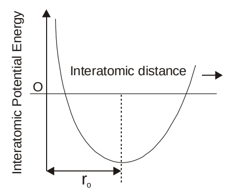
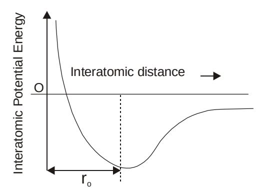
EXPANSION OF SOLIDS

20. Solids expand on heating due to increased atomic spacing.
21. A solid can be considered as periodic arrangement of atoms in the form of lattice.
22. At any particular temperature, the atoms are in a specific state of vibration about a fixed point called as equilibrium position in the lattice.
23. As the temperature increases, the amplitude of vibration of the atoms increases.



24. If the lattice vibrations are purely harmonic the potential energy curve is a symmetric parabola and there is not thermal expansion.
25. If the lattice vibrations are an harmonic, the potential energy of an oscillator is an asymmetric function of its position and thermal expansion is observed.



26. Coefficient of linear expansion (α) : The ratio of increase in length per one degree rise in temperature to its original length is called coefficient of linear expansion.

    $\alpha = \dfrac{l_2 - l_1}{l_1(t_2 - t_1)}$

    Unit of α is $c^{o^{-1}}$ or K^{-1}
27. The change in length is calculated using $\Delta L = L\ \alpha\ \Delta t$
28. Coefficient of area or superficial expansion (β) : The increase in area per unit area per one degree rise in temperature is called coefficient of areal expansion.

    $\beta = \dfrac{a_2 - a_1}{a_1(t_2 - t_1)}$

    Unit of β is $c^{o^{-1}}$ or K^{-1}
29. The change in area is calculated using formula $\Delta a = a\ \beta\ \Delta t$.
30. The coefficient of volume or cubical expansion (γ) is the increase in volume per unit volume per degree rise in temperature.

    $\gamma = \dfrac{V_2 - V_1}{V_1(t_2 - t_1)}$

    Unit of γ is $c^{o^{-1}}$ or K^{-1}
31. The change in volume is calculated using formula $\Delta V = V\ \gamma\ \Delta t$.
32. For all isotropic substances (solids which expand in the same ratio in all directions) $\alpha : \beta : \gamma = 1:2:3$ or $\gamma = 3\alpha$; $\beta = 2\alpha$; $\gamma = \alpha + \beta$.

33. If α_x, α_y and α_z represent the coefficients of linear expansion for an isotropic solids (solids which expand differently in different directions) in x, y and z directions respectively, then $\gamma=\alpha_x+\alpha_y+\alpha_z$ and the average coefficient of linear expansion $\alpha=\frac{\alpha_x+\alpha_y+\alpha_z}{3}$.
34. The numerical value of coefficient of linear expansion of a solid depends on the nature of the material and the scale of temperature used.
35. The numerical value of coefficient of linear expansion of a solid is independent of physical dimensions of the body and also on the unit of length chosen.
36. The increase in length or linear expansion of a rod depends on nature of material, initial length of rod and rise of temperature.
37. The numerical value of α or β or γ in the units of per °C is 9/5 times its numerical value in the units of per °F.
38. α per °F$=\frac{5}{9}.\alpha$ per °C.
39. α per °R$=\frac{5}{4}.\alpha$ per °C.
40. Variation of density with temperature : The density of a solid decreases with increase of temperature. $d_t=\frac{d_o}{1+\gamma t}$ or $d_t \approx d_o(1-\gamma t)$ where d_o is density at 0°C.
41. If R_1 and R_2 are the radii of a disc or a plate at t_1°C and t_2°C respectively then $R_2=R_1(1+\alpha(t_2-t_1))$.
42. A metal scale is calibrated at a particular temperature does not give the correct measurement at any other temperature.
    a) When scale expands correction to be made $\Delta l=L\ \alpha\ \Delta t$, correct reading$=L+\Delta l$
    b) When scale contracts correction to be made $\Delta l=L\ \alpha\ \Delta t$, correct reading$=L-\Delta l$. L=measured value.
    c) $L_{measured}=L_{true}[1-\alpha(\Delta t)]$
43. When a metal rod is heated or cooled and is not allowed to expand or contract thermal stress is developed.

    Thermal force $F=YA\ \alpha\ (t_2-t_1)$

    Thermal force is independent of length of rod.

