Structural Engineering Documents

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VIBRATIONS IN STRUCTURES

Induced by Man and Machines

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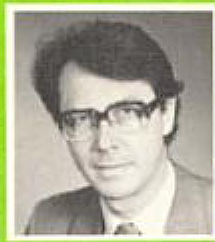

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1 SURVEY OF PROBLEMS IN STRUCTURAL DYNAMICS

In this chapter the different types of dynamic loads are characterized, followed by a description of their possible effects and existing measures for their limitation.

1.1 Characterization of Dynamic Loads

1.1.1 General Aspects

In broad terms, a major part of the loads encountered in civil engineering can be designated as dynamic because they vary with time. In practice, however, slowly varying loads can be treated as quasi-static since the inertia and damping forces are negligible. The presence of inertia and damping forces is in fact the important distinction between dynamic and static loading. These forces arise from the accelerations and velocities induced in the structural member, and they have to be included in the calculation of stress resultants and support reactions. The magnitude of these forces and their function against time depend both on the kind of excitation from outside and on the intrinsic dynamic behaviour of the structural member, i.e. its dynamic properties.

According to their time function, dynamic loads can be categorized (Figs. 1.1 and 1.2) as

- harmonic
- periodic
- transient
- impulsive.

Additional aspects and criteria, respectively, are the number of load cycles, the ensuing loading or strain rate in the affected member, the probability of occurrence of the load or the peak values it may attain (Tab. 1.1, Section 1.1.6).

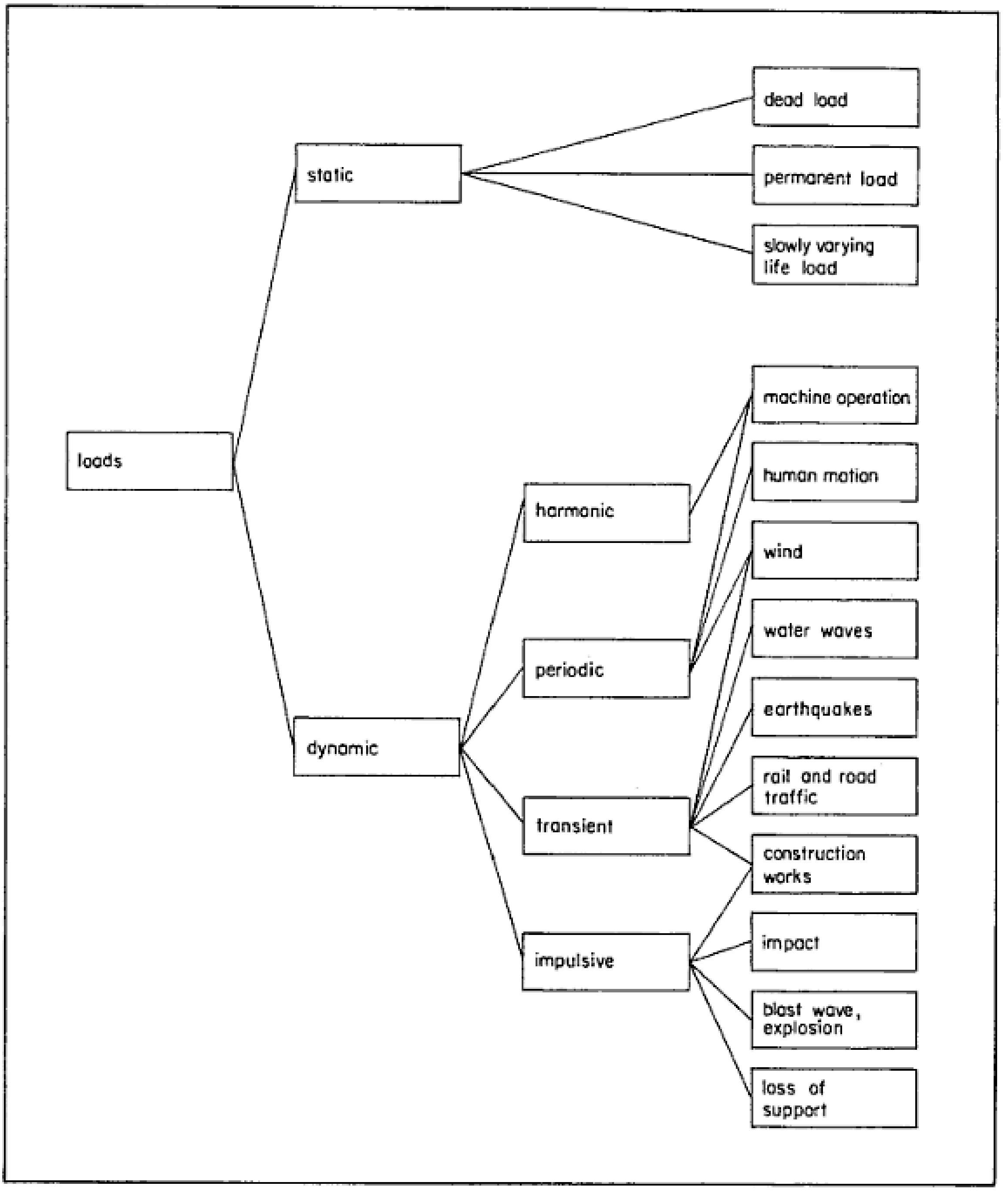


Fig. 1.1 Types of loading in civil engineering

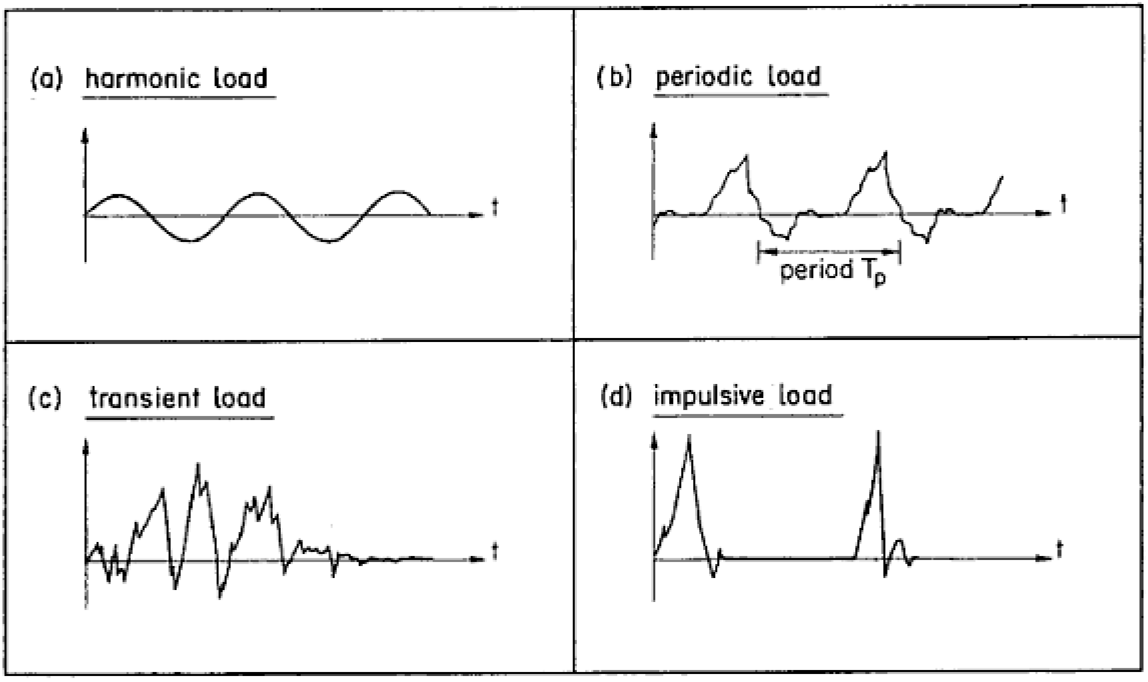


Fig. 1.2 Typical time functions of dynamic loads

1.1.2 Harmonic Loads

Harmonic loads vary according to a sine function with or without a certain phase shift. They affect the structure for a sufficiently long time to permit a steady-state vibration response (Fig. 1.2a, see also Appendix B 1). They may be caused by

- machines with synchronously rotating masses which are slightly out of balance (e.g. generators)
- machines with intentional out-of-balance forces (e.g. vibrators).

1.1.3 Periodic Loads

Periodic loads exhibit a time variation which is repeated at regular intervals called periods. Although the function within one period is arbitrary (Fig. 1.2b), its repetition allows it to be decomposed into a series of harmonic loads through a Fourier transformation («harmonic analysis», Appendix B 4). Again, the duration of the load is long enough for steady-state response to develop.

They may arise from

- human motion such as walking, skipping, dancing, deliberate excitation, etc.,
- machines having
  - more than one unbalanced mass (e.g. rotary presses, centrifugal pumps, blowers, centrifugal separators, vibrators, etc.)
  - oscillating parts (e.g. combustion engines, reciprocating compressors, textile machines, frame saws, bells, etc.)
  - periodically impacting parts (e.g. punching machines, presses, etc.).

Periodic loads can also be caused by wind, such as by vortex shedding, even though its general nature is transient.

1.1.4 Transient Loads

Transient loads exhibit an arbitrary time variation without any periodicity. Also the duration of the load is arbitrary (Fig. 1.2c). Such loads can be due to

- wind, namely in the wind direction as opposed to vortex shedding in the lateral direction
- water waves
- earthquakes
- rail or road traffic, either acting directly on a structure or interacting through the ground (ground vibrations)
- construction works (e.g. sheet or pile driving by ramming or drilling, use of vibrating rollers, blasting operations, etc.).

1.1.5 Impulsive Loads

Impulsive loads are also of a transient nature. Their duration, however, is so short that the structural member affected reacts in quite a different manner (Fig. 1.2d). This type of loading can result from

- machines operating in single impacts (e.g. forge hammers)
- construction works (e.g. in the immediate vicinity of blasting operations)
- impact (e.g. crash of vehicles, airplanes, ships, or projectiles hitting the structure, rock fall on a protective gallery, etc.)
- blast waves
- sudden collapse of load-bearing members (e.g. columns in buildings or piers of bridges).

1.1.6 Additional Loading Criteria

Apart from the load-time function, other features can be of importance (Tab. 1.1). These include

- the number of load cycles over a given period or over the entire life of the structure (N in Tab. 1.1)
- the strain rate ($\dot{\varepsilon}$ in Tab. 1.1) or rate of loading
- the peak values of the dynamic load and the rise time
- the probability of occurrence of exceptional loads, such as earthquakes, blast waves, etc., during the life time of the structure.

static and quasi-static		dynamic		
short term	long term	fatigue	low-cycle fatigue	impulsive
		$10^3 < N < 10^7 \div 10^8$	$10 < N < 10^3$	$1 < N < 20$
$\dot{\varepsilon} < 10^{-5}$	$\dot{\varepsilon} < 10^{-5}$	$10^{-6} < \dot{\varepsilon} < 10^{-3}$	$10^{-5} < \dot{\varepsilon} < 10^{-2}$	$10^{-3} < \dot{\varepsilon} < 10^{1} \div 10^{2}$
	• creep and relaxation • temperature	• traffic • machinery	• earthquake	• impact • blast wave • collapse of support

Tab. 1.1 Characteristics of static and dynamic loads:
N = number of load cycles, $\dot{\varepsilon}$ = strain rate [s^{-1}]

1.2 Source-Dependent Effects of Dynamic Loads and Countermeasures

1.2.1 Classification of Effects

Dynamic load effects can be divided into three categories, i.e.

- effects on structures
- effects on people
- effects on machinery and installations.

Effects on Structures

A structure can be affected in both its load-bearing capacity and its serviceability.

Impairment of the load-bearing capacity can take the following forms:

– Fatigue:
  The stress fluctuation associated with the dynamic load can critically impair the strength of the material after a certain number of cycles, all the more the higher the stress excursions are. Design against fatigue, as it is called, does not just depend on the type of loading, but also on the planned or expected life time of the structure. Distress due to large stress excursions, of which a few are sufficient to cause collapse, is termed low-cycle fatigue (Tab. 1.1).

– Local plastification:
  Under extraordinary high loads with a low probability of occurrence, such as vehicle crash against columns and earthquakes, localized or limited plastic deformation may be accepted in order to enable an economic design of the respective structural member.

– Alteration to the material properties under high-speed loading:
  A very short rise time of the load causes a high strain rate in the respective structural member (Tab. 1.1). The strength and stiffness parameters of most materials are sensitive to the strain rate (see e.g. [1.1] and [1.2]). For steel and concrete, the usual construction materials in civil engineering, the strain-rate effect is mostly beneficial to the load-bearing capacity of the structure.

Impairment of the serviceability entails damage mostly to nonstructural elements of a building, such as cracks in partitions, loss of cladding, etc.

Effects on People

The serviceability is impaired when the vibration of the building under dynamic load causes disturbance and discomfort to the occupants.

Effects on Machinery and Installations

The serviceability to manufacturing processes can be impaired by vibrations transmitted through the structure. Secondary vibration of the installations may hinder the production process, affect the proper functioning of machines, and cause associated material-technological problems.

1.2.2 Man-Induced Vibrations

Human motion is sufficient to cause various dynamic loads. They can be periodic in nature (e.g. due to walking, running, skipping, dancing, etc.) or transient (e.g. by jumping off a high-diving platform). Man-induced vibrations are thus to be generally expected in

- pedestrian structures
- gymnasia and sports halls
- high-diving platforms in swimming pools.

The resulting vibrations can lead to the following forms of distress:

- overstressing of the structure, in extreme cases to the loss of structural integrity
- damage to nonstructural elements (e. g. cladding on sports halls)
- intolerable vibration velocities and accelerations disturbing and discomforting the respective users
- excessive noise (e. g. due to reverberating equipment).

Of all possible countermeasures, the foremost idea is to avoid resonance phenomena, for instance by

- increasing the stiffness of the structure or the particular structural member
- installation of (tuned) vibration absorbers
- regulating the use to avoid critical loading.

1.2.3 Machine-Induced Vibrations

Machinery can affect buildings or structural members in several ways with quite different types of loading, depending on whether the critical forces arise from a rotating, oscillating or intermittent motion of the machine parts. Thus, machines can produce harmonic, periodic or transient loads (Fig. 1.1). The vibrations induced in the structure may cause

- failure of structural members in fatigue
- damage to nonstructural elements (e. g. partitions)
- intolerable vibration velocities and accelerations impairing the well-being of operating personnel and, possibly, persons in the neighbourhood
- excessive deformation interfering with the ongoing production (e. g. problems of tolerances in the manufactured items).

Again, avoidance of resonance is the primary precaution, with provision of additional damping to suppress remaining vibrations. Thus, the available countermeasures are

- stiffening of the structure or specific structural members
- disconnecting parts of the structure to cut off vibration transmission
- installation of spring(-damper) elements for isolation against vibration
- adjusting the speed of the machines (i. e. the period of loading)
- provision of tuned vibration absorbers.

1.2.4 Wind

Slender structures exposed to wind can essentially be subject to two different types of vibration:

- In the wind direction, vibrations are excited by gusts. The resulting structural vibrations interfere with the wind flow leading to motion-induced wind loads, which may enhance the vibrations.
- In the lateral direction, vibrations are excited by periodic vortex shedding on alternating sides of the structure.

These kinds of vibration are to be taken into consideration when designing

- high-rising and slender structures (e. g. towers, chimneys, skyscrapers)
- structures of large spans or slim profile (e. g. suspension bridges).

The wind-induced vibrations may affect these structures through

- deterioration caused by fatigue
- complete loss of load-bearing capacity
- failure of the fasteners of nonstructural elements (e.g. cladding)
- discomfort to occupants (e. g. excessive sway of a high-rise building).

To mitigate these effects, the following are the primary countermeasures taken:

- stiffening of the structure to avoid resonance phenomena
- roughening of its surface to moderate vortex shedding
- aerodynamic optimization of the profile
- installation of tuned vibration absorbers.

1.2.5 Water Waves

Water waves, caused by the interaction of the bottom air layer with the water surface, are characterized by their height, which determines the size of the loads on a structure, and their wave frequency or period. Vibrations due to water waves have to be considered in the case of the following structures:

- offshore structures (oil-drilling rigs)
- harbour structures.

Detrimental effects may include

- loss of load-bearing capacity
- excessive deformation, intolerable to the proper functioning of the appliances or the well-being of people.

The basic countermeasure is once again the stiffening of the structure to avoid resonance.

1.2.6 Earthquakes

Geotectonic activity in the earth's crust and the upper mantle trigger earthquakes, primarily of the dislocation type. Over a longer period of time elastic strain energy accumulates in a fault zone, often of historic origin, until the rock strength in the critical direction is reached. During the rupture a substantial part of the energy released is radiated as kinetic energy in the form of seismic waves. They induce in buildings both horizontal and vertical motions.

The possible damage due to seismic vibrations encompasses

- loss of the load-bearing capacity
- local irreversible deformation (plastic zones) of the primary structural members
- destruction of secondary elements, installations and equipment.

To control such damaging effects, the precautions have to include

- the choice of a suitable structural system with emphasis on symmetry
- appropriate seismic design loads for the structure
- careful detailing of structural members and nonstructural elements.

1.2.7 Rail and Road Traffic

Traffic-induced vibrations have various sources and emanate, for instance, from unevenness in rail track and road surfaces and the corresponding motions induced to vehicles. They occur in a directly loaded structure (e.g. a bridge) or are observed in buildings adjacent to traffic routes and railway lines. The vibrations can produce

- cracks in structural members or nonstructural elements
- disturbance to people on the structure or within the building.

Provisons may include

- stiffening of the structure or installation of tuned vibration absorbers in the case of direct loading
- repaving the roads and bridges
- for railway tracks, elastic support of the ties, of the concrete slab-track, etc.
- disconnecting parts of the structure to prevent transmission.

1.2.8 Construction Works

Construction works may be a source of vibrations when they include ramming or vibro-driving of piles and sheet piles, soil compaction with vibratory rollers, blasting, etc.

The effects of such vibrations may be

- cracks in nonstructural elements of buildings in the neighbourhood
- disturbance to people living in these buildings.

The vibrations can be kept within bounds by

- using a smaller pile hammer
- adjusting the frequency of the vibrator
- choice of an alternative method of construction (e.g. drilling instead of ramming)
- limiting the explosive charge in blasting
- restricting the time and duration of execution.

1.2.9 Impact

With regard to impact loads, local and global effects need to be clearly distinguished. While the local behaviour refers to influences in the zone of impact, the global behaviour describes the loading and vibrations spreading through individual structural members or the structure as a whole. Depending on the deformability of both the impacting object and the part of the structure directly affected, the impact may be characterized as hard or soft [1.2].

Impact loads may result from

- vehicles crashing into piers, parapets, etc.
- airplane crashes on buildings
- rockfall on protective galleries.

The possible effects comprise

- spalling of concrete in the impact area or on the reverse side
- punching failure
- failure of the impacted structural member in bending
- inadmissible vibrations in adjacent members.

Besides provisions to eliminate the chance of impact, measures to reduce the effects on a structural member include

- design for an appropriate impact load
- detailing for a large absorption capacity prior to failure (ductile behaviour)

– protection of the load-bearing structure with cushioning material (e.g. covering a rockfall-protection gallery with gravel).

1.2.10 Blast Waves and Explosions

Explosions generally produce a blast wave affecting the entire structure as a pressure front (gas explosion, nuclear warfare), but they may result alternatively in local impact if a charge is applied directly to a structural member.

Accordingly, the damage may involve

– local destruction of particular members due to direct hit (which may involve total loss of the structure by progressive collapse)
– development of plastic zones in the primary structural members (possibly to the point of failure) under large-scale surface pressure loading.

To avoid these kinds of damage, the following countermeasures may be introduced:

– design of the structure specifically against progressive collapse
– design of the structural members for the pressure wave load
– covering the structural members at risk with cushioning material.

1.2.11 Loss of Support

If a load-bearing member fails (e.g. a column, a wall, etc.), a step load is exerted on adjacent structural members previously supported (e.g. a floor slab). The effect can also be one of a sudden load removal.

Under these circumstances – often a direct consequence of an external impact –, the subsequent collapse of neighbouring structural members or the progressive collapse of the structure as a whole can only be prevented if these members have the ability to redistribute the forces quickly and to sustain the resulting stresses.

Apart from protective measures against impact (e.g. crash barriers around bridge piers), the structure could specifically be designed to have the capacity to survive the loss of a particular structural member and to redistribute the forces.

2 MAN-INDUCED VIBRATIONS

2.1 General Aspects

In an increasing number of cases, structures are found to be unduly responsive («lively») to human motion, resulting in disturbing or even harmful vibrations. Structures intended for moderate live loads of an assumed static nature are often designed with rather slender dimensions, the possibility that dynamic loading might govern the design being overlooked or underestimated. Another factor is the advent of new kinds of user activities such as fitness classes to the accompaniment of strongly rhythmic music in gymnasia, which may bring about a considerable dynamic loading. Although overloading of the structure itself is not the primary concern, secondary building elements (e.g. cladding, windows) can be damaged and the comfort of people impaired. They may feel alarmed enough to leave their places precipitately (see [2.1] and Case No. 7). One is usually more sensitive to vibrations induced by someone else than those which are partly due to one's own activity. Thus, man-induced vibrations are basically a problem of serviceability.

This chapter first describes the dynamic loading functions (force vs. time) resulting from various human motions. It then shows their effects. The countermeasures are classified according to categories of structures in which man-induced vibrations are to be expected.

2.2 Dynamic Loading from Human Motions

Man can cause various types of dynamic loads by his physical activity. The loads may be of periodic or transient nature (Fig. 1.2).

Periodic loads are mainly the result of the following forms of human motion:

- walking
- running
- skipping
- dancing.

Of course, this is just a crude categorization. Other forms of motion such as rhythmic skipping during fitness classes, jazz dance sessions, foot stamping, hand clapping and body rocking at a concert, etc., are included or may be a combination of those forms.

Transient loads result primarily from a jolting motion imparting a single impulse to a structural member (e.g. take-off from a diving platform, landing on a floor after jumping from an elevated position, bumping against a wall with the shoulder, etc.).

Periodic loads depend in their time function as well as in their frequency on the pace and the kind of motion. A closer look at the time function of the periodic loading reveals that considerable forces are not just transmitted in the actual frequency of the walking, skipping or dancing rhythm, but also in the frequencies of upper and lower harmonics.

A systematic categorization of man-induced loading is difficult. Apart from the aforementioned factors, the number, for example, of the people involved in the excitation plays a role as well. The following section deals first with periodic loads due to walking and running, skipping and dancing. Then the transient loads caused by a jolting motion are briefly discussed (Section 2.2.4).

2.2.1 Walking and Running

(a) General Characterization

The motion forms of walking and running may give rise to considerable dynamic loading on pedestrian structures, for example footbridges, overpasses, connecting passageways, etc. These forms can be characterized by the pacing rate, the forward speed and – specifically – by the time function of the loading.

Pacing Rate

The pacing rate (f_s) dominates the resulting dynamic load. It is sometimes given as footfalls per second [FF/s], but its nature as loading frequency is more adequately expressed in Hz. For normal walk on horizontal surface, both Matsumoto [2.2] and Schulze [2.3] found it to range between 1.5 and 2.5 Hz (Fig. 2.1). Assuming a Gaussian normal distribution around the mean of 2.0 Hz yields a standard deviation of 0.13 Hz [2.3] to 0.18 Hz [2.2]. Kramer [2.4] gives a slightly different average of 2.2 Hz with 0.3 Hz standard deviation.

For normal jog, the mean pace rate varies from 2.4 to 2.7 Hz [2.5], [2.10]. For sprinting it may be as high as 5.0 Hz [2.6]. On public pedestrian structures, however, pacing rates above 3.5 Hz are rare [2.5].

Forward Speed

The speed or velocity of pedestrian propagation (v_s) is coupled with the pacing rate (f_s) through the stride length (l_s). Naturally, different people may possess quite different stride lengths and paces for the same forward speed. Figure 2.2 gives average values for this interrelation from numerous tests [2.6], which are compiled in the following Tab. 2.1.

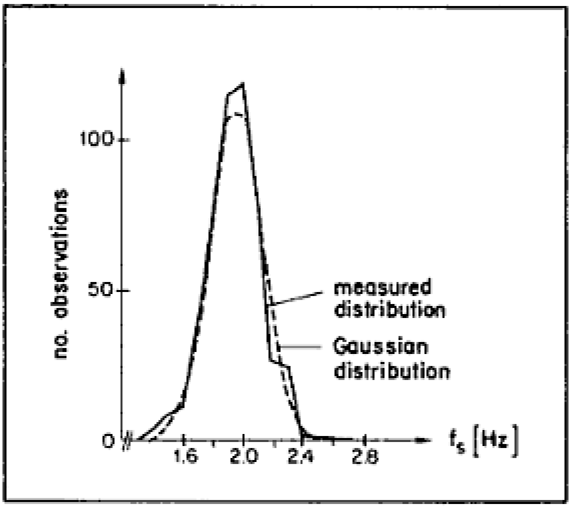


Fig. 2.1
Distribution of pacing rates for normal walk (from [2.2])

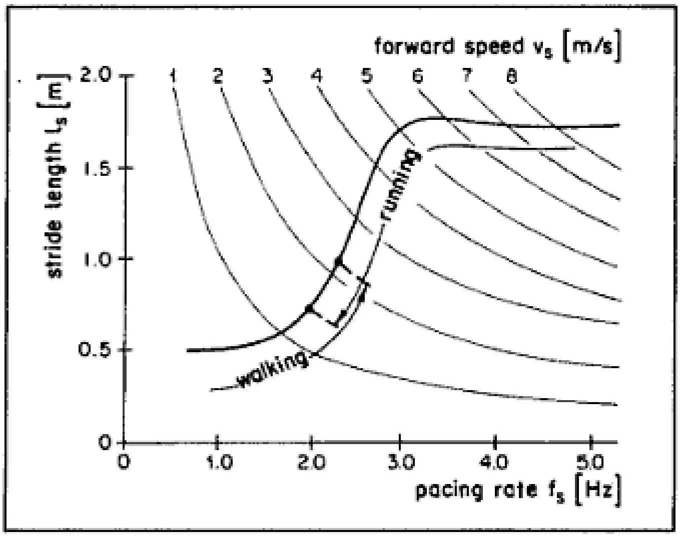


Fig. 2.2
Forward speed and stride length during walking and running (after [2.6])

	f_s [Hz]	v_s [m/s]	l_s [m]
slow walk	~ 1.7	1.1	0.60
normal walk	~ 2.0	1.5	0.75
fast walk	~ 2.3	2.2	1.00
slow running (jog)	~ 2.5	3.3	1.30
fast running (sprint)	> 3.2	5.5	1.75

Tab. 2.1 Correlation of pacing rate, forward speed and stride length for walking and running (average values after [2.6])

Load-Time Function

When walking or running, a person exerts a *vertical* and *horizontal* dynamic load (in forward and lateral direction), which, in fact, has the physical dimension of a force. The main parameters affecting the load-time function are the following (see also [2.10]):

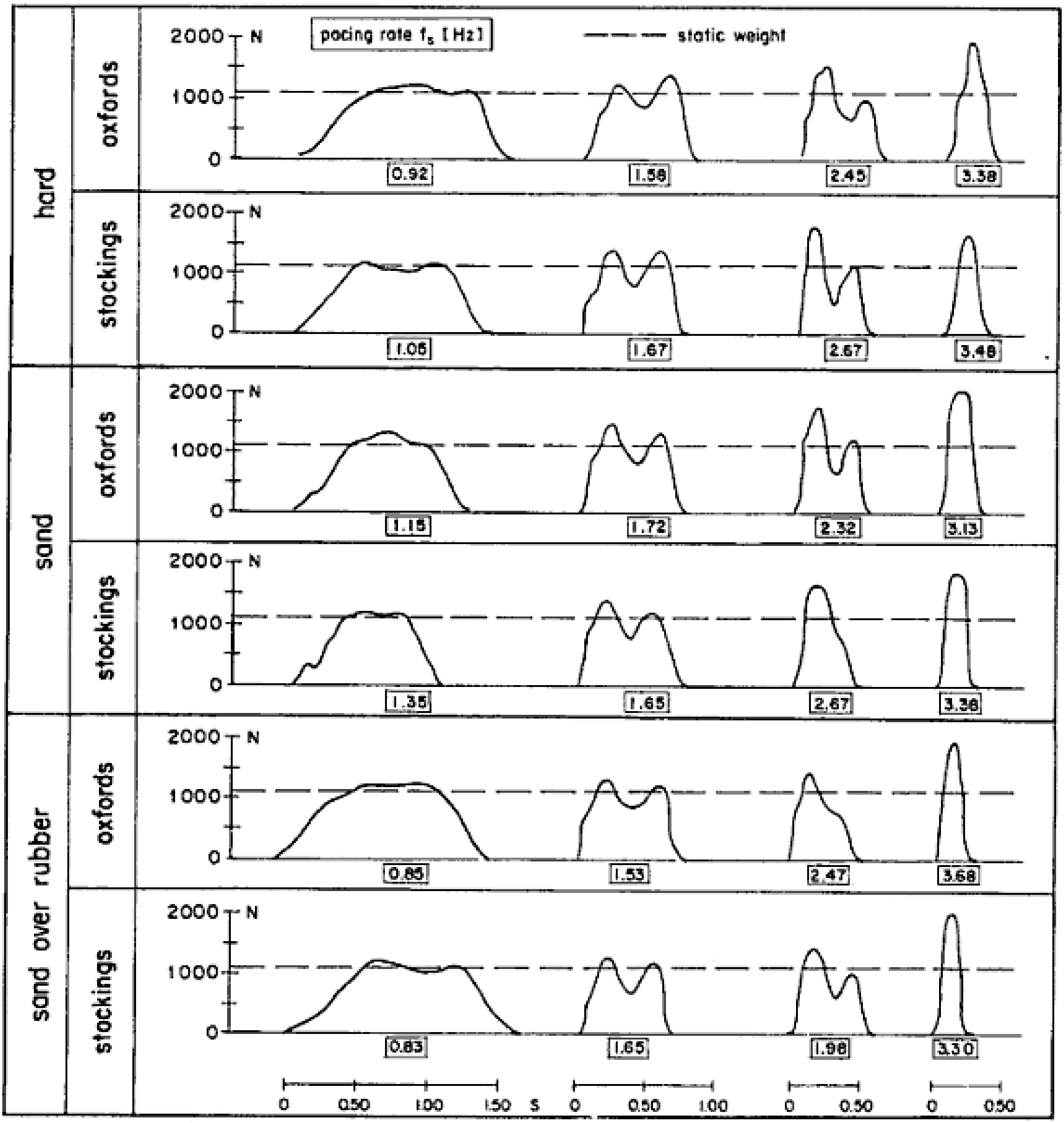


Fig. 2.3 Load-time functions for various pacing rates, footwear, and surface conditions (from [2.7])

- pacing rate
- stepping particularities (heel/ball contribution)
- person's weight
- person's sex
- type of footwear (or lack of it)
- floor surface condition (softness of cover).

(b) Time Function of the Vertical Load

Given the multitude of parameters, the results of different investigations vary greatly, influenced also by the test procedure and measuring technique adopted. The influence of pacing rate, footwear and floor surface on the development of the dynamic vertical load was thoroughly examined in [2.7]. As Fig. 2.3 reveals, the latter parameters are of minor importance compared to the pacing rate. The weight of the test person was 1100 N. The shape of the load-time function for walking with a medium pacing rate is more or less that of a saddle; the two observable load maxima result from stepping with the heel and pushing

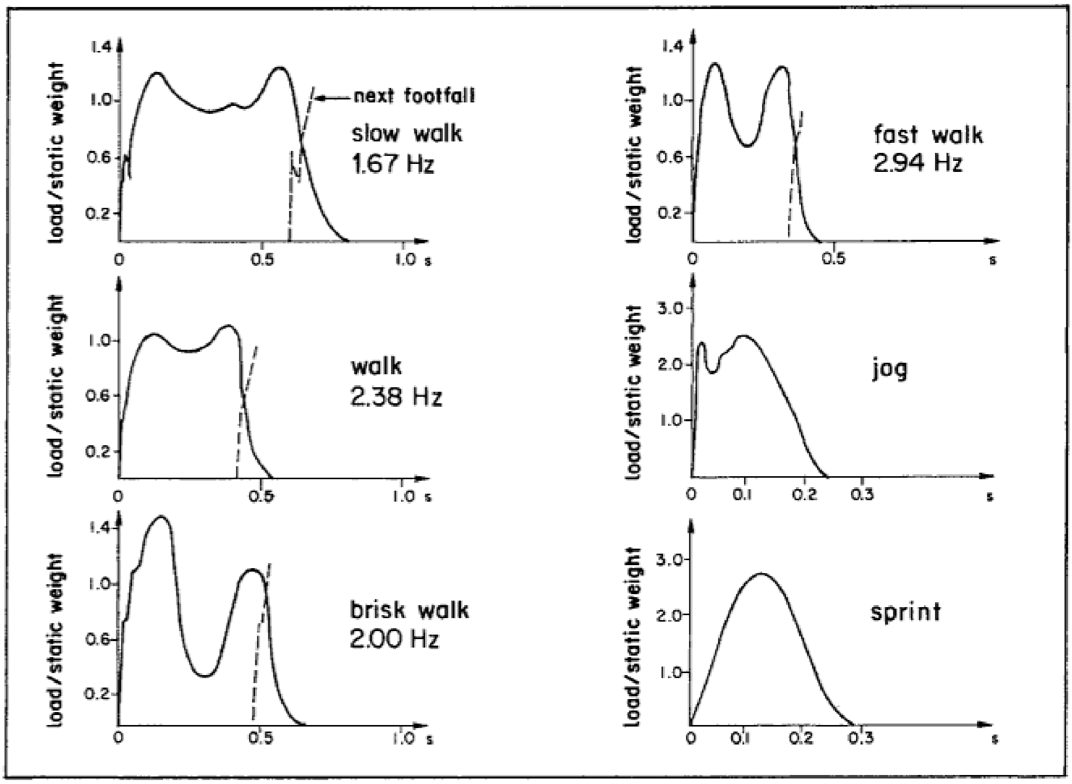


Fig. 2.4 Modification of load-time function with pacing rate [2.6]

off with the ball of the foot. This feature disappears with increasing pacing rate and degenerates to a single maximum of sharp rise and descent when the person is running. From low to high pacing rates, the width of the signal decreases, and the load maximum increases. While for strolling with a frequency below 1 Hz the maximum load hardly exceeds the weight of the person, it increases by a quarter or a third for 2 Hz and by a half around 2.5 Hz; at about 3.5 Hz the maximum reaches about double the weight of the test person.

For relatively high pacing rates (above ca. 2.5 Hz) somewhat larger maxima are indicated in Fig. 2.4, which is taken from Wheeler [2.6] and partly based on Harper's work [2.8]. For fast running the maximum load can increase to three times the weight (Fig. 2.5). The same figure shows a clear relation between the pacing rate and the duration of foot contact with the ground.

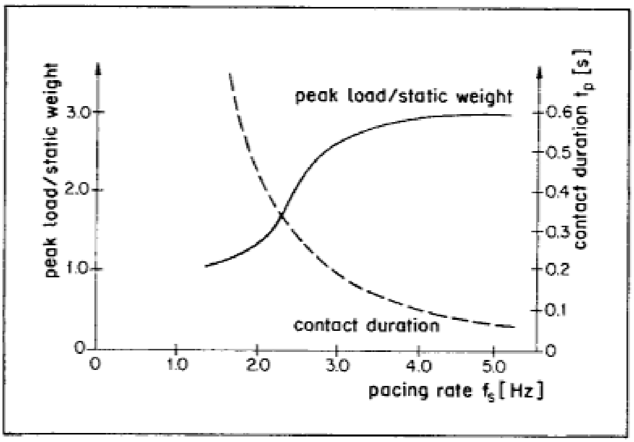


Fig. 2.5
Contact duration and peak load (rel. to static weight) as depending on the pacing rate (from [2.6])

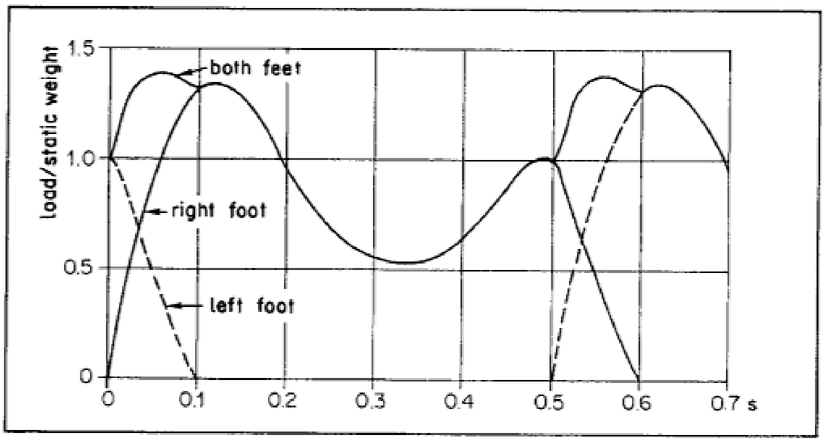


Fig. 2.6 Load-time function resulting from footfall overlap during walking (from [2.10])

During walking, one of the feet is always in contact with the ground at a given time (see in Fig. 2.4 the time function of the second foot in broken line), such that the contact duration of the two feet and thus the loads overlap. Figure 2.6 [2.10] shows the resulting time variation of the total dynamic load during walking. The load function clearly has components in the 2nd and 3rd harmonic, i. e. a pulsating load is also exerted at double and triple the pacing rate. The time function given in [2.3], although slightly different, exhibits basically the same phenomenon.

In contrast to the behaviour during walking, ground contact during running is interrupted, the ratio of contact duration to pace period becoming smaller with increasing pacing rate.

An idealized mathematical formulation of the dynamic load variation must take this difference between walking and running into account by choosing different approaches for «continuous ground contact» and «discontinuous ground contact».

Continuous Ground Contact

The load-time function for walking which exhibits an overlap of the individual contact times of either foot (Fig. 2.6) can be idealized by the following expression:

$$F_p(t) = G + \Delta G_1 \cdot \sin(2 \cdot \pi \cdot f_s \cdot t) + \Delta G_2 \cdot \sin(4 \cdot \pi \cdot f_s \cdot t - \varphi_2) + \Delta G_3 \cdot \sin(6 \cdot \pi \cdot f_s \cdot t - \varphi_3) \tag{2.1}$$

where:

G = weight of the person (generally assumed to $G = 800$ N)
ΔG_1 = load component (amplitude) of 1st harmonic
ΔG_2 = load component (amplitude) of 2nd harmonic
ΔG_3 = load component (amplitude) of 3rd harmonic
f_s = pacing rate
φ_2 = phase angle of the 2nd harmonic relative to the 1st harmonic
φ_3 = phase angle of the 3rd harmonic relative to the 1st harmonic.

The force component (i. e. the Fourier amplitude coefficient) of the 1st harmonic may be taken from the literature (e. g. [2.3], [2.6]) and from own test results [2.10] to be

$$\Delta G_1 = 0.4 \cdot G \quad \text{for} \quad f_s = 2.0\,\text{Hz} \tag{2.2}$$

$$\Delta G_1 = 0.5 \cdot G \quad \text{for} \quad f_s = 2.4\,\text{Hz} \tag{2.3}$$

with linear interpolation in between.

The load components of the 2nd and 3rd harmonic in the range of $f_s \cong 2$ Hz can be taken from [2.3], [2.10] as $\Delta G_2 \cong \Delta G_3 \cong 0.1 \cdot G$.

The phase angles φ_2 and φ_3 exhibit in reality a large scatter because of the many parameters they depend on [2.10]. In a computation they may be approximated to $\varphi_2 \cong \pi/2$ and $\varphi_3 \cong \pi/2$ (partly different values are given in [2.30]). If the most unfavourable combination of the different harmonics is to be captured, the phase angles need to be varied. In most cases, however, a forced vibration induced by walking is governed by just one harmonic, so that phase angles become immaterial.

Discontinuous Ground Contact

The load-time function for running, which is generally characterized by a single load maximum, can be expressed by a sequence of semi-sinusoidal pulses («half-sine model», Fig. 2.7a).

The function within one period is given by

$$F_p(t) = \begin{cases} k_p \cdot G \cdot \sin(\pi \cdot t/t_p) & \text{for} \quad t \leq t_p \\ 0 & \text{for} \quad t_p < t \leq T_p \end{cases} \tag{2.4}$$

with:

k_p = $F_{p,max}/G$ = dynamic impact factor
$F_{p,max}$ = peak dynamic load
G = weight of the jogger (generally assumed to $G = 800$ N)
t_p = contact duration
T_p = $1/f_s$ = pace period.

The impact factor k_p results from the condition of constant potential energy, i.e. that the integral of the load-time function over one pace period must equalize the load at rest (static weight). Figure 2.7 shows how k_p varies with the ratio t_p/T_p.

It is interesting to compare the theoretical values of k_p from the half-sine model (noting that $t_p \cdot f_s = t_p/T_p$ in Fig. 2.7b) with the average experimental values given in Fig. 2.5 for a certain contact duration t_p and pacing rate f_s: In the range of about 3 Hz they agree rather well, whereas for 2 Hz and 4 Hz the half-sine model produces roughly a 30% larger dynamic impact factor. Thus, the half-sine model can, in this case, be considered conservative with respect to the magnitude of the load. Experiments described in [2.10] yielded (for running) a k_p of more than factor three, which is slightly higher than theoretical values. Altogether, the half-sine model seems generally to be appropriate.

