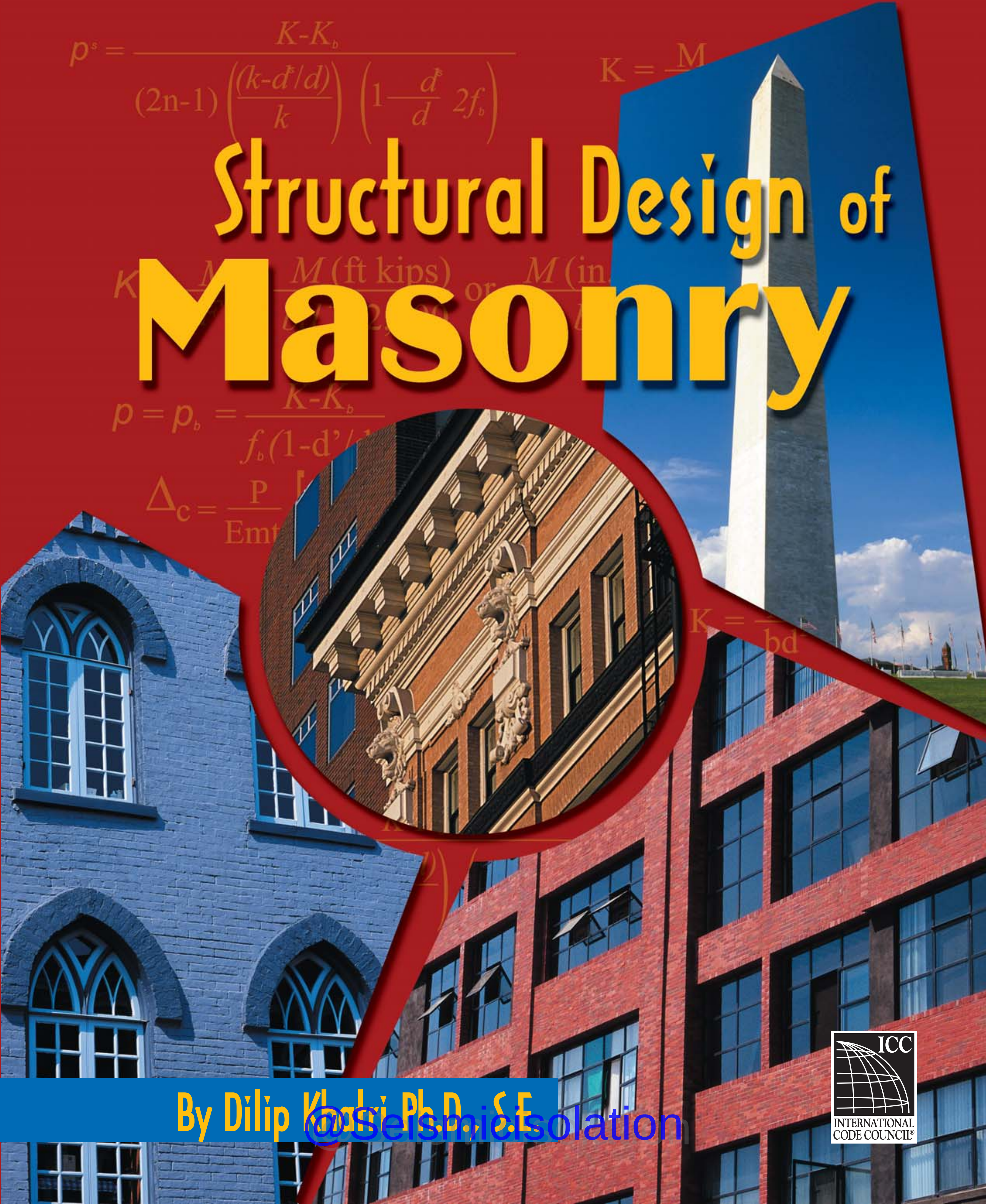
Structural Design of Masonry



By Dilip Khatri Ph.D., S.E.

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Preface

Structural Design of Masonry is intended to be a source of technical information for designers, builders, contractors, code officials, architects, and engineers: indeed, anyone involved with the business of masonry construction. Numerous sources, references, and technical experts have been consulted during its preparation.

The ability to solve structural design problems is a prime requisite for the success of any engineer and/or architect. To facilitate development of this ability, a collection of example problems accompanied by a series of practical solutions and structural engineering methodologies is included herein. These examples place special emphasis on detailed structural design of any portions of conventional structures for which masonry may be the designated material.

Since their introduction in the early 1960s, computer have enjoyed a phenomenal rise in popularity that has pushed members of the structural engineering profession to new heights driven by improved computational power and a growing need for new, safer buildings.

While older methods of structural design will remain useful, it becomes necessary to update the business of masonry design and accommodate to the pace of the construction industry in general.

To that end, recognizing the software capabilities of the Finite Element Method (FEM) when designing masonry buildings is essential. This text presents a series of problems/solutions to aid in the reader's understanding of the FEM. Specific reference is also made to Finite Element Analysis (FEA) as it concerns masonry structures and practical problem-solving techniques are included in the text.

The 1997 UBC and the 2000 IBC provide a fundamental source of information that supports the specific material contained herein. Both Working Stress Design and Strength Design methodologies are addressed, and specific code references are supplied where appropriate.

The CD accompanying this text contains the IBC and UBC chapters applicable to the subject of masonry construction.

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About the Author

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Dr. Khatri is a Registered Civil and Structural Engineer in the states of Illinois, New York, Virginia, and California, where he is also a licensed General Contractor.

His experience includes employment at NASA – JPL, Rockwell International, and the Pardee Construction Company. He has served as an expert witness for several construction-law firms and as an insurance/forensic investigator of structural failures. He served on the faculty of California State Polytechnic University in Pomona for seven years.

Dr. Khatri resides in Pasadena, California with his son, Viraj, to whom this book is dedicated.

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Note

In this document, certain numbers will appear in bold type at the right-hand margin of the text column. Such numbers will identify the sections, equations, formulas or tables appearing in the 2000 IBC and/or 1997 UBC that are referenced herein.

The 1997 UBC references are shown in parentheses

	IBC	UBC
Section	**000.0.0**	**(000.0.0)**
Equation	**Eq. 0-00**	**(Eq. 0-00)**
Formula	**F 0-0**	**(F 0-0)**
Table	**T 00.0**	**(T 00.0)**

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1

History of Masonry and Practical Applications

1.1 Brief History of Masonry

From the walls of Antioch to the Appian Way, from the Great Wall of China to the Pyramids of Giza, masonry has been used for fortifications, temples, roads, mosques, shrines, cathedrals, obelisks, and myriad other structures.

The Egyptians were among the first people in recorded history to use masonry, beginning construction on the massive pyramids at Giza circa 2500 BC. Historians and engineers still cannot determine how the ancient Egyptians could bring these raw materials together, cut them, move them, and place them where they are. The Temple of Khons, constructed at Karnak in the twelfth century BC, is another example of a massive Egyptian masonry undertaking.

The Egyptians were not the only civilization to discover the benefits of masonry. On the Yucatan Peninsula in Mexico, the Toltecs constructed El Castillo using the concept of masonry blocks in 1100 AD. And farther north, the Aztecs built their capital, Tenochtitlan, in 1325 AD; an entire city constructed using masonry technology.

In England, at about the same time the Toltecs were building El Castillo, William the Conqueror began construction on Windsor Castle. British castles had immediate practical use, providing the main line of defense against attackers. Even after the emergence of the Renaissance, castles were a functional part of British culture and continue to represent the history of the region.

In India, the magnificent Taj Mahal (Figure 1-1) was built over a span of twenty-two years, beginning in 1632 AD. It represents two important qualities in masonry: durability and architectural presence. Its marble, properly maintained, has shone for more than three centuries and will, presumably, continue to do so for centuries to come.

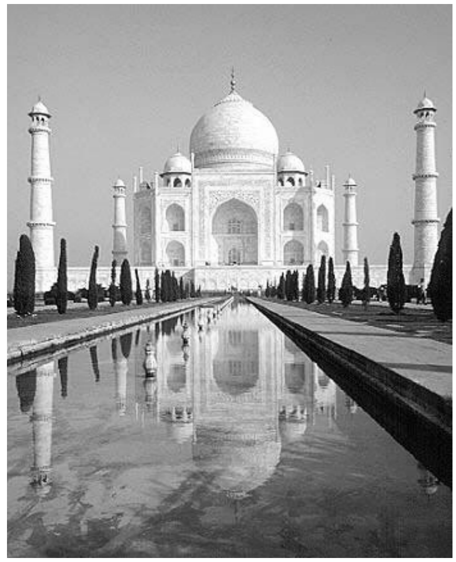


Figure 1-1
Taj Mahal

Masonry buildings comprised much of the early New York City skyline, (Figure 1-2). Among them, since demolished, was the Western Union Building in this 1911 photograph (Figure 1-3), which was constructed in 1872 and stood for over a century. The Evening Post Building (Figure 1-4) was another fixture of the New York skyline, and the Liberty Tower still stands as a landmark of masonry construction (Figure 1-5).

Collection of the New York Historical Society Negative No. 23366

Figure 1-2
Lower Manhattan, Bird's Eye View

Collection of the New York Historical Society Negative No. 48522

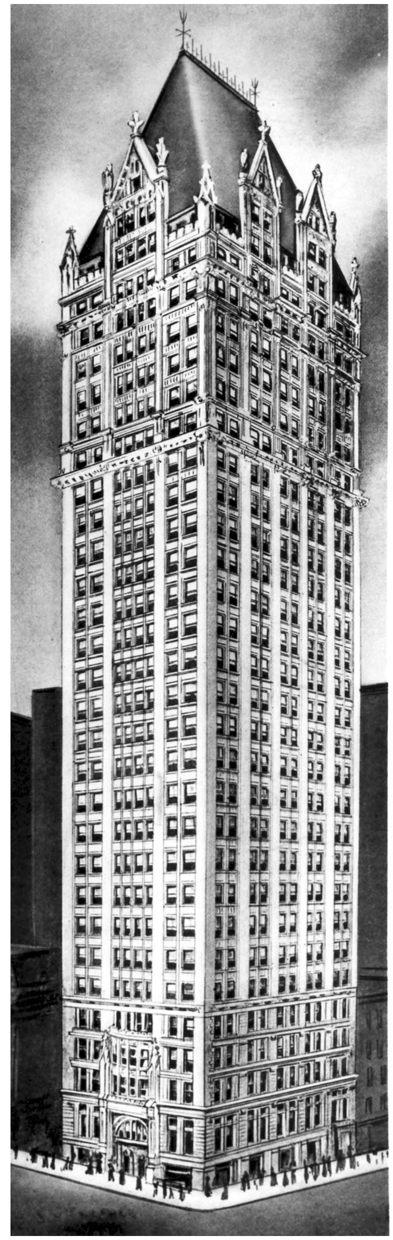
Figure 1-3
Western Union Building, Northwest Corner of Broadway and Dey Streets

Collection of the New York Historical Society Negative No. 75812

Figure 1-4
Evening Post Building

The Industrial Revolution brought steel and wood to the fore as construction materials, and during this time the use of concrete was perfected. However, masonry has always been the builders' choice because of three unique characteristics.

- Construction efficiency: masonry buildings use an automated process of assembling standard units (i.e., blocks). This allows for lower labor costs, ease of construction, and overall efficiency when compared to other modern methods.
- Fire endurance: masonry's long-term performance in fire resistance is unsurpassed. Only reinforced concrete structures can compare with reinforced masonry in this regard, but reinforced masonry has a lower construction efficiency rating.
- Strength and ductility: masonry has excellent compression properties that provide strength, and reinforcing steel provides ductility. Although reinforcement is a new concept in masonry--introduced in the twentieth century--the original characteristics of masonry were defined by weight. A mass of masonry creates a large vertical dead load that resists lateral loads. Ductility prevents collapse and, in areas prone to high seismic activity, provides insurance against damage from large-magnitude earthquakes.



Collection of the New York Historical Society
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Figure 1-5
Liberty Tower

1.2 Practical Aspects of Masonry

We have progressed from using large stones chiseled by hand to the 21st Century where not only have masonry construction elements changed but, to a great extent, the process and style of construction has also changed significantly. Today's masonry construction uses the building-block approach wherein each masonry unit is assembled into three basic structural elements that are then incorporated into a structural system. Figure 1-6 is a three-dimensional diagram of a typical masonry shear wall structure comprising pilasters and lintel beams.

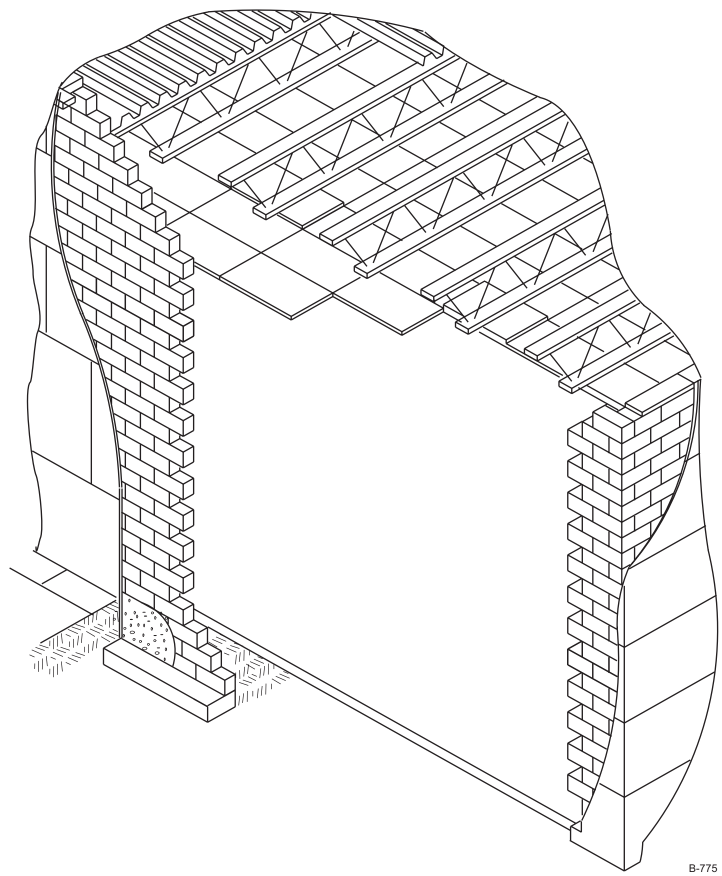

Figure 1-6

The three basic elements are

1) Walls: Structural and shear walls are designed to provide lateral stability both in-plane and out-of-plane (Figure 1-7).

2) Beams: Beams are designed for vertical transverse loads in bending (Figure 1-8).

3) Pilasters/Columns: These elements are designed for vertical axial loads (Figure 1-9).

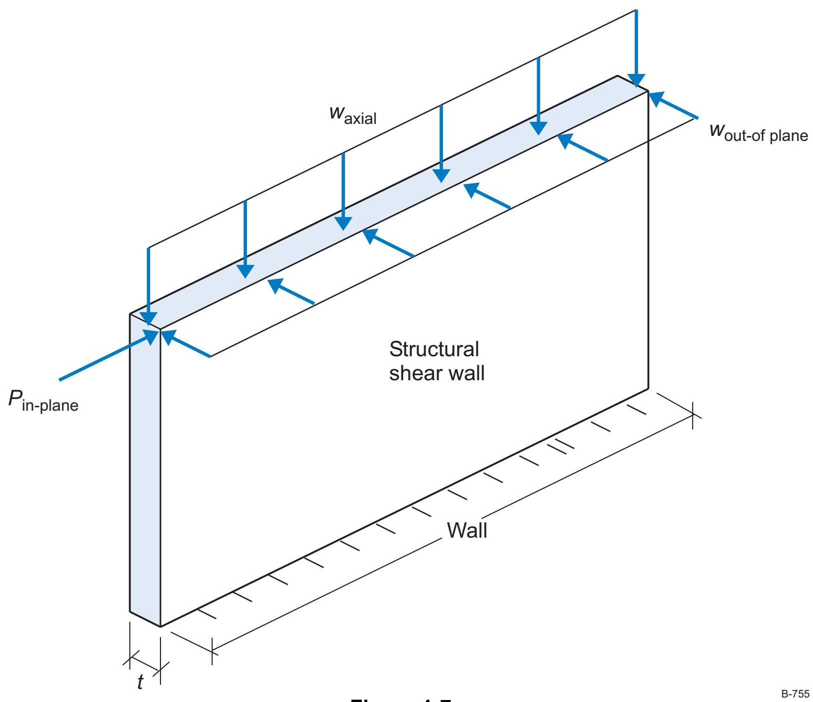

Figure 1-7

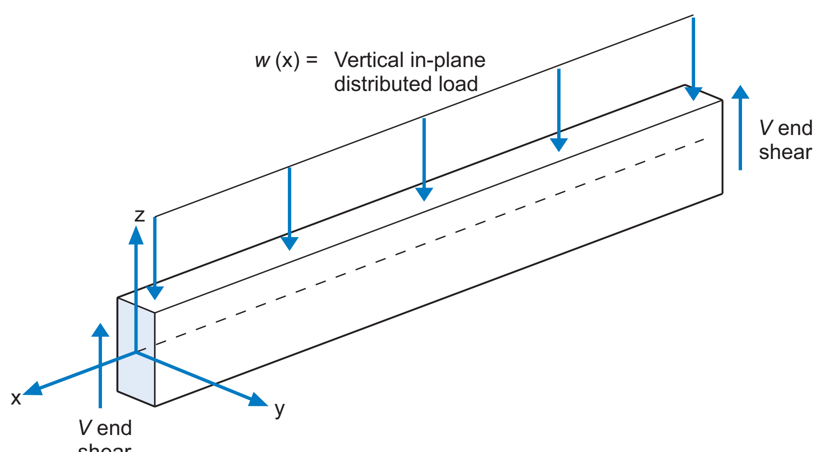

Figure 1-8

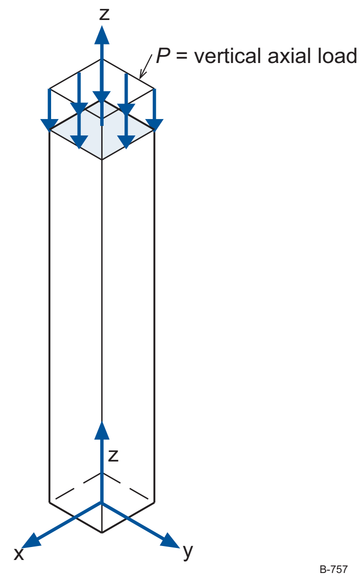

Figure 1-9

Structural engineers must master the design process of each of these components in order to assemble them into a building design. Every building can be broken down into the three elements.

The fundamental masonry units consist of blocks, which may be in the form of bricks or concrete masonry units (CMUs) manufactured by block plants that follow a standardized casting system subscribed to by the entire industry. Figure 1-10 shows some standardized shapes and sizes used for different masonry units. There are various construction methods for combining these masonry units into wall assemblies.

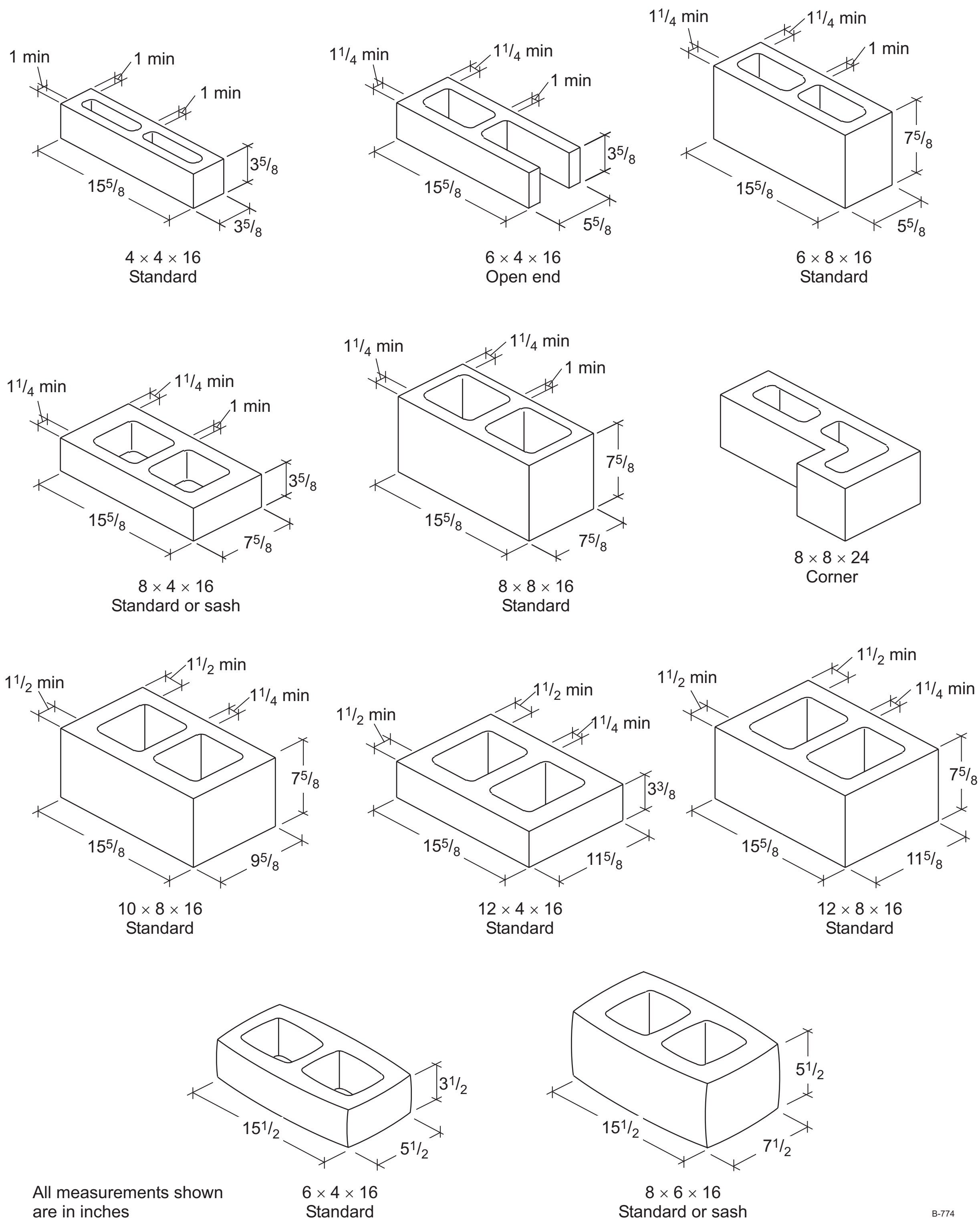

Figure 1-10

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Figure 1-11 is a three-dimensional sketch of a typical two-wythe brick wall, with two layers of brick assembled by using mortar between the brick elements. The cavity between the curtains of brick is filled with grout. The use of vertical and horizontal steel is incorporated in the design as a standard practice for earthquake-prone regions. This wall assembly is called reinforced grouted brick masonry. Architects use two-wythe walls when designing for a specific aesthetic appeal that can be achieved only with a brick exterior. Brick buildings were popular in the early 20th century because of their imposing presence but now are more costly to erect than those buildings that use other structural systems. However, brick facades convey a sense of ownership pride, and architects occasionally use the form where appropriate.

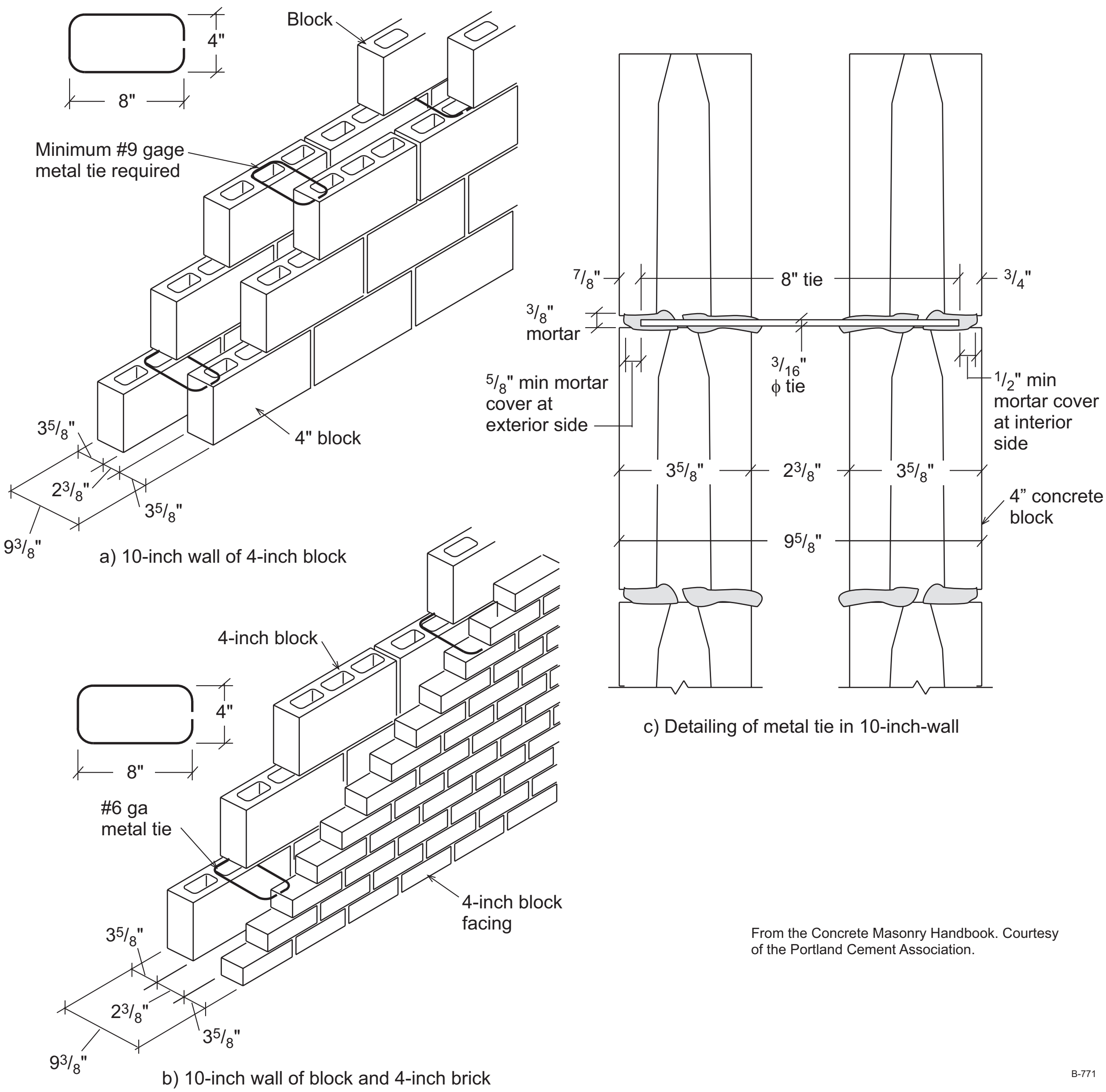


Figure 1-11

CMU construction uses the hollow masonry unit. Grout is placed within the masonry cell in reinforced units (Figure 1-12). Reinforcing steel serves two primary purposes: it provides bending resistance against out-of-plane loads, and also provides shear resistance against in-plane loads. In many areas of the world, the concept of reinforcing a masonry building is viewed as an unnecessary expense. This is certainly not the case for structures built in seismically active areas. Masonry possesses a strong compression resistance that is at least comparable to that of concrete, but with a lower construction cost. It would be foolish to construct any building solely out of concrete without steel reinforcement, and the same applies to masonry.

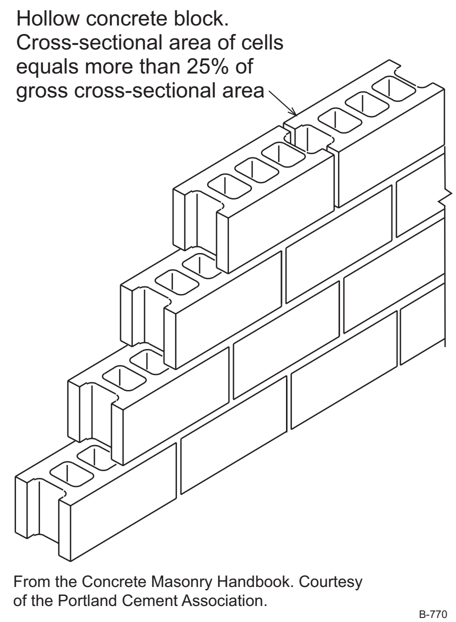


Figure 1-12

1.3 Practical Evaluation: Advantages, Disadvantages, and Cost Aspects

A structural engineer must quantify the practical value of masonry as a building material. Design parameters are required in order to facilitate decision-making early in the design process. Figure 1-13 is a flowchart that demonstrates the process of construction from start to finish. Every project is unique, but the intent of this flowchart is to present the thinking process and methodology usually followed in the industry today. As can be seen, once the choice of building material is made and the type of structural system is selected, it is nearly impossible to alter these decisions midway in the design process. Therefore, it is imperative that the design professional be acutely aware of all available choices and of the implications associated with the final selection.

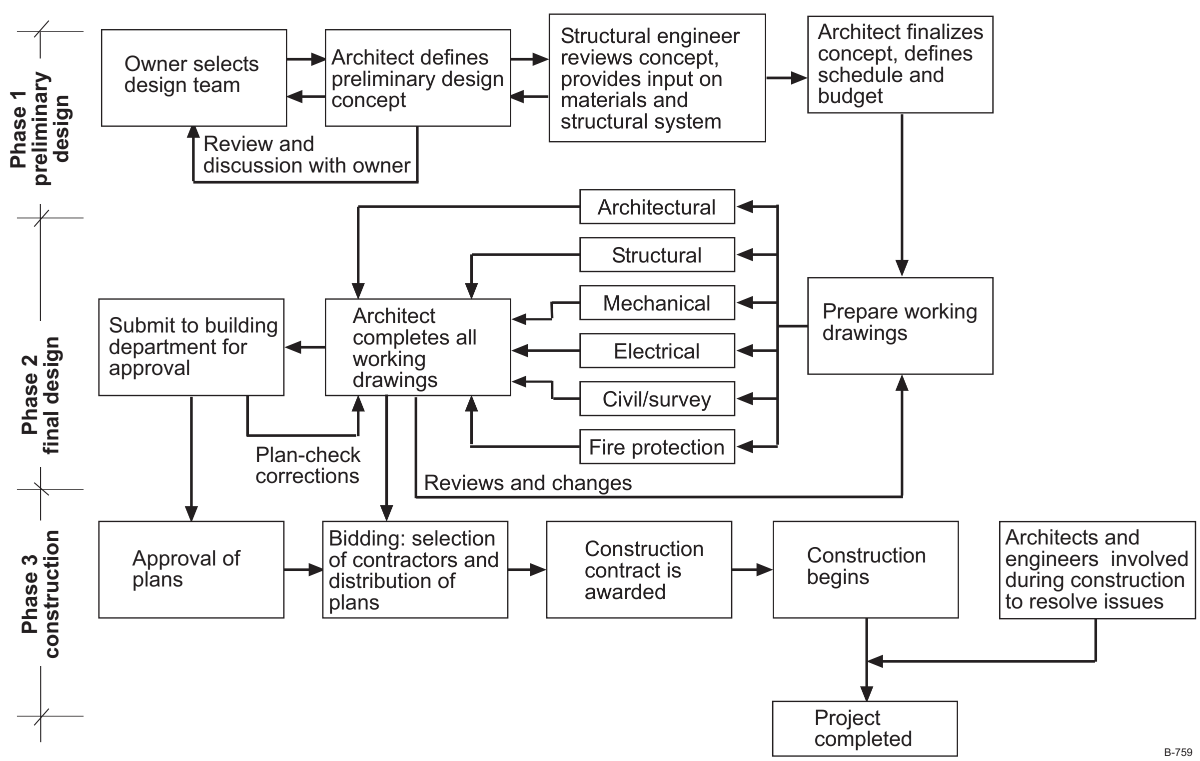


Figure 1-13

Reinforced masonry has been the choice for construction material in all seismically active areas of this country. Other parts of the nation that have traditionally relied on unreinforced masonry have been able to dispense with reinforcement because their local codes lack requirements for reinforcing. Until 1997, this procedure was acceptable. The recent codes (1997 UBC, 2000 IBC, and 2003 IBC) have extensive requirements for seismic evaluation and wind-resistant design that will change the status of plain masonry. Essentially, the trend is to move toward reinforced masonry as the standard. The following must be considered.

- Reinforced masonry has significant structural advantages over plain masonry. Even in seismically inactive areas there can be extreme demands on buildings: hurricane wind forces (74 to 140 mph), tornado wind forces (as high as 300 mph), and sudden wind gusts with peak velocities exceeding 110 mph. Since the IBC addresses these factors in detail, the requirement for reinforced masonry structures will increase nationwide.
- Reinforced masonry has both in-plane and out-of-plane shear and bending capacities. This will be further discussed in subsequent chapters because these capacities will affect the long-term durability of a masonry building.
- From a failure-analysis perspective, if the actual loads (demand) exceed design loads (capacity), structural engineers always have insurance in the design. This is known as the *factor of safety*. Reinforced masonry has an excellent built-in factor of safety because of the ductility value of the reinforcing steel. Figure 1-14 diagrams the terms associated with plastic/ductile performance of a reinforced masonry structure versus a plain masonry structure.

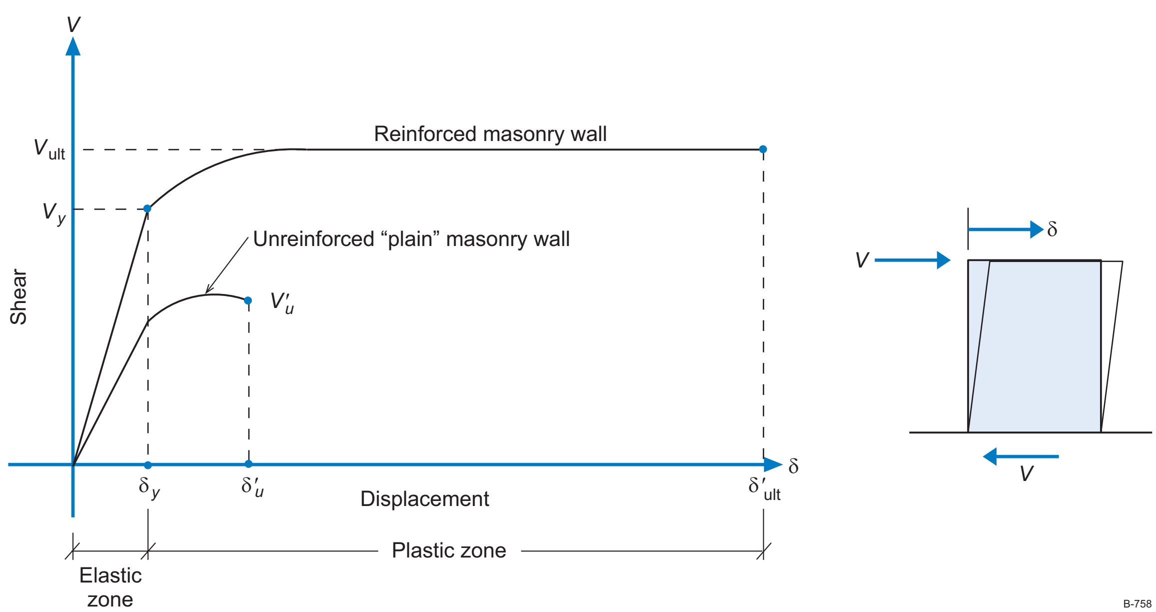


Figure 1-14

V_{ult} = ultimate lateral shear load = δ_{ult} = ultimate lateral deflection

V_y = yield shear load

δ_y = yield deflection

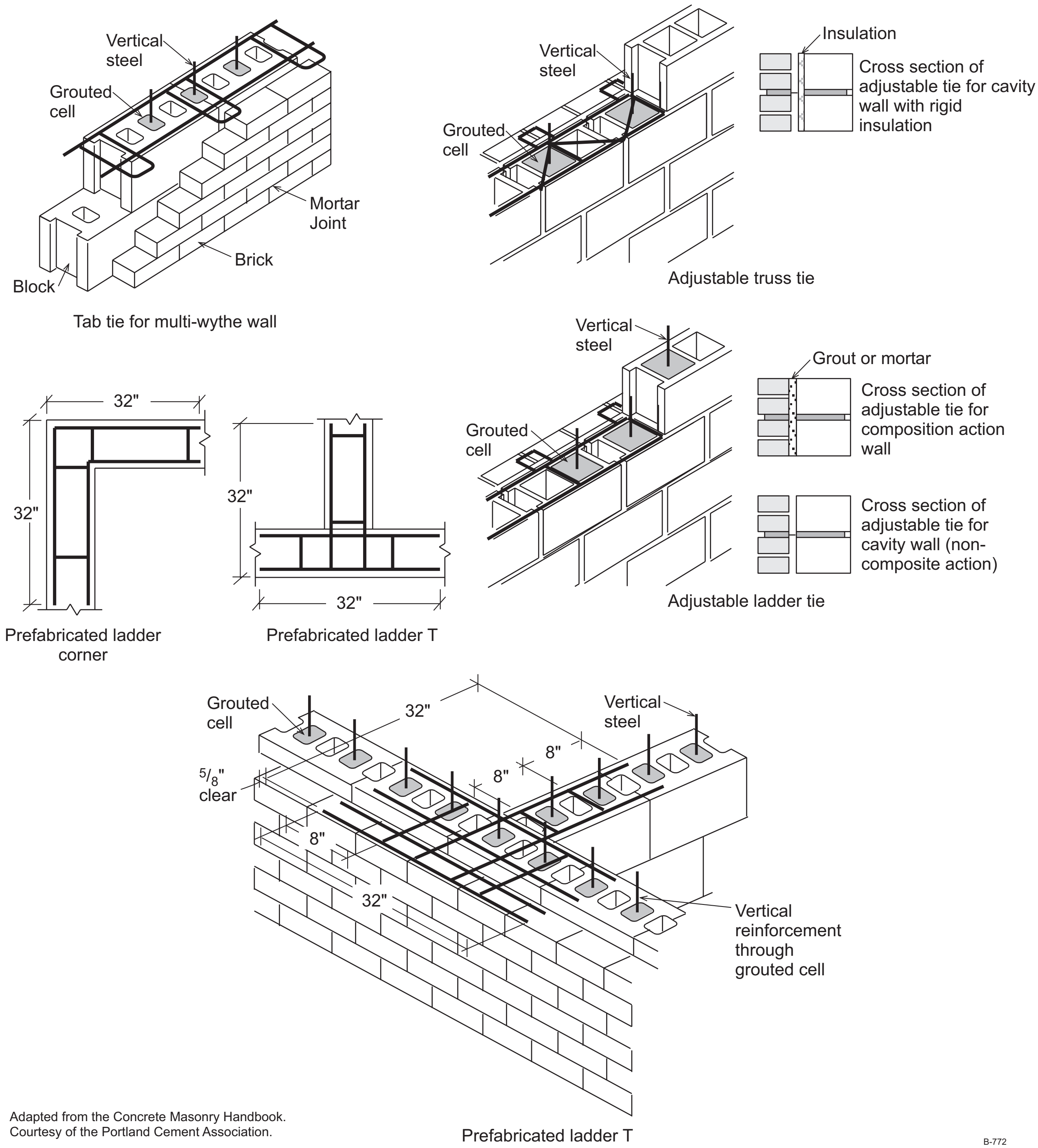
$\mu_\delta \quad = \dfrac{\delta'_{\text{ult}}}{\delta_y}$ = displacement ductility for reinforced masonry wall

$\mu'_\delta \quad = \dfrac{\delta'_u}{\delta_y}$ = displacement ductility for plain masonry wall

The additional cost of reinforced masonry includes the placement of steel reinforcement, the associated inspection, and construction time necessary to accomplish a quality job. The cost is nominal when compared to the numerous structural advantages, especially in seismically active locations. Opponents of reinforced masonry will usually argue that this is unnecessary over-design instituted by design professionals who must adhere to higher loading requirement regulations in other parts of the country. The factor-of-safety principle is equally important for all parts of the country and should be followed with uniformity. Every place in the world is subject to some form of natural disaster.

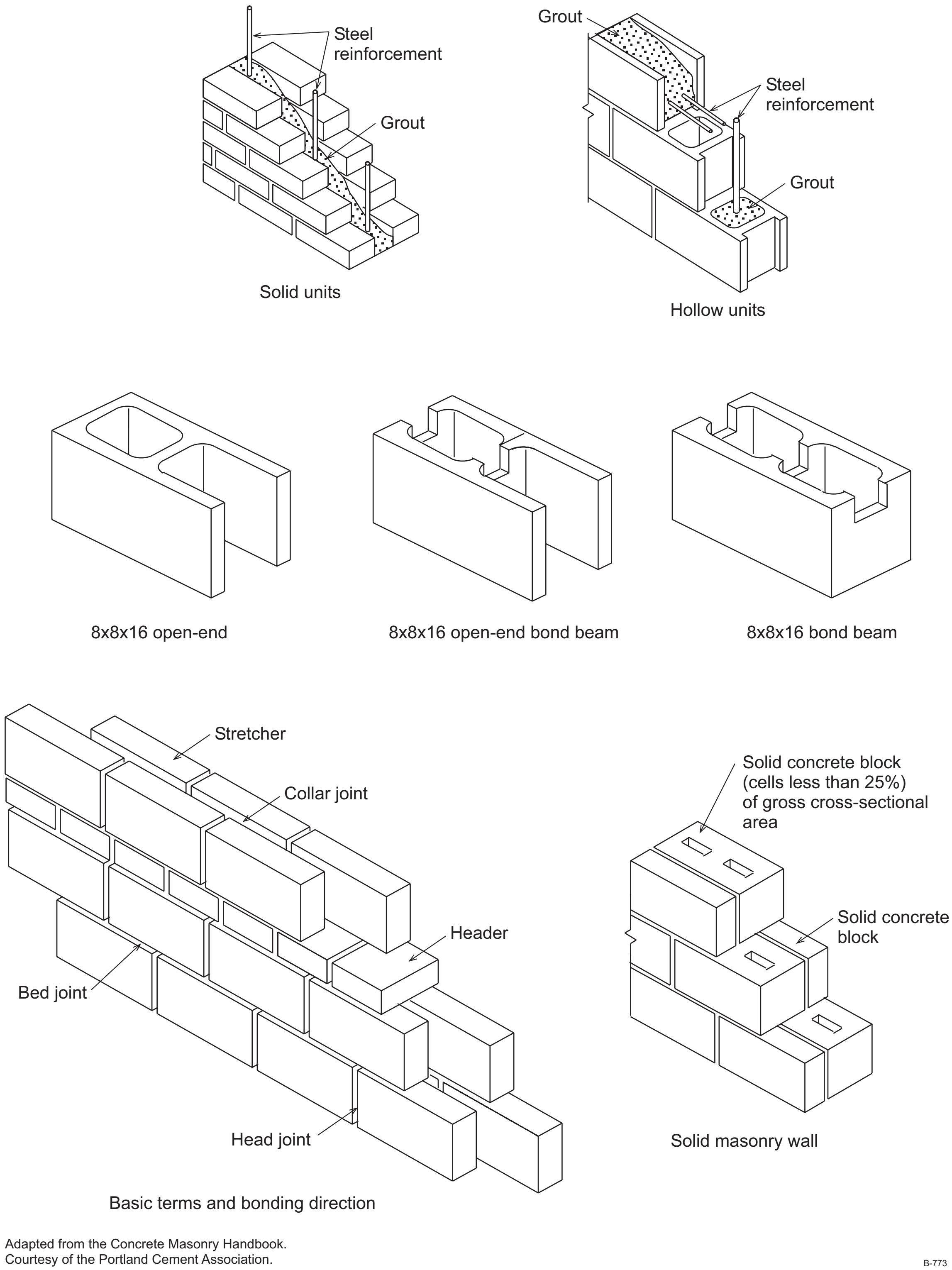
Masonry is produced in brick or concrete masonry units (CMUs). This allows for ease of placement and construction efficiency. Construction costs associated with reinforced concrete are heavily disproportionate toward the formwork. Formwork requires labor and materials in order to pour the concrete during the curing process, and it is a substantial part of the cost of reinforced concrete. In this lies the most powerful advantage of using reinforced masonry: no formwork. CMUs can be placed quickly, the steel positioned, inspection performed, and the grout placed in a matter of days (for a well-organized project). Figures 1-15 and 1-16 show examples of practical construction methods for reinforced masonry. These sample details show the practical aspects of actual wall construction. Construction efficiency has several advantages,

1) Projects can be kept on schedule allowing the contractor to manage the entire project without unexpected delays.
2) Costs are lowered, resulting in satisfaction for all concerned.
3) Material is more readily available. The length of time for ordering the product is reduced because block manufacturers have no shelf-life restrictions.



Adapted from the Concrete Masonry Handbook.
Courtesy of the Portland Cement Association.

Figure 1-15



Adapted from the Concrete Masonry Handbook.
Courtesy of the Portland Cement Association.

Figure 1-16

Another significant advantage of masonry over its competitors (wood and steel) is its fire rating. Fire rating and fire protection have assumed increased importance in the building industry because of large losses incurred by the insurance companies. From a designer's perspective, masonry offers a minimum 2-hour rating (per the UBC). This is far beyond the basic 1-hour rating provided by wood frame structures and fireproof/encased steel members. Figures 1-17 and 1-18 provide excerpts from the IBC on fire-resistance rating that demonstrate why masonry has such an excellent reputation in this area. In particular, grouted masonry walls are far superior to their steel and wood-frame counterparts because they approximate the performance of concrete shear walls.

IBC TABLE 719.1(2)
RATED FIRE-RESISTANCE PERIODS FOR VARIOUS WALLS AND PARTITIONS

MATERIAL	ITEM NUMBER	CONSTRUCTION	MINIMUM FINISHED THICKNESS FACE-TO-FACE (inches) 4 hour	3 hour	2 hour	1 hour
1. Brick of clay or shale	1-1.1	Solid brick of clay or shale[c]	6	4.9	3.8	2.7
	1-1.2	Hollow brick, not filled.	5.0	4.3	3.4	2.3
	1-1.3	Hollow brick unit wall, grout or filled with perlite vermiculite or expanded shale aggregate.	6.6	5.5	4.4	3.0
	1-2.1	4" nominal thick units at least 75 percent solid backed with a hat-shaped metal furring channel 3/4" thick formed from 0.021" sheet metal attached to the brick wall on 24" centers with approved fasteners, and 1/2" Type X gypsum wallboard attached to the metal furring strips with 1"-long Type S screws spaced 8" on center.	—	—	5	—
2. Combination of clay brick and load-bearing hollow clay tile	2-1.1	4" solid brick and 4" tile (at least 40 percent solid).	—	8	—	—
	2-1.2	4" solid brick and 8" tile (at least 40 percent solid).	12	—	—	—
3. Concrete masonry units	3-1.1	Expanded slag or pumice.	4.7	4.0	3.2	2.1
	3-1.2	Expanded clay, shale or slate.	5.1	4.4	3.6	2.6
	3-1.3	Limestone, cinders or air-cooled slag.	5.9	5.0	4.0	2.7
	3-1.4	Calcareous or siliceous gravel.	6.2	5.3	4.2	2.8
4. Solid concrete	4-1.1	Siliceous aggregate concrete.	7.0	6.2	5.0	3.5
		Carbonate aggregate concrete.	6.6	5.7	4.6	3.2
		Sand-lightweight concrete.	5.4	4.6	3.8	2.7
		Lightweight concrete.	5.1	4.4	3.6	2.5
5. Glazed or unglazed facing tile, nonload-bearing	5-1.1	One 2" unit cored 15 percent maximum and one 4" unit cored 25 percent maximum with 3/4" mortar-filled collar joint. Unit positions reversed in alternate courses.	—	6 3/8	—	—
	5-1.2	One 2" unit cored 15 percent maximum and one 4" unit cored percent maximum with 3/4" mortar-filled collar joint. Unit positions side with 3/4" gypsum plaster. Two wythes tied together every fourth course with No. 22 gage corrugated metal ties.	—	6 3/4	—	—
	5-1.3	One unit with three cells in wall thickness, cored 29 percent maximum.	—	—	6	—
	5-1.4	One 2" unit cored 22 percent maximum and one 4" unit cored 41 percent maximum with 1/4" mortar-filled collar joint. Two wythes tied together every third course with 0.030-inch (No. 22 galvanized sheet steel gage) corrugated metal ties.	—	—	6	—
	5-1.5	One 4" unit cored 25 percent maximum with 3/4" gypsum plaster on one side.	—	—	4 3/4	—
	5-1.6	One 4" unit with two cells in wall thickness, cored 22 percent maximum.	—	—	—	4

Figure 1-17 (Continued)

IBC TABLE 719.1(2) (Continued)
RATED FIRE-RESISTANCE PERIODS FOR VARIOUS WALLS AND PARTITIONS

MATERIAL	ITEM NUMBER	CONSTRUCTION	MINIMUM FINISHED THICKNESS FACE-TO-FACE (inches) 4 hour	3 hour	2 hour	1 hour
10. Hollow (studless) gypsum wallboard partition	10-1.1	One full-length layer of 5/8" Type X gypsum wallboard attached to both sides of wood or metal top and bottom runners laminated to each side of 1" x 6" full-length gypsum coreboard ribs spaced 24" on center with approved laminating compound. Ribs centered at vertical joints of face plies and joints staggered 24" in opposing faces. Ribs may be recessed 6" from the top and bottom.	—	—	—	2 1/4
	10-1.2	1" regular gypsum V-edge full-length backing board attached to both sides of wood or metal top and bottom runners with nails or 1 5/8" drywall screws at 24" on center. Minimum width of rumors 1 5/8". Face layer of 1/2" regular full-length gypsum wallboard laminated to outer faces of backing board with approved laminating compound.	—	—	4 5/8	—
11. Noncombustible studs—interior partition with plaster each side	11-1.1	3 1/4" x 0.044-inch (No. 18 carbon sheet steel gage) steel studs spaced 24" on center. 5/8" gypsum plaster on metal lath each side mixed 1:2 by weight, gypsum to sand aggregate.	—	—	—	4 3/4
	11-1.2	3 3/8" x 0.055-inch (No. 16 carbon sheet steel gage) approved nailable studs spaced 24" on center. 5/8" neat gypsum wood-fibered plaster each side over 3/8" rib metal lath nailed to studs with 6d common nails, 8" on center. Nails driven 1 1/4" and bent over.	—	—	5 5/8	—
	11-1.3	4" x 0.044-inch (No. 18 carbon sheet steel gage) channel-shaped steel studs at 16" on center. On each side approved resilient clips pressed onto stud flange at 16" vertical spacing, 1/4" pencil rods snapped into or wire tied onto outer loop of clips, metal lath wire-tied to pencil rods at 6" intervals, 1" perlite gypsum plaster, each side.	—	7 5/8	—	—
	11-1.4	2 1/2" x 0.044-inch (No. 18 carbon sheet steel gage) steel studs spaced 16" on center. Wood fibered gypsum plaster mixed 1:1 by weight gypsum to sand aggregate applied on 3/4-pound metal lath wire tied to studs, each side. 3/4" plaster applied over each face, including finish coat.	—	—	4 1/4	—
12. Wood studs interior partition with plaster each side	12-1.1[l,m]	2" x 4" wood studs 16" on center with 5/8" gypsum plaster on metal lath. Lath attached by 4d common nails bent over or No. 14 gage by 1 1/4" by 3/4" crown width staples spaced 6" on center. Plaster mixed 1:1 1/2 for scratch coat and 1:3 for brown coat, by weight, gypsum to sand aggregate.	—	—	—	5 1/8
	12-1.2[l]	2" x 4" wood studs 16" on center with metal lath and 7/8" neat wood-fibered gypsum plaster each side. Lath attached by 6d common nails, 7" on center. Nails driven 1 1/4" and bent over.	—	—	5 1/2	—
	12-1.3[l]	2" x 4" wood studs 16" on center with 3/8" perforated or plain gypsum lath and 1/2" gypsum plaster each side. Lath nailed with 1 1/8" by No. 13 gage by 19/64" head plasterboard blued nails, 4" on center. Plaster mixed 1:2 by weight, gypsum to sand aggregate.	—	—	—	5 1/4
	12-1.4[l]	2" x 4" wood studs 16" on center with 3/8" Type X gypsum lath and 1/2" gypsum plaster each side. Lath nailed with 1 1/8" by No. 13 gage by 19/64" head plasterboard blued nails, 5" on center. Plaster mixed 1:2 by weight, gypsum-to-sand aggregate.	—	—	—	5 1/4

Figure 1-17

Sound protection is an important design quality. Increasingly, owners are protesting the low sound-protection level of their buildings' finished product. Nowhere is this more evident than in multiple dwelling unit projects (i.e., apartments and condominiums). Traditional wood frame shear-wall construction has one recognized weakness: transmission of sound through the walls can reach a disproportionate level of annoyance to the unit residents. Designers have to compensate for this problem in wood frame walls by using insulation and/or creating a dual wall system; essentially, this is a double wall system with an open-air cavity between the walls.

Masonry's advantage in this area is due to its high sound transmission classification (STC) rating. The higher the STC rating, the better the sound protection. It is clear that hollow (ungrouted) walls have the lowest rated performance value, while fully grouted walls can reach STC values of 60. This is easily accomplished with masonry. As a point of comparison, the minimum STC rating requirement for the City of Los Angeles Building Department is 45. Figure 1-18 provides the STC rating for various wall assemblies.

The majority of sound walls constructed along freeways consist of fully grouted 8-inch masonry block.

Data from Sound Transmission Loss Tests (ASTM E90) of Concrete Masonry Walls*

Wall description	Test No. **	Wall weight, psf	STC
Unpainted walls:			
8-in. hollow lightweight-aggregate units, fully grouted, #5 vertical bars at approx. 40 in. o/c	1023-1-71	73	48
8-in. hollow lightweight aggregate units	1144-2-71	43	49
8-in. composite wall—4-in. brick, 4-in. lightweight hollow units	1023-4-71	58	51
8-in. dense-aggregate hollow units	1144-3-71	53	52
10-in. cavity wall—4-in. brick, 4-in. lightweight hollow units	1023-6-71	56	54
Walls painted on both sides with 2 coats of latex paint:			
4-in. hollow lightweight-aggregate units	1379-5-72	22	43
4-in. hollow dense-aggregate units	1379-3-72	28	44
6-in. hollow lightweight-aggregate units	933-2-70	28	46
6-in. hollow dense-aggregate units	1397-1-72	39	48
8-in. hollow lightweight-aggregate units, fully grouted, #5 vertical bars at approx. 40 in. o/c	1023-2-71	73	55
Walls plastered with $^{1}/_{2}$-in. gypsum plaster on both sides:			
8-in. composite wall—4-in. brick, 4 in lightweight hollow units	1023-10-71	61	53
8-in. hollow lightweight-aggregate units, fully grouted, #5 vertical bars at approx. 40 in. o/c	1023-9-71	79	56
10-in. cavity wall—4-in. brick, 4-in. lightweight hollow units.	1023-8-71	59	57
Walls covered with $^{1}/_{2}$-in. gypsum board on resilent channels:			
4-in. hollow lightweight-aggregate units	1379-4-72	26	47
4-in. hollow dense-aggregate units	1379-2-72	32	48
8-in. composite wall—4-in. brick, 4-in. lightweight hollow units	1023-5-71	60	56
8-in. hollow lightweight-aggregate units	933-1-70	40	56
10-in. cavity wall—4-in. brick, 4-in. lightweight hollow units.	1023-7-71	58	59
8-in. hollow lightweight-aggregate units, fully grouted, #5 vertical bars at approx. 40 in. o/c	1023-3-71	77	60

(Reproduced with permission from Concrete Masonry Handbook for Architects, Engineers, Builders - PCA EB008.05M)

Figure 1-18

Structural performance characteristics will be addressed in detail throughout this text, but in summary, reinforced masonry offers essential qualities to a designer. Reinforced masonry walls have excellent in-plane shear resistance. Even without reinforcement, the grout bond along with the weight of the walls creates a formidable entity with sufficient inertia to resist large loads. The addition of steel reinforcement provides an extra line of resistance that creates a rigid structure with excellent ductility. In-plane shear resistance is an important feature of structural design.

Additionally, reinforced masonry can perform as an overturning resisting element, similar to a steel moment frame; hence the term *masonry wall frame*. The overturning moment resistance allows masonry to be used for tall structures as a viable structural element in shear and bending. The significant difference between masonry wall frames and steel moment frames is in their ductility performance and overall stiffness. This area is debatable, but reinforced masonry walls are essentially rigid structural shear walls with excellent ductility. To allow steel moment frames to compensate for this factor, they must be braced to create the equivalent stiffness level. An additional advantage of reinforced masonry is that it offers out-of-plane resistance as well as in-plane shear and overturning. Therefore, the masonry wall frame is a functional structural element in three dimensions: vertical/axial load capacity, lateral in-plane shear and bending, and out-of-plane shear and bending. Steel-braced frames and wood frame shear walls cannot perform in this capacity. A similar structural element is a reinforced concrete shear wall.

No building material is perfect, and reinforced masonry does have its problems. A common perception is that masonry has quality control issues that relate to grout quantity, quality, and placement of steel reinforcement. Particularly in seismic activity zones, high quality and correct placement of steel reinforcement is of paramount concern. If the reinforcement is not properly applied, then the masonry wall has no more structural integrity than an ungrouted wall. There has been a continual problem with enforcement in the field to ensure that proper structural specifications are followed. This has sometimes led to the unfortunate assumption that masonry does not give the same quality performance as steel and reinforced concrete.

Quality control and adequate inspection of the finished product have been among the greatest challenges faced by the masonry industry. The 2000 IBC places responsibility on the design professional to facilitate and administer the inspection program.

The 2000 IBC is currently implemented in many states. Inspection should be required and administered by a design professional (architect or structural engineer). Numerous issues can arise from inadequate inspection practices that cause devaluation of property, lawsuits, and fragmentation of the construction process.

Another disadvantage in masonry construction is the limiting strength of the masonry assembly. Chapters 3 through 6 of this text address the structural engineering aspects of masonry in detail but, in general, designers will find that two principle factors control the strength of the assembly: steel yield strength (for Grade 60, normally 60 ksi), and masonry ultimate compressive strength, f'_m. Masonry compression strengths range from 1500 to 3000 psi, with the typical value of 3000 psi being the common choice. As structural walls get taller and require greater out-of-plane resistance, the limiting factor in the design process becomes the masonry compressive strength. If the upper limit of compression strength reached 5000 psi, the engineer would be forced to use larger block and hence construct a thicker wall. This creates limitations on using masonry for mid- to high-rise construction. Thus, many designers prefer to use steel moment frames or concrete shear walls. Their concern is based on a lack of testing on high-strength masonry as well as the poor quality control that has pervaded the masonry construction industry. If the upper limits of masonry can be raised and higher strength block/brick produced, then this product can be taken seriously as a usable component for large-scale structures.

The greatest problems found in masonry construction are cracking and waterproofing. There exists a history of difficulties associated with cracks radiating from the mortar joint line. These begin with temperature and shrinkage stresses that lead to separation points along the mortar bed joints. Although masonry walls seldom have structural failure, they often exhibit extensive cracking that creates the illusion of failure.

The Northridge, California earthquake (1994) produced thousands of minor mortar joint cracks in masonry walls. Some of the walls had true structural damage and required full replacement. However, in many instances the walls were only cosmetically damaged. Because of the owners' perceptions, though, insurance carriers replaced these walls to avoid litigation. There are no uniform standards defining the width of an acceptable crack to aid in determining whether the structure should be repaired or replaced. Engineers are left to their own judgment on these cases and must employ empirical design methodology to justify their decisions. This leads to extensive debate on the size and length of a crack and its effect on the structural integrity of a damaged wall.

Compounding the problem of cracking, water leakage through masonry walls is a common problem, particularly in subterranean structures with reinforced masonry retaining walls. Masonry is a porous block that water will permeate; therefore, the water-seepage problem is not a defect in the masonry unit (designers should be aware that this is characteristic of the block), but rather a failure in the waterproofing system. In this instance, ignorance is the culprit. Waterproofing of subterranean masonry should be accomplished with a water barrier and subdrain system.

1.4 Summary

Masonry’s long history as a construction material emphasizes several unique qualities that distinguish its capability: 1) durability, 2) fire resistance, 3) strength, and 4) ductility.

The modern application of masonry has evolved into the use of mass-produced units that are divided into two categories: bricks and concrete masonry units (CMUs). These are assembled into three basic structural elements: walls, beams, and pilasters. All masonry buildings use a combination of these elements. The basic elements of wall construction consist of the two-wythe brick wall and the hollow CMU wall with reinforcing in the grouted cells.

Masonry construction has both advantages and disadvantages that require evaluation before a designer makes a final choice of construction material. Among the advantages are cost savings, construction efficiency, strength, ductility, fire rating, sound insulation, and long-term durability. These provide excellent performance at a competitive price.

The disadvantages include poor quality control, limiting strength values, and long-term problems with cracking and water leakage. All these disadvantages are controllable and may be eliminated through adequate design, construction, and inspection. The advantages definitely outweigh the disadvantages and masonry is an excellent choice as a building material.

Assignments:

1. Select a masonry building in the nearby vicinity.

   1.1 Conduct a site visit, and document the following items:

   a) Research the architect, structural engineer, contractor, and date of construction.

   b) What was the main purpose of this building?

   c) Why was it built?

   d) How much did it cost?

   1.2 Photograph the structure from exterior and interior points, and document your field observations as to the type of brick/CMU used.

   1.3 Write a brief essay/report on your findings.

   a) Include six specific sections: Introduction, History, Construction, Exterior, Interior, and Conclusions.

   b) Provide your observations as to the architectural concept and floorplan layout.

   c) If you were designing this building, what would you have done differently, or what changes would you recommend to the present owner?

2. Several famous masonry structures in Washington D.C. were not presented in this chapter. Among these are the Capitol building, the Lincoln Memorial, the Jefferson Memorial, and the buildings of the Smithsonian Institution. Choose one of these structures and obtain photographs as well as specific information about the history, construction, and the architect. Compile the information into a brief report.

3. Define the following terms:

   a) grout

   b) shear wall

   c) mortar

   d) beam

   e) wall

4. With reference to the 1997 UBC and the 2000 IBC provisions on masonry prism testing to establish the f'_m value, research answers to the following:

a) Compare the prism test requirement from the 1997 UBC and the 2000 IBC. How many prisms are required in each?

b) What is the minimum area required to comply with the two codes? Is there any change in the requirements?

c) How does the engineer distinguish the f'_m value between the strength of the masonry unit (i.e., brick or CMU) versus the mortar strength versus the grout? These three components are all different; how does the f'_m value assess these quantities?

5. Research the definition of f'_m in the MSJC, and provide an explanation of the statistical model used to define this quantity. It is an identical definition to f'_c for concrete, but provide a detailed explanation of the concept of the probability of its being exceeded and the concept of the 10^{th} percentile.

6. Examine the differences between the two-wythe masonry brick wall and the CMU grouted wall.

   a) What is the primary difference between these two construction methods with respect to cost factors?

   b) From a practical standpoint, state three distinct advantages and three disadvantages of a two-wythe wall over a CMU block wall.

7. Research the 2000 IBC inspection requirements and provide a summary of the important points that distinguish this code from the 1997 UBC. What are the major differences between the two codes concerning inspection and quality control?

2

Masonry Components and Structural Engineering

2.1 Introduction

The three organizations responsible for developing the majority of building codes used in the United States have recently combined their efforts and formulated one standardized code for the entire nation: the 2000 *International Building Code*® (IBC)® for all buildings other than detached single family and two-family dwellings, which are covered by the *International Residential Code*® (IRC)®. This text focuses on the 1997 Uniform Building Code (UBC) and the 2000 IBC, as the standard documents for engineering practice. The IBC is updated every three years; therefore, readers are advised to keep up-to-date with the appropriate version.

Before beginning an in-depth analysis of masonry, it is necessary to understand the elements of a building. Familiarization with the terms of structural engineering is the first step toward comprehension of the entire building system.

2.2 Load Path

The term *load path* is used repeatedly in the structural engineering profession without a clear definition in any text, code, or standard. The engineer must understand this principle. Simply stated, *the load path is the structural system by which vertical and horizontal loads travel and distribute from their point of application to the foundation.* Whether they are wind forces, hurricane dynamic pressures, explosions, live loads, dead loads, or any variation on these factors, all loads should reach the foundation and then be transferred into the soil where they terminate. The line followed to reach the foundation becomes the load path.

Every building has its own distinct load path, but all buildings may be categorized into distinctive classes that differ on the basis of the load path system and the construction materials. Masonry buildings, for example, use a system of shear walls and pilasters to transfer vertical and horizontal loads to the foundation. Steel structures use beams and columns, referred to as moment frames, for their load path. Reinforced concrete buildings may be either moment frame or shear wall buildings. The first step for a structural engineer is to choose the type of load-path system for the building prior to commencing detailed design. Several key issues are inherent in the selection of the structural system, including the seismic vulnerability factor, construction cost, and architectural impact (defined as the effect on the building's elevation – an architect is concerned with the aesthetics of a building, and structural integrity affects this aspect).

2.2.1 Moment frame system [UBC 1629.6.4, IBC 1602.1]

Figure 2-1 shows a moment-frame structure in three dimensions; Figure 2-2 shows a two-dimensional moment-frame cross section.

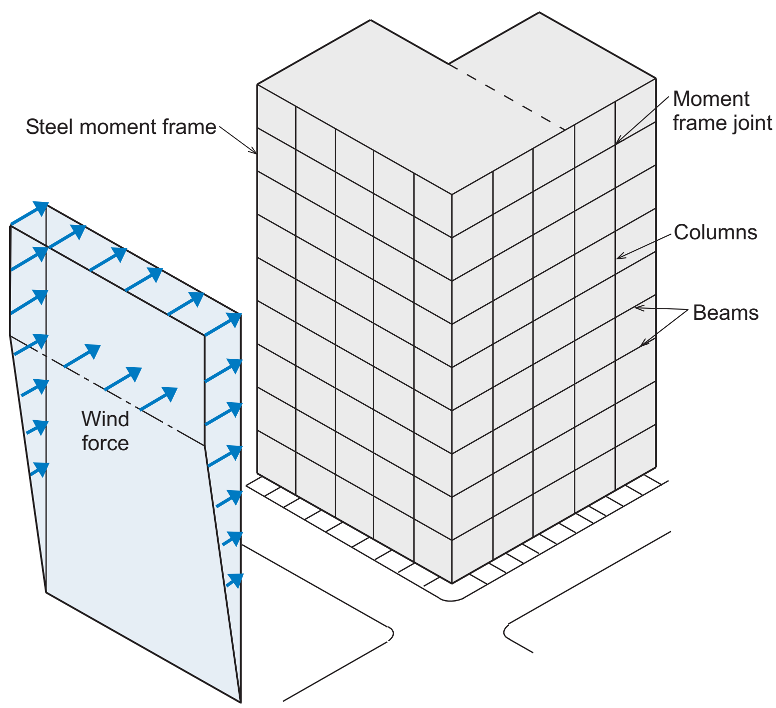


Figure 2-1

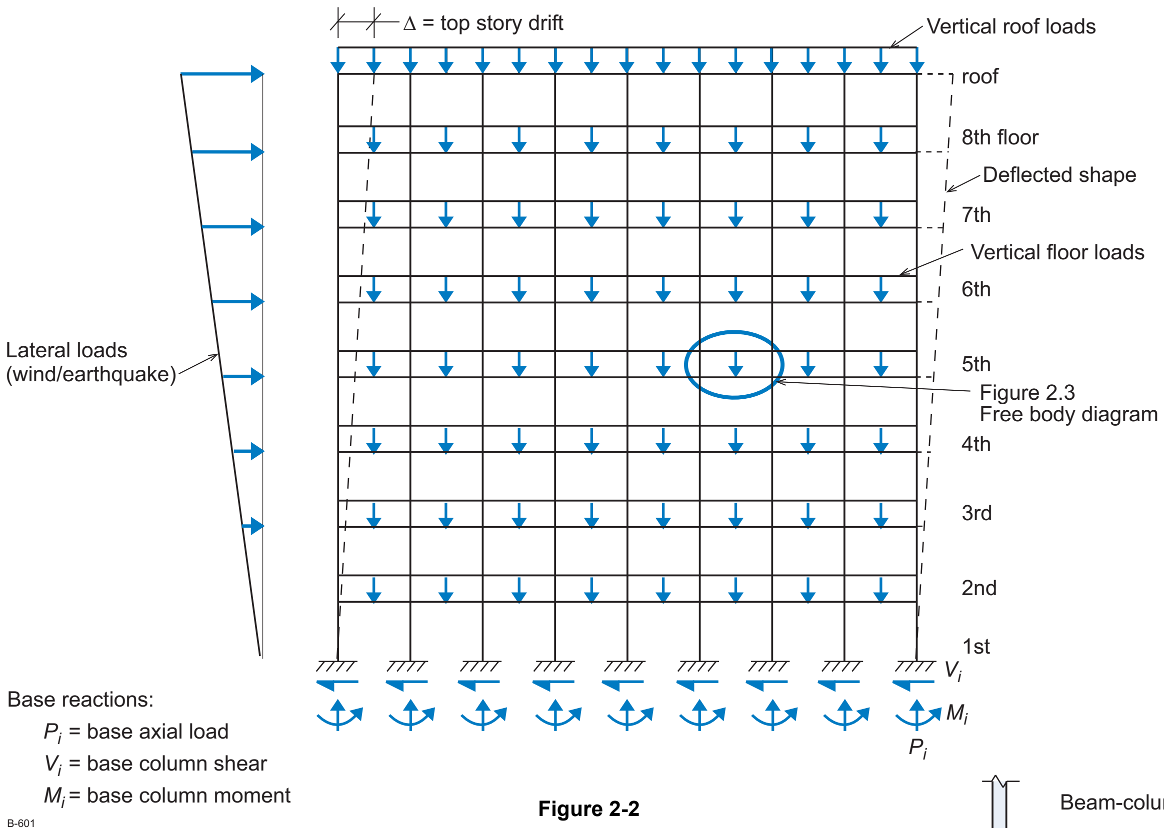


Figure 2-2

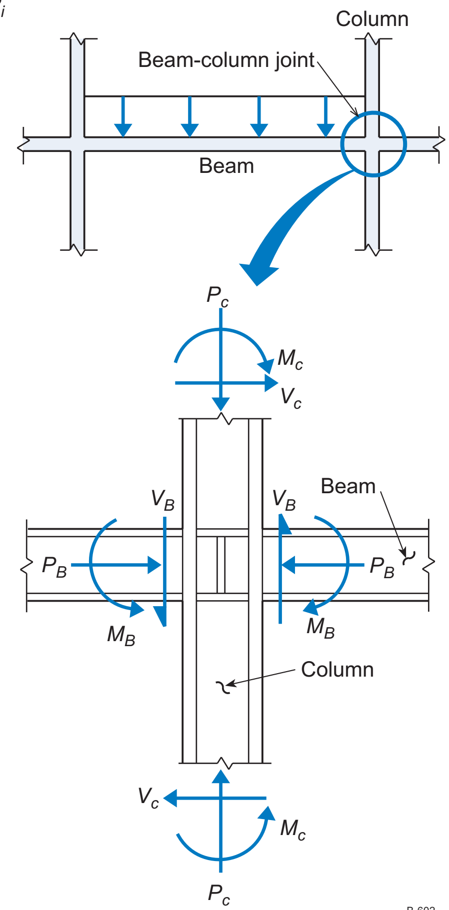


Figure 2-3

A moment frame transfers loads from each individual floor by using beams that rest on columns. The columns then transfer the vertical loads to the foundation elements, which could range from spread footings to piles to deep caissons. The resolution of horizontal and vertical loads is handled by a beam element that functions primarily in bending and the forces transfer to the column through bending and shear. Hence the term *moment frame*, because each beam-column joint absorbs enormous moments and resists rotation. Figure 2-3 diagrams the effect of the moments on a typical joint.

The modern steel moment-frame structure has been the primary choice for buildings exceeding 40 stories. All the major high-rise structures in New York, Hong Kong, Chicago, and Kuala Lumpur were built using the steel moment-frame system/concept. However, serious weld cracks that were observed after the Northridge, California earthquake of 1994 have challenged many notions concerning this building type's invincibility. It is worth noting that the moment-frame system is still active and may be used, but the reader is advised to keep current with the latest developments in steel moment-frame design. In particular, the Federal Emergency Management Agency (FEMA) is issuing several documents to modify the steel moment-frame connection design.

Within the moment-frame system are subcategories that comprise the ordinary moment frame (OMF), the special moment-resisting frame (SMRF), and the braced frame (i.e., Chevron, X-Brace, and K-Brace). These subclassifications fall under the general category of moment frames, and are discussed in detail in: *Steel Design* by Englekirk, and the *ASD Handbook* by AISC.

2.2.2 Bearing wall system [UBC 1629.6.2, IBC 1602.1]

Instead of using beams and columns, this system relies on vertical walls that are connected to either a flat slab concrete floor or a beams-with-girders system. Figure 2-4 shows the elements of the bearing wall structural system. The bearing wall system is the primary load path for masonry buildings.

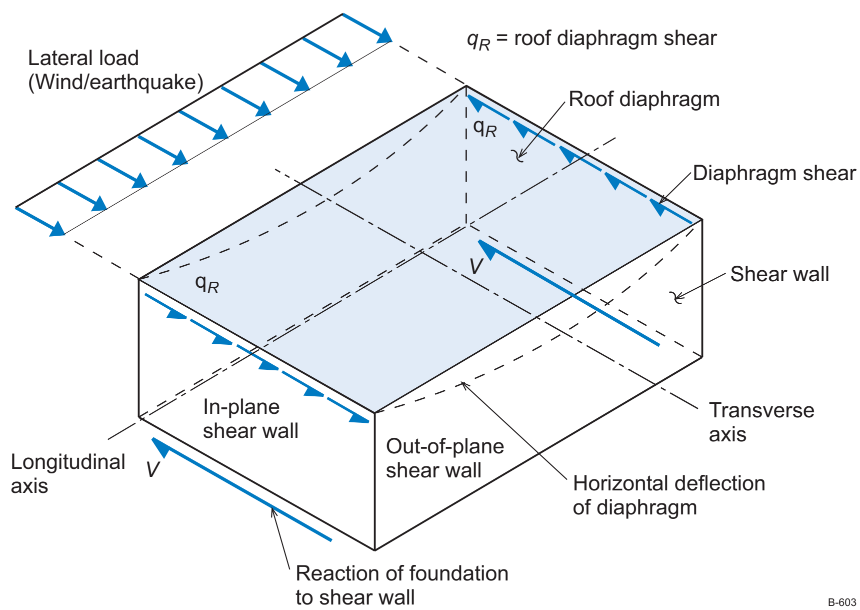
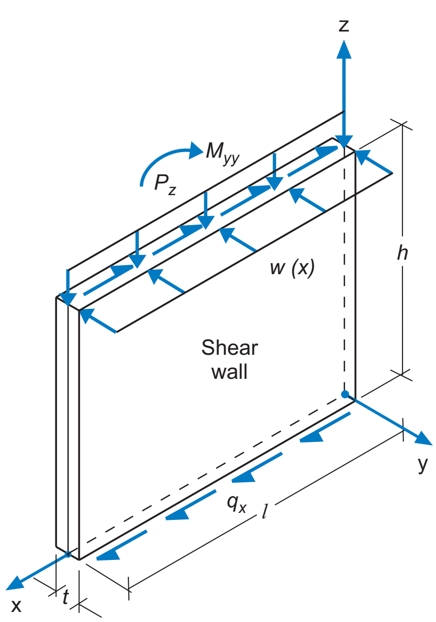


Figure 2-4



q_x = in-plane shear
P_z = in-plane axial load
M_{yy} = in-plane moment
$w(x)$ = out-of-plane load

Figure 2-5

A bearing wall building consists of two distinct components: vertical shear wall elements and horizontal diaphragms. Shear walls are vertical plane walls that resist in-plane forces (Figure 2-5). Loads are transferred through connections to the shear wall and then carried by virtue of bending, shear, and axial stress to the foundation. Shear walls can also resist out-of-plane forces, provided that construction materials are designed accordingly. For example, wood shear walls are not designed for out-of-plane loads because they are not structurally adequate for this application. However, reinforced masonry is structurally adequate and can be designed to resist these forces. Reinforced concrete shear walls may also be designed to resist out-of-plane forces, if necessary. Figure 2-6 shows an assembly of the bearing wall building. A complete definition of the building type is provided in UBC 1629 and in IBC 1602. (Both chapters are on the CD that accompanies this book.)

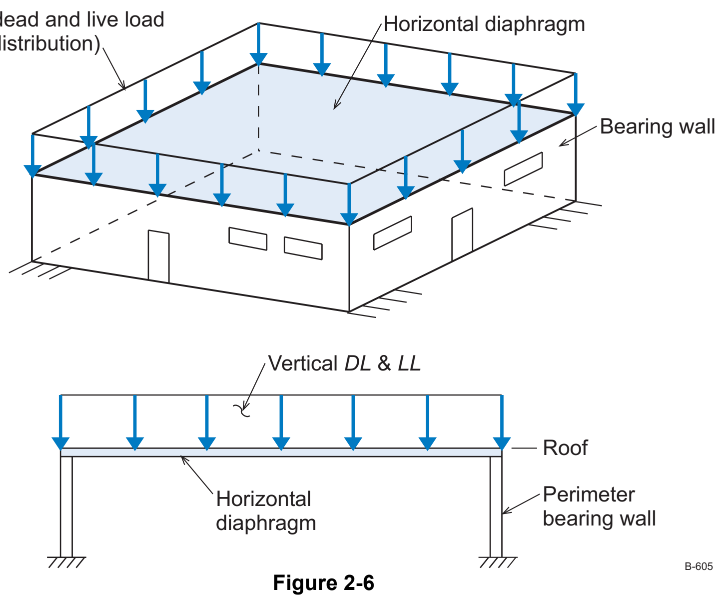


Figure 2-6

A diaphragm is a structural element that resists horizontal in-plane forces as does a shear wall, but with an additional benefit: a diaphragm also resists out-of-plane vertical loads, which are the service loads of the occupants, dead loads, and live loads (Figure 2-7). In a wood floor or steel frame diaphragm, the vertical loads are transferred by a system of joists and girders to supporting columns and/or shear walls. With a reinforced concrete floor system, the vertical loads are transferred by using a flat-plate slab supported by columns/walls. The lateral loads are resisted by means of the diaphragm's in-plane shear resistance and are transferred to the supporting shear walls. These walls absorb both in-plane and out-of-plane loads and are referred to as bearing walls because they function as more than shear walls. The term *shear wall* is used to describe bearing walls, but the IBC is very specific about this definition (IBC 1602, UBC 1629.6).

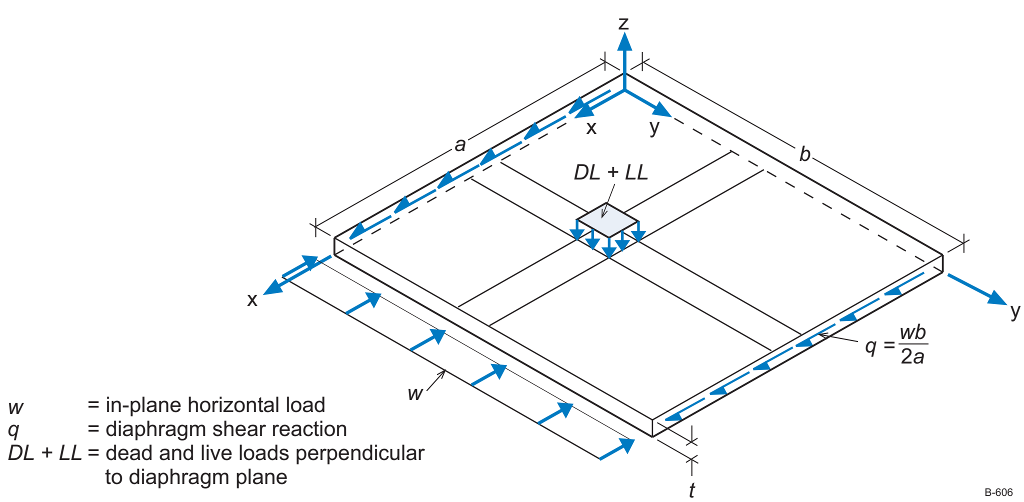


Figure 2-7

There are three types of diaphragms: rigid, semi-rigid, and flexible**.** Figures 2-8 and 2-9 demonstrate the conceptual difference between the two extremes of rigid and flexible.

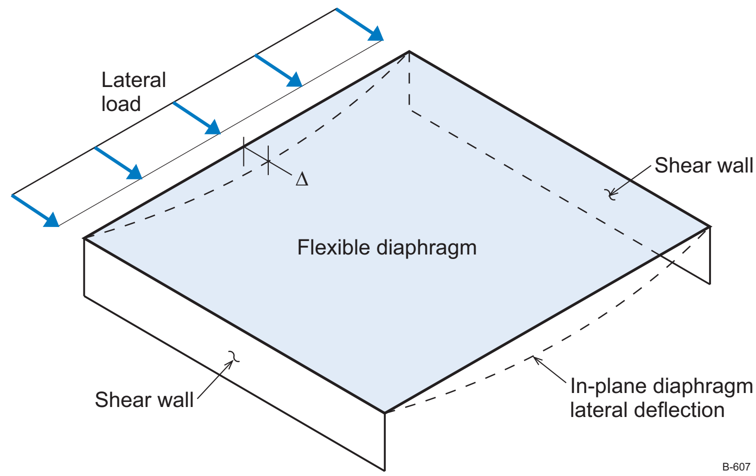


Figure 2-8

Wood diaphragms have traditionally been considered flexible diaphragms because of their wood joist-to-plywood connection. Flexible diaphragms work on the assumption that deflections within the horizontal plane are similar to a beam, and therefore result in shear/bending stresses.

A rigid diaphragm is assumed to be perfectly stiff with no in-plane deflections allowed. Figure 2-9 illustrates this concept and is particularly applicable to elevated reinforced concrete slabs such as parking structures, office buildings, and bridge decks. Rigid diaphragms do not allow for in-plane deflections, and therefore lead to deflection in the shear walls.

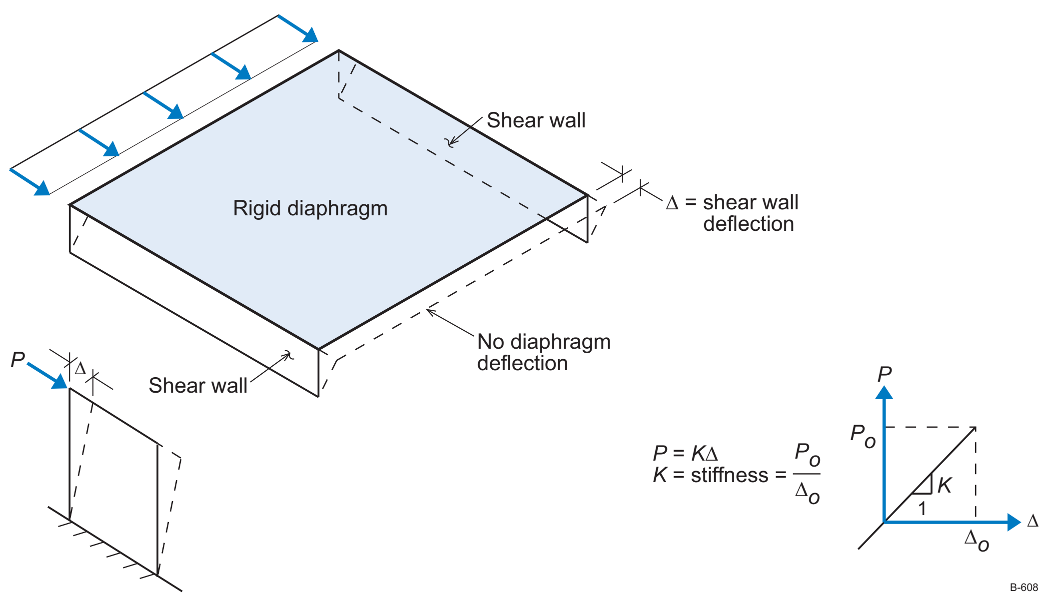


Figure 2-9

A semi-rigid diaphragm is a combination of the rigid and flexible versions. There are numerous practical examples of semi-rigid diaphragms. Among these are wood frame floor systems that have a 3- to 4-inch lightweight concrete decking. The lightweight concrete can reduce the sound transmission between floors and act as a fire retardant. It functions superbly in both capacities. Structurally, the performance of lightweight concrete also contributes to the in-plane stiffness of the wood diaphragm. This system may be regarded as a semi-rigid diaphragm. However, there are diverging opinions among practicing structural engineers on this point, and some may argue that this is a rigid diaphragm. Others will contend that the supporting structure is the wood framing and that the system is flexible. To further complicate this issue, there is no uniform design practice for dealing with semi-rigid diaphragms. Structural engineers will either use the flexible assumption and distribute the load based on tributary areas or will go to the other extreme of calling for a rigid diaphragm and use the stiffness assumption.

The important difference between the two systems is the load distribution. For flexible diaphragms, the load is distributed on the basis of tributary area concept. The principle of tributary area is simple; each wall absorbs the lateral load based on its equal share of span length. If two perimeter walls are separated by a 50-foot span and there are no interior shear walls, then the tributary width of each span to the wall is 25 feet. If one wall is added at mid span, then the perimeter wall will absorb $^1/_2 \times 25$ feet, which equals a $12^1/_2$-foot tributary width of the lateral load. The center shear wall will absorb $2 \times 12^1/_2$ feet, a 25-foot tributary width. Flexible diaphragms are used on a regular basis in wood frame construction, and the tributary area concept is fundamental to all wood buildings.

For rigid diaphragms, load distribution is based on the stiffness of each shear wall. This concept will be covered at greater depth in subsequent chapters, but the stiffness of each wall is defined and shown in Figure 2-9 at the lower right-hand corner.

A linear stiffness assumes that the wall load-to-deflection curve is straight with a slope, K. The stiffer the wall (i.e., higher value of K), the more load will be distributed to that shear wall. The less stiff (i.e., lower value of K), the less lateral load will be distributed to that shear wall. Nature has its own method of distributing less force to weaker walls and directing the larger forces to stronger or stiffer walls. It is natural for structures to distribute forces (i.e., energy) to their respective elements on the basis of stiffness. For example, a stiff wall will absorb more energy than a flexible wall. This follows the basic principles of physics and structural mechanics.

With rigid diaphragms, the load distribution will not be equal because each wall has its own strength properties. This leads to further structural engineering issues in floor plan designs for which the architect may require one elevation to be open and the other to be closed. Placing a weak wall next to a very stiff wall will lead to rotation of the diaphragm (a torsional moment). Rigid diaphragms pose such as challenge, and must be dealt with accordingly.

2.2.3 Building frame system (UBC 1629.6.3, IBC 1602.1)

The building frame system is a mixture of the moment-frame and bearing-wall systems. The moment frame provides for vertical loads. The bearing walls provide lateral resistance. When employing both systems, architects may use the moment frame for building portions requiring unrestricted views and the structural engineers' shear walls can be hidden inside the building. Figure 2-10 shows an example of such a floor plan.

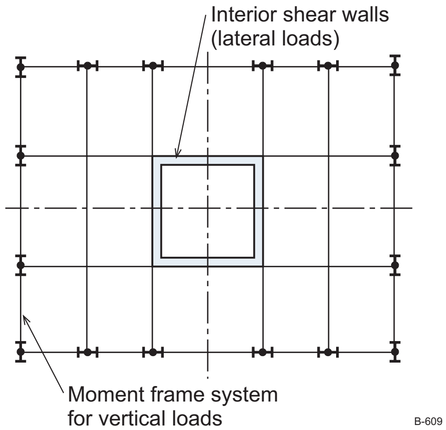


Figure 2-10

The loads are divided between the frame and the wall, which allows the architect more flexibility in configuring the layout. Building frame systems are commonly used with reinforced masonry shear walls to balance and/or stiffen a steel moment-frame structure. The advantage of reinforced masonry is that it can be constructed to be as stiff as reinforced concrete, but at a lower cost. Building frame systems usually will have a semi-rigid diaphragm having a steel floor system and lightweight concrete decking. The primary purpose of the lightweight concrete floor was described earlier, but in an office application, the lightweight concrete also minimizes floor deflections by adding to the vertical stiffness.

2.2.4 Dual system (UBC 1629.6.5, IBC 1602.1)

A dual system is similar to a building frame system with a separate moment frame and shear wall structure. The difference is in the lateral load formulation. Dual systems allow the lateral load to be resisted by either shear walls or moment frames and braced frames. This could be any combination of a special moment-resisting frame (SMRF), intermediate moment-resisting frame (IMRF), ordinary moment-resisting frame (OMRF), and moment-resisting wall frame (MRWF). Dual systems are common in mid- to high-rise buildings using steel for construction.

Figure 2-11 is a schematic cross section of the dual system concept.

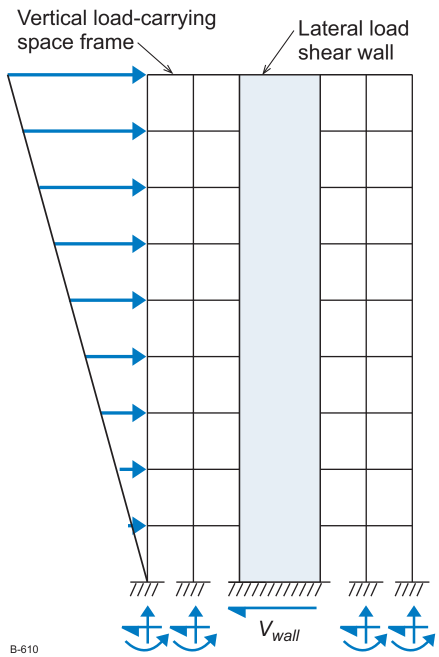


Figure 2-11

To the architect, the advantages of this system are the multi-purposes of achieving unrestricted views while having full flexibility to design an efficient tower structure for high-rise buildings. Dual systems are efficient because they can lower the structural dead-load distribution to an economical value (i.e., between 30 and 40 pounds per square foot). Remember that construction cost is directly proportional to the dead load: the heavier a structure, the higher the building cost. Therefore, any method or practice that reduces this single factor means savings on the construction budget.

