CHBE 552 Problem Set 1

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1 Question 1

1.1 Part a

$$F(\mathbf{x}) = (x_1 - 2)^4 + (x_1 - 2x_2)^2$$
$$\partial F_{x_1} = 2x_1 - 4x_2 + 4(x_1 - 2)^3$$
$$\partial F_{x_2} = -4x_1 + 8x_2$$

By solving each partial derivative (set equal to zero), the stationaty point is obtained to be

$$\begin{pmatrix} {x_1}^* \\ {x_2}^* \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

The Hessian matrix is determined to be

$$H_a = \begin{bmatrix} 12(x_1 - 2)^2 + 2 & -4 \\ -4 & 8 \end{bmatrix}$$

Then, the Hessian matrix at the stationary point is evaluated to be

$$H_{a,eva.} = \begin{bmatrix} 2 & -4 \\ -4 & 8 \end{bmatrix}$$

The eigen values of the evaluated Hessian matrix are computed to be $0, 10$, thus that stationary point is determined to be a local minimum since the function is Convex at this point

1.2 Part b

$$
\begin{aligned}
F(\mathbf{x}) &= 2x_1^3 + x_2^2 + x_1^2 x_2^2 + 4x_1 x_2 + 3 \\
\partial F_{x_1} &= 6x_1^2 + 2x_1 x_2^2 + 4x_2 \\
\partial F_{x_2} &= 2x_1^2 x_2 + 4x_1 + 2x_2
\end{aligned}
$$

By solving each partial derivative (set equal to zero), the stationaty point is obtained to be

$$
\begin{pmatrix} {x_1}^* \\ {x_2}^* \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}
$$

The Hessian matrix is determined to be

$$
H_b = \begin{bmatrix} 12x_1 + 2x_2^2 & 4x_1 x_2 + 4 \\ 4x_1 x_2 + 4 & 2x_1^2 + 2 \end{bmatrix}
$$

Then, the Hessian matrix at the stationary point is evaluated to be

$$
H_{b,eva.} = \begin{bmatrix} 0 & 4 \\ 4 & 2 \end{bmatrix}
$$

The eigen values of the evaluated Hessian matrix are computed to be $2, -16$, thus that stationary point is determined to be a saddle point

1.3 Part c

$$
\begin{aligned}
F(x) &= 12x^5 - 45x^4 + 40x^3 + 5 \\
F'(x) &= 60x^4 - 180x^3 + 120x^2
\end{aligned}
$$

By solving the first derivative (set equal to zero), the stationaty points are obtained to be

$$
x^* = 0, 1, 2
$$

The second derivative is determined to be

$$
F''(x) = 240x^3 - 540x^2 + 240x
$$

Then, the second derivative at the stationary point is evaluated to be

$$F''(x^*) = [0,\ -60,\ 240]$$

By checking the eigen values of the second derivative, those stationary points are determined to be a saddle point($x = 0$),a local maximun($x = 1$), and a local minimum($x = 2$)

2 Question 2

2.1 Part a

$$\begin{aligned} c = &\frac{58.0555555555556}{T^{0.3017}} + \frac{70.8612328738115T^{0.4925}}{q} \\ &+ \frac{2.82512546852169T^{0.7952}}{q} + \frac{19.8593073947553}{q^{0.1899}} \\ &+ 8.48356524677597 \cdot 10^{-5} q^{0.671} + 13.9 \\ &+ \frac{0.000332994515384452 \cdot (1700.0T + 162.162162162162q)}{q} \end{aligned}$$

the partial derivatives are found to be

$$\begin{aligned} \partial c_q &= -\frac{22.531393600576T^{0.4925}}{q^3} \\ \partial c_T &= -\frac{17.5153611111111}{T^{1.3017}} + \frac{5.54835567414184}{T^{0.5075}q^2} \end{aligned}$$

Since the Hessian matrix of the cost function could be obtained though its complicated and long

So with the implementation of Newton's Method, the optimum tanker size and refinery size are computed to be

$$\begin{aligned} q &= 174833 \text{bbl/day} \\ T &= 484726 \text{kL} \end{aligned}$$

And the minimun cost of oil is then computed to be

$$c_{min} = 17.8226$$

2.2 Part b

The profit is found to be

$$
\begin{aligned}
P &= 50 * Yp - \text{cost of additive of A per mol} - \text{cost of steam per mol} \\
&= -2000000S^2 + 0.005Sx_a + 0.047S - 20x_a^2 + 5.0x_a + 1.0
\end{aligned}
$$

And the corresponding Hessian matrix are calculated to be

$$
H_P = \begin{bmatrix} -40 & 0.005 \\ 0.005 & -4000000 \end{bmatrix}
$$

The eigen values are approximated to be -40 and -4000000, which are both negative. Therefore, the profit function is concave. This conclusion makes sense since it is rational to have a optimum input of material A and steam such that profit is maximized

2.3 (

Part c)

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