EXAMPLE SOLVING DIFF EQUN.

TIME DOMAIN

$$\frac{d^2y}{dt^2} + 12\frac{dy}{dt} + 32y = 32u(t) \qquad \begin{cases} y = y(t) \\ u = u(t) \end{cases} \quad [1]$$

LECTURE 2
CONTROL SYSTEM DESIGN

solution OMOGENEOUS + PARTICULAR

DEM

$$\begin{cases} y(0) = 0 \\ \dot{y}(0) = 0 \end{cases}$$

WE CAN WRITE.

$$s^2 Y(s) + 12\, Y(s) + 32\, Y(s) = \frac{32}{s}$$

$$\mathcal{L}(u(t)) = \frac{1}{s}$$

SOLVING $Y(s)$

$$Y(s) = \frac{32}{s(s^2+12s+32)} = \frac{32}{s(s+4)(s+8)}$$

$$Y(s) = \frac{k_1}{s} + \frac{k_2}{s+4} + \frac{k_3}{s+8}$$

$$k_1 = \left.\frac{\cancel{s}\ 32}{\cancel{s}(s+4)(s+8)}\right|_{s=0} = 1 \ \checkmark$$

$$k_2 = \left.\frac{\cancel{(s+4)}\ 32}{s\cancel{(s+4)}(s+8)}\right|_{s=-4} = -2 \ \checkmark$$

$$k_3 = \left.\frac{\cancel{(s+8)}\ 32}{s(s+4)\cancel{(s+8)}}\right|_{s=-8} = 1 \ \checkmark$$

[2]

$$Y(s) = \frac{1}{s} - \frac{2}{s+4} + \frac{1}{s+8}$$

$\mathcal{L}^{-1}$

$$y(t) = 1 - 2e^{-4t} + e^{-8t}$$ ✓

[3]

EX2

$$F(s) = \frac{10}{s(s+1)}$$ 1) APPLY FVT

2) VERIFY BY TAKING $\mathcal{L}^{-1}$

1) DEMONST. REMEMBER FVT $\mathcal{L}^{-1}$

$$\mathcal{L}[f(t)] = F(s) = \int_0^{\infty} e^{-st} f(t)\, dt$$

$$\mathcal{L}^{-1}[F(s)] = f(t) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} F(s)\, e^{st}\, ds$$

FVT

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} s F(s) \qquad F(s) = \mathcal{L}[f(t)]$$

$$\frac{10}{(s+1)s} = F(s) = \frac{A}{s} + \frac{B}{s+1} = \frac{A(s+1) + Bs}{s(s+1)}$$

Eq. THE NUMERATORS

$$10 = A(s+1) + Bs \qquad (A+B)s + A = 10$$

$$\begin{cases} A = 10 \\ A + B = 0 \quad A = -B \quad B = -10 \end{cases}$$

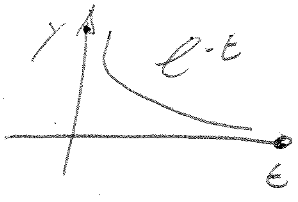
$$F(s) = \frac{10}{s} - \frac{10}{s+1} \quad \mathcal{L}^{-1} \quad f(t) = 10 - 10e^{-t}$$

[4]

FVT

$F(s)$ $\quad FVT = \lim_{s \to 0} s\,F(s) = \lim_{s \to 0} \cancel{s} \frac{10}{\cancel{s}(s+1)} = 10$ ✓

$f(t)$ $\quad FVT = \lim_{t \to \infty} 10 - 10e^{-t} = 10$ ✓



EX 3

DETERMINE $\begin{cases} f(0^+) \\ \dot{f}(0^+) \end{cases}$ using IVT $\qquad F(s) = \frac{1}{(s+2)^2}$

$F(s) = \frac{1}{(s+2)^2}$ MULTIPLE POLES

IVT $\quad \lim_{t \to 0^+} f(t) = \lim_{s \to \infty} s\,F(s) = \lim_{s \to \infty} s \frac{1}{(s+2)^2} = 0$

