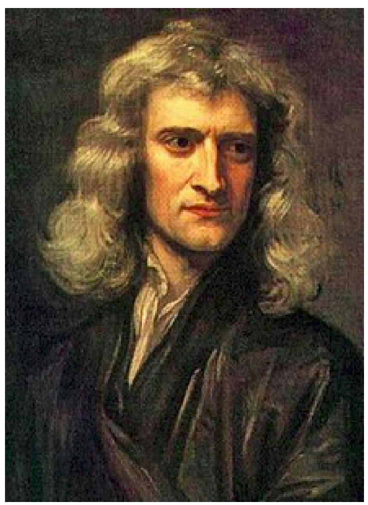
A Geometric Approach to Calculus

Conrad Crowley :: Maths Week+1 2017

“If a man's wit be wandering, let him study the mathematics.”

Brief History Of Calculus

- Both Sir. Issac Newton and Gottfried Leibniz are credited with the discovery of calculus. However most individual rules predate their discoveries. They both collected these rules and formalized them under Calculus.



Recent Independent Discoveries

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Ford-Green-Kongagih-Tao publish results on 21 Nov. 2013.

Gaps Between Primes

P_n = the nth prime

Gaps Between Primes

$$P_n \quad = \text{the nth prime}$$

They found an upper bound of

$$P_{n+1} - P_n > \frac{log(n)loglog(n)loglogloglog(n)}{logloglog(n)}$$

Back To Familiar Territories

Area of a circle =

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Area of a circle $= \pi r^2$

But mathematicians didn't pull this out of their backside did they?

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All we need to get this formula is :

Area of a triangle =

Perimeter of a circle =

Back To Familiar Territories

Area of a circle = πr^2

But mathematicans didn't pull this out of their backside did they?

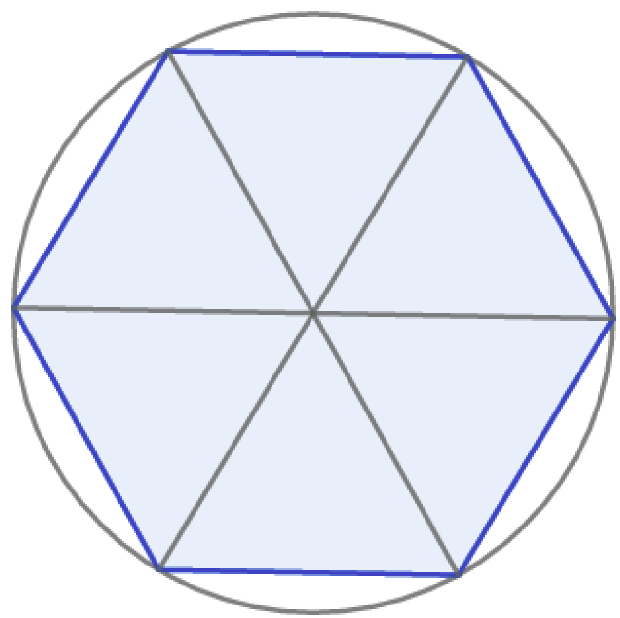
All we need to get this formula is :

