WAVE OPTICS

Huygen's Wave Theory

(1) Wave theory of light was given by Christian Huygen. According to this, a luminous body is a source of disturbance in a hypothetical medium ether. This medium pervades all space.

(2) It is assumed to be transparent and having zero inertia. The disturbance from the source is propagated in the form of waves through the space.

(3) The waves carry energy and momentum. Huygen assumed that the waves were longitudinal. Further when polarization was discovered, then to explain it, light waves were, assumed to be transverse in nature by Fresnel.

(4) This theory explains successfully, the phenomenon of interference and diffraction apart from other properties of light.

(5) The Huygen's theory fails to explain photo-electric effect, Compton's effect *etc*.

(6) The wave theory introduces the concept of wavefront.

Wavefront

(1) Suggested by Huygens

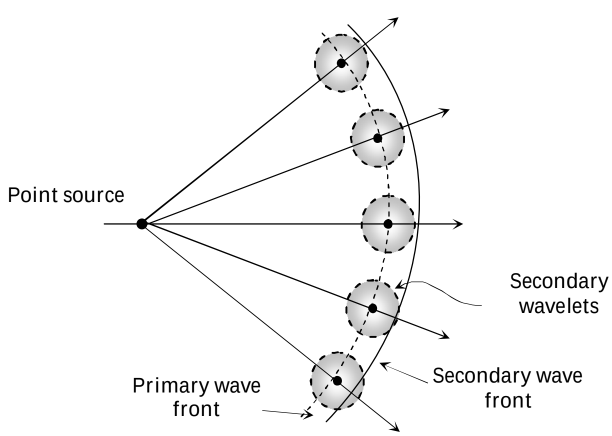
(2) The locus of all particles in a medium, vibrating in the same phase is called Wave Front (WF)

(3) The direction of propagation of light (ray of light) is perpendicular to the WF.

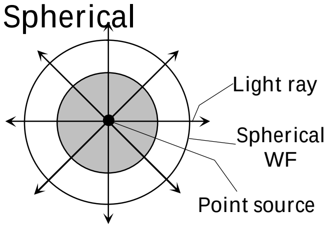
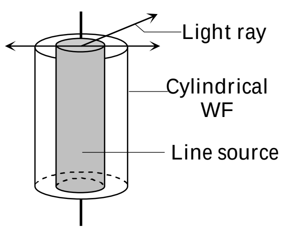
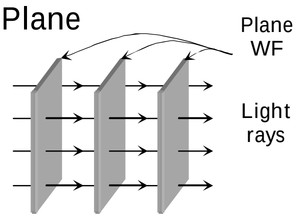
(4) Every point on the given wave front acts as a source of new disturbance called secondary wavelets which

travel in all directions with the velocity of light in the medium.

(5) A surface touching these secondary wavelets tangentially in the forward direction at any instant gives the new wave front at that instant. This is called secondary wave front

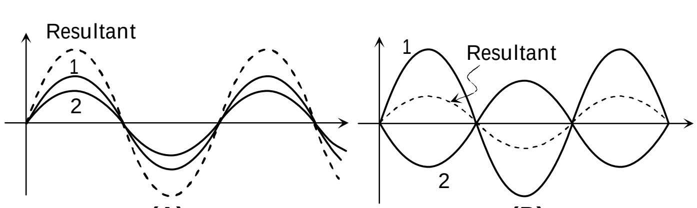


Different types of wavefront

Type of wavefront	Intensity	Amplitude
Spherical 	$I \propto \frac{1}{r^2}$	$A \propto \frac{1}{r}$
Cylindrical 	$I \propto \frac{1}{r}$	$A \propto \frac{1}{\sqrt{r}}$
Plane 	$I \propto r^0$	$A \propto r^0$

Super Position of Waves

When two or more than two waves superimpose over each other at a common particle of the medium then the resultant displacement (y) of the particle is equal to the vector sum of the displacements (y_1 and y_2) produced by individual waves. *i.e.* $\vec{y} = \vec{y}_1 + \vec{y}_2$



(A) **(B)**

Important Terms

(1) **Phase :** The argument of sine or cosine in the expression for displacement of a wave is defined as the phase. For displacement $y = a \sin \omega t$; term ωt = phase or instantaneous phase.

