HANDNOTES ON DYNAMICS

FOR A SYSTEM S THE STATE SPACE FORM

$$\begin{cases} \dot{x} = Ax + B \\ y = Cx + Du \end{cases}$$

$x(t) \in \mathbb{R}^m$

↓ VECTOR OF m COMPONENTS

$x(t)$ STATE OF THE SYSTEM S

$u(t)$ CAN BE ALSO A VECTOR OF INPUTS

IF THE SYSTEM HAS ONLY 1 INPUT $\Rightarrow$ $u(t) \in \mathbb{R}$ (1 DIMENSION)

HENCE $u(t)$ IS SCALAR.

$y(t)$ OUTPUT VECTOR $\in \mathbb{R}^m$

IN MOSTO OF OUR CASES AND EXAMPLE WE WILL HAVE $y(t) \in \mathbb{R}$ (SCALAR) (1 OUTPUT)

A, B, C, D MATRIX OR VECTOR

THEY ARE CARACTERIST OF THE SYSTEM

IF WE CONSIDER A SISTEM S AND THE DIFFERENTIAL EQNS

OF 1ST ORDER

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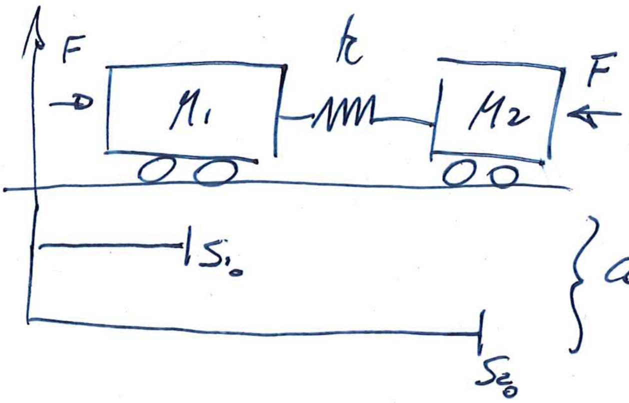
EXAMPLE

LET'S CONSIDER A 2 DoFs (DEGREES OF FREEDOM) SYSTEM

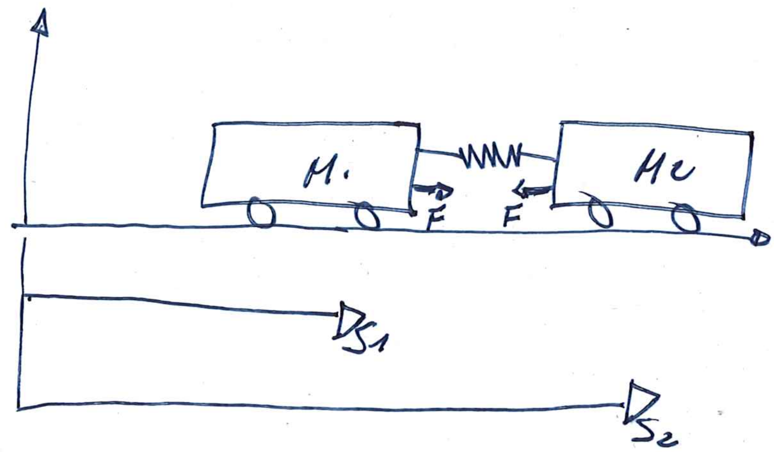


* WE DO NOT HAVE FRICTION BUT ONLY A SPRING k (STIFFNESS)

COORDINATES OF THE SYSTEM

S_{i_0} = POSITION AT REST (NO FORCE FROM THE SPRING)

LET'S MOVE THE TWO MASSES



LET'S APPLY NEWTON AND FIND THE DIFF EQNs OF MOTION

$$\begin{cases} M_1 \ddot{S}_1 = -k\left(S_1 - S_{10} - S_2 + S_{20}\right) + F \\ M_2 \ddot{S}_2 = k\left(S_1 - S_{10} - S_2 + S_{20}\right) - F \end{cases}$$

* SECOND ORD DIFF EQNs *

THE SYSTEM HAS TWO DoFs AND WE HAVE/NEED TWO VARIABLES TO DESCRIBE ITS STATE IN TIME (t)

WE CHOOSE THE STATE VECTOR $X = \begin{pmatrix} S_1 - S_{10} \\ S_2 - S_{20} \\ \dot{S}_1 \\ \dot{S}_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$

$$\frac{d}{dt} x(t) = \begin{pmatrix} \dot{s}_1 \\ \dot{s}_2 \\ \ddot{s}_1 \\ \ddot{s}_2 \end{pmatrix}$$

$\ddot{s}_1$ AND $\ddot{s}_2$ CAN BE OBTAINED FROM THE EQNS OF DYNAMICS

WHERE

$$\begin{cases} \ddot{s}_1 = -\dfrac{k}{M_1}(x_1 - x_2) + \dfrac{F}{M_1} \\[2ex] \ddot{s}_2 = \dfrac{k}{M_2}(x_1 - x_2) - \dfrac{F}{M_2} \end{cases}$$

$$\dot{x} = Ax + Bu$$

$$\dot{x} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k}{M_1} & \frac{k}{M_1} & 0 & 0 \\ \frac{k}{M_2} & -\frac{k}{M_2} & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1/M_1 \\ -1/M_2 \end{pmatrix} u$$

WHERE $u(t) = f(t)$
$= F(t)$

WHAT HAPPENS IF WE CHOOSE A DIFFERENT COORDINATE SYSTEM?