2.2.5 Cantilevered column system (UBC 1629.6.6, IBC 1602.1)

This system, commonly applied to buildings under five stories, uses a stiff vertical column with a fixed base connection into the foundation for the first (and perhaps second) story (Figure 2-12). It is also known as an inverted pendulum system. The vertical column is a bending element that resists lateral shear forces by virtue of its moment connection at the base. This is similar to a moment frame, except that the top connection point is hinged and allows for rotation/lateral deflection.

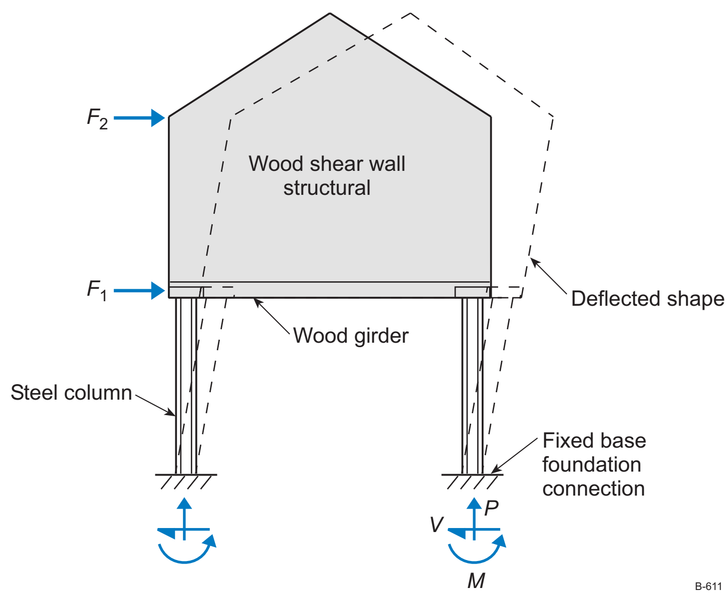


Figure 2-12

In certain wood frame buildings where shear walls are difficult to add because of architectural limitations on the floor plan design, the inverted pendulum concept allows a simple column element to substitute for an entire shear wall. The stiffness is derived from a fixed base column with free rotation at the top. Although this is a practical idea, the connection at the top allows for lateral deflection and can lead to secondary bending moments in the columns induced by vertical load—called the *P*-delta effect (Figure 2-13). This is the most dangerous factor and a proven disadvantage of this system: under large seismic loads the *P*-delta moments can result in column failure and eventual collapse of the building along the first-story line. Although not a frequent occurrence in past earthquakes, it has happened in situations where the column was not properly designed or was improperly connected or built.

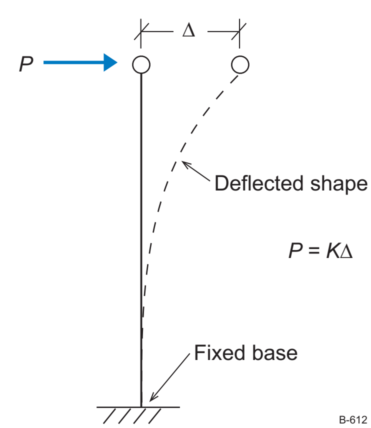


Figure 2-13

2.3 Vertical Load Analysis (UBC 1602, 1606, 1607, and IBC 1602)

Vertical loads are divided into two categories: dead loads and live loads. Dead loads are those inherent to the structure and are part of the permanent operation of the building. Live loads are derived from the occupancy of the building. Calculating these loads is the second step of a structural engineer's task. The formulae for these calculations are the product of many years of conservative assumption, application of factors of safety, and guesswork. The next step for the engineer is the distribution of loads, and application of such formulae to the tributary area concept for wood frame systems. For flat-plate concrete diaphragms, more in-depth analysis using one-way or two-slab analysis is required.

Structural engineers share a fear that haunts the profession: failure of the structural design. Even more worrisome is the potential loss of life resulting from that failure. The primary objective is to prevent such an event. All training, education, seminars, books, professional licenses, and building projects are devoted to the survival of the occupants. Both the IBC and UBC are dedicated to the promotion of better building construction and greater public safety.

In this regard, all load calculations are based on a factor of safety that allows permutations outside the normal range of loading. In particular, the live loads (UBC Table 16-A and IBC Table 1607.1) that are provided for various occupancy-use classifications have built-in factors of safety. For example, a 200-square-foot residential floor space with a 40-pound-per-square-foot live load equates to $200 \times 40 = 8000$ pounds. If that floor space were to be occupied by 20 football players at 300 pounds each, that would only be $20 \times 300 = 6000$ pounds. Admittedly, it would be quite difficult to fit 20 football players in a 200-square-foot space, but this is an indication of the over-estimation present in the codes. Another scenario involves vehicle weight. Assuming a medium size car is approximately 2000 pounds, the small 200-square-foot space can support almost four cars. Neither of these situations is attainable or realistic, but the floor system hypothesized is structurally adequate to support a high load capacity. Thus, the factor of safety is always part of the design because engineers would prefer to overestimate the load rather than underestimate it. Employing a similar design philosophy, all loads, wind, snow, and earthquake, are formulated on this basis. Although practicing engineers are sometimes accused of being overly conservative, the United States has a very low building risk-loss ratio when compared to other industrialized areas. (Building risk-loss ratio measures the percentage of a building’s total replacement value that could be lost in a catastrophic event.) Earthquakes in particular are notorious for wreaking economic havoc. Therefore, the practicality of overestimation and the factor of safety is evident in seismic design.

The load tables in Chapter 16 of the 2000 IBC are formulated with this in mind. All the design loads have overestimated the actual load. The strength properties of the lumber in the floor beams, shear walls, and foundation capacities are also estimated with a factor of safety, at both ends of the equation, which will serve to keep buildings safe and survivable during any forseeable catastrophic event.

Implicit in the vertical load design process is the concept of live-load reduction, which is an indirect method of reducing the over-estimation of live loads in a cumulative effect of multistory design. IBC Equation 16-1 is the live-load reduction formula. Live-load reduction (IBC 1607.9) will keep the design live loads realistic during the actual selection of structural elements.

2.4 Wind Load Design

Nowhere is factor of safety more necessary than in the wind and earthquake load provisions. The recurring force on a building is the wind pressure that will occur without interruption, and with 100-percent certainty.

The chart (Figure 2-14) shows the wind velocity-versus-time graph for a typical site. It is uneconomical to design for the peak wind velocity, because 99 percent of the time the structure undergoes lower forces; this is termed *over-design* or *conservative design.* At the other end of the spectrum, choosing a lower-than-average wind velocity would be dangerous because the bulk of the demand would be higher than the chosen design wind force. Under-design would cause an eventual structural failure, which is obviously counter productive.

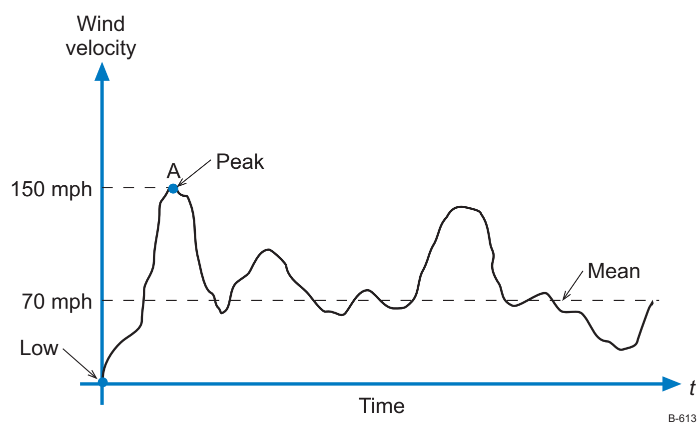


Figure 2-14

Economics plays a pivotal role in every decision. If structures were over-designed, the cost increase could be prohibitive. To maintain a balance, engineers must find a suitable compromise between over- and under-design. It is a process that is connected to mathematical probability analysis, judgment, politics, material performance, and permutations in load combinations. The UBC helps to solve this problem by requiring uniformity and standardization.

Wind provisions are contained in IBC 1609. The basic principle of wind design is to convert wind velocity into a force/pressure diagram. The 2000 IBC wind design provisions (1609) are based on ASCE 7-98. The fundamental equation for wind pressure is provided in ASCE 7-98. The 2000 IBC utilizes the Simplified Provisions for low-rise buildings (1609.6) that follow Tables 1609.6.2.1.(1) through (4).

Figure 2-15 shows the wind pressure diagram per IBC Figure 1609.6(3). The 2000 IBC and ASCE 7-98 tables are used to calculate the wind pressure.

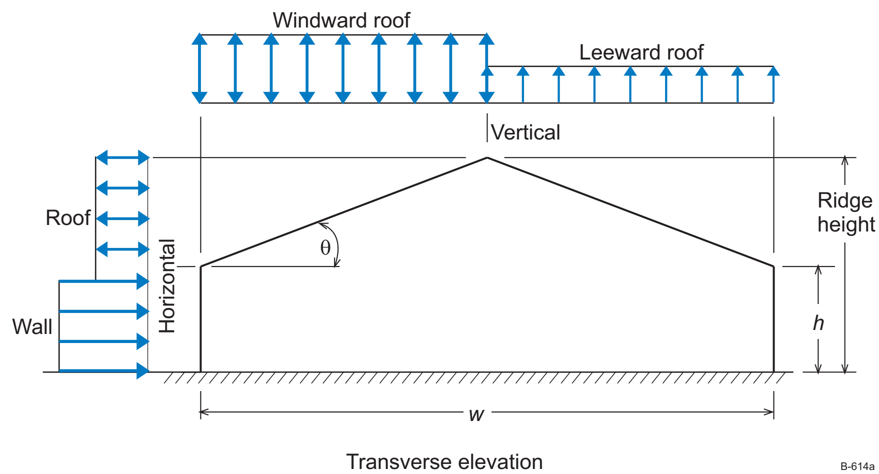


Figure 2-15

Wind pressure varies with height and is shown schematically in Figure 2-16.

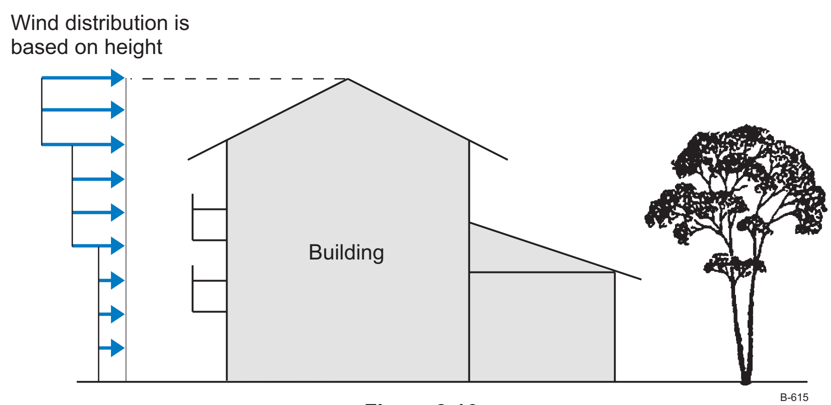


Figure 2-16

1) *Wind loading is inherently a dynamic effect.* Dynamic loading is induced by a wind force that varies in amplitude, frequency, and direction. As with any dynamic loading, frequency of the loading function on the structure is a prime consideration. This is not considered in the IBC static method, because the frequency issue is not as critical for low structures, which are generally stiff with very low periods of vibration that do not coincide with wind periods. The structural engineer must be aware that for tall structures, this is not the case. Dynamic frequency analysis is not considered in the static load procedure.

2) *Wind loads vary drastically across the country.* Nowhere is this issue more apparent than in the southeast where certain coastal zones may experience hurricane-force wind gusts in excess of 300 mph. The code attempts to draw a median line on this issue by using a design wind force based on a mean wind

velocity at the specified height of 30 feet. There have been numerous cases of wind load failures in hurricane zones for the simple reason that the actual wind force exceeded the code-prescribed wind load. Remember that the responsibility of the code is to develop a suitable set of standards that promotes life safety in a structure at a reasonable cost—it is not intended to save the structure. If the owner's intent differs from this perspective, then a Performance Based Design approach is in order.

3) *Wind load design is particularly important in buildings or structures with large projecting areas.* In such buildings, the engineer should ensure the quality of the structural design by allowing for a strong load path with a sufficient safety factor. Some designs fail because they are engineered to strict compliance with the code in a mistaken belief that the wind loads specified in the code are actual force equations. These static loads are code-prescribed forces based on recorded events. The engineer should rely on common sense and sufficient safety factors, and not depend on any single code for all the answers.

2.5 Earthquake Load Design

Earthquakes are among the most serious natural disasters faced by any structure because they are unpredictable, both in magnitude and location. Earthquakes originate deep under the earth's surface along fault lines that exist throughout the world. Certain areas are known to have a higher probability of fault ruptures than other locations. Figure 2-17 shows the fault location and source of a rupture occurring at a focal depth.

Figure 2-17 also shows the three types of waves, termed *Rayleigh waves,* produced by an earthquake event; the P-wave, SV-wave, and SH-wave.

Similar to an underground explosion, an earthquake sends shock waves traveling in all directions until they eventually reach the surface of the earth.

Dropping a rock into a calm lake causes waves that travel outward from the source (Figure 2-18). Earthquakes follow an identical pattern, except that they occur in three-dimensional space.

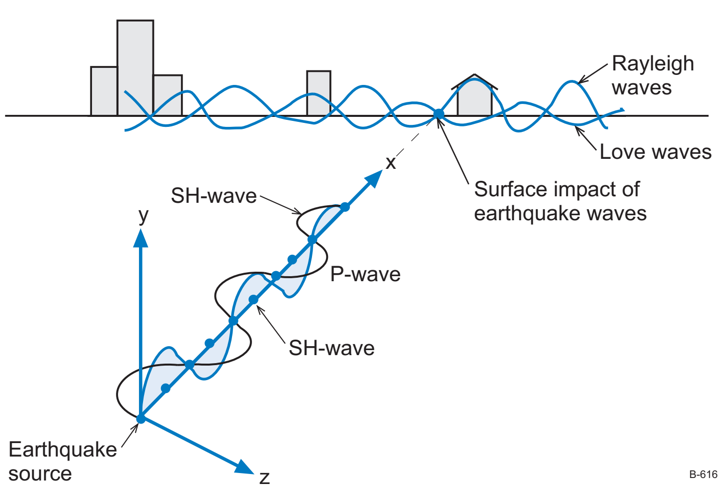


Figure 2-17

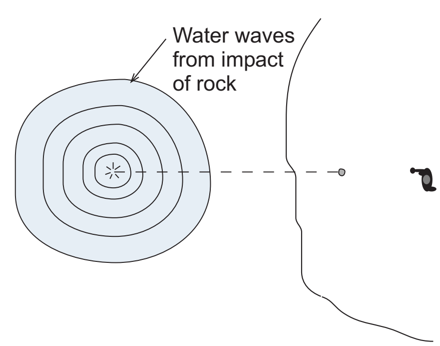


Figure 2-18

With an explosion at a focal depth (earthquake source), the energy release follows the wave distribution and eventually impacts the surface causing reflection, refraction, or traveling waves along the surface. The traveling waves depict vertical displacements (Rayleigh waves) and lateral displacements (Love waves). Essentially, the wave equation phenomenon is a three-dimensional event requireing complex mathematics to explain its behavior. An earthquake is not a simple problem involving only lateral acceleration, velocity, and displacement. The most damaging wave motions are the Rayleigh and Love waves, because they travel along the surface and cause substantial damage. Reflected waves travel away from the surface. Additional information on wave mechanics and particle dynamics is available in the *Earthquake Engineering Handbook* by W.F. Chen.

A structure experiences motion in three dimensions with dynamic forces and moments as shown in Figure 2-19. Along each axis (x, y, and z) are translational forces and moments. These are dynamic in reality and vary with time. For long-span structures (Figure 2-20), the problem of varying forces at different locations along the structure poses another challenge.

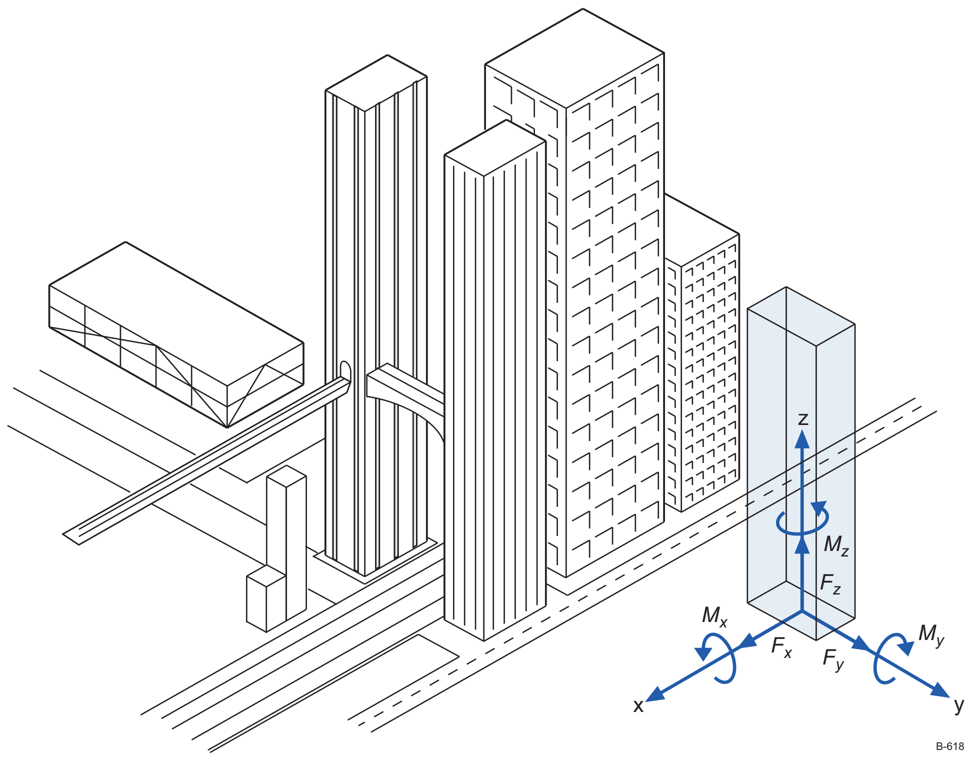


Figure 2-19

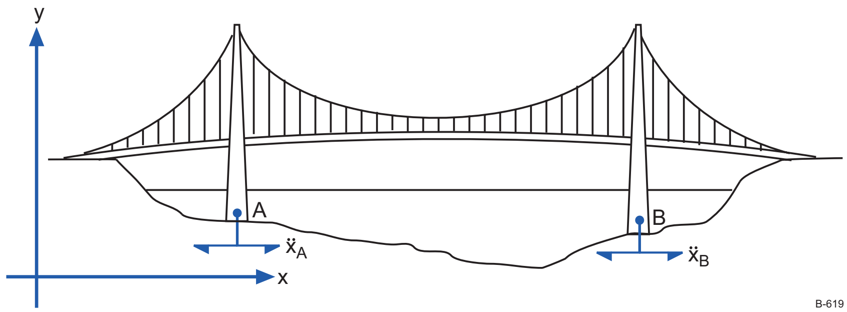


Figure 2-20

The real earthquake loading problem and structural stress analysis solution, considering the entire intricate wave propagation and three-dimensional issues, is complex. It is far beyond the technical requirement or necessity for most structural designs, and is therefore ignored for conventional building design. There is, however, a need to comprehend the "big picture" because there are specific structures for which these higher order effects cannot be ignored, and the IBC requirements are not applicable or must be extended appropriately. In summary, the limitations are:

1) The IBC static method is intended for structures approximately seven to ten stories high. It provides a simple approach that may be easily executed by practicing structural engineers and approved by the building departments in a reasonable time frame.

2) There are exceptions in which the building configuration and distribution of mass may classify the structure as "irregular." Such exceptions require a dynamic analysis to consider the structural vibration modes and mass participation factors. High-rise buildings in particular require extensive dynamic analysis modeling with wind tunnel testing to effectively deal with these issues.

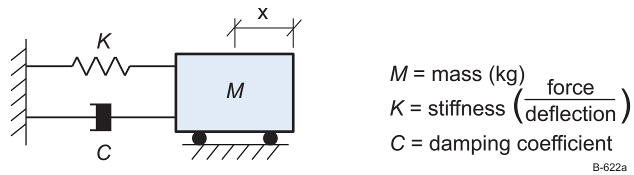
3) Subterranean structures (tunnels, storage tanks, missile silos) are prone to wave propagation effects and require further detailed analysis.

The Structural Engineers Association of California (SEAOC) has taken a key role in developing the static analysis procedure currently used by the profession. Although SEAOC does recommend using a dynamic analysis procedure where appropriate, it is only in specific circumstances that the structural dynamics of a building will require evaluation. The UBC adopted the provisions developed by SEAOC's Blue Book with some minor modifications. The dynamic analysis procedure may be examined in: *Earthquake Engineering* by W.F. Chen, or *Vector Mechanics for Engineers: Statics and Dynamics* by Beer and Johnson.

Before examining the IBC static analysis procedure, review how and on what basis this procedure was developed. But first, consider the basics of a spring-mass system, also referred to as a single-degree-of-freedom (SDOF) oscillator. All earthquake analysis of structural systems begins with the fundamentals of the SDOF spring-mass system.

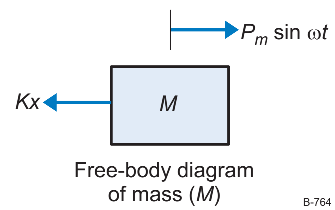
ANALYSIS OF SDOF:

$$M\frac{dx^2}{dt^2} + C\frac{dx}{dt} + kx = f(t)$$



Set $C = 0$ (undamped vibration)

$$M\ddot{x} + Kx = f(t) = P_m \sin \omega t$$



Forcing function $= P_m \sin \omega t$

$x \quad = x_p + x_n$

$x_p \quad$ = particular solution (i.e., solution ω/ forcing function)

$x_h \quad$ = homogenous solution (i.e., solution ω/ zero forcing function)

Solve for $x_h = M\ddot{x} + kx = 0$

$x_h \quad = A \sin \omega t + B \cos \omega t$

With $\omega^2 \quad = k/m$, substitute into

$\dot{x} \quad = v = A\omega \cos \omega t - B\omega \sin \omega t$

$\ddot{x} \quad = a = -A\omega^2 \sin \omega t - B\omega^2 \cos \omega t$

Substitute (Eq. 2-7) and (Eq. 2-5) into (Eq. 2-4)

Confirm x_n is solution to (Eq. 2-5)

$$\therefore \quad M(-A\omega^2 \sin \omega t - B\omega^2 \cos \omega t) + k(A \sin \omega t + B \cos \omega t) = 0$$

$$A \sin \omega t[k - M\omega^2] + B \cos \omega t[k - M\omega^2] = 0$$

$$M\omega^2 \quad = k \text{ or } \omega^2 = \sqrt{\frac{k}{m}}$$

Substitute for phase angle in Equations (2-5), (2-6), (2-7)

$x_h \quad = x_p \sin(\omega t + \phi)$

$x_p \quad$ = peak displacement

$\omega \quad = \sqrt{\frac{k}{m}}$ = natural frequency

$\phi \quad$ = phase angle

$\dot{x}_h \quad = v = x_p\omega \cos(\omega t + \phi)$

$\ddot{x} \quad = a = -x_p\omega^2 \sin(\omega t + \phi)$

$v_p \quad = \text{peak velocity} = x_p\omega$

$a_p \quad = \text{peak acceleration} = x_p\omega^2 = x_p v_p$

$$\omega = \sqrt{\frac{k}{m}} = \text{circular frequency (rad/sec)}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \text{frequency } (Hz = \text{cycles/sec})$$

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = \text{period (sec)}$$

Solving the basic equations of motion for an undamped SDOF system yields the solution for circular frequency, ω, cyclic frequency, f, and period, T. All three terms refer to the same quantity: the structure's mass-to-stiffness ratio. In the structural engineering arena, the term period, T, is used frequently for understanding the building's stiffness. Examine the equation for T and note the following.

1. Structures with high mass and low stiffness will have higher periods. These are called flexible structures. The longer the period, the more flexing or higher deflection it will experience. Flexible structures are those with periods exceeding 1.5 to 2.0 seconds. These are typical of mid- to high-rise buildings extending above 15 stories. The higher the period, the more flexible the structure.

2. Structures with low mass-to-height stiffness will have shorter periods and are classified as stiff structures. Short period structures are generally shear wall buildings with periods less than 1.0 second.

3. Structures with periods in the range of 1.0 to 2.0 seconds rank approximately mid-way on the spectrum from flexible to stiff.

These fundamentals are derived from the basic SDOF oscillator and are applied to multistory structures equally. This application is based on the long-tested hypothesis that multistory structures behave similar to an SDOF system (within a specific height and mass restriction).

Figure 2-21 demonstrates this assumption in a schematic showing the differences among a building, a cantilevered mass, and an SDOF oscillator. Engineers will approximate the building with an SDOF system and use the parameters (solved above) for determining the fundamental characteristics of the structure.

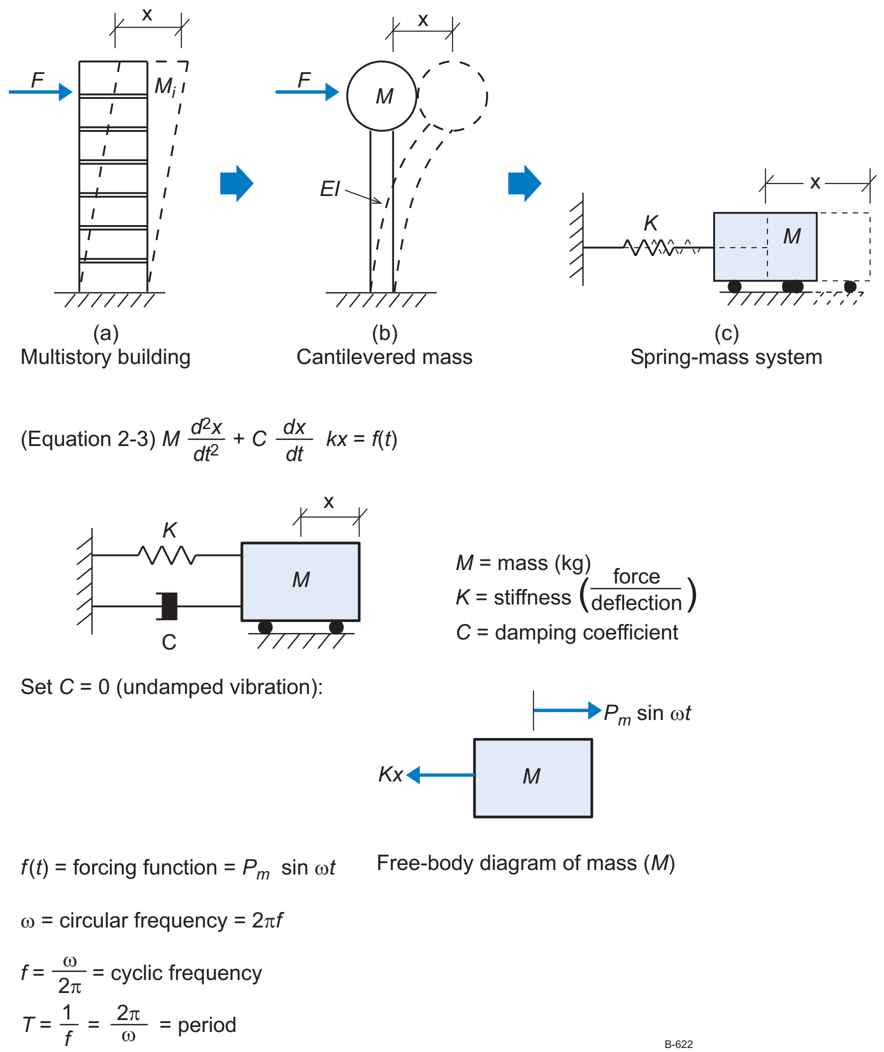


Figure 2-21

Every structure will respond to an earthquake ground motion in a series of mode shapes that are derived from the basic physical characteristics of the structure: mass distribution between the floor levels, floor-to-floor height, total height, and lateral stiffness. Approximating a multistory structure with a linear mass model simplifies the mathematics of modeling the details of the building, which is a reasonable assumption for buildings with uniform floor plans and equal mass distribution. Figure 2-22 shows this in a) a linear stick model, b) first mode shape, c) second mode shape, and d) third mode shape.

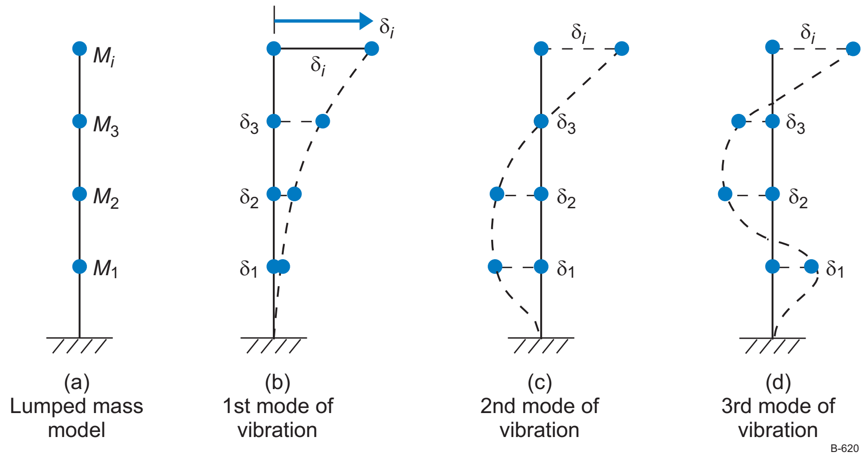


Figure 2-22

Structures that satisfy the UBC criteria for regular buildings (UBC 1629.5.2) will respond to an earthquake in the primary first mode of vibration [Fig. 2-22(b)]. The first mode shape is a simple vertical cantilevered deflection. This triangular elastic curve sets the precedent for the UBC static analysis procedure (1629.8.2).

2.5.1 UBC provisions

See Appendix A on the CD for replication of UBC 1629.8.3 and 1630.

2.5.2 2000 IBC provisions

Earthquake design takes a significant step forward in complexity in the 2000 IBC. The IBC requirements are based on the National Earthquake Hazard Reduction Program (NEHRP) funded by the Federal Emergency Management Agency (FEMA). Selected chapters of the IBC are provided in Appendix B**.** (Refer to the CD)

The essential differences between the 1997 UBC and the 2000 IBC concerning building design for earthquakes are summarized below.

1) The 2000 IBC emphasizes strength design. With particular attention to masonry, the IBC does allow for working stress design but eliminates provisions of allowable stress design for reinforced concrete.

2) In sharp contrast to the UBC, the IBC adopts references as nationally recognized standards. These references, which include ASCE 7-98, 1997 NEHRP, ACI 318-99, ACI 530/ASCE 5/TMS 402 and AISC LRFD (1993), are not included in the IBC document, but are referenced as part of the provisions.

3) The IBC accounts for a site-specific response spectrum and may require earthquake design provisions to be enforced in certain high-risk zones. By considering soil issues on a national scale, the IBC attempts to quantify risk by using seismic design category classifications.

4) Seismic Design Categories (SDCs) A through F determine the scope and level of analysis required (IBC Section 1616).

5) The IBC addresses wind load, seismic conditions, and geotechnical design considerations on a national level; the UBC focused on the western states.

A flowchart that outlines a detailed procedure for seismic design is provided in Appendix D4.

2.5.3 Dynamic analysis procedure

The 2000 IBC dynamic analysis procedure essentially duplicates the UBC provisions, except:

1. The IBC has stricter requirements in the *application* of the procedure. For example, buildings with irregularities may trigger a dynamic analysis procedure even though these structures may be less than three stories in height.

2. The IBC requires a dynamic analysis on certain structures in SDC F even though these buildings may not fall within standard earthquake-prone zones. As an example, a building located in Boston, MA, on soil type SF, subject to high wind loads, could also be subject to substantial lateral forces with dyamic effects. In many areas of the United States, the concept of a dynamic analysis procedure is unfamiliar. In areas of the midwest where buildings are under five stories, there is no need for a dynamic analysis. Therefore, the procedure is not applied.

3. With the added restrictive requirements in the IBC, engineers must upgrade their analysis skills to fully comprehend the intricacies of dynamic analysis and finite element procedures that, traditionally, have not been necessary for many civil engineering projects.

2.6 Snow Load Analysis

Snow loads are static load phenomena with drastic variations throughout the continental United States. As with any weather pattern, these loads follow a cycle that correlates with the seasons. Similar to rain and wind forces, snow accumulation follows a statistical pattern. Snow loads have several distinguishing characteristics.

1) *Snow accumulation is a quasi-static loading curve that occurs slowly over a period of time.* This is in sharp contrast to earthquake and wind forces that are dynamic forces with peak accelerations or wind-gust velocities that can severely damage a structure. Quasi-static loading implies that the load intensity can be considered a static application versus a dynamic load.

2) *Snow loads are cyclic with seasonal variations*. Seasonal variation can be a beneficial characteristic for inhabitants of certain geographical areas because records show that snow accumulation follows regional weather patterns, which are repetitive. These repetitive patterns result in statistical predictability, and thereby produce greater accuracy in weather-predicting calculations. Occasional snow seasons will have peak snowfalls that far surpass those of the previous 10 or 15 years. This means that engineers must select their comfort zones for a statistical factor of safety. ASCE 7-98, Chapter 7 discusses this and prescribes a 10-percent probability of being exceeded (PE) within 50 years. This is a conservative factor of safety, provided the ground snow load measurements are reliable. Local snow conditions vary drastically at higher elevations, and structural engineers must consider this during design formulation, understanding that it is better to overestimate snow loads with a factor of safety than to underestimate them and leave open the possibility of structure collapse.

3) *Roof conditions and site-specific structural design variations affect the final design values.* The IBC considers the issues of local architectural design modifications in its design formulation. Specific issues such as roof slope, warm-slope, cold-slope, and ponding instability are addressed in the IBC provisions and must be carefully studied.

Appendix D5 contains the flowchart for snow load design per the IBC.

2.7 Summary

Load path refers to the manner in which the vertical loads are transferred to the foundation from the point of application. There must be a continuous load path from the upper portions of the structure to its base. Every structure has a vertical load distribution, and is analyzed using the tributary area concept. The method of transferring vertical loads to the foundation follows the continuous load path. Wind loads are based on wind velocity data gathered though the National Weather Service. Using such wind velocity data, a tabulation of wind force/pressure was formulated: the basis for the UBC Wind Load Provisions. The wind load provisions for the 1997 UBC and 2000 IBC were based on the ASCE 7-98 provisions, which examine structural damage to buildings.

Ground acceleration earthquakes are created by deep ground-fault ruptures that cause seismic waves to travel to the surface. Such waves cause vertical, horizontal, and rotational displacements along the surface. The displacements institute ground accelerations, which then cause lateral responses in buildings that, in turn, lead to dynamic oscillation of the structures. These dynamic forces are converted into static lateral forces using the procedures described in the IBC and UBC. Static forces attempt to simulate the first mode of vibration of the building, and the building is analyzed for moments, shears, and axial forces on this basis.

Snow loads are highly static loads imposed by variable weather conditions. These are quasi-static monotonic loads that increase over a short period but must be modified for site-specific root building structure, and climatic considerations.

The IBC provides the dynamic analysis procedure for determining the seismic response of structures that do not fall within the category of conventional static analysis. Such buildings may fall within the class of irregular structural stiffness, tall (higher mode of vibration), or irregular-mass-distribution type buildings that require special analysis techniques. Additionally, buildings that are in close proximity to active faults with near-source seismic events could fall within this category. The 2000 IBC expands the use of dynamic analysis procedures to encompass buildings in varying seismic categories throughout the nation.

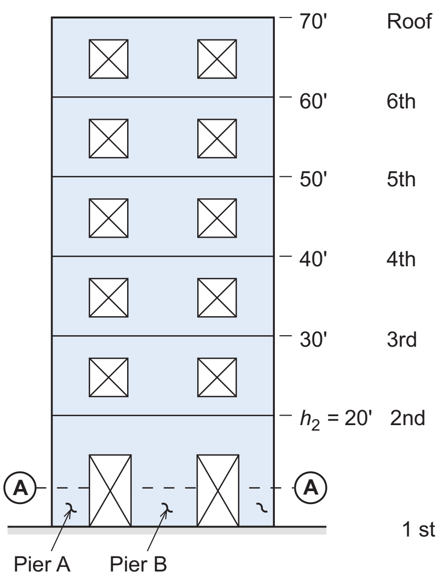
Example 2.1

The reinforced masonry shear wall building elevation is shown along with the floor plan design. This six-story structure with bearing walls is entirely supported with a 6-inch reinforced concrete diaphragm system connected to perimeter shear walls (Piers A and B).

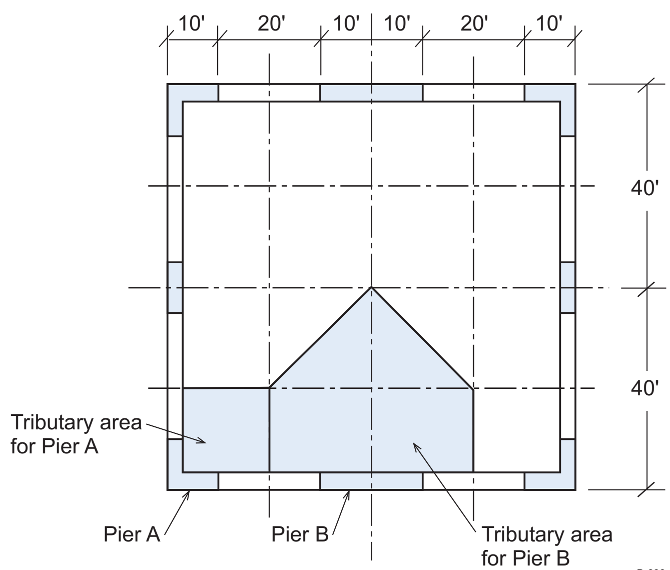
Given:

- Residential occupancy
- Flat roof
- 12-inch CMU walls, fully grouted,
- Wall weight = 115 psf

1. **Calculate the service loads on (a) the roof system and (b) the floor system per IBC 1607.11 and 1607.1.**
2. **Determine the load distribution to Pier A using the applicable live load reduction provisions, and calculate the basic service load combination.**



(a)



(b)

Solutions

1. (a) Roof system

Estimate dead load:

$$\text{DL} = \overset{\text{6-inch concrete slab}}{\left(\frac{6}{12}\right)}\ \overset{\text{weight density of concrete}}{(150\ \text{lb/ft}^3)} = \underline{75\ \text{lb/ft}^2}$$

Roof live load per IBC 2000, 1607.11.2

$$L_r = 20\, R_1 R_2 \qquad \textbf{Eq. 16-4}$$

Because the roof is flat,

and $R_1 = 1$ for $A_t < 200\ \text{ft}^2$

$R_2 = 1$ for $F \leq 4$

$\therefore\ L_r = \underline{20\ \text{psf}}$

$\therefore\ \text{Roof}_{\text{DL + LL}} = 75 + 20 = \underline{\underline{95\ \text{psf}}}$

(b) Floor system

Floor dead load = same as roof DL = 75 lb/ft^2

Floor live load = 40 psf [Table 1607.1, residential occupancy]

$\therefore\ \text{Floor}_{\text{DL + LL}} = 75 + 40 = 115\ \text{psf}$

2. (a) Calculate tributary area for Pier A

$$A_t = (20)(20) = 400\ \text{ft}^2/\text{floor}$$

(b) Determine total DL

$(\text{Pier A})_{\text{DL}} = 6(75\ \text{psf})(400\ \text{ft}^2) + \text{wall DL}$

The shear walls are 12-inch CMU, fully grouted, wall density of 115 psf

$$\text{Wall DL} = \overset{\text{Wall weight}}{(115\ \text{psf})}\underbrace{(10\ \text{ft} + 10\ \text{ft})}_{\text{Wall length}}\ (\overset{\text{1st floor}}{20\ \text{ft}} + \underbrace{10\ \text{ft} + 10\ \text{ft} + 10\ \text{ft} + 10\ \text{ft} + 10\ \text{ft}}_{\text{2nd – 6th floors}})$$

$\therefore$ Wall DL = 161,000 lb = 161 kips

$\therefore\ (\text{Pier A})_{\text{DL}} = 180 + 161 = 341\ \text{kips}$

(c) Floor live reduction (IBC 2000, 1607.9)

$$L = L_o\left(0.25 + \frac{15}{\sqrt{K_{\mathrm{LL}} A_T}}\right)$$ **Eq. 16-1**

From Table 1607.9.1

K_{LL} = 4 **Note:** Pier A classifies as a corner column without cantilevered span

A_T = (400)(6) = 2400 ft^2

$\therefore K_{\mathrm{LL}} A_T = 4(2400) = 9600$

$$\left(0.25 + \frac{15}{\sqrt{9600}}\right) = 0.40$$

$\therefore$ $L = 0.40 L_o$ This is at the maximum per 1607.9.1

L'_r = Reduced roof live load = 0.4(20) = 8 psf

L'_{rf} = Reduced floor live load = 0.4(40) = 16 psf

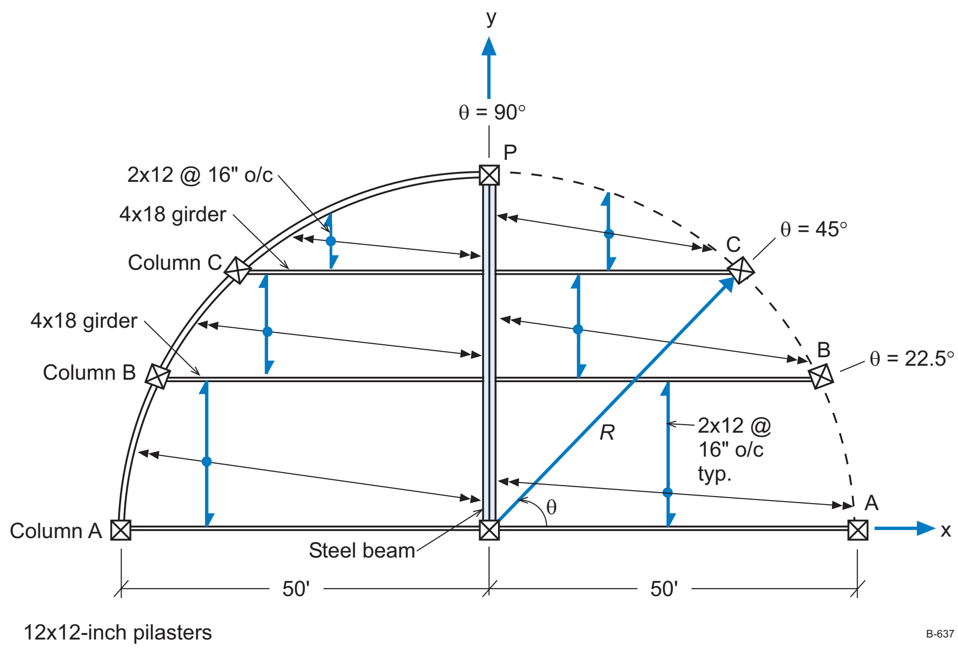
$(\text{Pier A})_{\mathrm{LL}} = 8(400\text{ ft}^2) + 16(400\text{ ft}^2)(5)$

$(\text{Pier A})_{\mathrm{LL}} = 3.2\text{ kips} + 32.0\text{ kips} = 35.2\text{ kips}$

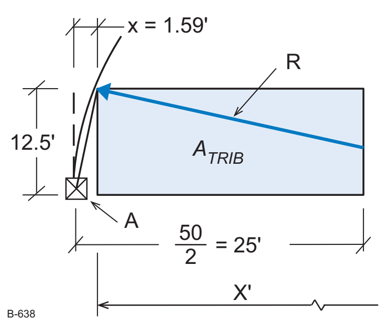
$\therefore$ $(\text{Pier A})_{\mathrm{DL+LL}} = 341. + 35.2 = \underline{\underline{376.2}}\text{ kips}$

Example 2.2

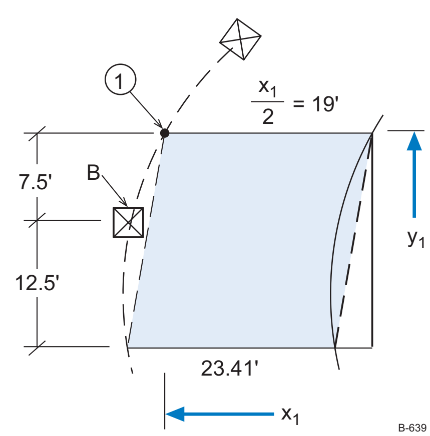
The semi-circular floor system comprises a system of steel beams/girders supported by a wood joist system as shown. At the column locations are reinforced masonry pilasters (i.e., columns A, B, and C). This architectural concept is intended to enable open panoramas with minimal obstruction. Therefore, long spans are used to transfer loads to the pilasters that are strategically placed to allow such unobstructed views.



(a)



(b)



(c)

Perform the following:

1. **Calculate the design live and dead loads per the 2000 IBC.**
2. **Determine the tributary load distribution to Column A.**
3. **Determine the tributary load distribution to Column B.**
4. **Determine the tributary load distribution to Column C.**

- Use approximatios for tributary area calculations\
- Neglect the roof system

Floor system is 2x12 joists with 1-inch-thick plywood sheathing

Occupancy is for assembly with movable seating, stages, and platforms

Solutions

1. (a) Live load per IBC 2000, Table 1607.1

L_f = Floor LL = 125 psf (No reduction)

(b) Dead load

1-inch plywood sheating	3.0 psf
2x12-inch joists @ 16 inches o/c	5.0 psf
Flooring	5.0 psf
Mechanical + electrical	3.0 psf
DL total =	16.0 psf

2. (a) Tributary area

$A_{\text{trib}} = (12.5)(25) = 312.5 \text{ ft}^2$

(b) Live load

from Table 1607.9.1,

$K_{\text{LL}} = 4$ (Exterior column without cantilevered slab)

$$L'_f = 125\left(0.25 + \frac{15}{\sqrt{(4)(312.5)}}\right) = 84.3 \text{ psf}$$

$\therefore (P_A)_{\text{LL}} = (84.3)(312.5) = 26.3$ kips

(c) Dead load

$(P_A)_{\text{DL}} = (312.5 \text{ ft}^2)(16.0 \text{ psf}) = 5.0$ kips

(d) Total DL + LL = $(P_A)_{\text{DL + LL}} = 5.0 + 26.3 = 31.3$ kips

3. (a) Live load = 125 psf

(b) Dead load = 16.0 psf

(c) $A_t = (12.5 + \frac{1}{2}(12.5)\left(\frac{1}{2}l_B\right)$

where $l_B = R\cos\theta = 50\text{ ft}\cos 22.5°$

$\therefore\ l_B = 46.2\text{ ft}$

$\therefore\ A_t = (18.75)\frac{1}{2}(46.2) = 433.1\text{ ft}^2$

$K_{LL} = 4$

$L'_f = 125\left(0.25 + \frac{15}{\sqrt{(4)(433.1)}}\right) = 76.3\text{ psf}$

(d) $(P_B)_{DL+LL} = 433.1(16.0) + 433.1(76.3) = \underline{\underline{40.0\text{ kips}}}$

4. (a) Live load = 125 psf

(b) Dead load = 16.0 psf

(c) $A_t = \left(\frac{1}{2}(12.5) + \frac{1}{2}(12.5)\right)\left(\frac{1}{2}l_c\right)$

$l_c = R\cos\theta = 50\cos 45° = 35.4\text{ ft}$

$\therefore\ A_t = (12.5)(35.4)\frac{1}{2} = 221.3\text{ ft}^3$

$K_u = 40$

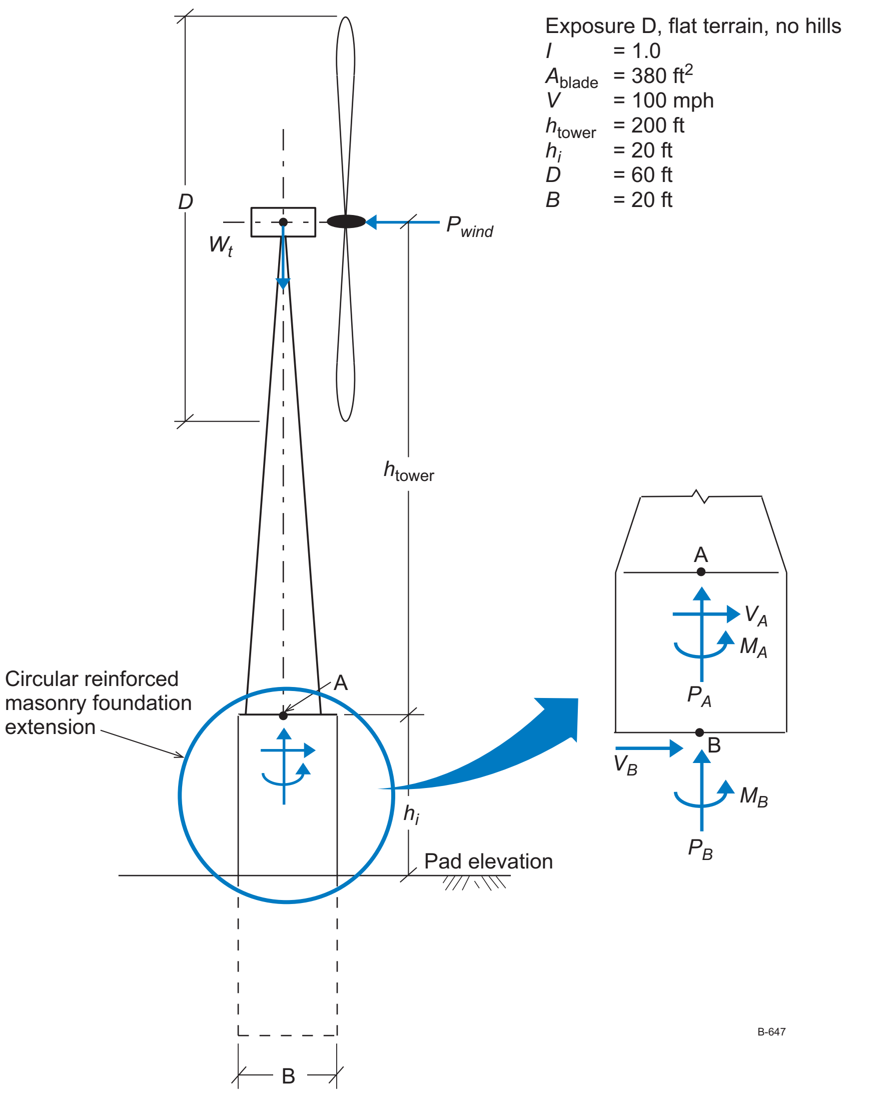
$L'_f = 125\left(0.25 + \frac{15}{\sqrt{(4)(221.3)}}\right) = 94.3\text{ psf}$

(d) $(P_C)_{DL+LL} = 221.3\ (16.0 + 94.3) = \underline{\underline{24.4\text{ kips}}}$

Example 2.3

The wind turbine tower shown is to be constructed on a circular reinforced masonry foundation system. The foundation design consists of a subgrade portion embedded in competent soil and an extension to support the tower structure. Such a design is used extensively in conventional wind tower structures.

1. **Calculate the wind force (P_{wind}) using ASCE 7-98, 6.5, Method 2 and consider this an open tower frame structure.**
2. **Determine the overturning moment at the base of the tower (i.e., point A).**
3. **Calculate the base shear reaction at pad elevation and overturning moment at point B.**
   - Ignore area of tower for wind force contribution



Solutions

1. From ASCE 7-98, Table 6-6

(a) V = 100 mph

K_d = 0.85 (Solid signs)

I = 1.0

(b) For exposure D, calculate K_z or K_h from Table 6-5

$h_{\text{total}} = 200 + 20 = 220$ ft

Case 2(b)

$$\therefore\ K_h = 1.61 + 20\left(\frac{1.68 - 1.61}{250 - 200}\right) = 1.64$$

$$\therefore\ K_h = \underline{\underline{1.64}}$$

(c) Topographic factor = $K_{zt} = (1 + K_1 K_2 K_3)^2$

From Figure 6-2, $K_1 = K_2 = K_3 = 0$ for flat terrain

$$\therefore\ K_{zt} = 1.0$$

(d) G_f (ASCE 7-98, 6.5.8)

Wind towers are flexible structures with typical frequency of $f_1 = 0.4\ H_z$ ($T_1 = 2.5$ seconds)

$$G_f = 0.925\left(\frac{1 + 1.7I_z\sqrt{g_Q{}^2 Q^2 + g_R{}^2 R^2}}{1 + 1.7g_r I_z}\right)$$

$$g_Q = g_v = 3.4$$

$$g_R = \sqrt{2ln(3600\eta_1)} + \frac{0.577}{\sqrt{2ln(3600\eta_1)}}$$

where $\eta_1 = 0.4\ Hz$

$$\therefore\ g_R = \sqrt{2ln(3600(0.4))} + \frac{0.577}{\sqrt{2ln(3600(0.4))}}$$

$$\therefore\ g_R = 3.814 + 0.296 = 4.110$$

$$R = \sqrt{\left(\frac{1}{\beta}R_\eta R_h R_B\right)(0.53 + 0.47R_L)}$$

where $R_n = \dfrac{7.47N_1}{(1 + 10.3N_1)^{5/3}}$

$$N_1 = \frac{\eta_1 L_{\bar{z}}}{\overline{V}_{\bar{z}}}$$

$$R_l = \frac{1}{\eta} - \frac{1}{2\eta^2}(1 - e^{-2\eta}) \text{ for } \eta > 0$$

From Table 6-4, for Exposure D

$$\alpha = 11.5$$

$$Z_g = 700$$

$$\hat{a} = 1/11\text{-}5,\ \hat{b} = 1.07,\ \bar{\alpha} = 1/9.0,\ \bar{b} = 0.80,\ C = 0.15$$

$$l = 650 \text{ ft},\ \bar{\varepsilon} = 1/8.0,\ Z_{min} = 7$$

for flat terrain $H = 0$

for this tower $B = 20$ ft, $L = 20$ ft

$$\bar{z} = 0.6h = 0.6(200) = 132 \text{ ft}$$

$$\therefore\ I_{\bar{z}} = c\left(\frac{33}{\bar{z}}\right)^{1/6} = 0.15\left(\frac{33}{132}\right)^{1/6} = 0.12$$

$$\therefore\ L_{\bar{z}} = I\left(\frac{\bar{z}}{33}\right)^{\bar{\varepsilon}} = \left(\frac{132}{33}\right)^{1/8.0} = 1.19$$

$$\therefore\ \bar{V}_{\bar{z}} = \bar{b}\left(\frac{\bar{z}}{33}\right)^{\bar{\alpha}} V\left(\frac{88}{60}\right)$$

$$\bar{V}_{\bar{z}} = 0.80\left(\frac{132}{33}\right)^{1/90} (100 \text{ mph})\left(\frac{88}{60}\right) = 136.9$$

$$N_1 = \frac{n_1 L_{\bar{z}}}{V_{\bar{z}}} = \frac{(0.4)(1.19)}{136.9} = 0.0035$$

$$R_n = \frac{7.47N_1}{(1 + 10.3N_1)^{5/3}} = \frac{7.47(0.0035)}{(1 + 10.3(0.0035))^{5/3}} = 0.025$$

$$R_l = R_n$$

$$\text{Set } n = 4.6n_1 h/\bar{V}_E = \eta = \frac{46(0.4)(132)}{136.9}$$

$$\therefore\ \eta = 1.77$$

$$R_h = \frac{1}{\eta} - \frac{1}{2\eta^2}(1 - \mathrm{e}^{-2n})$$

$$\therefore\ R_h = \frac{1}{1.77} - \frac{1}{2(1.77)^2}(1 - \mathrm{e}^{-2(1.77)}) = 0.41$$

$$R_l = R_B$$

$$\eta = \frac{4.6n_1 B}{\bar{V}_{\bar{z}}} = \frac{4.6(0.4)(20)}{136.9} = 0.27$$

$$R_B = \frac{1}{0.27} - \frac{1}{2(0.27)^2}\left(1 - e^{-2(0.27)}\right) = 0.84$$

$R_l = R_L$

$$\eta = \frac{15.4 n_1 L}{\bar{V}_{\bar{z}}} = \frac{15.4(0.4)(20)}{136.9} = 0.90$$

$$\therefore\ R_L = \frac{1}{0.90} - \frac{1}{2(0.90)^2}\left(1 - e^{-2(0.90)}\right) = 0.60$$

from Equation (6-4)

$$Q = \sqrt{\frac{1}{1 + 0.63\left(\frac{B+h}{L_{\bar{Z}}}\right)^{0.63}}}$$

$$\therefore\ Q = \sqrt{\frac{1}{1 + 0.63\left(\frac{20+220}{1.19}\right)^{0.63}}} = 0.23$$

$\beta = 0.05$ (5% of critical damping)

$$\therefore\ R = \sqrt{\frac{1}{\beta} R_n R_h R_B (0.53 + 0.47 R_L)}$$

$$\therefore\ R = \sqrt{\frac{1}{0.05}(0.025)(0.41)(0.84)(0.53 + 0.47(0.60))}$$

$$\therefore\ R = 0.37$$

$$\therefore\ G_f = 0.925\left\{\frac{1 + 1.7((0.12)\sqrt{3.4^2(0.37)^2 + (4.11)^2(0.37)^2})}{1 + 1.17(3.4)(0.12)}\right\}$$

$$\therefore\ G_f = \underline{\underline{0.88}}$$

(e) Open building/structure (ASCE 7-98, 6.5.13)

$$F = q_z G C_f A_f$$

$q_z = 0.00256\, K_z\, K_{zt}\, K_d\, V^2 I$ (ASCE 7-98, 6.5.6.3.2)

C_f is from Table 6-12 (ASCE 7-98)

$$\therefore\ q_z = 0.00256(1.0)(1.0)(0.85)(100)^2(1.0) = 21.76 \text{ psf}$$

Table 6-12, $D\sqrt{q_z} = 0.5\sqrt{21.76} = 2.35 < 2.5$

The blades are considered on "open sign" with $D = 0.5$ ft

$\varepsilon < 0.10 = 0.125$, rounded members

$$\therefore\ C_f = \underline{\underline{1.2}}$$

$$\therefore\ A_f = 380 \text{ ft}^2$$

$\therefore\ F \quad = P_{\text{WIND}} = (21.76)(0.88)(1.2)(380)$

$\therefore\ P_{\text{WIND}} = \underline{\underline{8.7}}$ kips

2. Moment at Point A

$\therefore\ M_A \quad = P_{\text{WIND}}\,(h_{\text{tower}}) = 8.7(200\text{ ft}) = \underline{\underline{1.740}}$ ft-kips

3. Shear and moment at Point B

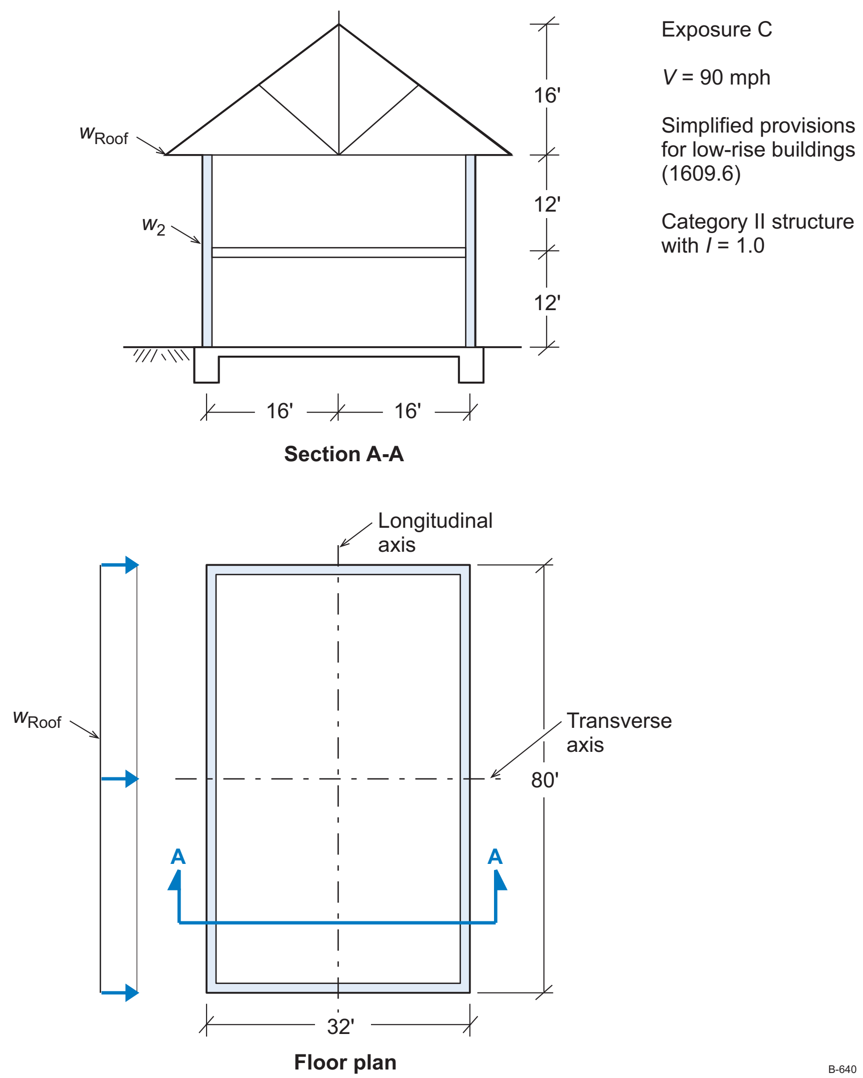
$\therefore\ M_B \quad = P_{\text{WIND}}\,h = 8.7\ (220\text{ ft}) = \underline{\underline{1.914}}$ ft-kips

$\therefore\ V_B \quad = P_{\text{WIND}} = \underline{\underline{8.7}}$ kips

Example 2.4

A cross section (Section A-A) and plan view for a rectangular building are shown. Using the IBC Wind Load Provisions, determine the following.

1. **Calculate the horizontal wind forces on the windward and leeward roof areas per Method 1, ASCE 7-98.**



Solutions

1. (a) h_{mean} = mean roof height = $12 + 12 + \frac{1}{2}16 = 32$ ft

(b) From IBC 2000, 1609.6.1

MWFRS, Table 1609.6.1(1)

$$\text{Roof angle} = \frac{16\,\text{ft}}{16\,\text{ft}} = 1.0 \qquad \theta = 45°$$

For transverse loads, from Figure 1609.6(3)

Roof = 7.9 psf (interior zone)

Wall = 11.5 psf (interior zone)

(c) Adjustment factor: Table 1609.6.2.1(4)

h_{mean} = 32 ft

Interpolate from table,

Exposure C:

$$\text{adj.} = 1.40 + (2)\left(\frac{1.45 - 1.40}{35 - 30}\right)$$

$$\text{adj.} = 1.42$$

$\therefore$ Roof wind pressure = 1.42(7.9) = 11.2 psf

$\therefore$ Wall wind pressure = 1.42(11.5) = 16.3 psf

(d) Wind loads

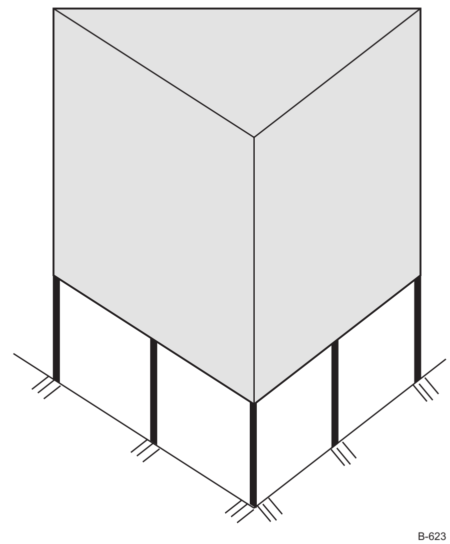
$$W_{roof} = 11.2\text{ psf (16 ft)} + 16.3\text{ psf}\left(\frac{12\text{ ft}}{2}\right)$$

$$\therefore\ W_{roof} = \underline{\underline{2.77}}\text{ lb/ft}$$

$$\therefore\ W_2 = 16.3\left(\frac{12}{2} + \frac{12}{2}\right) = \underline{\underline{195.6}}\text{ lb/ft}$$

Assignments

A mid-rise structure (16 stories) consists of a structural vertical-load-carrying space frame with shear walls for the upper stories. The ground floor consists of vertical columns over piles and grade beams. This building is to be constructed in Pasadena, CA. The intended occupancy is office space on all floors.

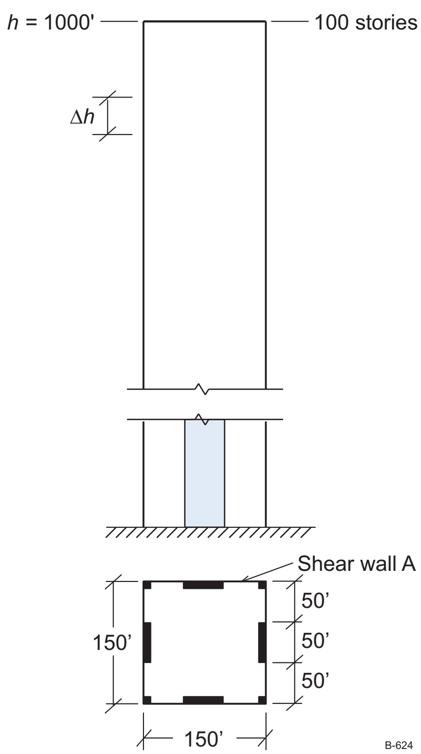


Assignment 1

1. Determine the appropriate structural system of this type of design.

   (1) What is the Omega factor?

   (2) Write a short paragraph on your observations regarding the structural integrity of this design? How would you modify this concept?

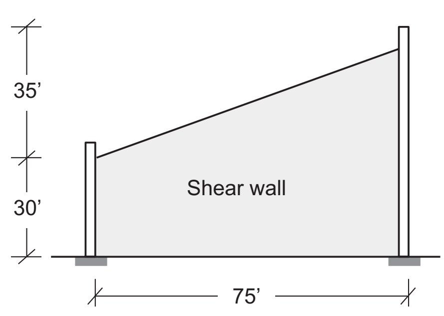


Assignment 2

2. A new reinforced 100-story residential building is designed using structural shear walls with the floor plan configuration shown. (Based on the 2000 IBC)

   (1) Calculate the design loads for the roof and typical floor plan.

   (2) Determine the vertical load distribution to shear wall A.



Assignment 3

3. A shear wall building has a tilted roof diaphragm as depicted in the elevation shown. This structure is located in North Dakota with Exposure Rating D.

   (1) Using the normal force method (Method 1), calculate the windward and leeward force pressures on the inclined roof diaphragm.

   (2) Using the projected area method (Method 2), calculate the force pressure distribution on the building.

   (3) Using the 2000 IBC wind provisions, calculate the total wind force and its distribution.

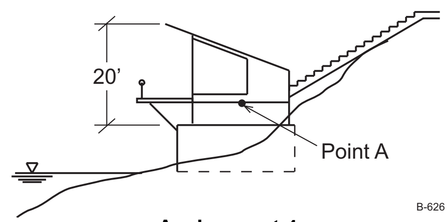
4. An architect has designed this lake-front property to be constructed in Lake Arrowhead, California. Assume a design wind speed of 100 miles per hour.

   (1) Determine the exposure category based on the sketch and description.

   (2) Calculate the wind pressure profile using 1997 UBC Method 2, and plot on a sketch.

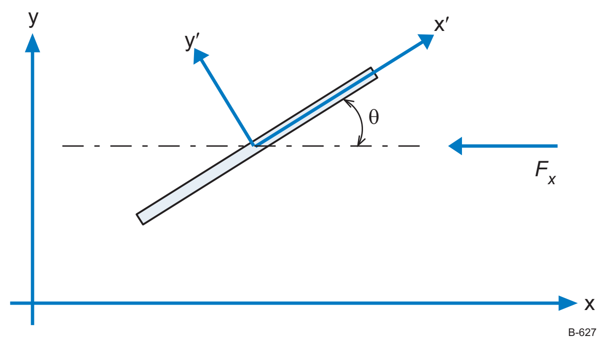
   (3) Calculate the overturning moment demand using service and factored loads at point A.

   (4) Calculate the wind pressure profile using the 2000 IBC



Assignment 4

5. The skewed shear wall shown at angle θ is subject to the force, F_x.



Assignment 5

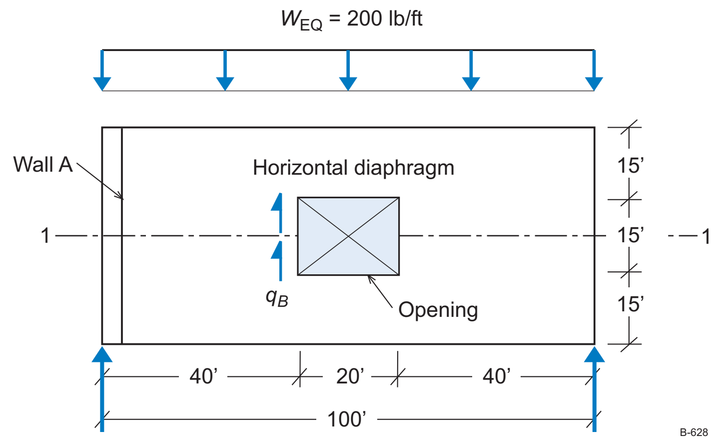
   (1) Calculate the component force distribution in terms of normal and shear forces on the wall.

   (2) Plot the shear force versus θ, from zero to 90 degrees (0 to π/2).

6. The flexible diaphragm has an opening cut as shown. The lateral-force distribution comprises earthquake forces (already factored).

   (1) Calculate the shear reaction at Wall A, and plot the shear diagram along the diaphragm line 1-1.

   (2) Determine the boundary shear force along the diaphragm opening (i.e., unit shear q_B).



Assignment 6

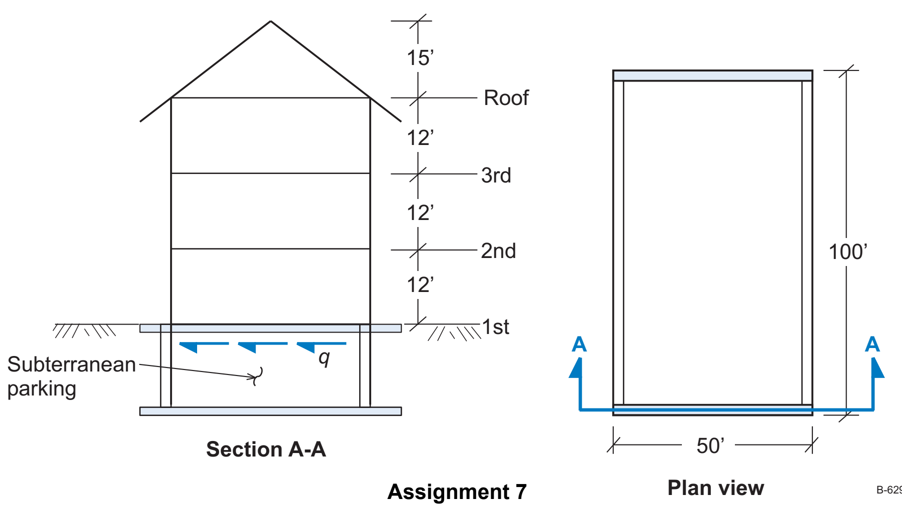
7. A three-story wood frame shear wall building is constructed over a subterranean reinforced masonry parking structure. This is typical of the apartment and condominium buildings constructed in various areas of the country (especially in southern California). This type of building experienced extensive damage in the Northridge earthquake (1994), but the subterranean structures remained largely intact. This structure is located in Seismic Zone 4, Exposure B.

   (1) Calculate the wind pressure profile using Method 2 of the UBC and the 2000 IBC Simplified Wind Provisions.

   (2) Calculate the seismic base shear using the IBC. Assume the structure is located in Los Angeles, California with a soil type SE.

   (3) Determine whether earthquake or wind will govern for the transverse direction. Use the 2000 IBC for this determination.

   (4) Calculate the critical shear along the reinforced masonry shear wall line A (as shown), and determine the unit shear at the first-floor line.



Assignment 7

3

Structural Engineering and Analysis

3.1 Working Stress Design Principles

Unlike other branches of engineering (i.e., aerospace, mechanical, electrical, and chemical), civil engineering deals with the construction and design of large projects that are difficult to test physically. A spacecraft can be prototyped and then tested in a controlled environment (i.e., a thermal vacuum chamber). An aircraft can also be prototyped and then tested in a wind tunnel or with a scale model. Similar examples exist for mechanical and electrical systems. However, a 150-story high-rise building is difficult, if not impossible, to test. There are experts who contend that a scale model test will produce reasonably accurate results for a large high-rise building, but this remains a controversial topic among both research and practicing structural engineers. How does one test an earth dam that is to hold 1,000,000 acre-feet of water and be subjected to a 7.0 Richter magnitude earthquake?

Because civil engineers are not able to test their theories with models, they must rely on analytical procedures. The concepts herein relating to working stress design (WSD) have been in existence since the 1930s, originating in theories developed for reinforced concrete. WSD has a large factor of safety and a nearly impeccable performance record.

3.1.1 Elastic zone and plastic zone

WSD is based on the concept that every structural design should have a basic factor of safety and that the structure should never yield. Figure 3.1, a typical stress versus strain curve, delineates the linear elastic and plastic zones.

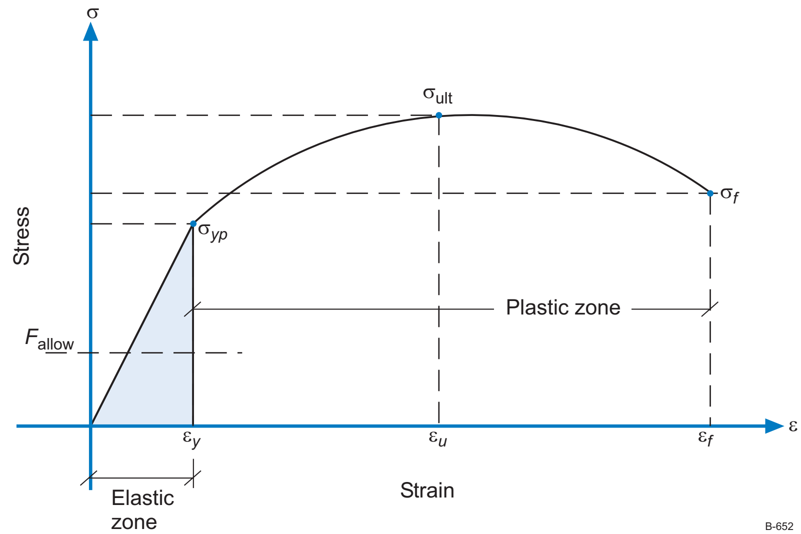


Figure 3-1

Although reinforced masonry does not have a clear linear zone, its yield point is defined by the steel reinforcement. Definitions of common WSD terms follow.

Yield Point: σ_{yp}, ε_{yp}
The stress-strain point at which the material undergoes permanent deformation. After passing the yield point, the material will never return to its original position.

Elastic Zone: $\varepsilon < \varepsilon_{yp}$
The portion of the stress-strain curve preceding the yield point. In this zone, all loads are reversible, and the structural material undergoes no permanent deformation.

Plastic Zone: $\varepsilon > \varepsilon_{yp}$
The portion of the stress-strain curve following the yield point. In this zone, all loads and deformations are irreversible, and the structural material has undergone permanent deformation.

Ultimate Stress: σ_{ult}, ε_{ult}
The peak stress on the stress-strain curve, occurring in the plastic zone. The corresponding strain is the ultimate strain.

Failure Stress: σ_f, ε_f
The amount of stress on the structure at the moment it collapses.

Design Stress/Allowable Stress: F_{allow}
Design limit for the material. This is obtained by dividing the yield stress by a factor of safety. The design/allowable stress is based on the standard of practice, which is derived from the applicable code provisions.

WSD relies on the yield stress divided by a factor of safety. This is different from ultimate strength design (SD), which is formulated on the ultimate stress and load factors applied to the working/service loads. SD will be discussed in Chapter 6.

An analysis of a single reinforced cross section is considered for reinforced masonry.

3.1.2 Analysis assumptions and structural behavior

Figures 3-2 and 3-3 show a typical shear wall with in-plane and out-of-plane loads, respectively.

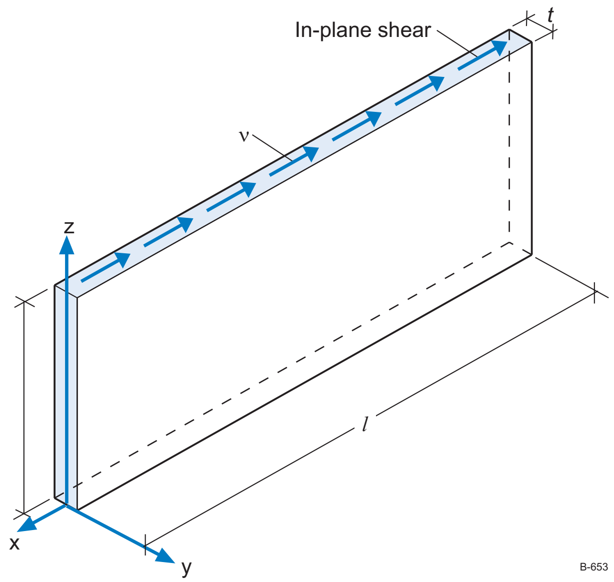


Figure 3-2

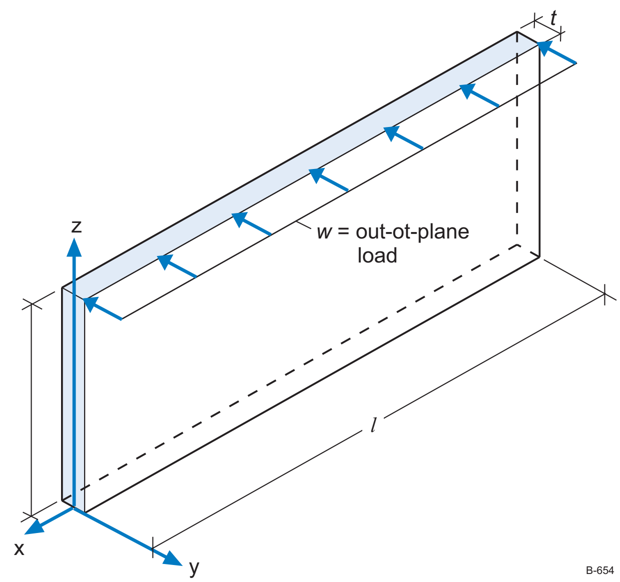


Figure 3-3

The analysis assumptions, based on linear elastic beam theory, are summarized here.

1) Plane sections remain plane. This concept is derived from the basic strength of materials regarding the curvature analysis of bending sections. It states that the longitudinal axis of the beam remains at a 90-degree angle to the cross-section plane (Figures 3-4 and 3-5).

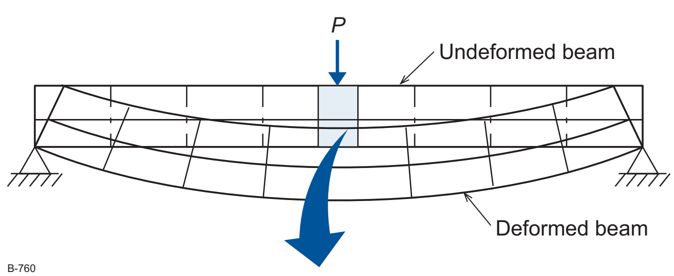


Figure 3-4

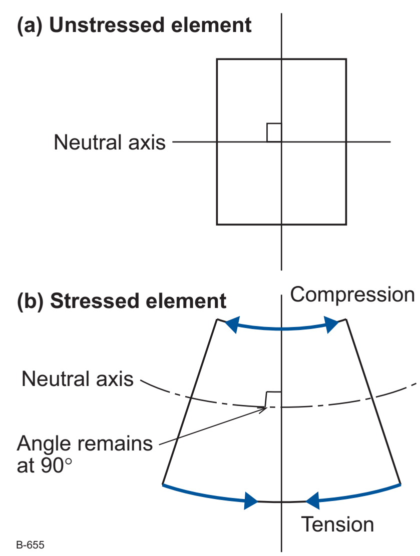


Figure 3-5

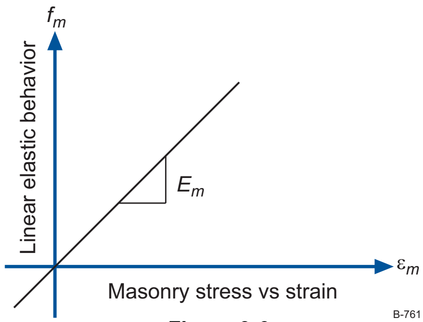


Figure 3-6

2) Stress is proportional to strain. Following the basic elements of Hooke's Law, there is perfect linear correlation between stress values and strain values (Figure 3-6).

3) Masonry has no tensile strength.

4) E_m and E_s remain unchanged throughout the loading process. Stress and strain follow Hooke's Law (Item 2, above).

5) Deformation is due largely to bending energy with minimal shear energy contribution. This is valid for beams with a long length-to-area ratio (i.e., slender beams) where the bending contribution is large (Figure 3-7).

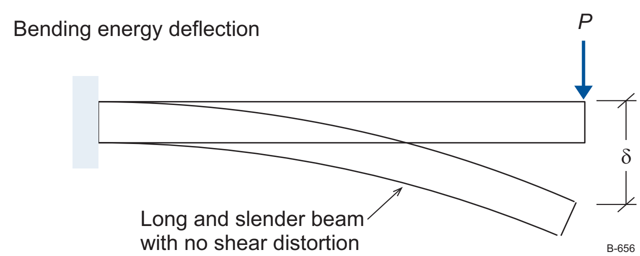


Figure 3-7

6) The member is prismatic; specifically, the cross section does not change.

7) Steel is assumed to behave elasto-plastically. Specifically, the stress-strain curve is assumed to be elastically perfectly plastic; that is, the curve has a zero tangent modulus, $E_t = 0$. The initial modulus (E_s = 29,000 ksi) remains constant until the yield strain, E_y, is reached (Figure 3-8).

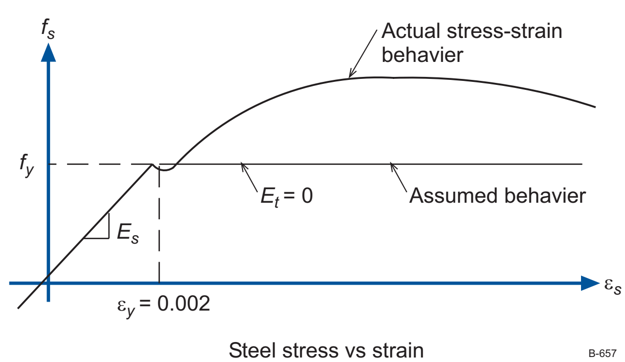


Figure 3-8

8) Masonry is assumed to reach failure at its design-allowable bending stress, F_b. $F_b = 0.33 F'_m$.

In the UBC, for structures without inspection, $\frac{1}{2}F'_m$ or $F_b = 0.33(\frac{1}{2}F'_m\ )r$.

9) No bond slip is considered. The steel reinforcing behaves integrally within the grouted section and actively resists the tensile stress (Figure 3-9).

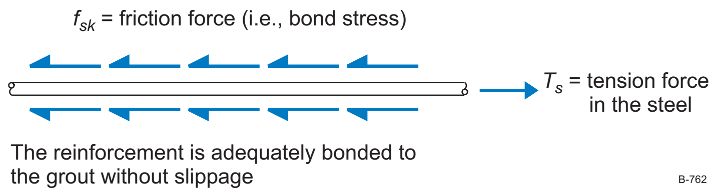


Figure 3-9

10) For ease in computation, shear stress is calculated as an average value without specific attention to maximum shear stress. This averaging works sufficiently well for masonry design because of the factor of safety (Figure 3-10).

3.1.3 Moment-curvature behavior

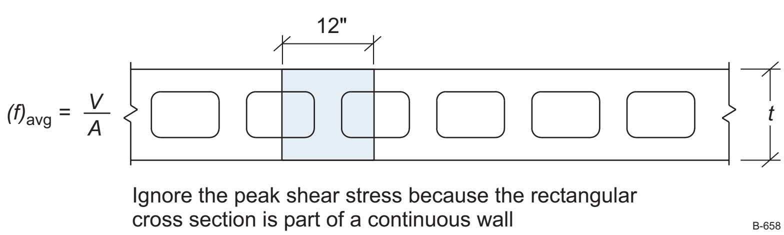


Figure 3-10

Figure 3-11 shows moment-curvature behavior in three phases. The behavior is applicable to walls acting in the out-of-plane mode with loads applied as in Figure 3.3. A vertical cross section is shown in Figure 3-12.

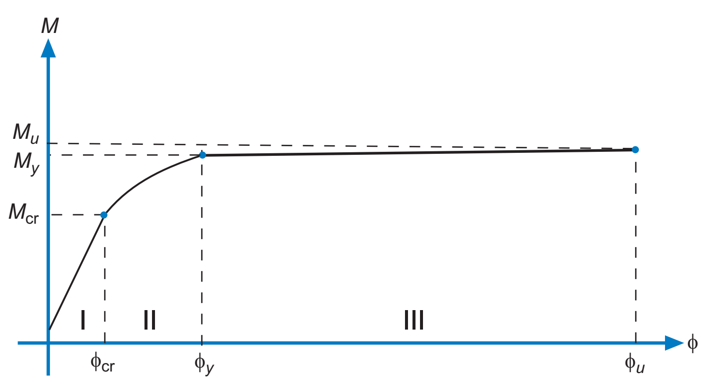


Figure 3-11

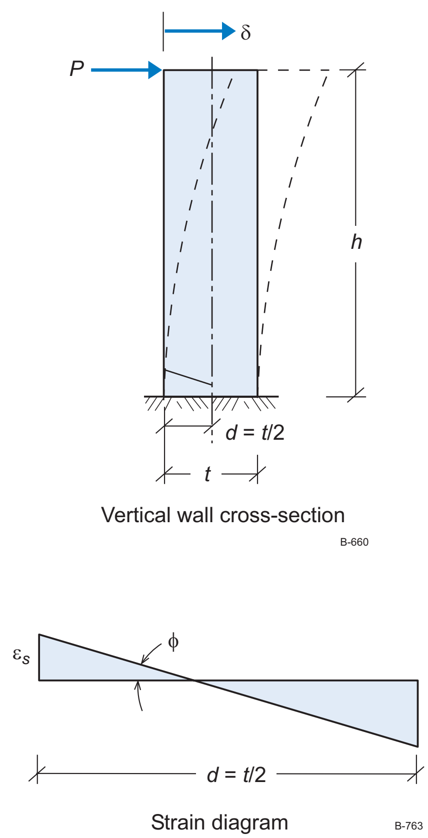


Figure 3-12

3.1.4 Stages of structural loading

1) Phase 1: Uncracked $M < M_{cr}$.
   The masonry has not reached its cracking capacity, so the linear relationships between both stress and strain and the moment curvature remain intact. Under normal service loads, Phase 1 is not expected to be exceeded.

2) Phase 2: Cracked section $M_{cr} < M < M_y$
   The masonry has cracked and steel stress is approaching yield value. The masonry moments are increasing and approaching the allowable bending. The steel strains are increasing toward yield value. The curvature (i.e., rotation and deformation) relationship is increasing over pre-cracking levels. Phase 2 occurs in such load conditions as earthquakes and high winds. The code prevents yielding of the reinforcement, which could cause permanent deflection.

3) Phase 3: Post-yield/plastic behavior $M < M_y$
   The steel reinforcement has yielded, and the deflections increase substantially with excessive cracking in the masonry. This is a nonlinear region with large displacements for small incremental loads. While beyond the designer's purview to consider this behavior, it is neccessary to understand the characteristics of the structure prior to its failure. Post-yield behavior results in severe strains or displacements until the ductility of the wall either reaches the collapse strain of the steel, or the masonry crushes under the large loads.

3.1.5 Structural performance and definitions

Structural Failure
Failure is the point at which the material exceeds either its allowable stress value or its design capacity. According to this definition, the structure is failing if steel stress exceeds yield.

Structural Collapse
This refers to the point at which the structure is approaching physical collapse or destruction. It is seldom analyzed in actual design practice, but it should be understood that structural failure does not necessarily imply structural collapse. For example, steel yielding may be termed a structural failure. However, because steel has excellent ductility and impressive energy absorption beyond yield, the actual collapse strain is far beyond its yield value.

Uniform Building Code and International Building Code Requirements
If the results of a structural analysis indicate that the applied loads (i.e., demand) exceed the structure's capacity, then the design is inadequate. The concept is the same for both working stress design and strength design, except that load factors are introduced in strength design. Code requirements are based on experience, empirically derived, and formulated to assist a structural engineer in the design process and are always prefaced with an explanation that these are minimum design loads. Some engineers mistakenly rely on these loads as the final word in practical design: not a safe practice. However,

no code can address every specific situation. Structural engineers will encounter many circumstances that must be addressed on a case-by-case basis.

