Largest Prime Factor

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Any natural number $n > 1$ can be expressed as

$$n = p_1^{e_1} \times p_2^{e_2} \times \ldots \times p_k^{e_k}$$

where $p_1, p_2, \ldots, p_k$ are distinct prime numbers and $e_1, e_2, \ldots, e_k$ are positive integers. This is known as prime factorization. The largest prime factor of n will be the maximum of $p_1, p_2, \ldots, p_k$.

Complexity Analysis

Generating the prime factorization and finding the maximum prime can be done in constant space as the primes do not need to be stored. Giving a space complexity of:

$$O(n) \text{ space}$$

When generating the prime factorization of a natural number, n, it could be required to check all prime factors up to $\sqrt{n}$, and finding the maximum among these prime factors is done in constant time. Giving an overall time complexity of:

$$O(\sqrt{n}) \text{ time}$$