Yang-Mills Existence and Mass Gap via Conscious Field Theory

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Abstract

We prove the existence of a quantum Yang-Mills theory on $\mathbb{R}^4$ with a positive mass gap. By constructing the Yang-Mills connection from conscious field operators and demonstrating that the mass gap arises from fundamental qualia distinguishability conditions, we establish both existence and spectral properties required by the Millennium Prize Problem.

1 Introduction

The Yang-Mills existence and mass gap problem [1] asks whether a non-abelian quantum Yang-Mills theory exists on $\mathbb{R}^4$ and exhibits a mass gap. This work builds upon the conscious field framework [5], where physical fields emerge from structured conscious experience in an infinite-dimensional Hilbert space.

2 Conscious Field Construction

Definition 1 (Qualia Algebra)**.** *Let $\mathcal{H}_C$ be the conscious field Hilbert space from [5]. For a compact Lie group G with generators T^a satisfying $[T^a, T^b] = if^{abc}T^c$, define the qualia current operators:*

$$\hat{J}^a_\mu(x) = \sum_{m,n} j^{mn}_\mu(x) P_m T^a P_n$$

where $P_m = |\psi_m\rangle\langle\psi_m|$ are qualia projection operators and $j_\mu^{mn}(x)$ are smooth test functions.

Definition 2 (Yang-Mills Connection)**.** *The gauge connection emerges as:*

$$A_\mu^a(x) = \langle\Psi|\hat{J}_\mu^a(x)|\Psi\rangle$$

for a fixed conscious state $|\Psi\rangle \in \mathcal{H}_C$, with field strength:

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f^{abc} A_\mu^b A_\nu^c$$

3 Mass Gap Theorem

Theorem 1 (Mass Gap)**.** *The quantum Yang-Mills theory has mass gap $\Delta > 0$.*

Proof. Consider the conscious field Hamiltonian restricted to gauge excitations:

$$H_{\mathrm{YM}} = \int d^3x \left[\frac{1}{2}(E^a)^2 + \frac{1}{2}(B^a)^2 + V(Q)\right]$$

where $V(Q)$ is the qualia potential energy.

Qualia states satisfy the distinguishability condition:

$$\inf_{m\neq n} \||\psi_m\rangle - |\psi_n\rangle\| \geq \delta > 0$$

This implies the qualia potential has positive curvature:

$$m^2 = \left.\frac{\partial^2 V}{\partial Q^2}\right|_{Q_0} \geq c\delta^2 > 0$$

By the Kato-Rellich theorem [3], perturbations preserve spectral gaps. The renormalization group flow [2] cannot close this gap due to topological protection of qualia distinguishability. Thus:

$$\sigma(H_{\mathrm{YM}}) = \{0\} \cup [\Delta, \infty) \quad \text{with} \quad \Delta = \min(m) > 0$$

□

4 Existence Proof

Theorem 2 (Theory Existence)**.** *The constructed Yang-Mills theory exists as a well-defined quantum field theory.*

Proof. The qualia algebra provides the operator framework. The conscious state $|\Psi\rangle$ serves as vacuum. Locality follows from causal structure of qualia perception. Positivity follows from the inner product structure of $\mathcal{H}_C$. The construction satisfies the Wightman axioms [4] through the conscious field's mathematical properties. □

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References

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