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A

1. $2\cdot\begin{pmatrix}1\\2\\3\end{pmatrix} - \begin{pmatrix}4\\5\\6\end{pmatrix} = \begin{pmatrix}-2\\-1\\0\end{pmatrix}$

2. $\hat{a} = \frac{1}{|a|} \quad \left(\frac{1}{\sqrt{14}} \quad \sqrt{\frac{2}{7}} \quad \frac{3}{\sqrt{14}}\right)$

3. $\|a\| = \sqrt{14}$, not two dimensional

4. $\cos\alpha = \frac{1}{\sqrt{14}} = \frac{\sqrt{14}}{14}$ $\quad \cos\beta = \frac{2}{\sqrt{14}} = \frac{\sqrt{14}}{7}$

$0.267 \qquad 0.534$

$\cos\gamma = \frac{3}{\sqrt{14}} = \frac{3\sqrt{14}}{14} \quad 0.801$

5. $|a| = \sqrt{14} \quad |b| = \sqrt{77}$

$\frac{u\cdot v}{|u|\cdot|v|} = \frac{32}{\sqrt{14}\sqrt{77}} = \frac{16\sqrt{22}}{77} = n$

$\frac{180\cos(n)}{\pi} = 13°$

6. $a\cdot b = \begin{bmatrix}1\\2\\3\end{bmatrix}\cdot\begin{bmatrix}4\\5\\6\end{bmatrix} = \begin{bmatrix}4\\10\\18\end{bmatrix} = 32$

$b\cdot a \begin{bmatrix}4\\5\\6\end{bmatrix}\cdot\begin{bmatrix}1\\2\\3\end{bmatrix} = \begin{bmatrix}4\\10\\18\end{bmatrix} = 32$

7. $\langle a,b\rangle \quad b^T a$

$[4\ 5\ 6]\begin{bmatrix}1\\2\\3\end{bmatrix} = [4\ 10\ 18] = [32]$

$\cos\theta = \frac{\langle a,b\rangle}{\|a\|\,\|b\|} = \frac{32}{\sqrt{14}\sqrt{77}} = \theta = \cos^{-1}\left(\frac{32}{7\sqrt{22}}\right)$

B. $2\begin{pmatrix}1&2&3\\4&-2&3\\0&5&-1\end{pmatrix}-\begin{pmatrix}1&2&1\\2&1&-4\\3&-2&1\end{pmatrix}=\begin{pmatrix}1&2&5\\6&-5&10\\-3&12&-3\end{pmatrix}$

2. $AB=\begin{pmatrix}14&-2&-4\\9&0&15\\7&7&-21\end{pmatrix}$

$BA=\begin{pmatrix}9&3&8\\6&-18&13\\-5&15&2\end{pmatrix}$

3. $AB^T=AB$ rotate $=\begin{pmatrix}14&9&7\\-2&0&7\\-4&15&-21\end{pmatrix}$

$B^TA^T=\begin{pmatrix}14&9&7\\-2&0&7\\-4&15&-21\end{pmatrix}$

4. $|A|=\begin{pmatrix}1&2&3\\4&-2&3\\0&5&-1\end{pmatrix}\to\begin{pmatrix}1&2&3\\0&-10&-9\\0&5&-1\end{pmatrix}\to$

$\begin{pmatrix}1&2&3\\0&-10&-9\\0&0&-\frac{11}{2}\end{pmatrix}=55$

$|C|=\begin{pmatrix}1&2&3\\4&5&6\\-1&1&3\end{pmatrix}\to\begin{pmatrix}1&2&3\\0&-3&-6\\-1&1&3\end{pmatrix}\to$

$\begin{pmatrix}1&2&3\\0&-3&-6\\0&0&0\end{pmatrix}=0$

5. $B=\begin{bmatrix}1&2&1\\2&1&-4\\3&-2&1\end{bmatrix}=$

$2+2-4=0$

$3-4+1=0$

$6-2-4=0$

orthogonal set

6. $A^{-1}=\left[\begin{matrix}1&2&3\\4&-2&3\\0&5&-1\end{matrix}\right]\left[\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}\right]=$

$\begin{bmatrix}-\frac{13}{55}&\frac{17}{55}&\frac{12}{55}\\\frac{4}{55}&-\frac{1}{55}&\frac{9}{55}\\\frac{4}{11}&-\frac{1}{11}&-\frac{2}{11}\end{bmatrix}$

11. $Bx = d$

$$B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} \quad \det(B) = -42$$

B^{-1} exists

$$x = B^{-1} d$$

$$\frac{1}{42} \begin{bmatrix} -7 & -9 & -9 \\ -14 & -2 & 6 \\ -7 & 8 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} =$$

$$x = \begin{bmatrix} 1 \\ 0 \\ -\frac{1}{3} \end{bmatrix}$$

$B^{-1} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$

$B^{-1} = \begin{bmatrix} \frac{1}{6} & \frac{2}{21} & \frac{3}{14} \\ \frac{1}{3} & \frac{1}{7} & -\frac{1}{7} \\ \frac{1}{6} & \frac{-4}{21} & \frac{1}{14} \end{bmatrix}$

7. C^{-1}

$\det(C) = 1\begin{vmatrix} 5 & 6 \\ 1 & 3 \end{vmatrix} - 2\begin{vmatrix} 4 & 6 \\ -1 & 3 \end{vmatrix}$
$+ 3\begin{vmatrix} 4 & 5 \\ -1 & 1 \end{vmatrix}$

$15 - 6 - 36 + 27 = 0$

$C^{-1} = \text{DNE}$

8. $Ad = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 14 \\ 9 \\ 7 \end{bmatrix}$

9. $\frac{\vec{a} \cdot \vec{b}}{\vec{b} \cdot \vec{b}} \cdot \vec{b} = \frac{14}{14} \langle 1,2,3 \rangle \; (1,2,3)$

10. Colums A

$1(1,4,0) +$
$2(2,-2,5) +$
$3(3,3,-1) =$
$(14,9,7)$

$\frac{4,-2,3 \; b}{b \cdot b} = \frac{9}{14} \langle 1,2,3 \rangle \; \frac{9}{14} \; \frac{9}{7} \; \frac{27}{14}$

$\frac{0,5,-1 \; b}{b \cdot b} = \frac{7}{14} \langle 1,2,3 \rangle$

$\frac{1}{2}, 1, \frac{3}{2}$

8. $b \cdot \hat{a} = b \cdot \frac{a}{|a|} = \frac{32}{\sqrt{14}}$

9. $s = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ $a \cdot s = (1,2,3) \cdot (1,1,-1)$

$1 + 2 + -3 = 0$

$a \cdot s = 0$

10. Not possible # col in a ≠ rows in b

11. $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} = \hat{i}(12-15) - \hat{j}(6-12) + \hat{k}(5-8)$

$-3\hat{i} + 6\hat{j} - 3\hat{k}$

12. $a = \frac{b+c}{3}$ $a = \frac{b}{3} + \frac{c}{3}$

$(1,2,3) = \frac{(4,5,6)}{3} + \frac{(-1,1,3)}{3}$

13. $a^T b = [1\ 2\ 3] \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = 32$

$ab^T = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} [4\ 5\ 6] = \begin{bmatrix} 4 & 5 & 6 \\ 8 & 10 & 12 \\ 12 & 15 & 18 \end{bmatrix}$