I CAN DESCRIBE THE SYSTEM WITH DIFFERENT COORDINATES

NOT ANYMORE S_1 AND S_2 WHICH ARE THE MASSES COORDINATES, BUT I CAN CONSIDER THE CENTER OF MASS AND THE RELATIVE MOTION

THE STATE VECTOR WILL BE $z = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix}$

AND THE REPRESENTATION

$$\dot{z}(t) = \tilde{A} z(t) + \tilde{B} u(t)$$

WHAT IS THE RELATION BETWEEN $z(t)$ AND $x(t)$?

FOR EXAMPLE LET'S START FROM THE ~~NEW~~ NEW COORDINATE ⇒ CENTER OF MASS OF M_1 AND M_2

$$z_1 = \frac{M_1 S_1 + M_2 S_2}{M_1 + M_2} - \frac{M_1 S_{10} + M_2 S_{20}}{M_1 + M_2}$$

IT IS A LINEAR COMBINATION OF S_1 AND S_2 THEREFORE ALSO THE TWO DESCRIPTIONS $\dot{z}(t)$, $\dot{x}(t)$ MUST BE LINEARLY RELATED.

IF WE HAVE

$$\begin{cases} \dot{x}(t) = A\,x(t) + B\,u(t) \\ y(t) = C\,x(t) + D\,u(t) \end{cases} \qquad \begin{cases} \dot{z}(t) = \tilde{A}\,z(t) + \tilde{B}\,u(t) \\ y(t) = \tilde{C}\,z(t) + \tilde{D}\,u(t) \end{cases}$$

EXISTS A NON-SINGULAR MATRIX (INVERTIBLE) T

$$z = T\,x \qquad x = T^{-1} z$$

$$\dot{z} = T\,\dot{x} = T\,A\,x(t) + T\,B\,u(t)$$

HENCE WE CAN WRITE

$$\begin{cases} \dot{z}(t) = T\,A\,T^{-1} z(t) + T\,B\,u(t) \\ y(t) = C\,T^{-1} z(t) + D\,u(t) \end{cases}$$

$\Rightarrow$ FROM WHICH WE DERIVE

$$\begin{cases} \tilde{B} = T\,B \\ \tilde{A} = T\,A\,T^{-1} \\ \tilde{C} = C\,T^{-1} \\ \tilde{D} = D \end{cases}$$

THE RESULT IN TERMS OF PHYSICAL MEANING MUST BE THE SAME.

DIGRESSION ON SOLUTIONS FOR THE STATE SPACE REPRESENTATION

IF WE HAVE $\dot{x} = A\,x(t) + B\,u(t)$

STATE SPACE FOR A GENERIC SYSTEM

IF WE CONSIDER FOR SIMPLICITY $A = a$, $B = b$ $\Rightarrow$ SCALARS (1 DOF SYSTEM)

$\dot{x}(t) = a\,x(t) + b\,u(t)$ WE HAVE TWO SOLUTIONS
1) HOMOGENEOUS
2) PARTICULAR

1) HOMOGENEOUS

$$\begin{cases} \dot{x} = a\,x(t) \\ x(0) = x_0 \end{cases} \Rightarrow x(t) = e^{at} x_0$$

2) IF WE CONSIDER ALSO $b\,u(t)$ THE GENERAL SOLUTION

$$(1)^* \quad x(t) = x_0\, e^{at} + \underbrace{\int_0^t e^{a(t-\tau)}\, b\,u(\tau)\, d\tau}_{\text{CONVOLUTION INTEGRAL}}$$

LET'S DEMONSTRATE THAT $(1)^*$ IS THE SOLUTION FOR

$$\dot{x}(t) = a\,x(t) + b\,u(t)$$

$\Rightarrow$ SOLUTION

LET'S APPLY A PROPERTY OF THE CONVOLUTION

$$\frac{d}{dt}\int_0^t f(t,\tau)\,d\tau = \int_0^t \frac{d}{dt} f(t,\tau)\,d\tau + f(t,\tau)\Big|_{\tau=t}$$

WE APPLY THIS PROPERTY TO (1*)

$$\frac{d}{dt} x(t) = \frac{d}{dt}\left[e^{\alpha t} x_0 + \int_0^t e^{\alpha(t-\tau)} b\, u(t)\, d\tau \right] =$$

$$\dot{x}(t) = \underbrace{\alpha X_0 e^{\alpha t} + \int_0^t \alpha\, e^{\alpha(t-\tau)} b\, u(t)}_{\alpha\, x(t)} + \underbrace{b\, u(t)}_{b\, u(t)}$$

IT HAS THE SAME FORM OF $\dot{x}(t) = \alpha(x(t)) + b\, u(t)$ #

THE PREVIOUS ASSUMPTION $A = \alpha$, $B = b$ WAS FOR A SYSTEM WITH 1 DoF, MEANING 1 DIFF EQN DESCRIBING ITS STATE IN TIME.

