MECH 503 Homework 2

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Q1

$$\mathbf{u} = \begin{bmatrix} x_1^3 + x_1x_2^2 + x_3^2 \\ -x_2^3 + x_3^2 \\ -x_1^3 + 2x_1x_2x_3 \end{bmatrix}$$

$$\nabla\mathbf{u} = \begin{bmatrix} 3x_1^2 + x_2^2 & 2x_1x_2 & 2x_3 \\ 0 & -3x_2^2 & 2x_3 \\ -3x_1^2 + 2x_2x_3 & 2x_1x_3 & 2x_1x_2 \end{bmatrix}$$

$$\nabla\mathbf{u}^T = \begin{bmatrix} 3x_1^2 + x_2^2 & 0 & -3x_1^2 + 2x_2x_3 \\ 2x_1x_2 & -3x_2^2 & 2x_1x_3 \\ 2x_3 & 2x_3 & 2x_1x_2 \end{bmatrix}$$

Thus, the infinitisimal strain tensor is determined to be:

$$\varepsilon = \begin{bmatrix} 3.0x_1^2 + 1.0x_2^2 & 1.0x_1x_2 & -1.5x_1^2 + 1.0x_2x_3 + 1.0x_3 \\ 1.0x_1x_2 & -3.0x_2^2 & 1.0x_1x_3 + 1.0x_3 \\ -1.5x_1^2 + 1.0x_2x_3 + 1.0x_3 & 1.0x_1x_3 + 1.0x_3 & 2.0x_1x_2 \end{bmatrix}$$

at point $\mathbf{x} = [1, 0, 1]^T$,

$$\varepsilon = \begin{bmatrix} 3.0 & 0 & -0.5 \\ 0 & 0 & 2.0 \\ -0.5 & 2.0 & 0 \end{bmatrix}$$

The vorticity tensor is

$$\omega = \begin{bmatrix} 0 & 1.0x_1x_2 & 1.5x_1^2 - 1.0x_2x_3 + 1.0x_3 \\ -1.0x_1x_2 & 0 & -1.0x_1x_3 + 1.0x_3 \\ -1.5x_1^2 + 1.0x_2x_3 - 1.0x_3 & 1.0x_1x_3 - 1.0x_3 & 0 \end{bmatrix}$$

at point $\mathbf{x} = [1, 0, 1]^T$,

$$\omega = \begin{bmatrix} 0 & 0 & 2.5 \\ 0 & 0 & 0 \\ -2.5 & 0 & 0 \end{bmatrix}$$

Deformation gradient is

$$\mathbf{F} = \begin{bmatrix} 3x_1^2 + x_2^2 + 1 & 2x_1x_2 & 2x_3 \\ 0 & 1 - 3x_2^2 & 2x_3 \\ -3x_1^2 + 2x_2x_3 & 2x_1x_3 & 2x_1x_2 + 1 \end{bmatrix}$$

The stretch ratio for the point $\mathbf{x} = [1, 0, 1]^T$ is

$$\lambda = \sqrt{22}$$

Thus, the relative length change is

$$\lambda_{rel} = -1 + \sqrt{22} \cong 3.69$$

The relative volum change at the given point is

$$\begin{aligned} \frac{\delta V}{\delta V_0} &= \mathrm{Tr}(\varepsilon) \\ &= 3 \end{aligned}$$

The deviatoric infinitisimal strain tensor is determined to be:

$$\begin{aligned} \mathbf{e} &= \varepsilon - \frac{1}{3}\mathrm{Tr}(\varepsilon) \\ &= \begin{bmatrix} 2.0 & 0 & -0.5 \\ 0 & -1.0 & 2.0 \\ -0.5 & 2.0 & -1.0 \end{bmatrix} \end{aligned}$$

Q2

$$
\begin{aligned}
\mathbf{E_L} &= \begin{bmatrix} 2 & 0.1 & 0 \\ 0.1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\
\mathbf{n} &= [\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0]^T \\
\varepsilon_L &= \mathbf{n}^T \mathbf{E_L} \mathbf{n} \\
&= 1.28
\end{aligned}
$$

Thus, the length of the fiber in the defromed configuration is

$$
\begin{aligned}
l &= \sqrt{1.28 \cdot 2 \cdot 5 + 5} \\
&\cong 4.219
\end{aligned}
$$

Q3

For each deformation gradient, their Jacobian values are computed to be:0.997 1.003 1.000 1.003 1.003. Also note that the angle of rotation given in the question is 0.1 degree which is too small to show in plots, so I changed it into 0.1 rads = 5.73 degree to have better view. The deformed configuration of the line and square elements under different deformation gradient is shown below in Figure 1. It seems like there is a typo in the question statement.

