Paired Samples t-test

Fikile Nkwanyana
Discipline of Public Health Medicine
25 July 2025

UKZN INSPIRING GREATNESS

Statistical tests for continuous data

Number of groups	**Dependent / Independent**	**Statistical test**
One	N/A	One- Sample test
Two	**Dependent**	**Paired samples t-test**
Two	Independent	Independent samples t-test
Three	Independent	One-way Analysis of Variance

Assume normal distribution

Steps in Hypothesis Testing

- ✓ Establish H_0 and H_a
- ✓ Set the significance level α (usually 0.05)
- ✓ Choose the appropriate statistical test
- ✓ Calculate the appropriate test statistic
- ✓ Read the relevant critical value from a stats table
- ✓ Compare the calculated statistics and the critical value.

Steps in Hypothesis Testing (cont)

- Make a decision regarding $H_{0:}$ If the calculated value is greater that the critical value, we reject H_0. If not we fail to reject the $H_{0.}$

- When the null hypothesis is rejected, the outcome is said to be "statistically significant"; when the null hypothesis is not rejected then the outcome is said be "not statistically significant."

- Draw a conclusion regarding your original research hypothesis based on your decision above.

Learning outcomes

- When to use paired samples t-test
- Assumptions
- Null and alternative hypotheses
- Calculate relevant statistics
- Interpret results

When to use
Paired samples t-test

Appropriate for comparing difference in means of **two related** groups.

- ✓ Useful in matched studies
- ✓ Useful in paired designs

Related groups also known as dependent groups or paired groups.

One person CAN be in both groups.

In matched studies – selection of participants in one group is influenced by characteristics of participants on the other group

Assumptions

1. The observations (*differences between two sets of values*) are independent of one another

2. The observations are continuous

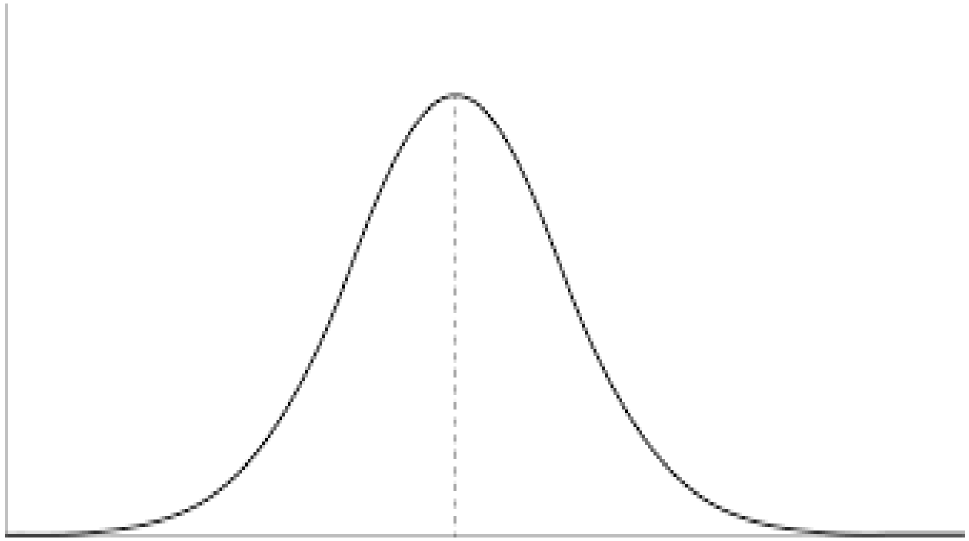
3. The observations are normally distributed

Assumptions

- Independence
  - Cannot be tested
  - Is assumed if the data collection process is random

Assumptions

- Normal distribution of a observations (differences between two sets of values)



mean = median = mode

Assumptions

Testing for Normality

- Shapiro-Wilks test and Kolmogorov-Smirnov test

(significant p-values imply that the data is NOT normally distributed)

Assumptions

If data are not normally distributed

- Transform data

- Use a non-parametric equivalent test - **Wilcoxon signed rank test**

(not for HMA exercises)

Null hypothesis

- Null hypothesis is a hypothesis of no difference, no association or no effect

Null hypothesis : the population mean difference is equal to zero

$$H_0\colon \boldsymbol{\delta} = 0$$

Alternative hypothesis

- Alternative hypothesis is a hypothesis of difference, association or effect

Alternative hypothesis : the population mean difference not equal to zero

$$H_a\text{: } \boldsymbol{\delta} \neq 0$$

Relevant statistics

- t-test

$$t_{n-1} = \frac{\bar{d} - \delta}{s_d/\sqrt{n}}$$

Paired Samples T-test Example

- A study was conducted to assess the effect of oral contraception on systolic blood pressure.
- Ten women were enrolled in the study.
- For each woman, systolic blood pressure was measured before taking the pill and after taking the pill.

