GRADUATE STUDENT STAT 840 A1

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Problem 2

(a)

We know that if $X \sim N(\theta, 1)$ then $\bar{X}_n \sim N(\theta, \frac{1}{n})$

Likelihood of $\bar{X}_{10}$:

$$f(x) = \frac{\sqrt{10}}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\sqrt{10}(x-\theta)\right)^2}$$

```
thetas = seq(from = -1, to = 6, length.out = 100)
lik_a = dnorm(3.5, thetas, 1/sqrt(10))
```

(b)

We know $\sqrt{n}(\bar{x} - \theta) \sim N(0, 1)$, which allows us to express the given condition as a probability statement:

$$\begin{aligned} 0 < \bar{x}_{10} < 5 \\ 0 - \theta < \bar{x}_{10} - \theta < 5 - \theta \\ -\sqrt{10}\theta < \sqrt{10}(\bar{x}_{10} - \theta) < \sqrt{10}(5 - \theta) \\ \Phi(\sqrt{10}(5 - \theta)) - \Phi(-\sqrt{10}\theta) \end{aligned}$$

```
lik_b = pnorm(sqrt(10)*(5 - thetas)) - pnorm(-sqrt(10)*thetas)
```

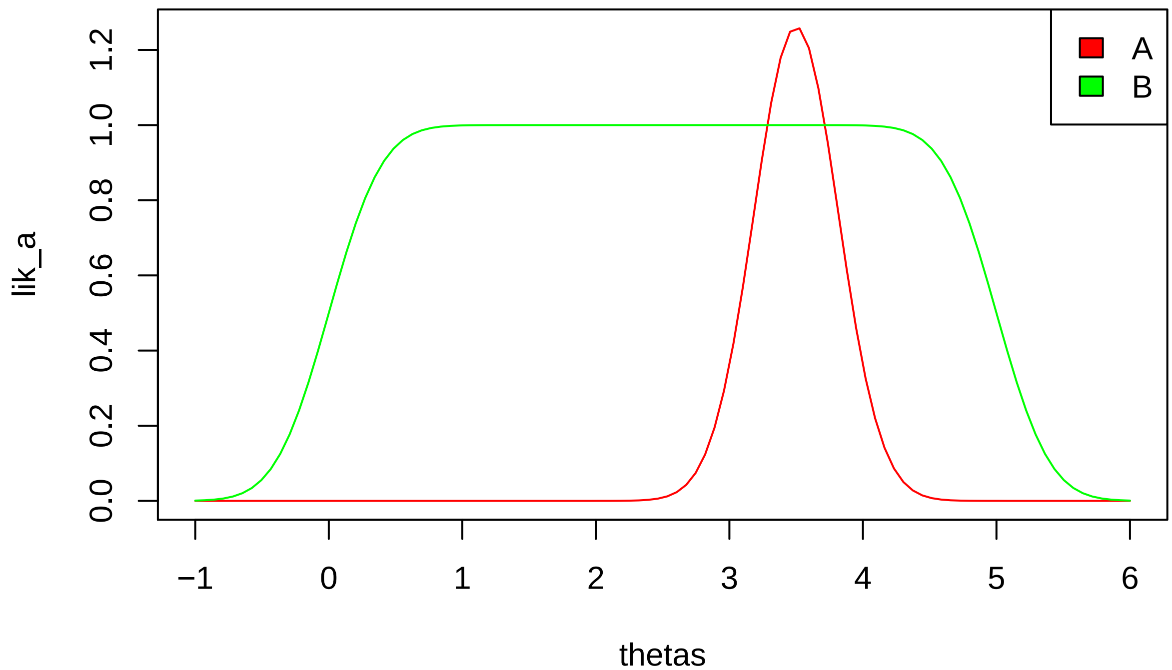
(c)

For part A, the likelihood is highest when $\theta = \bar{x}$, which is the usual MLE. It decreases around 3.5 in accordance to the sample mean's variance, which is $1/\sqrt{10} \approx 0.31$

For part B, we only know that the mean is in an interval. Standardizing this inequality gives us a probability statement. If θ was 0 then we have 50% chance the mean is to the right, and if θ is 5, 50% chance it is to the left. Since $\sigma = 0.31$, 3 standard deviations is approximately 1, we see that we quickly climb to 1 in the interval $(0, 5)$. This shows that if θ is here, then we are almost guaranteed to have $\bar{x}$ in $(0, 5)$, except near the individual points 0 and 5.

Naturally, having more precise information about $\bar{x}$ makes the likelihood that much more informative.

```
plot(thetas, lik_a, type='l', col='red')
lines(thetas, lik_b, col='green')
legend(x = 'topright',
       legend=c('A', 'B'),
       fill = c('red','green'))
```


lik_a
0.0
0.2
0.4
0.6
0.8
1.0
1.2
−1
0
1
2
3
4
5
6
thetas
A
B