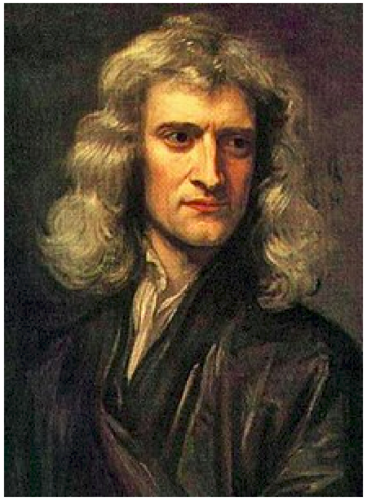
A Geometric Approach to Calculus

Conrad Crowley :: Maths Week 2017

“If a man's wit be wandering, let him study the mathematics.”

Brief History Of Calculus

- Both Sir. Issac Newton and Gottfried Leibniz are credited with the discovery of calculus. However most individual rules predate their discoveries. They both collected these rules and formalized them under Calculus.



Recent Independent Discoveries

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Gaps Between Primes

P_n = the nth prime

Gaps Between Primes

$$P_n \quad = \text{the nth prime}$$

They found an upper bound of

$$P_{n+1} - P_n > \frac{log(n)loglog(n)loglogloglog(n)}{logloglog(n)}$$

Back To Familiar Territories

Area of a circle =

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All we need to get this formula is :

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But mathematicans didn't pull this out of their backside did they?

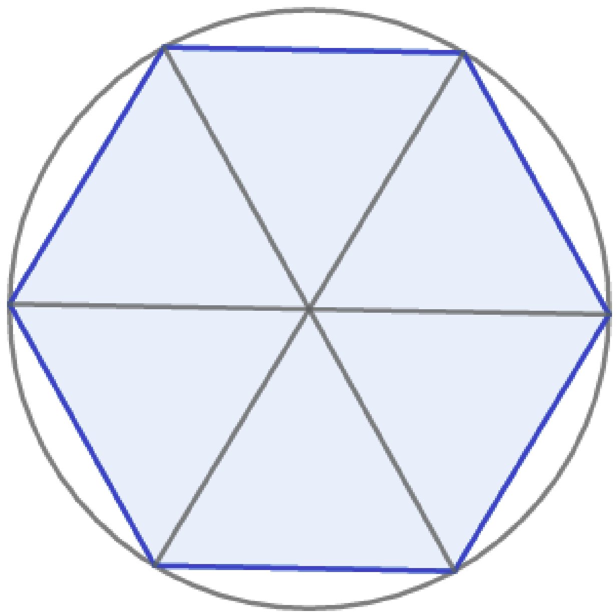
All we need to get this formula is :