The time function according to the half-sine model can be brought into the same format as Eq. (2.1), i.e. as the sum of static weight G and harmonic load components:

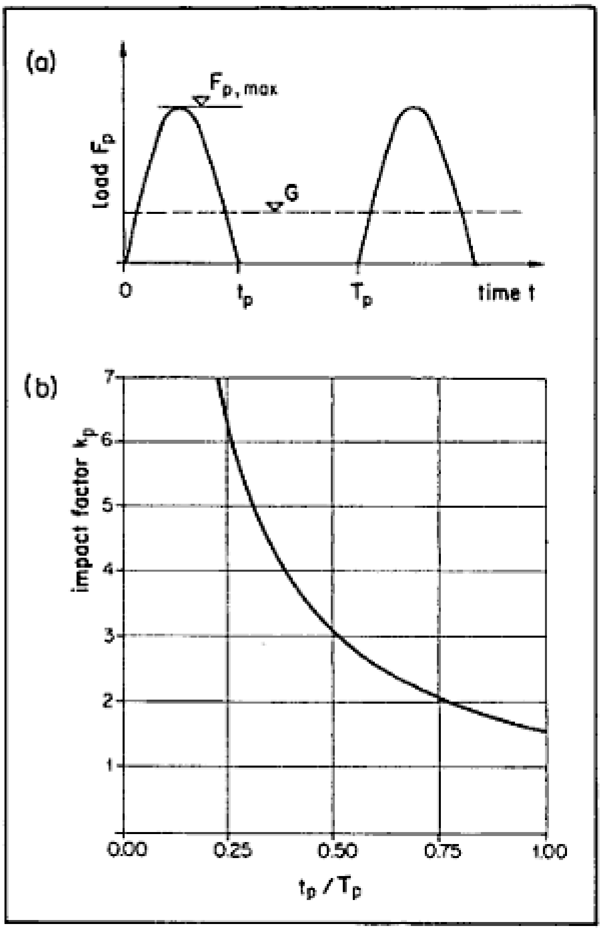


Fig. 2.7
Idealized load-time function for jogging and skipping:
(a) «half-sine model»
(b) impact factor depending on rel. contact duration

$$F_p(t) = G + \sum_{n=0}^{\infty} \Delta G_n \cdot \cos[2 \cdot \pi \cdot n \cdot f_s \cdot (t - \frac{t_p}{2 \cdot n})] \tag{2.5}$$

with:

G = weight of the person (generally assumed to G = 800 N)
ΔG_n = load component (amplitude) of n-th harmonic
n = number of n-th harmonic
f_s = pacing rate
t_p = contact duration.

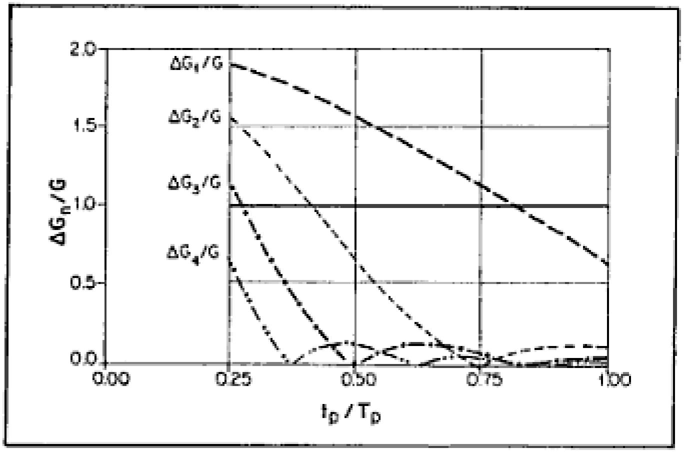


Fig. 2.8
Load components (amplitudes) of several harmonics according to the half-sine model

Figure 2.8 shows the load component amplitudes ΔG_n of the first four harmonics versus the contact duration ratio t_p/T_p as calculated by harmonic analysis (see Appendix B 4). Note that, depending on this ratio, higher harmonics may still contribute substantially to the loading. However, similar to walking, computation of a forced vibration is dominated by just one harmonic, so that the phase shift ($2 \cdot \pi \cdot f_s \cdot t_p/2$ for all harmonics) does not matter.

Loading due to walking or running can usually be assumed as stationary (i.e. spatially fixed) excitation. The relatively slow forward speed has hardly any effect on the vertical excitation of the structure as investigations have confirmed (e.g. [2.7] and [2.11]). If one were to consider the effect that the forward speed of a single walking or running person has on a forced vibration, the response would be transient with smaller amplitudes, since the steady state is not reached before the person leaves the structure.

(c) Time Function of the Horizontal Load

The loading from human walking or running is much smaller in the horizontal (longitudinal and lateral) directions than in the vertical direction. Even then it may become a problem for very flexible structures (see [2.12] and Case No. 4).

Investigations for a pacing rate of 2.0 Hz are presented in [2.3]. Looking at the time function of the lateral vibration displacement in Fig. 2.9, it becomes obvious that the lateral sway of the person's centre of gravity occurs with half the pacing rate (i.e. 1.0 Hz), whereas the longitudinal displacement is dominated by the full pacing rate as in the vertical direction.

A harmonic analysis of the pertinent loading according to Fig. 2.9, exerted by a relatively light person of 587 N, reveals more details, Fig. 2.10. The major lateral load

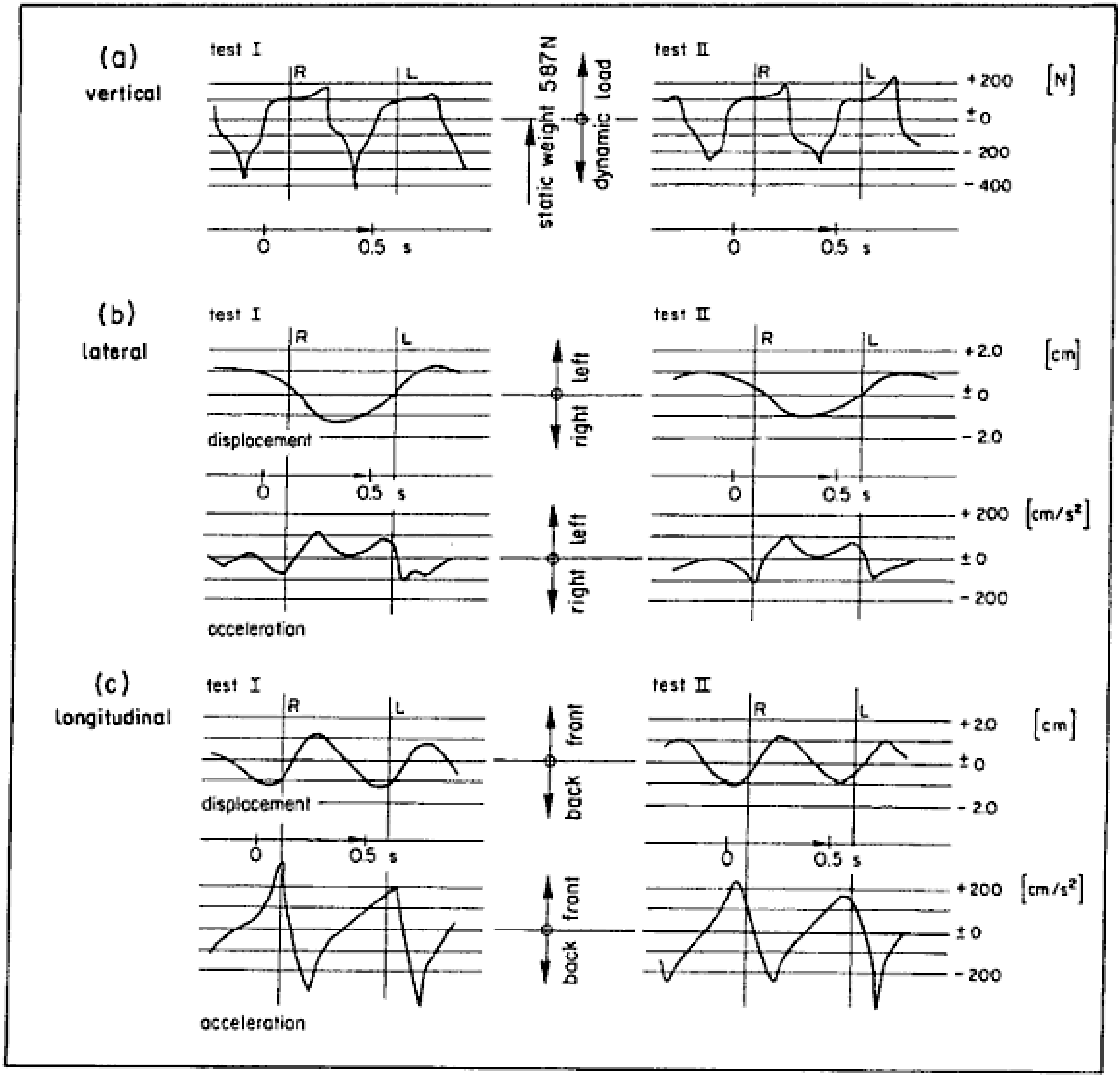


Fig. 2.9 Time function of the vertical dynamic loading with resulting horizontal vibration displacements and accelerations for a pedestrian walking with 2 Hz (from [2.3])

components are associated with frequencies of $f_s/2$ or $3{\cdot}f_s/2$; in the longitudinal direction the major components have f_s and $2{\cdot}f_s$, accompanied by a component in $f_s/2$ due to a more pronounced footfall on one side.

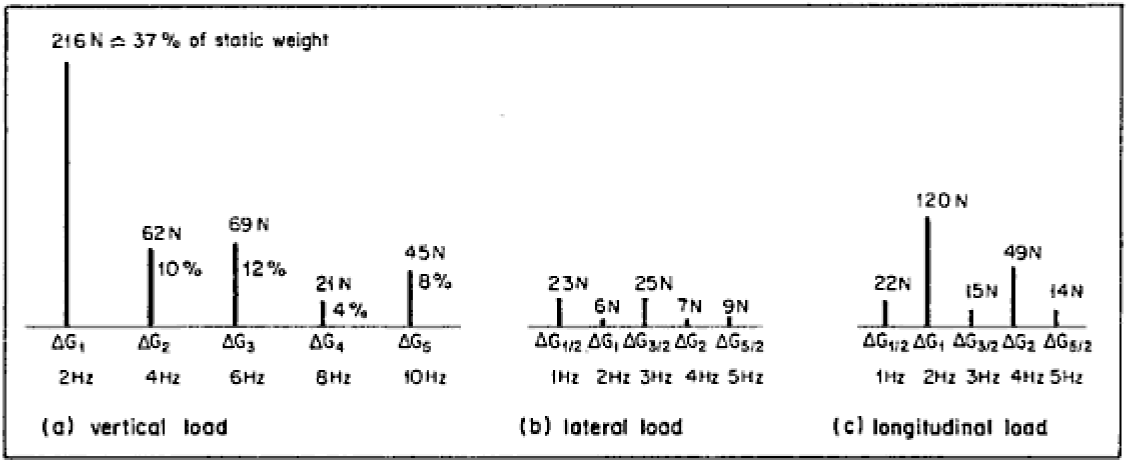


Fig. 2.10 Harmonic load components (Fourier amplitudes) of the directional load-time functions of Fig. 2.9 (from [2.3])

(d) Influence of the Number of People

The results above are for a single person. More interesting in practice is the combined effect of more than one person at a time, particularly in walking. For that purpose the following aspects have to be considered:

- The density of pedestrians is limited by the feasibility of uninhibited walking. According to [2.3] an upper limit is reached with 1.6 to 1.8 persons/m², which corresponds to a static load of about 1100 to 1400 N/m²; however, a value of 1 person/m² (as assumed in [2.2]) seems to be more realistic.
- Based on observations it is proposed in [2.3] that pedestrians walking initially with individual pace on a footbridge will try to adjust their step subconsciously to any vibration of the pavement. This phenomenon of feedback and synchronization becomes more pronounced with larger vibration of the structure. One observation in the case of a laterally swaying footbridge is reported in [2.12] (see Section 2.3). During tests by the authors [2.10] it appeared to be rather difficult to keep to one's usual step when disturbed by vertical vibration displacements of more than 10 or 20 mm; the person tends to fall out of step and adjusts more or less to the motion of the pavement. However, displacements have to be that large for the phenomenon to be noticeable.

The phenomena of more than one person walking can be described as follows: As long as just one person with a pacing rate between 1.5 and 2.5 Hz walks near the centre of a larger structural span, he will produce more or less a forced steady-state response (after an initial transient phase). A vibration produced by a differently walking second person will superimpose the first response so that at certain times the two vibration amplitudes will be additive or subtractive – depending on frequency and phase. Thus the amplitude of the bridge displacement will not be constant but exhibit a typical surge from interference of waves (see Case No. 1, Fig. A1.1a). With the number of pedestrians increasing and their effects enhancing one another at times, larger vibration amplitudes may occur. As mentioned before, the capacity of the bridge deck limits the number of people who can walk at a certain speed (about 1.5 m/s at $f_s = 2$ Hz) and with the necessary clearance (about 1 m in all directions at $f_s = 2$ Hz).

A mathematical description of excitation by more than one pedestrian is difficult. After Matsumoto [2.2] one can assume a Poisson distribution for the arrival probability of pedestrians and derive an enhancement factor (m) to be applied to the vibration amplitude caused by a single pedestrian in the centre of the span:

$$m = \sqrt{\lambda \cdot T_o} \tag{2.6}$$

where:

- λ = mean flow rate (persons/s over width of the deck) for a certain period of time (maximum $\lambda_{max} \cong 1.5$ person/s·m)
- T_o = time necessary to cross the bridge of length L at speed v_s, i.e. $T_o = L/v_s$
- $\lambda \cdot T_o$ = number of persons simultaneously on the bridge at the given mean flow rate.

This formula has not been verified in the field. Extensive computer studies [2.6], in which arrival times, individual weights and pace rates were varied at random, tended to confirm it, especially for bridges with a natural frequency near to the most probable pacing rate at 2 Hz. For example, the simulation with 100 persons/min. on a footbridge of 2 m width and 26 m span yielded m = 5.5, comparing well with m = 5.4 from Eq. (2.6) for v_s = 1.5 m/s. At the current state of knowledge the formula seems to hold true. In practice, the flow rates often will be relatively small and thus the factor m too. Possible infrequent excess of vibration acceptance criteria could be tolerated anyway.

Equation (2.6) may be directly applied to pedestrian structures with a natural frequency f_1 in the range of normal predominant pacing rate between 1.8 and 2.2 Hz (Fig. 2.1). Higher and lower pacing rates are quite rare; hence it is suggested that for structural frequencies f_1 between 2.2 Hz and 2.4 Hz and between 1.8 Hz and 1.6 Hz the enhancement factor is reduced linearly to $m_{min} = 2.0$. This figure corresponds to two people marching in step.

(e) Deliberate Excitation

Besides the response to one or more pedestrians walking at random, to joggers etc., there is the possibility that footbridges and similar structures are deliberately excited to resonance by a group of people («vandal loading»). This may happen in the vertical direction through marching in step or skipping to time, or in a horizontal direction through rhythmical shift of the centre of gravity. This aspect is given closer qualitative consideration in [2.9], for example.

The regulations [2.13] introduce a load case «extraordinary live loads» to cover groups marching in step, spectators rocking to music, and deliberate excitation. Equivalent static loads are prescribed, under which the behaviour of the structure has to be checked.

2.2.2 Skipping

(a) General Characterization

This kind of motion is likely to produce considerable dynamic loads particularly in gymnasia and sports halls, but it may also become critical in other types of buildings where jazz dance sessions are practised. The deliberate excitation of pedestrian structures may also be effected by skipping. Skipping can be characterized by the skipping frequency (f_s) and the time function of the dynamic load.

Skipping Frequency

More recently gymnastics to rhythmic music have become quite common. The items of music are chosen such that their rhythm suits various skipping, jumping and running exercises. During fitness classes, frequencies for different forms of motion ranging from 2.0 to 3.2 Hz were found [2.1]. Jazz dance sessions with skipping showed the same frequency range [2.14]. Tests with skipping for a longer time (1 to 2 min.) on the same spot revealed frequencies from 1.0 to 2.8 Hz [2.4]. It seems unlikely, that even for a short time (say 20 s) the human physiology would permit a frequency of more than 3.5 Hz. This leaves a range between 1.8 and 3.4 Hz for calculation purposes.

Load-Time Function

A person skipping exerts primarily a vertical load on the floor. The main parameters affecting the time function of this dynamic load are the following:

- skipping frequency
- intensity (moderate or maximum height)
- person's weight
- type of footwear
- floor surface condition (softness of cover).

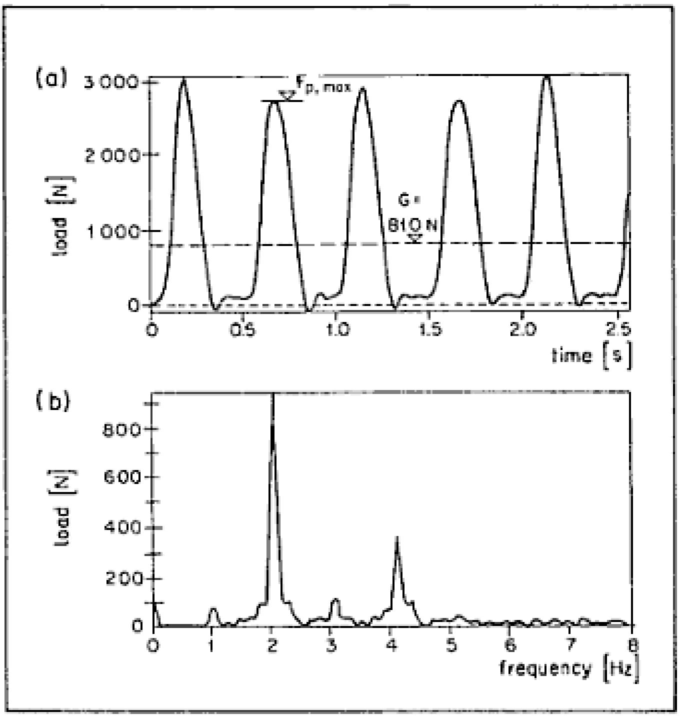


Fig. 2.11
Loading caused by a person's skipping (from [2.15]):
(a) Load-time function (low-pass filtered at 9 Hz)
(b) Fourier amplitude spectrum

(b) Load-Time Function

Although the range of possible frequencies is bounded as shown above, the variability of rhythmic exercises during gymnastics or other sporting events leaves much heterogeneity in terms of the load-time function. The following concentrates on «skipping on the spot» (both feet simultaneously) as a form of motion, which is easy enough to describe but still representative with view to the frequency range as well as to the maximum possible load amplitude.

Skipping of a single person with a frequency of $f_s = 2.1$ Hz is shown in Fig. 2.11 [2.15], both in terms of the resulting time function of the load and the associated Fourier amplitude spectrum. The latter exhibits a clear load component in the 2nd harmonic. The ratio of maximum load to the person's weight, i.e. the impact factor $k_p = F_{p,max}/G$ (see Section 2.2.1), reaches $k_p = 3.4$ in this example. The maximum peak load of a single person skipping is measured in [2.10] to be six times his weight (i.e. $k_p \cong 6$ with possible extension to $k_p \cong 7$ for extremely high skipping).

For an idealized mathematical description and calculation of a forced vibration, the «half-sine model» – a sequence of semi-sinusoidal pulses (Fig. 2.7a) – can be used again. Equations (2.4) and (2.5), which were derived for walking, are valid with T_p and f_s as skipping period and skipping frequency. The resulting impact factor increases with decreasing ratio t_p/T_p. According to experiments reported in [2.10], this contact ratio may vary between 0.25 and 0.6.

Because of individual differences in the motion, allowing for softer or harder bounce on the floor irrespective of the frequency, for skipping the relation between contact duration and frequency is not a priori defined as it was for walking and running (see Fig. 2.5). Hence, various contact durations t_p for the same frequency f_s (i.e. various t_p/T_p) yield a spectrum of impact factors k_p (see Fig. 2.7b). It was observed in [2.10] that physiology hardly permits a contact duration below $t_p \cong 0.15$ s. From that the minimum contact duration ratio is:

$$t_p/T_p = 0.15 \cdot f_s \tag{2.7}$$

Inserting the practical minimum skipping frequency of $f_s \cong 1.8$ Hz, one arrives at $t_p/T_p = 0.26$ and can take from Fig. 2.7b a factor of $k_p \cong 6$; this is the maximum impact factor reported in [2.10].

Going back to the results given in Fig. 2.11a, the half-sine approximation of the time function yields $t_p/T_p \cong 0.50$, i.e. for the measured $f_s \cong 2.1$ Hz an idealized contact duration of $t_p \cong 0.23$ s. Figure 2.7b gives a dynamic impact factor of $k_p \cong 3.1$, which compares well with the measured ratio of maximum dynamic load to weight of $k_p \cong 3.4$.

To further assess the validity of the half-sine model, a look at the Fourier amplitude spectrum of Fig. 2.11b is informative. Generally, skipping on the spot produces a major loading in the frequencies of the 1st, 2nd and 3rd harmonics, i.e. in f_s, $2 \cdot f_s$ and $3 \cdot f_s$. But the spectrum shows that loads can also be exerted in $f_s/2$ and $3 \cdot f_s/2$, or other «intermediate harmonics», as the time variation of the load is apparently not exactly periodic in the skipping frequency. This phenomenon can become much more pronounced, if the loads exerted from either foot differ, e.g. when leaping from one foot to the other. Complex physical exercises as occur during jazz dance sessions give rise to load-time functions considerably in variance with the half-sine model. Figure 2.12 shows the Fourier amplitude spectrum of measured accelerations of a gymnasium floor. Substantial response was observed at $3 \cdot f_s/2$ (besides $f_s = 2.3$ Hz and $2 \cdot f_s = 4.6$ Hz), which happened to coincide with the 3.5 Hz fundamental frequency of the floor structure. This kind of excitation can result, for instance, when one foot touches the floor with a slight time lag during skipping. Important as these subtleties may appear in certain cases, for general calculation purposes – forced vibrations in particular – the half-sine model is sufficiently accurate.

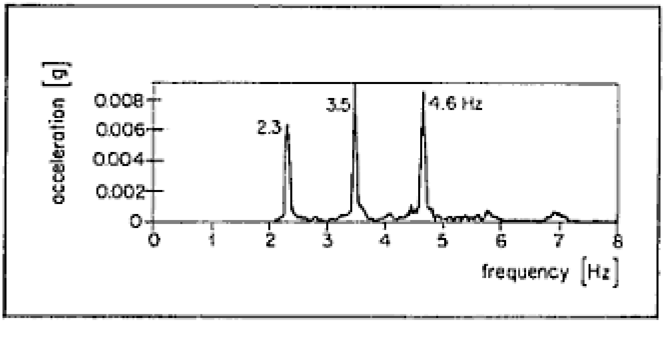


Fig. 2.12
Fourier amplitude spectrum of the accelerations of a gymnasium floor with $f_1 = 3.5$ Hz at $f_s = 2.3$ Hz (from [2.15] after [2.14])

(c) Influence of the Number of People

When comparing Fig. 2.13 to Fig. 2.11, one finds little difference in the load-time functions for skipping of one person or a group of eight persons. This is due to the remarkable synchronization by virtue of the accompanying music. A difference exists, however, with respect to the minimum possible duration of contact.

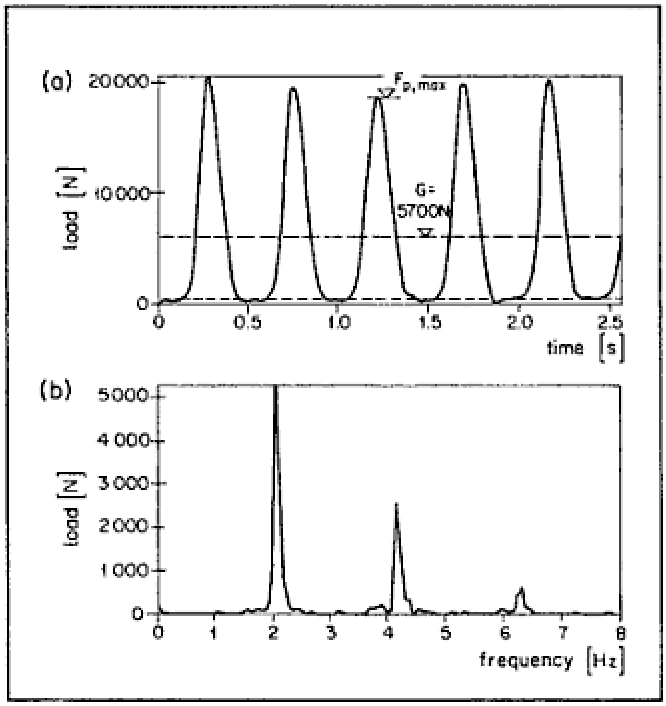


Fig. 2.13
Loading due to 8 persons skipping in phase (from [2.15]):
(a) Load-time function (low-pass filtered at 9 Hz)
(b) Fourier amplitude spectrum

Due to slight variations of the individual motions in a larger group of participants, which are never completely synchronized, the minimum contact duration increases to $t_p \cong 0.20$ s, resulting in a minimum contact duration ratio achievable by groups of

$$t_p/T_p = 0.20 \cdot f_s \tag{2.8}$$

Inserting $f_s \cong 1.8$ Hz as a realistic minimum frequency for skipping brings the contact duration ratio to $t_p/T_p = 0.35$ and leads in Fig. 2.7b to an upper bound of the impact factor of $k_p \cong 4.5$ for a larger group.

In contrast to the case of randomly walking pedestrians (see Section 2.2.1), where the dynamic load increases with the square root of the number of people involved, for skipping-type motion the dynamic load increases as a linear function. The upper limit of occupancy in gymnasia, as observed by the authors in fitness classes with skipping and jogging exercises, is about 1 person per 4 m², or 0.25 person/m². In a gymnasium of 20 x 30 m² floor area, this would amount to ca. 150 participants. A denser occupancy was assumed in [2.16] with 1 person per 2 m², or 0.5 person/m².

(d) Deliberate Excitation

Skipping is the easiest method to deliberately induce resonance in a structure vibrating vertically. In tests [2.6] on 22 footbridges, a group of two or three persons tried to excite the structure to the largest possible displacements in its fundamental frequency. In some cases, the attained displacement amplitudes hardly exceeded those due to normal pedestrian traffic, whereas in all the others they did not even approach them. Hence, vandal loading was not considered critical. The same conclusion was reached in [2.5]. However, our Case No. 1 deals with a footbridge of 40 m span which three persons, skipping continuously in the centre of the span, could excite to a displacement amplitude of 9.4 mm; this was more than double the amplitude of vibrations under a flow rate of 29 pedestrians per minute.

2.2.3 Dancing

During normal dance events, but also during concerts with a clapping, stamping and rocking audience (see Cases No. 10 and 11), serious dynamic loads may arise. Not much is known about the load-time functions of the resulting dynamic loading since it is rather difficult to measure these loads directly.

Dancing Frequency

Clapping of hands, foot stamping or body rocking occurs with the beat of the music, which may vary between 1.6 Hz for slow and 3 Hz for fast pieces. As the exerted forces usually vary from one beat to the other (unequal load on either foot, etc.), the load spectrum may also contain frequencies below the dancing frequency, for instance «semi-harmonic frequencies» as described in Section 2.2.2.

Load-Time Function

To measure directly the load-time function induced by dancing is difficult. Since floor contact usually is maintained, the maximum dynamic force is unlikely – according to Figs. 2.3 and 2.4 – to greatly exceed the static load. However, the total load can be substantial due to the dense occupancy of the floor, particularly at concerts. In [2.17] the density is estimated to be 2 persons/m², but can possibly increase locally to 6 persons/m².

For the purpose of calculating a forced vibration (see Section 2.4.4), an idealized mathematical load-time function can be taken from the motion type of walking with continuous ground contact (Section 2.2.1). The pertinent equation is (2.1). Following

[2.16], the amplitude coefficient of the 1st harmonic may be taken as $\Delta G_1 = 0.5 \cdot G$, that of the 2nd harmonic is estimated by the authors to be $\Delta G_2 \cong 0.15 \cdot G$. By the very nature of the diversity of dancing motion, this recommendation produces results which are approximate at best.

2.2.4 Jolting Motion

A person may exert a single impulse by taking off from a high-diving platform, by landing on a gymnasium floor after jumping from an elevated position, or by bumping against a wall with his shoulder. These loads are transient. The importance of these kinds of impact loading is minor and has been little investigated. The time function of the load is seldom available. A most extensive discussion of the subject is given in [2.18]. Further data is found in [2.19] or [2.20] and [2.21], wherein an equivalent spring stiffness is derived for a person jumping onto the floor.

2.3 Effects of Man-Induced Vibrations

Man-induced vibrations affect the structure and occupants in rather different ways.

For the structural integrity, they are rarely a problem. More seriously affected is the serviceability of the structure. Besides damage to nonstructural elements the major concern is for the wellbeing of the people affected. They tend to feel those vibrations to be particularly disturbing which are not caused by themselves. Section 4.3 gives acceptance criteria to evaluate the severity of possible discomfort.

Effects of man-induced vibrations on structures or structural members include

- loss of the load-bearing capacity; although rare, collapse may occur due to overloading of the structure (Hyatt Regency walkway in Kansas City [2.22]) or failure in fatigue;
- cracking of structural members, cladding and partitions, falling away of plaster, etc.

People may feel affected by man-induced vibrations in several ways:

- Mechanically as, for instance, by the vibrations of a footbridge or an intermediate floor in a gymnasium. High-diving platforms in swimming-pools are a special case, their vibration can irritate the athletes to the point that the platform becomes useless.
- Acoustically by the noise from reverberation and rattling of equipment; this effect can be quite dramatic, evoking pronounced anxiety and flight from the place (see Case No. 7).

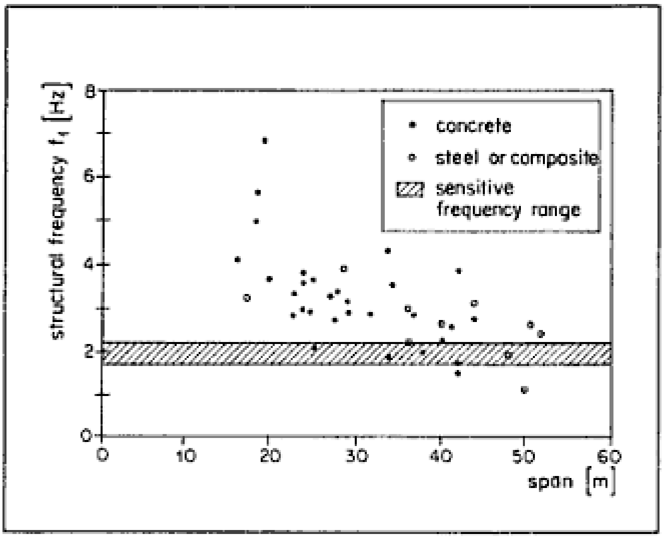


Fig. 2.14
Fundamental natural frequencies of footbridges of different span (from [2.5])

– Optically by impressions like irritatingly large deflections of the ceiling under a gymnasium hall or frightening sway of the diving platform from the viewpoint of the athlete.

The magnitude of man-induced vibrations in structures or of structural members depends primarily on the ratio between the dominant excitation frequency (see Section 3.3.1) and the fundamental natural frequency of the structure or structural member. The natural frequencies depend on the span and the statical system of the structure, its flexural rigidity and mass. Figure 2.14 gives a survey of spans and fundamental frequencies of 44 footbridges examined in Britain [2.5]. Generally, the fundamental frequency decreases with increasing span, but it can differ widely in structures of similar span, so that definite conclusions cannot be drawn.

The fundamental frequency, but also higher frequencies of a structure can be excited. A case is reported in [2.6] in which a steel footbridge with a natural frequency $f_1 = 8$ Hz exhibited resonance at a pacing rate of $f_s = 3.7$ Hz. The reason was that the 3rd harmonic of the dynamic load ($f = 3 \cdot 3.7 = 11.1$ Hz) excited the 2nd structural frequency $f_2 = 11.1$ Hz.

In some cases a footbridge can also get excited in longitudinal or lateral direction. This can be caused by dynamic loads with half the pace rate or a multiple of it (see Figs. 2.9 and 2.10). Note that the synchronization phenomenon described in Section 2.2.1 is more pronounced for lateral vibrations. It was held partly responsible in [2.12] for the observed amplification of substantial lateral vibrations of a steel bridge. Presumably, the pedestrian, having noticed the lateral sway, attempts to reestablish his balance by moving his body in the opposite direction; the load he thereby exerts on the pavement, however, is directed so as to enhance the structural vibration.

2.4 Measures Against Man-Induced Vibrations

As for machine-induced vibrations (see Chapter 3), the first and foremost countermeasure is to change the structural frequency (called frequency tuning, see Appendix B 5). Normally, this provides a sure solution. In some cases the calculation of a forced vibration is to be recommended. Special measures, e. g. installation of tuned vibration absorbers, are mainly suitable for improvement of existing structures (see Appendix B 7).

In the following, frequency tuning, calculation of a forced vibration, and special measures are discussed individually for each type of structure. High-diving platforms are an exception in that, besides the frequency, the stiffness of the structure enters as a second criterion. The goal of each measure is to suppress the structural vibrations to admissible upper bounds. Acceptance criteria for these bounds are given in Chapter 4 (for high-diving platforms, see Section 2.4.5). Appendix B 3 contains approximate formulas for determining the fundamental frequency of the respective structure, Appendix B 8 data on material properties.

2.4.1 Pedestrian Structures

The following countermeasures against man-induced vibrations are applicable to footbridges and similarly loaded structures such as overpasses, connecting passageways («skywalks»), long-span staircases, boat landing stages, etc.

(a) Frequency Tuning

Considering the statistical distribution of pacing rate (see Section 2.2.1), the largest risk of excitation in the *vertical direction* is for pedestrian structures having a natural frequency f_1 between ca. 1.6 and 2.4 Hz. Fundamental and higher frequencies should be kept out of this range by all means:

$$f_i \begin{cases} < 1.6\ \text{Hz} \\ > 2.4\ \text{Hz} \end{cases} \tag{2.9}$$

Moreover, since dynamic force components from walking are also exerted at double the pacing rate (2nd harmonic, see Section 2.2.1 and Case No. 3), bridge structures with relatively low stiffness and damping, particularly steel and composite steel bridges, should be designed not to have fundamental and higher frequencies in the range between ca. 3.5 and 4.5 Hz either:

$$f_i \begin{cases} < 3.5\ \text{Hz} \\ > 4.5\ \text{Hz} \end{cases} \tag{2.10}$$

The frequency clearance need not be as large as for the 1st harmonic (Eq. 2.9), as the forces excerted in the 2nd harmonic are smaller.

It is conservative in practically all cases to have a high tuning of the fundamental frequency to ca. 5 Hz, i.e.

$$f_1 > 5\,\text{Hz} \tag{2.11}$$

Compliance with the recommendations (2.9) and (2.10) does not eliminate occasional resonance in frequencies between the two clearance ranges, but this will be a rare loading for most structures. It could occur at brisk walk with $f_s \cong 2.5$ Hz, or if a jogger passed with $f_s = 3$ Hz. Other pedestrians, however, walking at the same time with $f_s \cong 2$ Hz would disturb the vibration thus tending to hinder resonance effects.

There rarely are structures with low stiffness and simultaneously low damping in the *lateral direction*, i.e. transverse to the general traffic axis (see Case No. 3, for example). Frequencies in the range 0.8 to 1.2 Hz should be avoided and, depending on circumstances, also in the range 2.6 to 3.4 Hz (see Fig. 2.10b).

Also in rare cases certain structures possess very low stiffness in the *longitudinal direction*, i.e. in the direction of traffic (e.g. a frame structure with rather flexible struts), or are supported on very soft bearings. Frequencies in the range 0.8 to 1.2 Hz should be avoided here as also in the second range from ca. 1.6 to 2.4 Hz (see Fig. 2.10c).

In the following, some regulations in codes of practice are drawn attention to:

- The British load specifications for bridges [2.25] waive vibration serviceability checks for pedestrian and cyclist overpasses if the fundamental frequency f_1 without live load lies above 5 Hz. See also Section 4.3.5.
- The Canadian code [2.16] adopts a frequency of $f_1 = 6$ Hz as threshold of checking requirements.
- The East-German code for bridges, footbridges and stairs [2.13] uses as a criterion the fundamental frequency (vertical, longitudinal and lateral) of the structure when loaded with 300 kg/m² additional mass, $f_{1,g+p}$. If this frequency is above 6 Hz, a check is not required; if it lies between 6 and 3 Hz, then the dynamic performance has to be evaluated for stairs but not for the other types of structure. For these checks in the vertical and longitudinal directions, the relevant pacing rates are taken as f_s and $2 \cdot f_s$, while in the lateral direction in addition $f_s/2$ and $3 \cdot f_s/2$ are investigated (as follows from the harmonic analysis of the load-time function of walking, see Fig. 2.10). The code specifies an equivalent static load procedure for computational purposes. For a range $1\,\text{Hz} < f_{1,g+p} < 3\,\text{Hz}$, pedestrian bridges have to be checked in the same way. Genuine dynamic investigation is stipulated for $f_{1,g+p} < 1$ Hz.

The natural frequencies of pedestrian structures should always be computed with conservative upper and lower bounds to account for secondary elements such as pavements or railings, the magnitude of the dynamic modulus of elasticity, the transition to the cracked state and the tension stiffening of the concrete between cracks in reinforced concrete structures, etc. (see also Appendix B 8).

In the cases of

- a very flexible structure
- very low damping properties (e.g. in welded steel structures)
- impossibility of frequency tuning as recommended
- high demands for the comfort of the user,

one should either calculate a forced vibration or – if that proves to be inadequate – adopt special measures.

(b) Calculation of a Forced Vibration

Calculating a forced vibration – in general at matched frequency as steady state response (see Appendix B 1) – will give an idea of the dynamic performance of the structure. The load-time function can be chosen for walking according to Eq. (2.1). Several uncertainties may arise: The often inaccurate determination of the fundamental structural frequency is one; others are the assumed damping properties (see Appendix B 8) and the influence of more than one pedestrian (see Section 2.2.1d); the latter is to be estimated from an average density of people and the mean flow rate of pedestrians respectively. The results of such a computation must be interpreted with reservation. They are to be compared to the acceptance criteria in Chapter 4.

(c) Special Measures

If the structural frequency cannot be properly shifted or the calculated forced vibration yields inadmissible amplitudes, special measures have to be considered. They are particularly appropriate to improve an existing structure.

Special measures may, for example, be

- installing a stiffer hand rail
- adding tie-down cables
- increasing the damping
- attaching a tuned vibration absorber.

The first two measures aim at stiffening the structure. For instance, a hand rail may be substituted or complemented by a truss construction. A pedestrian overpass across a motorway could be tied down in midspan. If symmetrically attached up to the middle third of the span, cables can effectively suppress horizontal vibrations in the 1st and 2nd mode shape. Arranged in an inclined plane, they simultaneously reduce vertical vibrations.

Substantially increasing the damping properties proves more difficult, unless the structure's intrinsic damping is very low. In structures with $\xi \cong 0.5\%$ (of critical damping), friction devices may be installed, for instance, in hand rails or at bearings. However, as soon as the intrinsic damping is close to $\xi \cong 1\%$, damping devices become too bulky for realization or too expensive.

Sometimes, the installation of one or more vibration absorbers is more promising. Such a vibration absorber consists of a mass-spring-damper system which is attached to the structure or its vibrating part at a selected spot and carefully tuned with respect to the primary structure's vibration characteristic. It is a typical improvement device. Some actual applications are reported in [2.2], [2.5], [2.11], [2.23] and [2.24]. How such an absorber works and how its behaviour is calculated, is shown in Appendix B 7. The effectiveness of the absorber is primarily determined by the ratio of the absorber mass to the (modal) mass of the structure and increases with smaller damping of the structure. This favours steel structures for possible application.

The extent to which encountered vibrations are to be reduced is given by the acceptance criteria in Section 4.3, especially those in Sections 4.3.5 (BS 5400) and 4.3.9 (values taken from the literature on pedestrian structures and not laid down in codes).

2.4.2 Office Buildings

The countermeasures discussed in the following sections concern vibrations caused by the random walk of people in office and administation buildings and disturbing personnel at their working places.

(a) Frequency Tuning

People spending some eight hours a day sitting or standing at their working places are rather sensitive to building vibrations. The tolerance of acceptable values is thus necessarily stricter (see e.g. [4.25]), and – in contrast to pedestrian structures, gymnasia and sports halls, etc. – the 3rd harmonic of the dynamic load-time function must be taken into consideration as well.

For floor slabs in office buildings (see Case No. 5), our present state of knowledge would suggest high tuning above the frequency of the 3rd harmonic of the load-time function (see Fig. 2.10a). Moreover, the stiffness, mass and damping of the type of structure in question must be taken into account.

