

Law of Conservation of Linear Momentum

If no external force acts on a system (called isolated) of constant mass, the total momentum of the system remains constant with time.

(1) According to this law for a system of particles $\vec{F} = \frac{d\vec{p}}{dt}$

In the absence of external force $\vec{F} = 0$ then $\vec{p} = \text{constant}$

i.e., $\vec{p} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \dots = \text{constant}$.

or $m_1 \vec{v}_1 + m_2 \vec{v}_2 + m_3 \vec{v}_3 + \dots = \text{constant}$

This equation shows that in absence of external force for a closed system the linear momentum of individual particles may change but their sum remains unchanged with time.

(2) Law of conservation of linear momentum is independent of frame of reference, though linear momentum depends on frame of reference.

(3) Conservation of linear momentum is equivalent to Newton's third law of motion.

For a system of two particles in absence of external force, by law of conservation of linear momentum.

$$\vec{p}_1 + \vec{p}_2 = \text{constant}.$$

$$\therefore m_1 \vec{v}_1 + m_2 \vec{v}_2 = \text{constant}.$$

Differentiating above with respect to time

$$m_1 \frac{d\vec{v}_1}{dt} + m_2 \frac{d\vec{v}_2}{dt} = 0 \Rightarrow m_1 \vec{a}_1 + m_2 \vec{a}_2 = 0 \Rightarrow \vec{F}_1 + \vec{F}_2 = 0$$

$$\therefore \vec{F}_2 = -\vec{F}_1$$

i.e. for every action there is an equal and opposite reaction which is Newton's third law of motion.

(4) Practical applications of the law of conservation of linear momentum

(i) When a man jumps out of a boat on the shore, the boat is pushed slightly away from the shore.

(ii) A person left on a frictionless surface can get away from it by blowing air out of his mouth or by throwing some object in a direction opposite to the direction in which he wants to move.

(iii) **Recoiling of a gun :** For bullet and gun system, the force exerted by trigger will be internal so the momentum of the system remains unaffected.



Let m_G = mass of gun, m_B = mass of bullet,

v_G = velocity of gun, v_B = velocity of bullet

Initial momentum of system = 0

Final momentum of system = $m_G \vec{v}_G + m_B \vec{v}_B$

By the law of conservation of linear momentum

$$m_G \vec{v}_G + m_B \vec{v}_B = 0$$

So recoil velocity $\vec{v}_G = -\frac{m_B}{m_G} \vec{v}_B$

(a) Here negative sign indicates that the velocity of recoil \vec{v}_G is opposite to the velocity of the bullet.

(b) $v_G \propto \frac{1}{m_G}$ i.e. higher the mass of gun, lesser the velocity of recoil of gun.

(c) While firing the gun must be held tightly to the shoulder, this would save hurting the shoulder because in this condition the body of the shooter and the gun behave as one body. Total mass become large and recoil velocity becomes too small.

Impulse

(1) When a large force works on a body for very small time interval, it is called impulsive force.

An impulsive force does not remain constant, but changes first from zero to maximum and then from maximum to zero. In such case we measure the total effect of force.

(2) Impulse of a force is a measure of total effect of force.

$$(3) \vec{I} = \int_{t_1}^{t_2} \vec{F} dt.$$

(4) Impulse is a vector quantity and its direction is same as that of force.

(5) Units : *Newton-second* or Kg-m-s^{-1} (S.I.)

Dyne-second or gm-cm-s^{-1} (C.G.S.)

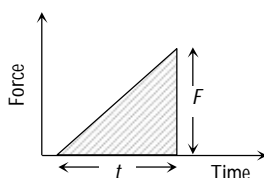
(6) Force-time graph : Impulse is equal to the area under $F-t$ curve.

If we plot a graph between force and time, the area under the curve and time axis gives the value of impulse.

$I =$ Area between curve and time axis

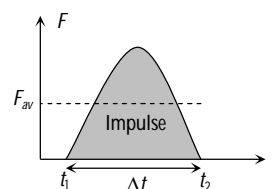
$$= \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$= \frac{1}{2} F t$$



(7) From Newton's second law

$$\Rightarrow \vec{I} = \vec{p}_2 - \vec{p}_1 = \Delta \vec{p}$$



i.e. The impulse of a force is equal to the change in momentum.

This statement is known as *Impulse momentum theorem*.

Examples : Hitting, kicking, catching, jumping, diving, collision *etc.*

In all these cases an impulse acts.

$$I = \int F dt = F_{av} \cdot \Delta t = \Delta p = \text{constant}$$

So if time of contact Δt is increased, average force is decreased (or diluted) and vice-versa.

(i) In hitting or kicking a ball we decrease the time of contact so that large force acts on the ball producing greater acceleration.

(ii) In catching a ball a player by drawing his hands backwards increases the time of contact and so, lesser force acts on his hands and his hands are saved from getting hurt.



(iii) In jumping on sand (or water) the time of contact is increased due to yielding of sand or water so force is decreased and we are not injured. However if we jump on cemented floor the motion stops in a very short interval of time resulting in a large force due to which we are seriously injured.

(iv) An athlete is advised to come to stop slowly after finishing a fast race, so that time of stop increases and hence force experienced by him decreases.

(v) China wares are wrapped in straw or paper before packing.