GRADUATE STUDENT STAT 840 A2

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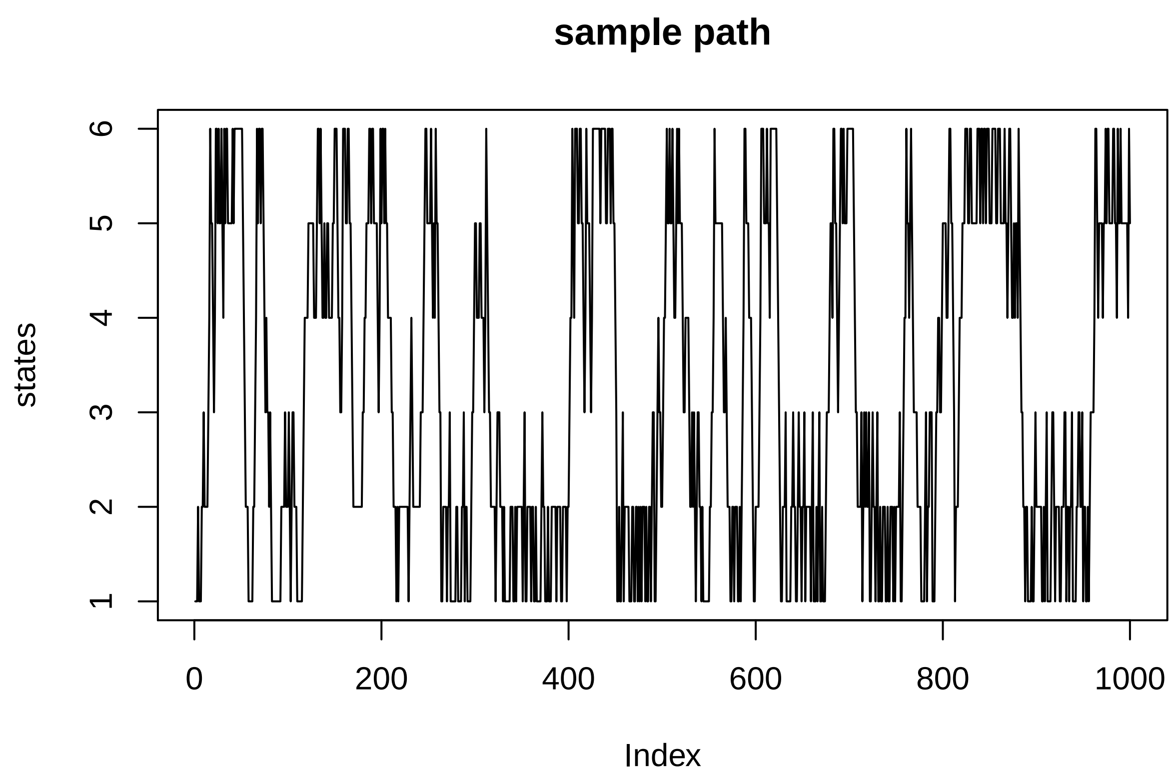
Problem 5

a)

Start with state 1. At time n, use `sample()` to randomly choose the next state X_n using the transition probabilities corresponding to state X_{n-1}. They are located in the row of the matrix corresponding to that state.

```
P = matrix(ncol = 6, nrow = 6)
P[1,] = c( .5,  .5,   0,   0,   0,   0)
P[2,] = c(.25,  .5, .25,   0,   0,   0)
P[3,] = c(.25, .25, .25, .25,   0,   0)
P[4,] = c(  0,   0, .25, .25, .25, .25)
P[5,] = c(  0,   0,   0, .25,  .5, .25)
P[6,] = c(  0,   0,   0,   0,  .5,  .5)
Pt = t(P)

n = 1000
states = rep(NA, n)
states[1] = 1

for (i in 2:n)
{
  # https://stats.stackexchange.com/questions/67911/how-to-sample-from-a-discrete-distribution
  states[i] = sample(x = c(1,2,3,4,5,6), size = 1, replace = T, prob = P[states[i-1],])
}

par(mfrow=c(1,1))
plot(states, type='l', main='sample path')
```



