FORCE and LAWS of MOTION

Point Mass

(1) An object can be considered as a point object if during motion in a given time, it covers distance much greater than its own size.

(2) Object with negligibly small dimension considered as a point mass.

Inertia

(1) Inherent property of all the bodies by virtue of which they cannot change their state of rest or uniform motion along a straight line by their own is called inertia.

(2) Inertia is not a physical quantity, it is only a property of the body which depends on mass of the body.

(3) Inertia has no units and no dimensions

(4) Two bodies of equal mass, one in motion and another is at rest, possess same inertia because it is a factor of mass only and does not depend upon the velocity.

Linear Momentum

(1) Linear momentum of a body is the quantity of motion contained in the body.

(2) It is measured in terms of the force required to stop the body in unit time.

(3) It is also measured as the product of the mass of the body and its velocity *i.e.*, Momentum = mass × velocity.

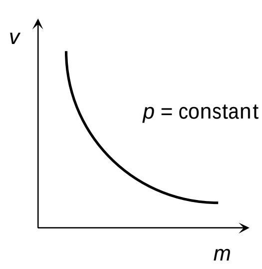
If a body of mass *m* is moving with velocity $\vec{v}$ then its linear momentum $\vec{p}$ is given by $\vec{p} = m\vec{v}$

(4) It is a vector quantity and it's direction is the same as the direction of velocity of the body.

(5) Units : *kg-m/sec* [S.I.], *g-cm/sec* [C.G.S.]

(6) Dimension : $[MLT^{-1}]$

(7) If two objects of different masses have same momentum, the lighter body possesses greater velocity.



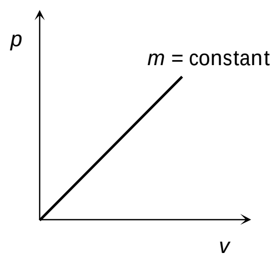
$p = m_1 v_1 = m_2 v_2 =$ constant $\therefore$ $\frac{v_1}{v_2} = \frac{m_2}{m_1}$

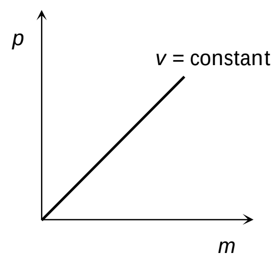
i.e. $v \propto \frac{1}{m}$

[As *p* is constant]

(8) For a given body $p \propto v$

(9) For different bodies moving with same velocities $p \propto m$





Newton's First Law

A body continue to be in its state of rest or of uniform motion along a straight line, unless it is acted upon by some external force to change the state.

(1) If no net force acts on a body, then the velocity of the body cannot change *i.e.* the body cannot accelerate.

(2) Newton's first law defines inertia and is rightly called the law of inertia. Inertia are of three types :

Inertia of rest, Inertia of motion and Inertia of direction.

(3) **Inertia of rest :** It is the inability of a body to change by itself, its state of rest. This means a body at rest remains at rest and cannot start moving by its own.

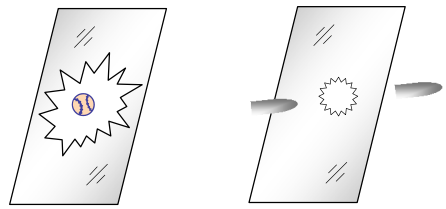
Example : (i) A person who is standing freely in bus, thrown backward, when bus starts suddenly.

When a bus suddenly starts, the force responsible for bringing bus in motion is also transmitted to lower part of body, so this part of the body comes in motion along with the bus. While the upper half of body (say above the waist) receives no force to overcome inertia of rest and so it stays in its original position. Thus there is a relative displacement between the two parts of the body and it appears as if the upper part of the body has been thrown backward.

(i) If the motion of the bus is slow, the inertia of motion will be transmitted to the body of the person uniformly and so the entire body of the person will come in motion with the bus and the person will not experience any jerk.

(ii) When a horse starts suddenly, the rider tends to fall backward on account of inertia of rest of upper part of the body as explained above.

