

MECH 503 Homework 1

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1 Question 1

(a)

$$\nu_i u_i = \sum_{i=1}^n \nu_i u_i$$

(b)

$$\delta_{ii} = \sum_{i=1}^n \delta_{ii} = n$$

(c)

$$\nu_{i,i} = \sum_{i=1}^n \frac{\partial \nu_i}{\partial x_i}$$

(d)

$$\delta_{ij,j} = \sum_{j=1}^n \frac{\partial \delta_{ij}}{\partial x_j} = 0$$

(e)

$$\epsilon_{ijk} v_j u_k = \sum_{j=1}^n \sum_{k=1}^n \epsilon_{ijk} v_j u_k$$

(f)

$$\delta_{mi} \delta_{mj} T_{ij} = \sum_{i=1}^n \sum_{j=1}^n \delta_{mi} \delta_{mj} T_{ij} = T_{mm}$$

(g)

$$Q_{ij}T_{ik}Q_{km} = \sum_{j=1}^n \sum_{k=1}^n Q_{ij}T_{ik}Q_{km}$$

(h)

$$\epsilon_{ijk}\sigma_{jk} = \sum_{j=1}^n \sum_{k=1}^n \epsilon_{ijk}\sigma_{jk}$$

2 Question 2

(a)

$$(u \times v) \cdot w = \varepsilon_{ijk}u_jv_kw_i$$

(b)

$$\det T = \varepsilon_{ijk}T_{1i}T_{2j}T_{3k}$$

(c)

$$(\nabla \times v)_i = \varepsilon_{ijk}\partial_jv_k$$

(d)

$$\nabla \cdot v = \partial_iv_i$$

(e)

$$(\nabla \times T)_{ij} = \varepsilon_{jkl}\partial_kT_{il}$$

(f)

$$(\nabla \times (\nabla T))_i = \varepsilon_{ijk}\partial_j(\partial_kT)$$

(g)

$$(\nabla \cdot T)_i = \partial_jT_{ij}$$

3 Question 3

Define a second order tensor C such that

$$\begin{aligned}\mathbf{C} &= \mathbf{A} \cdot \mathbf{B} \\ \iff C_{ij} &= A_{ik} \cdot B_{kj} \\ \det \mathbf{C} &= \epsilon_{ijk} C_{1i} C_{2j} C_{3k} \\ &= \epsilon_{ijk} (A_{1l} B_{li}) (A_{2m} B_{mj}) (A_{3n} B_{nk}) \\ &= \epsilon_{ijk} (A_{1l} A_{2m} A_{3n}) (B_{li} B_{mj} B_{nk})\end{aligned}$$

Now consider the determinant of A and B separately

$$\begin{aligned}\det \mathbf{A} &= \epsilon_{lmn} A_{1l} A_{2m} A_{3n} \\ \det \mathbf{B} &= \epsilon_{ijk} B_{li} B_{mj} B_{nk} \\ \det \mathbf{A} \cdot \det \mathbf{B} &= (\epsilon_{lmn} A_{1l} A_{2m} A_{3n}) (\epsilon_{ijk} B_{li} B_{mj} B_{nk})\end{aligned}$$

Thus, one can conclude the following

$$\det (\mathbf{A} \cdot \mathbf{B}) = \det \mathbf{A} \cdot \det \mathbf{B}$$

4 Question 4

$$\begin{aligned} Q_{ij}Q_{jk}^{-1} &= Q_{ij}Q_{kj}^T = \delta_{ik} \\ \implies Q_{jk}^{-1} &= Q_{kj}^T \\ (QQ^T)_{ij} &= Q_{ik}Q_{kj}^T \\ &= Q_{ik}Q_{jk} \\ &= \delta_{ij} \\ \implies QQ^T &= I \\ (Q^TQ)_{ij} &= Q_{ik}^TQ_{kj} \\ &= Q_{ki}Q_{kj} \\ &= \delta_{ji} \\ &= \delta_{ij} \\ \implies Q^TQ &= I \end{aligned}$$

5 Question 5