Mechanical Waves

(1) When a wave motion passes through a medium, particles of the medium only vibrate simple harmonically about their mean position. They do leave their position and move with the disturbance.

(2) In wave motion, the phase of particles of medium keeps on changing.

(3) The velocity of the particle during their vibration is different at different position.

(4) The velocity of wave motion through a particular medium is constant. It depends only on the nature of medium not on the frequency, wavelength or intensity.

(5) Energy is propagated along with the wave motion without any net transport of the medium.

(6) For the propagation of wave, a medium should have following characteristics.

(i) Elasticity : So that particles can return to their mean position, after having been.

(ii) Inertia : So that particles can store energy and overshoot their mean position.

(iii) Minimum friction amongst the particles of the medium.

(iv) Uniform density of the medium.

Types of Waves

Waves can be classified in a number of ways based on the following characteristics

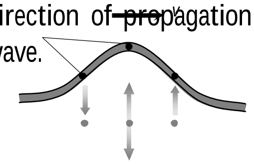
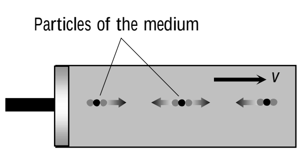
(1) **On the basis necessity of medium**

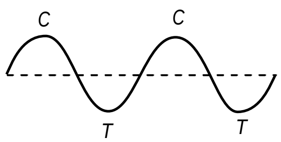
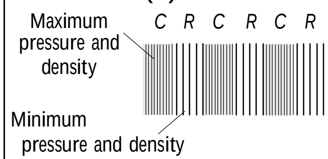
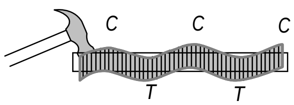
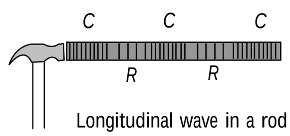
(i) **Mechanical waves :** Require medium for their propagation *e.g.* Waves on string and spring, waves on water surface, sound waves, seismic waves.

(ii) **Non-mechanical waves :** Do not require medium for their propagation are called *e.g,* Light, heat (Infrared), radio waves, γ- rays, X-rays *etc*.

(2) **On the basis of vibration of particle :** On the basis of vibration of particle of medium waves can be classified as transverse waves and longitudinal waves.

Transverse and longitudinal waves

Transverse waves	Longitudinal waves
Particles of the medium vibrates in a direction perpendicular to the direction of propagation of wave.  Transverse wave on a string	Particles of a medium vibrate in the direction of wave motion.  Longitudinal wave in a fluid

It travels in the form of crests (C) and troughs (T). 	It travels in the form of compression (C) and rarefaction (R). 
Transverse waves can be transmitted through solids, they can be setup on the surface of liquids. But they can not be transmitted into liquids and gases.  Transverse-wave in a rod	These waves can be transmitted through solids, liquids and gases because for these waves propagation, volume elasticity is necessary.  Longitudinal wave in a rod
Medium should posses the property of rigidity	Medium should posses the property of elasticity
Transverse waves can be polarised.	Longitudinal waves can not be polarised.
Movement of string of a sitar or violin, movement of the membrane of a Tabla or Dholak, movement of kink on a rope, waves set-up on the surface of water.	Sound waves travel through air, Vibration of air column in organ pipes Vibration of air column above the surface of water in the tube of resonance apparatus

(3) **On the basis of energy propagation**

(i) **Progressive wave :** These waves advances in a medium with definite velocity. These waves propagate energy in the medium *e.g.* Sound wave and light waves.

(ii) **Stationary wave :** These waves remains stationary between two boundaries in medium. Energy is not propagated by these waves but it is confined in segments (or loops) *e.g.*. Wave in a string, waves in organ pipes.

(4) **On the basis of dimension**

(i) **One dimensional wave :** Energy is transferred in a single direction only *e.g.* Wave propagating in a stretched string.

(ii) **Two dimensional wave :** Energy is transferred in a plane in two mutually perpendicular directions *e.g.* Wave propagating on the surface of water.

