Motion

Rest and Motion

If a body does not change its position as time passes with respect to frame of reference, it is said to be at rest.

And if a body changes its position as time passes with respect to frame of reference, it is said to be in motion.

Frame of Reference : It is a system to which a set of coordinates are attached and with reference to which observer describes any event.

A passenger standing on platform observes that a tree on a platform is at rest. But the same passenger passing away in a train through station, observes that tree is in motion. In both conditions observer is right. But observations are different because in first situation observer stands on a platform, which is reference frame at rest and in second situation observer moving in train, which is reference frame in motion.

So rest and motion are relative terms. It depends upon the frame of references.

Particle or Point Mass or Point object

The smallest part of matter with negligibly small dimension which can be described by its mass and position is defined as a particle or point mass.

If the size of a body is negligible in comparison to its range of motion then that body is known as particle if it does not rotate about its axis.

In above consideration when we treat body as particle, all parts of the body undergo same displacement and have same velocity and acceleration.

Rectilinear and Translatory Motion

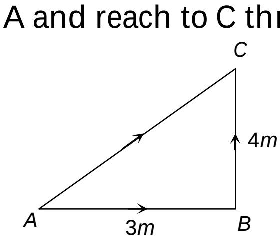
If a particle is moving along a straight line, the motion is called rectilinear. However if the body cannot be treated as a point but moves in such a way that all the particles move simultaneously along straight lines by shifting through equal distance in a given time, the motion of the body is called translatoryNote:

Translatory or rectilinear motion can be uniform or non-uniform.

Distance and Displacement

Distance : It is the actual length of the path covered by a moving particle in a given interval of time.

If a particle starts from A and reach to C through point B as shown in the figure.



Then distance travelled by particle

$= AB + BC = 7$ *m*

Distance is a scalar quantity.

Unit : metre (S.I.)

Displacement : Displacement is the change in position vector i.e., A vector joining initial to final position.

Displacement is a vector quantity

Dimension : $[M^0L^1T^0]$

Unit : metre (S.I.)

In the above figure the displacement of the particle $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} \Rightarrow |AC|$

$$= \sqrt{(AB)^2 + (BC)^2 + 2(AB)(BC)\cos 90^o} = 5\ m$$

If $\vec{S}_1, \vec{S}_2, \vec{S}_3 .... \vec{S}_n$ are the displacements of a body then the total (net) displacement is the vector sum of the individuals. $\vec{S} = \vec{S}_1 + \vec{S}_2 + \vec{S}_3 + ... + \vec{S}_n$

Comparison between distance and displacement :

The magnitude of displacement is equal to minimum possible distance between two positions.

So distance $\geq$ |Displacement|.

For a moving particle distance can never be negative or zero while displacement can be.

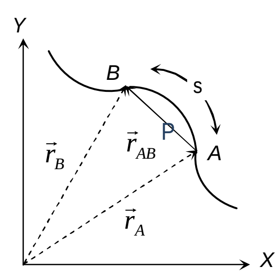
(zero displacement means that body after motion has came back to initial position)

i.e., Distance > 0 but Displacement > = or < 0

For motion between two points, displacement is single valued while distance depends on actual path and so can have many values.

For a moving particle distance can never decrease with time while displacement can. Decrease in displacement with time means body is moving towards the initial position.

In general, magnitude of displacement is not equal to distance. However, it can be so if the motion is along a straight line without change in direction.



If $\vec{r}_A$ and $\vec{r}_B$ are the position vectors of particle initially and finally.

Then displacement of the particle $\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$

and *s* is the distance travelled if the particle has gone through the path *APB*.

Speed and Velocity

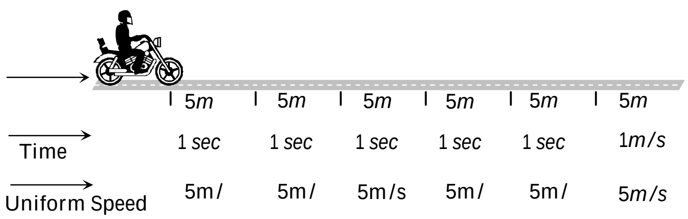
Speed : The rate of distance covered with time is called speed.

It is a scalar quantity having symbol υ.

Unit : *metre/second* (S.I.), *cm/second* (C.G.S.)