WHAT HAPPENS WITH MORE THAN 1 DoF? $(A \in \mathbb{R}^m)$

$$\underset{1\ \text{DoF}}{e^{\alpha t}} \Rightarrow \underset{\text{DoF} > 1}{e^{At}}$$

LET'S DEFINE THE EXPONENTIAL MATRIX

IF WE HAVE

$$e^{At} \simeq \underbrace{I + At + A^2\frac{t^2}{2} + \dots = \sum_{k=0}^{\infty} \frac{t^k A^k}{k!}}_{\text{SERIES EXPANSION}}$$

$A \in \mathbb{R}^{m}$

LET'S DIFFERENTIATE IN TIME $x(t) = x_0 e^{At}$

$$\frac{d}{dt}e^{At} = \left(0 + A + A^2 t + \dots\right)x_0 = A\left(I + At + A^2\frac{t^2}{2}\dots\right)x_0 =$$

$$= A\, e^{At} x_0 = A\, x(t)$$

IF WE LOOK AT $\dot{x}(t) = \underbrace{A\, x(t)}_{} + B\, u(t)$

THIS PART IS VERIFIED TO BE THE HOMOGENEOUS SOLUTION

$$\underbrace{\dot{x}(t) = A\, e^{At} x_0 = A\, x(t)}_{}$$

THE SOLUTION OF THE DIFF EQN WHEN NO INPUT ARE APPLIED.

~ WHAT HAPPEN IF WE HAVE A FORCE $B\,u(t)$? (INPUT)

WE KNOW THE SOLUTION IS

$$x(t) = \underbrace{e^{At} x_0}_{\text{HOMOGENEOUS SOLUTION}} + \int_0^t \underbrace{e^{A(t-\tau)} B\, u(\tau)\, d\tau}_{\text{PARTICULAR SOLUTION}}$$

$\begin{cases} A & \text{MATRIX} \\ B & \end{cases}$

THE OUTPUT

$$y(t) = C\, e^{At} x_0 + C \int_0^t e^{A(t-\tau)} B u(\tau)\, d\tau + D u(t)$$

$$= C \underbrace{\left[e^{At} x_0 + \int_0^t e^{A(t-\tau)} B u(\tau)\, d\tau \right]}_{x(t)} + D u(t)$$

CHARACTERISTICS OF A DYNAMIC SYSTEM

LET'S RECALL SOME PARTS OF LINEAR ALGEBRA

IF WE HAVE A SYSTEM REPRESENTED BY
$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

IF WE HAVE $A\,\mathbb{R}^n$ MORE THAN 1 DOF
$n > 1$

I WILL USE THE EXPONENTIAL MATRIX AND SOLVE THE DIFF EQNS.

$$e^{At} \simeq \sum_{k=1}^{\infty} \frac{A^k t^k}{k!}$$

WE CAN ALSO STUDY THE EVOLUTION OF THE SYSTEM IN TIME CONSIDERING THE EIGEN VALUE OF A.

IF $A(m \times m)$

WE KNOW THAT AN EIGENVALUE OF A λ_i

$$\lambda_i \ / \ A\mu_i = \lambda \mu_i$$

μ_i = EIGENVECTOR

TO GET EIGENVALUES OF A $\quad (\lambda I - A)\mu_i = 0$

IDENTITY MATRIX

$$\det(\lambda I - A) = p(\lambda)$$

$p(\lambda)$ = CHARACTERISTIC POLY : IF $A^{m\times m}$ / $p(\lambda)$ HAS m SOLUTIONS.

THIS m SOLUTIONS ARE EIGENVALUES OF A

$$p(\lambda) = \lambda^m + d_{m-1}\lambda^{m-1} + \ldots \lambda_0$$

THE SOLUTIONS CAN BE $\begin{cases} \text{REAL } \mathbb{R} \\ \text{COMPLEX CONJUGATE } \emptyset \\ \text{IMAGINARY } I \end{cases}$

~ ~ ~

* REAL AND DISTINCT EIGENVALUES

IF A $m\times m$ $\quad \lambda_1, \ldots m$ EIGENVALUES

WE HAVE DISTINCT EIGENVECTORS $\mu_1 \ldots \mu_m$

MATRIX FORM

$$\begin{cases} A\mu_i = \lambda_i \mu_i \\ \vdots \\ A\mu_m = \lambda_m \mu_m \end{cases} \Rightarrow AU = U\Lambda$$

$$\Lambda = \begin{bmatrix} \lambda_1 & & \emptyset \\ & \ddots & \\ \emptyset & & \lambda_m \end{bmatrix}$$ DIAGONAL

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AN IMPORTANT EFFECT IF Λ IS DIAGONAL IS THAT ALSO USING THE EXP e

$$e^{\Lambda t} = \begin{bmatrix} e^{\lambda_1 t} & & \emptyset \\ & \ddots & \\ \emptyset & & e^{\lambda_m t} \end{bmatrix}$$