(iii) **Three dimensional wave :** Energy in transferred in space in all direction *e.g.* Light and sound waves propagating in space.

(5) **Some other waves**

(i) **Matter waves :** The waves associated with the moving particles are called matter waves.

(ii) **Audible or sound waves :** Range 20 *Hz* to 20 *KHz.* These are generated by vibrating bodies such as vocal cords, stretched strings or membrane.

(iii) **Infrasonic waves :** Frequency lie below 20 *Hz* and wavelengths are greater than 16.6 *cm*. *Example* : waves produced during earth quake, ocean waves *etc.*

(iv) **Ultrasonic waves** : Frequency greater than 20 *KHz*. Human ear cannot detect these waves, certain creatures such as mosquito, dog and bat show response to these. As velocity of sound in air is 332 *m*/sec so the wavelength $\lambda < 1.66$ *cm*.

These waves are used for navigation under water (SONAR).

(v) **Shock waves** : When an object moves with a velocity greater than that of sound, it is termed as **Supersonic**. When such a supersonic body or plane travels in air, it produces energetic disturbance which moves in backward direction and diverges in the form of a cone. Such disturbance are called the shock waves.

The speed of supersonic is measured in Mach number. One mach number is the speed of sound.

Mach Number = $\dfrac{\text{Velocity of source}}{\text{Velocity of sound}}$.

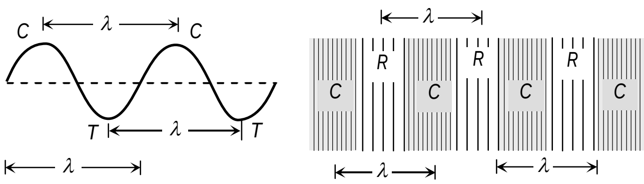
Important Terms Regarding Wave Motion

(1) **Amplitude** (*a*) : Maximum displacement of a vibrating particle of medium from it's mean position is called amplitude.

(2) **Wavelength** (λ) : It is equal to the distance travelled by the wave during the time in which any one particle of the medium completes one vibration about its mean position.

(i) Or distance travelled by the wave in one time period is known as wavelength.

(ii) Or is the distance between the two successive points with the same phase.



(A) Transverse wave **(B)** Longitudinal wave

(3) **Frequency** (*n*) : Frequency of vibration of a particle is defined as the number of vibrations completed by particle in one second.

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It is the number of complete wavelengths traversed by the wave in one second.

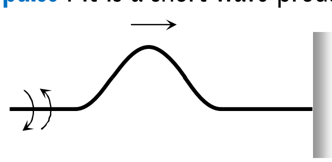
Units of frequency are hertz (*Hz*) and per second.

(4) **Time period** (*T*) : Time period of vibration of particle is defined as the time taken by the particle to complete one vibration about its mean position.

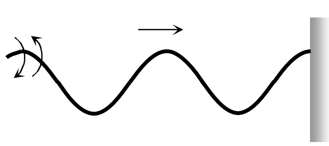
It is the time taken by the wave to travel a distance equal to one wavelength

Time period = 1/Frequency $\Rightarrow$ $T = 1/n$

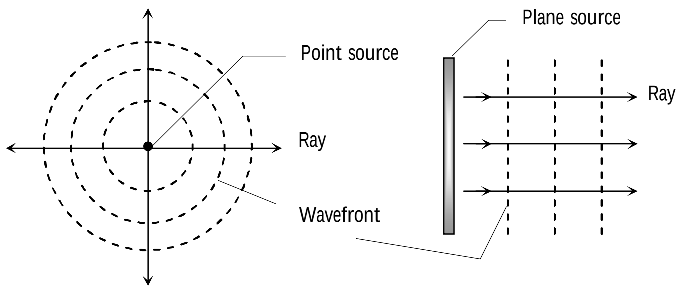
(5) **Wave pulse** : It is a short wave produced in a medium when the disturbance created for a short time.