As shown in Section 2.2.1, the walking pacing rate centres around 2 Hz and rarely reaches more than 2.4 Hz. Consequently, the structure should comply with the following fundamental frequencies:

- reinforced concrete $f_1 > 7.5$ Hz
- prestressed concrete $f_1 > 8.0$ Hz

- composite (in-situ concrete slab on steel girders) $f_1 > 8.5$ Hz
- steel (e.g. corrugated sheets with concrete infill on girders) $f_1 > 9.0$ Hz

The order of ascending frequencies is due to the accompanying decrease in stiffness, mass and damping.

Observing these lower frequency bounds should generally lead to an acceptable vibrational behaviour (see Chapter 4), although exceptionally they may prove insufficient for some structures with low stiffness and damping. On the other hand, massive concrete structures (e.g. large-span joist floors) may perform well with fundamental frequencies below these bounds. Massive reinforced concrete slabs have frequencies well above 7.5 Hz anyway. Hence the bounds given above for office buildings are mainly relevant for light-weight composite and, in particular, steel floor structures.

Calculating the fundamental frequency of slabs in office buildings calls for conservative estimates as to load sharing by the floor finish, the dynamic modulus of elasticity, the transition to the cracked state of reinforced concrete members and the tension stiffening of the concrete, etc. (see Appendix B 8). Stiffening due to nonstructural partitions should be neglected unless structural details such as load-carrying connections to the structure secure the participation.

Special cases, e.g.

- still higher demands of comfort to the user
- vibration-sensitive equipment or installations

may require raising the lower frequency bound above the given values.

Otherwise, one can attempt to calculate the expected amplitudes of a forced vibration, or one can resort to special measures.

(b) Calculation of a Forced Vibration

Random walk of people considered as a forcing function (see Appendices B 1 and B 2 for the general procedure) can be tackled in analogy to pedestrian structures. For the excitation from a single person, in most cases the dynamic load component of the 3rd harmonic (Eq. (2.1)) must be considered. The assumption of a matched-frequency loading, e.g. resonance steady state between this component and the fundamental frequency, will be safe in connection with a conservative estimate of the damping properties (see Appendix B 8). The damping effect even due to soft floor covering and partitions or other installations can be surprisingly small [2.26]. The influence of the number of people on the displacement amplitude can be approximately assessed using Matsumoto's formula (Eq. (2.6)).

(c) Special Measures

The special measures applicable to office floor slabs are

- stiffening the structure
- increasing the damping
- attaching tuned vibration absorbers (see Case No. 6).

Increasing the damping, however, is rather difficult to achieve.

2.4.3 Gymnasia and Sports Halls

The following measures to reduce man-induced vibrations in gymnasia and sports halls apply by and large also for places of similar use, such as jazz dance studios, although sometimes to a slightly relaxed standard (see Section 2.2.2).

(a) Frequency Tuning

Special attention has to be devoted to the design of the floors that are exposed to rhythmic skipping, jumping and running exercises (see Cases No. 7 and 8). The present state of knowledge suggests for this kind of building high tuning with respect to the 2nd harmonic of the load-time function, with «skipping on the spot» as relevant type of motion. Moreover, this has to be done with due consideration of stiffness, mass and damping, which vary with different floor designs.

Since critical frequencies for skipping lie between 2.8 and 3.4 Hz (see Section 2.2.2), the following values are recommended as lower bound for the fundamental structural frequency (which happen to coincide with those for office buildings before):

- reinforced concrete $f_1 > 7.5$ Hz
- prestressed concrete $f_1 > 8.0$ Hz
- composite (in-situ concrete slab on steel girders) $f_1 > 8.5$ Hz
- steel (e.g. corrugated sheets with concrete infill on girders) $f_1 > 9.0$ Hz

The order of ascending frequencies is due to the accompanying decrease in stiffness, mass and damping.

Under normal circumstances these bounds ensure that the vibrations remain within the acceptable range (e.g. with maximum accelerations below 5% g, see Chapter 4). In case of doubt, a forced vibration analysis should be carried out and the results compared with the acceptance criteria.

Following the relevant standards, gymnasium floors need only be designed for static, i.e. non-dynamic loading. One exception is the Dutch code [2.27], which requires a fundamental structural frequency of at least 5 Hz. The commentary on the Canadian code of 1980 [2.28] contains the same provision, but is being revised with more detailed recommendations in the new draft of the commentary [2.16]. They distinguish between kinds of use of the structure and possible excitations, the mass and type of design of the floor structure (e.g. also wooden floors), and the required level of comfort for the people affected. Similar recommendations are given in [2.15].

Again, the calculation of the fundamental floor frequency should be based on conservative estimates of participation of the floor finish, dynamic modulus of elasticity, transition to the cracked state and the tension stiffening in reinforced concrete (see Appendix B 8).

In cases in which

- the above frequency bounds cannot be assured with certainty or
- the demand for comfort is higher,

the computation of a forced vibration or special measures are advisable.

(b) Calculation of a Forced Vibration

Computing a forced vibration – generally for steady-state conditions, see Appendix B 1 – allows the dynamic behaviour of a gymnasium floor to be approximately assessed. As an input forcing function one can normally take the semi-sinusoidal loading function («half-sine model») presented in Fig. 2.7a for the motion type «skipping on the spot», starting from Eq. (2.8). To capture resonance conditions, the skipping frequency has to be chosen with respect to the fundamental structural frequency (an integer fraction in most cases).

The impact factor follows from Fig. 2.7b, while the Fourier amplitudes of the several harmonics of the loading function can be taken from Fig. 2.8. Note, however, that small Fourier amplitudes are subject to considerable uncertainties due to their sensitivity to small deviations of the actual loading function from the half-sine model. One should not use the values of DG_2, DG_3 and DG_4 to the right hand side of the first zero points. In tests [2.10] it proved difficult to excite a structure by the 3rd harmonic of the loading function from a single person's skipping. Unless the skipping frequency is kept precisely, the structural response drops substantially; this sensitivity is the more pronounced the higher the fundamental frequency of the structure. From our present knowledge it seems unlikely that structural frequencies above 7.5 Hz (to be matched by the 3rd harmonic of 2.5 Hz skipping) could be excited to strong vibrations even by a larger group of people.

The dynamic load should be based on the assumption of 1 person per 2 m^2 maximum, corresponding to 0.5 person/m^2 (see Section 2.2.2).

The damping properties to be used in the calculation may be taken from Appendix B 8. They are assumed to be somewhat higher than those for pedestrian structures because of the contributions of floor finish, cladding elements, etc. It has occasionally been mentioned that the damping may sometimes rise markedly (up to double its basic value) as soon as people are present on the floor (e.g. [2.16]); this is only true during decay measurements where the human body absorbs part of the vibration energy, but not for steady-state response to excitation by the person himself.

Results of these computations may be judged against the acceptance criteria of Chapter 4.

(c) Special Measures

In analogy to the sections on pedestrian structures and office buildings, possible special measures are

- stiffening the structure
- increasing the damping
- attaching tuned vibration absorbers.

Of course, critical activities like rhythmic skipping or running may simply be prohibited, but this would restrain the use of the building.

2.4.4 Dancing and Concert Halls

In the following, measures against the type of man-induced vibrations described in Section 2.2.3 are discussed. In concert halls, pop concerts that animate the audience to activities such as stamping, clapping, rocking, etc., are critical. Depending on circumstances, the same basic considerations could be applied to other types of buildings hosting similar events, for instance to grandstands in stadia.

(a) Frequency Tuning

The considerations are more or less those derived for gymnasia in the previous section, namely a request for high tuning with respect to the 2nd harmonic of the dynamic load. When comparing the extent to which the 2nd harmonic participates in skipping ($\Delta G_2 \cong 1.4 \cdot G$ for $t_p/T_p \cong 0.30$ from Fig. 2.8) and in dancing ($\Delta G_2 \cong 0.15 \cdot G$, see Section 2.2.3), one arrives still at the same magnitude if the very different possible density of people is accounted for as well (compare the respective sections on calculation of a forced vibration). As stated in Section 2.2.3, the dominant dancing frequencies lie between 2 and 3 Hz. Generally, the critical range of 3 to 3.4 Hz for gymnasia is avoided so that the 2nd harmonic as criterion yields lower bounds on the fundamental frequency as before. For different floor designs, the recommended bounds are now:

– reinforced concrete	$f_1 > 6.5$ Hz
– prestressed concrete	$f_1 > 7.0$ Hz
– composite (in-situ concrete slab on steel girders)	$f_1 > 7.5$ Hz
– steel (e. g. corrugated sheets with concrete infill on girders)	$f_1 > 8.0$ Hz

The increase in bounding frequency is again associated with the decrease in stiffness, mass and damping.

For calculation of the fundamental frequency of floors in dancing or concert halls, the body masses of the people on the floor have to be included in the correct way. Note, that particularly during pop concerts in halls without (fixed) seats, the audience may gather around the stage, leading to a loading density of up to 6 persons/m². This uneven distribution of people, together with the variety of their active or passive behaviour, may have a strong influence on the size of the moving mass and hence the floor frequency (see Case No. 10).

In addition to general conservativeness, the assumptions for the frequency computation should reflect the concentration of the added human mass in «active» and «passive» areas such that the given structural system is exposed to the most unfavourable mass distribution. The comments on frequency calculation given in Section 2.4.3 for gymnasia and sports halls should be remembered.

Although it may be appropriate in some cases to calculate a forced vibration, the uncertainty in the assumptions is much worse than for pedestrian structures or gymnasia. Special measures are therefore often preferred.

(b) Calculation of a Forced Vibration

For the above reasons, a steady-state computation (see Appendix B 1) for dancing and concert halls can yield just a crude approximation to the true dynamic behaviour. The load-time function would be assumed as in Section 2.2.3.

As regards the density of people, one can assume a maximum of 1 person per 0.17 m², corresponding to 6 persons/m², unseated and 2 persons/m² with fixed seats. Assumptions on density and distribution of the dynamic load are similar to the considerations previously made for the frequency computation.

Possible damping factors are to be found in Appendix B 8 and are generally identical with those for gymnasium floors.

The results of the computation can be judged against the acceptance criteria in Chapter 4.

(c) Special Measures

As in the case of gymnasia and sports halls the following measures might be considered, particularly for improvement of existing structures:

- stiffening the structure
- increasing the damping
- attaching tuned vibration absorbers
- restricting the use of the building.

2.4.5 High-Diving Platforms

Vibrations of high-diving platforms need to be suppressed not to hamper the athletes during their performance. They would be disturbed in their concentration by

- swaying of the shaft in a direction not necessarily coinciding with the direction of take-off
- rigid-body motion of the platform with similar effects
- vibration within the platform, which is particularly unpleasant as the athlete may be given an unwanted spin.

Following [2.29], two types of criteria are to be met:

- criteria of frequency equivalent to high tuning, necessitating a frequency computation
- criteria of stiffness to be checked by relatively simple static calculations.

Frequency Criteria

The bounds to be observed are listed in Tab. 2.2. They concern shaft sway, rigid-body motion and platform vibration. A major distinction is to be made whether or not a springboard for figure diving is mounted on the platform. The rhythmic skipping on the springboard contributes much to the excitation of the platform, so that stricter frequency bounds apply.

frequency bounds	without springboard	with springboard
shaft vibrations (all fundamental modes in nodding, lateral sway, twist)	$f_1 \geqslant 3.5$ Hz	$f_1 \geqslant 5$ Hz
rigid-body vibration (flexibility of foundation)	$f_1 \geqslant 7$ Hz	$f_1 \geqslant 10$ Hz
slab vibration	$f_1 \geqslant 10$ Hz	$f_1 \geqslant 10$ Hz

Tab. 2.2 Frequency criteria for high-diving platforms (from [2.29])

Stiffness Criteria

The spatial vector displacement of the front edge of the platform (Fig. 2.15) must remain under static loading (with or without springboard)

$$d\,(F_x, F_y, F_z) < 1\,\text{mm} \tag{2.12}$$

and the lateral front displacement alone

$$d_x = 0.5 \cdot d \quad < 0.5\,\text{mm} \tag{2.13}$$

for a set of forces of $2 \cdot F_x = F_y = F_z = 1$ kN.

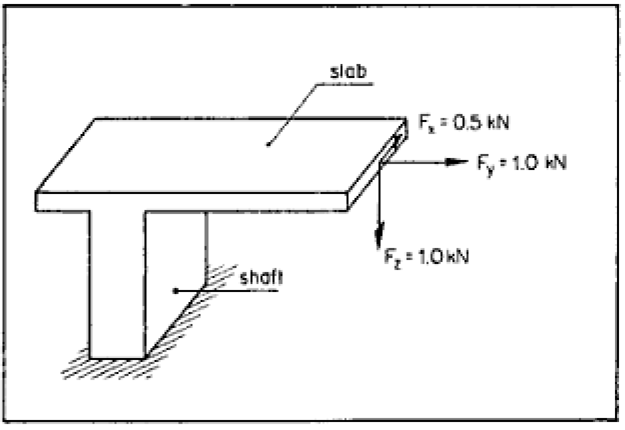


Fig. 2.15
Multiaxial static loading to check stiffness of diving platforms (after [2.29])

The stiffness criteria are relatively severe. According to practical experience, however, reinforced concrete platforms can be assumed to maintain their uncracked stiffness in bending as well as torsion. The listed bounds were derived for high-performance platforms suited to high and figure diving alike. For less professional demands in recreational indoor or outdoor swimming facilities, these bounds could well be lowered (see Cases No. 12, 13 and 14).

3 MACHINE-INDUCED VIBRATIONS

3.1 General Aspects

Machine-induced vibrations of buildings and their structural members have an increasing practical importance due to several reasons. A contribution from the structural side is the continuing trend to higher exploitation of material strength and larger spans; another part of the problem is the switch to larger machinery or one operated at higher speed. The observed vibrations may possibly impede or even prevent the opening of new production facilities. In most cases not the structural integrity and the overall safety are at risk, but either the personnel operating these machines is continuously or temporarily irritated, or production problems arise such as substandard goods due to unforeseen motion of the machinery. Machine-induced vibrations are thus – similar to man-induced vibrations – mainly a problem of serviceability.

The following chapter characterizes in its first part the dynamic loading exerted from various types of machinery. The second part deals with possible effects on buildings or structural members and with suitable countermeasures. The emphasis is thereby on small and medium size machinery which is typically installed on floors of buildings and liable to cause vibration problems there.

3.2 Dynamic Loads of Various Types of Machinery

Depending on its manufacturing purpose, state of maintenance and design details, a machine causes distinct loads on the structural member it rests on. The loads depend primarily on the type of motion the machine parts describe: whether it is rotating, oscillating, or of an impacting type. The following descripton of the dynamic loads is hence grouped into these three categories.

The time function of the exerted loading is periodic in general, sometimes even harmonic (see Section 1.1, Figs. 1.1 and 1.2). In any case can a periodic load be decomposed by means of a Fourier analysis into a number of harmonic components. The relevant theory is explained in Appendix B 4.

3.2.1 Machines with Rotating Parts

Dynamic loads may arise from rotating parts of machinery if they are insufficiently balanced or if electrodynamic fields are active.

Loads due to rotating parts may exhibit either of two time functions:

- quadratic excitation
- constant-load excitation.

Quadratic excitation is always related to out-of-balance forces and hence frequency dependent. It arises whenever the centre of mass of a rotating part does not coincide with the axis of rotation. The product of mass and eccentricity is called unbalance.

Out-of-balance forces may originate from:

- excessive tolerance in manufacturing the rotating parts
- insufficient flexural rigidity of the axle
- accidental influence on the operation (dust collecting on blower blades, etc.)
- intentionally eccentric masses (as in vibrators)
- bearing clearance (due to wear)
- passage through a critical speed
- accidental unbalance (e.g. failure of a turbine blade).

Machines with predominantly rotating parts are for instance:

- blowers and ventilators
- centrifugal separators
- vibrators
- washing machines
- lathes
- centrifugal pumps
- rotary presses
- plastics-moulding presses
- turbines
- generators.

The excitation due to out-of-balance forces is quadratic since the amplitude of the forces increases with the square of the excitation frequency. If a single rotating part is out of balance, it exerts a harmonic load in any direction in the plane perpendicular to the axis of rotation. The amplitude of this load is:

$$F_\omega = m' \cdot e \cdot \frac{4 \cdot \pi^2}{3600} \cdot n_b^2 = m' \cdot e \cdot 4 \cdot \pi^2 \cdot f_b^2 = m' \cdot e \cdot \omega^2 \tag{3.1}$$

where:

F_ω = centrifugal force
m' = mass of the rotating part (unbalanced fraction)
e = eccentricity of the unbalanced mass
n_b = speed of revolution of the part [r.p.m.]

f_b = excitation frequency ($f_b = n_b/60$)
ω = angular velocity of the rotating part.

In an arbitrary direction the harmonic load may thus be defined as

$$F(t) = F_\omega \cdot \sin(\omega \cdot t) = m' \cdot e \cdot \omega^2 \cdot \sin(\omega \cdot t) \tag{3.2}$$

which acts on the total mass m of the machine (incl. the mass m′ of the rotating part).

Taking a damped oscillator with a single degree of freedom (SDOF) as a rather simplified model, this dynamic load produces the following displacement of the total mass with time (see also Appendix B 1):

$$x(t) = \frac{m' \cdot e \cdot \omega^2}{k} \cdot C \cdot \sin(\omega \cdot t + \varphi) \tag{3.3}$$

where:

$$C = \frac{1}{\sqrt{(1-(\omega/\omega_1)^2)^2 + (2 \cdot \xi \cdot \omega/\omega_1)^2}}$$

$$\varphi = \arctan \frac{2 \cdot \xi \cdot \omega \cdot \omega_1}{\omega_1^2 - \omega^2} = \text{phase angle}$$

with:

x = displacement of the total mass m
ω_1 = circular eigenfrequency of the SDOF oscillator
k = stiffness of the SDOF oscillator
ξ = damping ratio (with respect to critical damping).

The displacement amplitude amounts to

$$x_{max} = \frac{m' \cdot e \cdot \omega^2}{k} \cdot \frac{1}{\sqrt{(1-(\omega/\omega_1)^2)^2 + (2 \cdot \xi \cdot \omega/\omega_1)^2}} \tag{3.4}$$

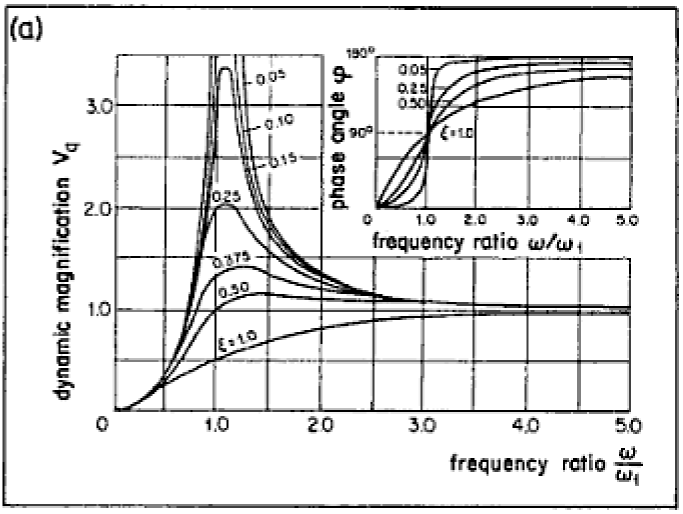
or rewritten to a dimensionless dynamic magnification factor on the displacements:

$$V_q = \frac{x_{max} \cdot m}{m' \cdot e} = \frac{(\omega/\omega_1)^2}{\sqrt{(1-(\omega/\omega_1)^2)^2 + (2 \cdot \xi \cdot \omega/\omega_1)^2}} \tag{3.5}$$

with:

$m = k/\omega_1^2$ = total mass.

Figure 3.1a shows the influence of the damping ratio on the magnification factor V_q and the accompanying lag in the phase angle between loading and displacement function.



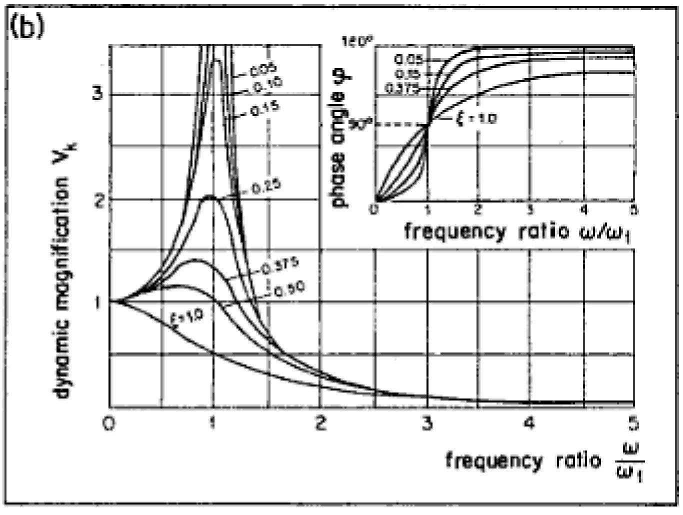


Fig. 3.1
Magnification factor and shift in phase angle for various damping ratios:
(a) V_q for quadratic excitation
(b) V_k for constant-load excitation

Engineering practice has the notion of an admissible unbalance. This figure depends on the type of machinery and its speed of revolution and is either provided by the manufacturer or can possibly be taken from guidelines (e.g. [3.1]). Data on admissible excentricities can also be found in [3.2].

If more rotating parts with individual unbalances are mounted on the same shaft, they rotate with identical speed but different phase angle, and hence they also produce a resultant harmonic load.

If several unbalanced parts rotate with different speeds, the superposition of their individual harmonic loads will, in general, not result in a strictly periodic load.

Not only unbalances but also alternating electromagnetic fields can give rise to periodic loads with considerable participation of higher and sometimes lower harmonics with respect to the operating frequency.

The forcing function of *constant-load excitation* is of the form

$$F(t) = F_o \cdot \sin(\omega \cdot t) \tag{3.6}$$

with:

F_o = amplitude of the force (constant).

The amplitude F_o remaining constant, $F(t)$ varies with $\sin(\omega \cdot t)$. If the circular frequency ω of the rotating part is constant, the quadratic excitation (3.2) degenerates to a constant-load excitation (3.6).

Taking again a damped SDOF oscillator as a rather simplified model of the structure or a member thereof, a constant-load excitation produces the following displacement of the mass with time (see Appendix B 1):

$$x(t) = \frac{F_o}{k} \cdot C \cdot \sin(\omega \cdot t + \varphi) \tag{3.7}$$

with C and φ as in Eq. (3.3).

The displacement amplitude amounts to

$$x_{max} = \frac{F_o}{k} \cdot \frac{1}{\sqrt{(1-(\omega/\omega_1)^2)^2 + (2 \cdot \xi \cdot \omega/\omega_1)^2}} \tag{3.8}$$

In relation to the static displacement, one obtains the dimensionless magnification factor

$$V_k = \frac{x_{max}}{x_{stat}} = \frac{x_{max}}{F_o/k} = \frac{1}{\sqrt{(1-(\omega/\omega_1)^2)^2 + (2 \cdot \xi \cdot \omega/\omega_1)^2}} \tag{3.9}$$

Figure 3.1b gives the magnification factor and the phase angle for various damping ratios under constant-load excitation. Note that the phase angle is the same for constant-load as for quadratic excitation.

3.2.2 Machines with Oscillating Parts

Oscillating parts of machines always excite dynamic loads. The causative motion can be translational, rotational with small angle, or a combination of both (pendular motion).

Machines with predominantly oscillating parts are for instance

- weaving machines
- piston engines
- reciprocating compressors
- reciprocating pumps
- emergency power generators (diesel engines)
- flat-bed printing presses
- frame saws
- crushing machines
- screening machines
- tool machines
- bells.

Although all reciprocating machines or engines exert primarily an oscillating force in the direction of the piston motion, they also give rise to a rotational load component due to the excentric hinging of the connecting rod to the crankshaft. Either component is of the quadratic excitation type. It is a question, however, of the number of pistons and of their suitable arrangement with respect to each other to minimize the resulting load (see e.g. [3.3]). Higher harmonics of the operating frequency sometimes become important.

Typical load-time functions from machines with oscillating parts are shown in Figs. 3.2a and 3.3. As for weaving, Fig. 3.2a gives for various types of machines the time function of the vertical load transmitted at the footings and Fig. 3.2b the pertinent spectra of Fourier amplitudes. It becomes apparent that only for air-jet machines and shuttle looms do the maxima of the transmitted loads coincide with the operating frequency (weft insertion rate). With all other types the maximum occurs in a higher harmonic frequency, with projectile machines, for instance, in the 4th harmonic (see Appendix B 4). This effect mainly reflects the mode of operation of the various types. Without sley stop the weft insertion rate governs the load-time function (Fig. 3.2a) and the Fourier amplitude spectrum, whereas with sley stop this influence is clearly superseded by higher harmonics (see [3.4]).

The static and dynamic footing loads of the four types of weaving machines in Fig. 3.2 are compiled in Tab. 3.1. Apart from the vertical, considerable dynamic load components result in warp thread direction (x-component), the ones in weft thread direction (y-component) being much smaller. As shown by Natke [3.4], the spectrum of a machine group is characterized by the individual spectrum of the respective type, i. e. no frequency shift results, for instance, from varying the number of machines on a common base slab.

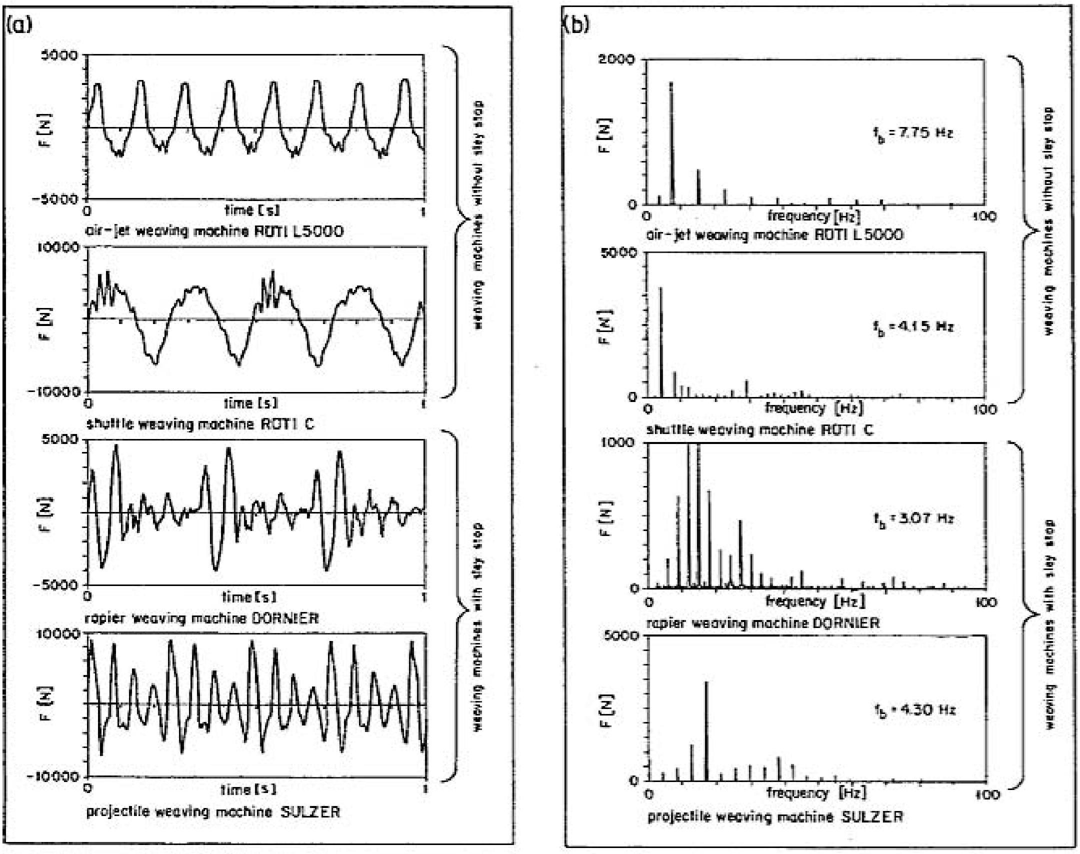
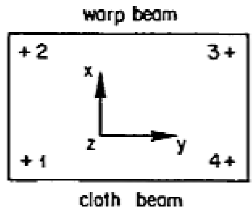


Fig. 3.2 Machines with predominantly oscillating parts, here the vertical loading from various types of weaving machines (from [3.4]):
(a) load-time function, (b) derived Fourier spectrum

Another example of a load due to an oscillating mass is the one exerted in horizontal direction by a bell. Figure 3.3 gives a load-time function and its Fourier amplitude spectrum.

In contrast to the load-time functions of most weaving machines, bells always exert the largest load component in the fundamental harmonic, i.e. in their swinging frequency of half the stroke rate. As the 2nd and 4th harmonics are vertical, the next structurally important harmonic of only slightly smaller amplitude is the 3rd. Load components of the 5th or higher harmonics are mostly negligible for bells with a swinging angle of 70 to 90°. Resonance phenomena may thus be encountered for bells with 60 to 65 strokes per minute, equivalent to a swinging frequency of about 0.5 Hz. In that case, the 3rd harmonic of the bell load ($f \cong 3 \cdot 0.5$ Hz = 1.5 Hz) may coincide with the fundamental frequency of the bell tower, which is most often one of rotational bending (due to elastic encastrement in the ground).



weaving machine Rüti shuttle	weft insertion 249/min	weft width 160 cm			range of spectrum [Hz]
foot no.	static loads [kN] F_z	dynamic loads [kN] F_z	F_x	F_y	
1	4.13	7.58	3.33	1.52	3–16
2	6.14	6.77	3.26	1.10	
3	8.26	6.26	3.46	1.07	
4	5.27	7.91	3.27	1.78	
total	23.80				

weaving machine Dornier rapier	weft insertion 184 / min	weft width 190 cm			range of spectrum [Hz]
foot no.	static loads [kN] F_z	dynamic loads [kN] F_z	F_x	F_y	
1	6.27	6.74	2.25	0.83	3-60
2	10.81	5.78	1.22	0.83	
3	6.07	6.18	2.12	0.70	
4	9.03	8.65	6.20	0.72	
total	32.18				

weaving machine Sulzer projectile	weft insertion 258/min	weft width 85°			range of spectrum [Hz]
foot no.	static loads [kN] F_z	dynamic loads [kN] F_z	F_x	F_y	
1	(n. a.)	6.81	4.45	0.75	3-40
2		9.58	5.52	0.56	
3		9.35	4.45	0.60	
4		6.23	5.17	1.01	

weaving machine Sulzer projectile	weft insertion 209/min	weft width 85°			range of spectrum [Hz]
foot no.	static loads [kN] F_z	dynamic loads [kN] F_z	F_x	F_y	
1	(n. a.)	4.55	2.94	0.41	3-40
2		6.26	3.57	0.39	
3		6.14	2.97	0.44	
4		4.94	3.96	0.74	

weaving machine Rüti air-jet	weft insertion 465/min	weft width 150 cm			range of spectrum [Hz]
foot no.	static loads [kN] F_z	dynamic loads [kN] F_z	F_x	F_y	
1	3.45	3.31	2.22	0.33	5-25
2	5.75	3.18	1.00	0.22	
3	10.35	2.66	2.04	0.47	
4	1.73	2.62	1.08	0.52	
total	21.28				

Tab. 3.1
Compilation of static and dynamic footing loads exerted by the weaving machine types of Fig. 3.2 (from [3.4])

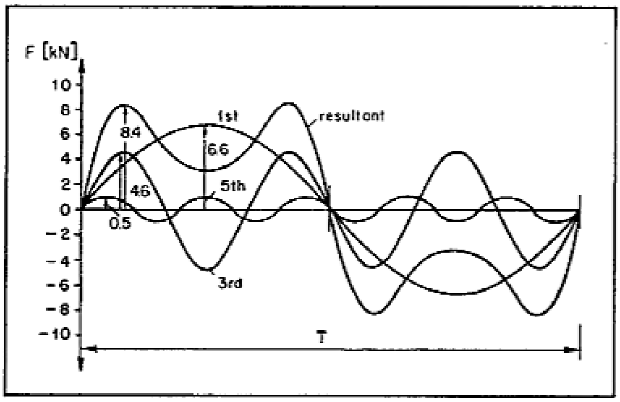


Fig. 3.3
Load-time function caused by bell ringing: harmonics 1, 3 and 5, and their superposition (from [3.5])

Bell assemblies with larger swinging angles (up to 360° for the English bell type) may well exhibit load components up to the 20th harmonic.

The two examples of weaving machines and bells sufficiently demonstrate that both the time function of the load and the pertinent Fourier spectrum depend strongly on the source. Additional and more detailed data on the load-time functions due to oscillating parts can be found, for instance, in [3.2], [3.5], [3.6], [3.7.], [3.8], [3.9], [3.10] and [4.24].

3.2.3 Machines with Impacting Parts

Machines with impacting parts often develop large intermittent dynamic forces. Skilful design, however, will attempt to balance (e.g. with a counterblow hammer) the major part of the force within the machine frame. This reduces the residual forces on the structure very much.

Machines with impacting parts are for instance

- moulding presses
- punching machines
- power hammers
- forging hammers.

While power and forging hammers exert a truly intermittent impact load, systems like moulding presses or punching machines have at least some oscillating parts. In contrast to true impact loading, these mixed dynamic loads of the second type of machine exhibit a periodic peak followed by a more or less free vibration decay of the impacted body. As the operating frequencies of these machines can be rather high, complete vibration decay between impacts may not always be possible.

For power and forging hammers, the load-time function is in most cases transient, i. e. rarely periodic. The exact shape of the function depends on the operating mode of the machine, but also on the moulding properties of the processed material. One can distinguish the loading phase (duration of impact) from the following, sometimes much longer, phase without load. In general, the load-free phase of vibration decay is not of constant duration, hence the nature of the loads is transient (single impulses). There are some machines on the market, though, which do have a constant load-free phase, resulting then in a periodic type of loading.

The loading can be characterized by the following parameters of the impact phase (Fig. 3.4):

- duration of impact (t_p)
- momentum (I)
- rise time of the load (t_a)
- peak force ($F_{p,max}$).

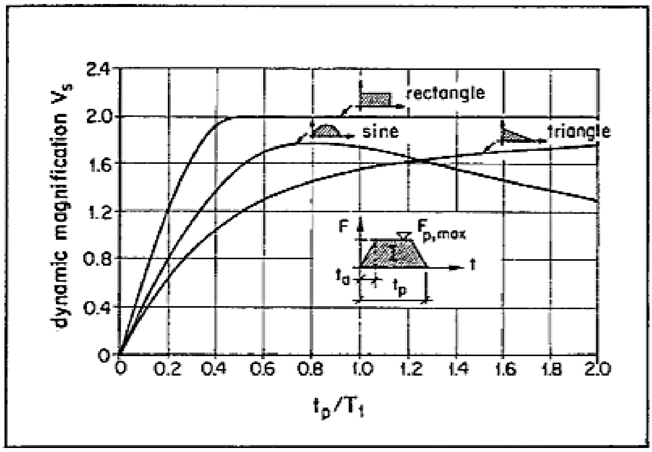


Fig. 3.4
Dynamic magnification for various loading functions as depending on the ratio between load duration and structural frequency

The combination of these parameters determines the shape of the load-time function, for instance semi-sinusoidal, triangular or even quite rectangular. In most cases just the maximum of a dynamic quantity resulting from the impact is of primary interest, e. g. the peak displacement x_{max} of the centre of mass of the affected structural member. Represented by an SDOF oscillator, Fig. 3.4 shows for various shapes of load-time functions the dynamic magnification factor V_s versus the ratio t_p/T_1 (duration of impact to the natural period of the oscillator). The dynamic magnification factor on the displacement is defined as in Eq. (3.9):

$$V_s = \frac{x_{max}}{F_{p,max}/k} \qquad (3.10)$$

where:

$F_{p,max}$ = peak force
k = stiffness of the impacted structural member
$F_{p,max}/k$ = static displacement under the peak force.

Figure 3.4 leads to the following conclusions:

(1) In the range $t_p/T_1 > 1$, the magnification factor V_s is dominated by the rise time. Instantaneously applied loads induce the highest possible value $V_s = 2.0$, whereas gradually applied loads lead to a lower bound on the magnification of $V_s = 1.0$.

(2) In the range $t_p/T_1 < 0.25$, the magnification V_s depends strongly on the shape of the load-time function, but the peak displacement is largely determined through the applied momentum. The relationship can be approximated to

$$x(\bar{t}) = \frac{1}{m \cdot \omega_1} \cdot I \cdot \sin(\omega_1 \cdot \bar{t}) \tag{3.11}$$

where:

m = mass of the SDOF oscillator (structural member)
ω_1 = circular eigenfrequency of the oscillator
$I = \int_0^{t_p} F(t) \cdot dt$ = applied momentum
$\bar{t} = t - t_p$

and for the peak displacement

$$x_{max} = \frac{1}{m \cdot \omega_1} \cdot I \tag{3.12}$$

Because of the typically very short impact duration t_p, considerable load components contribute over a wide frequency band. Structural damping does not have much effect under these loading conditions.

More details on the load-time functions can be found e.g. in [3.7].

3.3 Effects of Machine-Induced Vibrations

3.3.1 Inertial Vibrations

The effects of the vibrations do not depend qualitatively on whether they are due to rotational, oscillatory, or impacting type of motion; in this respect, no distinction is made in the following.

As mentioned in Section 1.2 the effects of structural vibrations can show a wide variety, the basic distinction being drawn between effects on structures or structural members on the one hand and those on people, installations, machinery and their products on the other. The severity of the effects depends primarily on the frequency and the amplitude of the vibration. Acceptance criteria for these have been compiled in Chapter 4; they mark thresholds of the vibrations being possibly detrimental to the structure, disturbing or even harmful to people, etc.

Effects of machine-induced vibrations on structures may include:

- appearance of cracking, crumbling plaster, loosening of screws, etc.
- problems of fatigue of steel girders or steel reinforcement with consequent damage, ultimately to the extent of collapse
- loss of the load-bearing capacity (in rare cases of overstressing).

People working temporarily or permanently near to the vibration emitting machine or the co-vibrating structural member are concerned to various degrees. The intensity may range from barely noticeable over slightly and severely disturbing to harmful. They can occur in three different ways:

- as mechanical effects (e.g. vibrations of floor or ceiling, see Cases No. 20 and 21)
- as acoustic effects (e.g. noise from reverberating installations and pieces of equipment, also structure-borne or air-borne sound, see Case No. 22)
- optical effects (e.g. visible motion of building elements, installations or objects).

Effects on machinery and installations can induce:

- problems of material behaviour in the machine itself (deformations, strength, fatigue)
- problems of material technology in the manufactured goods (e.g. excessive tolerance due to unplanned motion of tools and installations, see Cases No. 15, 17, 18 and 20).

The amplitude of structural vibrations due to dynamic loads from machinery depends strongly on the ratio of the natural frequencies of the structure to the dominant harmonics of the machinery loads. It may well happen that the largest vibrations do neither occur at the fundamental structural frequency nor at the operating frequency of the machine, but at a higher harmonic common to both the structural and the loading frequency spectra. For instance, the 3rd harmonic of a structural member may resonate with the 5th harmonic of the load from a weaving machine (depending on the type of machine).

It should be pointed out that, despite the distinct vertical direction in which the major part of the dynamic load is exerted on the structure, other spatial components of the load cannot be neglected without closer inspection. In particular, rather flexible structures may also undergo severe horizontal vibrations under horizontal dynamic load components.

3.3.2 Structure- and Air-Borne Acoustic Waves

Vibrations of machinery and its neighbourhood are also apt to induce acoustic waves in the structure or in air [3.11]. Air-borne acoustic waves are emitted from the machine (mostly from those running at higher speed) in the form of direct sound, which usually is sensed by the human ear as more or less disturbing noise (see Case No. 22).

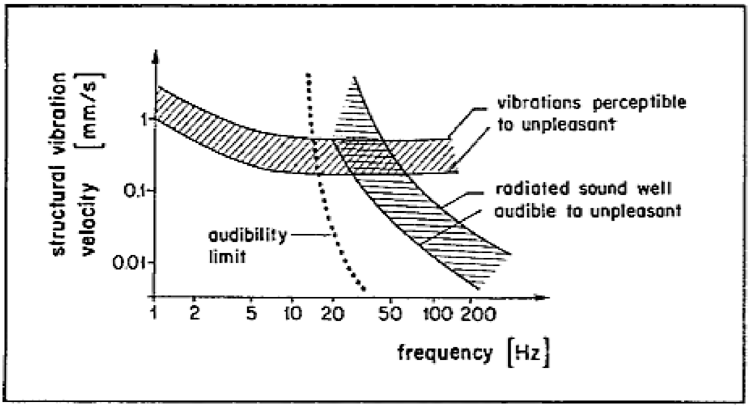


Fig. 3.5
Critical frequency domains for the structure's inertial vibration and the radiated, structure-borne sound (from [3.12])

On the other hand, vibrations may travel long distances through various transmitting media, such as columns, walls, foundations, piping, etc., in the form of structure-borne acoustic waves. They in turn can be radiated from the wall and ceiling surfaces of a room with an enclosed volume of air as indirect sound (called also secondary or radiated sound) and sensed by the ear as noise. This kind of indirect sound is often felt as particularly unpleasant because its source is mostly not locatable. Another and rather critical factor is the frequency content of the noise. The range of audibility is limited to about 20 Hz as lowest frequency. As Fig. 3.5 illustrates, the audibility increases with higher frequency, that is a lower sound level suffices for the same degree of perception. It also shows that in the range beyond 50 Hz the sound radiated from the structure becomes the dominating criterion for the disturbance of people, superseding the direct effect of vibration of the machine and its neighbourhood.