NECESSARY CONDITION IS THAT A MUST BE DIAGONAL AS WELL

IMPORTANT

IF WE HAVE A AND $\tilde{A}$ TWO SIMILAR AND EQUIVALENT STATE MATRICES $\Rightarrow$ THEN THEY HAVE SAME EIGENVALUES

THEY IN FACT MUST REPRESENT THE SAME DYNAMIC EVOLUTION IN TIME

IF WE ASSUME $\tilde{A} = \Lambda$ BUT WE KNOW $\tilde{A} = T A T^{-1}$

PREMULTIPLYING FO T^{-1} AND T WE HAVE

$$A = T^{-1} \Lambda T$$

AND THE EXP BECOMES

$$e^{At} = e^{(T^{-1}\Lambda T)t}$$

REMEMBERING THE EXPANSION OF $e^{At} \cong \sum_{k=0}^{\infty} \frac{t^k A^k}{k!}$

THEN WE CAN ALSO WRITE

$$e^{(T^{-1}\Lambda T)} \cong \sum_{k=0}^{\infty} \frac{t^k}{k!} (T^{-1}\Lambda T)^k$$

LET'S NOW DEMONSTRATE

$$(T^{-1}\Lambda T)^k = T^{-1}\Lambda^k T$$

LET'S SUPPOSE $k=2$

$$(T^{-1}\Lambda T)(T^{-1}\Lambda T) = T^{-1}\Lambda \underbrace{T\,T^{-1}}_{I}\Lambda T = T^{-1}\Lambda^2 T$$

AND SO ON FOR HIGHER POWERS 3-4-5 ETC

HENCE I CAN REWRITE

$$e^{At} = T^{-1} e^{\Lambda t} T = T^{-1}\left(\sum_{k=0}^{\infty} \frac{t^k}{k!}\Lambda^k\right) T$$

FURTHERMORE WE KNOW THAT $A U = U \Lambda$ ALSO $U^{-1} A U = \Lambda$

EIGEN VECTORS ~~MATRIX~~ EIGENVALUES

DOES OR WHICH IS A MATRIX T FOR WHICH $T A T^{-1} = \Lambda$?

THE ONLY CONDITION IS $T^{-1} = U$

MATRIX OF EIGEN VECTORS

HENCE I CAN WRITE $e^{At} = U e^{\Lambda t} U^{-1}$

$$\begin{bmatrix} u_1 & u_2 & u_m \\ | & | & | \\ | & | & | \\ | & | & | \end{bmatrix} \qquad \begin{bmatrix} v_1^T & - & - & - \\ v_i^T & - & - & - \\ v_n^T & - & - & - \end{bmatrix}$$

EIGEN VECTORS

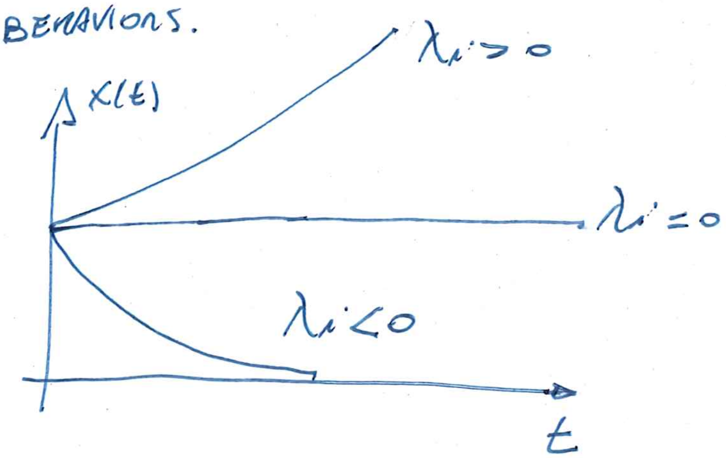
THEN WE CAN WRITE

$$e^{At} = T^{-1} e^{\Lambda t} T = U e^{\Lambda t} U^{-1} = \sum_{i=1}^{m} e^{\lambda_i t} \underbrace{u_i}_{\text{COLUMNS } U} \underbrace{v_i^T}_{\text{ROWS } U^{-1}}$$

FOR A SYSTEM THE HOMOGENEOUS SOLUTION WAS:

$$x(t) = e^{At} x_0 = \sum_{i=1}^{m} e^{\lambda_i} u_i v_i^T x_0$$ (HOMOGENEOUS RESPONSE)

DEPENDING ON $\lambda_i \gtrless 0$ THERE WILL BE DIFFERENT BEHAVIORS.



HENCE DIFFERENT RESPONSES.

* COMPLEX CONJUGATE EIGENVALUES.

$\lambda_i = \alpha_i \pm j\omega_i$

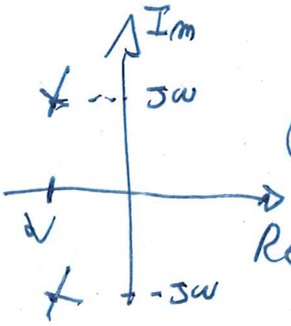
$$\Lambda = \begin{pmatrix} \alpha & \omega \\ -\omega & \alpha \end{pmatrix}$$

THIS IS THE CASE IN (2×2)

Λ IS NOT DIAGONAL ANYMORE.

