

Motion of Body Under Gravity (Free Fall)

The force of attraction of earth on bodies, is called force of gravity. Acceleration produced in the body by the force of gravity, is called acceleration due to gravity. It is represented by the symbol g .

In the absence of air resistance, it is found that all bodies (irrespective of the size, mass or composition) fall with the same acceleration near the surface of the earth. This motion of a body falling towards the earth from a small altitude ($h \ll R$) is called free fall.

An ideal example of one-dimensional motion is motion under gravity in which air resistance and the small changes in acceleration with height are neglected.

If a body is dropped from some height (initial velocity zero)

Equations of motion : Taking initial position as origin and direction of motion (*i.e.*, downward direction) as a positive, here we have

$$u = 0 \quad [\text{As body starts from rest}]$$

$$a = +g \quad [\text{As acceleration is in the direction of motion}]$$

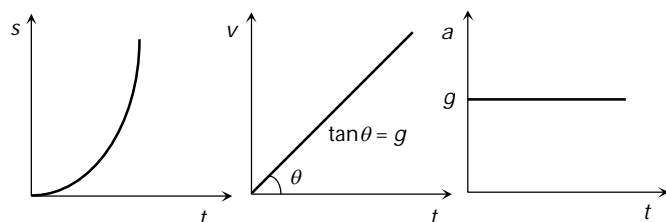
$$v = g t \quad \dots(\text{i})$$

$$h = \frac{1}{2} g t^2 \quad \dots(\text{ii})$$

$$v^2 = 2gh \quad \dots(\text{iii})$$

$$h_n = \frac{g}{2} (2n-1) \quad \dots(\text{iv})$$

Graph of distance, velocity and acceleration with respect to time :



As $h = (1/2)gt^2$, *i.e.*, $h \propto t^2$, distance covered in time $t, 2t, 3t$, *etc.*, will be in the ratio of $1^2 : 2^2 : 3^2$, *i.e.*, square of integers.

The distance covered in the n th sec, $h_n = \frac{1}{2} g (2n-1)$

So distance covered in 1st, 2nd, 3rd sec, *etc.*, will be in the ratio of $1 : 3 : 5$, *i.e.*, odd integers only.

If a body is projected vertically downward with some initial velocity

Equation of motion :

$$v = u + g t$$

$$h = ut + \frac{1}{2} g t^2$$

$$v^2 = u^2 + 2gh$$

$$h_n = u + \frac{g}{2}(2n-1)$$

If a body is projected vertically upward

Equation of motion : Taking initial position as origin and direction of motion (*i.e.*, vertically up) as positive

$a = -g$ [As acceleration is downwards while motion upwards]

So, if the body is projected with velocity u and after time t it reaches up to height h then

$$v = u - gt ; h = ut - \frac{1}{2}gt^2 ; v^2 = u^2 - 2gh ; h_n = u - \frac{g}{2}(2n-1)$$

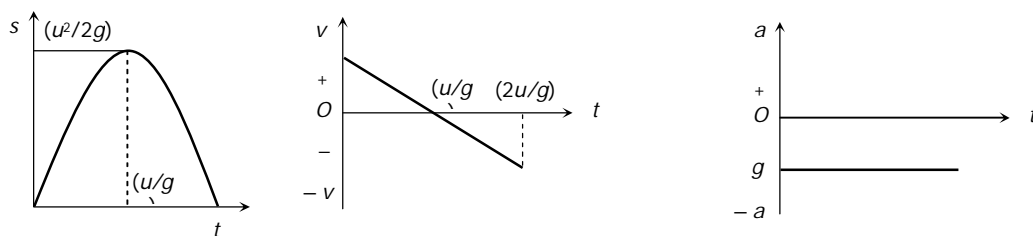
For maximum height $v = 0$

So from above equation $u = gt$,

$$h = \frac{1}{2}gt^2$$

$$\text{and } u^2 = 2gh$$

Graph of displacement, velocity and acceleration with respect to time :



The motion is independent of the mass of the body, as in any equation of motion, mass is not involved. That is why a heavy and light body when released from the same height, reach the ground simultaneously and with same velocity *i.e.*, $t = \sqrt{2h/g}$ and $v = \sqrt{2gh}$.

In case of motion under gravity, time taken to go up is equal to the time taken to fall down through the same distance. Time of descent (t_2) = time of ascent (t_1) = u/g

$$\therefore \text{Total time of flight } T = t_1 + t_2 = \frac{2u}{g}$$

In case of motion under gravity, the speed with which a body is projected up is equal to the speed with which it comes back to the point of projection.

As well as the magnitude of velocity at any point on the path is same whether the body is moving in upwards or downward direction.

A body is thrown vertically upwards. If air resistance is to be taken into account, then the time of ascent is less than the time of descent. $t_2 > t_1$

$$\text{Let } u \text{ is the initial velocity of body then time of ascent } t_1 = \frac{u}{g+a} \text{ and } h = \frac{u^2}{2(g+a)}$$

where g is acceleration due to gravity and a is retardation by air resistance and for upward motion both will work vertically downward.

For downward motion a and g will work in opposite direction because a always work in direction opposite to motion and g always work vertically downward.

$$\text{So } h = \frac{1}{2}(g - a)t_2^2$$

$$\Rightarrow \frac{u^2}{2(g + a)} = \frac{1}{2}(g - a)t_2^2$$

$$\Rightarrow t_2 = \frac{u}{\sqrt{(g + a)(g - a)}}$$

Comparing t_1 and t_2 we can say that $t_2 > t_1$

since $(g + a) > (g - a)$