Magnetic Effects of Electric Current

Oersted found that a magnetic field is established around a current carrying conductor.

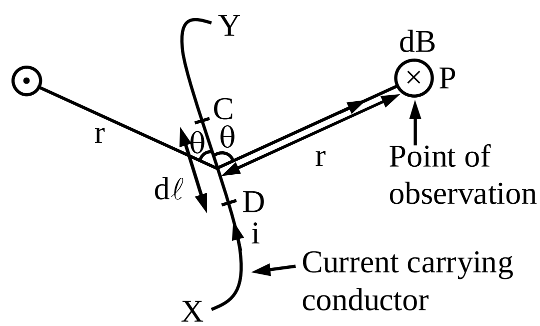
Magnetic field exists as long as there is current in the wire.

Biot-Savart's Law

Biot-Savart's law is used to determine the magnetic field at any point due to a current carrying conductor.

According to Biot-Savart Law, magnetic field at point '*P*' due to the current element $i\vec{dl}$ is given by the expression, $d\vec{B} = k\dfrac{i\,dl\sin\theta}{r^2}\hat{n}$ also $\vec{B} = \int d\vec{B} = \dfrac{\mu_0 i}{4\pi}.\int \dfrac{dl\sin\theta}{r^2}\hat{n}$

In C.G.S. $k = 1$ and in S.I. : $k = \frac{\mu_0}{4\pi}$



where μ_0 = Absolute permeability of air or vacuum $= 4\pi \times 10^{-7}\dfrac{Wb}{Amp - metre}$. It's other units are

$\dfrac{Henry}{metre}$ or $\dfrac{N}{Amp^2}$ or $\dfrac{Tesla - metre}{Ampere}$

Vectorially, $d\vec{B} = \dfrac{\mu_0}{4\pi}\cdot\dfrac{i(\vec{dl}\times\hat{r})}{r^2} = \dfrac{\mu_0}{4\pi}\cdot\dfrac{i(\vec{dl}\times\vec{r})}{r^3}$

Direction of Magnetic Field

The direction of magnetic field is determined with the help of the following simple laws

(1) **Maxwell's cork screw rule :** According to this rule, if

we imagine a right handed screw placed along the curre carrying linear conductor, be rotated such that the screw moves in the direction of flow of current, then the direction of rotationof the thumb gives the direction of magnetic lines of force.

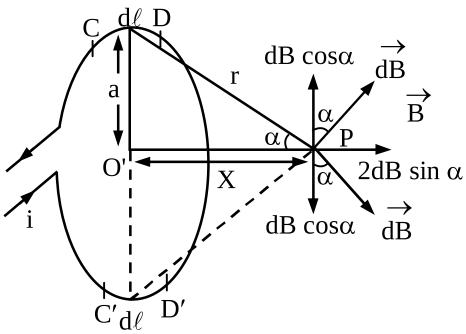
(2) **Right hand thumb rule :** According to this rule if a straight current carrying conductor is held in the right hand such that the thumb of the hand represents the direction of current flow, then the direction of folding fingers will represent the direction of magnetic lines of force.

(3) **Right hand thumb rule of circular currents :** According to this rule if the direction of current in circular conducting coil is in the direction of folding fingers of right hand, then the direction of magnetic field will be in the direction of stretched thumb.

(4) **Right hand palm rule**

If we stretch our right hand such that fingers point towards the point. At which magnetic field is required while thumb is in the direction of current then normal to the palm will show the direction of magnetic field.

Magnetic Field Due to Circular Current



$$\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{(2\pi a^2)i}{(a^2+x^2)^{3/2}}\right)\hat{i}$$

If number of turns in coil is 'n', then $B \propto n$

Therefore, $\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{(2\pi a^2)ni}{(a^2+x^2)^{3/2}}\right)\hat{i}$

Where $\hat{i}$ is the unit vector along x-axis which is the axis of coil in this case.

Special cases :

(a) x = 0 i.e. P is centre of coil

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2\pi a^2 i}{a^3} = \frac{\mu_0 i}{2r}\hat{i}$$

(b) $x = \pm a$,

$$B = \frac{\mu_0}{4\pi}\frac{(2\pi a^2)in}{(2a^2)^{3/2}} = \frac{\mu_0}{4\pi}\frac{2\pi a^2\, in}{2\sqrt{2}\,a^3}$$

$$= \frac{\mu_0 ni}{4\sqrt{2}a}, \text{Hence } \frac{B_{centre(x=0)}}{B_{(x=\pm a)}} = 2\sqrt{2}$$

(c) $x = \pm 0.766$ R

$$B = \frac{B_0}{2},\ B_0 = B_{center}$$

For fig (a) $B = \frac{\mu_0 i\alpha}{4\pi a}$ normal to the plane of paper downwards.

