Problems

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1 Maximum Likelihood

1.1 Problem

Given a Gaussian distribution $f(y, \mu, \sigma = 1) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{(y-\mu)^2}{2})$ and a set of observations y_1, y_2 ..., y_n

1. if we only have one observation $y_1 = 1$, what is the μ that will maximize the likelihood of the observation?
2. if $y_1 = 0$, what is the μ that will maximize the likelihood of the observation?
3. if we have observed both data points $y_1 = 0$ and $y_2 = 0$, what is the μ that will maximize the likelihood of the observations?

1.2 Solution

The likelihood function of the problem, $L = \frac{1}{(\sqrt{2\pi})^n} \exp\left(-\sum_{i=1}^{n} \frac{(y_i-\mu)^2}{2}\right)$ or the loglikelihood function $\ell = -\sum_{i=1}^{n} \frac{(y_i-\mu)^2}{2} + const$

1. when $y_1 = 1$, $\ell(y) = \frac{(y-\mu)^2}{2} + const$ is maximized if $\mu = 1$ as it gives $-\sum_{i=1}^{n} \frac{(1-\mu)^2}{2} = 0$, any μ that is different from $\mu = 1$ will leads to a negative first term.
2. when $y_1 = 0$, $\ell(y)$ is maximized if $\mu = 0$ as it gives $-\sum_{i=1}^{n} \frac{(0-\mu)^2}{2} = 0$, any μ that is different from $\mu = 0$ will leads to a negative first term.
3. if we have two observations $y_1 = 0$ and $y_2 = 0$, the corresponding loglikelihood function is $\ell(y_1 = 1, y_2 = 0) = \frac{(1-\mu)^2}{2} + \frac{(0-\mu)^2}{2}$, the μ that maximize the function satisfy the condtion $\frac{d\ell}{d\mu} = 0$, which is

$$(1 - \mu) + (0 - \mu) = 0$$

and therefore $\mu = \frac{1+0}{2} = 0.5$