Electric potential (V)

Electric potential at a point in a field is the amount of work done in bringing a unit +ve charge from infinity to the point.

(i) It is equal to the Electric potential energy of unit + ve charge at that point. It is a scalar

Unit and dimensional formula

S. I. unit : $\frac{Joule}{Coulomb} = volt$

C.G.S. unit : *Stat volt* (e.s.u.); **1 *volt*** $= \frac{1}{300}$ ***Stat volt***, Dimension : $[V] = [ML^2T^{-3}A^{-1}]$

(ii) Potential at a distance 'd' due to a point charge q in air or vacuum is

$$V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{d}$$

$$V = -\int \vec{E}.\vec{dx}$$

$$\vec{E} = -\frac{dv}{dx} \text{ (or) } V = Ed$$

(iii) **Potential of a system of point charges :** Consider P is a point at which net electric potential is to be determined due to several charges.

The net potential at P

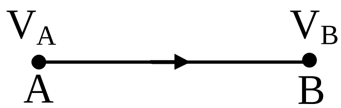
$$V = k\frac{Q_1}{r_1} + k\frac{Q_2}{r_2} + k\frac{Q_3}{r_3} + k\frac{(-Q_4)}{r_4} + \ldots$$

In general $V = \sum_{i=1}^{X} \frac{kQ_i}{r_i}$

(iv) A positive charge in a field moves from high potential to low potential where as electron moves from low potential to high potential when left free.

(v) Work done in moving a charge q through a potential difference V is $W = q\,V$ joule

(vi) Work done in moving a charge from one point to other in an electric field is equal to change in it's potential energy i.e. work done in moving Q from A to B = $qV_B - qV_A$ $= U_B - U_A$



(vii) If electric potential at a point is V then potential energy (PE) of a charge placed at that point will be qv.

(viii) A free charge moves from higher PE to lower PE state in an electric field. Hence

(a) a + ve charge will move form higher potential to lower potential while,

(b) a –ve charge will move form lower potential to higher potential

(ix) Electric potential due to a continuous charge distribution : The potential due to a continuous charge distribution is the sum of potentials of all the infinitesimal charge elements in which the distribution may be divided *i.e.*,

$$V = \int dV, \quad = \int \frac{dQ}{4\pi\varepsilon_0 r}$$

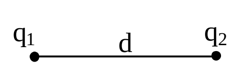
Potential Energy of System

Work done in bringing a charge from infinity to a point against the electric field is equal to the potential energy of that charge.

(i) The electric potential energy of a system of charges is the work that has been done in bringing those charges from infinity to near each other to form the system.

(ii) Energy of a system of two charges

$$PE = \frac{1}{4\pi \in_0} \frac{q_1 q_2}{d}$$



(iii) Energy of a system of three charges

$$PE = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + \frac{q_3q_1}{r_{31}}\right]$$

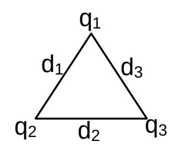
(iv) Energy of a system of n charges.

$$PE = \frac{1}{2}\cdot\frac{1}{4\pi\varepsilon_0}\left(\sum_{i=1}^{n} q_i\left(\sum_{\substack{j=1\\ i\neq j}}^{n}\frac{q_j}{r_{ij}}\right)\right)$$

(V) If two like charges (two protons or two electrons) are brought towards each other, the P.E of the system increases.

(vi) If two unlike charges (a proton and an electron) are brought towards each other, the P.E. of the system decreases.

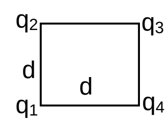
(vii) If three charges q_1, q_2 and q_3 are situated at the vertices of a triangle (as shown in the figure), the P.E. of the system is



$U = U_{12} + U_{23} + U_{31}$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{d_1} + \frac{q_2q_3}{d_2} + \frac{q_3q_1}{d_3}\right)$$

(viii) If four charges q_1, q_2 , q_3 and q_4 are situated at the corners of a square as shown in the figure, P.E of the system



$$\frac{1}{4\pi\varepsilon_0}\times\left(\frac{q_1q_2}{d} + \frac{q_2q_3}{d} + \frac{q_3q_4}{d} + \frac{q_4d_1}{d} + \frac{q_2q_4}{\sqrt{2}d} + \frac{q_1q_3}{\sqrt{2}d}\right)$$

(ix) **Work done in an electric field** -

In the field of a charge Q, if a charge q is moved against the electric field from a distance 'a' to a distance 'b' from Q, the work done W is given by

$$W = (V_b - V_a)q = \frac{1}{4\pi\varepsilon_o}\frac{Qq}{b} - \frac{1}{4\pi\varepsilon_o}\frac{Qq}{a} \qquad = \frac{Qq}{4\pi\varepsilon_o}\left[\frac{1}{b} - \frac{1}{a}\right] = \frac{Qq}{4\pi\varepsilon_o}\left[\frac{a-b}{ab}\right]$$

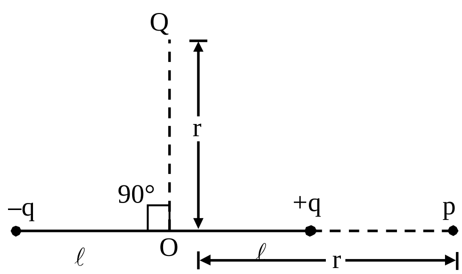
(ii) Total energy remains constant in an electric field i.e.