Area of a triangle = $\frac{1}{2}bh$

Perimeter of a circle = $2r\pi$

Approximations using triangles

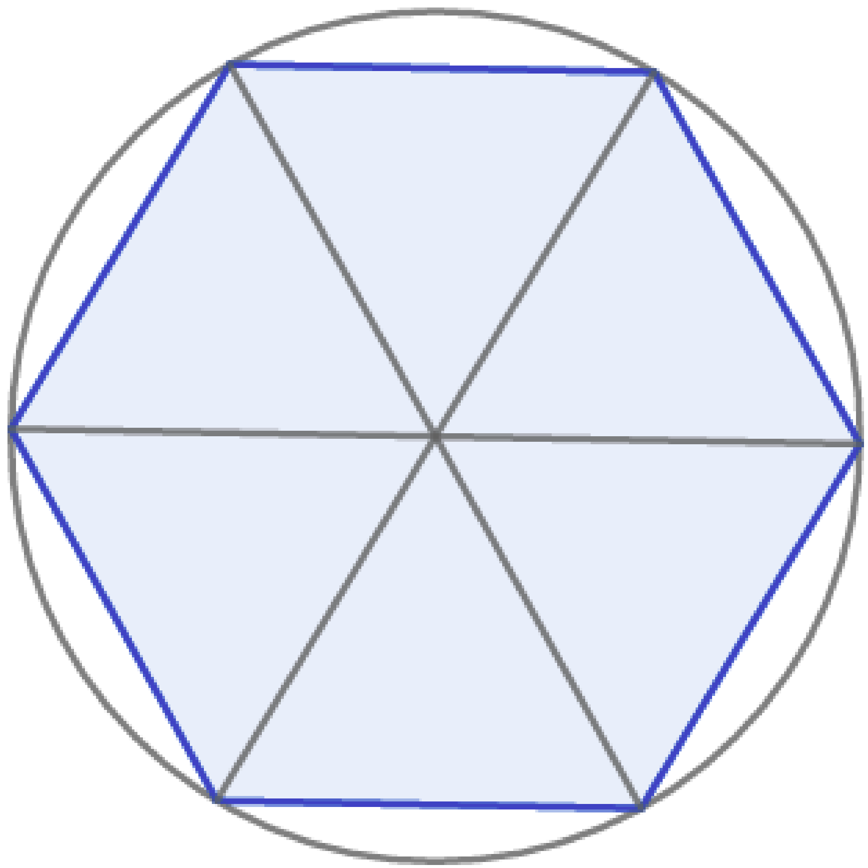
We can use six triangles to make a hexagon (6-gon) which fills up the space nicely.



Approximations using triangles

What is the area of our hexagon?

We can find it by adding up our triangles!



