Kepler's Laws of Planetary Motion

Kepler's laws confirm the helio–centric theory.

Kepler's first law of motion (Law of orbits) : All the planets revolve round the Sun in elliptical orbits with the Sun at one of the foci.

Planets are nine in number which revolve round the Sun and have self rotation. The order of the planets revolving round the Sun as we move away from the Sun is Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune and Pluto.

Jupiter is the biggest planet and Mercury is the smallest planet.

The planet which is nearest to the Earth is Venus.

The moon is the satellite of the earth. Acceleration due to gravity on the surface of the moon is 1.6 ms^{-2} or 1/6 that of the earth.

Mercury and Venus have no satellites, Earth and Pluto have each one satellite, Mars, Jupiter, Saturn, Uranus and Neptune have 2, 16, 22, 12 and 6 satellites respectively.

Kepler's second law : (Law of areas) : The radius vector joining a planet to the Sun sweeps out equal areas in equal intervals of time. ($I\omega$ = constant). This law is a direct consequence of the law of conservation of angular momentum.

a) A planet moves fastest when it is nearest to the Sun (perihelion or perigee) and moves slowest when it is farthest from the sun (aphelion or apogee).

b) The line joining the sun and the earth sweeps out equal areas is equal intervals of time i.e. a real velocity is constant.

c) A real velocity is $\frac{dA}{dt} = \frac{1}{2}r^2\omega$

$\frac{dA}{dt} = \frac{L}{2m}$ L is the angular momentum of the planet of mass m in the given orbit.

d) Kepler's second law is a consequence of law of conservation of angular momentum

e) According to second law a planet moves faster when it is nearer to sun and moves slower when it is far a way from the sun.

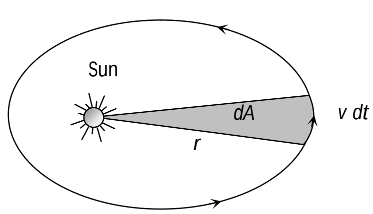
f) According to II law

$V_{max}\, r_{min} = V_{min}\, r_{max}$

Areal velocity $= \frac{dA}{dt} = \frac{1}{2}\frac{r(vdt)}{dt} = \frac{1}{2}rv$

$\therefore \frac{dA}{dt} = \frac{L}{2m}$

[As $L = mvr$; $rv = \frac{L}{m}$]



III law: Law of Periods:

f) Square of the period of any planet (T^2) about the sun is proportional to cube of the mean distance (R^3) of the planet from the sun.

$T^2 \propto R^3$ or T^2/R^3 = constant. $\frac{T_1^2}{R_1^3} = \frac{T_2^2}{R_2^3}$

g) According to third law, as the distance of the planet increases, duration of the year of the planet increases.

h) Kepler's laws supported heliocentric or Copernicus theory.

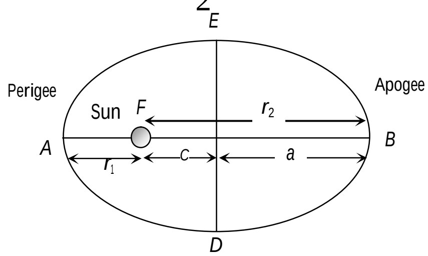
The period of revolution of the moon round the earth is equal to the period of its self rotation

The law of periods : The square of period of revolution (T) of any planet around sun is directly proportional to the cube of the semi-major axis of the orbit.

$$T^2 \propto a^3 \text{ or } T^2 \propto \left(\frac{r_1 + r_2}{2}\right)^3$$

Proof : From the figure $AB = AF + FB$

$$2a = r_1 + r_2 \quad \therefore a = \frac{r_1 + r_2}{2}$$



where a = semi-major axis

$r_1 =$ Shortest distance of planet from sun (perigee).

$r_2 =$ Largest distance of planet from sun (apogee)

Orbital Velocity of Satellite

Orbital velocity of a satellite is the velocity required to put the satellite into its orbit around the earth.

