MECH 503 Homework 1

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1   Question 1

(a)

$$\nu_i u_i = \sum_{i=1}^{n} \nu_i u_i$$

(b)

$$\delta_{ii} = \sum_{i=1}^{n} \delta_{ii} = n$$

(c)

$$\nu_{i,i} = \sum_{i=1}^{n} \frac{\partial \nu_i}{\partial x_i}$$

(d)

$$\delta_{ij,j} = \sum_{j=1}^{n} \frac{\partial \delta_{ij}}{\partial x_j} = 0$$

(e)

$$\epsilon_{ijk} v_j u_k = \sum_{j=1}^{n} \sum_{k=1}^{n} \epsilon_{ijk} v_j u_k$$

(f)

$$\delta_{mi} \delta_{mj} T_{ij} = \sum_{i=1}^{n} \sum_{j=1}^{n} \delta_{mi} \delta_{mj} T_{ij} = T_{mm}$$

(g)

$$Q_{ij}T_{ik}Q_{km} = \sum_{j=1}^{n}\sum_{k=1}^{n} Q_{ij}T_{ik}Q_{km}$$

(h)

$$\epsilon_{ijk}\sigma_{jk} = \sum_{j=1}^{n}\sum_{k=1}^{n} \epsilon_{ijk}\sigma_{jk}$$

2 Question 2

(a)

$$(u \times v) \cdot w = \varepsilon_{ijk}u_j v_k w_i$$

(b)

$$\det T = \varepsilon_{ijk}T_{1i}T_{2j}T_{3k}$$

(c)

$$(\nabla \times v)_i = \varepsilon_{ijk}\partial_j v_k$$

(d)

$$\nabla \cdot v = \partial_i v_i$$

(e)

$$(\nabla \times T)_{ij} = \varepsilon_{jkl}\partial_k T_{il}$$

(f)

$$(\nabla \times (\nabla T))_i = \varepsilon_{ijk}\partial_j(\partial_k T)$$

(g)

$$(\nabla \cdot T)_i = \partial_j T_{ij}$$

3   Question 3

Define a second order tensor C such that

$$\mathbf{C} = \mathbf{A} \cdot \mathbf{B}$$
$$\Longleftrightarrow C_{ij} = A_{ik} \cdot B_{kj}$$
$$\det \mathbf{C} = \epsilon_{ijk} C_{1i} C_{2j} C_{3k}$$
$$= \epsilon_{ijk} (A_{1l} B_{li})(A_{2m} B_{mj})(A_{3n} B_{nk})$$
$$= \epsilon_{ijk} (A_{1l} A_{2m} A_{3n})(B_{li} B_{mj} B_{nk})$$

Now consider the determinant of A and B separately

$$\det \mathbf{A} = \epsilon_{lmn} A_{1l} A_{2m} A_{3n}$$
$$\det \mathbf{B} = \epsilon_{ijk} B_{li} B_{mj} B_{nk}$$
$$\det \mathbf{A} \cdot \det \mathbf{B} = (\epsilon_{lmn} A_{1l} A_{2m} A_{3n})(\epsilon_{ijk} B_{li} B_{mj} B_{nk})$$

Thus, one can conclude the following

$$\det (\mathbf{A} \cdot \mathbf{B}) = \det \mathbf{A} \cdot \det \mathbf{B}$$