



Chapter 6: Formal Relational Query Languages

Database System Concepts, 6th Ed.

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Chapter 6: Formal Relational Query Languages

- Relational Algebra
- Tuple Relational Calculus
- Domain Relational Calculus



Formal Languages

- A **formal language** is a set of strings of **symbols** that may be constrained by **rules** that are specific to it.
- E.g.
 - Relational Algebra
 - Tuple Relational Calculus
 - Domain Relational Calculus



Relational Algebra

- Relational algebra received little attention outside of pure mathematics until the publication of E.F. Codd's relational model of data in 1970. Codd proposed such an algebra as a basis for database query languages.
- Procedural language
- Six basic operators
 - select: σ
 - project: Π
 - union: \cup
 - set difference: $-$
 - Cartesian product: \times
 - rename: ρ
- The operators take one or two relations as inputs and produce a new relation as a result.



Precedence of Relational Operators

- Precedence of relational operators
 1. $[\sigma, \Pi, \rho]$ (highest).
 2. $[x, \bowtie]$.
 3. \cap .
 4. $[\cup, -]$



Division Operator ***

- Inverse of Cartesian product
- Given relations $r(R)$ and $s(S)$, such that $S \subset R$, $r \div s$ is the **largest** relation $t(R-S)$ such that

$$t \times s \subseteq r$$

- E.g. Practical exercise 6.4
 - let $r(ID, course_id) = \Pi_{ID, course_id} (takes)$ and
 $s(course_id) = \Pi_{course_id} (\sigma_{dept_name="Biology"}(course))$
then $r \div s$ gives us students who have taken all courses in the Biology department
- Can write $r \div s$ as

$$temp1 \leftarrow \Pi_{R-S} (r)$$

$$temp2 \leftarrow \Pi_{R-S} ((temp1 \times s) - \Pi_{R-S,S} (r))$$

$$result = temp1 - temp2$$



Extended Relational-Algebra-Operations

- Generalized Projection
- Aggregate Functions



Aggregate Functions and Operations

- **Aggregate function** takes a collection of values and returns a single value as a result.

avg: average value

min: minimum value

max: maximum value

sum: sum of values

count: number of values

- **Aggregate operation** in relational algebra

$$G_1, G_2, \dots, G_n \mathcal{G}_{F_1(A_1), F_2(A_2), \dots, F_n(A_n)}(E)$$

E is any relational-algebra expression

- G_1, G_2, \dots, G_n is a list of attributes on which to group (can be empty)
- Each F_i is an aggregate function
- Each A_i is an attribute name

- Note: Some books/articles use γ instead of \mathcal{G} (Calligraphic G)



Result of Aggregate Operation

$$G_1, G_2, \dots, G_n \quad \mathcal{G}_{F_1(A_1), F_2(A_2), \dots, F_n(A_n)}(E)$$

- For each group (g_1, g_2, \dots, g_n) , the result has a tuple
 - $(g_1, g_2, \dots, g_n, a_1, a_2, \dots, a_n)$ where,
 - ▶ for each i , a_i is the result of applying the aggregate function F_i on the multiset of values for attribute A_i in the group.



Aggregate Operation – Example 1

- Relation r :

| A | B | C |
|----------|----------|-----|
| α | α | 7 |
| α | β | 7 |
| β | β | 3 |
| β | β | 10 |

- $G_{\text{sum}(c)}(r)$

| sum(c) |
|----------------|
| 27 |



Aggregate Operation – Example 2

- Find the total number of instructors who teach a course in the Spring 2010 semester
 - to eliminate duplicates, we use the hyphenated string “**distinct**” appended to the end of the function name (for example, **count-distinct**).

$$\mathcal{G}_{count-\textit{distinct}(ID)}(\sigma_{semester="Spring" \wedge year=2010}(teaches))$$