NOW WE NEED TO EVALUATE $\dot{f}(0^+)$

USE THE DIFFERENTIATION PROPERTY OF $\mathcal{L}$

$$\mathcal{L}[\dot{f}(t)] = s\,F(s) - f(0)\big|_{t=0^+}$$

[5]

$$\mathcal{L}[\dot{f}(t)] = sF(s) - f(t)\Big|_{t=0^+} = s\frac{1}{(s+2)^2} - 0$$

WE KNOW FROM PREVIOUS FVT

$f(0^+)=0$

$$\mathcal{L}[\dot{f}(t)] = \frac{s}{(s+2)^2}$$

FVT $\lim_{t\to 0^+} \dot{f}(t) = \lim_{s\to\infty} s\left\{\mathcal{L}[\dot{f}(t)]\right\} = \lim_{s\to\infty} s\,\frac{s}{(s+2)^2} =$

$$= \lim_{s\to\infty} \frac{s^2}{s^2+4s+4} = \lim_{s\to\infty} \frac{\cancel{s^2}}{\cancel{s^2}\left(1+\frac{4}{s}+\frac{4}{s^2}\right)} = 1 \checkmark$$

[6]

Ex 4

FIND $f(t)$ FOR

1) $F(s) = \dfrac{10}{s(s+1)(s+10)}$

2) $F(s) = \dfrac{1}{s(s+2)^2}$

DEMONS #1)

$F(s) = \dfrac{10}{s(s+1)(s+10)} = \dfrac{\alpha}{s} + \dfrac{\beta}{s+1} + \dfrac{\gamma}{s+10}$ PLAY WITH NUMERATOR

$\alpha(s+1)(s+10) + \beta(s+10)s + \gamma(s+1)s = 10$

$\alpha(s^2+10s+s+10) + \beta s^2 + \beta s\, 10 + \gamma s^2 + \gamma = 10$

$s^2(\alpha+\beta+\gamma) + s(11\alpha+10\beta+\gamma) + 10\alpha = 10$

$$\begin{cases} \alpha+\beta+\gamma = 0 \\ 11\alpha+10\beta+\gamma = 0 \\ 10\alpha = 10 \end{cases} \qquad \begin{cases} \alpha = 1 \\ \beta = -\dfrac{10}{9} \\ \gamma = \dfrac{1}{9} \end{cases}$$

$F(s) = \dfrac{1}{s} + \dfrac{-\frac{10}{9}}{s+1} + \dfrac{\frac{1}{9}}{s+10}$ $\mathcal{L}^{-1}$

$f(t) = 1 - \dfrac{10}{9} e^{-t} + \dfrac{1}{9} e^{-10t}$ ✓

7)

DEMONS #2) $F(s) = \dfrac{1}{s(s+2)^2}$ MULTIPLE POLES

$$F(s) = \frac{\alpha}{s} + \frac{\beta}{s+2} + \frac{\gamma}{(s+2)^2}$$ NUMERATOR EQUATING

$$\alpha(s+2)^2 + \beta s(s+2) + s\gamma = 1$$

$$s^2(\alpha+\beta) + s(4\alpha + 2\beta + \gamma) + 4\alpha = 1$$

$$\begin{cases} 4\alpha = 1 \\ \alpha + \beta = 0 \\ 4\alpha + 2\beta + \gamma = 0 \end{cases} \Rightarrow \begin{cases} \alpha = \frac{1}{4} \\ \beta = -\frac{1}{4} \\ \gamma = -\frac{1}{2} \end{cases}$$

$$F(s) = \frac{1}{4s} - \frac{1}{4(s+2)} - \frac{1}{\cancel{4}(s+2)^2 \cdot 2}$$

$\mathcal{L}^{-1} = \frac{1}{4}$

$\mathcal{L}^{-1} = \frac{1}{4} e^{-2t}$

$$\mathcal{L}^{-1} \Rightarrow \mathcal{L}\left[\frac{1}{(m-1)!} t^{m-1} e^{-\alpha t}\right] = \frac{1}{(s+\alpha)^m}$$

IN OUR CASE

$$\mathcal{L}^{-1}\left(\frac{1}{(s+2)^2}\right) = \frac{1}{(2-1)!} t^{2-1} e^{-2t} = t e^{-2t}$$

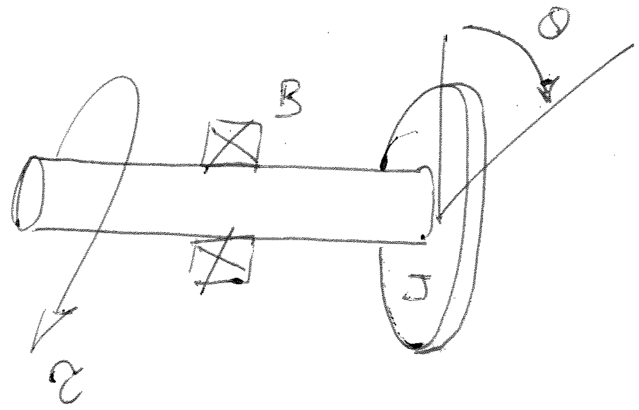
8) THEN

$$\mathcal{L}^{-1}\left[\frac{1}{S(S+2)^2}\right] = \frac{1}{4}\left\{1-e^{-2t}-2t\,e^{-2t}\right\} = f(t)$$

