Engineering Physics I

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1 Significant Figures

When multiplying or dividing, the result is as precise as the least precise input to the number of digits.
Example: $54.3 * 6.8991 = 374.62113$, truncated to 374 or rounded to 375.

When adding or subtracting, the result is as precise as the least precise input to the nuber of digits post decimal place.
Example: $10.65 + 3.0 = 13.65$, truncated to 13.6 or rounded to 13.7.

As a general rule (from the professor), round up down to the nearset even number, as always rounding up will accumulate more error.

2 Variables of Movement and Position

1. Position: Location in space with respect to another object or coordinate system.
   x, y, z
2. Displacement: Difference in position at two different times.
   $\Delta x, \Delta y, \Delta z, \Delta x = x_2 - X_1$
3. Average Velocity: Displacement divided by time.
   $v_{avg}, v_{avg} = \frac{\Delta x}{\Delta t}$
4. Speed: Total distance divided by time.
   $s, s \equiv \frac{d}{t}$
5. Instantanious Velocity: Velocity measured at a single time.
   $v, v = \lim_{t \to a} \frac{\Delta x}{\Delta t} = \lim_{t \to a} \frac{x_2 - x_1}{t_2 - t_1}$

3 Motion at Constant Velocity

$x = x_0 + vt$ The following computes a new position of x according to an object's initial position (x_0), velocity (v) and the given time passed (t). The equation is a slope-intercept formula.

4 Velocity at Constant Acceleration

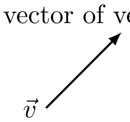
$v = v_0 + at$ The following computes a new velocity of v according to an object's initial velocity (v_0), acceleration (a) and the given time passed (t). The equation is a slope-intercept formula.

The following equations can be combined as a system to calculate constant acceleration with velocity, position and time. When given a final velocity v_f and an initial velocity v_0 one can deduce an average velocity (v or v_{avg})
$v_f = v_0 + at$ and $x = x_0 + vt \to x = x_0 + (\frac{v_0+v_f}{2})t \to x = x_0 + \frac{1}{2}(v_0 + v_0 + at)t \to x = x_0 + v_0 t + \frac{1}{2}at^2$ after simplification.

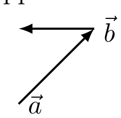
5 Vectors and Vector Math

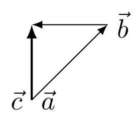
- Scalar $\equiv$ number, units $\equiv$ magnitude
- Vector $\equiv$ number, units, direction $\equiv$ magnitude, direction

Vectors allow for easier representation of position, velocity, and acceleration. A vector of velocity looks like $\vec{v}$ and can be drawn to look like the following.



Vector addition takes two vectors and aligns them tail to tip. For example, suppose the following vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ such that $\vec{a} + \vec{b} = \vec{c}$.





Vector addition is communative. $\vec{a} + \vec{b} = \vec{b} + \vec{a}$ Vector addition is associativ. $\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$

Unit Vectors are vectors of magnitude of 1 that extend in only the x, y, or z direction alone a 3D cartessian system. The vectors are $\hat{i}$, $\hat{j}$ and $\hat{k}$ respectively. Notation for vectors, when expanded, looks like $\vec{A} = (A_x\hat{i} - A_y\hat{j} + A_z\hat{k})$ which could be $= (12\hat{i} - 7\hat{j} + 14\hat{k})$. Vector addition becomes standard addition, and multplying vectors by scalars only needs to be distributed.

