Electrical capacity

(i) Electrical capacity of a conductor is its ability to store electric charge.

(ii) The potential acquired by a conductor is directly proportional to the charge given to it i.e., $V \propto Q$.

i.e., $Q \propto V$ or $Q = CV$ where the constant of proportionality 'C' is called the electrical capacity of the conductor.

(iii) Thus the capacity of a conductor is defined as the ratio of the charge to the potential.

(iv) Its SI unit is farad.

(v) 1 milli farad (1 mF) = 10^{-3} farad

1 micro farad (1 μF) = 10^{-6} farad

1 pico farad (1 pF) = 10^{-12} farad

(vi) The capacity of a spherical conductor in farad is given by $C = 4\pi\varepsilon_0 r$, where r = radius of the conductor.

(vii) If we imagine Earth to be a uniform solid sphere then the capacity of earth

$$C = 4\pi\varepsilon_0 R = \frac{6400 \times 10^3}{9 \times 10^9} = 711\mu F \cong 1 \text{ mF}$$

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Parallel plate Capacitor :

(i) Condenser (usually, a combination of two conductors) is a device by means of which larger amount of charge can be stored at a given potential by increasing its electric capacity.

(ii) Capacitance of a capacitor or condenser is the ratio of the charge on either of its plates to the potential difference between them.

(iii) Capacity of a parallel plate condenser without medium between the plates

$$C_0 = \frac{\varepsilon_0 A}{d}$$

A = area of each plate ; d = distance between the plates

(iv) With a medium of dielectric constant K completely filling the space between the plates $C = K\frac{\varepsilon_0 A}{d}$

(v) The **dielectric constant** of a dielectric material is defined as the ratio of the capacity of the parallel plate condenser with the dielectric between the plates to its capacity with air or vacuum between the plates.

$$K = \frac{C}{C_0} = \frac{\textit{Capacity of the condenser with dielectric medium between plates}}{\textit{Capacity of the same condeser with air as medium between plates}}$$

(vi) When a dielectric slab of thickness 't' is introduced between the plates $C = \frac{\varepsilon_0 A}{d - t + \frac{t}{k}} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{k}\right)}$

(vii) In this case the distance of separation decreases by $t\left(1 - \frac{1}{k}\right)$ and hence the capacity increases

(viii) To restore the capacity to original value the distance of separation is to be increased by $t\left(1 - \frac{1}{k}\right)$.

ix) a) If a metal slab of thickness t is introduced between the plates $C = \frac{\varepsilon_0 A}{d - t}$ because for metals K is infinity.

b) If a number of dielectric slabs are inserted between the plates, each parallel to plate surface, then equivalent capacity.

$$C = \frac{\epsilon_0 A}{d - t_1\left(1 - \frac{1}{K_1}\right) - t_2\left(1 - \frac{1}{K_2}\right) \ldots\ldots\ldots - t_n\left(1 - \frac{1}{K_n}\right)}.$$

If those slabs completely fill up the gap between the plates leaving without any air gap, $C = \frac{\varepsilon_0 A}{\left(\frac{t_1}{K_1} + \frac{t_2}{K_2} + \ldots\ldots\ldots + \frac{t_n}{K_n}\right)}$.

x) In a parallel plate capacitor, the electric field at the edges is not uniform and that field is called as the **fringing field**.

xi) Electric field between the plates is uniform electric intensity $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} = \frac{Q}{Cd}$. Here σ is the surface charge density on the plates = Q/A.

xii) Potential difference between the plates $V = E.d = \frac{Q}{\varepsilon_0 A}.d$

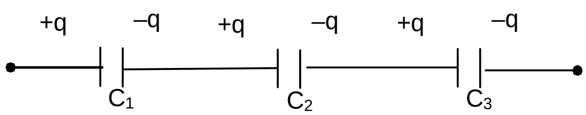
xiii) Force on each plate $F = \frac{1}{2}EQ = \frac{1}{2}\frac{Q^2}{Cd} = \frac{1}{2}\frac{CV^2}{d} = \frac{1}{2}\frac{Q^2}{\varepsilon_0 A} = \frac{1}{2}\varepsilon_0 AE^2$

xiv) Energy stored per unit volume of the medium = $\frac{1}{2}\varepsilon_0 E^2$

Combination of Capacitors

(i) When condensers are connected in series

All plates have the same charge in magnitude
Potential differences between the plates are different



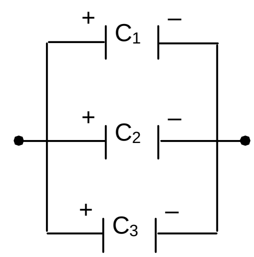
$$V_1 : V_2 : V_3 = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$$

Equivalent capacity is C then, $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

The equivalent capacity is less than the least individual capacity
Energies of the condensers $E_1 : E_2 : E_3 = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$

Total energy of the combination$=E_1+E_2 + E_3$.

(ii) When condensers are connected in parallel



P.D. across each condenser is same
Charge of each condenser is different
$Q_1 : Q_2 : Q_3 = C_1 : C_2 : C_3$
Equivalent capacity of the combination
$C = C_1 + C_2 + C_3$
The equivalent capacity is greater than the greatest individual capacity
Energies of the condensers $E_1 : E_2 : E_3 =$
$C_1 : C_2 : C_3$
Total energy of the combination$=E_1+E_2+E_3$

iii) When n identical condensers each of capacity C

Combined in series, the effective capacity $= C_s = C/n$

Combined in parallel, the effective capacity $C_p = nc$.
Ratio of the effective capacities $C_s:C_p=1: n^2$

iv) **Mixed group :**

If there are N capacitors each rated at capacity C and voltage V, by combining those we can obtain effective capacity rated at C^1 and voltage V^1. For this n capacitors are connected in a row and m such rows are connected in parallel.

Then $n = \frac{V^1}{V}$ and $m = \frac{nC^1}{C}$ where $mn = N$