Area of a triangle = $\frac{1}{2}bh$

Perimeter of a circle = $2r\pi$

Approximations using triangles

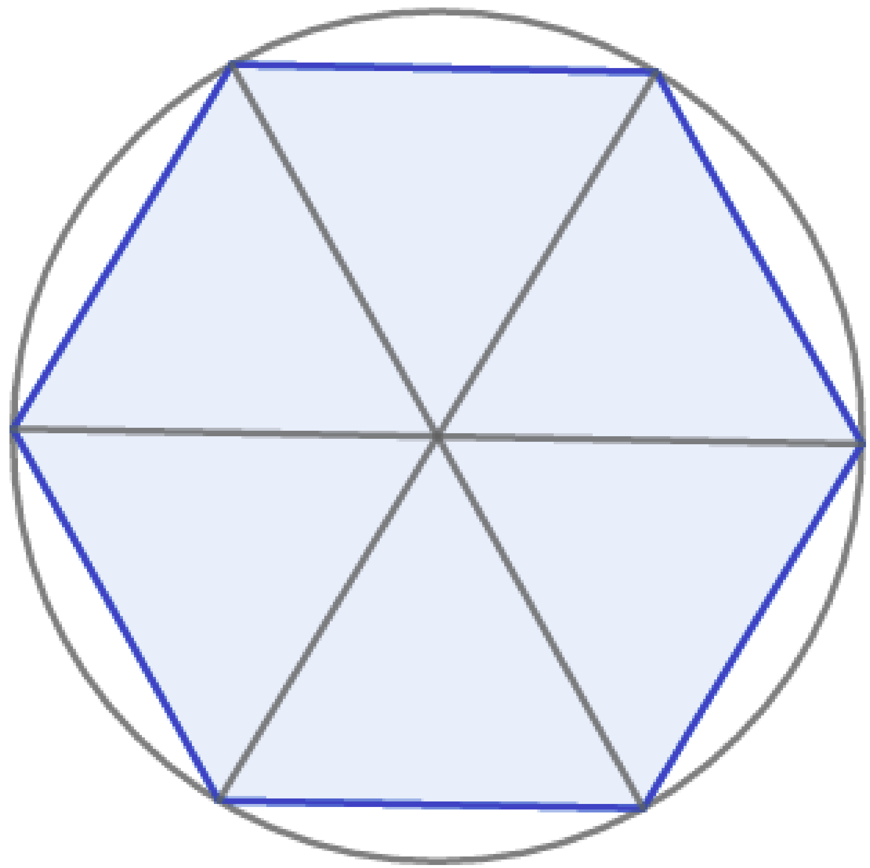
We can use six triangles to make a hexagon (6-gon) which fills up the space nicely.



Approximations using triangles

What is the area of our hexagon?

We can find it by adding up our triangles!