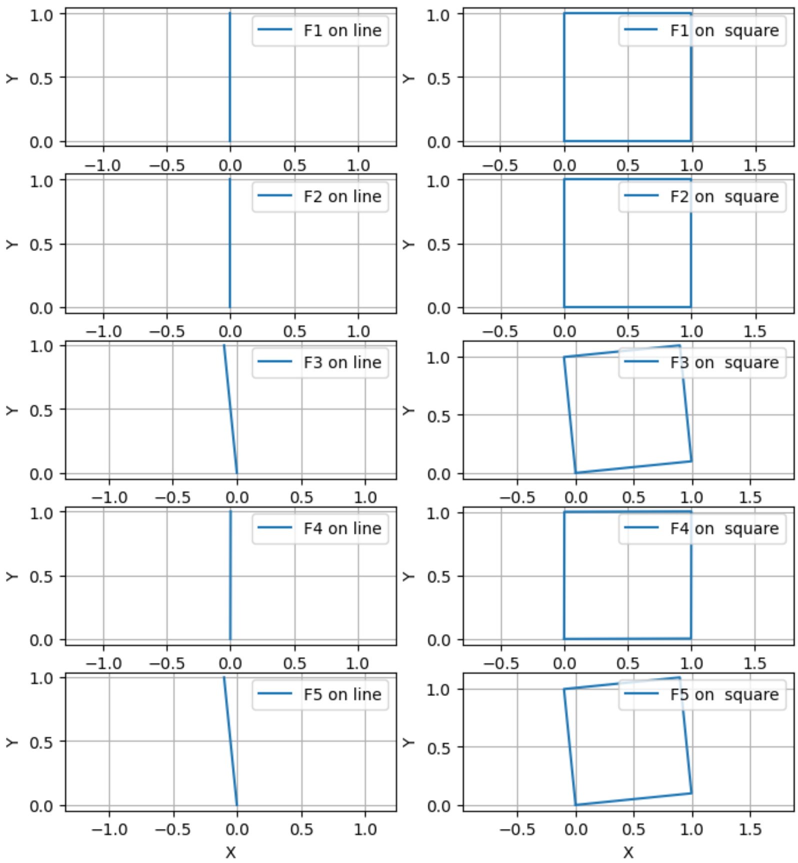

Figure 1: Deformation of Line and Square Element by Each $\mathbf{F}$

But the for the same conclusion. I chose to use $\mathbf{F_5}$ and $\mathbf{F_2F_3}$ to show the $\mathbf{F_5}$

can be decomposed into a rigid body rotation $\mathbf{F_3}$ and a stretch of element $\mathbf{F_2}$. The result is shown in Figure 2.

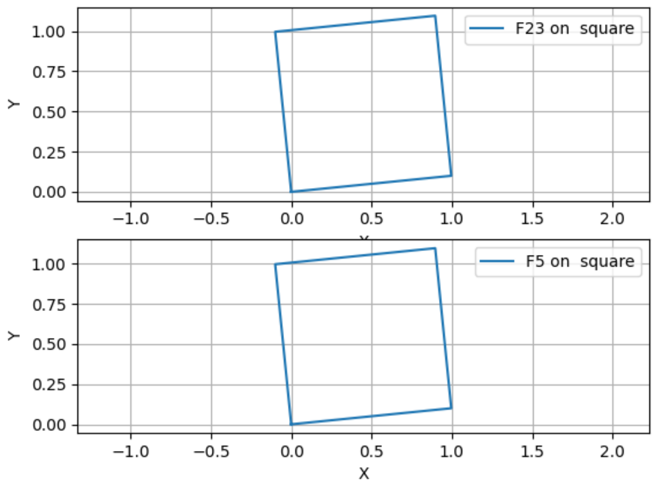

Figure 2: Deformation of the Square Element by $\mathbf{F_5}$ and $\mathbf{F_2 F_3}$

Q4

$$
\begin{aligned}
dl_{f1,F4} &= 0.002 \\
dl_{f2,F4} &= 0.001 \\
d\theta_{f1f2,F4} &= -0.00499 rads = 0.2807 degree
\end{aligned}
$$