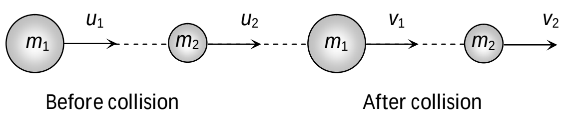
Perfectly elastic head on collision

Let two bodies of masses m_1 and m_2 moving with initial velocities u_1 and u_2 in the same direction and they collide such that after collision their final velocities are v_1 and v_2 respectively.



According to law of conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\Rightarrow \ m_1(u_1 - v_1) = m_2(v_2 - u_2) \qquad \text{-(eq-1)}$$

According to law of conservation of kinetic energy

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$\Rightarrow \ m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \ \text{-(eq-2)}$$

Dividing equation 2 with equation 1,we will get

$$v_1 + u_1 = v_2 + u_2$$

$$\Rightarrow \ u_1 - u_2 = v_2 - v_1$$

(Speed of separation after impact is equal to speed of approach before impact)

Substituting this value of v_2 in equation (i) and rearranging

we get, $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2}$ and $v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1u_1}{m_1 + m_2}$

Relative velocity of separation is equal to relative velocity of approach.

If projectile and target are of same mass *i.e.* $m_1 = m_2$

Since $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2}{m_1 + m_2}u_2$ and $v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1u_1}{m_1 + m_2}$

Substituting $m_1 = m_2$ we get

$v_1 = u_2$ and $v_2 = u_1$

It means when two bodies of equal masses undergo head on elastic collision, their velocities get interchanged.

(ii) **If massive projectile collides with a light target *i.e.* $m_1 >> m_2$**

Since $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2 u_2}{m_1 + m_2}$ and $v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1 u_1}{m_1 + m_2}$

Substituting $m_2 = 0$, we get

$v_1 = u_1$ and $v_2 = 2u_1 - u_2$

(iii) **If light projectile collides with a very heavy target *i.e.* $m_1 << m_2$**

Since $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2 u_2}{m_1 + m_2}$ and $v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1 u_1}{m_1 + m_2}$

Substituting $m_1 = 0$, we get $v_1 = -u_1 + 2u_2$ and $v_2 = u_2$