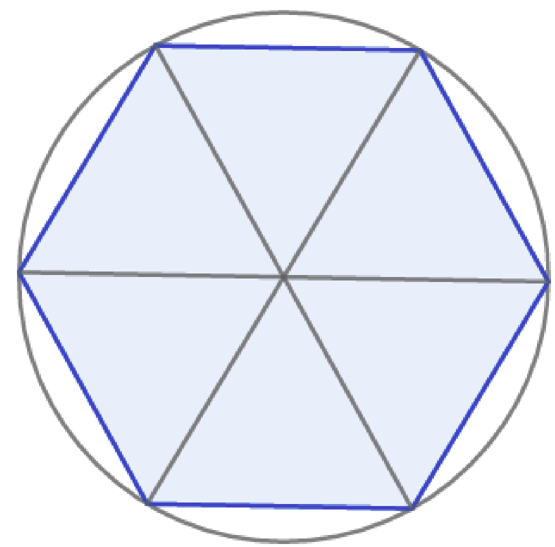
Approximations using triangles

Base $= \dfrac{p}{6}$ Height $= r\cos 30$

Number of triangles $= 6$

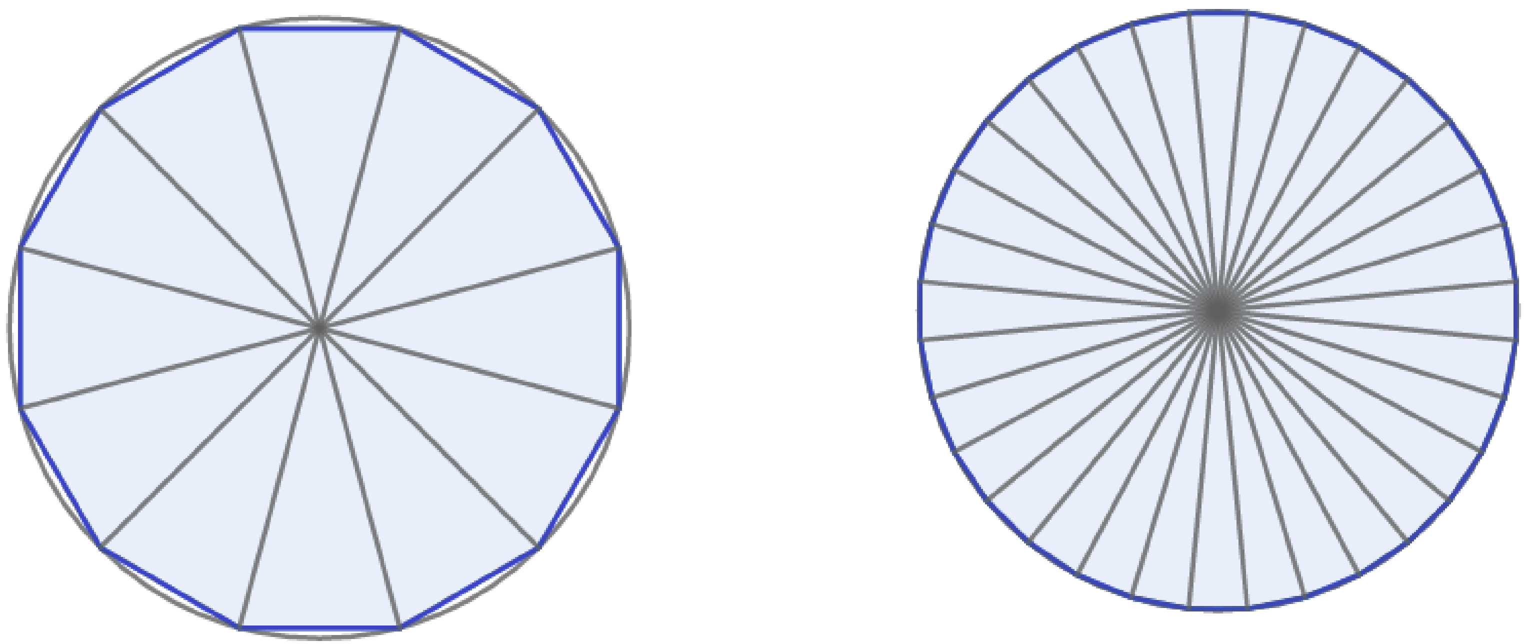
Area $=$

$$6 \cdot \frac{1}{2} \cdot \frac{p}{6} \cdot r\cos 30$$



Approximations using triangles

But we can do better, below is a 12-gon and a 32-gon.



Approximations using triangles

I hate approximations.

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Like really hate them.

Approximations using triangles

So how do we get it to be the exact area ?

Approximations using triangles

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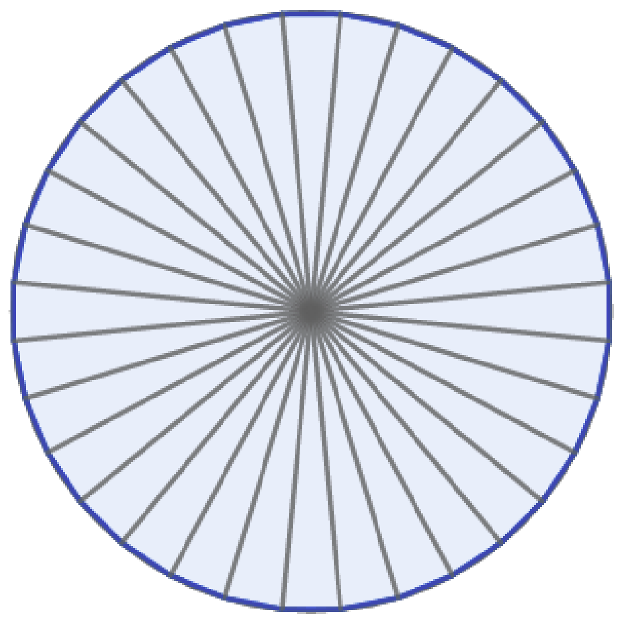
We put in infinitely many triangles!



