Gravitation

All forces in nature can be classified under three categories depending upon their relative strengths. They are

(i) gravitational force,

(ii) electromagnetic force and

(iii) nuclear force.

Fundamental forces of the universe:

a) **Gravitational Force:**

i) It is the weakest of all the forces but has the longest range.

ii) It is because of attraction between particles due to the property of mass.

iii) Since it is a weak force, the force effects are considerable only when the interacting objects are massive.

iv) It provides the large scale structure for the universe.

b) **Electromagnetic force:**

i) It is a strong force between two charged particles and has a long range.

ii) It acts through electric and magnetic fields.

iii) It can be attractive as well as repulsive.

iv) According to quantum field theory electromagnetic force between two charges is mediated by exchange of **Photons.**

c) **Nuclear force:**

i) They are a short range, strong force of attraction between nucleons, which provides stability to the nucleons.

ii) It is the strongest of all the fundamental forces and has a range of 1 fermi = 10^{-15} m.

a) **Order of Range**

Range of Gravitational force > Range of Electromagnetic force > Range of nuclear force.

b) **Order of strength:**

Nuclear force > Electromagnetic force > Gravitational force

The ratio of relative strengths of nuclear, electromagnetic and gravitational forces is $1 : 10^{-15} : 10^{-35}$.

Aryabhat in his famous book "Aryabhatiyam" suggested that earth is a solid sphere and it spins around itself.

In Rigveda, paths of planets in solar system were suggested to be elliptical.

In "Thaithireeya Aruna Patham", existence of several solar systems moving under the influence of a great central force was suggested.

The geo–centric theory was proposed by **Ptolemy**. According to this theory, all the planets revolve round the earth in circular orbits, the earth being at the centre

The helio–centric theory was proposed by **Copernicus**. According to this theory, all the planets revolve round the Sun in circular orbits, the Sun being at the centre

Newton's law of universal gravitation : Every two bodies in the universe attract each other with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

If m_1 and m_2 are the masses of two bodies and d is the distance between them, the gravitational force of attraction F which each exerts on the other is given by

$F = G \cdot \frac{m_1 m_2}{d^2}$ where G is called universal gravitational constant and is equal to 6.67×10^{-11} Nm^2kg^{-2}.

G was first accurately determined by Cavendish. It is a scalar quantity.

Properties of gravitational force:

a) The gravitational force of attraction between two particles from an action and reaction pair, ie equal in magnitude and opposite in direction.

b) Gravitational force is a central force i.e. it acts along the line joining the two particles.

c) Gravitational force between two particles is independent of the properties of intervening medium.

d)Gravitational force between two particles is independent of the presence of other particles

e) **Principle of superposition:** If a no. of particles interact with each other, the net force acting on a given particle is the vector sum of the forces acting upon it, due to its interaction with each of the other particles.

f) They are long range attractive forces.

If two identical spheres each of radius r are kept in contact with each other, the gravitational force F between them is proportional to r^4.

If two identical spheres each of radius r are separated by a certain distance and the distance between the spheres is maintained constant, the gravitational force F between them is proportional to r^6.

Newton's third law of motion do not apply when (i) velocities of moving bodies are comparable to velocity of light and (ii) gravitational fields are very strong, e.g. gravitational field between objects whose masses are greater than the mass of sun.

Universal law of gravitation cannot explain the reason for gravity between objects and force of attraction between two bodies even when they are not in physical contact

Acceleration Due to Gravity

The relation between g and G is given by $g = \frac{GM}{R^2} = \frac{4}{3}\pi R\rho G$ *nhyg*

Variations of g

are due to i) shape of the earth (Pear shaped, more flattened at the S–pole than at the N–pole),
ii)Spin of the earth,
iii)Latitude,
iv) Altitude and
v) Local conditions.

Earth is flat at the poles and some what bulky at the equator. The polar radius is lesser than the equatorial radius by 21 km. Hence g is greater at the polar regions than at the equatorial region.

Because of the spin of the earth, more centrifugal force acts on bodies near the equator. Hence g value is less at the equator.
Variation of g due to rotation of the earth is given by $g_1 = g - R\,\omega^2\cos^2\lambda$ where λ= latitude angle

Spin of the earth does not affect the value of g at the poles.