For revolution of satellite around the earth, the gravitational pull provides the required centripetal force.

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{r}}$$

$$v=\sqrt{\frac{gR^2}{R+h}}=R\sqrt{\frac{g}{R+h}}$$

[As $GM=gR^2$ and $r=R+h$]

Orbital velocity is independent of the mass of the orbiting body and is always along the tangent of the orbit *i.e.*, satellites of diferent masses have same orbital velocity, if they are in the same orbit.

Orbital velocity depends on the mass of central body and radius of orbit.

For a given planet, greater the radius of orbit, lesser will be the orbital velocity of the satellite $\left(v\propto 1/\sqrt{r}\right)$.

Orbital velocity of the satellite when it revolves very close to the surface of the planet

$$v=\sqrt{\frac{GM}{r}}=\sqrt{\frac{GM}{R+h}} \quad \therefore \; v=\sqrt{\frac{GM}{R}}=\sqrt{gR}$$

[As $h=0$ and $GM=gR^2$]

For the earth $v=\sqrt{9.8\times 6.4\times 10^6}\;=7.9\,km/s\approx 8\,km/\sec$

Close to the surface of planet $v=\sqrt{\frac{GM}{R}}$

[As $v_e=\sqrt{\frac{2GM}{R}}$]

$$\therefore \; v=\frac{v_e}{\sqrt{2}} \qquad i.e.,\; v_{escape}=\sqrt{2}\;\; v_{orbital}$$

It means that if the speed of a satellite orbiting close to the earth is made $\sqrt{2}$ times (or increased by 41%) then it will escape from the gravitational field.

Time Period of Satellite

It is the time taken by satellite to go once around the earth.

$$\therefore \; T=\frac{\text{Circumference of the orbit}}{\text{orbital velocity}}$$

$$\Rightarrow \qquad T=\frac{2\pi r}{v}=2\pi r\sqrt{\frac{r}{GM}} \qquad \text{[As } v=\sqrt{\frac{GM}{r}}\text{]}$$

$$\Rightarrow \; T=2\pi\sqrt{\frac{r^3}{GM}}=2\pi\sqrt{\frac{r^3}{gR^2}} \quad \text{[As } GM=gR^2\text{]}$$

$$\Rightarrow \; T=2\pi\sqrt{\frac{(R+h)^3}{g\,R^2}}\;=2\,\pi\sqrt{\frac{R}{g}}\left(1+\frac{h}{R}\right)^{3/2} \quad \text{[As } r=R+h\text{]}$$

From $T=2\pi\sqrt{\frac{r^3}{GM}}$, it is clear that time period is independent of the mass of orbiting body and depends on the mass of central body and radius of the orbit

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

$$\Rightarrow \; T^2 = \frac{4\pi^2}{GM} r^3 \;\; i.e., \; T^2 \propto r^3$$

This is in accordance with Kepler's third law of planetary motion r becomes a (semi major axis) if the orbit is elliptic.

Time period of nearby satellite,

From $T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{R^3}{gR^2}} = 2\pi\sqrt{\frac{R}{g}}$ [As $h = 0$ and $GM = gR^2$]

For earth $R = 6400\, km$ and $g = 9.8 m/s^2$

$T = 84.6$ minute $\approx 1.4\ hr$

(iv) Time period of nearby satellite in terms of density of planet can be given as

$$T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{R^3}{GM}} = \frac{2\pi (R^3)^{1/2}}{\left[G.\frac{4}{3}\pi R^3 \rho\right]^{1/2}} = \sqrt{\frac{3\pi}{G\rho}}$$

(v) If the gravitational force of attraction of the sun on the planet varies as $F \propto \frac{1}{r^n}$ then the time period varies as $T \propto r^{\frac{n+1}{2}}$

If there is a satellite in the equatorial plane rotating in the direction of earth's rotation from west to east, then for an observer, on the earth, angular velocity of satellite will be $(\omega_S - \omega_E)$. The time interval between the two consecutive appearances overhead will be

$$T = \frac{2\pi}{\omega_s - \omega_E} = \frac{T_S T_E}{T_E - T_S} \qquad \left[\text{As } T = \frac{2\pi}{\omega}\right]$$

If $\omega_S = \omega_E$, $T = \infty$ *i.e.* satellite will appear stationary relative to earth. Such satellites are called geostationary satellites.

