TIME RESPONSE OF AN UNDERDAMPED 2nd ORDER SYSTEM. #1

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$R(s) \longrightarrow \boxed{G(s)} \longrightarrow C(s)$$

IF $R(s) = \frac{1}{s}$

$C(s) = G(s) \cdot R(s)$

$$C(s) = \frac{1}{s} \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

LET'S DERIVE THE $\frac{d}{dt} c(t)$

$$= \frac{1}{s} - \frac{(s+\xi\omega_n) + \frac{\xi}{\sqrt{1-\xi^2}} \omega_n \sqrt{1-\xi^2}}{(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)}$$

TIME

$c(t) = \mathcal{L}^{-1}[C(s)]$ I CAN EXPRESS $C(s)$ IN:

$$C(s) = \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2 + \omega_d^2} - \frac{\xi\omega_n}{(s+\xi\omega_n)^2 + \omega_d^2}$$

WHERE

$\omega_d = \omega_n\sqrt{1-\xi^2}$ I OBTAIN:

$$c(t) = 1 - e^{-\xi\omega_n t}\left(\cos\omega_d t + \frac{\xi}{\sqrt{1-\xi^2}} \sin\omega_d t\right)$$

NOW I WILL DIFFERENTIATE IT.

#2

$$\frac{d}{dt}\left[f(t)\,g(t)\right] = \frac{d}{dt}f(t)\cdot g(t) + f(t)\cdot\frac{d}{dt}g(t)$$

$$\frac{dc(t)}{dt} = \xi\omega_n e^{-\xi\omega_n t}\left(\cos\omega_d t + \frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_d t\right) +$$

$$+ e^{-\xi\omega_n t}\left(\omega_d \sin\omega_d t - \frac{\xi\overset{\swarrow\,\omega_n\sqrt{1-\xi^2}}{\omega_d}}{\sqrt{1-\xi^2}}\cos\omega_d t\right).$$

$$= \xi\omega_n e^{-\xi\omega_n t}\left(\overset{\times}{\cos}\omega_d t + \frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_d t\right)$$

$$+ e^{-\xi\omega_n t}\left(\omega_d \sin\omega_d t - \frac{\xi\cdot\omega_n\cancel{\sqrt{1-\xi^2}}}{\cancel{\sqrt{1-\xi^2}}}\overset{\times}{\cos}\omega_d t\right)$$

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THE COSINES CANCEL EACH OTHER!

HENCE

$$\dot{c}(t) = \xi\omega_n e^{-\omega_n\xi t}\cdot\frac{\xi}{\sqrt{1-\xi^2}}\cdot\sin\omega_d t +$$

$$+ e^{-\xi\omega_n t}\cdot\omega_d \sin\omega_d t.$$

#3

PUTTING IN EVIDENCE:

$$\dot{c}(t) = e^{-\xi w_n t} \sin w_d t \underbrace{\left[\frac{\xi w_n \xi}{\sqrt{1-\xi^2}} + w_n \sqrt{1-\xi^2} \right]}_{\frac{\cancel{\xi^2 w_n} + w_n - \cancel{\xi^2 w_n}}{\sqrt{1-\xi^2}}}$$

$$\Rightarrow \dot{c}(t) = \frac{w_n}{\sqrt{1-\xi^2}} e^{-\xi w_n t} \sin w_d t$$

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