Arbitrary rotations

Problem: Create rotations to solve

1. Rotate any vector A to the Z-axis.
2. Rotate Z-axis to any vector A.
   (Follows easily from 1: Just invert the transformation.)
3. Rotate any vector A to another given vector B.
   (Follows easily from 1 and 2: Product of $A \rightarrow Z$ followed by $Z \rightarrow B$.)

Rotation via 3x3 matrix R:

Given an arbitrary vector A, form a three vector ortho-normal basis around A by

$\bar{A} = normalize(A)$
$\bar{B} = normalize(V \times \bar{A})$ where V is **any** vector not parallel to A . (Hint: Try Z, then X.)
$\bar{C} = \bar{A} \times \bar{B}$

and form a 3x3 matrix with those vectors as rows.

$$R = \begin{bmatrix} \bar{B} \\ \bar{C} \\ \bar{A} \end{bmatrix}.$$

Properties of R:

Notice that, as constructed, the three vectors are

unit length: $1 = \bar{A}\cdot\bar{A} = \bar{B}\cdot\bar{B} = \bar{C}\cdot\bar{C}$, and
mutually perpendicular: $0 = \bar{A}\cdot\bar{B} = \bar{B}\cdot\bar{C} = \bar{C}\cdot\bar{A}$

It is easy to see that R maps the three vectors to the X, Y, and Z axes.

$$\begin{bmatrix} \bar{B} \\ \bar{C} \\ \bar{A} \end{bmatrix}\begin{bmatrix} \bar{B} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} \bar{B} \\ \bar{C} \\ \bar{A} \end{bmatrix}\begin{bmatrix} \bar{C} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} \bar{B} \\ \bar{C} \\ \bar{A} \end{bmatrix}\begin{bmatrix} \bar{A} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Fact: This **is** a rotation matrix.
Proof … TBD …

The inverse of R is just the transpose

That is $R^{-1} = R^{T}$ because:

$$R\,R^{T} = \begin{bmatrix} \bar{B} \\ \bar{C} \\ \bar{A} \end{bmatrix}\begin{bmatrix} \bar{B} & \bar{C} & \bar{A} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

In GLM:

Use mat3(…) constructor from 3 vectors,
Beware: The matrix is built in column major form!
So build $R^{-1} = \text{glm::mat3}(\bar{B},\ \bar{C},\ \bar{A})$
then $R = \text{glm::transpose}(R^{-1})$