(iii) A bullet fired on a window pane makes a clean hole through it, while a ball breaks the whole window. The bullet has a speed much greater than the ball. So its time of contact with glass is small. So in case of bullet the motion is transmitted only to a small portion of the glass in that small time. Hence a clear hole is created in the glass window, while in case of ball, the time and the area of contact is large. During this time the motion is transmitted to the entire window, thus creating the cracks in the entire window.



Cracks by the ball   Hole by the bullet

(iv) In the arrangement shown in the figure :

(a) If the string B is pulled with a sudden jerk then it will experience tension while due to inertia of rest of mass M this force will not be transmitted to the string A and so the string B will break.

(b) If the string B is pulled steadily the force applied to it will be transmitted from string B to A through the mass M and as tension in A will be greater than in B by Mg (weight of mass M), the string A will break.

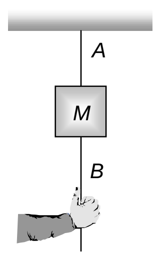


Fig : 4.5

(v) If we place a coin on smooth piece of card board covering a glass and strike the card board piece suddenly with a finger. The cardboard slips away and the coin falls into the glass due to inertia of rest.

(vi) The dust particles in a carpet falls off when it is beaten with a stick. This is because the beating sets the carpet in motion whereas the dust particles tend to remain at rest and hence separate.

(4) **Inertia of motion :** It is the inability of a body to change by itself its state of uniform motion *i.e.,* a body in uniform motion can neither accelerate nor retard by its own.

Example : (i) When a bus or train stops suddenly, a passenger sitting inside tends to fall forward. This is because the lower part of his body comes to rest with the bus or train but the upper part tends to continue its motion due to inertia of motion.

(ii) A person jumping out of a moving train may fall forward.

(iii) An athlete runs a certain distance before taking a long jump. This is because velocity acquired by running is added to velocity of the athlete at the time of jump. Hence he can jump over a longer distance.

(5) **Inertia of direction :** It is the inability of a body to change by itself it's direction of motion.

Example : (i) When a stone tied to one end of a string is whirled and the string breaks suddenly, the stone flies off along the tangent to the circle. This is because the pull in the string was forcing the stone to move in a circle. As soon as the string breaks, the pull vanishes. The stone in a bid to move along the straight line flies off tangentially.

(ii) The rotating wheel of any vehicle throw out mud, if any, tangentially, due to directional inertia.

(iii) When a car goes round a curve suddenly, the person sitting inside is thrown outwards.

Newton's Second Law

(1) The rate of change of linear momentum of a body is directly proportional to the external force applied on the body and this change takes place always in the direction of the applied force.

(2) If a body of mass *m*, moves with velocity $\vec{v}$ then its linear momentum can be given by $\vec{p} = m\vec{v}$ and if force $\vec{F}$ is applied on a body, then

$$\vec{F} \propto \frac{d\vec{p}}{dt} \Rightarrow F = K\frac{d\vec{p}}{dt}$$

or $\vec{F} = \frac{d\vec{p}}{dt}$ (K = 1 in C.G.S. and S.I. units)

or $\vec{F} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a}$

(As $a = \frac{d\vec{v}}{dt} =$ acceleration produced in the body)

$\therefore \vec{F} = m\vec{a}$

Force = mass $\times$ acceleration

Force

(1) Force is an external effect in the form of a push or pull which

(i) Produces or tries to produce motion in a body at rest.

(ii) Stops or tries to stop a moving body.

(iii) Changes or tries to change the direction of motion of the body.

Units : Absolute units : (i) *Newton* (S.I.) (ii) *Dyne* (C.G.S)

Gravitational units : (i) Kilogram-force (M.K.S.) (ii) Gram-force (C.G.S)

Newton : One Newton is that force which produces an acceleration of $1 m/s^2$ in a body of mass 1 *Kilogram*.

$\therefore 1$ *Newton* $= 1 kg\text{–}m/s^2$

Dyne : One dyne is that force which produces an acceleration of $1 cm/s^2$ in a body of mass 1 *gram*.

$\therefore$ 1 *Dyne* $= 1 gm\ cm/\sec^2$

Relation between absolute units of force 1 *Newton* $= 10^5$ *Dyne*

Kilogram-force : It is that force which produces an acceleration of $9.8 m/s^2$ in a body of mass 1 *kg*.

