

## Homework # 2

August 17, 2019

### 1 Optimization I: Linear Regression

#### 1.1 Single Variable Regression

Analyze the data set "**single\_variate\_optimization.csv**" and understand the relationship between the target variable  $Y$  and the predictor  $X$

- (1) Calculate the variance, standard deviation of  $X$  and  $Y$
- (2) Calculate the covariance, correlation coefficient of  $X$  and  $Y$
- (3) Using the `LinearRegression` from `sklearn` package in python and build a linear regression model  $Y \sim X$ , what are  $\beta_0$  and  $\beta_1$ ?
- (4) Can you replicate  $\beta_1$  using the calculation from (1) and (2)? How about  $\beta_0$ ?

#### 1.2 Multivariate Regression

Analyze the data set "**multi\_variate\_optimization.csv**" and understand the relationship between the target variable  $Y$  and the predictor  $X$ s

- (1) Using the `LinearRegression` from `sklearn` package in python and build a linear regression model  $Y \sim X$ , what are the  $\beta$ s?
- (2) Replicate the  $\beta$ s using matrix operation. hint:  $\hat{\beta} = (X^T X)^{-1} X^T Y$ . Make sure you replicate  $\beta_0$ .

### 2 Optimization II: Logistic Regression

Analyze the data set "**logistic\_regression\_optimization.csv**" and understand the relationship between the target variable  $Y$  and the predictor  $X$ s

- (1) Using the `LogisticRegression` from `sklearn` package in python and build a linear regression model  $Y \sim X$ , what are the  $\beta$ s? Hint: make sure that you set the regularization parameter to be large e.g.  $C = 1e4$ . See the code below for your reference

```
from sklearn.linear_model import LogisticRegression
logisticRegr = LogisticRegression(C=1e4)
```

- (2) Replicate the  $\beta$ s using Newton-Raphson method. Hint: initialize  $\beta$ s with some random values e.g. 0.5, update  $\beta$ s using the formula below until converge. What is your convergence condition? How are your  $\beta$ s compare to problem (1)

$$\vec{\beta}^{(m+1)} \leftarrow \vec{\beta}^{(m)} + (\mathbf{X}^T \mathbf{W} \mathbf{X})^{-1} \mathbf{X}^T (\mathbf{Y} - \hat{\mathbf{Y}})$$

Here,

$$\mathbf{W} = \begin{bmatrix} \hat{y}^1(1 - \hat{y}^1) & & \\ & \ddots & \\ & & \hat{y}^n(1 - \hat{y}^n) \end{bmatrix}$$