(6) **Wave train** : A series of wave pulse is called wave train.



nnnnnnnn(7) **Wave front** : A wave front is a line or surface on which the disturbance has **the** same phase at all points. If the source is periodic, it produces a succession of wave front, all of the same shape. Ripples on a pond are the example of wave fronts.



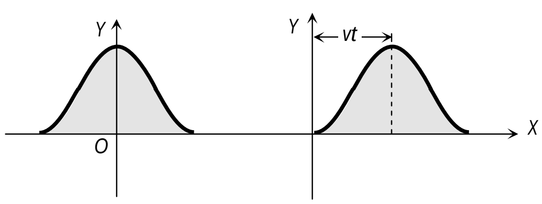
(A) Spherical wavefront   **(B)** Plane wavefront

(8) **Wave function** ; It is a mathematical description of the disturbance created by a wave. For a string, the wave function is a displacement for sound waves. It is a pressure or density fluctuation where as for light waves it is electric or magnetic field.

Now let us consider a one dimensional wave travelling along x-axis. During wave motion, a particle with equilibrium position x is displaced some distance y in the direction perpendicular to the x-axis. In this case y is a function of position (x) and time (t).

i.e. $y = f(x, t)$. This is called wave function .

Let the wave pulse is travelling with a speed v, after a time t, the pulse reaches a distance vt along the +x-axis as shown. The wave function now can be represented as $y = f(x - vt)$



(A) Pulse at time t = 0   **(B)** Pulse after time t

If the wave pulse is travelling along – x-axis then $y = f(x + vt)$

If order of a wave function to represent a wave, the three quantities x, v, t must appear in combinations ($x + vt$) or ($x - vt$)

Thus $y = (x - vt)^2$, $\sqrt{(x - vt)}$, $Ae^{-B(x-vt)^2}$ *etc.* represents travelling waves while $y = (x^2 - v^2t^2), (\sqrt{x} - \sqrt{vt})$, $A \sin (4x^2 - 9t^2)$ *etc.* doesn't represents a wave.

(9) **Harmonic wave** : If a travelling wave is a sin or cos function of $(x \pm vt)$ the wave is said to be harmonic progressive wave.

(10) **The wave equation** : All the travelling waves satisfy a differential equation which is called the wave equation. It is given by $\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$; where $v = \frac{\omega}{k}$

It is satisfied by any equation of the form $y = f(x \pm vt)$

(11) **Angular wave number or propagation constant (k)** : Number of wavelengths in the distance 2π is called the angular wave number or propagation constant *i.e.* $k = \frac{2\pi}{\lambda}$.

It is unit is *rad/m*.

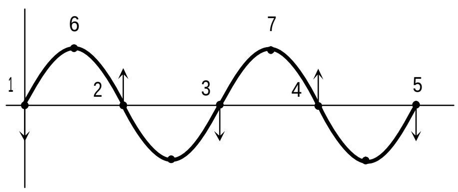
(12) **Wave velocity** (v) : It is the distance travelled by the disturbance in one time period. It only depends on the properties of the medium and it independent of time and position.

$$v = n\lambda = \frac{\lambda}{T} = \frac{\omega}{2\pi} = \frac{\omega}{k}$$

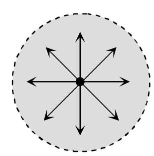
(13) **Group velocity** (v_g) : The velocity with which the group of waves travels is known as group velocity or the velocity with which a wave packet travels is known as group velocity $v_g = \frac{d\omega}{dk}$.

(14) **Phase** (ϕ) : The quantity which express at any instant, the displacement of the particle and it's direction of motion is called the phase of the particle.

If two particles of the medium, at any instant are at the same distance in the same direction from their equilibrium positions and are moving in the same direction then they are said to be in same phase *e.g.* In the following figure particles 1, 3 and 5 are in same phase and point 6, 7 are also in same phase.