(2) **Phase difference (ϕ) :** The difference between the phases of two waves at a point is called phase difference *i.e.* if $y_1 = a_1 \sin \omega t$ and $y_2 = a_2 \sin(\omega t + \phi)$ so phase difference = ϕ

(3) **Path difference (Δ) :** The difference in path length's of two waves meeting at a point is called path difference between the waves at that point. Also $\Delta = \frac{\lambda}{2\pi} \times \phi$

(4) **Time difference (*T.D.*) :** Time difference between the waves meeting at a point is $T.D. = \frac{T}{2\pi} \times \phi$

Resultant Amplitude and Intensity

Let us consider two waves that have the same frequency but have a certain fixed (constant) phase difference between them. Their super position shown below

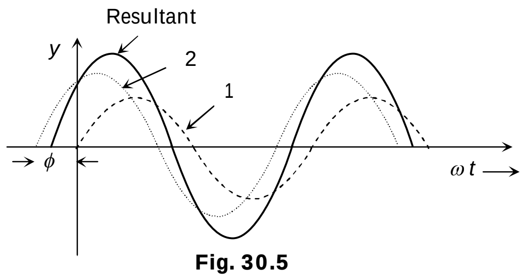


Fig. 30.5

Let the two waves are

$y_1 = a_1 \sin \omega t$ and $y_2 = a_2 \sin(\omega t + \phi)$

where $a_1, a_2 =$ Individual amplitudes,

ϕ = Phase difference between the waves at an instant when they are meeting a point.

(1) **Resultant amplitude :** The resultant wave can be written as $y = A \sin(\omega t + \theta)$

where A = resultant amplitude $= \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos \varphi}$

(2) **Resultant intensity :** As we know intensity $\propto$ (Amplitude)^2

$\Rightarrow I_1 = ka_1^2, I_2 = ka_2^2$ and $I = kA^2$ (k is a proportionality constant). Resultant intensity $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$

For two identical source $I_1 = I_2 = I_0 \Rightarrow I = I_0 + I_0 + 2\sqrt{I_0 I_0} \cos \phi$

$= 4I_0 \cos^2 \frac{\phi}{2}$ [$1 + \cos\theta = 2\cos^2 \frac{\theta}{2}$]

Coherence

The phase relationship between two light waves can very from time to time and from point to point in space. The property of definite phase relationship is called coherence.

(1) **Temporal coherence :** In a light source a light wave (photon) is produced when an excited atom goes to the ground state and emits light.

(i) The duration of this transition is about 10^{-9} to 10^{-10} *sec*. Thus the emitted wave remains sinusoidal for this much time. This time is known as coherence time (τ_c).

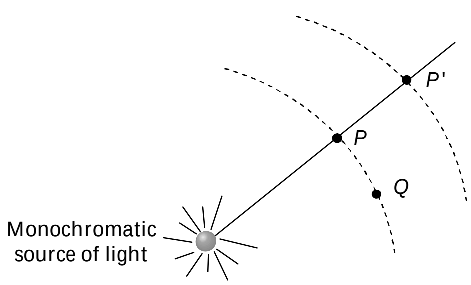
(ii) Definite phase relationship is maintained for a length $L = c\tau_c$ called coherence length. For neon λ = 6328 Å, $\tau_c \approx 10^{-10}$ *sec* and L = 0.03 *m*.

