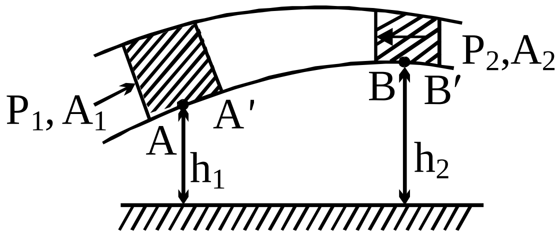
Bernoulli's Theorem

According to this theorem the total energy (pressure energy, potential energy and kinetic energy) per unit volume or mass of an incompressible and non-viscous fluid in steady flow through a pipe remains constant throughout the flow, provided there is no source or sink of the fluid along the length of the pipe.



Mathematically for unit volume of liquid flowing through a pipe.

$P + \rho gh + \frac{1}{2}\rho v^2 =$ constant

To prove it, consider a liquid flowing steadily through a tube of non-uniform area of cross-section as shown in fig. If P_1 and P_2 are the pressures at the two ends of the tube respectively, work done in pushing the volume V of incompressible fluid from point B to C through the tube will be

$W = P_1V - P_2V = (P_1 - P_2)V$ ...(i)

This work is used by the fluid in two ways.

(a) In changing the potential energy of mass m (in the volume V) from mgh_1 to mgh_2,

i.e., $\Delta U = mg(h_2 - h_1)$ ...(ii)

(b) In changing the kinetic energy from $\frac{1}{2}mv_1^2$ to $\frac{1}{2}mv_2^2$,

i.e., $\Delta K = \frac{1}{2}m(v_2^2 - v_1^2)$ ...(iii)

Now as the fluid is non-viscous, by conservation of mechanical energy

$W = \Delta U + \Delta K$

i.e., $(P_1 - P_2)V = mg(h_2 - h_1) + \frac{1}{2}m(v_2^2 - v_1^2)$

or $$P_1 - P_2 = \rho g(h_2 - h_1) + \frac{1}{2}\rho(v_2^2 - v_1^2) \qquad [\text{As } \rho = m/V]$$

or $$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

or $$P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}$$

This equation is the so called Bernoulli's equation and represents conservation of mechanical energy in case of moving fluids.

(i) Bernoulli's theorem for unit mass of liquid flowing through a pipe can also be written as:

$$\frac{P}{\rho} + gh + \frac{1}{2}v^2 = \text{constant}$$

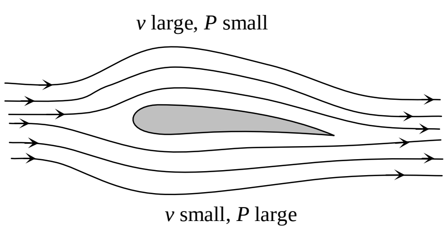
(ii) Dividing above equation by g we get $\frac{P}{\rho g} + h + \frac{v^2}{2g} = \text{constant}$

Here $\frac{P}{\rho g}$ is called pressure head, h is called gravitational head and $\frac{v^2}{2g}$ is called velocity head. From this equation Bernoulli's theorem can be stated as.

"In stream line flow of an ideal liquid, the sum of pressure head, gravitational head and velocity head of every cross section of the liquid is constant."

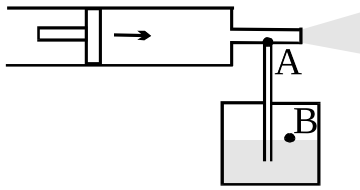
Applications of Bernoulli's Theorem

(i) Working of an aeroplane



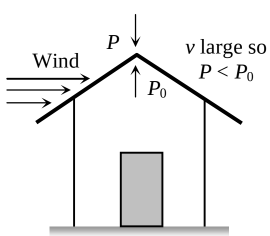
This is also based on Bernoulli's principle. The wings of the aeroplane are of the shape as shown in fig. Due to this specific shape of wings when the aeroplane runs, air passes at higher speed over it as compared to its lower surface. This difference of air speeds above and below the wings, in accordance with Bernoulli's principle, creates a pressure difference, due to which an upward force called 'dynamic lift' (= pressure difference × area of wing) acts on the plane. If this force becomes greater than the weight of the plane, the plane will rise up.

(ii) Action of atomizer



The action of carburetor, paint-gun, scent-spray or insect-sprayer is based on Bernoulli's principle. In all these, by means of motion of a piston *P* in a cylinder *C*, high speed air is passed over a tube *T* dipped in liquid *L* to be sprayed. High speed air creates low pressure over the tube due to which liquid (paint, scent, insecticide or petrol) rises in it and is then blown off in very small droplets with expelled air.

(iv) Blowing off roofs by wind storms



During a tornado or hurricane, when a high speed wind blows over a straw or tin roof, it creates a low pressure (P) in accordance with Bernoulli's principle.

However, the pressure below the roof (*i.e.*, inside the room) is still atmospheric ($= P_0$). So due to this difference of pressure, the roof is lifted up and is then blown off by the wind.

(v) **Magnus effect :** When a spinning ball is thrown, it deviates from its usual path in flight. This effect is called Magnus effect and plays as important role in tennis, cricket and soccer, etc. as by applying appropriate spin the moving ball can be made to curve in any desired direction.

If a ball is moving from left to right and also spinning about a horizontal axis perpendicular to the direction of motion as shown in fig. then relative to the ball, air will be moving from right to left.

The resultant velocity of air above the ball will be $(v + r\omega)$ while below it $(v - r\omega)$. So in accordance with Bernoulli's principle pressure above the ball will be less than below it. Due to this difference of pressure an upward force will act on the ball and hence the ball will deviate from its usual path OA_0 and will hit the ground at A_1 following the path OA_1 *i.e.*, if a ball is thrown with back-spin, the pitch will curve less sharply prolonging the flight.