Problem 1 : Multiples of 3 or 5

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The number of multiples of Y that are less than or equal to X is given as

$$\left\lfloor \frac{X}{Y} \right\rfloor$$

Therefore the number of multiples of 3 less than 1000 is equal to $\left\lfloor \frac{1000-1}{3} \right\rfloor$, and the number of multiples of 5 less than 1000 is equal to $\left\lfloor \frac{1000-1}{3} \right\rfloor$. However the sum of $\left\lfloor \frac{1000-1}{3} \right\rfloor$ and $\left\lfloor \frac{1000-1}{5} \right\rfloor$ will count numbers which are multiples of both 3 and 5 twice. Every number which is a multiple of both 3 and 5 is a multiple of the lowest common multiple of 3 and 5. Since $LCM(3,5) = 15$, the number of multiples being counted twice is equal to $\left\lfloor \frac{1000-1}{15} \right\rfloor$.

Giving a final answer of:

$$\left\lfloor \frac{1000-1}{3} \right\rfloor + \left\lfloor \frac{1000-1}{5} \right\rfloor - \left\lfloor \frac{1000-1}{15} \right\rfloor$$

Complexity Analysis

This runs in constant time and constant space.