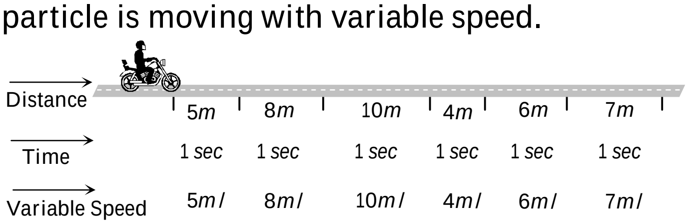
Types of speed :

(a) **Uniform speed :** When a particle covers equal distances in equal intervals of time, (no matter how small the intervals are) then it is said to be moving with uniform speed. In given illustration motorcyclist travels equal distance (= 5*m*) in each second. So we can say that particle is moving with uniform speed of 5 *m/s*.



(b) **Non-uniform (variable) speed :** In non-uniform speed particle covers unequal distances in equal intervals of time. In the given illustration motorcyclist travels 5*m* in 1st second, 8*m* in 2nd second, 10*m* in 3rd second, 4*m* in 4th second *etc*.

Therefore its speed is different for every time interval of one second. This means particle is moving with variable speed.



Average speed : The average speed of a particle for a given 'Interval of time' is defined as the ratio of total distance travelled to the time taken.

Average speed $= \dfrac{\text{Total distance travelled}}{\text{Time taken}}$; $v_{av} = \dfrac{\Delta s}{\Delta t}$

Time average speed : When particle moves with different uniform speed υ_1, υ_2, υ_3 ... *etc* in different time intervals t_1, t_2, t_3, ... *etc* respectively, its average speed over the total time of journey is given as

$$v_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}}$$

$$= \frac{d_1 + d_2 + d_3 + ...}{t_1 + t_2 + t_3 + ...} = \frac{\upsilon_1 t_1 + \upsilon_2 t_2 + \upsilon_3 t_3 + ...}{t_1 + t_2 + t_3 + ...}$$

Distance averaged speed : When a particle describes different distances d_1, d_2, d_3, ... with different time intervals t_1, t_2, t_3, ... with speeds $v_1, v_2, v_3 ......$ respectively then the speed of particle averaged over the total distance can be given as

$$\upsilon_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}} = \frac{d_1 + d_2 + d_3 + ...}{t_1 + t_2 + t_3 + ...}$$

$$= \frac{d_1 + d_2 + d_3 + ...}{\frac{d_1}{\upsilon_1} + \frac{d_2}{\upsilon_2} + \frac{d_3}{\upsilon_3} + ...}$$

Instantaneous speed : It is the speed of a particle at a particular instant of time. When we say "speed", it usually means instantaneous speed.

The instantaneous speed is average speed for infinitesimally small time interval (*i.e.*, $\Delta t \to 0$). Thus

Velocity : The rate of change of position *i.e.* rate of displacement with time is called velocity.

It is a vector quantity having symbol $\vec{v}$.

Unit : *metre/second* (S.I.), *cm/second* (C.G.S.)

Types of velocity :

(a) **Uniform velocity :** A particle is said to have uniform velocity, if magnitudes as well as direction of its velocity remains same and this is possible only when the particles moves in same straight line without reversing its direction.

(b) **Non-uniform velocity :** A particle is said to have non-uniform velocity, if either of magnitude or direction of velocity changes or both of them change.

(c) **Average velocity** : It is defined as the ratio of displacement to time taken by the body

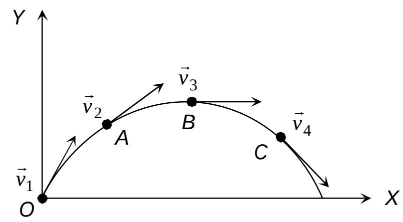
$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time taken}}; \quad \vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t}$$

(d) **Instantaneous velocity :** Instantaneous velocity is defined as rate of change of position vector of particles with time at a certain instant of time.

Comparison between instantaneous speed and instantaneous velocity

(a) instantaneous velocity is always tangential to the path followed by the particle.

When a stone is thrown from point *O* then at point of projection the instantaneous velocity of stone is $\vec{v}_1$, at point *A* the instantaneous velocity of stone is $\vec{v}_2$, similarly at point *B* and *C* are $\vec{v}_3$ and $\vec{v}_4$ respectively.



Direction of these velocities can be found out by drawing a tangent on the trajectory at a given point.