Moreover, acoustic vibrations may also impair the structural members and installations through which they are transmitted. Even though the resulting alternating stresses are in most cases of small amplitude, their very high number of cycles may still present a certain danger in terms of «acoustic fatigue».

3.4 Measures Against Machine-Induced Vibrations

Vibrations due to rotational and oscillatory motions are more or less similar in cause, effects, and hence also in countermeasures; thus they are treated together. Fundamentally different are alone vibrations due to intermittent motion of machinery with impacting parts, which require special measures.

As for man-induced vibrations (see Chapter 2), separating the frequencies by tuning is the most effective measure (see Appendix B 5). Some cases may need the calculation of a forced vibration or warrant special measures. However, priority should always be given to suppressing vibrations at their source, for instance by keeping out-of-balance forces to a minimum, reducing impact, etc.

Frequency tuning between machine and structure or structural members is commonly referred to as vibration isolation. In the present context of preventing load vibrations from being transmitted to neighbouring zones, one speaks of active isolation. The opposite action of preventing external vibrations from reaching an object is termed accordingly passive isolation.

3.4.1 Machines with Rotating and Oscillating Parts

Adjusting the fundamental frequency so as not to coincide with the operating frequency nor with a higher harmonic of the load-time function is effective in most cases of rotary or oscillatory motion (see Cases No. 15, 21 and 22). Different measures should be chosen according to the respective type of machine, characterized in the following by its operating frequency. Of the three categories given below, a certain overlap between the first two cannot be avoided.

Group 1:
Low to medium operating frequency ($n_b = 1 \div 600$ r.p.m., $f_b = 0.02 \div 10$ Hz); examples are reciprocating pumps and compressors, weaving machines, rotary presses, etc. Machines with low operating frequency ($f_b < 1$ Hz) can be expected to produce only small dynamic loads, the exception being bells (see Section 3.2.2).

Group 2:
Medium to high operating frequency ($n_b = 300 \div 900$ r.p.m., $f_b = 5 \div 15$ Hz); e.g. large diesel engines, blowers, certain weaving machines, etc.

Group 3:
High operating frequency ($n_b > 1000$ r.p.m., $f_b > 15 \div 20$ Hz); e.g. turbines, small diesel engines, centrifugal separators, vibrators, etc.

(a) Frequency Tuning

Machines with predominantly rotational or oscillatory motion allow, in general, low tuning as well as high tuning. Since the objective is to completely avoid any state of resonance, the following parameters become important:

– the dominant natural frequencies of the structure as a whole and of the member immediately under the machine, taking into account existing spring(-damper) elements or a base slab (called in the following the machine base, see Fig. 3.6)

– the frequencies of the dominant dynamic load components (operating frequency of the machine and relevant higher harmonics).

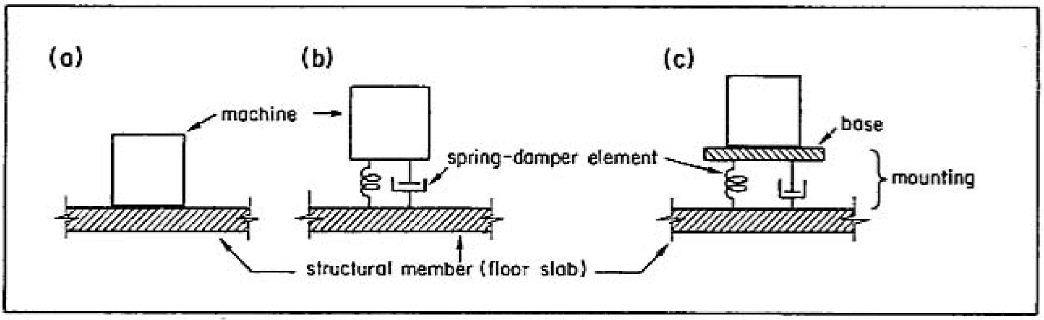


Fig. 3.6 Low-tuning of machinery with rotational or oscillatory motion:
(a) direct mounting on a structural member
(b) support on spring(-damper) elements
(c) mounting on a vibration-isolated base

If the fundamental frequency f_1 of the isolated system falls clearly below the operating frequency f_b of the machine, *low tuning* materializes. For effective *high tuning*, the fundamental frequency f_1 of the isolated system must be established well above the highest frequency component with an appreciable contribution to the dynamic loading (higher harmonics of the operating frequency). A short excursion into the theory of low and high tuning will elucidate the difference (see also Appendix B 5).

Theoretical Background

The load a machine with quadratic or constant-load excitation exerts on its base can be visualized as composed of a spring force and a damping force. The maximum reaction force at the base becomes for *quadratic excitation:*

$$R_{max} = m' \cdot e \cdot \omega^2 \cdot V_k \cdot \sqrt{1 + (2 \cdot \xi \cdot \omega/\omega_1)^2} \tag{3.13}$$

for *constant-load excitation:*

$$R_{max} = F_o \cdot V_k \cdot \sqrt{1 + (2 \cdot \xi \cdot \omega/\omega_1)^2} \tag{3.14}$$

whereby V_k is in both equations the dynamic magnification factor of Eq. (3.9). The frequency ratio ω/ω_1 (or f/f_1) in this context is called the tuning ratio (or sometimes the tuning factor).

The ratio of the maximum reaction force to the maximum load coming from the machine gives the so-called transmissibility V_R with

$$V_R = \frac{R_{max}}{F_o} = \frac{R_{max}}{m' \cdot e \cdot \omega^2} = V_k \cdot \sqrt{1 + (2 \cdot \xi \cdot \omega/\omega_1)^2} \tag{3.15}$$

This transmissibility V_R is plotted in Fig. 3.7 for various damping ratios. Note that the influence of damping is advantageous only up to the tuning ratio $\omega/\omega_1 \leq \sqrt{2}$, whereas damping reduces the isolation effect for tuning ratios higher than that. Besides, the transmissibility is also the same for passive isolation, so that Fig. 3.7 can be used for both kinds of isolation alike.

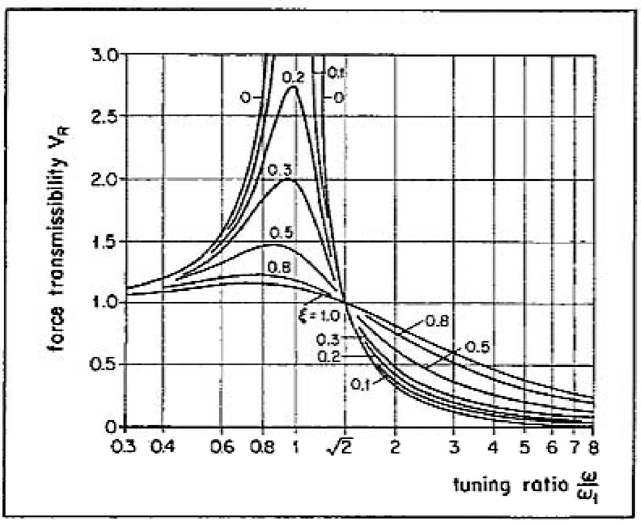


Fig. 3.7
Transmissibility for various damping ratios as depending on the tuning ratio between operating and structural frequency

Instead of the transmissibility V_R one may prefer to work with the complementary part, which is the isolation effectiveness:

$$1-V_R = 1 - \frac{\sqrt{1+(2\cdot\xi\cdot\omega/\omega_1)^2}}{\sqrt{(1-(\omega/\omega_1)^2)^2+(2\cdot\xi\cdot\omega/\omega_1)^2}} \tag{3.16}$$

Under the assumption of negligibly small damping one obtains for $\omega/\omega_1 > \sqrt{2}$:

$$1-V_R = 1 + \frac{1}{1-(\omega/\omega_1)^2} = \frac{(\omega/\omega_1)^2-2}{(\omega/\omega_1)^2-1} \tag{3.17}$$

The eigenfrequency $\omega_1 = \sqrt{k/m}$ of the isolated system can be related through $m = G/g$ to the combined weight of machine and base and thus to the static deflection produced in the spring(-damper) elements [3.13]:

$$f = \frac{1}{2\cdot\pi}\cdot\sqrt{\frac{g}{x_{stat}}\cdot\frac{2-(1-V_R)}{1-(1-V_R)}} \tag{3.18}$$

where:

f = frequency of the dynamic loading
g = acceleration of gravity (9.81 m/s²)
x_{stat} = static displacement under the weight of machine and base.

The evaluation of (3.18) in Fig. 3.8 allows to determine from the known excitation frequency (f) the support displacement (x_{stat}) necessary to achieve any desired level of isolation effectiveness ($1-V_R$), assuming that the isolators have little damping.

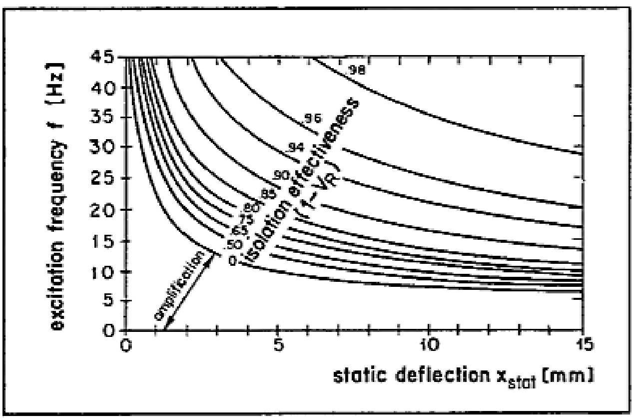


Fig. 3.8
Effectiveness of vibration isolation depending on the static displacement due to machine weight and operating frequency

Low Tuning

On the one hand, low tuning reduces the reaction forces substantially (see the trend of the transmissibility function in Fig. 3.7), while on the other the resulting soft support yields large displacements already under static load. Certain conditions must be fulfilled for low tuning to be effective:

(1) The machine should be operated at a speed of more than 4 to 6 Hz.
(2) Higher structural eigenfrequencies must not coincide with the dominating harmonic of the dynamic load.
(3) The soft base of the machine must not cause manufacturing problems.
(4) On passing the fundamental and higher natural frequencies of the base system during start-up and shut-down of the machine, no intolerably large vibrations may occur.

Condition (1) reflects the constructional limitation on the degree of softening of the base (Fig. 3.6) that can be achieved. The limit is often reached which a stiffness resulting in an isolation-system frequency of ca. 1.5 to 3 Hz; this necessitates a minimum operating frequency of ca. 4 to 6 Hz. The idea is to achieve a tuning ratio of about 2 (see Appendix B 5). However, a tuning ratio between 2.5 and 4 is to be preferred since a larger ratio yields smaller reaction forces.

Condition (2) prevents resonance phenomena in the higher frequencies of the base.

Condition (3) results from production requirements, as a softer support yields larger displacements of the machine.

Condition (4) can often be ensured by precise instructions on start-up and shut-down of the machinery, possibly in combination with braking devices in the slowing-down phase or enhanced damping of the base.

In some cases of employing a reduced tuning ratio (owing to extremely soft spring (-damper) elements, a large base mass, etc.), the recommended operating frequency need not be adhered to if condition (3) is observed with particular care.

Of the three machine frequency groups mentioned at the beginning of the chapter, the third, i. e. machines with high operating frequencies, is with respect to low tuning the most suited. Accepting a tuning ratio near the lower limit ($2.0 \div 2.5$) opens the possibility of low tuning also to the second group of medium to high-frequency machines.

If applicable, low tuning can be put into practice in three possible ways (Fig. 3.6):

- *Direct mounting of the machine on the structural member (Fig. 3.6a)*

  As a structure or its structural members cannot be softened at will, this type of mounting is appropriate only for high-speed machines (group 3). If the structure or structural members were to be constructed with a softer response ($2 \div 3$ Hz), the ensuing displacement amplitudes might then become so large as to critically affect operation by the personnel and the functioning of the machine itself.

- *Mounting of the machine on spring(-damper) elements (Fig. 3.6b)*

For this variant the frame of the machine needs to be designed sufficiently stiff in order to equalize the differences in motion of the individual support points. In addition, the basic vibration behaviour of the machine must not be substantially altered (e.g. introducing new torsional vibrations). Often the machine manufacturer himself will design for this solution, because it has the advantage that the spring(-damper) elements could easily be modified or exchanged should the isolation prove ineffective.

The spring(-damper) elements differ with the machine characteristics (its dimensions, mass, genuine stiffness, operating frequency, etc.). The different types on the market comprise steel springs, elastomeric or rubber pads, and air suspension.

Steel springs can be used in the frequency range between 1 and 8 Hz. One distinguishes between helical, cup, and laminated springs. The latter two types are in principle one-dimensional elements, whereas helical springs may be used in spatial isolation and are hence often preferred for relatively low-frequency excitation due to unbalanced rotating masses. All springs are attached either on one side to the machine frame alone (with guard against self-movements) or on both sides to machine and building structure.

Elastomeric and rubber pads have a range of applicability from about 5 to 20 Hz. Mounting can be effected in three ways:

- floating support:
  The machine is continuously bedded on elastomeric or rubber mats. (This type of support also uses cork or felt above frequencies of at minimum 10 Hz with cork and 20 Hz with felt.)
- one-sided fixation at the machine:
  The spring(-damper) elements are attached locally to the machine allowing for construction tolerances and unevenness of the base.
- two-sided fixation at both machine and floor:
  Attaching the spring(-damper) elements on both sides still allows adjustment in height, but is additionally suitable for resisting some limited horizontal load components.

Air suspension is employed for frequencies from 0.5 to 5 Hz. This is the range of very low frequencies in which steel springs would yield too large displacements.

- *Mounting on a vibration-isolated base (Fig. 3.6c)*

This measure is taken into consideration for machines with

- very large dynamic loads
- insufficient stiffness for isolation according to Fig. 3.6b
- very small mass which would demand extremely soft springs for low tuning
- very eccentric centre of mass.

Due to the additional mass of the base one can employ stiffer spring(-damper) elements than one could in direct support, thereby reducing the vibration amplitudes. The stabilizing mass should have at least the same weight as the mass of the machine alone.

Mounting on a machine base is very advantageous for isolation but is relatively costly. It may also interfere with the production process (impeded access, problematic level difference for supply of raw materials and removal of end products, etc., even when several manufacturing machines are mounted on the same base). Sometimes, however, an over-thick floor slab or foundation enables the stabilizing mass to be accommodated in a deep cavity thus ensuring free access. Consideration must be given to a sufficient stiffness of the base itself.

High Tuning

In general, high tuning is only feasible with a rigid connection to the structure and is chosen especially in those cases in which low tuning is impractical. The requirements for high tuning are:

(1) The frequency of the highest relevant load component does not exceed about 20 Hz.
(2) Production demands that machine vibrations be kept small.

The first requirement results from the fact that a structure or structural member cannot be designed for high stiffness at will within a given expenditure. Since the computational prediction of the structural frequency contains some uncertainty, the tuning ratio is augmented by an additional safety factor. The lower bound to the fundamental frequency of the structure or the respective structural member thus becomes

$$f_1 \geqslant s_f \cdot n_{max} \cdot f_b \cdot \omega_1/\omega \tag{3.19}$$

with:

f_1 = required fundamental structural frequency
f_b = operating frequency of the machine (r.p.m./60)
n_{max} = order of the highest relevant harmonic of the loading
ω_1/ω = reciprocal of the tuning ratio (see Appendix B 5)
s_f = safety factor (usually taken as 1.1 to 1.2).

For calculating the expected frequency of stiffly designed structures one usually cannot neglect the flexibility of the ground, which may under some circumstances substantially reduce the combined frequency of the soil-structure system [3.4], [3.5].

Figure 3.7 demonstrates that the forces under high tuning always have a factor $V_R > 1$ with respect to rigid base (equivalent to the static case). High tuning is the preferred measure for all machines of the first (lowest) frequency group (see Case No. 15).

(b) Calculation of a Forced Vibration

If the success of frequency tuning is somewhat doubtful (e.g. because of a wide frequency spectrum of the dynamic loading, etc.), the calculation of the structural behaviour under forced vibration is to be recommended. Such a calculation yields information on the maximum expected vibration amplitudes, which then could be compared with the acceptance criteria given in Chapter 4. However, such a calculation is itself subject to considerable uncertainty, especially as far as the actual dynamic loads from the machine and realistic damping properties are concerned. The procedure for such a computation is outlined in the Appendices B 1, B 4 and B 5. Appendix B 8 suggests appropriate damping coefficients.

(c) Special Measures

A further option for the reduction of vibration is – just as for man-induced vibrations – the provision of tuned vibration absorbers. They are chosen mainly for the reduction of vibrations in existing structures with little damping or relatively small mass, when the dynamic loads cannot be reduced and when alternative measures (such as stiffening the structure) would lead to unreasonable costs. One should realize, however, that any tuned vibration absorber is a mechanical system vibrating at a considerable amplitude and that it needs proper maintenance. The theoretical basis and the various construction types of absorbers are described in Appendix B 7.

3.4.2 Machines with Impacting Parts

The following countermeasures apply to machinery with an intermittent kind of motion, giving rise to either periodic or single impulses (transient load, see Appendix B 6). The solution of a practical problem is largely determined by the load-time function (see Cases No. 16, 17, 19 and 20). As discussed previously in Section 3.2.3, the important parameters are the duration of the impulse, the momentum and the peak force from the loading.

(a) Frequency Tuning

Vibrations of machines with intermittent motion are best counteracted by low tuning, i.e. by providing low stiffness of supports. High tuning, in contrast, is most likely not an alternative as the Fourier spectrum of the loading is typically rather wide, i.e. relevant load components are exerted over a wide frequency band.

Low tuning is applicable under the following conditions:

(1) The resulting displacement amplitudes of relatively large magnitude must not interfere with production requirements.
(2) The loading on the structure must be accommodated without difficulty.

Condition (1) is frequently not met for machines in the metal-working or plastics-producing industry because of the usual production requirement of a very stiff mounting. This problem can be mitigated considerably by using a stabilizing mass and thereby rendering low tuning feasible. Low tuning finds a common application in machine hammer design, often in combination with a stabilizing mass.

Condition (2) can normally be achieved by an appropriate design of the structure or the respective structural member.

The above conditions being met, low tuning of machines with predominantly intermittent motion can proceed along the same lines as discussed previously in Section 3.4.1 for rotating or oscillating motion.

Of the feasible solutions depicted in Fig. 3.6 solution (c), suggesting mounting on an isolated base, is the best for low tuning. The resulting displacements can thereby be kept substantially smaller than in solution (b), using only spring(-damper) elements as supports.

Direct mounting of the machine on a relatively soft structure or member is only feasible for small impacting loads. Attention has then to be given to the possibility of substantial excitation of higher structural frequencies.

(b) Calculation of a Forced Vibration

If the success of frequency tuning cannot be guaranteed, resort may again be made to the approximate computation of a forced vibration. For impacting loads and direct mounting (Fig. 3.6a), an appropriate method is outlined in Appendix B 6.

A periodic loading can be decomposed by Fourier analysis into its constitutive harmonics, of which subsequently a small number are selected as forcing functions.

If the loading is of a transient nature and exhibits strongly the features of Section 3.2.3, then the computation of a forced vibration must proceed in two phases (see Appendix B6):

Phase 1: Computation for the effective duration of the loading (impact).

Phase 2: Computation of the subsequent free vibration (after the loading has ceased), for which the current values of the state variables of motion at the end of phase 1 serve as initial conditions for evaluating the integration constants of phase 2.

4 ACCEPTANCE CRITERIA

4.1 General Aspects

In civil engineering practice, measured or calculated vibration magnitudes (e.g. accelerations, velocities, displacements) need to be evaluated as to whether or not the vibration effects can be tolerated. To this end, the following acceptance criteria are introduced. Magnitudes exceeding these values or falling short of them do not necessarily indicate an inadmissible state of vibration. In only a few cases the acceptance criteria represent more or less agreed bounds; much more often they indicate a practicable order of magnitude with a certain scatter. Nevertheless, these criteria may be used as bounds of admissibility, for instance if made a part of the contract or some other form of obligation.

Criteria for vibrations of buildings and pedestrian structures may be stipulated with respect to the following effects:

- overstressing of structural members (deformation, fatigue, strength)
- physiological effects on people (mechanical, acoustic, optical)
- impediment of production processes (problems of product tolerances, etc.) as well as overstressing of machinery (deformation, fatigue, strength).

How to derive the acceptance criteria, is quite a complex problem. Tolerable values for vibration effects on machinery and installations and – still more so – for the physiological effects on people are most difficult to agree upon, implying a considerable range of discretion. Somewhat more reliable are the values of tolerable stressing of structures, because their vibrations are relatively easy to measure and to assess.

Vibration bounds may be given as physical quantities such as

- displacement amplitude
- velocity amplitude
- acceleration amplitude

or as

- empirically derived quantities (e.g. KB intensity [4.1]).

The above categorization of the various vibration effects leads to the following division into

- structural criteria
- physiological criteria
- production-quality criteria.

A concise compilation of criteria as this does not really permit their direct application. Rather more, the reader should routinely consult the complete edition of the respective regulation or reference literature, in the choice of which he will – hopefully – be assisted by this survey.

The last paragraph of this chapter recommends global criteria derived for the most critical vibration effect, depending on types of buildings and their use. They may suffice for feasibility studies, in cases with large uncertainties regarding the dynamic loads or the structural properties, or whenever no specific regulation is to be adhered to.

4.2 Structural Criteria

Vibrations induced by man, machinery, traffic, construction works, etc., may cause deformations and smaller or larger forms of distress in buildings, their structural members and – particularly – nonstructural elements. Forms of distress may be

- cracking of walls and slabs
- aggravation of existing cracking in structural members and nonstructural elements
- falling down of equipment or cladding thereby endangering occupants.

Continuous vibration, however, can also lead to problems of fatigue and overstress in principal load-bearing members.

Acceptance criteria must take account of the following parameters (among others):

- type and quality of the building material (especially its ductility)
- type of construction
- properties of the building foundation
- main dimensions of the principal load-bearing members
- age of the building
- duration of the vibration effects
- characterization of vibration (frequency, etc.).

Figure 4.1 shows the amount of expected structural damage to depend on various parameters. Although the acceptance criteria are taken to be independent of frequency, the quantity most suitable as indicator differs over the frequency range: While the bound should be on vibration velocities for low frequencies, it should be on accelerations for high frequencies. This notwithstanding, most regulations define their bounds in terms of velocity.

The following gives a concise survey of various regulations, codes of practice and some reference literature. Another, more extensive survey on acceptance criteria used in different countries is given in [4.3] with emphasis on vibrations of buildings due to nearby blasting operations.

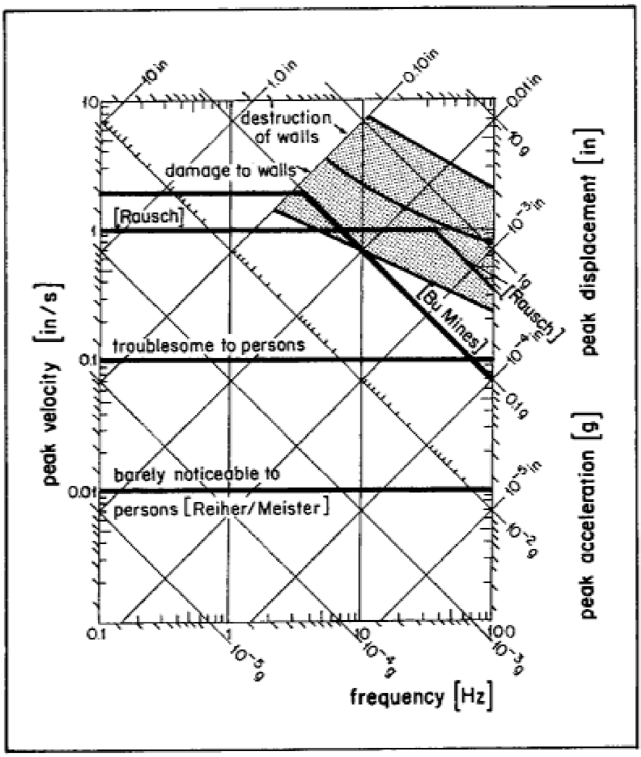


Fig. 4.1
Response spectrum for vibration effects on people and structure (from [4.21]);
1 in. = 25.4 mm
Origin of data:
U.S. Bureau of Mines
Rausch [4.25]
Reiher/Meister [4.23]

4.2.1 **Standard DIN 4150, Part 3 (1983)** [4.4]

The third part of Standard DIN 4150 treats effects on buildings and structural members due to an internal or external source of vibration. For their assessment, the vibration velocities or – if necessary – the stresses due to dynamic loads are to be compared with the given criteria. With predominant use of (mostly measured) vibration velocities, different criteria are used for

- short-term structural vibrations (transient)
- steady-state structural vibrations
- steady-state vibrations, particularly of floor slabs.

Transient vibrations, as excited e. g. by blasting operations, pile driving, etc., are to be limited in terms of maximum foundation velocities, the values of which depend on the accepted degree of damage, Fig. 4.2: 20 ÷ 50 mm/s (from $f \leq 10$ Hz to $f = 100$ Hz) to avoid severe damage, 5 ÷ 20 mm/s to avoid slight damage, and 3 ÷ 5 mm/s for particularly sensitive environments. For steady-state structural vibrations, floor slabs in particular, 10 mm/s is considered admissible, even though it does not entirely preclude slight damage such as cracking.

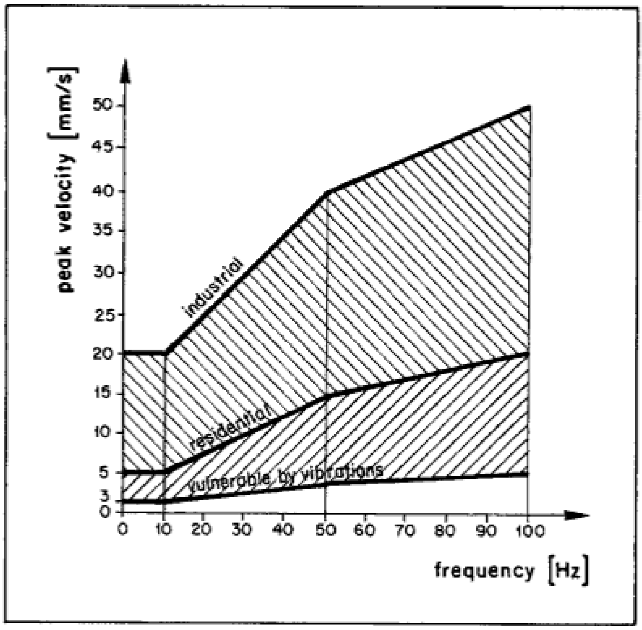


Fig. 4.2
Bounds on the foundation vibration velocities for building categories of DIN 4150 Part 3 (from [4.4])

4.2.2 Standard SN 640312 (1978) [4.6]

The Association of Swiss Highway Engineers distinguishes in their Standard SN 640312 four different categories of buildings, mainly according to the type of construction, Tab. 4.1. The acceptance criteria are again peak vibration velocities, which are quoted in Tab. 4.2. Two different groups of vibration sources are considered: The first group comprises machinery, traffic and construction equipment, the second group blasting operations assumed to occur less frequently and thus permitted for higher bounds. The criteria are defined for two frequency ranges and are based on the spatial vector of the vibration velocity.

4.2.3 Directive KDT 046/72 (1972) [4.7]

The Directive issued by the Chamber of Technology of the German Democratic Republic also distinguishes four categories of buildings and defines admissible vibration velocities in function of frequency, Tab. 4.3. Bounds on vibrations due to blasting for the same categories are presented in Fig. 4.3. The Directive is discussed e.g. in [4.8].

4.2.4 Draft ISO/DIS 4866 (1984) [4.9]

The draft of Standard ISO/DIS 4866 deals primarily with types and methods of vibration measurement on structures and does not contain any acceptance criteria. Interesting in the present context, however, is the distinguishing of 14 different building categories accord-

structural category	definition
I	reinforced-concrete and steel structures (without plaster) such as industrial buildings, bridges, masts, retaining walls, unburied pipelines; underground structures such as caverns, tunnels, galleries, lined and unlined
II	buildings with concrete floors and basement walls, above-grade walls of concrete, brick or ashlar masonry; ashlar retaining walls, buried pipelines; underground structures such as caverns, tunnels, galleries, with masonry lining
III	buildings with concrete basement floors and walls, above-grade masonry walls, timber joist floors
IV	buildings which are particularly vulnerable or worth protecting

Tab. 4.1 Structural categories according to SN 640312 (from [4.6])

structural category	source M		source S	
	f [Hz]	v_{max} [mm/s]	f [Hz]	v_{max} [mm/s]
I	10 ÷ 30 30 ÷ 60	12 12 ÷ 18*	10 ÷ 60 60 ÷ 90	30 30 ÷ 40**
II	10 ÷ 30 30 ÷ 60	8 8 ÷ 12*	10 ÷ 60 60 ÷ 90	18 18 ÷ 25**
III	10 ÷ 30 30 ÷ 60	5 5 ÷ 8*	10 ÷ 60 60 ÷ 90	12 12 ÷ 18**
IV	10 ÷ 30 30 ÷ 60	3 3 ÷ 5*	10 ÷ 60 60 ÷ 90	8 8 ÷ 12**

source M: machinery, traffic, construction works – (*) the lower value applies to 30 Hz, the upper to 60 Hz, with interpolation in between.

source S: blasting operations – (**) the lower value applies to 60 Hz, the upper to 90 Hz, with interpolation in between.

Tab. 4.2 Acceptance criteria of SN 640312 for the structural categories of Tab. 4.1 (from [4.6]) ing to type of construction, foundation, ground condition, and importance of the building. The usefulness of such a detailed categorization in view of the many uncertainties remains questionable.

	building category	$v_{z,adm}$ [mm/s]
I	historical monuments	2
II	half-timbered houses	5
III	wall construction (e.g. buildings of slab walls, blocks, masonry)	10
IV	framed construction (e.g. buildings of steel, reinforced concrete, timber)	30

Tab. 4.3 Building categories and admissible velocities in KDT 046/72 (from [4.8])

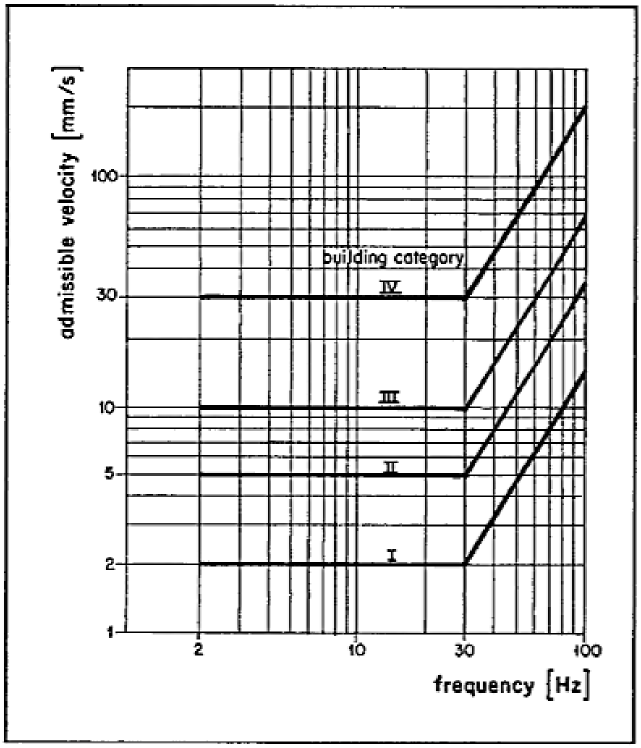


Fig. 4.3
Bounds on vibration velocities due to blasting in function of building category (Tab. 4.3) in KDT 046/72 (from [4.8])

4.3 Physiological Criteria

In principle, the human sensitivity to mechanical vibrations is very subtle. As an example, the human body notices vibration displacement amplitudes of only 0.001 mm, but finger-tips can even detect amplitudes 20 times smaller. However, the human reaction to a given vibration depends very much on circumstances. Personal discomfort is realized to a different degree if, for instance, one sits at an office desk, operates a machine, or drives a car. Obviously the person's attitude to the vibration source matters – whether it is his own purposeful activity or external – as well as the degree of accustoming.

Parameters influencing the human sensitivity are the following:

- position of the affected person (standing, sitting, lying)
- direction of incidence with respect to the human spine
- activity of the affected person (resting, walking, running)
- community (existence of fellow-sufferers)
- age
- sex
- frequency of occurrence and time of day
- duration of decay (damping).

The intensity of perception depends, of course, also on the objective physical vibration parameters

- displacement amplitude
- velocity amplitude
- acceleration amplitude
- duration of effect (exposure time)
- vibration frequency.

This is illustrated in the following figures.

Figure 4.4 shows the human sensitivity spectra of displacement amplitudes of different frequency (from [4.10]). According to Fig. 4.5, also the acceptance criteria for velocity amplitudes depend on frequency (from [4.11]). Table 4.4 gives acceptance criteria for effects on people in function of velocity and acceleration amplitudes, which apply to harmonic vibrations with displacement amplitudes of less than 1 mm ([3.2] after [4.12]).

Different codes of practice and numerous publications have attempted to find the most realistic criteria for physiological vibration effects. Concepts and some figures of several codes are cited and discussed in the following.

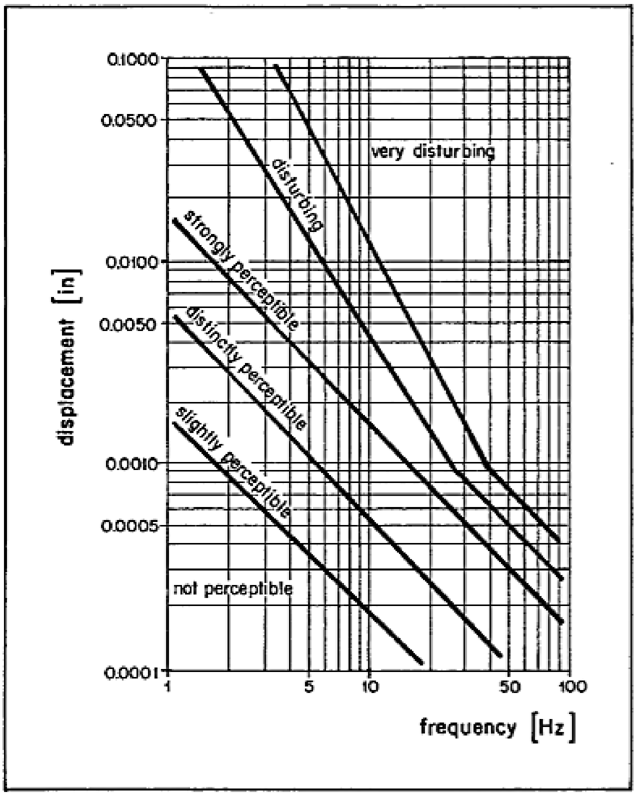


Fig. 4.4
Frequency dependence of the human perception of vibration displacements (from [4.10]); 1 in. = 25.4 mm

vibration effects on people	frequencies 1 ÷ 10 Hz a_{max} [mm/s²]	frequencies 10 ÷ 100 Hz v_{max} [mm/s]
imperceptible	10	0.16
just perceptible	40	0.64
clearly perceptible	125	2.0
annoying	400	6.4
unpleasant, painful if lasting	1000	16.0
harmful	> 1000	> 16.0

Tab. 4.4 Acceptance criteria for physiological vibration effects (from [3.2] after [4.12])

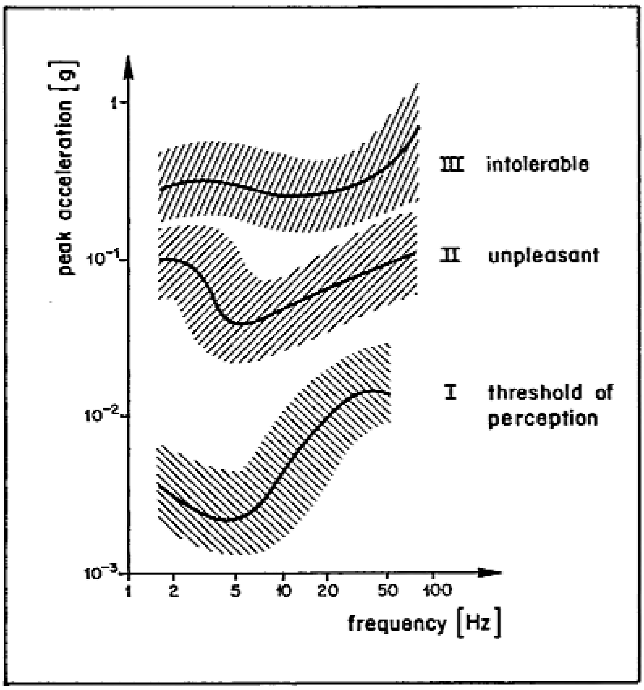


Fig. 4.5
Frequency dependence of the human perception of vibration accelerations (from [4.11])

4.3.1 Standard DIN 4150, Part 2 (1975) [4.1]

The second part of Standard DIN 4150 deals with the effects of vibrations from mostly external sources on people in residental buildings. The frequencies considered range from 1 to 80 Hz. Measured vibration quantities (displacement, velocity, acceleration) serve as input for an empirically derived intensity of perception, called «KB value»:

$$KB = d \cdot \frac{0.8 \cdot f^2}{\sqrt{1 + 0.032 \cdot f^2}} \tag{4.1}$$

where:

d = displacement amplitude [mm]
f = vibration frequency [Hz].

The KB value can alternatively be formulated in terms of velocity (v) or acceleration (a) of the vibration, which – if the vibration is harmonic – are interrelated by

$$d = \frac{v}{2 \cdot \pi \cdot f} = \frac{a}{4 \cdot \pi^2 \cdot f^2} \tag{4.2}$$

The thus calculated KB value of the examined vibration is to be compared with the reference value in the code according to

- use of the building
- frequency of occurrence
- duration of effects
- time of day of occurrence.

Figure 4.6 depicts one of the double-logarithmic diagrams in [4.1]. Table 4.5 gives acceptance criteria for assessing vibrations in apartments and similar environments.

building zone (actual utilization and development of the estate within radius of vibration emission)	time	acceptable KB intensity	
		continuous or repeatedly	infrequent
purely residential, housing estate, holiday resort	day	0.2 (0.15*)	4
	night	0.15 (0.1*)	0.15
village and small business, town-centres	day	0.3 (0.2*)	8
	night	0.2	0.2
business and trade (incl. offices)	day	0.4	12
	night	0.3	0.3
industrial	day	0.6	12
	night	0.4	0.4
exceptional areas (acc. to residential content)	day	0.1 ÷ 0.6	4 ÷ 12
	night	0.1 ÷ 0.4	0.15 ÷ 0.4
(*) Values in brackets should be complied with if buildings are exited horizontally with a frequency below ~ 5 Hz.			

Tab. 4.5 Acceptable KB intensities for residential buildings (from [4.1])

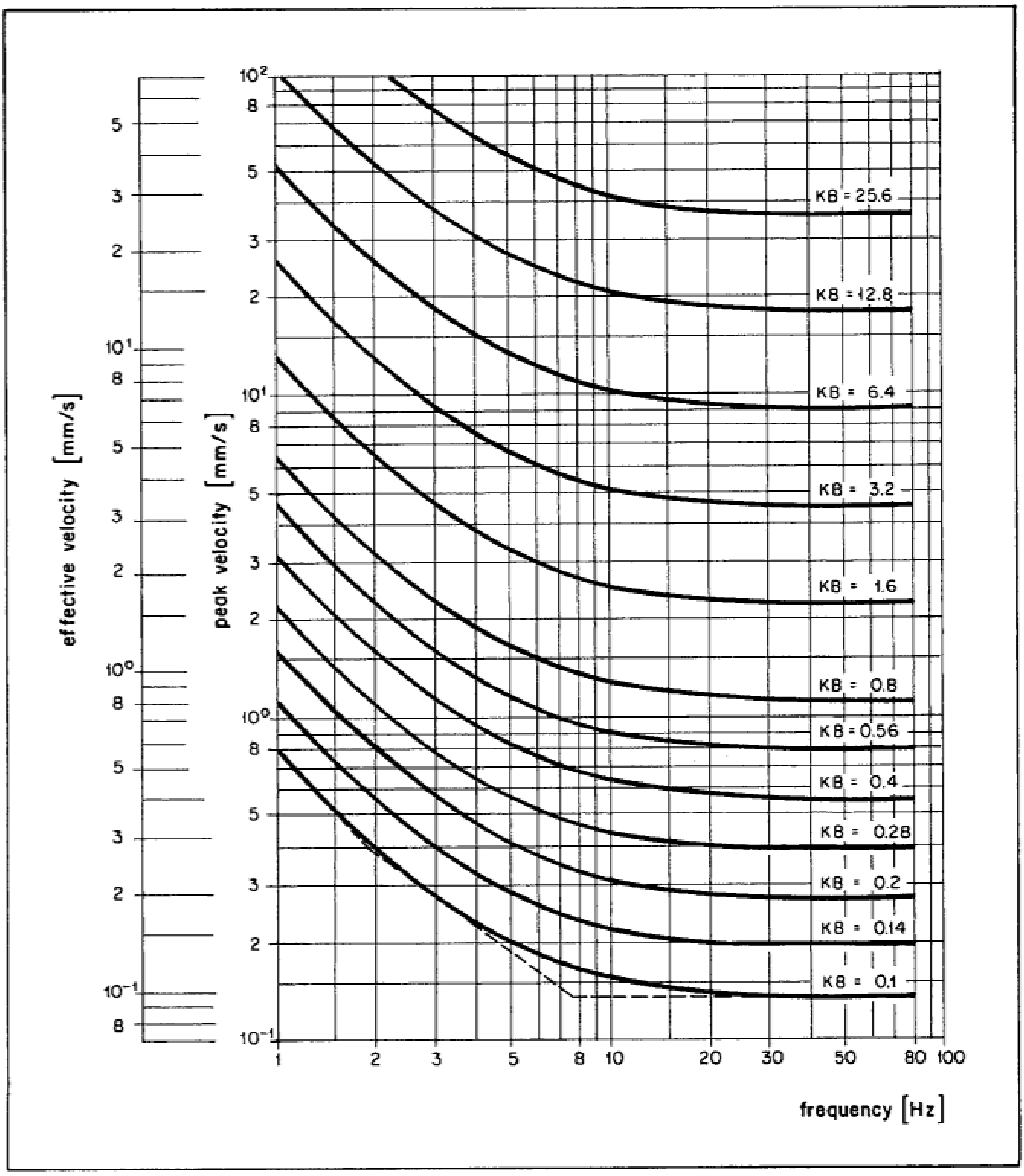


Fig. 4.6 Iso-perception curves in function of frequency (from [4.1])

4.3.2 Standard ISO 2631 (1980) [4.13]

The Standard ISO 2631 covers all effects on people from periodic or transient vibrations in the frequency range of 1 to 80 Hz. Three different levels of human discomfort are distinguished:

- The «reduced comfort boundary» applies to tolerable disturbance during activities like eating, reading or writing.
- The «fatigue-decreased proficiency boundary» describes the level at which recurrent vibrations cause fatigue to personnel with consequent reduction of efficiency; this occurs at about three times the first discomfort level.
- The «exposure limit» defines the maximum vibration tolerable with respect to human health and safety and is about six times higher than the first level.