Approximations using triangles

All the bases must add up to $2r\pi$

The heights become r

Adding all the triangle's area this means :

Area of Circle $=$

Approximations using triangles

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The heights become r

Adding all the triangle's area this means :

$$\text{Area of Circle} = \frac{1}{2} \cdot 2\pi r \cdot r$$

$$= \pi r^2$$

Real World Applications

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A good example is car moving with constant acceleration.

Its distance traveled at a time t is given by:

$$S = ut + \frac{1}{2}at^2$$

Real World Applications

We get $$\frac{ds}{dt} = u + at$$

Which is velocity!

And what is the definition of acceleration?

More quotes than my english essays

Hilbert was told that the student had left the university to become a poet. Hilbert:"I can't say I'm surprised. I never thought he had enough creativity to be a mathematician."

Geometry and Calculus

In class we have done the algebraic methods behind calculus, but this is not the only way to look at this!

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Deeeeeeeeeeeeeeeeeeeeeeeeeeex

No it's delta x

Geometry and Calculus

We are going to try and invoke geometry into our problems.

Geometry and Calculus

We will initially focus on the $y = x^2$ differentiating the rather trivial equation

We must stop and ponder, how can we represent x^2 in geometry?

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Of course! As the area of a square!

Geometry and Calculus

We now move to the nicely animated portion of this presentation.

Infinity is bigger than you think

It is easy to be misled into thinking that there is only one size of infinity.

Infinity is bigger than you think

It is easy to be misled into thinking that there is only one size of infinity.

There is actually infinitely many sizes of infinity.

We will look at the two familiar ones.

To look at these sizes, we need to learn how to compare the sizes of sets.

Consider the sets:

$A = \{1,3,4,5,6,7,8,12583\}$

$B = \{2,5,88,43,32,55,77,88\}$

To look at these sizes, we need to learn how to compare the sizes of sets.

Consider the sets:

A = {1,3,4,5,6,7,8,12583}

B = {2,5,88,43,32,55,77,88}

How can we check if they are the same size <u>without</u> counting them?

This trick will help us compare infinitely large sets and see if they are the same size.

Consider two new sets, the Natural Numbers and the Even Numbers.
N = {0,1,2,3,4,5,6,7,8,9,10...}

E = {0,2,4,6,8,10,12,14,16,18,20}

Which one is larger?

Wow what a result!
Now consider another two sets, the Natural Numbers and the Integers:

$N = \{0,1,2,3,4,5,6,7,8,9,10...\}$

$Z = \{..-4,-3,-2,-1,0,1,2,3,4,5..\}$

Now which one is bigger?

This infinity is in fact the smallest infinity and is denoted $\aleph_0$ (Aleph-null).

We will continue by considering the sets of natural numbers and the rational numbers (fractions)

Unfortunately our trick doesn't work here and we need to think on our feet?

Any ideas?

	1	2	3	4	5	6	7	8	
1	1/1	1/2	1/3	1/4	1/5	1/6	1/7	1/8	⋯
2	2/1	2/2	2/3	2/4	2/5	2/6	2/7	2/8	⋯
3	3/1	3/2	3/3	3/4	3/5	3/6	3/7	3/8	⋯
4	4/1	4/2	4/3	4/4	4/5	4/6	4/7	4/8	⋯
5	5/1	5/2	5/3	5/4	5/5	5/6	5/7	5/8	⋯
6	6/1	6/2	6/3	6/4	6/5	6/6	6/7	6/8	⋯
7	7/1	7/2	7/3	7/4	7/5	7/6	7/7	7/8	⋯
8	8/1	8/2	8/3	8/4	8/5	8/6	8/7	8/8	⋯
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋱

Now I wasn't just comparing sets for the craic, I was leading up to this comparison:

Finally consider the following sets, The Natural Numbers, and The Real Numbers.

Which one is larger ?