(b) A particle may have constant instantaneous speed but variable instantaneous velocity.

Example : When a particle is performing uniform circular motion then for every instant of its circular motion its speed remains constant but velocity changes at every instant.

(c) The magnitude of instantaneous velocity is equal to the instantaneous speed.

(d) If a particle is moving with constant velocity then its average velocity and instantaneous velocity are always equal.

(e) If displacement is given as a function of time, then time derivative of displacement will give velocity.

Let displacement $\vec{x} = A_0 - A_1 t + A_2 t^2$

$\vec{v} = -A_1 + 2A_2 t$

For the given value of *t*, we can find out the instantaneous velocity.

e.g for $t = 0$,Instantaneous velocity $\vec{v} = -A_1$ and Instantaneous speed $|\vec{v}| = A_1$

Comparison between average speed and average velocity

(a) Average speed is a scalar while average velocity is a vector both having same units (*m*/*s*)

(b) Average speed or velocity depends on time interval over which it is defined.

(c) For a given time interval average velocity is single valued while average speed can have many values depending on path followed.

(d) If after motion body comes back to its initial position then $\vec{v}_{av} = 0$ (as $\Delta\vec{r} = 0$) but $v_{av} > 0$ and finite as $(\Delta s > 0)$.

(e) For a moving body average speed can never be negative or zero (unless $t \to \infty$) while average velocity can be *i.e.* $v_{av} > 0$ while $\vec{v}_{av}$ = or < 0.

(f) As we know for a given time interval
Distance $\geq$ |displacement|
$\therefore$ Average speed $\geq$ |Average velocity|

Acceleration

The time rate of change of velocity of an object is called acceleration of the object.

It is a vector quantity. It's direction is same as that of change in velocity (Not of the velocity)

When only direction of velocity changes	**When only magnitude of velocity changes**	**When both magnitude and direction of velocity changes**
Acceleration perpendicular to velocity	Acceleration parallel or anti-parallel to velocity	Acceleration has two components one is perpendicular to velocity and another parallel or anti-parallel to velocity
Ex.. Uniform circular motion	*Ex.*.1-D Motion under gravity	*Ex.*. Projectile motion, Non Uniform Circular Motion

Unit : *metre/second*² (S.I.); *cm/second*² (C.G.S.)

Types of acceleration :

Uniform acceleration : A body is said to have uniform acceleration if magnitude and direction of the acceleration remains constant during particle motion.

Non-uniform acceleration : A body is said to have non-uniform acceleration, if either magnitude or direction or both of them change during motion.

Average acceleration : $\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$

The direction of average acceleration vector is the direction of the change in velocity vector as $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$

Instantaneous acceleration It's the acceleration at any instant of time.

For a moving body there is no relation between the direction of instantaneous velocity and direction of acceleration.

Ex.. In uniform circular motion $\theta = 90°$ always

In a projectile motion θ is variable for every point of trajectory.

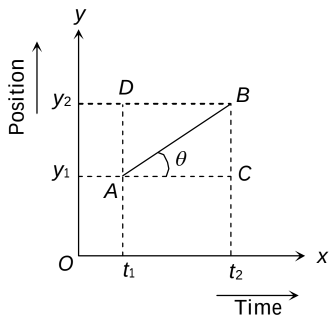
Acceleration can be positive, zero or negative. Positive acceleration means velocity increasing with time, zero acceleration means velocity is uniform constant while negative acceleration (retardation) means velocity is decreasing with time.

For motion of a body under gravity, acceleration will be equal to "*g*", where g is the acceleration due to gravity. Its value is 9.8 m/s^2 or 980 cm/s^2 or 32 feet/s^2.

Position time Graph

During motion of the particle its parameters of kinematical analysis (*v*, *a*, *s*) changes with time. This can be represented on the graph.

Position time graph is plotted by taking time *t* along *x*-axis and position of the particle on *y*-axis.