IF A 2×2 AND Λ IS LIKE NON DIAGONAL

WE HAVE EIGENVALUES $\begin{cases} \lambda = \alpha + j\omega \\ \lambda^* = \alpha - j\omega \end{cases}$ COMPLEX CONJUGATE



(IF $\alpha < 0$)

LET'S USE THE PROPERTY OF $e^{A_1 t} \cdot e^{A_2 t} =$

$= e^{(A_1 + A_2)t}$

LET'S DEFINE

$$\tilde{A} = \underset{[1]}{\begin{pmatrix} \alpha & 0 \\ 0 & \alpha \end{pmatrix}} + \underset{[2]}{\begin{pmatrix} 0 & \omega \\ -\omega & 0 \end{pmatrix}}$$

$$e^{\tilde{A}t} = e^{([1]+[2])t} = \underbrace{e^{\alpha t} I}_{e^{\begin{pmatrix} \alpha t & 0 \\ 0 & \alpha t \end{pmatrix}}} \; e^{\begin{pmatrix} 0 & \omega \\ -\omega & 0 \end{pmatrix} t}$$

NOW LET'S FOCUS ON $e^{\begin{pmatrix} 0 & \omega \\ -\omega & 0 \end{pmatrix} t}$: HOW CAN I SIMPLIFY?

REMEMBER THE TAYLOR EXPANSION

$$e^{At} = I + At + A^2\frac{t^2}{2} \cdots \simeq I + \underbrace{\begin{pmatrix} 0 & \omega \\ -\omega & 0 \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} -\omega^2 & 0 \\ 0 & -\omega^2 \end{pmatrix} t^2 + \cdots}$$

IF WE SUM UP TERM BY TERM WE OBTAIN THE TAYLOR SERIES OF TRIGONOMETRIC FUNCTION $\sin(\omega)$ AND $\cos(\omega)$

HENCE

$$e^{\begin{pmatrix} 0 & \omega \\ -\omega & 0 \end{pmatrix} t} = \begin{pmatrix} \cos\omega t & \sin\omega t \\ -\sin\omega t & \cos\omega t \end{pmatrix}$$

HENCE

$$e^{\tilde{A}t} = e^{\alpha t}\begin{pmatrix} \cos\omega t & \sin\omega t \\ -\sin\omega t & \cos\omega t \end{pmatrix}$$

AND ALSO

$$e^{At} = T^{-1} e^{\tilde{A}t} T \quad \#$$

WHAT HAPPENS IF NOW WE STUDY:

$$x(t) = x_0 \cdot e^{At} = x_0 T^{-1} e^{\tilde{A}t} T \; ?$$

$\downarrow$

WHAT HAPPENS AND HOW THE SYSTEM EVOLVES ?

AFTER WE HAVE DEMONSTRATED HOW TO REPRESENT AN EXPONENTIAL IN CASE A HAS COMPLEX CONJUGATE EIGEN VALUES ($\lambda_i = \alpha_i \pm J\omega_i$), LET'S SEE WHAT THIS MEANS IN TIME EVOLUTION FOR OUR SYSTEM.

$$x(t) = x_0 e^{At} = x_0 T^{-1} e^{\tilde{A}t} T = x_0 T^{-1} e^{\alpha t} \begin{pmatrix} \cos & \sin \\ -\sin & \cos \end{pmatrix} T$$

IF WE POSE $T \cdot x_0 = \begin{pmatrix} C_\alpha \\ C_b \end{pmatrix}$ AND $m = \sqrt{C_\alpha^2 + C_b^2}$

$$\sin\varphi = \frac{C_\alpha}{m} \qquad \cos\varphi = \frac{C_b}{m}$$

I WILL HAVE

$$x(t) = e^{At} x_0 = T^{-1} e^{\alpha t} \begin{pmatrix} \cos & \sin \\ -\sin & \cos \end{pmatrix} \underbrace{m \cdot \begin{pmatrix} \sin\varphi \\ \cos\varphi \end{pmatrix}}_{T x_0}$$

IF $T^{-1} = \begin{pmatrix} \vdots & \vdots \\ u_\alpha & u_b \\ \vdots & \vdots \end{pmatrix}$ EIGEN VECTORS

$u_1 = u_{\alpha 1} + J u_{b1}$

$u_2 = u_{\alpha 2} + J u_{b2}$

$\lambda_1 = \alpha_1 + J\omega_1$ EIGENVALUES

$\lambda_2 = \alpha_1 - J\omega_1$

$$T^{-1} = \begin{pmatrix} u_{\alpha 1} & u_{b1} \\ u_{\alpha 2} & u_{b2} \end{pmatrix}$$

