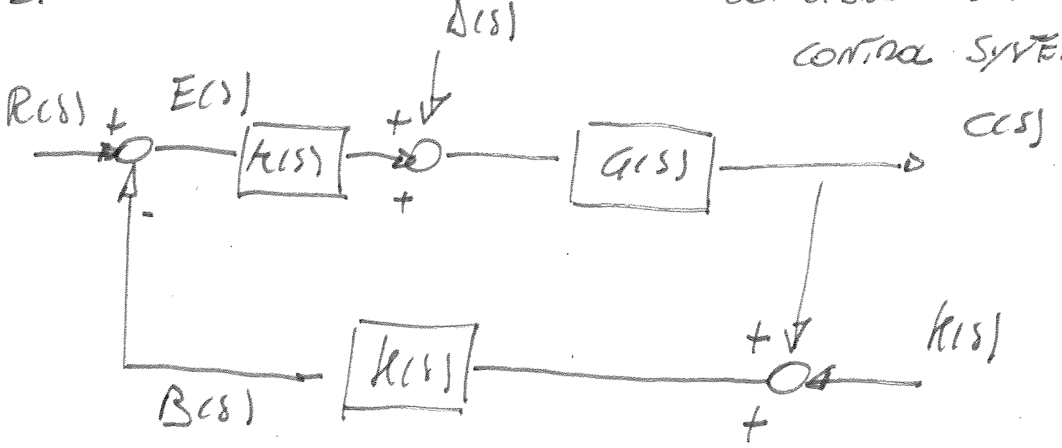
EXAMPLE 1

CONSIDER THE FOLLOWING CONTROL SYSTEM



R(s) REF INPUT D(s) DISTURBANCE C(s) OUTPUT

K(s) CONTROLLER G(s) PLANT / SYSTEM N(s) = NOISE

H(s) SENSOR E(s) ERROR B(s) = FEEDBACK

EVALUATE THE FOLLOWING TRANSFER FUNCTION

	IN	OUT
1	R(s)	C(s)
2	N(s)	C(s)
3	D(s)	C(s)

Hp: WE ARE IN THE LINEAR DOMAIN HENCE WE CAN USE THE SUPERPOSITION METHODS

THE OUTPUT (TOTAL) IS GIVEN BY THE CONTRIBUTION OF ALL THE INPUTS (R(s), N(s), D(s))

1) $C_R(s) \rightarrow R(s)$ $(D(s) = N(s) = 0)$

$$\frac{C_R(s)}{R(s)} = \frac{K(s)\,G(s)}{1 + H(s)\,K(s)\,G(s)}$$

#2

2) $G_D(s) \to D(s) \qquad (R(s) = N(s) = 0)$

$$\frac{C_D(s)}{D(s)} = \frac{G(s)}{1+G(s)H(s)K(s)}$$

3) $G_N(s) \to N(s) \qquad (R(s) = D(s) = 0)$

$$\frac{C_N(s)}{N(s)} = \frac{-H(s)G(s)K(s)}{1+G(s)H(s)K(s)}$$

DEMONSTRATION

$$C = (N+C)(-HKG)$$

$$C = -NHKG - CHKG$$

$$C(1+HKG) = -NHKG$$

$$\frac{C}{N} = \frac{-HKG}{1+HKG}$$

NOW WE USE THE SUPERPOSITION PRINCIPLE AND WE FIND THE TOTAL RESPONSE.

$$C_{TOT}(s) = G_R(s) + G_D(s) + G_N(s) =$$

$$= \frac{[R(s)K(s)G(s)] + [G(s)D(s)] - [K(s)G(s)H(s)N(s)]}{1+K(s)G(s)H(s)}$$

#

#3

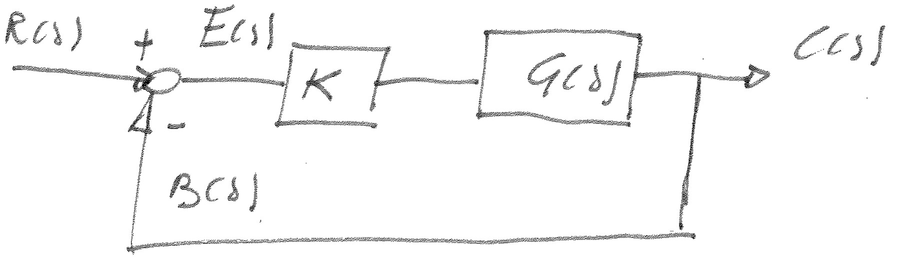
ADDITIONAL QUESTION: ASSUMING THAT
$$\begin{cases} H(s) = 1 \\ A(s) = k \\ N(s) = 0 \\ D(s) = 0 \end{cases}$$

DERIVE A SIMPLE EXPRESSION FOR THE ERROR

$E(s) = R(s) - B(s)$ IN TERM OF $R(s)$, $G(s)$ AND k.

