

Sum Square Difference

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The sum of the first n natural numbers squared can be written as:

$$\left(\sum_{i=1}^{i \leq n} i\right)^2 = \left\lfloor \frac{n^2 + n}{2} \right\rfloor^2$$

The sum of the squares of the first n natural numbers can be written as:

$$\sum_{i=1}^{i \leq n} i^2 = \left\lfloor \frac{n^2 + n}{2} \right\rfloor \cdot \left\lfloor \frac{2n + 1}{3} \right\rfloor$$

Hence the difference between the the sum squared and the sum of squares is equal to:

$$\left\lfloor \frac{n^2 + n}{2} \right\rfloor^2 - \left\lfloor \frac{n^2 + n}{2} \right\rfloor \cdot \left\lfloor \frac{2n + 1}{3} \right\rfloor$$

Complexity Analysis

The difference is caluclated in both constant space and constant time.