# MECH 570C-FSI: Code Project 1
# Fluid-structure interaction of a rigid circular cylinder

Jincong Li
60539939

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## Abstract

## Introduction

This project is designed to develop foundational knowledge in the field of fluid-structure interaction, focusing on a paradigmatic issue of vortex-induced vibration caused by flow around a smooth circular cylinder. The fluid dynamics are characterized using the incompressible, two-dimensional Navier-Stokes equations, formulated within the arbitrary Lagrangian-Eulerian (ALE) framework. The analysis of the rigid body structure, which possesses a two-degree-of-freedom translational capability, is conducted employing the Lagrangian description. At the interface between the fluid and structure, both velocity continuity and traction equilibrium conditions are rigorously maintained.

## Methodology

Given that a substantial portion of the code has been provided, two primary objectives have been delineated as follows: The initial goal involves the execution of the IntegratedOutput function by completing the requisite equations to calculate the tractions and forces exerted on the cylinder's surface. The secondary task is to formulate the code for the Arbitrary Lagrangian-Eulerian (ALE) mesh, thereby enabling the accurate representation of the mesh's movement. Detailed instructions for the implementation of these two tasks will be elaborated upon in subsequent subsections. Moreover, the overall logic of the entire code is discussed in the last subsection.

## Implementing IntegratedOutput

The IntegratedOutput function initiates the process by identifing the indices of elements located on the cylinder's surface, subsequently extracting the global coordinates of each node from the matrix containing global coordinate information for subsequent application. Following this, the Gauss Quadrature and Galerkin Projection methods, in conjunction with the shape function, are applied to discretize the governing equation of the stress tensor along the boundary of each element on the cylinder, represented as:

$$\sigma = -p\mathbf{I} + \mu(\nabla\mathbf{U} + \nabla\mathbf{U}^T),$$

whereafter the integrals are computed for each element. The traction in x-direction ($\mathbf{t_x}$) is the first element in $\sigma$ and the traction in y-direction ($\mathbf{t_y}$) is the second elements. Thus the code-wise view is:

$$\begin{aligned} tx(:,p) &= ((2 * fluid.visc. * locgradUx - locP). * normal(:,1) \\ &\quad + fluid.visc. * (locgradUy + locgradVx). * normal(:,2)); \\ ty(:,p) &= ((2 * fluid.visc. * locgradVy - locP). * normal(:,2) \\ &\quad + fluid.visc. * (locgradUy + locgradVx). * normal(:,1)); \end{aligned}$$

The next step is to multiply those tractions for each quadrature point as well as for each element on the cylinder boundary with their corresponding surface area and then report the forces as the output of the IntegratedOutput function.

The verification of this objective is discussed in the Result section.

## Implementing ALEmesh

The alemesh function starts from initializing the ALE displacement of each nodes in the global domain and forming the local to global map. Boundary values of ALE displacement are identified to be zero on the outter boundaries and setted equal to the corresponding displacement on the cylinder boundary. Then with the similar implementing strategy as shown in navierstoke function, Galerkinterms function, and Poisson example, the governing equation of ALE mehs displacement:

$$\begin{aligned} \nabla \cdot \sigma^{\mathbf{m}} &= 0, \in \Omega^s \\ \sigma &= \nabla\eta + \nabla\eta^T + (\nabla \cdot \eta)\mathbf{I} \end{aligned}$$

is firstly transformed into weak form:

$$\int_\Omega \big(\nabla v \cdot \nabla\eta + \nabla v \cdot (\nabla\eta)^T + (\nabla \cdot \eta)(\nabla \cdot v)\big)\, d\Omega = 0$$

Then by applying the Gauss Quadrature and Galerkin Projection methods with the shape function, the corresponding terms are represented by stiffness matrix (please refer to the code for detail information) and the displacement for free nodes are solved and returned as the output of alemesh function. Note the velocity of ALE mesh movement is also computed since the navierstoke function requires it.

## Overall Logic

The overall logic of the code is explained as follows: In each timestep in main function, the IntegratedOutput function computes the force on the cylinder surface at current location, and then the rigidbody function convert the forces into displacement of the cylinder boundary. ALEmesh function then computed new displacement for all nodes other than the outter boundaries and cylinder boundary according to the displacement of the cylinder boundary and compute the velocity of mesh movement. Thus, the global coordinates of each nodes are updated in the main function. Finally, the navierstoke function takes the updated coordinates information and mesh velocity as input, it returns corresponding fluid velocity and pressure distribution.

