Исследовать функцию на условный экстремум

1) $u = 3 - 8x + 6y \qquad x^2 + y^2 = 36$

$L(\lambda_1, x, y) = -8x + 6y + 3 + \lambda_1 (x^2 + y^2 - 36)$

$L'_x = -8 + \lambda_1 2x$

$L'_y = 6 + \lambda_1 2y$

$L_{\lambda_1} = x^2 + y^2 - 36$

$$\begin{cases} x = \dfrac{8}{2\lambda_1} \\ y = -\dfrac{6}{2\lambda_1} \\ \left(\dfrac{8}{2\lambda_1}\right)^2 + \left(-\dfrac{6}{2\lambda_1}\right)^2 = 36 \end{cases}$$

$$\begin{cases} x = \dfrac{8}{2\lambda_1} \\ y = -\dfrac{6}{2\lambda_1} \\ \dfrac{64}{4\lambda_1^2} + \dfrac{36}{4\lambda_1^2} = 36 \Rightarrow \dfrac{25}{\lambda_1^2} = 36 \Rightarrow \lambda_1^2 = \dfrac{25}{36} \end{cases}$$

$\left(\frac{5}{6}; \frac{24}{5}; -\frac{18}{5}\right), \left(-\frac{5}{6}; -\frac{24}{5}; \frac{18}{5}\right)$

$L''_{xx} = 2\lambda_1$

$L''_{yy} = 2\lambda_1$

$L''_{\lambda_1\lambda_1} = 0$

$L''_{xy} = 0$

$L''_{x\lambda_1} = 2x$

$L''_{y\lambda_1} = 2y$

$$\left|\begin{matrix} 0 & 2x & 2y \\ 2x & 2\lambda_1 & 0 \\ 2y & 0 & 2\lambda_1 \end{matrix}\right| = 0 \cdot \begin{vmatrix} 2\lambda_1 & 0 \\ 0 & 2\lambda_1 \end{vmatrix} -$$

$$2x \begin{vmatrix} 2x & 0 \\ 2y & 2\lambda_1 \end{vmatrix} + 2y \begin{vmatrix} 2x & 2\lambda_1 \\ 2y & 0 \end{vmatrix} =$$

$$-8\lambda_1 (x^2 + y^2)$$

3) Найти производную ф-ции $U = x^2 + y^2 + z^2$ по направлению вектора $\vec{c}(-9, 8, -12)$ в точке $M(8; -12; 9)$

$U'_x = 2x \Rightarrow 16$

$U'_y = 2y \Rightarrow -24$

$U'_z = 2z \Rightarrow 18$

$|\vec{c}| = \sqrt{x_0^2 + y_0^2 + z_0^2} = \sqrt{(-9)^2 + 8^2 + (-12)^2} =$

$= \sqrt{81 + 64 + 144} = 17$

$\vec{c_0} = \frac{\vec{c}}{|\vec{c}|} = \left(-\frac{9}{17}; \frac{8}{17}; -\frac{12}{17}\right)$

$U'_{\vec{c}}\Big|_{(-9, 8, -12)} = 16 \cdot \left(-\frac{9}{17}\right) + (-24) \cdot \frac{8}{17} + 18 \left(-\frac{12}{17}\right.$

$= -\frac{144}{17} - \frac{192}{17} - \frac{216}{17} = -\frac{552}{17}$

4) Найти производную ф-ции $U = e^{x^2+y^2+z^2}$ по направлению вектора $\vec{d} = (4, -13, -16)$ в точке $L(-16; 4; -13)$

$U'_x = 2x \cdot e^{x^2+y^2+z^2} \Rightarrow -32 \cdot e^{441}$

$U'_y = 2y\, e^{x^2+y^2+z^2} \Rightarrow 8e^{441}$

$U'_z = 2z \cdot e^{x^2+y^2+z^2} \Rightarrow -26 \cdot e^{441}$

$|\vec{d}| =$

$\sqrt{4^2 + (-13)^2 + (-16)^2} = 21$

$\vec{d}_b = \frac{\vec{d}}{|\vec{d}|} = \left(\frac{4}{21}; -\frac{13}{21}; -\frac{16}{21}\right)$

$U'_{\vec{d}}\Big|_{(4,-13,-16} = -\frac{32 \cdot e^{441} \cdot 4}{21} + \frac{8e^{441} \cdot (-13)}{21} + \left(\frac{-26 \cdot e^{441} \cdot (-16)}{21}\right) =$

$= -\frac{128 \cdot e^{441}}{21} - \frac{24 \cdot e^{441}}{21} + \frac{416 \cdot e^{441}}{21} = \frac{264\, e^{441}}{21}$