NOW THAT WE HAVE ALL THE COMPONENTS WE RUN THE MULTIPLICATIONS

REPEATING ALL THE PASSAGES

$$x(t) = e^{At} x_0 = T^{-1} e^{\tilde{A}t} T x_0 =$$

$$= T^{-1} e^{\alpha t} \begin{pmatrix} \cos & \sin \\ -\sin & \cos \end{pmatrix} m \begin{pmatrix} \cos\varphi \\ \sin\varphi \end{pmatrix} =$$

$$= \begin{pmatrix} u_{a1} & u_{b1} \\ u_{a2} & u_{b2} \end{pmatrix} e^{\alpha t} \begin{pmatrix} \cos & \sin \\ -\sin & \cos \end{pmatrix} m \begin{pmatrix} \cos\varphi \\ \sin\varphi \end{pmatrix} =$$

$$= m\, e^{\alpha t} \left[\sin(\omega t + \varphi)\, \underset{\downarrow}{u_a} + \cos(\omega t + \varphi)\, \underset{\downarrow}{u_b} \right]$$

$$u_a = \begin{bmatrix} u_{a1} \\ u_{a2} \end{bmatrix} \qquad u_b = \begin{bmatrix} u_{b1} \\ u_{b2} \end{bmatrix}$$

WHERE

$$\begin{cases} m\, e^{\alpha t} \sin(\omega t + \varphi) \\ m\, e^{\alpha t} \cos(\omega t + \varphi) \end{cases}$$ VIBRATIONAL DAMPED MODES