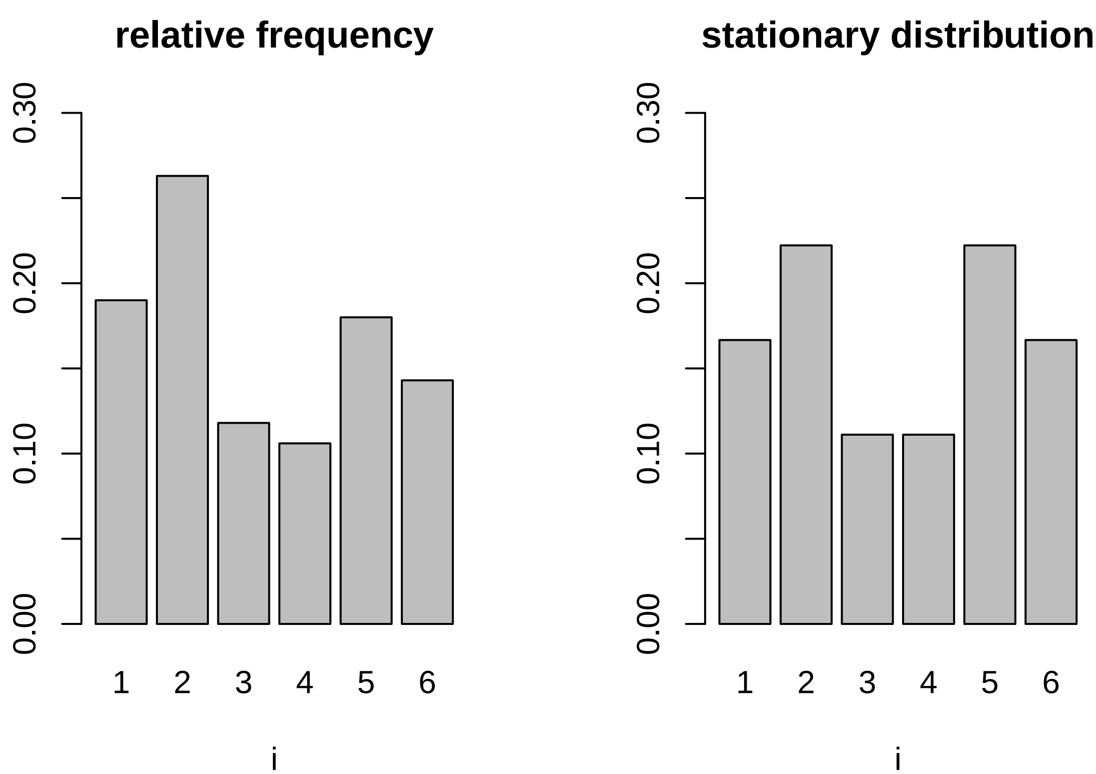
b)

Compute relative frequencies by counting the proportion of time spent in each state. We can find the stationary distribution π by solving the equation $P^T \pi = \pi$ which equates to finding the eigenvector with eigenvalue 1.

```
# compute relative frequency of simulation
freq = rep(NA,6)
for (i in 1:6)
{
  freq[i] = sum(states == i) / n
}

# guess value of stationary distribution pi
eig_ = eigen(Pt)

# the eigenvalue 1 is in first place
for (i in 1:6)
  print(eig_$values[i])
```

```
## [1] 1+0i
## [1] 0.9330127+0i
## [1] 0.25+0i
## [1] 0.25-0i
## [1] 0.0669873+0i
## [1] 1.971719e-16+0i
```

```
# its corresponding eigenvector
pi_ = matrix(nrow=6, ncol=1)
pi_[,1] = as.numeric(eig_$vectors[,1])
pi_ = pi_ / sum(pi_) # probabilities need to sum to 1

# compare relative freq to stationary dist
par(mfrow=c(1,2))
barplot(freq,
        ylim=c(0,.3),
        names.arg = c(1,2,3,4,5,6),
        main='relative frequency',
        xlab='i')
barplot(pi_[,1],
        ylim=c(0,.3),
        names.arg = c(1,2,3,4,5,6),
        main='stationary distribution',
        xlab='i')
```



c)

Check that the distribution is stationary because taking a step, meaning multiplying by P^T, does not change the distribution.

```
# compare equality up to numeric error
(pi_ - Pt %*% pi_) < 0.0000000000000001
```

```
##      [,1]
```

```
## [1,] TRUE
## [2,] TRUE
## [3,] TRUE
## [4,] TRUE
## [5,] TRUE
## [6,] TRUE
```