Assignement 1 Ph121c

Constantin Cedillo-Vayson de Pradenne
ccedillo@caltech.edu

1 CODE

The github code can be found **here**.
The google colab code can be found **here**

2 4.1 DENSE EXACT DIAGONALIZATION

The only non-zero matrix elements $H_{\alpha\beta}$ are such that $\alpha = \beta$ or β and α differ by a single bit flip. Thus we can use the operators ^ and « to implement this. In this part, we first initialized our Hamiltonian, a zero matrix of dimension $2^L \times 2^L$. For each β (b in our code), we can create α (a) such that they differ by a single bit flip. This was done using $1 << j-1$ which shifts the value of 1 by $j-1$ spots. Since β contains L bits, j ranges from 1 to $L+1 \Rightarrow j-1$ ranges from 0 to L. We then want to compute $\langle\alpha| H |\beta\rangle$ when the term $\sum_{i=0}^{L} \sigma_i^x$ operates, hence only one bit has value 1. Since σ_x will flip each bit one by one using our for loop, applying H to β ultimately results in α, so $\langle\alpha| H |\beta\rangle = \langle\alpha|\alpha\rangle = 1$. Hence we store $-h$ for $H_{\alpha\beta}$ (factor of $-h$ in the Hamiltonian). The next step is to find the non-zero elements on which $\sum_{i=0}^{L} \sigma_i^z \sigma_{i+1}^z$ act on. We are looking for the α's that have two consecutive bits with value 1, which we did using the ^ , « , and AND & operators as a&(1«j) essentially checks to see if the jth bit has value 1. We equal this with (a&(1«j-1))*2 as both integers differ by a factor of two. We also make sure to add the corresponding terms if we choose to have periodic=True.

Our second block plots the ground state energies of H for h between -2 and 2 for open and periodic systems and for L=[8,10,12,14].

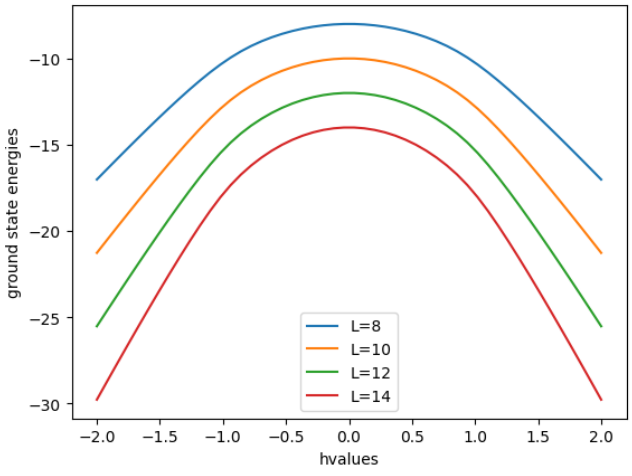

Figure 1: Ground state energies for h ranging from -2 to 2 using a dense matrix for the Hamiltonian

3 4.2 SPARSE MATRIX COMPUTATION

For the implementation of the sparse matrix, we follow the same logic using bitwise operations but store onyl non zero values in the form of tuples ($H_{\alpha\beta}$, α, β). I was able to push this to L=20. I had however not saved the output and was forced to limit myself to L=14 due to time constraints. However, using the sparse matrix, L=20 takes around 45 minutes to run which is a significant improvement from the dense diagonlization method (which took around 8 hours for L=14)