For cadmium λ = 6438 Å, $\tau_c = 10^{-9}$ *sec* and L = 0.3 *m*

For Laser $\tau_c = 10^{-5}$ *sec* and L = 3 *km*

(iii) The spectral lines width $\Delta\lambda$ is related to coherence length L and coherence time τ_c. $\Delta\lambda \approx \frac{\lambda^2}{c\tau_c}$ or $\Delta\lambda \approx \frac{\lambda^2}{L}$

(2) **Spatial coherence :** Two points in space are said to be spatially coherence if the waves reaching there maintains a constant phase difference

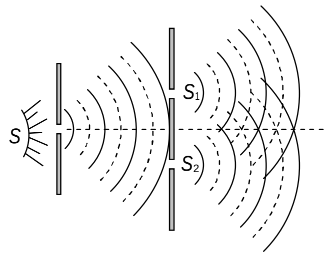


Points P and Q are at the same distance from S, they will always be having the same phase. Points P and P' will be spatially coherent if the distance between P and P' is much less than the coherence length *i.e.* $PP' << c\tau_c$

(3) **Methods of obtaining coherent sources :** Two coherent sources are produced from a single source of light by two methods (i) By division of wavefront and (ii) By division of amplitude

(i) **Division of wave front :** The wave front emitted by a narrow source is divided in two parts by reflection, refraction or diffraction.

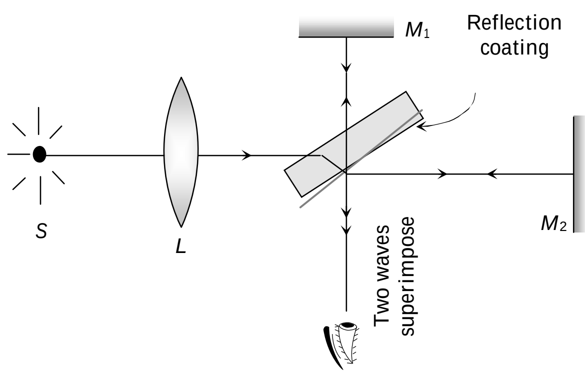
The coherent sources so obtained are imaginary. There produced in Fresnel's biprism, Llyod's mirror Youngs' double slit *etc.*



(ii) **Division of amplitude :** In this arrangement light wave is partly reflected (50%) and partly transmitted (50%) to produced two light rays.

The amplitude of wave emitted by an extend source of light is divided in two parts by partial reflection and partial refraction.

The coherent sources obtained are real and are obtained in Newton's rings, Michelson's interferrometer, colours in thin films.



Interference of Light

When two waves of exactly same frequency (coming from two coherent sources) travels in a medium, in the same direction simultaneously then due to their superposition, at some points intensity of light is maximum while at some other points intensity is minimum. This phenomenon is called Interference of light. It is of following two types

(1) **Constructive interference :** When the waves meets a point with same phase, constructive interference is obtained at that point (*i.e.* maximum light)

(i) Phase difference between the waves at the point of observation $\phi = 0^o$ or $2n\pi$

(ii) Path difference between the waves at the point of observation $\Delta = n\lambda$ (*i.e.* even multiple of $\lambda/2$)

(iii) Resultant amplitude at the point of observation will be maximum $A_{max} = a_1 + a_2$

If $a_1 = a_2 = a_0 \Rightarrow A_{max} = 2a_0$

(iv) Resultant intensity at the point of observation will be maximum $I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

If $I_1 = I_2 = I_0 \Rightarrow I_{\max} = 4I_0$

(2) **Destructive interference :** When the wave meets a point with opposite phase, destructive interference is obtained at that point (*i.e.* minimum light)

(i) Phase difference $\phi = 180^o$ or $(2n-1)\pi$; n = 1, 2, ...

or $(2n+1)\pi$; $n = 0,1,2.....$

(ii) Path difference $\Delta = (2n-1)\frac{\lambda}{2}$ (*i.e.* odd multiple of $\lambda/2$)

(iii) Resultant amplitude at the point of observation will be minimum $A_{\min} = a_1 - a_2$

