Notation

Notation for Differential Equations and Finite Elements

Ω	open set in $\mathbb{R}^n$
Γ	$=\partial\Omega$
Γ_D	part of the boundary on which Dirichlet conditions are prescribed
Γ_N	part of the boundary on which Neumann conditions are prescribed
Δ	Laplace operator
L	differential operator
a_{ik}, a_0	coefficient functions of the differential equation
$[\,\cdot\,]_*$	difference star, stencil
$L^2(\Omega)$	space of square-integrable functions over Ω
$H^m(\Omega)$	Sobolev space of L_2 functions with square-integrable derivatives up to order m
$H_0^m(\Omega)$	subspace of $H^m(\Omega)$ of functions with generalized zero bounary conditions
$C^k(\Omega)$	set of functions with continuous derivatives up to order k
$C_0^k(\Omega)$	subspace of $C^k(\Omega)$ of functions with compact support
γ	trace operator
$\|\cdot\|_m$	Sobolev norm of order m
$|\cdot|_m$	Sobolev semi-norm of order m
$\|\cdot\|_\infty$	supremum norm
ℓ_2	space of square-summable sequences
H'	dual space of H
$\langle\cdot,\cdot\rangle$	dual pairing
$|\alpha|$	$=\sum\alpha_i$, order of multiindex α
∂_i	partial derivative $\frac{\partial}{\partial x_i}$
∂^α	partial derivative of order α
D	(Fréchet) derivative
α	ellipticity constant
ν, n	exterior normal
$\partial_\nu, \partial/\partial\nu, \partial/\partial n$	derivative in the direction of the exterior normal
∇f	$(\partial f/\partial x_1, \partial f/\partial x_2, \ldots, \partial f/\partial x_n)$
$\operatorname{div} f$	$\sum_{i=1}^n(\partial f/\partial x_i)$
S_h	finite element space
ψ_h	basis function in S_h
$\mathcal{T}_h$	partition of Ω
T	(triangular or quadrilateral) element in $\mathcal{T}_h$
T_{ref}	reference element

h_T, ρ_T	radii of circumscribed circle and incircle of T, respectively
κ	shape parameter of a partition
$\mu(T)$	area (volume) of T
$\mathcal{P}_t$	set of polynomials of degree $\leq t$
$\mathcal{Q}_t$	polynomial set (II.5.4) w.r.t. quadrilateral elements
$\mathcal{P}_{3,\mathrm{red}}$	cubic polynomial without bubble function term
Π_{ref}	set of polynomials which are formed by the restriction of S_h to a (reference) element
s	$= \dim \Pi_{\mathrm{ref}}$
Σ	set of linear functionals in the definition of affine families
$\mathcal{M}^k, \mathcal{M}^k_s, \mathcal{M}^k_{s,0}$	polynomial finite element spaces in L_2, H^{s+1} and H^{s+1}_0
$\mathcal{M}^1_{*,0}$	set of functions in $\mathcal{M}^1$ which are continuous at the midpoints of the sides and which satisfy zero boundary conditions in the same sense
RT_k	Raviart–Thomas element of degree k
I, I_h	interpolation operators on Π_{ref} and on S_h, respectively
A	stiffness or system matrix
$\delta_{..}$	Kronecker symbol
e	edge of an element
$\\|\cdot\\|_{m,h}$	mesh-dependent norm
$\ker L$	kernel of the linear mapping L
$V^\perp$	orthogonal complement of V
V^0	polar of V
$\mathcal{L}$	Lagrange function
M	space of restrictions (for saddle point problems)
β	constant in the Brezzi condition
$H(\mathrm{div}, \Omega)$	$:= \{v \in L_2(\Omega)^d;\ \mathrm{div}\, v \in L_2(\Omega)\}, \Omega \in \mathbb{R}^d$
$L_{2,0}(\Omega)$	set of functions in $L_2(\Omega)$ with integral mean 0
B_3	cubic bubble functions
$\eta_{\dots}$	error estimator

Notation for the Method of Conjugate Gradients

∇f	gradient of f (column vector)
$\kappa(A)$	spectral condition number of the matrix A
$\sigma(A)$	spectrum of the matrix A
$\rho(A)$	spectral radius of the matrix A
$\lambda_{\min}(A)$	smallest eigenvalue of the matrix A
$\lambda_{\max}(A)$	largest eigenvalue of the matrix A
A^t	transpose of the matrix A
I	unit matrix
C	preconditioning matrix
g_k	gradient at the actual approximation x_k

d_k	direction of the correction in step k
V_k	$= \mathrm{span}[g_0, \ldots, g_{k-1}]$
$x'y$	Euclidean scalar product of the vectors x and y
$\|x\|_A$	$= \sqrt{x'Ax}$ (energy norm)
$\|x\|_\infty$	$= \max_i \|x_i\|$ (maximum norm)
T_k	k-th Chebyshev polynomial
ω	relaxation parameter

