CHBE 552 Problem Set 3

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Question 1

By implementing SA method with following parameters: bounds = $(-1, 1)$, initial temperature = 10000, cooling rate = 0.95, stopping temperature = 0.001, and max iterations = 1000, three tests are conducted and the results are shown as following:

Test 1:

Best x_1 and x_2 are $[0.00016685, -0.00014399]$ and the best function value is $1.074285020 \cdot 10^{-6}$.

Test 2:

Best x_1 and x_2 are $[0.00019128, 0.00056343]$ and the best function value is $1.08674897771 \cdot 10^{-5}$.

Test 3:

Best x_1 and x_2 are $[0.00023714, 0.00051509]$ and the best function value is $9.45038628186 \cdot 10^{-6}$.

Thus, one can conclude the best x_1 and x_2 are $[0, 0]$ and the best function value is 0. The tiny error might comes from the numerical uncertainty of float computation in Python.

Question 2

By implementing Luus-Jaakola method accoording to the reference with such parameters:

$$\text{Initial guess of each } x_j : x_{bounds} = [(0, 1)]$$
$$\text{max iterations} : N = 10000$$
$$\text{Initial search area} : V_{initial} = 0.1$$
$$\text{Search area reduction factor} : V_r = 0.999$$

Solution for variables x_1 to x_{10} are found to be

$$x_1 = 0.0406700206$$
$$x_2 = 0.147744627$$
$$x_3 = 0.783102950$$
$$x_4 = 0.00141387372$$
$$x_5 = 0.485248112$$
$$x_6 = 0.000691530012$$
$$x_7 = 0.0273983722$$
$$x_8 = 0.0179474168$$
$$x_9 = 0.0373039831$$
$$x_{10} = 0.0969432948$$

And the optimal objective function value is computed to be -47.7610, which generally agrees with the value provided in the reference paper.

Note that the number of iterations are setted to be very such that a global minimum could be found, and the search area reduction fractor are close to 1 such that the search area only reduces a little bit in each iteration, thus, the solution of x_j could have more digits.

Question 3

By implementing Luus-Jaakola method accoording to the reference with same parameters stated in question 2, except for initial guess of x_j since the bounds should follow the indicated constraint in the paper:

Best Solution for independent variable:
$$x_1 = 1727.77228$$
$$x_7 = 94.25960$$
$$x_8 = 10.41761$$
Dependent Variables:
$$x_2 = 16000$$
$$x_3 = 99.4398201$$
$$x_4 = 3054.94307$$
$$x_5 = 1999.25826$$
$$x_6 = 90.8309867$$
$$x_9 = 2.56910919$$
$$x_{10} = 149.778787$$
Best Objective Function Value:1161.45677

The results generally agrees with the value provided in the reference paper.

Question 4&5

By implementing Nelder-Mead algorithm in Python, for function in part 1: the minimum point is $[0.99999871 \approx 1, 0.99999707 \approx 1]$, and the minimum function value is $1.3556651538450528e - 11 \approx 0$.

For function in part 2: the minimum point is $[6.50737582e - 04 \approx 0, -6.52767435e - 05 \approx 0, -6.74455549e - 04 \approx 0, -6.76241743e - 04 \approx 0]$, and the minimum function value is $5.379451007433241e - 11 \approx 0$.

Again, The tiny error might comes from the numerical uncertainty of float computation in Python.

Question 6

The work done by the first compressor $C - 1$ is given by:

$$W_{C-1} = \frac{RT_1}{\gamma} \left(\left(\frac{P_2}{P_1} \right)^{\gamma} - 1 \right) \tag{1}$$

The work done by the second compressor $C - 2$, compressing from P_2 to P_4, is (since $P_2 = P_3$):

$$W_{C-2} = \frac{RT_1}{\gamma} \left(\left(\frac{P_4}{P_2} \right)^{\gamma} - 1 \right) \tag{2}$$

The total work W_{tot} is the sum of the work done by both compressors:

$$W_{tot} = W_{C-1} + W_{C-2} \tag{3}$$

$$W_{tot} = \frac{RT_1}{\gamma} \left(\left(\frac{P_2}{P_1} \right)^{\gamma} - 1 \right) + \frac{RT_1}{\gamma} \left(\left(\frac{P_4}{P_2} \right)^{\gamma} - 1 \right) \tag{4}$$

Differentiate W_{tot} with respect to P_2 to find the minimum:

$$\frac{dW_{tot}}{dP_2} = \frac{RT_1\gamma}{\gamma} \left(\frac{P_2}{P_1} \right)^{\gamma - 1} \frac{1}{P_1} - \frac{RT_1\gamma}{\gamma} \left(\frac{P_4}{P_2} \right)^{\gamma} \frac{1}{P_2^2} \tag{5}$$

Set the derivative equal to zero and solve for P_2:

$$0 = \frac{RT_1\gamma}{\gamma} \left[\left(\frac{P_2}{P_1} \right)^{\gamma - 1} \frac{1}{P_1} - \left(\frac{P_4}{P_2} \right)^{\gamma} \frac{1}{P_2^2} \right] \tag{6}$$

Simplify and solve for P_2:

$$0 = P_2^{\gamma-1}\frac{1}{P_1^{\gamma}} - P_4^{\gamma}\frac{1}{P_2^{\gamma+1}}$$

$$P_4^{\gamma} = P_2^{2\gamma}\frac{1}{P_1^{\gamma}}$$

$$P_2^{2\gamma} = P_1^{\gamma}P_4^{\gamma}$$

$$P_2^2 = P_1 P_4$$

Take the square root of both sides to find P_2:

$$P_2 = \sqrt{P_1 P_4} \tag{7}$$

This result indicates that the optimal intermediate pressure P_2 is the geometric mean of the inlet pressure P_1 and the final pressure P_4.