CS411 COMPUTER GRAPHICS FALL 2021 BRACK TURNER

Brack Turner
CS411
AS4 Problems

1a $(1,1)$ $u=0$ $(4,7)$ $u=1$

$x = x_0 + (x_1 - x_0)\cdot u$ $\quad 1+(4-1)\cdot(0.3) = 1.9$

$y = y_0 + (y_1 - y_0)\cdot u$ $\quad 1+(7-1)\cdot 0.3 = 2.8$

$= (1.9, 2.8)$

b. Parametric continuity defines curves as a function. Geometric continuity defines the curve of its shape. Using piecewise interpolation is harder to complete with curves with many points. By using lower order polynomials there is an advantage of piecing the curve into smaller pieces

c. $(0,0)$ $(2,2)$ tangents $(1,1)$ $(1,-1)$

$$P(0) = P_k \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$P(1) = P_{k+1} \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$P'(0) = dP_k \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$P'(1) = dP_{k+1} \begin{bmatrix} 3 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}}_{\text{Matrix Hermite}} \underbrace{\begin{bmatrix} 0 \\ 2 \\ 1 \\ 1 \end{bmatrix}}_{P} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

d. $(2,2)$ $(4,2)$ tangent $(1,1)$ $(1,-1)$

$u=0$ $u=1$ $P(0.5) = [0.5^3\ 0.5^2\ 0.5\ 1]\, M_H P$

Hermite

$$x(0.5) = [0.5^3\ 0.5^2\ 0.5\ 1]\, M_H \begin{bmatrix} 2 \\ 4 \\ 1 \\ 1 \end{bmatrix}$$

$$y(0.5) = [0.5^3\ 0.5^2\ 0.5\ 1]\, M_H \begin{bmatrix} 2 \\ 2 \\ 1 \\ -1 \end{bmatrix}$$

$$x(0.5) = [0.125\ 0.25\ 0.5\ 1] \begin{bmatrix} -2 \\ 3 \\ 1 \\ 2 \end{bmatrix}$$

$$-0.25 + 0.75 + 0.5 + 2 = 3$$

$$y(0.5) = [0.125\ 0.25\ 0.5\ 1] \begin{bmatrix} 0 \\ -1 \\ 1 \\ 2 \end{bmatrix}$$

$$-0.25 + 0.5 + 2 = 2.25$$

$$x,y(0.5) = (3, 2.25)$$

c. $$b(u) = M_H^T u = \begin{bmatrix} 2u^3 - 3u^2 + 1 \\ -2u^3 + 3u^2 \\ u^3 - 2u^2 + u \\ u^3 - u^2 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.125 \\ -0.125 \end{bmatrix}$$

$$x(0.5) = 0.5(2) + 0.5(4) + 1(0.125) + -1(0.125) = 3$$

$$y(0.5) = 0.5(2) + 0.5(2) + 0.125(1) - 0.125(-1) = 2.25 = (3, 2.25)$$

f $(1,1)(2,2)(4,2)(5,1)$ $u=0.5$

$u=0$ $u=1$ tension $=0.5$

$$M_c = \begin{bmatrix} -0.5 & 1.5 & -1.5 & 0.5 \\ 1 & -2.5 & 2 & -0.5 \\ -0.5 & 0 & 0.5 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

x coord $= [1,2,4,5]$

y coords $= [1,2,2,1]$

$M_cX = M_c X X$ $M_cY = M_c X Y$

$M_{cX} = [-1, 1.5, 1.5, 2]$

$M_cY = [0, -0.5, 0.5, 2]$

$U_x = U_y = [0.125, 0.25, 0.5, 1]$

$M_cX \times U_x = -0.125 + 0.375 + 0.75 + 2 = 3$

$M_cY \times U_y = 0 + (-0.125) + 0.25 + 2 = 2.125$

$u=0.5$ $(3, 2.125)$

g. $c_0(u) = -su^3 + 2su^2 - su = -0.0625$

$c_1(u) = (2-s)u^3 + (s-3)u^2 + 1 = 0.5625$

$c_2(u) = (s-2)u^3 + (3-2s)u^2 + su = 0.5625$

$c_3(u) = su^3 - su^2 = -0.0625$

$$x(u) = C_0(u) P_{k-1} + C_1(u) P_k + C_2(u) P_{k+1} + C_3(u) P_{k+2}$$

$$x(u) = -0.0625(1) + 0.5625(2) + 0.5625(4) + -0.0625(5) = 3$$

$$y(u) = -0.0625(1) + 0.5625(2) + 0.5625(2) + -0.0625(1) = 2.125$$

$(3, 2.125)$

H. $(1,1)\ (2,2)\ (4,2)\ (5,1)$ $u=0.5$

$u=0$ $u=1$

$$P(u) = U^T M_B P$$

$$M_B = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad U^T = [0.125\ 0.25\ 0.5\ 1]$$

$$P_x = \begin{bmatrix} 1 \\ 2 \\ 4 \\ 5 \end{bmatrix} \quad P_y = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}$$

$M_B P_x = [2, 3, 3, 1]$

$M_B P_y = [0, -3, 3, 1]$ $> U^T$

$U^T M_B P_x = 3$ $U^T M_B P_y = 1.75$

$(x,y)_{(u=0.5)} = (3, 1.75)$

I. $b_u = M_B^T u = \begin{bmatrix} (1-u)^3 \\ 3u(1-u)^2 \\ 3u^2(1-u) \\ u^3 \end{bmatrix} = \begin{bmatrix} 0.125 \\ 0.375 \\ 0.375 \\ 0.125 \end{bmatrix}$

$P_x = 0.125(1) + 0.375(2) + 0.375(4) + 0.125(5)$

$P_y = 0.125(1) + 0.375(2) + 0.375(2) + 0.125(1)$

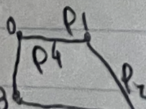
$P(x,y)_{(u=0.5)} = (3, 1.75)$

J.

K. Bezier curves are easier

a. The zeroes or roots are at $u=0$ and/or $u=1$. For each blending polynomial without a zero in its interval it must be smooth.

All the polynomials must be inside its convex hull.



And they cannot be far from their control points.

l. (1,1) (2,2) (4,2) (5,1)

knot vector [0,1,2,3,4,5,6]

$u=2$

$(x,y)_{(u=2)}$ (1.54, 1.54)

m. $B_0^2(u) =$