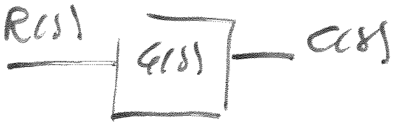
EXAMPLE

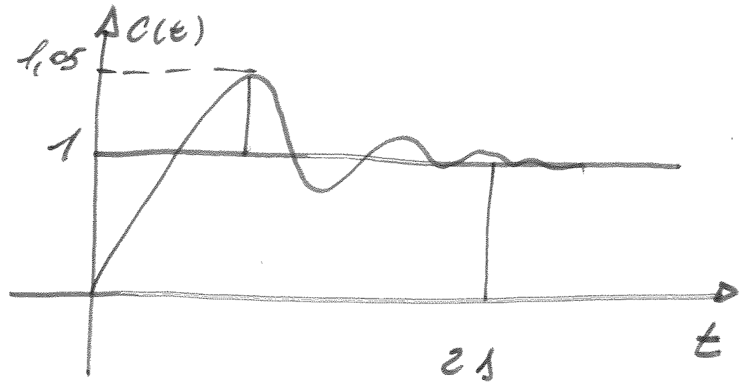
$$T(s) = G(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \qquad \frac{C(s)}{R(s)} = G(s) \qquad [1]$$



DETERMINE ξ ? AND ω_n ? $\begin{cases} M_p\ OV\% = 5\% \\ T_s = 2\ s \end{cases}$

— DEM —

$R(s) = \frac{1}{s}$



GENERAL

$$c(t) = 1 - \frac{1}{\sqrt{1-\xi^2}}\, e^{-\xi\omega_n t} \cos\left(\omega_n\sqrt{1-\xi^2}\, t - \phi\right)$$

SETTLING TIME $\quad \dfrac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} = 0{,}02 \qquad T_s = \dfrac{-\ln\left(0{,}02\sqrt{1-\xi^2}\right)}{\xi\omega_n}$

$$T_s \simeq \frac{4}{\xi\omega_n} \quad \Rightarrow T_s = 2 = \frac{4}{\xi\omega_n} \quad \Rightarrow \xi\omega_n = 2$$

PAG AFTER LET'S FIND THE OS%

[2]

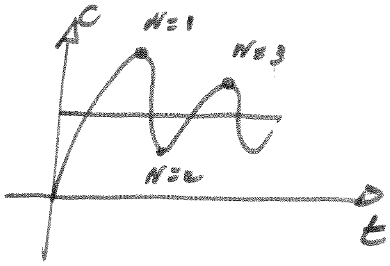
WE NEED TO REFER TO THE TIME PEAK (MAX)

$$c(t) = 1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi \omega_n t} \cos\left(\omega_n \sqrt{1-\xi^2}\, t - \phi\right)$$

$$\dot{c}(t) = \frac{dc(t)}{dt} = \frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi \omega_n t} \sin \omega_n t \sqrt{1-\xi^2}$$

TO FIND MAX $\dot{c}(t) = 0$ FOR $\omega_n \sqrt{1-\xi^2}\, t = m\pi$

$m = 1$ WE HAVE THE FIRST PEAK



$$T_p = \frac{\pi}{\omega_n \sqrt{1-\xi^2}}$$

NOW FOCUS ON OS% = 0,05

$$c(T_p) = 1 - e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} \left(\underbrace{\cos\pi}_{1} + \frac{\xi}{\sqrt{1-\xi^2}} \underbrace{\sin\pi}_{0}\right)$$

$$e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} = 0,05 \qquad \frac{\xi\pi}{\sqrt{1-\xi^2}} = -\ln(0,05) = 2,9957$$

[3.]

SOLVING IN ξ

$$\xi^2 \pi^2 = (2,9957)^2 (1-\xi^2)$$

$$\Rightarrow \xi = 0,69$$

$$(0 < \xi < 1)$$

WE HAD ALSO $\xi \omega_m = 2$ $\omega_m = \frac{2}{\xi} = \frac{2}{0,69} = 2,9 \ RAD/s$

$$\begin{cases} M_p = 0,05 \\ t_s = 2 \end{cases} \Rightarrow \begin{cases} \xi = 0,69 \\ \omega_m = 2,9 \ RAD/s \end{cases}$$

141.

EXAMPLE 2

DETERMINE k & $\underline{k}$ $\Rightarrow \begin{cases} \xi = 0{,}7 \\ \omega_m = 4 \text{ RAD/s} \end{cases}$

$$\frac{C(s)}{R(s)} = \frac{k}{\underbrace{s^2 + (k\underline{k} + 2)s + k}}$$

$$\Downarrow$$

FOR ANALOGY TWO EQNS.

$$s^2 + 2\xi\omega_m s + \omega_m^2$$

$$\begin{cases} 2\xi\omega_m = (k\underline{k} + 2) \\ \omega_m^2 = k \end{cases}$$

$4^2 = k \Rightarrow k = 16$

$$2\xi\omega_m = 2 + k\underline{k}$$

$$2 \cdot 0{,}7 \cdot 4 = 2 + 16\underline{k} \Rightarrow \underline{k} = 0{,}225$$