If $a_1 = a_2 \Rightarrow A_{\min} = 0$

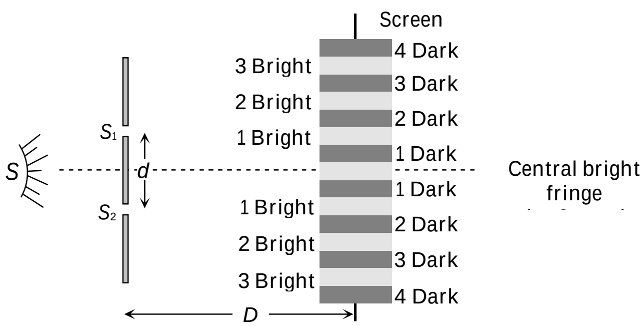
(iv) Resultant intensity at the point of observation will be minimum $I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

If $I_1 = I_2 = I_0 \Rightarrow I_{\min} = 0$

(3) **Super position of waves of random phase difference :** When two waves (or more waves) having random phase difference between them super impose, then no interference pattern is produced. Then the resultant intensity is just the sum of the two intensities. $I = I_1 + I_2$

Young's Double Slit Experiment (YDSE)

Monochromatic light (single wavelength) falls on two narrow slits S_1 and S_2 which are very close together acts as two coherent sources, when waves coming from two coherent sources (S_1, S_2) superimposes on each other, an interference pattern is obtained on the screen. In YDSE alternate bright and dark bands obtained on the screen. These bands are called Fringes.



d = Distance between slits
D = Distance between slits and screen
λ = Wavelength of monochromatic light emitted from source

(1) Central fringe is always bright, because at central position $\phi = 0^o$ or $\Delta = 0$

(2) The fringe pattern obtained due to a slit is more bright than that due to a point.

(3) If the slit widths are unequal, the minima will not be complete dark. For very large width uniform illumination occurs.

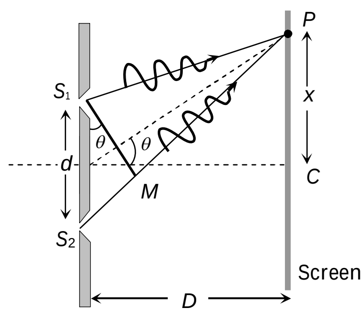
(4) If one slit is illuminated with red light and the other slit is illuminated with blue light, no interference pattern is observed on the screen.

(5) If the two coherent sources consist of object and it's reflected image, the central fringe is dark instead of bright one.

Important Results

(1) **Path difference :** Path difference between the interfering waves meeting at a point *P* on the screen is given by

$\Delta = \Delta_i + \Delta_f$; where $\Delta_i =$ initial path difference between the waves before the slits and $\Delta_f =$ path difference between the waves after emerging from the slits. In this case $\Delta_i = 0$ (Commonly used condition). So $\Delta = \Delta_f = \frac{xd}{D} = d \sin\theta$



where *x* is the position of point *P* from central maxima.

For maxima at *P* : $\Delta = n\lambda$; where $n = 0, \pm 1, \pm 2, ...$

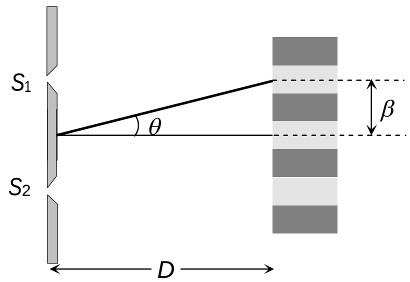
and For minima at *P* : $\Delta = \frac{(2n-1)\lambda}{2}$; where $n = \pm 1, \pm 2, ...$

(2) **Location of fringe :** Position of n^{th} bright fringe from central maxima $x_n = \frac{n\lambda D}{d} = n\beta$; $n = 0, 1, 2 ....$

Position of n^{th} dark fringe from central maxima

$$x_n = \frac{(2n-1)\lambda D}{2d} = \frac{(2n-1)\beta}{2}\ ;\ n = 1, 2, 3 \ldots.$$

(3) **Fringe width (β) :** The separation between any two consecutive bright or dark fringes is called fringe width. In *YDSE* all fringes are of equal width. Fringe width $\beta = \frac{\lambda D}{d}$.



and angular fringe width $\theta = \frac{\lambda}{d} = \frac{\beta}{D}$

(4) In YDSE, if n_1 fringes are visible in a field of view with light of wavelength λ_1, while n_1 with light of wavelength λ_2 in the same field, then $\boldsymbol{n_1\lambda_1 = n_2\lambda_2}$.

