GRADUATE STUDENT STAT 840 A1

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Problem 1

(a) + (b)

A likelihood:

$$f_A(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x} = \frac{\beta^3}{\Gamma(3)} x^{3-1} e^{-\beta x} = \frac{\beta^3}{2} x^2 e^{-\beta x} = \frac{\theta^3}{2} x^2 e^{-\theta x}$$

$$f_A(2) = 2\theta^3 e^{-2\theta}$$

B likelihood:

$$f_B(x) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{2^x \theta^x e^{-2\theta}}{x!}$$

$$f_B(3) = \frac{8}{6} \theta^3 e^{-2\theta}$$

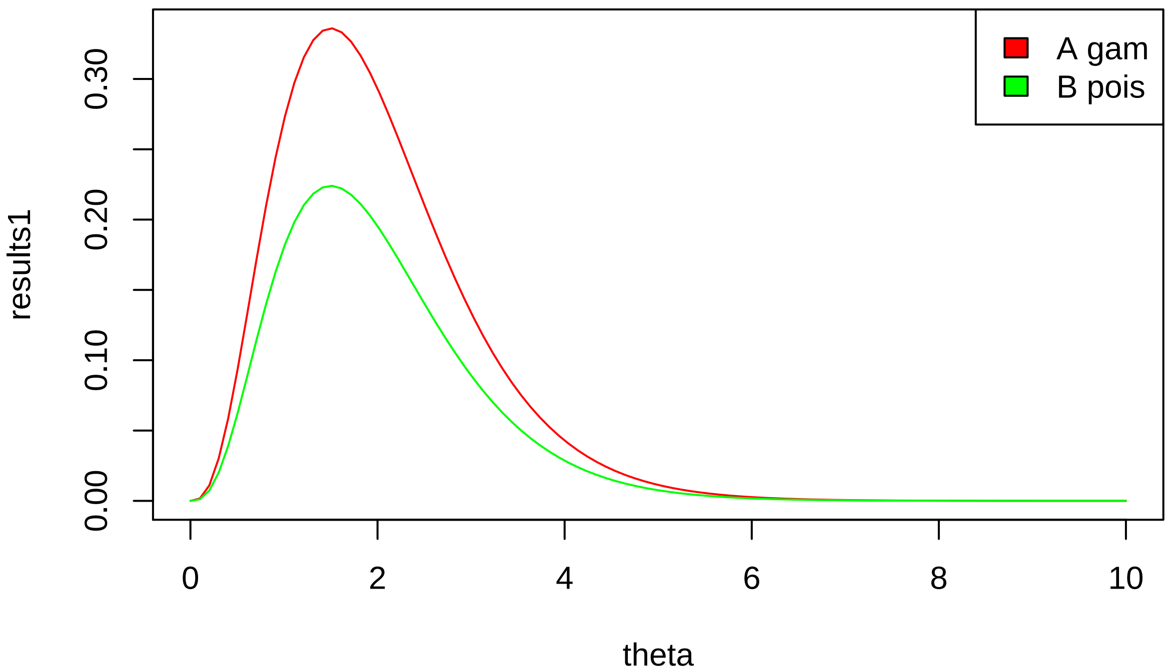
In particular,

$$f_A(2) = \frac{6}{4} f_B(3)$$

```r
n = 100
results1 = rep(0, n)
results2 = rep(0, n)
theta = seq(from = 0, to = 10, length.out = n)

for (i in 1:n)
{
  results1[i] = dgamma(2, 3, theta[i])
  results2[i] = dpois(3, theta[i]*2)
}

plot(theta, results1, type='l', col='red')
lines(theta, results2, col='green')
legend(x = 'topright',
       legend=c('A gam', 'B pois'),
       fill = c('red','green'))
```



(c)

Comparing the likelihoods, we see that L_A is proportional to L_B as a function of θ. The MLE will thus be the same, yielding $\theta^{\star} = 3/2$.

$$loglik = c + 3log\theta - 2\theta$$
$$\frac{d}{d\theta}(c + 3log\theta - 2\theta) = 0$$
$$\frac{3}{\theta} = 2$$
$$\theta = \frac{3}{2}$$

Discussion:

According to the Strong Likelihood Principle, the likelihood function contains all information x has about θ. If x, y are two observations (possibly from different models) satisfying $L_A = cL_B$, for every θ, they carry the same information about θ and must lead to identical inference. The constraints are satisfied in this situation, meaning both experimenters A and B have the same inference about θ, given their chosen models and the observations x, y. Their likelihood functions are proportional. So A and B make identical inferences about θ.