Aggregate Operation – Example 3

- Find the average salary in each department

dept_name \bar{G} **avg**(salary) (*instructor*)

| <i>ID</i> | <i>name</i> | <i>dept_name</i> | <i>salary</i> |
|-----------|-------------|------------------|---------------|
| 76766 | Crick | Biology | 72000 |
| 45565 | Katz | Comp. Sci. | 75000 |
| 10101 | Srinivasan | Comp. Sci. | 65000 |
| 83821 | Brandt | Comp. Sci. | 92000 |
| 98345 | Kim | Elec. Eng. | 80000 |
| 12121 | Wu | Finance | 90000 |
| 76543 | Singh | Finance | 80000 |
| 32343 | El Said | History | 60000 |
| 58583 | Califieri | History | 62000 |
| 15151 | Mozart | Music | 40000 |
| 33456 | Gold | Physics | 87000 |
| 22222 | Einstein | Physics | 95000 |

| <i>dept_name</i> | <i>avg_salary</i> |
|------------------|-------------------|
| Biology | 72000 |
| Comp. Sci. | 77333 |
| Elec. Eng. | 80000 |
| Finance | 85000 |
| History | 61000 |
| Music | 40000 |
| Physics | 91000 |



Aggregate Functions (Cont.)

- Result of aggregation does not have a name
 - Can use rename operation to give it a name
 - For convenience, we permit renaming as part of aggregate operation

dept_name \mathcal{G} **avg**(salary) **as** *avg_sal* (*instructor*)



Modification of the Database

- The content of the database may be modified using the following operations:
 - Deletion
 - Insertion
 - Updating
- All these operations can be expressed using the assignment operator
 - See Appendix in the end of this Chapter



Multiset Relational Algebra

- Pure relational algebra removes all duplicates
 - e.g. after projection
- Multiset relational algebra retains duplicates, to match SQL semantics
 - SQL duplicate retention was initially for efficiency, but is now a feature
- Multiset relational algebra defined as follows
 - selection: has as many duplicates of a tuple as in the input, if the tuple satisfies the selection
 - projection: one tuple per input tuple, even if it is a duplicate
 - cross product: If there are m copies of $t1$ in r , and n copies of $t2$ in s , there are $m \times n$ copies of $t1.t2$ in $r \times s$
 - Other operators similarly defined
 - ▶ E.g. union: $m + n$ copies, intersection: $\min(m, n)$ copies
difference: $\max(0, m - n)$ copies



SQL and Relational Algebra

- **select** A_1, A_2, \dots, A_n
from r_1, r_2, \dots, r_m
where P

is equivalent to the following expression in multiset relational algebra

$$\Pi_{A_1, \dots, A_n} (\sigma_P (r_1 \times r_2 \times \dots \times r_m))$$

- **select** $A_1, A_2, \text{sum}(A_3)$
from r_1, r_2, \dots, r_m
where P
group by A_1, A_2

is equivalent to the following expression in multiset relational algebra

$$A_1, A_2 \text{ } G_{\text{sum}(A_3)} (\sigma_P (r_1 \times r_2 \times \dots \times r_m))$$



SQL and Relational Algebra

- More generally, the non-aggregated attributes in the **select** clause may be a subset of the **group by** attributes, in which case the equivalence is as follows:

```
select  $A_1$ , sum( $A_3$ )  
from  $r_1, r_2, \dots, r_m$   
where  $P$   
group by  $A_1, A_2$ 
```

is equivalent to the following expression in multiset relational algebra

$$\Pi_{A_1, \text{sum}A_3} (\mathcal{G}_{A_1, A_2} \text{sum}(A_3) \text{ as } \text{sum}A_3 (\sigma_P (r_1 \times r_2 \times \dots \times r_m)))$$



Tuple Relational Calculus



Tuple Relational Calculus

- A nonprocedural query language, where each query is of the form

$$\{t \mid P(t)\}$$

- It is the set of all tuples t such that predicate P is true for t
 - t is a *tuple variable*,
 - $t[A]$ denotes the value of tuple t on attribute A
 - $t \in r$ denotes that tuple t is in relation r
 - P is a *formula* similar to that of the predicate calculus
- **Nonprocedural** query language:
 - It describes the desired information **without giving a specific procedure** for obtaining that information.