(5) **Separation** (Δx) **between fringes**

(i) Between n^{th} bright and m^{th} bright fringes $(n > m)$

$$\Delta x = (n-m)\beta$$

(ii) Between n^{th} bright and m^{th} dark fringe

(a) If $n > m$ then $\Delta x = \left(n - m + \frac{1}{2}\right)\beta$

(b) If $n < m$ then $\Delta x = \left(m - n - \frac{1}{2}\right)\beta$

(6) **Identification of central bright fringe :** To identify central bright fringe, monochromatic light is replaced by white light. Due to overlapping central maxima will be white with red edges. On the other side of it we shall get a few coloured band and then uniform illumination.

If the whole YDSE set up is taken in another medium then λ changes so β changes

e.g. in water $\lambda_w = \frac{\lambda_a}{\mu_w} \Rightarrow \beta_w = \frac{\beta_a}{\mu_w} = \frac{3}{4}\beta_a$

Condition for Observing Interference

(1) The initial phase difference between the interfering waves must remain constant. Otherwise the interference will not be sustained.

(2) The frequency and wavelengths of two waves should be equal. If not the phase difference will not remain constant and so the interference will not be sustained.

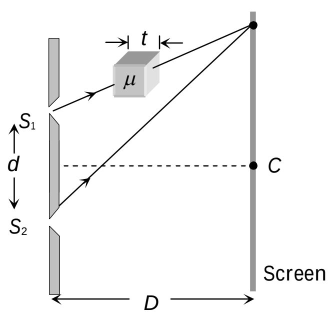
(3) The light must be monochromatic. This eliminates overlapping of patterns as each wavelength corresponds to one interference pattern.

(4) The amplitudes of the waves must be equal. This improves contrast with $I_{\max} = 4I_0$ and $I_{\min} = 0$.

(5) The sources must be close to each other. Otherwise due to small fringe width $\left(\beta \propto \frac{1}{d}\right)$ the eye can not resolve fringes resulting in uniform illumination.

Shifting of Fringe Pattern in YDSE

If a transparent thin film of mica or glass is put in the path of one of the waves, then the whole fringe pattern gets shifted towards the slit in front of which glass plate is placed.



(1) $\text{Fringe shift} = \frac{D}{d}(\mu - 1)t = \frac{\beta}{\lambda}(\mu - 1)t$

(2) Additional path difference $= (\mu - 1)t$

(3) If shift is equivalent to n fringes then $n = \frac{(\mu - 1)t}{\lambda}$ or $t = \frac{n\lambda}{(\mu - 1)}$

(4) Shift is independent of the order of fringe (*i.e.* shift of zero order maxima = shift of n^{th} order maxima.

(5) Shift is independent of wavelength.

Fringe Visibility (*V*)

With the help of visibility, knowledge about coherence, fringe contrast an interference pattern is obtained.

$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = 2\frac{\sqrt{I_1 I_2}}{(I_1 + I_2)}$ If $I_{min} = 0$, $V = 1$ (maximum) *i.e.*, fringe visibility will be best.

Also if $I_{max} = 0, V = -1$ and If $I_{max} = I_{min}, V = 0$

Missing Wavelength in Front of One Slit in YDSE

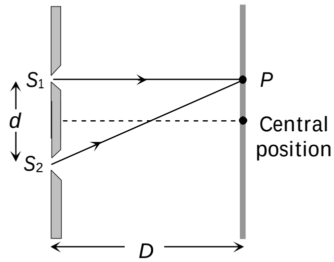
Suppose *P* is a point of observation infront of slit S_1 as shown

