

GRADUATE STUDENT STAT 840 A1

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Problem 7

(a)

```
d = function(x)
{
  return((x^3)*exp(x))
}

get_SE = function(x) # return variance of sample mean
{
  n = length(x)
  mu = sum(x) / n
  s_sq = sum((x - mu)^2) / ((n-1)*n) # notes p 30
  return(sqrt(s_sq))
}

get_CI = function(xbar, se, conf=.95)
{
  a = 1 - conf
  L = xbar + qnorm(a/2)*se
  U = xbar + qnorm(1 - a/2)*se
  return(c(L,U))
}

n = 100000
z = rnorm(n)

deltas = d(z)
theta_hat = mean(deltas)
SE = get_SE(deltas)
CI = get_CI(theta_hat, SE)
# whenever reporting a MC estimate, report n, SE of estimate, and CI
c(n, theta_hat, SE, CI)

## [1] 1.000000e+05 6.880651e+00 1.729775e-01 6.541621e+00 7.219681e+00
```

(b)

This is adapted from example 22 in the notes. It yields the exact values produced there. In this problem, the correlation seems to be negligible.

```
# antithetic
n = 100000
```

```

z = rnorm(n/2)
deltas = (d(z) + d(-z))/2
theta_as = mean(deltas)
se_as = get_SE(deltas)
ci_as = get_CI(theta_as, se_as)
c(n, theta_as, se_as, ci_as)

## [1] 1.000000e+05 6.765762e+00 2.533267e-01 6.269251e+00 7.262273e+00

cor(d(z), d(-z))

## [1] -0.01024348

```

(c)

We see that the variances are about the same. In this case, we do not get any variance reduction using antithetic sampling because the correlation between the estimators θ_1 and θ_2 is near 0.

```

nn = 1000
mc_simple = rep(0, nn)
mc_as = rep(0, nn)

for (i in 1:nn)
{
  n = 10000

  # standard
  z1 = rnorm(n)
  theta_hat = mean(d(z1))

  # antithetic
  z2 = rnorm(n/2)
  theta_as = mean((d(z2) + d(-z2))/2)

  mc_simple[i] = theta_hat
  mc_as[i] = theta_as
}

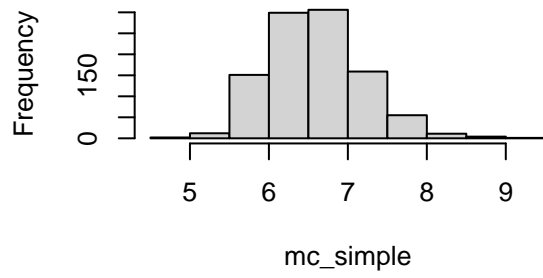
par(mfrow = c(2,2))
p1 = hist(mc_simple)
p2 = hist(mc_as)
plot( p1, col=rgb(0,0,1,1/4), xlim=c(0,10))
plot( p2, col=rgb(1,0,0,1/4), xlim=c(0,10), add=T)

c(var(mc_simple), var(mc_as))

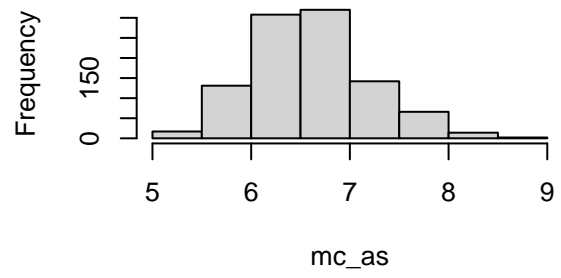
## [1] 0.3542654 0.3449027

```

Histogram of mc_simple



Histogram of mc_as



Histogram of mc_simple