(15) **Intensity of wave** : The wave intensity is defined as the average amount of energy flow in medium per unit time and per unit of it's cross-sectional area. It's unit is W/m^2



Hence intensity (I) $= \frac{\text{Energy}}{\text{Area} \times \text{Time}} = \frac{\text{Power}}{\text{Area}} = 2\pi^2 n^2 a^2 \rho v$

$\Rightarrow I \propto a^2$ (when v, ρ = constant)

where a = Amplitude, n =Frequency, v = Wave velocity,

ρ = Density of medium.

At a distance r from a point source of power P the intensity is given by $I = \frac{P}{4\pi r^2} \Rightarrow I \propto \frac{1}{r^2}$

The human ear can hear sound of intensity up to 10^{-12} W/m^2. This is called **threshold of intensity**. The upper limit of intensity of sound which can be tolerated by human ear is 1 W/m^2. This is called **threshold of pain**.

(16) **Energy density** : The energy associated with unit volume of the medium is defined as energy density

Energy density $= \frac{\text{Energy}}{\text{Volume}} = \frac{\text{Intensity}}{\text{Velocity}} = \frac{2\pi^2 n^2 a^2 \rho v}{v} = 2\pi^2 n^2 a^2 \rho$

Velocity of Transverse Wave

The velocity of a transverse wave in a stretched string is given by $v = \sqrt{\frac{T}{m}}$; where T = Tension in the string; m = Linear density of string (mass per unit length).

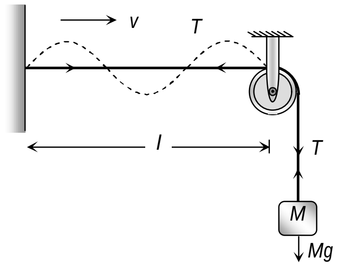
(1) If A is the area of cross-section of the wire then $m = \rho A$

$\Rightarrow v = \sqrt{\frac{T}{\rho A}} = \sqrt{\frac{S}{\rho}}$; where S = Stress $= \frac{T}{A}$

(2) If string is stretched by some weight then

$T = Mg$

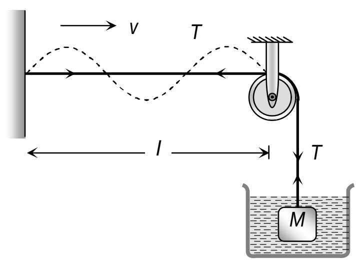
$\Rightarrow v = \sqrt{\frac{Mg}{m}}$



(3) If suspended weight is immersed in a liquid of density σ and ρ = density of material of the suspended load then

$T = Mg\left(1 - \frac{\sigma}{\rho}\right)$

$\Rightarrow v = \sqrt{\frac{Mg(1 - \sigma / \rho)}{m}}$

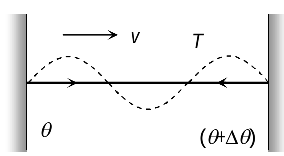


(4) If two rigid supports of stretched string are maintained at temperature difference of $\Delta\theta$ then due to elasticity of string.

$T = YA\,\alpha\,\Delta\theta$

$\Rightarrow v = \sqrt{\frac{YA\,\alpha\Delta\theta}{m}}$

$= \sqrt{\frac{Y\alpha\Delta\theta}{d}}$



where Y = Young's modulus of elasticity of string, A = Area of cross section of string, α = Temperature coefficient of thermal expansion, d = Density of wire $= \frac{m}{A}$

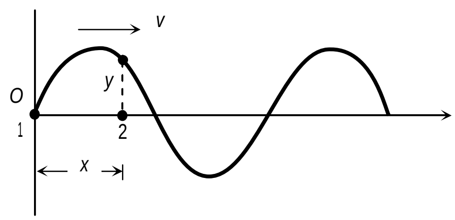
(5) In a solid body : $v = \sqrt{\frac{\eta}{\rho}}$

where η = Modulus of rigidity; ρ = Density of the material.