We prove by the following:

We assign each natural number a real number friend.

$$1 = 0.10101010101010101010$$
$$2 = 0.01010101010101010101$$
$$3 = 0.10001010101010101010$$
$$4 = 0.11212121212121212122$$
$$5 = 0.12121245543534535535$$
$$6 = 0.21125522123131312434$$
$$\vdots$$

We prove by the following:

Now we construct a number that is different from every other number in the list.

$$1 = 0.10101010101010101010$$
$$2 = 0.01010101010101010101$$
$$3 = 0.10001010101010101010$$
$$4 = 0.11212121212121212122$$
$$5 = 0.12121245543534535535$$
$$6 = 0.21125522123131312434$$
$$\vdots$$
$$\mathrm{R} =$$

This infinity is called $\aleph_1$(Aleph One)

Now that problem is out of the way we can play around with infinty safely and without risk of mistake.....

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Now that problem is out of the way we can play around with infinty safely and without risk of mistake.....

Or maybe not..

Infinity Trolling

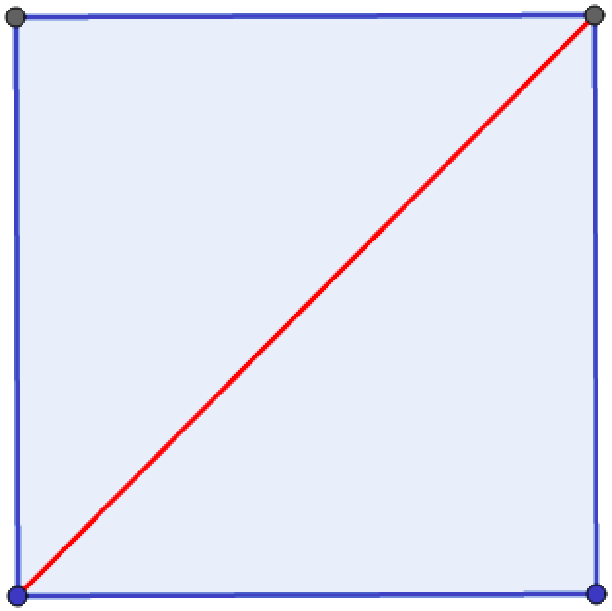
While infinitely small and infinitely big numbers can be exciting and useful, there can be some less than honest uses of these quantities.

Infinity Trolling

We start with a problem in geometry: The infinite staircase problem appears to show that $\sqrt{2} = 2$. Hopefully you can see that such a result is mildy problematic.

Infinity Trolling

Consider a square with side length 1.



What is the length of the red line?