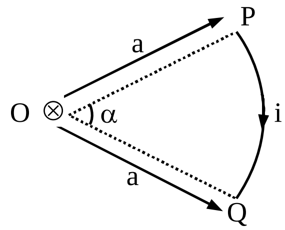
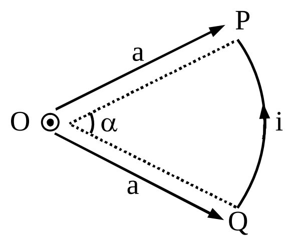
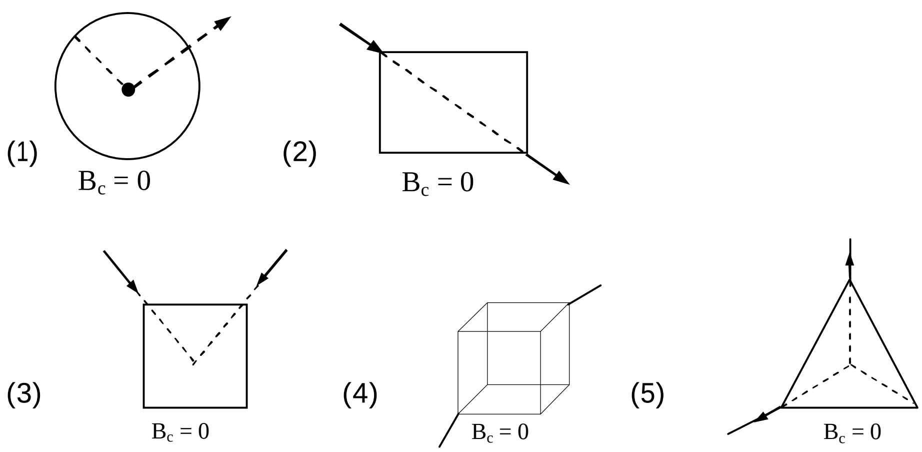


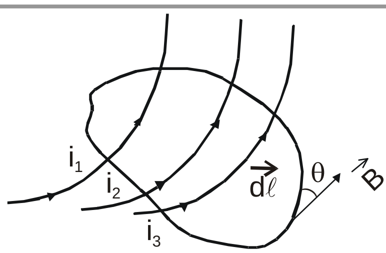
Fig fig (b) $B = \frac{\mu_0 i\alpha}{4\pi a}$, normal to the plane of paper upwards.



If in a symmetrical geometry, current enters from one end and exit from the other, then magnetic field at the center is zero.



Ampere's Law



Amperes law gives another method to calculate the magnetic field due to a given current distribution.

Line integral of the magnetic field $\vec{B}$ around any closed curve is equal to μ_0 times the net current *i* threading through the area enclosed by the curve *i.e.*

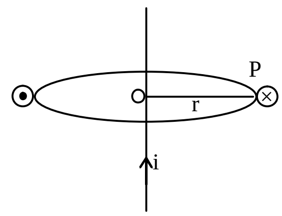
$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \sum i = \mu_0 (i_1 + i_3 - i_2)$$

Total current crossing the above area is $(i_1 + i_3 - i_2)$. Any current outside the area is not included in net current. (Outward ⊙ → +*ve*, Inward ⊗ → – *ve*)

Application of Ampere's law

(a) Magnetic field of a long straight wire :

Let OP = r



By Ampere's Law, $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 i$

Since B is constant through out the circle, Let it be B.