Design Load and Allowable Stress
The design load is the code-prescribed load or demand on the structure. It is termed a service load in allowable/working stress design (ASD/WSD) because it represents the actual loads on the building. These service loads have factors of safety already placed in them by code writers. Similarly, ASD values represent the design capacity of the structural member using code-prescribed values. The WSD philosophy relies on the yield value divided by a factor of safety (usually between 2 and 4, depending on the material). Therefore, the design or service load and the allowable stress each have a built-in factor of safety.

ASCE Minimum Design Loads
The American Society of Civil Engineers (ASCE) developed a national guideline (ASCE 7-98) that has been adopted by reference into the 2000 IBC. This document represents the cooperative efforts of the academic, consulting, and industrial sectors of the engineering profession to present a uniform national standard of design loads. The term "minimum" is used in the title to emphasize that the design professional always retains final authority for the product and its application. In other words, if the designer observes situations that require higher design loads, it is within her or his discretion and authority to impose those higher loads, because ACSE 7-98 represents the minimum standard.

Objectives of Working Stress Design (WSD)
Although strength design is becoming the new standard and has replaced WSD in the areas of reinforced concrete and steel for large-scale projects, WSD is the design practice that has been used for decades. Therefore, it is imperative to understand its inherent philosophy.

1) WSD is a simplified methodology for structural application that describes suitable factors of safety against the yield of the reinforcement. These factors of safety are conservative and seek to prevent plastic behavior in the reinforced masonry element.

2) WSD controls displacements with the factors of safety against yield. The structure is kept elastic with linear behavior. By definition, in WSD, the member fails if it exceeds its allowable design, which is always lower than the yield value.

3) Given the large displacement ductility of steel, WSD is a proven, safe method of masonry design that has an excellent performance record.

3.1.6 Derivation of analysis equations

WSD equations are formulated using the principles of Hooke’s Law, which is derived from the Bernoulli-Euler Beam Equation.

C = compression force in masonry section

T = tensile force in steel

f_m = masonry stress

f_s = steel stress

kd = length of compression-stress zone

jd = length of moment arm

A_s = steel reinforcement area

ε_m = masonry strain

ε_s = steel strain

E_m = modulus of elasticity, masonry

E_s = modulus of elasticity, steel

n = modular ratio = $\dfrac{E_s}{E_m}$

nA_s = equivalent steel area (transformed steel area)

C = T

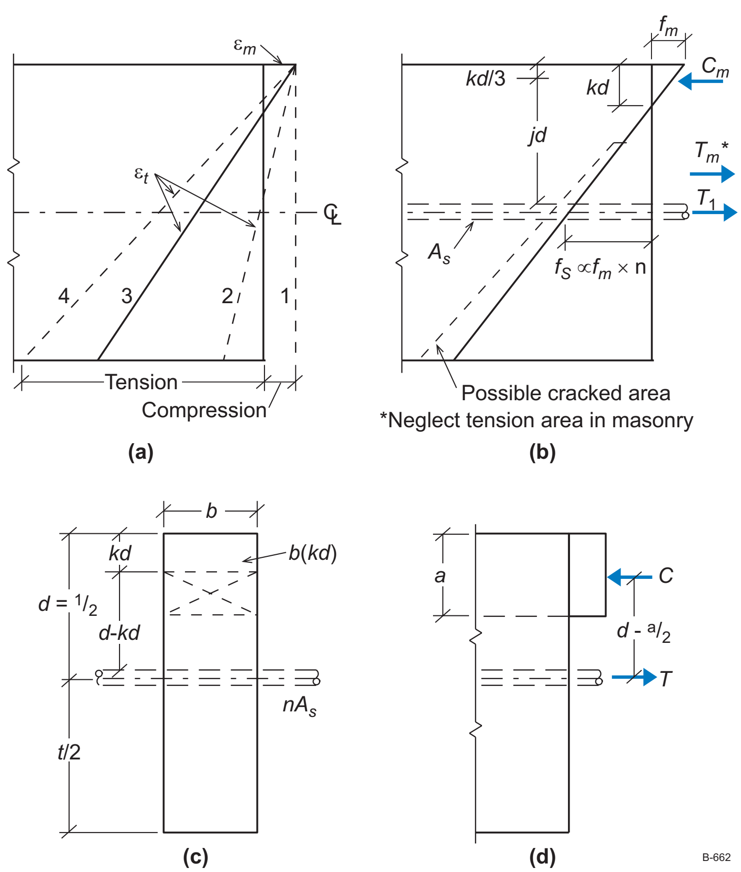


Figure 3-13

$$C = \frac{1}{2} f_m kdb$$

$$T = f_s A_s$$

$$\frac{1}{2} f_m kdb = f_s A_s$$

where:

$$f_m = \varepsilon_m E_m$$

$$f_s = \varepsilon_s E_s$$

From the strain diagram (Figure 3-14)

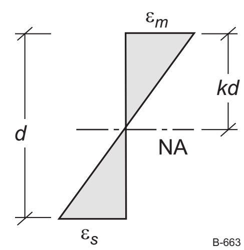


Figure 3-14

$$\frac{\varepsilon_m}{kd} = \frac{\varepsilon_s}{d - kd}$$

$$\frac{\varepsilon_m}{\varepsilon_s} = \frac{kd}{d - kd}$$

Substitute into Equation 3-4

$$\frac{1}{2} \varepsilon_m E_m kdb = \varepsilon_s E_s A_s$$

$$\frac{1}{2}\left(\frac{\varepsilon_m}{\varepsilon_s}\right) kdb = \left(\frac{E_s}{E_m}\right) A_s$$

$$\frac{1}{2}\left(\frac{kd}{d - kd}\right) = kd(b) = nA_s$$

$$\frac{1}{2} b(kd)^2 - nA_s(d - kd) = 0$$

$$\text{Define } \rho = \frac{A_s}{bd} = \text{steel ratio}$$

$$A_s = \rho bd$$

$$\frac{1}{2} b(kd)^2 - n\rho bd(d - kd) = 0$$

$$\frac{1}{2} bk^2d^2 - \rho nbd^2 + \rho nbkd^2 = 0$$

Divide with bd^2

$$\frac{1}{2} k^2 - \rho n(1 - k) = 0$$

Multiply by 2

$$k^2 - 2\rho n + 2\rho nk = 0$$

or

$$k^2 + 2\rho nk - 2\rho n = 0 \text{ (quadratic equation)}$$

$$\therefore \quad k = \frac{-2\rho n \pm \sqrt{(2\rho n)^2 - 4(1)(-2\rho n)}}{2} \Rightarrow k = \sqrt{(np)^2 + 2np} - \rho n$$

Location of neutral axis (NA)

$$k = \sqrt{(np)^2 + 2np} - n\rho$$

jd = moment arm between C and T

$$jd = d - \frac{kd}{3}$$

$$\therefore j = 1 - \frac{k}{3} = \text{moment arm coefficent}$$

M_s = steel reinforcement moment capacity

$$M_s = Tjd$$

$$M_s = f_s A_s jd$$

$$\therefore \quad M_s = \rho bd f_s jd = \text{steel moment capacity}$$

or

$$f_s = \frac{M_s}{\rho bdjd} = \frac{M_s}{\rho jbd^2}$$

M_m = masonry moment capacity

$$M_m = Cjd$$

$$M_m = \frac{1}{2} f_m bkd(jd)$$

$$\therefore \quad M_m = \frac{1}{2} f_m jkbd^2 = \text{masonry moment capacity}$$

or

$$f_m = \frac{M_m}{2jkbd^2}$$

Balanced steel ratio

$$\rho_b = \frac{A_{sb}}{bd}$$

1) Reinforcing steel should yield before failure because the masonry is forced to crack in compression. Reinforced masonry should be "under reinforced" with reinforcement that is below the balanced failure ratio.
2) Masonry cracking is expected and creates a visible indicator of structural collapse before collapse actually occurs. This provides a built-in early warning system for occupants, similar to cracking in reinforced concrete structures.
3) Structures are designed with this concept of visible cracking, so that any observer may notice the cracks and exit the facility before it collapses.

F_b = allowable bending stress of masonry

f_y = yield stress of steel

From the balanced stress-strain diagram, Figure 3-15

$\therefore\ \varepsilon_{mu}$ = masonry strain at $F_b = F_b / E_m$

$\therefore\ \varepsilon_y$ = F_y / E_s = steel strain at F_y (yield strain)

T_{sb} = tensile force at balanced failure

$T_{sb} = A_{sb} F_y$

C_{sb} = compression force at balanced failure

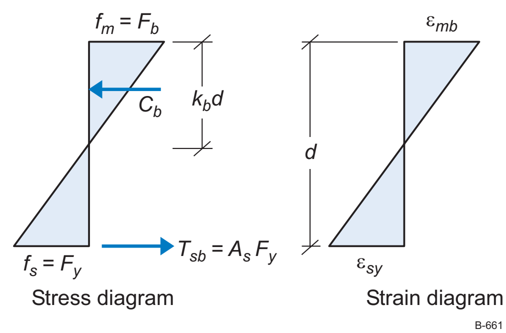


Figure 3-15

At balanced failure, ε_{sy} occurs simultaneously as $\varepsilon_m = \varepsilon_{mb}$

or

$f_m = F_b$ simultaneously as $f_s = F_y$

Therefore,

$$C_b = T_{sb}$$

$$\frac{1}{2} F_b k_b bd = F_y A_{sb}$$

$$\frac{1}{2} F_b k_b bd = F_y \rho_b bd$$

$$\therefore\ \rho_b = \frac{F_b k_b}{2F_y}$$

A_{sb} = balanced steel area

$\rho_b = \dfrac{A_{sb}}{bd}$ = balanced steel ratio

The concept of balanced steel ratio is important for an understanding of WSD. However, in the IBC, this concept is replaced with a maximum steel ratio (ρ_{max}). The ρ_{max} provisions are for strength design (IBC 2108.9.2.13).

To determine k_b, refer to the strain diagram

$$\frac{\varepsilon_{mb}}{k_b d} = \frac{\varepsilon_{sy}}{d - k_b d}$$

$$\varepsilon_{mb} = E_m F_b$$

$$\varepsilon_{sy} = E_s F_y$$

$$\frac{\varepsilon_{mb}}{\varepsilon_{sy}} = \frac{F_b / E_m}{F_y / E_s} = \frac{nF_b}{F_y} \quad (n = \frac{E_s}{E_m})$$

$$\varepsilon_{mb}(d\text{-}k_b d) = \varepsilon_{sy}(k_b d)$$

$$\frac{nF_b}{F_y}(d\text{-}k_b d) = k_b d$$

$$k_b(1 + \frac{nF_b}{F_y}) = \frac{nF_b}{F_y}$$

$$\mathrm{k_b} = \frac{n(F_b / F_y)}{\left(1 + \dfrac{nF_b}{F_y}\right)}$$

Balanced failure equations

$$k_b = \frac{n}{n + F_y / F_b}$$ Balanced condition. Location of neutral axis (k_b)

Substrate into ρ_b

$$\rho_b = \frac{F_b}{2F_y}\left(\frac{n}{n + F_y / F_b}\right)$$ Balanced steel ratio

Sizing of masonry members and use of tables

Let: $bd^2 = \dfrac{M}{K}$

Then: $K = \dfrac{M}{bd^2} = \dfrac{1}{2} f_m jk$ at failure

K	$=\frac{1}{2}F_b jk$	Masonry K factor for sizing elements

3.1.7 Design procedure

WSD is a simplified methodology for estimating the strength of a reinforced masonry design. With practice, a structural engineer will develop a sense for the numbers and can then provide a suitable design with reasonable accuracy based on judgment and experience. Given the advances in computer technology, using design tables is no longer the only protocol, but may be retained as an option.

There are three design procedures for structural engineers to consider.

Design procedure by hand calculation: This is the method presented in textbooks and taught in many classrooms. It is also used in actual practice to verify results and plays an important part in the process of design development.

Design procedure by tables and charts: This method uses standardized tables and charts from available textbooks such as the *Reinforced Masonry Engineering Handbook* (RMEH).

Design procedure using spreadsheets/software: Specialized spreadsheets (not included in this text) have been developed to assist practicing engineers in solving masonry problems efficiently without sacrificing accuracy. Such spreadsheets represent the quickest and simplest method for direct, practical application. Several analysis packages are available such as RISA, ETABS, M-STRUDL, etc.

Notes:

1) Masonry capacity usually governs. There are two moment capacities calculated, M_m and M_s. Of these two, usually the M_m capacity will govern the design. Steel can always be increased in size. Cost increases are minimal when changing the reinforcement from a #4 to a #6 bar, but increasing the masonry block from an 8-inch to a 12-inch-wide concrete masonry unit (CMU) could prove expensive. Therefore, masonry capacity is generally the upper boundary in the final design.
2) Deflection will govern in high walls. For retaining walls and large shear walls with high lateral loads (out-of-plane), the lateral stiffness will be controlled by deflection considerations. This concept is presented in Section 3.3 and further discussed in the examples.
3) Code requirements change over time, but engineering fundamentals remain constant. Code values will fluctuate, and jurisdictions may vary building policy, but engineering judgment and the laws of physics remain the same.

4) Remember to double-check the analysis by hand and use common-sense engineering regardless of the design procedure chosen. If it does not make sense on paper, it certainly will not work in the field.

3.2 In-plane Shear Analysis

3.2.1 Concept

Shear walls have in-plane loads that run along the longitudinal axis of the wall (see Figure 3-2).

The in-plane traction force diagram depicts a linear shear load (force/length) that is absorbed by the wall along the x-axis. In-plane force is the basis of shear wall behavior and its presence prescribes the use and application of reinforced concrete and reinforced masonry. It is important to differentiate between true shear and flexural behavior. While many engineers use the term "shear wall" loosely, there is a subtle difference between a true shear wall and a structural/flexural wall.

Refer to Figure 3-16, which shows that the shear wall has a predominantly in-plane diagonal shear stress across 45-degree angles. Classic shear failure occurs along this 45-degree axis as described by the Mohr shear stress concept of maximum shear.

Shear stress in masonry walls follows the mortar joint line along approximate 45-degree lines as the theory suggests. The mortar joint line is the weakest part of a masonry shear wall. Reinforcement is the strongest element: the next strongest elements are the grout material and the masonry face shell. Therefore, any shear or flexural failure will occur in the mortar first.

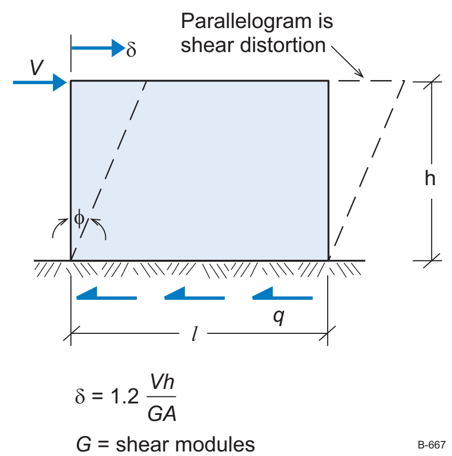


Figure 3-16

3.2.2 Definitions

Shear Behavior
In-plane shear deflection is the result of energy distortion of a rectangle. Shear energy distortion is the deflection pattern similar to the parallelogram shown in Figure 3-16. The rectangle deforms into a parallelogram by lateral displacement of the top fiber from the in-plane shear force. The shear angle (phi or ϕ) is based on fundamental equations from strength of materials.

Bending Behavior (Figure 3-17)
Bending energy deflection is the result of lateral displacement that forces curvature bending of the beam. This energy displacement is based on the standard beam behavior bending theory and follows Hooke's Law. Bending energy is addressed in Chapter 6 under Wall-Frame Analysis.

The diagram shows the bending strain energy with plane sections remaining plane. The lateral deflection is $\delta = \dfrac{Vh^3}{3EI}$

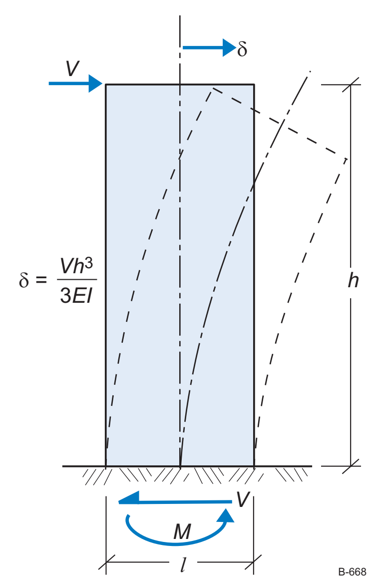


Figure 3-17

$f_r \quad = \dfrac{V}{t_{eff} \times L_{eff}}$ = in-plane shear stress

t_{eff} = *effective wall thickness*

Grout spacing determines the effective thickness of the wall. For example, in a fully grouted wall there are no voids or cavities, resulting in an effective thickness equivalent to the wall thickness. For a fully grouted 8-inch CMU, the effective thickness is $7^5/_8$ or 8 inches. Partially grouted walls do not have all vertical cells filled with grout, thereby reducing the effective wall thickness as given by the following tables. (See also Figure 3-18.)

(a) Hollow Concrete Masonry Walls

Grout spacing	Equivalent solid thickness Nominal unit thickness (inches)			In-plane shear area—in^2/ft
	6	8	12	(8-inch block)
Solid grouted	5.6	7.6	11.6	88.5
Grout at 16 in o/c	4.5	5.8	8.5	60.9
24	4.1	5.2	7.5	50.5
32	3.9	4.9	7.0	45.3
40	3.8	4.7	6.7	42.3
48	3.7	4.6	6.5	40.5
No grout	3.4	4.0	5.5	

(b) Hollow Clay Brick Walls

Grout spacing	Equivalent solid thickness Nominal unit thickness (inches)			In-plane shear area—in^2/ft
	5	8	5	(8-inch block)
Solid grouted	4.5	7.5	54.0	90.0
Grout at 12 in o/c	3.5	4.9	41.5	58.7
24	3.0	3.7	35.8	44.3
36	2.8	3.3	33.8	39.6
48	2.7	3.1	32.9	37.2

*Some special shapes will vary from the figures shown

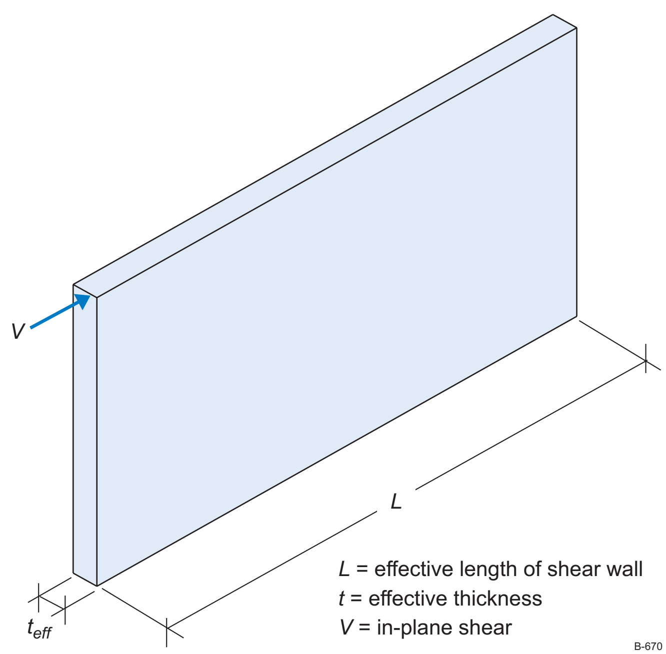


Figure 3-18

L = effective wall length
The length of shear wall is calculated from boundary steel to boundary steel. Each edge of the wall has a boundary element with reinforcement steel. This is used to resist overturning moments through axial compression/tension in the boundary steel.

Shear Area
This is the product of the effective wall thickness and the effective length to calculate the net area of the wall to resist in-plane shear forces. For in-plane, loads the shear is distributed along the wall; for out-of-plane forces, the loads are resisted by the vertical reinforcement in conjunction with grout and face shells.

Shear Area = $t_{eff} \times L_{eff}$ = effective area

Allowable shear stress (F_v)

From 1997 UBC (2107.2.8) based on WSD: MSJC 2.3.5

$$F_v = 1.0\sqrt{f'_m} < 50 \text{ psi (no shear reinforcement)}$$

With shear reinforcement

$$F_v = 3.0\sqrt{f'_m} < 150 \text{ psi}$$

From 1997 UBC (2107.2.9): MSJC 2.3.5

With in-plane flexural reinforcement

For $\dfrac{M}{Vd} < 1$

$$F_v = \frac{1}{3}\left(4 - \frac{M}{Vd}\right)\sqrt{f'_m} < \left(80 - 45\frac{M}{Vd}\right)$$

With in-plane shear reinforcements

For $\dfrac{M}{Vd} < 1$

$$F_v = \frac{1}{2}\left(4 - \frac{M}{Vd}\right)\sqrt{f'_m} < \left(120 - 45\frac{M}{Vd}\right)$$

For $\dfrac{M}{Vd} > 1$

$$F_v = 1.5\sqrt{f'_m} < 75 \text{ psi}$$

Flexural Steel for in-plane forces
Boundary elements are provided to resist overturning moments that cause force couples to exist in the edge of the wall. Such vertical forces are either compression or tension loads that are transferred into the foundation. These are referred to as *flexural steel* because the boundary steel provides the primary bending resistance. For out-of-plane loads, flexural steel refers to conventional vertical reinforcement.

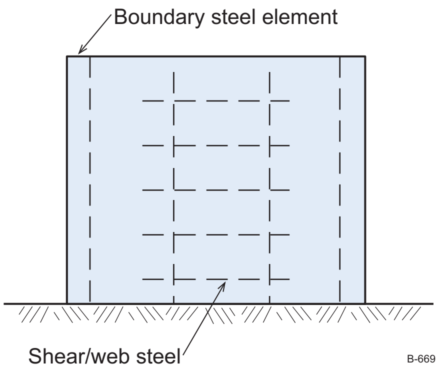


Figure 3-19

Shear Steel
The steel contained within the web of the wall provides in-plane shear resistance and out-of-plane bending resistance. For in-plane loads, this is referred to as *web steel* or *shear steel*. The redundancy of the shear steel resists the lateral deflections of the wall. An example of shear steel is provided by Figure 3-19.

The following is from the 2000 IBC (Strength design).

2108.9.3.5.2 Nominal shear strength. Nominal shear strength, V_n, shall be computed as follows:

$$V_n = V_m + V_s \qquad \text{Eq. 21-27}$$

For $M/Vd_v < 0.25$

$$V_n = 6A_n\sqrt{f'_m} \qquad \text{Eq. 21-28}$$

For $M/Vd_v > 1.00$

$$V_n = 4A_n\sqrt{f'_m} \qquad \text{Eq. 21-29}$$

where:

A_n = Net cross-sectional area of masonry, square inches (mm^2)

f'_m = Specified compressive strength of masonry at age of 28 days psi (MPa)

V_m = Shear strength provided by masonry, pounds (N)

V_n = Nominal shear strength, pounds (N)

V_s = Shear strength provided by shear reinforcement, pounds (N)

Value of M/Vd_v between 0.25 and 1.0 is permitted to be interpolated.

2108.9.3.5.2.1 Nominal masonry shear strength. Shear strength, V_m, provided by the masonry shall be as follows:

$$V_m = \left[4.0 - 1.75\left(\frac{M}{Vd_v}\right)\right]A_n\sqrt{f'_m} + 0.25P \qquad \text{Eq. 21-30}$$

where:

M/Vd_v need not be taken greater than 1.0 and

For SI

$$V_m = 0.83\left[4.0 - 1.75\left(\frac{M}{Vd_v}\right)\right]A_n\sqrt{f'_m} + 0.25P$$

where:

A_n = Net cross-sectional area of masonry, square inches (mm^2)

d_v = Length of member in direction of shear force, inches (mm)

f'_m = Specified compressive strength of masonry at age of 28 days, psi (MPa)

M = Moment on a masonry section due to unfactored loads, inch-pound (N-mm)

P = Axial force on a masonry section due to unfactored loads, pounds (N)

V = Shear on a masonry section due to unfactored loads, pounds (N)

V_m = Shear strength provided by masonry, pounds (N)

2108.9.3.5.2.2 Nominal shear strength provided by reinforcement. Nominal shear strength, V_s, provided by reinforcement shall be as follows:

$$V_s = 0.5\left(\frac{A_v}{s}\right) f_y d_v \qquad \textbf{Eq. 21-31}$$

where:

A_v = cross-sectional area of shear reinforcement, square inches (mm^2)

d_v = length of member in direction of shear force, inch (mm)

f_y = specified yield stress of the reinforcement or the anchor bolt, psi (MPa)

S = spacing of stirrups or of bent bars in direction parallel to that of main reinforcement, inches (mm)

V_s = shear strength provided by shear reinforcement, pounds (N)

2108.9.3.6 Reinforcement.

1. Where transverse reinforcement is required, the maximum spacing shall not exceed one-half the depth of the member nor 48 inches (1219 mm).
2. Flexural reinforcement shall be uniformly distributed throughout the depth of the element.
3. Flexural elements subjected to load reversals shall be symmetrically reinforced.
4. The nominal moment strength at any section along a member shall not be less than one-fourth the maximum moment strength.
5. The maximum flexural reinforcement ratio shall be determined by Section 2108.9.2.13.
6. Lap splices shall comply with the provisions of Section 2108.9.2.11.

7. Welded splices and mechanical splices that develop at least 125 percent of the specified yield strength of a bar may be used for splicing the reinforcement. Not more than two longitudinal bars shall be spliced at a section. The distance between splices of adjacent bars shall be at least 30 inches (762 mm) along the longitudinal axis.

8. Specified yield strength of reinforcement shall not exceed 60,000 psi (414 MPa). The actual yield strength based on mill tests shall not exceed 1.3 times the specified yield strength.

3.3 Out-of-plane Bending

3.3.1 Concept

Lateral loads applied to a shear wall cause displacement along the top of the wall (Figure 3-20). The forces are applied normal to the surface of the wall and bring about lateral movement. This results in out-of-plane bending and deflection. Vertical steel rods spaced along the wall length reinforce structural capacity. It is also possible to develop out-of-plane forces with the top restrained.

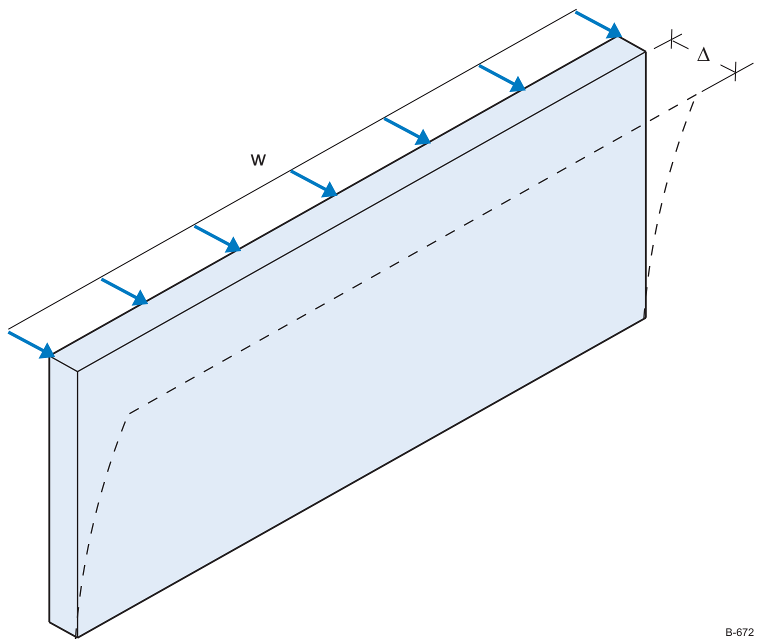


Figure 3-20

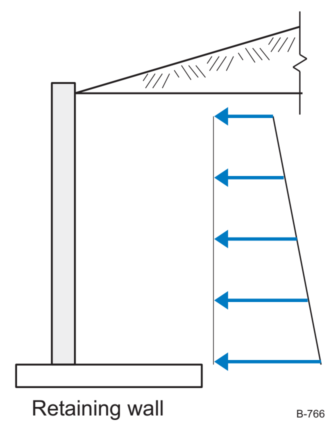


Figure 3-21

3.3.2 Practical example

3.3.2.1 Retaining Walls

In Figure 3-21, retaining walls provide lateral resistance against earth fills/soil pressure. The lateral force on the wall results in bending moments resisted by vertical reinforcement stee.

3.3.2.2 Wind Force on Structural wall

Wind forces generate pressures on the structural shear wall (Figure 3-22). Lateral displacements within the shear wall will occur along the wall's mid height to create bending moments and shears. These are resisted in the same manner as a retaining wall.

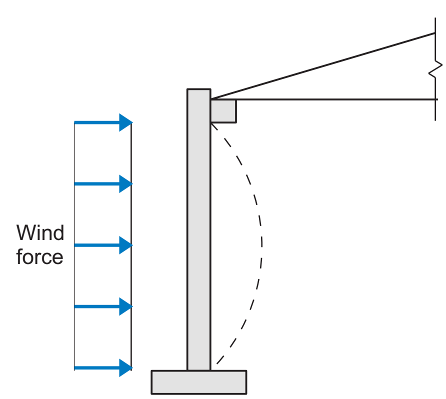


Figure 3-22

3.3.3 Analysis equations

Fully grouted section
Figure 3-23 shows a cross section of a masonry wall with grout contained in every cell. This creates a fully effective section but also increases the dead load of the wall.

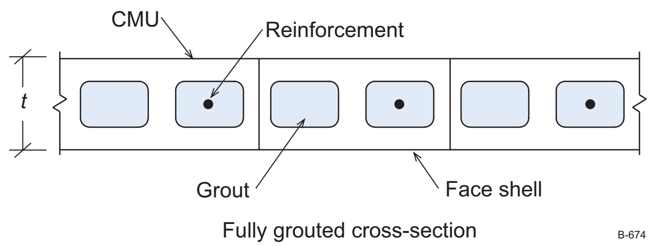


Figure 3-23

T-beam analysis (partially grouted)
Figure 3-24 shows a partially grouted wall with lateral spacing of the reinforcement. Partially grouted walls may reduce the overall construction cost of the wall. Reinforcement is placed at equal intervals in the grouted cells.

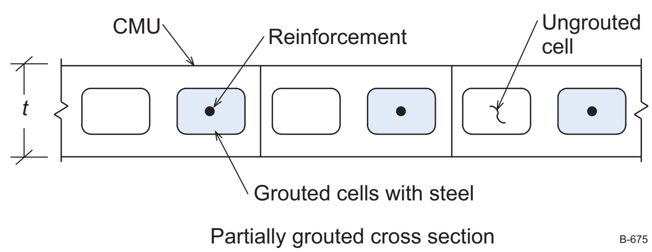


Figure 3-24

Double reinforced analysis (full or partially grouted)
Figure 3-25 illustrates the use of two layers of reinforcement to add compression reinforcement.

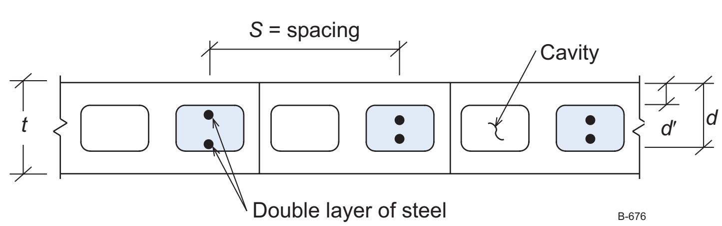


Figure 3-25

Deflection analysis
Lateral deflection is essential to drift considerations and displacement control. Deflection calculations account for the stiffness of the wall and estimate the total stiffness degradation by reducing the gross moment inertia to a cracked section.

3.3.4 Analysis of T-beam section

Figure 3-26 shows a laterally loaded wall. Figure 3-27 shows that wall in cross section, while a compression stress diagram is illustrated in Figure 3-28.

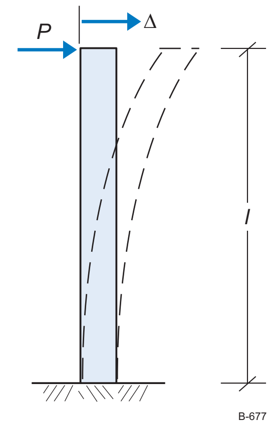


Figure 3-26

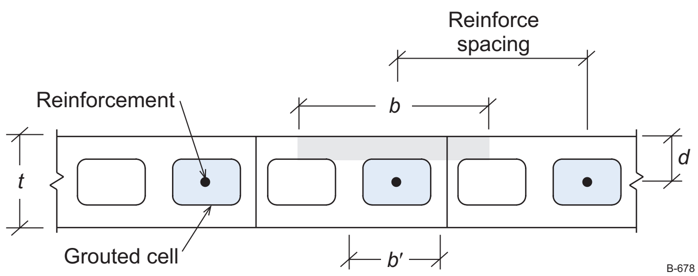


Figure 3-27

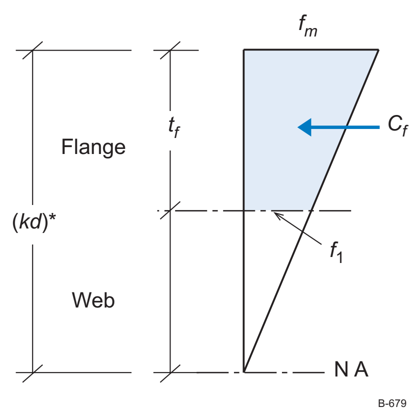


Figure 3-28

b = effective flange width – that portion of the flange or face shell of the masonry block intended to accept compression forces

b' = web width – the grouted cell portion with reinforcement that will accept a portion of the compression force

d = T-beam depth to reinforcement

t = thickness of masonry unit

Analysis Assumptions

Effective Flange Width
The effective flange width comprises the face shell portion between the reinforcement spacing. The cross section is assumed to act compositely with the reinforcement accepting tension forces and the face shell compression forces.

Grouted Cell
Reinforcement is placed in the grouted cell to create composite action with the tension steel. The grouted cell acts as a web element in the T-beam cross section.

Plane Sections Remain Plane
All prior assumptions pertaining to basic beam bending theory are applicable in this analysis. The primary mode of deflection is due to bending energy versus shear energy. This automatically implies lateral deflection in conformance with beam bending theory.

Linear Elastic Behavior
The out-of-plane analysis follows the assumptions of Hooke's Law and linear elastic load versus deflection behavior.

Masonry Tension Stress
The tensile stress value of masonry is assumed to be zero.

T-beam

From stress diagram and similar triangles (Figures 3-29 and 3-30)

$$f_i = \left(\frac{kd - t_f}{kd}\right) f_m \text{ and } f_f = \text{average flange stress}$$

$$f_f = \frac{1}{2}(f_i + f_m)$$

$$f_f = \frac{1}{2}\left(f_i + \frac{kd - t_f}{kd} f_m\right)$$

$$C_f = f_f A_f$$

$$\therefore\ C_f = (f_f)(bt_f) = f_m\left(\frac{2kd - t_f}{2kd}\right) bt_f$$

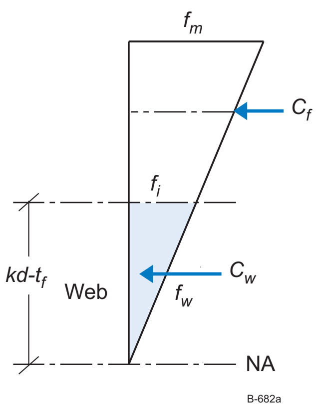


Figure 3-29

Calculate average web stress, f_w

$$f_w = \frac{1}{2} f_i$$

$$f_w = \frac{1}{2} f_m \left(\frac{kd - t_f}{kd}\right) = \text{average web stress}$$

$$\therefore\ C_w = f_w = ((kd\text{-}t_f)b')$$

$$C_w = \frac{f_m}{2}\left(\frac{kd - t_f}{kd}\right) b'(kd - t_f)$$

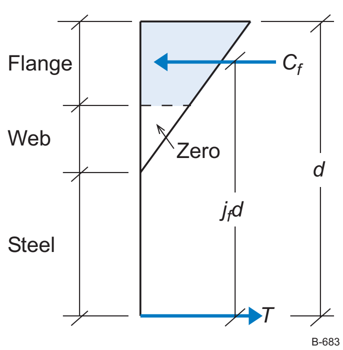


Figure 3-30

Moment of flange compression force

$$M_f = C_f j_f d = f_m\left(\frac{2kd - t_f}{2kd}\right) bt_f(j_f d)$$

Moment of web compression force

$$M_w = C_w j_w d = \frac{1}{2} f_m \left(\frac{kd - t_f}{kd}\right) b'(kd - t_f)(j_w d)$$

The contribution of the web in compression can be ignored because it is a minor force. ($C_w = 0$)

Therefore,

$$T = C_f \text{ (let } C_w = 0)$$

$$\Rightarrow \rho bdf_s = f_m\left(\frac{2kd - t_f}{2kd}\right) bt_f$$

From strain compatibility (Figures 3-31 and 3-32)

$$\frac{kd}{d} = \frac{\varepsilon_m}{\varepsilon_m + \varepsilon_s}$$

or

$$k = \frac{1}{1 + \dfrac{\varepsilon_s}{\varepsilon_m}}$$

$$k = \frac{1}{1 + \left(\dfrac{f_s}{E_s}\right)\left(\dfrac{E_m}{f_m}\right)} = \frac{1}{1 + \left(\dfrac{f_s}{f_m}\right)\left(\dfrac{1}{n}\right)} = \frac{n}{n + \dfrac{f_s}{f_m}}$$

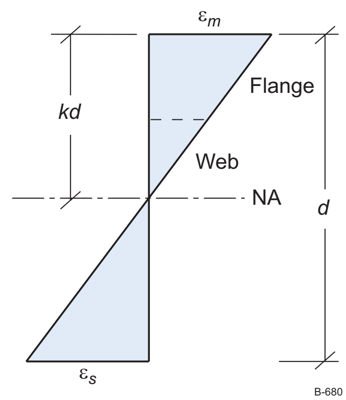


Figure 3-31

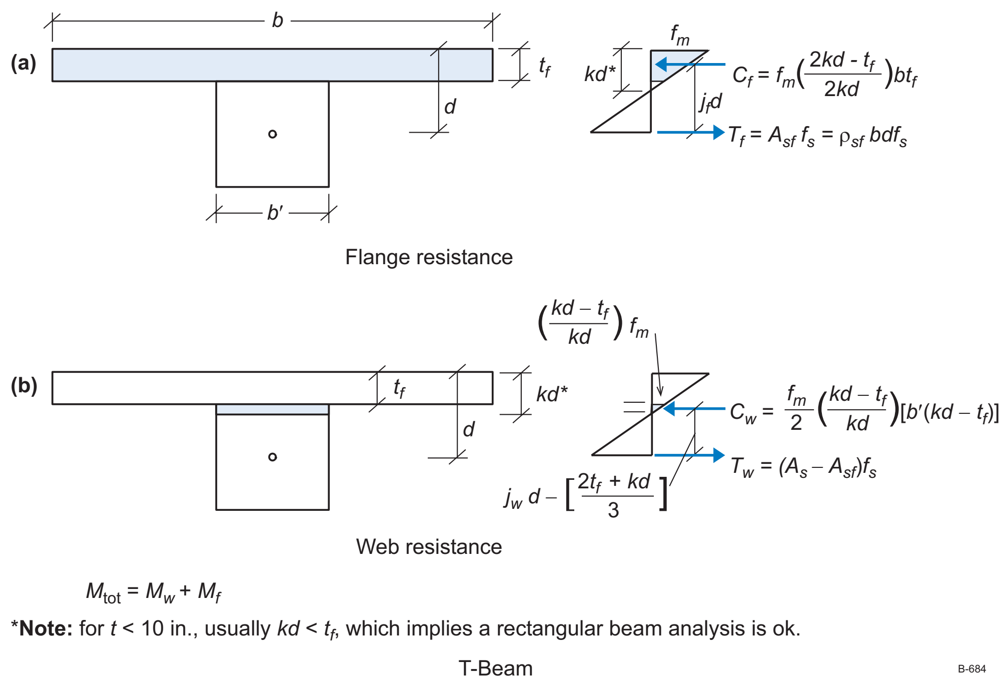


Figure 3-32

Solve for f_m,

$$\therefore\ f_m = f_s\left(\frac{k}{n(1-k)}\right)$$

Substitute into $T = C_f$

$$\rho b d f_s = f_s\left(\frac{k}{n(1-k)}\right)\left(\frac{2kd - t_f}{2kd}\right) b t_f$$

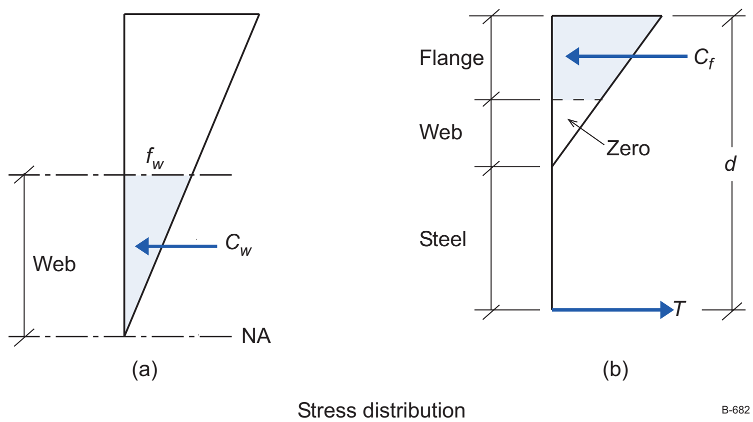
Solve for k:

$$\rho n\,(1-k)(2kd)(d) = k(2kd - t_f)\,t_f$$

$$k^2(2t_f d - 2\rho b d^2) = \mathrm{k}\,(2\rho b d^2 + t_f^2)$$

$$\therefore\ k \quad = \frac{\rho n + \frac{1}{2}\left(\frac{t_f}{d}\right)^2}{\rho n + \left(\frac{t_f}{d}\right)}$$

Determine location of C_f (flange compression force) from the top fiber (Figures 3-33 and 3-34).



Stress distribution

Figure 3-33

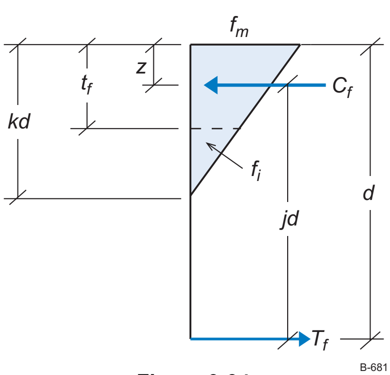


Figure 3-34

$$z \quad = \frac{3kd - 2t_f}{2kd - t_f}\left(\frac{t_f}{3}\right)$$

$$\therefore\ jd \quad = d - z$$

$$\Rightarrow \mathrm{j} \quad = \frac{6 - 6\left(\frac{t_f}{d}\right) + 2\left(\frac{t}{d_f}\right)^2 + \left(\frac{t_f}{d}\right)^3\left(\frac{1}{2pn}\right)}{6 - 3\left(\frac{t_f}{d}\right)}$$

$$\therefore\ M_s \quad = \text{Steel moment capacity} = A_sF_sjd$$

$$\therefore\ M_m \quad = \text{Masonry moment capacity} = f\left(l - \frac{t_f}{2kd}\right) = bt_f\,jd$$

Select M_{cap} = Lower of M_s or M_m

For design calculation,

$$M_s = A_sF_s\left(d - \frac{t_f}{2}\right)$$

$$M_m = \frac{1}{2}f_mbt_f\left(d - \frac{t_f}{2}\right)$$

3.3.5 Analysis of a double reinforced section

There are two elements of strength derived from a reinforced masonry wall: steel tensile strength and masonry compression capacity. One of these two elements will control the overall capacity. *The structure is only as strong as its weakest element.* Therefore, if the steel fails at a lower capacity, the overall wall strength will be governed by the steel reinforcement. If the masonry compression capacity controls the design, the masonry block will govern the final capacity.

In certain situations it is not practical to increase the block size or strength. This could be for a variety of reasons: architectural requirements may govern, access to the site may pose difficulties, construction implementation problems with several layers of block may be an issue, and cost factors are always a consideration. When it is not practical to increase the block size or strength, a double reinforced section (with two layers of steel) can significantly enhance a wall's structural capacity. The depth of the section is increased because both layers of steel lie closer to the face shell, which further increases the bending capacity of the wall. Compression reinforcement adds to the compression force of the masonry, which is further balanced by the tension reinforcement. The combined effect is a higher capacity wall with an efficient cross section. In summary, the following advantages are obtained.

1) Moment capacity is increased.
2) Higher lateral stiffness against deflection is increased, which results in a stiffer structure.
3) Double reinforcement is less expensive than increasing the block width. For example, constructing a wall with two 12-inch CMU sections costs

more than constructing the same wall with one 12-inch CMU using two layers of steel.

The primary disadvantage of a double reinforced section is the placement of the reinforcement. Placing two layer of steel into a confined cell is difficult, so it is advisable to check this design proposal with a qualified masonry contractor to assure that it is both practical and feasible.

Double reinforced section (Figures 3-35 and 3-36)

Analysis equations

M_m = masonry moment capacity

M_2 = additional capacity due to compression reinfrocement

$M_{tot} = M_m + M_2$

Based on balanced failure condition

$$M_m = \frac{1}{2} F_b j_b k_b b d^2$$

where:

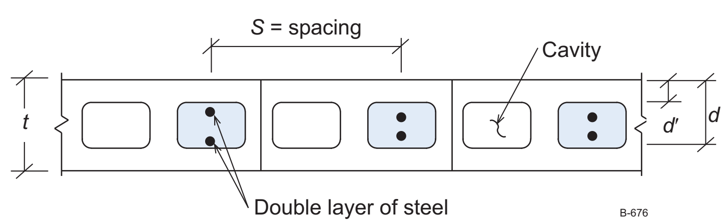


Figure 3-35

$$k_b = \frac{n}{n + F_s / F_b}$$

$$j_b = 1 - \frac{k_b}{3}$$

A_{sm} = tension steel to balance masonry compression force (C_m)

$$A_{sm} = \frac{M_m}{F_s j_b d}$$

A'_s = compression steel

A_{s2} = tension steel required to balance (C_s)

A_s = total tension steel = $A_{sm} + A_{s2}$

$\therefore\ A_{s2} = A_s - A_{sm}$

$$M_2 = A'_s f'_s (d - d')$$

or use the lesser value

$$M_2 = A_{s2} f_s (d - d')$$

$$M_{tot} = M_m + M_2$$

For additional compression steel

$$M_m = \frac{1}{2} F_b k_b j_b b d^2 = K_b b d^2$$

and

$$A_{sm} = \frac{M_m}{f_s j_b d}$$

$$M_2 = \text{steel moment capacity} = M_{\text{req'd}} - M_m$$

$$A_{s2} = \text{steel area rquired to develop } M_2$$

$$A_{s2} = \frac{M_2}{f_s (d - d')}$$

$$\therefore\ A_s = A_{sm} + A_{s2}$$

Calculate compression steel stress from the strain diagram

$$\frac{\varepsilon_s}{\varepsilon_s} = \frac{d - kd}{kd - d'} = \frac{f_s}{f'_s}$$

$$f'_s = f_s \left(\frac{k - \frac{d'}{d}}{1 - k} \right) \le f_s$$

Compression reinforcement

$$M_2 = A'_s f'_s (d - d')$$

$$\therefore\ A'_s = \frac{M_2}{f'_s (d - d') \left(\frac{n - 1}{n} \right)}$$

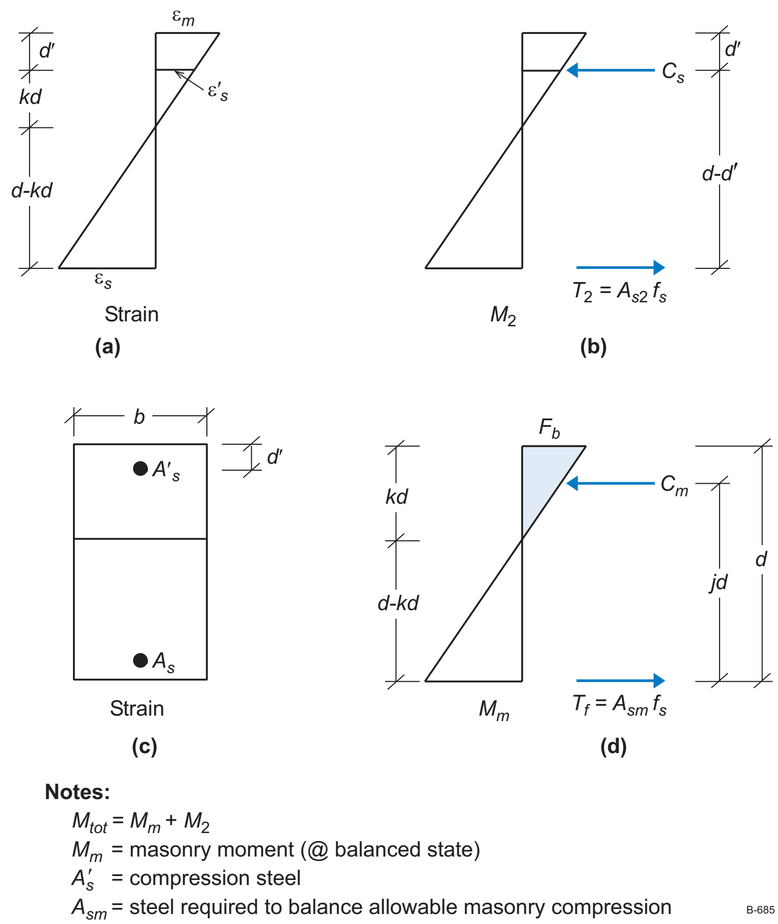


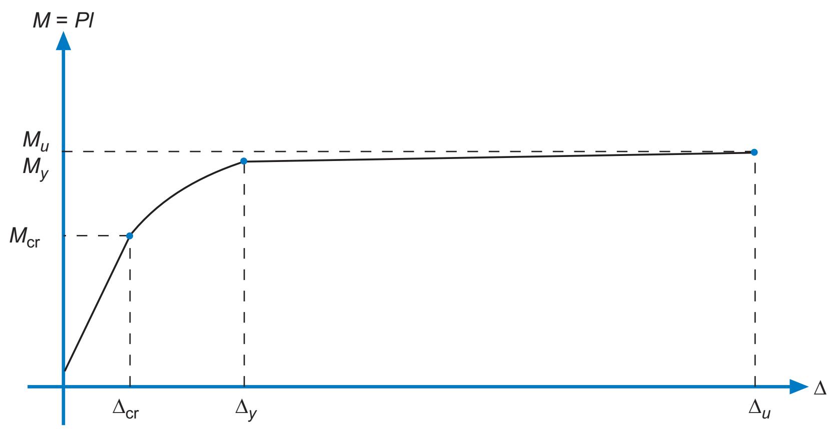
Figure 3-36

3.3.6 Analysis of deflection

1. *Purpose of Estimating Deflection*
   Structures will experience deflection or deformation under both normal and extreme load conditions. The design target is to prevent excessive deflection over the life span of a structure. Excessive deflection can result in external moments and forces beyond the original capacity of the structural member. For example, secondary bending moments can develop from *P*-delta effects that could cause a collapse. Long-term risk of failure or collapse is directly related to deflection and drift, so these must be controlled.

2. *Engineering Assumptions*
   Several assumptions are necessary to simplify the analysis in order to determine the maximum deflection. Figure 3-37 shows the nonlinear moment versus deflection curve for a simple cantilevered wall with an out-of-plane load, *P*. This wall is shown in Figure 3-38. The elastic behavior stops at M_y; the wall then deflects in an inelastic manner with reduced stiffness, *EI*. The reduced stiffness is also termed *stiffness degradation* and is the combined

result of steel yielding along with cracking of the masonry block. Therefore, the gross stiffness (*EI* gross) must be reduced to account for these effects. It is common practice to divide the gross stiffness by 2 and term the result *the effective stiffness*: $EI_{eff} = (EI)$ gross/2.



Notes:

M_{cr} = cracking moment
Δ_{cr} = deflection at M_{cr}
M_y = yield moment capacity
Δ_y = deflection at M_y
M_u = ultimate moment capacity
Δ_u = deflection at M_u
μ_u = displacement ductility = $\dfrac{\Delta_u}{\Delta_y}$

Figure 3-37

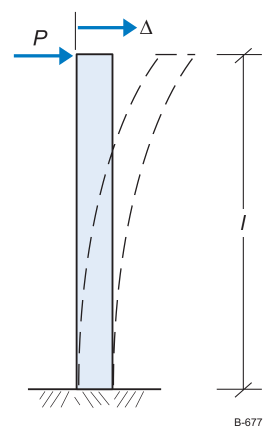


Figure 3-38

The deflection curve defines the yield deflection at the yield moment. At this point the structure behaves in an inelastic manner and moves to an ultimate deflection that is the point of failure. The ratio of ultimate deflection to yield deflection is the displacement ductility. Displacement ductility provides an approximate factor of safety on the structural performance of the member. Using displacement ductility values between 3 and 5 for reinforced masonry members is generally accepted.

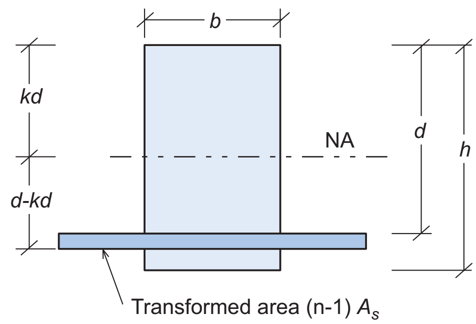
3. *Stiffness (EI)*
Stiffness is the product of the modulus of elasticity and the moment of inertia, representing the structural capacity of the member. As this value decreases because of steel yielding and masonry cracking, the structure undergoes stiffness degradation that results in further displacement (Figures 3-39 and 3-40).

4. *Practical Considerations*
Displacement and drift calculations are approximate at best, so the standard structural engineering philosophy is to implement a conservative approach to displacement control. It is better to control deflection at small values than to allow for large displacements that could result in potential structural failure. For this reason, the codes are conservative in their allowances for deflection, especially concerning mid- to high-rise buildings. Following the codes for

these buildings results in a *displacement controlled design* that governs the final stiffness calculation and member selection.

For masonry shear wall structures, displacement control is not an issue about in-plane forces. Shear walls possess a high degree of stiffness along the plane but are weak in the out-of-plane direction. In all building plans, other shear walls in the perpendicular direction restrain the out-of-plane force versus deflection curve. In theory, this prevents lateral displacement in the out-of-plane direction. From a practical perspective, the actual floor plan must have a balanced design to allow a reasonable distribution of forces without excessive distance between shear walls.

Beer and Johnston's *Mechanics of Material* contains useful deflection tables.



Uncracked section

Figure 3-39

Section properties

Unlocked section

Locate centroid

$\Sigma M_{NA} = 0 \circlearrowleft +$

$$b(kd)\frac{(kd)}{2} = (n-1)A_s(d-kd) + b(h-kd)\left(\frac{h-kd}{2}\right)$$

Solve for *kd*

$$\frac{b(kd)^2}{2} = (n-1)A_s d - (n-1)A_s kd + \frac{b}{2}\underbrace{(h-kd)^2}_{h^2 - 2hkd + (kd)^2}$$

$$(kd)^2\underbrace{\left[\frac{b}{2} - \frac{b}{2}\right]}_{\text{zero}} + kd[-(n-1)A_s + bh] = (n-1)A_s d + \frac{bh^2}{2}$$

$$kd = \frac{\frac{bh^2}{2} + (n-1)A_s d}{bh - (n-1)A_s}$$

Uncracked section properties

I_g = gross moment of inertia

S_g = gross section modules

$$I_g = \frac{bh^3}{12}$$

$$S_g = \frac{I_g}{C} = \frac{bh^2}{6}$$

$$M_{cr} = S_g f_r$$

f_r = modulus of rupture

Cracked section properties

$$b(kd)\left(\frac{kd}{2}\right) = nA_s(d - kd)$$

$$A_s = \rho bd$$

$$\frac{b(kd)^2}{2} = \rho nbd(d - kd)$$

$$k = \sqrt{2\rho n + (\rho n)^2} - \rho n$$

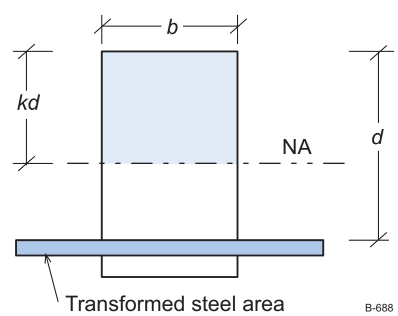


Figure 3-40

Recall,

I_{cr} = cracked moment of inertia

$$\therefore\ I_{cr} = \frac{b(kd)^3}{3} + \rho nbd\,(d - kd)^2$$

$$\therefore\ S_m = \frac{I_{cr}}{kd} \qquad \therefore\ S_{ST} = \frac{I_{cr}}{d - kd}$$

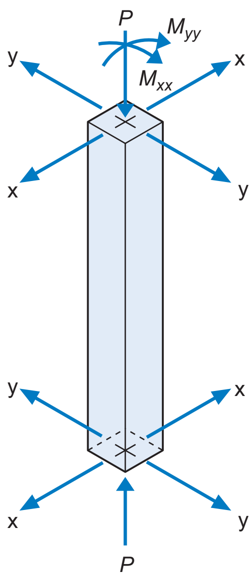
$$\therefore\ f_m = \frac{M}{S_m} \qquad \therefore\ f_{ST} = \frac{M}{S_{ST}}$$

3.4 Axial Compression and Buckling

3.4.1 Column analysis

Column Definition: Figure 3-41 shows the loads, forces, and moments on a typical column.

Axial compression is directed along the axis of the column, with biaxial moments about the x and y axes.



Axial load: the vertical axial force (P) is directed along the axis of the column.
M_{xx} = bending moment about the x axis
M_{yy} = bending moment about the y axis
B-767

Figure 3-41

3.4.2 Structural failure modes

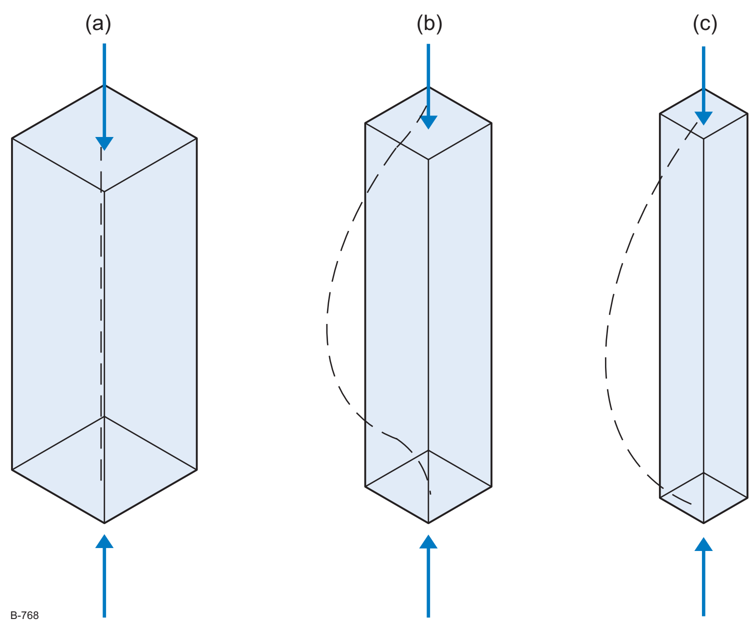


Figure 3-42

Short Columns
Figure 3-42(a) diagrams a column that experiences failure because of compression stress. A short column fails because of plastic stress along the column axis without a buckling possibility.

Intermediate columns
Figure 3-42(b) diagrams the combination failure mode of buckling and compression crushing.

Long Columns
Figure 3-42(c) shows a column with the primary failure mode in buckling. Buckling failure occurs at a critical axial load at which the column experiences instability and fails elastically because of secondary moments generated by the axial load.

Elastic Failure (Buckling)
This failure is due to instability caused by secondary moments generated from the axial load.

Plastic Failure (Inelastic Yielding)
Compression/crushing failure of the column is the result of inelastic yielding of the column.

3.4.3. Euler formula for pin-ended columns

Column Loading and Assumptions
The axial load on column creates a buckled mode shape as shown in Figure 3-43. Secondary moments generated from the axial load are multiplied by the lateral deflection, *y*, to create a moment about the column. The free body diagram (Figure 3-44) shows the forces on the column at section and depicts the buckled mode shape.

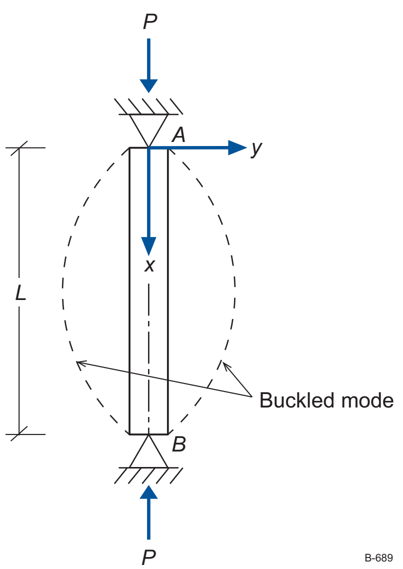


Figure 3-43

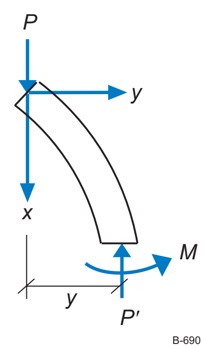


Figure 3-44

Bernoulli-Euler beam equation

This is a linear homogeneous differential equation with constant coefficients.

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

From free body diagrams

$$M = -Py$$

$$\frac{d^2y}{dx^2} = \frac{M}{EI} = \frac{-Py}{EI}$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = 0$$

The general solution is

$$y = A \sin \omega x + B \cos \omega x$$

with

$$\omega^2 = \frac{P}{EI}$$

$$\frac{d^2y}{dx^2} + \omega^2 y = 0$$

Boundary conditions

at $x = 0$, $y = 0$ and $y = 0$ at $x = \mathrm{L}$

Substitute boundary conditions

$A \sin \omega L = 0$

$\Rightarrow \mathrm{A} = 0$ . . . Case 1: straight column

or

$\sin \omega L = 0$ . . . Case 2: buckled column

$$\omega L = n\pi$$

$$\therefore \omega^2 = \frac{n^2\pi^2}{L^2}$$

$$\therefore \frac{P}{EI} = \frac{n^2\pi^2}{L^2} \Rightarrow P = \frac{n^2\pi^2 EI}{L^2}$$

Lowest mode shape is with $n = l$

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$f_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EAr^2}{AL^2}$$

where $\mathrm{r} = \sqrt{\frac{I}{A}}$ = radius of gyration

$$f_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{\left(\frac{L}{r}\right)^2}$$

For general column analysis (Figure 3-45)

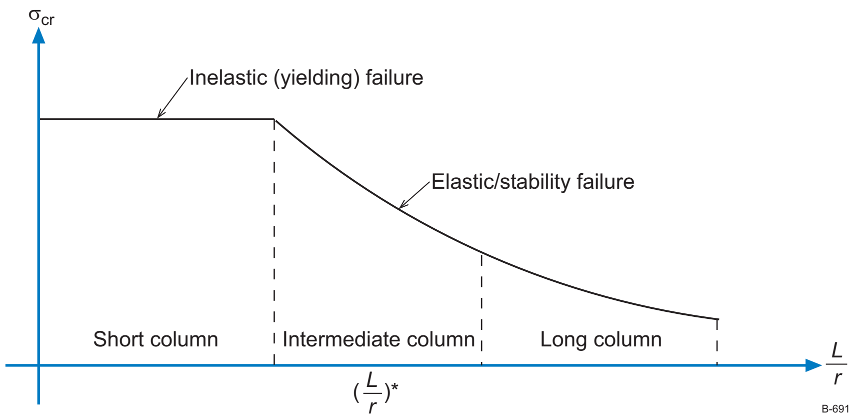


Figure 3-45

$\frac{L}{r}$ = slenderness ratio

$\left(\frac{L}{r}\right)^*$ = critical slenderness ratio defining the difference between a short and long column

3.4.4 Euler column formula for variation on end conditions

To correctly interpret end conditions and stiffness requirements at the support points, the effective length factor, K, takes into account the buckled mode shape with end restraints. The American Institute of Steel Construction (AISC) has tabulated K for various column-end conditions and axial loads in *Manual of Steel Constrction: Allowable Stress Design*. These K tables give the results of testing and analysis, enabling ready application by the structural engineer.

3.4.5. Practical/Field considerations

Field conditions present challenges to the structural engineer. Figure 3-46 shows a typical support condition of the glue laminated timber (glulam) beam over a 12-inch-square masonry pilaster that is anchored with reinforcing steel into a square footing.

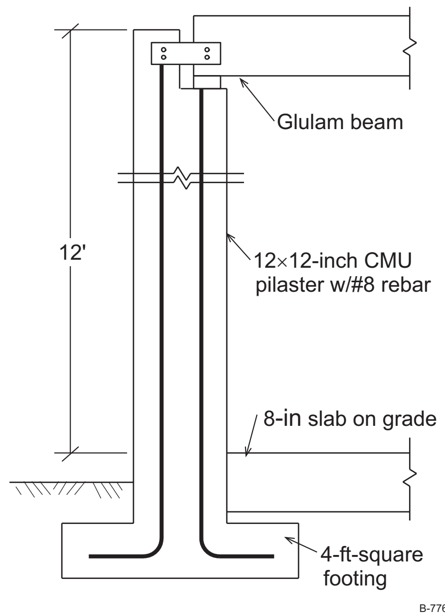


Figure 3-46

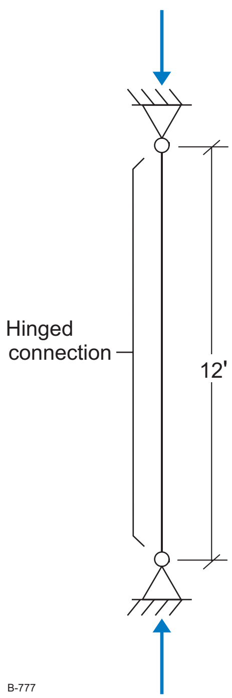


Figure 3-47

By examining both the end rotation potential of the glulam beam-to-pilaster connection and the footing detail, one may conclude that the hinge connection is the most conservative assumption. The hinged connection used in a strut-analysis formulation is shown in Figure 3-47. Although the reinforcement detail does provide some rotational resistance, the conservative assumption is that the rotational stiffness is zero. This approach allows for a simplified evaluation of the column as a hinged connection in order to estimate the allowable buckling load.

Engineering Analysis Assumptions:

End Rotation
The end rotation is taken as a hinged connection point for the purpose of this derivation. In real-world construction this will not always be the case, but the end-hinged connection allows for a conservative estimate of the allowable buckling load.

Fixed End vs Partial Fixity
Certain column connections will have moment-resisting capacity, but could be considered to have partial fixity, which indicates an end rotation-restrained connection that has rotational stiffness. The *Allowable Stress Design Manual of Steel Construction* contains specific tables covering this type of situation. For the purpose of masonry construction, use a simplified approach and assume either full fixity or zero fixity (hinged).

Field Interpretation
The structural engineer must evaluate the field conditions and determine whether the end fixity should be either fully restrained or hinged. This decision-making ability is developed through experience and requires a comprehensive understanding of the field connection detail in order to assess the stiffness quality. The most efficacious approach is to assume a hinge type connection and work with the zero stiffness assumption.

3.4.6 Secant loading: secant formula and *P*-delta effects

Eccentric loading conditions occur in the field on a regular basis; they are formulated with an assumed eccentricity value of *e* that is offset from the axis of the column (Figure 3-48). Eccentric loading: secant formula and *P*-delta effects are pictured.

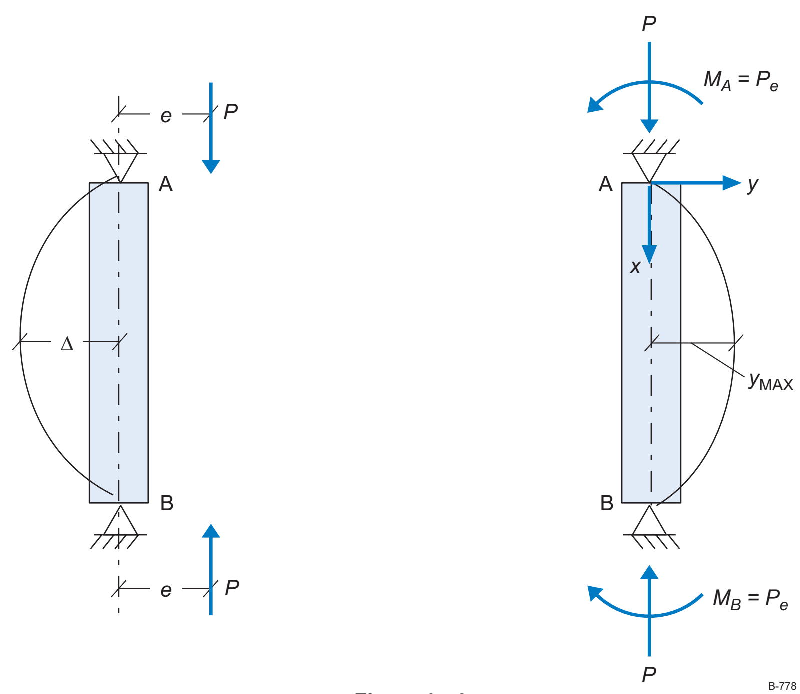


Figure 3-48

This creates a secondary moment about the center axis of the column. A derivation of this evaluation considers the eccentric loading case using the secant formula with *P*-delta effects.

Eccentric Loading
The cause of eccentric loading is linked to material deficiencies, construction problems, and inherent design eccentricity. Every column has a built-in eccentricity that will lead to secondary moments about the center column axis.

P-delta Effect

The P-delta (P-Δ) effect occurs during actual buckling and results in secondary bending moments about the center axis of the column. The product of the axial load and the lateral deflection of the column causes the secondary moments.

Example

The glulam beam-to-pilaster column connection is a typical example of eccentric loading on the column/masonry shear wall. This connection contains a built-in eccentricity that results in bending moments about the masonry pilaster/shear wall. In addition to the built-in load eccentricity, secondary P-delta moments occur about the shear wall. Figures 3-49 and 3-50 illustrate this concept.

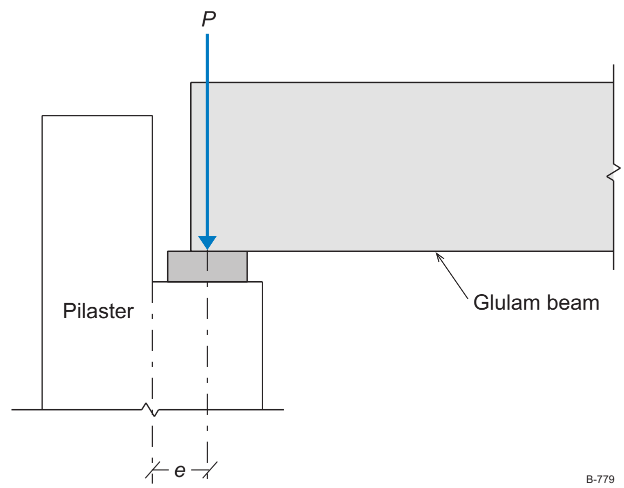


Figure 3-49

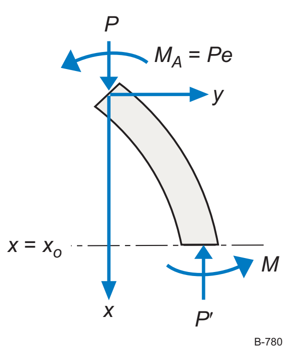


Figure 3-50

Free body diagram

$$\circlearrowleft + \Sigma M_{X=0} = 0$$

$$M = -Py - M_A = -Py - Pe$$

From Bernoulli Euler beam equation

$$\frac{d^2y}{dx^2} = \frac{M}{EI} = -\frac{P}{EI}y - \frac{P}{EI}e$$

Substitute $\omega^2 = \dfrac{P}{EI}$

$$\frac{d^2y}{dx^2} + \omega^2 y = -\omega^2 e$$

General solution:

$$y = A \sin \omega x + B \cos \omega x - e$$

Solve with boundary conditions

(1) At $x = 0, y = 0 \Rightarrow B = e$

(2) At $x = \mathrm{L}, y = 0 \Rightarrow A \sin \omega L = e\,(1 - \cos \omega L)$

From trigonometry identity

$$\sin \omega L = 2 \sin\frac{\omega L}{2} \cos\frac{\omega L}{2}$$

$$1 - \cos \omega L = 2 \sin^2\frac{\omega L}{2}$$

Substitute into boundary condition (2)

$$2 A \sin\frac{\omega L}{2} \cos\frac{\omega L}{2} = 2e \sin^2\frac{\omega L}{2}$$

$$\therefore \; A = e \tan\frac{\omega L}{2}$$

Substitute for A and B into the solution

$$y = e(\tan\frac{\omega L}{2} \sin \omega x + \cos \omega x - 1)$$

To calculate maximum deflection, set $x = \frac{L}{2}$ (i.e., column mid-height)

$$\therefore \; y_{\max} = e\left(\tan\frac{\omega L}{2} \sin\frac{\omega L}{2} + \cos\frac{\omega L}{2} - 1\right)$$

$$\therefore \; y_{\max} = e\left\{\frac{5\ \text{in}^2\frac{\omega L}{2} + \cos_2\frac{\omega L}{2}}{\cos\frac{\omega L}{2}} - 1\right\}$$

$$\therefore \; y_{\max} = e\left(\sec\frac{\omega L}{2} - 1\right)$$

$$\Rightarrow \; y_{\max} = e\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - 1\right]$$

To calculate maximum stress, $\sigma_{\max}$, first determine the location of $M_{\max}$

$$M_{\max} = P y_{\max} + M_A = P(y_{\max} + e)$$

Therefore, the maximum stress represented in Figure 3-51 is

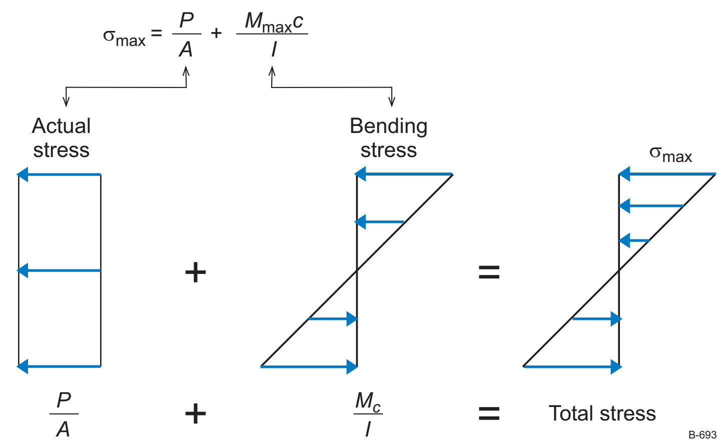


Figure 3-51

Use $I = A_r^2$

$$\therefore\ \sigma_{\max} = \frac{P}{A}\left[1 + \frac{(y_{\max} + e)C}{r^2}\right]$$

Substitute for $y_{\max}$

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\right]$$

or (in terms of Euler load)

$$\sigma_{\max} = \frac{P}{A}\left(1 + \frac{ec}{r^2}\sec\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)$$

This combined stress formulation accounts for two distinct secondary moments:

(1) Eccentric load = $(Pe) = M_A$

where

P = axial load

e = eccentricity due to construction and/or design

(2) Secondary moment due to stability failure affect = $P\Delta$

where

P = axial load

Δ = lateral defection at the column mid height

(3) The practical appliction must consider both the eccentric load and $P\Delta$ effect for a combined axial load stability calculation

$$\frac{P}{A} = \frac{\sigma_{max}}{1 + \frac{ec}{r^2}\sec\left(\frac{1}{2}\sqrt{\frac{P}{EA}}\frac{Le}{r}\right)}$$

3.4.7 Combined axial and flexural stress

In actual design practice, axial loads cannot be separated from flexural loads into individual cases; these load types usually occur in combined frameworks to affect the structure with both axial and bending loads simultaneously. This combination is referred to as *combined axial and flexural stress* and is depicted in Figure 3-52.

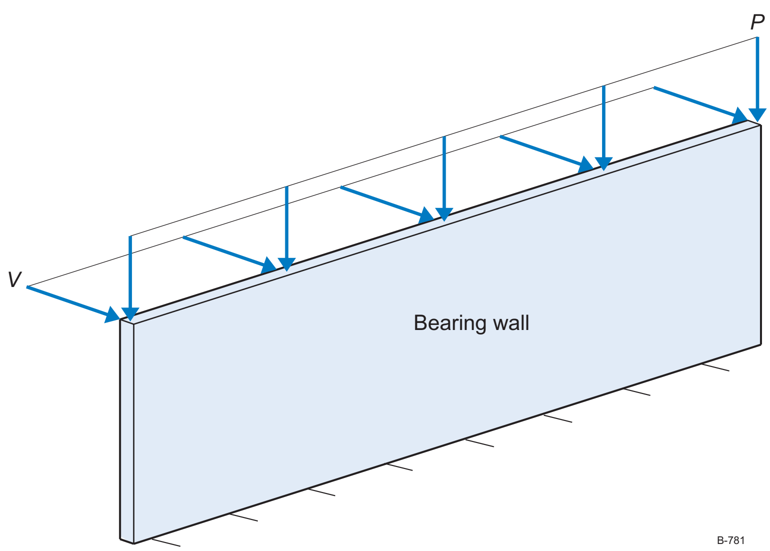


Figure 3-52

Combined Stresses
Axial and flexural stresses result from in-plane axial forces occurring simultaneously with out-of-plane bending/shear forces. These combined stresses must be accounted for in the engineering analysis process in order to correctly estimate the structural safety of the wall or column.

Engineering Considerations
Bearing walls always have combined stresses and must be evaluated with this in mind. Buckling capacity must be considered along with bending stress. Lateral deflection in order to limit drift values to those prescribed by the code.

Analysis Assumptions
The vertical axial load capacity is evaluated using the Euler buckling load capacity equation, and lateral bending capacity is evaluated using the same analysis equations presented for single reinforced sections. The two evaluations are performed independently and then combined into a standard interaction formula. The result is a conservative evaluation of the total combined stress capacity.

In-plane Loads
The vertical axial loads, P, are the in-plane forces that act along the vertical axis of the wall. These distributed forces create a vertical compression on the bearing wall. Buckling, secondary moments, and eccentric loading must be included in the analysis of these loads.

Interaction Formula
This formula follows the same premise as the AISC steel formula and the reinforced concrete column formula. It assumes that each force occurs independent of the other, but the combined demand-to-capacity ratio must be less than unity. Figure 3-53 shows this graphically.

The interaction formula is based on fundamental principles.

$$\frac{f_a}{F_a} \leq 1 \text{ and } \frac{f_b}{F_b} \leq 1$$

$$\therefore \ \frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1$$

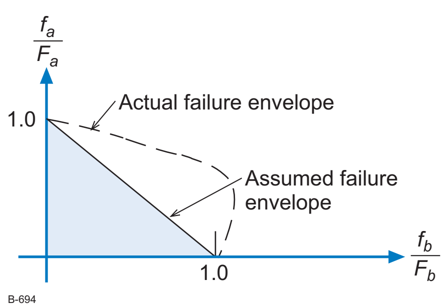


Figure 3-53

By using 1, the equation is conservative.

In terms of loads and moments

$$\frac{P}{P_{\text{all}}} + \frac{M}{M_{\text{all}}} \leq 1$$

where:

P = imposed axial load

P_{all} = allowable axial load

M = imposed moment

M_{all} = allowable moment capacity (based on analysis)

Figure 3-54 illustrates combined bending and direct stress.

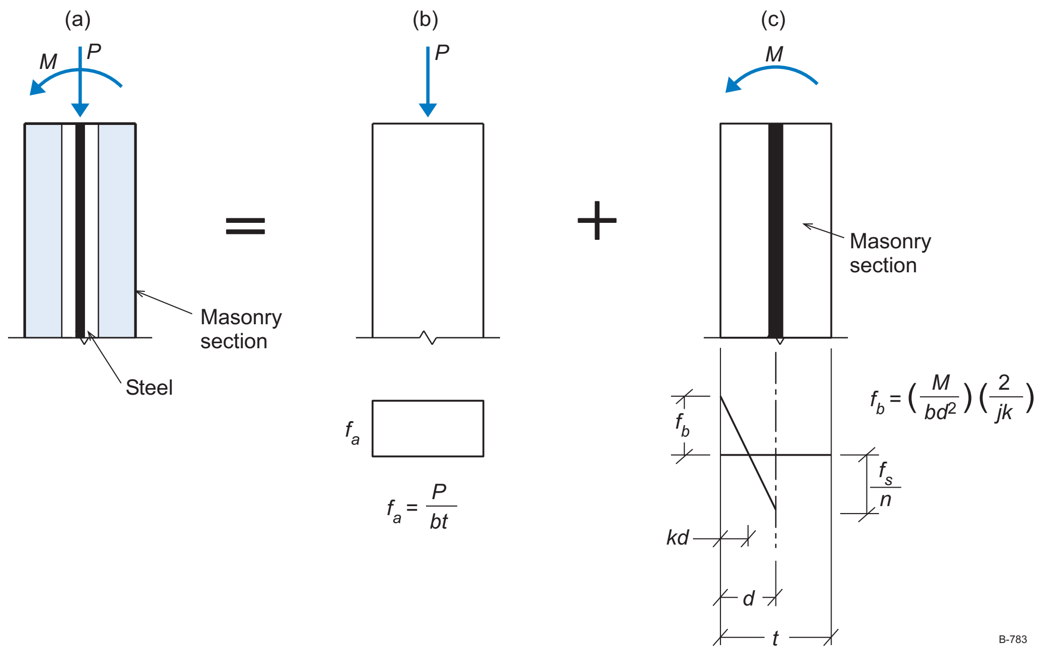


Figure 3-54

Interaction formula

$$f_m = \frac{P}{A} + \frac{M_c}{I} = \frac{P}{A} + \frac{M}{S}$$ = maximum compression in masonry; considering combined stream condition

Divide by f_m

$$1 = \frac{P}{Af_m} + \frac{M}{Sf_m}$$

$$\text{or} \Rightarrow \frac{f_a}{f_m} + \frac{M/S}{f_m} = \frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0$$

3.5 Practical Evaluation of Buildings

The structural capacity of masonry buildings has an excellent record. There are few failures resulting from actual overload during seismic or wind events. Following are the typical failure patterns of masonry walls.

Structural cracking of the masonry bed and mortar joints
This is the first sign that a wall may be compromised. Cracking of the mortar joints does not necessarily imply a failed wall, but it can indicate structural problems with that wall.

Drift problems
Any structural element that has exceeded allowable drift values should be investigated further for possible structural defects. Masonry walls are rigid elements with high stiffness and limited lateral deflection. Therefore, any drift beyond the limits can cause structural yielding or failure of the wall and should be investigated.

Diagonal shear cracks
These indicate shear stress failure of the masonry wall and could suggest lateral movement in the plane of the wall.

3.6 Summary

The working stress design (WSD) formula represents the prevailing methodology for analysis of the structure of masonry buildings and components. This design philosophy postulates that structures should not yield or behave plastically. Factors of safety have been introduced over the years which, when followed, allow the elastic behavior of masonry buildings without reinforcement yielding. Formulas and analysis assumptions exist for single reinforced, double reinforced, and T-beam analysis equations.

In-plane shear stress results from in-plane shear forces on masonry walls. The longitudinal in-plane forces are evaluated using the equivalent solid thickness of the wall, and the allowable shear stresses are provided in the 1997 UBC and the 2000 IBC. In addition to bending stresses, out-of-plane forces also create shear forces in walls. Procedures exist for analyzing each of these forces. Walls are subject to out-of-plane forces that create bending moments and shear stresses. Out-of-plane forces also cause lateral drift and deflection. The criteria of bending stress, shear, and deflection must all be evaluated to determine the structural integrity of both partially grouted and fully grouted bearing walls.

Axial compression creates buckling forces on walls and columns. Built-in eccentricity along with the lateral deflection of the column creates secondary moments in the wall or column. Analysis equations exist for walls and columns to account for load eccentricity and secondary moments created by P-delta effects. Secondary moments are evaluated using the secant formula. K values are evaluated with end rotation and fixity considerations.

Assignments

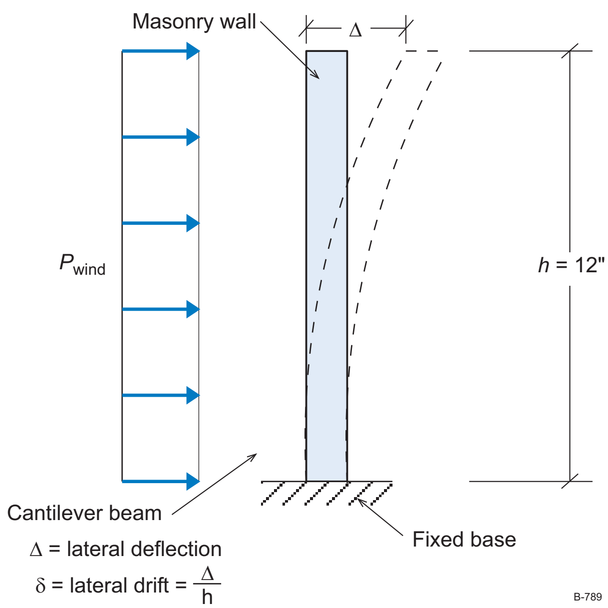
1. A masonry wall is subject to a lateral out-of-plane load based on the design wind speed of 100 mph. Given the criteria noted, complete the following items.

   a) Calculate and draw the shear and moment diagrams for the wall.

   b) Calculate the lateral out-of-plane deflection with an assumed EI_{gross} value for the stiffness of the wall.

   c) Perform (b) with half the gross stiffness, $(EI)_{eff} = \frac{1}{2}(EI)_{gross}$.

   d) Calculate the drift limits of the wall using the criteria from the 2000 IBC.



2. The retaining wall shows the lateral load on a 12-inch CMU wall at the base. Given the criteria shown, evaluate the following items.

   a) Calculate and draw the shear and moment diagrams for the wall.

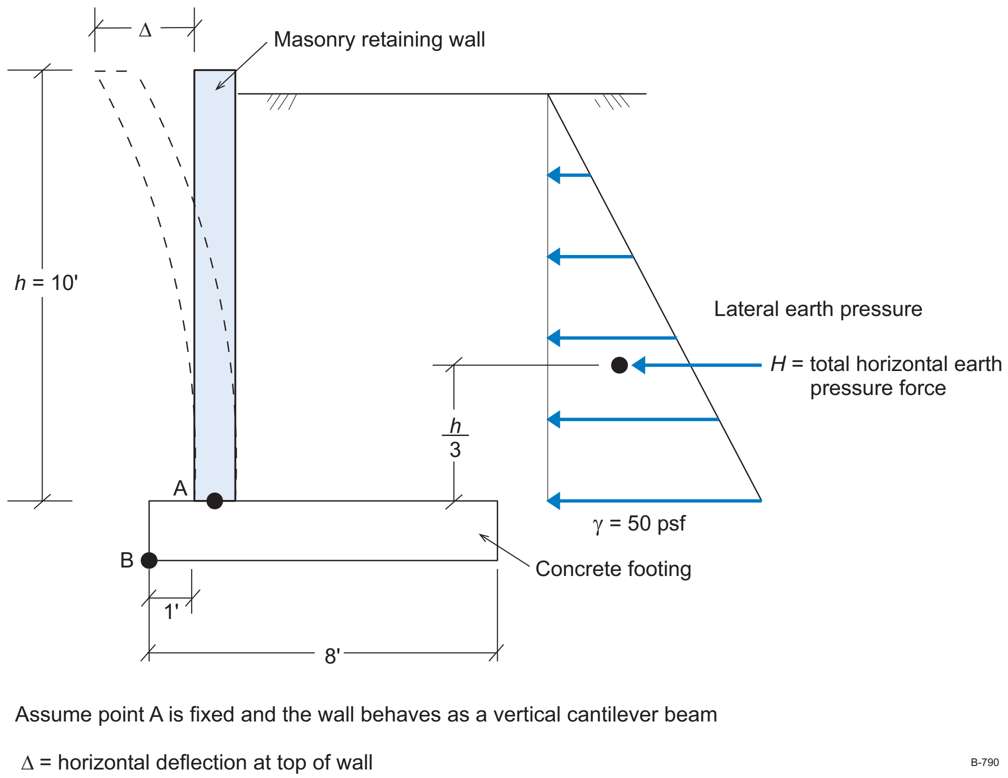
   b) Calculate the lateral out-of-plane deflection with an assumed EI_{gross} value for the stiffness of the wall, and with half the gross stiffness, $(EI)_{eff} = \frac{1}{2}(EI)_{gross}$.

   c) Evaluate the overturning moments (OTMs) at points A and B.

   d) Calculate the in-plane shear at point A.

   e) Calculate the in-plane shear at point B.

   f) Calculate the resisting moments at A and B. Determine whether this wall is structurally safe or should be modified further.

