s = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R^2} = \frac{\rho R}{3\varepsilon_0}$$

(iii) **Inside the sphere :** At a distance r from the centre

$$E_{in} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qr}{R^3} = \frac{\rho r}{3\varepsilon_0} \quad \{E_{in} \propto r\}$$

Graph