    Thermal stress $\sigma=Y\ \alpha\ (t_2-t_1)$

    Y=Young's modulus

    α=coefficient of linear expansion

    t_2-t_1=difference of temperature

A=area of cross-section of the metal rod.

For same thermal stress in two different rods heated through the same rise in temperature, $Y_1\alpha_1=Y_2\alpha_2$.

44. Barometer with brass scale :

    Relation between faulty and actual barometric height is given by $h_2=h_1[1+(\alpha_s-Y_{Hg})(t_2-t_1)]$

    h_1=height of barometer at t_1°C where the scale is marked

    h_2=height of barometer at t_2°C where the measurement is made

    γ_{Hg}=real coefficient of expansion of mercury

    α_s=coefficient of linear expansion of scale

45. Pendulum clocks lose or gain time as the length increases or decreases respectively.

    The fractional change=$\frac{\Delta T}{T}=\frac{\alpha\Delta t}{2}$.

    The loss or gain per day=$\frac{\alpha\Delta t}{2}$x86400 seconds.

46. The condition required for two rods of different materials to have the difference between the lengths always constant is $L_1\alpha_1=L_2\alpha_2$.
47. A hole in a metal plate expands on heating just like a solid plate of the same size.
48. A cavity of a solid object expands on heating just like a solid object of the same volume.
49. If a hollow pipe and a solid rod of same dimensions made of same material are heated to the same rise in temperature, both expand equally.
50. If a thin rod and a thick rod of same length and material are heated to same rise in temperature, both expand equally.
51. If a thin rod and a thick rod of same length and material are heated by equal quantities of heat, thin rod expands more than thick rod.
52. A rectangular metal plate contains a circular hole. If it is heated, the size of the hole increases and the shape of the hole remains circular.
53. A metal plate contains two holes at a certain distance apart from each other. If the plate is heated, the distance between the centers of the holes increases.
54. The change in the volume of a body, when its temperature is raised, does not depend on the cavities inside the body.

Applications of linear expansion

55. Platinum (or monel) is used to seal inside glass because both have nearly equal coefficients of linear expansion.
56. Iron or steel is used for reinforcement in concrete because both have nearly equal coefficients of expansion.
57. Pyrex glass has low α. Hence combustion tubes and test tubes for hating purpose are made out of it.
58. Invar steel (steel+nickel) has very low α. So it is used in making pendulum clocks, balancing wheels and measuring tapes. (Composition of invar steel is 64% steel and 36% nickel).
59. Metal pipes that carry steam are provided with bends to allow for expansion.
60. Telephone wires held tightly between the poles snap in winter due to induced tensile stress as a result of prevented contraction.
61. Thick glass tumbler cracks when hot liquid is poured into it because of unequal expansion.
62. Hot chimney cracks when a drop of water falls on it because of unequal contraction.
63. A brass disc snuggly fits in a hole in a steel plate. To loosen the disc from the hole, the system should be cooled.
64. To remove a tight metal cap of a glass bottle, it should be warmed.
65. While laying railway tracks, small gaps are left between adjacent rails to allow for free expansion without affecting the track during summer. Gap to be left $(\Delta l)=\alpha l\Delta t$=expansion of each rail.
66. Concrete roads are laid in sections and expansion channels are provided between them.
67. Thermostat is a device which maintains a steady temperature.
68. Thermostats are used in refrigerators, automatic irons and incubators.
69. Thermostat is a bimetallic strip made of iron and brass. The principle involved is different materials will have different coefficients of linear expansion.
70. A bimetallic strip is used in dial-type thermometer.
71. If an iron ring with a saw-cut is heated, the width of the gap increases.
72. Barometric scale which expands or contracts measures wrong pressure. On expansion the true pressure is less than measured pressure.
    $P_{true}=P_{measured}[1-(\gamma-\alpha)t]$

where γ=coefficient of cubical expansion of mercury
α=coefficient of linear expansion of the material used in making the scale
t=rise of the temperature

73. When a straight bimetallic strip is heated it bends in such a way that the more expansive metal lies on the outer side. If d is the thickness of the each strip in a bimetallic strip, then the radius of the compound strip is given by $R=\frac{d}{(\alpha_2-\alpha_1)\Delta t}$.