α IS THE MAIN PARAMETER $\lambda_i = \alpha \pm j\omega$

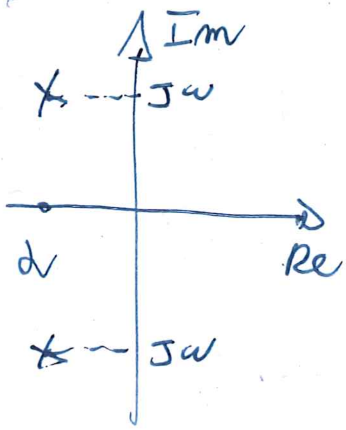
$\downarrow$

$\alpha \gtrless 0$

3 CASES

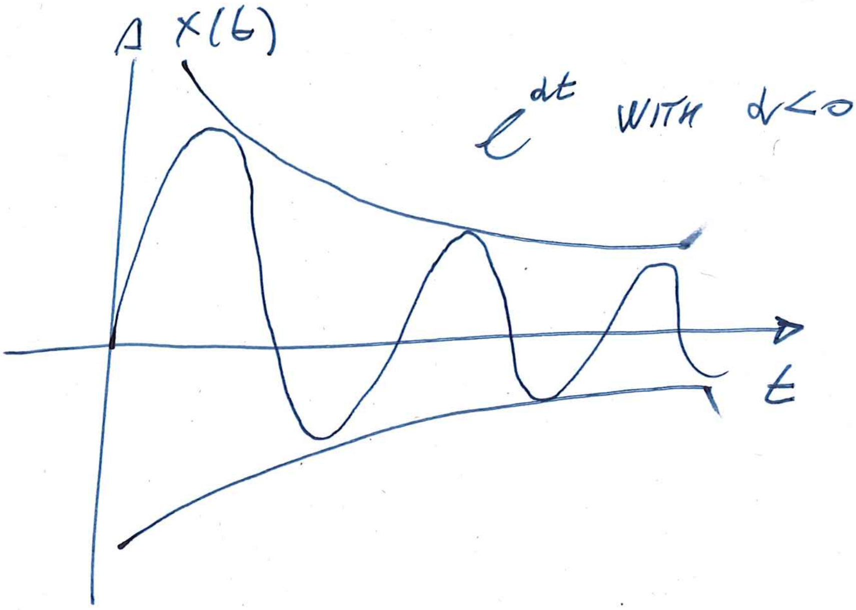
$$\lambda_i = \alpha \pm j\omega$$

$\boxed{\text{CASE 1} \quad \alpha < 0}$

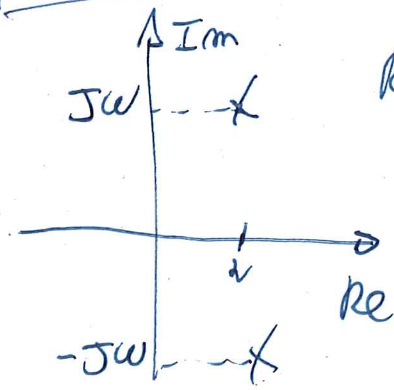


$Re[\lambda_i] < 0$

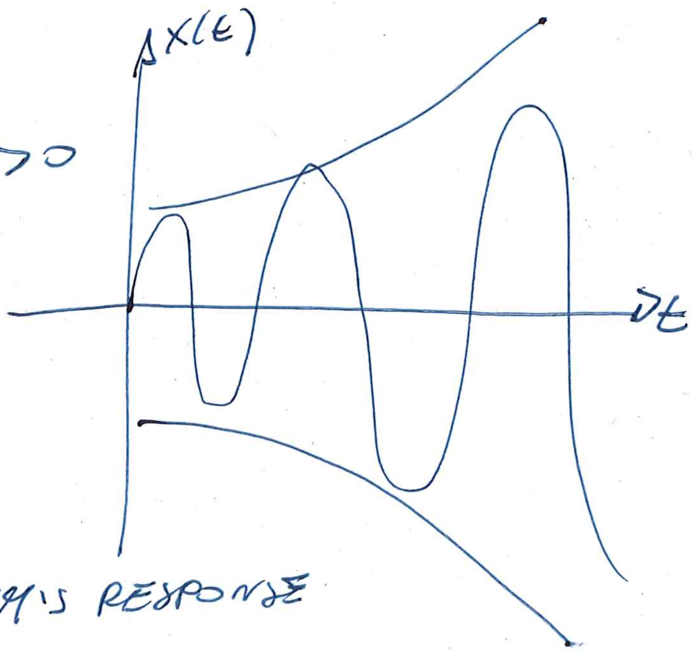
RESPONS OF THE SYSTEM CONVERGES



$\boxed{\text{CASE 2} \quad \alpha > 0}$



$Re[\lambda_i] > 0$



THE SYSTEM'S RESPONSE DIVERGE

$e^{\alpha t}$ WITH $\alpha > 0$

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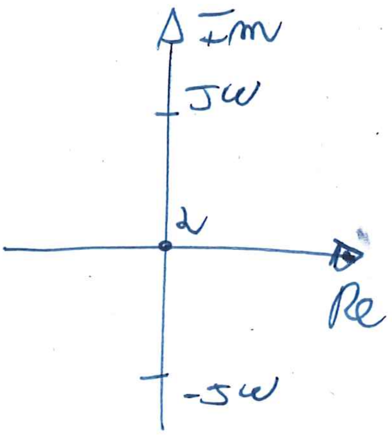
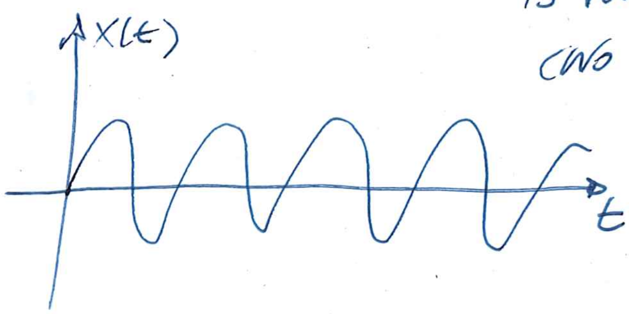
$\boxed{\text{CASE 3} \quad \alpha = 0}$



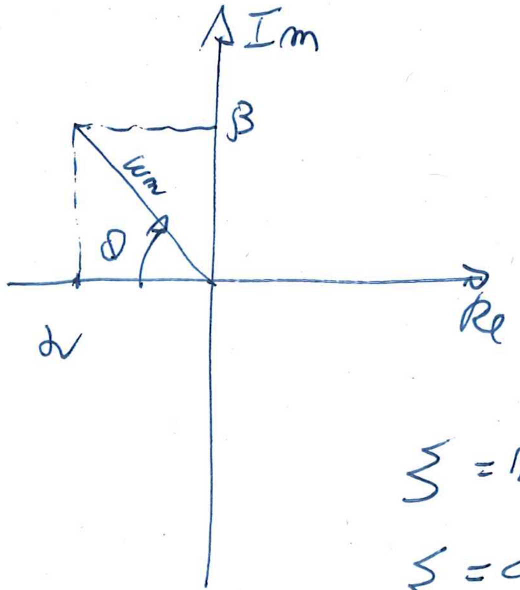
$$e^{\alpha t} = 1 \quad \alpha = 0$$

$$\lambda_1 = 0 \pm j\omega$$



THE SYSTEM RESPONSE IS PURELY PERIODIC (NO DAMPING)

IMPORTANCE OF EIGEN VECTOR/VALUE IN THE SYSTEM RESPONSE



$$\lambda_i = \alpha \pm j\beta = Re \pm j Im$$

W_m = NATURAL FREQUENCY

$$\omega_m = \sqrt{\alpha^2 + \beta^2} = \sqrt{Re^2 + Im^2}$$

ξ = DAMPING FACTOR

$$\xi = \cos\theta \qquad \theta = 0 \quad \xi = 1$$

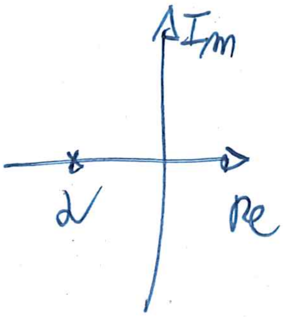
$$\theta = 90 \quad \xi = 0 \qquad 0 \le \xi \le 1$$

THE REAL OSCILLATORY BEHAVIOR OF THE SYSTEM IS

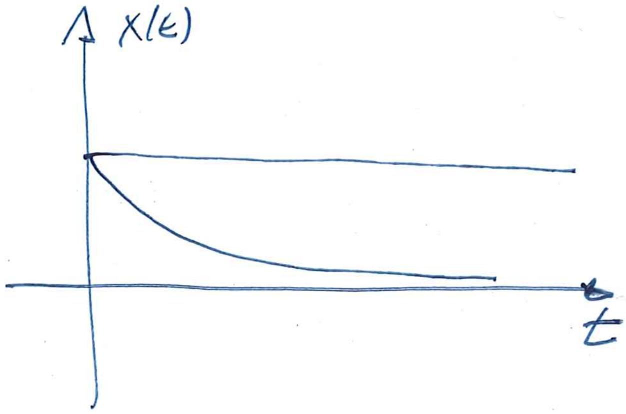
$$\omega_d = \sqrt{1 - \xi^2} \cdot \omega_m$$ DEPENDS ON ξ

