GRADUATE STUDENT STAT 840 A4

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Problem 3

a)

Each Y_i is pointed to by X, Z_i. This yields:

$$f(Y_1 \mid X, Z_1)$$
$$f(Y_2 \mid X, Z_2)$$
$$f(Y_3 \mid X, Z_3)$$
$$f(Y_4 \mid X, Z_4)$$

Each Z_i is not pointed to by anything. This yields:

$$f(Z_1)$$
$$f(Z_2)$$
$$f(Z_3)$$
$$f(Z_4)$$

Likewise, X is not pointed to by anything, this yields:

$$f(X)$$

Altogether, this tells us that the joint probability is:

$$f(X, Y_1, Y_2, Y_3, Y_4, Z_1, Z_2, Z_3, Z_4) = f(X)f(Z_1)f(Z_2)f(Z_3)f(Z_4)f(Y_1 \mid X, Z_1)f(Y_2 \mid X, Z_2)f(Y_3 \mid X, Z_3)f(Y_4 \mid X, Z_4)$$

b)

The Markov property says that

$$W \perp \tilde{W} \mid \pi_W$$

where π_W are the parents of W, and $\tilde{W}$ are all other variables except the parents and descendants of W (Lec 21, slide 11). Now, X is not a parent nor descendant of Z_j for any j. Additionally, Z_j is not a parent nor descendant of X for any j (it is a collider). Thus we can substitute Z_j for W, and substitute X for $\tilde{W}$ in the above result. Since Z_j has no parents, π_{Z_j} is the nullset and we substitute the nullset for π_W in the result. This yields $X \perp Z_j$.