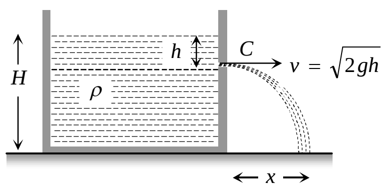
Velocity of Efflux

If a liquid is filled in a vessel up to height H and a hole is made at a depth h below the free surface of the liquid as shown in fig. then taking the level of hole as reference level (*i.e.*, zero point of potential energy) and applying Bernoulli's principle to the liquid just inside and outside the hole (assuming the liquid to be at rest inside) we get

$$\therefore \quad (P_0 + h\rho g) + 0 = P_0 + \frac{1}{2}\rho v^2 \text{ or } v = \sqrt{2gh}$$

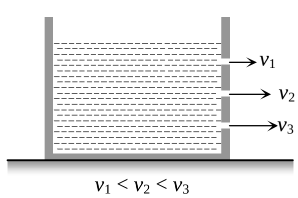


Which is same as the speed that an object would acquire in falling from rest through a distance h and is called velocity of efflux or velocity of flow.

This result was first given by Torricelli, so this is known as Torricelli's theorem.

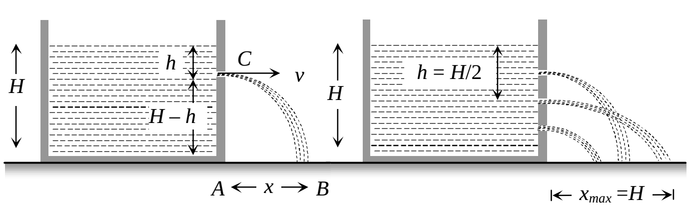
(i) The velocity of efflux is independent of the nature of liquid, quantity of liquid in the vessel and the area of orifice.

(ii) Greater is the distance of the hole from the free surface of liquid, greater will be the velocity of efflux [*i.e.*, $v \propto \sqrt{h}$]



(iii) As the vertical velocity of liquid at the orifice is zero and it is at a height $(H-h)$ from the base, the time taken by the liquid to reach the base-level $t = \sqrt{\frac{2(H-h)}{g}}$

(iv) Now during time t liquid is moving horizontally with constant velocity v, so it will hit the base level at a horizontal distance x (called range) as shown in figure.



Such that $x = vt = \sqrt{2gh} \times \sqrt{[2(H-h)/g]} = 2\sqrt{h(H-h)}$

For maximum range $\frac{dx}{dh} = 0$

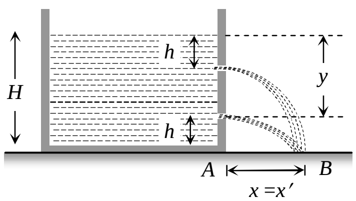
$\therefore \quad h = \frac{H}{2}$

i.e., range *x* will be maximum when

$h = \frac{H}{2}$.

$\therefore$ Maximum range $x_{\max} = 2\sqrt{\frac{H}{2}\left[H - \frac{H}{2}\right]} = H$

(v)



If the level of free surface in a container is at height *H* from the base and there are two holes at depth *h* and *y* below the free surface, then

$x = 2\sqrt{h(H-h)}$ and $x' = 2\sqrt{y(H-y)}$

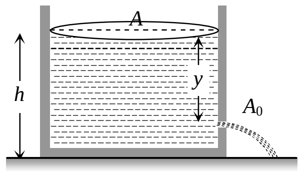
Now if $x = x'$, *i.e.*, $h(H-h) = y(H-y)$

i.e., $y^2 - Hy + h(H-h) = 0$

or $y = \frac{1}{2}[H \pm (H-2h)]$,

i.e., $y = h$ or $(H-h)$

i.e., the range will be same if the orifice is at a depth *h* or $(H-h)$ below the free surface. Now as the distance $(H-h)$ from top means $H-(H-h) = h$ from the bottom, so the range is same for liquid coming out of holes at same distance below the top and above the bottom.

(vi) 

If A_0 is the area of orifice at a depth y below the free surface and A is that of container, the volume of liquid coming out of the orifice per second will be $(dV/dt) = vA_0 = A_0\sqrt{2gy}$ [As $v = \sqrt{2gy}$]

Due to this, the level of liquid in the container will decrease and so if the level of liquid in the container above the hole changes from y to $y - dy$ in time t to $t + dt$ then $-dV = A\,dy$

So substituting this value of dV in the above equation

$$-A\frac{dy}{dt} = A_0\sqrt{2gy}$$

i.e., $\int dt = -\frac{A}{A_0}\frac{1}{\sqrt{2g}}\int y^{-1/2}dy$

So the time taken for the level to fall from H *to* H'

$$t = -\frac{A}{A_0}\frac{1}{\sqrt{2g}}\int_H^{H'} y^{-1/2}dy = \frac{A}{A_0}\sqrt{\frac{2}{g}}[\sqrt{H} - \sqrt{H'}]$$

If the hole is at the bottom of the tank, time t to make the tank empty :

$$t = \frac{A}{A_0}\sqrt{\frac{2H}{g}}$$ [As here $H' = 0$]