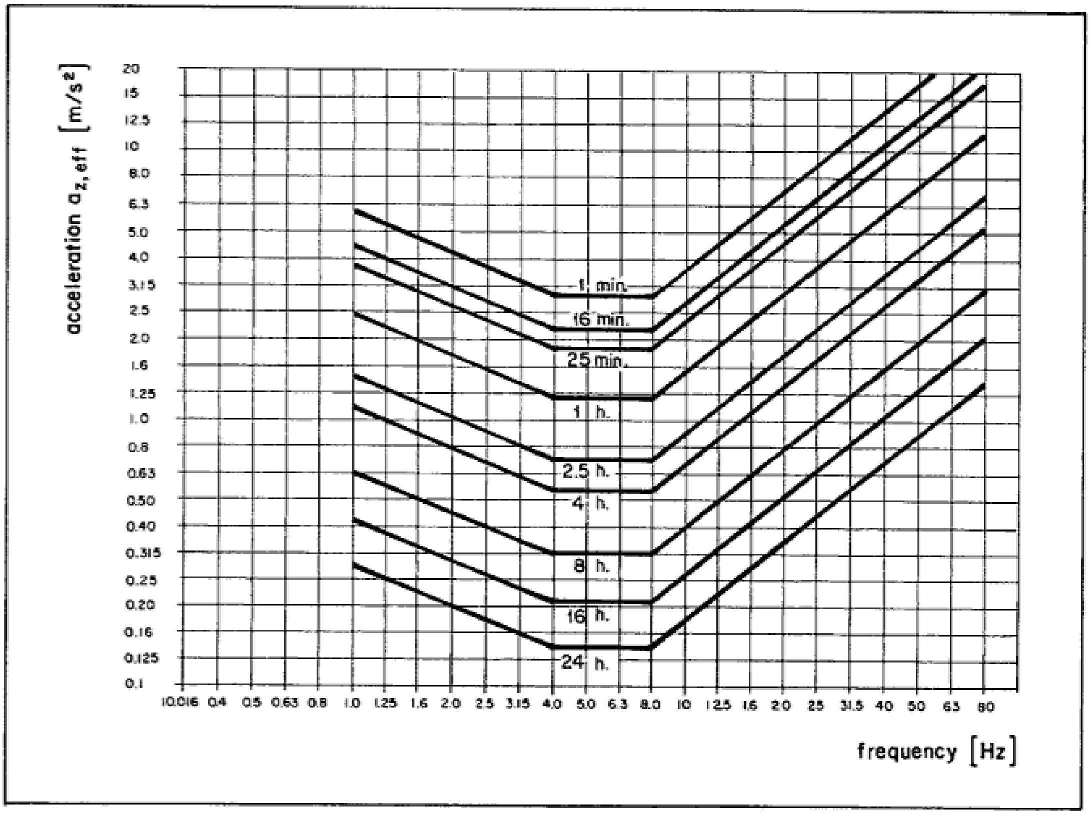


Fig. 4.8 Bounds on z-acceleration for «fatigue-decreased proficiency» depending on exposure duration (from [4.13]); the «exposure limit» is obtained by multiplying by 2, the «reduced-comfort boundary» by dividing by 3.15

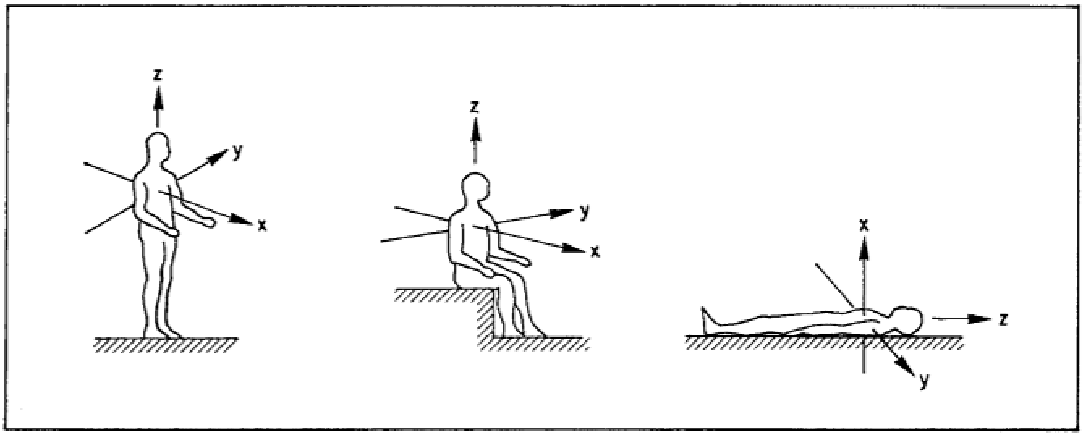


Fig. 4.7 Reference coordinate system for vibration effects on man (from [4.13])

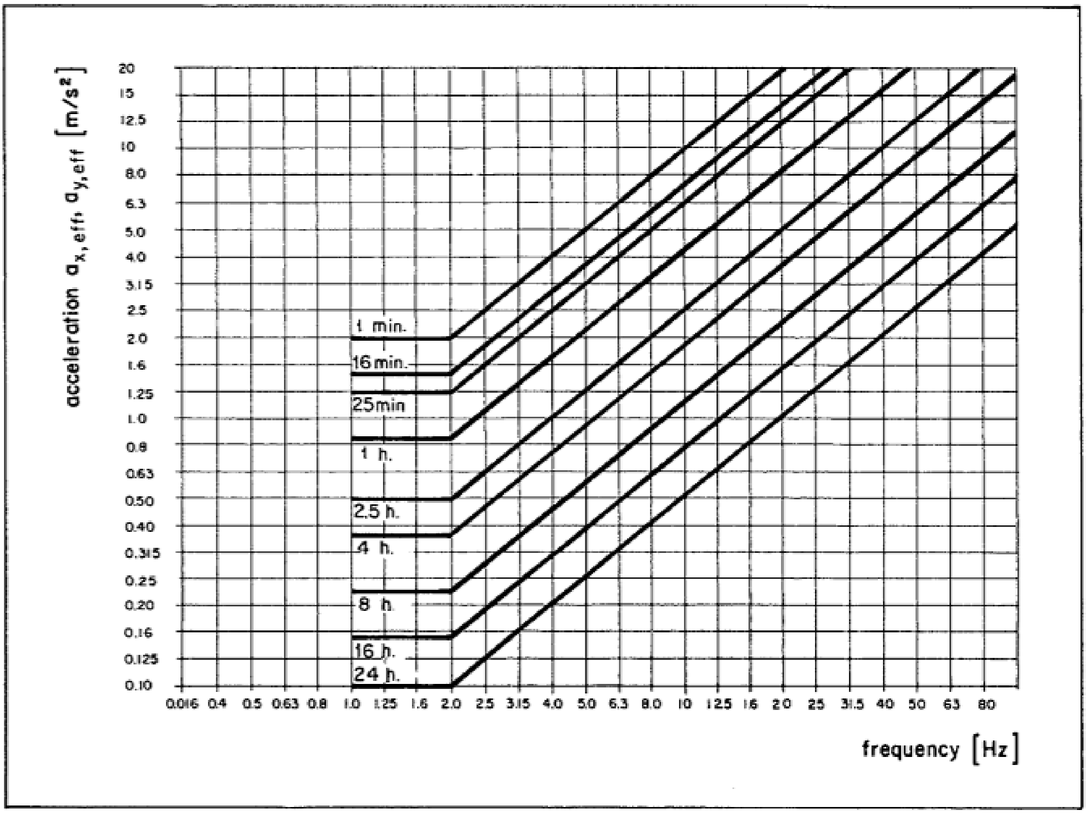


Fig. 4.9 Bounds on x- and y-accelerations for «fatigue-decreased proficiency» depending on exposure duration (from [4.13]);
other bounds are obtained by the same factors as in Fig. 4.8

The bounds given in tables and diagrams depend on the direction of incidence to the human body, using a reference coordinate system with the z-axis in the direction of the human spine, Fig. 4.7.

Criteria are formulated in terms of an effective acceleration which is defined as the root-mean-square value over the exposure time T:

$$a_{eff} = \sqrt{\frac{1}{T} \cdot \int_0^T a^2(t) \cdot dt} \tag{4.3}$$

Figure 4.8 gives the bounds on the effective acceleration in the spine direction, Fig. 4.9 those on the effective accelerations perpendicular to the spine. The values represent the «fatigue-decreased proficiency boundary», from which the values for the two other levels can be derived by division or multiplication with the aforementioned factors.

4.3.3 Directive VDI 2057 (1983/1981/1979) [4.14]

The Directive VDI 2057 comprises four Parts (Folios), of which only Part 2 has been passed whereas the others exist in draft form. Particularly Parts 1 and 2 are similar to ISO 2631 [4.13], so that the reader is referred to the outline of the latter above. One main difference lies in the definition of the vibration intensity: The effective accelerations of ISO 2631 are already weighted with frequency and direction of incidence (see Fig. 4.7) to derive direction-dependent KX, KY and KZ values. Given for various durations of vibration effects, these bounds can now be applied irrespective of frequency and represent the same discomfort levels as in ISO 2631.

4.3.4 BRE Digest 278 (1983) [4.15]

The British Building Research Establishment defined K intensities in borrowing the KB levels of perception from DIN 4150 part 2 [4.1]. Table 4.6 attributes the different K values to the respective perception intensities, Tab. 4.7 gives the acceptable K intensities for different exposure times and categories of buildings.

4.3.5 British Standard BS 5400, Part 2 (1978) [2.25]

Part 2 of the British Standard BS 5400 limits for pedestrian bridges the acceleration amplitude for matched-frequency loading by a notional pedestrian. The upper bound is specified as $0.5 \cdot \sqrt{f_1}$ [m/s²], amounting to 0.7 m/s² at 2 Hz and to 1.1 m/s² at 5 Hz.

human perception	K value	commentary
not felt	< 0.1	Values are applicable to both horizontal and vertical vibrations. K values 25 and 63 of DIN 4150 were here not adopted as their effects on people can hardly be distinguished.
threshold of perception	0.1	
barely noticeable	0.25	
noticeable	0.63	
easily noticeable	1.6	
strongly detectable	4	
very strongly detectable	10	

Tab. 4.6 Relation between K values and human perception of motion (from [4.15])

building category	time	acceptable K intensity		
		continuous	repeatedly	occasionally
hospitals and nursing homes	day	0.1	0.1	2.5
	night			0.1
residential	day	0.1	0.2 (0.1*)	4
	night		0.1	0.1
city residential and business	day	0.3 (0.15*)	0.63 (0.3*)	8
	night	0.1	0.1	0.1
industrial	day	0.63 (0.3*)	0.8 (0.4*)	12
	night	0.63 (0.3*)	0.8 (0.4*)	12
(*) Values in brackets apply to cases where the frequency of vibration is below 15 Hz.				

Tab. 4.7 K intensity in function of building category and exposure time (from [4.15])

4.3.6 British Standard BS 6472 (1984) [4.25]

The British Standard BS 6472 serves to assess vibration effects on people in workshops, offices, residential buildings and particularly sensitive environments such as operating theatres. Similar to DIN 4150 [4.1] it categorizes according to vibration frequency and exposure time and defines criteria either in velocity or acceleration, depending on the frequency range. Accelerations are given as root-mean-square effective values (confer Eq. (4.3)), whereas the velocities are specified as peak values.

4.3.7 NBC Canada, Commentary A (1985) [2.16]

The National Building Code of Canada gives in the pertinent Commentary A frequency-independent acceptance criteria for accelerations in the following environments:

- dancing and dining (close to a dance floor) 2% g
- concert and sport events 5% g
- fitness classes 5% g

4.3.8 Regulation SBA 123 (1982) [2.13]

The Regulation SBA 123 [2.13] of the State Construction Supervision Board of the German Democratic Republic requires an explicit check on liveliness for pedestrian bridges, stair cases, and similar structures. Essentially, the Regulation defines a physiological stress value K, both for vertical and horizontal (longitudinal or lateral) vibration:

$$K = x_{stat} \cdot f^2 \tag{4.4}$$

where:

x_{stat} = displacement of a characteristic point under equivalent static load [mm]
f = excitation frequency [Hz].

The upper bound for acceptance is $K_{adm} = 10$.

4.3.9 Criteria from the Literature

The available literature contains several suggested acceptance criteria for various types of structure, especially pedestrian structures. As the vibration effects on the structure (e.g. fatigue) are virtually negligible, the objective is here too a limit of discomfort to people.

Criteria for pedestrian structures are always to be used with care. It makes a large difference whether the magnitude of vibration to be assessed

- occurs frequently, rarely or exceptionally
- has been actually measured (on an existing structure)
- has been computed from more or less vague data, and then whether it is a mean expected value or a conservative upper bound.

Acceptance criteria should pay regard to circumstances, such as the user-accepted or design level of comfort, etc.

The generally used indicator is the maximum acceleration:

- In the opinion of [2.2], 10% g is a threshold value above which pedestrians feel insecure and the structural serviceability is impaired.
- Report [2.5] advocates for footbridges that the acceleration bound stipulated by BS 5400 [2.25] be relaxed to $1.0 \cdot \sqrt{f_1}$ [m/s^2], provided f_1 is outside the critical range of 1.7 ÷ 2.2 Hz. This criterion amounts to a bound of 13% g at 1.7 Hz and 15% g at 2.2 Hz.
- Reference [2.6] suggests the peak vibration velocity as criterion and points to the different intensity of perception during walking and standing: Vibration frequencies above ca. 3 Hz are less bearable when standing than when walking, while it is the opposite for slower vibrations. A walking pedestrian is said to sense as unpleasant a velocity of 24 mm/s. On the basis of a harmonic vibration, this criterion can be transformed to a bound on the displacement or the acceleration amplitude; it then corresponds to 3% g at 2 Hz and 7.5% g at 5 Hz, i. e. much smaller values than in the previous references.

4.4 Production-Quality Criteria

For industrial or scientific work, bounds to vibration effects need to be formulated with respect to the following problems:

- problem of production technology apparent in the manufactured goods (e. g. problems of tolerance on lathes, milling machines, weaving machines, extrusion presses, etc.)
- diminished performance of apparatus (e. g. electron microscope, computer, etc.)
- problems of material technology of the machine itself (e. g. wear on shaft bearings, excessive deformation, fatigue and strength limits of machinery parts or at supports, etc.).

Universally applicable criteria cannot be given but need to be specified individually for types of machinery. Therefore, codes of practice and literature contain either very general statements or values of rather limited applicability.

General statements try mostly to group various types of machinery in different sensitivity classes, such as the ones in Tab. 4.8. Depending on the frequency range, either the peak vibration velocity or the peak acceleration is used as critical quantity, Tab. 4.9.

apparatus category	machinery and equipment
I	optical instruments, such as microscopes, interferometer, optimeter, etc.; mechanical measuring instruments in the micron-range, apparatus for precision scale calibration; finishing of optical lenses; precision cutters; rotor-balancing machines and other heavy precision machinery; machine control stations
II	machinery for grinding of ball bearings, cogwheels, razor blades, etc.; coordinate grinding machines, milling and turning machinery to a precision of some hundreths mm
III	metal-working machinery for turning, cutting, drilling, milling, etc., to usual precision; spinning, weaving and sewing machinery; printing presses, etc.
IV	rotary machines such as blowers, centrifugal separators, electric engines, etc.; stamping machines and presses in light metal-working industry; precision drilling machines; vibratory machines such as vibrators, jarring plates, riddlers, strewing machines, etc.

Tab. 4.8 Categories of apparatus sensitivity (from [3.2])

apparatus category	sensitivity to harmonic vibrations	frequ. $1 \div 10$ Hz a_{max} [mm/s^2]	frequ. $10 \div 100$ Hz v_{max} [mm/s]
I	highly sensitive	6.3	0.1
II	normally sensitive	63	1
III	little sensitive	250	4
IV	insensitive	>250	>4

Tab. 4.9 Acceptance criteria for the categories defined in Tab. 4.8 (from [3.2])

More detailed data are to be found in the following standards and recommendations:

- ISO 2372, 1974 [4.16]
- ISO 2373, 1974 [4.17]
- VDI 2056, 1964 [4.18]
- VDI 2063, 1982 [4.19]
- ISA-Transactions, 1964 [4.20].

References [4.21] and – more so – [4.22] quote further national codes and recommendations where acceptance criteria with regard to machinery can be found. Specific data on individual types of machinery are given e.g. in [4.24] and [4.2].

4.5 Suggested Overall Acceptance Levels

As is apparent from what has been said thus far, the assessment or the design limitation of vibrations has to observe bounds to the (simultaneous) effects on the structure and its structural members, on people, and sometimes also on machinery and equipment. Depending on the type of structure and its use, the three objectives may yield rather different bounds, let alone the discrepancies in recommendations of different codes and publications for the same type.

Therefore, it may be helpful in some cases – and particularly for provisional studies – to operate with crude but simple global criteria. They can be quite appropriate in the absence of exact data on the dynamic loads or in combination with relatively rough estimates on the structural properties, necessary to calculate a forced vibration. In the light of such uncertainties, frequency-dependent values and more sophisticated definitions (e.g. effective values) are hardly practicable. Instead, of the three objectives of vibration limitation, as mentioned above, only the most critical one is selected and observed. The resulting overall bounds may also be used sensibly in cases of not designing for compliance with a specific regulation.

With this in mind, the authors undertook to derive from their own experience and knowledge of the literature criteria for various environments, Tab. 4.10. The comments indicate the limitations of their applicability.

structure	acceptance level	comments	cases no.
pedestrian structures	$a \leqslant 5 \div 10\%\ g$	the lower value does normally not produce discomfort	1, 2, 3, 4
office buildings	$a \leqslant 2\%\ g$	DIN 4150 and BS 6472 may yield quite different values	4, 6
gymnasia (sports halls)	$a \leqslant 5 \div 10\%\ g$	the higher value only if • acoustic effect small • only participants on or near the vibrating floor	7, 8, 9
dancing and concert halls	$a \leqslant 5 \div 10\%\ g$	same as for gymnasia	10
factory floors	$v \leqslant \sim 10$ mm/s	e.g. for conventional loom; high-quality production needs much stricter bounds	15

Tab. 4.10 Overall acceptance levels for various types of environments

Appendix A

CASE REPORTS

In the following, several practical cases are described where vibrations induced by man or machines were observed in structures. Most cases involve problems with interesting features, cases in which there had been complaints and for which in part improvement measures have been carried out. For the sake of the practical lessons they teach, these cases are described in a practice-oriented manner, and more abstract aspects are deliberately left out.

Some cases stem from the authors' own expertise, others are cited from the literature or were made available by colleagues of the engineering community with their permission for publication. Their courtesy is specially acknowledged. The authors would also be grateful for their readers' reporting further interesting cases, because they feel that in learning from experience, designs which were found inadequate in hindsight are often the most valuable.

No. 1: Footbridge of 1.92 Hz

Designed as a simply supported beam with a 40 m span, the bridge vibrated vertically with clearly perceptible amplitude when used under normal conditions (pedestrian walking) [A1.1]. In order to clarify the nature and intensity of the vibrations, tests and measurements were made.

Figure A1.1a gives an excerpt of a typical plot of the vertical vibration velocity. Characteristic is the surging effect which was observed under normal walking conditions as well as under deliberate excitation by three persons skipping continuously in midspan. Figure A1.1b shows a similar excerpt of a decay measurement following a single impulse due to the synchronous jumping of three persons.

As results of two different readings, the peak displacement amplitude for a rate of ca. 29 people crossing per minute was ±4.6 mm, corresponding to an acceleration of 6.7% g. The damping ratio was evaluated from three decay measurements to 2.3%, 1.8% and 2.4%, i.e. an average of 2.2% of critical damping. The three persons skipping produced a displacement amplitude of ±9.4 mm, more than double the one at crossing of 29 persons/min.

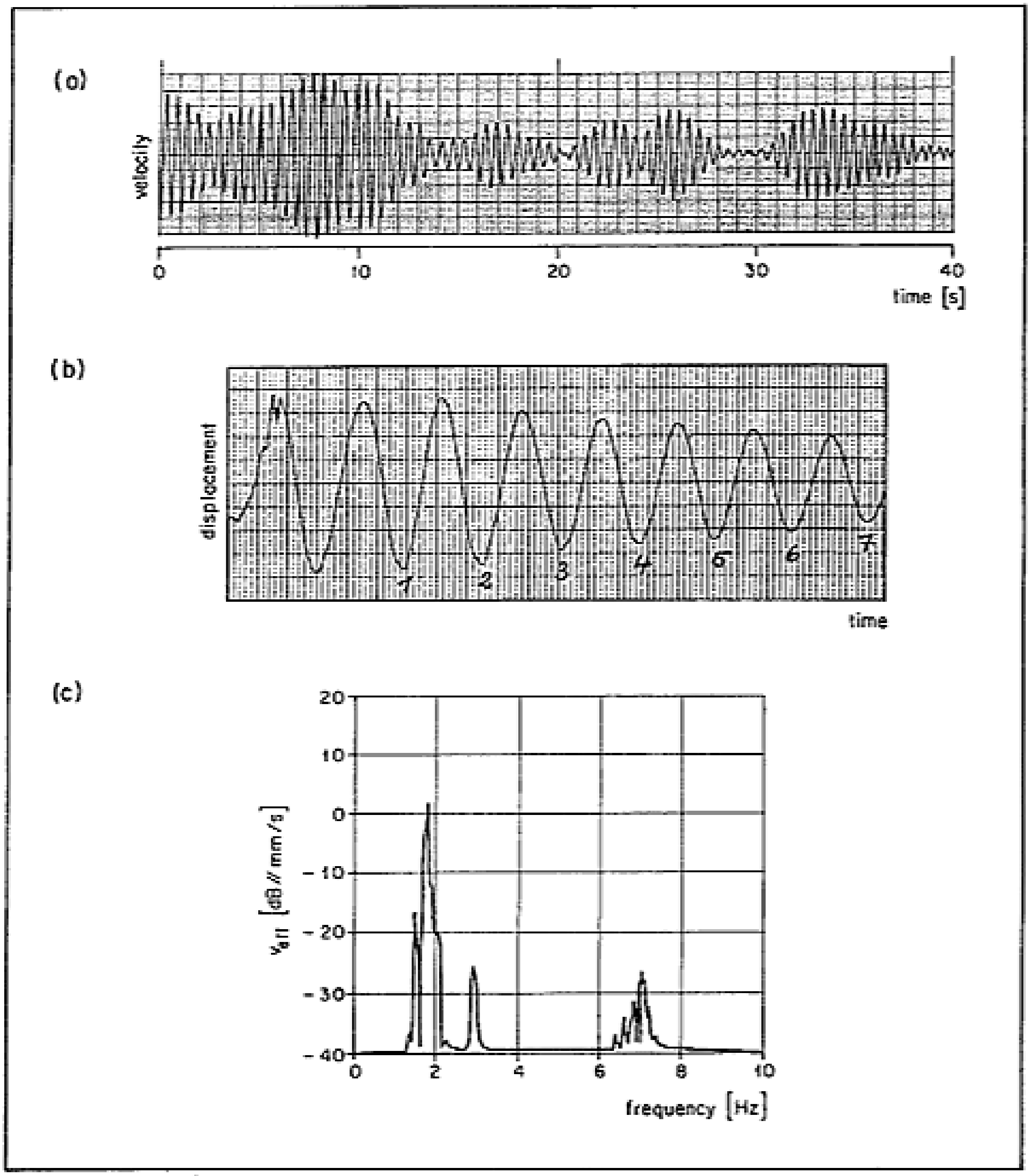


Fig. A1.1 Footbridge with 1.92 Hz fundamental frequency (from [A1.1]):
(a) typical surge phenomenon in midspan vibration velocity
(b) displacement decay in an impulse loading test
(c) amplitude spectrum of quarter-span vibration velocity

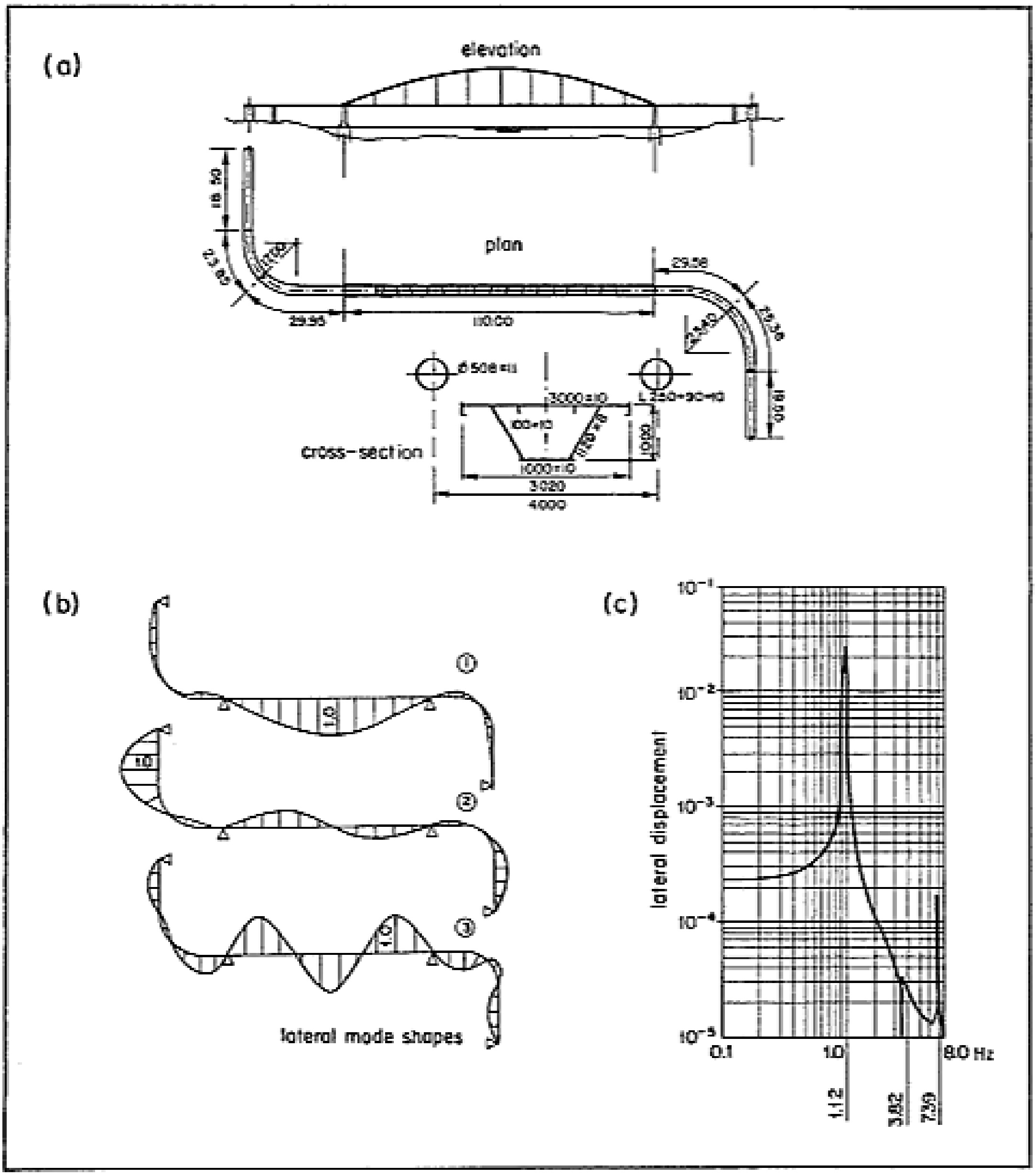


Fig. A4.1 Footbridge with lateral vibrations at 1.1 Hz (from [2.12]):
(a) elevation, plan and cross-section
(b) first three mode shapes of lateral vibration
(c) transfer function for the lateral displacement in midspan

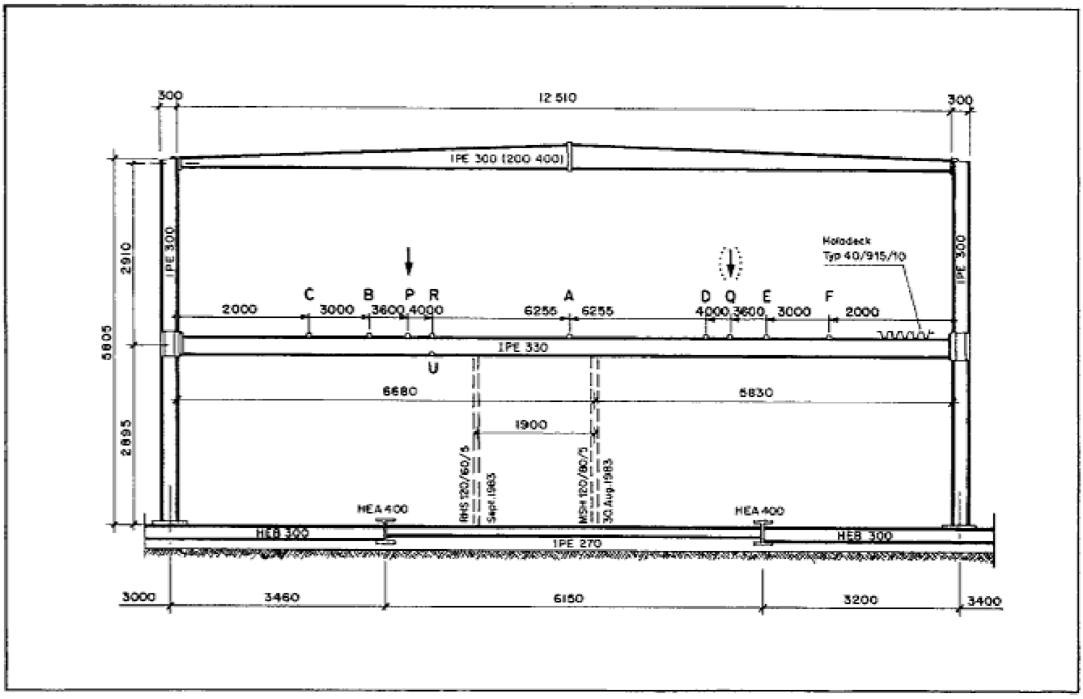


Fig. A5.1 Office building (from [2.26]): Section through a typical frame, supported elastically by the steel girder grid

sates the smaller impact factor at higher frequency (see Section 2.2.2) and imparts more energy to the system.

The vibration due to three persons walking on the floor was also largely dominated by the fundamental structural frequency of 5.85 Hz and had a peak acceleration of 10% g. The serviceability criteria of both ISO 2631 [4.13] and VDI 2057 [4.14] indicated that normal office work could not be performed on the intermediate floor without improvement measures. The much stricter bounds of DIN 4150 [4.1] were exceeded by far, but also according to BS 6472 [4.25] complaints were certainly to be expected.

Improvement was attempted primarily by fitting a number of additional columns MSH 120/80/5 into the lower floor of the extension (see Fig. A5.1). This raised the fundamental frequency by a factor of almost two to 11.25 Hz and reduced the amplitudes noticeably, in particular the maximum acceleration (due to three persons walking) to ~6.6% g. Judging the new state by the aforementioned codes leads to diverging opinions: ISO 2631 is not really applicable to buildings, VDI 2057 seems to lack the necessary distinctions for this problem, according to DIN 4150 remaining vibrations are still far from being admissible, and BS 6472 indicates that user complaints might be possible but not very probable.

Since then, a second row of additional columns RHS 120/60/5 has been fitted (Fig. A5.1), and – among other extra measures – a specially soft floor cover has been provided. People walking in offices and corridors still cause vibrations of clearly perceptible magnitude, which, however, when questioned, the personnel in the extension storeys did not regard as disturbing. Hence, in cases like this, the British Standard BS 6472 [4.25] seems to be best applicable.

No. 6: Exhibition Pavilion

In a two-storey exhibition pavilion strong vibrations occurred whenever people walked on the intermediate floor, restricting considerably the serviceability of the building [2.23]. The structure vibrated both vertically and horizontally.

Designed as a lightweight steel frame, the loading from the intermediate floor is carried via corrugated steel sheets and single-span cross-beams to two main longitudinal beams of section IPB 200, which run continuously over several spans.

Measuring the vibrations due to walking gave the frequency spectrum of Fig. A6.1a. Peak values can be seen to occur around 6 Hz and 27 Hz. Under impulse loading the lowest frequency of the corrugated sheet floor was determined to be 27 Hz, that of the cross-beams to be 18.5 Hz and that of the longitudinal beams to be 6.2 Hz. In a similar manner to that of the office building (Case No. 5), mainly the fundamental frequency of the system was excited by the 3rd harmonic of the dynamic load-time function (about 2 Hz pacing rate).

Architectural reasons did not permit any visible alteration of the structure, so that the stiffness could not be raised by fitting additional columns or strengthening the longitudinal beams. The only practicable way of improvement was to install tuned vibration absorbers, eight in total, carefully tuned and damped with a mass of 65 kg each. Figure A6.1b gives the constructional details. They reduced the displacement amplitude of the main beams by a factor of six.

No. 7: Gymnasium Building

A two-storey gymnasium building showed severe vibrations soon after its opening [2.1]. They occurred regularly whenever the upper hall was used by modern-style fitness classes, performing skipping, jumping and running exercises to strongly rhythmic music. They became manifest mainly in the lower hall, where the vertical swinging of the intermediate floor under the direct loading due to the fitness class was clearly noticed and where the glazed exterior walls vibrated horizontally.

The secondary effects were also rather dramatic: rattling of the entrance doors and the shutters to the apparatus store-room, clattering of equipment attached to floor and walls, and a strong, alternating draught due to compression and decompression of the air volume in the lower hall, which one could feel when standing near the open entrance door. Several times these combined phenomena caused people to leave the lower hall hastily during fitness classes going on above.

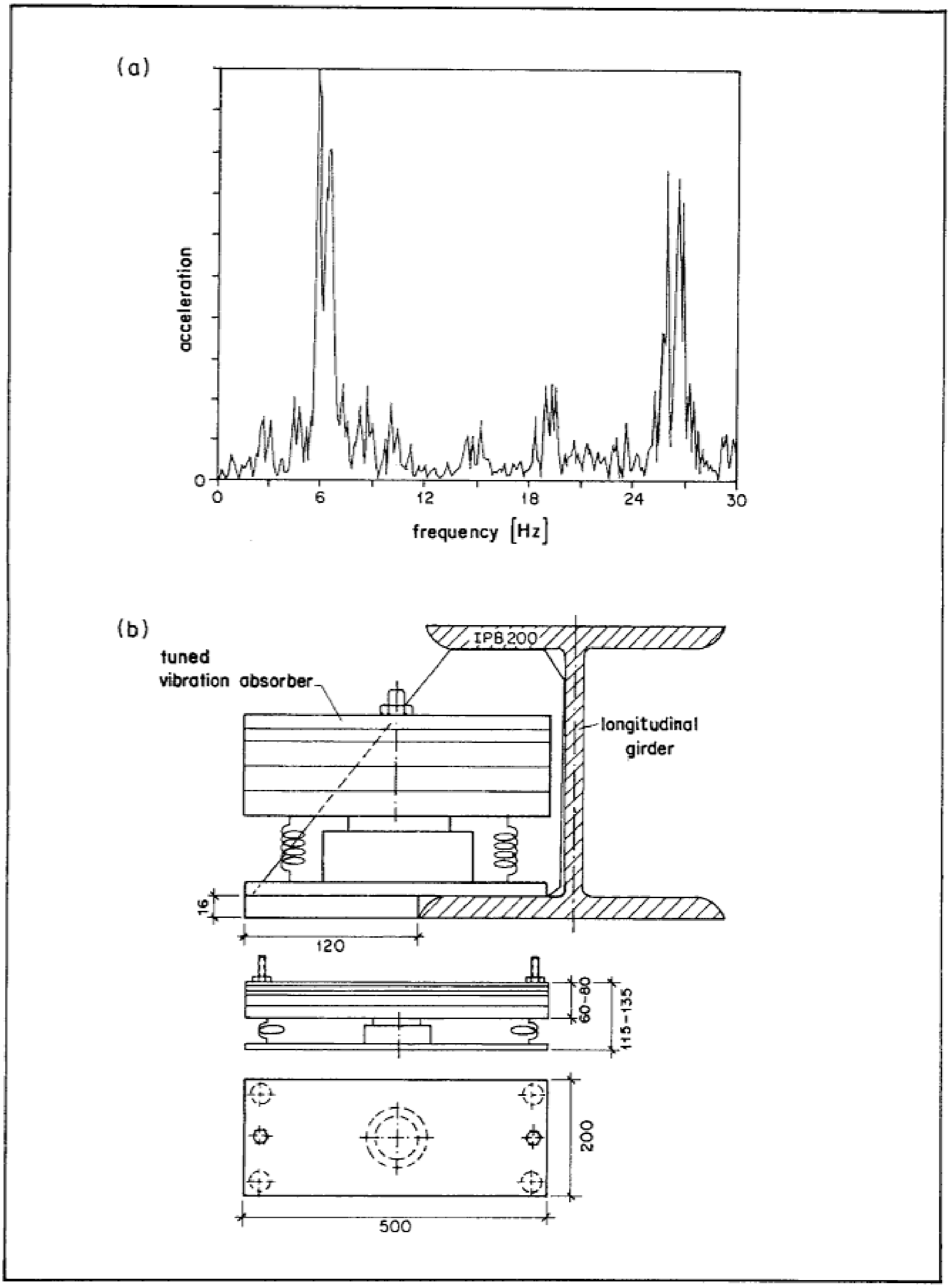


Fig. A6.1 Exhibition pavilion (from [2.23]):
(a) acceleration frequency spectrum
(b) details of the vibration absorbers on the main girders

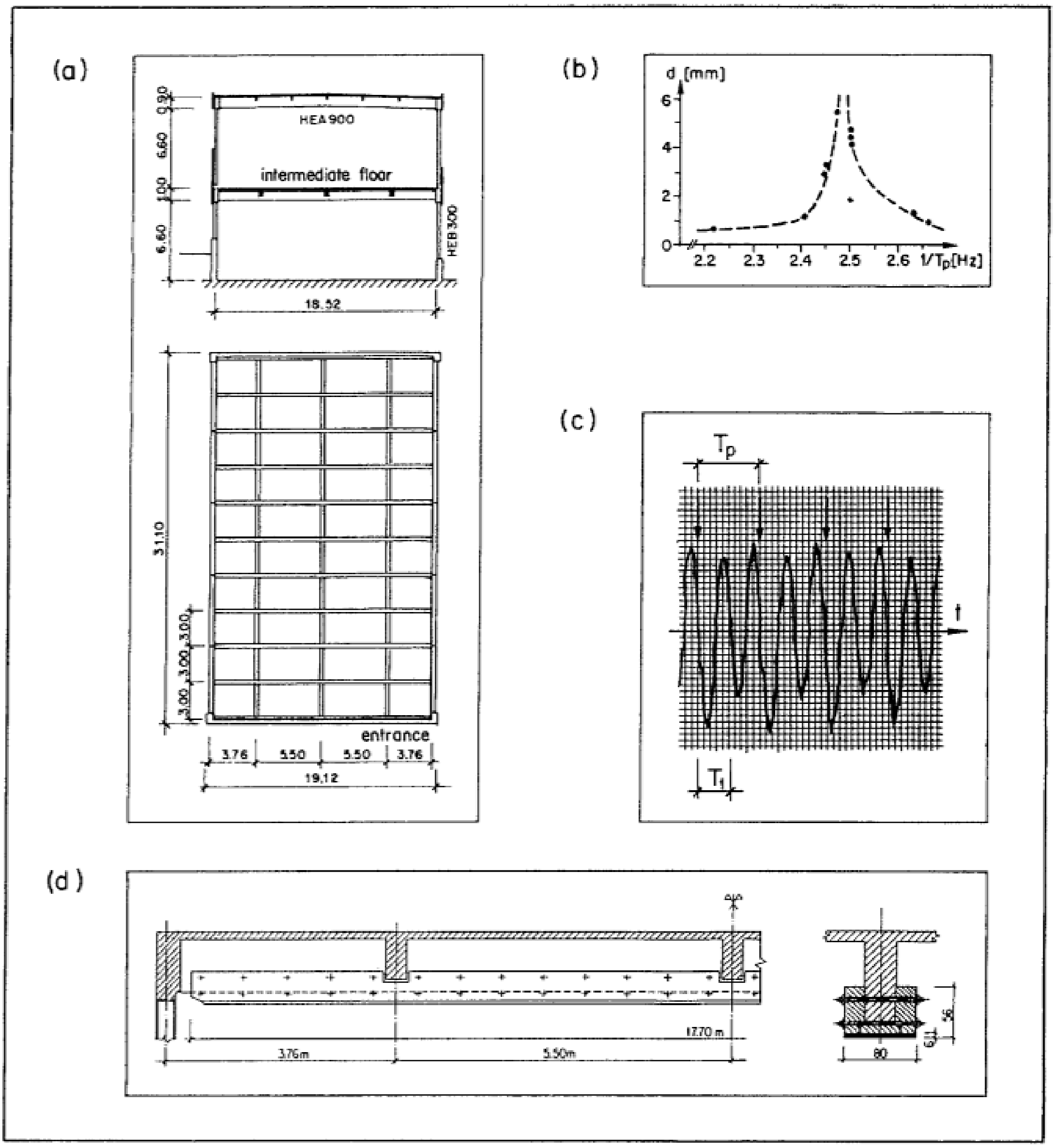


Fig. A7.1 Two-storey gymnasium hall (from [2.1]):
(a) cross-section of building and plan of the intermediate floor
(b) resonance curve of displacements
(c) detail of the displacement plot with the supposed incidents of loading with period T_p and structural period $T_1 = T_p/2$
(d) elevation and cross-section of the improvement carried out

Figure A7.1a shows the building in plan and section. The intermediate floor is constructed as reinforced concrete beams of 1 m height, 18.5 m span, and at 3 m spacing. They rest on edge beams supported by steel columns at 6 m intervals.