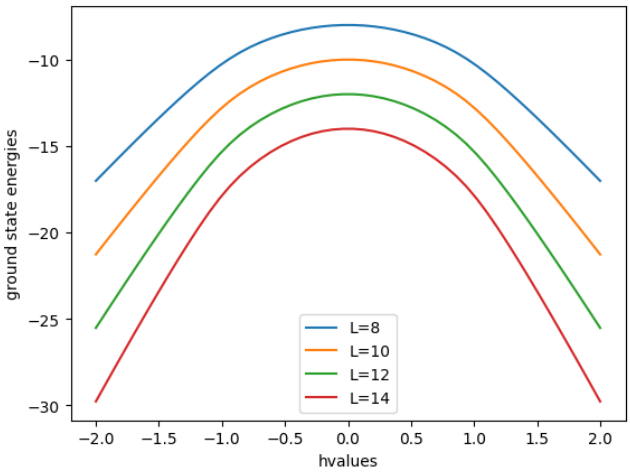

Figure 2: Ground state energies for h ranging from -2 to 2 using a sparse matrix

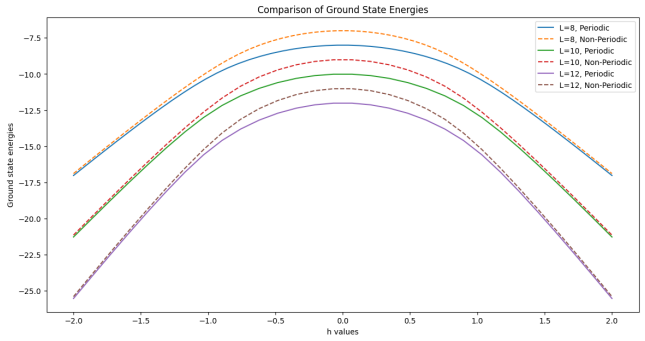

Figure 3: Comparison of open and periodic boundaries for L=12

4 4.3 STUDY OF CONVERGENCE WITH SYSTEM SIZE

We plotted E_{gs}/L for open and periodic systems and L={1,...,20} to study the thermodynamic limit $L \mapsto \infty$

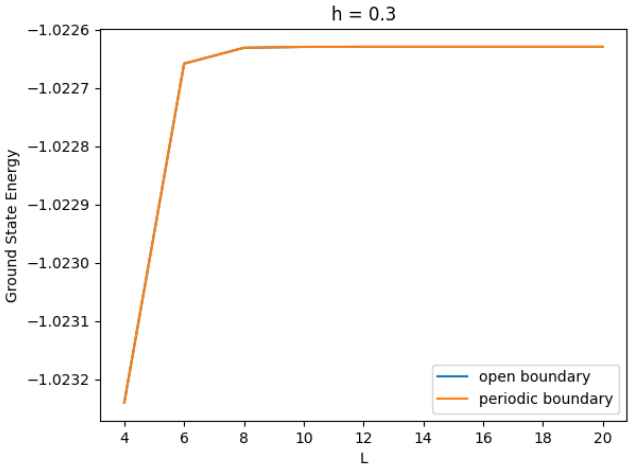

Figure 4: Convergence of system for L={4, 6, 8, 10, 12, 14, 16, 18, 20} and h=0.3

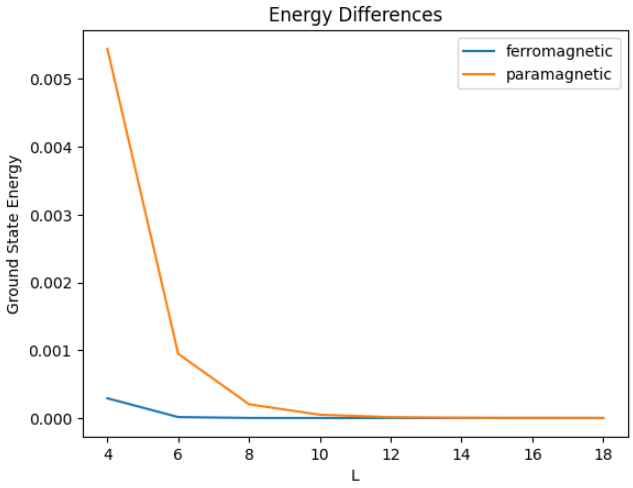

Figure 6: Energy difference for L={4, 6, 8, 10, 12, 14, 16, 18, 20} in ferromagentic and paramagnetic phase

5 4.4 FINDING THE QUANTUM PHASE TRANSITION

We used the different methods proposed in the assignement to find the quantum phase transition. The critical exponent nu for the first excited state is 0.9853933505650097

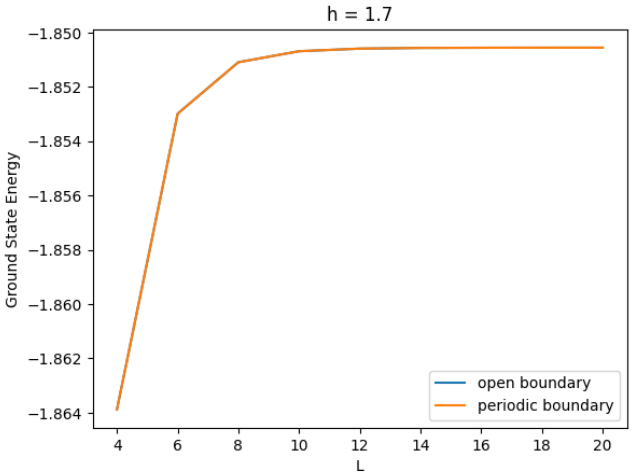

Figure 5: Convergence of system for L={4, 6, 8, 10, 12, 14, 16, 18, 20} and h=01.7

