CHBE 552 Problem Set 1

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1 Question 1

1.1 Part a

$$
\begin{aligned}
F(\mathbf{x}) &= (x_1 - 2)^4 + (x_1 - 2x_2)^2 \\
\partial F_{x_1} &= 2x_1 - 4x_2 + 4\left(x_1 - 2\right)^3 \\
\partial F_{x_2} &= -4x_1 + 8x_2
\end{aligned}
$$

By solving each partial derivative (set equal to zero), the stationaty point is obtained to be

$$
\begin{pmatrix} {x_1}^* \\ {x_2}^* \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}
$$

The Hessian matrix is determined to be

$$
H_a = \begin{bmatrix} 12\left(x_1 - 2\right)^2 + 2 & -4 \\ -4 & 8 \end{bmatrix}
$$

then, the Hessian matrix at the stationary point is evaluated to be

$$
H_{a,eva.} = \begin{bmatrix} 2 & -4 \\ -4 & 8 \end{bmatrix}
$$

by checking the eigen values of the evaluated Hessian matrix, that stationary point is determined to be a saddle point

1.2 Part b

$$
\begin{aligned}
F(\mathbf{x}) &= 2x_1^3 + x_2^2 + x_1^2 x_2^2 + 4x_1 x_2 + 3 \\
\partial F_{x_1} &= 6x_1^2 + 2x_1 x_2^2 + 4x_2 \\
\partial F_{x_2} &= 2x_1^2 x_2 + 4x_1 + 2x_2
\end{aligned}
$$

By solving each partial derivative (set equal to zero), the stationaty point is obtained to be

$$
\begin{pmatrix} {x_1}^* \\ {x_2}^* \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}
$$

The Hessian matrix is determined to be

$$
H_b = \begin{bmatrix} 12x_1 + 2x_2^2 & 4x_1 x_2 + 4 \\ 4x_1 x_2 + 4 & 2x_1^2 + 2 \end{bmatrix}
$$

then, the Hessian matrix at the stationary point is evaluated to be

$$
H_{b,eva.} = \begin{bmatrix} 0 & 4 \\ 4 & 2 \end{bmatrix}
$$

by checking the eigen values of the evaluated Hessian matrix, that stationary point is determined to be a saddle point

1.3 Part c

$$
\begin{aligned}
F(x) &= 12x^5 - 45x^4 + 40x^3 + 5 \\
F'(x) &= 60x^4 - 180x^3 + 120x^2
\end{aligned}
$$

By solving the first derivative (set equal to zero), the stationaty points are obtained to be

$$
x^* = 0, 1, 2
$$

The second derivative is determined to be

$$
F''(x) = 240x^3 - 540x^2 + 240x
$$

then, the second derivative at the stationary point is evaluated to be

$$F''(x^*) = [0,\ -60,\ 240]$$

by checking the eigen values of the second derivative, those stationary points are determined to be a saddle point($x = 0$),a local maximun($x = 1$), and a local minimum($x = 2$)