In the next timestep, all information from the last timestep is stored as "previous" version and repeat the procedure indicated above.

# Results

## Verification of IntegratedOutput

By setting Reynolds number to 100 (viscosity of the fluid is set to be 5 instead of 10) and disable the rigidbody function and alemesh function (equivalent to a stationary cylinder), the code runs for 2000 timesteps with $\Delta t = 0.1s$. The following is the coefficient analysis and their plots. Since the simulated

Table 1: IntegratedOutput Result Comparison

| | Reference Values | Simulated Values |
|---|---|---|
| $\bar{C}_d$ | 1.375 | 1.3753 |
| $C_l^{\max}$ | 0.3352 | 0.3359 |
| $\bar{C}_l^{rms}$ | 0.2368 | 0.2375 |

values agree with the reference values, one can conclude the implementation of the IntegratedOutput function is correct. Also the plots of coefficient of lift and drag are shown below in Figure 1 & 2.

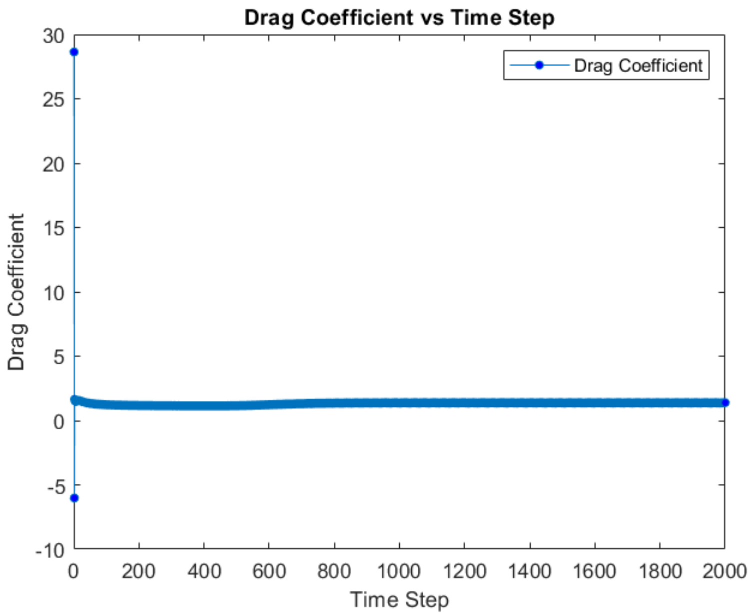


Figure 1: Drag Coefficient

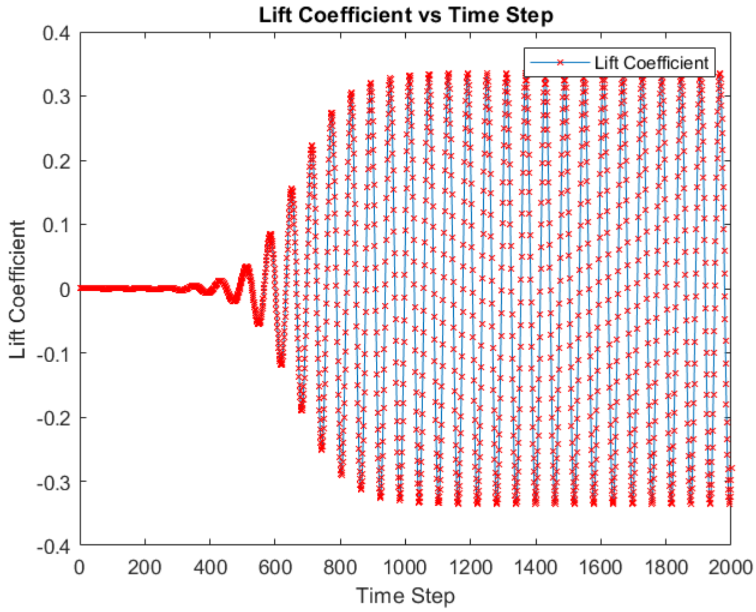


Figure 2: Lift Coefficient

## Verification of ALE mesh

The alemesh function is tested by executing a simulation for a two-degree-of-freedom vortex-induced vibration (VIV) scenario of the cylinder, devoid of any damping coefficient, at a Reynolds number ($Re$) of 200, featuring a mass ratio $m^* = \frac{4m}{\rho f D^2} = 10$; and a reduced velocity $U_r = \frac{U}{f_n D} = 5$, where $m$ denotes the mass of the cylinder, and $f_n = \frac{\sqrt{k/m}}{2\pi}$ represents the natural frequency of the elastically mounted cylinder with $k = k_x = k_y$ being the spring stiffness. The outcomes of this simulation can be compared with the results shown in the Table 2. Note that the simulation of the VIV problem