6 Circular Motion

Angular vectors use radius unit vector ($\hat{r}$) and an inclination unit vector ($\hat{\theta}$). Position around a circle can be determined if given an average velocity and time.
Let $T = \frac{2\pi r}{v}$ where r is the radius and v is the velocity.
Position: $\vec{r}(t) = r\cos\frac{2\pi r}{T}\hat{\imath} + r\sin\frac{2\pi r}{T}\hat{\jmath}$
Velocity (derivative of position): $\vec{v}(t) = -r\frac{2\pi}{T}\sin\frac{2\pi r}{T}\hat{\imath} + r\frac{2\pi}{T}\cos\frac{2\pi r}{T}\hat{\jmath}$
Acceleration (derivative of velocity): $\vec{a}(t) = -r\frac{4\pi^2}{T^2}\cos\frac{2\pi r}{T}\hat{\imath} - r\frac{4\pi^2}{T^2}\sin\frac{2\pi r}{T}\hat{\jmath}$
Or
$\vec{a}_c = \frac{v^2}{r}\hat{r}$
Angular velocity can also be defined in terms of angular momentum (ω): $\vec{a}_c = r\omega^2$

7 Relativity

Motion and position are dependant on a specific inertial reference frame - some body with no apparent acceleration. It was believed by Aristotle that the Earth was the reference frame for everything, i.e. it was the center of the universe. A reference frame is represented mathematically as the origin.
All motion and laws of motion must work and provide the same result across refernce frames. Using this fact, you can calculate the time it takes for an arrow to hit a wall when the reference frame is the archer, and come out with the same result when the wall hits the arrow when the arrow is the reference frame.

8 Forces

Forces can be given / found a variety of different ways. It is important to note first how these forces act on bodies. The intuitive definition of a force is that they are "pushes" or "pulls" that change the motion and position of bodies. Each force has a scalar and vector component, mass and a vector acceleration.

8.1 Variables used in Forces

- m: the mass of an object in kilograms.
- $\vec{g}$: the gravitational constant.
- μ_s: coeffecient of static friction. (Between 0 and 1)
- μ_k: coeffecient of kinetic friction. (Between 0 and 1)
- C: drag coeffecient
- ρ: fluid density
- A: cross sectional area

8.2 Common Forces

The final, or net force, on an object is given by the summation of all relevant forces. $F_{net} = \sum F_n$

- External Forces: any force that is added to a system but did not originate there. $\vec{F}_n$ where n is arbitrary to help define and organize many forces. Thus, n can be a number, name, or replaced with one of the following internal forces.

- Weight: Force given from gravity. Points towards down in the vertical dimension. $\vec{w} = m\vec{g}$

- Normal: The observered force that prevents boxes from falling through tables. The x component of this vector is typically to $|\vec{g}|$. The normal force is always perpendicular to the surface an object sits on. $\vec{N} = -m\vec{g}$

- Tension: A pulling force that often conteracts gravity. $\vec{T} = \vec{w} = m\vec{g}$

- Friction: Other forces on an object must cause the static frition inequality to be false in order for an object to be displaced. Once this threshold is met, friction force travels parallel to the surface and opposite of motion.
  Static: given by $\vec{f_s} \leq \mu_s \vec{N}$. This implies that static friction cannot exceed the maximum. $\vec{f_s}(max) = \mu_s \vec{N}$.
  Kinetic: given by $\vec{f_k} = \mu_k \vec{N}$.

- Centripetal: the "center-seeking" force. $F_c = m\frac{\vec{v}^2}{r} = mr\vec{\omega}^2$

- Drag: a force proportional to speed ($F_D \propto v^2$) that is gained when fluids reduce speed (like falling through air). $F_D = \frac{1}{2}C\rho Av^2$

9 Terminal Velocity

$F_{net} = mg - F_D = ma = 0$ thus $mg = F_D$. Using the equation for drag, we get $mg = \frac{1}{2}C\rho Av^2$. Solving for terminal velocity gives us $v_T = \sqrt{\frac{2mg}{\rho CA}}$.

10 Equations for Centripetal Acceleration

- $\omega = \frac{v}{r}$

- $\omega = 2\pi f$

- $a_c = \frac{v^2}{r} = \omega^2 r$

11 Equations with Work and Force

- $F = \frac{E_K}{d}$ where E_k is kinetic energy and d is distance.
- $F = kx$ where x is the displacement from a spring and k is a spring constant.
- $W = \frac{1}{2}kx^2$
- $\Delta E_K = F * \Delta x$