

# Analytical Geometry and Calculus II

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## Review of Theorems for Limits

Let  $a$  and  $c$  be any number, if  $F = \lim_{x \rightarrow a} f(x)$  and  $G = \lim_{x \rightarrow a} g(x)$  then

1.  $\lim_{x \rightarrow a} (f(x) + g(x)) = F + G$
2.  $\lim_{x \rightarrow a} (f(x) - g(x)) = F - G$
3.  $\lim_{x \rightarrow a} (c * f(x)) = c * F$
4.  $\lim_{x \rightarrow a} \left( \frac{f(x)}{g(x)} \right) = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$  except when  $G = 0$
5.  $\lim_{x \rightarrow a} (f(x))^c = F^c$

## Limits Involving Infinity while $c \neq 0$

These are templates, where  $x$  is either taken to  $\infty$  or  $0$ .

1.  $c * (\pm\infty) = \pm\infty$  Example:  $\lim_{x \rightarrow \infty} 5x = \infty$
2.  $\frac{c}{\pm\infty} = 0$  Example:  $\lim_{x \rightarrow \infty} \frac{5}{x} = 0$
3.  $\frac{c}{0} = \pm\infty$  Example:  $\lim_{x \rightarrow 0} \frac{5}{x} = \infty$
4.  $\frac{\pm\infty}{c} = \pm\infty$

## Limits that can't be dealt with

- $\frac{0}{0}$
- $\frac{\infty}{\infty}$
- $\infty - \infty$
- $0 * \infty$

## Limits of Rational Functions

Theorem: Given a rational function  $\frac{f(x)}{g(x)}$ , the following is be true. Let  $d$  represent the degree of  $f(x)$  and  $e$  represent the degree of  $g(x)$ .

1. If  $d > e$  then  $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = \pm\infty$
2. If  $d < e$  then  $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = 0$
3. If  $d = e$  then  $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = \frac{a}{b}$  where  $a$  is the leading term of  $f(x)$  and  $b$  is the leading term of  $g(x)$

Example: To evaluate  $\lim_{x \rightarrow \infty} \frac{10x^3 - 3x^2 + 8}{\sqrt{25x^6 + x^4 + 2}}$

1. Reduce all terms by  $x$  of the leading coefficient's degree.
2. Then take the limit of each term.  $\frac{10 - 3x^{-1} + 8x^{-3}}{\sqrt{25 + x^{-2} + 2x^{-6}}}$
3. Simplify.  $\frac{10 - 0 + 0}{\sqrt{25 + 0 + 0}}$
4.  $\frac{10}{\sqrt{25}} = \frac{10}{5} = 2$

## Inverse Trig Functions

Definition:  $y = \sin^{-1}x$  is the value of  $y$  such that  $x = \sin y$ .

Domain:  $-1 \leq x \leq 1$

Range:  $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

Definition:  $y = \cos^{-1}x$  is the value of  $y$  such that  $x = \cos y$ .

Domain:  $-1 \leq x \leq 1$

Range:  $0 \leq y \leq \pi$

## Inverse Trig Identities

- $\sin(\sin^{-1}x) = x$
- $\cos(\cos^{-1}x) = x$
- $\sin^{-1}(\sin x) = x$  only if  $x$  is in range of  $\sin^{-1}$
- $\cos^{-1}(\cos x) = x$  only if  $x$  is in range of  $\cos^{-1}$

Example:  $\sin^{-1}(\sin \pi) = \sin^{-1}(0) = 0 \neq \pi$

Example:  $\cos(\sin^{-1}x)$

1. Let  $y = \sin^{-1}x$  so that  $x = \sin y$  and  $\cos(\sin^{-1}x) = \cos y$
2. Recall that  $\sin = \frac{\text{opposite}}{\text{hypotenuse}}$
3. Let  $\text{hypotenuse} = 1$  and  $\text{opposite} = b$  where  $b$  has yet to be determined.

4. Recall that  $\cos y = \frac{\text{adjacent}}{\text{hypotenuse}}$
5.  $\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{1}$ , and therefor  $\cos y = b$
6. Use the Pythagorean Theorem to solve:  $x^2 + b^2 = 1^2$
7.  $b^2 = 1 - x^2$
8.  $b = \sqrt{1 - x^2}$
9. Therefor  $\cos(\sin^{-1}x) = \sqrt{1 - x^2}$