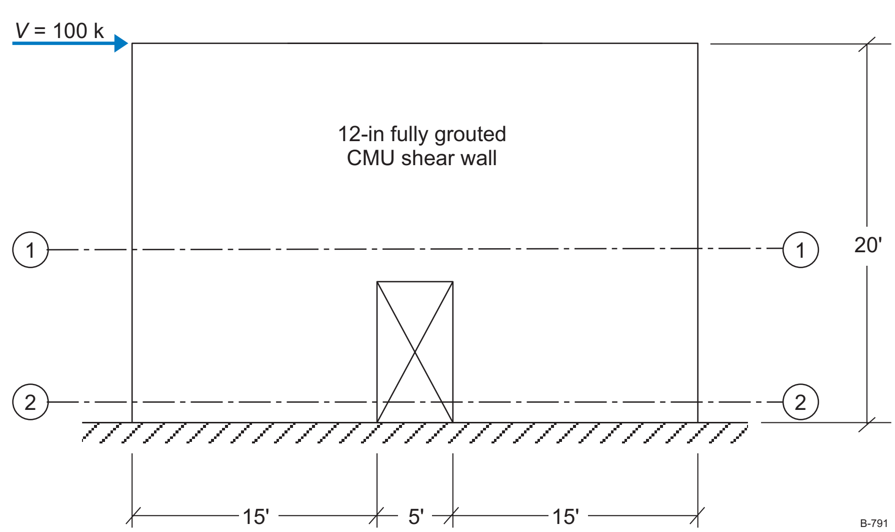


3. The shear wall shown has one opening. For the criteria shown, evaluate the following items.

   a) Determine the shear stress along line 1.

   b) Determine the shear stress along line 2.

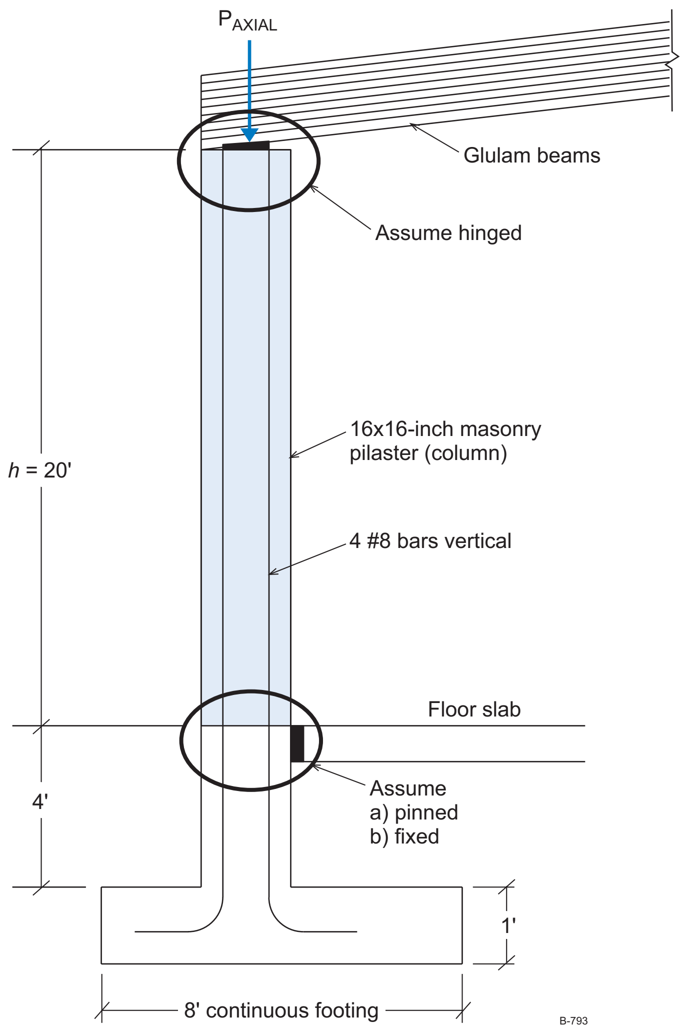
   c) Calculate the allowable shear stress using the 2000 IBC and evaluate along Lines 1 and 2.



4. The column shown supports a roof diaphragm comprising glulam beams. A schematic diagram of the footing is also shown.

   a) Calculate the axial load capacity of the column with hinged end conditions using the 2000 IBC criteria.

   b) Evaluate the eccentric load stress using the secant formula.



4

Shear Wall Buildings with Rigid Diaphragms

4.1 Introduction

In the United States, there are three categories of reinforced masonry shear wall buildings, each distinguished by its structural diaphragm. Although there are numerous combinations of reinforced masonry with steel moment-frame systems and even reinforced concrete shear walls, the structural characteristic that distinguishes these buildings is the performance of the diaphragm. There are three possibilities:

1. Reinforced masonry buildings with flexible (wood frame) diaphragms

These buildings comprise a large percentage of light industrial facilities. Automobile repair buildings, industrial and storage facilities, shopping malls, and office buildings under three stories are included in this classification. The concept is illustrated schematically in Figure 4-1.

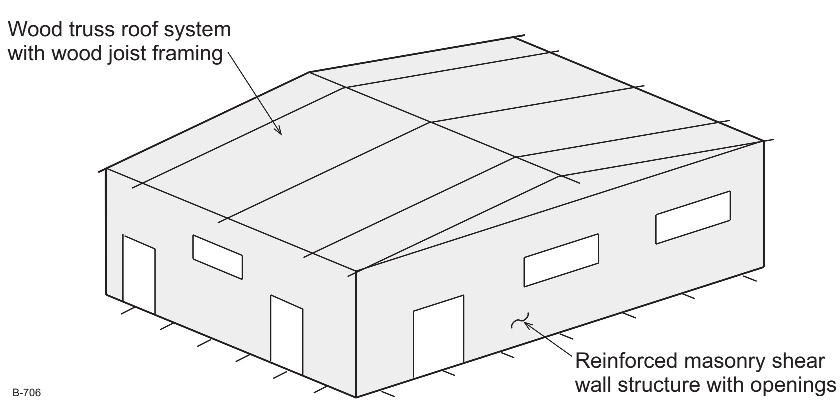


Figure 4-1

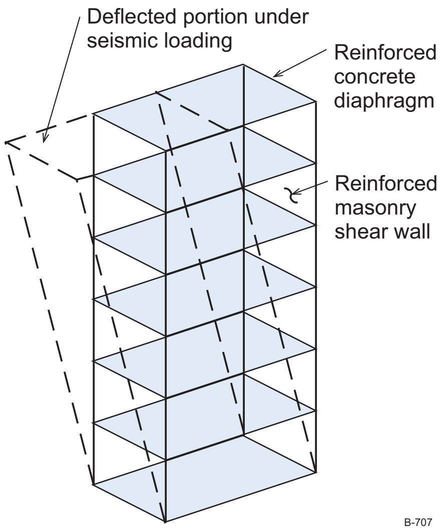


Figure 4-2

2. Reinforced masonry buildings with rigid (reinforced concrete) diaphragms

Traditionally, these buildings are used in the mid- to high-rise residential market and are found predominantly in high-density population areas throughout the world. They have perimeter shear walls with reinforced masonry construction and a rigid reinforced concrete diaphragm. In the United States, such buildings accommodate multistory residential needs on tight land requirements. This classification of buildings has excellent fire resistance and has demonstrated superior performance during seismic events. Although these structures may have declined in popularity in some parts of the United States where the population in suburban communities prefers tract housing to high-rise residential living, they remain the mainstay of development in other geographical areas. This type of construction is also popular throughout much of Asia (India, China, Malaysia, Japan, Singapore), Central and South America, and much of the Middle East. The concept is illustrated in Figure 4-2.

3. Reinforced masonry buildings with semi-rigid diaphragms (Figure 4-3)

These buildings use a steel joist system to support a lightweight concrete floor/deck. Common occupancy types include industrial, office, and manufacturing facilities.

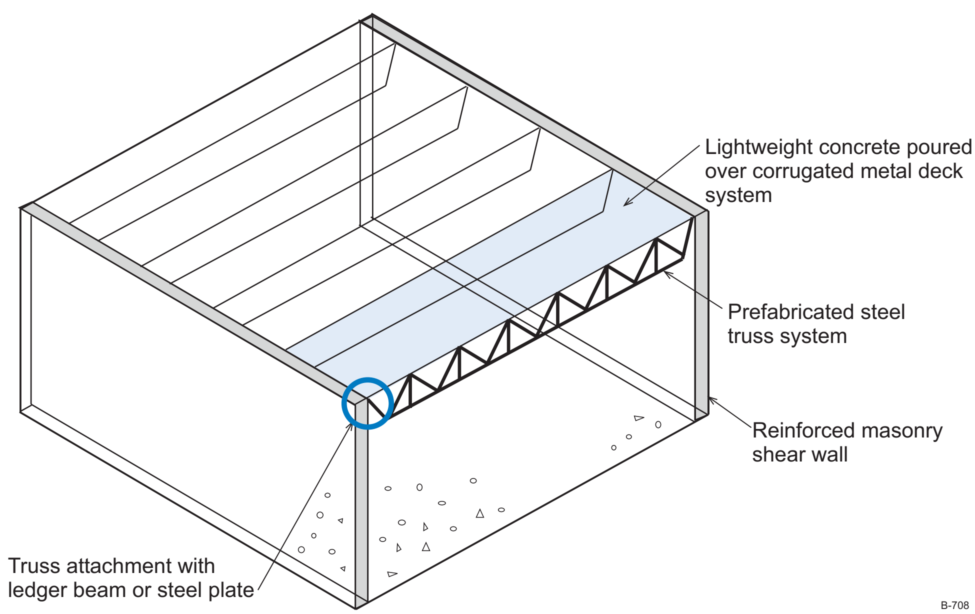


Figure 4-3

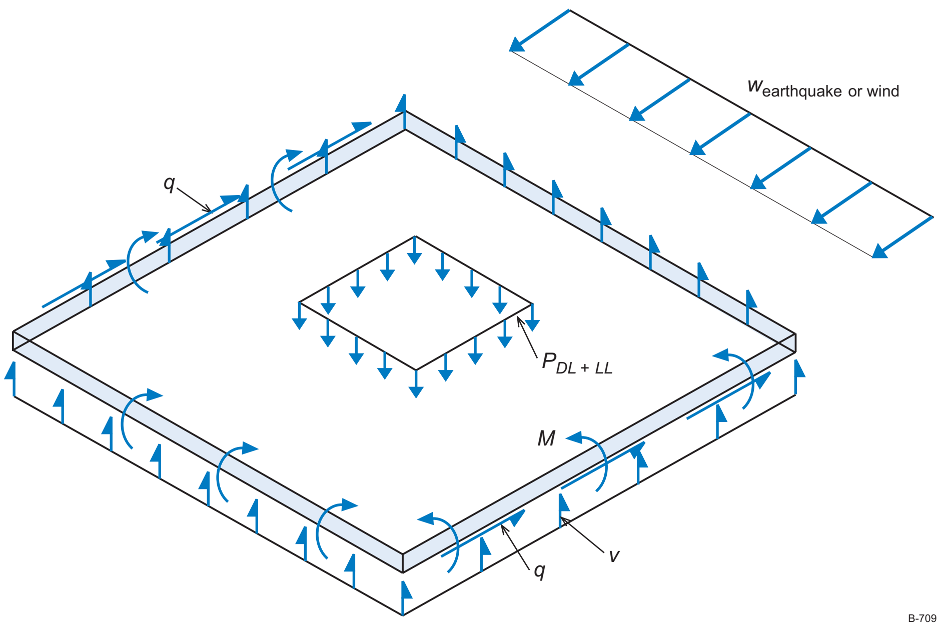
4.2 Diaphragm Behavior

The horizontal diaphragm is the structural element responsible for two important functions:

a) It transfers vertical (out-of-plane) loads to vertical load-carrying elements such as columns or shear walls.

b) It transfers lateral (in-plane) loads to the respective lateral-force-resisting system (LFRS), which may consist of shear walls, moment frames, or vertical truss systems.

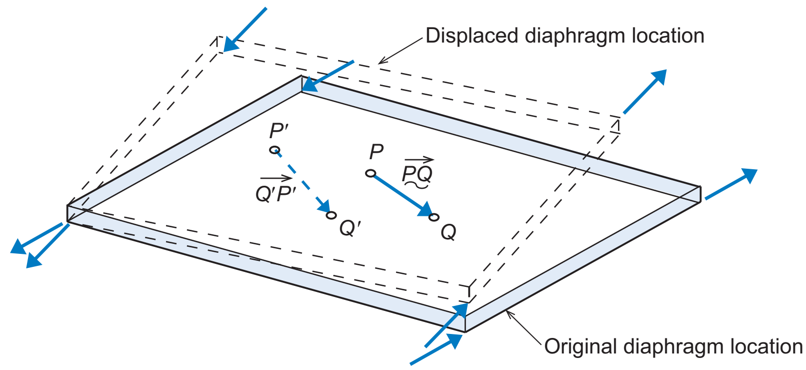
Figure 4-4 is a three-dimensional sketch of a horizontal diaphragm with relevant free body forces along its boundary. The diaphragm is essentially a plate structure serving to resist both out-of-plane vertical loads and in-plane lateral/shear forces. Diaphragm behavior is characterized as 1) flexible, 2) rigid, or 3) semi-rigid. This characterization is based on in-plane deformation behavior.



w	= lateral load imposed by earthquake or wind forces.
q	= lateral shear transfer at the edge of the diaphragm. This edge force is transferred to the perimeter shear wall or moment frame system.
P_{DL+LL}	= vertical $DL + LL$ applied to the diaphragm. This would comprise the people, equipment, structural dead load, and associated surcharge load.
v	= perimeter shear force carrying the vertical load (p) to the shear wall or moment frame system.
M	= edge resisting moments. These would occur in a fixed-end connection with a shear wall system, and would resist the vertical load (P) and the lateral load (W).

Figure 4-4

A rigid diaphragm (Figure 4-5) will not deflect in the force plane. No structure has infinite stiffness/rigidity, but from a practical standpoint the relative displacement is so small that it is considered rigid. In practical engineering analysis, this assumption is effective for structural analysis purposes. Note that the rigid diaphragm behavior applies only to in-plane characteristics and not to out-of-plane (vertical) deflection.



$\underset{\sim}{PQ}$ = vector between points P and Q

$\underset{\sim}{P'Q'}$ = vector between points P' and Q'

$|\overrightarrow{PQ}|$ = $|\overrightarrow{P'Q'}|$ = distance between any two points, P and Q, never changes within the rigid diaphragm

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Figure 4-5

A flexible diaphragm (Figure 4-6) will deflect under normal in-plane shear forces. Flexible diaphragms comprise mostly wood frame joist construction with plywood sheathing. They deflect as a simple beam loaded in a horizontal plane and consequently distribute shear forces on a tributary area basis. (The concept of tributary area loading was introduced in Chapter 2.) Recent earthquakes in California have brought to the fore the question of whether wood frame diaphragms are truly flexible. Although this concept is under dispute, the practical analysis method of tributary area loading remains the professional standard, and is used in this text. A few situations where the rigid diaphragm concept may apply to wood frame structures are discussed.

w = in-plane lateral load

q = diaphragm shear = $\dfrac{wl}{2b}$

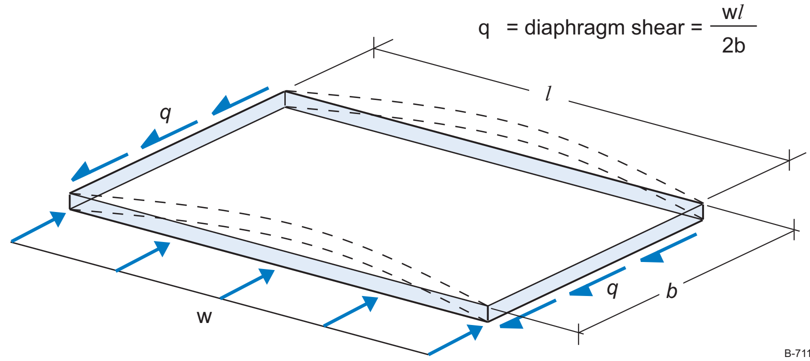
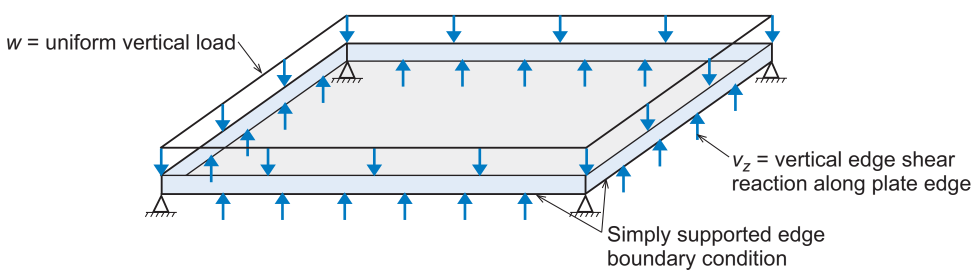


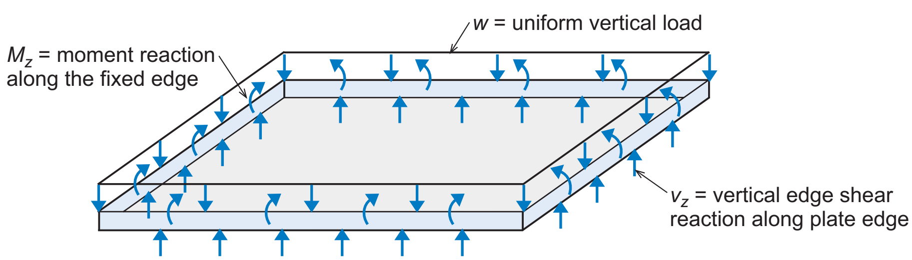
Figure 4-6

A semi-rigid diaphragm employs both a rigid and a flexible diaphragm. In this combination the in-plane stiffness is difficult to assess quantitatively between purely rigid versus purely flexible. In most practical situations, the structural engineer will define a diaphragm as either rigid or flexible and then continue with the analysis. A semi-rigid diaphragm would require a structural analysis using semi-rigid plate elements with a lateral stiffness value for the in-plane force deflection behavior to model the diaphragm. This effort requires further evaluation of the diaphragm to establish its in-plane stiffness. Unfortunately for structural engineering consulting firms, it is not economical to develop true diaphragm stiffness by structural testing and finite element analysis. It is, of course, possible to perform such a level of detailed testing and analysis for structures that warrant such expense and effort. In a majority of practical design circumstances, the problem is resolved by making the conservative assumption of a rigid diaphragm. It is rare to find an actual semi-rigid diaphragm analysis because of the complexity.

Although diaphragms are assumed to be either rigid or flexible, understanding the fundamentals of elastic plate theory helps to comprehend the technical characteristics of these structural elements. Texts on elasticity and plate theory by Timoshenko, *Theory of Elasticity*, 1934, and Goodier, *Theory of Plates and Shells*, 1959 may be of some help. Plate theory focuses on the out-of-plane deformation of various plate configurations subjected to a variety of load and boundary conditions. Two sample configurations are pictured in Figure 4-7 to illustrate the concept. This out-of-plane behavior covers the performance of reinforced concrete diaphragms subject to transverse loading and is normally outside the scope of typical engineering design texts. This brief explanation is included to familiarize the reader with the technical characteristics associated with load distribution and stress/strain in the diaphragm. In-plane and out-of-plane stiffness are proportional to each other through a three-dimensional stiffness matrix. Therefore, a mathematical relationship between the in-plane diaphragm stiffness and the out-of-plane elastic stiffness exists. The in-plane shear force distribution is contingent upon this definition and further analysis. The out-of-plane elasticity behavior of the plates is presented in Figure 4-8.



(a) Simply-supported rectangular plate with uniform vertical load



(b) Fixed edge rectangular plate with uniform vertical load

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Figure 4-7

Bending of thin rectangular plates

Define the middle plane of the plate (Figure 4-8) at the center of the plate thickness.

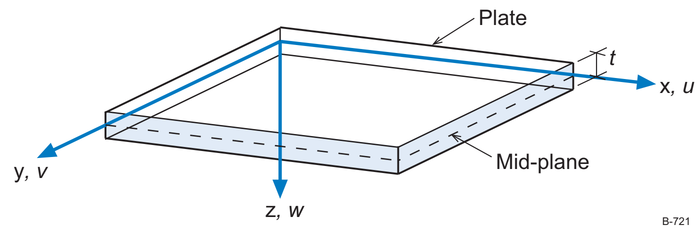


Figure 4-8

x, y, z = coordinate system defined at the mid-plane of the plate

u, v, w = deflection/displacements along the x, y, and z axes, respectively

Consider an arc length along plate (Figures 4-9 and 4-10)

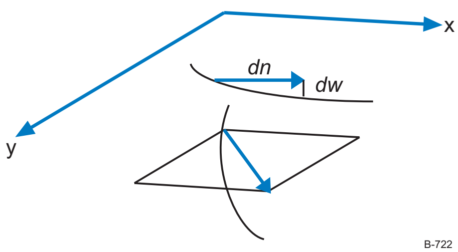


Figure 4-9

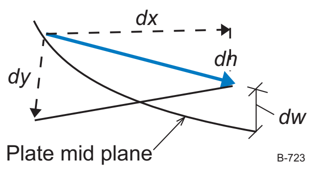


Figure 4-10

where:

$dn \quad = dx + dy$

$$dw = \frac{\partial w}{\partial x}dx + \frac{\partial w}{\partial y}dy$$

$w \quad$ = vertical deflection measured positive downward

$\frac{\partial w}{\partial n}$ = slope of the plate along the n (arbitrary) axis

$\frac{\partial^2 w}{\partial n^2}$ = curvature of the plate along the n axis

r_n = radius of curvature along the n vector

r_x = radius of curvature along the x axis

$$\frac{1}{r_n} = -\frac{\partial}{\partial n}\left(\frac{\partial w}{\partial n}\right) = -\frac{\partial^2 w}{\partial n^2}$$

$$\frac{1}{r_x} = \frac{\partial^2 w}{2x^2}$$

μ = poisson's ratio

γ_{xy} = shear strain

and for the y axis

$$\frac{1}{r_y} = \frac{\partial^2 w}{2y^2}$$

For a complete derivation, refer to Timoshenko, *Theory of Elasticity.*

Figure 4-11 represents the definitions of strain and stress

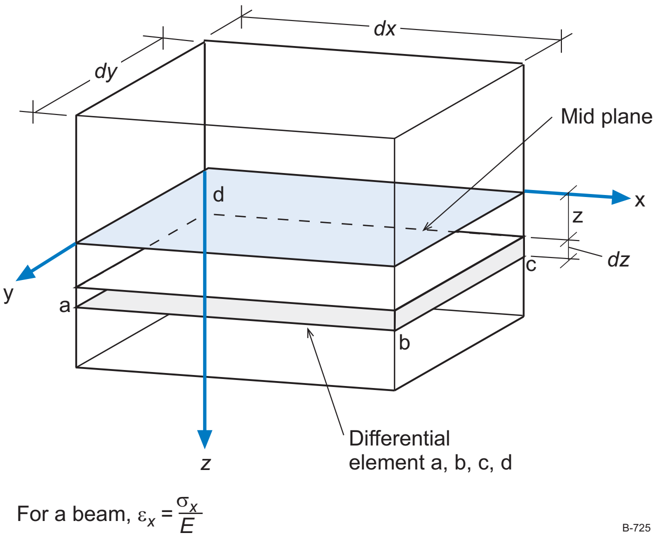


Figure 4-11

The mid-plane is the neutral surface (z = 0)

From elementary beam theory (Figure 4-12), recall

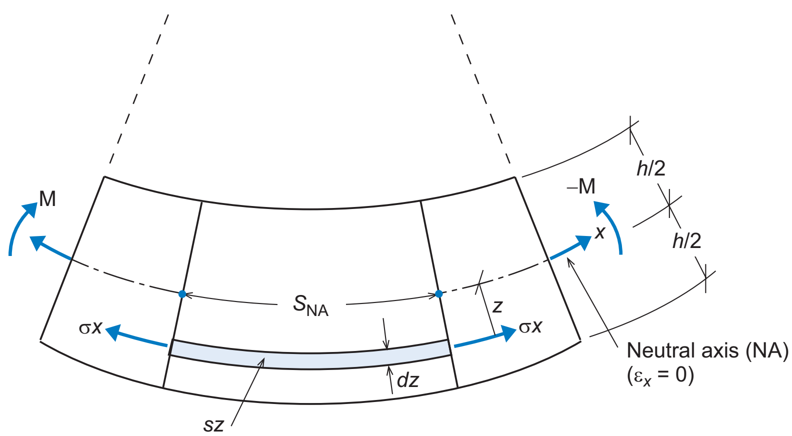


Figure 4-12

For a plate:

$$\varepsilon_x = \frac{\sigma_x}{E} - \upsilon\frac{\sigma_y}{E}$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \upsilon\sigma_x)$$

and

$$\sigma_x = \frac{E}{1-\upsilon}(\varepsilon_x + \upsilon\varepsilon_y)$$

$$\sigma_y = \frac{E}{1-\upsilon}(\varepsilon_y + \upsilon\varepsilon_x)$$

$$\sigma_y = \frac{E}{2(1+\upsilon)}\gamma_{xy}$$

Using basic geometry we arrive at Figure 4-13.

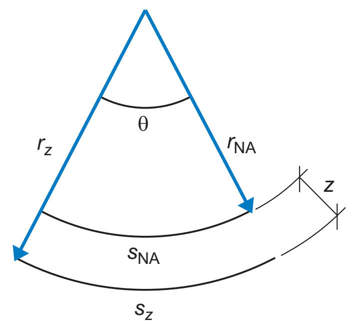


Figure 4-13

$$z = r_z - r_{NA}$$

$$\frac{S_{NA}}{r_{NA}} = \theta \Rightarrow S_{NA} = \theta r_{NA}$$

$$\frac{S_z}{r_z} = \theta \Rightarrow S_z = \theta r_z$$

$$\varepsilon_x = \frac{S_z - S_{NA}}{S_{NA}} = \frac{\theta(r_z - r_{NA})}{\theta r_{NA}}$$

$$\therefore \varepsilon_x = \frac{z}{r_{NA}}$$

or

$$\varepsilon_x = \frac{z}{r}$$

$$\therefore \varepsilon_Y = \frac{z}{r_y}$$

Therefore,

$$\sigma_x = \frac{E_z}{l-v^2}\left(\frac{1}{r_x} + v\frac{1}{r_y}\right)$$

$$\sigma_y = \frac{E_z}{l-v^2}\left(\frac{1}{r_y} + v\frac{1}{r_x}\right)$$

To calculate the bending moments, this is integrated across the cross section.

$$M_x dy = \int_{\frac{-h}{2}}^{\frac{+h}{2}} \sigma_x z dy dz$$

$$M_y dx = \int_{\frac{-h}{2}}^{\frac{+h}{2}} \sigma_y z dx dz$$

Timoshenko presents extensive mathematical formulation to define the shear strains, stresses, as well as other theoretical definitions. See Goodier's *Theory of Plates and Shells* for further information.

From a practical standpoint, the conclusions are the important element, so this text skips the derivations.

1. The maximum principle stress occurs at the extreme fiber location, $z = \pm\frac{h}{L}$, and is

$$(\sigma_x)_{\max} = \frac{6M_x}{h^2}, (\sigma_y)_{\max} = \frac{6M_y}{h^2}$$

2. The maximum shear stress occurs on the 45° plane between the x-z and y-z planes

$$\tau_{\max} = \frac{1}{2}(\sigma_x - \sigma_y) = \frac{3(M_x - M_y)}{h^2}$$

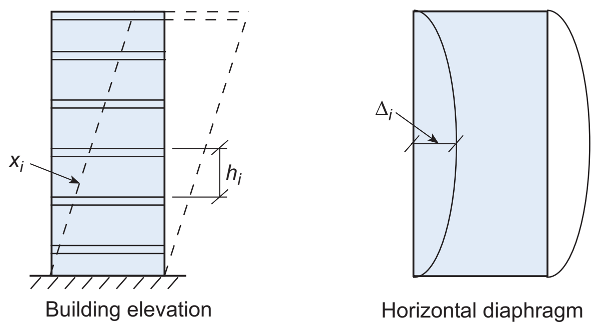
The elastic zone ($P < P_y$) is characterized by a linear stiffener (K = constant). The plastic zone ($P > P_y$) is the nonlinear portion of a force-displacement curve. Structural engineers design in the elastic zone with a concern for inelastic behavior, but the overall goal is to maintain elastic response in their structures. Inelastic displacement is not desirable but may occur under extreme load conditions.

4.2.1 Flexible and rigid diaphragms

The issue of diaphragm flexibility is debated in many forums. The following definitions are contained in the 2000 IBC.

DIAPHRAGM, FLEXIBLE. A diaphragm is flexible for the purpose of distribution of story shear and torsional moment when the lateral deformation of the diaphragm is more than two times the average story drift of the associated story, determined by comparing the computed maximum in-plane deflection of the diaphragm itself under lateral load with the story drift of adjoining vertical-resisting elements under equivalent tributary lateral load.

DIAPHRAGM, RIGID. A diaphragm that does not conform to the definition of flexible diaphragm (Figure 4-14).



x_i = i^{th} story deflection
h_i = i^{th} story height
$x_{i\text{-}1}$ = i-1 story deflection
Δ_i = diaphragm deflection
Δx_i = story drift = $x_i - x_{i\text{-}1}$
If $\Delta_i > 2\Delta x_y$, then the diaphragm is flexible
Consequently, if $\Delta_i < 2(\Delta x_i)$, then it is a rigid diaphragm

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Figure 4-14

The 1997 UBC and the SEAOC Blue Book are consistent with the 2000 IBC definition. Conceptually, the definitions of flexible and rigid diaphragms are not difficult to comprehend; the real challenge lies in quantifying the diaphragm behavior, specifically the in-plane force deflection relationship (i.e., in-plane stiffness). There are established formulas in the 1997 UBC (UBC Standard 23-2).

However, the available data on physical testing of wood diaphragms is limited. The SEAOC Blue Book contains a detailed discussion on this topic.

A large percentage of masonry buildings are constructed with wood diaphragms. For international projects, the rigid diaphragm (i.e., reinforced concrete) is the mainstay and is implemented in many countries. In the United States, the majority of masonry structures are designed using a wood frame diaphragm with the flexible assumption. For a rigid diaphragm, the reinforced concrete diaphragm is an obvious choice. Following the 1994 Northridge, California earthquake, SEAOC began to question the validity of the flexible diaphragm for various aspect ratios.

4.3 Shear Wall Stiffness

A shear wall is a vertical bearing wall that resists in-plane lateral loads. This concept, was introduced in Chapter 1, is further quantified in this section. A principle concern is to delineate the structural characteristics of a shear wall.

Figure 4-15 (a) is defined as a structural wall because its aspect ratio (height to length) is greater than 2. The primary source of lateral deflection is from bending strain energy. Figure 4-15 (b) is a true shear wall because its primary source of lateral deflection is shear strain energy. Differentiating between shear and bending strain energy is accomplished by examining the theoretical development of the deflection equation for beams.

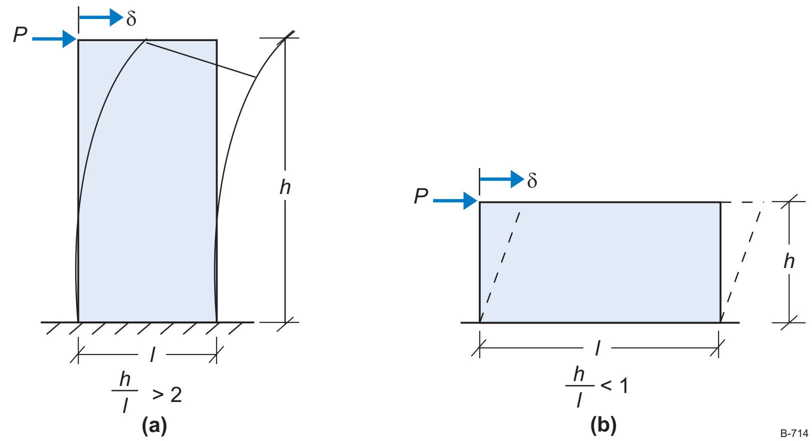


Figure 4-15

Recall from basic structural theory the formulas for elastic strain energy, (Figure 4-16); the formulaltions are:

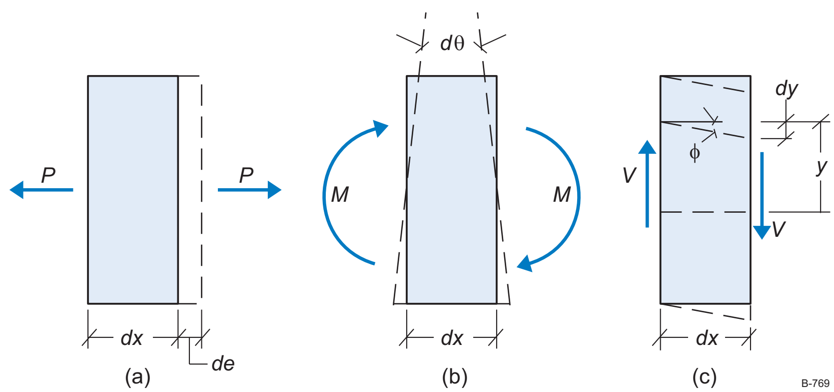


Figure 4-16

E = Module of elasticity

A = Cross-section area

I = Moment of inertia

Z = Shape factor for axial and bending

G = Shear modulus

J = Torsional moment of inertia

K = Shape factor for shear energy

Axial force:	$W_{int} =$	Internal strain energy due to axial force (P)	$= \int_o^l \frac{P^2}{2EA} dx$
Bending:	$W_{int} =$	Internal strain energy due to bending moment (M)	$= \int_o^l \frac{M^2}{2EI} dx$
Shear force:	$W_{int} =$	Internal strain energy due to shear force (V)	$= K\int_o^l \frac{V^2}{2EI} dx$
Twist/torsion:	$W_{int} =$	Internal strain energy due to torsional moment (T)	$= \int_o^l \frac{T^2}{2GJ} dx$

The equations may be used in their virtual work form

Virtual axial energy:
- P = real force
- p = virtual force

$$\delta W_I = \int_o^l \frac{Pp}{EA} dx$$

Virtual bending energy:
- M = real moment
- m = virtual moment

$$\delta W_I = \int_o^l \frac{Mm}{EI} dx$$

Virtual shear energy:
- V = real shear force
- v = virtual shear force
- k = shape factor

$$\delta W_I = k\int_o^l \frac{Vv}{GA} dx$$

Virtual torsional energy:
- T = real torsional moment
- t = virtual torsional moment

$$\delta W_I = \int_o^l \frac{Tt}{GJ} dx$$

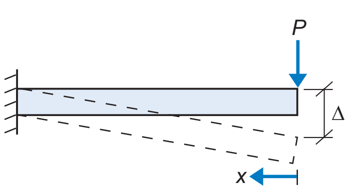


Figure 4-17

In reference to shear walls, the basic equation of beam bending and shear energy to drive the force-displacement relationship for a cantilever beam follows (Figures 4-17 and 4-18).

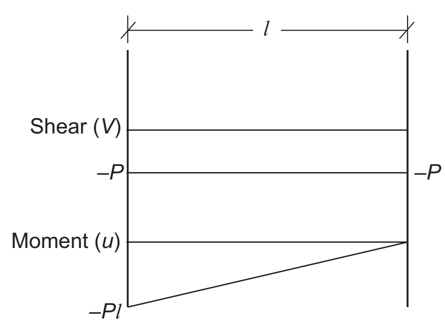


Figure 4-18

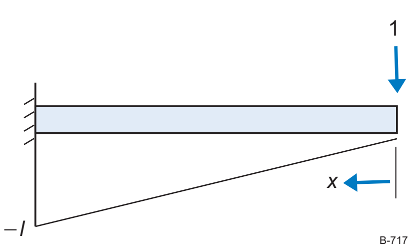


Figure 4-19

Using the virtual work method, the unit load is applied at the free end to obtain (Figure 4-19).

$$\therefore \ \Delta = \int_o^l \frac{Mm}{EI}dx + K\int_o^l \frac{Vv}{GA}dx$$

where:

$$M = -Px \quad m = -x$$

$$V = -P \quad v = -1$$

$$\Delta = \frac{1}{EI}\int_o^l (-Px)(-x)dx + \frac{K}{GA}\int_o^l (-P)(-1)dx$$

$$\Delta = \frac{1}{EI}\left[\frac{Px^3}{3}\right]_o^l + \frac{K}{GA}[Px]_o^l$$

$$\Delta = \frac{Pl^3}{3EI} + \frac{KPl}{GA}$$

For rectangular cross sections

$$\Delta = \frac{Pl^3}{3EI} + \frac{1.2Pl}{GA}$$

Following are two extreme versions of shear walls.

Shear wall A: (Figure 4-20) l =10 feet, h = 30 feet, $\frac{h}{l} = 3$

$$\Delta_A = \frac{Ph_A^3}{3(EI)_A} + \frac{1.2Ph_A}{GA_A}$$

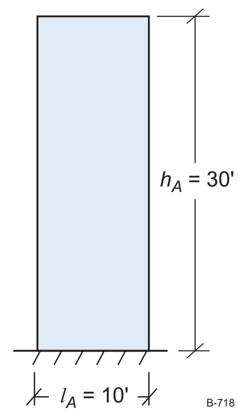


Figure 4-20

Shear wall B: (Figure 4-21) l = 30 feet, h = 10 feet, $\frac{h}{3} = 1/3$

$$\Delta_B = \frac{Ph_B^3}{3(EI)_B} + \frac{1.2Ph_B}{GA_B}$$

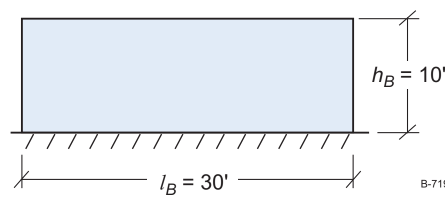


Figure 4-21

For typical masonry shear walls:

8-inch CMU wall

t = 7.63 in

f'_m = 2000 psi

$E_m = 750f'_m$ = 1500 ksi

$G_m = 0.4\,E_m = 0.4(1500)$ = 600 ksi

l_A = 10 ft = 120 in $\quad h_A$ = 360 in $\quad A_A$ = (120 in)(7.63) 915.6 in^2 (fully grouted)

l_B = 30 ft = 360 in h_B = 10 in = 120 in A_B = (360 in)(7.63) = 2746.8 in^2

$$\text{Shear wall A} = I_A = \frac{bl^3}{12} = \frac{(7.63)(120)^3}{12} = 1{,}098{,}720\ \text{in}^4$$

$$\Delta_A = P\left(\frac{360}{3(1{,}5000{,}000)(1{,}098{,}720)}\right) + 1.2P\left(\frac{360}{(600{,}000)(915.6)}\right)$$

Let P = 10 k = 10,000 lb

$$\therefore\ \Delta_A = \underbrace{0.0944}_{\Delta_M = \text{bending}} + \underbrace{0.0079}_{\Delta_y = \text{shear}} = 0.1023$$

$$\text{Bending deflection} = \frac{\Delta_m}{\Delta_A} = \frac{0.0944}{0.1023} = 92\%$$

$$\text{Shear deflection} = \frac{\Delta_v}{\Delta_A} = \frac{0.0079}{0.1023} = 8\%$$

$$\text{Shear wall B} = I_B = \frac{bl^3}{12} = \frac{(7.63)(360)^3}{12} = 2.97 \times 10^7\ \text{in}^4$$

$$\Delta_B = P\left(\frac{120}{3(1{,}500{,}00)(2.97 \times 10^7)^4}\right) + 1.2P\left(\frac{120}{(600{,}000)(2747)}\right)$$

Let P = 10,000 lb

$$\therefore\ \Delta_B = \underbrace{1.29 \times 10^{-4}}_{\Delta_M = \text{bending}} + 8.74 \times 10^{-4} = 10^{-3}\ \text{in} = 0.001\ \text{in}$$

Because shear wall B is clearly more stiff, try P =100 k

$$\therefore\ \Delta_B = 10^{-2} = 0.01\ \text{inch with } P = 100\ \text{k}$$

The example produces three important conclusions.

1. The bending energy component is important for high-aspect-ratio shear walls $\left(\frac{h}{l} > 2\right)$
2. The shear energy component is the primary deflection for short (low-aspect-ratio) shear walls $\left(\frac{h}{l} < 1\right)$
3. Both deflection components should be analyzed for the intermediate shear wall $\left(1 < \frac{h}{l} < 2\right)$

This concept is extended further to calculate the stiffness/rigidity, K, of shear walls.

$$K = \text{stiffness} = \frac{P}{\delta} = \frac{\text{force}}{\text{deflection}}$$

For a typical shear wall, the force versus displacement relationship is shown in Figure 4-22.

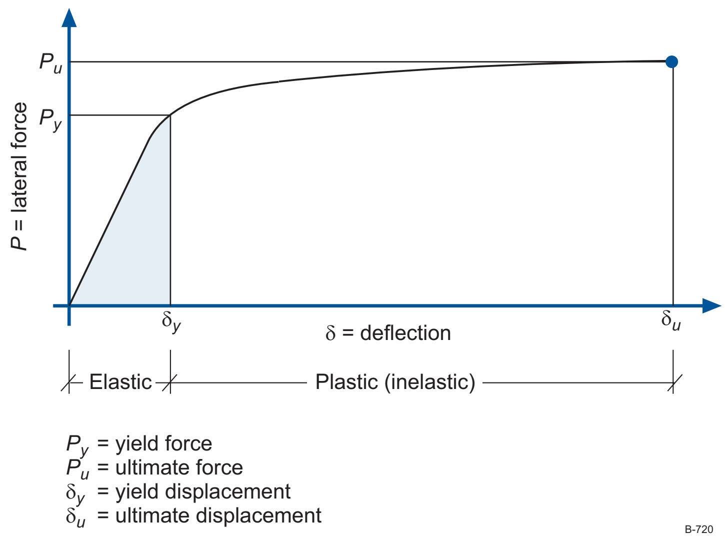


Figure 4-22

Wall stiffness also depends on the end constraint condition. For example, the free-end cantilever behaves like a cantilever beam (Figure 4-23).

$$\Delta = \Delta_m + \Delta_y = \frac{Ph^3}{3EI} + \frac{1.2Ph}{GA}$$

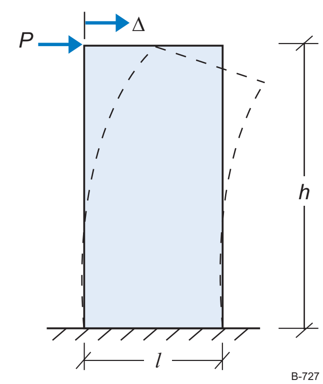


Figure 4-23

The fixed-fixed end condition has a higher wall stiffness (Figure 4-24)

$$\Delta_{\text{fixed}} = \Delta_m + \Delta_x = \frac{Ph^3}{12EI} + \frac{1.2Ph}{GA}$$

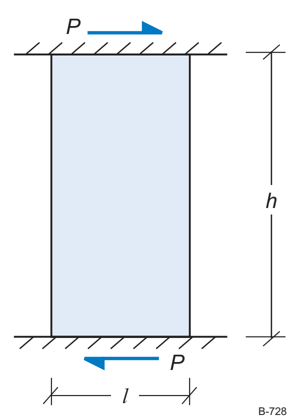


Figure 4-24

Substituting for E = 1,000,000 psi, t = 1 in. and P = 100 k

$$\Delta_{\text{cant}} = \Delta_m + \Delta_y = \frac{(100)(h^3)}{3(1000)(1)(l)^3/12} + \frac{1.2(100)(h)}{0.4(1000)(1)(l)}$$

$$\Delta_{\text{cant}} = 0.4\left(\frac{h}{l}\right)^3 + 0.3\left(\frac{h}{l}\right)$$

For a fixed-end

$$\Delta_{\text{fixed}} = \frac{100\,(h)^3}{12(1000)(1)\left(\frac{l^3}{12}\right)} + \frac{1.2(100)h}{0.4(1000)(1)l}$$

$$\Delta_{\text{fixed}} = 0.1\left(\frac{h}{l}\right)^3 + 0.3\left(\frac{h}{l}\right)$$

Relative stiffness is the most important characteristic in building design. Absolute stiffness affects displacements and ductility behavior; but for rigid diaphragms, the relative stiffness values determine whether there is torsional behavior.

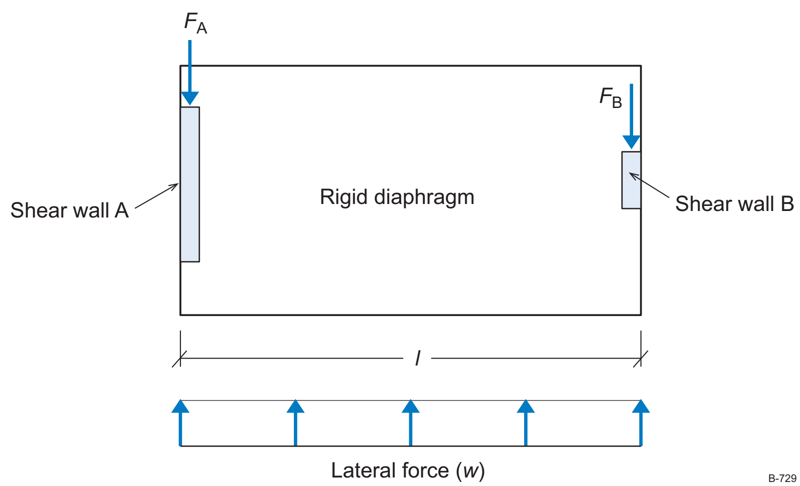


Figure 4-25

The force distribution (F_A and F_B) between shear walls A and B (Figure 4-25) depends on the relative stiffness between the two shear walls and their geometric placement. This concept is similar to a spring-mass model (Figures 4-26, 4-27, and 4-28).

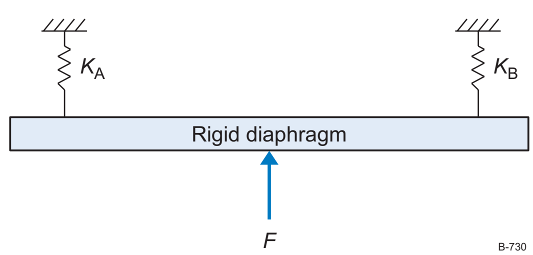


Figure 4-26

If $K_A >> K_B$,

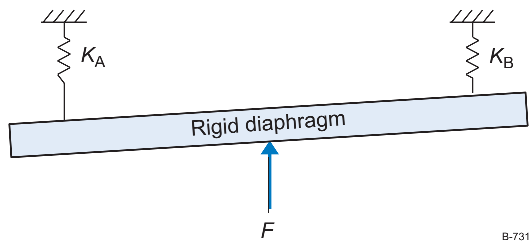


Figure 4-27

If $K_B >> K_A$,

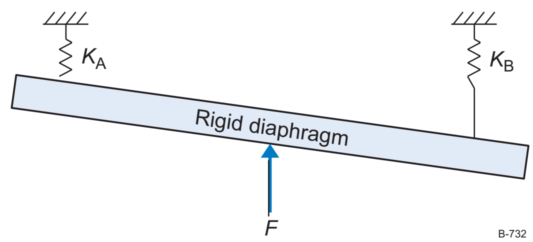


Figure 4-28

The total stiffness is $K_{total} = K_A + K_B$

Therefore, the relative stiffness is

$$\text{wall A} = \frac{K_A}{K_{A1} + K_B}$$

$$\text{wall B} = \frac{K_B}{K_A + K_B}$$

$$\text{For } n \text{ walls } K_i = \frac{K_i}{\sum_{i=1}^{n} K_i}$$

For rigid slabs, the diaphragm is treated as purely rigid (having infinite stiffness), and any mild flexibility is ignored. For reinforced concrete slab construction, pure rigidity is a reasonable assumption. For wood frame diaphragms, the flexibility assumption is the standard convention.

Wall stiffness (rigidity) values may be calculated using the *Reinforced Masonry Engineering Handbook, 5th edition updated*, pages 397-403, Tables 1a through 1g.

Multistory shear walls with openings require specific analysis depending on the wall geometry. For example, see Figure 4-29.

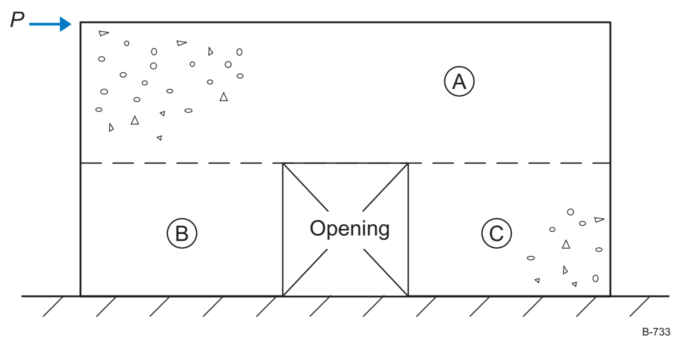


Figure 4-29

This shear wall has three components: A, B, and C. Elements B and C refer to piers B and C. To calculate the combined stiffness of the wall, it must be broken down into three free body elements as shown in Figure 4-30.

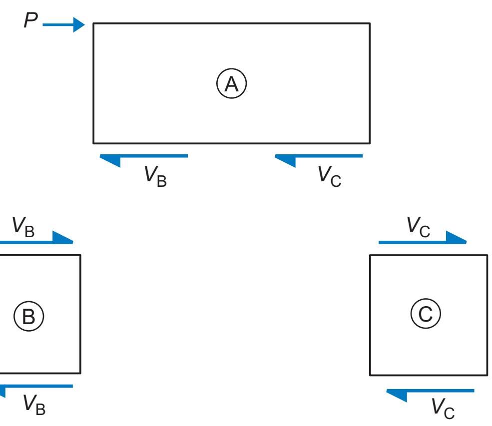


Figure 4-30

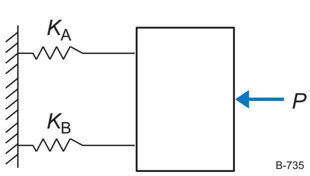


Figure 4-31

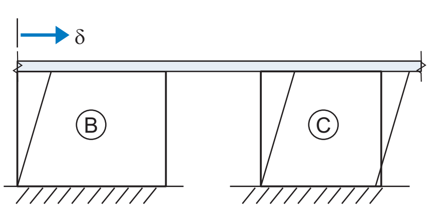


Figure 4-32

Clearly $V_B + V_C = P$

The wall stiffnesses of piers B and C are additive

$$K_B + K_C = K_{BC}$$

Now, consider the two piers as two springs in parallel (Figure 4-31)

$$K_{total} = K_{AB} = K_A + K_B$$

For the two shear walls, the top deflection is constrained to be equal (Figure 4-32)

$$K\delta = (K_B + K_C)\,\delta$$

$$\therefore \quad K_{BC} = K_B + K_C$$

The wall fixity is a fixed-fixed connection (Figure 4-33)

$$\delta_B = \frac{V_B h^3}{12EI} + \frac{1.2 V_B h}{GA}$$

or, for simplicity

$$\Delta_B = 0.1\left(\frac{h}{l}\right)^3 + 0.3\left(\frac{h}{l}\right)$$

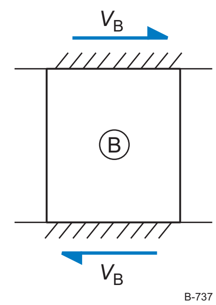


Figure 4-33

From a stiffness perspective

$$K = \frac{P}{\delta}$$

$$K_B = \frac{12EI}{h^3{}_B} + \frac{1.2GA}{h}$$

or,

$$R_B = \text{rigidity of wall pier B} = \frac{1}{\delta_B}$$

$$\therefore\ R_{BC} = R_B + R_C$$

To combine this with wall element A (Figure 4-34)

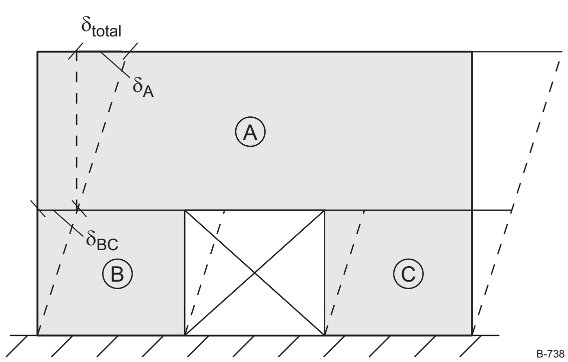


Figure 4-34

$$\delta_{total} = \delta_A + \delta_{BC}$$

where:

δ_A = relative deflection of wall element A

δ_{BC} = deflection of the combined wall elements B and C

By adding these two deflection components, the total wall stiffness/rigidity is obtained

$$\delta_{total} = \delta_A + \delta_{BC}$$

$$\therefore\ R_{total} = \frac{1}{\delta_{total}} = \frac{1}{\delta_A + \delta_{BC}}$$

4.4 Center of Rigidity and Center of Gravity

Once the shear wall stiffness values are calculated, the diaphragm's center of rigidity (CR) must be established. Figure 4-35 is a three-dimensional perspective of a shear wall building indicating the concept of center of rigidity. The lateral forces are either wind or earthquake forces, although there are rare applications of explosion forces (pressure waves) that apply to military structures.

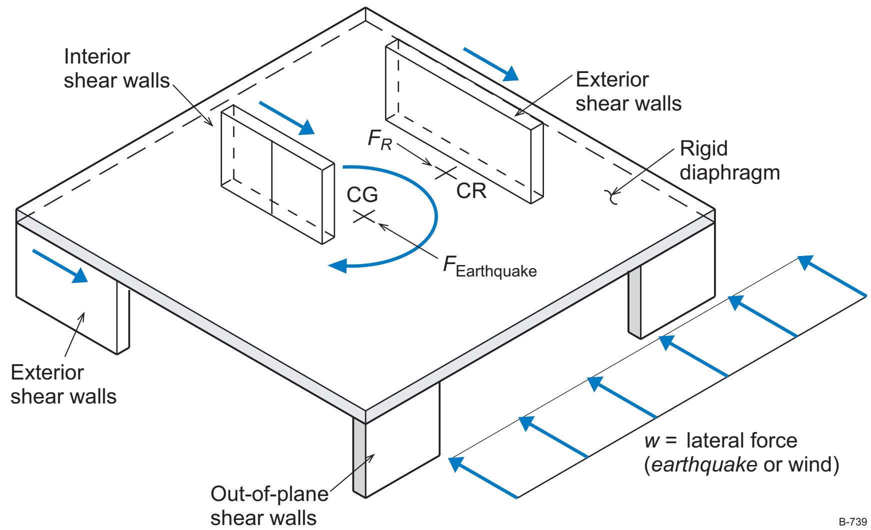


Figure 4-35

These lateral forces are concentrated along the center of gravity of the diaphragm, which is calculated using the laws of physics and statics (Figure 4-36).

To review:

Center of gravity (CG) or centroid – To calculate:

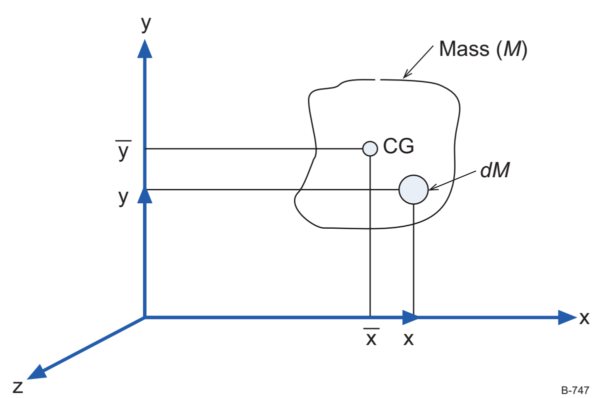


Figure 4-36

Center of gravity is defined mathematically as

$$\bar{x}M = \int x dM \qquad \bar{y}M = \int y dM \qquad \bar{z}M = \int z dM$$

Therefore,

$$\bar{x}_{CG} = \frac{\int_M \bar{x} dM}{\int_M dM} \qquad \bar{y}_{CG} = \frac{\int_M \bar{y} dM}{\int_M dM} \qquad \bar{z}_{CG} = \frac{\int_M \bar{z} dM}{\int_M dM}$$

The two-dimensional case with equal thickness simplifies to

$$\bar{x}_{CG} = \frac{\int_A \bar{x} dA}{\int_A dA}$$

A practical version of discrete area summation becomes

$$\bar{x}_{CG} = \frac{\sum_{i=1}^{n} x_i A_i}{\sum_{i=1}^{n} A_i} \qquad \bar{y}_{CG} = \frac{\sum_{i=1}^{n} y_i A_i}{\sum_{i=1}^{n} A_i}$$

Definition: *Center of gravity defines the location where the inertial forces are concentrated.*

Center of Rigidity (CR)

While the CG pinpoints the location of inertial force, the CR pinpoints the location of resistance forces (Figure 4-37).

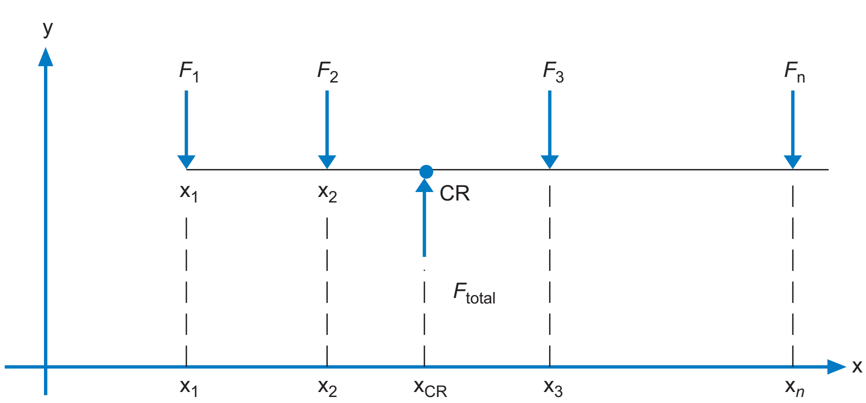


Figure 4-37

$$F_{\text{total}} = \sum_{i=1}^{n} F_i$$

$$x_{\text{CR}}\, F_{\text{total}} = \sum_{i=1}^{n} F_{yi} x_i$$

CR is defined as

$$\therefore x_{\text{CR}} = \frac{\sum_{i=1}^{n} x_i F_{yi}}{\sum_{i=1}^{n} F_{yi}}, \quad y_{\text{CR}} = \frac{\sum_{i=1}^{n} y_i F_{xi}}{\sum_{i=1}^{n} F_{xi}}$$

Force and stiffness/rigidity are related quantities

$$F = K\delta$$

$$\therefore K = \frac{F}{\delta}$$

For equal displacements, the force is directly proportional to stiffness

$$K_i = \frac{F_i}{\delta_i} \text{ or } F_i = K_i\,\delta_i, \text{ and } R_i = \frac{1}{\delta_i}$$

Rigidity and stiffness are the same concept, except $R_i = (K_i)_{F_i = 1}$

Substitute into CR definition above

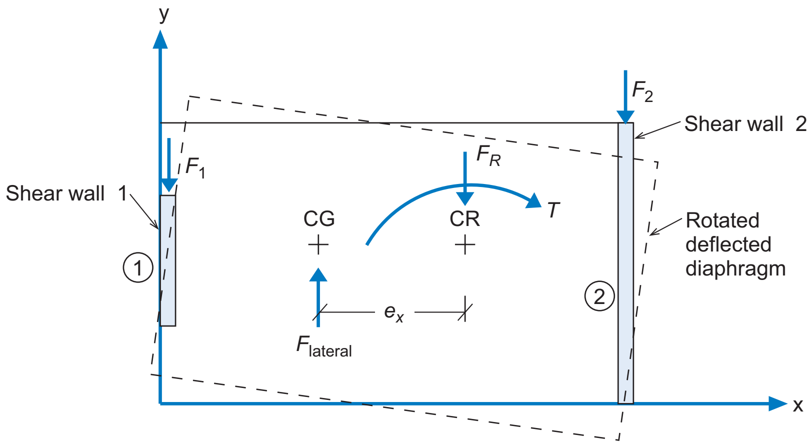
$$x_{\text{CR}} = \frac{\sum_{i=1}^{n} x_i K_{yi} \delta_{yi}}{\sum_{i=1}^{n} K_{yi} \delta_{yi}}$$

Since δ_i = deflection and $K_i = R_i = \frac{F_i}{\delta_i}$ with $F_i = 1$ (wall rigidity), then replace with R_i

$$x_{\text{CR}} = \frac{\sum_{i=1}^{n} x_i R_{yi}}{\sum_{i=1}^{n} R_{yi}}, \quad y_{\text{CR}} = \frac{\sum_{i=1}^{n} y_i R_{xi}}{\sum_{i=1}^{n} R_{xi}}$$

4.5 Torsion of a Rigid Diaphragm

If the center of rigidity (CR) and mass/gravity (CG) do not coincide, then the imposed lateral forces will create an in-plane (torsional) moment within the diaphragm, as illustrated in Figure 4-38. The lateral force (F_{lateral}) acts at the center of rigidity and creates an in-plane torsional moment ($T = F_{\text{lateral}} \times e$). The figure illustrates an x-axis eccentricity and also shows that the torsional moment causes diaphragm rotation in addition to displacement.



F_{lateral} = Lateral force acting on the diaphragm

F_R = Resisting force from the structure

e = Eccentricity between the CR and CG

$e_x = | x_{CG} - x_{CR} |$

$e_y = | y_{CG} - y_{CR} |$

1 = Shear wall 1 with resisting force, F_1

2 = Shear wall 2 with resisting force, F_2

$F_R \equiv F_1 + F_2$

T_y = torsional moment = $(F_{\text{lateral}})(e_x)$

B-753

Figure 4-38

A supplemental shear force (torsional shear) is produced by the rotation. This force is distributed to the shear walls using the walls' relative rigidity. The walls with higher stiffness accept a proportionally higher load. Torsion produces shear stress in the diaphragm, which seldom results in damage because rigid diaphragms are mostly reinforced concrete and have excellent factors of safety against shear failure.

Figure 4-39 shows the general equation for two-dimensional torsional and shear force distribution.

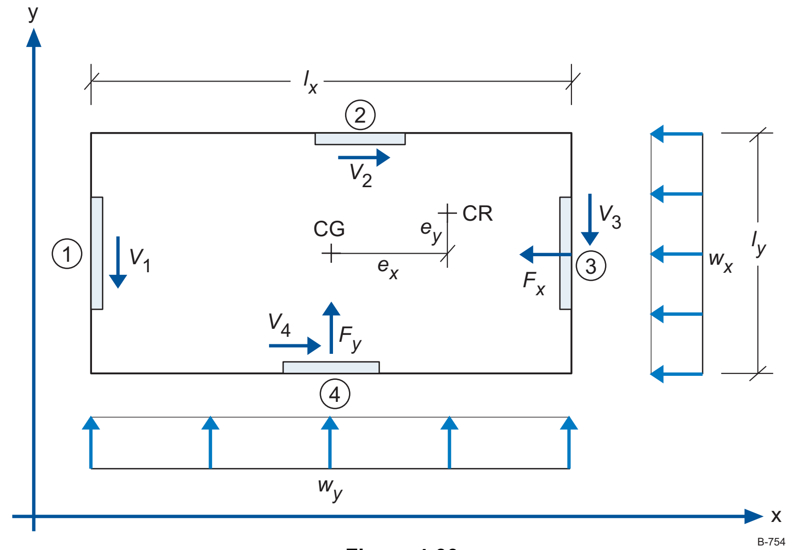


Figure 4-39

The basic equations for distributing the lateral forces to the shear walls are:

w_x, w_y = distributed lateral forces along x and y axes, respectively

F_x = $w_x l_y$ = concentrated equivalent force along the x-axis

F_y = $w_y l_x$ = concentrated equivalent force along the y-axis

V_1, V_2, V_3, V_4 ... V_i = shear wall forces in the i^{th} wall, (i = 1,2,3,4 for this example)

e_x = $|x_{CR} - x_{CG}|$ = torsional eccentricity along the x-axis

e_y = $|y_{CR} - y_{CG}|$ = torsional eccentricity along the y-axis

T_x = torsional moment for x-axis = $F_x e_y$

T_y = torsional moment for y-axis = $F_y e_x$

$(V_{vx})_i$ = shear wall force in the i^{th} wall due to shear along the x-axis

$(V_{vy})_i$ = shear wall force in the i^{th} wall due to shear along the y-axis

$$(V_{vx})_i = F_x \underbrace{\left(\frac{R_{xi}}{\sum_{i=1}^{n} R_{xi}} \right)}_{\text{relative rigidity distribution factor along the x-axis}}$$

$$(V_{vy})_i = F_y \underbrace{\left(\frac{R_{yi}}{\sum_{i=1}^{n} R_{yi}} \right)}_{\text{relative rigidity distribution factor along the y-axis}}$$

$(V_{tx})_i$ = shear wall force in i^{th} wall due to torsional shear along the x-axis (i.e., T_x)

$(V_{ty})_i$ = shear wall force in i^{th} wall due to torsional shear along the y-axis (i.e., T_y)

d_{xi} = distance of i^{th} shear wall to the CR along the x-axis

d_{yi} = distance of i^{th} shear wall to the CR along the y-axis

$$(V_{tx})_i = T_x \left(\frac{R_{xi} d_{xi}}{\sum_{i=1}^{n} R_{xi} d_{xi}} \right)$$

$$(V_{ty})_i = T_y \left(\frac{R_{yi} d_{yi}}{\sum_{i=1}^{n} R_{yi} d_{yi}} \right)$$

V_{xi} = total shear force in i^{th} wall along the x-axis = $(V_{vx})_i + (V_x)_i$

V_{yi} = total shear force in i^{th} wall along the y-axis = $(V_{vy})_i + (V_y)_i$

Therefore, the general equations for torsional shear force distribution are

$$V_{xi} = F_x \left(\frac{R_{xi}}{\sum_{i=1}^{n} R_{xi}} \right) + T_x \left(\frac{R_{xi} d_{xi}}{\sum_{i=1}^{n} R_{xi} d_{xi}} \right)$$

$$V_{yi} = F_y \left(\frac{R_{yi}}{\sum_{i=1}^{n} R_{yi}} \right) + T_y \left(\frac{R_{yi} d_{yi}}{\sum_{i=1}^{n} R_{yi} d_{yi}} \right)$$

This process can also be demonstrated using the example in Section 4.3.

Both the 1997 UBC and the 2000 IBC have requirements for torsional force distribution.

The definition of torsional irregularity is discussed in UBC Table 16-M. Be sure to amplify the accidental torsion by A_x, as defined in Equation 30-16.

The purpose of accidental torsion is to account for higher irregular stiffness distribution in the vertical wall elements. For example, as shown in Figure 4-38, the rotation of the diaphragm is pronounced along the shear wall. This would require an accidental torsion amplification factor.

The technical requirements are identical to the 1997 UBC.

4-6 Summary

This chapter has presented an examination of diaphragm-to-shear wall behavior prevalent in masonry structures. The three types of diaphragms considered are 1) rigid, 2) semi-rigid, and 3) flexible. Quantifying the structural characteristics and load distribution criteria of each type before proceeding with the evaluation of the structure is essential.

The structural provisions of the 1997 UBC and the 2000 IBC strongly suggest that a majority of the design community considers wood frame diaphragms to be flexible. However, this is not the case in Southern California where some jurisdictions have adopted the rigid-diaphragm concept for wood-frame floor systems.

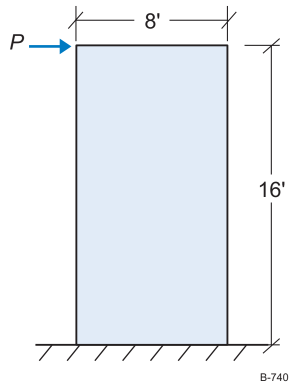
The concept of relative rigidity and wall in-plane stiffness has been discussed in detail. For rigid diaphragm behavior, the lateral forces are distributed on the basis of the relative stiffness/rigidity of each shear wall.

If the shear walls are not symmetrically oriented, an offset is created between the center of gravity and the center of rigidity. This is termed the *eccentricity* because it creates torsional shear in both the diaphragm and the shear wall. The eccentricity must be evaluated for non-symmetric shear-wall plans because torsional behavior plays a significant role in the dynamic response of any structure.

Evaluating the rigid diaphragm and torsional shear behavior will confirm whether or not the structural system has adequate lateral capacity

Example 4-1

Calculate the wall rigidity and deflection for $P = 20$ k



12-inch CMU

$f'_m \quad = 2500$ psi

Fully grouted

$E_m \quad = 750 f'_m = 750\,(2500) = 1875$ ksi

$t \quad = 11.875$ in

Wall rigidity and deflection –

Cantilevered wall/pier

$$\frac{h}{d} = 16/8 = 2$$

$$\Delta = \frac{P}{E_m t}\left[4\left(\frac{h}{d}\right)^3 + 3\left(\frac{h}{d}\right)\right]$$

$$\Delta = \frac{20{,}000}{(1{,}875{,}000)(11.875)}\left[4(2)^3 + 3(2)\right] = 0.034 \text{ in}$$

$$\text{Rigidity} = \frac{1}{\Delta} = 29.3$$

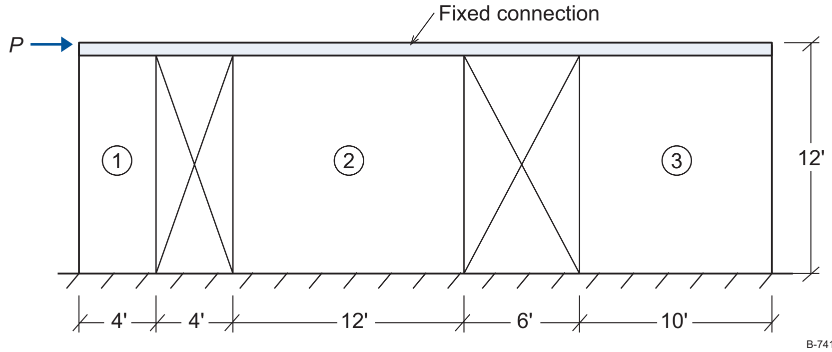
Or, from the table,

$$\frac{h}{d} = 2 \quad \Rightarrow \quad \Delta_c = 3.8$$

Adjust for actual load and thickness

$$\Delta = (3.8)\left(\frac{20{,}000}{100{,}000}\right)\left(\frac{1}{11.875}\right)\left(\frac{1000}{1875}\right) = 0.034 \text{ in (same)}$$

Example 4-2



8-inch CMU, fully grouted, $\therefore$ $t = 7.63$ in

$$f'_m = 2000 \text{ psi}$$

$$E_m = 750 f'_m = 1500 \text{ ksi}$$

Calculate the combined wall stiffness of piers 1, 2, and 3.

Pier	h/d	Δ_F	R_F
1	12/4 = 3.0	3.600	0.278
2	12/12 = 1.0	0.400	2.500
3	12/10 = 1.2	0.533	0.951

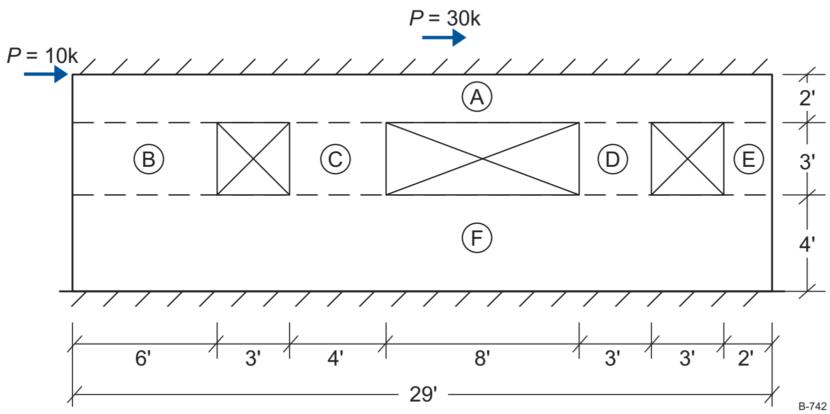
Since these three wall piers are connected by a fixed top plate

$$R_{\text{total}} = \sum_{i=1}^{3} R_i = 0.278 + 2.5 + 0.951 = 3.729$$

Adjust for E, t

$$R = 3.729\left(\frac{1500}{1000}\right)\left(\frac{7.63}{1}\right) = 42.68 \text{ in/100 k} = 0.427 \text{ in/k}$$

Example 4-3



12-inch CMU

f'_m = 3000 psi

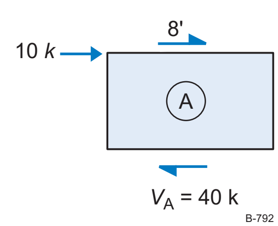
E_m = 2250 ksi

The base section of multistory shear wall is shown with the applied loads. This section represents the lower story of a tall shear wall in a mid-rise structure. Therefore, the second floor line is considered fixed.

1) Calculate the relative rigidities of the wall pier to determine the shear force distribution within the wall.

Pier	h/d	Δ_F	R_F
A	2/29 = 0.070	0.021	48.230
B	3/6 = 0.50	0.163	6.154
C	3/4 = 0.75	0.267	3.743
D	3/3 = 1.00	0.400	2.500
E	3/2 = 1.50	0.788	1.270
F	4/29 = 0.148	0.042	23.655

Pier A



$$\Delta_F = 0.1\,(0.069)^3 + 0.3\,(0.069) = 0.0207$$

$$V_A = 30\text{ k} + 10\text{ k}$$

$$V_A = 40\text{ k}$$

$$R_{BCDE} = \Sigma R_i = R_B + R_C + R_D + R_E$$

$$R_{BCDE} = 6.154 + 3.743 + 2.500 + 1.270 = 13.667$$

Distribute forces within piers

$$V_B = \frac{R_B}{\Sigma R_i} V_A = \left(\frac{6.154}{13.667}\right) 40 = 18.0\text{ k}$$

$$V_C = \left(\frac{3.74}{13.667}\right) 40 = 11.0 \text{ k}$$

$$V_D = \left(\frac{2.50}{13.667}\right) 40 = 7.3 \text{ k}$$

$$V_E = \left(\frac{1.27}{13.667}\right) 40 = 3.7 \text{ k}$$

$\Rightarrow$ Check $\Sigma V_i = 40 \ldots$ **OK**

$$V_F = V_A = 40 \text{ k}$$

2) Determine the overall stiffness/rigidity and drift for the given load.

$$\text{Since } R_{BCDE} = 13.667 \quad \Rightarrow \quad \Delta_{BCDE} = \frac{1}{13.667} = 0.0732$$

$$\Delta_{total} = \Delta_F + \Delta_{BCDE} + \Delta_A$$

$$\Delta_{total} = 0.042 + 0.073 + 0.021 = 0.136$$

$$\Delta_{total} = 0.136$$

$$R_{total} = \frac{1}{\Delta_{total}} = 7.358$$

Evaluate for;

$$P = 40 \text{ k}$$

$$E_m = 2250 \text{ ksi}$$

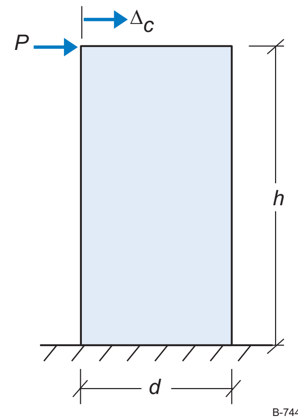
$$t = 11.63 \text{ in}$$

$$\Delta = (0.1359)\left(\frac{40}{100}\right)\left(\frac{1000}{2250}\right)\left(\frac{1}{11.63}\right) = 0.0021 \text{ in}$$

Example 4-4

For a general cantilevered shear wall, the deflection equation is given as

$$\Delta_C = \frac{P}{E_m t}\left[4\left(\frac{h}{d}\right)^3 + 3\left(\frac{h}{d}\right)\right]$$



1. Let $\frac{h}{d} = \alpha$, and graph the stiffness $\left(k = \frac{P}{\Delta}\right)$ curve for $0 < \alpha < 20$.
2. Define equation in terms of stiffness as a function of $\alpha = \frac{h}{d}$. Differentiate this equation to obtain $\frac{dk}{d\alpha}$, and determine at what point does $\frac{dk}{d\alpha}$ approach zero.
3. Drift is defined as deflection over the height

$$\delta = \text{drift ratio} = \frac{\Delta}{h}$$

Rearrange the equation in terms of drift ratio vs. aspect ratio (i.e., $\delta = f(\alpha)$).

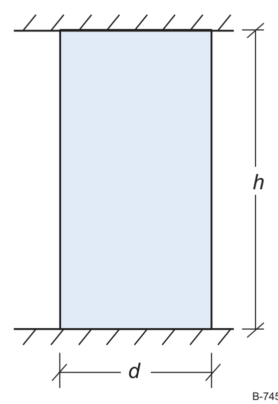
Solution

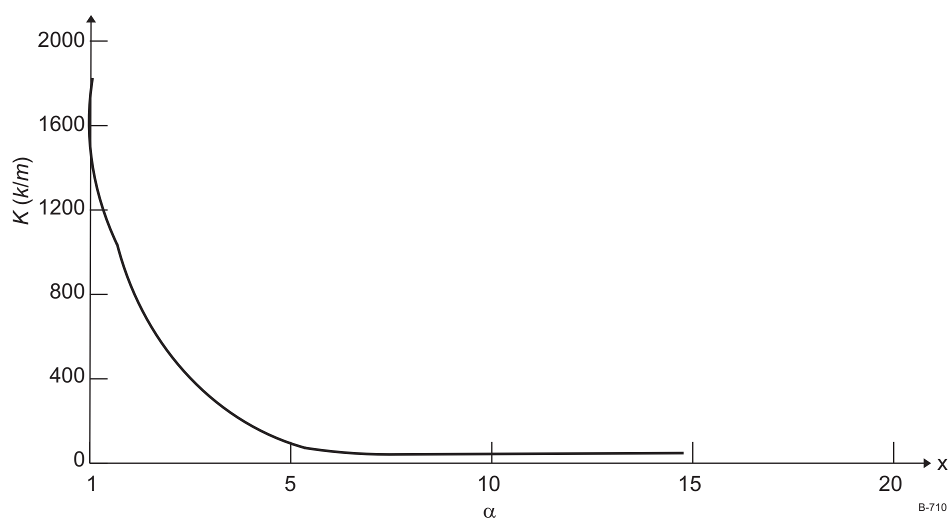
1. $$\frac{h}{d} = \alpha \Rightarrow \Delta_C = \frac{P}{E_m t}[4\alpha^3 + 3d]$$

Using $\quad P = K\Delta_C$

$$\Rightarrow K = \frac{P}{\Delta_c} = \frac{E_m t}{4\alpha^2 + 32}$$

$$\therefore K = \frac{E_m t}{\alpha(4\alpha^2 + 3)}$$





Solution with $E_m = 900(3000) = 2700$ ksi and $t = 7.63$ in.

$$\therefore\ E_m t = 20{,}600 \text{ k/in}$$

$$\alpha = 1 \quad K = \frac{E_m t}{4(1)^3 + 3} = \frac{E_m t}{12} = 0.0833\, E_m t$$

$$\therefore\ K = 1716 \text{ k/in.}$$

$$\alpha = 5 \quad K = \frac{E_m t}{4(5)^3 + 3(5)} = \frac{E_m t}{515} = 0.00194\, E_m t$$

$$\therefore\ K = 40 \text{ k/in.}$$

$$\alpha = 10 \quad K = \frac{E_m t}{4(10)^3 + 3(10)} = \frac{E_m t}{4030}$$

$$\therefore\ K = 5.1 \text{ k/in.}$$

$$\alpha = 20 \quad K = \frac{E_m t}{4(20)^3 + 3(20)} = \frac{E_m t}{32,060}$$

$$\therefore\ K = 0.64 \text{ k/in.}$$

2. $K = \dfrac{E_m t}{\alpha(4\alpha^2 + 3)}$, using calculus $\quad d\left(\dfrac{u}{v}\right) = \dfrac{vdu - udv}{v^2}$

$$d\,(uv) = udv + vdu$$

$$\frac{dk}{dd} = E_m t\left[\frac{[\alpha(B\alpha) + (4\alpha^2 + 3)]}{\alpha^2(4\alpha^2 + 3)^2}\right]$$

$$\therefore\ \frac{dk}{dd} = -E_m t\left[\frac{12\alpha^2 + 3}{\alpha^2(4\alpha^2 + 3)^2}\right]$$

K approaches zero as $\alpha \to \infty$

$$\text{limit } \alpha \to \infty \quad \frac{E_m t}{\alpha(4\alpha^2 + 3)} \to 0$$

and $\dfrac{dk}{dd} \to 0$

$$12\,\alpha^2 = 3$$

$$\alpha^2 = 4$$

$\therefore\ \alpha = 2 \Rightarrow$ before and after $\alpha = 2$, the slope is negative

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3. Rearrange the deflection equation

$$\frac{\Delta_c}{h} = \frac{P}{E_m t}\left[4\left(\frac{\alpha}{h}\right)^3 + 3\left(\frac{\alpha}{h}\right)\right]$$

$$\delta = \frac{P}{E_m t h}[4\alpha^3 + 32]$$

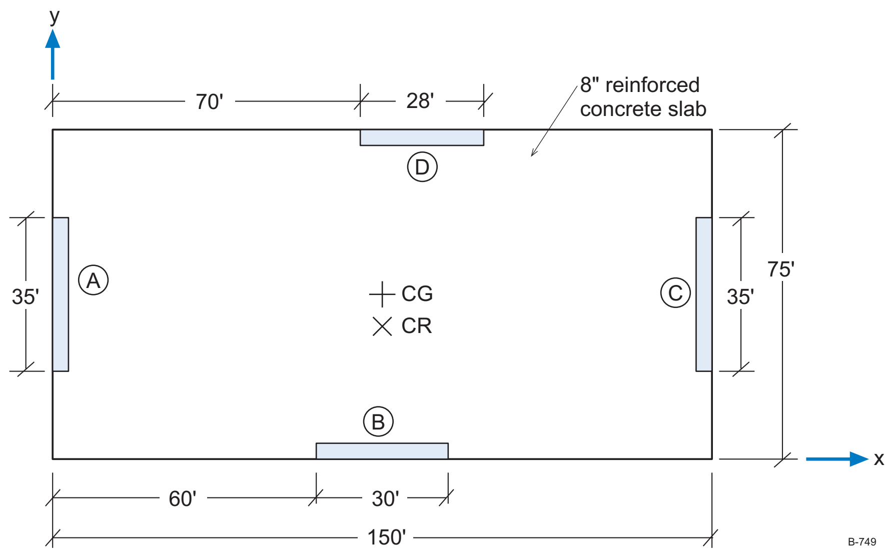
Example 4-5

The rigid diaphragm has four shear walls along the perimeter (A, B, C, and D).

8-inch CMU, fully grouted with reinforcement

$f_m = 2500$ psi, $h = 12$ feet

Determine the location of the center of gravity (CG) and the center of rigidity (CR).



1. Calculate the location of the center of gravity for a reinforced, rectangular concrete slab.

$$\bar{x} = \frac{150}{2} = 75 \text{ ft}$$

$$\bar{y} = \frac{75}{2} = 37.5 \text{ ft}$$

2. Calculate the center of gravity for the shear wall system.

Shear wall	h/l	R_i	x_i	y_i	$x_i R_{yi}$	$y_i R_{xi}$
A	12/35 = 0.34	9.44	0.0	–	0	–
B	12/30 = 0.40	7.91	–	0.0	–	0.0
C	12/35 = 0.34	9.44	150.0	–	1416	–
D	12/28 = 0.43	7.30	–	75.0	–	547.5
					$\Sigma = 1416$	$\Sigma = 547.5$

Notes:

(1) The shear wall is part of a multistory structure. Therefore, end fixity is taken as fixed-fixed.

(2) R_i can be determined from the *RMEH, 5th edition*, page 397-403, Table 1a through 1g. Only the relative values are important, so no adjustments are necessary.

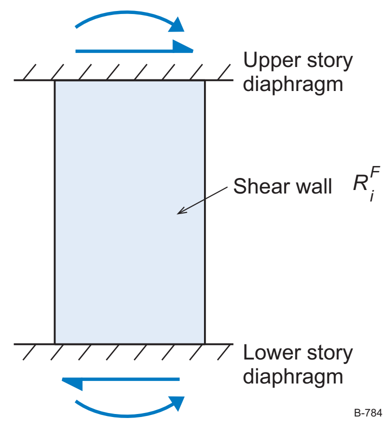
(3) The shear wall coordinaate for resistance in the y direction is x_i. The shear wall coordinate for forces in the x direction is y_i.

$$(\Sigma R_{yi}) = R_A + R_C = 9.44 + 9.44 = 18.88$$

$$\Sigma R_{xi} = R_B + R_D = 7.91 + 7.30 = 15.21$$

$$\therefore \quad \bar{x}_{CR} = \frac{\sum_{i=1}^{n} x_i R_{yi}}{\sum_{i=1}^{n}} = \frac{1416.0}{18.88} = 75$$

$$\therefore \quad \bar{y}_{CR} = \frac{\sum_{i=1}^{n} y_i R_{xi}}{\sum_{i=1}^{n} R_{xi}} = \frac{547.5}{15.21} = 36$$



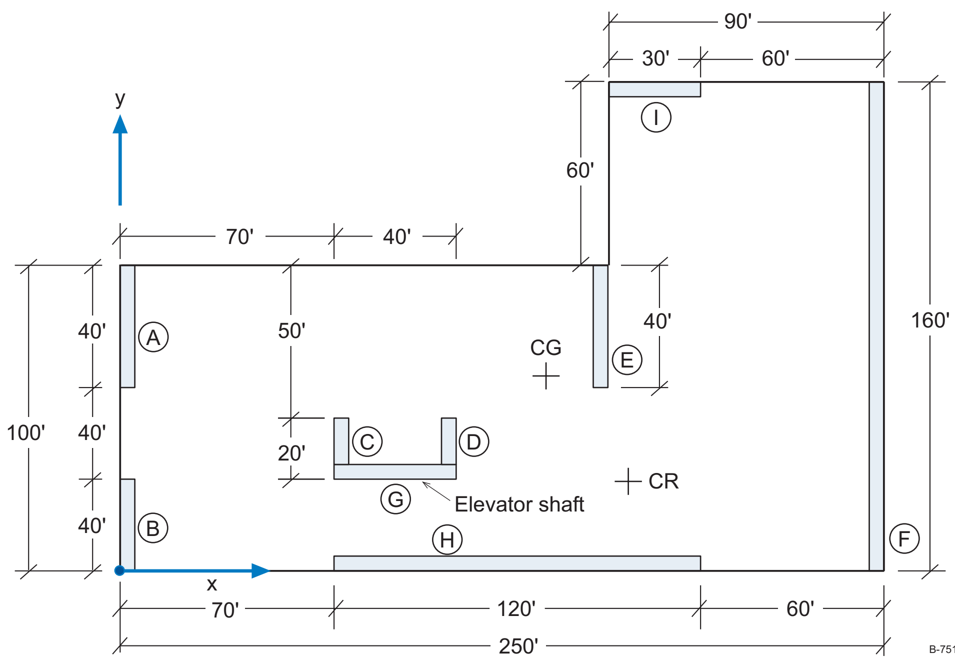
1) The location of $\bar{x}_{CR}$ would be at the geometric center because the two shear walls (A and B) are of equal rigidity and balance each other.
2) In the x direction, shear walls B and D are not equal and have a modest difference in rigidity values. Therefore, the $\bar{y}_{CR}$ = 36.0 is closer to shear wall B because this is the stronger shear wall.
3) An engineering solution should be evaluated for practical accuracy before implementing it in actual practice. Good judgment combined with experience is an invaluable asset and should never be underestimated.

Example 4-6

The L-shaped rigid diaphragm has nine shear walls (A through I). The reinforced concrete diaphragm is 10 inches thick (to support heavy mechanical and electrical equipment).

1. Determine the location of the center of gravity (CG).

2. Determine the location of the center of rigidity (CR).



Masonry walls in x direction are G, H, and I

Masonry walls in y direction are A, B, C, D, E, and F

All walls have fixed-fixed end connections

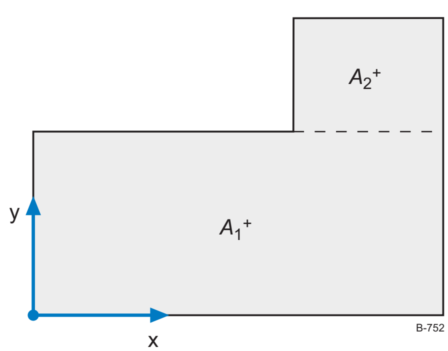
Wall height = 14 ft

12-inch CMU walls

$F'_m = 3000$ psi

1) To calculate location of the CG

Divide the diaphragm into two rectangular areas



$$A_1 = (250)\,(100) = 25{,}000 \text{ sq ft}$$

$$\bar{x}_1 = \frac{1}{2}(250) = 125 \text{ ft}$$

$$\bar{y}_1 = \frac{1}{2}(100) = 50 \text{ ft}$$

$$A_2 = (90)(60) = 5400 \text{ sq ft}$$

$$X_2 = 250 - 45 = 205 \text{ ft}, \qquad y_2 = 100 + \frac{1}{2}(60) = 130 \text{ ft}$$

i	A_i	$\bar{x}_i$	$\bar{y}_i$	$\bar{x}_i A_i$	$\bar{y}_i A_i$
1	25,000	125.0	50.0	3,125,000	1,250,000
2	5,400	205.0	130.0	1,107,000	702,000
	$\Sigma A_i = 30{,}400$			$\Sigma \bar{x}_i A_i = 4{,}232{,}000$	$\Sigma \bar{y}_i A_i = 1{,}952{,}000$

$$\therefore \; \bar{x}_{CG} = \frac{\Sigma x_i A_i}{\Sigma A_i} = \frac{4{,}232{,}000}{30{,}400} = 139.21 \text{ ft}$$

$$\therefore \; \bar{y}_{CG} = \frac{\Sigma y_i A_i}{\Sigma A_i} = \frac{1{,}952{,}000}{30{,}400} = 64.21 \text{ ft}$$

The location is plotted on the figure.