Statistical Tests - Examples

✓Establish H_0 and H_a

$$H_0: \quad \delta = 0$$

$$H_a: \quad \delta \neq 0$$

✓Set the significance level α (usually 0.05)

Statistical Tests - Examples

✓ Choose the appropriate statistical test

Type of variable (systolic BP) – continuous
Distribution of systolic BP– normal
Number of groups – two
Independent or dependent – dependent

Appropriate test statistic: Paired samples t test

Statistical Tests - Examples

- Calculate the appropriate test statistic

$$t = \frac{\bar{d} - \delta}{s_d / \sqrt{n}}$$

Paired Samples T-test Example

before	after
115	128
112	115
107	106
119	128
115	122
138	145
126	132
105	109
104	102
115	117

Paired Samples T-test Example

before	after	difference
115	128	13
112	115	3
107	106	-1
119	128	9
115	122	7
138	145	7
126	132	6
105	109	4
104	102	-2
115	117	2

Paired Samples T-test Example

- $mean_{before} = 115{,}6\ mmHg$
- $mean_{after} = 120{,}4\ mmHg$
- $\bar{d} = 4{,}8$
- $s_d = 4{,}56$

Paired Samples T-test Example

$$t = \frac{\bar{d} - \delta}{s_d/\sqrt{n}}$$

$$t = \frac{4{,}8 - 0}{4{,}56/\sqrt{10}}$$

$$t = \frac{4{,}8}{4{,}56/_{3{,}16}}$$

$$= \frac{4{,}8}{1{,}44} = 3{,}33$$

- Check the critical value associated with the test statistics in the table

Read critical value of t from the stats tables for t-statistics.

TABLE 3—Percentage Points of Student's t Distribution

Degrees of Freedom	.25	.20	.15	.10	.05	.025	.01	.005	.0005
			Level of Significance for a One-Tailed Test						
(n-1)	.50	.40	.30	.20	.10	.05	.02	.01	.001
			Level of Significance for a Two-Tailed Test						
1	1.000	1.376	1.963	3.078	6.314	12[illegible]06	31.821	63.657	636.619
2	.816	1.061	1.386	1.886	2.920	4[illegible]03	6.965	9.925	31.598
3	.765	.978	1.250	1.638	2.353	3[illegible]82	4.541	5.841	12.924
4	.741	.941	1.190	1.533	2.132	2[illegible]76	3.747	4.604	8.610
5	.727	.920	1.156	1.476	2.015	2[illegible]71	3.365	4.032	6.869
6	.718	.906	1.134	1.440	1.943	2[illegible]47	3.143	3.707	5.959
7	.711	.896	1.119	1.415	1.895	2[illegible]65	2.998	3.499	5.408
8	.706	.889	1.108	1.397	1.860	[illegible]	2.896	3.355	5.041
9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.262	2.821	3.250	4.781
10	.700	.879	1.093	1.372	1.812	[illegible]	2.764	3.169	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.718	3.106	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.681	3.055	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.650	3.012	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.624	2.977	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.602	2.947	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.583	2.921	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.567	2.898	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.552	2.878	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.539	2.861	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.528	2.845	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.518	2.831	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.508	2.819	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.500	2.807	3.767
24	.685	.857	1.059	1.318	1.711	2.064	2.492	2.797	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.485	2.787	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.479	2.779	3.70
27	.684	.855	1.057	1.314	1.703	2.052	2.473	2.771	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.467	2.763	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.462	2.756	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.457	2.750	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.423	2.704	3.551
60	.679	.848	1.046	1.296	1.671	2.000	2.390	2.660	3.460
120	.677	.845	1.041	1.289	1.658	1.980	2.358	2.617	3.373
∞	.674	.842	1.036	1.282	1.645	1.960	2.326	2.576	3.291

$$t_{9;0,05} = 2{,}26$$

Since t_{calc} =3,33 is greater than 2,26;

Reject the null hypothesis *(the mean difference of systolic blood pressure before and after taking a contraceptive pill is equal to zero)* if the calculated value of t is greater that the critical value of t.

Conclusion

- The mean difference of systolic blood pressure before and after taking a contraceptive pill is not equal to zero

Therefore, systolic blood pressure changes after taking a contraceptive pill.

Estimate range of a p-value

- Check where the calculated value of t lie at the relevant degrees of freedom and level of significance

t_{calc} =3,33

- Check where 3,33 lie at 9 degrees of freedom

TABLE 3—Percentage Points of Student's t Distribution

Degrees of Freedom	Level of Significance for a One-Tailed Test								
	.25	.20	.15	.10	.05	.025	.01	.005	.0005
(n-1)	Level of Significance for a Two-Tailed Test								
	.50	.40	.30	.20	.10	.05	.02	.01	.001
1	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657	636.619
2	.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	31.598
3	.765	.978	1.250	1.638	2.353	3.182	4.541	5.841	12.924
4	.741	.941	1.190	1.533	2.132	2.776	3.747	4.604	8.610
5	.727	.920	1.156	1.476	2.015	2.571	3.365	4.032	6.869
6	.718	.906	1.134	1.440	1.943	2.447	3.143	3.707	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.998	3.499	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.896	3.355	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.821	3.250	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.764	3.169	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.718	3.106	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.681	3.055	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.650	3.012	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.624	2.977	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.602	2.947	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.583	2.921	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.567	2.898	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.552	2.878	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.539	2.861	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.528	2.845	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.518	2.831	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.508	2.819	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.500	2.807	3.767
24	.685	.857	1.059	1.318	1.711	2.064	2.492	2.797	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.485	2.787	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.479	2.779	3.70
27	.684	.855	1.057	1.314	1.703	2.052	2.473	2.771	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.467	2.763	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.462	2.756	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.457	2.750	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.423	2.704	3.551
60	.679	.848	1.046	1.296	1.671	2.000	2.390	2.660	3.460
120	.677	.845	1.041	1.289	1.658	1.980	2.358	2.617	3.373
∞	.674	.842	1.036	1.282	1.645	1.960	2.326	2.576	3.291

- At 9 degrees of freedom; 3,33 lie between 0,01 and 0,001.

$$0{,}001 < p < 0{,}01$$

Questions ?