中山大学本科生期末考试

考试科目：《概率统计》（A 卷）

学年学期：**2016** 学年第一学期　　姓　　名：_____

学 院/系：数据科学与计算机学院　　学　　号：_____

考试方式：闭卷　　年级专业：_____

考试时长：120 分钟　　班　　别：_____

警示 《中山大学授予学士学位工作细则》第八条：“考试作弊者，不授予学士学位。”

-------以下为试题区域，共 9 道大题，总分 100 分,考生请在答题纸上作答--------

1. Lengths of bus routes for any particular transit system will typically vary from one route to another. Here gives the following information on lengths (km) for one particular system: (the symbol ‘a-<b’ means the interval [a,b)) (15 points)

Length	6-<8	8-<10	10-<12	12-<14	14-<16	16-<18	18-<20	20-<22
Frequency	6	23	30	35	32	48	42	40
Length	22-<24	24-<26	26-<28	28-<30	30-<35	35-<40	40-<45	
Frequency	28	27	26	14	27	11	2	

(a) Roughly draw a histogram corresponding to these frequencies;
(b) What proportion of these routes have lengths of at least 30?
(c) Roughly what is the median route length?

2. An individual has 3 different email accounts. Most of her messages, in fact 70%, come into account #1, whereas 20% come into account #2 and the remaining 10% into account #3. Of the messages into account #1, only 1% are spam, whereas the corresponding percentages for accounts #2 and #3 are 2% and 5%, respectively. (10 points)
   a) What is the probability that a randomly selected message is spam?
   b) What is the probability that a selected spam coming into account #2?

3. If a publisher of nontechnical books takes great pains to ensure that its books are free of typographical errors, so that the probability of any given page containing at least one such error is .005 and errors are independent from page to page, what is the probability that one of its 400-page novels will contain at most three pages with errors? (10 points)
   a) Show the formula for calculating the probability with Binomial distribution.
   b) Approximate this probability with Poisson distribution.

4. Suppose only 75% of all drivers in a certain state regularly wear a seat belt. A random sample of 500 drivers is selected. Show the probability between 360 and 400 (inclusive) of the drivers in the sample regularly wear a seat belt? (Hint: the Normal Approximation to the Binomial Distribution) (10 point)

5. The time that it takes to serve a customer at the cash register in a minimarket is a random variable having an exponential distribution with parameter λ. Suppose X_1 and X_2 are service times for two different customers, assumed independent of each other. (10 points)
   a) Derive the cdf of $T_0=X_1+X_2$.
   b) Derive the pdf of $T_0=X_1+X_2$.

6. Let $X_1,\cdots X_n$ be a random sample from a normal distribution. (15 points)
   a) Derive the MLEs of μ and σ^2, respectively.
   b) Derive the Moment estimators of μ and σ^2, respectively.
   c) Are they unbiased estimators?

7. The alternating current breakdown voltage of an insulating liquid indicates its dielectric strength. Here gave the accompanying sample observations on breakdown voltage of a particular circuit under certain conditions (10 points)

   62, 50, 53, 57, 41, 53, 55, 61, 59, 64, 50, 53, 64, 62, 50, 68
   54, 55, 57, 50, 55, 50, 56, 55, 46, 55, 53, 54, 52, 47, 47, 55
   57, 48, 63, 57, 57, 55, 53, 59, 53, 52, 50, 55, 60, 50, 56, 58

   a) Draw the boxplot of the above data
   b) Calculate a 95% confidence interval for the population mean

8. A manufacturer of sprinkler systems used for fire protection in office buildings claims that the true average system-activation temperature is 130°. A sample of n=9 systems, when tested, yields a sample average activation temperature of 131.08°F. If the distribution of activation times is normal with standard deviation 1.5°F, (10 points)
   a) Does the data contradict the manufacturer's claim at significance levelα=0.01?
   b) Let μ' denote a particular value of μ that exceeds the null value μ_0, please derive β(μ'=132) .

9. Let X_1, X_2… X_n be a random sample from a distribution with mean μ and variable δ^2. Then the estimator

$$\frac{\sum_{i=1}^{n}(X_i-\overline{X})^2}{n-1}$$

   is an unbiased estimator of δ^2 (10 points)

Table 1：Standard Normal Curve Areas (*cont*.)

$$\Phi(z)=\int_{-\infty}^{z}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}dx \qquad (z\geq 0)$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9430	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9648	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9700	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9762	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9874	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986