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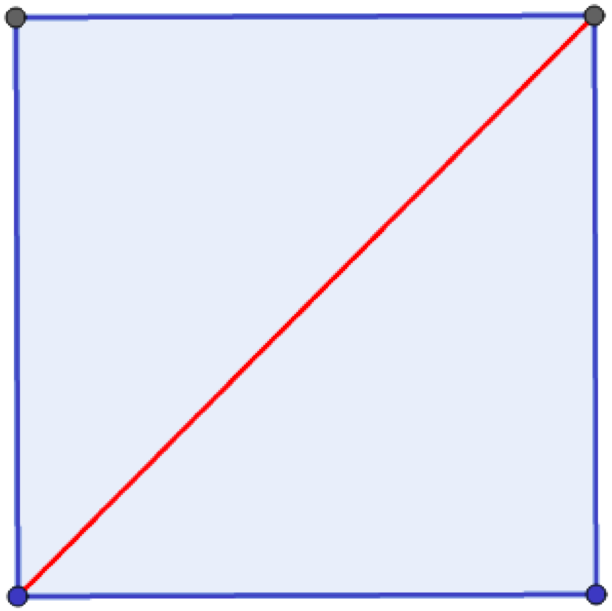
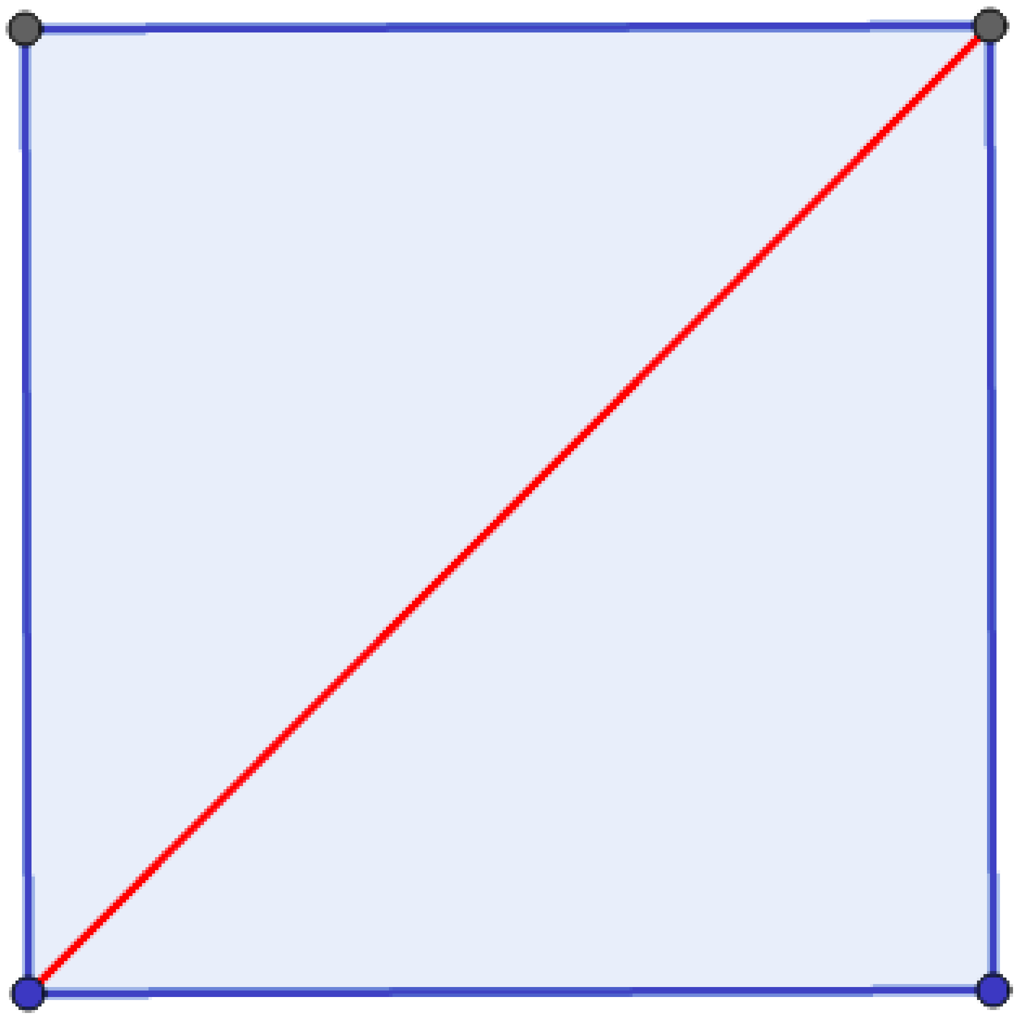


What is the length of the red line?

Hint: it's $\sqrt{2}$

Infinity Trolling



Infinity Trolling

We continue with a problem in probability:

Suppose you throw a dart at a circular dartboard. There is a 100% chance the dart hits the dartboard. What is the probability that the dart hits a given point?

Infinity Trolling

We can use the probability formula:

$$P(x) = \frac{\text{Number of desirable outcomes}}{\text{Number of possible outcomes}}$$

What are these equal to?

Conclusion

Now would be a good time to ask questions about anything.

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Last chance before I go back to being socialy inept.

Conclusion

Now would be a good time to ask questions about anything.

Hopefully you've seen the fun one can have bringing geometry into everything.

All material used on board available at:
www.stuffconradsaid.tk

You can also leave feedback about this presentation.

This final slide is trivial and left as an excercise to the viewer