The dynamic properties of the floor and the magnitude of loading was measured during controlled tests in which fitness classes of up to 130 participants were instructed to perform various exercises at specified frequencies, synchronized by the loudspeaker-amplified beat of a metronome.

Resonance curves were evaluated for approximately 20 seconds during skipping excitations with frequencies varied from 2.0 to 3.2 Hz (Fig. A7.1b). The strongest vibration occurred at ~2.48 Hz. Since the fundamental frequency of the floor was ~4.9 Hz, resonance was excited by the 2nd harmonic of the load-time function as though the gymnasts were loading the floor on every other downward swing (with a certain phase shift) (Fig. A7.1c). The peak amplitudes were measured to ±5.5 mm vertical displacement, ±167 mm/s corresponding velocity, and 5.15 m/s^2 acceleration, i.e. 52% g (!).

The damping ratio was determined as 2.4% of critical damping by measuring the vibration decay after a commanded sudden standstill of the group. (The damping ratio would have been smaller without people on the floor.)

Improvement of the structure was imperative because the high stress level entailed the risk of fatigue deterioration of the reinforcing steel and of other damage; moreover, the serviceability was much reduced as in the upper hall fitness classes with the usual number of participants would have to be discontinued.

In order to avoid resonance with the 2nd harmonic of all possible load-time functions, the fundamental floor frequency had to be brought to some 7.5 Hz. The beams were stiffened by adding a bottom flange, made of a steel plate 800 mm x 60 mm and two side plates and filled with concrete. For a good bond the surface of the concrete beam was chipped and the flange with the concrete infill pressed against the beam by means of prestressed cross-bolting (Fig. A7.1d).

New excitation tests with fitness classes as before yielded a much reduced vertical displacement amplitude of ±0.3 mm, i.e. only 5% of the value prior to improvement, and did not produce resonance. The new fundamental frequency was measured to 7.3 ± 0.1 Hz and the damping ratio to 2%. As this was done in a drop-test with a 100 kg sandbag from a height of 1.8 m, this damping value cannot directly be compared to the previous one, in which many more people were on the floor. Further details, discussion of theory and improvement variants can be found in [2.1].

No. 8: Sports Hall Adjacent to Grandstand

The design project of a large two-storey sports hall involved 3 m high and 42 m spanning prestressed concrete beams (Fig. A8.1a). On one side the columns had also to support a grandstand.

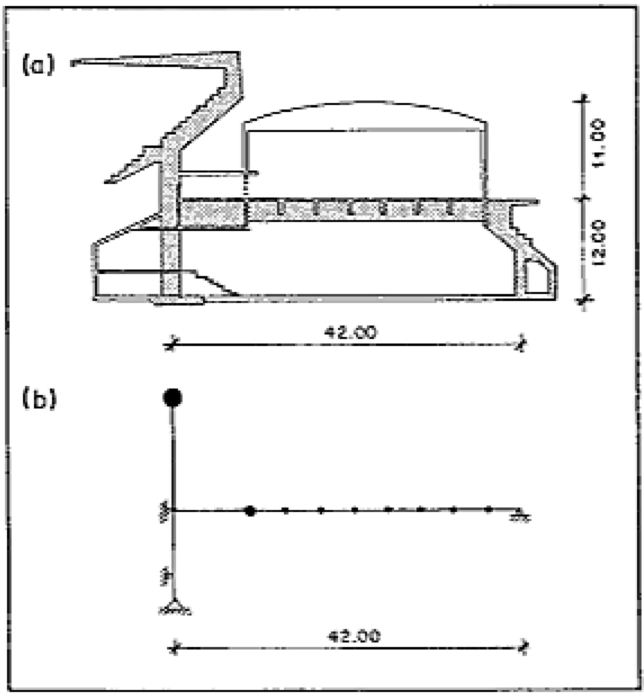


Fig. A8.1
Sports hall connected to grandstand:
(a) cross-section
(b) simple dynamic model

For a first assessment of the vibrational performance under rhythmic excitation as during fitness classes, dynamic calculations were applied to a simplified but conservative model structure (Fig. A8.1b) and yielded a fundamental frequency of ~2.4 Hz. The low value was mainly due to a very adverse contribution of the horizontally swinging grandstand mass. For the resonance case of the excitation frequency being equal to the structural frequency, a forced vibration computation with 1% damping and a user density of one person skipping per 4 m^2 indicated amplitudes of ±34 mm displacement, ±560 mm/s velocity, and 8.6 m/s^2 (!) acceleration.

The high stress level was expected to lead to damage or even structural collapse. Therefore, one had to consider substantial design modifications and strengthening with the goal of reaching a fundamental frequency of the floor of about 8 Hz.

No. 9: Gymnastics Room

In a high-rise building with a reinforced-concrete framed structure, gymnastic exercises held in the uppermost storey regularly produced considerable vibrations [A9.1]. They were felt in the uppermost six storeys, i.e. over about a quarter of the building height, and led to complaints of the office employees. At a skipping frequency of 2.2 Hz, maximum floor accelerations were measured to be some 1% g, a magnitude which can be quite disturbing.

According to a preliminary investigation, the office floors vibrate mostly with 4.4 Hz, even though their fundamental frequency lies around 10 Hz. This strange phenomenon found some explanation in the observation that the 4.4 Hz vibrations originate from the concrete columns acting as extensional springs, an effect which also could be confirmed through calculation. The vibration is excited by the 2nd harmonic of the loading function and migrates through the columns into the lower floors. This rather interesting finding needs further investigation, but any economical improvement would certainly be rather difficult to conceive.

No. 10: Concert Hall with Pop Music

A medium-size hall of 32.8 m x 55 m area exhibited strong floor vibration during pop concerts [A10.1]. These events were attended by up to 4000 people, a part of the audience was always enthused to activities such as clapping of hands, body rocking or light bouncing up and down to the music. Several hall areas without fixed seats brought about differences in density of the audience with a particular grouping (estimated 6 persons/m^2 maximum) towards the stage whenever the musicians were on show.

The plan and the sections of the floor structure in Fig. A10.1 reveal a hollow-block slab of 0.45 m thickness and about 450 kg/m^2 mass. The floor is supported by two rows of columns spaced 6.62 m apart, across which runs in longitudinal direction an internal joist (a cast-in strengthened steel section DIE 38). The structural action can thus be described as a predominantly crosswise-spanning continuous plate over three openings of 8.6 m / 14 m / 8.6 m. The floor is bisected by a dilatational joint. Designated originally as an exhibition hall, the building had merely been designed for static loads, so that the new problems are to be attributed to the unanticipated utilization for concerts.

Measurements were made at several concerts, showing that the vibration intensity depended more or less on the kind of music influencing the activities of the audience, and on its beat frequency f_m. For example, soft pop music (Band «Simple Minds») with $f_m \cong 2.0 \div 2.5$ Hz and 3500 relatively quiet spectators yielded at $f_m \cong 2 \div 3$ Hz a peak vibration amplitude of ±4.6 mm vertical displacement and 1.66 m/s^2 acceleration. During hard rock music (Band «Status Quo»), however, with a beat of $f_m \cong 2.1 \div 3$ Hz and an audience of ca. 2000 (of which about one third was quite electrified), the displacement at $f_m \cong 2.1$ Hz reached ±9.5 mm, the peak velocity ±177 mm/s and the maximum acceleration 2.7 m/s^2.

When excited by a local impulse, the unloaded floor showed a fundamental frequency of ~6.2 Hz. Dynamic loadings on a larger area would probably bring out the inherent structural action in the crosswise direction of the building and – with the additional mass of the audience – a fundamental frequency in the range of double the beat frequency of the music (ca. $4.0 \div 4.5$ Hz). This indicates a resonance-like excitation of the fundamental frequency of the structure, as found for the gymnasium floor of Case No. 7, by the 2nd harmonic of the audience's bouncing-time function.

The measured vibration magnitudes and concluded stressing of the floor exceeded the admissible values by far so that the hall had to be closed for pop concerts. For improvement, the floor would have to be supported by additional beams in the cross-direction, or temporary props would have to be fitted and removed before and after every concert, if the use of the lower storey was not to be greatly restricted. Finally, the discussion became inconsequential as another location for such concerts was found with a much higher floor frequency of about 12 Hz (unloaded).

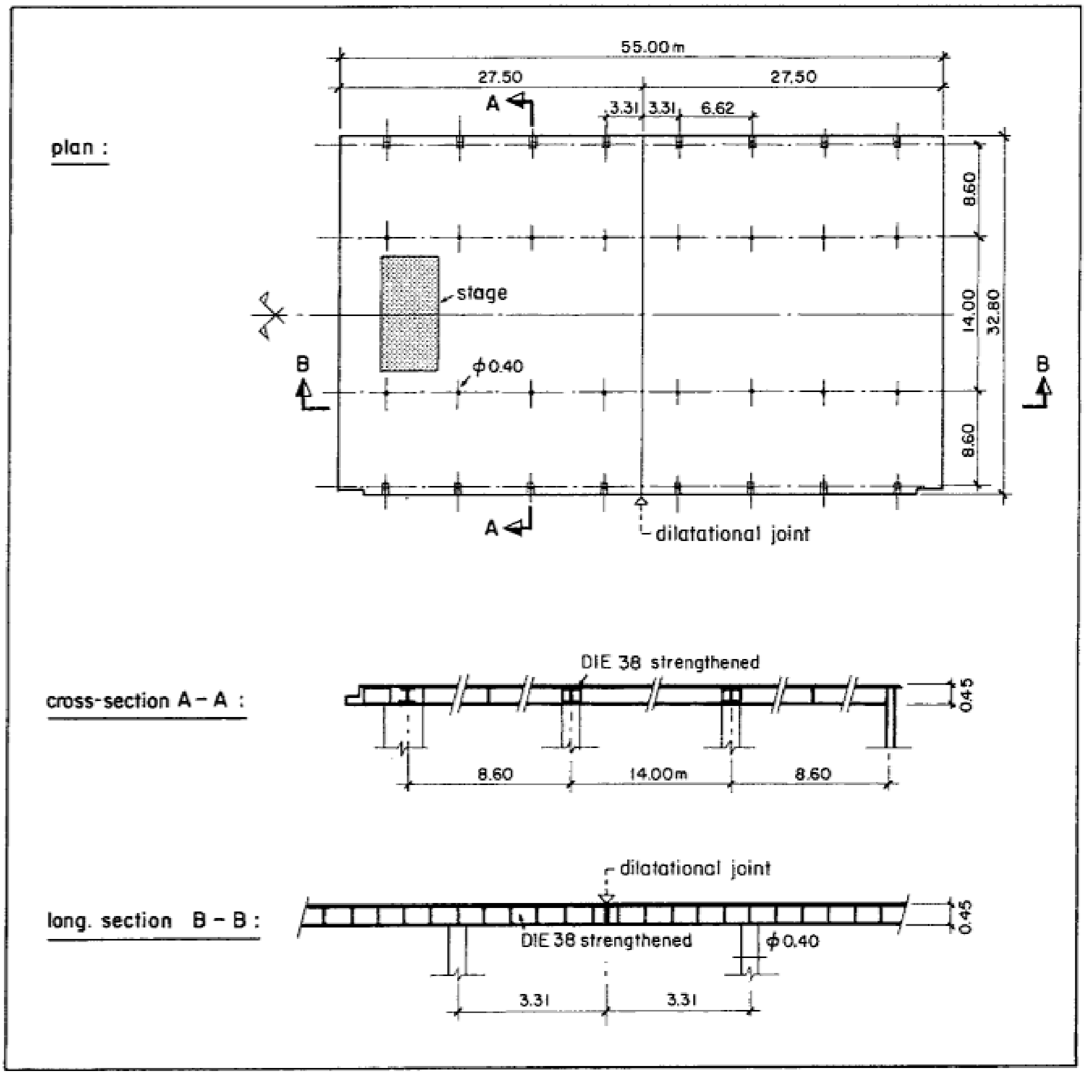


Fig. A10.1 Concert hall with hollow-cast floor structure (from [A10.1])

No. 11: Open-Air Theatre

An open-air amphitheatre housed a religious gathering of the «New Life People». While music was played through the loudspeakers, now and then the delighted audience on the main stand rose from their seats clapping their hands. Considerable vibrations occurred in the stand structure which, according to unsophisticated measurements, had a fundamental frequency of 2.5 Hz and a damping ratio of 1.5%. Installing additional columns improved the situation.

No. 12: 10 m Diving Platform

The high-diving platform of Fig. A12.1 showed severe vibrations when used by visitors to the pool, so it had to be closed and modified [A1.1].

Before and after the structural modifications, test persons excited controlled vibrations for velocity measurements at the two points P_1 and P_2 both during build-up and decay of the motion. From a Fourier power spectrum, the following frequencies of the fundamental modes were established:

	before modification	after modification
shaft nodding (P_1)	2.12 Hz	3.27 Hz
lateral shaft sway (P_1)	2.45 Hz	3.67 Hz
shaft twist (P_2)	2.37 Hz	3.71 Hz

After modification, much smaller vibration amplitudes were obtained allowing the facility to be reopened to the public.

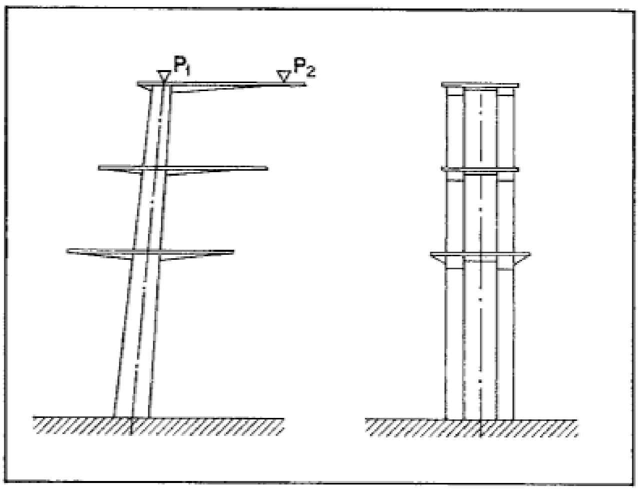


Fig. A12.1
10 m diving platform with measurement points P_1, P_2 (from [A1.1])

Of interest is the comparison between the post-modification frequencies and the lower frequency bound of 3.5 Hz recommended in Section 4.5. The shaft nodding mode is just below this bound. Neither rigid-body vibration nor slab vibration seem to have been critical, probably due to favourable ground conditions and the existing platform joist, respectively.

No. 13: 5 m Diving Platform

The shaft of a 3 m and a 5 m platform showed intolerably large twisting vibration soon after completion. The two reinforced-concrete platforms are each supported by two steel columns with cantelever beams, resulting in an eccentric layout of the facility with the four columns forming the corners of an open rectangular shaft. The columns were braced horizontally only near the foundation.

The design engineer, although he had calculated the fundamental bending frequency and had found it uncritical, must have overlooked the low torsional rigidity of his design. Subsequently, two concrete wall members were erected between the foundation and each platform parallel to the flights of stairs, combining an architecturally satisfying solution with a much improved dynamic behaviour.

No. 14: 10 m Indoor-Diving Platform

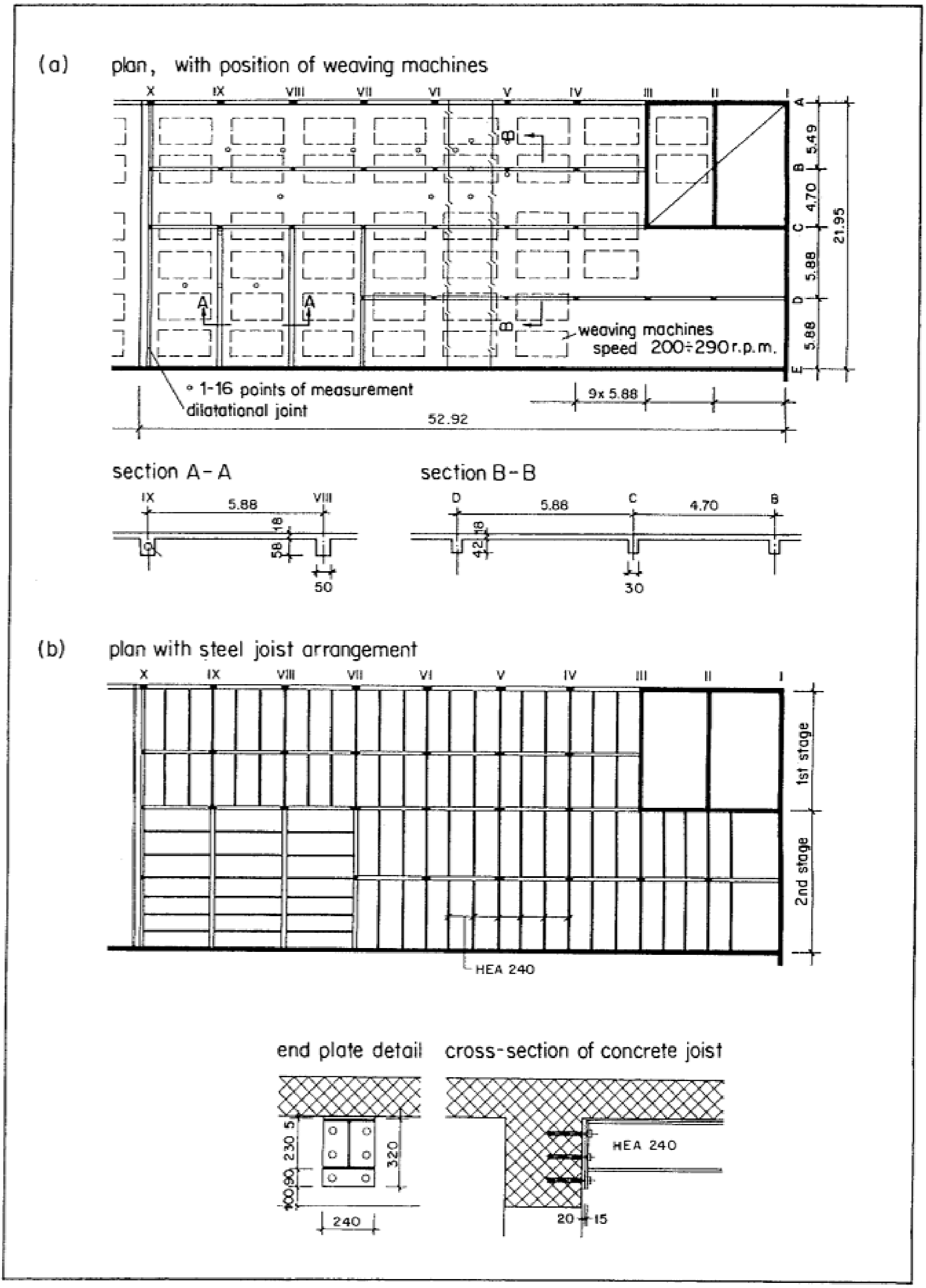
An indoor swimming pool with a 10 m platform of high architectural beauty in very slender construction turned out to be useless because of its pronounced responsiveness. Although many variants of design modifications were studied, no satisfactory modification could be found, and thus the platform had to be demolished in favour of a much stiffer structure.

More cases of inadequate platforms are described in [2.29].

No. 15: Weaving Factory Building

Strong vibrations occurred in the factory floor (with a basement underneath) of a twenty-year old weaving mill after modernization to faster running weaving machines [A15.1].

Thorough measurements found vibration velocities of the floor structure under the machines of up to 24 mm/s, exceeding by far the commonly used acceptance criteria of 10 mm/s of DIN 4150. The machines themselves vibrated horizontally with a double amplitude of 1.2 mm where only about 1 mm was tolerable. This limitation was not demanded by the machine construction but by the risk of weaving flaws in the production of delicate fabric due to weft irregularity.


(a) plan, with position of weaving machines
X IX VIII VII VI V IV III II I
A B C D E
5.49
4.70
5.88
5.88
21.95
B
B
A
A
weaving machines
speed 200÷290 r.p.m.
∘ 1-16 points of measurement
dilatational joint
9x 5.88
52.92
section A-A
IX 5.88 VIII
18
58
50
section B-B
D 5.88 C 4.70 B
18
42
30
(b) plan with steel joist arrangement
X IX VIII VII VI V IV III II I
1st stage
2nd stage
HEA 240
end plate detail
230
5
100
80
320
240
cross-section of concrete joist
HEA 240
20
15

The floor structure consists of an 18 cm thick reinforced-concrete slab spanning continuously 4.70 ÷ 5.88 m over joists, which in turn are supported at 5.88 m spacing, in a smaller area as prestressed joists at twice this spacing (Fig. A15.1a). The fundamental frequency of the floor was found to be ~21 Hz. Prior to modernization, the weaving machines had a speed of about 200 r.p.m., equivalent to a frequency of ~3.3 Hz. The new machines, however, run – depending on the produced fabric – at 240 to 290 r.p.m., equivalent to ~4.8 Hz at maximum. Apparently, the floor vibrations were excited mainly by higher (4th and 5th) harmonics of the load-time function, details of which are unknown.

Low tuning by mounting the machines on soft spring(-damper) elements was impossible – despite reduction in floor vibrations – because this would have aggravated the weaving machine vibrations beyond specified quality standards. Therefore, the floor was strengthened by fitting wide-flange steel sections every 2 m between the existing concrete joists (Fig. A15.1b). The upper flanges were glued with resin mortar to the underside of the concrete slab, and the section end plates were glued and dowelled to the concrete joists.

The improvement was successfull in that subsequent measurements showed the vibration velocity to exceed virtually nowhere the bound of 10 mm/s (with the exception of a cracked old concrete construction joint which demanded special attention). The weaving machine vibrations were also much smaller than before, and no further production-quality problems occurred.

No. 16: Factory Building with Plastics-Forming Machines

A newly erected factory building of about 17 m x 100 m in plan had two floors below street level, the gound floor, and one upper floor, with a distinct design change at street level (Fig. A16.1): The floor structure of the basement is a joistless flat slab of 22 cm structural thickness and 3 cm cement topping, supported at 5.20 m column spacing in the longitudinal direction, and at 5.60 m / 5.20 m / 5.70 m spacing in the lateral direction of the building. The floor structure above the ground floor, however, is a 10 cm corrugated-sheet steel composite construction, supported by steel frames of sections HEA 700 as primary and by 140 mm steel sections as secondary girders. The frames (without intermediate props shown in Fig. A16.1) span about 16.5 m, while the secondary girders lie 2.30 m apart.

After completion, various plastics-production machines were installed in the ground floor, among others extrusion presses, cutters and hot-moulding presses, which all caused strongly felt floor vibrations. A measuring campaign was initiated, since the factory owner intended to install similar machinery on the first floor as well, despite its planned use as material store room and for the location of lighter machinery only.

Fig. A15.1 Weaving factory building (from [A15.1]):
(a) floor plan with location of machines and cross-sections
(b) lay-out and details of installed steel girders

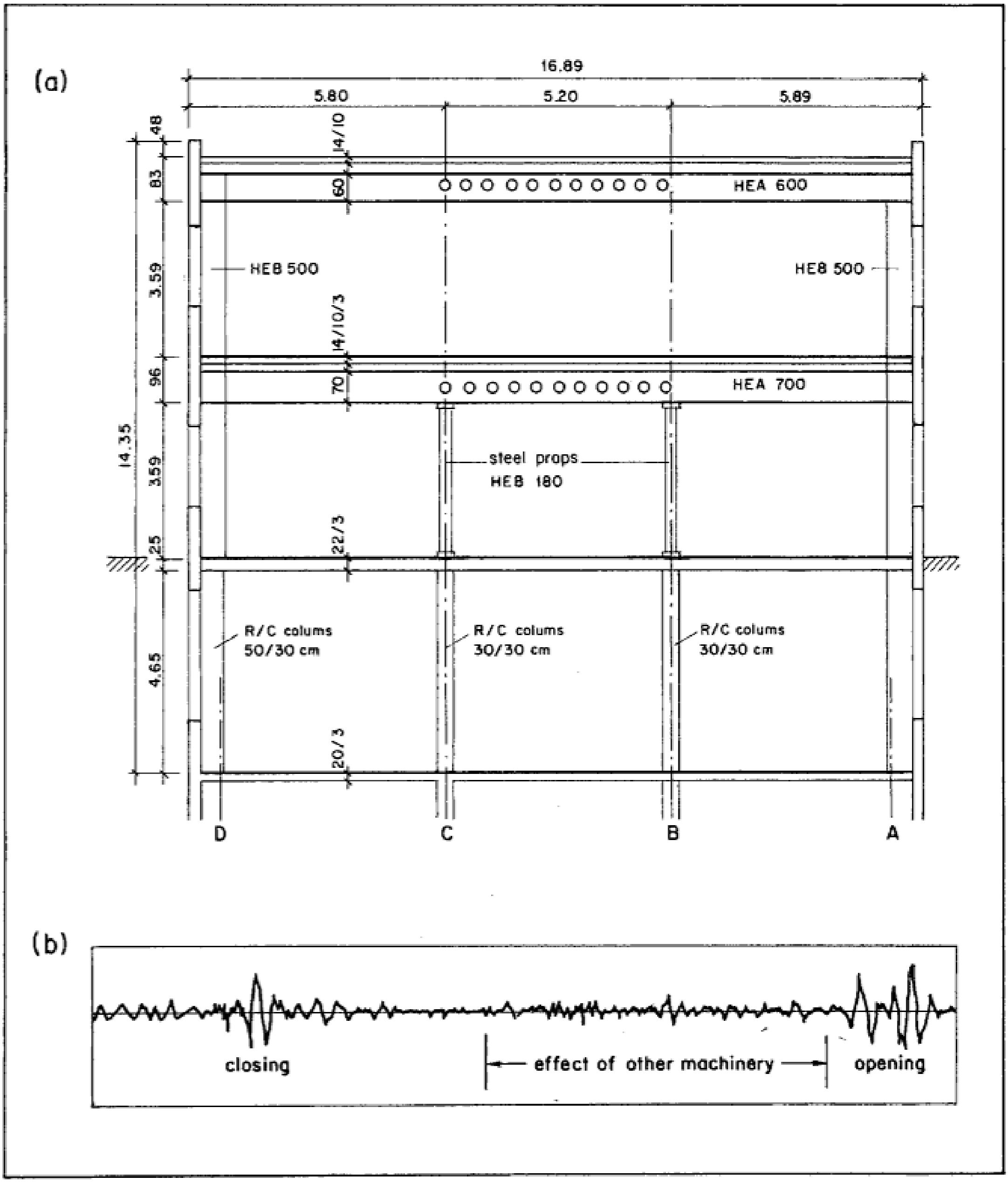


Fig. A16.1 Factory building with plastics-forming machines:
(a) cross-section
(b) induced vibration of the floor slab above the basement

It soon became obvious that the main sources of vibration was the stamping action of the hot-moulding presses producing egg-boxes, cups, containers and similar products: In a double motion, a table of e. g. 80 cm x 80 cm area moves upwards and is met by the stamp coming downwards, resulting in a strong impact at closing as well as opening of the moulds. The lifting rate was counted to 13 ÷ 21 per minute, i. e. 26 ÷ 42 impacts per minute, that is a period of 1.4 ÷ 2.3 s.

The concrete floor, including the weight of the machinery, had a fundamental frequency of ~19 Hz, i. e. a period of ~0.05 s. With this large difference in period, the floor slab is not excited to a forced periodic vibration but loaded more or less transiently by single thrusts with a free vibration phase (in a conglomerate of frequencies) to almost complete decay before the next impact. Hence, the physical effect is that of single impulses (Fig. A16.1b), even though the rhythmic repetition suggests a periodic loading.

The velocity of the floor vibration amounted to 5 mm/s, which was irrelevant to the floor structure's integrity. The K intensity, however, which describes the human perception according to VDI Directive 2057 [4.14], can be calculated to 3.5 (see Section 4.3). This intensity, when one is exposed to it for hours, is categorized as strongly noticeable and unpleasant.

The investigation of the floor structure above the ground floor revealed frequencies of 6.3 Hz for the frame and ~12 Hz for the secondary composite structure, much less stiff than the basement floor. Installing there plastics hot-moulding presses was impossible without extensive strengthening. After careful consideration, the planned improvement was to install internal columns in the ground floor (at intervals of about one third of the span) with the consequence of restricted production area, and to provide a 20 cm concrete slab between the footings of the critical machines and the composite floor structure (Fig. A16.1a).

No. 17: Factory Building with Metal-Forming Press

The reinforced-concrete floor slab of a factory building had been designed for static loads according to yield-line theory [A10.1]. Thus, the 5.90 m (resp. 5.31 m) x 7.40 m large floor bays between joists had a thickness of just 18 cm (Fig. A17.1).

The floor vibrated considerably under the impacts of a metal-forming press it supported, the peak vibration displacements being measured to ±1.44 mm at a floor frequency of ~13 Hz. Fears that the structure might be overstressed turned out to be unfounded, but the vibration of the press led to violation of specified product tolerances.

The required strengthening was realized in a first step by gluing two steel girders HEA 300 to the underside of the floor slab, reducing the displacement amplitude to about 60%. As the remaining vibrations were still too large, the steel girders were replaced by concrete beams with dowel connection to the floor, and the spans were reduced by haunching the existing columns (Fig. A17.1). This brought the floor displacement amplitude down to about 40% of its initial value.

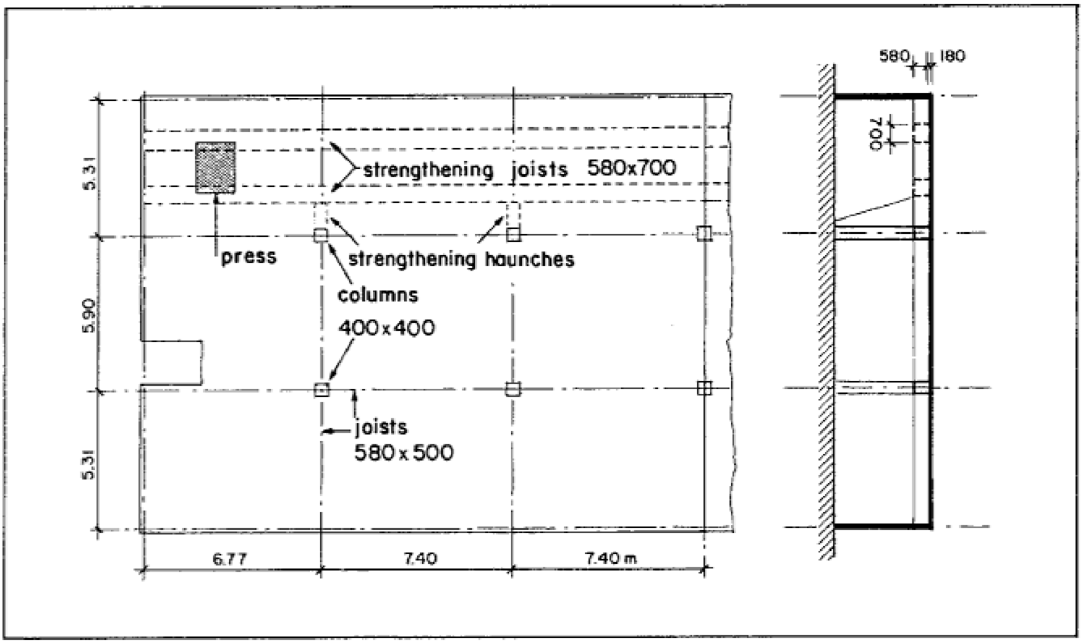


Fig. A17.1 Plan and section of a factory floor with metal-forming press (from [A10.1])

No. 18: Factory Building with Automatic Boring Machine

Boring machine stations in the metal-working industry process the clamped workpiece with a moveable boring head which is programme-controlled in spatial motion, and its tools are changed automatically by a robot arm.

In this case, a prototype was put in operation and exhibited vibration of the boring head which caused the working tolerances to be exceeded. To reduce the head vibration, the machine shaft was filled with concrete, with the effect that the machine's intrinsic vibration was reduced but that now the enlarged mass excited severely the reinforced-concrete flat floor slab on which the machine stood; this in turn again led to machine vibrations with tolerance violation.

The vibrations were reduced by doubling the time scale of the programme which controlled the working motions. In the long run, however, a 50% reduction in productivity was not acceptable, therefore a strengthening of the floor structure was envisaged. Owing to refined programming in which the motion was only slowed down during the phase of abrupt thrusting motion of the boring head, strengthening of the structure was «electronically» circumvented and postponed to the time when the installation of more automatic stations would call for a comprehensive review of the vibration problem.

No. 19: Dairy with Butter Churns

On the first floor of a dairy building four butter churns were installed. The floor slab supporting them is a 52 cm thick prestressed hollow-block slab spanning about 13 m. The blocks are formed by top and bottom plates of 10 cm thickness and 25 cm wide ribs (in the principal load-carrying direction) at 1.0 m spacing. These ribs also contain the prestressing steel.

Each of the four butter churns rotates during production around a horizontal axis and exerts thereby dynamic loads via their trestle to the floor. One cycle of 1.5 to 1.75 hours produces between 1.3 and 2.5 tons of butter. The largest load results from 10 to 20 minutes of kneading as the last process of the cycle: At every revolution of the churn of 3000 litre capacity of cream (one of the four machines even had 6000 litres), the solidifying mass of the nearly finished butter drops from the cover onto the bottom; this exerts an impulse on the trestle and the floor.

For an investigation of the floor response, four acceleration pickups were installed to transmit the vibration during production, from which the floor velocity was obtained by integration. Its peak value was 6 mm/s near the smaller butter churns, while near the big one several cycles exceeded 10 mm/s, the largest peak reaching 21 mm/s. A frequency analyzer extracted two predominant floor frequencies from the records, the first at 12.5 Hz and the second at 23.5 Hz.

As regards potential overstressing of the floor structure, Part 3 of DIN 4150 [4.4] points out that vibration velocities of more than 10 mm/s may cause deterioration, so that a calculational check of the stress level in the structural member is needed.

No. 20: Factory Building with Stamping Machines

In a factory building, in which stamping machines with automatic retraction were operated, the floor vibrated to a degree that the owner was afraid of damage to the reinforced-concrete structure [4.20]. The vibrations occurred although the machines were mounted on spring(-damper) elements (according to Fig. 3.6b), which, however, could not be tuned low enough as then the machines would malfunction.

The vibrations were measured to clarify whether the structural integrity was at risk and what kind of vibration isolation might be feasible. The fundamental floor frequency was 17 Hz, while the presses worked at 43 and 27 lifts/min, respectively. It was found that every lift exerted an impulse to the floor slab which subsequently vibrated freely in its fundamental mode.

The peak velocities of the (predominantly vertical) floor vibrations were measured as 3.21 mm/s at 43 lifts/min and as 1.60 mm/s at 27 lifts/min. To these values, the Standard DIN 4150 (Part 2) [4.1] assigns perception intensities of KB = 2.2 and KB = 1.1, respectively (see Section 4.3). The conclusions according to the Standard were the following:

(1) The magnitude of floor vibrations did not jeopardize the integrity of the structure.

(2) An intensity of KB = 2.2 is qualified as easily noticeable, but it would not impair efficiency or well-being of the operating personnel.

(3) Reduction of the floor vibration without enhancing the machine vibration would necessitate low tuning with a stabilizing mass (see Fig. 3.6c).

(4) Some reduction could also have been achieved by overhauling the machines (elimination of bearing clearances, optimizing the shape of cams, etc.).

No. 21: Production and Assembly Building with Milling and Grinding Machines

The operation of a milling machine in an upper floor of a production and assembly building caused considerable vibration over a large area of the reinforced-concrete floor slab having a fundamental frequency of ~10 Hz [A20.1].

Close to the milling machine the vertical vibration velocity of the floor was measured as 1.3 ÷ 1.6 mm/s, depending on the cutting depth varying between 3 and 6 mm. At the spot where a new grinding machine was intended to be installed, the velocities were still 0.2 ÷ 0.3 mm/s, which was considered too large for an undisturbed independent operation.

The vibration transmission could be efficiently curbed by supporting the milling machine on spring(-damper) elements (see Fig. 3.6b): Close to the machine the peak vertical floor velocities were reduced to 0.45 and 0.60 mm/s for cutting depths of 3 and 6 mm, respectively, and at position of the grinding machine to 0.5 mm/s. When in operation, no interference of the two machines was observed.

No. 22: School Building with Heat Pump

The vibrations and noise emanating from the heating system in the basement of a school building disturbed the classes [A20.1].

Measurements of vibration and noise showed that the vibration, mainly of the floor above the basement, was largely caused by insufficient isolation of a heat pump and that part of the noise had been also due to transmission of evaporator pump vibrations through its supports and the pipework. No isolation to structure-borne noise was provided between the heat reservoirs and the building. The vibration isolation of the heat pump consisted of a rubber mat between the structure and the concrete base on which the pump assembly was mounted (see Fig. 3.6c).

The peak vibration velocities were measured as 10 mm/s on this base, 0.31 mm/s on the floor adjacent to the base, and 0.21 mm/s on the floor above the basement. For the dominant frequency of 16.5 Hz – which turned out to be the operating frequency of the

Appendix B

FUNDAMENTALS OF VIBRATION THEORY

Appendix B provides a summary of fundamental structural dynamics sufficient to understand the material presented in Chapters 2 and 3. Examples are given for common practical problems. Problems beyond this scope will necessitate the consultation of standard textbooks on the subject.

The following is divided into the sections:

B 1 Systems with a single degree of freedom (SDOF)
B 2 Systems with many degrees of freedom (MDOF)
B 3 Systems with distributed parameters
B 4 Harmonic analysis
B 5 Frequency tuning
B 6 Structural behaviour under impulsive loading
B 7 Tuned vibration absorbers
B 8 Material properties under dynamic loading.

The differential equation of motion is usually written in complex state variables which include implicitly the phase angle. In some applications, however, the classical notation with sine and cosine functions is preferable. The agreed nomenclature for displacement is accordingly

$u(t)$ = complex displacement
$x(t)$ = real displacement.

B 1 SDOF SYSTEMS

As an introduction to more complicated systems, we consider a concentrated mass acting on a spring(-damper) element, Fig. B1.1. It helps to illustrate some very fundamental relations of general applicability.

B 1.1 Free Vibration

One speaks of a free vibration if a dynamic system undergoes a transient motion without external excitation. This state, assuming that damping is proportional to the vibration velocity, is described by the differential equation:

$$m \cdot \ddot{x} + c \cdot \dot{x} + k \cdot x = 0 \tag{B1.1}$$

where:

x = displacement
m = mass
c = damping constant
k = spring stiffness.

The terms in Eq. (B1.1) represent forces standing in equilibrium:

$m \cdot \ddot{x}$ = inertial force
$c \cdot \dot{x}$ = viscous damping force
$k \cdot x$ = elastic restoring (spring) force.