Missing wavelength at *P*

$\lambda = \frac{d^2}{(2n-1)D}$

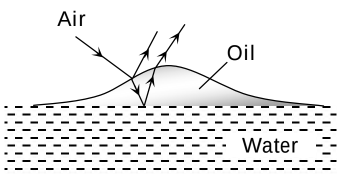
By putting $n = 1, 2, 3 ....$ Missing wavelengths are

$\lambda = \frac{d^2}{D}, \frac{d^2}{3D}, \frac{d^2}{5D} ....$



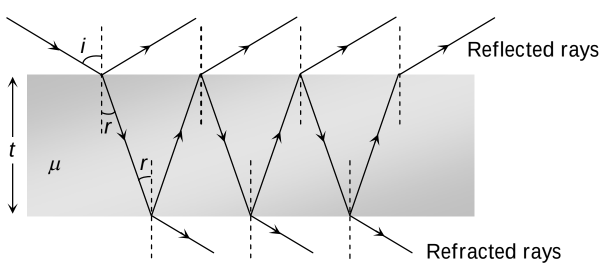
Interference in Thin films

Interference effects are commonly observed in thin films when their thickness is comparable to wavelength of incident light (If it is too thin as compared to wavelength of light it appears dark and if it is too thick, this will result in uniform illumination of film). Thin layer of oil on water surface and soap bubbles shows various colours in white light due to interference of waves reflected from the two surfaces of the film.



Oil film on water surface

In thin films interference takes place between the waves reflected from it's two surfaces and waves refracted through it.



(1) **Interference in reflected light** : Condition of constructive interference (maximum intensity)

$$\Delta = 2\mu\ t\cos r = (2n-1)\frac{\lambda}{2}.$$

For normal incidence $r = 0$ so $2\mu t = (2n-1)\frac{\lambda}{2}$

Condition of destructive interference (minimum intensity)

$\Delta = 2\mu t\cos r = (2n)\frac{\lambda}{2}$. For normal incidence $2\mu t = n\lambda$

(2) **Interference in refracted light** : Condition of constructive interference (maximum intensity)

$\Delta = 2\mu t\cos r = (2n)\frac{\lambda}{2}$. For normal incidence $2\mu t = n\lambda$

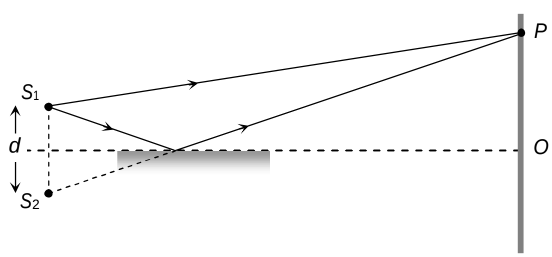
Condition of destructive interference (minimum intensity)

$$\Delta = 2\mu t\cos r = (2n-1)\frac{\lambda}{2}$$

For normal incidence $2\mu t = (2n-1)\frac{\lambda}{2}$

Lloyd's Mirror

A plane glass plate (acting as a mirror) is illuminated at almost grazing incidence by a light from a slit S_1. A virtual image S_2 of S_1 is formed closed to S_1 by reflection and these two act as coherent sources. The expression giving the fringe width is the same as for the double slit, but the fringe system differs in one important respect.



The path difference $S_2P - S_1P$ is a whole number of wavelengths, the fringe at P is dark not bright. This is due to 180° phase change which occurs when light is reflected from a denser medium. At grazing incidence a fringe is formed at O, where the geometrical path difference between the direct and reflected waves is zero and it follows that it will be dark rather than bright.