$KE_A + PE_A = KE_B + PE_B$

KE = Kinetic energy

PE = Potential energy

Electric Potential due to a dipole



$$V = \frac{1}{4\pi\varepsilon_0}.\frac{q(2\ell)\cos\theta}{r^2}$$

$$= \frac{1}{4\pi\varepsilon_0}.\frac{P\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}.\frac{\vec{P}.\vec{r}}{r^2}$$

$$= \frac{1}{4\pi\varepsilon_0}.\frac{\vec{P}.\hat{r}}{r^3}$$

where θ is the angle between $\vec{P}$ and $\vec{r}$.

V can also be written as

$$V = -\frac{1}{4\pi\varepsilon_0}\vec{P}.\nabla\left(\frac{1}{r}\right) \text{because } \nabla\left(\frac{1}{r}\right) = -\frac{\hat{r}}{r^2}$$

(i) If $\theta = 0$, $V_{axis} = \frac{P}{4\pi\varepsilon_0 . r^2}$

(ii) If $\theta = 90^\circ$, $V_{equator} = 0$

(iii) Here we see that $V = 0$ but $E \neq 0$ for points at equator

(iv) Again, if $r >> 2\ell$ is not true and $d = 2\ell$,

$$V_{axis} = \frac{1}{4\pi\varepsilon_0}.\frac{P}{(r^2 - \ell^2)}$$

$$V_{equator} = 0$$

Note :

(i) This is not essential that at a point, where E = 0 , V will also be zero there eg. inside a uniformly charged sphere, E = 0 but V≠ 0

(ii) Also if V = 0 , it is not essential for E to be zero eg. in equatorial position of dipole V = 0, but E ≠ 0

Electric dipole in uniform electric field

(i) Potential energy of the dipole

$U = -PE\cos\theta = -\vec{P}.\vec{E}$

Cases :

(a) If $\theta = 0^o$, i.e. $\vec{P} \parallel \vec{E}$

$U = -PE$, dipole is in the minimum potential energy state and no torque acting on it and hence it is in the stable equilibrium state.

(b) For $\theta = 180^o$, i.e. $\vec{P}$ and $\vec{E}$ are in opposite direction, then $U = PE$ which is maximum potential energy state. Although it is in equilibrium but it is not a stable state and a slight perturbation can disturb it.

(c) $\theta = 90^0$, i.e. $\vec{P} \perp \vec{E}$, then $U = 0$

Note :

There is no net force acting on the dipole in a uniform electric field.

Work done in rotating the dipole from θ_1 to θ_2 in an uniform electric field

$W = PE(\cos\theta_1 - \cos\theta_2)$

Equipotential surface:

(i) The surface which is the locus of all points which are at the same potential is known as equipotential surface

(ii) No work is required to move a charge from one point to another on the equipotential surface.

(iii) No two equipotential surfaces intersect

(iv) The direction of electric lines of force or direction of electric field is always normal to the equipotential surface.

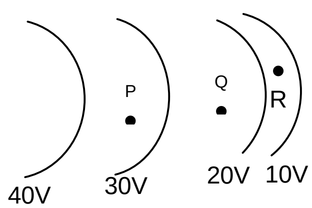
(v) Inside a hollow charged spherical conductor the potential is constant. This can be treated as equipotential volume. No work is required to move a charge from the centre to the surface.

(vi) For an isolated point charge, the equipotental surface is a sphere. i.e. concentric spheres around the point charge are different equipotential surfaces.

(vii) In a uniform electric field any plane normal to the field direction is an equipotential surface.

(viii) The spacing between equipotential surfaces enables us to identify regions of strong and weak field.

$$E = -\frac{dV}{dr} \Rightarrow E \propto \frac{1}{dr}$$



$E_P < E_Q < E_R$

Relation Between Electric Field and Potential

(i) In an electric field rate of change of potential with distance is known as **potential gradient.**

(ii) Potential gradient is a vector quantity and it's direction is opposite to that of electric field.

(iii) Potential gradient relates with electric field according to the following relation $\boldsymbol{E} = -\frac{dV}{dr}$; This relation gives another unit of electric field is $\frac{volt}{meter}$.

(iv) In the above relation negative sign indicates that in the direction of electric field potential decreases.

(v) Negative of the slope of the *V*-*r* graph denotes intensity of electric field *i.e.* $\tan\theta = \frac{V}{r} = -E$

(vi) In space around a charge distribution we can also write $\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$

where $E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y}$ and $E_z = -\frac{\partial V}{\partial z}$

(vii) With the help of formula $E = -\frac{dV}{dr}$, potential difference between any two points in an electric field can be determined by knowing the boundary conditions

$$dV = -\int_{r_1}^{r_2} \vec{E}.\vec{dr} = -\int_{r_1}^{r_2} E.dr\cos\theta$$

. Dielectric materials, Polar and non polar molecules :

(i) *Dielectric material* : Any material that do not allow the electrical charges to easily pass through them is called insulator or dielectric material or simply a dielectric.
Dielectric is a technical term for an insulator.

(ii) Non *-polar molecule :*
In certain kind of materials, ordinarily the molecules will have symmetric charge distributions.
Such kind of molecules are called non-polar molecules.
In the absence of any external electric field, a non-polar molecule will have its centre of positive charge coinciding with centre of negative charge.

(iii) *Polar molecule :*
Certain dielectrics like water, hydrogenchloride and alcohol are made of molecules that have a non uniform distribution of electric charge.
In such molecules, the positive charge centre will not coincide with the negative charge centre, even in the absence of any external field.
The molecules are polarized even in the absence of any external electric field.
Such kind of molecules are called Polar molecules.