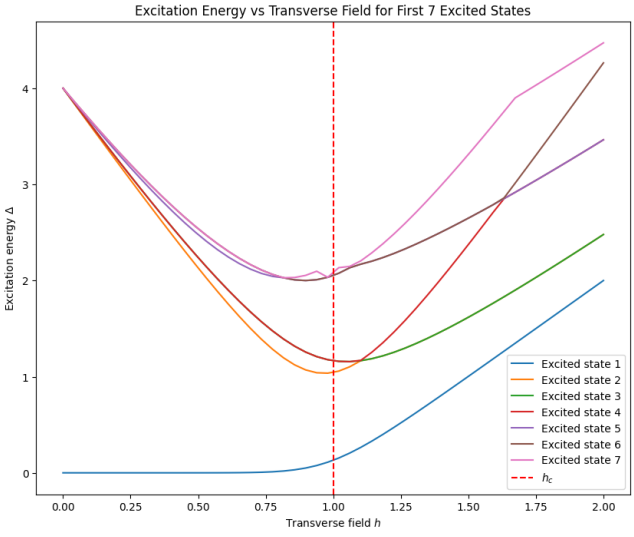

Figure 7: Excitation energy vs tranverse field

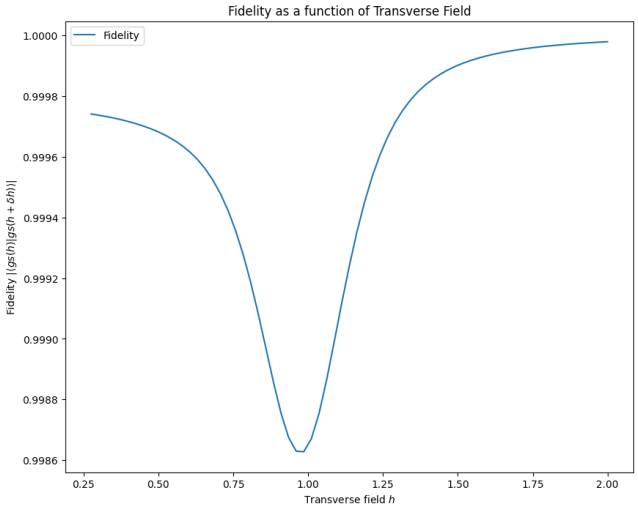

Figure 8: Fidelity vs tranverse field

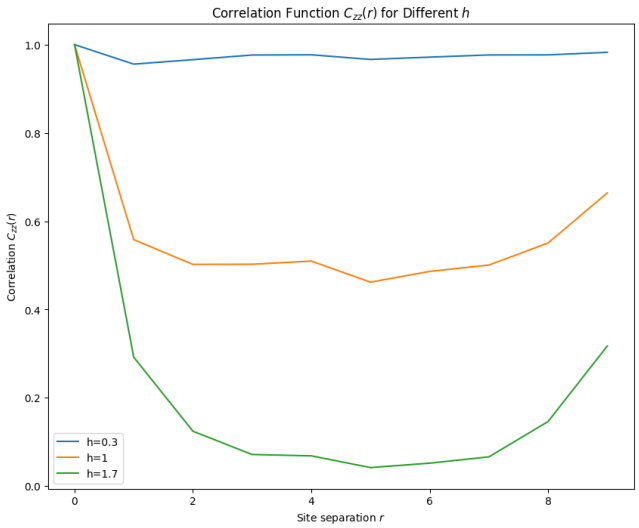

Figure 10: Correlation $C_{zz}(r)$ as a function of r for h={ 0.3, 1, 1.7 } and L=10

6 4.5 STUDY OF MAGNETIC ORDERING

We plotted $C_{zz}(r)$ as a function of r for multiple system sizes L. This is symmetric and goes back up towards 1 when the site seperation r becomes larger.

