CHBE 552 PROBLEM SET 2 Assigned: January 27, 2024 Total marks: 12

Q1 (3 marks). Solve the following problems

a) Apply *Newton's* method for the minimization to find the minimum of the following two functions

1. F(x)=$1 + x_1 + x_2 + x_3 + x_4 + x_1x_2 + x_1x_3 + x_1x_4 + x_2x_3 + x_2x_4 + x_3x_4 + x_1^2 + x_2^2 + x_3^2 + x_4^2$ with initial guess (-3, -30, -4, -0.1). and also (0.5, 1.0, 8.0, -0.7)
2. F(x)=$8x_1^2 + 4x_1x_2 + 5x_2^2$ starting at (10,10)

b) Use the *Fletcher-Reeves* method to find the minimum of the following objective function F(x)=$4(x_1 - 5)^2 + (x_2 - 6)^2$

c) Apply the *DFP method* to the minimization of F(x)=$x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ with a starting point (0,0).

Q2 (1 mark). Solve the following problems

a) Minimize $F(x_1, x_2) = x_1^2 + x_2^2 + 10x_1 + 20x_2 + 25$ *subject to* $x_1 + x_2 = 0$ using the Langrange multiplier method. Calculate the optimum values of x_1, x_2, λ and F. Also, using sensitivity analysis, determine the increase in F_{opt} when the constraint is changed to $x_1+x_2=0.01$.

b) Determine the dimensions of a cylindrical can of maximum volume subject to the condition that the total surface area be equal to 24π. Show that the answer indeed corresponds to a maximum (x_1=radius, x_2=height). The problem is equivalent to

Minimize F(x_1,x_2)=$-\pi x_1^2 x_2$ *such that* $2\pi x_1^2 + 2\pi x_1 x_2 = A_0 = 24\pi$

Q3 (1 mark). A problem in *chemical equilibrium* is to minimize the following objective function

$$f(\boldsymbol{x}) = \sum_{i=1}^{n} x_i \left(w_i + lnP + ln \frac{x_i}{\sum_{i=1}^{n} x_i} \right)$$

subject to the material balances

$x_1+2x_2+2x_3+x_6+x_{10}=2$

$x_4+2x_5+x_6+x_7=1$

$x_3+x_7+x_8+2x_9+x_{10}=1$

given P=750 and w_i

i	w_i	i	w_i
1	-10.021	6	-18.918
2	-21.096	7	-28.032
3	-37.986	8	-14.640
4	-9.846	9	-30.594
5	-28.653	10	-26.111

What is $\boldsymbol{x}^*$ and $f(\mathbf{x}^*)$?

Q4 (1 mark). Find the point on $z^2=4x^2+2y^2$ which is nearest to the point (2,0,1). Since the minimizer of the distance d is also the minimizer of d^2 we write the problem as

Minimize F(x,y,z)=$(x-2)^2+y^2+(z-1)^2$

Subject to $z^2-4x^2-2y^2=0$

Q5 (3 marks). Solve the following problems

a) *Minimize* $F(x_1, x_2) = x_1^2 + x_2^2 - 14x_1 - 6x_2 - 7$ $such\ that\ x_1 + x_2 \leq 2. and\ x_1 + 2x_2 \leq 3$
b) *Minimize* F(**x**)= $x_1^2 + 2(x_2 + 1)^2\ such\ that - x_1 + x_2 = 2,\ and\ \ - x_1 - x_2 - 1 \leq 0.$

Use the Langrange multiplier method. Calculate the optimum values of x*, λ*, u*, F*. Demonstrate the sufficiency conditions.

Q6 (3 marks). Solve the following problems

a. Minimize $F(x_1, x_2) = x_1^2 + \frac{3}{2}x_2^2 - 4x_1 - 7x_2 + x_1x_2 + 9 - \ln(x_1) - \ln(x_2)$
b. Minimize $F(x_1, x_2) = x_1^2 + \frac{3}{2}x_2^2 - 4x_1 - 7x_2 + x_1x_2 + 9 - \ln(x_1) - \ln(x_2)$
$subject\ to\ 4 - x_1x_2 \leq 0.$
c. Minimize $F(x_1, x_2) = x_1^2 + \frac{3}{2}x_2^2 - 4x_1 - 7x_2 + x_1x_2 + 9 - \ln(x_1) - \ln(x_2)$
$subject\ to\ 4 - x_1x_2 \leq 0. and\ 2x_1 - x_2 = 0.$

Use the Langrange multiplier method. Calculate the optimum values of x*, λ*, u*, F*. Demonstrate the sufficiency conditions.