Predicate Calculus Formula

1. Set of attributes and constants
2. Set of comparison operators: (e.g., $<$, \leq , $=$, \neq , $>$, \geq)
3. Set of connectives: and (\wedge), or (\vee), not (\neg)
4. Implication (\Rightarrow): $x \Rightarrow y$, if x is true, then y is true

$$x \Rightarrow y \equiv \neg x \vee y$$

5. Set of quantifiers:

- ▶ Existential: $\exists t \in r (Q(t)) \equiv$ "there exists" a tuple t in relation r such that predicate $Q(t)$ is true
- ▶ Universal: $\forall t \in r (Q(t)) \equiv Q$ is true "for all" tuples t in relation r
 - ▶ $Q(t_1) \wedge Q(t_2) \wedge \dots \wedge Q(t_n)$



Example Queries

- Find the *ID*, *name*, *dept_name*, *salary* for instructors whose salary is greater than \$80,000

$$\{t \mid t \in \text{instructor} \wedge t[\text{salary}] > 80000\}$$

- As in the previous query, but output only the *ID* attribute value

$$\{t \mid \exists s \in \text{instructor} (t[\text{ID}] = s[\text{ID}] \wedge s[\text{salary}] > 80000)\}$$

Notice that a relation on schema (*ID*) is **implicitly defined** by the query



Example Queries

- Find the names of all instructors whose department is in the Watson building

$$\{t \mid \exists s \in \text{instructor} (t[\text{name}] = s[\text{name}] \\ \wedge \exists u \in \text{department} (u[\text{dept_name}] = s[\text{dept_name}] \\ \wedge u[\text{building}] = \text{"Watson"}))\}$$

- Find the set of all courses taught in the Fall 2009 semester, **or** in the Spring 2010 semester, or both

$$\{t \mid \exists s \in \text{section} (t[\text{course_id}] = s[\text{course_id}] \wedge \\ s[\text{semester}] = \text{"Fall"} \wedge s[\text{year}] = 2009) \\ \vee \exists u \in \text{section} (t[\text{course_id}] = u[\text{course_id}] \wedge \\ u[\text{semester}] = \text{"Spring"} \wedge u[\text{year}] = 2010) \}$$



Expressive Power of Languages

- For every relational-algebra expression using only the basic operations, there is an equivalent expression in the tuple relational calculus
 - The basic operations:
 - ▶ $\cup, -, \times, \sigma$, and ρ ,
 - ▶ without the extended relational operations such as generalized projection and aggregation
- For every tuple-relational-calculus expression, there is an equivalent relational algebra expression.



Domain Relational Calculus



Domain Relational Calculus

- A **nonprocedural** query language equivalent in power to the tuple relational calculus
- Each query is an expression of the form:

$$\{ \langle x_1, x_2, \dots, x_n \rangle \mid P(x_1, x_2, \dots, x_n) \}$$

- x_1, x_2, \dots, x_n represent **domain variables**
- P represents a formula similar to that of the predicate calculus



Example Queries

- Find the *ID*, *name*, *dept_name*, *salary* for instructors whose salary is greater than \$80,000
 - $\{ \langle i, n, d, s \rangle \mid \langle i, n, d, s \rangle \in \text{instructor} \wedge s > 80000 \}$
- As in the previous query, but output only the *ID* attribute value
 - $\{ \langle i \rangle \mid \langle i, n, d, s \rangle \in \text{instructor} \wedge s > 80000 \}$
- Find the names of all instructors whose department is in the Watson building
 - $\{ \langle n \rangle \mid \exists i, d, s (\langle i, n, d, s \rangle \in \text{instructor} \wedge \exists b, a (\langle d, b, a \rangle \in \text{department} \wedge b = \text{"Watson"})) \}$



End of Chapter 6

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Practical Exercises

Textbook-6th edition:

- **6.1**
- **6.11**



Appendix ***

- Deletion in relational algebra ***
- Insertion in relational algebra ***
- Update in relational algebra ***



Deletion in Relational Algebra

- A delete request is expressed similarly to a query, except instead of displaying tuples to the user, the selected tuples are removed from the database.
- Can delete only whole tuples; cannot delete values on only particular attributes
- A deletion is expressed in relational algebra by:

$$r \leftarrow r - E$$

where r is a relation and E is a relational algebra query.



