Adiabatic process

Heat supplied or taken by the system is zero i.e. the system is well insulated so that no heat enters or leaves the system.

Process equation is,

$$PV^{\gamma} = \text{Constant}$$

Using PV = nRT and $P = \frac{nRT}{M}$ we have,

(a) $TV^{\gamma-1} = \text{Constant}$ (b) $P^{1-\gamma} T^{\gamma} = \text{Constant}$

(c) $d^{1-\gamma} T = \text{Constant}$ (d) $Pd^{-\gamma} = \text{Constant}$

If a system undergoes change from A to B in an adiabatic process, then

$$A \xrightarrow[\text{Process}]{\text{Adiabatic}} B$$

$$(P_1, V_1, T_1) \quad (P_2, V_2, T_2)$$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} = Nr \quad \text{and} \quad P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

Heat supplied to a gas is zero i.e.

$$\delta Q = 0$$

The change in internal energy

$$dU = nC_V(T_2 - T_1) = \frac{nR(T_2 - T_1)}{\gamma - 1}$$

Work done by the gas is

$$dW = -dU$$

(from I[st] law of thermodynamics)

$$= -\left[\frac{nR(T_2 - T_1)}{\gamma - 1}\right]$$

Specific heat for an adiabatic process is zero.

Bulk modulus of adiabatic process = γ P

The equation of first law of thermodynamics is,

$$nCdT = nC_VdT + PdV$$

for adiabatic process dQ = nCdT = 0

$\therefore \quad nC_V dT + PdV = 0 \quad \text{...(1)}$

Now $\quad PV = nRT$ (gives)

$$P = \frac{nRT}{V}$$

Substitute the value of P in eqn. (1)

$$nC_V dT + \frac{nRT}{V} dV = 0$$

$$nC_V \frac{dT}{T} + nR\frac{dV}{V} = 0$$

But $\quad C_V = \frac{R}{\gamma - 1}$

$$\therefore \quad \frac{nR}{\gamma-1}.\frac{dT}{T} + nR\frac{dV}{V} = 0$$

$$nR\frac{dT}{T} + (\gamma-1)nR\frac{dV}{V} = 0$$

$$nR\frac{dT}{T} = (1-\gamma)nR\frac{dV}{V}$$

On integrating the above expression

$$\int nR\frac{dT}{T} = (1-\gamma)\, nR\frac{dV}{V}$$

$$\ell n\, T = (1-\gamma)\, \ell n\, V + C$$

$$= \ell n\, V^{1-\gamma} + C$$

$$\ell n\, \frac{T}{V^{1-\gamma}} = C$$

$$\ell n\, TV^{\gamma-1} = C$$

$$V^{\gamma-1}T = \text{Constant}$$

or $\quad PV^{\gamma} = \text{Constant}$

(x) Work done is given by $W = \int_{v_1}^{v_2} PdV$

But, $\quad PV^{\gamma} = K$

$$P = \frac{K}{V^{\gamma}}$$

$$\therefore \quad W = \int_{v_1}^{v_2} \frac{K}{V^{\gamma}}\, dV$$

$$= K \left[\frac{V^{1-\gamma}}{1-\gamma}\right]_{V_1}^{V_2}$$

$$= \frac{K}{1-\gamma} \left[V_2^{1-\gamma} - V_1^{1-\gamma}\right]$$

As $P_1V_1^{\gamma} = P_2V_2^{\gamma} = K$

$$\therefore \quad W = \frac{P_2V_2 - P_1V_1}{1-\gamma} = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$