

MECH 503 Final Exam

by

Jincong Li

M.Eng, The University of British Columbia, 2024

April 19, 2024

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1 Question 1

$$BC_1 = \sigma_{xy}(x, y = \pm \frac{h}{2}) = 0$$

$$BC_2 = \sigma_y(x, y = \frac{h}{2}) = 0$$

$$BC_3 = \sigma_y(x, y = \frac{-h}{2}) = -q$$

$$BC_4 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x(x = \pm L, y) dy = 0$$

$$BC_5 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x(x = \pm L, y) y dy = 0$$

$$BC_6 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{xy}(x = \pm L, y) dy = \pm Lq$$

One could observe the boundary conditions are satisfied in classical and weak sense. For the normal stress in x-direction:

$$\sigma_x(x = \pm L, y) = \pm \frac{qy(3h^2 - 20y^2)}{5h^3}$$

which is symmetric about the y-axis, thus, the moment is balanced. The distribution of shear

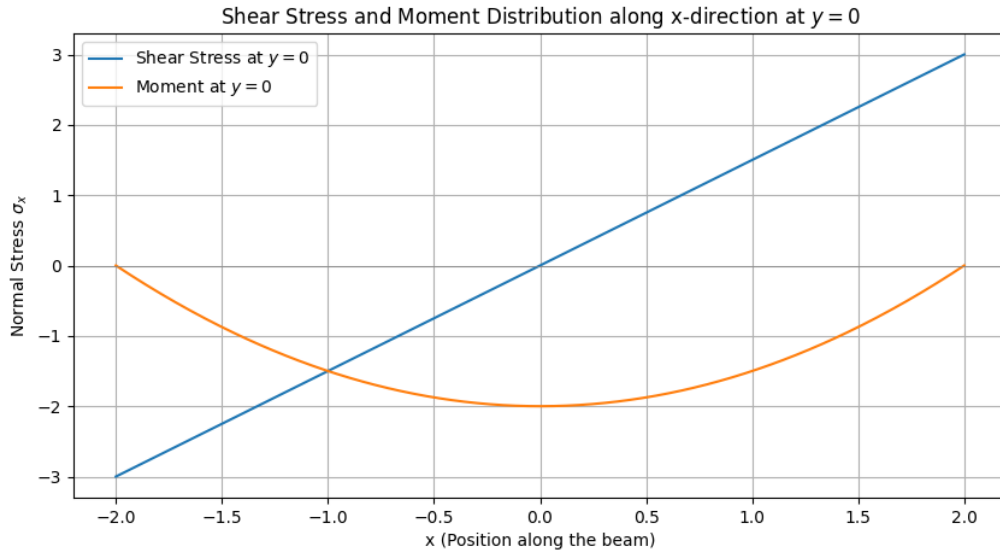


Figure 1: Shear Stress and Moment Distribution along x-direction at $y = 0$

stress and moment is shown in figure 1, which agrees with the theory.

The Python code is provided in the Appendix file.

2 Question 2

Part 1

$$\begin{aligned}\sigma_{\theta\theta}(r = a) &= 0 \\ \sigma_{r\theta}(r = a) &= 0\end{aligned}$$

Part 2

$$\begin{aligned}\sigma_{rr}\left(\frac{r}{a} = \infty\right) &= \sigma_0 \sin^2(\theta) \\ \sigma_{r\theta}\left(\frac{r}{a} = \infty\right) &= -\frac{\sigma_0 \sin(2\theta)}{2} \\ \sigma_{\theta\theta}\left(\frac{r}{a} = \infty\right) &= \sigma_0 \sin^2(\theta)\end{aligned}$$

Part 3

$$\begin{aligned}\sigma_{xx}\left(\frac{r}{a} = \infty\right) &= -\sigma_0 \sin^2(\theta) \cos(2\theta) \\ \sigma_{yy}\left(\frac{r}{a} = \infty\right) &= -\frac{\sigma_0 \sin(4\theta)}{4} \\ \sigma_{xy}\left(\frac{r}{a} = \infty\right) &= -\sigma_0 \sin^2(\theta) \cos(2\theta)\end{aligned}$$