Approximations using triangles

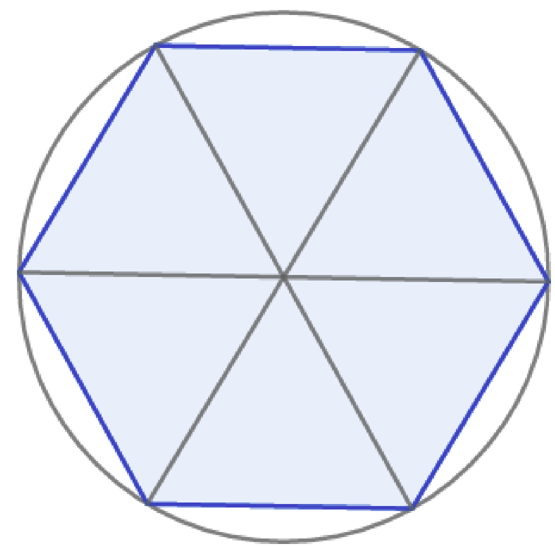
Base $= \dfrac{p}{6}$

Height $= r\cos 30$

Number of triangles $= 6$

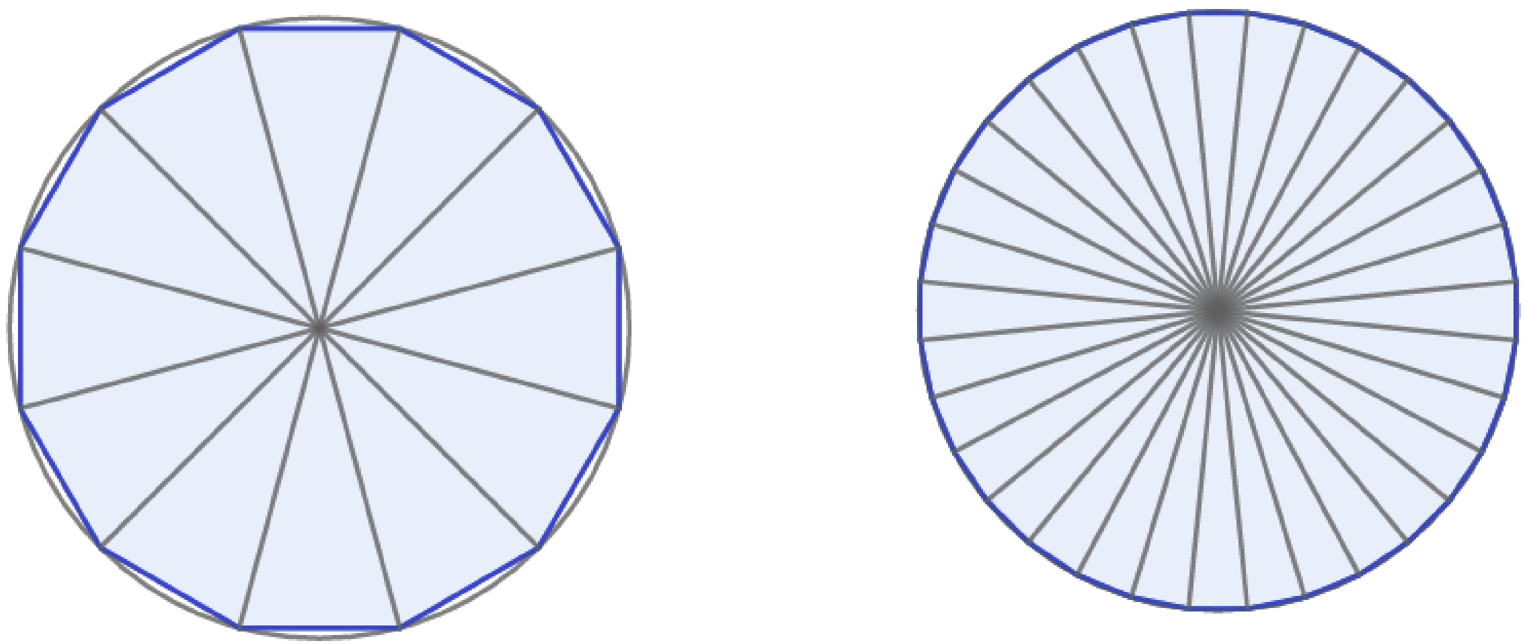
Area $=$

$$6 \cdot \frac{1}{2} \cdot \frac{p}{6} \cdot r\cos 30$$



Approximations using triangles

But we can do better, below is a 12-gon and a 32-gon.



Approximations using triangles

I hate approximations.

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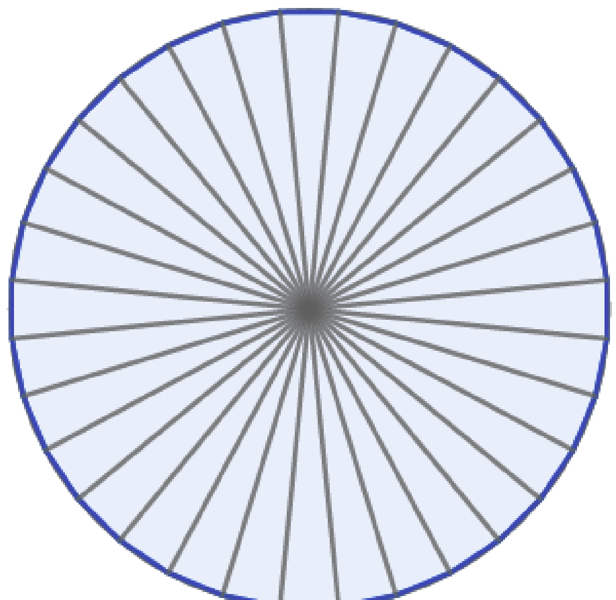
Like really hate them.

Approximations using triangles

So how do we get it to be the exact area ?

Approximations using triangles

So how do we get it to be the exact area?
We put in infinitely many triangles!



Approximations using triangles

All the bases must add up to $2r\pi$

The heights become r

Adding all the triangle's area this means :

Area of Circle $=$

Approximations using triangles

All the bases must add up to $2r\pi$

The heights become r

Adding all the triangle's area this means :

$$\text{Area of Circle} = \frac{1}{2} \cdot 2\pi r \cdot r$$

$$= \pi r^2$$

Real World Applications

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A good example is car moving with constant acceleration.

