18329015

赤阶俗讳

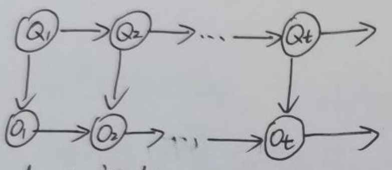
12.6

(a) $\because \alpha_{t+1}(i) = P_r(O_1:t+1, Q_{t+1}=S_i|\lambda)$

$= P_r(O_1:t, Q_{t+1}=S_i|\lambda)\cdot b_i(O_t+1)$

又 $\because P_r(O_1:t, Q_{t+1}=S_i|\lambda) = \sum_{j=1}^{N} P_r(O_1:T-1, Q_T=S_i, Q_{T-1}=S_j|\lambda)$

$\because$ HMM模型为



由d-分离条件可知，$O_1:T-1, Q_T, Q_{T-1}$ 独立

$\therefore$ 上式 $= \sum_{j=1}^{N} P_r(O_1:T, Q_T=S_j|\lambda) P_r(Q_T=S_i|Q_{T+1}=S_j, \lambda)$

$= \sum_{j=1}^{N} P_r(O_1:T, Q_T=S_j|\lambda) A_{ji}$

$\therefore \alpha_{t+1}(i) = \left(\sum_{j=1}^{N} \alpha_t(j) A_{ji}\right) b_i(O_{t+1})$

(b) $\beta_t(i) = P_r(O_{t+1}:T|Q_t=S_i, \lambda)$

$= P_r(O_{t+1}, O_{t+2}:T|Q_t=S_i, \lambda)$

$= \sum_{j=1}^{N} P_r(Q_{t+1}=S_j|Q_t=S_i, \lambda) P_r(O_{t+1}, O_{t+2}:T|Q_{t+1}=S_j, \lambda)$

$= \sum_{j=1}^{N} A_{ij} b_j(O_{t+1}) \beta_{t+1}(j)$

(c) $P_r(O_1:T|\lambda) = P_r(O_1, O_2:T|\lambda)$

$$= \sum_{i=1}^{N} P_r(O_1, O_2:T|Q_1 = S_i, \lambda) P_r(Q_1 = S_i)$$

$$= \sum_{i=1}^{N} \pi_i \cdot b_i(O_1) \beta_1(i)$$

14.2

(a) $P(O_{t_1} = S_{q_1}, \cdots, O_{t_T} = S_{q_T}) = P(Q_1 = S_{q_1})$

$$= \prod_{t=1}^{T-1} A_{q_t q_{t+1}} \cdot \prod_{t=1}^{T} b_{q_t}(O_t)$$

$\therefore \ln P(O_{t_1} = S_{q_1}, \cdots, O_{t_T} = S_{q_T})$

$$= \ln \pi_{q_1} + \sum_{t=1}^{T-1} \ln A_{q_t q_{t+1}} + \sum_{t=1}^{T} \ln b_{q_t}(O_t)$$

(b) $E_{Q_1:T}[\ln \pi_{\theta_1}] = \sum_{i=1}^{N} r_1(i) \cdot \ln \pi_i$

(c)(d)(e)(f)(g)不会