2) To calculate the location of the CR.

i	h/l	Rx_i	y_i	Ry_i	x_i	$y_i Rx_i$	$x_i Ry_i$
A	14/40 = 0.35	–	–	9.15	0.0	–	0.0
B	14/30 = 0.47	–	–	6.606	0.0	–	0.0
C	14/20 = 0.70	–	–	4.093	70.0	–	286.5
D	14/20 = 0.70	–	–	4.093	110.0	–	450.2
E	14/40 = 0.35	–	–	9.150	160.0	–	1464.0
F	14/160 = 0.09	–	–	37.780	250.0	–	9445.0
G	14/40 = 0.35	9.150	30.0	–	–	274.5	–
H	14/120 = 0.12	27.645	0.0	–	–	0.0	–
I	14/30 = 0.47	6.606	160.0	–	–	1057.0	–
		$\Sigma R_{xi} = 43.4$		$\Sigma R_{yi} = 70.9$		$\Sigma y_i R_{xi} = 1331.5$	$\Sigma x_i R_{yi} = 11{,}645.7$

$$*\Delta_F^E = 0.1(0.088)^3 + 0.3(0.088) = 0.0265$$

$$\therefore \; F_F^E = \frac{1}{\Delta_F^E} = 37.78$$

$$\therefore \; x_{CR} = \frac{\sum_{i=1}^{n} x_i R_{yi}}{\sum_{i=1}^{n} R_{yi}} = \frac{11{,}645.7}{70.9} = 164.3 \text{ ft}$$

$$\therefore \; y_{CR} = \frac{\sum_{i=1}^{n} y_i R_{xi}}{\sum_{i=1}^{n} R_{xi}} = \frac{1331.5}{43.4} = 30.7 \text{ ft}$$

5

Working Stress Design

5.1 Introduction

The applications, construction methods, and design methodologies of masonry have altered over time, but it remains an economical choice for construction material. This chapter presents the technical design provisions and analysis methods appropriate for masonry structures, particularly the technological advancements of finite element technology and computer software.

Study the relevant code and design manuals that dictate the guidelines that engineers must follow. The flowchart in Figure 5-1 shows the path whereby standards are developed, reviewed, and eventually approved by the ICC.

The fundamentals of masonry design have not changed for the past 40 years. However, during the late 1960s, the design process was streamlined by the introduction of computers and software. This has propelled the engineering profession toward more elaborate and advanced techniques that were impossible when using conventional hand-calculation methods. These include finite-element technology, dynamic analysis, response-spectrum analysis, and the simplification of calculations using basic spreadsheet software. Nevertheless, the old system of relying on standardized tables and charts is still the mainstay of masonry design and remains the choice for most structural engineers. Engineers must check their computer results and, for this reason, hand calculations and the basic theory of structural behavior should never be removed from the engineering curriculum.

This chapter introduces the fundamentals of Working Stress Design (WSD) with specific practical engineering applications. Notice that while WSD remains the prevalent design method for masonry, there is strong support among engineers to shift to Strength Design. This dispute may continue for several years, but it is unlikely that WSD will ever disappear from the scene. As an example, the American Institute of Steel Construction (AISC) has been promoting Load and Resistance Factor Design (LRFD) since the late 1960s, but it continues to deal with issues from engineers who insist on using the older but still viable, WSD method.

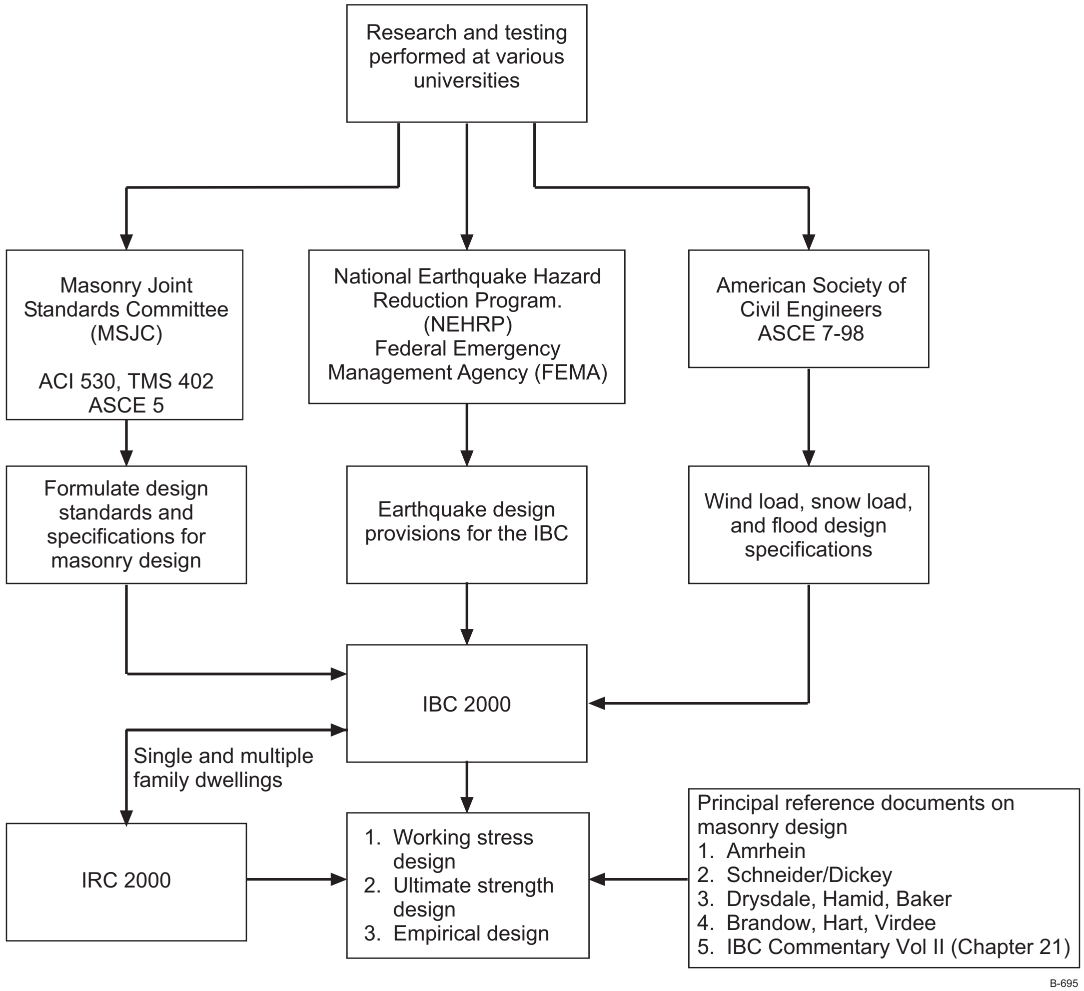


Figure 5-1

5.2 Analysis of Beams and Lintels

The basic concepts of beam analysis discussed in Chapter 3 also apply to this section. The major change since the 1997 UBC is that there are now two national codes governing the requirements for masonry design: the *International Building Code* (2000 IBC) and the *International Residential Code* (2000 IRC). In terms of application, both codes emanate from the basic reference for masonry, ACI 530/ ASCE 5/TMS 402. A schematic chart shows the flow of code information.

Using the 2000 IBC as a base, certain jurisdictions will make modifications to the umbrella requirements. The importance of these amendments to specific applications must be acknowledged. Some jurisdictional variations are shown in Figure 5-2. However, the purpose of this text is to focus on the engineering fundamentals and leave specific application details to the practicing professional.

The 2000 IRC simplifies the building design process for a large category of structures: single- and multiple-family dwelling units. These include single-family detached housing, town homes, and condominium structures not exceeding three stories. Of particular interest is that the 2000 IRC R606.1.1 specifically states, "When the empirical design provisions of ACI 530/ASCE 5/TMS 402 Chapter 5 or the provisions of this section are used to design masonry, project drawings, typical details, and specifications are not required to bear the seal of the architect or engineer responsible for design, unless otherwise required by the state law of the jurisdiction having authority." This gives tremendous latitude to the building community to develop documents based on the 2000 IRC. Because the 2000 IRC is an empirically based design code that minimizes the technical effort, it is referred to for completeness and clarity.

Figures 5-3a and 5-3b summarize masonry WSD design equations and specific code requirements. The chart extracts the applicable code provisions that are specific to beam design. The shear wall design sections are addressed separately in Section 5.3.

Essentially, the 2000 IBC has adopted the general provisions of ACI 530/ASCE 5/TMS 402 with only minor modifications.

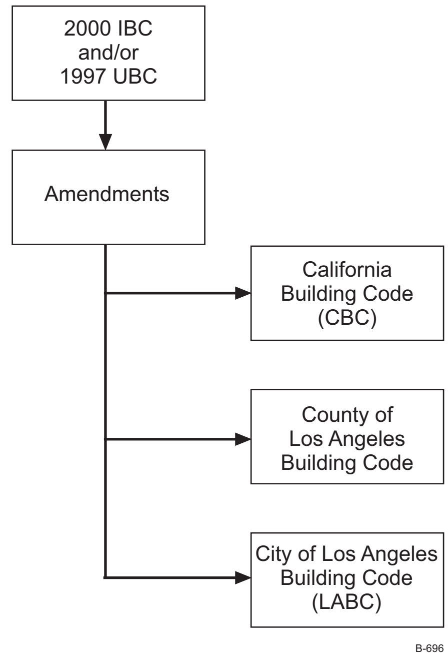


Figure 5-2

1 **Determine working loads and load conditions**

	Formula/number	
•	D	**16-7**
	$D + L$	**16-8**
	$D + L\ (L_r$ or S or $R)$	**16-9**
	$D + (W$ or $0.7E) + L + (L_r$ or S or $R)$	**16-10**
	$0.6D + W$	**16-11**
	$0.6D + 0.7E$	**16-12**
•	$D + L + (L_r$ or S or $R)$	**16-13**
	$D + L + (\omega W)$	**16-14**
	$D + L + \omega W + S/2$	**16-15**
	$D + L + S + \omega W/2$	**16-16**
	$D + L + S + E/1.4$	**16-17**
	$0.9D + E/1.4$	**16-18**

↓ 2

B-785

Figure 5-3a (continued)

2 **Preliminary section design**

1. Select f'_m – Design parameters $1500 < f'_m < 3000$ psi
2. Select b, d, A_s – Use 1% grade 60 steel, F_s = 2.4 ksi
   – select block/brick size
3. Calculate allowable stresses

F_b = 0.33 f'_m, 2000 psi (13.8 MPa) maximum
(use $^1/_2\, f'_m$ for no inspection)

F_s = 24 ksi (grade 60 steel)

or F_s = 20 ksi (grade 40 steel)

↓

3 **Summary of flexural formulas**

n = modular ratio = $\dfrac{E_s}{E_m}$

where E_s = 29,000 ksi
E_m = 750 f'_m, 3,000,000 psi (20.5 GPa) max
for clay, shale, or concrete unit **(Eq. 6-3)**
(Eq. 6-4)

ρ = steel area = $\dfrac{A_s}{bd}$

$k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n$

$j = 1 - k/3$, $K_s = f_s \rho j$ where $f_s = F_s$ = allowable steel stress
= 24 ksi (grade 60 steel)

$T_s = A_s f_s \Rightarrow$ moment capacity of steel section $= M_s = K_s bd^2$
moment capacity of masonry section $= M_m = {}^1/_2\, F_b kbd^2$
or define $= K_m = {}^1/_2\, F_b k$
$\therefore\ M_m = K_m bd^2$

↓ 4

B-786

Figure 5-3a (continued)

4 **Perform analysis for moment, shear, and deflection**

1) M_s, M_m ← Select M_{allow} = Min [M_s, M_m]

2) Compare M >/< M_{allow}

if $M > M_{allow}$, go to 2 — No → 2

Yes ↓

3) Shear capacity

$F_v = 1.0\sqrt{f'_m}$, 50 psi max — UBC **(7-19)**

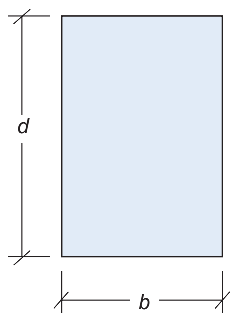
if $f_r > F_r$, go to 2 — No → 2

Yes ↓

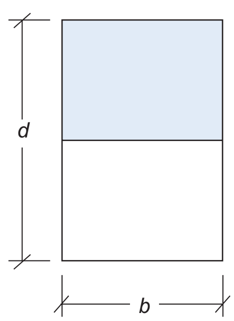
4) Deflection analysis

$(EI)_{eff} = {}^1/_2\,(E_m I_m)$

or $(EI)_{eff} = E_m I_{cr}$ ← cracked section modules



$I_g = bh^3/12$ $S_g = bh^2/6$



$$I_{cr} = \frac{b(kd)^3}{3} + (\rho nbd)(d - kd)^2$$

$(S_{cr})_n = I_{cr}/kd$ and $(S_{cr})_S = I_{cr} = I_{cr}/(d - kd)_n$

Check $\Delta < \Delta_{allow}$

if $\Delta > \Delta_{allow}$, go to 2 — No → 2

B-787

Figure 5-3a (continued)

(5a) **UBC special seismic load combinations**

$$1.2D + f_1L + 1.0\,E_m \qquad \textbf{(12-17)}$$

$$0.9D \pm 1.0\,E_m \qquad \textbf{(12-18)}$$

or

(5b) **2000 IBC, Seismic design category requirements**

2107.2.3 ACI 530/ASCE 5/TMS 402, Section 2.1.8.6.1.1, lap splices. The minimum length of lap splices for reinforcing bars in tension or compression, l_{ld} shall be calculated by Equation 21-2, but shall not be less than 15 inches (380 mm).

Eq. 21-2

$$l_{ld} \quad \frac{0.16 d_b^2 f_y \gamma}{K\sqrt{f'_m}}$$

For SI: $$l_{ld} \quad \frac{1.95 d_b^2 f_y \gamma}{K\sqrt{f'_m}}$$

where:

d_b = Diameter of reinforcement, inches (mm).

f_y = Specified yield stress of the reinforcement or the anchor bolt, psi (MPa).

f'_m = Specified compressive strength of masonry at age of 28 days, psi (MPa).

or $$\left.\begin{array}{l} E_m = 700\,f'_m \text{ for clay and shale} \\ E_m = 900\,f'_m \text{ for concrete units} \end{array}\right\} \text{IBC}$$

l_{ld} = Minimum lap splice length inches (mm)

K = The lesser of the masonry cover, clear spacing between adjacent reinforcement, or 5 times d_d, inches (mm).

γ = 1.0 for No. 3 through No. 5 reinforcing bars.
1.4 for No. 6 and No. 7 reinforcing bars
1.5 for No. 8 through No. 9 reinforcing bars.

2107.2.4 ACI 530/ASCE 5/TMS 402, maximum bar size. The bar diameter shall not exceed one-eighth of the nominal wall thickness and shall not exceed one-quarter of the least dimension of the cell, course, or collar joint in which it is placed.

2107.2.5 ACI 530/ASCE 5/TMS 402, splice for large bars. Reinforcing bars larger than No. 9 shall be spliced using mechanical connectors in accordance with ACI 530/ASCE 5/TMS 402, Section 2.1.8.6.3.

B-788

Figure 5-3a

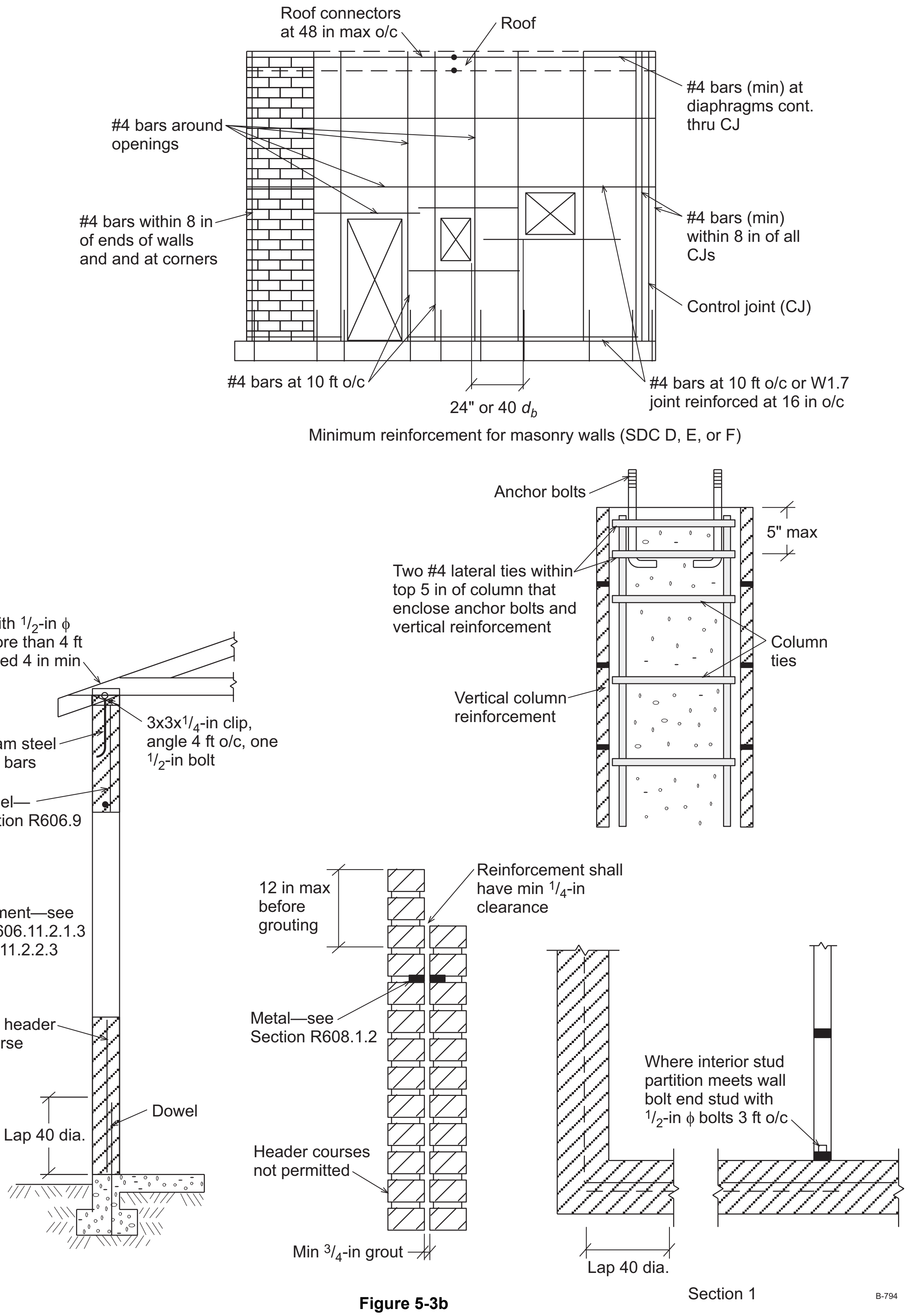

Roof connectors at 48 in max o/c
Roof
#4 bars around openings
#4 bars (min) at diaphragms cont. thru CJ
#4 bars within 8 in of ends of walls and and at corners
#4 bars (min) within 8 in of all CJs
Control joint (CJ)
#4 bars at 10 ft o/c
#4 bars at 10 ft o/c or W1.7 joint reinforced at 16 in o/c
24" or 40 d_b
Minimum reinforcement for masonry walls (SDC D, E, or F)
Anchor bolts
5" max
Two #4 lateral ties within top 5 in of column that enclose anchor bolts and vertical reinforcement
Column ties
Vertical column reinforcement
2-in plate with $^1/_2$-in ϕ bolts not more than 4 ft o/c embedded 4 in min
3x3x$^1/_4$-in clip, angle 4 ft o/c, one $^1/_2$-in bolt
Bond beam steel two $^1/_2$-in bars
Lintel steel—See Section R606.9
Reinforcement—see Section R606.11.2.1.3 and R606.11.2.2.3
Not header course
Dowel
Lap 40 dia.
12 in max before grouting
Reinforcement shall have min $^1/_4$-in clearance
Metal—see Section R608.1.2
Header courses not permitted
Min $^3/_4$-in grout
Where interior stud partition meets wall bolt end stud with $^1/_2$-in ϕ bolts 3 ft o/c
Lap 40 dia.
Section 1
B-794

Figure 5-3b

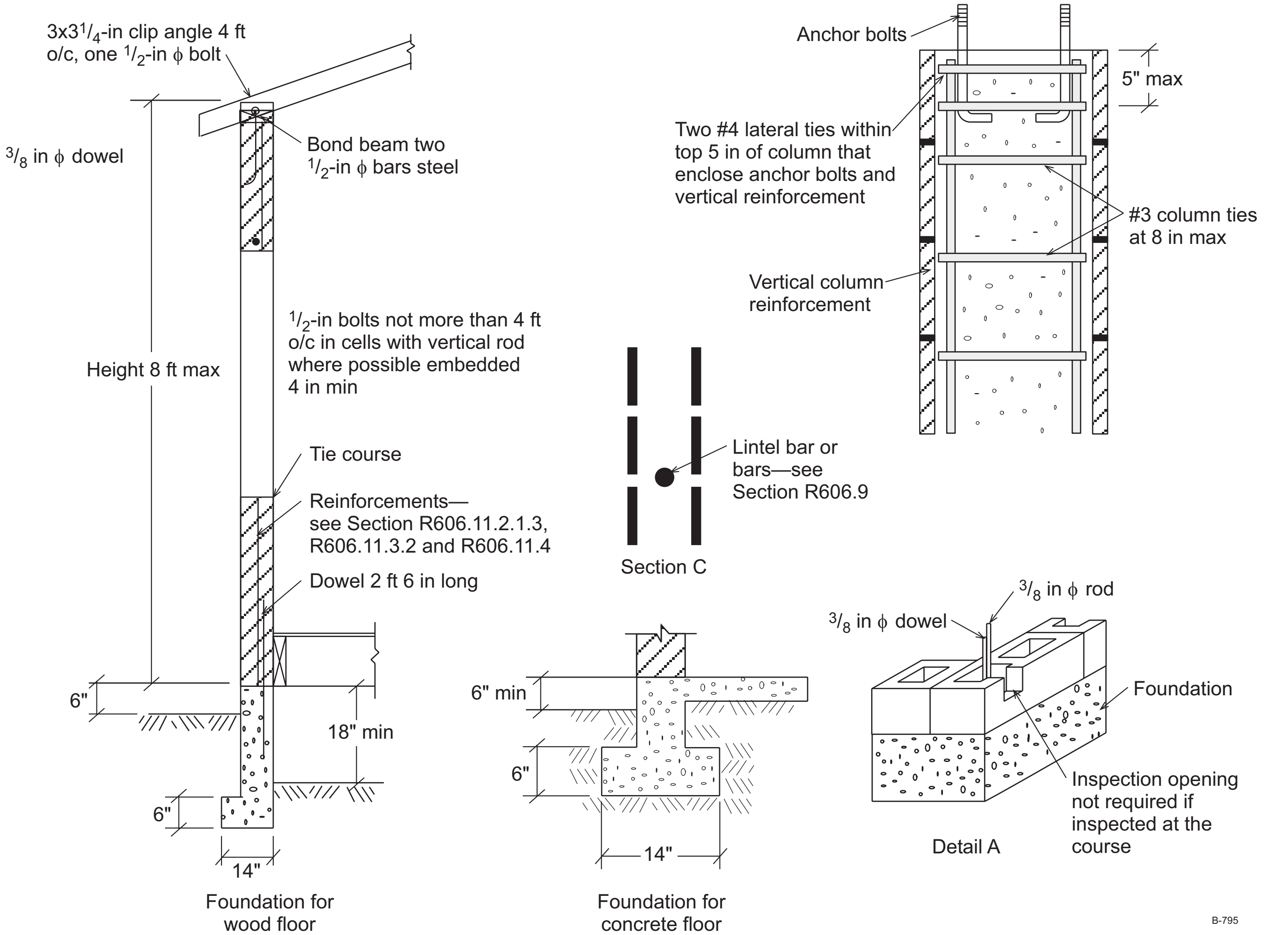


Figure 5-3b

Figure 5-4 is a three-dimensional view of a reinforced masonry beam showing stress and strain distribution. The WSD equations were discussed in Chapter 3; therefore, the beam design process is only summarized below.

Moment Capacity Analysis

Shear Analysis

Deflection Analysis Figure

Specific Code Provisions

1997 UBC

2000 IBC

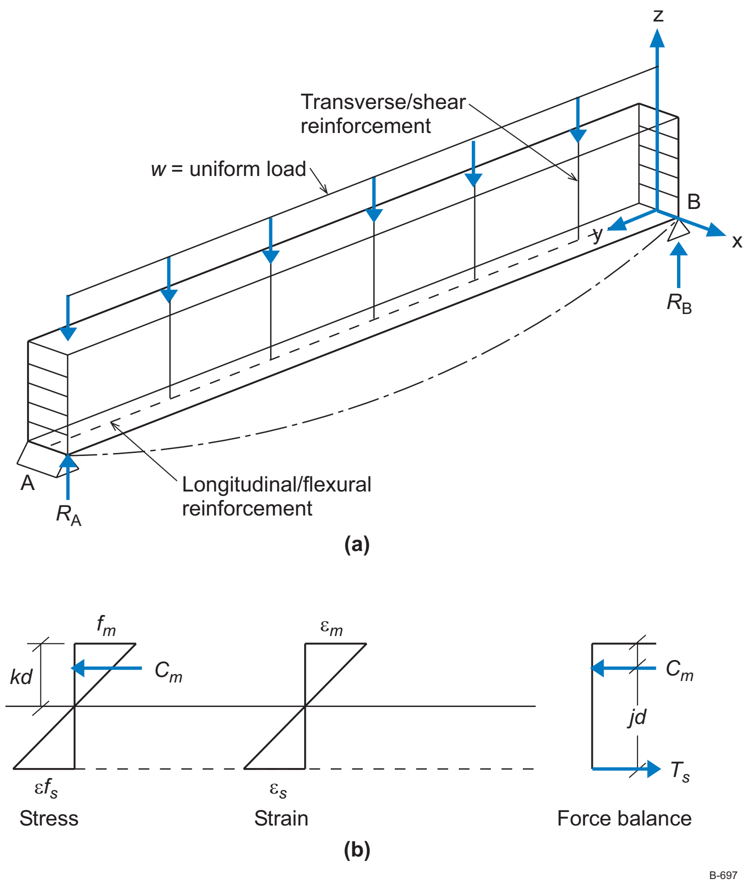

z
Transverse/shear
reinforcement
w = uniform load
B
y
x
R_B
A
R_A
Longitudinal/flexural
reinforcement
(a)
f_m
C_m
kd
ε_m
C_m
jd
T_s
εf_s
ε_s
Stress
Strain
Force balance
(b)
B-697

Figure 5-4

5.3 Shear Wall Analysis

All the relevant research and documentation on shear walls cannot be presented in one chapter because so much analytical and structural testing has been conducted during many years of international effort. Shear walls have been studied extensively by researchers, practitioners, and testing agencies. Briefly, shear wall buildings, which include both reinforced masonry and concrete, have been outstanding in their resistance to earthquakes, hurricanes, tornadoes, high winds, explosions, fire, and age. In many areas, the concept of bearing walls is used in lieu of shear walls. Bearing walls are designed for vertical loads. Lateral loads are excluded.

Because of this excellent record, structural engineers have recently been adopting more liberal design equations in order to use the full capacity of shear wall sections. This translates into the principles of Strength Design. The main structural features that contribute to the success of shear wall elements are summarized as:

high displacement and strain ductility performance

excellent in-plane and out-of-plane shear and moment resistance

superb nonlinear reserve capacity prior to actual failure

long term fire resistance

superior load-bearing capacity

A brief examination of each of these features reveals that:

1. Shear walls have excellent displacement and strain ductility (Figure 5-5). Parts (a), the displacement ductility, μ_Δ, and (b), the strain ductility, μ_ϕ, depict a shear wall's ductility performance. This concept was detailed in Chapters 3 and 4.

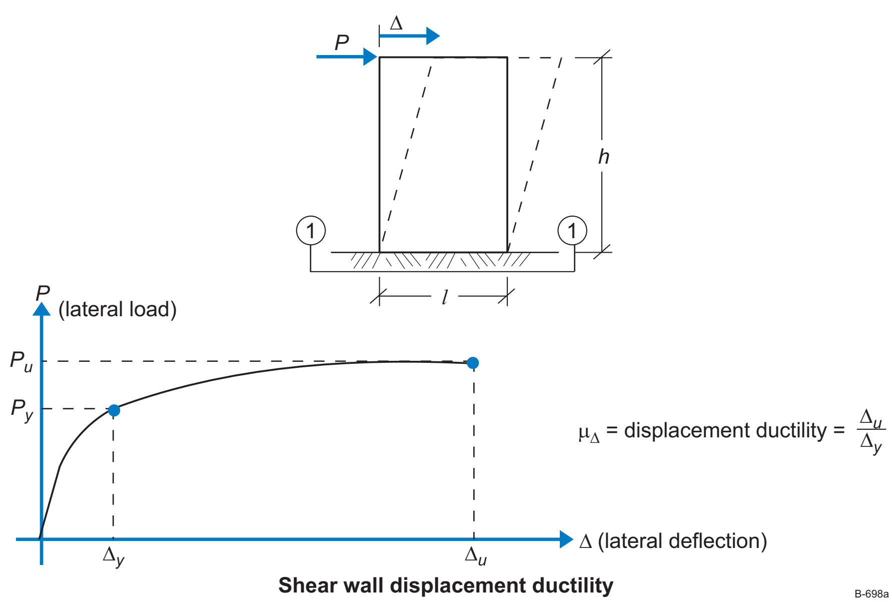


Figure 5-5a

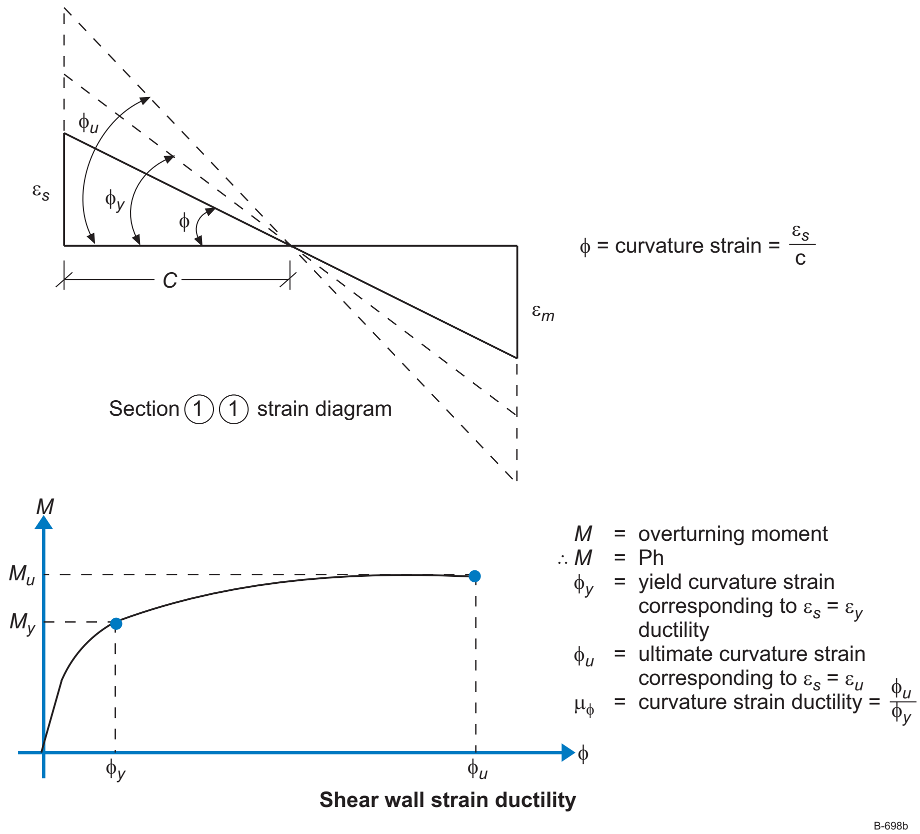


Figure 5-5b

2. In-plane and out-of-plane moment capacity is noteworthy because there are few structural elements other than a reinforced concrete shear wall that can compare with the resistance of masonry. This principle is shown in Figure 5-6.

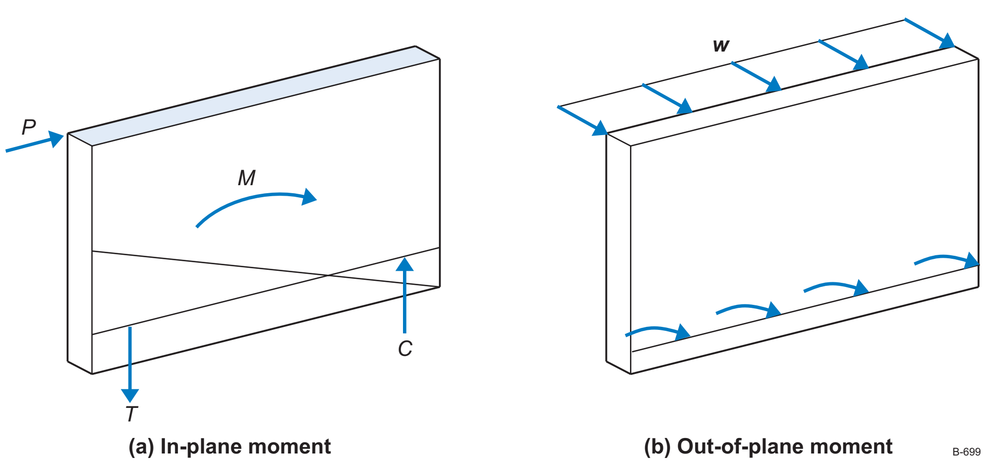


Figure 5-6

3. Nonlinear reserve capacity refers to the excess strain capacity beyond yield. This is similar to displacement ductility, but goes one step further in defining the actual failure (collapse) load. The collapse load for a reinforced masonry shear wall extends far beyond the ultimate strain point because steel fracture strains are as high as 7 to 8 percent ($\varepsilon_{fracture} - 0.07$ or 0.08). The excellent ductility of steel reinforcement has contributed to numerous cases of *preventive collapse*, a coined term that refers to a structure's ability to resist complete destruction. Ductility is an important element of this property, but preventive collapse is a result of ingenious design.

Figure 5-7 is a shear wall design-and-analysis flowchart. There are two distinct paths on the chart: in-plane and out-of-plane analysis. Of course, both are required because of geometric loading conditions. The flowchart organizes the existing design methodology, but it is not intended to replace engineering judgment.

The out-of-plane analysis segment covers the areas of technical evaluation presented in Chapter 3. The WSD equations are identical to those for beam analysis, except that the orientation is vertical rather than horizontal. Double reinforced sections are rare in shear walls but prevalent in retaining-wall design where the depth (i.e., wall thickness) must be limited to reduce material and construction cost.

Code requirements are divided into three categories: the 1997 *Uniform Building Code*, the 2000 *International Building Code*, and the 2000 *International Residential Code*. All derive from one parent document, the ASCE 5.

Several WSD methods for out-of-plane analysis are presented. Four of these come from the classic *Reinforced Masonry Engineering Handbook* (RMEH) and are based on fundamental concepts of reinforced concrete shear wall analysis.

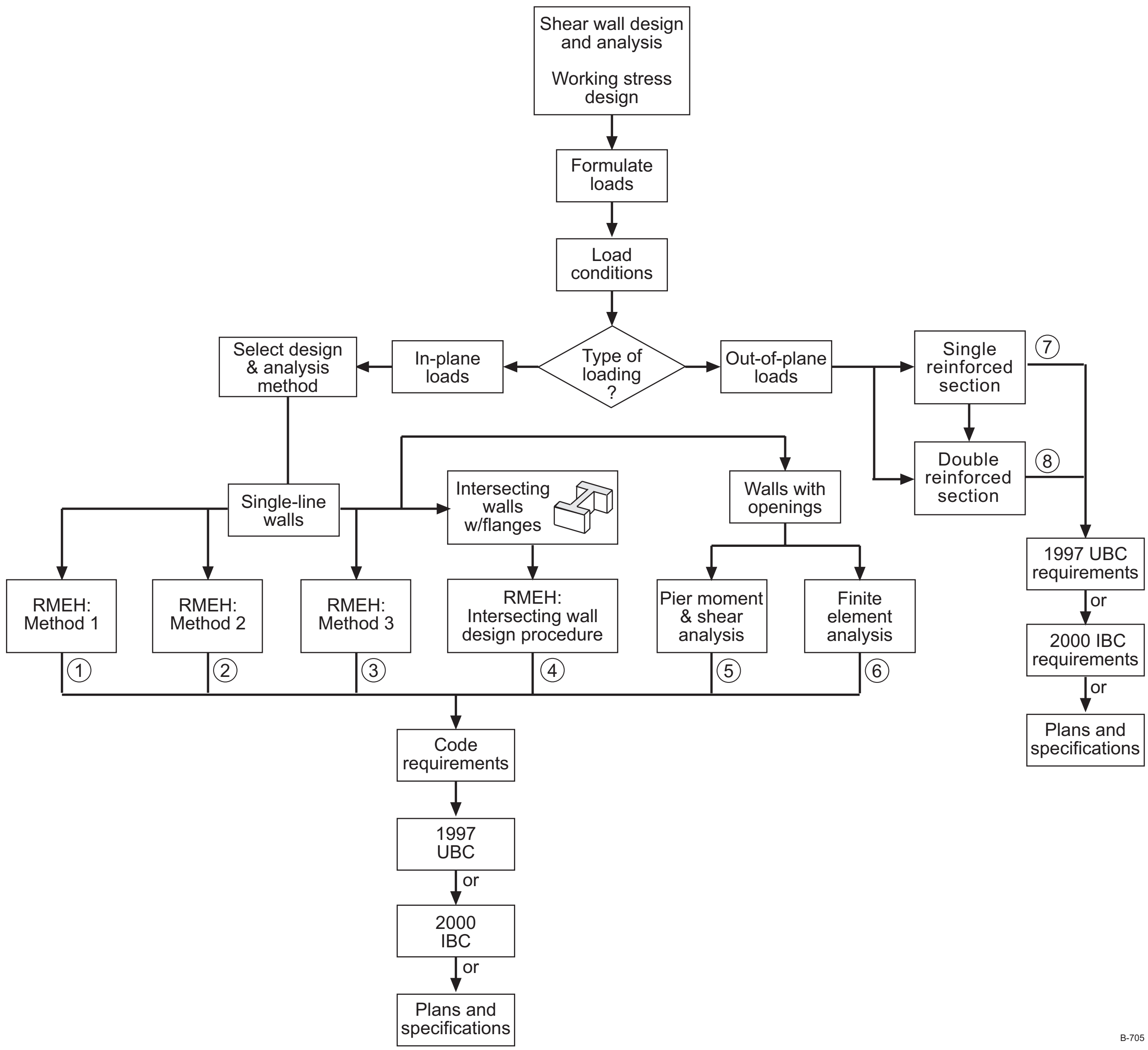

Shear wall design and analysis
Working stress design
Formulate loads
Load conditions
Type of loading ?
In-plane loads
Select design & analysis method
Out-of-plane loads
Single reinforced section
Double reinforced section
7
8
1997 UBC requirements
or
2000 IBC requirements
or
Plans and specifications
Single-line walls
Intersecting walls w/flanges
Walls with openings
RMEH: Method 1
RMEH: Method 2
RMEH: Method 3
RMEH: Intersecting wall design procedure
Pier moment & shear analysis
Finite element analysis
1
2
3
4
5
6
Code requirements
1997 UBC
or
2000 IBC
or
Plans and specifications
B-705

Figure 5-7

5.4 Finite Element Analysis of Shear Walls

The concept of Finite Element Analysis (FEA) has been known for more than 100 years. Originally, the early mathematicians (Euler, LaGrange, Newton, Leibnitze) formulated solutions to complex analytical problems using numerical methods. The Finite Difference Method was one of the first numerical analysis procedures implemented to solve complex differential equations. The concept of using finite difference methods is the genesis of the Finite Element Method that was later developed into a comprehensive procedure for structural analysis, temperature analysis, fluid flow, aerodynamics, and heat transfer, and for solving a multitude of technical problems.

With the swift advancement in computer technology, A platform has evolved that supercedes the technological capability of previous generations by several exponents. In the early 1960s, FEA was viewed as an esoteric technology reserved for the defense industry, to be used only by aerospace companies that could afford to maintain the computer capacity necessary to solve big structural problems. This view modified slowly during the 1970s and 1980s because of the expenses associated with mainframe computer systems. That particular situation was forever changed by the introduction of the personal computer (PC), which has brought this once specialized technology to the doorstep of the practicing structural engineering community at an affordable price.

To understand the pace of progress, compare the cost of performing an FEA in 1983 and in 2002. In 1983, a structural analysis program offered by MacNeil Shwindler Corporation (MSC), called MSC–NASTRAN, was the supreme lead software product in the industry with virtually no competition. The use of MSC-NASTRAN was charged for according to Computer Processing Unit (CPU) time. A problem of approximately 20,000 degrees of freedom (DOF) would take roughly two days of CPU time, and would cost $50,000 (1983 dollars). This fee covered the CPU time and the software license. One structural problem required 250,000 DOF, took 5 days to run, and cost the company $250,000 (1983 dollars) per execution. Imagine the cost of making a numerical error on the input file.

In 2003, the cost of a comparable PC is well under $2,000. Dozens of FEA packages are available. The typical price of an FEA software program for use with structural shear wall problems is around $1,000, with the more advanced programs selling for approximately $5,000 to $6,000. Design software programs sell for between $50 and $600, depending on the scope of the software. Keep in mind that these software and hardware prices are for the purchase of the program/equipment with unlimited usage. Some software companies still sell their FEA programs on a lease basis, but this is increasingly discouraged because of formidable competition in the industry. The 20,000-DOF static analysis problem can be solved on a Pentium PC (700 MHz) with a 40-GB harddrive in about 40 minutes. Add up the costs of the hardware and software, and the total cost of being fully equipped with a basic FEA system is about $4,000. For an advanced software package, the total cost is under $12,000. A structural engineer can design a 50-story building using just a basic software package.

Very little training on this subject has been available to structural engineers because it is still viewed as an advanced topic. This section presents a simplified and practical look at Finite Element Technology as it applies to masonry structures.

5.4.1 Finite element basics

To understand the basic premise of the Finite Element Method, take this example of the Quadratic equation

$$ax^2 + by + c = 0$$

which has the following solution

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This analytical answer is derived from the original problem statement. It represents two roots to the second degree polynomial.

There are many polynomials that may not have a straightforward analytical solution, such as the general third-degree equation

$$f(x) = ax^3 + bx^2 + cx + d = 0 \qquad \textbf{(Eq. 5-3)}$$

Extend this concept further to incorporate a general (n^th^) degree polynomial

$$f(x) = a_n x^n + a_{n+1} x^{n-1} + \ldots + a_1\,\mathrm{x} + a_0 = 0 \qquad \textbf{(Eq. 2-3)}$$

There is no direct solution for Equation 5-4, but a solution may be represented as shown in Figure 5-8.

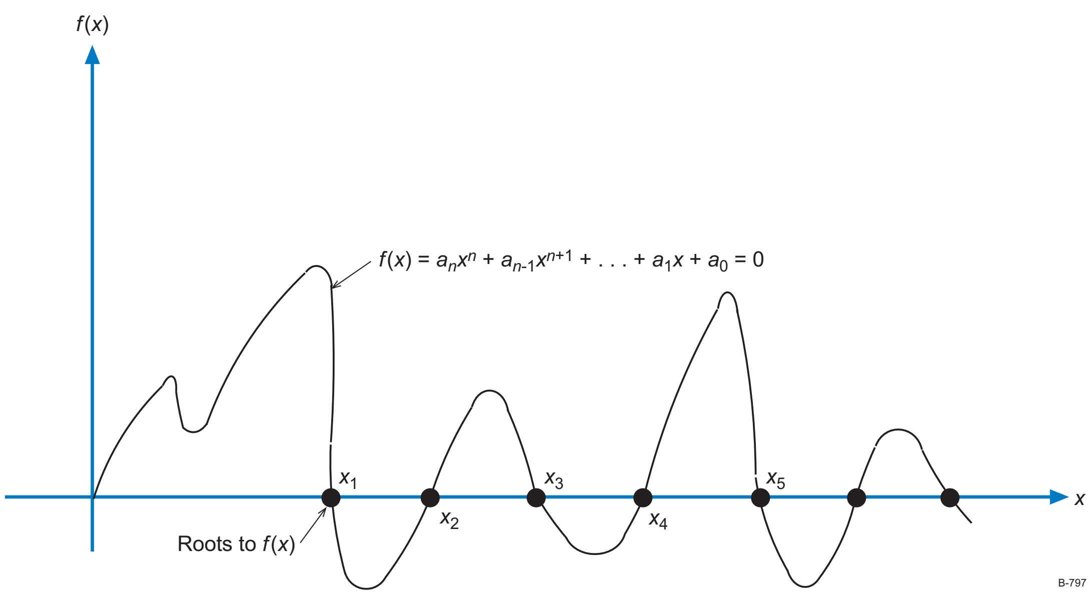


Figure 5-8

The process of solving Equation 5-4 using a numerical procedure involves dividing the function into segments (i.e., elements), and then systematically solving for each individual root, x_I. This procedure, dividing the problem statement into segments/elements and then solving each segment individually, is the cornerstone of the finite element process. The individual solutions can then be integrated to form a final answer.

Solving an n^{th} degree polynomial involves one of two numerical methods:

1) Newton-Raphson Method

2) Newton Method

Both involve an iterative equation that converges on a solution. (Figure 5-9)

$$X_{\eta+1} = X_{\eta} - \frac{f(x_{\eta})}{f'(x_{\eta})}$$

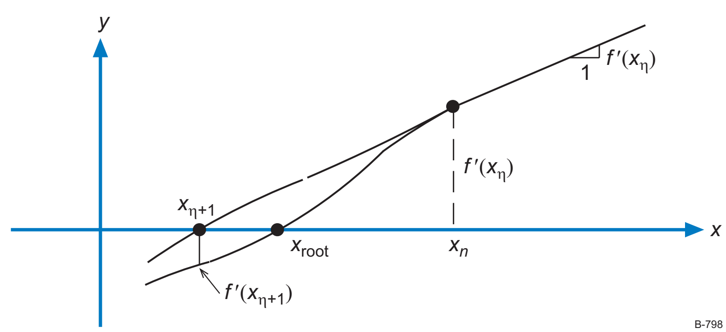


Figure 5-9

The process of iteration is termed *convergence*. At some point, a reasonable solution based on the difference (accuracy) between the calculated solution and the exact answer is obtained

$$\text{accuracy} = 100\% - \left(\frac{x_{\text{root}} - x_{\eta+1}}{x_{\text{root}}}\right) 100\%$$

Since the exact answer (x_{root}) is unknown in complicated problems, the accuracy may be expressed through convergence criteria

$$\varepsilon \equiv \frac{x_{\eta+1} - x_{\eta}}{x_{\eta}} \times 100\%$$

This leads to the practical definition of accuracy

$$\text{accuracy} = (100 - \varepsilon)\ \%$$

5.4.1.1 Structural analysis

An understanding of matrix methods and stiffness matrix concepts is necessary, and details may be found in several references.

The fundamental force-deflection equation is based on the stiffness matrix concept

$$\{F\} = [K]\,\{u\}$$

where

$\{F\}$ = $1 \times n$ force vector

$[K]$ = $n \times n$ stiffness matrix

$\{u\}$ = $1 \times n$ displacement vector

which may be written in expanded form as

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ . \\ . \\ . \\ F_n \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & \dots & K_{in} \\ K_{21} & & & & K_{in} \\ K_{31} & & & & K_{in} \\ & & . & & \\ & & . & & \\ & & . & & \\ K_{nl} & \dots & \dots & \dots & K_{nn} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ . \\ . \\ . \\ . \\ u_n \end{Bmatrix}$$

The basic assumptions associated with this fundamental equation must be emphasized.

1) Material is linearly elastic and follows

   $\{F\} = [K]\,\{x\}$

   Hooke's law: $\sigma = E\varepsilon$

2) Material is isotropic with linear elastic behavior about the three axes

   $E_x = E_y = E_z$

3) The limit of the applied loads is restricted to the yield point.

The force vector $\{F\}$ is linearly proportional to the displacement vector $\{x\}$. This proportionality factor is the stiffness matrix $[K]$. The basis for using elastic finite elements is conceptually derived from dividing the structure into pieces (i.e., finite elements).

Figure 5-10 is a simple example of a cantilever beam, with 6 nodes.

The analytical solution for a cantilever beam is

$$\delta = \frac{PL^3}{3EI}$$

$$M = PL$$

$$V = P$$

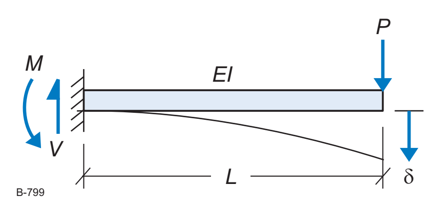


Figure 5-10

To analyze this problem with separate elements would involve dividing the beam into specific elements (Figure 5-11).

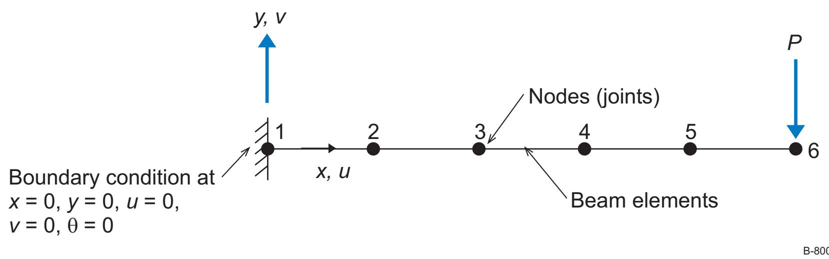


Figure 5-11

x = horizontal coordinate $\quad$ u = displacement along the x-axis

y = vertical coordinate $\quad$ v = displacement along the y-axis

EI = beam stiffness component

Because this finite element model is composed of six nodes, the two-dimensional grid provides for three degrees of freedom (DOF) per node.

Total DOF	= 6 nodes × 3 DOF	= 18
Restrained DOF	= 1 node × 3 DOF	= 3
Translational DOF	= 5 nodes × 1 DOF	= 5
Total DOF to be solved		= 10

The restrained DOF occur at the fixed connection (node 1) where the translation and rotation are fixed. (u, v, and θ are set to zero.)

Translational DOF along the x-axis (u) are set to zero because the longitudinal direction will experience small displacements (i.e., $u \approx 0$).

This leaves the total DOF to be solved equal to 10. The formulation of the problem is with a global stiffness matrix that is 10 × 10.

The displacement vector is

$$\{u\}_{1 \times 10} = \begin{Bmatrix} u_2 \\ \theta_2 \\ u_3 \\ \theta_3 \\ u_4 \\ \theta_4 \\ u_5 \\ \theta_5 \\ u_6 \\ \theta_6 \end{Bmatrix}$$

The force vector is

$$\{F\} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \\ F_7 \\ F_8 \\ F_9 \\ F_{10} \end{Bmatrix} = \begin{Bmatrix} F_{2x} \\ F_{2y} \\ F_{3x} \\ F_{3y} \\ F_{4x} \\ F_{4y} \\ F_{5x} \\ F_{5y} \\ F_{6x} \\ F_{6y} \end{Bmatrix}$$

The global stiffness matrix is formulated

$$\{F\} = 1 \times 10 \begin{bmatrix} K_{11} \ldots\ldots K_{1,10} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ K_{10,1} \ldots\ldots K_{10,10} \end{bmatrix}_{10 \times 10} \{u\}_{1 \times 10}$$

This matrix is an assembly of the local element matrix for each individual beam element (Figure 5-12). Each beam element has a force-displacement relationship based on linear elastic beam equations.

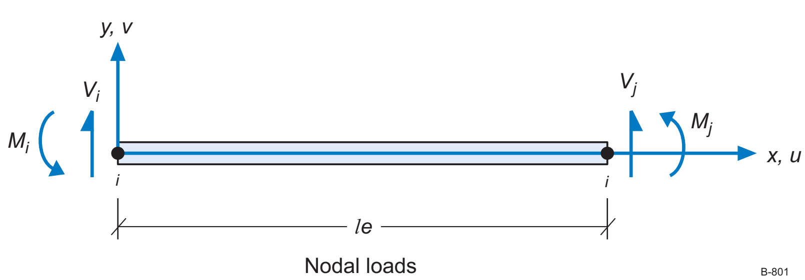


Figure 5-12

Local beam element displacements are represented in Figure 5-13.

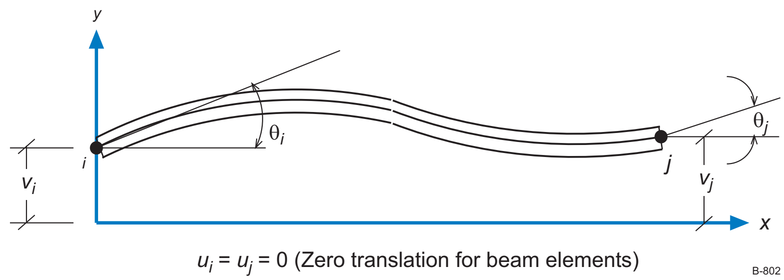


Figure 5-13

The stiffness matrix for the beam element is

$$[K]^e = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

The solution for the cantilever beam is found by setting the model load, $F_{10} = P$, and combining the global stiffness matrix to take the inverse and solve for the displacements $\{u\}$

(a) $\{F\} = [K]\{u\}$

(b) $\therefore \{u\} = [K]^{-1}\{F\}$

$$[K]^{-1} = \frac{[\tilde{K}]^T}{|K|}$$

where $[\tilde{K}]$ = cofactor matrix of $[K]$

and $[K]^T$ = transpose matrix

so, $[\tilde{K}]^T$ = transpose of the cofactor matrix $[K]$

$|k|$ = determinant of matrix $[K]$

The key to the solution of (a) is (b).

Performing these computations by hand is cumbersome; they may be accomplished systematically by using a computer.

For a complete definition of $[\tilde{K}]$, $[K]^T$, $[K]^{-1}$, and $|K|$, refer to Laursen, *Structural Analysis*.

Finite element problems may be solved using specialized software.

The following sample output is assembled in the local element axis as

$$\begin{Bmatrix} V_i \\ M_i \\ V_j \\ M_j \end{Bmatrix} = [K]^e \begin{Bmatrix} V_i \\ \theta_i \\ V_j \\ \theta_j \end{Bmatrix}$$

From a practical standpoint

1) Beam elements with lateral and longitudinal stiffness are a combination of truss and beam elements. These are normally used for structural members that support vertical transverse loads (beams, girders, lintels).

2) For masonry structures, shear wall elements are used for the in-plane and out-of-plane loads. These are modeled using plate elements.

The plate element configurations, shown in Figure 5-14 and 5-15 are classified as linear elastic isoparametric shell elements i.e., purely elastic stiffness elements. The use of FEA in masonry design is gaining acceptance slowly and will likely continue to do so.

Consider the following:

1) Elastic shell elements do not have a tensile failure envelope. Therefore, in zones that show tensile stress, such elements require steel reinforceing.

2) Mortar joints are weak points in masonry structures and are not physically modeled in a conventional FEA. Maintain this phisical restriction of the allowable shear stress when performing an FEA.

3) Eventually, nonliner analysis methods will become prevalent in masonry design. This will provide more powerful tools for analyzing masonry structures and considering tensile failure envelops, steel reinforcement interaction, mortar joint failure, and plastic behavior.

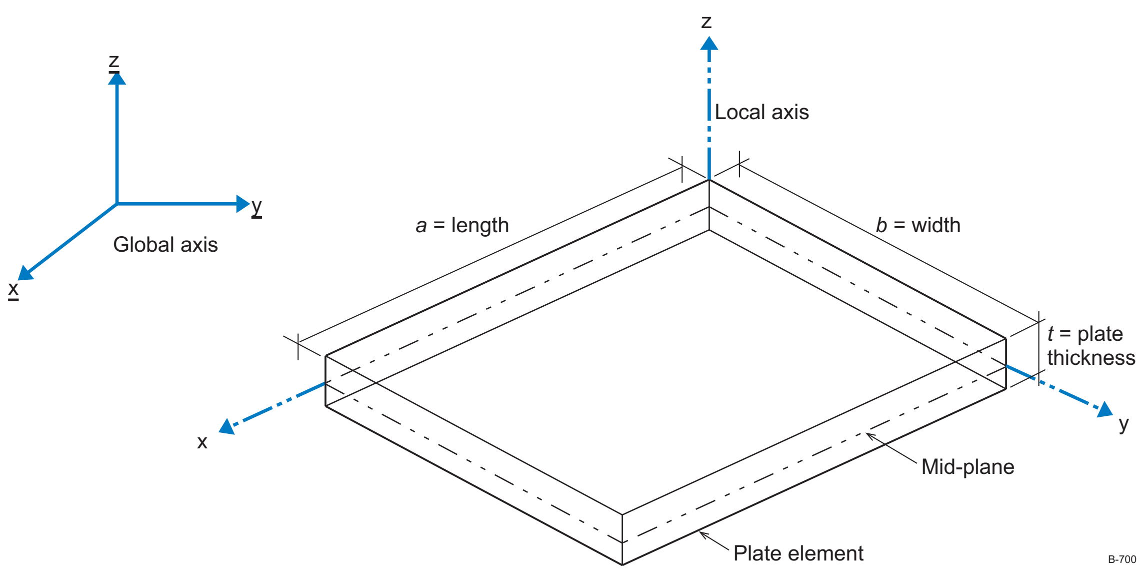


Figure 5-14

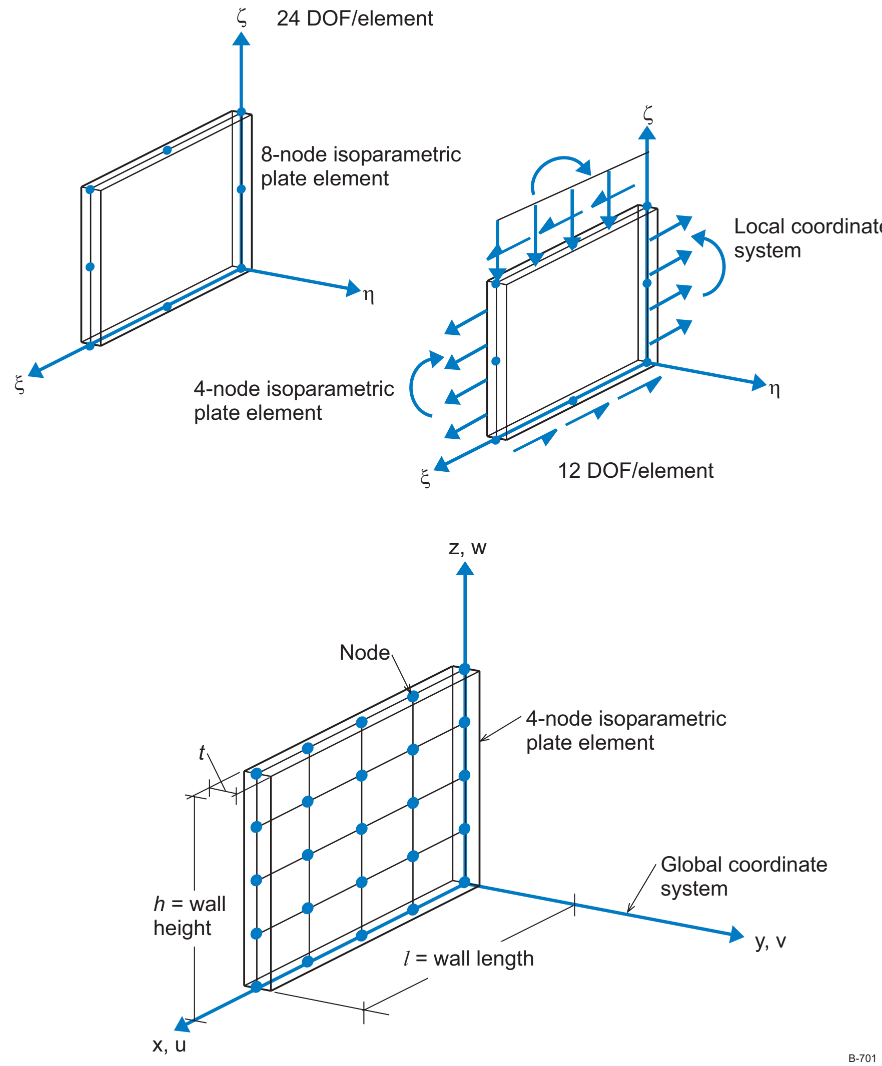


Figure 5-15

5-5. Practical Engineering Evaluation and Application

From the standpoint of structural safety, masonry buildings are in a low-risk category when compared to other buildings of similar height, size, and occupancy. In the United States, masonry buildings comprise a large percentage of warehouse, commercial, industrial, and office space occupancies that are combined with wood frame diaphragms. Typically, these structures seldom exceed three stories but have large floor-plan dimensions. The masonry shear walls perform in both the lateral force and vertical load carrying systems. A typical masonry building is framed as pictured in Figure 5-14, which shows the shear wall system connected to the floor diaphragm, and a typical foundation support consisting of continuous footings. Registered design professionals are interested in defining the areas of critical concern for designers, contractors, and owners.

Fortunately for structural engineers, problems with masonry buildings are seldom, if ever, structurally related. But architects and contractors must be concerned with long-term liability issues as outlined here in the order of their importance.

- **WATERPROOFING AND MOISTURE CONTROL**

Water penetration is of primary concern to those involved with masonry construction. Therefore, the following is brought to your attention.

1. Masonry structures are porous and will absorb large amounts of water if not adequately waterproofed. This absorption can result in long-term white deposits on the inside of a wall. While unsightly, this usually does not create a structural hazard unless allowed to continue over a long period of time (25 years or so).
2. The obvious solution to the problem of water absorption is waterproofing or dampproofing with a membrane material: a standard procedure requiring careful application.
3. Two issues relate to the use of membrane material as waterproofing: a) specifying the correct product, and b) ensuring its correct installation.
4. In cold, harsh winter environments, masonry walls that are exposed to repetitive freeze-thaw cycles with extreme temperature variations accompanied by moisture leakage will crack and demonstrate signs of damage. Such difficulties are especially prevalent in the northeastern United States because of the constant exposure to moisture.
5. Refer to Figure 5-15 for details.

- **TEMPERATURE AND SHRINKAGE CRACKING**

Expansion and contraction are caused by temperature fluctuation. Steel and masonry expand with rising temperatures and contract as the temperature lowers. Cracks develop in masonry walls as a result of this process, particularly along the mortar/bed joints (Figure 5-16). Standard practice required by the 1997 UBC and 2000 IBC is to employ reinforcing material in the form of shrinkage steel. This reinforcement creates a minimum bond strength that prevents the masonry units from separating along the joint lines.

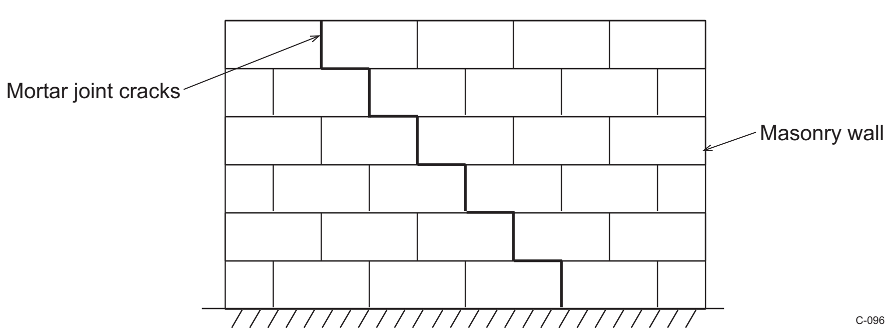


Figure 5-16

Shrinkage cracking is not a structural concern unless water seepage causes degrading of steel reinforcement.

To prevent reinforcement from becoming an issue after the fact, the design specifications must call for the installation of adequate shrinkage steel, and proper inspection of that installation must be guaranteed.

1. A solid design package will include clear specifications for placement of the steel (both shrinkage and structural). Construction notes must be concise and complete in every detail.
2. Periodic on-site inspection is recommended to assure construction quality and compliance with the design plans. This became a standard in the 2000 IBC where QA is the responsibility of the design professional and QC is a field issue.

- **FIRE DAMAGE**

Fire resistance and long term durability of reinforced masonry surpasses that of steel and wood and is uniformly comparative to that of reinforced concrete. The long-term effects on reinforced masonry resulting from fire are minimal and the end effect on structural capacity is usually of no consequence. Reinforced masonry has an excellent fire performance rating.

- **DISCOLORATION OF MASONRY AND BRICK**

Discoloration is the result of water penetration and moisture. Although not a structural issue and few structural engineers would care about this problem, it is a serious architectural issue. Therefore, providing sufficient waterproofing to avoid this occurence is necessary.

- **EARTHQUAKE DAMAGE – SHEAR CRACKING**

Cracking is often most pronounced around window and door openings. It is caused by shear stress concentration at the corner joint. The 90-degree corner joint creates a peak stress point and causes a high concentration of shear stress. This was demonstrated in the Finite Element Analysis in the Von Mises and Tau Stress distribution plots.

The solution is to provide adequate steel reinforcement with boundary elements. Thicker walls result in less damage, more steel provides greater ductility, so the detail connection points are a key element in the design of the building.

- **ROOF DIAPHRAGM CONNECTION FAILURE**

This kind of structural failure is common during earthquakes. The lateral displacement of the roof diaphragm causes out-of-plane pull-out forces on the diaphragm anchorage. These pull-out forces may cause separation of the roof from the shear wall. While technically not part of masonry shear wall design, it is a primary structural connection because failure results in near total collapse. There are two practical points for structural engineers to remember.

1. *A properly designed masonry shear wall does not preclude the failure of the roof diaphragm anchorage connection. Consideration of all the elements of the structure is necessary. The truth of the statement "The chain is only as strong as its weakest link" is readily apparent.*

2. *Load analysis of the out-of-plane anchorage requires careful design consideration: particularly the embedment depth and torque requirements of the bolt connection.*

- **TORSION OF THE RIGID DIAPHRAGM**

The problem of torsion has pervaded the wood shear industry during the past decade because of the issues posed around wood diaphragms. It is reassuring to note that in masonry shear wall structures the problem/solution has long been part of the design protocol. Therefore, its consideration is important to the use of reinforced concrete diaphragm systems.

Structural failures related to torsion are usually the result of high aspect ratios in diaphragms. Keeping diaphragm aspect ratios under 2.0 prevents problems with torsion, and having balanced lateral stiffness in the shear wall design is imperative.

- **SETTLEMENT/SINKING DUE TO INADEQUATE GEOTECHNICAL ENGINEERING**

Surprisingly, many buildings are constructed without adequate soils investigation. A structural engineer may assume soil conditions based on typical bearing pressure values from the code and then present a design for approval.

1. *Get a soils report! This may cost the owner, but it is worth it.*
2. *The geotechnical engineer must review the foundation design.*
3. *The civil engineer must review the on-site grading, site plan, and drainage concept.*

- **INSPECTION AND QUALITY CONTROL**

The 2000 IBC contains a detailed list of requirements for the registered design professional. Chapter 17 covers proper field inspection and quality control specifications.

1. *Follow the requirements of the 2000 IBC Chapter 17 and draft a detailed Quality Control/Assurance (QC/QA) program for the project.*
2. *Retain the services of a Licensed Deputy Inspector to perform these tasks and report directly to the structural engineer and/or architect. Do not let the contractor perform this task!*
3. *Retain the services of a qualified test lab to perform random field tests of the masonry materials, reinforcement, mortar and grout, and evaluate construction practice.*
4. *Require a final inspection report from both the test lab and inspector directed to the owner.*
5. *Enforce these requirements and be prepared to stop the work if necessary.*
6. *Communicate regularly with the local building department/official regarding inspections and approvals.*

Example 5-1

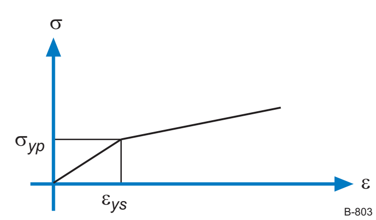
Reply to the following.

1. **What are the basic assumptions in elastic design of a flexural member?**
2. **Is strain compatible with stress? What is its significance with respect to compression steel?**
3. **What is the modular ratio? What is its significance?**
4. **Explain the function of the flexural coefficient, *K*. How does it vary from an under-reinforced section to an over-reinforced section?**
5. **Given: a 10-inch (nominal) reinforced concrete masonry retaining wall, cantilevered with vertical steel #6 bars 24 inches on center.**
   a. **What is the maximum *d* value for which this wall could be designed?**
   b. **Locate the neutral axis by means of transformed areas if this wall is solid grouted and f'_m is 2500 psi.**
   c. **If the reinforcing steel has a maximum allowable stress of 24,000 psi, what is the allowable moment for the section?**

****

Solutions

1. Basic assumptions
   a. Stress is proportional to strain, and Hooke's Law prevails

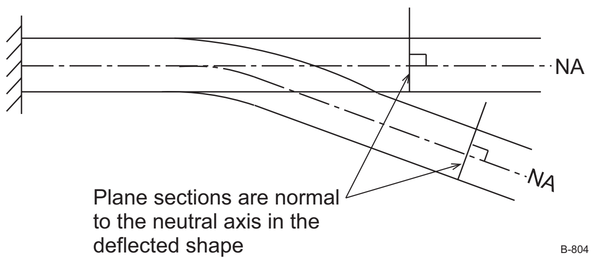


This implies the linear stress-strain curve for the reinforcement and the masonry.

   b. Masonry has zero tension capacity

$$f_t = 0$$

   c. Plane sections remain plane before and after bending

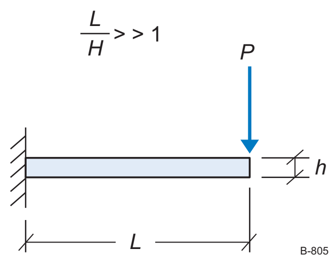


d. Modulus of elasticity is constant. Specifically, linear elastic behavior

$$E_m = \text{constant}$$

$$E_s = \text{constant}$$

e. Large span-to-depth ratio



f. Homogenous and isotropic behavior

g. External and internal force and moment equilibrium

$$\Sigma F_x = 0, \quad \Sigma F_y = 0, \quad \Sigma F_z = 0$$

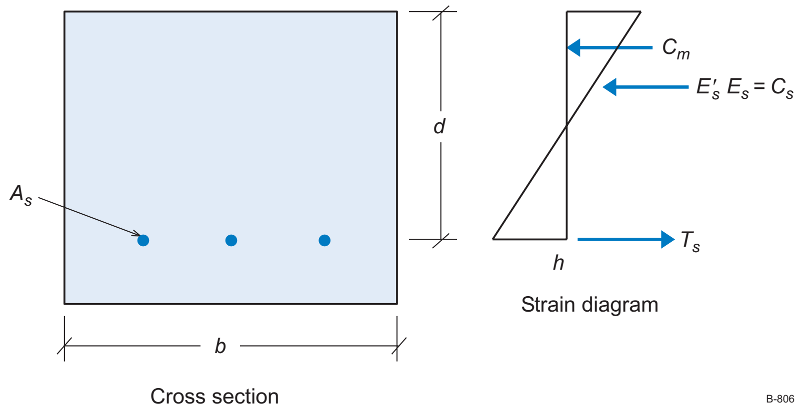
$$\therefore \Rightarrow \Sigma M_x = 0, \quad \Sigma M_y = 0, \quad \Sigma M_z = 0$$

h. Member is a uniform cross section

2. Yes, because $f_s = E_s \varepsilon_s$

Therefore, the strains are compatible with stresses to satisfy the basic linear assumptions.

With regard to compression steel, this affects the force balance equation because the compression force must be included with the equilibrium equations.



3. $n = E_s/E_m$ = modular ratio

   based on E_s = 29,000 ksi

   and E_m = $750f'_m$ (UBC)

   for 2000 IBC

   E_m = $900f'_m$ (concrete block)

   or $700f'_m$ (clay block)

   n is the basic parameter for masonry design and affects all the results for selection of steel and spacing.

   Selection of f'_m, f_s, loads, and dimensions constitutes the overall design problem, and they are selected by the designers.

4. K = flexural coefficient = $\rho j F_s$ (for steel)

   $K_s = \rho j F_s$

   $K_m = \frac{1}{2} kjFb$

   where $\rho = A_s/bd$, $j = 1 - \frac{k}{3}$

   and $k = \sqrt{(pn)^2 + 2pn} - (pm)$

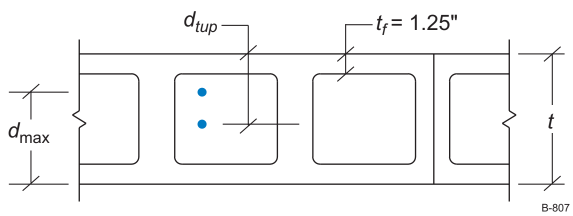
   The flexural coefficient provides a "short cut" solution to calculate masonry moment capacity using established tables, charts, or spreadsheets.

   $M_s = K_s bd^2$

   $M_m = K_m bd^2$

5. a. Maximum d value depends on the block dimensions.

A typical d value is to place the reinforcement at the center of the cavity



For a 10-inch CMU, $t = 10 - \frac{3}{8}$ in $= 9\frac{5}{8}$ in $= 9.625$

typical d value $= \frac{t}{2} = \frac{9.625}{2} = 4.81$ in

To calculate the maximum d, subtract the face shell thickness and assume #9 bars and $^1/_2$-inch grout.

$$d_{max} = t - t_f - \frac{1}{2}(d_b)$$

$$\therefore\ d_{max} = 9.625 - 1.25 - \frac{1}{2}(1) = 7.88 \text{ in}$$

Assume the reinforcement is at the center.

b. To locate the neutral axis (NA), calculate n and solve for k

$E_s = 29{,}000$ ksi, $F_s = 24$ ksi

$E_m = 750\ (2500) = 1875$ ksi **(UBC)**

$$\therefore\ n = \frac{E_s}{E_m} = \frac{29{,}000}{1875} = 15.5$$

Calculate ρ for #6 bar @ 24 inches o/c, solid grouted.

$$A_s = \frac{0.44}{2\ \text{ft}} = 0.22\ \text{in}^2/\text{ft}$$

$$\rho = \frac{0.22\ \text{in}^2}{(12)(4.81)} = 0.0038\ (0.38\%)$$

$$\therefore\ \rho n = 15.5 \times \rho n = 0.0589$$

$$\therefore\ k = \sqrt{(0.0589)^2 + 2(0.0589)} - (0.0589)$$

$$\therefore\ k = 0.29$$

Neutral axis location = kd

$$\therefore\ kd = 0.29\ (4.81) = 1.89 \text{ in}$$

c. Allowable moment capacity

$$J = 1 - \frac{0.29}{3} = 0.90; \quad F_b = 0.33f'_m = 0.33\ (2500) = 825\ \text{psi}$$

$$M_m = \frac{1}{2}\ F_b jk\ bd^2\ \frac{1}{2}(0.29)(0.90)(0.825)(12)(4.81)^2$$

$$M_m = 29.9\ \text{in-k}$$

$$M_s = pj\ F_s\ bd^2\ (0.0038)(0.90)(24)(12)(4.81)^2$$

$$M_s = 22.8\ \text{in-k steel governs}$$

$$\therefore\ M_{allow} = M_s = 31.6\ \text{in-k}$$

2000 IBC solution

$$E_m = 900f'_m = 2250\ \text{ksi}$$

$$n = \frac{29{,}000}{2250} = 12.9$$

$$\rho n = -\ 0.0032 \times 12.9 = 0.04902$$

$$k = \sqrt{0.04902^2 + 2(0.04902)} - 0.04902 = 0.29$$

Neutral axis location = $kd = 0.27(4.81) = \underline{\underline{1.30}}$ in

$$j = 1 - \frac{k}{3} = 0.91$$

$$F_b = 0.33f'_m = 0.825\ \text{ksi}$$

$$M_m = \frac{1}{2}\ F_b jk\ bd^2$$

$$M_m = \frac{1}{2}(0.27)(0.91)(0.825)(12)(4.91)^2 = 28.1\ \text{in-k}$$

$$M_s = F_s\ pj\ bd^2$$

$$M_s = (0.0038)(0.91)(24)(12)(4.81)^2$$

$$M_s = \underline{\underline{23.0}}\ \text{in-k . . . steel governs}$$

$$M_{allow} = 23.0\ \text{in-k}$$

Example 5-2

Complete the following exercise.

1. **From basic principles, establish the following values for a rectangular section for f'_m = 2250 psi, F_s = 18,000 psi, F_y = 60 ksi, CMU block with ε_m = 0.003**

   a. balanced steel ratio, p_b

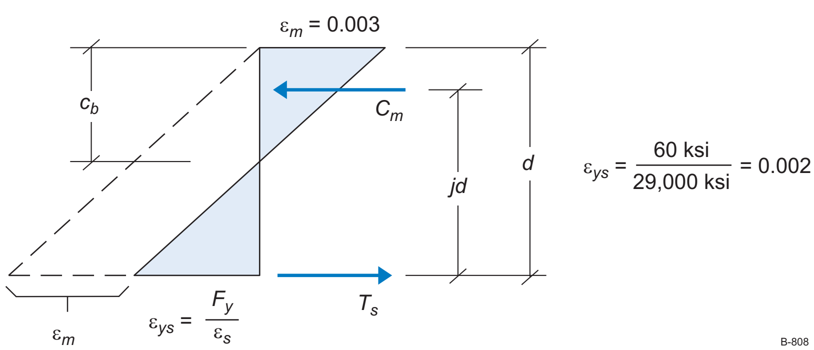
   b. j_b value for balanced condition

   c. balanced flexural coefficient, k_b

****

Solutions

By definition, balanced failure is:



From the strain diagram

$$\frac{\varepsilon_m}{c_b} = \frac{\varepsilon_{ys}}{d - c_b}$$

$$c_b\,\varepsilon_{ys} = \varepsilon_m\,(d - c_b)$$

$$\therefore\; c_b\,(\varepsilon_{ys} + \varepsilon_m) = \varepsilon_m d$$

$$\therefore\; c_b = \frac{\varepsilon_m d}{\varepsilon_{ys} + \varepsilon_m}$$

$$\therefore\; c_b = \left(\frac{0.003}{\frac{F_y}{\varepsilon_s} + 0.003}\right) d$$

$$a_b = 0.85\,c_b = \beta c_b$$

$$C_{mb} = T_{sb}$$

$$0.85f'_m\, a_b b = A_{sb}F_y$$

$$A_{sb} = \frac{0.85f'_m b}{F_y}$$

$$\rho_b = \frac{A_{sb}}{bd} = \frac{\left(\frac{0.85f'_m b}{F_y}\right)(\beta)\left(\frac{0.003}{\frac{F_y}{\varepsilon_s} + 0.003}\right)d}{bd}$$

a. $\Rightarrow$ Balanced Steel Ratio $\therefore\ \rho_b = \dfrac{0.85f'_m(0.003)}{F_y\left(0.003 + \frac{F_y}{\varepsilon_s}\right)}$

for $F_y = 40$ ksi, $\beta = 0.85$, $f'_m = 2250$, $\varepsilon ys = \dfrac{40{,}000}{29{,}000{,}000} = 0.0014$

$$\rho_b = \frac{(0.85)(0.85)(2250)(0.003)}{40{,}000\ (0.0014)} = 0.087$$

b. $k_b = \rho_b j_b F_s$ **2000 IBC Solution**

$$n_s = \frac{E_s}{E_m} = \frac{29{,}000}{900f'_m} = \frac{29{,}000}{2025} = 14.3$$

$$\rho_b n = 0.087(14.3) = 1.2441$$

$$k_b = \sqrt{(1.2441)^2 + 2(1.2441)} - 1.2441$$

c. $k_b = 0.77$

$$j_b = 1 - \frac{0.77}{3} = 0.75$$

Example 5-3

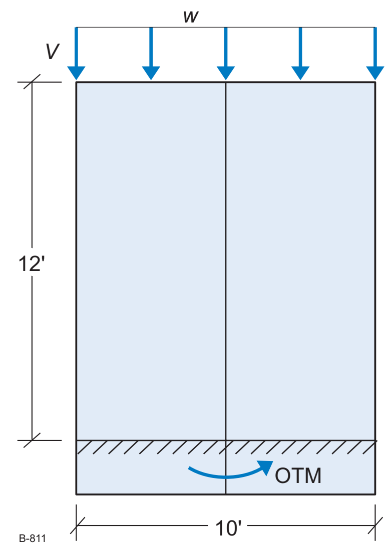
Using the information supplied, perform the following exercises.

1. **Design the tension reinforcing steel and specify the minimum allowable strength of masonry, f'_m , for a wall subjected to axial load and seismic overturning moment.**

   The wall is a nominal 10 inches thick, 10 feet long, and 12 feet high

   $F_s = 24{,}000$ psi

   Axial load = 100 kips, overturning moment = 300 ft-kips parallel to the wall



2. **Based on Method 2 (RMEH), calculate the steel reinforcement requirement. Check this against the IBC minimum steel requirements.**

   Given:

   In-plane loads

   $P = wl = 100$ k

   OTM = 300 ft-k

   t_{nominal} = 10-in CMU block

   $t = 9.63$ in

****

Solutions

1. Specify minimum f'_m **2108.7.4**

   $f'_m = 1500$ psi

2. Step 1 – using Method 2

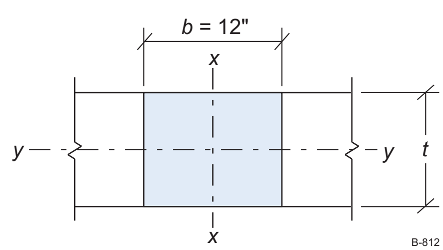
$$f_a = \frac{P}{lt} = \frac{100}{(10 \times 12)(9.63)} = 0.087 \text{ ksi}$$

$$I_y = \frac{bt^3}{12}$$

$$A = bt$$

$$\therefore \frac{I}{A} = \left(\frac{bt^3}{12}\right)\left(\frac{1}{bt}\right) = \frac{t^2}{12}$$

$$\therefore r = \sqrt{\frac{I}{A}} = \frac{t}{\sqrt{12}} = 0.29t$$



In the case of shearwall, b is replaced with L.

For 8-inch CMU $t = 7.63 \quad \therefore \quad r = 0.29\,(7.63)$

$r = 2.21$

10-inch CMU $t = 9.63 \quad \therefore \quad r = 0.29\,(9.63) = 2.80$

12-inch CMU $t = 11.63 \quad \therefore \quad r = 0.29\,(11.63) = 3.37$

$$\frac{h'}{r} = \frac{12 \times 12}{2.30} = 51.4$$

$$F_a = 0.25(1500)\left[1 - \left(\frac{144}{140 \times 2.80}\right)^2\right] = 324 \text{ psi}$$

Step 2 – Evaluate per interaction formula

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.33 \quad \text{. . . (equals Allowable Stress increase per IB 1605.3.2)}$$

$$\therefore\ F_b = F_b\left\{1.33 - \frac{f_a}{F_a}\right\}$$

where

$$F_b = 0.33f'_m = 0.33(1500) = 500 \text{ psi} = 0.5 \text{ ksi}$$

$$\frac{f_a}{F_a} = \frac{0.087}{0.326} = 0.27$$

$$F_b = 0.5\ \{1.33 - 0.27\} = 0.53 \text{ ksi}$$

$$f_m = f_b + f_a = 0.53 + 0.087 = 0.617 = 617 \text{ psi}$$

Step 3 – Solve quadratic formula

$$a = \frac{1}{6}tf_m = \frac{1}{6}(9.63)(0.617) = 0.99$$

$$b = -\frac{1}{2}tf_m\,(l - d') = -\frac{1}{2}(9.63)(0.617)(108) = -321$$

assume $d' = 0.1L = 12$ in

$l - d' = 120 - 12 = 108$ in

$$c = P\left(\frac{l}{2} - d\right) + M$$

$$c = 100\left(\frac{120}{2} - 12\right) + 300\ (12)$$

$$c = 8400$$

$$kd = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

@Seismicisolation

$$\therefore\ kd = \frac{+321 - \sqrt{321^2 - 4(0.99)(8400)}}{2(0.99)}$$

$$\therefore\ kd = 28.4 \text{ in}$$

Step 4 - Solve for force distribution

$$C_m = \frac{1}{2} tkd\, f_m = \frac{1}{2}(9.63)(28.4)(0.617)$$

$$C_m = 84.4 \text{ k}$$

$$T = C_m - P = 84.4 - 100 = -15.6 \text{ k}$$

No tension reinforcing steel required

Step 5 – Specify minimum steel (per IBC)

This depends on the seismic design category (SDC)

i. SDC A, B -No minimum steel

ii. SDC C - IBC 2106.4.2.3.1

iii. SDC D - IBC 2106.5.2

iv. SDC E,F - IBC 2106.6.2

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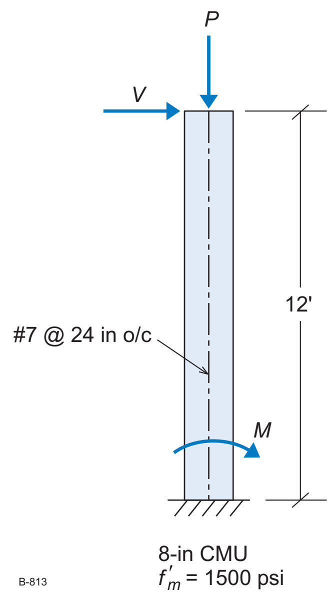
Example 5-4



Reply to the following question.

1. **What is the maximum moment that can be applied perpendicular to the wall if the depth (a) of the reinforcement is**
   a. **3.75 inches**
   b. **5.25 inches**

Given: An 8-inch concrete masonry wall, solid grouted, 12 feet high and reinforced with #7 bars at 24 inches o/c

Axial load, $P = 3$ k/ft

$f'_m = 1500$ psi

special inspection is provided (2000 IBC)

****

Solutions

a. Evaluate using Method 1 ($d = 3.75$ in)

Step 1

$$c_k = \frac{t}{6} = \frac{7.63}{6} = 1.27 \text{ in}$$

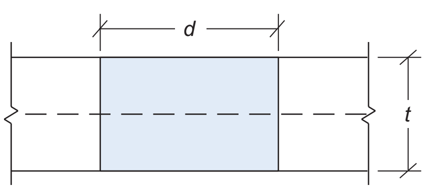
Step 2 Interaction formula; M is due to wind/earthquake

$$h' = 12 \text{ ft} = 144 \text{ in}$$

$$I = \frac{bt^3}{12}; \quad A = bt$$

$$\frac{I}{A} = \frac{bt^3}{12(bt)} = \frac{t^2}{12}$$

$$r = 0.29\sqrt{t} \quad \Rightarrow \quad r = 0.29\sqrt{7.63} = 0.80 \text{ in}$$



Step 3

$$\frac{h'}{r} = \frac{144}{0.80} = 180 > 99$$

$$\frac{h}{t} = \frac{144}{7.63} = 18.9$$

Step 4

$$F_a = 0.25 f'_m \left(\frac{70r}{h}\right)^2$$

$$F_a = 0.25(1500)\left(\frac{70 \times 0.80}{144}\right)^2 = 56.7 \text{ psi}$$

$P_a = F_a A_e = 56.7(12 \times 7.63) = 5.2$ k/in

$$\frac{P}{P_a} = \frac{3}{5.2} = 0.58$$

Step 5

$$\frac{P}{P_a} + \frac{M}{M_a} < 1.33$$

$F_b = 0.33 f'_m = 0.33\ (1500) = 500$ psi $= 0.5$ ksi

Step 6

$$M < M_{\text{allow}}\left(1.33 - \frac{P}{P_a}\right)$$

$M_{\text{allow}} = \min\ \{M_s, M_n\}$

Evaluate moment capacity: Steel #7 bars @ 24 in o/c

$$A_s = \frac{0.6}{2} = 0.3 \text{ in}^2/\text{ft}$$

$M_s = F_s A_s jd = K_s bd^2$

$$\rho = \frac{A_s}{bd} = \frac{0.3}{12 \times 3.75} = 0.0067$$

$E_s = 29{,}000$ ksi

$E_m = 900 f'_m$ **2108.7.2**

$E_m = 900\ (1.5) = 1350$ ksi

$$n = \frac{E_s}{E_m} = \frac{29{,}000}{1350} = 21.5$$

$\rho n = 0.0067 \times 21.5 = 0.144$

$$k = \sqrt{(\rho n)^2 + 2\rho n} - (\rho n)$$

$$\Rightarrow k = 0.41, \quad j = l - \frac{k}{3} = 0.86, \quad d = 3.75 \text{ in}$$

$M_s = (24)(0.30^2)(0.86)(3.75) = 23.2$ in-k

$$M_m = \frac{1}{2} F_b kjbd^2$$

$$M_m = \frac{1}{2}(0.5)(0.41)(0.86)(12)(3.75)^2 = \underline{\underline{14.9}} \text{ in-k} \qquad \text{Controls}$$

$M_{\text{all}} = 14.9$ in-k

$M_{\text{cap}} < 14.9\ (1.33 - 0.58) = \underline{\underline{11.2}}$ in-k

$V \times h = M_{\text{cap}} = 11.2$ in-k

$$V_{cap} = \frac{M_{cap}}{H} = \frac{11.2}{144} = 0.077 = \underline{\underline{77}} \text{ lb}$$

Maximum horizontal shear at top of wall

b. Increase $d = 5.25$ in

$$\rho = \frac{0.30}{12(5.25)} = 0.0047$$

$$\rho n = 0.102$$

$$k = \sqrt{0.102^2 + 2(0.102)} - 0.102 = 0.36$$

$$j = 1 - \frac{k}{3} = 0.88$$

$$M_m = \frac{1}{2}(0.5)(0.36)(0.88)(12)(5.25)^2 = \underline{\underline{26.2}} \text{ in-k} \qquad \text{Governs}$$

$$M_s = (24)(0.3)(0.88)(5.25) = 33.3 \text{ in-k}$$

$$M_{cap} = 26.2(1.33 - 0.58) = \underline{\underline{19.7}} \text{ in-k}$$

$$V_{cap} = \frac{19.7}{144} = 0.137 \sim \underline{\underline{137}} \text{ lb}$$

Example 5-5

Flanged walls are structural shear wall elements with intersecting cross walls at their respective ends. These wall elements have the added advantage of "flange" to increase the moment of inertia and to provide greater bending moment capacity and shear resistance. They are analogous to steel W-shape sections because they resemble the I-section geometry.

