Equations of Kinematics (Integration approach)

These are the various relations between *u*, *v*, *a*, *t* and *s* for the particle moving with uniform acceleration where the notations are used as :

u = Initial velocity of the particle at time t = 0 *sec*

v = Final velocity at time t *sec*

a = Acceleration of the particle

s = Distance travelled in time t *sec*

s_n = Distance travelled by the body in n^{th} *sec*

When particle moves with zero acceleration

(i) It is a unidirectional motion with constant speed.

(ii) Magnitude of displacement is always equal to the distance travelled.

(iii) $v = u$, $s = u\,t$ [As $a = 0$]

When particle moves with constant acceleration

Acceleration is said to be constant when both the magnitude and direction of acceleration remain constant.

There will be one dimensional motion if initial velocity and acceleration are parallel or anti-parallel to each other.

Equations of motion (in scalar from)	Equation of motion (in vector from)
$\upsilon = u + at$	$\vec{v} = \vec{u} + \vec{a}t$
$s = ut + \frac{1}{2}at^2$	$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$
$\upsilon^2 = u^2 + 2as$	$\vec{v}.\vec{v} - \vec{u}.\vec{u} = 2\vec{a}.\vec{s}$
$s = \left(\frac{u+v}{2}\right)t$	$\vec{s} = \frac{1}{2}(\vec{u} + \vec{v})t$
$s_n = u + \frac{a}{2}(2n-1)$	$\vec{s}_n = \vec{u} + \frac{\vec{a}}{2}(2n-1)$

We have to do the integration for the following cases of variable Acceleration:

a) If we know the acceleration - time relation and we want to find the velocity-time relation .

$$\left(\int dv = \int a dt\right)$$

b) If we know the velocity - time relation and we want to find the displacement-time relation .

$$\left(\int ds = \int v dt\right)$$

c) If we know the acceleration - displacement relation and we want to find the velocity - displacement relation

$$a = v\frac{dv}{ds}$$

or vdv = ads

$$\int vdv = \int ads$$

d. If acceleration is a function of time

$a = f(t)$ then $v = u + \int_0^t f(t)\,dt$

and $s = ut + \int_0^t \left(\int f(t)\,dt\right)dt$

e. If acceleration is a function of distance

$a = f(x)$ then $v^2 = u^2 + 2\int_{x_0}^{x} f(x)\,dx$

f. If acceleration is a function of velocity

$a = f(v)$ then $t = \int_u^v \frac{dv}{f(v)}$ and $x = x_0 + \int_u^v \frac{vdv}{f(v)}$