Equation of a Plane Progressive Waves

(1) If during the propagation of a progressive wave, the particles of the medium perform SHM about their mean position, then the waves is known as a harmonic progressive wave.

(2) Suppose a plane simple harmonic wave travels from the origin along the positive direction of *x-axis* from left to right as shown in the figure.



The displacement y of a particle 1 at O from its mean position at any time t is given by $y = a \sin \omega t$.

The wave reaches the particle 2 after time $t = \frac{x}{v}$. Hence displacement y of a particle 2 is given by

$$y = a \sin \omega \left(t - \frac{x}{v} \right) = a \sin(\omega t - kx) \qquad \left(\because k = \frac{\omega}{v} \right)$$

The general equation of a plane progressive wave with initial phase is

Displacement   Amplitude   Oscillating term   Phase

$$y(x, t) = a \sin (\omega t \pm kx \pm \phi_0)$$

Angular frequency   Initial phase   Position   Propagation constant

(3) Various forms of progressive wave function.

(i) $y = a \sin (\omega t - kx)$

(ii) $y = a \sin (\omega t - \frac{2\pi}{\lambda} x)$

(iii) $y = a \sin 2\pi \left[\frac{t}{T} - \frac{x}{\lambda} \right]$

(iv) $y = a \sin \frac{2\pi}{T} \left(t - x \frac{T}{\lambda} \right)$

(v) $y = a \sin \frac{2\pi}{\lambda} (v t - x)$

(vi) $y = a \sin \omega \left(t - \frac{x}{v} \right)$

(4) **Particle velocity** : The rate of change of displacement y *w.r.t.* time t is known as particle velocity

Hence from $y = a \sin (\omega t - kx)$

Particle velocity $v_p = \frac{\partial y}{\partial t} = a\omega \cos(\omega t - kx)$

Maximum particle velocity $(v_p)_{\max} = a\omega$

Also $\frac{\partial y}{\partial t} = -\frac{\omega}{k} \times \frac{\partial y}{\partial x}$ $\Rightarrow v_p = -v \times$ Slope of wave at that point

(5) **Important relations for numerical solving**

(i) Angular frequency ω = co-efficient of t

(ii) Propagation constant k = co-efficient of x

Wave speed $v = \frac{\text{co - efficient of } t}{\text{co - efficient of } x} = \frac{\omega}{k}$

(iii) Wave length $\lambda = \frac{\text{co - efficient of } x}{2\pi}$

(iv) Time period $T = \frac{2\pi}{\text{co - efficient of } t}$

(v) Frequency $n = \frac{\text{co - efficient of } t}{2\pi}$

(vi) $(v_p)_{\max} = a\omega = a(2\pi n) = \frac{a2\pi}{T}$

(vii) If the sign between t and x terms is negative the wave is propagating along positive X-axis and if the sign is positive then the wave moves in negative X-axis direction.

(viii) Co-efficient of sin or cos functions *i.e.* Argument of sin or cos function is represented by phase *i.e.* $(\omega t - kx)$ = Phase.

(ix) **Phase difference and path difference** : At any instant t, if ϕ_1 and ϕ_2 are the phases of two particles whose distances from the origin are x_1 and x_2 respectively then $\phi_1 = (\omega t - kx_1)$ and $\phi_2 = (\omega t - kx_2)$ $\Rightarrow$ $\phi_1 - \phi_2 = k(x_2 - x_1)$

$\Rightarrow$ Phase difference $(\Delta\phi) = \frac{2\pi}{\lambda}.\text{Path difference } (\Delta x)$

(x) **Phase difference and time difference** : If the phases of a particle distance x from the origin is ϕ_1 at time t and ϕ_2 at time t_2, then $\phi_1 = (\omega t_1 - kx)$ and $\phi_1 = (\omega t_2 - kx)$ $\Rightarrow$ $\phi_1 - \phi_2 = \omega(t_1 - t_2)$

$\Rightarrow$ Phase difference $(\Delta\phi) = \frac{2\pi}{T}.\text{Time difference } (\Delta t)$