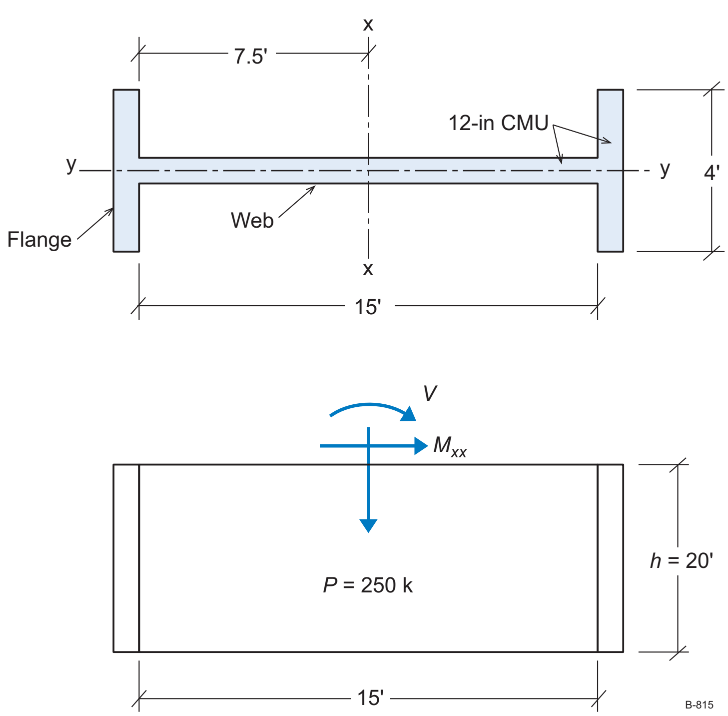
To analyze a flanged shear wall for in-plane moment and shear capacity:

1. **Calculate the section properties of the flanged shear wall: I_x, I_y, S_x, A**
2. **Determine the virtual eccentricity of the load-moment relationship.**
3. **Calculate the stresses for bending and axial forces.**
4. **Calculate the horizontal shear stress distribution resulting from the applied overturning moment.**

12-inch CMU fully grouted wall

f'_m = 3000 psi, full inspection

(M_{xx}) = 2600 ft-k



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Solutions

1. Section properties of flanged shear wall.

   Section is symmetric

$$I_x = \frac{(11.63)(15 \times 12)^3}{12} + 2(48)(11.63)(90 \text{ in} + \frac{11.63}{2})^2$$

$$\bar{I}_o = \frac{th^3}{12} \text{ web section} \qquad Ad^2 \text{ flange section}$$

$$I_x = 5{,}652{,}180^4 + 10{,}249{,}861^4$$

$$\Rightarrow \quad I_x = \underline{\underline{15{,}902{,}041}} \text{ in}^4$$

$$S_x = \frac{I_x}{C} = \frac{I_x}{(d/2)} = \frac{15{,}902{,}041}{(90 + 11.63)} = \underline{\underline{156{,}470}} \text{ in}^3$$

$$A = (15 \times 12)(11.63) + 2(11.63)(48) = \underline{\underline{3210}} \text{ in}^2$$

2. Virtual eccentricity

a. $e = \frac{M}{P} = \frac{2600}{250} = 10.4 \text{ ft} = 124.8 \text{ in}$

b. Kern distance $= e_k = \frac{S_x}{A} = \frac{156{,}470}{3210} = \underline{\underline{48.7}}$ in

"Kern" – middle area of a shear wall where loads will produce only compression stress. If a vertical load is outside the kern, tension stress will develop.

c. Since $e > e_k \Rightarrow$ there will be tension stress; therefore, tension steel is necessary

3. Stress calculations

a. Axial stress

$$f_a = \frac{P}{A} = \frac{250}{3210^2} = 0.078 \text{ ksi}$$

b. $\frac{h'}{r} = \frac{(20)(12)}{3.48} = 69 << 99$

$(r = 0.30t)$

$$F_a = 0.25 f'_m \left[1 - \left(\frac{h'}{140r}\right)^2\right]$$

$$F_a = 0.25\,(3)\left[1 - \left(\frac{69}{140}\right)^2\right] = 0.57 \text{ ksi}$$

c. Flexural stress

$$F_b = \frac{M}{S} = \frac{2600\,(12)}{156{,}470^3} = 0.20 \text{ ksi}$$

d. Allowable stress

$$F_b = \frac{1}{3}f'_m = \frac{1}{3}(3000) = 1 \text{ ksi}$$

e. Interaction equation

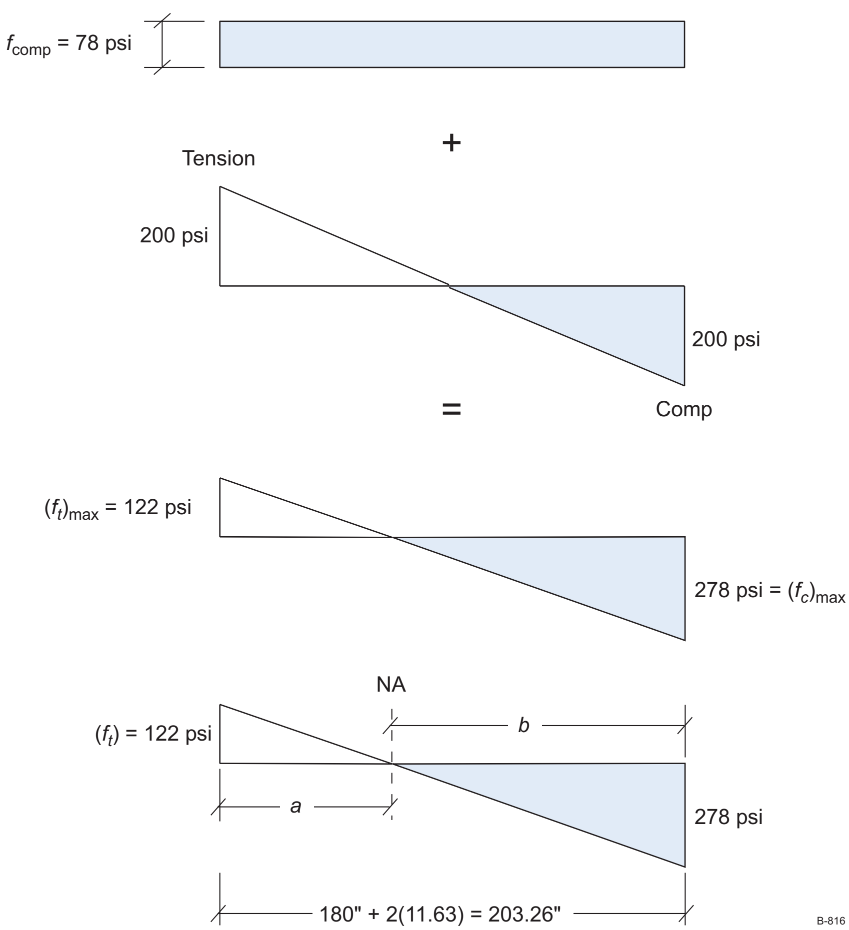
$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \le 1.33 \ \text{(wind or seismic)}$$

$$\left(\frac{0.078}{0.57}\right) + \left(\frac{0.200}{1.0}\right) \le 1.33$$

$0.14 + 0.20 = 0.34 << 1.33$ **OK**

The 1.33 factor is allowed per IBC 1605.3.1.2

f. Combine stresses



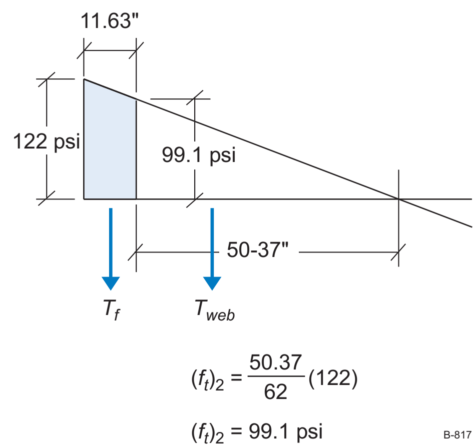
Location of neutral axis (NA)

$$a = \frac{122}{122 + 278}(203.26) = 62 \text{ in}$$

$$b = \frac{278}{278 + 122}(203.26) = 141.3 \text{ in}$$

Check $a + b = 203.26$ **OK**

g. Tension force



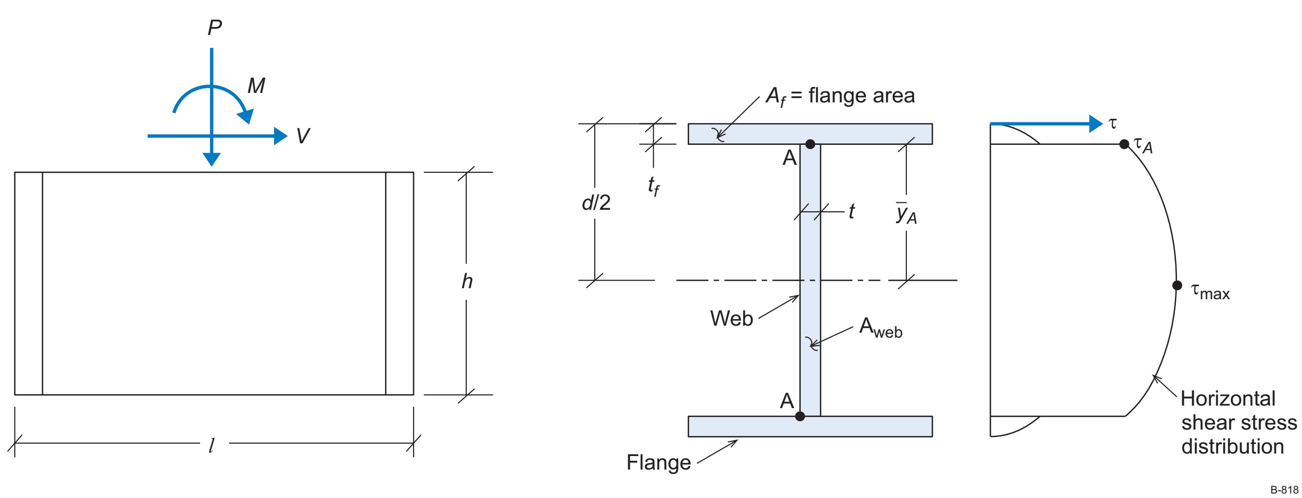
$$T_s = T_{\text{flange}} + T_{\text{web}}$$

$$T_s = (A_{\text{flange}})(f_{\text{flange}}) + (A_{\text{web}})(f_{\text{web}})$$

$$T_s = \frac{1}{2}(122 + 99.1)(11.63)(48) + (11.63)(50.37)\left(\frac{1}{2}\right)(99.1)$$

$$T_s = 59{,}453 + 29{,}027 = 88{,}480 \text{ lb} \sim 88.5 \text{ kips}$$

4. Shear stress distribution



where

$V = \frac{M}{h}$ = horizontal shear force

τ = horizontal shear stress

A = point of shear stress concentration

I = moment of inertia

t_w = thickness of web

Q = moment of area about centroidal axis

$\therefore\ Q_A = A_f\bar{y}$ for point A

and $\bar{y} = \frac{1}{2}(d - tf)$

Horizontal shear stress

Horizontal shear stress distribution is based on cross-section geometry, and is calculated on the basis of shear flow theory from equation $\tau = \frac{VQ}{It_w}$. The flange sections provide the axial force couple to create a bending moment resistance, and web provides the in-plane shear resistance. At point A where these two elements coincide, there is a shear stress concentration. In addition, the wall web shear stress must be calculated considering the maximum shear stress at the neutral axis. The difference between the average shear and the horizontal shear stress distribution is calculated from equation $\tau = \frac{VQ}{It_w}$ and cannot be neglected. The reinforcement steel may be determined from the shear stress calculation. Note that the calculation must be performed twice for nonsymmetrical sections.

a. For this example, τ_A

$$A_f = (48)(11.63) = 558.24 \text{ in}^2$$

$$\bar{y} = [(15)(12) - 11.63]\ \frac{1}{2} = 84.2$$

$$y = \frac{M}{h} = \frac{2600}{20} = 130 \text{ k}$$

$$Q_A = A_f\bar{y} = (558.24)(84.1) = 47.003 \text{ in}^3$$

$$\therefore\ \tau_A = \frac{(130)(47.003)}{(15{,}902{,}041^4)(11.63)} = 0.033 \text{ ksi} = 33 \text{ psi}$$

b. For peak shear stress, a conservative estimate is $\tau_{max} = \frac{VQ}{It} = 1.5\left(\frac{V}{A_{web}}\right)$

$$\tau_{max} = 1.5\left(\frac{130}{(180)(11.63)}\right) = 0.093 \text{ ksi} = 93 \text{ psi}$$

c. Allowable shear stress

$$\frac{M}{Vd} = \frac{2600}{(130)\left(\frac{203.26}{12}\right)} = 1.18 > 1.0$$

$$F_v = 1.5\sqrt{f'_m} < 75 \text{ psi max}$$

$$(F_v)_{allow} = (1.33)(75) = 100 \text{ psi for seismic or wind}$$

The calculation for maximum shear stress is based on the distribution of the shear to be 100 percent in the web of the wall. This is a conservative estimate because the flange areas do have a contribution. However, this contribution is small and the extra effort in the calculation process is not worth the meager benefit. Typically, 95 percent of the shear force is taken through the web, so the calculation for peak shear stress is based on the formulation of rectangular sections. The 1.5(V/A) is derived in several strength-of-materials tests.

Shear stress at point A is calculated to determine the necessary steel reinforcement to assure adequate bond of the flange to the web. This joint reinforcement is a primary area of structural failure and requires careful detailing to assure adequate connection strength.

Allowable stresses are calculated from the basic WSD equations (UBC 2107.2.9 and 2000 IBC 1605.3.2). The 33-percent increase is allowed for either wind or earthquake forces.

Example 5-6

When performing finite element analysis (FEA), certain definitions require clarification – Von Mises Stress, Shear Stress, Principle Stresses, and Maximum Shear Stress. These are covered in this example, which also considers analysis objectives and the purposes of conducting FEA.

1. **Define the analysis objectives.**
2. **Discuss the definitions of Von Mises Stress, Shear Stress, Principal Stresses, and Maximum Shear Stress.**
3. **What practical applications of the four stress quantities can be found in the field of structural engineering?**
4. **What are three limitations in finite element applications regarding reinforced masonry shear wall analysis?**

****

Solutions

1. Analysis objectives
   a. To calculate the bending and shear stresses caused by the imposed loads
   b. To determine the accuracy and calculation efficiency of the FEA
2. These definitions must be clarified so that full understanding of the computer results can be achieved.

σ_{VM} = Von Mises stress

τ = Shear stress

σ_1, σ_2 = Principle stresses

τ_{max} = Maximum shear stress

Theory of Elasticity (Timoshenko, Budynas) gives the following.

$$\sigma_{VM} = \sqrt{\frac{1}{2}\{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma^3 - \sigma^1)^2\}} = \sigma_y$$

τ_{ij} = shear stress along the ij plane

σ_1 = principle stress along principle axis (i = 1, 2, 3)

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = 0.5\,\tau_y$$

where σ_y = yield point stress of the material

There are three key strength-failure theories used for ductile materials

Tresca Theory – based on the maximum shear stress calculations

Von Mises Theory – based on the maximum energy distortion theory

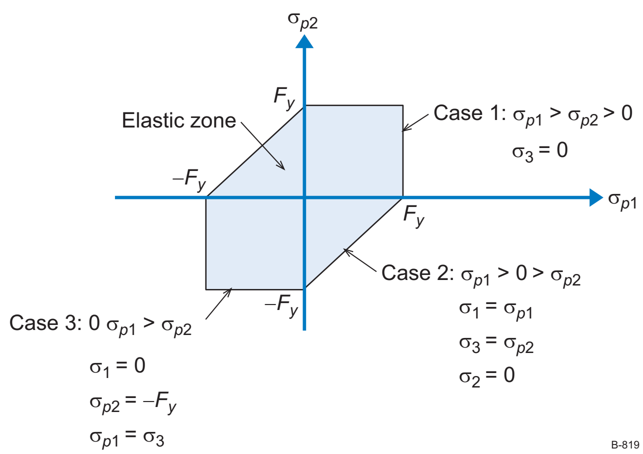
Mohr-Coulomb Theory – for brittle materials

The first two theories are applied for shear wall analysis, and Mohr-Coulomb is used mostly for geotechnical applications.

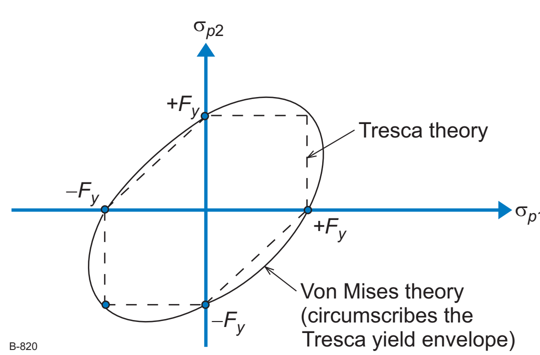
The Tresca Theory is based on the following formulation

$$\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_3) \leq 0.5\, F_y$$

where F_y = yield stress of the material shown graphically below



The Von Mises theory is based on the formulation of maximum distortion energy



The failure criterion is

$$\sqrt{\sigma_{p1}^2 + \sigma_{p2}^2 - \sigma_{p1}\sigma_{p2}} \leq F_y$$

$$\sigma_{VM} = \text{Equivalent Von Mises Stress} = \sqrt{\sigma_{p1}^2 + \sigma_{p2}^2 - \sigma_{p1}\sigma_{p2}}$$

3. Practical applications

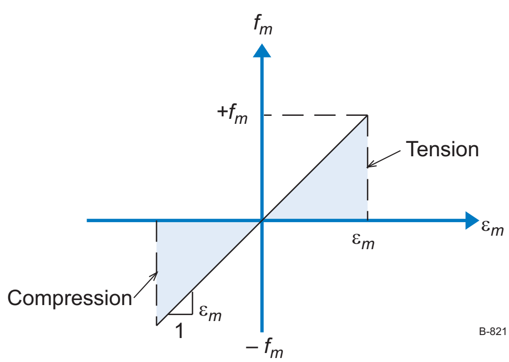
   a) A reinforced wall of complex geometry may be analyzed with these definitions because the FE programs are equipped to handle such problems.

   b) Areas of stress concentration around doors, windows, or portions of structures with high load transfer can be evaluated more accurately.

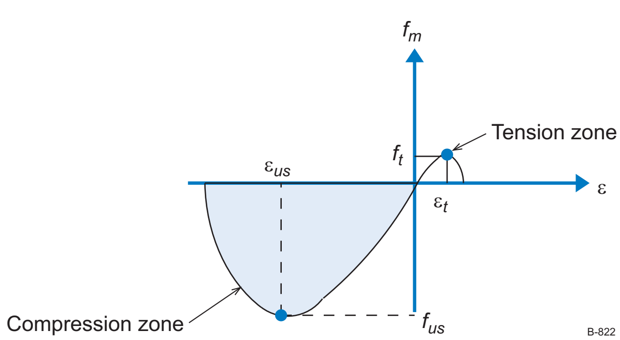
   c) Structural engineers can evaluate multiple design scenarios without employing manual calculations to verify/confirm a design. The structural properties and the geometry can be adjusted quickly and easily in the FE model.

4. Every technology must be applied with attention to the prevailing limitations. Keep the following issues in mind.

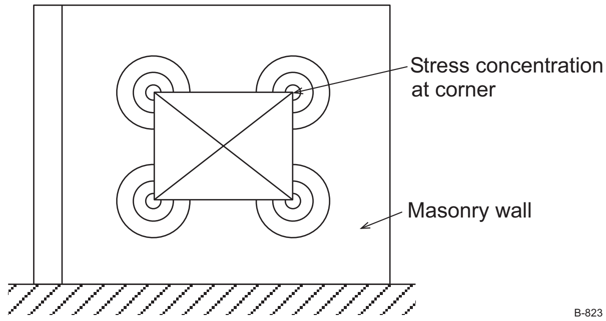
   a) Linear elastic plate elements assure perfect isotropic behavior. The FE formulation stress-strain curve for the masonry is linear in both tension and compression.



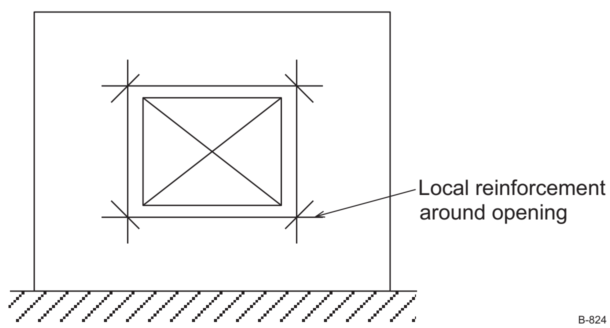
This assures that tensile stresses are taken by the steel. Steel reinforcement must be placed in the correct locations. Furthermore, the linear elastic isotropic behavior does not necessarily occur in the real world. Symmetry is shown in the ideal stress-strain diagram, while the actual stress-strain diagram for masonry is not symmetrical.



b) The FE model may over- or under-predict stress concentration values around openings.



This is caused by the high mesh generation required for accurate analysis of corners, and the fact that diagonal reinforcement is used in the actual structure.



Specify that stress concentrations should be investigated on a case-by-case basis.

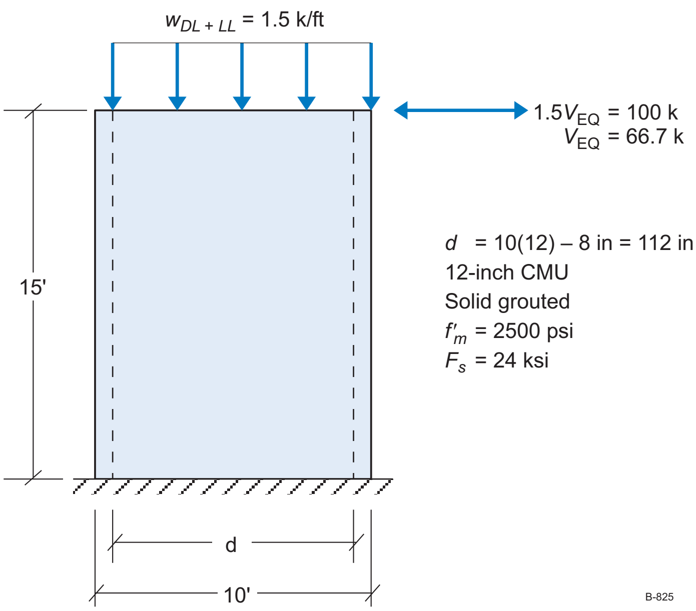
c) Cross-check results. Probably the most important part of utilizing high-powered sophisticated programs. – CHECK THE ANSWERS

The engineer must verify program results.

Example 5-7

Using Method 1 (from the RMEH), perform the following tasks.

1. **Calculate the working stress design overturning moment (WSD-OTM) capacity of the shear wall.**
2. **Calculate the allowable shear capacity of the wall, and design the shear reinforcement.**
3. **Design the tension reinforcing based on the 2000 IBC WSD procedure.**
4. **Sketch the detail of the boundary element.**
5. **Provide three design recommendations for strengthening the OTM capacity of the wall.**



****

Solutions

1. OTM = 100 × 15 = 1500 ft-k = 18,000 in-k
2. Allowable shear capacity

$$\frac{M}{Vl} = \frac{Vh}{Vl} = \frac{h}{l} = \frac{15}{10} = 1.5 > 1.0$$

with no reinforcement

$$F_{v_m} = 1.0\sqrt{f'_m} < 50 \text{ psi}$$

$$F_{v_m} = 1.0\sqrt{2500} < 50 \text{ psi}$$

Actual shear stress = f_{rm}

$$f_{rm} = \frac{1.5V_{EQ}}{tl} = \frac{100}{(11.6)(120)} = 72 \text{ psi} > 50 \text{ psi}$$

Provide shear reinforcement for all shear

$$f_{vm} = 1.5\sqrt{f'_m} = 1.5\sqrt{2500} = 75 \text{ psi} > 72 \text{ psi} \qquad \textbf{OK}$$

For earthquake loads, per alternate load combinations (IBC 1605.3.2)

$$(f_{vm})_{EQ} = \frac{4}{3} \times 75 = 99.8 \text{ psi} > 72 \text{ psi} \qquad \textbf{OK}$$

Assume 16-inch o/c spacing for reinforcement

$$A_{vs} = \frac{(f_{vm})ts}{F_s} = \frac{(72)(11.63)(16)}{1.33 \times 24{,}000} = 0.42 \text{ in}^2$$

Specify #6 bar @ 16 inches o/c

3. Evaluate per RMEH 5th edition, Method 2

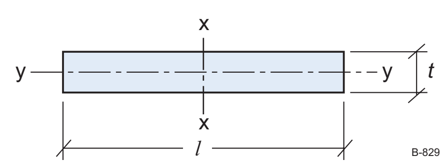
$$f_a = \frac{P}{lt}$$

$$P = 1.5(10) = 15 \text{ k}$$

$$\therefore f_a = \frac{15}{(120)(11.63)} = 0.011 \text{ ksi}$$

$$F_a = 0.25\,(2500)\left[1 - \left(\frac{12 \times 15 \text{ ft}}{140r}\right)^2\right]$$

$r = \sqrt{\frac{I}{A}}$ = radius of gyration about y-axis



$$r = \left[\left(\frac{lt^3}{12}\right)\left(\frac{1}{tl}\right)\right]^{\frac{1}{2}} = \frac{t}{\sqrt{12}}$$

$$r = 0.29t$$

For 12-inch CMU, $t = 11.63$ $\qquad$ r = 3.36

$$F_a = 533.5 \text{ psi} \sim 534$$

$$\frac{f_a}{F_a} = \frac{11}{534} = 0.02$$

$F_b \quad = 0.33f'_m = 0.33(2500) = 825 \text{ psi}$

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \le 1.33 \ldots \text{EQ/wind}$$

$f_b \quad = 825\ (1.33 - 0.02) = 1081 \text{ psi} = 1.08 \text{ ksi}$

$f_m \quad = f_a + f_b = 0.011 + 1.08 = 1.09 \text{ ksi}$

Solve quadratic equation

$$a = \frac{1}{6} t f_m = \frac{1}{6}(11.63)(1.09) = 2.11$$

$$b = -\frac{1}{2} t f_m\,(l - d') = -\frac{1}{2}(11.63)(1.09)(120 - 8)$$

$$b = -709.9$$

$$c = P\left(\frac{l}{2} - d\right) + \text{M}$$

$$c = 15\text{ k}\left(\frac{120}{2} - 8\right) + 18{,}000 = 19{,}680 \text{ in-k}$$

$$kd = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{+709.9 - \sqrt{709.9^2 - 4(2.11)(19{,}680)}}{2(2.11)}$$

$$kd = 30.5$$

$$d = 112 \text{ in}$$

$$k = \frac{30.5}{112} = 0.27$$

$$j = 1 - \frac{k}{3} = 0.91$$

$$c = \frac{1}{2} t k d f_m = \frac{1}{2}(11.63)(30.5)(1.09) = 193.3 \text{ k}$$

$$T = C - P = 193.3 - 15 = 178.3 \text{ k} \ldots \Rightarrow \text{must reinforce for tension}$$

$$A_s = \frac{T}{f_s} = \frac{178.3 \text{ k}}{f_s} \quad \ldots f_s = \left(\frac{l - k}{k}\right) n f_m$$

$$E_m = 900 f'_m = 900\ (2.5) = 2250$$

$$n = \frac{E_s}{E_m} = \frac{29{,}000}{2250} = 12.89$$

$$\therefore\ f_s = \left(\frac{1 - 0.27}{0.27}\right) 12.89\ (1.09) = 38 \text{ ksi} > 1.33\ F_s$$

$$1.33\ F_s = 24 \times 1.33 = 32 \text{ ksi}$$

Must reduce

$$f_m = 1.0 \text{ ksi}$$

Recompute

$$a = \frac{1}{6} tf_m = 1.94$$

$$b = -\frac{1}{2} tf_m (l - d') = -651.3$$

$$c = 19{,}680$$

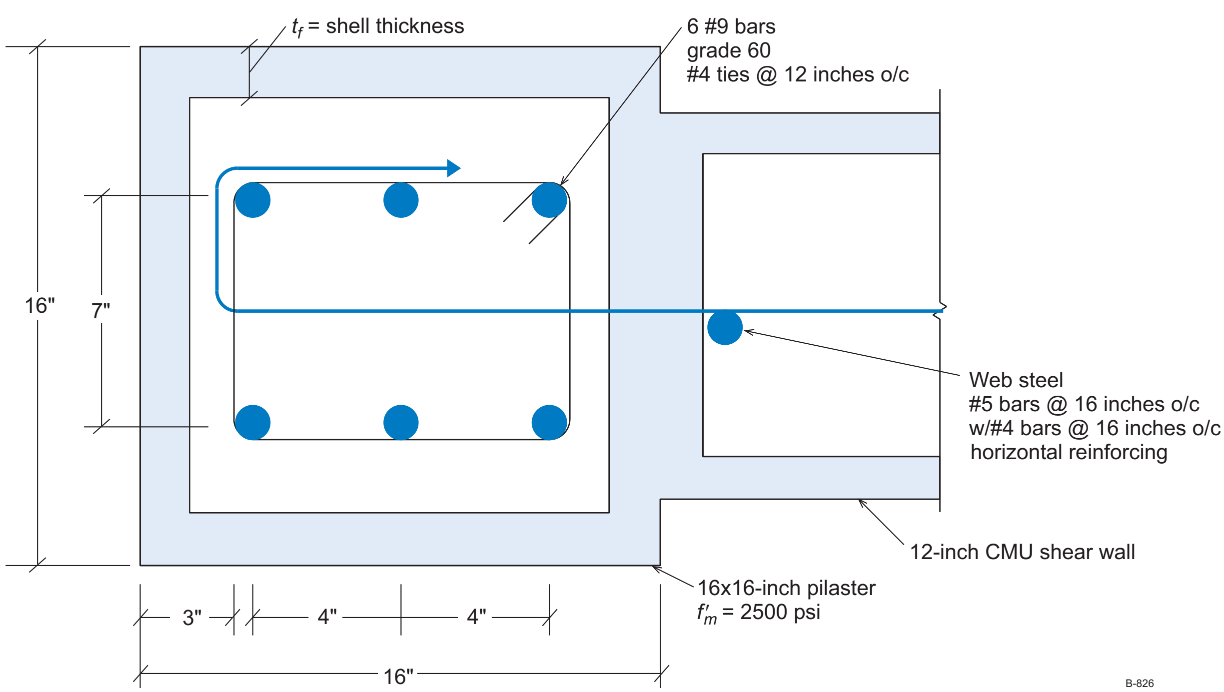
$$kd = 33.6$$

$$k = \frac{33.6}{112} = 0.3$$

$$f_s = \left(\frac{1 - 0.3}{0.3}\right)(12.89)(1.0) = 30.1 \text{ ksi} < 32 \text{ ksi} \qquad \textbf{OK}$$

$$A_s = \frac{T}{f_s} = \frac{178.3}{30.1} = \underline{\underline{5.92}} \text{ in}^2$$

4. Because the A_s is quite large, the shear wall should be designed with boundary elements as shown in the cross section below. $A_s = 6 \text{ in}^2$

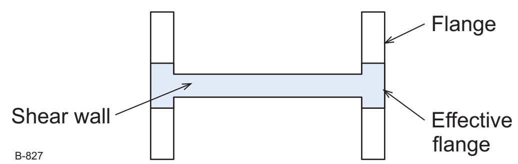


5. Recommendations

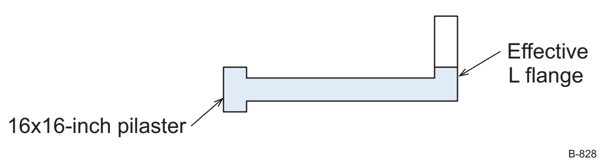
   a. Because the shear wall has large OTMs, the strength of the masonry should be at least 3000 to 4000 psi. Owners refrain from these high values because of the additional cost, but there is a benefit.

   b. The use of a flanged wall section is significant to improving OTM capacity as shown in the following examples.

H configuration



L configuration



   c. Plan to use strength design when looking for economy. (Explained and illustrated in Chapter 6.)

Example 5-8

Use the finite element method with the structural analysis program RISA-3D to analyze a 10x10-foot shear wall using 8-inch CMUs.

1. **Perform an FEA of the shear wall. How many nodes are in the model? What is the total size of the stiffness matrix? How many degrees of freedom (DOF) are in the solution?**
2. **Explain the boundary conditions along the base of the shear wall. How does the program handle the out-of-plane displacement?**
3. **Prepare a "shear stress color contour plot." Explain what shear stress is. Why is this chart important? What are the structural engineering conclusions derived from this color contour chart?**
4. **Prepare a "principle stress color contour plot." Explain what principle stress is. Why is this chart important? What are the structural engineering conclusions derived from this color contour chart?**
5. **Describe how an FEA of a shear wall can enable a structural engineer to efficiently prepare a structural design. What are the practical benefits of this level of analysis?**

****

Solutions

1. Create a three-dimensional perspective view of the masonry shear wall with plate elements. The base nodes will be fixed. A series of linear point loads will appear on the top of the shear wall. These are shown with 4-kip nodal vectors on N6, N12, N18, N24, N30, and N36.

   There are 36 nodes

   $$N_{\text{total}} = 36$$

   $$N_{\text{fixed}} = 6$$

   $$N_{\text{free}} = 36 - 6 = 30$$

   Because the translation in the Z direction is free (out-of-plane), this could be assumed zero.

   Each node has 6 DOF (u, v, w, θ, α, β)

   $$N_{\text{DOF}} = 30 \times 6 = 180 \text{ DOF (3-D model)}$$

   If we exclude the Z translation

   $$N_{\text{DOF}} = 30 \times 5 = 150 \text{ DOF (2-D model)}$$

2. The boundary conditions along the base of the shear wall are fixed (Nodes N1, N7, N13, N25, N31).

   u, v, w = displacements along x, y, z axes, respectively

   $$\therefore \quad u \equiv v \equiv w \equiv 0 \text{ along the base}$$

   Out-of-plane displacements do not occur in the solution because the loads are entirely in plane.

The rotations are also fixed.

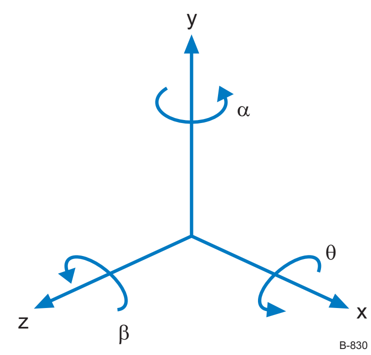
Define the following

θ = rotation about the x-axis

α = rotation about the y-axis

β = rotation about the z-axis

$\therefore\ \theta \equiv \alpha \equiv \beta \equiv 0$ along nodes at the base.



This program handles out-of-plane displacement through the plate stiffness matrix. This is important to note because certain FE programs do not have plate elements with out-of-plane (i.e., 3-D) stiffness.

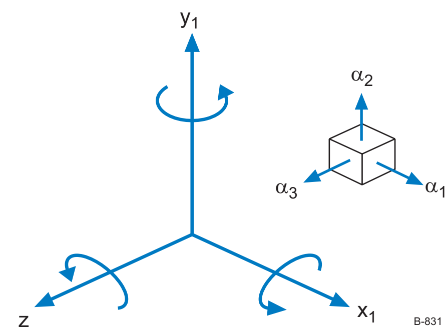
3. Shear stress is defined in accordance with the definition from Tresca and Mohr

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$

a) The shear stresses reach maximum at the corners

$$\tau_{max} = 0.064 \text{ ksi} = 64 \text{ psi}$$

b) The areas of high shear stress correspond to the requirement for boundary steel (per IBC and UBC).

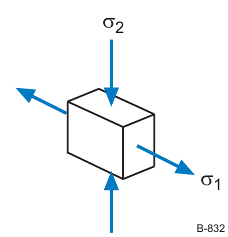


The solution for the shear stress. It shows a symmetric distribution of shear stress (t_{top}) with peak values occurring at the base corner points. The results are convergent with the basic theory of shear walls. Peak shear stresses will occur around openings and at corner points. The top corner points will also show peak values with the maximum opening occurring along the base.

4. a) Principle stresses are according to the definitions from Mohr's Circle: σ_1, σ_2, σ_3 for a plate element

b) These two element plots will assist in determining areas of maximum compression and tension stress.

c) The stress concentration is at the corners



σ_1 = +0.154 ksi at the left (tension)

σ_2 = –0.154 ksi at the right (compression)

It is perfectly symmetrical.

The principle stress contour plots for σ_1 and σ_2 will be mirror images of each other. The σ_1 contours show peak tension stress (154 psi) occurring on the lower left corner point. This coincides with the placement of boundary steel for resistance to overturning. This is also shown in σ_2, which has the localized compression stress (154 psi) at the opposite corner.

Output can be extensive or tailored for specific results in accordance with RISA-3D software requirements. plate corner forces, plate forces, and stress (both principal and shear) values. RISA provides an excellent arrangement to control the output quantity from a typical analysis. This gives the engineer/analyst the final decision on the quantity of output.

Conclusions:

- This example illustrates the practical benefit of finite element analysis (FEA) in masonry shear wall design. The stress concentration points are clearly indicated in the plots.
- From the shear stress contour plot, we can conclude that the peak shear stress value of 0.064 ksi (64 psi) occurs at the lower corner points of the wall. In similar fashion, the principle stress contours indicate the maximum tension and compression zones at the same points.
- A major advantage of FEA solutions is that the wall geometry can be altered, structural properties changed, and wall thickness adjusted to reflect a modified design. This process would normally require many hours to complete but, with a computer, is accomplished within a relatively short time.

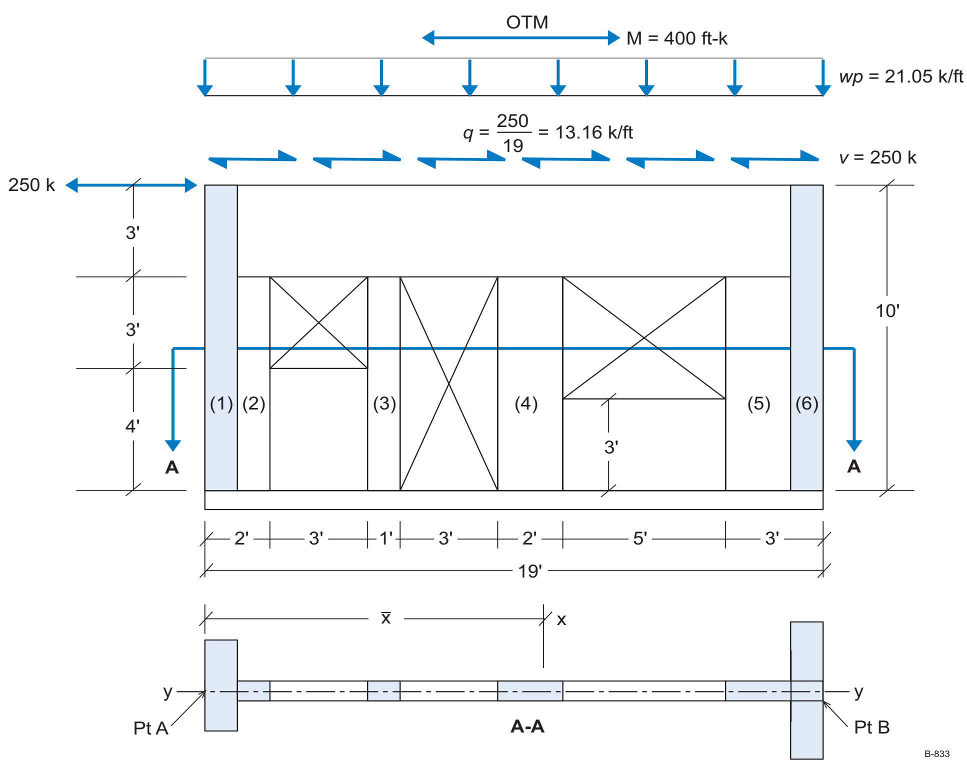
Example 5-9

The flanged shear wall is shown with service loads consisting of an overturning moment, M or OTM, and axial compression force, p. Using RMEH Method 3, WSD, the 2000 IBC, and considering the effect of the openings, complete the following.

1. **Calculate the centroid of the section and the section properties I and S.**
2. **Evaluate the bending stresses at points A and B. Which is critical?**
3. **Using the interaction equation, evaluate the structural capacity of the wall.**
4. **a. Combine the stresses and draw the interaction effects of the axial compression coupled with the bending stresses for only one direction (point A in tension, B in compression).**

   b. Calculate the required area of steel for the tension section (point A).

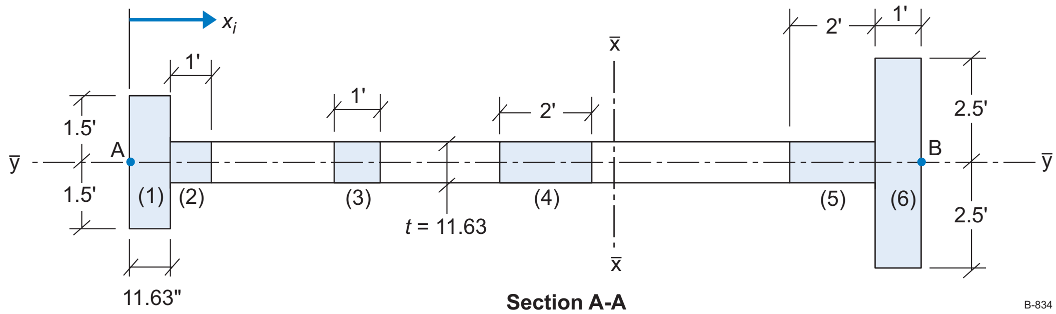
   c. Calculate the overturning moment capacity.
5. **Provide structural engineering conclusions from this analysis and decide whether the wall is over designed. What effect do the openings have on the wall design capacity?**



12-inch CMU, f'_m = 3000 psi, fully grouted

Solutions

1. Calculate centroid section properties



Section A-A

	(In)^2 A_i	(In) $\bar{x}_i$	(In)^3 $A_i\bar{x}_i$	(In^4) I_i	(In) $d_i = \bar{x} + \bar{x}_i$	(In)^4 Ad_i^2
1	418.7	5.82	2437.7	4,719.1	133.3	7.44×10^6
2	135.3	17.50	2367.8	1,524.5	121.6	2.00×10^6
3	135.3	66.00	8929.8	1,524.5	73.1	7.24×10^6
4	279.1	120.00	33,492.0	13,397.8	19.3	1.02×10^5
5	279.1	204.00	56,936.4	13,397.8	64.9	1.18×10^5
6	837.4	222.00	185,902.8	9,438.2	82.9	5.96×10^6
	$\Sigma = 2084.9$ in^2		$\Sigma = 290{,}066.0$ in	$\Sigma = 44{,}002.0$ in^4		$\Sigma = 2.27 \times 10^7$

Calculations;

1. $t = 11.63$ in^2

$$A_1 = 11.63 \times 36 = 418.7 \text{ in}^2$$

$$\bar{x}_1 = \frac{11.63}{2} = 5.82 \text{ in}$$

$$I_1 = \frac{(36)(11.63)^3}{12} = 4719.1 \text{ in}^4$$

2. $A_2 = 11.63^2 = 135.3$ in^2

$$\bar{x}_2 = 11.63 + \frac{11.63}{2} = 17.5 \text{ in}$$

$$\bar{I}_2 = \frac{(11.63)^4}{12} = 1524.5 \text{ in}^4$$

3. $A_3 = A_2$ same as 2 $\quad \bar{x}_3 = 5.5$ ft $= 66$ in

4. $A_4 = 24\,(11.63) = 279.1\ \text{in}^2$

$I_4 = \dfrac{11.63(24)^3}{12} = 13{,}397.8\ \text{in}^4$

$\bar{x}_4 = 10\ \text{ft} = 120\ \text{in}$

5. same as 4

$\bar{x}_5 = 19 - 2 = 17\ \text{ft} = 204\ \text{in}$

6. $A_6 = (72\ \text{in})(11.63) = 837.4\ \text{in}^2$

$I_6 = 72\,(11.63)\dfrac{3}{12} = 9438.2$

$\bar{x}_6 = 19\ \text{ft} - 0.5\ \text{ft} = 18.5\ \text{ft} = 222\ \text{in}$

$\Sigma A_i = 2094.9\ \text{in}^2$

$\Sigma A_i\bar{x}_i = 290{,}066.5\ \text{in}^3$

$$\therefore\ \bar{x} = \frac{\Sigma A_i\bar{x}_i}{\Sigma A_i} = \frac{290{,}066.5}{2084.9} = \underline{\underline{139.13}}\ \text{in}$$

$$\bar{x} = \underline{\underline{11.6}}\ \text{ft}$$

After calculating the centroid, the section properties are determined

$d_i = \bar{x} - \bar{x}_i$

$\Sigma \bar{I}_i = 44{,}002\ \text{in}^2$

$d_1 = \bar{x} - \bar{x}_i = 139.13 - 5.82 = 133.3\ \text{in}$ $\quad A_1d_1 = 418.7 \times 133.32 = 7.44 \times 10^6\ \text{in}^4$

$d_2 = 139.13 - 17.5 = 121.6\ \text{in}$ $\quad A_2d_2^{\,2} = 2.00 \times 10^6\ \text{in}^4$

$d_3 = 139.13 - 66 = 73.1\ \text{in}$ $\quad A_3d_3^{\,2} = 135.3\,(73.1)^2$

$A_3d_3^{\,2} = 7.24 \times 10^5\ \text{in}^4$

$d_4 = -120 + 139.13 = 19.13\ \text{in}$ $\quad A_4d_4^{\,2} = 1.02 \times 10^5\ \text{in}^4$

$d_5 = 204 - 139.13 = 64.9\ \text{in}$ $\quad A_5d_5^{\,2} = 279.1\,(64.9)^2 = 1.18 \times 10^6$

$d_6 = 222 - 139.13 = 829\ \text{in}$

$A_6d_6^{\,2} = 837.4\,(82.9)^2 = 5.76 \times 10^6\ \text{in}^4$

$I_{\text{wall}} = \Sigma I_i + = \Sigma A_i d_i^{\,2} = 44{,}002 + 2.27 \times 10^7 = \underline{\underline{2.27 \times 10^7}}\ \text{in}^4$

Section Properties

$$A = 2084.9 \text{ in}^2$$

$$I_x = 2.27 \times 10^8 \text{ in}^4 \qquad c_A = \bar{x} = 139.13 \text{ in}$$

$$c_B = 19\,(12) - 139.13 = 88.9 \text{ in}$$

$$(S_x)_A = \frac{I_x}{C_A} = \frac{2.27 \times 10^7}{139.13} = \underline{\underline{1.63 \times 10^5}} \text{ in}^3 \quad \text{. . . Controls}$$

$$(S_x)_B = \frac{I_x}{C_B} = \frac{2.27 \times 10^7}{88.9} = 2.55 \times 10^5 \text{ in}^3$$

2. Evaluate bending stresses at points A and B

$$(f_b)_A = \frac{M}{S_A} \leq F_b = 0.33 f'_m = 1 \text{ ksi}$$

$$M = 4000 \text{ ft-k} = 48{,}000 \text{ in-k}$$

Since S_A is critical, this supercedes the stresses at point B

$$(f_b)_A = \frac{48{,}000}{1.63 \times 10^5} = 0.300 \text{ ksi}$$

$$(f_b)_B = \frac{48{,}000}{2.55 \times 10^3} = 0.190 \text{ ksi}$$

3. Interaction equation

$$\frac{(f_b)_A}{F_b} + \frac{f_a}{F_a} \leq 1.33 \quad \text{. . . Seismic/wind condition (2000 IBC, 1605.3.2)}$$

$$wp = 21.05 \text{ k/ft}$$

$$P = 400 \text{ kips}$$

$$f_a = \frac{P}{A} = \frac{400}{2084.9} = 0.192 \text{ ksi}$$

$$F_a = 0.25 f'_m \left(1 - \left(\frac{h}{140r}\right)^2\right)$$

$$r = 0.29t = 3.37 \text{ in}$$

$$\frac{h'}{r} + \frac{120}{3.37} = 35.6 < 99$$

$$\therefore\ F_a = 0.25\,(3)\left(1 - \left(\frac{35.6}{140}\right)^2\right) = 0.701 \text{ ksi}$$

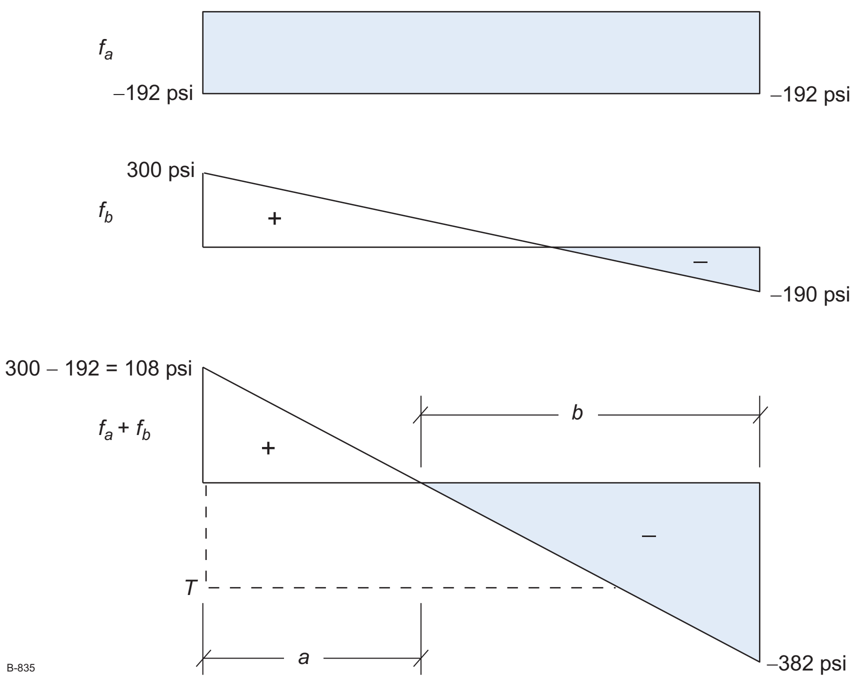
$$M = 4000 \text{ ft-k} = 48{,}000 \text{ in-k}$$

$$\frac{(f_b)_A}{(F_b)} = \frac{300}{1000} = 0.30$$

$$\frac{f_a}{F_a} = \frac{0.192}{0.701} = 0.28$$

$$\frac{(f_b)_A}{F_b} + \frac{f_a}{F_a} = 0.30 + 0.28 = 0.58 < 1.33 \qquad \textbf{OK}$$

4. a) Combine stresses



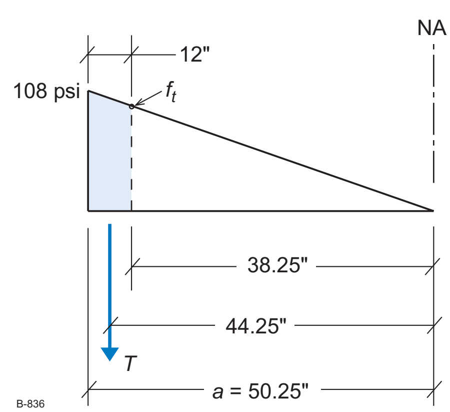
$$e = \frac{M}{P} = \frac{4000}{400} = 10 \text{ ft} = 120 \text{ in}$$

By similar triangles, l = 19 ft × 12 = 228 in

$$a = \frac{108}{108 + 382}(228) = 50.25 \text{ in}$$

$$b = 228 - 50.25 = 177.8 \text{ in}$$

b) Tension steel requirement



$$f_t = \left(\frac{38.25}{50.25}\right) 105 = 82.2 \text{ psi}$$

$$T = \frac{1}{2} f_t t_{\text{wall}} (a - t_{\text{wall}})$$

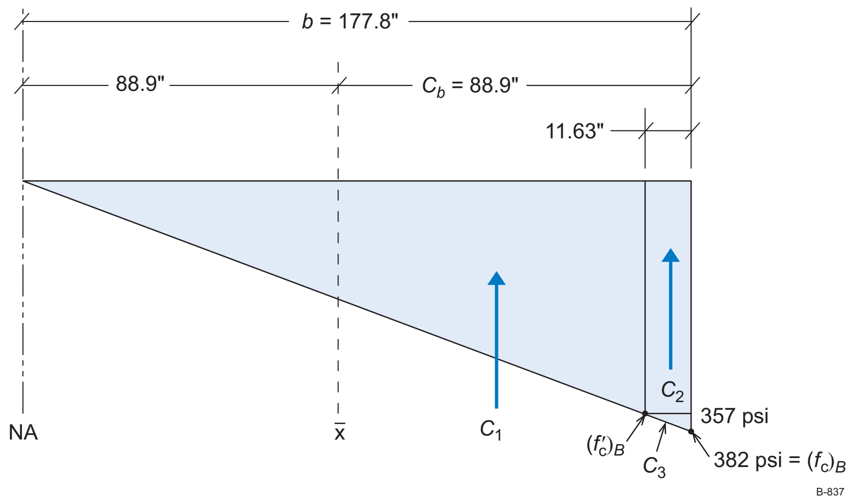
$$T = \frac{1}{2}(82.2)(11.63)(38.25)$$

$$T \quad = 18{,}283 \text{ lb}$$

$$A_s \quad = \frac{T}{F_s} = \frac{18{,}283}{1.33 \times 24 \text{ ksi}}$$

Tension steel area $A_s = 0.57 \text{ in}^2$

c) Overturning moment capacity



$$(f_c)_B \quad = 382 \text{ psi}$$

$$(f_c)_{B'} \quad = 382\left(\frac{177.8 - 11.63}{177.8}\right) = 357 \text{ psi}$$

$$C_1 \quad = \frac{1}{2}(0.357 \text{ ksi})\underbrace{(177.8 - 11.63)}_{166.17}(11.63) = 345 \text{ k}$$

$$C_2 \quad = 11.63\ (0.357)(11.63) = 48 \text{ k}$$

$$C_3 \quad = \frac{1}{2}(0.382 - 0.357)(11.63)^2 = 1.7$$

$$C_{\text{total}} = \Sigma C_i = 394.7 \text{ k}$$

$$\Sigma F_{\text{vert}} = P + T - C = +18.3 \text{ k} - 394.7 = 23.6 \text{ k}$$

Add compression steel effect

$$C_s \quad = T_s = 18.3 \text{ k (equal steel area)}$$

$$\Sigma F_{\text{vert}} = P + T - C - C_s = 5.3 \Rightarrow 1.3\%$$

$$\text{error} = \frac{53}{400} = 1.3\% \text{ error}$$

OK

$$\Sigma M_{\text{tension}} = \Sigma C_i(x_i)_T = C_1(x_1)_T + C_2(x_2)_T + C_3(x_3)_T + (C_s)(x_s)_T$$

$$= 345 \text{ k} \left(\frac{1}{3} \times 166.17 + 44.25\right)$$

$$+ 48\,(172 + 44.25) + 18.3\,(172 + 44.25) + 1.7$$

$$+ \ldots + 1.7\,(166.17 + \left(\frac{2}{3}\right)(11.63) + 44.25)$$

$$\Sigma M_{\text{tension}} = 34.185 + 10{,}380 + 3957$$

$$+ 371 = \underline{\underline{48{,}893}} \approx \underline{\underline{48{,}000}} \text{ in-k}$$ **OK**

(5) Recommendations and Conclusion

The wall's openings reduce the section modulus. This is obvious because the openings reduce the effective area and, consequently, lead to reduced stiffness. As a rule of thumb, the greater the number of openings, the weaker the wall.

Overturning moment (OTM) capacity is not a problem for this shear wall because the demand is satisfied with nominal steel reinforcement. Therefore, increasing the OTM capacity by adding tension and compression steel in the boundary elements and/or flanges is not difficult. There is sufficient area to place the steel.

Since the demand-to-capacity ratio is 0.58, which is lower than 1.33 with sufficient margin of safety, it is possible to consider reducing the compressive strength from 3000 psi to 2500 psi. This would necessitate a recalculation, but could be worth the extra effort in reduced construction costs. There is also a possibility of reducing the block size from a 12-inch CMU to a 10- or 8-inch CMU. Each of these options would require the same level of detailed analysis; another good reason to perform these calculations using an Excel spreadsheet.

Another design element that warrants further investigation is the shear stresses in the piers. While not part of this particular exercise, these should be further examined because of the slender shear/pier elements in this wall. The in-plane drift could also pose a problem. Both of these analysis problems can be solved through standard formulations or by implementing a finite element model.

Example 5-10

A three-story shear wall with openings is presented. Perform a finite element analysis of the wall using the RISA-3D program to demonstrate the analysis methodologies and then reply to the following questions.

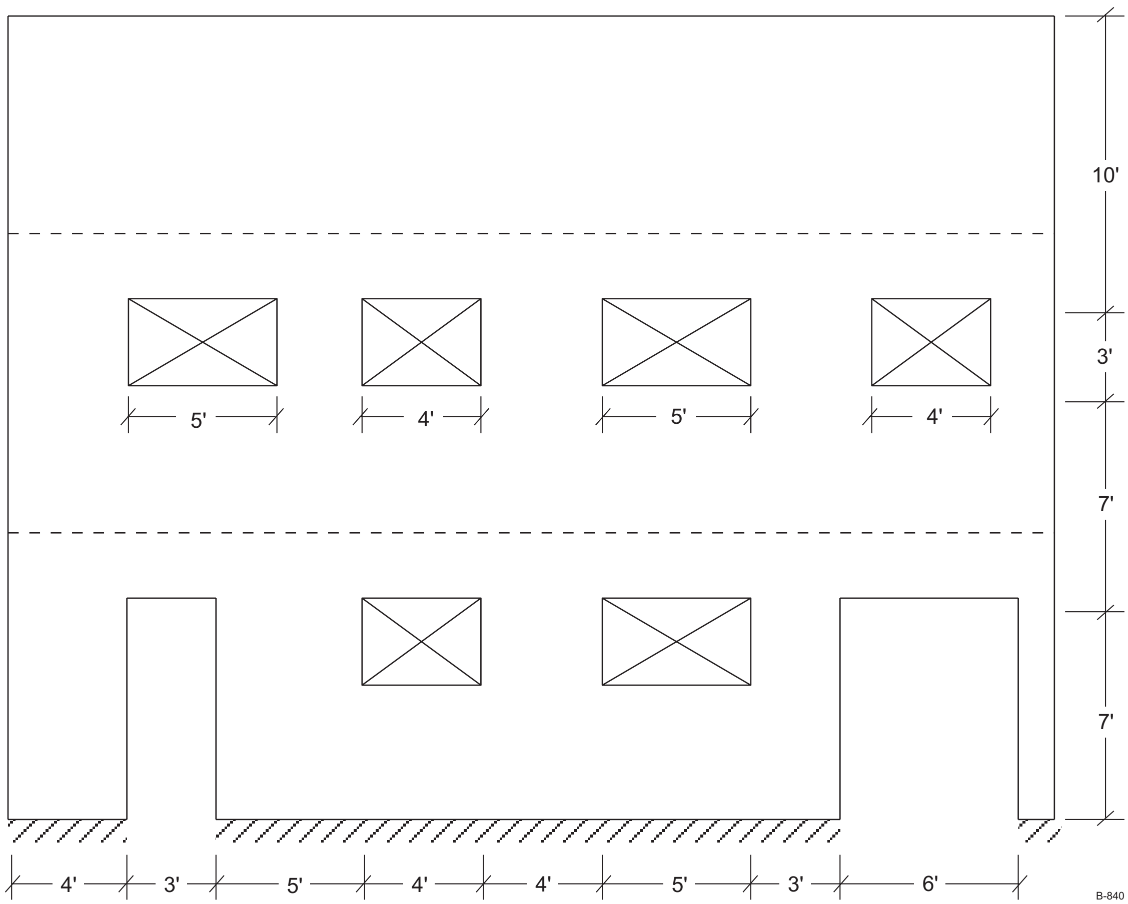


Figure E5-10(a)

1. **How many plate elements, nodes, and degrees-of-freedom (DOF) are in this model?**
2. **What areas of the model require higher mesh generation and perhaps greater discretization?**
3. **From a shear stress contour plot, what areas are subject to potential failure? Identify these zones and describe the structural engineering solution.**
4. **From the principle stress contours, what areas are subject to high tension and compression stresses? What is the solution for these zones?**

5. **For the door opening on the first floor (right-hand side), provide a structural detail from a schematic design that could be utilized in this area. What are the primary considerations for this pier?**

6. **How are the Von Mises stress contours different from shear stress contours. Are there any important conclusions derived from the Von Mises stress contours?**

7. **With the $F_{x,y}$ force plot option, show the force plots and describe the relevance these contours have on the structural design of the wall. What are the major conclusions from these contour plots?**

****

1. The geometric profile of the wall is shown along with dimensions for door and window openings [Figure E5-10(a)]. The FEA will discretize this into a finite element mesh. Use a mesh generation of 1- by 1-foot plate elements. In previous FEAs, the problem of high-mesh discretization led to expensive solution-processing times (programs were charged by the CPU). Now, with high-powered PC technology, the mesh size is no longer an obstacle.

   From the RISA-3D output files:

   DOF = 5,506

   PLATES = 801

   NODES = 917

   The lateral loads will be applied along the floor/diaphragm lines with nodal loads in the horizontal direction.

2. The task is to identify the global performance characteristics of the wall, and since the wall size is not large, the size of the model is solvable within a few seconds (run time less than 10 sec) on a Pentium Pro 600 Mhz. If a more detailed answer is necessary for stress concentration effects, then the areas around the door opening next to the pier (right-hand side) will require more finite elements. The areas around the doors and windows have normal pier sizes and do not need extra geometric definition. The deflected shape plot will provide a magnified view of the static response of a three-story wall. Distortion of the masonry block elements can be clearly observed in areas around the window and door openings. If an analysis of specific concentration effects were necessary, the "fine mesh" model of those specific areas would be required. However, in this example, the objective is to analyze the overall wall capacity and performance.

3. A shear stress contour plot will clarify the structural engineering judgement by showing high stress values around the right side of the door opening and on the left lower corner of the wall. This is expected because: a) the right side absorbs the maximum compression force and is a small dimension pier, and b) the left side has the high tension force. The piers between the openings also have high shear stress values.

   The shear stress contour plot enables the structural engineer to identify these areas and determine the appropriate level of reinforcement necessary. For example, the peak shear stress of approximately 4.0 ksi is well above the permitted maximum of 75 psi. Therefore, with a thicker section, a boundary element may be necessary, and will definitely require the placement of boundary steel reinforcement.

Keep in mind that this example has only the lateral shear loads in the model, whereas the axial compression forces would neutralize the high shear values noted. This example demonstrates the capabilities of the finite element analysis method.

4. The principal stress contours can be used to locate high compression and tension zones. Sigma 1 principal stress shows high tension stresses at the left corner. This represents the tension steel requirement at the corner boundary element. Sigma 2 shows high compression values at the right-hand side of the wall. These values may be interpreted and used for determining the level of steel reinforcement required.

5. The door-opening area on the right-hand side will certainly require a boundary element with, possibly, a flange detail. This is not modeled in the example, but is sketched in the diagram. The flange detail will increase the bending moment capacity and axial compression values. Since RISA-3D has three-dimensional capability, it is easy to include the flange detail in the model by adding a wall projecting along the z-axis.

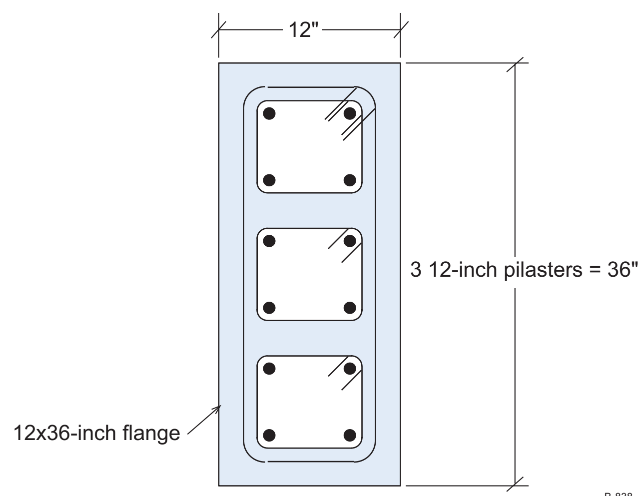


Figure E5-10(b)

6. The Von Mises stress follows the Von Mises plasticity yielding model. Therefore, the conclusions and values of these two contour plots are very similar. A close examination will show almost identical values with areas of stress concentration matching closely.

7. Force plots agree with the stress contours (as expected), but provide actual force distributions. The direction of each of the force plots is shown in the attached sketches. Essentially, this is just another approach to solving the same issues outlined in parts 1 through 6. Certain engineers prefer to work with force values rather than stresses. It's a matter of choice.

The F_y (Y-force) plots are indicative of compression and tension stress zones. The F_x (X-force) plots show the horizontal shear stress zones. The F_{xy} (XY-force) is the in-plane shear stress zones. A close examination of the force plots shows agreement with the stress contours. [Figure E5-10(h)].

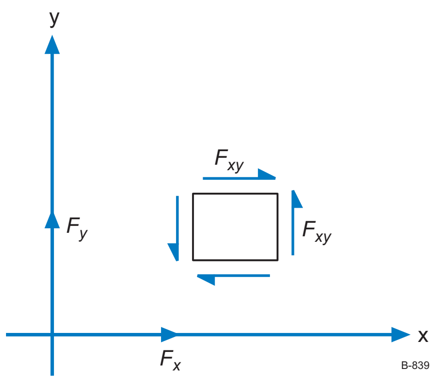


Figure E5-10(c)

The important fact is that the analysis results are in agreement with structural engineering judgment and can be verified through hand calculations.

6

Strength Design of Shear Walls and Masonry Wall Frames

6.1 Introduction

The concept of strength design has been in development for many years. It was originally crafted out of research in the steel and concrete industries. After much technical debate, and while still contested by many seasoned engineers, the strength design provisions have succeeded in displacing many of the working stress design philosophies applied to other structural materials. It is readily apparent that the thrust toward strength design with unifying load factors is becoming the focus of discussion, with working stress design gradually being omitted from the design process.

For example, in reinforced concrete design, the working stress design (WSD) approach has been almost totally replaced by ultimate strength design (USD) in textbooks and code reference documentation. In most texts, USD is featured with only a mention of WSD in the appendix/reference section. In steel design, load and resistance factor design (LRFD) is slowly shedding the allowable stress design (ASD) procedure and will eventually become the mainstay of the profession. The 2000 IBC is heavily slanted toward strength design (SD) in all materials, but does leave the door open to the continued use of WSD in masonry and wood structures.

The main difference between SD and WSD is the potential saving of material. All construction costs are proportionate to material cost. Therefore, if engineers can reduce the quantity of material required, it should translate to lower total project costs. LRFD/USD/SD could save between 5 and 10 percent of the anticipated cost of a construction project. This is the industry estimate used to promote strength design. The principles of strength design for large-scale projects are recommended where the additional design time/effort is warranted. However, for smaller projects, the extra effort may not be warranted given the low volume of materials versus the additional design effort required.

For example – If a structural engineer is designing a block wall for a freeway application that requires more than 10,000 linear feet of block wall (2 miles), the final cost might be \$30/linear foot × 10,000 = \$300,000. Using a strength design approach can reduce the reinforcement, block size, strength size, and spacing requirements to save 5 percent of \$300,000 (\$15,000). This saving, applied on an entire length of freeway retaining wall system of 50 to 100 miles or more, quickly reveals the benefits of using strength design.

Take another example. A structural engineer designs a block wall for the owner of an apartment complex where the total length of the wall may reach 100 linear feet. [100 ft × \$50/ft (\$5,000)]. The unit cost is higher because although the quantity of wall is lower, the fixed cost of installation does not change. In this example the savings may be 5 percent of \$5,000 (\$250). The amount of design work required of the engineer is the same in both cases – yet in the first example, the structural engineer can command a fee for value engineering the final design concept, whereas in the second case, there is little incentive to perform any value engineering at all.

Therefore, there exists a split in the practice of structural engineering regarding SD versus WSD. Those engineers who graduated before 1980 are fully trained in WSD and find SD to be a nuisance. Recent graduates, on the other hand, are equally well trained in SD and find it to be second nature. It can be concluded that each method is useful and practical depending on the circumstances as outlined in the examples above.

In summary:

1. Strength design relies on the evaluation of the factored load combinations versus the ultimate strength/capacity.
2. Strength design is based on the ultimate stress/strain condition of the masonry section.

   (Load factors are provided in Chapter 16 of the 1997 UBC and the 2000 IBC.)

Figure 6-1 is a flowchart that outlines the evaluation process for three structural components using strength design:

1. Shear walls with in-plane loads
2. Shear walls with out-of-plane loads
3. Masonry wall frames with in-plane loads

The key parameter for using the SD methods rather than the WSD is the height-thickness ratio, h/t. With WSD, the h/t is limited to between 25 and 30, depending on the end restraint and base fixity. By using the SD procedures and slender wall equations, this limit can be exceeded. There is no technical upper bound on the h/t in SD, but the analysis procedures in SD can be used as a means of control.

The same principle holds true for in-plane load considerations. SD has a consideration for the strain distribution within the shear wall. This strain-compatibility approach considers the web steel to be part of the overturning moment capacity, which leads to a greater value for the in-plane wall capacity.

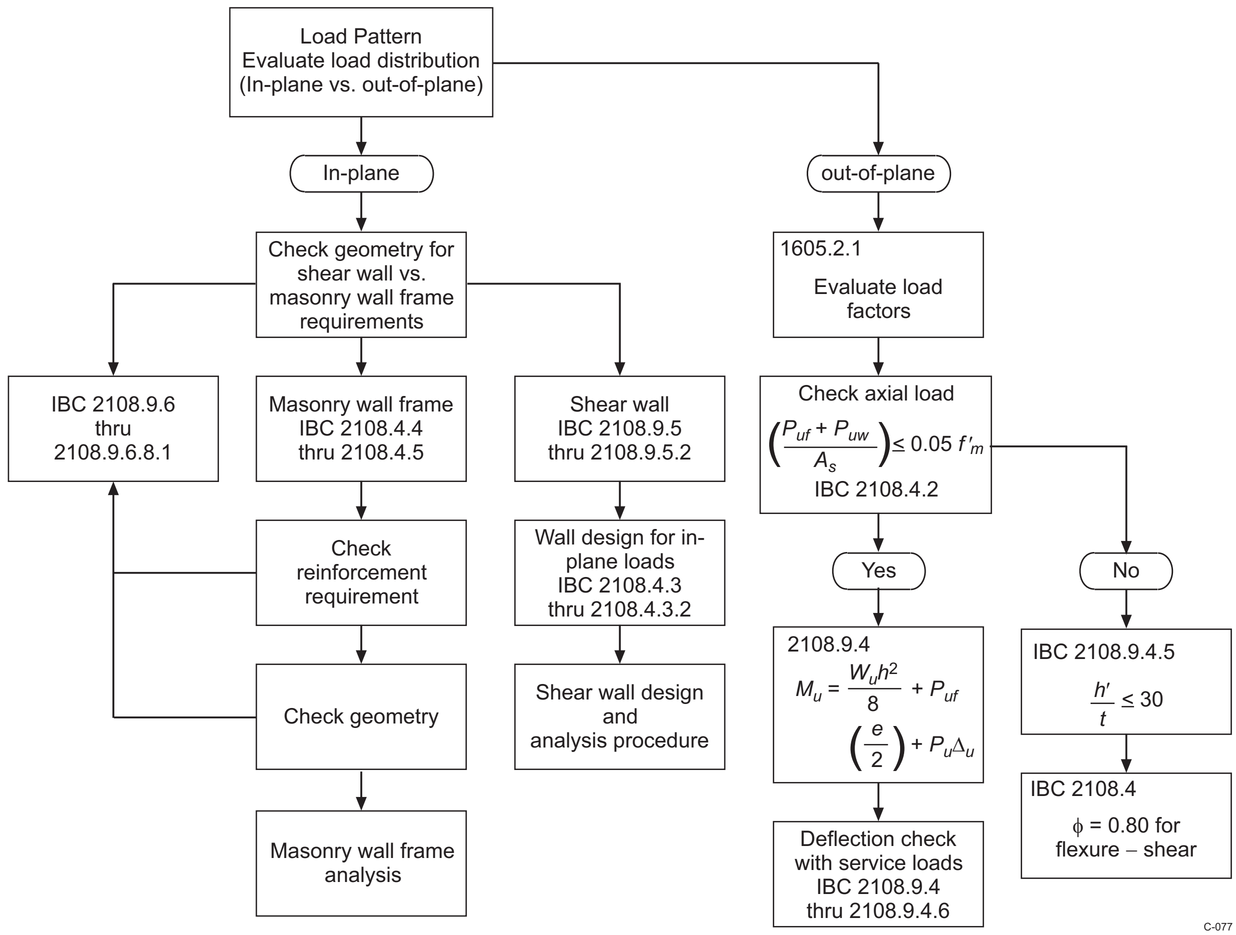


Figure 6-1

Strain compatibility is illustrated in Figure 6-2.

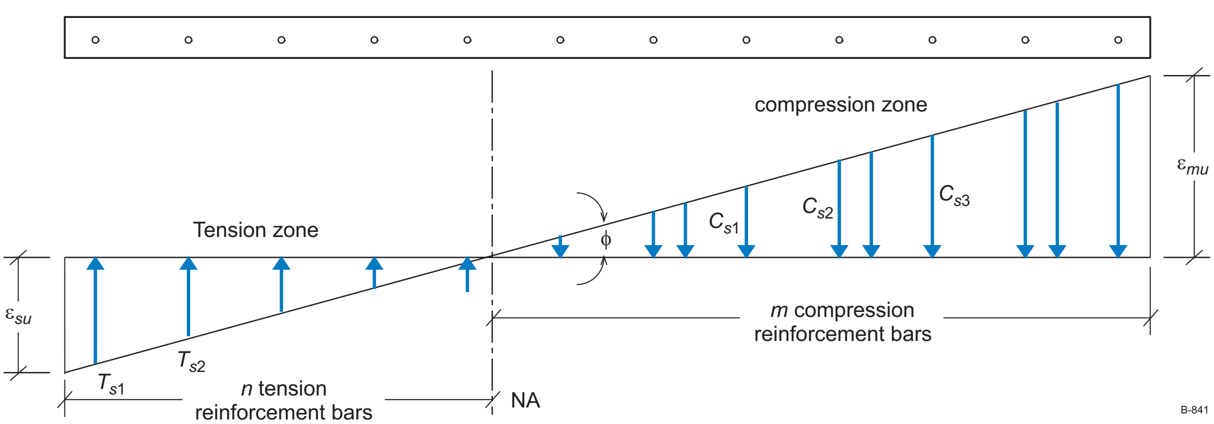


Figure 6-2

Strain compatibility:

$$\underbrace{\sum_{i=1}^{n} T_{si}}_{\text{Tension reinforcement force}} = \underbrace{\sum_{i=1}^{m} C_{si}}_{\text{Compression reinforcement force}} + \underbrace{C_m}_{\text{Compression masonry force}}$$

where

$$T_{si} = (A_{sTe})(f_{si})$$

$$\therefore\ T_{si} = (A_{si})(E_s)(\varepsilon_{si})$$

For compression zone:

$$C_{si} = (A_{sci})(f_{sci})$$

$$\therefore\ C_{si} = (A_{sci})(E_s)(E_{sci})$$

and

$$C_m = 0.85 f'_m\ ab$$

The key part of this solution is to find the location of the neutral axis, which allows for strain compatibility and force balance

$$\sum_{i=1}^{n} T_{si} = \sum_{i=1}^{m} C_{si} + C_m$$

$$\sum_{i=1}^{n} A_{si} E_s \varepsilon_{si} = \sum_{i=1}^{m} A_{sci} E_s \varepsilon_{sci} + 0.85 f'_m\, ab$$

where $a = \beta c$

Single reinforced sections

The fundamental strength design equations for conventional single reinforced sections, are derived as in Figure 6-3.

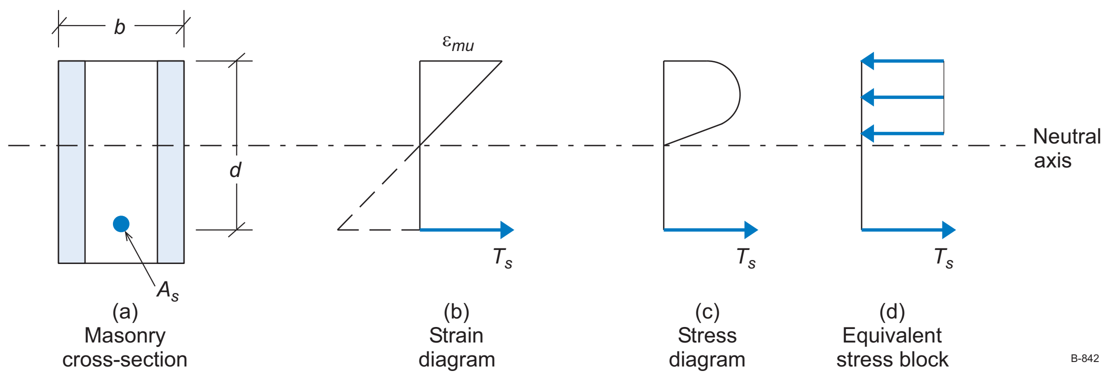


Figure 6-3

Tension Reinforcement equations

From the strain diagram in Figure 6-3, the stresses are computed using the equivalent stress block theory.

Two modifications from the 2000 IBC are the ultimate compression strain and calculation ε_m

ε_{mu} = maximum masonry compression strain

= 0.0035 clay masonry **2108.9**

= 0.0025 concrete block **2108.7.2**

$\varepsilon_m = 700 f'_m$ for clay masonry **2108.7.2**

$\varepsilon_m = 900 f'_m$ for concrete masonry

The design assumptions from IBC 2108.9.1 are included here for clarity.

2108.9 Design assumptions.

The following assumptions apply:

1. Masonry carries no tensile stress greater than the modulus of rupture.
2. Reinforcement is completely surrounded by and bonded to masonry material so that they work together as a homogeneous material.
3. Nominal strength of singly reinforced masonry wall cross sections for combined fixture and axial load shall be based on applicable conditions of equilibrium and compatibility of strains. Strain in reinforcement and masonry walls shall be assumed to be directly proportional to the distance from the neutral axis.
4. The maximum usable strain, ε_{mu}, at the extreme masonry compression fiber shall be assumed to be 0.0035 inch/inch (mm/mm) for clay masonry and 0.0025 inch/inch (mm/mm) for concrete masonry.

5. Strain in reinforcement and masonry shall be assumed to be directly proportional to the distance from the neutral axis.

6. Stress in reinforcement below specified yield strength f_y, for grade of reinforcement used shall be taken as E_s times steel strain. For strains greater than that corresponding to f_y, stress in reinforcement shall be considered independent of strain and equal to f_y.

7. Tensile strength of masonry walls shall be neglected in flexural calculations of strength, except when computing requirements for deflection.

8. Relationship between masonry compressive stress and masonry strain may be assumed to be rectangular.

9. Masonry stress of $0.85f'_m$ shall be assumed uniformly distributed over an equivalent compression zone bounded by edges of the cross section and a straight line located parallel to the neutral axis at a distance $a = 0.85c$ from the fiber of maximum strain to the neutral axis shall be measured in a direction perpendicular to the axis. (Figure 6-4)

Two formulations are calculated; one for clay and one for concrete masonry.

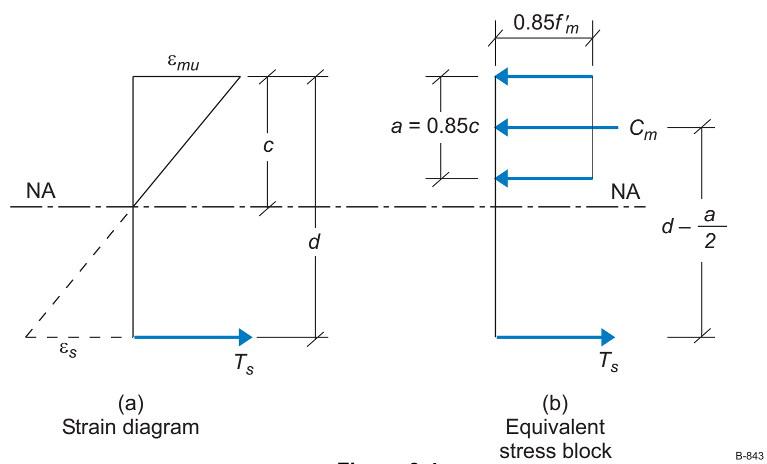


Figure 6-4

$$\Sigma F_x \equiv 0$$

C_m = compressive force in the masonry = $0.85f'_m ab$

T_s = tensile force in the reinforcement = $A_s F_y$

$$C_m = T_s$$

$$085f'_m ab = A_s F_y$$

$$\therefore\ a = \frac{A_s F_y}{0.85\, f'_m b}$$

Define $\rho = \dfrac{A_s}{bd}$ = steel reinforcement ratio.

$$A_s = \rho bd$$

$$\text{then } a = \frac{\rho bdF_y}{0.85f'_m b} = \frac{\rho dF_y}{0.85f'_m b}$$

Moment capacity of the section

$$M_n = C_m\left(d - \frac{a}{2}\right) = T_s\left(d - \frac{a}{2}\right)$$

$$M_n = 0.85f'_m ab\left(d - \frac{a}{2}\right) \quad \text{Masonry capacity}$$

$$M_n = A_s F_y\left(d - \frac{a}{2}\right) \quad \text{Steel capacity}$$

and,

$$\phi M_n = \phi 0.85f'_m ab\left(d - \frac{a}{2}\right) \quad \text{Masonry capacity}$$

$$\phi M_n = \phi A_s F_y\left(d - \frac{a}{2}\right) \quad \text{Steel capacity}$$

In terms of steel ratio,

$$M_n = 0.85f'_m b\left(\frac{\rho dF_y}{0.85f'_m}\right)\left(d - \frac{\rho dF_y}{2(0.85)f'_m}\right)$$

$$M_n = \rho F_y bd^2\left(1 - (0.59)\frac{\rho F_y}{f'_m}\right)$$

Define K_n = Flexural coefficient = $\rho F_y\left(1 - 0.59\dfrac{\rho F_y}{f'_m}\right)$

$$M_n = K_n bd^2$$

$$\phi M_n = \phi K_n bd^2 \text{ with } K_n = \rho F_y\left(1 - 0.59\frac{\rho F y}{f'_m}\right)$$

Balanced steel ratio

Define as

$$\rho_b = \frac{A_{sb}}{bd}$$

By definition, a balanced failure is a strain condition where the masonry and the steel reach their respective failure strains simultaneously.

Figure 6-5 shows three strain diagrams that correspond to varying failure patterns.

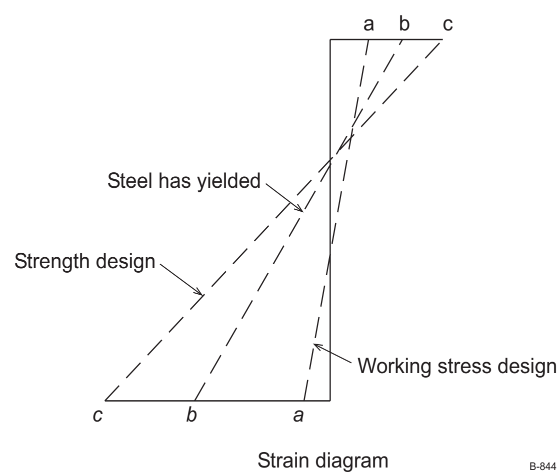


Figure 6-5

Balanced steel failure

Three conditions are shown in Figure 6-5.

a-a: This stress-strain condition corresponds to the WSD condition. The steel strain has not yielded and the masonry strain is below its critical yield. Although masonry does not have a specific yield point, this condition corresponds to $0.33f'_m$, which is the allowable compressive stress in the masonry.

b-b: This stress-strain condition corresponds to the intermediate step between WSD and the SD condition of **c-c**. The steel has reached its yield strain before the masonry has reached its compressive strain limit, ε_{mu}. The significance of **b-b** is that the steel is yielding and causing tension reinforcement to deform. This produces cracking on the tension face of the member. Examples of this type of failure are important to understand because this is the essential reason that reinforced masonry (and concrete) are designed with steel to yield prior to failure.

c-c: This stress-strain value corresponds to an SD condition. The tension steel has yielded and produced prolonged deformation, and the masonry has reached its maximum compressive strain, ε_{mu}. In previous codes, this was defined as 0.003. However, the 2000 IBC now defines it as either 0.0035 (clay masonry) or 0.0025 (concrete masonry).

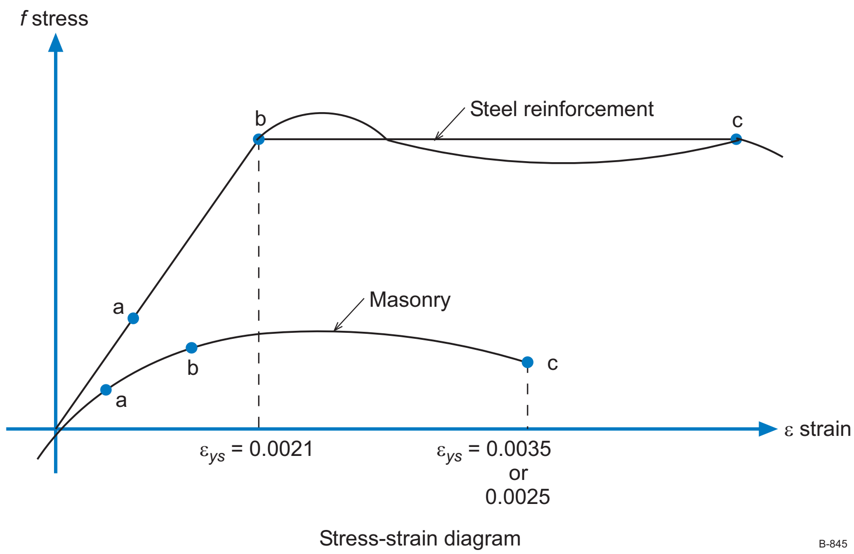


Figure 6-6

As the cross section develops its ultimate strength from **a-a** to **c-c**, it undergoes stress and strain transformations. Understanding the reasons for having the steel reinforcement yield before the compressive strain in the masonry reaches its ultimate value is of major importance. Figure 6-6 represent the stress vs. strain for masonry and steel.

1. Steel yielding. Steel yielding before the masonry allows for deformation on the masonry tension face creates visible deflection in the structure.
2. Masonry is nonductile. Because masonry has nonductile behavior beyond its peak compressive strength, the possibility of a sudden failure is likely. Therefore, this would pose a dangerous scenario for occupants. By imposing the requirement of steel yielding first, the masonry cannot fail without extensive signs of tension cracking.
3. Cracking/warning signs. Cracks on the tension face of the masonry structure provide clear signals to occupants that the structure may be close to collapse. Warning signs have been present in numerous cases of actual structures that first experienced failure stresses and then physically collapsed. This is the principle behind the under-reinforcement of a section (providing less steel than the balanced steel ratio).

Retaining wall failure

The tension stress and strain occur on the inside face of the wall (i.e., soil side). A progressive collapse scenario would entail increasing tensile strains that lead to large deflections at the top of the wall. Although no direct signs of cracking are observed on the compression face of the wall, the "leaning effect" can be noted by those same observers. This creates an indirect warning sign to occupants that the wall is undergoing unusually high stress. (Figure 6-7)

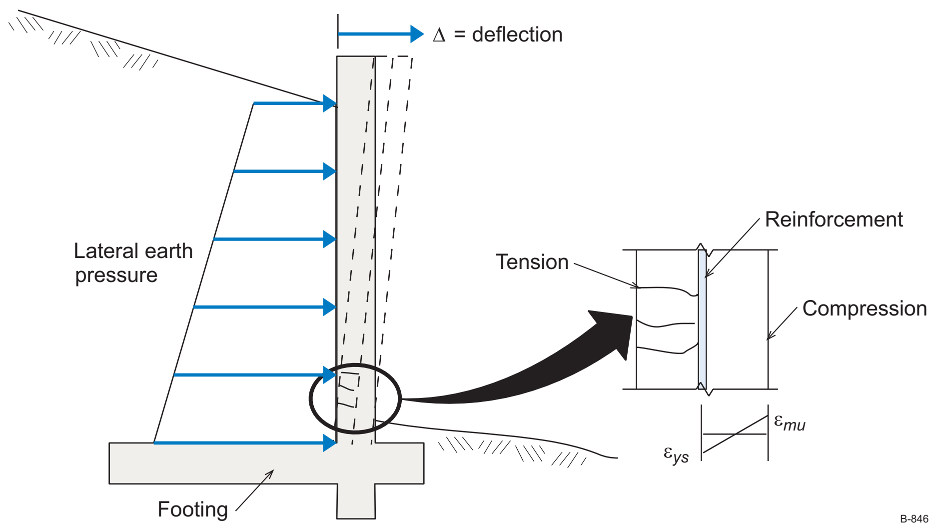


Figure 6-7

The high ductility of the steel reinforcement allows large deformations to develop and the wall to bend without collapsing. This prevents immediate collapse of the structure, but allows a slow progressive collapse for which temporary shoring may be provided. In any case, occupants can remain clear and safe.