$\therefore$ 1 *kg-f* = 9.80 *Newton*

Gram-force : It is that force which produces an acceleration of $980 cm/s^2$ in a body of mass 1*gm*.

$\therefore$ 1 *gm-f* = 980 *Dyne*

$\vec{F} = m\vec{a}$ formula is valid only if force is changing the state of rest or motion and the mass of the body is constant and finite.

If *m* is not constant $\vec{F} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$

If force and acceleration have three component along *x*, *y* and *z* axis, then

$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ and $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

From above it is clear that $F_x = ma_x,\ F_y = ma_y, F_z = ma_z$

No force is required to move a body uniformly along a straight line with constant speed.

$\vec{F} = m\vec{a}$ $\qquad \therefore \vec{F} = 0$ (As $\vec{a} = 0$)

When force is written without direction then positive force means repulsive while negative force means attractive.

Example : *Positive force* – Force between two similar charges

Negative force – Force between two opposite charges

Out of so many natural forces, for distance 10^{-15} *metre*, nuclear force is strongest while gravitational force weakest. $F_{\text{nuclear}} > F_{\text{electromag netic}} > F_{\text{gravitatio nal}}$

Constant force : If the direction and magnitude of a force is constant. It is said to be a constant force.

Variable or dependent force :

(i) *Time dependent force* : In case of impulse or motion of a charged particle in an alternating electric field force is time dependent.

(ii) *Position dependent force* : Gravitational force between two bodies $\frac{Gm_1m_2}{r^2}$

or Force between two charged particles $= \frac{q_1q_2}{4\pi\varepsilon_0 r^2}$.

(iii) *Velocity dependent force* : Viscous force $(6\pi\eta rv)$

Force on charged particle in a magnetic field $(qvB\sin\theta)$

(13) Central force : If a position dependent force is directed towards or away from a fixed point it is said to be central otherwise non-central.

Example : Motion of Earth around the Sun. Motion of electron in an atom. Scattering of α-particles from a nucleus.

(14) Conservative or non conservative force : If under the action of a force the work done in a round trip is zero or the work is path independent, the force is said to be conservative otherwise non conservative.

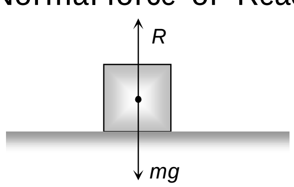
Example : Conservative force : Gravitational force, electric force, elastic force.

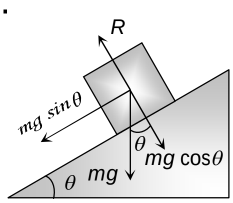
Non conservative force : Frictional force, viscous force.

(15) Common forces in mechanics :

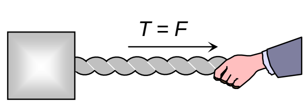
(i) *Weight* : Weight of an object is the force with which earth attracts it. It is also called the force of gravity or the gravitational force.

(ii) *Reaction or Normal force :* When a body is placed on a rigid surface, the body experiences a force which is perpendicular to the surfaces in contact. Then force is called 'Normal force' or 'Reaction'.

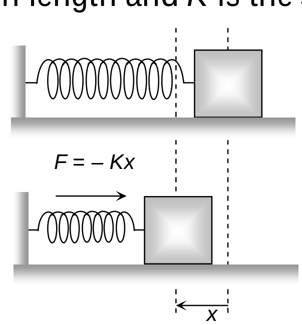




(iii) *Tension* : The force exerted by the end of taut string, rope or chain against pulling (applied) force is called the tension. The direction of tension is so as to pull the body.



(iv) *Spring force :* Every spring resists any attempt to change its length. This resistive force increases with change in length. Spring force is given by $F = -Kx$; where x is the change in length and K is the spring constant (unit N/m).



Equilibrium of Concurrent Force

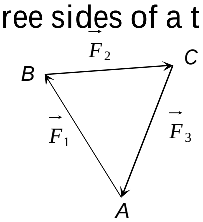
(1) If all the forces working on a body are acting on the same point, then they are said to be concurrent.

