Siblings paper
Exploded logit

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1 Model

1.1 Utility

$$u_{ijk} = \omega_y u_{y_i j} + (1 - \omega_y) u_{o_i k} + \gamma \mathbb{1}[j = k] + \varepsilon_{ijk} \tag{1}$$

where j is the school-assignment of the younger children y in family i, and k is the school-assignment of the older children o. ω_y is the weight that the family places on the utility of the younger children.

$$u_{y_i j} = \beta_y dist_{y_i j} + \delta_y quality_j \cancel{+ \varepsilon_{y_i j}} \tag{2}$$

$$u_{o_i k} = \beta_o dist_{o_i k} + \delta_o quality_k \cancel{+ \varepsilon_{o_i k}} \tag{3}$$

where β_y, β_o, δ_y and δ_o are the parameters that govern preferences of the younger and older siblings respectively, and ε_y and ε_o are idiosyncratic i.i.d. preference shocks.

I'm ignoring the distance between schools for now.

Assumptions:

- When families report marginal applications, it is like they were only applying to one child.
- ε_{ijk} are i.i.d. type-I extreme value.

1.2 Probabilities

Families with more than one common-school applied were asked between,

a) Worst-joint (WJ)
b) Best-older-solo (BOS) and best-younger-solo (BYS)

Families with only one common-school applied were asked between,

a) "Best"-joint (BJ)
b) BOS & BYS

Let's define,

$$V_{ijk} \equiv \omega_y u_{y_i j} + (1-\omega_y) u_{o_i k} + \gamma \mathbb{1}[j=k]$$

Hence,

$$V_i^J \equiv V_{ijj} \qquad \text{(Assigned together)}$$
$$V_i^S \equiv V_{ilk} \qquad \text{(Split assignment)}$$

The probability that family i prefers being assigned together is,

$$P_i \equiv \Pr\left[V_i^J + \varepsilon_{ijj} > V_i^S + \varepsilon_{ilk}\right] = \frac{\exp\left(V_i^J\right)}{\exp\left(V_i^J\right) + \exp\left(V_i^S\right)}$$

1.3 Log-likelihood: Joint vs Split

Let $y_i \in \{0,1\}$ be the observed answer (1 is prefer joint, 0 is prefer split),

$$L_i^{JS}(\theta) = P_i^{y_i}(1-P_i)^{1-y_i} \quad ,\theta = (\beta_y, \beta_o, \delta_y, \delta_o, \omega_y, \gamma)$$

Assuming independent families $i = 1, ..., N$

$$\mathcal{L}^{JS}(\theta) = \prod_{i=1}^{N} L_i^{JS}(\theta)$$
$$\ell^{JS}(\theta) = \sum_{i=1}^{N} y_i \log P_i + (1-y_i)\log(1-P_i)$$

1.4 Marginal Likelihoods

We extend the model to include marginal ranking information for each sibling. Let $\mathcal{J}_i^y$ and $\mathcal{J}_i^o$ denote the choice sets (schools applied to) for the younger and older siblings in family i, respectively.

1.4.1 Younger Sibling Marginal

For the younger sibling's marginal ranking, let $j_i^* \in \mathcal{J}_i^y$ denote the first-choice school (orden = 1). The probability of this choice follows the exploded logit form:

$$P_i^y(j_i^*) = \frac{\exp(u_{y_i j_i^*})}{\sum_{j \in \mathcal{J}_i^y} \exp(u_{y_i j})}$$

where $u_{y_i j} = \beta_y dist_{y_i j} + \delta_y quality_j$ is the utility of school j for the younger sibling.

The marginal log-likelihood contribution from the younger sibling in family i is:

$$\ell_i^y(\theta) = \log P_i^y(j_i^*)$$

1.4.2 Older Sibling Marginal

Similarly, for the older sibling with first-choice school $k_i^* \in \mathcal{J}_i^o$:

$$P_i^o(k_i^*) = \frac{\exp(u_{o_i k_i^*})}{\sum_{k \in \mathcal{J}_i^o} \exp(u_{o_i k})}$$

where $u_{o_i k} = \beta_o dist_{o_i k} + \delta_o quality_k$.

The marginal log-likelihood contribution is:

$$\ell_i^o(\theta) = \log P_i^o(k_i^*)$$

1.5 Full Log-likelihood

Assuming independence of the marginal choices conditional on type (with Type-I Extreme Value idiosyncratic shocks), the full likelihood for family i is:

$$L_i(\theta) = L_i^{JS}(\theta) \times L_i^y(\theta) \times L_i^o(\theta)$$

In log-likelihood form across all N families:

$$\begin{aligned}\ell(\theta) &= \sum_{i=1}^{N} \left[\ell_i^{JS}(\theta) + \ell_i^y(\theta) + \ell_i^o(\theta)\right] \\ &= \ell^{JS}(\theta) + \sum_{i=1}^{N} \ell_i^y(\theta) + \sum_{i=1}^{N} \ell_i^o(\theta)\end{aligned}$$

Implementation note: The extended model uses three data sources:

- `survey_responses.csv`: Joint vs split scenarios (BJ, WJ, BOS, BYS)
- `marginal_applications_older.csv`: Complete rankings for older sibling
- `marginal_applications_younger.csv`: Complete rankings for younger sibling

2 Data

From survey responses, we have 4 types of schools:

1. Best-joint (BJ)
2. Worst-joint (WJ)
3. Best-older-solo (BOS)
4. Best-younger-solo (BYS)

From application data, we can say:

- survey showed 10 options max for each, so some lists were trimmed (0,5% sample approx).
- BOS and BYS: 6% selected schools were not found in application data.

Characteristics of schools:

- Specific for each sibling, even for BJ/WJ (since lat-long not always the same between siblings and between campus, quality depends on grade).
- 3% - 6% schools without info of school quality.
- 6% we don't have information of distance for BOS/BYS

3 Results

$N = 9,231$ (from original $12,917$).