Deletion Examples

- Delete all account records in the Perryridge branch.

$account \leftarrow account - \sigma_{branch_name = "Perryridge"}(account)$

- Delete all loan records with amount in the range of 0 to 50

$loan \leftarrow loan - \sigma_{amount \geq 0 \text{ and } amount \leq 50}(loan)$

- Delete all accounts at branches located in Needham.

$r_1 \leftarrow \sigma_{branch_city = "Needham"}(account \bowtie branch)$

$r_2 \leftarrow \Pi_{account_number, branch_name, balance}(r_1)$

$r_3 \leftarrow \Pi_{customer_name, account_number}(r_2 \bowtie depositor)$

$account \leftarrow account - r_2$

$depositor \leftarrow depositor - r_3$



Insertion in Relational Algebra

- To insert data into a relation, we either:
 - specify a tuple to be inserted
 - write a query whose result is a set of tuples to be inserted
- in relational algebra, an insertion is expressed by:

$$r \leftarrow r \cup E$$

where r is a relation and E is a relational algebra expression.

- The insertion of a single tuple is expressed by letting E be a constant relation containing one tuple.



Insertion Examples

- Insert information in the database specifying that Smith has \$1200 in account A-973 at the Perryridge branch.

$$account \leftarrow account \cup \{("A-973", "Perryridge", 1200)\}$$
$$depositor \leftarrow depositor \cup \{("Smith", "A-973")\}$$

- Provide as a gift for all loan customers in the Perryridge branch, a \$200 savings account. Let the loan number serve as the account number for the new savings account.

$$r_1 \leftarrow (\sigma_{branch_name = "Perryridge"}(borrower \bowtie loan))$$
$$account \leftarrow account \cup \Pi_{loan_number, branch_name, 200}(r_1)$$
$$depositor \leftarrow depositor \cup \Pi_{customer_name, loan_number}(r_1)$$



Updating in Relational Algebra

- A mechanism to change a value in a tuple without changing *all* values in the tuple
- Use the generalized projection operator to do this task

$$r \leftarrow \Pi_{F_1, F_2, \dots, F_l}(r)$$

- Each F_i is either
 - the i^{th} attribute of r , if the i^{th} attribute is not updated, or,
 - if the attribute is to be updated F_i is an expression, involving only constants and the attributes of r , which gives the new value for the attribute



Update Examples

- Make interest payments by increasing all balances by 5 percent.

$account \leftarrow \Pi_{account_number, branch_name, balance * 1.05} (account)$

- Pay all accounts with balances over \$10,000 6 percent interest and pay all others 5 percent

$account \leftarrow \Pi_{account_number, branch_name, balance * 1.06} (\sigma_{BAL > 10000} (account))$
 $\cup \Pi_{account_number, branch_name, balance * 1.05} (\sigma_{BAL \leq 10000} (account))$



Example Queries

- Find the names of all customers who have a loan and an account at bank.

$$\Pi_{customer_name} (borrower) \cap \Pi_{customer_name} (depositor)$$

- Find the name of all customers who have a loan at the bank and the loan amount

$$\Pi_{customer_name, loan_number, amount} (borrower \bowtie loan)$$



Example Queries

- Find all customers who have an account from at least the “Downtown” and the Uptown” branches.

- Query 1

$$\Pi_{customer_name} (\sigma_{branch_name = \text{“Downtown”}} (depositor \bowtie account)) \cap \\ \Pi_{customer_name} (\sigma_{branch_name = \text{“Uptown”}} (depositor \bowtie account))$$

- Query 2

$$\Pi_{customer_name, branch_name} (depositor \bowtie account) \\ \div \rho_{temp(branch_name)} (\{(\text{“Downtown”}), (\text{“Uptown”})\})$$

Note that Query 2 uses a constant relation.



Bank Example Queries

- Find all customers who have an account at all branches located in Brooklyn city.

$$\begin{aligned} & \Pi_{customer_name, branch_name} (depositor \bowtie account) \\ & \div \Pi_{branch_name} (\sigma_{branch_city = \text{"Brooklyn"}} (branch)) \end{aligned}$$