Its distance traveled at a time t is given by:

$$S = ut + \frac{1}{2}at^2$$

Real World Applications

We get $$\frac{ds}{dt} = u + at$$

Which is velocity!

And what is the definition of acceleration?

More quotes than my english essays

Hilbert was told that the student had left the university to become a poet. Hilbert:"I can't say I'm surprised. I never thought he had enough creativity to be a mathematician."

Geometry and Calculus

In class we have done the algebraic methods behind calculus, but this is not the only way to look at this!

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Deeeeeeeeeeeeeeeeeeeeeeeeeeex

No it's delta x

Geometry and Calculus

We are going to try and invoke geometry into our problems.

Why?

It's becomes much more obvious what happens when we diffrentiate the function

Geometry and Calculus

We will initially focus on the $y = x^2$ differentiating the rather trivial equation

We must stop and ponder, how can we represent x^2 in geometry?

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Of course! As the area of a square!

Geometry and Calculus

We now move to the nicely animated portion of this presentation.

Infinity Trolling

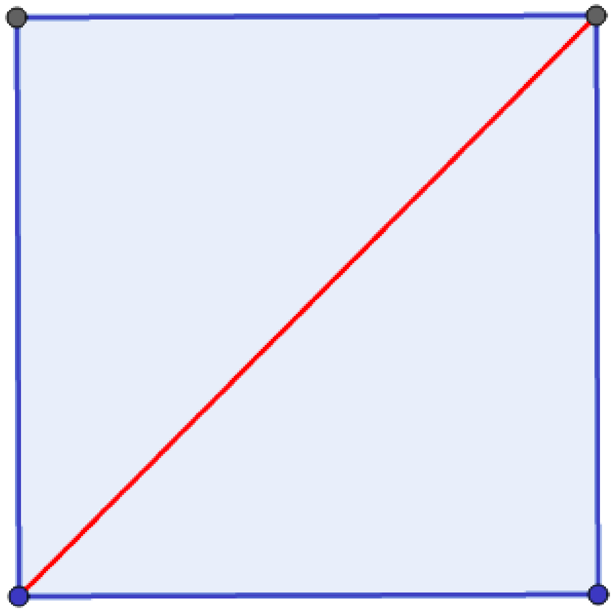
While infinitely small and infinitely big numbers can be exciting and useful, there can be some less than honest uses of these quantities.

Infinity Trolling

We start with a problem in geometry: The infinite staircase problem appears to show that $\sqrt{2} = 2$. Hopefully you can see that such a result is mildy problematic.

Infinity Trolling

Consider a square with side length 1.



What is the length of the red line?

