WORK

Work is said to be done when the point of application of force has some displacement in the direction of the force.

The amount of work done is given by the dot product of force and displacement.

$$\vec{F}.\vec{s} = Fs\cos\theta$$

Work is independent of the time taken and is a scalar.
If the force and displacement are perpendicular to each other, then the work done is zero.

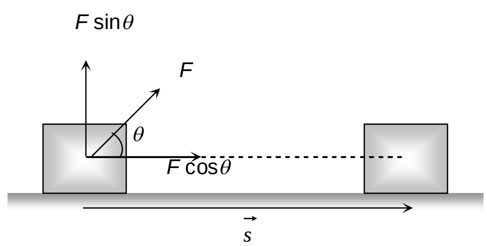
Work Done by a Constant Force

Let a constant force $\vec{F}$ be applied on the body such that it makes an angle θ with the horizontal and body is displaced through a distance s

By resolving force $\vec{F}$ into two components :

(i) $F\cos\theta$ in the direction of displacement of the body.

(ii) $F\sin\theta$ in the perpendicular direction of displacement of the body.



Since body is being displaced in the direction of $F\cos\theta$, therefore work done by the force in displacing the body through a distance s is given by

$$W = (F\cos\theta)s = Fs\cos\theta$$

or $$W = \vec{F}.\vec{s}$$

Thus work done by a force is equal to the scalar (or dot product) of the force and the displacement of the body.

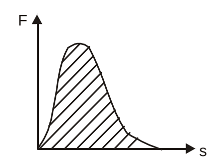
If a number of forces $\vec{F}_1, \vec{F}_2, \vec{F}_3 ......\vec{F}_n$ are acting on a body and it shifts from position vector $\vec{r}_1$ to position vector $\vec{r}_2$ then $W = (\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...\vec{F}_n).(\vec{r}_2 - \vec{r}_1)$

Work Depends on Frame of Reference

A person rowing a boat upstream is at rest with respect to an observer on the shore. According to the observer the person does not perform any work. However, the person performs work against the flow of water. If he stops rowing the boat, the boat moves in the direction of flow of water and work is performed by the force due to flow, as there is displacement in the direction of flow.

If the work is done by a uniformly varying force such as restoring force in a spring, then the work done is equal to the product of average force and displacement.

If the force is varying non–uniformly, then the work done $=\int \vec{F}.\overrightarrow{ds} = \int F.ds.\cos\theta$.



The area of F–s graph gives the work done.

SI unit of work is **joule**. Joule is the work done when a force of one newton displaces a body through one metre in the direction of force.

CGS unit of work is erg; 1 J = 10^7 ergs.

The work done in lifting an object of mass m through a height 'h' is equal to mgh.

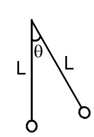
When a body of mass m is raised from a height h_1 to height h_2, then the work done $= mg(h_2 - h_1)$.

Let a body be lifted through a height 'h' vertically upwards by a force 'F' acting upwards. Then, the work done by the resultant force is W = (F – mg)h.

The work done on a spring in stretching or compressing it through a distance x is given $W = \frac{1}{2}kx^2$ where k is the force constant or spring constant.

Work done in changing the elongation of a spring from x_1 to x_2 is $W = \frac{1}{2}k(x_2^2 - x_1^2)$.

a) The work done in pulling the bob of a simple pendulum of length L through an angle θ as shown in the figure is



$W = mgL(1 - \cos\theta) = 2mgL\sin^2(\theta/2)$

b) the velocity acquired by it when released from that position is $v = \sqrt{2gl(1-\cos\theta)}$

Inclined plane :

i) Work done in moving a block of mass 'm' up a smooth inclined plane of inclination 'θ' through a distance 's' is

$W = Fs = mg\sin\theta\, s$

ii) if the plane is rough, then

$W = mg(\sin\theta + \mu_k \cos\theta)s$