

CHBE 552 Problem Set 1

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Jan 24th

1 Question 1

1.1 Part a

$$\begin{aligned}F(\mathbf{x}) &= (x_1 - 2)^4 + (x_1 - 2x_2)^2 \\ \partial F_{x_1} &= 2x_1 - 4x_2 + 4(x_1 - 2)^3 \\ \partial F_{x_2} &= -4x_1 + 8x_2\end{aligned}$$

By solving each partial derivative (set equal to zero), the stationaty point is obtained to be

$$\begin{pmatrix} x_1^* \\ x_2^* \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

The Hessian matrix is determined to be

$$H_a = \begin{bmatrix} 12(x_1 - 2)^2 + 2 & -4 \\ -4 & 8 \end{bmatrix}$$

then, the Hessian matrix at the stationary point is evaluated to be

$$H_{a,eva.} = \begin{bmatrix} 2 & -4 \\ -4 & 8 \end{bmatrix}$$

by checking the eigen values of the evaluated Hessian matrix, that stationary point is determined to be a saddle point

1.2 Part b

$$\begin{aligned}F(\mathbf{x}) &= 2x_1^3 + x_2^2 + x_1^2x_2^2 + 4x_1x_2 + 3 \\ \partial F_{x_1} &= 6x_1^2 + 2x_1x_2^2 + 4x_2 \\ \partial F_{x_2} &= 2x_1^2x_2 + 4x_1 + 2x_2\end{aligned}$$

By solving each partial derivative (set equal to zero), the stationaty point is obtained to be

$$\begin{pmatrix} x_1^* \\ x_2^* \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

The Hessian matrix is determined to be

$$H_b = \begin{bmatrix} 12x_1 + 2x_2^2 & 4x_1x_2 + 4 \\ 4x_1x_2 + 4 & 2x_1^2 + 2 \end{bmatrix}$$

then, the Hessian matrix at the stationary point is evaluated to be

$$H_{b,eva.} = \begin{bmatrix} 0 & 4 \\ 4 & 2 \end{bmatrix}$$

by checking the eigen values of the evaluated Hessian matrix, that stationary point is determined to be a saddle point

1.3 Part c

$$\begin{aligned}F(x) &= 12x^5 - 45x^4 + 40x^3 + 5 \\ F'(x) &= 60x^4 - 180x^3 + 120x^2\end{aligned}$$

By solving the first derivative (set equal to zero), the stationaty points are obtained to be

$$x^* = 0, 1, 2$$

The second derivative is determined to be

$$F''(x) = 240x^3 - 540x^2 + 240x$$

then, the second derivative at the stationary point is evaluated to be

$$F''(x^*) = [0, -60, 240]$$

by checking the eigen values of the second derivative, those stationary points are determined to be a saddle point($x = 0$), a local maximum($x = 1$), and a local minimum($x = 2$)

2 Question 2

2.1 Part a

$$c = \frac{58.05555555555556}{T^{0.3017}} + \frac{56.32848400144T^{0.4925}\alpha}{q^2} + c_c + c_i + c_x$$