The differential equation (B1.1) is tried to solve with an exponential type of solution, which will turn out later to be complex when damping is present:

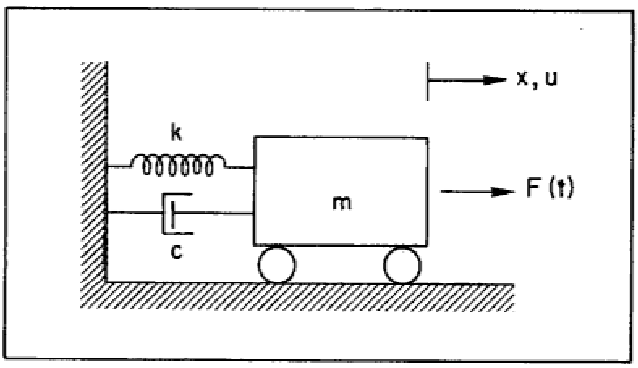


Fig. B1.1
Dynamic system with a single degree of freedom (SDOF oscillator)

$$x(t) = u(t) = U \cdot e^{s \cdot t} \tag{B1.2}$$

Inserting the trial solution into Eq. (B1.1), yields the characteristic polynomial

$$m \cdot s^2 + c \cdot s + k = 0 \tag{B1.3}$$

The following parameters are substituted:

$$\omega_1^2 = \frac{k}{m} \tag{B1.4}$$

$$\xi = \frac{c}{2 \cdot m \cdot \omega_1} \tag{B1.5}$$

Their physical meaning is that ω_1 is the *undamped circular eigenfrequency* and ξ the *damping ratio*. (Note that here and in the following, the subscript «1» refers to the SDOF system and does not necessarily imply an equivalence to the fundamental frequency of an actual structure.)

Then the polynomial (B1.3) becomes

$$s^2 + 2 \cdot \xi \cdot \omega_1 \cdot s + \omega_1^2 = 0 \tag{B1.6}$$

with the two roots:

$$s = -\xi \cdot \omega_1 \pm \sqrt{(\xi \cdot \omega_1)^2 - \omega_1^2} = \omega_1 \cdot (-\xi \pm \sqrt{\xi^2 - 1}) \tag{B1.7}$$

The damping ratio ξ relates the existing damping to the *critical damping*

$$c_{crit} = 2 \cdot m \cdot \omega_1$$

at which a vibration decays from the maximum displacement straight to the rest position without actually performing a vibration at all («aperiodic vibration»). Typical damping ratios in civil engineering are

$$\xi \ll 1$$

Thus, the roots of (B1.7) are complex:

$$s = \omega_1 \cdot (-\xi \pm i \cdot \sqrt{1 - \xi^2}) \tag{B1.8}$$

with:

$$i = \sqrt{-1}$$

The frequency $\omega_D = \omega_1 \cdot \sqrt{1-\xi^2}$ is the eigenfrequency of the damped SDOF system and is always smaller than the undamped eigenfrequency ω_1. As the difference is small for common damping ratios up to $\xi = 10 \div 15\%$, one usually sets (although not used in the following development):

$$\omega_D = \omega_1 \cdot \sqrt{1-\xi^2} \cong \omega_1 \tag{B1.9}$$

If Eq. (B1.8) is substituted into (B1.2), the transient displacement solution is obtained:

$$u(t) = e^{-\xi \cdot \omega_1 \cdot t} \cdot (U_A \cdot e^{i \cdot \omega_D \cdot t} + U_B \cdot e^{-i \cdot \omega_D \cdot t}) \tag{B1.10}$$

with two integration constants U_A and U_B to be determined from the initial conditions.

With aid of the Eulerian equations, the exponential function can be expressed in two harmonic terms:

$$e^{\pm i \cdot \omega_D \cdot t} = \cos(\omega_D \cdot t) \pm i \cdot \sin(\omega_D \cdot t) \tag{B1.11}$$

so that (B1.10) can alternatively be written as

$$x(t) = e^{-\xi \cdot \omega_1 \cdot t} \cdot \left(U_A \cdot [\cos(\omega_D \cdot t) + i \cdot \sin(\omega_D \cdot t)] + U_B \cdot [\cos(\omega_D \cdot t) - i \cdot \sin(\omega_D \cdot t)] \right)$$

By substituting the complex vibration constants U by real constants X

$$U_A = \frac{X_B - i \cdot X_A}{2} \quad \text{and} \quad U_B = \frac{X_B + i \cdot X_A}{2}$$

the displacement may be rewritten in real format x(t) as

$$x(t) = e^{-\xi \cdot \omega_1 \cdot t} \cdot \left(X_A \cdot \sin(\omega_D \cdot t) + X_B \cdot \cos(\omega_D \cdot t) \right) \tag{B1.12a}$$

The physical meaning of this equation may be elucidated by introducing:

$$X_1 = \sqrt{X_A^2 + X_B^2} = \text{amplitude constant}$$

$$\varphi_1 = \arctan \frac{X_B}{X_A} = \text{phase angle}$$

or alternatively by the inverse:

$$T_1 = \frac{1}{f_1} = 2 \cdot \pi \cdot \sqrt{\frac{m}{k}} \tag{B1.15}$$

which is the *fundamental period* of vibration of the structure.

B 1.2 Forced Vibration

A forced vibration results when an external transient load F(t) acts on the mass, extending the differential Eq. (B1.1) by a known forcing term on the right-hand side:

$$m \cdot \ddot{x} + c \cdot \dot{x} + k \cdot x = F(t) \tag{B1.16}$$

The free vibration (B1.10) now constitutes only one part, the *homogeneous solution*, on which the force-dependent, *particular solution* is superimposed.

The particular solution becomes especially simple if the time function of the force is harmonic.

Harmonic Loading

The harmonic loading function may be caused by machinery with rotating parts. For the so-called constant-load excitation (Section 3.2.1), the harmonic force takes the form

$$F(t) = F_o \cdot \sin(\omega_o \cdot t)$$

in which:

F_o = amplitude of the loading
ω_o = circular frequency of the loading.

The differential equation (B1.16) of forced motion of a damped SDOF system then becomes:

$$m \cdot \ddot{x} + c \cdot \dot{x} + k \cdot x = F_o \cdot \sin(\omega_o \cdot t) \tag{B1.17}$$

The homogeneous solution corresponds to Eqs. (B1.10) and (B1.12), respectively, the particular solution is of the form:

$$x(t) = X_C \cdot \sin(\omega_o \cdot t) + X_D \cdot \cos(\omega_o \cdot t) \tag{B1.18}$$

The second term with X_D is necessary as x(t) and F(t) usually are not in phase.

In contrast to (B1.17), the loading function of (B1.22) now contains an imaginary contribution as well (remember the Eulerian equation (B1.11)). The homogeneous solution is again Eq. (B1.10), while the particular solution becomes

$$u(t) = \sum_{j=0}^{\infty} U_j \cdot e^{i \cdot \omega_j \cdot t} \tag{B1.23}$$

For linear-elastic vibrational systems the total vibration response can be obtained by *superposition of the responses of individual harmonic components*. (This is implied in the following without writing the sum over all j.)

Inserting this with use of ω_1 (B1.4) and ξ (B1.5) for the SDOF system properties, the differential equation (B1.22) becomes for the j-th forcing component:

$$-\omega_j^2 \cdot U_j \cdot e^{i \cdot \omega_j \cdot t} + 2 \cdot \xi \cdot \omega_1 \cdot i \cdot \omega_j \cdot U_j \cdot e^{i \cdot \omega_j \cdot t} + \omega_1^2 \cdot U_j \cdot e^{i \cdot \omega_j \cdot t} = \frac{A_j}{m} \cdot e^{i \cdot \omega_j \cdot t} \tag{B1.24}$$

The common exponential term dropping out, the complex displacement amplitude U_j is derived to

$$U_j = \frac{A_j}{m} \cdot \frac{1}{\omega_1^2 - \omega_j^2 + 2 \cdot \xi \cdot i \cdot \omega_1 \cdot \omega_j} \tag{B1.25}$$

Dividing both numerator and denominator by ω_1^2 yields

$$U_j = \frac{A_j}{k} \cdot \frac{1}{1 - \omega_j^2/\omega_1^2 + 2 \cdot \xi \cdot i \cdot \omega_j/\omega_1} \tag{B1.26}$$

the amplitude of which becomes for positive real A_j

$$|U_j| = \frac{A_j}{k} \cdot \frac{1}{\sqrt{(1 - \omega_j^2/\omega_1^2)^2 + (2 \cdot \xi \cdot \omega_j/\omega_1)^2}} \tag{B1.27}$$

The phase angle to the j-th component of the loading is

$$\varphi_j = \arctan \frac{2 \cdot \xi \cdot \omega_j \cdot \omega_1}{\omega_1^2 - \omega_j^2} \tag{B1.28}$$

One obtains the total solution, considering all forcing functions, by superposing the particular part (B1.23) on the homogeneous part (B1.10):

$$u(t) = e^{-\xi \cdot \omega_1 \cdot t} \cdot (U_A \cdot e^{i \cdot \omega_D \cdot t} + U_B \cdot e^{-i \cdot \omega_D \cdot t}) + \sum_{j=0}^{\infty} U_j \cdot e^{i \cdot \omega_j \cdot t} \tag{B1.29}$$

As already mentioned for Eq. (B1.21), the homogeneous solution accounts for the initial conditions. As this part decays with the factor $e^{-\xi \cdot \omega_1 \cdot t}$ exponentially due to the damping ξ, the transient vibrational behaviour becomes irrelevant in many practical cases, leaving just the particular part

$$u(t) = U_j \cdot e^{i \cdot \omega_j \cdot t} \tag{B1.30}$$

as the steady-state vibration under the j-th component of a general periodic forcing function.

With the aid of the response amplitude (B1.27), one can define a dynamic magnification factor to the static displacement under the load component A_j:

$$V_k(\omega_j) = \frac{|U_j|}{A_j/k} = \frac{1}{\sqrt{(1 - \omega_j^2/\omega_1^2)^2 + (2 \cdot \xi \cdot \omega_j/\omega_1)^2}} \tag{B1.31}$$

This dynamic-to-static ratio of peak displacement depends on the damping ratio ξ and on the ratio between the loading and the oscillator frequency, ω_j/ω_1. Plotting the dynamic magnification factor versus ω_j/ω_1 for various damping ratios ξ yields the so-called *frequency response curves*, Fig. B1.3. They allow the rapid calculation of the expected displacement amplitude for a loading at a given frequency or, respectively, of the critical structural frequency range.

If the damping is small, the dynamic magnification has its maximum at resonance, i.e. when $\omega_j = \omega_1$:

$$V_{k,max} = V_k(\omega_j = \omega_1) \cong \frac{1}{2 \cdot \xi} \tag{B1.32}$$

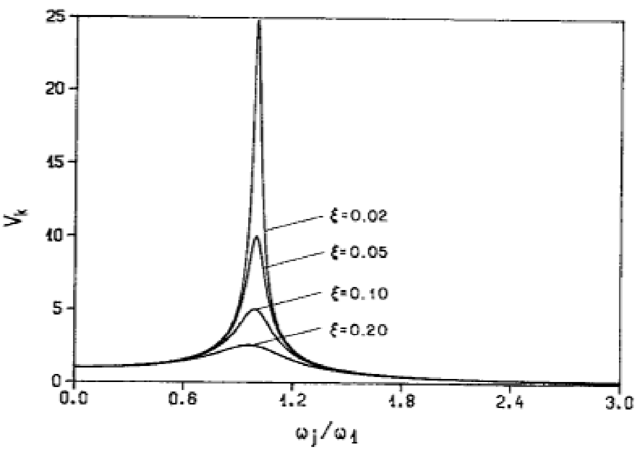


Fig. B1.3
Dynamic magnification with rel. excitation frequency for various damping ratios ξ (frequency response curves)

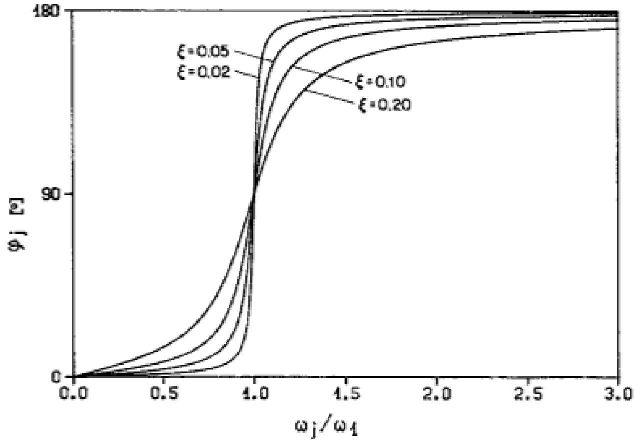


Fig. B1.4
Phase shift of the displacement of an SDOF oscillator with respect to the harmonic load

Figure B1.4 shows the phase angle as defined in Eq. (B1.28). Note that at resonance ($\omega_j = \omega_1$) the loading and the displacement response exhibit a phase shift of $\varphi_j = 90°$.

General Loading

If the loading follows a general periodic (but non-harmonic) or a transient time function, then a closed-form particular solution cannot be found.

One way to calculate the SDOF response to such a loading is to work in the *frequency domain*. Firstly, the loading is decomposed into its harmonic components by Fourier analysis (see Appendix B 4), transforming the loading function into the frequency domain with complex Fourier amplitudes A_j. Then, the appropriate values of the frequency response curve (from Eq. (B1.26)) are multiplied with the Fourier amplitudes of the loading and superimposed with the individual phase angles of the components. Having thus obtained the displacement response in the frequency domain, the inverse Fourier transformation yields the response in the time domain.

Example B1.1

The SDOF oscillator of Fig. B1.1 is excited by a half-sine impulse loading as used in the later Example B4.3 (Fig. B4.1) with $T_p = 0.4$ s, resulting e.g. from a gymnast skipping with $f_s = 2.5$ Hz. The steady-state displacement response in the time domain is to be found.

The system has the properties:

$$
\begin{aligned}
m &= 10{,}000 \text{ kg} \\
k &= 10 \text{ kN/m} \\
\xi &= 2\ \% \\
f_1 &= 5 \text{ Hz} \qquad \text{i.e. } \omega_1 = 2\cdot\pi\cdot f_1 = 31.3 \text{ rad/s.}
\end{aligned}
$$

For simple hand calculation purposes, one may assume that the response is dominated by the loading component corresponding to resonance with the structural system. Then from Fig. B4.1c the respective Fourier amplitude at ω_1 is $A_2 = 0.42$ kN, which is superimposed to the static displacement (at $\omega_o = 0$) under $A_o = 0.64$ kN and yields

$$u_{max} = \frac{1}{k} \cdot \left(A_o + \frac{1}{2 \cdot \xi} \cdot A_2\right) = \frac{1}{10{,}000} \cdot \left(0.64 + \frac{0.42}{2 \cdot 0.02}\right) = 0.00111\ \mathrm{m} \tag{B1.33}$$

Note that about 94% of the total displacement is due to the dynamic effect, giving $u_{dyn} \cong \pm 1.05$ mm. Because of the rather wide spread of frequencies in this case, the total response amplitude (Fig. B1.7) is reasonably well approximated by Eq. (B1.33).

An equivalent approach for determining the response to a general loading function is based on Duhamel's *convolution integral*. It describes the response of the SDOF oscillator to a series of infinitely short impulses (Dirac functions). The response velocity due to such an impulse of duration t_p can be calculated from the equation of momentum to be

$$\dot{x}(t_p) = \frac{1}{m} \cdot \int_o^{t_p} F(t) \cdot dt \tag{B1.34}$$

As the duration of the impulse is infinitely short, i.e.

$$\bar{t} = t - t_p \cong t \tag{B1.35}$$

and the displacement $x(t{=}t_p)$ is assumed to be zero (i.e. $X_B = 0$ at $\bar{t} = 0$), the subsequent free response $x(t)$ can be calculated from Eq. (B1.12a) with Eq. (B1.34) to

$$x(t) \cong x(\bar{t}) = e^{-\xi \cdot \omega \cdot \bar{t}} \cdot X_A \cdot \sin(\omega_D \cdot \bar{t}) = e^{-\xi \cdot \omega \cdot \bar{t}} \cdot \frac{\dot{x}(t_p)}{\omega_1} \cdot \sin(\omega_D \cdot \bar{t})$$

$$x(t) \cong e^{-\xi \cdot \omega_1 \cdot t} \cdot \frac{1}{m \cdot \omega_1} \cdot \left(\int_o^{t_p} F(t) \cdot dt\right) \cdot \sin(\omega_1 \cdot t) \tag{B1.36}$$

while neglecting the damping influence on the SDOF eigenfrequency according to Eq. (B1.9).

The integral contains the momentum of the impuls, which for the Dirac function has the value «one» with the appropriate units.

B 2 MDOF SYSTEMS

For the calculation of vibrational response, a structure has to be abstracted to an appropriate simpler model with a reduced number of degrees of freedom. A beam, for example, can be modelled as a series of lumped masses along its central axis, or a framed structure with a lumped mass for each storey on a vertical bar. This is denoted as a multi-degree of freedom (MDOF) system.

B 2.1 Free Vibration

Without external excitation, an MDOF system vibrates in its *natural modes* each of which is analogous to an SDOF oscillator. The differential equation of motion for free vibration is written in real matrix-form as follows:

$$[M]\cdot\{\ddot{x}\} + [C]\cdot\{\dot{x}\} + [K]\cdot\{x\} = \{0\} \tag{B2.1}$$

where:

$[M]$ = mass matrix (diagonal for lumped masses)
$[K]$ = stiffness matrix (symmetric)
$[C]$ = viscous damping matrix (symmetric and orthogonal, see below)
$\{x\}$ = vector of displacement $x(t)$.

The differential equation is of the same form as the one for the SDOF oscillator Eq. (B1.1), except that now the single variables are substituted by vectors and matrices.

In the absence of damping, Eq. (B2.1) is simplified to:

$$[M]\cdot\{\ddot{x}\} + [K]\cdot\{x\} = \{0\} \tag{B2.2}$$

Assuming the solution to be of the form (analogous to Eq. (B1.2)):

$$\{x(t)\} = \{u(t)\} = \sum_{k=1}^{n} \{\psi_k\}\cdot e^{i\cdot\omega_k\cdot t} \tag{B2.3}$$

with:

$\{\psi_k\}$ = vector of the k-th *eigenmode* at frequency ω_k

one obtains a system of homogeneous equations:

$$([K]-\omega_k^2\cdot[M])\cdot\{\psi_k\}=\{0\} \tag{B2.4}$$

which represents a dynamic eigenvalue problem. It has non-trivial solutions (in the sense of $\{\psi_k\} \neq \{0\}$ for all k) only if its determinant vanishes:

$$\left|[K]-\omega^2\cdot[M]\right|=0 \tag{B2.5}$$

This requirement leads to the characteristic equation of the problem with a polynomial of degree n, which generally yields n different eigenvalues ω_k, the circular eigenfrequencies. From (B2.5) then follow n eigenvectors $\{\psi_k\}$, the mode shapes, which form together the matrix $[\Psi]$ of the eigenvectors, the *modal matrix*. (For solution methods of the eigenvalue problem, see e.g. [B1.1].)

Example B2.1

Figure B2.1 shows the shapes of the eigenmodes for a simplified three-storey building with three degrees of freedom. The parameters are:

$k_1 = 600$ kN/m	$m_1 = 10$ t	$\omega_1 = 4.58$ rads/s
$k_2 = 1200$ kN/m	$m_2 = 15$ t	$\omega_2 = 9.83$ rads/s
$k_3 = 1800$ kN/m	$m_3 = 20$ t	$\omega_3 = 14.50$ rads/s

giving the stiffness and the mass matrix of the system to

$$[K] = \begin{bmatrix} 600 & -600 & 0 \\ -600 & 1800 & -1200 \\ 0 & -1200 & 3000 \end{bmatrix} [\text{kN/m}], \qquad [M] = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 20 \end{bmatrix} [\text{t}]$$

B 2.2 Forced Vibration

For a forced vibration the differential equation of free vibrations is augmented by the forcing function vector $\{F(t)\}$:

$$[M]\cdot\{\ddot{x}\}+[C]\cdot\{\dot{x}\}+[K]\cdot\{x\}=\{F(t)\} \tag{B2.6}$$

The solution can be obtained:

– in the frequency domain
– by modal analysis
– by direct integration in the time domain.

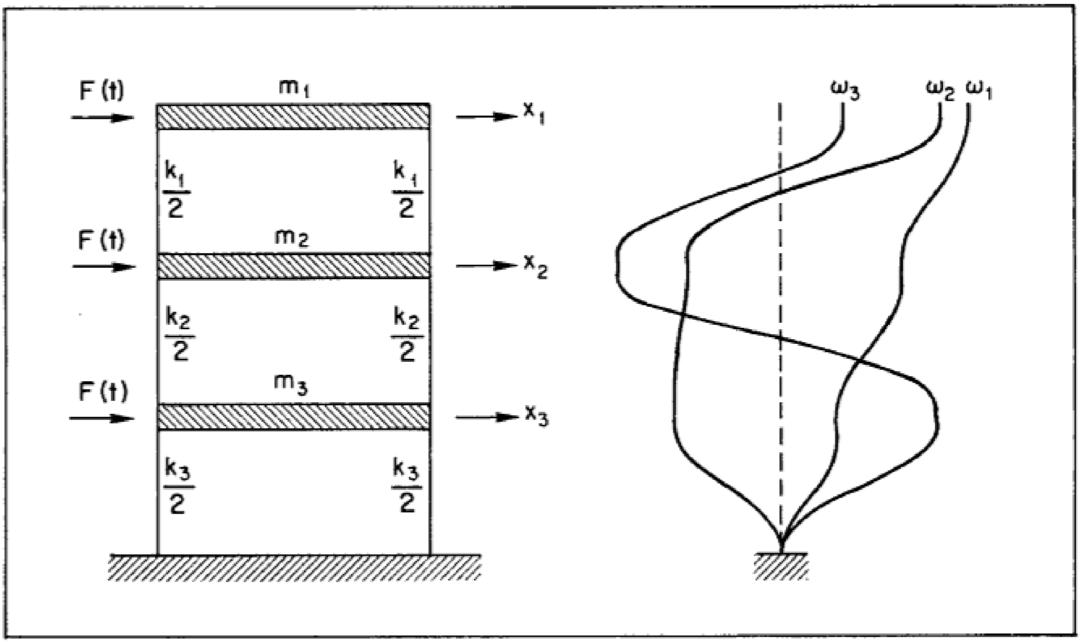


Fig. B2.1 Example B2.1:
Idealization of a three-storey building as 3-DOF system

Solution in the Frequency domain

In generalization of Eqs. (B1.22), (B1.23), the forcing function F(t) and the solution x(t) can be transformed into the frequency domain. Periodic loading results in discrete frequencies ω_j, whereas aperiodic loading has to be described by a continuous frequency band.

For the simplest case of a *harmonic excitation*, the Fourier transforms (see also Appendix B 4) read

$$\{u(t)\} = \{U(\omega)\} \cdot e^{i \cdot \omega \cdot t} \tag{B2.7}$$

$$\{F(t)\} = \{A(\omega)\} \cdot e^{i \cdot \omega \cdot t}$$

and the differential equation (B2.6) becomes after substitution:

$$([K] + i \cdot \omega \cdot [C] - \omega^2 \cdot [M]) \cdot \{U(\omega)\} \cdot e^{i \cdot \omega \cdot t} = \{A(\omega)\} \cdot e^{i \cdot \omega \cdot t} \tag{B2.8}$$

Collecting the matrices in the bracket above into the so-called dynamic stiffness matrix

$$[S] = [K] + i \cdot \omega \cdot [C] - \omega^2 \cdot [M] \tag{B2.9}$$

the equation systems in the frequency domain becomes:

$$[S]\cdot\{U(\omega)\} = \{A(\omega)\} \tag{B2.10}$$

This permits direct solution for the displacement amplitudes:

$$\{U(\omega)\} = [S]^{-1}\cdot\{A(\omega)\} \tag{B2.11}$$

By applying the inverse Fourier transformation one obtains the displacement amplitude in the time domain. Because of the high computational effort, this method is feasible only for systems with a moderate number of degrees of freedom.

Example B2.2:

For the 3-DOF system of the previous example, Fig. B2.2 gives the frequency response curve of the displacements for the case that the same loading is synchronously applied at all lumped masses. Then Eq. (B2.11) becomes

$$\{U(\omega)\} = [S]^{-1}\cdot\begin{Bmatrix}1\\1\\1\end{Bmatrix}\cdot A(\omega) \tag{B2.12}$$

The displacements in Fig. B2.2 are normalized with respect to $A(\omega)$. Obviously, only the first eigenmode matters under such a loading. The higher modes are of greater importance, if a load excites only the mass closest to the base (Fig. B2.3). In this calculation the damping matrix [C] was assumed proportional to the mass matrix ($\alpha_o = 0.0625$) and the stiffness matrix ($\alpha_1 = 0.00139$) as so-called Rayleigh damping (see Eq. (B2.21) in the following section).

Modal Analysis

This method allows to *decouple* the system of differential equations of motion into a set of independent differential equations of SDOF oscillators. To achieve this, the displacement variables are transformed with the modal matrix $[\Psi]$ to *modal coordinates*:

$$\{x(t)\} = \sum_{k=1}^{n} \{\psi_k\}\cdot y_k = [\Psi]\cdot\{y\} \tag{B2.13}$$

where:

- n = number of degrees of freedom considered
- $\{y\}$ = vector of the modal coordinates y_k
- $\{\psi_k\}$ = k-th eigenmode vector
- $[\Psi]$ = matrix of the n eigenmode vectors.

The eigenmodes of the damped system are identical with those of the undamped system.

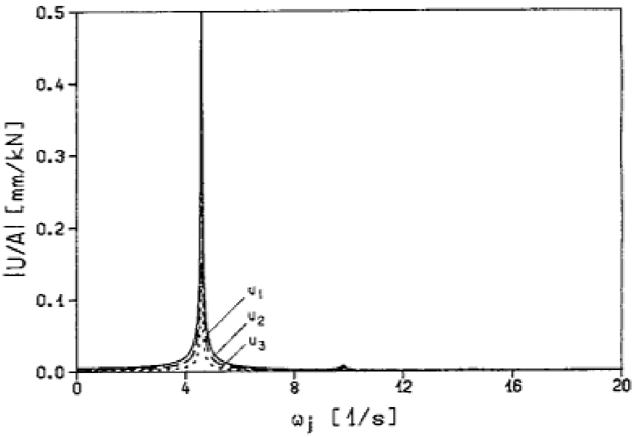


Fig. B2.2
Example B2.1:
Frequency response curve of displacements under synchronous loading at all lumped masses with Rayleigh damping of $\alpha_o = 0.0625$, $\alpha_1 = 0.00139$

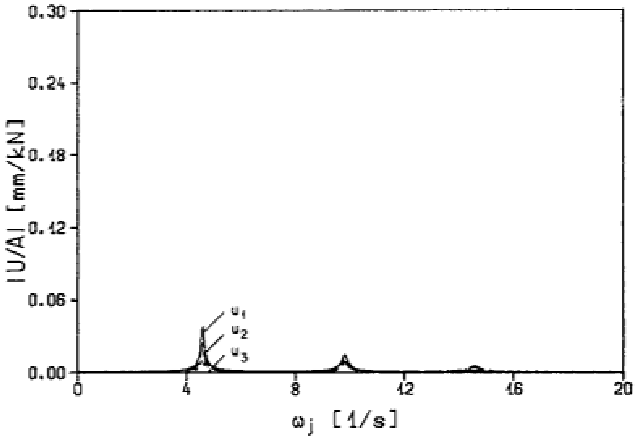


Fig. B2.3
Example B2.1:
Frequency response curve of displacements as in Fig. B2.2, but due to only one time-varying load acting on the lowest mass

Substituting the physical coordinates in Eq. (B2.6) by the modal coordinates (B2.13):

$$[M]\cdot[\Psi]\cdot\{\ddot{y}\} + [C]\cdot[\Psi]\cdot\{\dot{y}\} + [K]\cdot[\Psi]\cdot\{y\} = \{F(t)\} \tag{B2.14}$$

and premultiplying all terms with $[\Psi]^T$ yields

$$[\Psi]^T\cdot[M]\cdot[\Psi]\cdot\{\ddot{y}\} + [\Psi]^T\cdot[C]\cdot[\Psi]\cdot\{\dot{y}\} + [\Psi]^T\cdot[K]\cdot[\Psi]\cdot\{y\} = [\Psi]^T\cdot\{F(t)\} \tag{B2.15}$$

The matrix products are termed *generalized matrices*, i.e.

$$[\Psi]^T\cdot[M]\cdot[\Psi] = [M^*] \tag{B2.16}$$

$$[\Psi]^T\cdot[C]\cdot[\Psi] = [M^*]\cdot 2\cdot\xi\cdot[\Omega] = [C^*] \tag{B2.17}$$

$$[\Psi]^T\cdot[K]\cdot[\Psi] = [M^*]\cdot[\Omega^2] = [K^*] \tag{B2.18}$$

[M*] is the generalized mass matrix, [C*] the generalized damping matrix, and [K*] the generalized stiffness matrix. [Ω] is a diagonal matrix with the circular eigenfrequencies. The foregoing operation renders all three generalized matrices diagonal with the following relation for the k-th eigenmode (as k-th entries in the matrices):

$$M_k^* = \{\psi_k\}^T \cdot [M] \cdot \{\psi_k\} \tag{B2.19}$$

$$K_k^* = M_k^* \cdot \omega_k^2 \tag{B2.20}$$

To decouple the damping matrix [C], it is mostly assumed to be a linear combination of mass and stiffness matrix, the so-called *Rayleigh damping* [3.13]:

$$[C] = \alpha_o \cdot [M] + \alpha_1 \cdot [K] \tag{B2.21}$$

or in generalized form

$$[\Psi]^T \cdot [C] \cdot [\Psi] = \alpha_o \cdot [M^*] + \alpha_1 \cdot [K^*] = [C^*] \tag{B2.22}$$

The k-th generalized damping term becomes, due to (B2.17),

$$C_k^* = 2 \cdot \xi_k \cdot \omega_k \cdot M_k^* \tag{B2.23}$$

and a *modal damping ratio* ξ_k can be evaluated:

$$\xi_k = \frac{1}{2} \cdot \left(\frac{\alpha_o}{\omega_k} + \alpha_1 \cdot \omega_k\right) \tag{B2.24}$$

Damping defined in the (mutually orthogonal) eigenmodes of the undamped structure is more generally termed orthogonal or classical damping. In principle, a different damping ratio could be specified for every mode. In many cases, however, one damping ratio common to all modes suffices.

Now, inserting the above generalized matrices into (B2.15) yields the differential equation of motion in modal coordinates {y}:

$$[M^*] \cdot \{\ddot{y}\} + [C^*] \cdot \{\dot{y}\} + [K^*] \cdot \{y\} = [\Psi]^T \cdot \{F(t)\} \tag{B2.25}$$

As all matrices are diagonal, the individual differential equations are decoupled and can be considered separately, i.e. the k-th mode becomes:

$$M_k^* \cdot \ddot{y}_k + C_k^* \cdot \dot{y}_k + K_k^* \cdot y_k = \{\psi_k\}^T \cdot \{F(t)\} \tag{B2.26}$$

Note, that this equation has the form of Eq. (B1.16) of an SDOF oscillator and can be solved in the same way (see Appendix B 1). The results of all equations are the modal coordinates $y_k(t)$ which serve as multipliers to the eigenmode shape vectors $\{\psi_k\}$ to yield the total displacement vector $\{x(t)\}$ by superposition according to Eq. (B2.13).

Modal analysis is motivated by the advantage of being able to consider only the *dominant eigenmodes* of vibration and thus to reduce the degrees of freedom to the number of modal coordinates essential to capture the system's behaviour.

Direct Integration in the Time Domain

The direct integration discretizes the system of differential equations (B2.6) in time with either implicit or explicit operators. For the solution procedure, the reader is referred to [3.13] and [B1.1].

B 3 DISTRIBUTED-PARAMETER SYSTEMS (EIGENFREQUENCIES)

True continuous mass distribution can be used for evaluating the eigenfrequencies of beams, frames, arches and plates. They are calculated from the partial differential equation of motion for free vibration.

The following is a compilation of eigenfrequencies of several structural systems. Finally it will be shown how to calculated approximately a forced vibration on continuous systems by using the analogy of an equivalent SDOF oscillator.

B 3.1 Single-Span Beams

Consider the partial differential equation of the simply supported and freely vibrating beam with continuous distribution of mass:

$$E \cdot I \cdot \frac{\partial^4 u}{\partial x^4} = -\mu \cdot \frac{\partial^2 u}{\partial t^2} \tag{B3.1}$$

in which (slightly different from the notation in previous chapters):

- u = dynamic deflection of the beam
- x = longitudinal coordinate along beam axis
- t = time
- μ = mass per unit length
- $E \cdot I$ = flexural rigidity (assumed constant).

The problem is solved by separating spatial and temporal variables in product form:

$$u(x,t) = w(x) \cdot T(t) \tag{B3.2}$$

The time-dependent term follows to

$$T(t) = T_1 \cdot e^{i \cdot \omega \cdot t} \tag{B3.3}$$

and the spatial shape of the vibration becomes

$$w(x) = W_A \cdot \cos(\beta \cdot x) + W_B \cdot \sin(\beta \cdot x) + W_C \cdot \sinh(\beta \cdot x) + W_D \cdot \cosh(\beta \cdot x) \tag{B3.4}$$

with:

$$\beta^4 = \frac{\mu \cdot \omega^2}{E \cdot I} \tag{B3.5}$$

Introducing the appropriate boundary conditions yields – from the frequency equation – the eigenfrequencies ω_n, the constants in the shape equation W_i, β_n, and finally $w(x,\omega_n)$, i. e. the vibration shape dependent on the eigenfrequency in question.

Eigenfrequencies have been compiled in various tables and textbooks. Figure B3.1 compares the free vibrations of single-span beams with different boundary conditions, expressed in the variable φ_n. Then the eigenfrequencies result from the common formula

$$\omega_n = \frac{\varphi_n^2}{l^2} \cdot \sqrt{\frac{E \cdot I}{\mu}} \tag{B3.6}$$

support conditions	factor φ_n
	φ_1 = 4.73 φ_2 = 7.85 φ_3 = 11.00 φ_4 = 14.14
	$\varphi_n = n \cdot \pi$ $n = 1, 2, \ldots$
	φ_1 = 4.73 φ_2 = 7.85 φ_3 = 11.00 φ_4 = 14.14
	φ_1 = 1.88 φ_2 = 4.69 φ_3 = 7.85 φ_4 = 11.00
	φ_1 = 3.93 φ_2 = 7.07 φ_3 = 10.21 φ_4 = 13.35

Fig. B3.1
Eigenfrequency coefficients φ_n for flexural vibration modes of single-span beams with various support conditions, Eq. (B3.6)

Fig. B3.2
Eigenfrequency coefficients φ_n for other vibration modes of single-span beams with various support conditions, Eq. (B3.7)

support condition	factor φ_n
	$\varphi_n = n \cdot \pi$ $n = 1, 2, \ldots$
	$\varphi_n = -\frac{\pi}{2} + n\pi$ $n = 1, 2, \ldots$

If an additional mass is attached to the beam, the fundamental flexural eigenfrequencies can be taken from Fig. B3.3. The beam itself may be treated as massless or as having a uniform mass distribution (with $m_b = \mu \cdot l$).

B 3.2 Continuous Beams

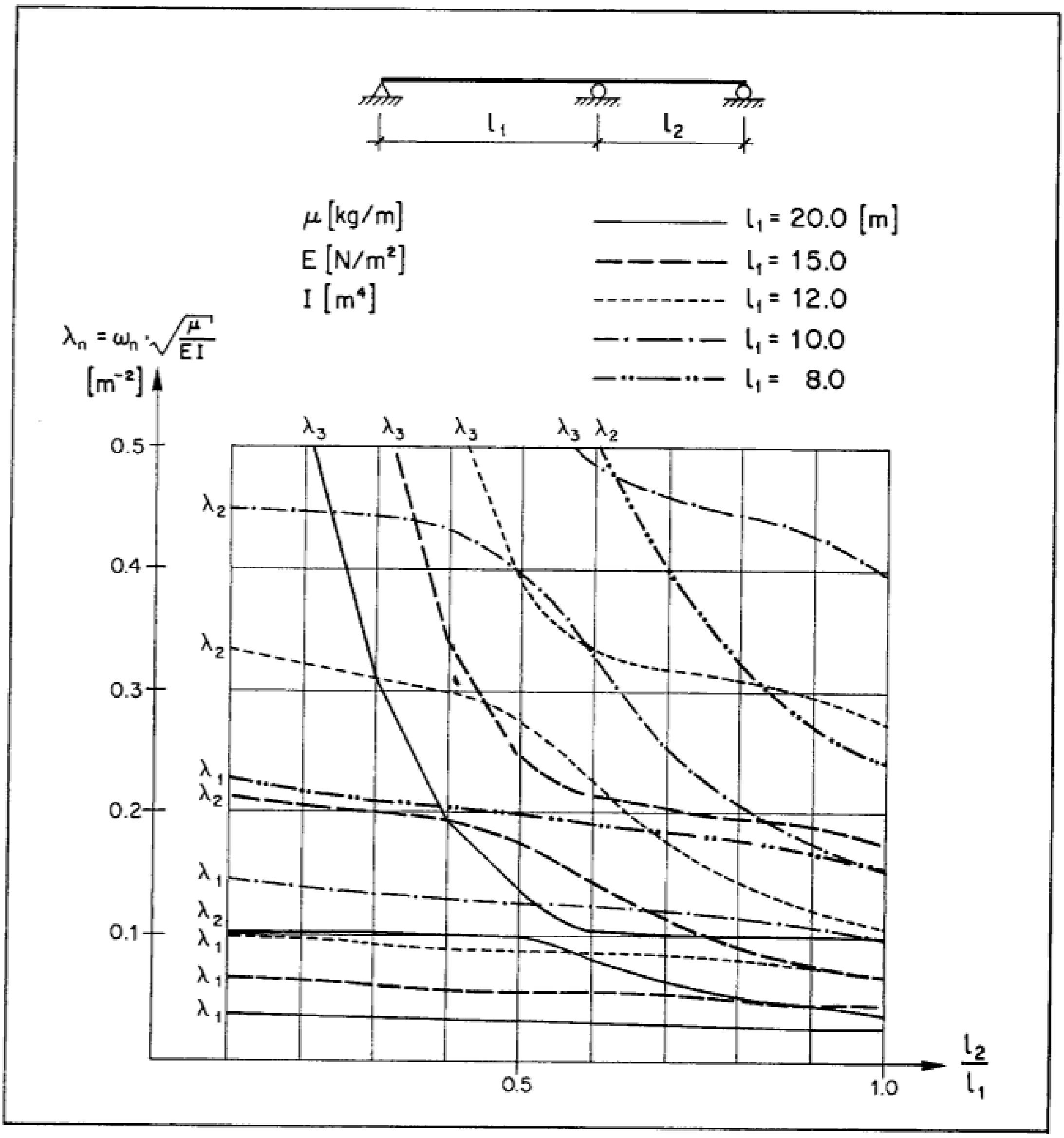


Fig. B3.4a Chart for eigenfrequencies of two-span beams (after [B3.1])

The eigenmodes and frequencies of continuous beams are subject to too many parameters to permit a comprehensive survey in this booklet.

As an example, however, Figs. B3.4a/b show diagrams for finding the fundamental eigenfrequencies of beams over two or three spans. They are based on [B3.1], where many more tables of other cases may also be found.