Shear wall failure

The shear wall failure scenario shown in Figure 6-8 is illustrated with diagonal tension field cracks. These diagonal cracks follow the theory of elasticity principles. Specifically, the maximum shear stress occurs at a 45-degree angle to the principle stress, as seen in structures that have exhibited this type of failure.

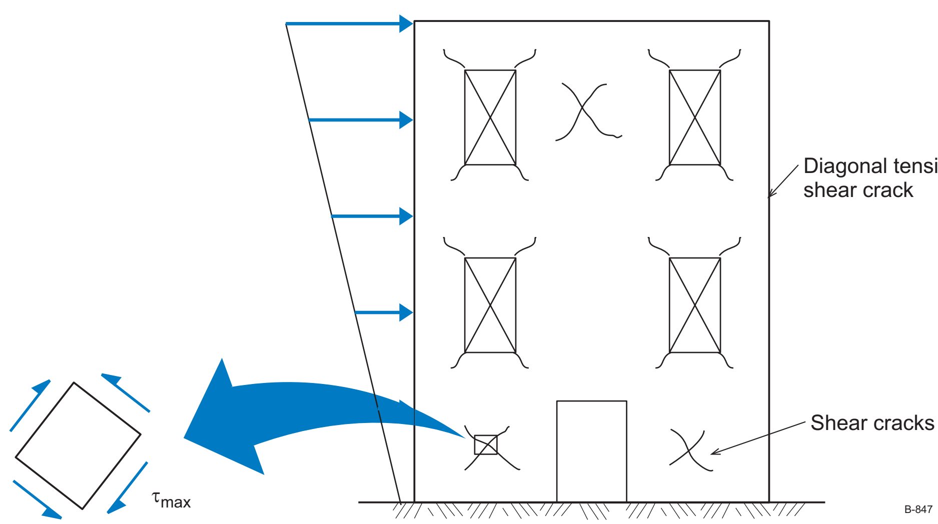


Figure 6-8

Once again the concept of steel yielding allows the presence of significant cracking before actual failure/collapse occurs. This gives suitable warning time for occupants to vacate the structure. The ductility of the steel reinforcement is important to the structural performance of reinforced masonry.

Balanced steel condition

Balanced Failure – *the steel reinforcement percentage that corresponds to the simultaneous failure of the steel and the masonry.* The balanced failure condition is a hypothetical strain diagram for calculating the balanced steel ratio. This ratio is used to define the maximum steel reinforcement permitted in a reinforced masonry structure. The reason for requiring a maximum steel ratio is clear – It is not advisable to over-reinforce a section.

Three reinforcement conditions may exist in a masonry structure.

1. *Over-reinforcement* – an undesirable condition not permitted by either the IBC or UBC, occurs when the quantity of steel exceeds the balanced steel ratio. As illustrated in this chapter's examples, over-reinforcement could cause a sudden collapse because the steel would not yield sufficiently to create significant warning signs.
2. *Under-reinforcement* – is the preferred condition by both the IBC and UBC. It occurs when the steel is less than the maximum percentage value allowed by the code. This condition creates a ductile failure of the masonry structure.
3. *Balanced reinforcement* – The hypothetical situation for calculating the balanced steel ratio. It is not allowed in practice because it would correspond to a nonductile failure.

The derivation of the balanced steel ratio is

$$\therefore\ \rho_b = \frac{0.85(0.531)f'_m}{(d)F_y}d = 0.451\left(\frac{f'_m}{F_y}\right) \text{ clay masonry}$$

$$\therefore\ \rho_b = \frac{0.85(0.462)f'_m}{F_y} = 0.393\left(\frac{f'_m}{F_y}\right) \text{ concrete masonry}$$

Maximum reinforcement ratio, 1997 UBC

The 1997 UBC provisions limit the steel reinforcement to 0.5ρ, which may be calculated directly from the basic equations of ρ_{bal}, but using a maximum masonry strain of $\varepsilon_{mu} = 0.003$ for both clay and concrete. These equations are provided for clarity.

Maximum reinforcement ratio, 2000 IBC

The 2000 IBC has altered the approach to involve the strain diagram of the cross section. The basic equations for maximum reinforcement are now replaced per IBC 2108.9.2.13.

The derivation of the maximum reinforcement is based on the strain diagram and is divided into two methods: A and B. Both are acceptable, and are presented here accompanied by useful tables.

METHOD A

Masonry

Story drift $= \delta = \frac{A}{h} \leq 0.01\, n_{sx}$

$\varepsilon_{mu} < 0.0035$ for clay masonry

$\varepsilon_{mu} < 0.0025$ for concrete masonry

In-plane forces (Figure 6-9)

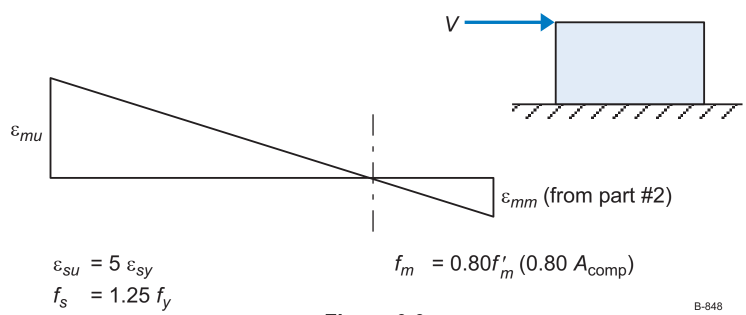


Figure 6-9

Out-of-plane forces (Figure 6-10)

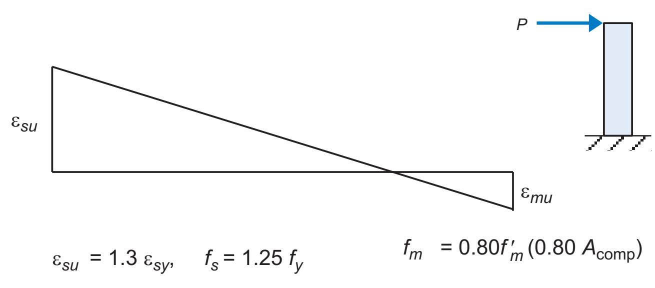


Figure 6-10

Steel

To calculate maximum steel ratio

In-plane forces

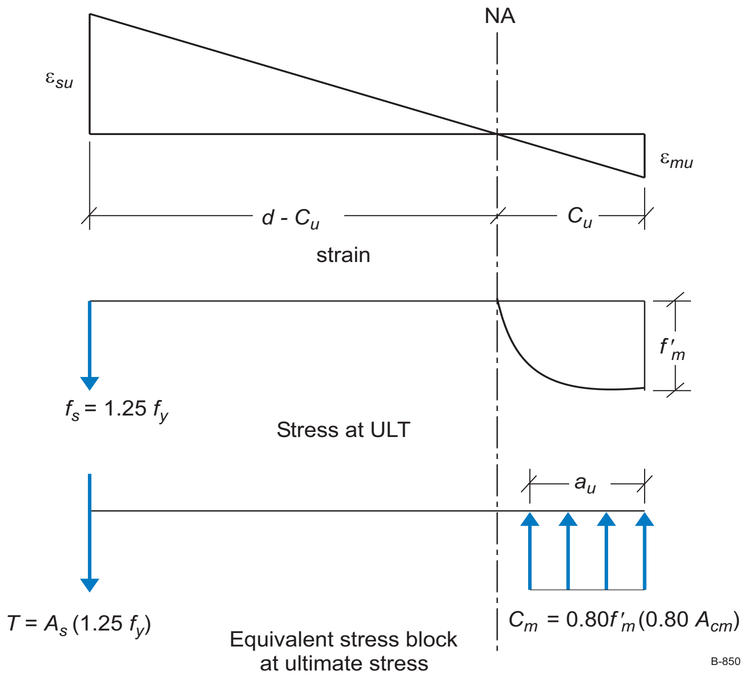


Figure 6-11

Locate NA from strain diagram

$$\frac{\varepsilon_{mu}}{c_u} = \frac{\varepsilon_{su} + \varepsilon_{mu}}{d}$$

$$c_u = d\left(\frac{\varepsilon_{mu}}{\varepsilon_{su} + \varepsilon_{mu}}\right)$$

$$A_{cm} = c_u b$$

$$C_m = 0.80 f'_m \,(0.8 c_u b) \qquad \textbf{2108.9.2.13.1}$$

$$T_s = 1.25 A_s f_y$$

$$T_s = C_m$$

$$1.25\, A_s f_y = (0.80)^2 f'_m \, c_u b$$

$$(A_s)_{\max} = \frac{(0.80)^2 f'_m \, c_u b}{1.25\, f_y}$$

where $c_u = d\left(\frac{\varepsilon_{mu}}{\varepsilon_{su} + \varepsilon_{mu}}\right)$

$$\rho_{max} = \frac{(A_s)_{max}}{bd} = \frac{(0.80)^2 f'_m}{1.25 f_y} \left\{ \frac{\varepsilon_{mu}}{\varepsilon_{su} + \varepsilon_{mu}} \right\}$$

For clay masonry

$$\varepsilon_{mu} = 0.0035$$

$$\varepsilon_{su} = 5\varepsilon_{sy} = 5 \times 0.002 = 0.01$$

with grade 60 steel

$$f_y = 60 \text{ ksi}$$

where $\varepsilon_{sy} = \dfrac{60}{29{,}000} = 0.002$

For clay masonry with grade 60 steel

$$\rho_{max} = \frac{(0.80)^2 f'_m}{1.25 f_y} \left(\frac{0.0035}{0.01 + 0.0035} \right)$$

$$\rho_{max} = 0.26 \frac{f'_m}{f_y}$$

In-plane shear, clay masonry, grade 60 steel (f_y =60 ksi)

f'_m	In-plane ρ_{max}
1500	0.00650
2000	0.00867
2500	0.01083
3000	0.01300
3500	0.01517
4000	0.01733
4500	0.01950
5000	0.02667

For concrete masonry with grade 60 steel

$$\rho_{max} = \frac{(0.80)^2}{1.25} \left(\frac{f'_m}{f_y} \right) \left(\frac{(0.0025)}{0.01 + 0.0025} \right)$$

$$\rho_{max} = 0.20 \left(\frac{f'_m}{f_y} \right)$$

f'_m	In-plane ρ_{max}
1500	0.00500
2000	0.00427
2500	0.00533
3000	0.00640
3500	0.00747
4000	0.00853
4500	0.00960
5000	0.01067

Out-of-plane

Same stress and strain diagram with equations, except the ultimate steel strain is 1-3 ε_{sy}

Clay masonry

$$\varepsilon_{mu} = 0.0035$$

$$\varepsilon_{sy} = 1.3\,(0.002) = 0.0026$$

$$\rho_{max} = \frac{(0.80)^2}{1.25}\left(\frac{f'_m}{f_y}\right)\left(\frac{0.0035}{0.0026 + 0.0035}\right)$$

$$\rho_{max} = 0.2938\left(\frac{f'_m}{f_y}\right) \text{ clay masonry}$$

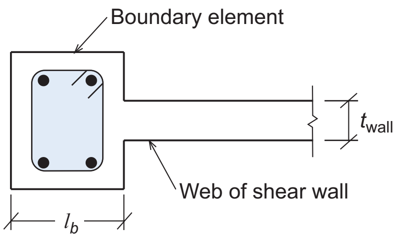
Concrete masonry

$$\rho_{max} = \frac{(0.80)^2}{1.25}\left(\frac{f'_m}{f_y}\right)\left(\frac{0.0025}{0.0026 + 0.0025}\right)$$

$$\rho_{max} = 0.2510\left(\frac{f'_m}{f_y}\right) \text{ concrete masonry}$$

Maximum reinforcement
Out-of-plane, f_y = 60 ksi

f'_m (psi)	ρ_{max} Concrete masonry ε_{mu} = 0.0025	ρ_{max} Clay masonry ε_{mu} = 0.0035
1500	0.006275	0.007345
2000	0.008367	0.009793
2500	0.010460	0.012242
3000	0.012550	0.014690
3500	0.014642	0.017380
4000	0.016733	0.019590
4500	0.018825	0.022040
5000	0.020917	0.024480



l_b = length of boundary elements

Figure 6-12

METHOD B

Maximum reinforcement

$$\text{Story drift} = \delta = \frac{\Delta}{h} \leq 0.13 h_{sx} \qquad \textbf{1617-3}$$

In-plane requirement (Figure 6-12)

If ε_{cu} = compressive strain in shear wall > 0.002,

then

boundary members are required in the shear wall

$(l_B)_{min} = 3(t_{wall})$ and must include areas

where $\varepsilon_{cu} > 0.002$

1. Lateral reinforcement for boundary elements is minimum of #3 closed ties @ 8 inches o/c
2. The grouted care must develop an ultimate compressive strain of 0.006 ($\varepsilon_{mu} = 0.006$)

$$(\rho_{max})_{longitudinal} \leqq 0.15 \frac{f'_m}{f_y}$$

In-plane Shear

Concrete and clay masonry $(\rho_{max})_{longitudinal} \leqq 0.15 \frac{f'_m}{f_y}$

f'_m	ρ_{max}
1500	0.00375
2000	0.00500
2500	0.00625
3000	0.00750
3500	0.00875
4000	0.01000
4500	0.01125
5000	0.01250

Double reinforced sections

A double reinforced section contains both tension and compression steel. This type of structural element is seldom used in reinforced masonry, but is important for analysis purposes. In the 1997 UBC, the analysis of this section was simple because the ultimate strain value of the masonry was 0.003, and the maximum reinforcement ratio was based on 0.5 p_{bal}. This is no longer the case.

For the 2000 IBC, the maximum strain value is two possible numbers and the maximum reinforcement ratio is a complex equation. Therefore, the fundamental concepts of strength design are the same, but the final equations are left in a format that may be applied to either clay or concrete masonry. Engineers must be careful to enter the correct strain and maximum reinforcement value into these equations. A spreadsheet can be created to assist with these calculations.

A derivation of the basic double reinforced equations is provided and is based on the Principle of Superposition. Two sections are created. One has a single reinforced element and the other is a tension-compression steel section. The two sections are analyzed and the final moment capacity is added at the end. The phi (ϕ) factor is applied at the end, and is calculated from IBC 2108.4.1.

The sections pictured in Figure 6-13 have tension and compression steel.

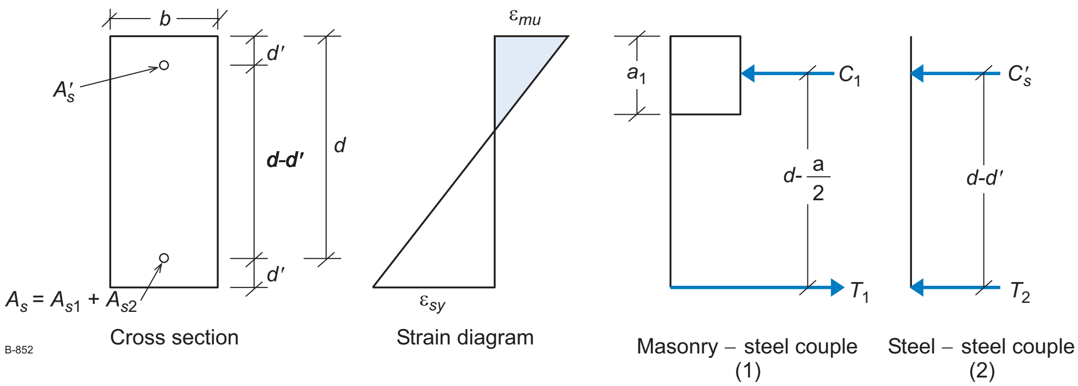


Figure 6-13

Moment capacity $= M_n = M_1 + M_2$

where: M_1 = Moment capacity of the masonry-steel force couple system

M_2 = Moment capacity of the compression-tension steel couple

Based on the Principle of Superposition, these two force couple systems are added to calculate the total resisting force-couple of a double reinforced systems.

The ϕ factor is applied to determine the ultimate moment capacity

$$\phi M_n = \phi(M_1 + M_2)$$

Masonry – Steel Couple

$$M_1 = T_1\left(d - \frac{a}{2}\right)$$

To calculate M_1 (based on the maximum steel ratio (ρ_{max}))

$$A_{S1} = \rho_{max} bd$$

$$\therefore \; T_{S1} = A_{S1}F_y$$

and the neutral axis location is determined from the single reinforced section

$$\rho_{max} = \frac{(0.80)^2 f'_m}{1.25F_y}\left(\frac{\varepsilon_{mu}}{\varepsilon_{mu} + \varepsilon_{su}}\right)$$

$$a_1 = 0.85c_1 = \frac{A_{s1}F_y}{0.85f'_m b}$$

and,

$$M_1 = T_{s1}\left(d - \frac{a_1}{2}\right)$$

$$M_1 = A_{s1}F_y\left(d - \frac{a}{2}\right)$$

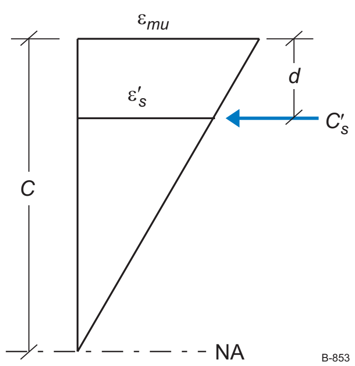


Figure 6-14

Steel – Steel Couple

$$A_{s2} = A_s - A_{s1}$$

and $A'_s = A_{s2}$

Tension steel $A_{s2}, A_{s2} = F_y$

Compression steel $A'_s \; ; f_s' < F_y$

To calculate the stress and strain in the compression steel, use similar triangles (Figure 6-14).

$$f'_s = \varepsilon'_s E_s$$

$$f'_s = \varepsilon_{mu}\left(\frac{c - d'}{c}\right) E_s$$

and ε_{mu} = 0.0025 concrete masonry

= 0.0035 clay masonry

and $c = \dfrac{a_1}{0.85}$

6.2 Shear Wall Analysis

The load factors for strength design (SD) are found in IBC Section 1605.

Formula/number	
$1.4D$	16-1
$1.2D + 1.6L + 0.5(L_r$ or S or $R)$	16-2
$1.2D + 1.6(L_r$ or S or $R) + (f_1L$ or $0.8W)$	16-3
$1.2D + 1.6W + f_1L + 0.5\ (L_r$ or S or $R)$	16-4
$1.2D + 1.0E + f_1L + f_2S$	16-5
$0.9D + (1.0E$ or $1.6W)$	16-6

For masonry shear walls, there is an additional multiplier of 1.1 that is added in the 1997 UBC but omitted in the IBC. The strength design of shear walls may be considered in the following categories.

- *Axial and lateral load*: Axial compression of the wall from vertical dead and live loads and out-of-plane lateral loads (Figure 6-15)

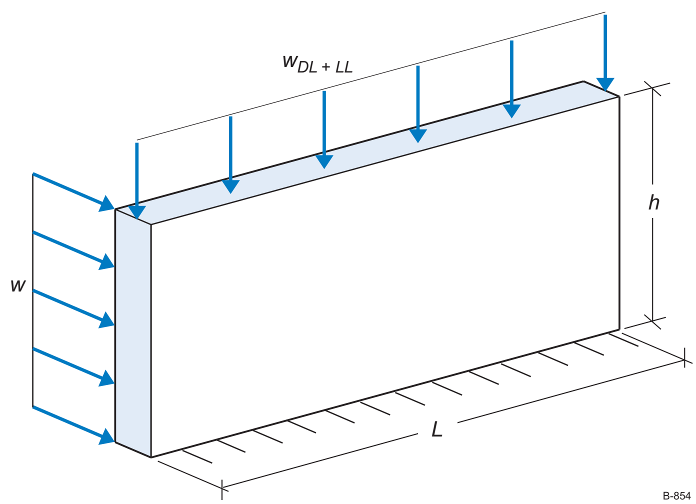


Figure 6-15

For this category, the axial load analysis considering the slender wall provision from the IBC is applicable. The critical loads are multiplied by the appropriate load factor and then the slender wall analysis procedure is used.

- *Axial and shear (in-plane) loads*: Axial compression plus in-plane shear load Figure 6-16.

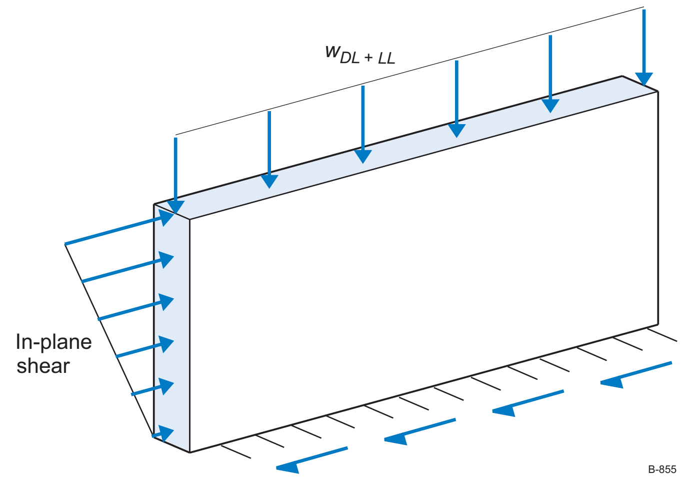


Figure 6-16

For this category of loading, the axial and in-plane load analysis is performed in the context of a load-moment interaction curve (i.e., P-M diagram). There are several established methods for this type of analysis. Two methods are taken from the RMEH, 5th edition, by Amrhein.

- *Case 1a: Slender wall analysis procedure* (2000 IBC) (Figure 6-17)

  This design procedure is intended for walls with high (h/t) ratios. *Slender* is roughly defined as $h/t > 30$.

The method accounts for post-bucking stability failure of the wall by considering the P-delta (P-Δ) effect of the wall. This procedure is adopted (with minor modification) from the 1997 UBC into the 2000 IBC.

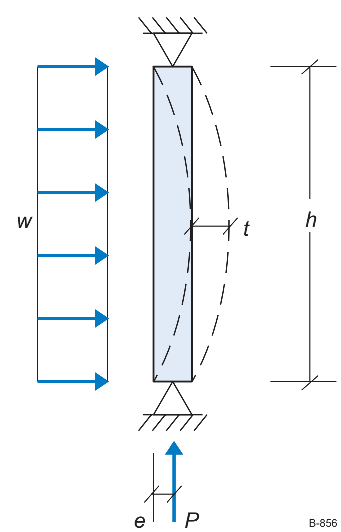
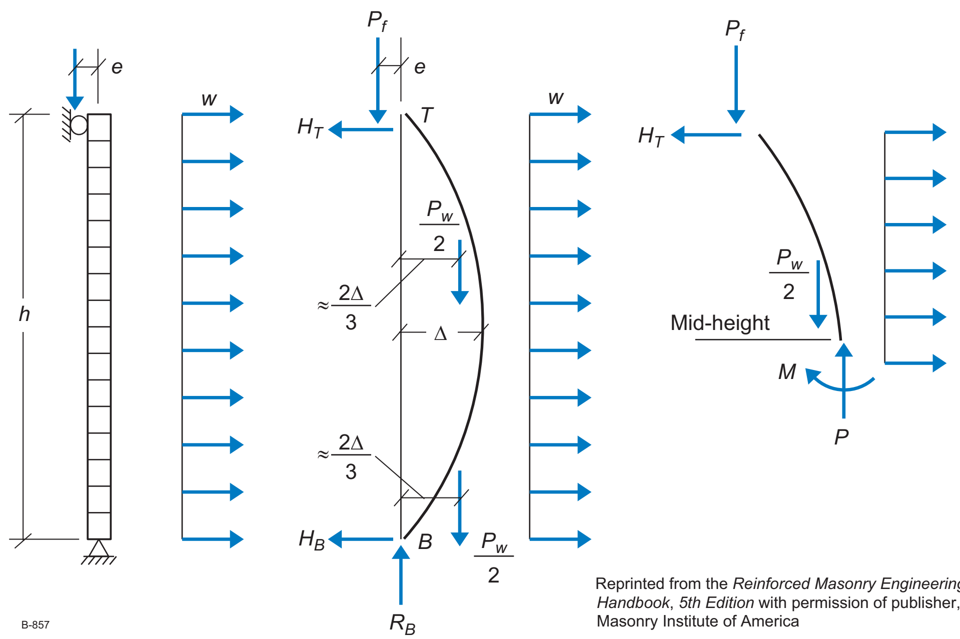


Figure 6-17

- *Case 1b: Shearwall design procedure* (Figure 6-18) **2108.9.4**



Reprinted from the *Reinforced Masonry Engineering Handbook*, *5th Edition* with permission of publisher, Masonry Institute of America

Figure 6-18

1. Check axial stress requirement

$$\left(\frac{P_{uw} + P_{uf}}{A_g}\right) \leq 0.05f'_m$$ **Eq. 21-32**

P_{uw} = factored wall weight

P_{uf} = factored floor load

A_g = gross area of wall

If this axial stress limitation is not satisfied, increase the cross section or limit $(h/t) < 30$ provided $(P_{uw} + P_{uf}/A_g) < 0.20f'_m$ **2108.9.4.5**

2. Minimum wall size is 6 inches

3. Maximum reinforcement = ρ_{max} **2108.9.2.13**

4. Δ_{max} = maximum lateral deflection **2108.9.4.6**

Δ_{max} = $0.007h$ based on service loads

5. Strength analysis check

M_u = factored moment capacity at midheight

$$M_u = \frac{W_u h^2}{8} + P_{uf}\frac{e}{2} + P_u\Delta_u$$ **Eq. 21-33**

Eq. 21-34

Eq. 21-34

$$\text{where } P_u = P_{uw} + P_{uf} \qquad \textbf{Eq. 21-34}$$

$$\phi M_n = \text{wall capacity} = A_{se} f_y \left(d - \frac{a}{2}\right) \qquad \textbf{Eq. 21-36}$$

$$A_{se} = \frac{A_s f_y + P_u}{f_y} \qquad \textbf{Eq. 21-37}$$

$$a = \frac{(P_u + A_s f_y)}{0.85 f'_m b} \qquad \textbf{Eq. 21-38}$$

The design-analysis is performed iteratively as shown in Figure 6-19.

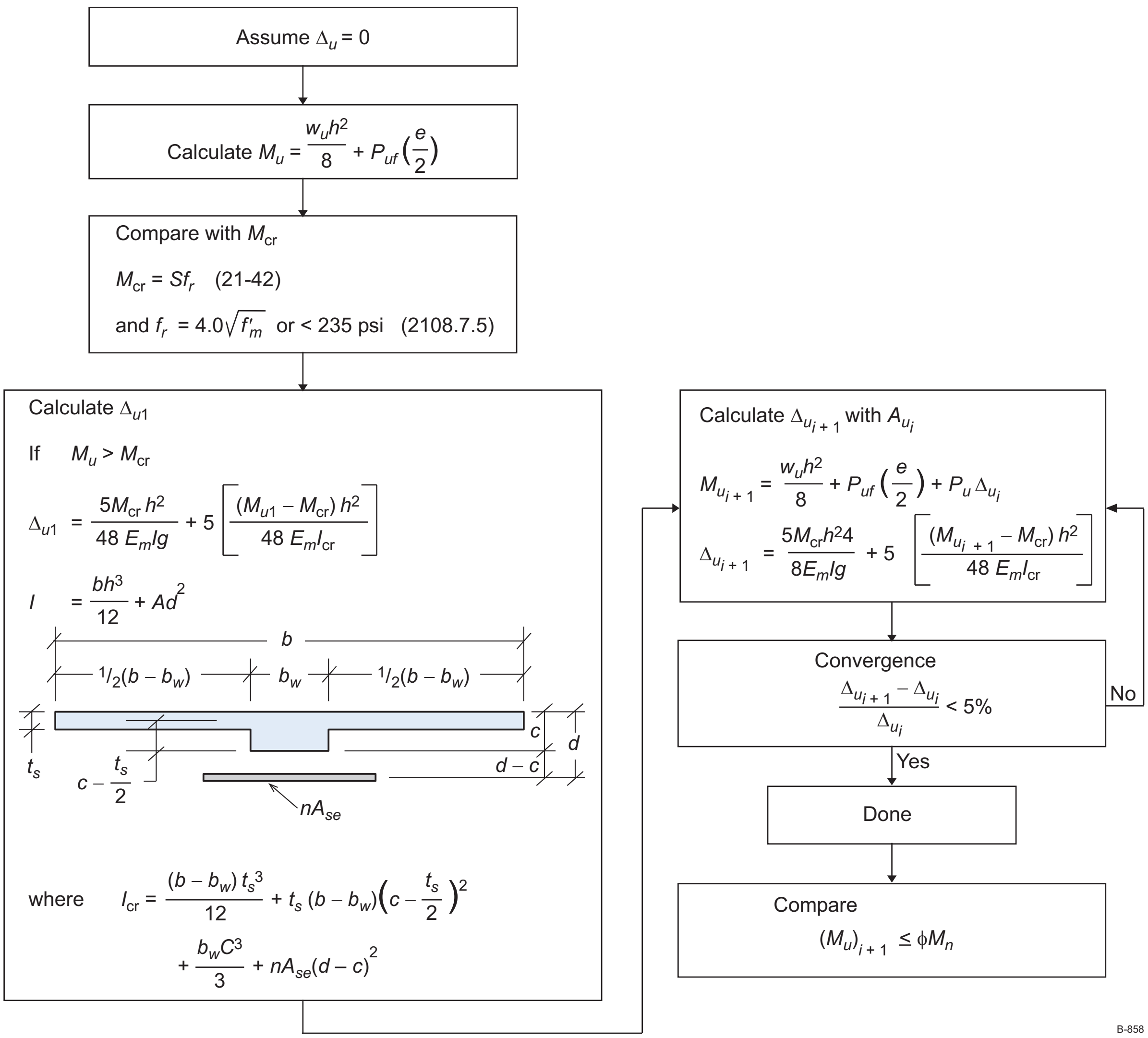


Figure 6-19

6. Deflection analysis for service loads is represented in Figure 6-20.

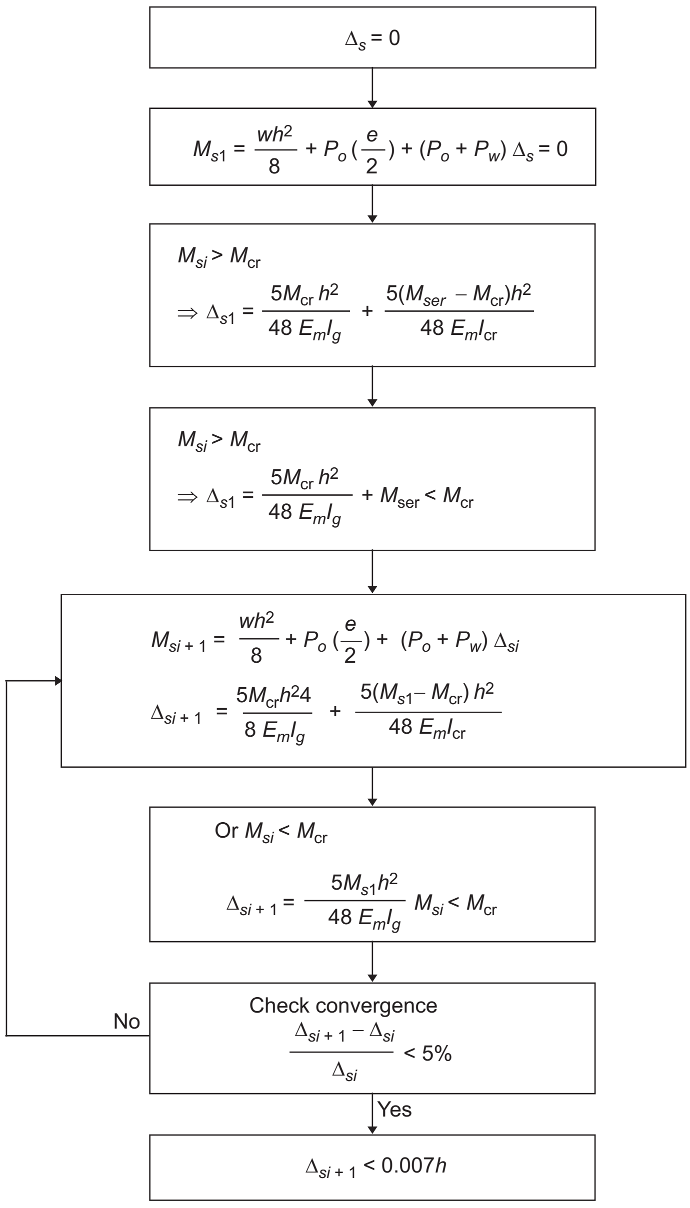


Figure 6-20

Case II: Axial and in-plane loads (Figure 6-21)

1. Per 2000 IBC 2108.9.5.2, reinforcement shall be in accordance with the following:
   1. When the shear wall failure mode is in flexure, the nominal flexural strength of the shear wall shall be at least 1.5 times the cracking moment strength of a fully grouted wall or 3.0 times the cracking moment strength of a partially grouted wall from Equation 21-42.
   2. The amount of vertical reinforcement shall not be less than one-half the horizontal reinforcement.
   3. The maximum reinforcement ratio shall be determined by Section 2108.9.2.13.
2. Capacity reduction factor

   $\phi = 0.65$ for axial load and flexure

   $\phi = 0.80$ for shear
3. Axial strength **2108.9.5.4**

   $P_o = 0.85 f'_m (A_e - A_s) + f_y A_s$ **Eq. 21-43**

   $\phi P_n = \phi\, 0.80 P_o$

   where $\phi = 0.65$
4. The purpose of the P-M diagram is to analyze the three critical points on the interaction diagram, below.

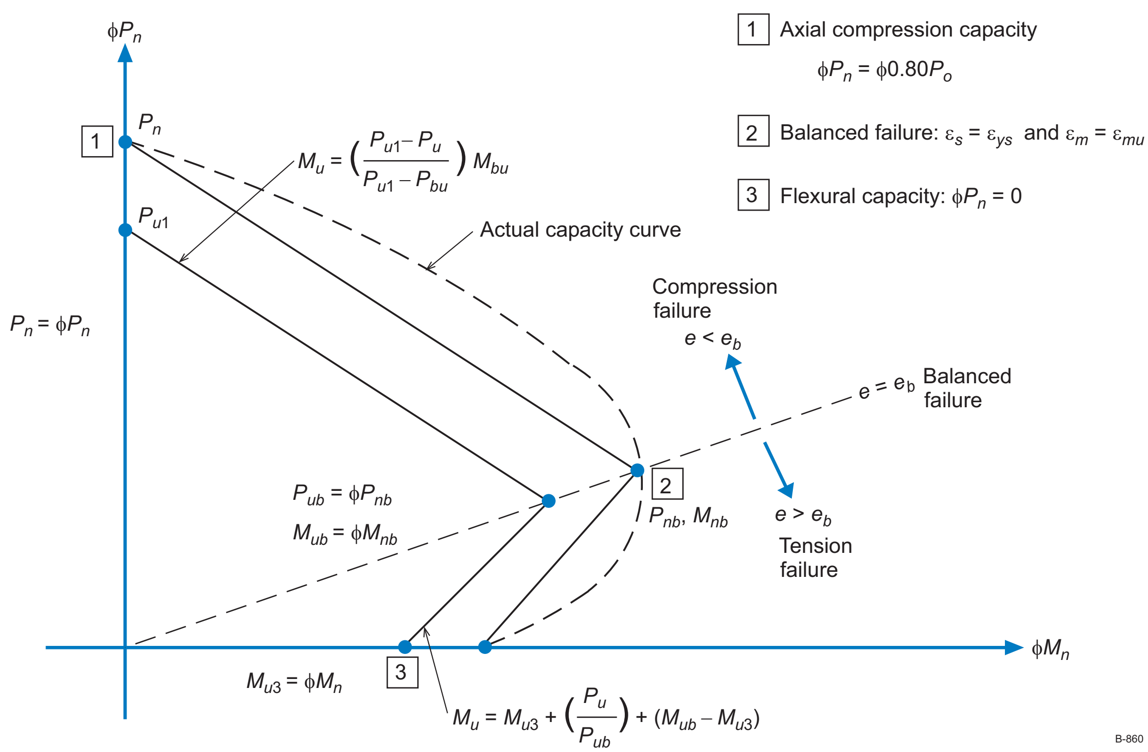


Figure 6-21

2 Balanced failure analysis, (Figure 6-22)

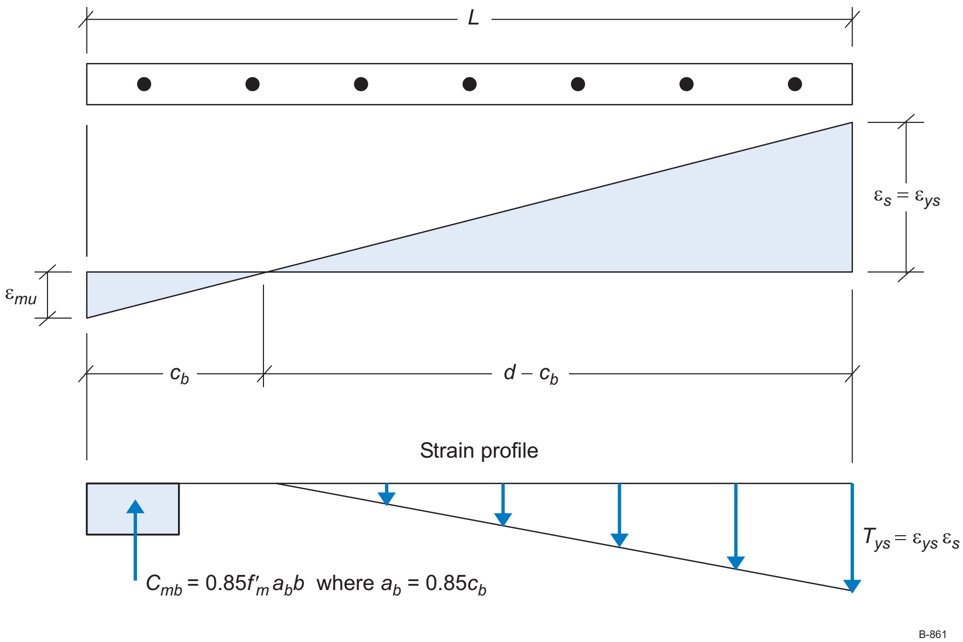


Figure 6-22

$$c_b = \left(\frac{\varepsilon_{mu}}{\varepsilon_{mu} + \frac{fy}{E_s}}\right) d$$

The value of $\varepsilon_{mu} = \left[\begin{array}{l} 0.0025 \text{ concrete masonry} \\ 0.0035 \text{ clay masonry} \end{array}\right]$

$$C_{\text{total}} = A_s f_s + 0.85 f'_m\, a_b b = C_b$$

$$C_{\text{total}} = \sum_{\substack{\text{non-yield} \\ \text{zone}}} A_{si} f_{si} + \sum_{\substack{\text{yield} \\ \text{zone}}} A_{si} f_{ys} = T_b$$

$$P_{nb} = C_b - T_b$$

$$\therefore\ P_{ub} = \phi P_{nb} = 0.65(P_{nb})$$

$$M_b = \sum_{\substack{\text{non-yield} \\ \text{zone}}} A_s f_s \ (\text{moment arm}) + 0.85 f'_m\, a_b b$$

and $M_{ub} = \phi M_b = 0.65 M_b$

3 Flexural analysis (Figure 6-23)

$$M_n = \sum T \text{ (moment arm)} - \sum C \text{ (moment arm)}$$

where $C = T$

$$C = C = A_s f_s + 0.85 f'_m \, a_b$$

$$T = \sum_{\substack{\text{yield}\\\text{zone}}} A_s f_y + \sum_{\substack{\text{non-yield}\\\text{zone}}} A_s f_s$$

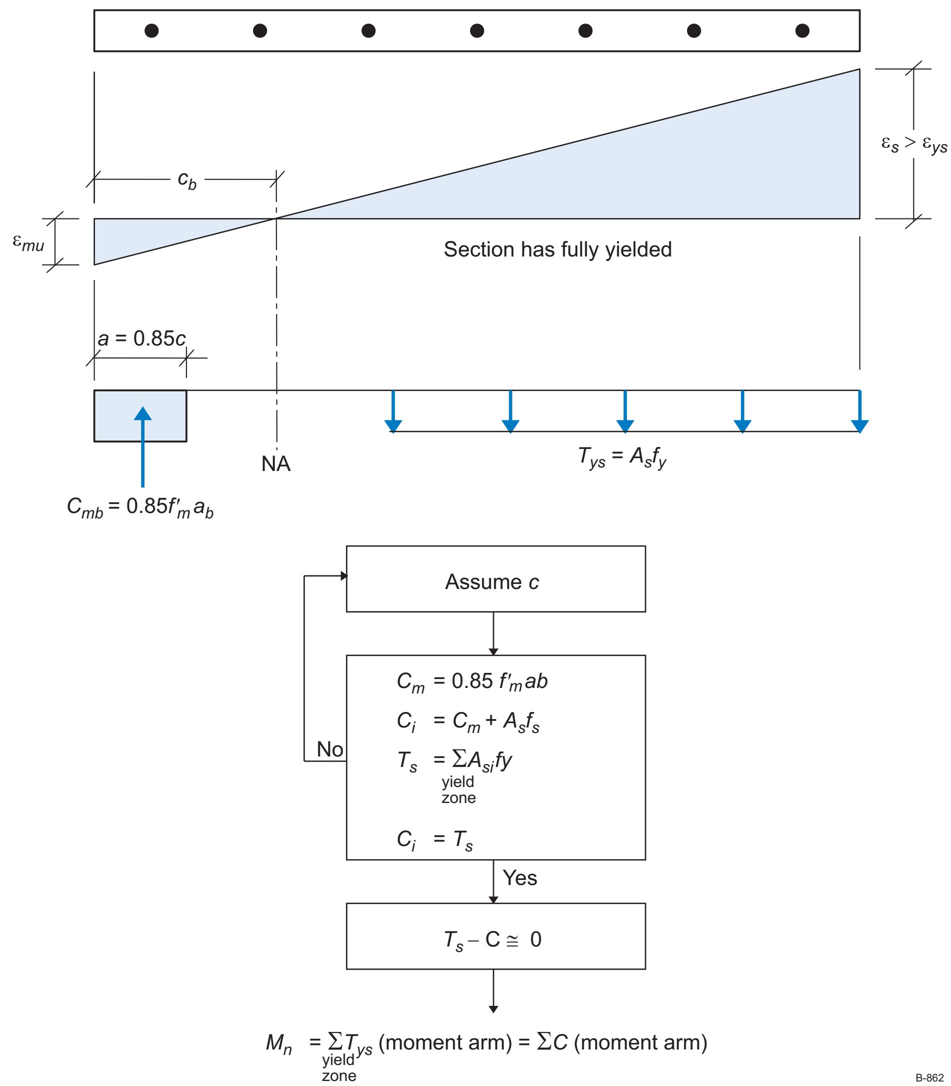


Figure 6-23

6.3 Finite Element Analysis of Shear Walls Using Strength Design

Finite Element Methods/Analysis (FEM/FEA) have been commercially available since the 1960s, but were considered too expensive and computationally intense for civil and structural engineering applications. A practical application to illustrate the benefits for the structural engineer is included in this text.

FEM divides the structure into individual elements that are then assembled into the global system. For example, a beam may be analyzed by using individual beam elements connected at node locations. After the model is constructed in the computer, any variety of static and dynamic loads with differing combinations and load factors can be analyzed quickly. The computer can solve eight load conditions in the same runtime it takes an engineer to solve one load condition.

Advantages of Finite Element Analysis

1. Computationally efficient to solve large, complex problems.
2. FEM can be used to solve unusual wall configurations with estimates of stress concentrations around door/window openings that cannot be done quickly with normal hand-calculation procedures.
3. The global model of the structure can be altered quickly to experiment with structural design options. The FEA provides more chances to examine a wider spectrum of designs without investing excessive time to redesign elements.
4. The output is of professional quality with color charts that can be used effectively to show the structure undergoing displacement, shear stress, compression stress, and stress concentration areas. This is useful when explaining the complex behavior of structures to a nontechnical audience (i.e., architects, owners, clients).

However, with every great advance in technology, there can be disadvantages. It is wise to examine the possible pitfalls of using a sophisticated technology such as FEA because they can be diminished or avoided by proper planning.

Disadvantages of Finite Element Analysis

1. Computers have a tendency to make the user too comfortable, and structural engineers may forget (or are not trained) to check results. Answers must be verified by running a parallel analysis or performing careful hand calculations.
2. Output files from an FEA program are often extensive—The resulting bulk may be misconstrued as useful.
3. Practical insight may be overcome by a comfortable analysis program. No computer can replace good judgement.

Finite Element Analysis Programs

RISA 3D, ETABS 2000, and STAAD are among the available programs. All have strong practical features that make them easy to implement and their output demonstrates professional quality.

For sophisticated nonlinear analysis, more powerful packages are available: SAP2000, COSMOS, and ADINA. Nonlinear analysis, which is not covered here, can be of practical value for evaluation of existing buildings.

Tau (Tresca) Shear Stress, Von Mises Stress, and Principle Stress are defined later in this text.

Study the free body diagram of a three-dimensional cube in elastic stress space (Figure 6-24). The normal and shear stresses are shown. FEA is based on understanding the distribution of these stresses in a continuum material.

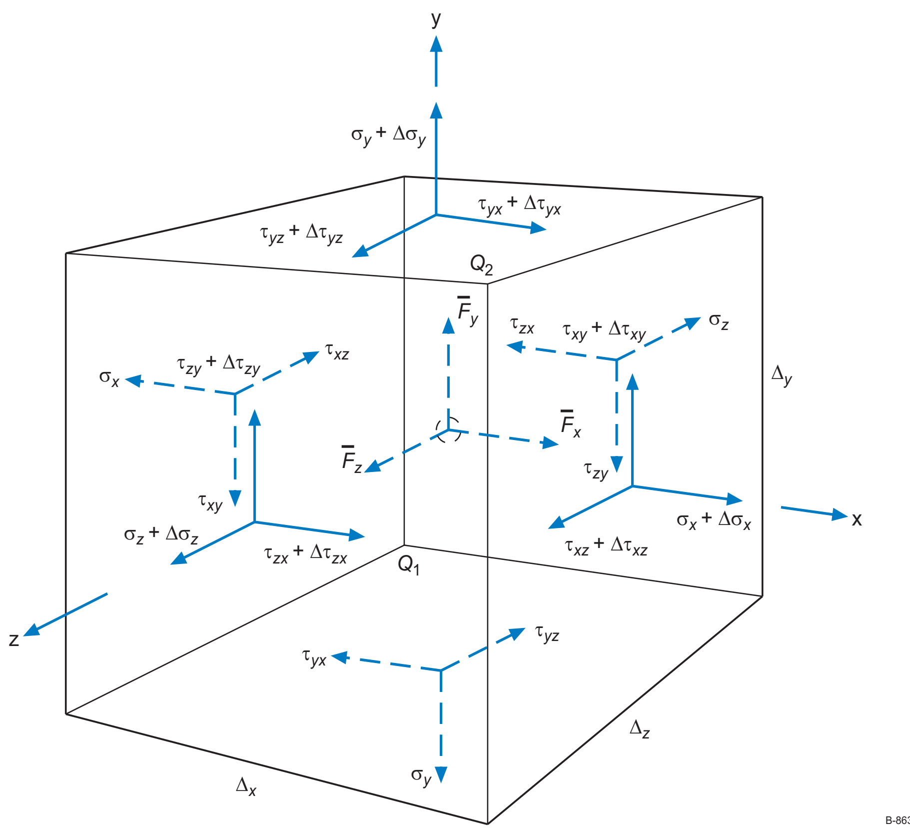


Figure 6-24

Shear stress distribution is a major concern with masonry shear walls. Figure 6-25 illustrates the distortion of the cube element, which is shown in two dimensions.

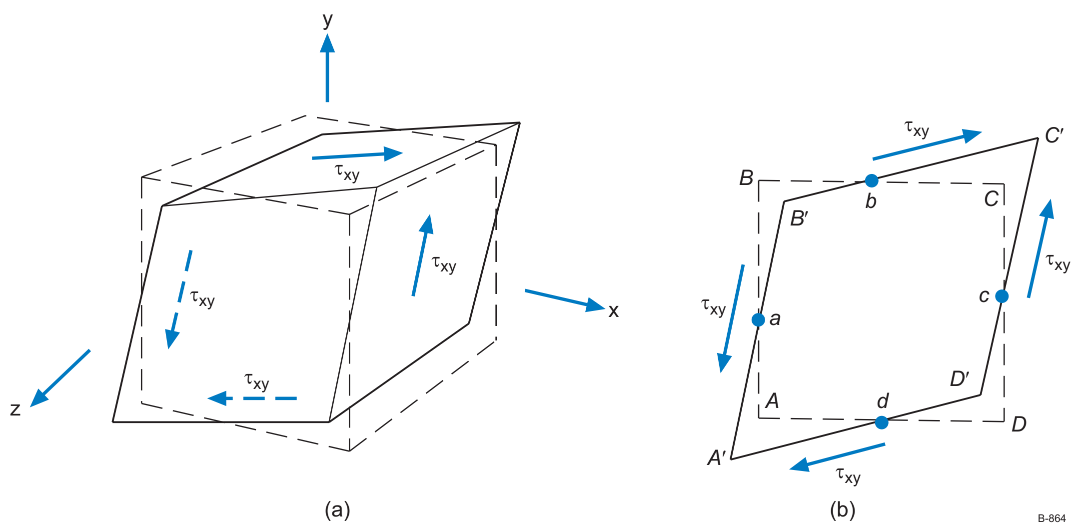


Figure 6-25

Stress deformation through distortion is calculated using the Tresca and Von Mises failure theories.

Based on FEM results, it is possible to identify the failure and its orientation within a plate element.

Correct failure surface based on 3-D analysis is shown in Figure 6-26.

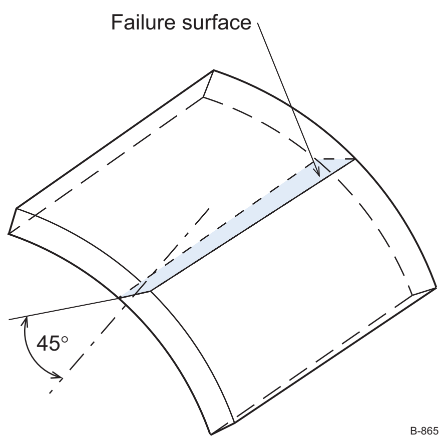


Figure 6-26

The maximum/minimum principal and shear stresses are defined from the Mohr's Circle Theory (Figure 6-27).

$$\sigma_{\frac{max}{min}} = \sigma_{avg} \pm R = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{\frac{max}{min}} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

The Tresca (Tau) Shear Stress is

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{1}{2}\sigma_{vm}$$

$$t_{tresca} = M_{max}$$

The Von Mises Stress is

$$\sigma_{max} = \sqrt{\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2(\sigma_3 - \sigma_1)^2]}$$

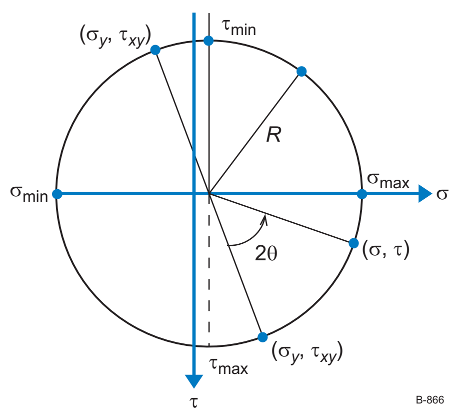


Figure 6-27

The limitations of the FEM for masonry shear walls are:

1. Masonry bond strength. The mortar joint and shear failure mechanism is a common point of failure. This is difficult to model in an FEA. There are specialized programs with this level of complex analysis, but they involve sophisticated nonlinear analysis methods.

2. Reinforcement steel is part of the model. In conventional FEA, the reinforcement is not part of the FEM. Plate elements are used with masonry properties. There are advanced programs that can model the reinforcement steel. This is generally not considered in conventional linear elastic FEA.

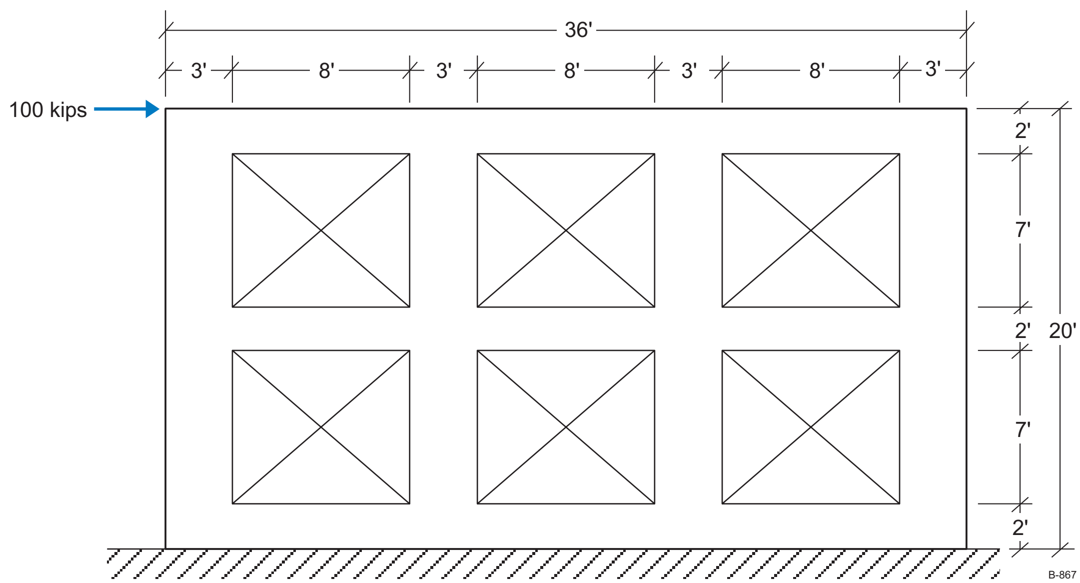
3. For buildings, the FEM can produce useful practical results within the linear elastic range. However, in structures where extensive cracking is expected (i.e., plastic behavior), nonlinear analysis FEA is warranted.

6.4 Reinforced Masonry Wall Frames

Reinforced masonry wall frames have become more prevalent in the practical construction arena over the past 10 years because they allow for openings in the wall frame. These openings give architects the freedom to design buildings with proper accessibility, window area, and create an open concept for the building.

The wall frame is essentially a moment-frame system where the masonry elements are divided into beam and column elements. The analysis procedure is similar to a moment-frame design of a steel or reinforced concrete structure. This example illustrates the effectiveness of finite element software to analyze such structures quickly.

Figure 6-28 shows the elevation of a reinforced masonry wall frame.



f'_m = 3000 psi. Use solid grouted 12-inch CMU

Figure 6-28

Requirements:

Analysis check list

1. FEA using RISA-3D
2. Analyze existing design per 2000 IBC.
3. Evaluate seismic reinforcement detailing per 2000 IBC.
4. Prepare a cross-section diagram of typical beam and column connection

Solution

Step 1: FEA with RISA-3D

The FEA uses a system of beam-column elements to represent the structure. The material properties are calculated and the key assumptions are outlined.

a. The full modulus of elasticity is assumed for this analysis. This is acceptable for strength evaluation. For deflection analysis, the cracked modulus or a separate, effective EI value is calculated.

b. The lateral drift values are estimated from the FEA, but should be checked using an approximate method such as the Portal Method or equivalent stiffness approximation.

c. The high-stress zones occur at the beam-column joints and these correspond to the points of peak bending moments and shears.

Finite element model

The finite element model of the masonry wall frame consists of a series of members attached at the node points (the dotted lines in Figure 6-29 show the location of members). The boundary conditions are N_1, N_8, N_9, N_{10}, N_{12}, – fully fixed.

Material properties

f'_m = 3000 psi

$E_m = 900f'_m = 900 \times 3000 = 2700$ ksi

Density = 0.135

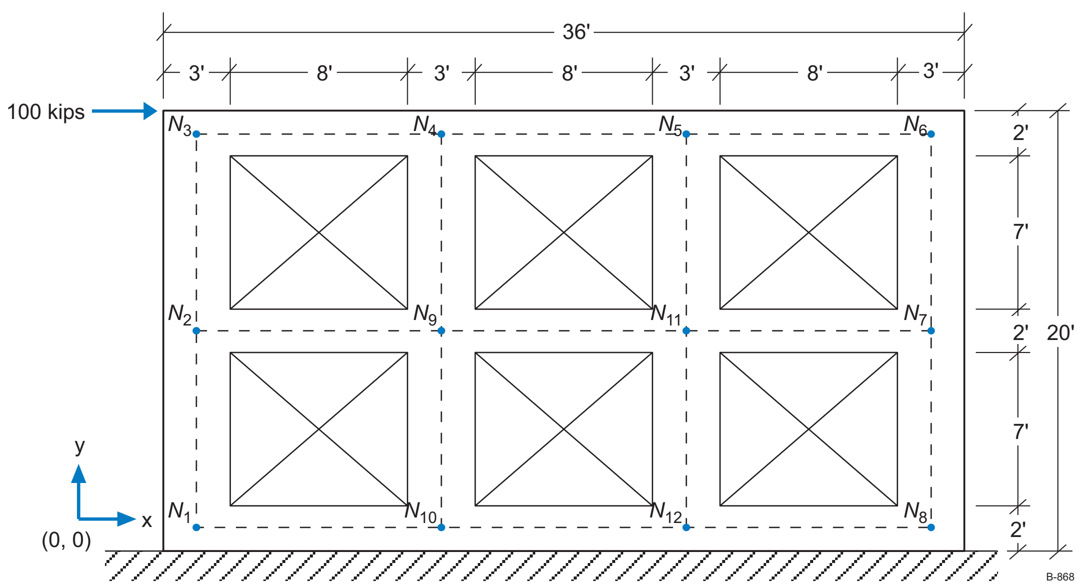


Figure 6-29

Cross section of beam/column elements

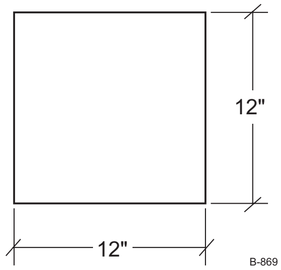


Figure 6-30

Loads

(1) Dead load

(2) 100 kips applied at N_3 $\xrightarrow[\text{X}]{\text{100 kips}}$

Load combinations – 1997 UBC WSD (alternate basic load combinations)

(1) 1.4*D*	Load condition 1	**(F 12-1)**
(2) 1.4*D* + 1.0*E*	Load condition 2	**(F 12-5)**

These load conditions are analyzed individually. It is also possible to have them combined in RISA-3D by using the options of *envelope of marked combinations.*

Step 2: Analyze existing design per the 2000 IBC

The FEA, utilizing the RISA-3D program, is demonstrated in this example. The moment frame model is similar to standard beam-column systems for steel structures. With exceptions, this model has masonry properties for its beams and columns. The section properties are adjusted in the RISA-3D input file to produce the final version. The horizontal loads represent the factored earthquake loads, and the vertical loads are the factored dead loads.

Use the moment diagram for the critical load case (1.2 *DL* + 1.0 *EQ*). The joint moments are easily evaluated, and the critical beam and column sections can be designed accordingly.

The shear diagram for the critical load case shows the shear forces clearly marked. In the lateral deflection plot, maximum deflections are less than 0.1 inch.

Step 3: Design process

The design process for masonry wall frames is outlined in Figures 6-33 through 6-41 with specific code sections and the design equation. each connection must satisfy fhese provisons.

The 1997 UBC and 2000 IBC are identical in their requirements for wall frames. Typically, the worst case load condition is identified and one typical design is produced for the beam section, column section, and standard beam-column joint.

A formal set of calculations is provided for this design to demonstrate the process.

Masonry wall frame design (Figure 6-31)

1. Dimensional limits for 12-inch units **(2108.2.6.1.2)**

   Beam $d_{\text{bm}} = 24$ in **OK**

   Piers $d_{\text{col}} = 36 \text{ in} > 32$ in **OK**

   Joints $f'_g = 3000$ psi with #5 bar **(2108.2.6.9)**

   Beam depth $h_b \geq \dfrac{1800 d_{bp}}{\sqrt{f'_g}}$ **(Eq. 8-47)**

   $$h_b \geq \frac{1800(0.625)}{\sqrt{3000}} = 21 \text{ in}$$

   $h_b = 24 \text{ in} > 21$ in **OK**

   Pier depth $h_b \geq \dfrac{4800 d_{bb}}{\sqrt{f'_g}} = 55 \text{ in} > 36$ in

   with $f'_g = 5000 \text{ psi} \, h_p = \dfrac{4800(0.625)}{\sqrt{5000}} = 42.5$ in **No Good**

   Try #4 bars $h_b = \dfrac{4800(0.5)}{\sqrt{5000}} = 34 \text{ in} < 36$ in **OK**

   For piers, use $f'_g = 5000$ psi w/#4 bars

2. Frame analysis results can be found in the RISA-3D plots.

3. Beam design

   $U = (1.2D + 1.0E + 0.5L)\ 1.1$

   $M_u = (27.1)\ 1.1 = 29.8 \text{ ft-k} = 357.6 \text{ in-k}$

   $V_u = (4.8)(1.1) = 5.3$ kips

   Use #5 bars

   $f'_m = 3000$ psi

   $A_s = 0.31 \text{ in}^2$

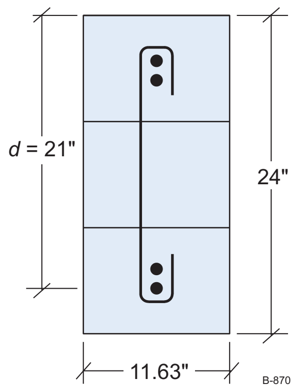


Figure 6-31

$$\rho = \frac{0.31}{(21)(11.63)} = 0.00127$$

$$T = A_s f_y = 0.31(60) = 18.6 \text{ k}$$

$$a = \frac{A_s f_y}{0.85 f'_m b} = \frac{18.6}{0.85(3)(11.63)} = 0.63 \text{ in}$$

$$\phi M_n = (0.85) A_s f_y \left(d - \frac{a}{2}\right)$$

$$\phi M_n = 0.85(18.6)\left(21 - \frac{0.63}{2}\right) = 327 \text{ in-k} < 385 \text{ in-k}$$

$\Rightarrow$ use 2 #5 bars

$$A_{st} = 2(0.31) = 0.62 \text{ in}^2$$

$$A_s f_y = 37.2 \text{ k}$$

$$a = \frac{37.2}{0.85(3)(11.63)} = 1.26 \text{ in}$$

$$\phi M_n = 0.85(37.2)\left(21 - \frac{1.26}{2}\right) = 644 \text{ in-k} < 385 \text{ in-k}$$ **OK**

$$M_n = \frac{644}{0.85} = 785 \text{ in-k}$$

$$M_n = 63 \text{ k-ft}$$

Specify = 2 #5 bars top and bottom

4. Shear strength requirement

V_u = 5.3 kips (computer output)

$$V_n \geq 1.4\left[\frac{\Sigma M_n(\text{beam})}{L} \pm \frac{D+L}{2}\right]$$

$$Vn \geq 1.4\left[\frac{2(63)}{8 \text{ ft}} + \frac{2 \text{ k}}{2}\right] = 23.5 \text{ k}$$

ϕV_n = 0.80(23.5) 18.8 k > V_u = 5.3 k

provide for ϕV_n = 18.8 k $\Rightarrow$ V_n = 23.5 k

At the beam face, $V_m = 0$

$$V_n = V_s = A_{mv}\,\rho_n f_y$$

$$V_n = \frac{A_v f_y h}{S}$$

where $S \leq \frac{h}{4} = \frac{24}{4} = 6$ in

$$A_v = \frac{V_n S}{f_y h} = \frac{(23.5)(6)}{(60)(24)} = 0.098 \text{ in}^2$$

A_v = 0.20 in^2 w/#4 bar

Use #4 ties @ 6 inches o/c

Check that #4 @ 16 inches o/c is adequate at center span

$$\rho = \frac{A_y}{ts} = \frac{0.20}{11.63\ (16)} = 0.0011$$

$A_{mv} \approx ht$ = 24(11.63) = 279.12 in^2

$V_s = A_{mv}\,\rho_n f_y$ = (279 in^2)(0.001) 60 ksi

V_s = 18.4 kips

In the center of span; $V_s = 1.2\, A_{mv} \sqrt{f'_m}$

$V_m = 1.2(279)\sqrt{3000}$ = 18.34 kips

$V_n = V_m + V_s$ = 18.34 + 18.4 = 3674 kips > 23.5 kips

5. Pier design (Figure 6-32)

Use P_u = 13.0 kips

V_u = 8.7 kips

M_u = 60.6 ft-k = 727.2 in-k

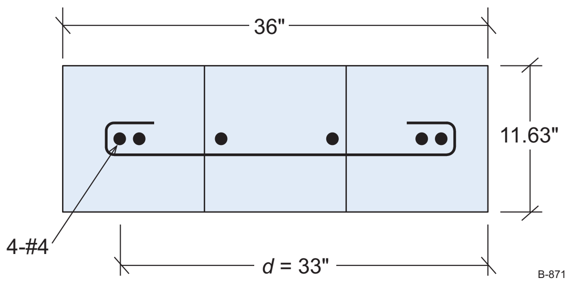


Figure 6-32

With P_n $= 0$

Select reinforcement using #4 bars

A_s $= 0.20 \times 2 = 0.40\ \text{in}^2$

$A_s f_y$ $= 2(0.20)(60) = (12)(2) = 24\ \text{k}$

M_n $\approx A_s f_y\ (0.9d) = 24\ (0.9 \times 33)\ 712.8\ \text{in-k}$

Increase d $= 34.5\ \text{in}$

M_n $\approx 24\ (0.9 \times 34.5)\ 745.2\ \text{in-k}$

Dimensional limits (Figure 6-33)

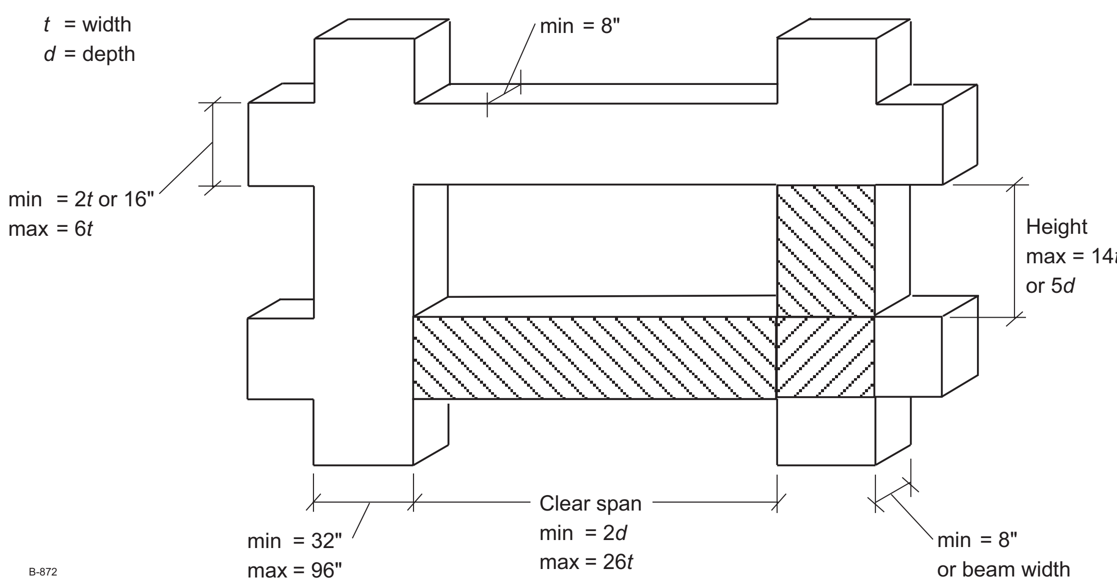
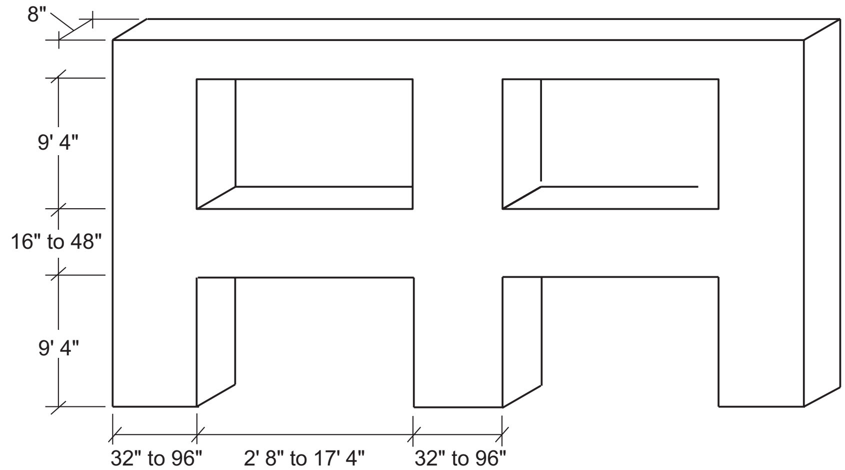


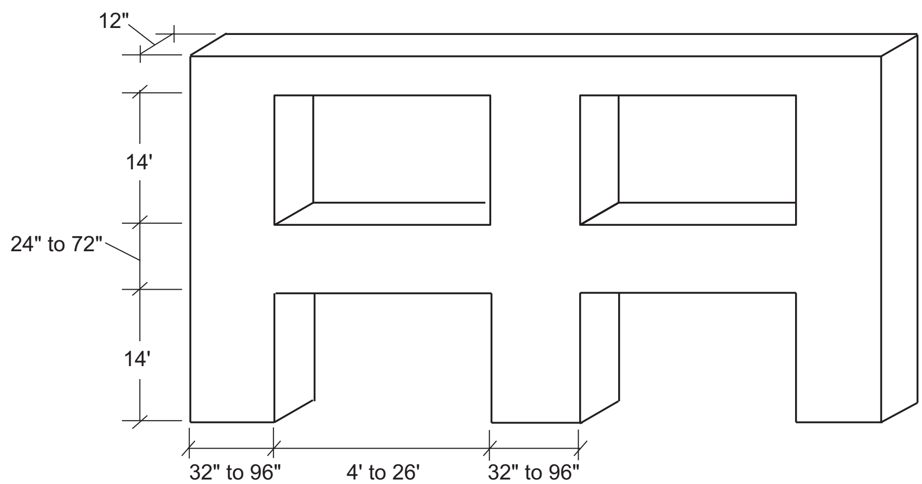
Figure 6-33

@Seismicisolation

Architectural considerations (Figure 6-34)



Dimension limits for 8-inch units



Dimension limits for 12-inch units

B-873

Figure 6-34

Beam (Figure 6-35)

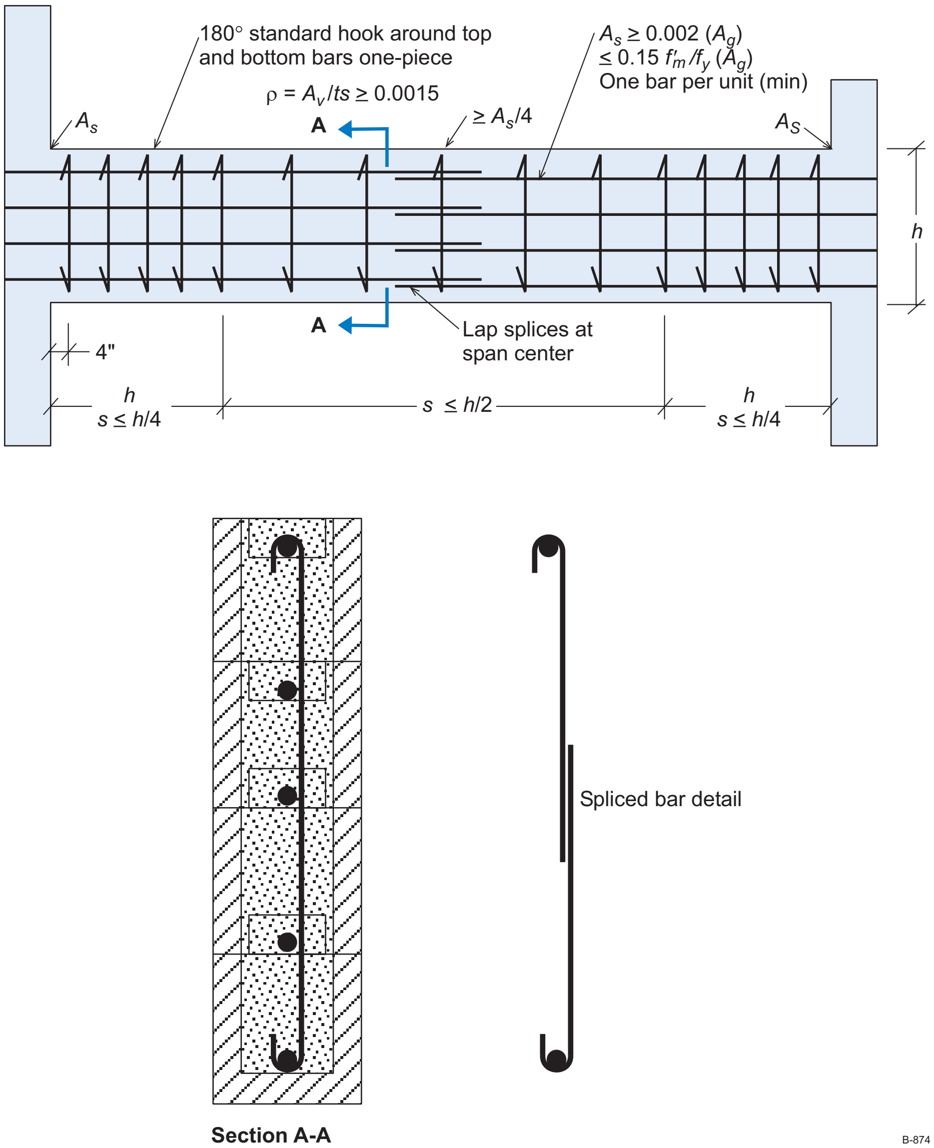


Figure 6-35

Piers (Figure 6-36)

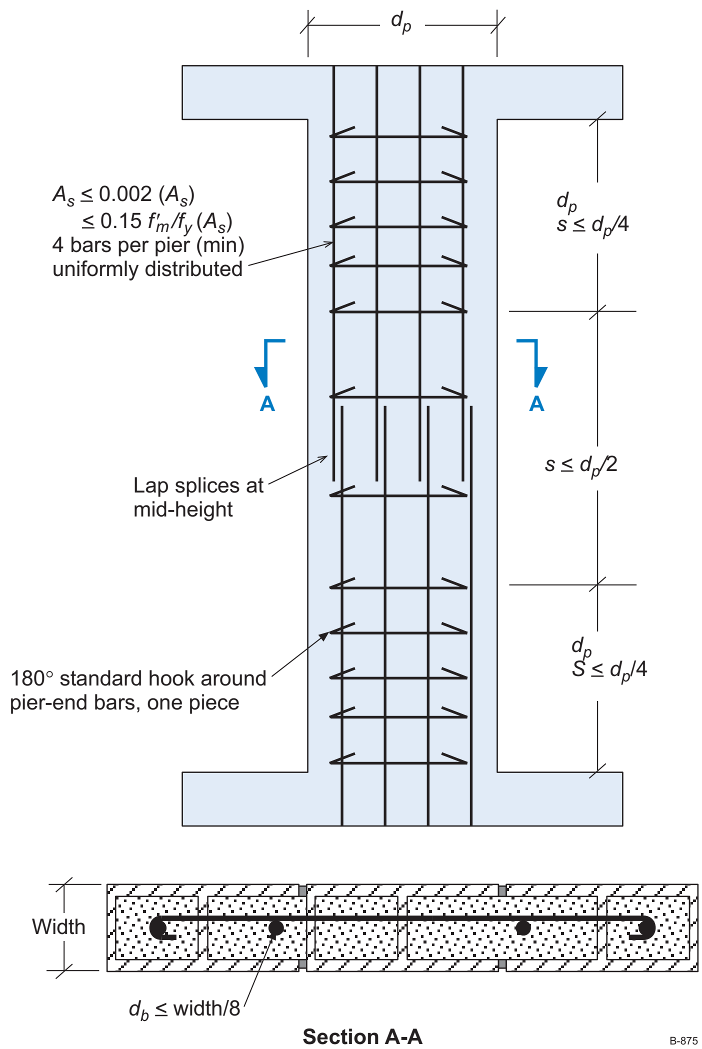


Figure 6-36

Material strength limitations **(2108.2.6.2.3)**

$1500 \leq f'_m \geq 4000$ psi

Specified $f_y \leq 60{,}000$ psi

Actual $\leq 78{,}000$ psi . . . ($1.4\,f_y$ max)

(Use A706 low-alloy steel)

Strength reduction factors **(2108.1.4.4)**

Flexure; $\phi = 0.85$

Axial load with flexure: $\phi = 0.85 - 2\left(\frac{p_u}{A_n f'_m}\right) \geq 0.65$

$P_u = 0 \ldots \phi = 0.85$

$P_u \geq 0.1\, A_n f'_m \ \ldots \phi = 0.65$

Shear: $\phi = 0.80$

Pier design forces (Figure 6-37) **(2108.2.6.2.7)**

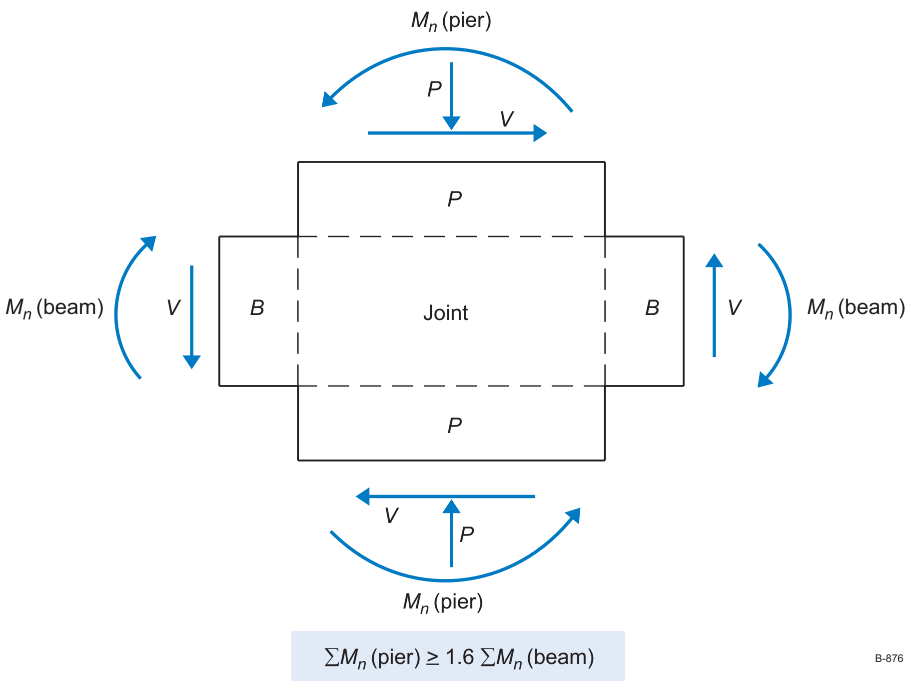


Figure 6-37

Weak-beam/strong-pier response . . . to ensure a ductile mechanism forming in the beam and maintaining a strong column to support vertical loads.

Pier axial load P_u, including factored dead and live loads, must not exceed $P_u \leq 0.15\, A_n f'_m$.

Beam and pier shear strength (figure 6-38)

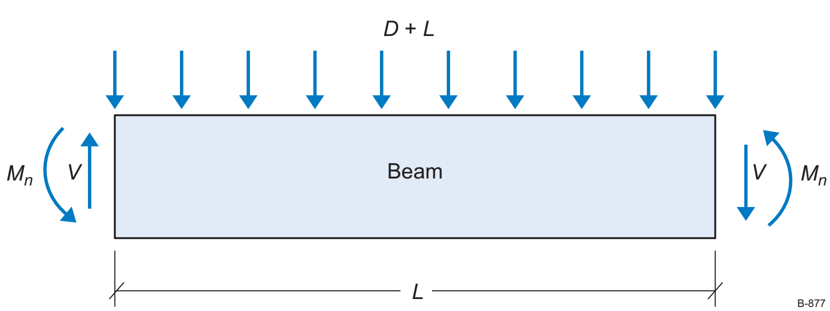


Figure 6-38

$$V = \frac{\Sigma M_n}{L} \pm \frac{(D+L)}{2}$$

M_n = beam flexural yielding (nominal moment strength)

$D + L$ = tributary gravity load

Beam and pier nominal shear strength not less than 1.4 × shear corresponding to development of beam flexural yielding.

$$V_n\,(\text{beam}) \text{ and } V_n\,(\text{pier}) \geq 1.4\left[\frac{\Sigma M_n(\text{beam})}{L} \pm \frac{(D+L)}{2}\right]$$

Beam shear strength (Figure 6-39) **(2108.2.6.2.8)**

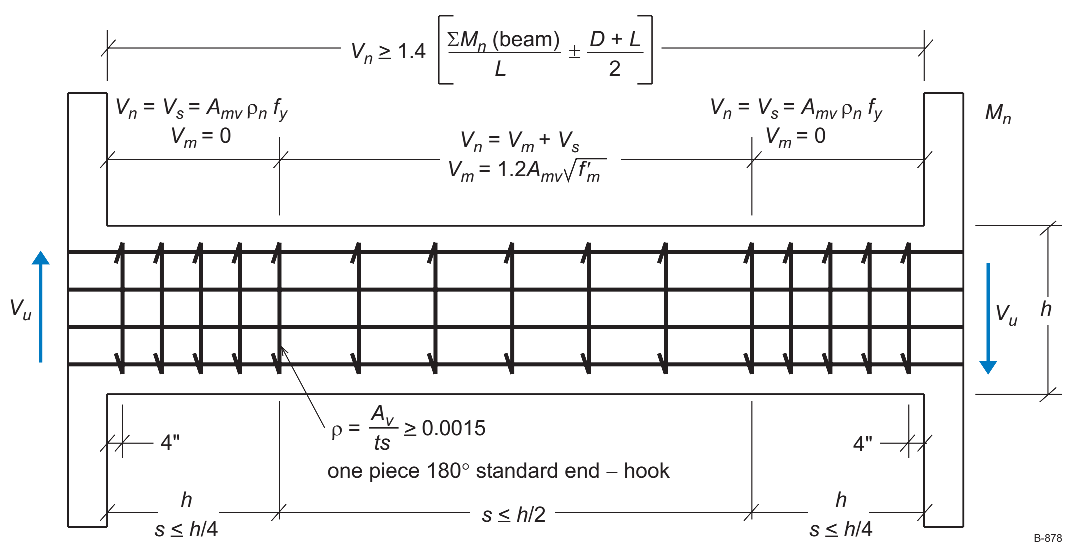


Figure 6-39

$$V_n \leq \phi - V_n$$

$$V_n \geq 1.4\left[\frac{\Sigma M_n(\text{beam})}{L} \pm \frac{(D+L)}{2}\right] \leq \phi(4A_{mv}\sqrt{f'_m}) \quad \textbf{(Eq. 8-45)}$$

$$V_n = V_m + V_s \quad \textbf{(Eq. 8-41)}$$

$$V_m = 1.2A_{mv}\sqrt{f'_m} \quad \textbf{(Eq. 8-44)}$$

V_m = 0 . . . within one beam depth from pier faces

$$V_s = A_{mv}\rho_n f_y$$

ϕ $= 0.80$

A_{mv} $\approx th$

ρ_n $= \dfrac{A_v}{ts}$

Pier shear strength (Figure 6-40)

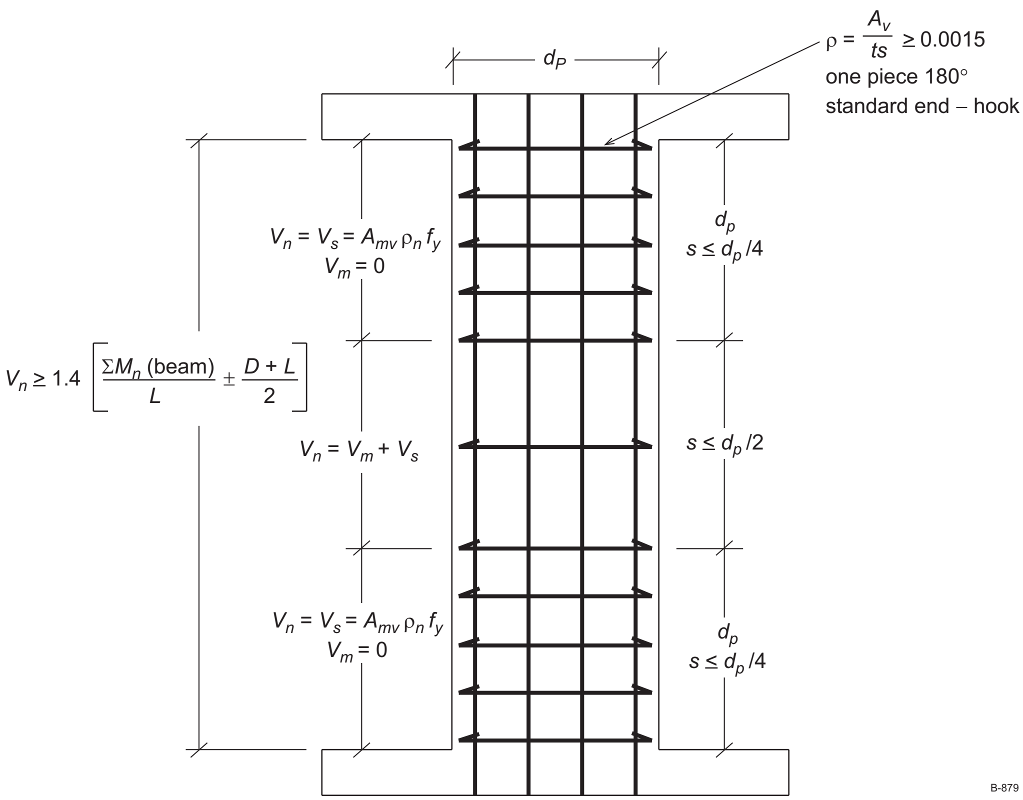


Figure 6-40

V_u $\leq \phi\, V_n$ **(Eq. 2-3)**

V_n $= V_m + V_s$ **(Eq. 8-41)**

V_n $\geq 1.4\left[\dfrac{\Sigma M_n(\text{beam})}{L} \pm \dfrac{(D+L)}{2}\right]$ **(2108.2.6.2.8)**

V_m $= C_d\sqrt{f'_m}\,A_{mv}$ **(Eq. 8-42)**

V_m $= 0$ . . . within one pier depth from beam faces

V_s $= A_{mv}\,\rho_n f_y$ **(Eq. 8-43)**

where

ϕ $= 0.80$ **(2108.1.4.4.2)**

$$C_d \quad = 2.4 \ldots \text{for} \frac{M}{Vd} \le 0.25 \qquad \textbf{(21-K)}$$

$$C_d \quad = 1.2 \ldots \text{for} \frac{M}{Vd} \le 1.0$$

$$V_n \quad \le 6.0\sqrt{f'_m}A_{mv} \ldots \text{for} \frac{M}{Vd} \le 0.25 \qquad \textbf{(T 21-J)}$$

$$V_n \quad \le 4.0\sqrt{f'_m}A_{mv} \ldots \text{for} \frac{M}{Vd} \le 1.0$$

$$A_{mv} \quad \approx d_{pt}$$

$$\rho_n \quad = \frac{A_v}{ts}$$

Beam pier joints (Figure 6-41) **(2108.2.6.2.9)**

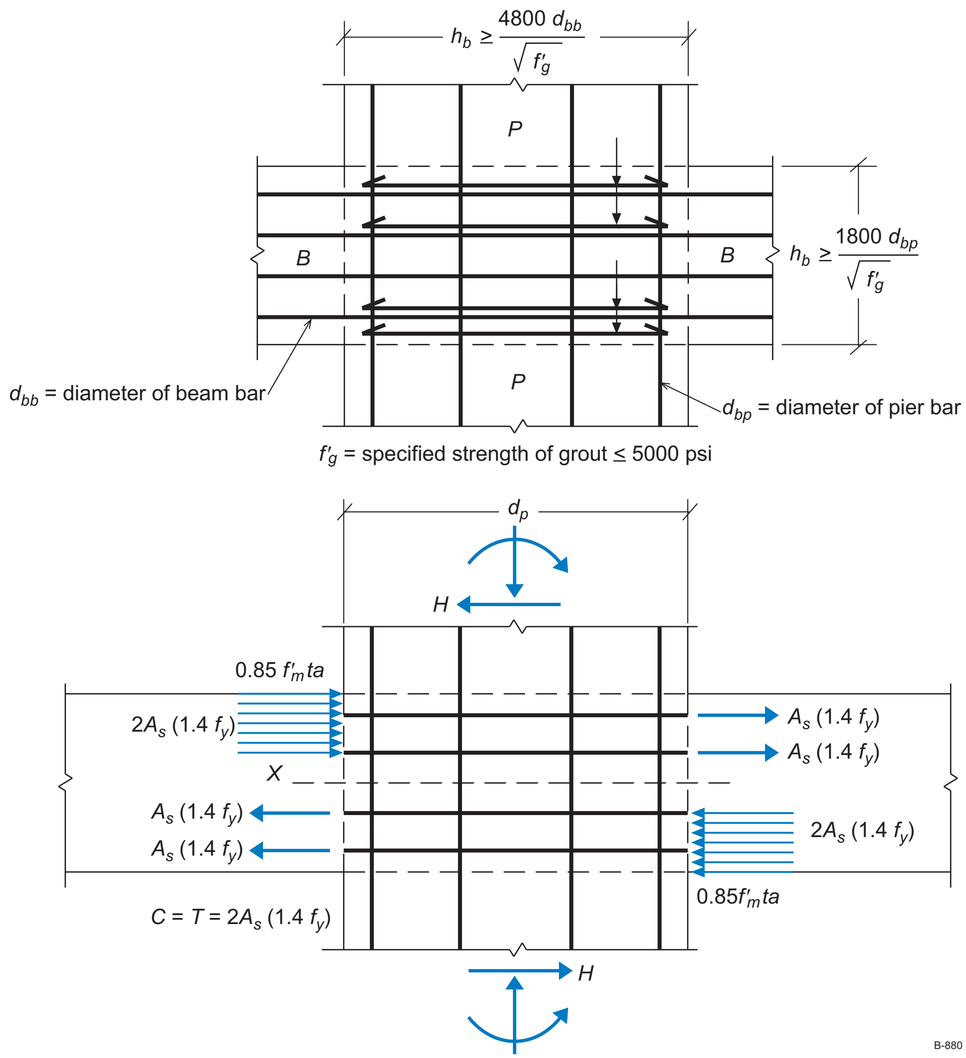


Figure 6-41

At x-x

$$V_{jh} = 4A_s(1.4f_y) - H$$

Shear strength

$$V_{jh} \leq \phi - V_n$$

$$V_{jh} \leq \phi(7\sqrt{f'_m}\,d_p t) \leq \phi(350 d_p t)$$

6.5 Earthquake Damage

Diagonal shear cracks appear in this reinforced concrete moment frame building. The 45° shear cracks at the beam-column joints indicate shear failure at the joint connection. This is characteristic of plastic moment failure in concrete frame structures, particularly near the first/ground level.

