MECH 503 MT Feb. 29th

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Q1. The statement is correct. $\underline{\underline{E}}^* = \frac{1}{2}(\underline{\underline{I}} - \underline{\underline{F}}^{-T}\underline{\underline{F}}^{-1})$

Q2. a) $\underline{\underline{F}} = \frac{dy}{dx} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -2 \\ -2 & 2 & 1 \end{bmatrix}$

$\underline{\underline{C}} = \underline{\underline{F}}^T \cdot \underline{\underline{F}} = \begin{bmatrix} 5 & -4 & 0 \\ -4 & 5 & 0 \\ 0 & 0 & 9 \end{bmatrix}$

b) $\cos\theta = \dfrac{\underline{\underline{C}} \cdot \underline{m} \cdot \underline{n}}{(\underline{\underline{C}} \cdot \underline{m} \cdot \underline{m})^{1/2} (\underline{\underline{C}} \cdot \underline{n} \cdot \underline{n})^{1/2}} = -0.8$ so not perpendicular.

c) $dl = l_0 (\underline{\underline{C}} \cdot \underline{n}_0 \cdot \underline{n}_0)^{1/2} = 2.8284$ so the element stretches.

d) $\underline{\underline{E}}_L = \frac{1}{2}(\underline{\underline{C}} - \underline{\underline{I}}) = \begin{bmatrix} 2 & -2 & 0 \\ -2 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

$\underline{\underline{E}}_E = \frac{1}{2}(\underline{\underline{I}} - \underline{\underline{C}}^{-1}) = \begin{bmatrix} 0.222 & -0.222 & 0 \\ -0.222 & 0.222 & 0 \\ 0 & 0 & 0.4444 \end{bmatrix}$

e) $\underline{\underline{U}}^2 = \underline{\underline{C}}$ and by SVD, so $\underline{\underline{U}} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is symmetric.

$\underline{\underline{R}} = \underline{\underline{F}} \cdot \underline{\underline{U}}^{-1} = \begin{bmatrix} 0.6667 & 0.3333 & 0.6667 \\ 0.3333 & 0.6667 & -0.6667 \\ -0.6667 & 0.6667 & 0.3333 \end{bmatrix}$

by checking $R^T = R^{-1}$ and $\det(R) = 1$, R is orthonormal.

$R^T = \begin{bmatrix} 0.6667 & 0.3333 & -0.6667 \\ 0.3333 & 0.6667 & 0.6667 \\ 0.6667 & -0.6667 & 0.3333 \end{bmatrix} = R^{-1} = \begin{bmatrix} 0.6667 & 0.3333 & -0.6667 \\ 0.3333 & 0.6667 & 0.6667 \\ 0.6667 & -0.6667 & 0.3333 \end{bmatrix}$

the principle stresses are computed from eigen values of U

so the priciple stresses are $[3, 1, 3]$ and their directions are

$\begin{bmatrix} 0.7071 \\ -0.7071 \\ 0 \end{bmatrix}, \begin{bmatrix} -0.7071 \\ -0.7071 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$ accordingly.

Q3. $\underline{\underline{\sigma}} = \begin{bmatrix} -50 & 50 & 70 \\ 50 & 100 & 0 \\ 70 & 0 & 70 \end{bmatrix}$

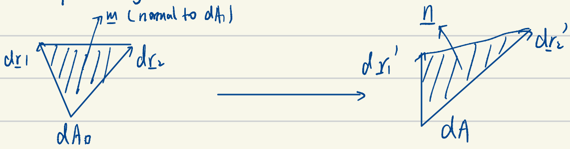
a) $\underline{n} = [1, 0, 0.5]^T / \text{sqrt}(1+0.5^2)$

$$\underline{t} = \underline{\underline{\sigma}} \cdot \underline{n} = \begin{bmatrix} -13.4164 \\ 44.7214 \\ 93.9149 \end{bmatrix}$$

b) components of $\underline{t}$ in $\underline{n}$ direction means the traction in $\underline{n}$ direction can be decomposed into three principle value t_1, t_2, t_3, which represent the decomposed traction components in each orthogonal direction. such as x, y, z.

c). principle stresses are: -93.0118 85.7477 127.2641 all in MPa.

Q4. ref. config.



$$dA_0 \underline{m} = d\underline{r}_1 \times d\underline{r}_2 = \varepsilon_{ijk} dr_{1i} dr_{2j} \qquad dA \underline{n} = d\underline{r}_1' \times d\underline{r}_2' = \varepsilon_{ijk} dr_{1i}' dr_{2j}'$$

since $dr_{1i}' = F_{il} dr_{1l}$ $\quad dr_{2j}' = F_{jm} dr_{2m}$

$$dA \underline{n} = \varepsilon_{ijk} \underbrace{F_{il} F_{jm}}_{= J / F^T} dr_{1l} dr_{2m}$$

so $dA \underline{n} = J F^{-T} dA_0 \underline{m}$

Q5. $\underline{\underline{F}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \gamma \\ 0 & 0 & 1 \end{bmatrix}$

considering a simple fiber (0,0,1) from (0,0,0)

$\underline{m}' = \underline{\underline{F}} \cdot \underline{m} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \gamma \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \gamma \end{bmatrix}$ $\underline{m}'$ is the deformed fiber. θ is the angle between

so. $\theta = \arctan(\gamma)$ it is a simple shear.