Lecture Note - Support Vector Machine

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April 5, 2020

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1 Loss Function of Support Vector Machine

$$L = \frac{1}{2}|w|^2 - \sum_{i=1}^{N} \alpha_i \left[y^i(\vec{w} \cdot \vec{x}^i + b) - 1 \right] \tag{1}$$

The optimal condition is when $\vec{w}$ is chosen at the point where

$$\begin{aligned}
\frac{\partial L}{\partial w_j} &= \frac{1}{2}\frac{\partial}{\partial w_j}|w|^2 - \frac{\partial}{\partial w_j}\sum_{i}^{N} \alpha_i \left[y^i(\vec{w} \cdot \vec{x}^i + b) - 1 \right] \\
&= w_j - \sum_{i=1}^{N} \alpha_i y^i x_j^i \\
&= 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial L}{\partial b} &= \frac{1}{2}\frac{\partial}{\partial b}|w|^2 - \frac{\partial}{\partial b}\sum_{i}^{N} \alpha_i \left[y^i(\vec{w} \cdot \vec{x}^i + b) - 1 \right] \\
&= -\sum_{i=1}^{N} \alpha_i y^i \\
&= 0
\end{aligned}$$

Or equivalently

$$\begin{cases} \vec{w} = \sum_{i=1}^{N} \alpha_i y^i \vec{x}^i \\ \sum_{i=1}^{N} \alpha_i y^i = 0 \end{cases} \tag{2}$$

Insert equation (2) back into equation (1)

We have

$$\begin{aligned} L &= \sum_{i,j}^{N} \frac{1}{2}\alpha_i\alpha_j y^i y^j \vec{x}^i \cdot \vec{x}^j - \sum_{i=1}^{N} \alpha_i y^i (\sum_{k} \alpha_j y^j \vec{x}^i \cdot \vec{x}^j) - \sum_{i=1}^{N} \alpha_i y^i b + \sum_{i=1}^{N} \alpha_i \\ &= \sum_{i=1}^{N} \alpha_i - \sum_{i,j}^{N} \frac{1}{2}\alpha_i\alpha_j y^i y^j \vec{x}^i \cdot \vec{x}^j \end{aligned}$$

The optimal problem is now

$$L = \sum_{i=1}^{N} \alpha_i - \sum_{i,j}^{N} \frac{1}{2}\alpha_i\alpha_j y^i y^j \vec{x}^i \cdot \vec{x}^j, \text{ s.t. } \sum_{i=1}^{N} \alpha_i y^i = 0 \tag{3}$$