Table 2: VIV Problem Result Comparison

| | Reference Values[1] | Simulated Values |
|---|---|---|
| $\bar{C}_d$ | 2.0551 | 1.1369 |
| $\bar{C}_l^{rms}$ | 0.0893 | 0.5062 |

is not quite correct. In the simulation, the cylinder tends to flutter after 50 seconds, so the simulated value shown in Table 2 is computed from the first 500 steps. The overall coefficients plots are shown in Figure 4 & 5 below.

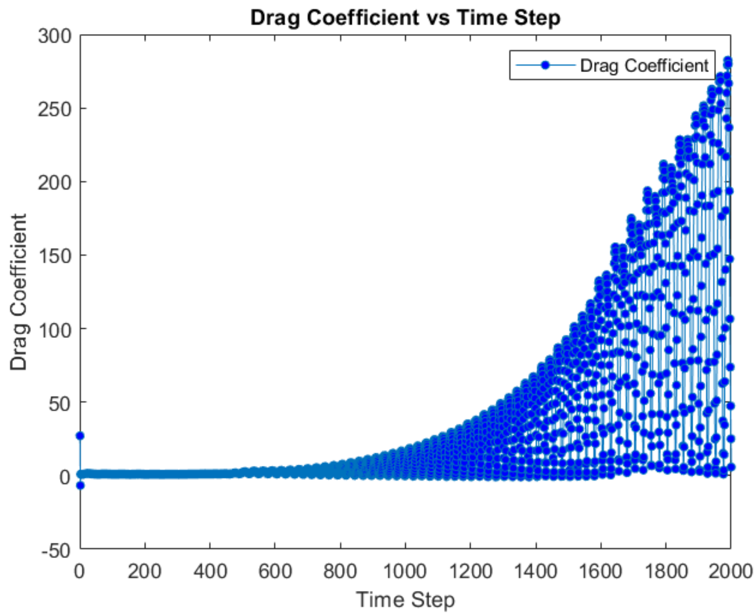


Figure 3: Drag Coefficient

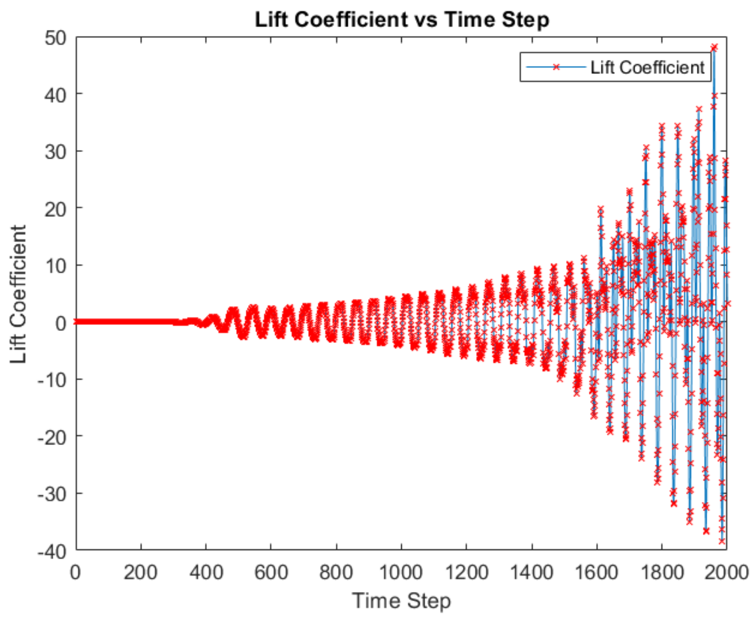


Figure 4: Lift Coefficient

# Conclusion

# Reference

[1]:"On the vortex-induced oscillations of a freely vibrating cylinder in the vicinity of a stationary plane wall" L. Zhong, W. Yao, K. Yag, R. Jaiman, B. C. Khoo, Journal of Fluids and Structures, 65, Pg. 495-526 (2016).