MECH 570C-FSI: Coding Project 2
Fluid-structure interaction with nonlinear hyperleastic structure

Winter Term 2, Year 2023-2024

The assignment 1 of the course gave exposure on a simple canonical geometry of a rigid cylindrical structure interacting with the flow field. The structure was assumed to be elastically mounted on a spring-damper system which simplified the structural equation drastically. As a next step in this assignment, your ultimate goal is to simulate a flexible hyperelastic structure such as a flag by modeling the three-dimensional nonlinear structural equation.

Denoting the structural displacement by $\boldsymbol{\eta}^{\mathrm{s}}$, the structural equation in the Lagrangian description is given by

$$\rho^{\mathrm{s}} \frac{\partial^2 \boldsymbol{\eta}^{\mathrm{s}}}{\partial t^2} = \nabla \cdot (\boldsymbol{\sigma}^{\mathrm{s}}) + \rho^{\mathrm{s}} \boldsymbol{g}, \text{ in } \Omega^{\mathrm{s}}, \tag{1}$$

where $\boldsymbol{\sigma}^{\mathrm{s}}$ for a St. Venant-Kirchhoff material is given by

$$\boldsymbol{\sigma}^{\mathrm{s}} = \lambda^{\mathrm{s}} (\mathrm{tr}\boldsymbol{E}) \boldsymbol{F} + 2\mu^{\mathrm{s}} \boldsymbol{F}\boldsymbol{E}, \tag{2}$$

where $\boldsymbol{F} = \boldsymbol{I} + \nabla \boldsymbol{\eta}^{\mathrm{s}}$ is the deformation gradient tensor and $\boldsymbol{E}$ is the Green-Lagrange strain tensor given by

$$\boldsymbol{E} = \frac{1}{2}(\boldsymbol{F}^T \boldsymbol{F} - \boldsymbol{I}). \tag{3}$$

The density of the structure in the undeformed configuration is ρ^{s}. The material is characterized by the Poisson's ratio ν^{s} and the Young's modulus E. The Lamé coefficients λ^{s} and μ^{s} (shear modulus) are related to E and μ^{s} by the following expressions:

$$\mu^{\mathrm{s}} = \frac{E}{2(1+\nu^{\mathrm{s}})}, \tag{4}$$

$$\lambda^{\mathrm{s}} = \frac{\nu^{\mathrm{s}} E}{(1+\nu^{\mathrm{s}})(1-2\nu^{\mathrm{s}})}. \tag{5}$$

The above equations define the structural characteristics. Most of the numerical codes from Assignment 1 will be utilized in continuation to this assignment. The assignment consists of the following tasks:

1. Understand the structural equation given above and write it in discretized finite element form.

2. Code the equation in a file with name "hyperelasticMaterial.m" with a similar format as what has been carried out for the Navier-Stokes equations (navierStokes.m). Note the nonlinearity in the structural equation. Remember to use Newton-Raphson technique to solve for the increments in displacement and then update the displacements.

3. Carry out the isolated structural model tests without any fluid forces using the Benchmark on a cylinder-bar problem (Section 4.2 CSM Tests in "Proposal for Numerical Benchmarking of Fluid-Structure Interaction between an Elastic Object and Laminar Incompressible Flow," S. Turek and J. Hron, Part of the Lecture Notes in Computational Science and Engineering book series titled "Fluid-structure interaction. Modelling, Simulation, Optimization" by Springer, Pg. 371-385, 2006). Note that for this case, you will not be needing any fluid forces, but the mesh will be deforming, i.e., your kinematic equilibrium will be satisfied at the fluid-structure interface with dynamic equilibrium being satisfied trivially as fluid forces are zero.

4. Once the structural solver is giving satisfactory results, focus on transferring the fluid forces correctly to the structural elements. You will need to take help from the IntegratedOutput.m file that you coded in Assignment 1.

5. Once the transfer of forces is satisfactory, perform the FSI test FSI2 in the reference given above and compare your solutions with the results provided.

6. Apply the validated FSI solver to simulate the flapping filament problem at $Re = 1000$, $m^* \in [0.025, 0.1, 0.2]$ and $K_B = 0.0001$. Compare the results with that in the following reference: "A stable second-order scheme for fluid-structure interaction with strong added-mass effects," J. Liu, R. K. Jaiman, P. S. Gurugubelli, *Journal of Computational Physics*, Vol. 270, Pg. 687-710, 2014. Plot the flapping response like in Fig. 8 and compare the results obtained with different types of response in Fig. 15 in the reference. [NOTE: Let us know if the solver is diverging for these cases but converged for the FSI Turek Benchmark for cylinder-bar problem.]

The required mesh data is provided for the two problems as:

1. Turek cylinder-bar benchmark: Data_cylinder_bar.mat

2. Flapping filament: Data_flexible_filament.mat