# MECH 570C Code Project 1

### February 18, 2024

#### 1 main

Crd has the id of each points on the entire domian, the second colume is the x-coord of each point and the thrid is the y-coord.

Cnn (=conn) defines every elements in the entire domain, for instance, 6->173->1238->464 is the first element.

BCCyl/Top/bot are not boundary conditions, they are the id of elements on the corresponding boundary.

C = unique(A) returns the same data as in A, but with no repetitions. C is in **sorted** order.

Sol.u has number of elements as row and 2 columns, 1 is the x direction and 2 is the y direction.

pmc is the time integration parameters.

idx2(:,1) means which elements has the indicated number as the first coord. Eg: the 4249th element has the 4th points as the first coord which is on the boundary of the circle.

So each local elements/poins shown in BCCyl is found in the global cnn matrix with global id. Then form new cnnCyl by local id and global 4 coords.

cnnCylnew reorders the elements on the cylinder.

## 2 IO

is member: [Lia, Locb] = ismember() also returns an array, Locb, using any of the previous syntaxes. Generally, Locb contains the lowest index in B for each value in A that is a member of B. Values of 0 indicate where A is not a member of B.

If the 'rows' option is specified, then Locb contains the lowest index in B for each row in A that is also a row in B. Values of 0 indicate where A is not a row of B

If A and B are tables or timetables, then Locb contains the lowest index in B for each row in A that is also a row in B. Values of 0 indicate where A is not a row of B.

## 3 Poisson2D

10 elements in each direction, so 11 points and 11x11 = 121 sets of coord info, from left to right, bot to top.

Bcx0y0xLyL has the id of nodes on each boundary 11/.

Bcn has left right bot and top in chain.

Why pmc is needed in ALE?

#### 4 N-S

#### **Function Definition**

 $\rho u_t + \rho u.divu - div(mu * gradu) + gradp = fin\Omega$ , divu = 0 in  $\Omega$ , Dirichlet boundary condition  $u = g_D on \Gamma_D$ , Neumann boundary condition  $du/dn - np = g_N on \Gamma_N$ .

- Function Name: navierStokes
- Inputs:

- solver, fluid, pmc: Structures containing various solver settings, fluid properties, and numerical method parameters.
- Sol: Structure holding the solution vectors.
- cnn, crd: Connectivity and coordinate matrices for the mesh.
- elemType, ndof, nen, nElem, BCCyl: Parameters defining the type of elements used, degrees of freedom, number of nodes per element, total number of elements, and boundary condition information.

## Quadrature Rules

- Sets up quadrature points (gP) and weights (gW) based on the element type (triangular or quadrilateral). These are used for numerical integration.
- Defines shape functions (N) and their derivatives (Nx, Ny) for the finite elements.

## **Boundary Conditions**

• Applies Dirichlet and Neumann boundary conditions to the solution vector Sol.u.

# Interpolation for Alpha Values

• Interpolates values for the generalized-alpha method, a numerical technique for time integration in transient problems.

# **Navier-Stokes Equations**

• Prepares variables (xxf, yyf, ux, uy, etc.) for assembling the finite element matrices. These include coordinates, velocities, pressure, and additional variables.

### Assembly of Galerkin and Petrov-Galerkin Terms

- Calls functions to form the left-hand side (LHS) and right-hand side (RHS) of the Navier-Stokes equations. This involves complex operations based on the finite element method.
- The Galerkin method is used for discretizing the problem, while Petrov-Galerkin is an enhanced approach for stability and accuracy.

### Solving the Linear System

- Determines the free nodes not constrained by boundary conditions.
- Solves the linear system for the unknowns (Increment) using the assembled LHS and RHS.

## Update and Output

- Updates the solution vectors (Sol.u, Sol.uDot, Sol.p) with the new increments.
- Calculates a norm (NSnormIndicator) to indicate the convergence or error of the current iteration.
- Outputs the updated solution structure Sol and the convergence/error indicator.

# Summary of Data Flow

- 1. **Input Processing:** Takes in initial conditions, mesh information, and solver parameters.
- 2. **Setup:** Establishes quadrature rules and boundary conditions.
- 3. **Equation Assembly:** Forms the Navier-Stokes equations using finite element discretization.
- 4. **Solution Update:** Solves for increments and updates the solution.
- 5. Output: Returns the updated solution and a convergence/error metric.

# 5 Integraded Output

#### **Function Definition**

- Function Name: IntegratedOutput
- Inputs:
  - Sol: A structure containing the solution vectors (velocity, pressure, etc.).
  - crd: The coordinates of the mesh nodes.
  - BCCyl: Boundary condition data.
  - fluid: A structure containing fluid properties.
  - cnn: Connectivity matrix for the mesh elements.

#### Initialization

- Sets the number of element nodes (nen) for a 2D element (4 for a quadrilateral element).
- Swaps the columns of BCCyl for further processing.
- Determines the number of elements (nElem) and degrees of freedom (ndof) involved in the boundary condition.

#### **Boundary Layer Elements**

- Identifies elements corresponding to the first layer of the boundary using cnn and BCCyl.
- Reorders the element points for reduced integration.

# Quadrature Integration Setup

- Defines Gauss points (gP) and weights (gW) for numerical integration.
- Sets up shape functions (N) and their derivatives (Nx, Ny).

## Localizing Data

• Extracts local coordinates (xxf, yyf), velocities (ux, uy), and pressure (pres) for each element.

### **Integration Process**

- Loops over quadrature points to:
  - Calculate the Jacobian matrix J for coordinate transformation.
  - Compute the volume and normal vectors for each element.
  - Evaluate pressure and velocity gradients (locgradUx, locgradUy, locgradVx, locgradVy).
  - Compute the length/area (A0) of the line/surface integral.

#### Calculation of Forces

- The loop sums up the contributions from all quadrature points to calculate the integrated force over the element surface.
- The force components (X and Y directions) are to be computed within the loop (currently not implemented in the provided code).

## Output

• Returns the total length of the integrated area (Length) and the force components (Force).