If the earth stops spinning, g increases slightly near the equator.

If the earth shrinks without change in its mass, g increases.

The reduction in value of ‘g’ at the equator is 0.034 ms^{-2} due to the rotation of earth ($\because$ $R\square^2$ = 0.034)

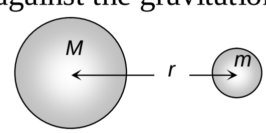
If the earth spins at 17 times the present speed, g becomes zero at the equator.
The angular velocity of rotation of the earth is $7.27\times10^{\square 5}$ $rads^{\square 1}$. The linear velocity of a body at the equator is 0.465 $kms^{\square 1}$.
With the help of isograms, mineral deposits and mineral oils are located.
Isograms are the lines joining the places of equal g on the earth.

As the height from the surface of the earth increases, the value of g decreases.
If g is the acceleration due to gravity on the surface of earth and g_h at a height h above the earth, then $g_h = g(1-\frac{2h}{R})$ approximately or $g_h = \frac{gR^2}{(R+h)^2}$ exactly.
As the depth from the surface of the earth increases, the value of g decreases.
If d is the depth below the surface, then $g_d = g(1-\frac{d}{R})$.

Gravitational Potential Energy

The gravitational potential energy of a body at a point is defined as the amount of work done in bringing the body from infinity to that point against the gravitational force.

$$W = \int_{\infty}^{r} \frac{GMm}{x^2}\,dx = -GMm\left[\frac{1}{x}\right]_{\infty}^{r}$$



$$W = -\frac{GMm}{r}$$

This work done is stored inside the body as its gravitational potential energy

$$\therefore U = -\frac{GMm}{r}$$

Potential energy is a scalar quantity.

Unit : Joule

Dimension : $[ML^2T^{-2}]$

Gravitational potential energy is always negative in the gravitational field because the force is always attractive in nature.

As the distance r increases, the gravitational potential energy becomes less negative i.e., it increases.

If $r = \infty$ then it becomes zero (maximum)

In case of discrete distribution of masses

Gravitational potential energy

$$U = \sum u_i = -\left[\frac{Gm_1m_2}{r_{12}} + \frac{Gm_2m_3}{r_{23}} + ...\right]$$

If the body of mass m is moved from a point at a distance r_1 to a point at distance $r_2 (r_1 > r_2)$ then change in potential energy $\Delta U = \int_{r_1}^{r_2} \frac{GMm}{x^2} dx = -GMm\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$

or $\Delta U = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

As r_1 is greater than r_2, the change in potential energy of the body will be negative. It means that if a body is brought closer to earth it's potential energy decreases.

Relation between gravitational potential energy and potential $U = -\frac{GMm}{r} = m\left[\frac{-GM}{r}\right]$

$$\therefore = 2\pi \sqrt{\frac{R}{g}}\left(1 + \frac{h}{R}\right)^{3/2}$$

(x) Gravitational potential energy at the centre of earth relative to infinity.

$$U_{centre} = mV_{centre} = m\left(-\frac{3}{2}\frac{GM}{R}\right) = -\frac{3}{2}\frac{GMm}{R}$$

Escape Velocity

The minimum velocity with which a body must be projected up so as to enable it to just overcome the gravitational pull, is known as escape velocity.

The work done to displace a body from the surface of earth ($r = R$) to infinity ($r = \infty$) is

$$g = \frac{4}{3}\pi\rho GR = -GMm\left[\frac{1}{\infty} - \frac{1}{R}\right]$$

$$\Rightarrow \qquad W = \frac{GMm}{R}$$

This work required to project the body so as to escape the gravitational pull is performed on the body by providing an equal amount of kinetic energy to it at the surface of the earth.

If v_e is the required escape velocity, then kinetic energy

which should be given to the body is $\frac{1}{2}mv_e^2$

$$\therefore \therefore \qquad \frac{1}{2}mv_e^2 = \frac{GMm}{R} \Rightarrow v_e = \sqrt{\frac{2GM}{R}}$$

$\Rightarrow \quad v_e = \sqrt{2gR}$ [As $GM = gR^2$]

Escape velocity is independent of the mass and direction of projection of the body.