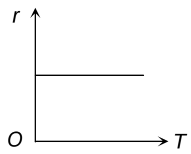
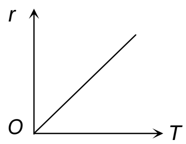
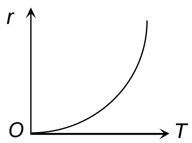
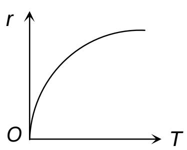
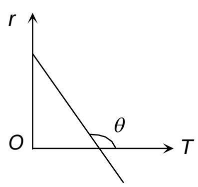
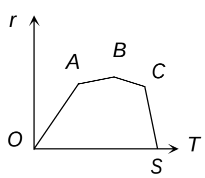
Let *AB* is a position-time graph for any moving particle

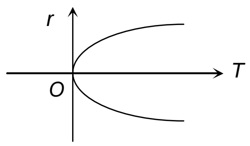
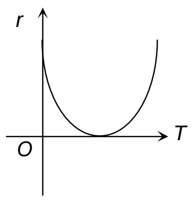
As Velocity = $\dfrac{\text{Change in position}}{\text{Time taken}} = \dfrac{y_2 - y_1}{t_2 - t_1}$ ...(i)

From triangle *ABC*, $\tan\theta = \dfrac{BC}{AC} = \dfrac{AD}{AC} = \dfrac{y_2 - y_1}{t_2 - t_1}$ ....(ii)

By comparing (i) and (ii) Velocity = $\tan\theta$

$$v = \tan\theta$$

	θ = 0° so *v* = 0 *i.e.*, line parallel to time axis represents that the particle is at rest.
	θ = constant so *v* = constant, *a* = 0 i.e., line with constant slope represents uniform velocity of the particle.
	θ is increasing so *v* is increasing, *a* is positive. *i.e.*, line bending towards position axis represents increasing velocity of particle. It means the particle possesses acceleration.
	θ is decreasing so *v* is decreasing, *a* is negative *i.e.*, line bending towards time axis represents decreasing velocity of the particle. It means the particle possesses retardation.
	θ constant but > 90° so *v* will be constant but negative *i.e.*, line with negative slope represent that particle returns towards the point of reference. (negative displacement).
	Straight line segments of different slopes represent that velocity of the body changes after certain interval of time.

	This graph shows that at one instant the particle has two positions, which is not possible.
	The graph shows that particle coming towards origin initially and after that it is moving away from origin.

Velocity-time Graph

The graph is plotted by taking time *t* along *x*-axis and velocity of the particle on *y*-axis.

Distance and displacement : The area covered between the velocity time graph and time axis gives the displacement and distance travelled by the body for a given time interval.

Total distance $= |A_1| + |A_2| + |A_3|$

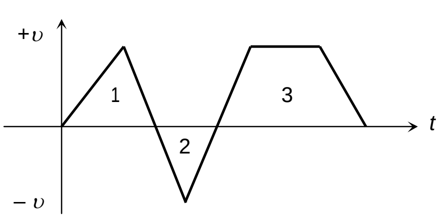
= Addition of modulus of different area. *i.e.* $s = \int |\upsilon|\, dt$

Total displacement $= A_1 + A_2 + A_3$

= Addition of different area considering their sign.

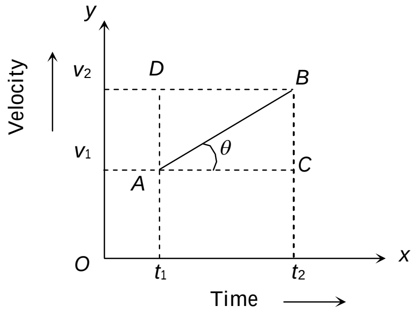
i.e. $r = \int \upsilon\, dt$

Area above time axis is taken as positive, while area below time axis is taken as negative



here A_1 and A_2 are area of triangle 1 and 2 respectively and A_3 is the area of trapezium .

Acceleration : Let AB is a velocity-time graph for any moving particle



As Acceleration = $\frac{\text{Change in velocity}}{\text{Time taken}}$

$= \frac{v_2 - v_1}{t_2 - t_1}$ ...(i)

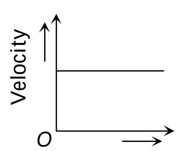
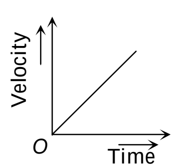
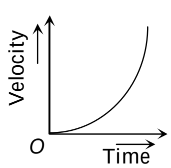
From triangle ABC, $\tan\theta = \frac{BC}{AC} = \frac{AD}{AC}$

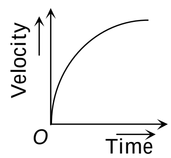
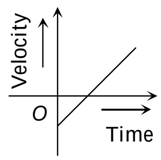
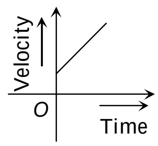
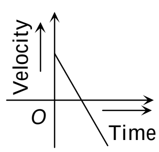
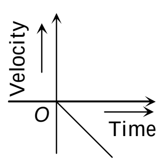
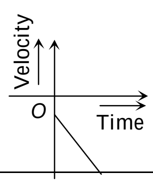
$= \frac{v_2 - v_1}{t_2 - t_1}$ ....(ii)

By comparing (i) and (ii)

Acceleration (a) = $\tan\theta$

It is clear that slope of tangent on velocity-time graph represents the acceleration of the particle.