#DEMONSTRATION

THE NEW SYSTEM



$C(s) = B(s)$ BECAUSE $H(s) = 1$

$$E(s) = R(s) - B(s) = R(s) - C(s)$$

BUT $C(s) = \dfrac{k\,G(s)}{1 + k\,G(s)} \cdot R(s)$ THEREFORE WE HAVE

$$E(s) = R(s)\left[1 - \frac{k\,G(s)}{1 + k\,G(s)}\right] = R(s)\left[\frac{1 + k\,G(s) - k\,G(s)}{1 + k\,G(s)}\right]$$

$$\frac{E(s)}{R(s)} = \frac{1}{1 + k\,G(s)}$$

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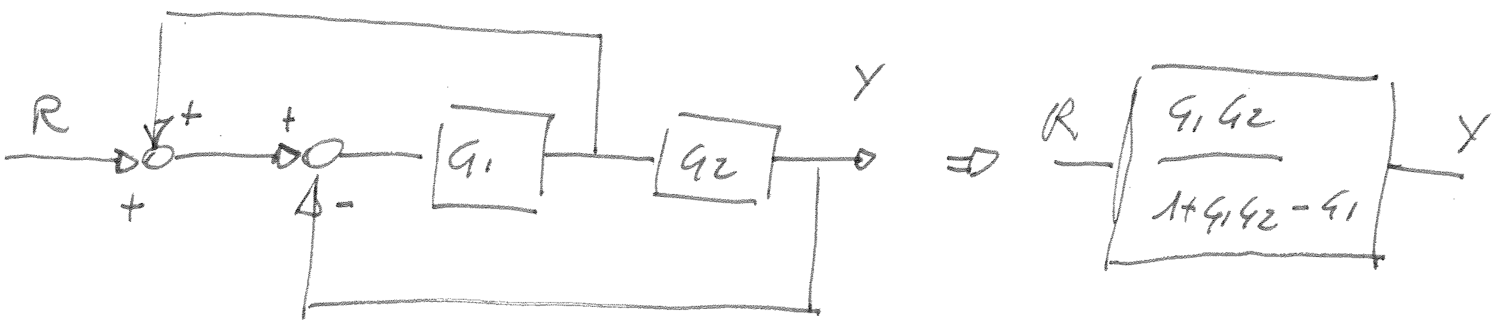
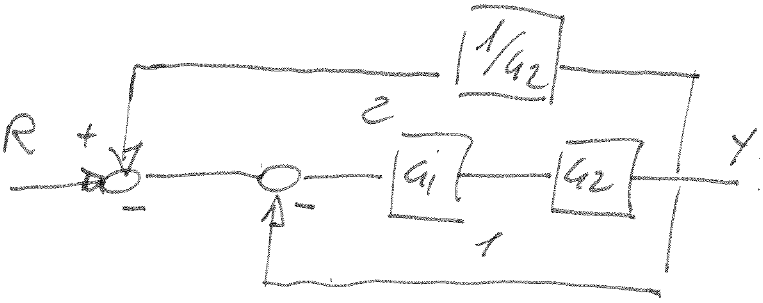
#4 EXAMPLE N2

DEMONSTRATE THE EQUIVALENCE OF THESE TWO SYSTEMS

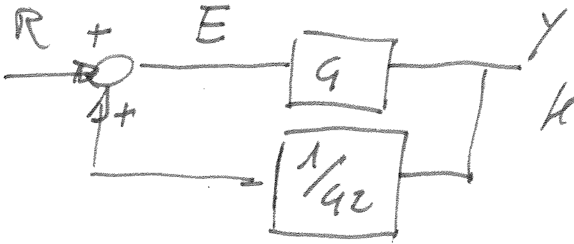


#DEMONSTRATION

WE CAN REWRITE THE ABOVE SCHEME.



THE LOOP 1 CAN BE NOW REDUCED $\quad G(s) = \dfrac{G_1 G_2}{1+G_1G_2}$



HOW TO EXPRESS $Y = (\ldots)R$?

LET'S CONSIDER $E = R + Y/G_2$ ALSO $Y = E\,G = E\,\dfrac{G_1G_2}{1+G_1G_2}$

COMBINING THE EQUATIONS by SUBSTITUTING E IN

$$Y = \left(R + \frac{Y}{G_2}\right)\left(\frac{G_1G_2}{1+G_1G_2}\right)$$

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$$Y = \left(R + \frac{Y}{G_2}\right)\left(\frac{G_1G_2}{1+G_1G_2}\right) = \frac{G_1G_2R}{1+G_1G_2} + \frac{Y\,G_1G_2}{G_2(1+G_1G_2)}$$

MANIPULATING THE EQUATIONS $\Rightarrow$

$$Y - Y\frac{G_1}{1+G_1G_2} = \frac{G_1G_2R}{1+G_1G_2}$$

$$Y + YG_1G_2 - YG_1 = G_1G_2R$$

$$Y(1 + G_1G_2 - G_1) = G_1G_2R$$

$$\boxed{Y = \frac{G_1G_2}{1+G_1G_2-G_1} \cdot R}$$

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