-2

4) Найти предел

а) $\lim\limits_{x\to 0} \frac{3x^3-2x^2}{4x^2} = \left(\frac{0}{0}\right) =$

$= \frac{2x^2\left(\frac{3}{2}x-1\right)}{4x^2} = \frac{\frac{3}{2}x-1}{2} = -\frac{1}{2}$

в) $\lim\limits_{x\to 0} \frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1} = \left(\frac{0}{0}\right) =$

$= \lim\limits_{x\to 0} \frac{\sqrt{1+x}^2-1}{\sqrt[3]{1+x}^3-1} \cdot \frac{\sqrt{1+x}+1}{\sqrt{1+x}+1} \cdot \frac{(1+x)^2+(1+x)\cdot 1+1}{(1+x)^2+(1+x)+1} =$

$\lim\limits_{x\to 0} \frac{1+x-1}{1+x-1} \cdot \frac{(1+x)^2+1+x+1}{\sqrt{1+x}+1} =$

$= \lim\limits_{x\to 0} \frac{1+x-1}{1+x-1} \cdot \frac{1+2x+x^2+1+x+1}{\sqrt{1+x}+1} =$

$= \frac{3}{2}$

c) $\lim\limits_{x\to\infty}\left(\frac{x+3}{x}\right)^{4x+1} =$

$= \lim\limits_{x\to\infty}\left(1+\frac{3}{x}\right)^{4x+1} =$

$\lim\limits_{x\to\infty}\left(1+\frac{3}{x}\right)^{1} \cdot \lim\limits_{x\to\infty}\left(1+\frac{3}{x}\right)^{4x} =$

$= \lim\limits_{x\to\infty}\left(1+\frac{3}{x}\right)^{4x}$

$t = \frac{x}{3}$

$\lim\limits_{x\to\infty}\left(1+\frac{1}{t}\right)^{12t} = e^{12}$

t

a) $\lim_{x \to 0} \frac{\sin 2x}{4x} = \frac{\sin 2x}{2 \cdot 2x} = \frac{1}{2}$

b) $\lim_{x \to 0} \frac{x}{\sin x} = \frac{1}{}$

c) $\lim_{x \to 0} \frac{x}{\arcsin(x)} =$

$$= \lim_{x \to 0} \frac{\frac{d}{dx} x}{\frac{d}{dx} \arcsin(x)} = \frac{1}{\frac{1}{\sqrt{1-x^2}}}$$

$$= \lim_{x \to 0} \sqrt{1-x^2} = 1$$

d) $\lim_{x\to\infty}\left(\frac{4x+3}{4x-3}\right)^{6x} =$

$\lim_{x\to\infty}\frac{(4x-3)+6}{4x-3} =$

$\lim_{x\to\infty}\left(1+\frac{6}{4x-3}\right)^{6x}$

$t=\frac{4x-3}{6}$

$x=\frac{6t+3}{4}$

$6x=\frac{\cancel{36}t+18}{4}=$

$6x=9t+\frac{9}{2}$

$\lim_{x\to\infty}\left(1+\frac{1}{t}\right)^{9t+\frac{9}{2}}=$

$=\lim_{x\to\infty}\left(1+\frac{1}{t}\right)^{\frac{9}{2}}\cdot\lim_{x\to\infty}\left(1+\frac{1}{t}\right)^{9t}=$

$=\lim_{x\to\infty}\left(1+\frac{1}{t}\right)^{9t}=e^{9}$