

Program Verification with Dafny (Part 2)

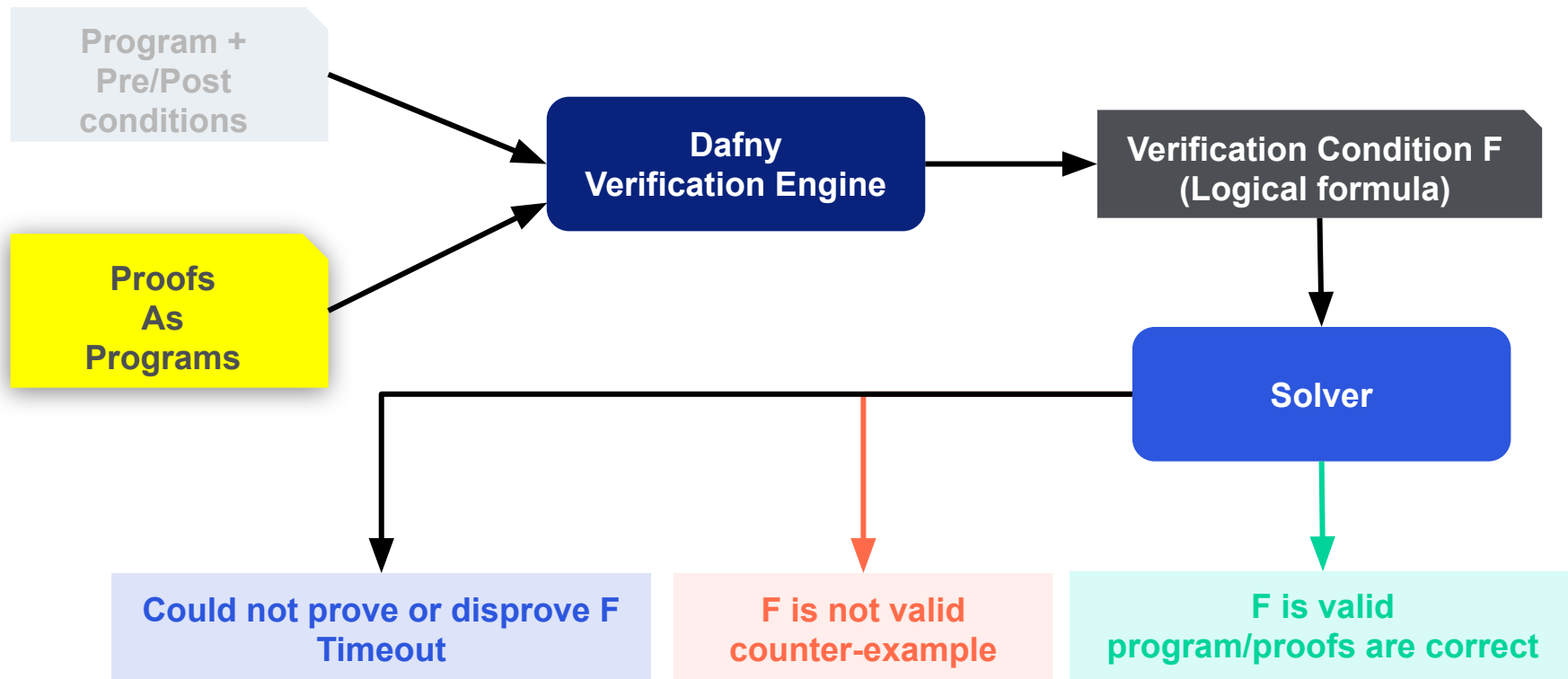
Franck Cassez

Trustworthy Smart Contracts Team, ConsenSys Software R&D

<https://franck44.github.io/>

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Dafny – A Verification-Friendly Programming Language



Objectives of this session

1. **Write functional/recursive specifications**
2. **Write proofs as programs (lemmas)**
 - 2.1. **Calculations**
 - 2.2. **Induction**
3. **Define/Use Abstract Data Types: Trees**
 - 3.1. **Write type and (functional) algorithm**
 - 3.2. **Write proofs (induction)**
4. **More induction: Lists**
 - 4.1. **Define lists, length, append, reverse functions**
 - 4.2. **Prove idempotence theorem for reverse**

Examples in VSCode

Basic CLI commands (Reminder)

#help

dafny /help

Compile (in memory) and execute Main:

dafny /noVerify /compile:4 training1.dfy

Verify a file, don't compile

dafny /dafnyVerify:1 /compile:0 training1.dfy

Further reading for this session:

<https://cseweb.ucsd.edu/~npolikarpova/publications/vstte13.pdf>

Example 0

```
function powerOf2 (n: nat) :  
nat  
  decreases n  
{  
  if n == 0 then  
    1  
  else  
    2 * powerOf2 (n - 1)  
}
```

Task 1

1. Prove $2^n + 2^n = 2^{(n + 1)}$

Use (Verified) Calculations

Task 2

1. Monotonicity: $n \leq m \Rightarrow 2^n \leq 2^m$

2. Identity1: $2^n * 2^m == 2^{(n + m)}$

Use (Verified) Induction

Example 1.a

```
datatype Tree =  
  Leaf  
  | Node(left: Tree, right: Tree)
```

```
function height(root : Tree) : nat  
  ensures height(root) >= 1  
  decreases root  
{  
  match root  
    case Leaf => 1  
    case Node(lc, rc) => 1 + max(height(lc), height(rc))  
}
```

Task

1. Define `nodesCount(root)`
2. Define `leavesCount(root)`
3. Prove
 $\text{nodesCount}(\text{root}) \leq 2^{(\text{height}(\text{root}) - 1)}$
4. Prove
 $\text{leavesCount}(\text{root}) \leq 2^{(\text{height}(\text{root}) - 1)}$

Example 1.b (Optional)

```
datatype Tree =  
  Leaf  
| Node(left: Tree, right: Tree)
```

Task

1. Define complete (or perfect) trees
2. Prove
 $\text{nodesCount}(\text{root}) == 2^{(\text{height}(\text{root}) - 1)}$
3. Prove
 $\text{leavesCount}(\text{root}) == 2^{(\text{height}(\text{root}) - 1)}$

Example 2 – Inductive/Generic type

```
datatype List<T> =  
  Nil  
  | Cons (... , ...)
```

Task

1. Define lists
2. Define length of lists
3. Define append/concatenate two lists
4. Define reverse list
5. Prove
 $\text{reverse}(\text{append}(l1, l2)) == \text{append}(\text{reverse}(l2), \text{reverse}(l1))$
6. Prove $\text{reverse}(\text{reverse}(l)) == l$