Factorisation of Polynomials:

(i) Factor Theorem:

If p(x) is a polynomial of degree $n \ge 1$ and a is any real number, then

(a)
$$x - a$$
 is a factor of $p(x)$, if $p(a) = 0$

(b)
$$p(a) = 0$$
, if $x - a$ is a factor of $p(x)$

For Example: Check whether (x + 1) is factor of $p(x) = x^3 + x^2 + x + 1$.

As per Factor Theorem, (x + 1) is factor of $p(x) = x^3 + x^2 + x + 1$, if p(-1) = 0.

Therefore, $p(-1) = (-1)^3 + (-1)^2 + (-1) + 1 = -1 + 1 - 1 + 1 = 0$.

Thus, (x + 1) is factor of $p(x) = x^3 + x^2 + x + 1$.

ii) Factorisation of Quadratic Polynomials- Splitting the middle term

Factorisation of the polynomial $ax^2 + bx + c$ by splitting the middle term is as follows:

Step 1: We split the middle term by finding two numbers such that their sum is equal to the coefficient of x and their product is equal to the product of the constant term and the coefficient of x^2 .

For example, for the quadratic polynomial $(x^2 + 5x + 6)$ the middle term can be split as,

$$x^2 + 2x + 3x + 6$$

Here, 2 + 3 = 5 and $2 \times 3 = 6$.

Step 2: Now, we factorise by pairing the terms and taking the common factors.

$$x^2 + 2x + 3x + 6$$

$$= x (x + 2) + 3(x + 2)$$

$$= (x + 2) (x + 3)$$

Thus, x + 2 and x + 3 are factors of $x^2 + 5x + 6$.