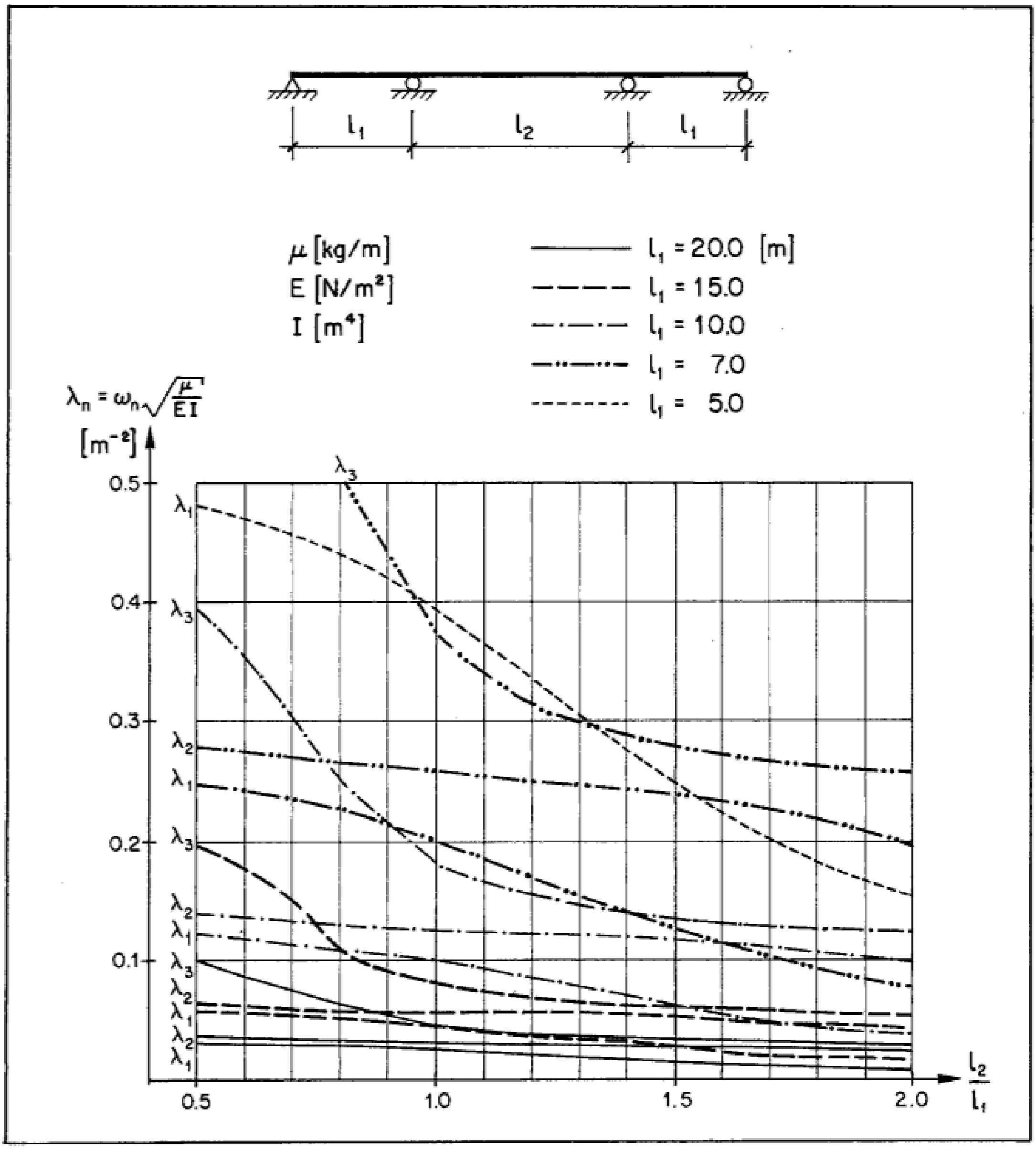


Fig. B3.4b Chart for eigenfrequencies of three-span beams (after [B3.1])

B 3.3 Frames

For the vibrational properties of frames, the reader is referred to [B3.2], where numerous cases are tabulated.

B 3.4 Arches

The fundamental eigenfrequencies for radial and flexural vibrations, i. e. for symmetric and antimetric vibrations, of a *circular arch* are (from [B3.3]):

$$\text{radial} \quad \omega_1 = \frac{\varphi_R}{r^2} \cdot \sqrt{\frac{E \cdot I}{\mu}} \qquad \text{flexural} \quad \omega_1 = \frac{\varphi_B}{\alpha^2 \cdot r^2} \cdot \sqrt{\frac{E \cdot I}{\mu}} \tag{B3.8}$$

with:

α = sector angle [radians]
r = radius [m].

The values φ_R and φ_B can be taken from Fig. B3.5, where:

A = cross-sectional area [m²].

	support conditions	factor φ
radial mode		$\varphi_R = \sqrt{1 + \frac{\pi^4 I}{\alpha^4 \cdot r^2 \cdot A}}$
radial mode		$\varphi_R = \sqrt{1 + \frac{500.5 \cdot I}{\alpha^4 \cdot r^2 \cdot A}}$
flexural mode		$\varphi_B = \frac{4\pi^2 - \alpha^2}{\sqrt{1 + \frac{3\alpha^2}{4\pi^2}}}$
flexural mode		$\varphi_B = \sqrt{\frac{3803.2 - 92.101\,\alpha^2 + \alpha^4}{1 + 0.06054 \cdot \alpha^2}}$

Fig. B3.5
Eigenfrequency coefficients of circular arches, Eq. (B3.8) (from [B3.3])

Following [B3.3], a similar formula applies to *parabolic arches:*

$$\omega_1 = \frac{\varphi_a}{l^2} \cdot \sqrt{\frac{E \cdot I}{\mu}} \tag{B3.9}$$

with the φ_a values to be taken from Fig. B3.6.

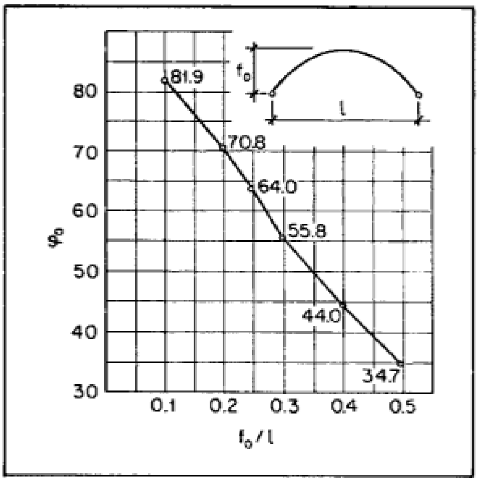


Fig. B3.6
Eigenfrequency coefficients of parabolic arches, Eq. (B3.9) (from [B3.3])

B 3.5 Plates and Slabs

Quadratic and Circular Plates

The eigenfrequencies ω_n for quadratic and circular plates with various boundary conditions are determined from the formula given in the caption of Fig. B3.7 with the aid of the tabulated coefficient B_n.

Rectangular Plates

Similar for rectangular plates, the two lowest eigenfrequencies ω_n for various boundary conditions follow from the formula in the caption of Fig. B3.8, for which the coefficients φ_n are to be evaluated in function of the side aspect ratio γ.

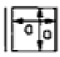
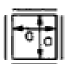
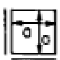
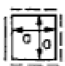
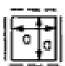
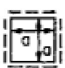
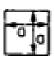
support conditions	B_1	B_2	B_3	B_4	B_5	B_6	B_7
	11.84	24.61	40.41	46.14	103.12		
	6.09	10.53	14.19	23.80	40.88	44.68	61.38
	4.35	24.26	70.39	138.85			
	5.90						
	1.01	2.47	6.20	7.94	9.01		
	10.40	21.21	31.29	38.04	38.22	47.73	
	2.01	6.96	7.74	13.89	18.25		
	6.83	14.94	16.95	24.89	28.99	32.71	
	8.37	15.82	20.03	27.34	29.54	37.31	
	5.70	14.26	22.82	28.52	37.08	48.49	
	4.07	5.94	6.91	10.39	17.80	18.85	

Fig. B3.7
Eigenfrequencies of circular and quadratic plates for various support conditions (after [3.8]):

$$\omega_n = B_n \cdot \sqrt{\frac{E \cdot t^2}{\varrho \cdot a^4 \cdot (1-\nu^2)}}$$

with

E = Young's modulus [N/m²]
ν = Poisson's ratio
t = slab thickness [m]
a = diameter, side length [m]
ϱ = mass per volume [kg/m³]

for the support conditions

——— free edge
– – – hinged edge
═══ fixed edge

Continuous Plates

For the eigenfrequencies of continuous plates, if not to be approximated as continuous beams (Fig. B3.4), the reader is referred to [3.6].

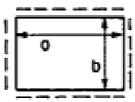
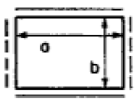
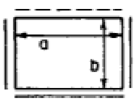
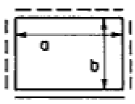
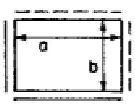
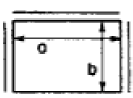
support conditions	
	$\varphi_{1,1} = 1.57\,(1+\gamma^2)$ $\varphi_{2,1} = 6.28\,(1+0.25\gamma^2)$ $\varphi_{1,2} = 1.57\,(1+4\gamma^2)$
	$\varphi_{1,1} = 1.57\sqrt{1+2.5\gamma^2+5.14\gamma^4}$ $\varphi_{2,1} = 6.28\sqrt{1+0.625\gamma^2+0.321\gamma^4}$ $\varphi_{1,2} = 1.57\sqrt{1+9.32\gamma^2+39.06\gamma^4}$
	$\varphi_{1,1} = 1.57\sqrt{5.14+2.92\gamma^2+2.44\gamma^4}$ $\varphi_{2,1} = 9.82\sqrt{1+0.266\gamma^2+0.0625\gamma^4}$ $\varphi_{1,2} = 1.57\sqrt{5.14+10.86\gamma^2+25.63\gamma^4}$
	$\varphi_{1,1} = 1.57\sqrt{1+2.33\gamma^2+2.44\gamma^4}$ $\varphi_{2,1} = 6.28\sqrt{1+0.582\gamma^2+0.152\gamma^4}$ $\varphi_{1,2} = 1.57\sqrt{1+8.69\gamma^2+25.63\gamma^4}$
	$\varphi_{1,1} = 1.57\sqrt{2.44+2.72\gamma^2+2.44\gamma^4}$ $\varphi_{2,1} = 7.95\sqrt{1+0.395\gamma^2+0.095\gamma^4}$ $\varphi_{1,2} = 1.57\sqrt{2.44+10.12\gamma^2+25.63\gamma^4}$
	$\varphi_{1,1} = 1.57\sqrt{5.14+3.13\gamma^2+5.14\gamma^4}$ $\varphi_{2,1} = 9.82\sqrt{1+0.298\gamma^2+0.132\gamma^4}$ $\varphi_{1,2} = 1.57\sqrt{5.14+11.65\gamma^2+39.06\gamma^4}$

Fig. B3.8
Eigenfrequencies of rectangular plates for various support conditions (after [B3.6]):

$$\omega_n = 2\cdot\pi\cdot\frac{\varphi_n}{a^2}\cdot\sqrt{\frac{E\cdot t^3}{\mu\cdot 12\cdot(1-\nu^2)}}$$

with

$\varphi_1 = \varphi_{1,1}$
$\varphi_2 = \text{Min.}\,\{\varphi_{1,2};\varphi_{2,1}\}$

and

E = Young's modulus [N/m²]
ν = Poisson's ratio
t = slab thickness [m]
γ = aspect ratio a/b
a,b = side length [m]
μ = mass per area [kg/m²]

B 3.6 Substitute SDOF Systems

In certain cases, the system to be calculated, such as a slab or a beam, can be approximated by a substitute SDOF oscillator. For it to be equivalent, its properties have to be chosen such as yield to the same eigenfrequency of interest and the same maximum displacement amplitude – under a substitute force $\tilde{F}(t)$ – as the original system. The

$$\omega = \sqrt{\tilde{k}/\tilde{m}} \tag{B3.10}$$

Besides the factors φ_L, φ_M, φ_{LM} and the equivalent spring stiffness $\tilde{k}$, the Figs. B3.9 and B3.10 contain information on the load capacity of the respective system and on the magnitude of its dynamic support reactions.

Example B3.1

The fundamental eigenfrequency of a single-span, simply supported beam is to be derived from an equivalent substitute system. The result can be compared with the original beam frequency given in Fig. B3.1.

Parameters of the original structure are:

length l, mass μ per m length, flexural rigidity $E \cdot I$.

Conversion factors for the substitute SDOF system from Fig. B3.9:

$$\varphi_L = 0.640, \qquad \varphi_M = 0.50, \qquad k = \frac{384 \cdot EI}{5 \cdot l^3}$$

giving

$$\omega = \sqrt{\frac{\tilde{k}}{\tilde{m}}} = \sqrt{\frac{\varphi_L \cdot k}{\varphi_M \cdot \mu \cdot l}} = \sqrt{\frac{0.64 \cdot 384 \cdot E \cdot I}{0.50 \cdot \mu \cdot l \cdot 5 \cdot l^3}} = 9.92 \cdot \sqrt{\frac{E \cdot I}{\mu \cdot l^4}}$$

In this simple case, the SDOF approximation compares very well with the fundamental eigenfrequency from Fig. B3.1, which yields the factor

$$\varphi_i^2 = \pi^2 = 9.87 \cong 9.92$$

in front of the square root.

B 4 HARMONIC ANALYSIS

Any *periodic loading* F(t) can be decomposed into a constant part and several harmonic load contributions which, when superimposed, result in the total load-time function given. This harmonic decomposition is a *discrete Fourier transformation* of the loading and is therefore also called a Fourier analysis.

The basic relation is

$$F(t) = a_o + \sum_{n=1}^{\infty} a_n \cdot \cos(n \cdot \frac{2 \cdot \pi}{T} \cdot t) + \sum_{n=1}^{\infty} b_n \cdot \sin(n \cdot \frac{2 \cdot \pi}{T} \cdot t) \tag{B4.1}$$

with the Fourier coefficients

$$a_o = \frac{1}{T} \cdot \int_o^T F(t) \cdot dt \tag{B4.2}$$

$$a_n = \frac{2}{T} \cdot \int_o^T F(t) \cdot \cos(n \cdot \frac{2 \cdot \pi}{T} \cdot t) \cdot dt \tag{B4.3}$$

$$b_n = \frac{2}{T} \cdot \int_o^T F(t) \cdot \sin(n \cdot \frac{2 \cdot \pi}{T} \cdot t) \cdot dt \tag{B4.4}$$

in which $\omega = 2 \cdot \pi / T$ is the lowest circular frequency of the loading.

The frequencies of the harmonic components are multiples of the loading frequency ω. An alternative way of writing the n-th component of the load is

$$F_n(t) = a_n \cdot \cos(n \cdot \frac{2 \cdot \pi}{T} \cdot t) + b_n \cdot \sin(n \cdot \frac{2 \cdot \pi}{T} \cdot t) = A_n \cdot \sin(n \cdot \frac{2 \cdot \pi}{T} \cdot t + \varphi_n) \tag{B4.5}$$

with the amplitude A_n and the phase angle φ_n of the n-th component. (For the static component $n = 0$ is $F_o = a_o$.) They are:

$$A_n = \sqrt{a_n^2 + b_n^2} \tag{B4.6}$$

$$\varphi_n = \arctan \frac{a_n}{b_n} \tag{B4.7}$$

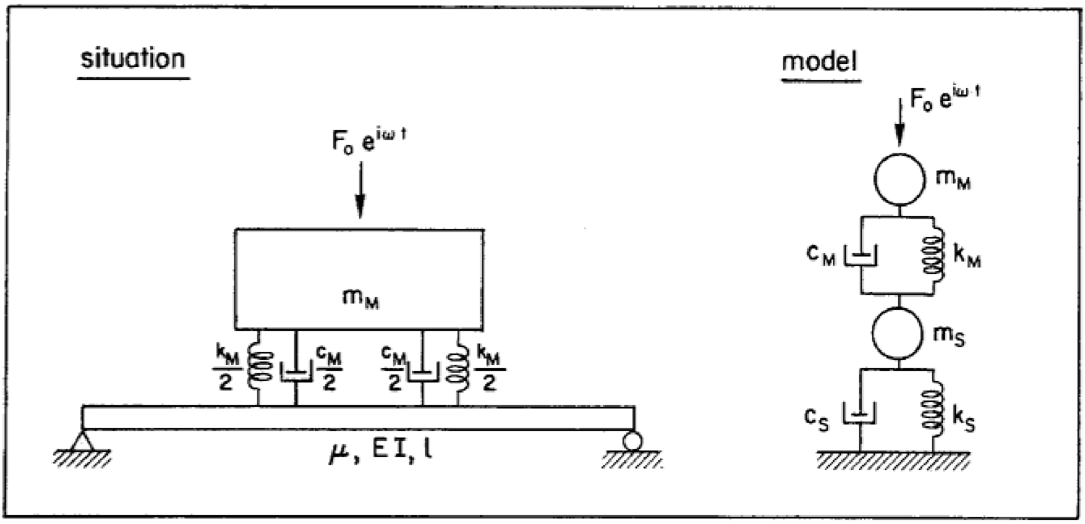


Fig. B5.5 Dynamic model of a machine mounted on a structural member under harmonic excitation

of a vibration-isolated base («type (c)» design, Fig. 3.6). The stiffness and damping coefficients k_M and c_M are those of the isolating spring(-damper) elements. The mass m_S and the stiffness k_S are the parameters of the equivalent SDOF systems substituting the structural member (see Appendix B 3.6). The loading from the machine is assumed to be harmonic.

The major question is whether the resulting 2-DOF system, Fig. B5.5, can be treated as two independent SDOF oscillators, i.e. whether the two spring-mass systems can be decoupled. To this end, Fig. B5.6 takes the ratio m_M/m_S as parameter for a set of curves, which give the *coupled eigenfrequencies* ω_1 and ω_2 (relative to the structural eigenfrequency ω_S) versus the ratio of the decoupled eigenfrequencies of machine and structure. It can be concluded that decoupling (i.e. ω_S not influenced by the existence of m_M and vice versa) is feasible only if the individual eigenfrequencies of the two partial systems are spaced apart and if the mass m_M (machine including stabilizing mass) is small compared to the equivalent mass m_S of the structural member. With m_M approaching the size of m_S, the coupled frequencies divert quickly from the decoupled values.

The eigenfrequencies ω_1 and ω_2 of the coupled 2-DOF system are:

$$\omega_{1,2}^2 = \frac{\omega_S^2}{2} \cdot \left\{1 + \frac{k_M \cdot m_S}{k_S \cdot m_M} \cdot \left(1 + \frac{m_M}{m_S}\right) \mp \sqrt{\left[1 + \frac{k_M \cdot m_S}{k_S \cdot m_M} \cdot \left(1 + \frac{m_M}{m_S}\right)\right]^2 - 4 \cdot \left[\frac{k_M \cdot m_S}{k_S \cdot m_M}\right]}\right\} \tag{B5.1}$$

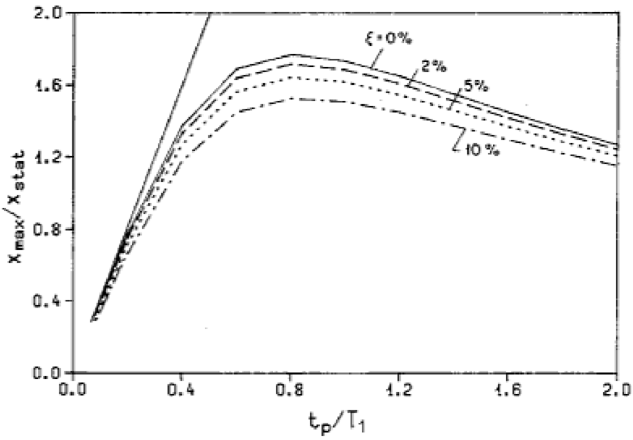


Fig. B6.2
Response spectra to a semi-sinusoidal impulse for various SDOF periods and damping ratios

in which I is the *momentum*

$$I = \int_0^{t_p} F(t) \cdot dt \tag{B6.2}$$

Note that for a very short relative duration of the impact, the maximum displacement depends only on the momentum of the impulse irrespective of its shape. According to Fig. B6.2, this approximation may be used for relative durations up to $t_p/T_1 \cong 0.1 \div 0.2$.

For calculating a *2-DOF system* under impact load, the primary question is that of possible decoupling as discussed in Section B5.2 (Fig. B5.6).

If the system can be decoupled, the directly hit mass can be treated first according to the procedure outlined above. The resulting reaction forces are then imposed as loading on the second mass for calculating the latter's reaction forces and displacements. In any case, however, a check on the decoupling assumption is recommended.

If the system cannot be decoupled, then the complete system must be analyzed with two degrees of freedom. For a very short impact in relation to the system frequency ($t_p/T_1 < \sim 0.1$), the magnitude of the momentum can again be assumed to determine the response. Otherwise, the impact loading has to be followed in the time domain.

B 6.2 Periodic Impact

In most vibration problems of human or mechanical origin the impact loading is periodic, that is a sequence of single impulses occurring with a return period T_p.

In Fig. B6.3, the dynamic displacement magnification of an SDOF oscillator is plotted against the ratio of impulse duration to structural frequency for various impulse shapes (Fig. B6.1), assuming an impact period of $T_p = 2 \cdot t_p$ and $\xi = 5\%$ damping.

For the particular case of a half-sine impulse, the Figs. B6.4 and B6.5 show the influence of the relative impact period for two different damping ratios of $\xi = 2\%$ and $\xi = 5\%$, respectively.

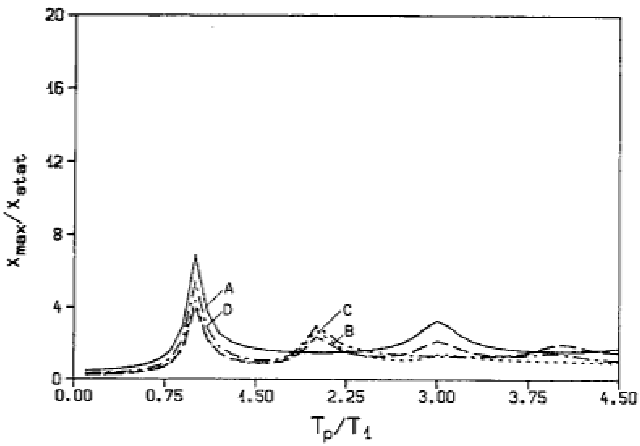


Fig. B6.3
Response spectra to periodic impact loads ($t_p/T_p = 0.5$) of the impulse shapes in Fig. B6.1 for 5% damping

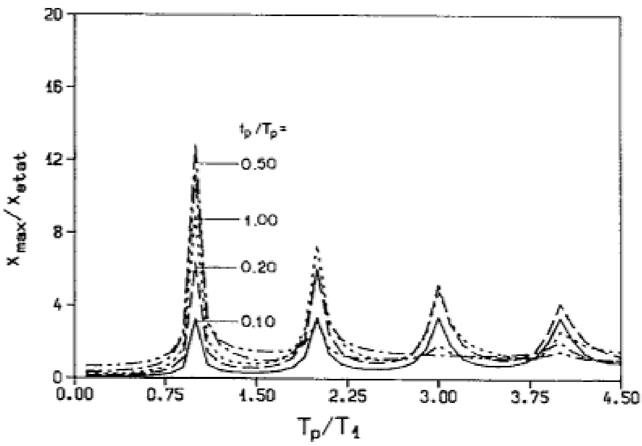


Fig. B6.4
Response spectra to periodic semi-sinusoidal impact loads of various durations t_p/T_p for 2% damping

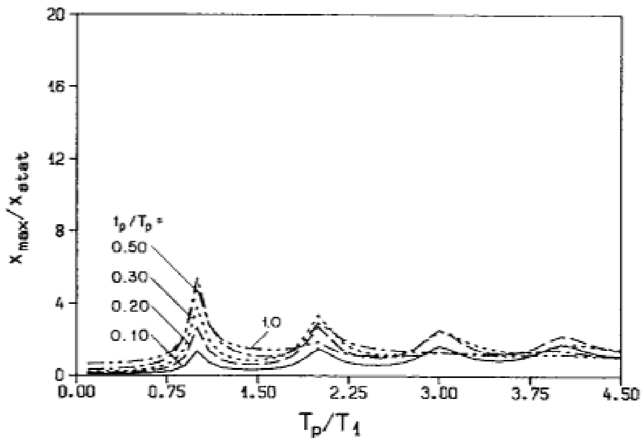


Fig. B6.5
Response spectra as in Fig. B6.4, but for 5% damping

B 7 VIBRATION ABSORBERS

B 7.1 Definition

A vibration absorber is a vibratory subsystem attached to and tuned with respect to a primary vibrating system (a machine, a structure or structural member). Tuning accurately the frequency of the absorber results in induced inertial forces, which counteract the vibration of the primary system. While the primary vibration amplitudes can thus be suppressed to a large degree, considerable displacement amplitudes must be accepted in the absorber system.

B 7.2 Categories

Vibration absorbers fall into one of two categories (see for example [3.2]):

- linear vibration absorbers (tuned spring-mass-damper system)
- shock absorbers.

In each category, sketched in Fig. B7.1, several further distinctions are possible.

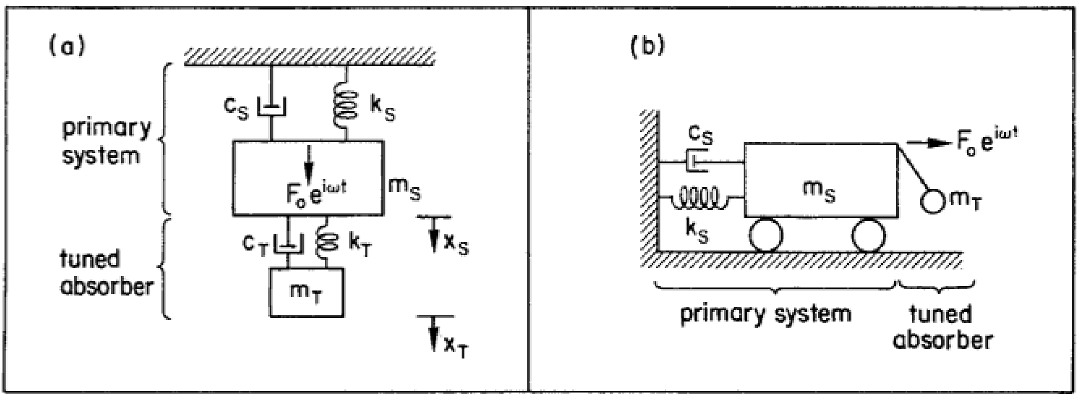


Fig. B7.1 Dynamic model of a linear absorber and a shock absorber

In the following only the working principles of the two absorber types are discussed. It is recommended that designing a vibration absorber should be based on the relevant literature.

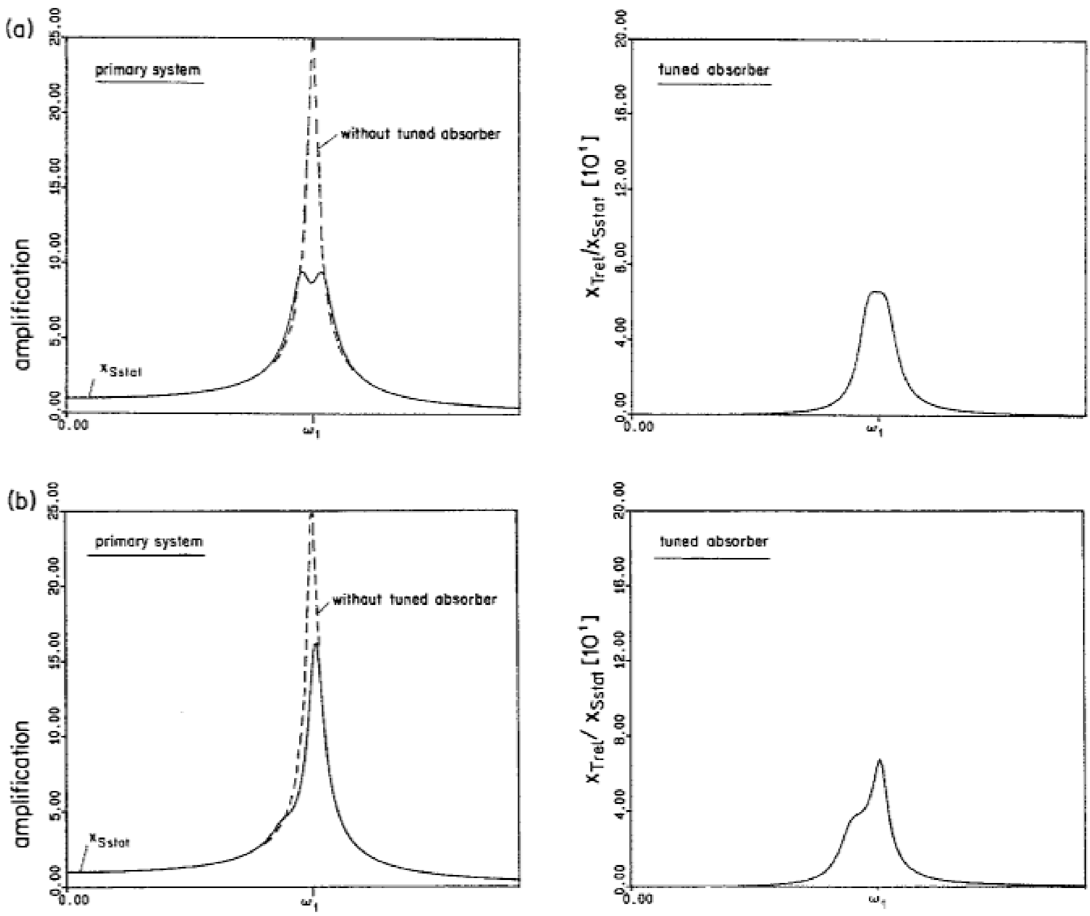


Fig. B7.2 Frequency response curves of an SDOF oscillator ($\xi = 2\%$) with a damped dynamic absorber of mass $m_T = \frac{1}{100} \cdot m_S$:
(a) optimum tuning: $f_T/f_S = 0.987$, $\xi_T = 6.6\%$
(b) poor tuning: $f_T/f_S = 0.900$, $\xi_T = 6.6\%$

The *optimum damping ratio* can be determined from

$$\xi_{opt} = \sqrt{\frac{3 \cdot (m_T/m_S)}{8 \cdot (1 + m_T/m_S)^3}} \tag{B7.5}$$

or through stepwise variation of the numerical value of ξ. Figure B7.3 gives an example of the effect of damping on the displacements of the primary system. In special cases, damping need not be provided at all, in particular for a constant excitation frequency near or exactly at the resonance point. See [3.2] for details.

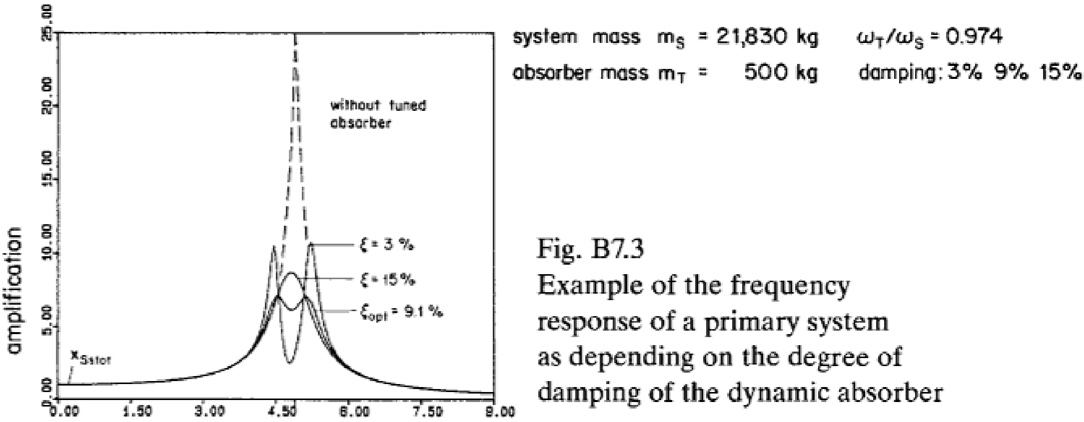


Fig. B7.3
Example of the frequency response of a primary system as depending on the degree of damping of the dynamic absorber

Vibration absorption is more efficient, the larger the *mass of the absorber* is compared to that of the primary system and the smaller the primary damping. An appropriate choice is about 1/20 of the primary mass [2.23], a larger absorber mass resulting in a negligible further reduction of displacements. It follows that absorbers are particularly well suited to reduce resonating vibrations of structures or structural members with not too large a mass and small genuine damping; they will prove inadequate for structures having a large mass or vibrating strongly despite substantial genuine damping.

As an absorber of the type shown above is only effective over a narrow frequency band and tuned to a particular structural frequency, it is not satisfactory in structures with several closely spaced eigenfrequencies which are all more or less excited by the dynamic loading in question.

The application of vibration absorbers will be considered whenever critical dynamic loading cannot be avoided, especially in existing structures which are difficult to stiffen economically. It should also be borne in mind, that an absorber is a mechanical system with large vibration amplitudes, which needs adequate maintenance.

B 7.4 Shock absorbers

Shock absorbers are nonlinear in behaviour and found on machines, chimneys, masts, etc. As sketched schematically in Fig. B7.1b, they consist of a mass attached to the primary vibrating system so as to strike on the latter one or two times in each vibration period. If the absorber force is to act at the right moment to counteract the primary vibration, such a system also requires careful tuning and maintenance.

B 8 DYNAMIC MATERIAL PROPERTIES

B 8.1 Strength and Ductility

Under dynamic loading most material properties – such as Young's modulus, strength and strain limits – change to a greater or lesser extent, when compared to the values characteristic for slow, quasi-static loading. The change accelerates with the rate of loading and is usually expressed in function of the *strain rate*

$$\dot{\varepsilon} = \frac{d\varepsilon}{dt} \tag{B8.1}$$

Most often an average strain rate of loading can be considered. In inertial vibration problems it hardly exceeds $\dot{\varepsilon} \cong 0.1\ s^{-1}$, so that the expected change in properties is moderate [1.2]. A much larger influence is to be noted for strain rates $\dot{\varepsilon} \cong 1 \div 10\ s^{-1}$, as occur typically for high impact loads and will generally lead to plastic deformation in the structure or the member affected.

Dynamic loads may also influence the fatigue resistance of the structural material, even at a low number of cycles if the load is large enough (low-cycle fatigue).

The property changes for concrete and steel are briefly discussed in the following. Usually they describe a straight line when plotted against the strain rate in semi-logarithmic scale and are given in form of a factor with respect to the quasi-static value. This approach will be followed here to indicate the general trend of change; a more extensive and quantitative discussion can be found in [1.2]. As the influence of the strain rates encountered for typical dynamic loads (disregarding strong impact, blast loading, etc.) is limited to a few percent, it may be neglected in most design cases.

Tensile strength of concrete

The effect of the strain rate is relatively large. For instance, a rate $\dot{\varepsilon} = 0.1\ s^{-1}$ will result in some 60% increase compared to the quasi-static tensile strength. Above ca. $\dot{\varepsilon} = 0.3\ s^{-1}$ the increase with strain rate is even more rapid.

Compressive strength of concrete

The compressive strength increases with higher strain rate at a smaller percentage than the tensile strength. The increase at $\dot{\varepsilon} = 0.1\ s^{-1}$ amounts to about 10%.

$$E_c = E_{c,dyn} \cong 1.10 \cdot E_{c,stat}$$
$=$ dynamic modulus of elasticity of concrete (see Section B 8.1)

I_i^I = moment of inertia of the uncracked section (possibly including reinforcing bars)

– *completely cracked state* («state II» without tension stiffening):

$$B^{II} = E_c \cdot I_i^{II} \tag{B8.2}$$

in which:

I_i^{II} = moment of inertia of the cracked section (including the reinforcing bars in the tension zone)

– *effective flexural rigidity* of the cracked section with tension stiffening by the concrete in between cracks:

$$B_{eff}^{II} = \frac{B^{II}}{\varkappa} \tag{B8.3}$$

in which the so-called bond coefficient $\varkappa$ is computed as

$$\varkappa \cong 1 - \left[\frac{M_{cr}}{M_{stat}}\right]^2 \cdot \left[1 - \frac{B^{II}}{B^{I}}\right] \leqslant 1 \tag{B8.4}$$

where:

M_{cr} = cracking moment of the cross-section (uncracked stress distribution according to the tensile strength of concrete)

M_{stat} = bending moment due to static loading (at rest).

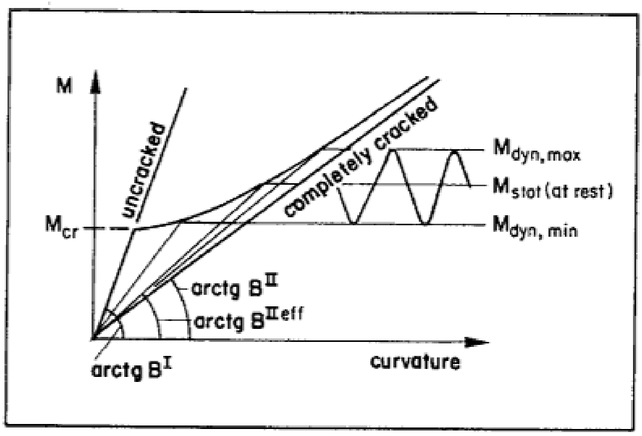


Fig. B8.1
Moment-curvature relation and secant stiffnesses of a cracked reinforced-concrete beam element under dynamic loading (starting from static load)

The flexural rigidity will hardly fall below B^{II}_{eff} (towards B^{II}) unless the cracks are spaced very closely (e.g. due to very small spacing of stirrups) or the bond is destroyed over long distances due to large bar diameters and a high number of load cycles. Regarding the moment-curvature diagram in Fig. B8.1, the secant values of the flexural rigidity can be quite different for the upper and lower load maximum in a cycle. As a reasonable approximation, one may take the average rigidity corresponding to the static load level when the structure is at rest.

In a refined analysis one may take into account the existence of uncracked zones where the stressing is low, e.g. toward the supports, although these zones do not contribute much to the total deformation.

B 8.3 Damping

Inherent structural damping is very important for the dynamic behaviour. Several mechanisms can be distinguished (e.g. [B8.2]):

- *Material damping* denotes the energy dissipation inside a monolithic structure.
- *System damping* results from energy dissipation in appurtenances, for instance friction in sliding bearings and joints, energy dissipation in nonstructural elements.
- *Radiation damping* describes the effect that energy is transmitted from the structure through the foundation and subsequently radiated into the halfspace of the ground in the form of wave motion.

Each of these mechanisms contribute to the damping value (logarithmic decrement δ) measured in a vibration decay test on structures in situ. Cracking in reinforced concrete structures may also contribute a large share [B8.1], [B8.2], [B8.3]. The damping is further raised by the presence of many people on the vibrating structural member (see Section 2.2).

In the present context the main distinction is to be drawn between the damping in

- pedestrian structures (overpasses, walkways, stairs, etc.)
- buildings (gymnasia and sports halls, dancing and concert halls, industrial buildings, etc.).

Pedestrian structures are often lightly damped because of the few nonstructural elements (surfacing, railing) which could contribute to system damping. Figure B8.2 gives a distribution of the damping values (logarithmic decrement) found in 44 investigated pedestrian bridges [2.5]. The damping values for the basic material, for beams with frictionless bearings and for bridges, respectively, are given in Tab. B8.1. Another investigation gave similar results [2.6]. Despite a considerable scatter the difference in construction produces

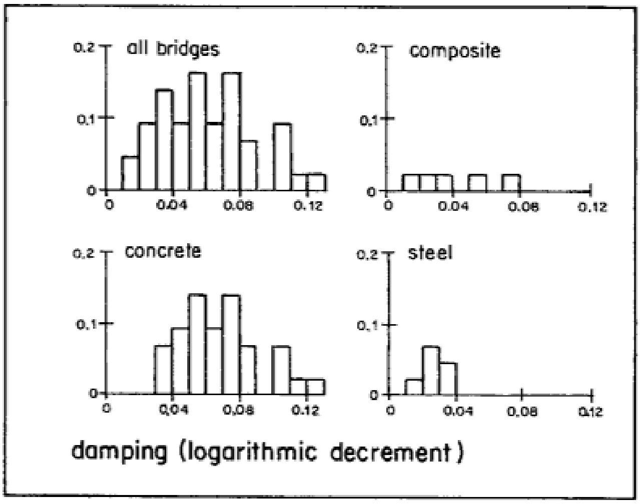


Fig. B8.2
Statistical distribution of the degree of damping found in 44 different footbridges (after [2.5]): total, and groups according to construction material

a systematic trend from non-prestressed concrete structures with the highest damping to welded steel structures with the lowest damping.

	basic material	beams	bridges
steel	$0.002 \div 0.008$	$0.004 \div 0.03$	$0.02 \div 0.06$
concrete	$0.01 \div 0.06$	$0.02 \div 0.06$	$0.02 \div 0.2$

Tab. B8.1 Damping in various construction materials and in structures due to contributions of system damping (from [2.5]); values are logarithmic decrements δ (Eq. (B1.13))

In the absence of better information, the following damping ratios may be assumed for calculation purposes:

- reinforced concrete structures $\xi = 0.7\%$
- prestressed and composite (concrete slab on steel girder) $\xi = 0.5\%$
- steel structures $\xi = 0.3\%$
- timber structures $\xi = 1.5\%$

In the vast majority of cases these values should give conservative lower bounds, the average values being possibly twice as high.

The larger damping found in *buildings* is due to energy dissipating floor cover, cladding, nonstructural partitions, installations, stored goods, etc. However, the high damping values quoted in handbooks are stated for earthquake excitation and should never be used

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Information for the practising engineer by means of documents of a high scientific and technical level on structural engineering themes.

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Vibrations in Structures
Induced by Man and Machines

«Vibrations in Structures» concentrates on vibrations in structures as excited by human motion or machine operation. *Man-induced* vibrations may arise from walking, running, skipping, dancing, etc. They occur mostly in pedestrian structures, office buildings, gymnasia and sports halls, dancing and concert halls, stadia, etc. Existing publications treat by and large some isolated aspects of the problem; the present one attempts, for the first time, a systematic survey of man-induced vibrations. *Machine-induced* vibrations occur during the operation of all sorts of machinery and tools with rotating, oscillating or thrusting parts. The study concentrates rather on small and medium size machinery placed on floors of industrial buildings and creating a potential source of undesirable vibrations. The associated questions have rarely been tackled to date; they entail problems similar to those of man-induced vibrations.

The book is consciously intended *to serve the practising structural engineer* and not primarily the dynamic specialist. It should be noted that its aim is not to provide directions on how to perform comprehensive dynamic computations. Instead, it attempts the following:

1. to show where dynamic problems could occur and where a word of caution is good advice;
2. to further the understanding of the phenomena encountered as well as of the underlying principles;
3. to impart the basic knowledge for assessing the dynamic behaviour of the structures or structural elements;
4. to describe suitable measures, both preventive to be applied in the design stage and remedial in the case of rehabilitation.