

# MECH 570C Code Project 1

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## 1 main

Crđ has the id of each points on the entire domian,the second colume is the x-coord of each point and the thrid is the y-coord.

Cnn defines every elements in the entire domain, for instance, 6->173->1238->464 is the first element.

BCCyl/Top/bot are not boundary conditions, they are the id of elements on the corresponding boundary.

$C = \text{unique}(A)$  returns the same data as in A, but with no repetitions. C is in **sorted** order.

Sol.u has number of elements as row and 2 columns, 1 is the x direction and 2 is the y direction.

pmc is the time integration parameters.

idx2(:,1) means which elements has the indicated number as the first coord. Eg: the 4249th element has the 4th points as the first coord which is on the boundary of the circle.

So each local elements/poins shown in BCCyl is found in the global cnn matrix with global id. Then form new cnnCyl by local id and global 4 coords.

cnnCylnew reorders the elements on the cylinder

## 2 IO

`ismember`:  $[Lia, Locb] = ismember()$  also returns an array, `Locb`, using any of the previous syntaxes. Generally, `Locb` contains the lowest index in `B` for each value in `A` that is a member of `B`. Values of 0 indicate where `A` is not a member of `B`.

If the ‘rows’ option is specified, then `Locb` contains the lowest index in `B` for each row in `A` that is also a row in `B`. Values of 0 indicate where `A` is not a row of `B`.

If `A` and `B` are tables or timetables, then `Locb` contains the lowest index in `B` for each row in `A` that is also a row in `B`. Values of 0 indicate where `A` is not a row of `B`.

## 3 N-S

### Function Definition

$\rho u_t + \rho u \cdot \text{div}u - \text{div}(mu * \text{grad}u) + \text{grad}p = fin\Omega$ ,  $\text{div}u = 0$  in  $\Omega$ , Dirichlet boundary condition  $u = g_D$  on  $\Gamma_D$ , Neumann boundary condition  $du/dn - np = g_N$  on  $\Gamma_N$ .

- **Function Name:** `navierStokes`
- **Inputs:**
  - `solver`, `fluid`, `pmc`: Structures containing various solver settings, fluid properties, and numerical method parameters.
  - `Sol`: Structure holding the solution vectors.
  - `cnn`, `crd`: Connectivity and coordinate matrices for the mesh.
  - `elemType`, `ndof`, `nen`, `nElem`, `BCCyl`: Parameters defining the type of elements used, degrees of freedom, number of nodes per element, total number of elements, and boundary condition information.

## Quadrature Rules

- Sets up quadrature points ( $\mathbf{gP}$ ) and weights ( $\mathbf{gW}$ ) based on the element type (triangular or quadrilateral). These are used for numerical integration.
- Defines shape functions ( $\mathbf{N}$ ) and their derivatives ( $\mathbf{N_x}$ ,  $\mathbf{N_y}$ ) for the finite elements.

## Boundary Conditions

- Applies Dirichlet and Neumann boundary conditions to the solution vector `Sol.u`.

## Interpolation for Alpha Values

- Interpolates values for the generalized-alpha method, a numerical technique for time integration in transient problems.

## Navier-Stokes Equations

- Prepares variables (`xxf`, `yyf`, `ux`, `uy`, etc.) for assembling the finite element matrices. These include coordinates, velocities, pressure, and additional variables.

## Assembly of Galerkin and Petrov-Galerkin Terms

- Calls functions to form the left-hand side (LHS) and right-hand side (RHS) of the Navier-Stokes equations. This involves complex operations based on the finite element method.
- The Galerkin method is used for discretizing the problem, while Petrov-Galerkin is an enhanced approach for stability and accuracy.

## Solving the Linear System

- Determines the free nodes not constrained by boundary conditions.
- Solves the linear system for the unknowns (**Increment**) using the assembled LHS and RHS.

## Update and Output

- Updates the solution vectors (**Sol.u**, **Sol.uDot**, **Sol.p**) with the new increments.
- Calculates a norm (**NSnormIndicator**) to indicate the convergence or error of the current iteration.
- Outputs the updated solution structure **Sol** and the convergence/error indicator.

## Summary of Data Flow

1. **Input Processing:** Takes in initial conditions, mesh information, and solver parameters.
2. **Setup:** Establishes quadrature rules and boundary conditions.
3. **Equation Assembly:** Forms the Navier-Stokes equations using finite element discretization.
4. **Solution Update:** Solves for increments and updates the solution.
5. **Output:** Returns the updated solution and a convergence/error metric.

## 4 Integrated Output

### Function Definition

- **Function Name:** `IntegratedOutput`
- **Inputs:**

- **Sol**: A structure containing the solution vectors (velocity, pressure, etc.).
- **crd**: The coordinates of the mesh nodes.
- **BCCyl**: Boundary condition data.
- **fluid**: A structure containing fluid properties.
- **cnn**: **Connectivity matrix for the mesh elements.**

## Initialization

- Sets the number of element nodes (**n<sub>en</sub>**) for a 2D element (4 for a quadrilateral element).
- Swaps the columns of **BCCyl** for further processing.
- Determines the number of elements (**nElem**) and degrees of freedom (**ndof**) involved in the boundary condition.

## Boundary Layer Elements

- Identifies elements corresponding to the first layer of the boundary using **cnn** and **BCCyl**.
- Reorders the element points for reduced integration.

## Quadrature Integration Setup

- Defines Gauss points (**gP**) and weights (**gW**) for numerical integration.
- Sets up shape functions (**N**) and their derivatives (**N<sub>x</sub>**, **N<sub>y</sub>**).

## Localizing Data

- Extracts local coordinates (**xxf**, **yyf**), velocities (**ux**, **uy**), and pressure (**pres**) for each element.

## Integration Process

- Loops over quadrature points to:
  - Calculate the Jacobian matrix  $J$  for coordinate transformation.
  - Compute the volume and normal vectors for each element.
  - Evaluate pressure and velocity gradients (`locgradUx`, `locgradUy`, `locgradVx`, `locgradVy`).
  - Compute the length/area (`A0`) of the line/surface integral.

## Calculation of Forces

- The loop sums up the contributions from all quadrature points to calculate the integrated force over the element surface.
- The force components (X and Y directions) are to be computed within the loop (currently not implemented in the provided code).

## Output

- Returns the total length of the integrated area (`Length`) and the force components (`Force`).