(2) A body, under the action of concurrent forces, is said to be in equilibrium, when there is no change in the state of rest or of uniform motion along a straight line.

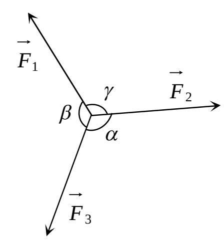
(3) The necessary condition for the equilibrium of a body under the action of concurrent forces is that the vector sum of all the forces acting on the body must be zero.

(4) Mathematically for equilibrium $\sum \vec{F}_{\text{net}} = 0$ or $\sum F_x = 0$; $\sum F_y = 0$; , $\sum F_z = 0$

(5) Three concurrent forces will be in equilibrium, if they can be represented completely by three sides of a triangle taken in order.



(6) Lami's Theorem : For three concurrent forces in equilibrium $\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$



Newton's Third Law

To every action, there is always an equal (in magnitude) and opposite (in direction) reaction.

(1) When a body exerts a force on any other body, the second body also exerts an equal and opposite force on the first.

(2) Forces in nature always occurs in pairs. A single isolated force is not possible.

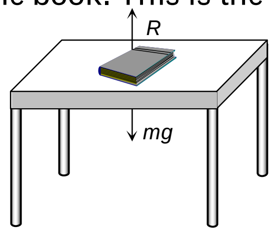
(3) Any agent, applying a force also experiences a force of equal magnitude but in opposite direction. The force applied by the agent is called '*Action*' and the counter force experienced by it is called '*Reaction*'.

(4) Action and reaction never act on the same body. If it were so, the total force on a body would have always been zero *i.e.* the body will always remain in equilibrium.

(5) If $\vec{F}_{AB}$ = force exerted on body *A* by body *B* (Action) and $\vec{F}_{BA}$ = force exerted on body *B* by body *A* (Reaction)

Then according to Newton's third law of motion $\vec{F}_{AB} = -\vec{F}_{BA}$

(6) Example : (i) A book lying on a table exerts a force on the table which is equal to the weight of the book. This is the force of action.



The table supports the book, by exerting an equal force on the book. This is the force of reaction.

As the system is at rest, net force on it is zero. Therefore force of action and reaction must be equal and opposite.

(ii) Swimming is possible due to third law of motion.

(iii) When a gun is fired, the bullet moves forward (action). The gun recoils backward (reaction)

(iv) Rebounding of rubber ball takes place due to third law of motion.

(v) While walking a person presses the ground in the backward direction (action) by his feet. The ground pushes the person in forward direction with an equal force (reaction). The component of reaction in horizontal direction makes the person move forward.

(vi) It is difficult to walk on sand or ice.

(vii) Driving a nail into a wooden block without holding the block is difficult.

Impulse

(1) When a large force works on a body for very small time interval, it is called impulsive force.

An impulsive force does not remain constant, but changes first from zero to maximum and then from maximum to zero. In such case we measure the total effect of force.

(2) Impulse of a force is a measure of total effect of force.

(3) $\vec{I} = \int_{t_1}^{t_2} \vec{F}\, dt$.

(4) Impulse is a vector quantity and its direction is same as that of force.

(5) Units : *Newton-second* or *Kg-m-*s^{-1} (S.I.)

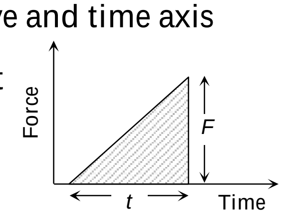
Dyne-second or *gm-cm-*s^{-1} (C.G.S.)

(6) Force-time graph : Impulse is equal to the area under *F-t* curve.

If we plot a graph between force and time, the area under the curve and time axis gives the value of impulse.