Thus, whenever there exists a phase difference of a π between the two interfering beams of light, conditions of maximas and minimas are interchanged, *i.e.*, $\Delta x = n\lambda$ (for minimum intensity)

and $\Delta x = (2n-1)\lambda/2$ (for maximum intensity)

Fresnel's Biprism

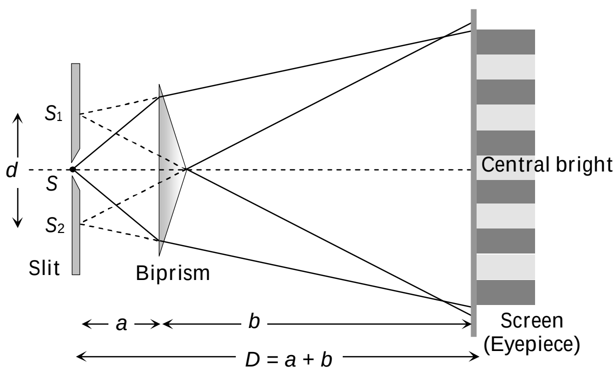
(1) It is an optical device of producing interference of light Fresnel's biprism is made by joining base to base two thin prism of very small angle

(2) Acute angle of prism is about $1/2^o$ and obtuse angle of prism is about 179^o.

(3) When a monochromatic light source is kept in front of biprism two coherent virtual source S_1 and S_2 are produced.

(4) Interference fringes are found on the screen placed behind the biprism interference fringes are formed in the limited region which can be observed with the help eye piece.

(5) Fringe width is measured by a micrometer attached to the eye piece. Fringes are of equal width and its value is $\beta = \frac{\lambda D}{d}$



(6) Let the separation between S_1 and S_2 be d and the distance of slits and the screen from the biprism be a and b respectively *i.e.* $D = (a + b)$. If angle of prism is α and refractive index is μ then $d = 2a(\mu - 1)\alpha$

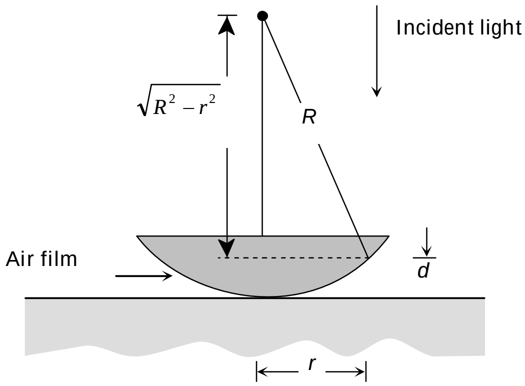
$$\therefore \quad \lambda = \frac{\beta[2a(\mu-1)\alpha]}{(a+b)} \quad \Rightarrow \quad \beta = \frac{(a+b)\lambda}{2a(\mu-1)\alpha}$$

(7) If a convex lens is mounted between the biprism and eye piece. There will be two positions of lens when the sharp images of coherent sources will be observed in the eyepiece. The separation of the images in the two positions are measured. Let these be d_1 and d_2 then $d = \sqrt{d_1 d_2}$ $\quad \therefore \ \lambda = \frac{\beta d}{D} = \frac{\beta\sqrt{d_1 d_2}}{(a+b)}$.

Newton's Rings

(1) If we place a plano-convex lens on a plane glass surface, a thin film of air is formed between the curved surface of the lens and plane glass plate.

(2) If we allow monochomatic light to fall normally on the surface of lens, then circular interference fringes of radius *r* can be seen in the reflected light. This circular fringes are called Newton rings.



(3) The central fringe is a dark spot then there are alternate bright and dark fringes (Ring shape).

(4) Radius of n^{th} dark ring $r_m \simeq \sqrt{\lambda R}$

$n = 0, 1, 2, ...,$ R = Radius of convex surface

(5) Radius of n^{th} bright ring $r_n = \sqrt{\left(n + \frac{1}{2}\right)\lambda R}$

(6) If a liquid of ref index μ is introduced between the lens and glass plate, the radii of dark ring would be $r_n = \sqrt{\frac{n\lambda R}{\mu}}$

(7) Newton's ring arrangement is used of determining the wavelength of monochromatic light. For this the diameter of n^{th} dark ring (D_n) and $(n + p)^{th}$ dark ring (D_{n+p}) are measured then

$D^2_{(n+p)} = 4(n+p)\lambda R$ and $D^2_n = 4n\lambda R$ $\Rightarrow$ $\lambda = \frac{D^2_{n+p} - D^2_n}{4pR}$