$B \int d\vec{\ell} = \mu_0 i$

$B.\ 2\pi r = \mu_0 i$, Hence, $B = \frac{\mu_0 i}{2\pi r}$

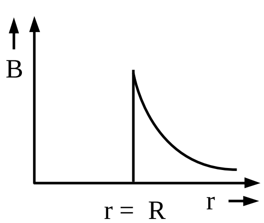
$B \propto \frac{1}{r}$ hence $B_\infty = 0$

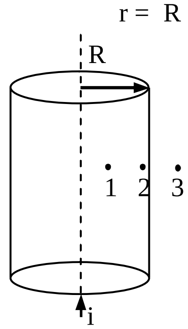
(b) Magnetic field due to a current carrying cylinder :

(i) Cylindrical shell :

$B_1 = 0$

$B_2 = \frac{\mu_0 I}{2\pi R}$

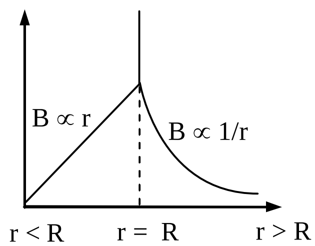


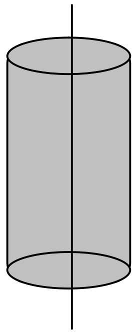


$B_3 = \frac{\mu_0 I}{2\pi r}$

(ii) Rigid Cylinder :

$$B_1 = \frac{\mu_0 Ir}{2\pi R^2} \qquad B_2 = \frac{\mu_0 I}{2\pi R} \qquad B_3 = \frac{\mu_0 I}{2\pi r}$$

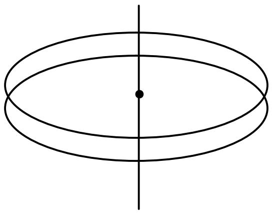




(c) Magnetic field due to a charged ring

Let the frequency of ring = n and charge on ring = q. Then

(a) current I = nq



(b) Magnetic field at the centre

$$= \frac{\mu_0 I}{2R} = \frac{\mu_0 nq}{2R} = \frac{\mu_0 \omega q}{2\pi R} \quad (\omega = 2\pi n)$$

(c) Magnetic moment

$$M = i\,A = (qn)\,(\pi R^2)$$

$$\text{Hence, } M = \pi qnR^2 \text{ or } M = \frac{q\omega R^2}{2}$$

SOLENOID

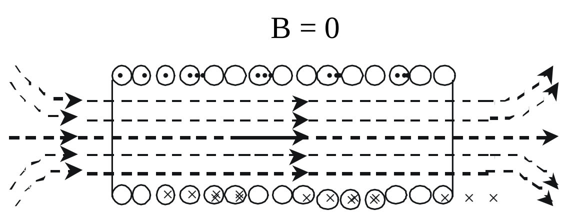
It is a long cylindrical helix made by winding closely a large number of turns of insulated copper wire over a card board or china clay.

(b) The winding or wire is uniform.
(c) A magnetic field is produced around and within the solenoid. The magnetic field within the solenoid is uniform and parallel to the axis of Solenoid.
(d) Magnetic field outside the solenoid is negligible.
(e) For an ideal solenoid, length is very-very greater than it's radius.

If i is the current flowing in solenoid then magnetic field inside the solenoid

$$B = \mu_0 n i$$

Wheren = number of turns in unit length

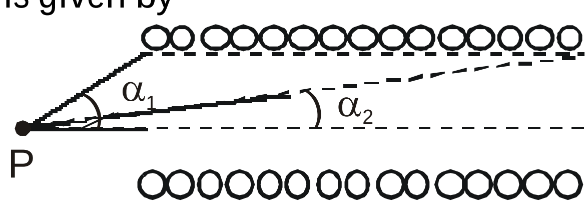


(g) If length of solenoid = L(>> r) and number of turns = N, then

$n = \frac{N}{L} \Rightarrow B = \mu_0 \frac{N}{L} i$

(h) Magnetic field at the center is twice the magnetic field at an end of the solenoid i.e. $B_{end} = \frac{\mu_0 n i}{2}$

(i) The magnetic field at any point P due to a solenoid of finite length is given by



$$B = \frac{1}{2}\mu_0 n i (\cos\alpha_2 - \cos\alpha_1)$$

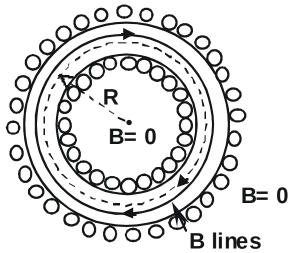
TOROID

This is a solenoid bent round in the form of a closed ring and carrying a current.

(b) Magnetic field within the toroid is uniform and outside it is zero.
(c) Magnetic field inside the toroid

$$B = \mu_0 ni = \mu_0 \frac{N}{2\pi R} i$$

Where N = Total number of turns.
R = Radius of toroid.



(d) If magnetic permeability of material of toroid, μ then

$$B = \mu ni$$

Here, $\mu = \mu_0 \mu_r$