CHBE 552 Problem Set 1

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Question 1

By implementing SA method with following parameters: bounds = $(-1, 1)$, initial temperature = 10000, cooling rate = 0.95, stopping temperature = 0.001, and max iterations = 1000, three tests are conducted and the results are shown as following:

Test 1:

Best x_1 and x_2 are $[0.00016685, -0.00014399]$ and the best function value is $1.074285020 \cdot 10^{-6}$.

Test 2:

Best x_1 and x_2 are $[0.00019128, 0.00056343]$ and the best function value is $1.08674897771 \cdot 10^{-5}$.

Test 3:

Best x_1 and x_2 are $[0.00023714, 0.00051509]$ and the best function value is $9.45038628186 \cdot 10^{-6}$.

Thus, one can conclude the best x_1 and x_2 are $[0, 0]$ and the best function value is 0. The tiny error might comes from the numerical uncertainty of float computation in Python.

Question 2

By implementing Luus-Jaakola method accoording to the reference with such parameters:

$$
\begin{aligned}
\text{Initial guess of each } x_j\text{:} x_{bounds} &= [(0, 1)] \\
\text{max iterations:} N &= 10000 \\
\text{Initial search area:} V_{initial} &= 0.1 \\
\text{Search area reduction factor:} V_r &= 0.999
\end{aligned}
$$

Solution for variables x_1 to x_{10} are found to be

$$\begin{aligned}
x_1 &= 0.0406700206 \\
x_2 &= 0.147744627 \\
x_3 &= 0.783102950 \\
x_4 &= 0.00141387372 \\
x_5 &= 0.485248112 \\
x_6 &= 0.000691530012 \\
x_7 &= 0.0273983722 \\
x_8 &= 0.0179474168 \\
x_9 &= 0.0373039831 \\
x_{10} &= 0.0969432948
\end{aligned}$$

And the optimal objective function value is computed to be -47.7610, which generally agrees with the value provided in the reference paper.

Note that the number of iterations are setted to be very such that a global minimum could be found, and the search area reduction fractor are close to 1 such that the search area only reduces a little bit in each iteration, thus, the solution of x_j could have more digits.

Question 3

By implementing Luus-Jaakola method accoording to the reference with same parameters stated in question 2, except for initial guess of x_j since the bounds should follow the indicated constraint in the paper:

$$\begin{aligned}
\text{Best Solution for independent variable:} & \\
x_1 &= 1727.77228 \\
x_7 &= 94.25960 \\
x_8 &= 10.41761 \\
\text{Dependent Variables:} & \\
x_2 &= 0.000160000000 \\
x_3 &= 0.994398201 \\
x_4 &= 0.00305494307 \\
x_5 &= 0.00199925826 \\
x_6 &= 0.908309867 \\
x_9 &= 2.56910919 \\
x_{10} &= 0.0149778787 \\
\text{Best Objective Function Value:}1161.45677 &
\end{aligned}$$

The results generally agrees with the value provided in the reference paper.

Question 4&5

By implementing Nelder-Mead algorithm in Python, for function in part 1: the minimum point is $[0.99999871 \approx 1, 0.99999707 \approx 1]$, and the minimum function value is $1.3556651538450528e-11 \approx 0$.

For function in part 2: the minimum point is $[6.50737582e-04 \approx 0, -6.52767435e-05 \approx 0, -6.74455549e-04 \approx 0, -6.76241743e-04 \approx 0]$, and the minimum function value is $5.379451007433241e-11 \approx 0$.

Again, The tiny error might comes from the numerical uncertainty of float computation in Python.

Question 6

The work done by the first compressor $C-1$ is given by:

$$W_{C-1} = \frac{RT_1}{\gamma}\left(\left(\frac{P_2}{P_1}\right)^{\gamma} - 1\right) \tag{1}$$

The work done by the second compressor $C-2$, compressing from P_2 to P_4, is (since$P_2 = P_3$):

$$W_{C-2} = \frac{RT_1}{\gamma}\left(\left(\frac{P_4}{P_2}\right)^{\gamma} - 1\right) \tag{2}$$

The total work W_{tot} is the sum of the work done by both compressors:

$$W_{tot} = W_{C-1} + W_{C-2} \tag{3}$$

$$W_{tot} = \frac{RT_1}{\gamma}\left(\left(\frac{P_2}{P_1}\right)^{\gamma} - 1\right) + \frac{RT_1}{\gamma}\left(\left(\frac{P_4}{P_2}\right)^{\gamma} - 1\right) \tag{4}$$

Differentiate W_{tot} with respect to P_2 to find the minimum:

$$\frac{dW_{tot}}{dP_2} = \frac{RT_1\gamma}{\gamma}\left(\frac{P_2}{P_1}\right)^{\gamma-1}\frac{1}{P_1} - \frac{RT_1\gamma}{\gamma}\left(\frac{P_4}{P_2}\right)^{\gamma}\frac{1}{P_2^2} \tag{5}$$

Set the derivative equal to zero and solve for P_2:

$$0 = \frac{RT_1\gamma}{\gamma}\left[\left(\frac{P_2}{P_1}\right)^{\gamma-1}\frac{1}{P_1} - \left(\frac{P_4}{P_2}\right)^{\gamma}\frac{1}{P_2^2}\right] \tag{6}$$

Simplify and solve for P_2:

$$
\begin{aligned}
0 &= P_2^{\gamma-1}\frac{1}{P_1^{\gamma}} - P_4^{\gamma}\frac{1}{P_2^{\gamma+1}} \\
P_4^{\gamma} &= P_2^{2\gamma}\frac{1}{P_1^{\gamma}} \\
P_2^{2\gamma} &= P_1^{\gamma}P_4^{\gamma} \\
P_2^2 &= P_1P_4
\end{aligned}
$$

Take the square root of both sides to find P_2:

$$P_2 = \sqrt{P_1P_4} \tag{7}$$

This result indicates that the optimal intermediate pressure P_2 is the geometric mean of the inlet pressure P_1 and the final pressure P_4.