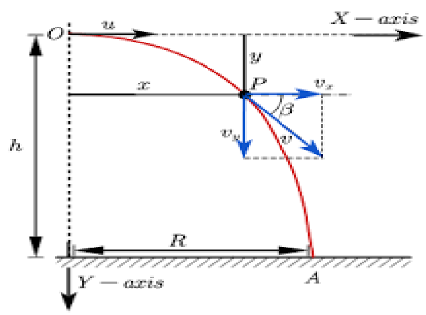
Horizontal Projectile

A body is projected horizontally from a certain height 'h' vertically above the ground with initial velocity *u*. If air resistance is considered to be absent, then there is no other horizontal force which can affect the horizontal motion..

The horizontal velocity therefore remains constant and so the object covers equal distance in horizontal direction in equal intervals of time.

Trajectory of horizontal projectile : The horizontal displacement *x* is governed by the equation



$x = ut \Rightarrow t = \frac{x}{u}$ .... (i)

The vertical displacement *y* is given by

$y = -\frac{1}{2}gt^2$ .... (ii)

(since initial vertical velocity is zero)

By substituting the value of *t* in equation (ii) $y = -\frac{1}{2}\frac{g\,x^2}{u^2}$

Displacement of Projectile $(\vec{r})$: After time *t*, horizontal displacement $x = ut$ and vertical displacement $y = -\frac{1}{2}gt^2$.

So, the position vector $\vec{r} = ut\,\hat{i} - \frac{1}{2}gt^2\,\hat{j}$

Therefore $r = ut\sqrt{1 + \left(\frac{gt}{2u}\right)^2}$ and $\alpha = \tan^{-1}\left(-\frac{gt}{2u}\right)$

$$\alpha = \tan^{-1}\left(-\sqrt{\frac{gy}{2}}\Big/u\right) \qquad \left(\text{as } t = \sqrt{\frac{2y}{g}}\right)$$

Instantaneous velocity : Throughout the motion, the horizontal component of the velocity is $v_x = u$.

The vertical component of velocity increases with time and is given by

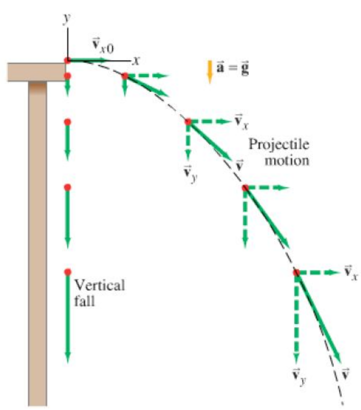
$v_y = 0 - g\,t = -\,g\,t$ (From $v = u + g\,t$)

So, $\vec{v} = v_x\hat{i} + v_y\hat{j}$ $= u\,\hat{i} - g\,t\,\hat{j}$

i.e. $v = \sqrt{u^2 + (gt)^2}$ $= u\sqrt{1 + \left(\frac{gt}{u}\right)^2}$

Again $\vec{v} = u\hat{i} - \sqrt{2gy}\,\hat{j}$

i.e. Speed $v = \sqrt{u^2 + 2gy}$



Direction of instantaneous velocity : $\tan\phi = \frac{v_y}{v_x}$

$$\Rightarrow \quad \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-\sqrt{2gy}}{u}\right) \text{ or } \phi = \tan^{-1}\left(-\frac{gt}{u}\right)$$

Where ϕ is the angle of instantaneous velocity from the horizontal.

Time of flight : If a body is projected horizontally from a height h with velocity u and time taken by the body to reach the ground is T, then

$-h = 0 - \frac{1}{2}gT^2$ (for vertical motion)

$T = \sqrt{\frac{2h}{g}}$

Horizontal range : Let R is the horizontal distance travelled by the body

$R = uT + \frac{1}{2} 0 T^2$ (for horizontal motion)

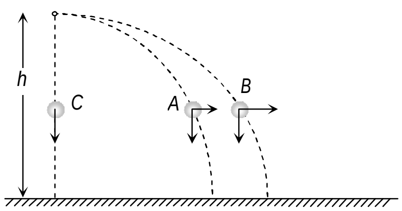
$R = u\sqrt{\frac{2h}{g}}$

If projectiles A and B are projected horizontally with different initial velocity from same height and third particle C is dropped from same point then

(i) All three particles will take equal time to reach the ground.

(ii) Their net velocity would be different but all three particle possess same vertical component of velocity.

(iii) The trajectory of projectiles A and B will be straight line *w.r.t.* particle C.



A shell released from horizontally flying airplane is an example of horizontal projectile.