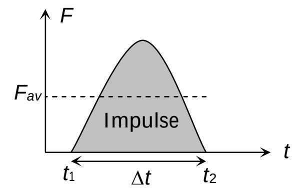
$I =$ Area between curve and time axis

$= \frac{1}{2} \times$ Base $\times$ Height



$= \frac{1}{2} F\, t$

(7) From Newton's second law



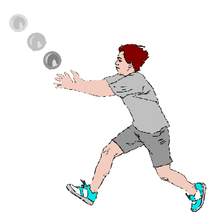
$\Rightarrow \vec{I} = \vec{p}_2 - \vec{p}_1 = \overrightarrow{\Delta p}$

i.e. The impulse of a force is equal to the change in momentum.
This statement is known as *Impulse momentum theorem*.
Examples : Hitting, kicking, catching, jumping, diving, collision *etc*.
In all these cases an impulse acts.
$I = \int F\, dt = F_{av} . \Delta t = \Delta p =$ constant

So if time of contact Δt is increased, average force is decreased (or diluted) and vice-versa.

(i) In hitting or kicking a ball we decrease the time of contact so that large force acts on the ball producing greater acceleration.

(ii) In catching a ball a player by drawing his hands backwards increases the time of contact and so, lesser force acts on his hands and his hands are saved from getting hurt.



(iii) In jumping on sand (or water) the time of contact is increased due to yielding of sand or water so force is decreased and we are not injured. However if we jump on cemented floor the motion stops in a very short interval of time resulting in a large force due to which we are seriously injured.

(iv) An athlete is advised to come to stop slowly after finishing a fast race, so that time of stop increases and hence force experienced by him decreases.

(v) China wares are wrapped in straw or paper before packing.

Law of Conservation of Linear Momentum

If no external force acts on a system (called isolated) of constant mass, the total momentum of the system remains constant with time.

(1) According to this law for a system of particles $\vec{F} = \frac{d\vec{p}}{dt}$

In the absence of external force $\vec{F} = 0$ then $\vec{p} =$ constant

i.e., $\vec{p} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + ... =$ constant.

or $m_1 \vec{v}_1 + m_2 \vec{v}_2 + m_3 \vec{v}_3 + ... =$ constant

This equation shows that in absence of external force for a closed system the linear momentum of individual particles may change but their sum remains unchanged with time.

(2) Law of conservation of linear momentum is independent of frame of reference, though linear momentum depends on frame of reference.

(3) Conservation of linear momentum is equivalent to Newton's third law of motion.

For a system of two particles in absence of external force, by law of conservation of linear momentum.

$\vec{p}_1 + \vec{p}_2 =$ constant.

$\therefore m_1\vec{v}_1 + m_2\vec{v}_2 =$ constant.

Differentiating above with respect to time

$$m_1 \frac{d\vec{v}_1}{dt} + m_2 \frac{d\vec{v}_2}{dt} = 0 \Rightarrow m_1\vec{a}_1 + m_2\vec{a}_2 = 0 \Rightarrow \vec{F}_1 + \vec{F}_2 = 0$$

$\therefore \vec{F}_2 = -\vec{F}_1$

i.e. for every action there is an equal and opposite reaction which is Newton's third law of motion.

(4) Practical applications of the law of conservation of linear momentum

(i) When a man jumps out of a boat on the shore, the boat is pushed slightly away from the shore.

(ii) A person left on a frictionless surface can get away from it by blowing air out of his mouth or by throwing some object in a direction opposite to the direction in which he wants to move.

(iii) **Recoiling of a gun :** For bullet and gun system, the force exerted by trigger will be internal so the momentum of the system remains unaffected.

Let $m_G =$ mass of gun, $m_B =$ mass of bullet,

$v_G =$ velocity of gun, $v_B =$ velocity of bullet

Initial momentum of system = 0

Final momentum of system $= m_G\vec{v}_G + m_B\vec{v}_B$

By the law of conservation of linear momentum

$$m_G\vec{v}_G + m_B\vec{v}_B = 0$$

So recoil velocity $\vec{v}_G = -\frac{m_B}{m_G}\vec{v}_B$

(a) Here negative sign indicates that the velocity of recoil $\vec{v}_G$ is opposite to the velocity of the bullet.

(b) $v_G \propto \frac{1}{m_G}$ *i.e.* higher the mass of gun, lesser the velocity of recoil of gun.

(c) While firing the gun must be held tightly to the shoulder, this would save hurting the shoulder because in this condition the body of the shooter and the gun behave as one body. Total mass become large and recoil velocity becomes too small.

$$v_G \propto \frac{1}{m_G + m_{\text{man}}}$$