Notation for the Multigrid Method

$\mathcal{T}_\ell$	triangulation on the level ℓ
$S_\ell = S_{h_\ell}$	finite element space on the level ℓ
A_ℓ	system matrix on the level ℓ
N_ℓ	$= \dim S_\ell$
$\mathcal{S}$	smoothing operator
$r, \tilde{r}$	restrictions
p	prolongation
$x^{\ell,k,m}, u^{\ell,k,m}$	variable on the level ℓ in the k-th iteration step and in the m-th substep
ν_1, ν_2	number of presmoothings or postsmoothings, respectively
ν	$= \nu_1 + \nu_2$
μ	$= 1$ for V-cycle, $= 2$ for W-cycle
q	$= \ell_{\max}$
ψ_ℓ^j	j-th basis function on the level ℓ
ρ_ℓ	convergence rate of $\mathbf{MGM}_\ell$
ρ	$= \sup_\ell \rho_\ell$
$\|\|\| \cdot \|\|\|_s$	discrete norm of order s
β	measure of the smoothness of a function in S_h
$\mathcal{L}$	nonlinear operator
$\mathcal{L}_\ell$	nonlinear mapping on the level ℓ
$D\mathcal{L}$	derivative of $\mathcal{L}$
λ	homotopy parameter for incremental methods

Notation for Solid Mechanics

u	displacement
ϕ	deformation
id	identity mapping
C	$= \nabla\phi^T \nabla\phi$ Cauchy–Green strain tensor
E	strain
ε	strain in a linear approximation
t	Cauchy stress vector
T	Cauchy stress tensor
T_R	first Piola–Kirchhoff stress tensor
Σ_R	second Piola–Kirchhoff stress tensor

$\hat{T}$	$= \hat{T}(F)$ response function for the Cauchy stress tensor
$\hat{\Sigma}$	$= \hat{\Sigma}(F)$ response function for the Piola–Kirchhoff stress tensor
$\tilde{\Sigma}$	$\tilde{\Sigma}(F^T F) = \hat{\Sigma}(F)$
$\bar{T}$	$\bar{T}(FF^T) = \hat{T}(F)$
σ	stress in linear approximation
S^2	unit sphere in $\mathbb{R}^3$
$\mathbb{M}^3$	set of 3×3 matrices
$\mathbb{M}^3_+$	set of matrices in $\mathbb{M}^3$ with positive determinants
$\mathbb{O}^3$	set of orthogonal 3×3 matrices
$\mathbb{O}^3_+$	$= \mathbb{O}^3 \cap \mathbb{M}^3_+$
$\mathbb{S}^3$	set of symmetric 3×3 matrices
$\mathbb{S}^3_>$	set of positive definite matrices in $\mathbb{S}^3$
ι_A	$= (\iota_1(A), \iota_2(A), \iota_3(A))$, invariants of A
$\wedge$	vector product in $\mathbb{R}^3$
$\mathrm{diag}(d_1, \ldots, d_n)$	diagonal matrix with elements $d_1, \ldots, d_n$
λ, μ	Lamé constants
$\mathtt{E}$	modulus of elasticity
ν	Poisson ratio
n	normal vector (different from Chs. II and III)
$\mathcal{C}$	$\sigma = \mathcal{C}\,\varepsilon$
$\hat{W}$	energy functional of hyperelastic materials
$\tilde{W}$	$\tilde{W}(F^T F) = \hat{W}(F)$
$\varepsilon : \sigma$	$= \sum_{ij} \varepsilon_{ij}\sigma_{ij}$
Γ_0, Γ_1	parts of the boundary on which u and$\sigma \cdot n$ are prescribed, respectively
Π	energy functional in the linear theory
$\nabla^{(s)}$	symmetric gradient
$as(\tau)$	skew-symmetric part of τ
$H^s(\Omega)^d$	$= [H^s(\Omega)]^d$
$H^1_\Gamma(\Omega)$	$:= \{v \in H^1(\Omega)R;\ v(x) = 0 \text{ for } x \in \Gamma_0\}$
$H(\mathrm{div}, \Omega)$	$:= \{\tau \in L_2(\Omega);\ \mathrm{div}\,\tau \in L_2(\Omega)\}$, τ is a vector or a tensor
$H(\mathrm{rot}, \Omega)$	$:= \{\eta \in L_2(\Omega)^2;\ \mathrm{rot}\,\eta \in L_2(\Omega)\},\ \Omega \subset \mathbb{R}^2$
$H^{-1}(\mathrm{div}, \Omega)$	$:= \{\tau \in H^{-1}(\Omega)^d;\ \mathrm{div}\,\tau \in H^{-1}(\Omega)\},\ \Omega \subset \mathbb{R}^d$
θ, γ, w	rotation, shear term, and transverse displacement of beams and plates
t	thickness of a beam, membrane, or plate
ℓ	length of a beam
$W_h, \Theta_h, \Gamma_h, Q_h$	finite element spaces in plate theory
π_h	L_2-projector onto Γ_h
R	restriction to Γ_h
P_h	L_2-projector onto Q_h