$\boxed{\xi = 1} \Rightarrow \beta = 0 \quad \theta = 0$

$\lambda_i = \alpha \pm 0 = \alpha$ EIGEN VALUE REAL NEGATIVE



RESPONSE OF THE SYSTEM = EXPONENTIAL WITH NO OSCILLATIONS

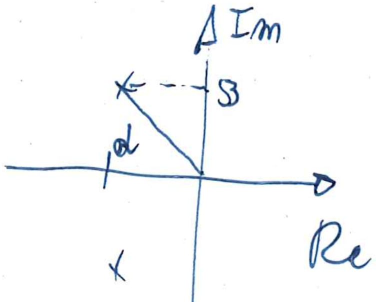


$\omega_d = \omega_m \sqrt{1-\xi^2} =$
$= \omega_m \sqrt{1-1} = \emptyset$

$\boxed{0 < \xi < 1}$

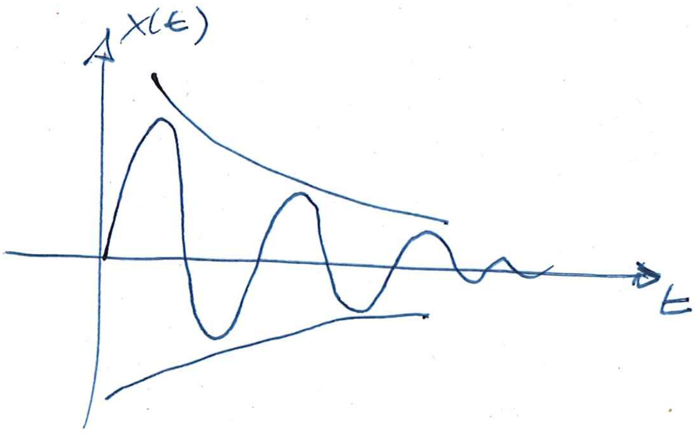
$\beta \neq 0 \quad 0 < \theta < 90$

$\lambda_i = \alpha \pm j\beta$



$$x(t) = X_0 \, e^{-\xi \omega_m t} \cos(\underbrace{\sqrt{1-\xi^2}\,\omega_m}_{\omega_d} t)$$

DAMPED FREQUENCY

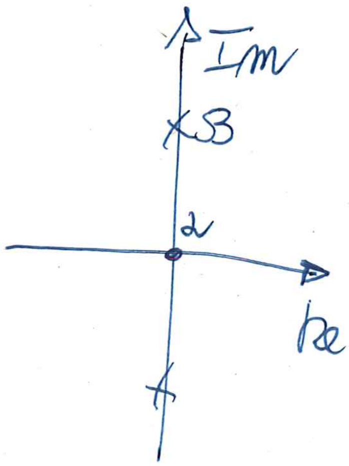


$$\boxed{\xi = 0}$$

$\delta = 90$

$\xi = 0 \quad \beta \neq 0 \qquad \lambda_i = 0 \pm j\beta$

$\alpha = 0$

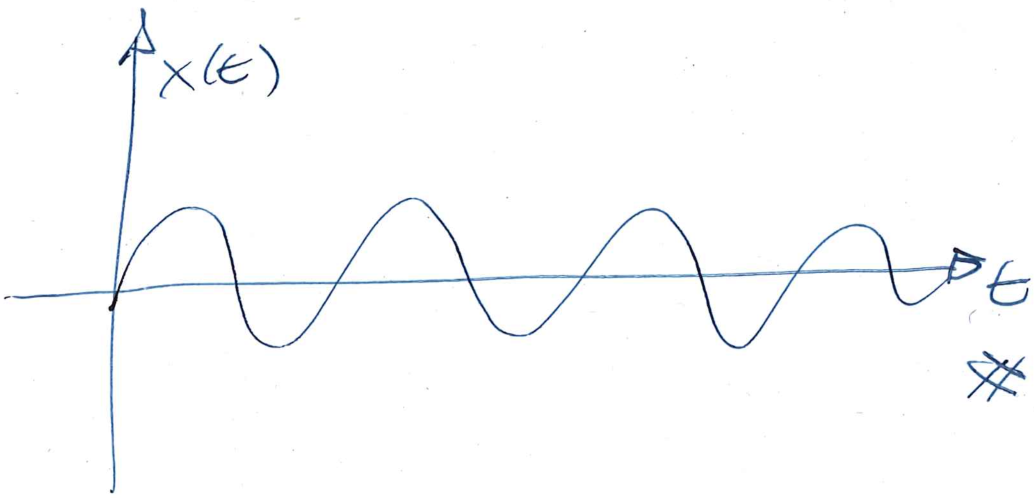


$$\omega_d = \omega_n \sqrt{1 - 0} = \omega_n$$

$$\omega_n = \sqrt{\alpha^2 + \beta^2} = \sqrt{\beta^2} = \beta$$

$$x(t) = X_0 \, e^{0t} \cos\left(\sqrt{1-0}\, \omega_n t\right) =$$

$$x(t) = X_0 \cos(\omega_n t)$$



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