Part 4

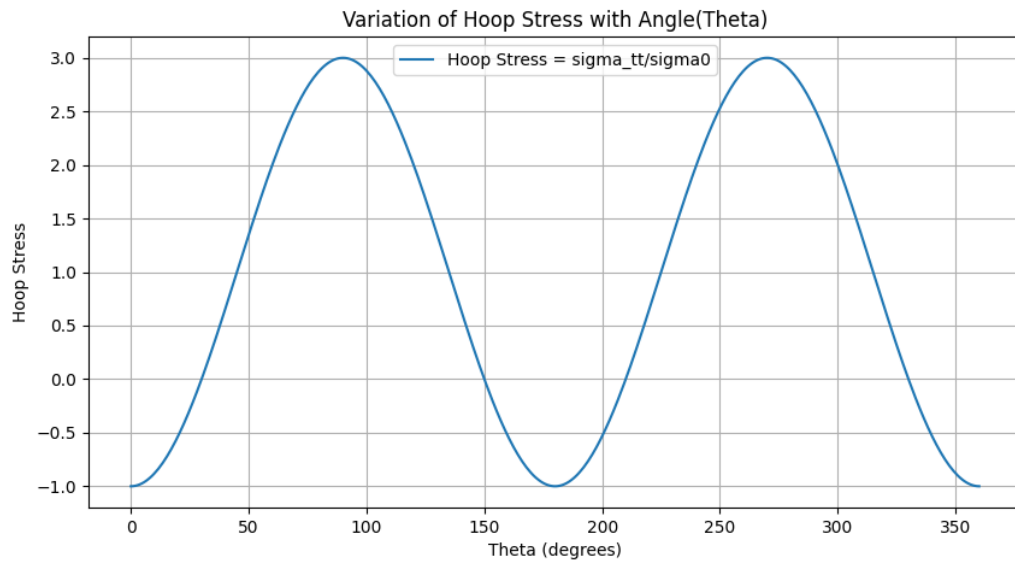


Figure 2: Hoop Stress vs. Angle(θ)

The maximum value of hoop stress is 3, which occurs at $\theta = 90.0$ degrees.

2.1 Part 5

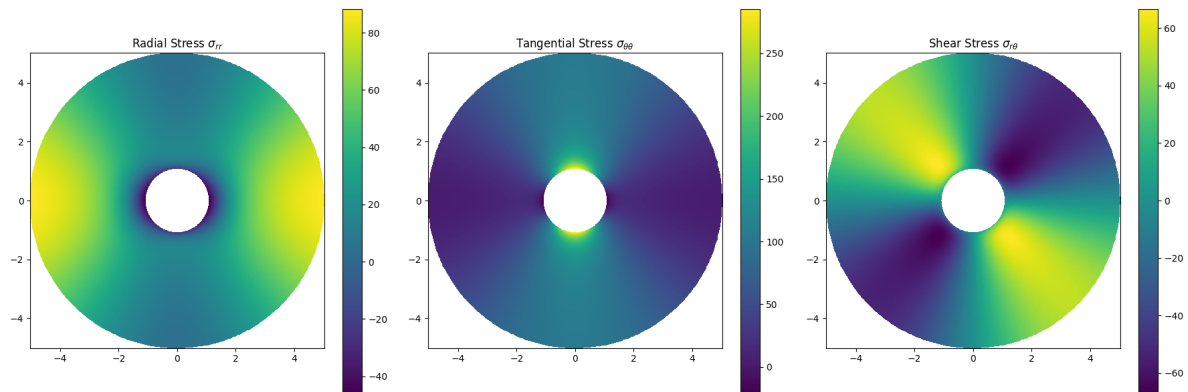


Figure 3: Stress Distribution around the Hole

3 Question 3

4 Question 4

This question is coded in Python entirely, thus, for detail information, please refer to the code provided in Appendix file. And also note that the computation process is modified according to the MATLAB code posted on Canvas.

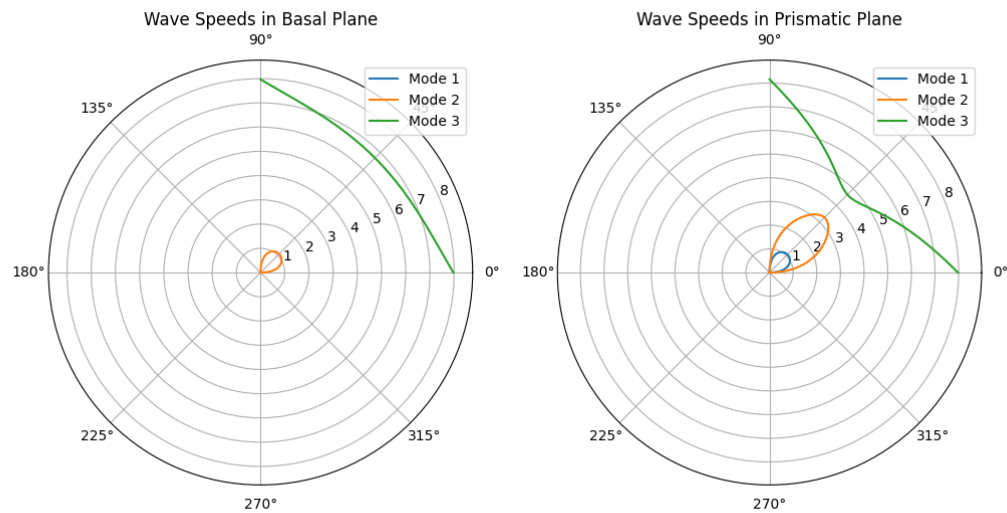


Figure 4: Wave Speed Distribution in Polar Plot

5 Question 5

6 Question 6

Q3

Q4

Q5

Q6