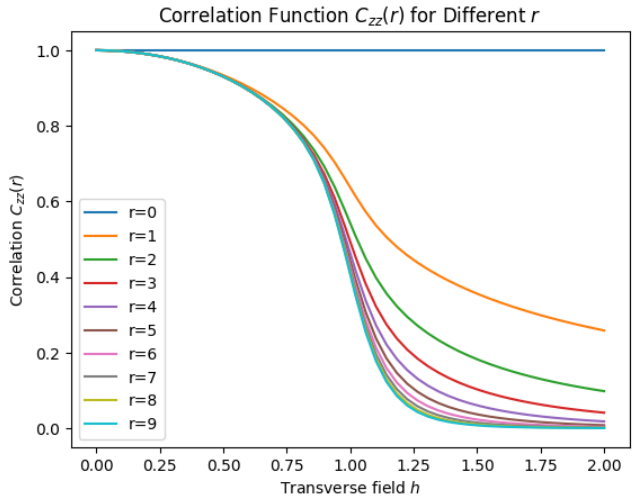

Figure 9: Correlation $C_{zz}(r)$ as a function of h for 9 values of r

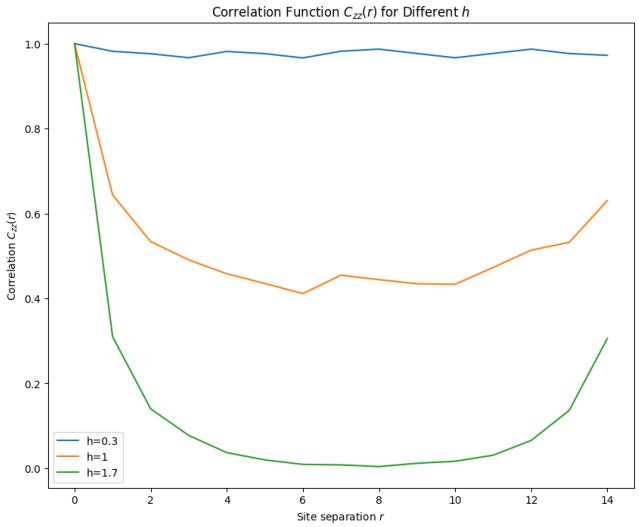

Figure 11: Correlation $C_{zz}(r)$ as a function of r for h={ 0.3, 1, 1.7 } and L=15

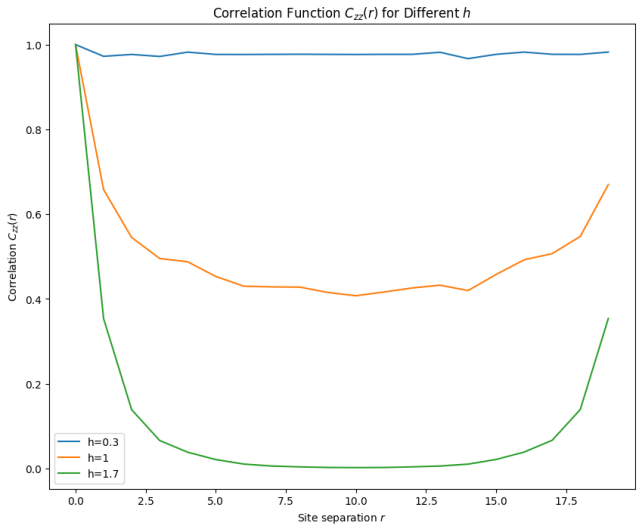

Figure 12: Correlation $C_{zz}(r)$ as a function of r for h={ 0.3, 1, 1.7 } and L=20

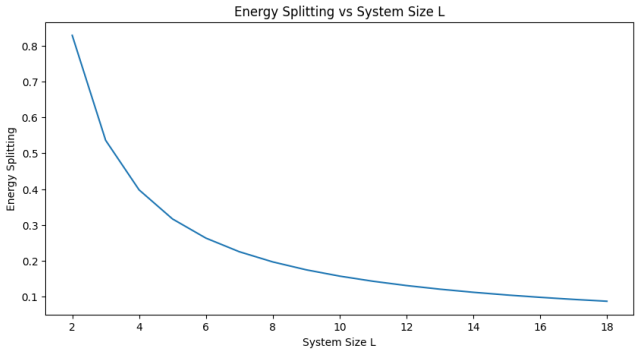

Figure 14: Energy splitting as a function of system size

7 4.7 ISING SYMMETRY

We used the Ising symmetry to make the computation more efficient. We used the fact that $H = H_+ \oplus H_-$. In a Hamiltonian of a given parity, if we know L-1 of the bits of a state (in the integer representation), we know the Lth using the fact that H_-, H_+ has eigenvalues repectively -1, 1. Thus we can reduce the size of our matrice by 2. The implementation is very similar to what has been done throughout the rest of the assignement. This makes our code significantly faster. Due to lack of time before due date of the assignement, I did not have time to let the code run for L={8, 10, 12, 14, 16, 18, 20, 22} and stopped at L=16, although going to L=22 would have worked fine. We also plotted the energy splitting as a function of L, L going from 2 to 18.

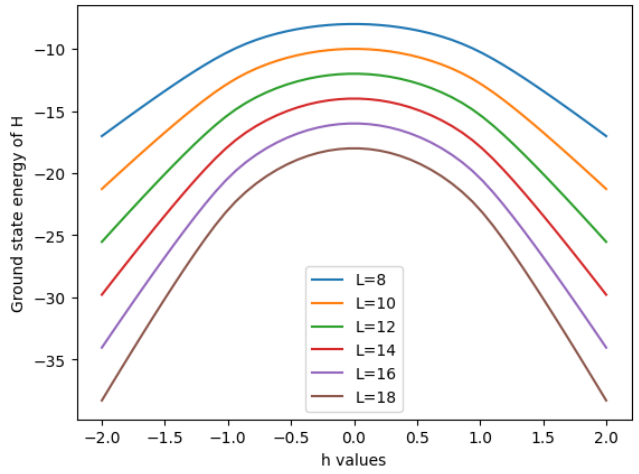

Figure 13: Ground state energy as a function of L