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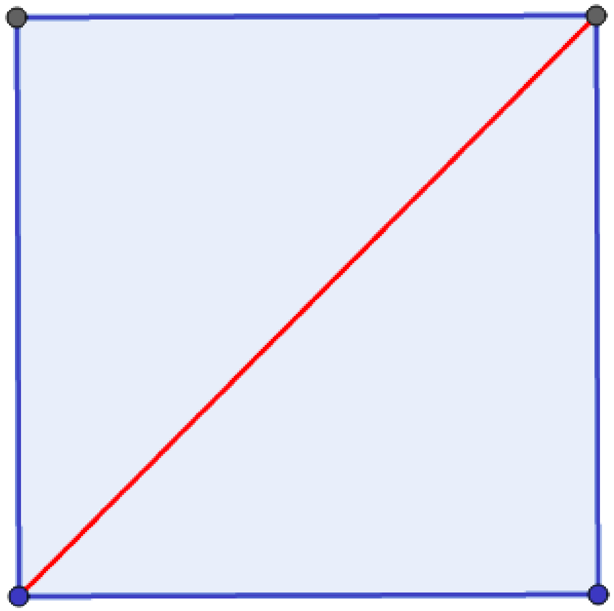
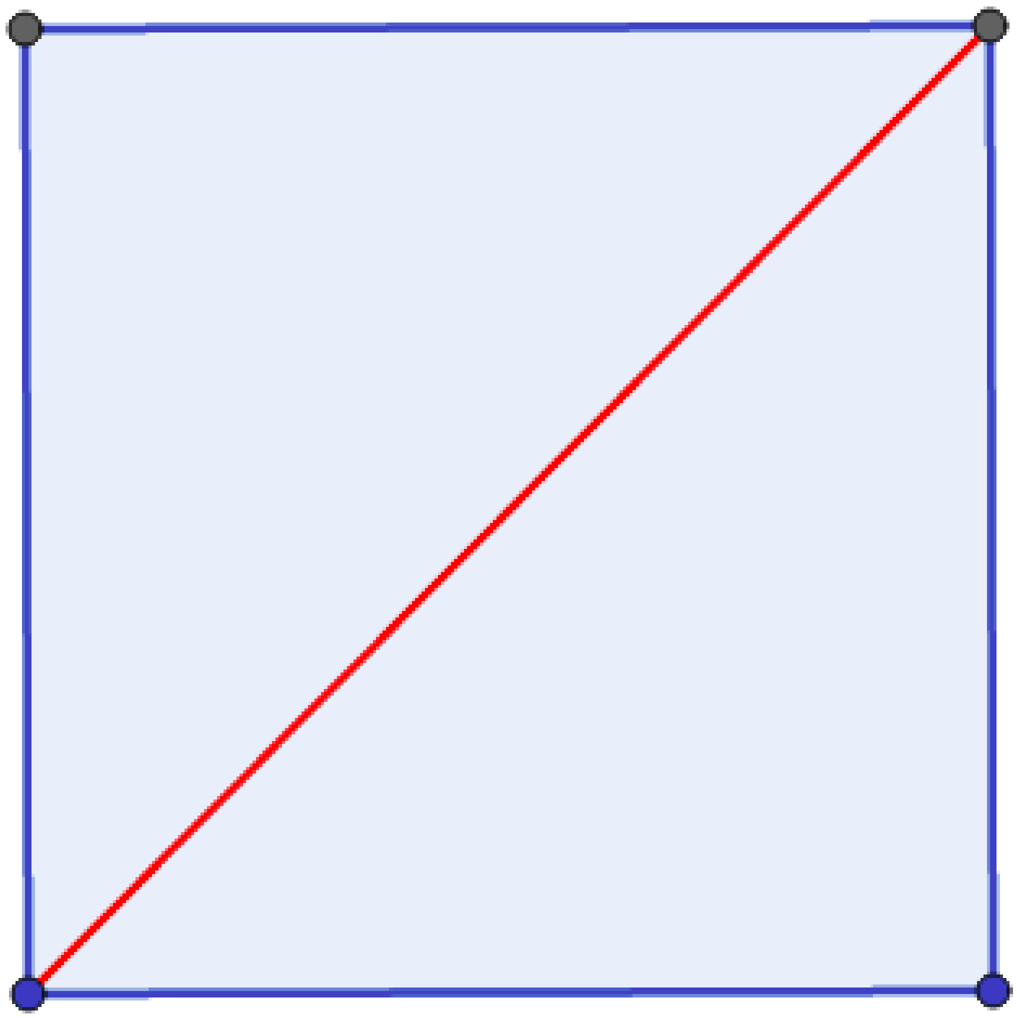


What is the length of the red line?

Hint: it's $\sqrt{2}$

Infinity Trolling



Infinity Trolling

We continue with a problem in probability:

Suppose you throw a dart at a circular dartboard. There is a 100% chance the dart hits the dartboard. What is the probability that the dart hits a given point?

Infinity Trolling

We can use the probability formula:

$$P(x) = \frac{\text{Number of desirable outcomes}}{\text{Number of possible outcomes}}$$

What are these equal to?

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Conclusion

Now would be a good time to ask questions about anything.

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Hopefully you've seen the fun one can have bringing geometry into everything.

All material used on board available at:

www.stuffconradsaid.tk

You can also leave feedback about this presentation.

This final slide is trivial and left as an excercise to the viewer