CHBE 552 Problem Set 1

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Question 1

Part a

$$\begin{aligned}
\mathbf{F_1} &= x_1^2 + x_1x_2 + x_1x_3 + x_1x_4 + x_1 \\
&+ x_2^2 + x_2x_3 + x_2x_4 + x_2 + x_3^2 + x_3x_4 \\
&+ x_3 + x_4^2 + x_4 + 1 \\
\mathbf{H_{F_1}} &= \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} \\
\mathbf{F_2} &= 8x_1^2 + 4x_1x_2 + 5x_2^2 \\
\mathbf{H_{F_2}} &= \begin{bmatrix} 16 & 4 \\ 4 & 10 \end{bmatrix}
\end{aligned}$$

By implementating Newton's method, the corresponding minimums are found to be:

$$\begin{aligned}
\mathbf{x_{F1}} &= \begin{bmatrix} -0.2 \\ -0.2 \\ -0.2 \\ -0.2 \end{bmatrix} \text{ or } \begin{bmatrix} -0.2 \\ -0.2 \\ -0.2 \\ -0.2 \end{bmatrix} \\
\mathbf{x_{F2}} &= \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\end{aligned}$$

Note that for $\mathbf{F_1}$, two initial guesses are provided the value of $\mathbf{x_{F1}}$ shown above are derived from thoes two initial guesses and they are actually the same.

Part b

$$\begin{aligned}\mathbf{F} &= 4\left(x_1-5\right)^2+\left(x_2-6\right)^2\\ \nabla\mathbf{F} &= \left[8x_1-40,\ 2x_2-12\right]\end{aligned}$$

By implementating Fletcher Reeves's method, the minimum is found to be:

$$\mathbf{x}=\begin{bmatrix}5.11\\2.48\end{bmatrix}$$

Part c

$$\begin{aligned}\mathbf{F} &= 2x_1^2+2x_1x_2+x_1+x_2^2-x_2\\ \nabla\mathbf{F} &= \left[4x_1+2x_2+1,\ 2x_1+2x_2-1\right]\end{aligned}$$

By implementating DFP method, the minimum is found to be:

$$\mathbf{x}=\begin{bmatrix}-0.65\\0.98\end{bmatrix}$$

Question 2

Part a

$$\begin{aligned}F &= x_1^2+10x_1+x_2^2+20x_2+25\\ L &= \lambda\left(x_1+x_2\right)+x_1^2+10x_1+x_2^2+20x_2+25\\ \partial F_{x_1} &= \lambda+2x_1+10\\ \partial F_{x_2} &= \lambda+2x_2+20\\ \partial F_{\lambda} &= x_1+x_2\end{aligned}$$

By setting those partial derivative equations to zero, the optimum values are found to be:

$$
\begin{aligned}
x_{1,opt} &= \frac{5}{2} \\
x_{2,opt} &= -\frac{5}{2} \\
\lambda_{opt} &= -15
\end{aligned}
$$

At this optimum point the original function is evaluated to be:

$$
F_{opt} = \frac{25}{2}
$$

For a sensitivity test the constraint condition is changed to $x_1 + x_2 = 0.01$, then the previous steps are repeated:

$$
\begin{aligned}
L_{new} &= \lambda\left(x_1 + x_2 - 0.01\right) + x_1^2 + 10x_1 + x_2^2 + 20x_2 + 25 \\
\partial F_{x_1} &= \lambda + 2x_1 + 10 \\
\partial F_{x_2} &= \lambda + 2x_2 + 20 \\
\partial F_{\lambda} &= x_1 + x_2 - 0.01 \\
x_{1,opt} &= 2.505 \\
x_{2,opt} &= -2.495 \\
\lambda_{opt} &= -15.01 \\
F_{new,opt} &= 12.65005
\end{aligned}
$$

Thus, the increment of the function value is computed to be

$$
\Delta F = 0.15
$$

Part b

$$
\begin{aligned}
F &= -\pi x_1^2 x_2 \\
L &= \lambda\left(2\pi x_1^2 + 2\pi x_1 x_2 - 24\pi\right) - \pi x_1^2 x_2 \\
\partial F_{x_1} &= \lambda\left(4\pi x_1 + 2\pi x_2\right) - 2\pi x_1 x_2 \\
\partial F_{x_2} &= 2\pi\lambda x_1 - \pi x_1^2 \\
\partial F_{\lambda} &= 2\pi x_1^2 + 2\pi x_1 x_2 - 24\pi \\
\text{Solution} &= [(2,\ 4)]
\end{aligned}
$$