✓

9)

ROTATIONAL SYSTEMS



DATA

$$\begin{cases} J = 100 \ kg\,m^2 \\ B = 10 \ Nm/(RAD/s) \end{cases}$$

QUESTIONS

1) FIND THE TF $\dfrac{\Omega(s)}{T(s)}$

WHERE

$$\Omega(s) = \mathcal{L}[w(t)]$$

2) FIND USING FVT

$w(t)$ FINAL VALUE FOR

$$\tau = 100 \ Nm$$

– DEMONSTRATION

LET'S USE NEWTON Eq.

$$\sum \tau = J\dot{w}$$

SUM OF THE TORQUES ON THE SYSTEM

$$\tau - Bw = J\dot{w}$$

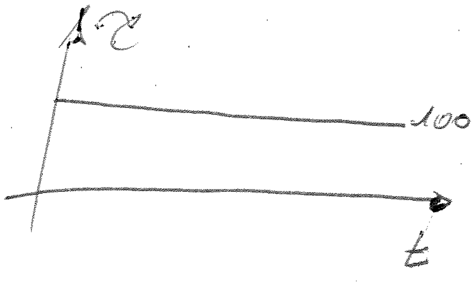
$$\tau - Bw(t) = J\dot{w}(t)$$

$$J\dot{w}(t) + Bw(t) = \tau(t)$$

$$\mathcal{L} \Rightarrow \quad Js\,\Omega(s) + B\,\Omega(s) = T(s)$$

$$\frac{\Omega(s)}{T(s)} = \frac{1}{Js+B} = \frac{1}{100s+10}$$

NOW WE APPLY A TORQUE $\tau = 100 \ Nm$



$$\Rightarrow T(s) = \mathcal{L}[\tau(t)] = \frac{100}{s}$$

10)

$$\frac{\Omega(s)}{T(s)} = \frac{1}{100s+10} \qquad \Omega(s) = \frac{1}{100s+10}\cdot T(s)$$

$$\Omega(s) = \frac{1}{100s+10}\cdot\frac{100}{s}$$ USING THE FVT

$$\omega_{FIN} = \lim_{t\to\infty}\omega(t) = \lim_{s\to 0} s\,\Omega(s) =$$

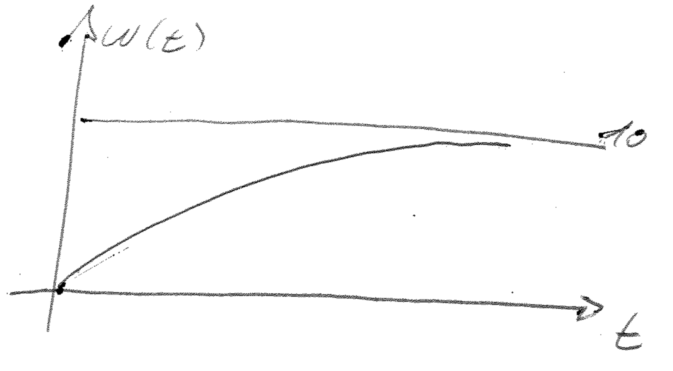
$$= \lim_{s\to 0} s\,\frac{1}{100s+10}\cdot\frac{100}{s} = \lim_{s\to 0}\frac{100}{100s+10} = 10 \ \mathrm{RAD}/s$$

IF WE DO USING $f(t) \Rightarrow \omega(t)$

$$\Omega(s) = \frac{1}{100s+10}\cdot\frac{100}{s} = \frac{A}{s} + \frac{B}{100s+10} \Rightarrow \begin{cases} A = 10 \\ B = -1000 \end{cases}$$

$$\Omega(s) = \frac{10}{s} - \frac{100}{10s+1} \rightarrow \mathcal{L}^{-1} \rightarrow \omega(t) = 10 - 10\,e^{-\frac{t}{10}}$$

$$\omega_{FIN} = \lim_{t\to\infty}\omega(t) = 10 \ \mathrm{RAD}/s$$



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