Geostationary Satellite

The satellite which appears stationary relative to earth is called geostationary or geosynchronous satellite, communication satellite.

A geostationary satellite always stays over the same place above the earth such a satellite is never at rest. Such a satellite appears stationary due to its zero relative velocity *w.r.t.* that place on earth.

The orbit of a geostationary satellite is known as the parking orbit.

(i) It should revolve in an orbit concentric and coplanar with the equatorial plane.

(ii) Its sense of rotation should be same as that of earth about its own axis *i.e.*, in anti-clockwise direction (from west to east).

(iii) Its period of revolution around the earth should be same as that of earth about its own axis.

$\therefore\ T = 24\ hr = 86400$ sec

Height of geostationary satellite

As $T = 2\pi\sqrt{\dfrac{r^3}{GM}} \Rightarrow 2\pi\sqrt{\dfrac{(R+h)^3}{GM}} = 24\,hr$

Substituting the value of G and M we get $R + h = r = 42000\ km = 7R$

$\therefore$ height of geostationary satellite from the surface of earh $h = 6R = 36000\ km$

Orbital velocity of geo stationary satellite can be calculated by $v = \sqrt{\dfrac{GM}{r}}$

Substituting the value of G and M we get $v = 3.08\ km/\sec$

Energy of Satellite

When a satellite revolves around a planet in its orbit, it possesses both potential energy (due to its position against gravitational pull of earth) and kinetic energy (due to orbital motion).

Potential energy : $U = mV = \dfrac{-GMm}{r} = \dfrac{-L^2}{mr^2}$

$\left[\text{As } V = \dfrac{-GM}{r}, L^2 = m^2GMr\right]$

Kinetic energy : $K = \dfrac{1}{2}mv^2 = \dfrac{GMm}{2r} = \dfrac{L^2}{2mr^2}$ $\left[\text{As } v = \sqrt{\dfrac{GM}{r}}\right]$

Total energy :

$$E = U + K = \frac{-GMm}{r} + \frac{GMm}{2r} = \frac{-GMm}{2r} = \frac{-L^2}{2mr^2}$$

Kinetic energy, potential energy or total energy of a satellite depends on the mass of the satellite and the central body and also on the radius of the orbit.

From the above expressions we can say that

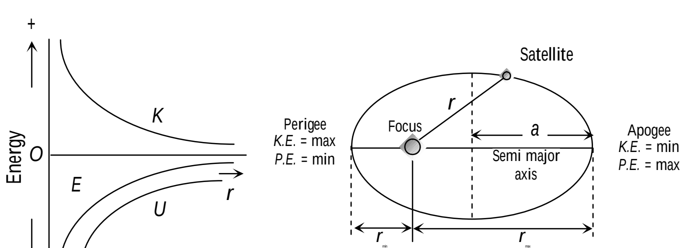
Kinetic energy (K) = – (Total energy)

Potential energy (U) = 2 (Total energy)

Potential energy (K) = – 2 (Kinetic energy)

(iii) Energy graph for a satellite

(iv) Energy distribution in elliptical orbit



(A) (B)

If the orbit of a satellite is elliptic then

Total energy $(E) = \frac{-GMm}{2a} =$ constant ; where a is semi-major axis .

Binding Energy : Total energy of a satellite in its orbit is negative. Negative energy means that the satellite is bound to the central body by an attractive force and energy must be supplied to remove it from the orbit to infinity. The energy required to remove the satellite from its orbit to infinity is called Binding Energy of the system, *i.e.*,

Binding Energy (B.E.) $= -E = \frac{GMm}{2r}$