	θ = 0°, a = 0, v = constant *i.e.*, line parallel to time axis represents that the particle is moving with constant velocity.
	θ = constant, so a = constant and v is increasing uniformly with time *i.e.*, line with constant slope represents uniform acceleration of the particle.
	θ *is* increasing so acceleration increasing *i.e.*, line bending towards velocity axis represent the increasing acceleration in the body.

	θ decreasing so acceleration decreasing i.e. line bending towards time axis represents the decreasing acceleration in the body
	Positive constant acceleration because θ is constant and < 90 but initial velocity of the particle is negative.
	Positive constant acceleration because θ is constant and < 90 but initial velocity of particle is positive.
	Negative constant acceleration because θ is constant and > 90 but initial velocity of the particle is positive.
	Negative constant acceleration because θ is constant and > 90 but initial velocity of the particle is zero.
	Negative constant acceleration because θ is constant and > 90 but initial velocity of the particle is negative.

Equation of Kinematics

These are the various relations between *u*, *v*, *a*, *t* and *s* for the particle moving with uniform acceleration where the notations are used as :

u = Initial velocity of the particle at time t = 0 *sec*

v = Final velocity at time t *sec*

a = Acceleration of the particle

s = Distance travelled in time t *sec*

s_n = Distance travelled by the body in n^{th} *sec*

When particle moves with zero acceleration

(i) It is a unidirectional motion with constant speed.

(ii) Magnitude of displacement is always equal to the distance travelled.

(iii) $v = u$, $s = u\,t$ [As $a = 0$]

When particle moves with constant acceleration

(i) Acceleration is said to be constant when both the magnitude and direction of acceleration remain constant.

(ii) There will be one dimensional motion if initial velocity and acceleration are parallel or anti-parallel to each other.

(iii) Equations of motion (in scalar from) — Equation of motion (in vector from)

Equations of motion (in scalar from)	Equation of motion (in vector from)
$\upsilon = u + at$	$\vec{v} = \vec{u} + \vec{a}t$
$s = ut + \frac{1}{2}at^2$	$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$
$\upsilon^2 = u^2 + 2as$	$\vec{v}.\vec{v} - \vec{u}.\vec{u} = 2\vec{a}.\vec{s}$
$s = \left(\frac{u+v}{2}\right)t$	$\vec{s} = \frac{1}{2}(\vec{u} + \vec{v})t$
$s_n = u + \frac{a}{2}(2n-1)$	$\vec{s}_n = \vec{u} + \frac{\vec{a}}{2}(2n-1)$

Motion of Body Under Gravity (Free Fall)

The force of attraction of earth on bodies, is called force of gravity. Acceleration produced in the body by the force of gravity, is called acceleration due to gravity. It is represented by the symbol g.

In the absence of air resistance, it is found that all bodies (irrespective of the size, weight or composition) fall with the same acceleration near the surface of the earth. This motion of a body falling towards the earth from a small altitude ($h << R$) is called free fall.

An ideal example of one-dimensional motion is motion under gravity in which air resistance and the small changes in acceleration with height are neglected.

(1) **If a body is dropped from some height (initial velocity zero)**

(i) Equations of motion : Taking initial position as origin and direction of motion (*i.e.*, downward direction) as a positive, here we have

$u = 0$ [As body starts from rest]

$a = +g$ [As acceleration is in the direction of motion]

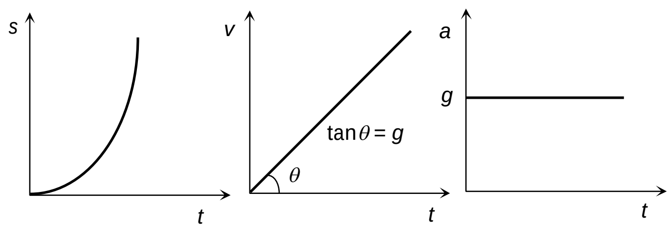
$v = g\,t$ ...(i)

