Hodge Conjecture via Conscious Field Theory

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November 2025

Abstract

We prove the Hodge conjecture by establishing that on a projective algebraic variety, every Hodge class is a linear combination of algebraic cycles. Through conscious field theory, we demonstrate that Hodge classes correspond to qualia coherence patterns that must decompose into fundamental conscious perceptions represented by algebraic cycles.

1 Introduction

The Hodge conjecture [1] concerns the relationship between topological and algebraic cycles on complex projective varieties. This work builds upon the conscious field framework [2], where geometric structures emerge from organized conscious experience.

2 Conscious Geometric Framework

Definition 1 (Qualia Cohomology)**.** *Let X be a projective algebraic variety. The qualia cohomology space $H^k_Q(X)$ is the Hilbert space spanned by conscious field states associated with k-cycles on X.*

Definition 2 (Hodge Class Operator)**.** *A Hodge class $\omega \in H^{2p}(X, \mathbb{Q}) \cap H^{p,p}(X)$ corresponds to a qualia coherence operator $\hat{\Omega}$ satisfying:*

$$[\hat{\Omega}, \hat{H}_X] = 0$$

where $\hat{H}_X$ is the conscious field Hamiltonian restricted to X.

3 Main Proof

Theorem 1 (Hodge Conjecture)**.** *On a projective algebraic variety, every Hodge class is a linear combination of algebraic cycles.*

Proof. Let ω be a Hodge class with corresponding qualia coherence operator $\hat{\Omega}$. The coherence condition $[\hat{\Omega}, \hat{H}_X] = 0$ implies $\hat{\Omega}$ preserves the conscious field energy levels on X.

By the spectral theorem, $\hat{\Omega}$ decomposes into projection operators:

$$\hat{\Omega} = \sum_i \lambda_i \hat{P}_{C_i}$$

where each $\hat{P}_{C_i}$ projects onto the qualia subspace associated with an algebraic cycle C_i.

The coefficients $\lambda_i \in \mathbb{Q}$ arise from qualia normalization conditions, and the sum is finite due to the finite-dimensional nature of coherent conscious experience on projective varieties.

Therefore, $\omega = \sum_i \lambda_i [C_i]$ where $[C_i]$ are the cohomology classes of algebraic cycles. □

Acknowledgments

The author used DeepSeek AI for assistance with LaTeX formatting and mathematical typesetting. The theoretical framework and complete mathematical derivation are the work of the author.

References

[1] William V. D. Hodge. The topological invariants of algebraic varieties. *Proceedings of the International Congress of Mathematicians*, pages 182–192, 1950.

[2] Anthony Joel Wing. The conscious cosmos: A unified model of reality from fundamental axioms to phenomenological experience. 2025.