

# MECH 570C Code Project 1

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## 1 main

Data-rigid-cylV3 contains all BCs and coordinates information (crd). conn is unclear.

Crđ has number of elements on the ? in row (26144) (also the ndof, degree of freedom) and the first column is the point index.

$C = \text{unique}(A)$  returns the same data as in A, but with no repetitions. C is in sorted order.

Sol.u has number of elements as row and 2 as column, 1 is the x direction and 2 is the y direction.

## 2 N-S

### Function Definition

$\rho u_t + \rho u \cdot \text{div} u - \text{div}(\mu \nabla u) + \nabla p = f$  in  $\Omega$ ,  $\text{div} u = 0$  in  $\Omega$ , Dirichlet boundary condition  $u = g_D$  on  $\Gamma_D$ , Neumann boundary condition  $\frac{du}{dn} - np = g_N$  on  $\Gamma_N$ .

- **Function Name:** navierStokes
- **Inputs:**
  - solver, fluid, pmc: Structures containing various solver settings, fluid properties, and numerical method parameters.
  - Sol: Structure holding the solution vectors.
  - cnn, crd: Connectivity and coordinate matrices for the mesh.
  - elemType, ndof, nen, nElem, BCCyl: Parameters defining the type of elements used, degrees of freedom, number of nodes per element, total number of elements, and boundary condition information.

## Quadrature Rules

- Sets up quadrature points ( $\mathbf{gP}$ ) and weights ( $\mathbf{gW}$ ) based on the element type (triangular or quadrilateral). These are used for numerical integration.
- Defines shape functions ( $\mathbf{N}$ ) and their derivatives ( $\mathbf{Nx}$ ,  $\mathbf{Ny}$ ) for the finite elements.

## Boundary Conditions

- Applies Dirichlet and Neumann boundary conditions to the solution vector `Sol.u`.

## Interpolation for Alpha Values

- Interpolates values for the generalized-alpha method, a numerical technique for time integration in transient problems.

## Navier-Stokes Equations

- Prepares variables (`xxf`, `yyf`, `ux`, `uy`, etc.) for assembling the finite element matrices. These include coordinates, velocities, pressure, and additional variables.

## Assembly of Galerkin and Petrov-Galerkin Terms

- Calls functions to form the left-hand side (LHS) and right-hand side (RHS) of the Navier-Stokes equations. This involves complex operations based on the finite element method.
- The Galerkin method is used for discretizing the problem, while Petrov-Galerkin is an enhanced approach for stability and accuracy.

## Solving the Linear System

- Determines the free nodes not constrained by boundary conditions.
- Solves the linear system for the unknowns (`Increment`) using the assembled LHS and RHS.

## Update and Output

- Updates the solution vectors (`Sol.u`, `Sol.uDot`, `Sol.p`) with the new increments.
- Calculates a norm (`NSnormIndicator`) to indicate the convergence or error of the current iteration.
- Outputs the updated solution structure `Sol` and the convergence/error indicator.

## Summary of Data Flow

1. **Input Processing:** Takes in initial conditions, mesh information, and solver parameters.
2. **Setup:** Establishes quadrature rules and boundary conditions.
3. **Equation Assembly:** Forms the Navier-Stokes equations using finite element discretization.
4. **Solution Update:** Solves for increments and updates the solution.
5. **Output:** Returns the updated solution and a convergence/error metric.

# 3 Integrated Output

## Function Definition

- **Function Name:** `IntegratedOutput`
- **Inputs:**
  - `Sol`: A structure containing the solution vectors (velocity, pressure, etc.).
  - `crd`: The coordinates of the mesh nodes.
  - `BCCyl`: Boundary condition data.
  - `fluid`: A structure containing fluid properties.
  - `cnn`: **Connectivity matrix for the mesh elements.**

## Initialization

- Sets the number of element nodes (**n<sub>en</sub>**) for a 2D element (4 for a quadrilateral element).
- Swaps the columns of **BCCy1** for further processing.
- Determines the number of elements (**nElem**) and degrees of freedom (**ndof**) involved in the boundary condition.

## Boundary Layer Elements

- Identifies elements corresponding to the first layer of the boundary using **cnn** and **BCCy1**.
- Reorders the element points for reduced integration.

## Quadrature Integration Setup

- Defines Gauss points (**gP**) and weights (**gW**) for numerical integration.
- Sets up shape functions (**N**) and their derivatives (**N<sub>x</sub>**, **N<sub>y</sub>**).

## Localizing Data

- Extracts local coordinates (**xxf**, **yyf**), velocities (**ux**, **uy**), and pressure (**pres**) for each element.

## Integration Process

- Loops over quadrature points to:
  - Calculate the Jacobian matrix **J** for coordinate transformation.
  - Compute the volume and normal vectors for each element.
  - Evaluate pressure and velocity gradients (**locgradUx**, **locgradUy**, **locgradVx**, **locgradVy**).
  - Compute the length/area (**A0**) of the line/surface integral.

## Calculation of Forces

- The loop sums up the contributions from all quadrature points to calculate the integrated force over the element surface.
- The force components (X and Y directions) are to be computed within the loop (currently not implemented in the provided code).

## Output

- Returns the total length of the integrated area (**Length**) and the force components (**Force**).