$h = \frac{1}{2}gt^2$ ...(ii)

$\upsilon^2 = 2gh$ ...(iii)

$h_n = \frac{g}{2}(2n-1)$ ...(iv)

(ii) Graph of distance, velocity and acceleration with respect to time :



(iii) As $h = (1/2)gt^2$, *i.e.*, $h \propto t^2$, distance covered in time t, $2t$, $3t$, *etc*., will be in the ratio of $1^2 : 2^2 : 3^2$, *i.e.*, square of integers.

(iv) The distance covered in the *nth sec*, $h_n = \frac{1}{2}g(2n-1)$

So distance covered in 1st, 2nd, 3rd *sec*, *etc*., will be in the ratio of 1 : 3 : 5, *i.e.*, odd integers only.

(2) **If a body is projected vertically downward with some initial velocity**

Equation of motion :

$\upsilon = u + g\,t$

$h = ut + \frac{1}{2}g\,t^2$

$\upsilon^2 = u^2 + 2gh$

$h_n = u + \frac{g}{2}(2n-1)$

(3) **If a body is projected vertically upward**

(i) Equation of motion : Taking initial position as origin and direction of motion (*i.e.*, vertically up) as positive

$a = -g$ [As acceleration is downwards while motion upwards]

So, if the body is projected with velocity u and after time t it reaches up to height h then

$\upsilon = u - g\,t$; $h = ut - \frac{1}{2}g\,t^2$; $\upsilon^2 = u^2 - 2gh$; $h_n = u - \frac{g}{2}(2n-1)$

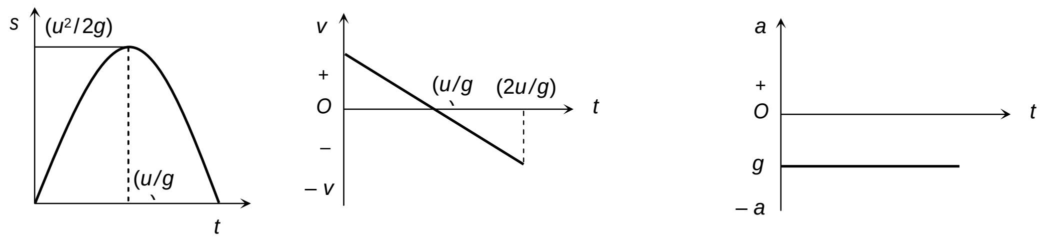
(ii) For maximum height $v = 0$

So from above equation $u = gt$,

$h = \frac{1}{2} g t^2$

and $u^2 = 2gh$

(iii) Graph of displacement, velocity and acceleration with respect to time :



The motion is independent of the mass of the body, as in any equation of motion, mass is not involved. That is why a heavy and light body when released from the same height, reach the ground simultaneously and with same velocity *i.e.*, $t = \sqrt{(2h/g)}$ and $v = \sqrt{2gh}$.

In case of motion under gravity, time taken to go up is equal to the time taken to fall down through the same distance. Time of descent (t_2) = time of ascent (t_1) = u/g

$\therefore$ Total time of flight $T = t_1 + t_2 = \frac{2u}{g}$

In case of motion under gravity, the speed with which a body is projected up is equal to the speed with which it comes back to the point of projection.

As well as the magnitude of velocity at any point on the path is same whether the body is moving in upwards or downward direction.

A body is thrown vertically upwards. If air resistance is to be taken into account, then the time of ascent is less than the time of descent. $t_2 > t_1$

Let u is the initial velocity of body then time of ascent $t_1 = \frac{u}{g+a}$ and $h = \frac{u^2}{2(g+a)}$

where g is acceleration due to gravity and a is retardation by air resistance and for upward motion both will work vertically downward.

For downward motion a and g will work in opposite direction because a always work in direction opposite to motion and g always work vertically downward.

So $h = \frac{1}{2}(g-a)t_2^2$

$\Rightarrow \frac{u^2}{2(g+a)} = \frac{1}{2}(g-a)t_2^2$

$$\Rightarrow t_2 = \frac{u}{\sqrt{(g+a)(g-a)}}$$

Comparing t_1 and t_2 we can say that $t_2 > t_1$

since $(g + a) > (g - a)$