The Long Beach Earthquake (1933) caused extensive damage to this reinforced masonry structure. Notice the shear walls that have fallen away from the building. Subsequent modifications to the building code were created to prevent this type of failure by requiring adequate diaphragm anchorage.

Structural failure/collapse of the exterior perimeter shear walls demonstrates the importance of diaphragm anchorage. Although the interior walls are still standing, this building could easily collapse from an aftershock.

An earthquake in Nicaragua caused this damage to an unreinforced stone house. The cracking is evident through the mortar and stone joints.

Shear failure of the 1st story column connection from joint shear cracking is pictured. This type of failure emphasizes the importance of proper strength design throughout the height of the building because the upper floors appear to be intact. This does not alter the overall vulnerability of the building; the first floor column joint failed and put the entire building at risk.

Vertical and lateral road displacements are evident in the photo above. Earthquakes release energy through ground displacement, velocity, and acceleration. The total displacement could be substantial. This picture demonstrates the potential permanent displacement that can be exerted on a building's foundation.

Diagonal shear cracks on interior shear walls have caused almost total failure of the wall. This follows the structural theory of Mohr's circle by emulating the classic 45° shear plane. The walls do not fail in compression or tension, but in diagonal shear – as predicted in the structural textbooks. The interior steel reinforcement provides the yielding mechanism to prevent total collapse.

The worse possible structural failure – total collapse, is precisely the type of failure every structural engineer wants to avoid. The entire building has collapsed with the pancake effect of the floor diaphragms.

Aerial photograph of the ground ruptures caused by the Alaska Earthquake (1965). Ground ruptures are the result of faulting at the earth's surface, and may occur anywhere. Recent changes in the building codes now require structural and geotechnical engineers to search for site-specific faults that may cause ruptures.

Structural failure of a reinforced concrete column connection with full bending failure. The lateral movement of the floor diaphragm was substantial and caused the column to yield well into the plastic zone. However, it is worth noting that the steel has not ruptured and is still providing internal bonding. Even at large strains, the steel reinforcement provides a level of structural resistance.

Example 6-1

A slender wall is located in San Francisco. Seismic Design Category E

$S_s = 1.0\text{g},\ S_1 = 0.5\text{g}$

- Determine the adequacy of a 6-inch hollow brick wall that is 25 feet between horizontal supports.

Wall is solid grouted

$f'_m = 5000$ psi

$f_y = 60$ ksi

$t = 5.5$ in

$d = \frac{1}{2} = 2.75$ in

Strength reduction factor $\phi = 0.8$

Solution (based on 2000 IBC)

1. Vertical loads – for 6-inch hollow brick wall

Wall weight = 61 psf

Roof dead load (DL) = 400 plf

$$P_{\text{wall}}\ (\text{at mid-height}) = 61\left(\underbrace{2.5}_{\text{parapet}} + \underbrace{\frac{2.5}{2}}_{^1/_2\text{ of wall}}\right) = 915\text{ lb/ft}$$

P_{uw} = factored wall $DL = 1.2D = 1.2(915) = 1098 - 61$ k/ft

$P_{uf} = 1.2(400) = 480$ lb/ft

$A_g = 5.5(12) = 66\text{ in}^2$

$$\frac{P_{uw} + P_{uf}}{A_g} \le 0.05f'_m = 0.05(5000) = 250\text{ psi}$$ **2108.9.4.4**

$$\frac{P_{uw} + P_{uf}}{A_g} = \frac{1098 + 480}{66} = 23.9 < 250$$ **OK**

2. Lateral loads – wind

For San Francisco, assume 85-mph wind speed. For the main wind-force-resisting system (MWFRS), the maximum pressure for exposure B is 11.5 psf. **T 1609.2.1(1)**

3. Seismic loads – Seismic Design Category E

$F_P = 0.40 I_E S_{DS} W_{\text{wall}}$ **1620.1.7** where

$$S_{DS} = \frac{2}{3} S_{MS} \text{ and } S_{MS} = F_a S_S$$

From Table 1615.1.2(1)

for $S_S = 1.0g$

$F_a = 0.9$

$S_{MS} = 0.9(10) = 0.9g$

$S_{DS} = \frac{2}{3}(0.9) = 0.6g$

$I_E = 1.0$ **T 1604.5**

$F_P = 0.4(10)(0.60)(61) = 14.6 \text{ psf} > 11.5 \text{ psf}$

Seismic governs but the load combinations must be evaluated.

4. Factored load combination

Evaluate per load conditions of 1605.1

Formula/number	
$1.4D$	16-1
$1.2D + 1.6L$	16-2
$1.2D + 1.6(L_r) + (f_1 L \text{ or } 0.8W)$	16-3
$1.2D + 1.6W + f_1 L + 0.5(L_r)$	16-4
$1.2D + 1.0E + f_1 L + f_2 S$	16-5
$0.9D + (1.0E \text{ or } 1.6W)$	16-6

For axial compression only, Formula (16-1) governs.

However, for consideration of the interaction effect of the vertical and lateral load, it is necessary to evaluate all the combinations of formulas (16-2) through (16-6) systematically.

$1.2D + 1.6W \Rightarrow W_u = 1.6\,(11.5) = 18.4 \text{ psf}$

(11.5 is the working stress load from the IBC)

(18.4 is the factored wind load)

In this example $L = 0$, $L_r = 0$

Formula (16-2) supersedes (16-3) and (16-4).

$1.2\,D + 1.0\,E$ **F 16-5**

$1.0\,E_u = 14.6 \text{ psf} < 1.6W = 18.4 \text{ psf}$
(Wind load governs over seismic)

$0.9\,D + (1.0\,E \text{ or } 1.6W)$ **F 16-6**

($1.6W$ governs here, but this condition is superseded by (16-2)

∴ Load condition (16-2) governs.

$W_u \quad = 18.4$ psf

$P_u \quad = P_{uw} + P_{uf} = 1098 + 480 = 1578$ lb/ft ~ 1.6 k/ft

5. Start with preliminary design: #6 @ 16 inches o/c, solid grouted

$$A_s \quad = 0.44 \times \frac{12}{16} = 0.33 \text{ in}^2\text{/ft}$$

$$\text{Gross steel ratio} = \rho_g = \frac{A_s}{bt} = \frac{0.33}{12\,(5.5)} = 0.005$$

$$\text{Structural steel ratio} = \rho = \frac{A_s}{bd} = d = \frac{t}{2}$$

$$r \quad = \frac{0.33}{(12)(2.75)} = 0.01$$

6. Check against maximum steel ratio

This is out-of plane loading, so Method A for clay masonry applies. (clay-brick)

From the maximum reinforcement table in out-of-plane Method A, and equation

$$\rho_{max} \quad = 0.2938\left(\frac{f'_m}{f_y}\right)$$

For $f_y \quad = 60$ ksi, $f'_m = 5$ ksi

$\rho_{max} \quad = 0.02448 > 0.01$ **OK**

7. Calculate E_m and n **2108.7.2**

$E_m \quad = 700 f'_m$ for clay masonry

$E_m \quad = 700(5) = 3500$ ksi

$$n \quad = \frac{E_s}{E_m} = \frac{29{,}000}{3500} = 8.29$$

8. Modulus of rupture, from 2108.7.5

$f_r \quad = 4.0\sqrt{f'_m}$ for solid-grouted hollow unit masonry **Eq. 21-21**

$f_r \quad = 4.0\sqrt{5000} = 282.84$ psi ~ 283 psi

9. Gross moment of inertia $= I_g = \dfrac{bt^3}{12}$

$$I_g \quad = \frac{(12)(5.5)^3}{12} = 166.4 \text{ in}^3$$

10. Moment at crack condition

$$M_{cr} = f_r\left(\frac{2I_g}{t}\right)$$

$$M_{cr} = (283)\left(\frac{2 \times 166.4}{5.5}\right) = 17.124 \text{ in-lb}$$

$$M_{cr} = \underline{\underline{17.1}} \text{ in-k}$$

11. Cracked moment of inertia

$$I_{cr} = nA_{se}(d-c)^2 + \frac{bc^3}{3}$$

$$A_{se} = \text{effective area of steel} = \frac{P_u + A_s f_y}{f_y}$$

$$A_{se} = \frac{1.6 + (0.33)(60)}{60}$$

$$A_{se} = 0.36 \text{ in}^2$$

12. Depth of rectangular stress block

$$a = \frac{P_u + A_s f_y}{0.85 f'_m b} = \frac{1.6 + 0.33(60)}{0.85\ (5)(12)}$$

$$a = 0.42 \text{ in}$$

13. C = distance to neutral axis = $\dfrac{a}{0.85}$

$$C = \frac{a}{0.85} = 0.49 \text{ in}$$

14. Cracked section moment of inertia and section modules

$$I_{cr} = nA_{se}(d-c)^2 + \frac{bc^3}{3}$$

$$I_{cr} = 8.29(0.36)^2(2.75 - 0.49)^2 + \frac{12(0.49)^3}{3}$$

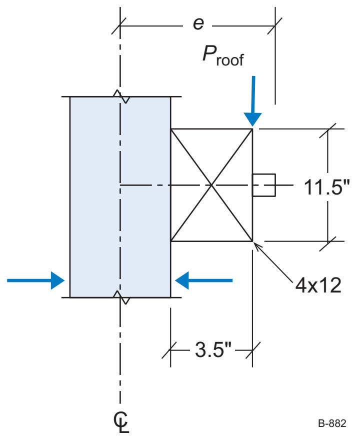
$$I_{cr} = 15.24 + 0.47 = 15.71 \text{ in}^4$$

$$S_{cr} = \frac{I_{cr}}{c} = \frac{15.71}{0.49} = 32.1 \text{ in}^3$$

15. Eccentricity of ledger roof load:

Ledger member = 4 × 12

$$e = \frac{1}{2}(5.5) + 3.5 = \underline{6.25} \text{ in}$$



16. Mid-height moment and lateral deflection due to service loads

$$i = 1$$

(a) First iteration

$$A_s = 0$$

$$M_{S1} = \frac{Wh^2}{8} + P_o\left(\frac{e}{2}\right) + (P_o + P_w)\,\Delta_s$$

For service load considerations,

Ultimate seismic lateral load → $F_p = 14.6$

$$(F_p)_{sl} = \frac{14.6}{1.4} = 10.42 \text{ psf}$$

$$W_{\text{wind}} = 11.5 \text{ psf (service load)}$$

∴ Wind governs for service load.

$$M_{s1} = \frac{11.5(25)^2 12}{8} + 400\left(\frac{6.25}{2}\right) + (400 + 915)(0)$$

$$= (10{,}781 \text{ in-lb}) + 1250 + 0 = 12{,}031 \text{ in-lb}$$

$$\Delta_{s1} = \frac{5M_{cr}h^2}{48E_mI_g} + \frac{5(M_s - M_{cr})h^2}{48E_mI_{cr}}$$

$$\Delta_{s1} = \frac{5(17.1 \text{ in-k})(25 \times 12)^2}{48\,(3500 \text{ ksi})(166.4 \text{ in})} + \frac{5(12.00 - 17.1)h^2}{48E_mI_{cr}} = 0$$

$$\Delta_{s1} = 0.275 \sim 0.28 \text{ in}$$

(b) Second iteration

$$\Delta_s = 0.28 \text{ in}$$

$$M_{s2} = \frac{Wh^2}{8} + P_o\left(\frac{e}{2}\right) + (P_o + P_w)\,\Delta_s$$

$$M_{s2} = \underbrace{M_{s1}}_{12{,}031\text{ in-lb}} + \underbrace{(400 + 915)(0.28)}_{368.2\text{ in-lb}} = 12{,}399.2\text{ in-lb}$$

$$\Delta_{S2} = \frac{5M_{cr}h^2}{48E_mI_g} + \frac{5(M_s - M_{cr})h^2}{48E_mI_{cr}}$$

M_{S2} = 12.4 in-k << M_{cr} = 17.1 in-k $\Rightarrow$ uncracked section

Since $M_{service} < M_{cr}$ $\Rightarrow \Delta_s = \dfrac{5M_{ser}h^2}{48\,E_mI_g} = 0.20$ in

$\Delta_{s2} = \Delta_{s1}$ **OK**

17. Check allowable deflection

Δ_s = 0.20 in

$\Delta_{allow} = 0.007h = 0.007(25 \times 12) = 2.10$ in >> Δ_s **OK**

18. Strength calculation

Mid-height moment under factored loads

$$M_u = \frac{W_uh^2}{8} + P_{uf}\left(\frac{e}{2}\right) + \underbrace{(P_{wu} + P_{uf})}_{P_u}\,\Delta_u$$ **Eq. 21-33**

Solve for load condition, $1.2D + 1.6W + f_1L + 0.5(L_r)$ **F 16-4**

W_u = 18.4 psf (for wind load, F 16-2, step 4)

P_{uf} = 400 plf × 1.2 (roof dead load factor) = 480 lb/ft

Since there is no specified live load; $L \equiv 0$, $L_r \equiv 0$

P_{wu} = 1.2(915) = 1.1 k/ft

P_u = 1.1 + 0.48 = 1.58 k/ft ~ 1.6 k/ft

(a) Assume $\Delta_u = 0.0$

$$M_{u1} = \frac{(12)18.4(25)^2}{8} + 480\frac{6.25}{2} + (1580)\,(\phi)$$

M_{u1} = 17,250 + 1500 = 18,750 in-lb

$$\Delta_{u1} = \frac{5M_{cr}h^2}{48\,E_mI_g} + \frac{5(M_u - M_{cr})h^2}{48\,E_mI_{cr}}$$

M_u = 18.75 in-k

M_{cr} = 17.1 in-k

$$\Delta_{u1} = \frac{5(17.1)(25 \times 12)^2}{48(3500)(166.1)} + \frac{5(18.75 - 17.1)(25 \times 12)^2}{48(3500)(15.71)}$$

= 0.28 + 0.28 = 0.56 in

(b) Δ_{s2} = 0.56 in

$M_{u2} = M_{u1}$ + 1580 (0.56) = 19,634 in-k

$$\Delta_{u2} = \underbrace{0.28}_{\Delta_{cr}} + \underbrace{\frac{5(19.6 - 17.1)(25 \times 2)^2}{48(3500)(15.71)}}_{0.43 \text{ in}}$$

Δ_{u2} = 0.71 in

(c) Δ_{s3} = 0.71 in

$M_{u3} = M_{u1}$ + 1580 (0.71) = 19.972 in-k

M_{u3} = 19.9 in-k

$$\Delta_{u3} = \underbrace{0.28 \text{ in}}_{\Delta_{cr}} + \underbrace{\frac{5(19.9 - 17.1)(25 \times 2)^2}{48(3500)(15.71)}}_{0.48}$$

Δ_{u3} = 0.76 in

Check convergence

(d) $\dfrac{M_{u3} - M_{u2}}{M_{u2}} = \dfrac{19.9 - 19.6}{19.6} = 1.5\%$ **OK**

(e) $\dfrac{\Delta_{u3} - \Delta_{u2}}{\Delta_{u2}} = \dfrac{0.76 - 0.71}{0.71}$ = 7.1 (exceed 5%)

(f) Δ_{u4} = 0.76 in

$$M_{u4} = \underbrace{M_{u1}}_{18,750} + \underbrace{1580\ (0.76)}_{1201} = 19.95 \text{ in-k}$$

M_{u4} = 19.95 in-k

$$\Delta_{u4} = 0.28 + \frac{5(19.95 - 17.1)(25 \times 12)^2}{48(3500)(15.71)}$$

$$\Delta_{u4} = 0.77 \text{ in}$$

$$\frac{\Delta_{u4} - \Delta_{u3}}{\Delta_{u3}} = 0.008 < 1\% \qquad \textbf{OK}$$

$$\Rightarrow M_u = 19.95 \text{ in-k}$$

19. Check strength/capacity

$$M_u \le \phi M_n$$

$$M_n = A_{se} f_y \left(d - \frac{a}{2}\right)$$

where

$$A_{se} = \frac{(A_s f_y + P_u)}{f_y} = \frac{(0.33\ )(60) + 1.6}{60}$$

$$A_{se} = 0.36 \text{ in}^2$$

$$a = \frac{P_u + A_s f_y}{0.85 f'_m b} = \frac{1.6 + 0.33(60)}{0.85(5)(12)} = 0.42 \text{ in}$$

$$M_n = 0.36\ (60)\left(2.75 - \frac{0.42}{2}\right) = 54.9 \text{ in-k}$$

ϕM_n, evaluate ϕ from 2108.4.1

$$\phi = 0.8 - \frac{P_u}{A_e f'_m} = 0.8 - \left(\frac{16}{(12 \times 5.5)5}\right) \approx 0.80$$

$$\phi M_n = 0.8(54.9) = \underline{\underline{43.9 \text{ in-k}}} >> \underline{\underline{19.95 \text{ in-k}}} \qquad \textbf{OK}$$

Example 6-2

A gymnasium wall is 24 feet between lateral supports, and has a 4-foot parapet. It is constructed of 8-inch hollow clay blocks and is located in San Francisco, California.

Roof load

P_o = 400 lb/ft = $P_L + P_{RDL}$ = 200 lb/ft + 200 lb/ft

f'_m = 2500 psi

F_y = 60 ksi

8-inch nominal thickness, t_{wall} = 7.5 in

The IBC base accelerations are

S_s = 1.0g

S_1 = 0.6g, Seismic Design Category E and 90-mph wind speed, Exposure B.

- Design the wall using the slender wall design procedure per 2000 IBC requirements.
- Evaluate the reinforcement requirements per the 2000 IBC.

Solution

1. Evaluate axial loads

Wall DL = 75 psf (solid grouted)

$$P_w = 75\left(4 + \frac{24}{2}\right) = 1200 \text{ lb/ft @ mid-height}$$

P_o = Roof load = 400 plf

From load conditions, 1605.2

P_{uw} = 1.44 k/ft

P_{uf} = 1.2(1200) + 0.5(200) = 0.34 k/ft

$$\frac{P_{uw} + P_{uf}}{A_g} = \frac{1.44 + 0.34}{(7.5)(12.0)} = 0.020 \text{ ksi}$$ **2108.9.4.4**

$0.05f'_m$ = 0.05(2.5) = 0.125 ksi > 0.020 ksi

Therefore IBC 2108.9.4.4 will govern.

2. Wind load

For 90 mph, from Table 1609.6.21(1), for the MWFRS, the maximum pressure for exposure B is 11.5 psf.

$W_{wind} = 11.5$ psf (service load)

$(W_u)_{wind} = 1.6(11.5) = 18.4$ psf

3. Seismic load

Seismic Design Category E

$F_p = 0.40 I_E S_{DS} w_{wall}$ **Eq. 16-63**

where $S_{DS} = \frac{2}{3} S_{MS}$ and $S_{MS} = F_a S_s$

From Table 1615.2(1)

for $S_s = 1.0$g

$F_a = 0.9$

$S_{MS} = 0.9\,(1.0) = 0.9$g

$S_{DS} = \frac{2}{3}(0.9) = 0.60$g

$I_E = 1.0$ **T 1604.5**

Evaluate factored load combinations from 1605.1. From inspection of (16-1) through (16-6), the roof live load must be included.

Formula/number	
$1.4D$	16-1
$1.2D + 1.6L$	16-2
$1.2D + 1.6(L_r) + (f_1L \text{ or } 0.8W)$	16-3
$1.2D + 1.6W + f_1L + 0.5(L_r)$	16-4
$1.2D + 1.0E + f_1L + f_2S$	16-5
$0.9D + (1.0E \text{ or } 1.6W)$	16-6

Since $L = 0$

$L_r = 200$ lb/ft

For strength design

$1.6W = 18.4$ psf ← Wind governs

$1.0E = F_p = 0.4(1.0)(0.60)(75) = 18.0$ psf

For service load considerations, (i.e., lateral deflection)

$W = 11.5$ psf

$\frac{F_p}{1.4} = 12.9$ psf ← Seismic governs

Load condition Formula (16-4) is critical.

@Seismicisolation

4. Selected reinforcement: #8 @ 16 inches o/c, solid grouted

$$A_s = \frac{0.789 \times 12}{16} = 0.59 \text{ in}^2/\text{ft}$$

$$\rho_{gross} = \text{gross steel ratio} = \frac{A_s}{bt} = \frac{0.59}{12 \times 7.5} = 0.0066$$

$$\rho_s = \frac{A_s}{bd} \qquad \text{where } d = \frac{t}{2} = \frac{7.5}{2} = 3.75 \text{ in}$$

$$\rho_s = \text{steel ratio} = \frac{0.5}{3.75 \times 12} = 0.013 \sim 1.3\%$$

5. Check maximum steel ratio, Method A, **2108.9.2.13**

For clay masonry

$$E_{mu} = 0.0035$$

$$f'_m = 2500 \text{ psi}$$

Out-of plane

$\rho_{max} = 0.02872 > 0.013$ **OK**

proceed with #8 @ 16 inches o/c

6. Section properties

$$A_{se} = \frac{P_u + A_s f_y}{f_y} = \frac{1.78 + 0.5(60)}{60} = 0.53 \text{ in}^2$$

$$a = \frac{P_u + A_s f_y}{0.85 f'_m b} = \frac{1.78 + 30}{0.85(2.5)(12)} = 1.25 \text{ in}$$

$$c = \frac{a}{0.85} = \frac{1.25}{085} = 1.47 \text{ in}$$

7. Nominal moment capacity

$$M_n = A_{se} f_y \left(d - \frac{a}{2}\right) = 0.53(60)\left(3.75 - \frac{1.25}{2}\right) = 99.4 \text{ in-k}$$

$$\phi M_n = 0.8\ (99.4) = 79.5 \text{ in-k}$$

8. Modulus of elasticity **2108.7.2**

$$E_m = 700 f'_m$$

$$E_m = 700(2500) = 1750 \text{ ksi}$$

$$n = \text{modular ratio} = \frac{E_s}{E_m} = \frac{29{,}000}{1750} = 16.6$$

9. Cracked moment of inertia, I_{cr}

$$I_{cr} = nA_{se}(d-c)^2 + \frac{bc^3}{3}$$

$$I_{cr} = 16.6(0.53)(3.75 - 1.47)^2 + \frac{12(1.43)^3}{3}$$

$$I_{cr} = 45.7 + 12.7 = 58.4 \text{ in}^4$$

10. Gross moment of inertia, I_g

$$I_g = \frac{bt^3}{12} = \frac{12(7.5)^3}{12} = 421.9 \text{ in}^3$$

11. Cracking moment, M_{cr}

$$M_{cr} = f_r\left(\frac{2I_g}{t}\right)$$

where $f_r = 4.0\sqrt{f'_m}$ **Eq. 21-21**

$$f_r = 4\sqrt{2500} = 200 \text{ psi}$$

$$M_{cr} = 200\left(\frac{2 \times 421.9^4}{7.5}\right) = \underline{22.5} \text{ in-k}$$

12. Eccentricity

$$e = \frac{7.5}{2} + 3.5 = \underline{\underline{7.25}} \text{ in}$$

13. Ultimate moment demand = M_u

$$M_u = \frac{W_u h^2}{8} + P_{uf}\left(\frac{e}{2}\right) + P_u(\Delta_u)$$

for Δ_u, calculate from $\phi M_n = 79.5$ in-k

$$\Delta_u = \frac{5(\phi M_n)h^2}{48E_m I_{cr}} = \frac{5(79.5)(24 \times 12)^2}{48(1750)(58.4)}$$

$$\Delta_u = 6.72 \text{ in}$$

$$M_u = \frac{18.4\,(24)^2 12}{8} + 340\frac{7.25}{2} + 1780\,(6.72 \text{ in})$$

$$M_u = 15{,}898 + 1233 + 11{,}962 = \underline{\underline{29.1}} \text{ in-k}$$

Since $M_u << \phi M_n$, the wall section is adequate.

14. Service load deflection

$$M_s = \frac{W_s h^2}{8} + P_o\left(\frac{e}{2}\right) + (P_o + P_w)\,\Delta_s$$

Δ_s = allowable service load deflection = $0.007h$

= 0.007(24 × 12) = 2.02 in

Δ_s = 2.02 in

W_s = 12.9 psf (seismic governs for service load)

P_o = 400 lb/ft

P_w = 1200 lb/ft

$$M_s = \frac{12.9(24)^2 12}{8} + 400\left(\frac{7.25}{2}\right) + (1600)(2.02\text{ in})$$

M_s = 11,146 + 1450 + 3232 in-lb

$M_s = \underline{\underline{15.9}}$ in-k $< M_{cr}$

Deflection at service

$$\Delta_{service} = \frac{5M_{service}h^2}{48E_mI_g} = \frac{5(15.9)(24\times 12)^2}{48(1750)(1.75)(421.9)} = 0.19\text{ in}$$

$\Delta_{service}$ = 0.19 in << 2.02 in **OK**

Example 6.3

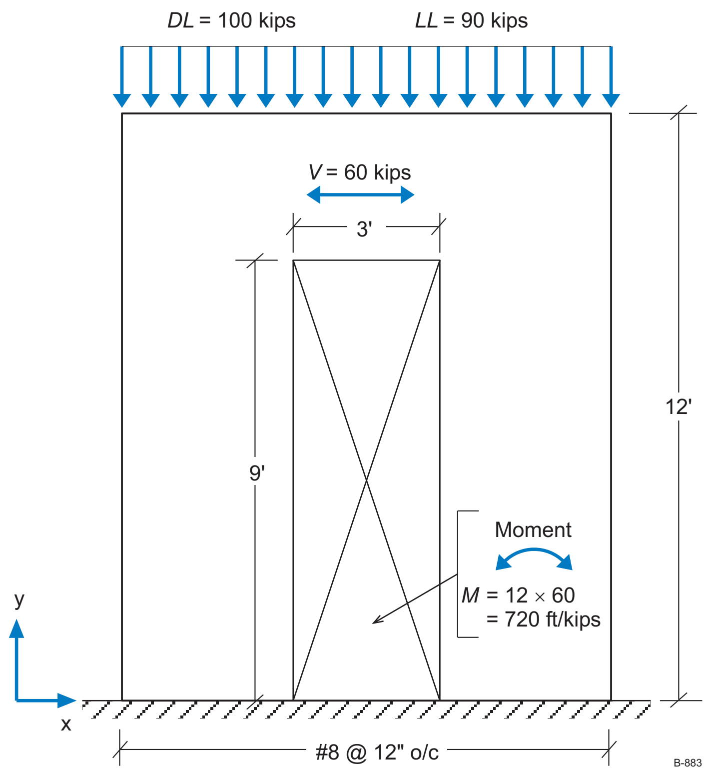


Figure E6-3(1)

Given:

Nominal 8-inch CMU shear wall

Solid grouted

12 feet high, 10 feet long

f'_m	= 2500 psi	f_y	= 60 ksi
Lateral seismic sheer		V	= 60 kips
Vertical dead load		P_{DL}	= 100 kips
Vertical live load		P_{LL}	= 90 kips
Seismic moment		M	= 720 kips

Reinforced with 9 #8 bars

- Plot the interaction diagram and determine if wall and reinforcing is adequate for the loads and moments imposed.

Solution

f'_m = 3000 psi

f_y = 60,000 psi

$E_m = 900f'_m = 27 \times 10^5$ psi [for concrete masonry (2108.7)]

$E_s = 29 \times 10^6$ psi

$$n = \frac{E_s}{E_m} = \frac{29 \times 10^6}{27 \times 10^5} = 10.74$$

Modulus of rupture **2108.7.5**

$f_r = 4\sqrt{f'_m} = 219$ psi, 235 psi maximum
(for fully grouted hollow masonry unit) **Eq. 21-21**

Maximum usable masonry strain for concrete masonry

= 0.0025 in/in (eM_u) **2108.9.1(4)**

Strength reduction factors

ϕ = 0.80 shear **2108.4.2**

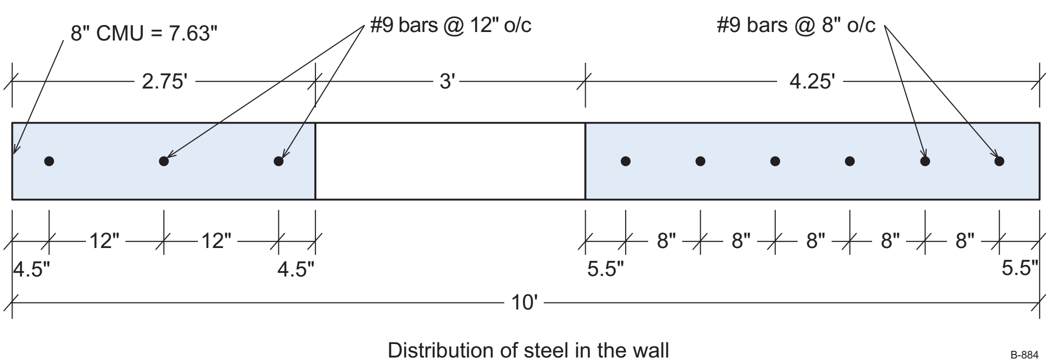
ϕ = 0.80 flexure **2108.4.2**

ϕ = 0.65 axial load only **2108.4.3.1**

ϕ = 0.65 axial load and flexure **2108.4.3.1**

Load combinations (strength design)

Formula/number	
$1.4D$	16-1
$1.2D + 1.6L + 0.5(L_r, S, \text{ or } R)$	16-2
$1.2D + 1.6(L_r, S, \text{ or } R) + (f_1L \text{ or } 0.8W)$	16-3
$1.2D + 1.6W + f_1L + 0.5(L_r, S, \text{ or } R)$	16-4
$1.2D + 1.0E + f_1L + f_2S$	16-5
$0.9D + (1.0E \text{ or } 1.6W)$	16-6



Distribution of steel in the wall

Figure E6-3(2)

Check for maximum steel

$$\varepsilon_{\max} = \frac{A_{s\max}}{bd} = \frac{(0.80)^2 f'_m}{1.25 f_y}\left(\frac{\varepsilon_{mu}}{\varepsilon_{su} + \varepsilon_{mu}}\right)$$

$$= \frac{(0.80)^2(3000\text{ psi})}{1.25(60{,}000\text{ psi})}\left(\frac{0.0025}{0.0026 + 0.0025}\right)$$

$$\varepsilon_{mu} = 0.0025 \text{ (for concrete masonry)}$$

$$\varepsilon_{su} = 1.3\,\varepsilon_{sy} = 1.3(0.002) = 0.0026$$

$$e_{\max} = 0.0125$$

$$e_{\text{provided}} = \frac{A_s}{bd} = \frac{1}{(7\text{ ft} \times 1\text{ ft} \times 3.815\text{ in})} = 0.00312$$

$$e_{\text{provided}} < e_{\max}$$ **OK**

Estimate vertical steel requirement for overturning moment

$$A_s = \frac{M}{df_y} = \frac{720 \times \underbrace{1.0}_{\text{load factor}} \times 12}{9.167 \times 12 \times 66} = 1.31\text{ in}^2 \text{ (neglecting the opening)}$$

Try

#9 bars @ 8 inches for 4.25-ft pier

$$A_s = 1.5\text{ in}^2$$

#9 bars @ 12 inches for 2.75-ft pier

$$A_s = 1\text{ in}^2$$

Analyze the shear wall

1. Plot the interaction diagram for the wall
2. Determine the cracking moment, $M_n >> M_{cr}$
3. Check loading conditions for vertical load and moment
4. Check the requirements for boundary members and confinement
5. Determine shear reinforcing

1. Plot interaction diagram

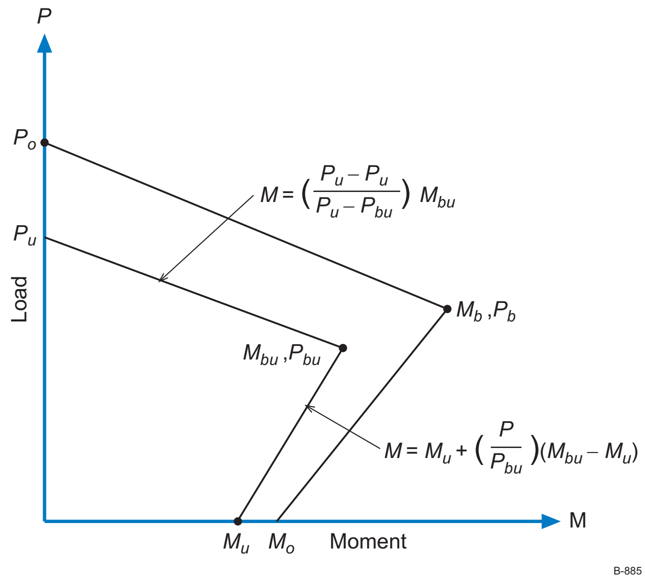


Figure E6-3(3)

where

P_o = nominal axial strength

P_u = ϕ times the nominal axial strength or factored axial load on the wall

M_n = nominal moment strength

M_y = ϕ times the nominal moment strength or factored moment on the wall

P_b = balanced axial strength

P_{bu} = ϕ times the balanced axial strength

M_b = balanced moment strength

M_{bu} = ϕ times balanced moment strength

a. Nominal axial load P_o **2108.9.3.5**

$P_o = 0.85 f'_m (A_g - A_s) + f_y A_s$ **Eq. 21-26**

$= 0.85(3)\{(4.25 \text{ ft} - 2.75)\ 12\ (7 - 625) - (1 \times 9)\}$

$+ 60(9 \times 1) = 2150.3$ kips

where

$A_g - A_s$ = net cross-sectional area of masonry (in^2)

A_s = effective cross-sectional area of reinforcement (in^2)

f'_m = specified yield stress of reinforcement (psi)

F_y = specified compressive strength of masonry at 28 days (ksi)

b. Factored axial load, P_u

$P_u = 1.2D + 1.6L = 1.2(100) + 1.6(90) = 264$ k

Check $P_u \leq 0.65\ P_o$

$264 \leq 0.65(0.80)\ 2150.3$

$264 < 1118.15$ kips **OK**

c. Nominal moment strength, M_n

Solve for location of NA so that the sum of the vertical forces equals zero.

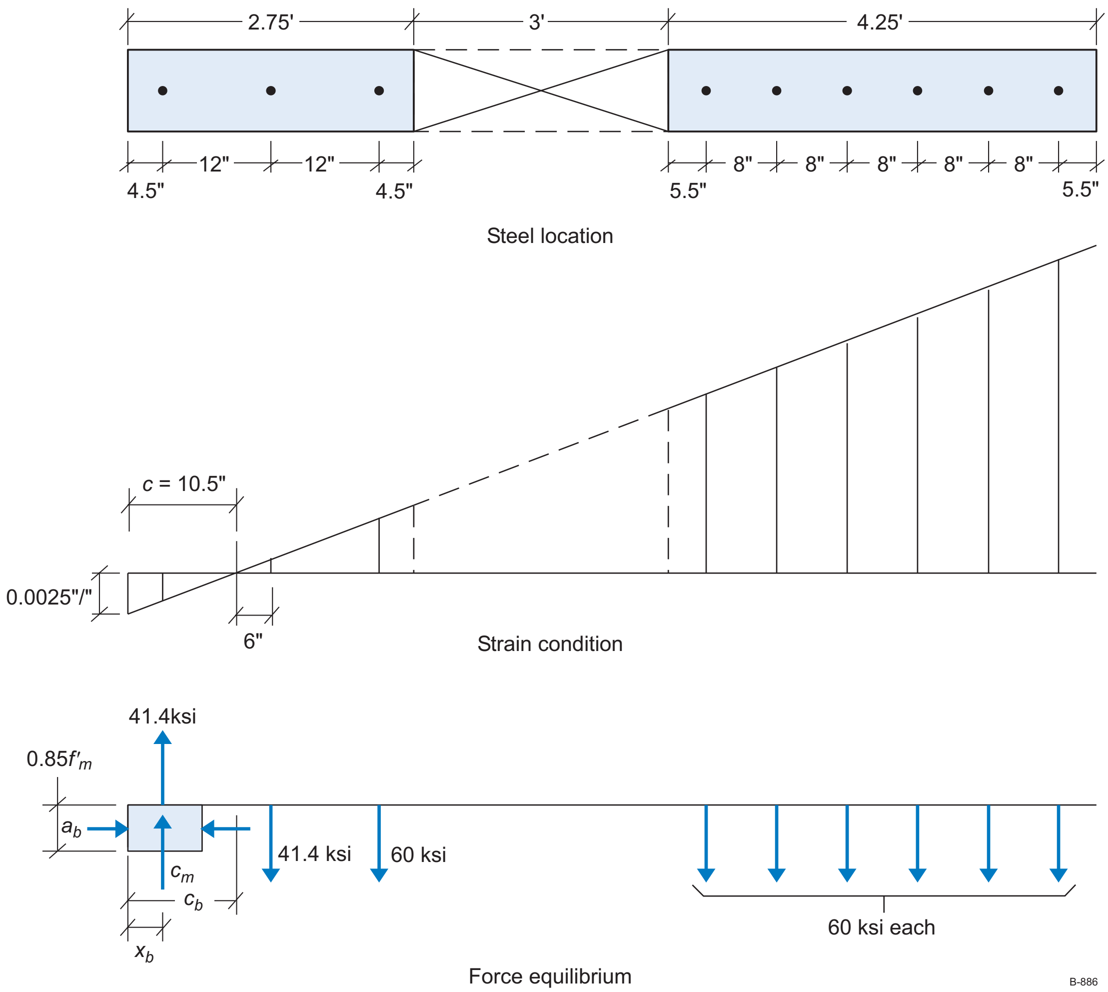


Figure E6-3(4)

Strain in compression steel, $\dfrac{0.0025}{10.5} = \dfrac{\varepsilon_{sc}}{(10.5 - 4.5)}$

$\varepsilon_{sc} = 0.00142$ in/in

$\dfrac{\text{Stress}}{\text{Strain}} = E_s = 29 \times 10^6$ psi

Stress in compression steel $= 0.00142 \times 29 \times 10^6 = 41.4$ ksi

a = depth of compression stress block

$= 0.85c = 0.85(10.5) = 8.93$ in

$x = 0.5a = 4.4625$ in

Tension force $= A_s f_y = (1)(7)\ 60 + (1)(1)\ 41.4$

$= 461.4$ 105kips

Compression force $= A_s f_s + 0.85 f'_m\ ba$

$= (1)(41.4) + 0.85(3)(7.625)(8.925)$

$= 41.4 + 173.53 = 214.94$ kips

$T - C = 461.4 - 214.935 = 246$ kips

Change location of neutral axis

Use $C = 22.5$ in

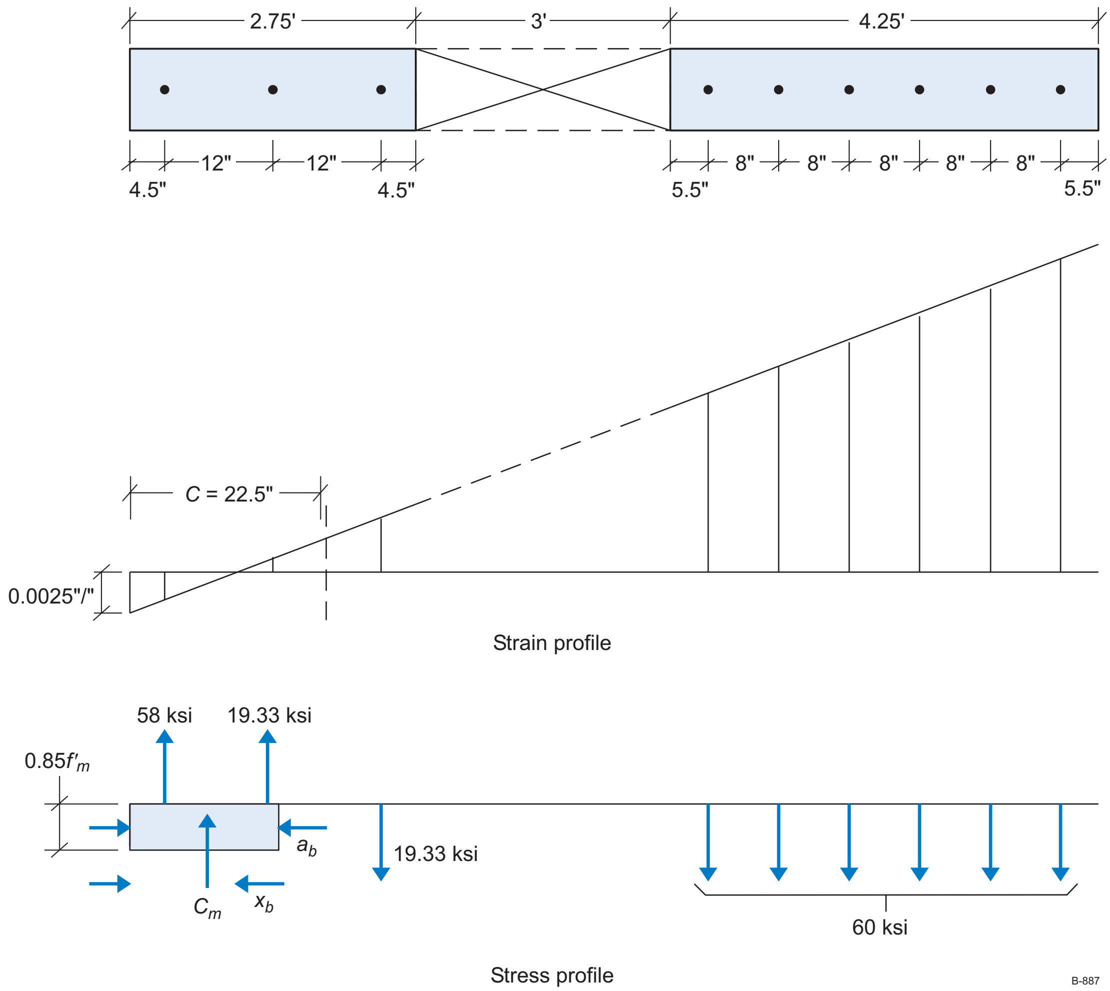


Figure E6-3(5)

$$a \quad = 0.85c = 0.85(22.5) = 19.13 \text{ in}$$

$$x \quad = 0.5a = 9.56$$

$$T \quad = A_s f_s = (1)\,6(60) + 19.33(1)(1) = 379.33 \text{ kips}$$

$$\text{Compression force } = A_s f_s + 0.85 f'_m\, ba$$

$$= 1(58) + (1)19.33 + 0.85(3)(7.625)\ 19\text{-}125$$

$$(77.33 + 371.86) = 449.19 \text{ kips}$$

$$T - C = 379.33 - 449.19 = -69.86 \text{ kips}$$ **No Good**

Use $C = 16.5$ in

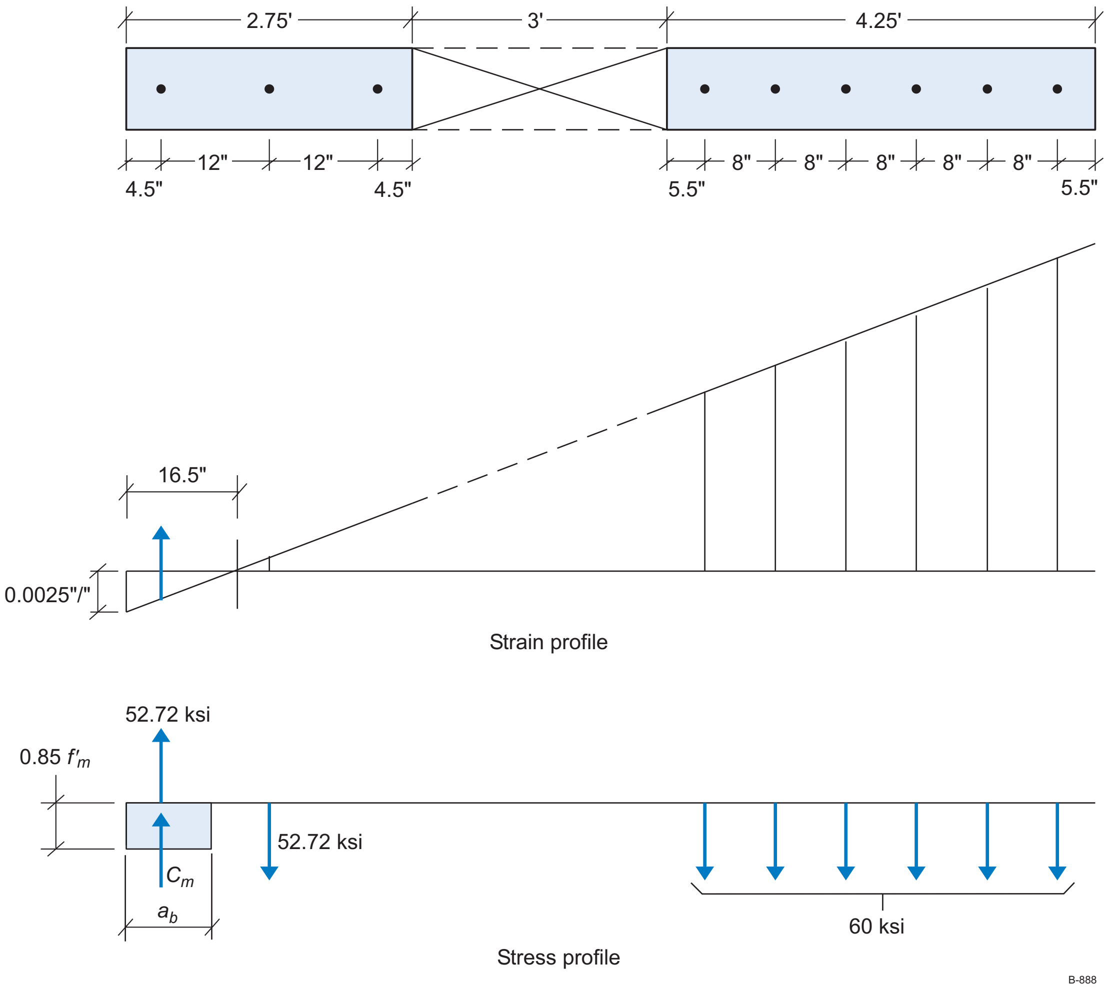


Figure E6-3(6)

$a \quad = 0.85C = 0.85(16.5) = 14.025$ in

$x \quad = 0.5a = 7.013$ in

$T \quad = A_s f_y = (52.72)(1) + 60(1)(6) = 412.72$ kips

$C \quad = A_s f_s + 0.85\, f'_m\, ab$

$\quad\;\; = (52.72)(1) + 0.85(3)\, 14.025\, (7.625)$

$\quad\;\; = 52.72 + 272.69 = 325.418$ kips

$T - C = 412.72 - 325.418 = \underline{87.30}$ kips **No Good**

Try $c = 19.5$

$a \quad = 0.85C = 0.85(19.5) = 16.575$ in

$x \quad = 0.5a = 8.288$ in

$T \quad = A_s f_s = 60(1)(6) + 33.46(1) = 393.46$ kips

$$C = A_s f_s + 0.85 f'_m ab$$

$$= (1)\ 11.15 + (1)\ 55.76 + 0.85(3)(16.575)(7.625)$$

$$= 11.15 + 55.76 + 322.28 = 389.19 \text{ kips}$$

$$T - C = 393.46 - 389.19 = 4.26 \text{ kips}$$

Use $C = 19.5$ in

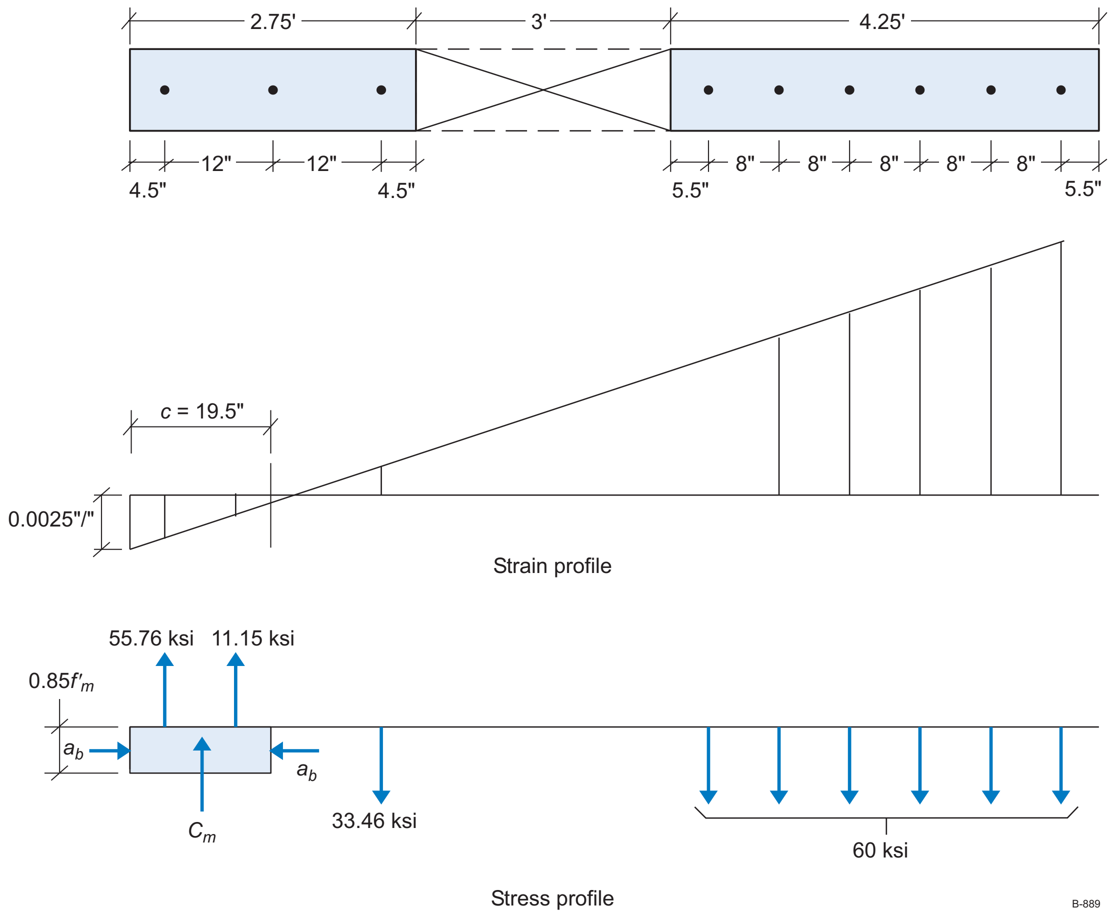


Figure E6-3(7)

Nominal bending moment, M_n

Sum of moments about left edge of wall

$$M_n = T\ (\text{moment arm}) - C\ (\text{moment arm})$$

$$= A_s f_y\ (\text{moment arm}) - \{0.85 f'_m ab\ (\text{moment arm}) + A_s f_s\ (\text{moment arm})\}$$

$$= 1(60)(114.5 + 106.5 + 98.5 + 90.5 + 82.5 + 74.5) + (10)(33.46)(28.5) - \{(0.85(3)\ 16.575(7.625)\ (8.288) + (1)\ 55.76(4.5) + [(11.15)(16.5)]$$

$$= 34973.6 - 2670.896 - 250.92 - 183.975$$

$$= 31{,}876.81 \text{ in-kips} = 2656 \text{ ft-kips}$$

d. Design bending moment, M_u

$$M_u = \phi M_n = 0.80(2656) = 2125 \text{ ft-kips}$$

e. Nominal balanced design axial load, P_b

Compression capacity, $C_m = 0.85 f'_m a_b b$

Where balanced stress block, $a_b = 0.85C$

$$c_b = \left(\frac{\varepsilon_{mu}}{\varepsilon_{mu} + \frac{f_y}{E_s}}\right) d = \left(\frac{0.0025}{0.0025 + \frac{60{,}000}{29 \times 10^6}}\right) d = 0.547d$$

$$c_b = 0.547(114.5) = 62.63 \text{ in (NA for balanced design)}$$

$$a_b = 0.85C_b = 0.85(62.63) = 53.23 \text{ in}$$

(depth of compression stress block)

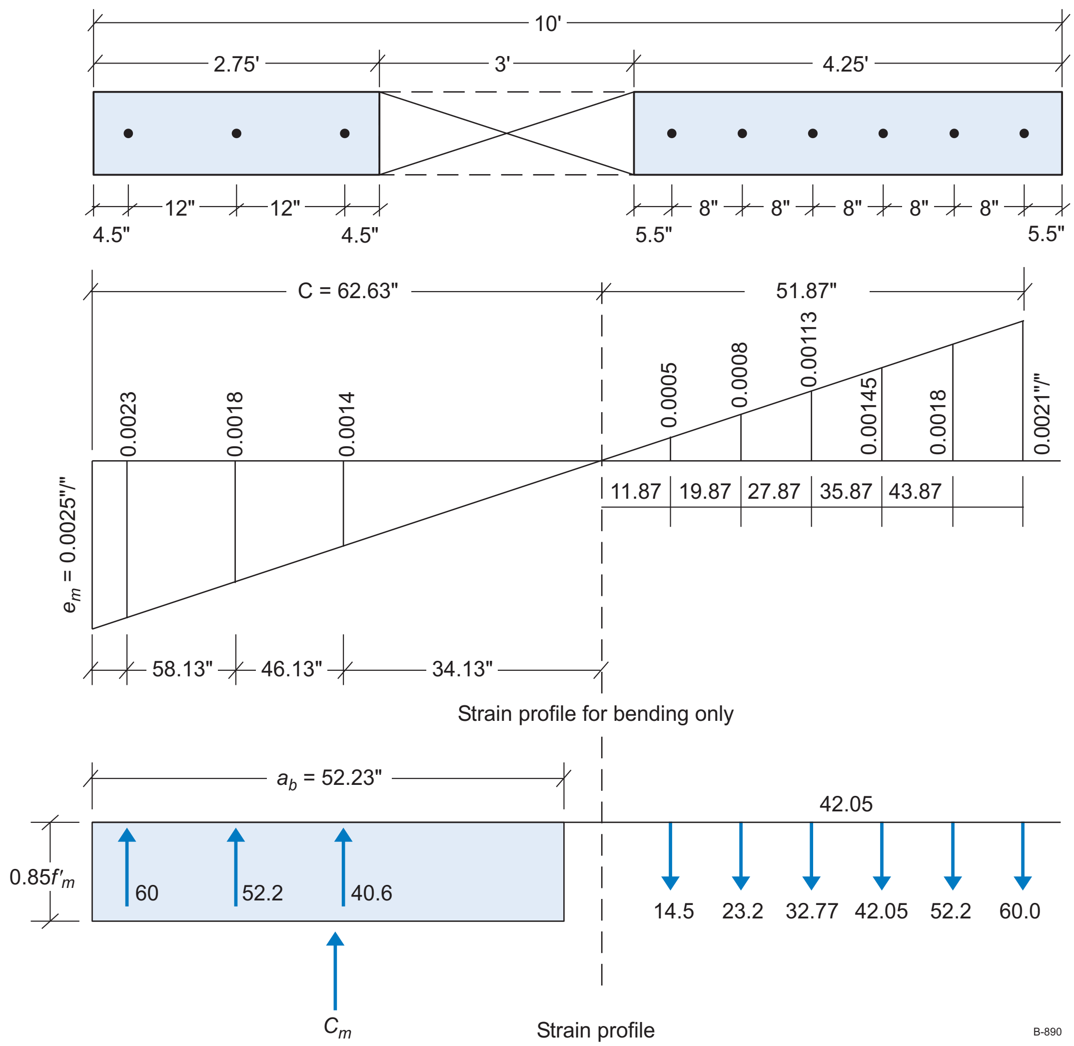


Figure E6-3(8)

Tension force T $= A_s f_s$

$= 1(14.5 + 23.2 + 32.77 + 42.05 + 52.2 + 60)$

$= 224.72$ kips

Compression force C $= A_s f_s + 0.85 f'_m a_b b$

$= 1(60 + 52.2 + 40.6) + 0.85(3)(53.23)(7.625)$

$= 152.8 + 1034.99 = 1188$ kips

Sum of vertical forces

$P_b = \text{C} - \text{T} = 1188 - 224.72 = 963$ kips

f. Design balanced axial load, P_{bu}

$P_{bu} = \phi P_b = 0.65(963) = 626$ kips

g. Nominal balanced design moment strength, M_b

Take moments about neutral axis

$M_b = A_s f_s\,(\text{moment arm}) + 0.85 f'_m a_b x_b$

$= 1\,((60(51.87) + 52.2(43.87) + 42.05(35.87) + 32.77(27.80)$
$+ 23.2(19.87) + 14.5(11.87) + 40.6(34.13) + 52.2(46.1)$
$+ 60\,(58.13)) + 0.85(3) + 53.23(37.015)$
$= 20344.78$ k-in

$= 1695$ ft-kips

h. Design balanced moment strength, M_{bu}

$M_{bu} = \phi M_b$

$= 0.65(1695) = 1102$ ft-kips

i. Plot interaction diagram

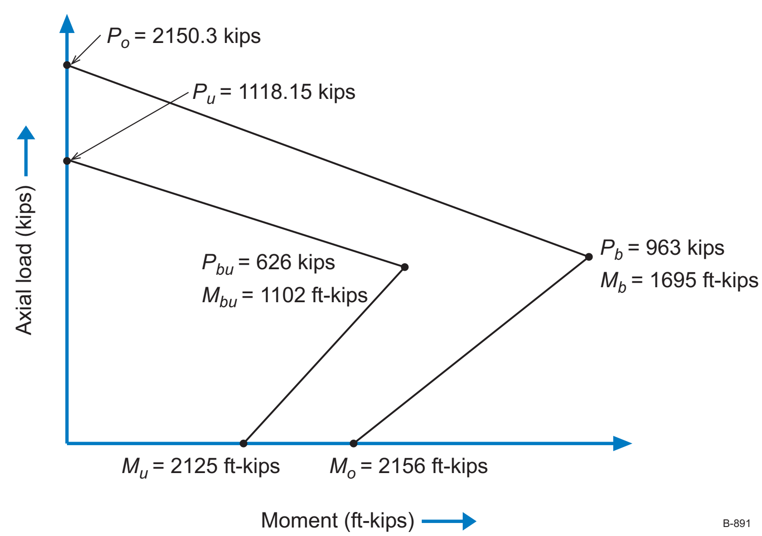


Figure E6-3(9)

2. Cracking moment, M_{cr}

Using gross section properties and linear elastic theory

$$f_r = \frac{M_{cr}}{S} - \frac{P}{A}$$

where

A = area of cross section, $bl = 84(12)\ 7.63 = 640.92\ \text{in}^2$

$$S = \frac{bl^2}{6} = \frac{7.625(7 \times 12)^2}{6} = 8967\ \text{in}^3$$

$$f_r = 4\sqrt{f'_m} = 4\sqrt{3000} = 219\ \text{psi}$$

ρ = axial load = dead load + live load = 190 kips

$$M_{cr} = S\left(\frac{P}{A} + f_r\right)$$

$$= 8967\left(\frac{190{,}000}{640.92} + 219\right)\frac{1}{1000} = 4622.03\ \text{in-kips}$$

$$= 385.2\ \text{ft-kips}$$

3. Check loading conditions

Load condition 1

$1.2D + 1.6L$

$= 1.2(100) + 1.6(90) = 264$ kips

$P_u = 264\ \text{kips} < P_{bu} = 626\ \text{kips}$

Determine the nominal moment strength, M_n, for $P_u = 264$ kips

$$\frac{P_{bu}}{(M_{bu} - M_u)} = \frac{P_u}{M_x}$$

$$M_x = \left(\frac{P_u}{P_{bu}}\right)(M_{bu} - M_u)$$

$$M_n = (M_u = M_{ol})/\phi \qquad \phi = 0.65$$

$$= \left\{2125 + \frac{264}{626}(1102 - 2125)\right\}/0.65$$

$$= 2605.5\ \text{ft-kips}$$

Check ratio of nominal moment to cracking moment

$$\frac{M_n}{M_{cr}} = \frac{2605.5}{385.3} = 6.76$$

Nominal moment is 576% greater than cracking moment

Load condition 2

$1.2D + 1E = 1.2(100) + 1(720 \text{ ft-kips}) \rightarrow$ (seismic moment)

$= 120 + 720$ ft-kips

Use factored loads to compare to values on interaction diagram

$P_u = 120 \text{ kips} < P_{bu} = 626 \text{ kips}$

Nominal moment strength, M_n

for $P_n = 120$ k

$\phi = 0.65$

$$M_n = \left\{M_u + \left(\frac{P_u}{P_{bu}}\right)(M_{bu} - M_u)\right\}/\phi$$

$$= 2125 + \frac{120}{626}(1102 - 2125)/0.65 = 2968 \text{ ft-kips}$$

$$\frac{M_n}{M_{cr}} = \frac{2968}{3853} = 7.7$$

4. Check requirements for boundary members

Check for stress in masonry, $f_m > 0.2f'_m$

Assume linear elastic conditions and an uncracked section

f_m = maximum extreme compression fiber stress

$$f_m = \frac{P_u}{A} + \frac{M_u}{S}$$

$$= \frac{264 \times 10^3}{640.5} + \frac{720 \times 12{,}000}{8967} = 1376 \text{ psi} > 0.4f'_m$$

$0.2f'_m = 0.2 \times 3000 = 600$ psi

$0.4f'_m = 0.4 \times 3000 = 1200$ psi

Boundary members are required when compressive strains in the wall exceed 0.0015

5. Determine shear reinforcing

Shear design

a. Shear requirement

$V_u = 1 - V_s = 1 \times 6 = 60$ kips

b. Nominal shear strength **2108.9.3.5.2**

$V_n = V_m + V_s$ **Eq. 21-27**

$M_u = 720$ ft-kips

$V_u = 60$ kips

d_v = 114.5 in

$$\frac{M_u}{V_u d_v} = \frac{720}{60\left(\frac{114.5}{112} - 3\text{ ft}\right)} = 1.83 > 1$$

For $M/V_u d_v > 1.00$

$V_n = 4A_n\sqrt{f'_m} = 4(640.5)\sqrt{3000} = 140.3$ kips **Eq. 21-29**

A_n = net cross sectional area of masonry $= 84 \times 7.63$

$= 640.9\text{ in}^2$

$V_n = 0.60\ (140.3) = 84.18$ kips

$V_u < \phi V_n$; 84.18 kips > 60 kips

No shear reinforcement is required

Appendix A

(refer to the CD)

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Appendix B

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Appendix C

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Appendix C1: Nonlinear Analysis of Shear Walls

Buildings with shear walls have an excellent record for surviving natural disaster events. Working Stress design (WSD) and Strength Design (SD) have governed the shear wall design process. They are the recognized design procedures referenced in the 1997 UBC and the 2000 IBC

The advent of high-capacity computers and sophisticated Finite Element Methods (FEM) software has furthered the proliferation of Finite Element Analysis (FEA) tools for structural engineers. FEA tools are discussed in Chapter 5 but information is focused on liner elastic analysis.

The method for predicting structural failure capacity is termed *Limit Analysis of Structures* and is grounded in plasticity theory entailing the nonlinear effects of structural materials, geometry, and basic statics. There are practical benefits to considering the nonlinear effects and analyzing for plasticity behavior in reinforced/unreinforced masonry shear walls:

1. In certain structures, ultimate failure will be a major concern. For example, military and national defense facilities, emergency response facilities, and other structures having critical demands. The ultimate failure scenario is important because the probability of higher-than-normal demand loads exceeding basic prescriptive code criteria must be considered. The principle extends to all structures.

2. For existing masonry structures that rely on the performance capability of shear walls, the plasticity capacity (i.e., reserve capacity prior to failure) may be critical when deciding whether the structure requires a retrofit/upgrade. The design codes do not necessarily apply to many historic structures because of the types of materials implemented. Therefore, a specific analysis that considers both the materials and geometric nonlinear characteristics is necessary in order to formulate a suitable demand-capacity analysis.

   For example, an historic unreinforced masonry building could take advantage if nonlinear finite element analysis to determine the strength of the structure during extreme wind and seismic loads. The potential damage scenario could be evaluated and then various retrofit design options and their respective economic benefits could be considered.

3. Certain clients may wish to evaluate the risk exposure of their properties. Risk management involves an evaluation of buildings to determine the effectiveness of their structural system during disaster events. The nonlinear analysis methods could be beneficial in this context.

Nonlinear Analysis Methodologies

The two categories of limit analysis are 1) Analytical Methods and 2) Finite Element Methods. Complete coverage of nonlinear analysis is beyond the scope of this text, but a brief comment on its practical benefits follows.

Analytical Method

There are many sources of theoretical equations and high-level theories of elasticity dealing with nonlinear problems. One of the most helpful references is *Limit Analysis and Concrete Plasticity*, 2ndedition, M.P. Neilsen, CRC Press, New York, 1999. Although the title refers to concrete plasticity, the same principles of mechanics apply to reinforced masonry because of similarity in the composite structure. Likewise, the principles of reinforced concrete are relied on in reinforced masonry strength design.

The text takes a practical approach by employing notations familiar to structural engineers. Equations are developed for a typical shear wall with lateral and axial loads. To predict limit capacity, the plasticity theory relies on the strut-and-tie concept, thus dividing shear wall into basic elements. Lateral load is resisted primarily by the diagonal compression strut and web shear. The top beam is a transfer mechanism to absorb the shear and axial force into the wall. Vertical axial load adds to the overturning moment resistance while also increasing the strut compression force. This formulation applies to a general concept for shear walls, but more complicated configurations with openings and end flanges require analysis using FEMs with a nonlinear shear wall element.

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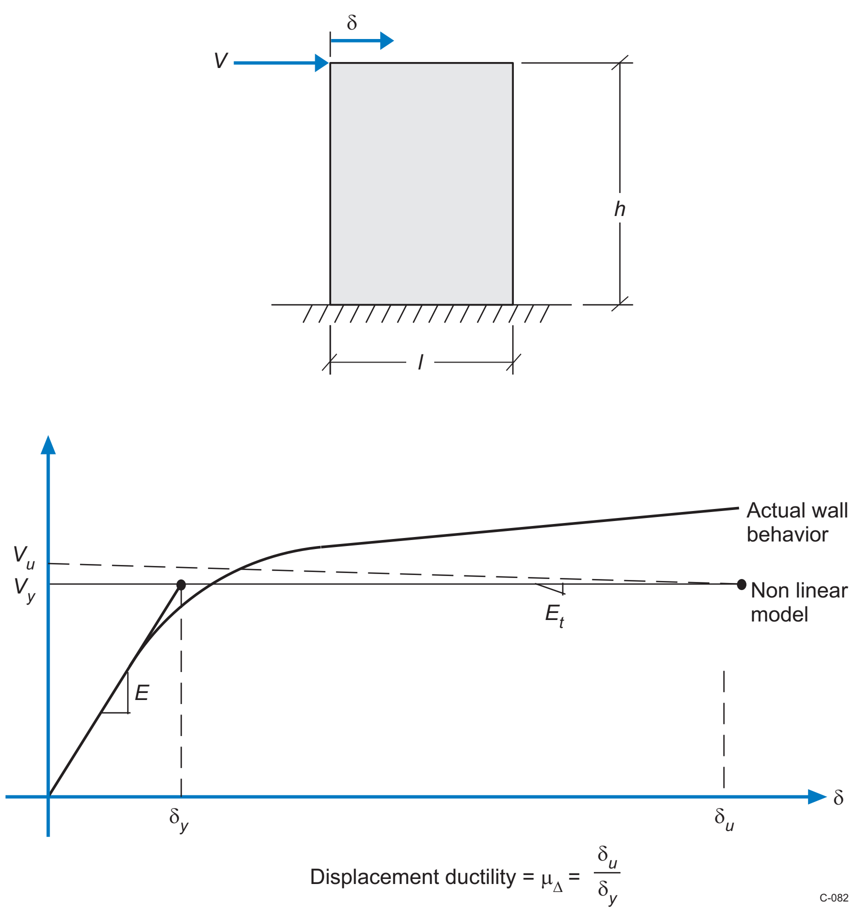
Nonlinear Finite Element Methods for Shear Walls

Great strides have been made in nonlinear finite elements and there are many enhancements to existing software technology. Traditionally, FEA has been viewed as an arena for research restricted to those interested parties who are not practicing engineers. Things are changing.

Recently the FEA programs began offering standard nonlinear analysis protocols in their basic packages. And because of the progress in the computer hardware field, the power of the PC/MAC has been considerably enhanced, with greater advances to come.

Nonlinear Finite Element Basics

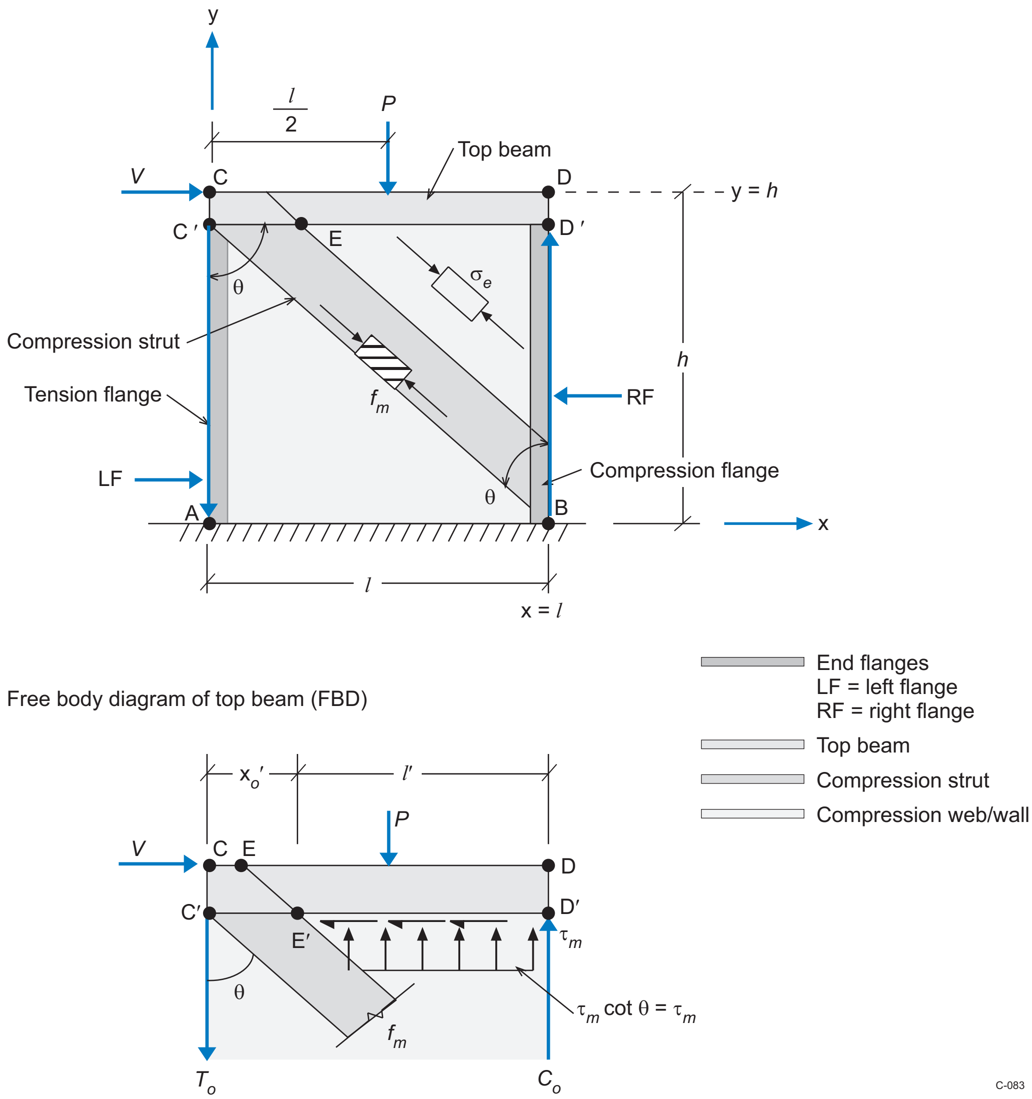
A nonlinear analysis is intended to incorporate either geometric or material nonlinearity, or both. The focus of shear wall analysis is on incorporating and accounting for the stiffness degradation in the wall.



This section will provide several simplified equations and some technical insight so that structural engineers can implement the analysis procedure. Two methods for solution are discussed.

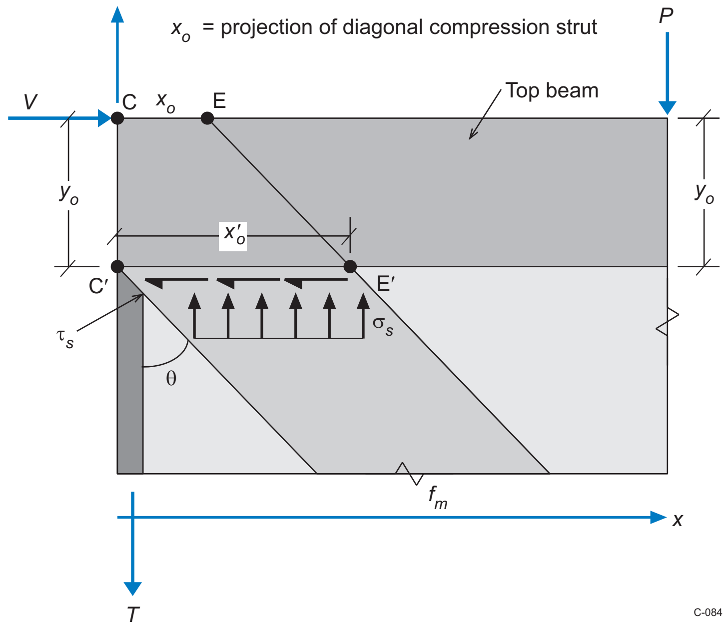
1. Analytical formulation based on theories of limit analysis
2. Computation formulation using nonlinear finite element analysis

The principles of analytical formulation apply to reinforced masonry shear walls, provided they have adquate horizontal and vertical steel to justify homogeneous behavior of CMU, steel, and grout. The formulation is based on the popular strut-and-tie model used for concrete plasticity.



The diagonal compression strut absorbs the bulk of the shear force in the form of compressive stress. This is tranferred through the top beam. The end flanges function in tension (left flange, LF) and compression (right flange, RF).

The free-body diagram of the top beam shows the LF (T_o) forces and RF (C_o) forces. The diagonal compression strut geometry is shown here in greater detail.



x_o = projection of diagonal compression strut onto top beam x-axis ≡ CE

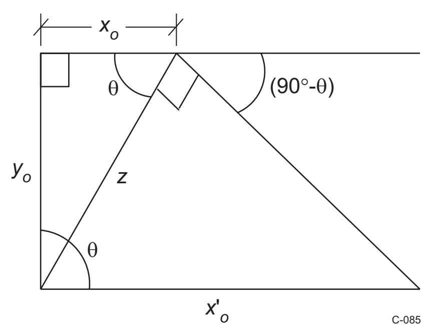
y_o = projection of diagonal compression strut onto top beam y-axis ≡ CC'

x'_o = horizontal end bearing of diagonal compression strut onto top beam ≡ $C'E'$

From geometry

$$\frac{x_o}{z} = \cos\theta$$

$$\Rightarrow z = \frac{x_o}{\cos\theta}$$



$$\frac{z}{x'_o} = \cos\theta \qquad \boxed{\Rightarrow x'_o = \frac{x_o}{\cos^2\theta}}$$

σ_s = normal (vertical) end bearing stress

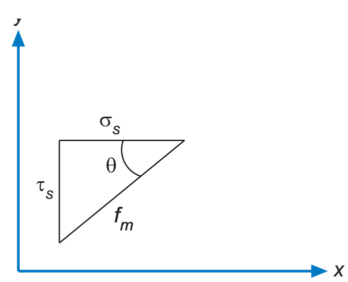
τ_s = shear (horizontal) end bearing stress

By stress transformation

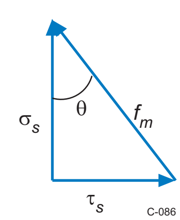
$$\therefore\ \sigma_s = f_m \cos^2\theta$$

$$\therefore\ \tau_s = f_m \cos^2\theta \sin\theta$$

$$\Sigma M_{D'} = 0$$



Areas of stress



Force vectors

$$T_o = \frac{(V)(y_o)}{l} + \frac{x'_o t}{l}\left(l - \frac{1}{2}x'_o\right) f_m \cos^2\theta - \frac{1}{2}P$$

$$C_o = T_o - x'_o \; t f_m \cos^2\theta + \mathrm{P}$$

$$(F_s)_{\max} = \text{Maximum strut compression force} = \frac{1}{2} tl\, f_m\left[\sqrt{1 + \left(\frac{h}{l}\right)^2} - \frac{h}{l}\right]$$

$$\tau_m = \frac{V - (F_s)_{\max}}{t(l - x'_o)} \qquad \text{where } x'_o = \frac{x_o}{\cos^2\theta}$$

$$\sigma_m = \tau_m(\tan\theta + \cot\theta)$$

The masonry stress is $\sigma_m \cot\theta$ and the steel stress is $\tau_m \tan\theta = \rho_x F_y$

Define

$$\tau_{\text{avg}} = \frac{V}{lt} = \text{applied horizontal shear stress}$$

$$f'_m = \text{maximum compressive stress in the masonry}$$

$$\rho_4 = \text{reinforcement ratio in the verticle direction (y)} = \frac{(A_s)_{\text{vert}}}{lt}$$

$$\rho_x = \text{reinforcement ratio in the horizontal (x) direction}$$

$$\rho_x = \frac{(A_s)_{\text{horiz}}}{ht}$$

$$F_{ys} = \text{yield stress of steel reinforcement}$$

The required vertical reinforcement is based on

$$\rho_4 F_{ys} = \tau_s \cot\theta$$

The required horizontal reinforcement is based on

$$\rho_x F_{ys} = \tau_s \tan\theta$$

Define $\Psi = \Phi_x = \rho_x \frac{F_{ys}}{f'_m}$

By combining the applied horizontal average shear stress with the maximum strut force equation, we may obtain a formulation of the maximum average stress

$$\frac{\tau_{\text{avg}}}{f'_m} = \frac{1}{2}\left[1 + \left(\frac{h}{l}\right)^2 - \left(\frac{h}{l}\right)\right] + \psi\left(\frac{h}{l}\right)$$

Where $\Psi = \rho_x \frac{F_{ys}}{f'_m}$

$$\therefore (\rho_{\text{avg}})_{\max} = \frac{(lt)f'_m}{2}\left[\sqrt{1 + \left(\frac{h}{l}\right)^2}\right] + tf'_m\,\psi(h)$$

if $\tau_m = f'_m$ then, $\rho_x F_{ys} = f'_m \sin^2\theta$

and $\sin\theta = \sqrt{\psi}$

therefore,

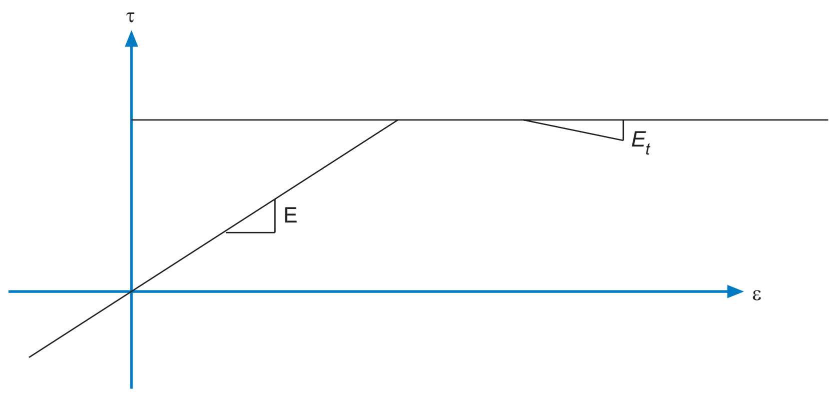
$$\frac{\tau_{avg}}{f'_m} = \sqrt{4(1-\psi)}$$

Finite element software with nonlinear elements

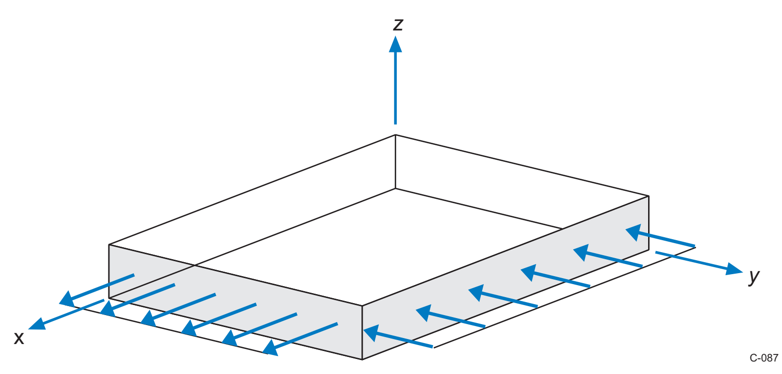
Four distinctive programs have nonlinear shear/plate element capabilities.

1. ADINA – Automatic Dynamic Incremental Nonlinear Analysis
2. COSMOS/M – Structural Analysis and Research Corporation
3. SAP2000 – Structural Analysis Program (produced by computers and Structures, Berkeley, CA
4. MSCNASTRAN – MacNeal-Schwendler Corporation

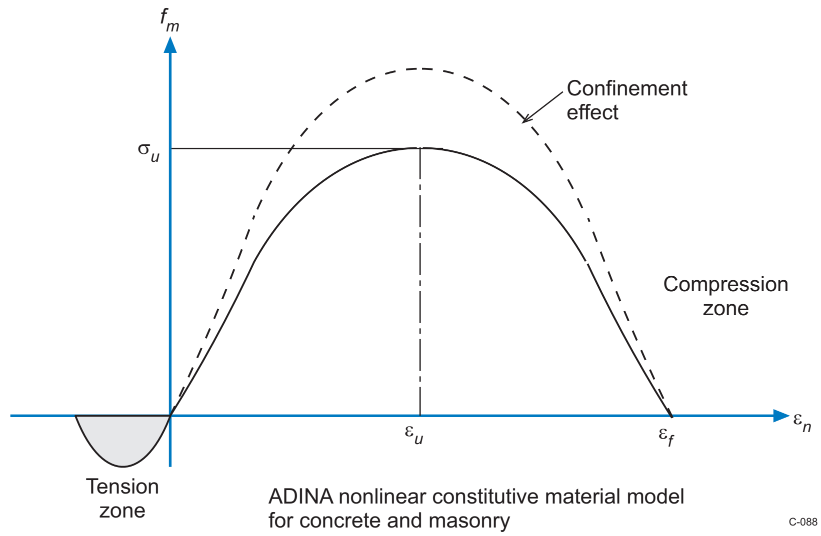
The essential nonlinear element is modeled with a bi-linear isotropic material model with strain hardening using E_t = tangent modules.



Bi-linear material/constitive model



By employing SAP2000 and COSMOS, the basic bilinear material model can be used for reinforced concrete and masonry. For a more sophisticated approach, ADINA has developed a nonlinear constitutive material model to simulate actual behavior with confinement effects, nonlinear strain hardening, and reinforcement.



ADINA nonlinear constitutive material model for concrete and masonry

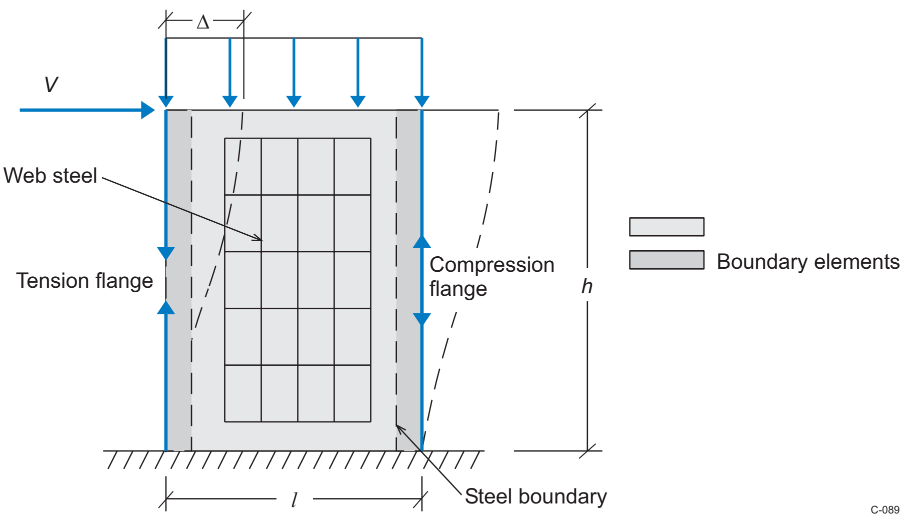
These programs all have three-dimensional nonlinear static and dynamic capabilities. The increasing computational power of PCs makes these tools available to practical engineering applications. By evaluating the existing nonlinear capacities of buildings, more efficient designs with greater practical value can be provided.

APPENDIX C2: Deflection Analysis of Structural Walls

Deflection analysis is part of every structural evaluation. This section presents the basic equations and assumptions behind the deflection calculations. Deflection analysis of shear walls is separated into two categories: in-plane, and out-of plane analysis. Both of these computations contribute to lateral drift that must be controlled within the parameters of the 2000 IBC. Both lateral drift and interstory drift are of concern in high-rise buildings and, if reinforced masonry shear walls form the structural system, the methodology of deflection analysis is important.

1. In-plane Deflection Analysis

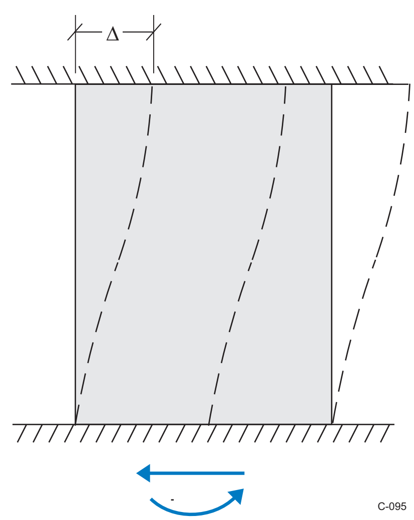
Deflection analysis of walls:



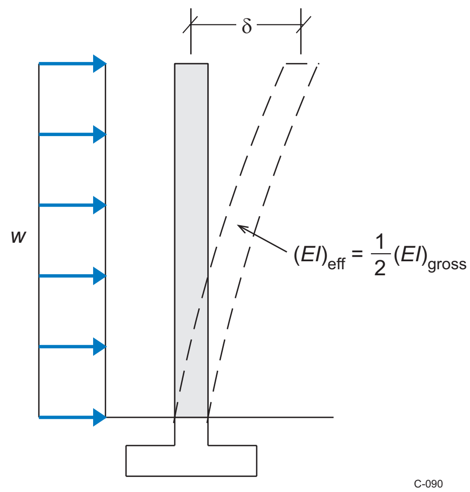
$$\Delta = \frac{Vl^3}{3(EI)_{eff}} + \frac{12V}{GA}$$

$(EI)_{eff} = \frac{1}{2}(EI)_{gross}$ Stiffness assumption

$$\Delta = \frac{Vl^3}{12(EI)_{eff}}$$



2. Out-of-plane Deflection Analysis

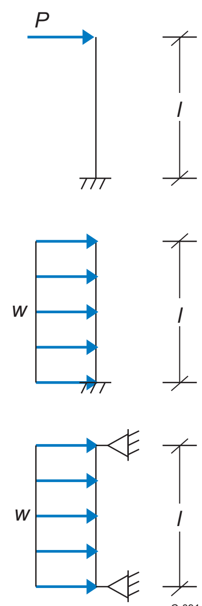


The out-of-plane deflection analysis assumes that the wall will behave like a cantilever beam fixed at the base.

$$\delta = \frac{Pl^3}{3EI} \text{ for concentrated load}$$

$$\delta = \frac{wl^4}{8EI} \text{ for uniform, load}$$

$$\delta = \frac{5wl^4}{384EI} \text{ for a simple support condition}$$



Out-of-plane analysis follows the same formulation as that for elastic beams with the exception that the stiffness degradation (i.e., cracking) must be accounted for by using an appropriate, effective *EI* value.

3. Finite Element Formulations

As in the analytical equations, finite element models may be used for calculating deflection and drift values by adjusting for the stiffness in the material model. This tends to over-predict the displacements but is reasonable for the purpose of controlling drift.

Remember:

1. The $^1/_2$ EI_{gross} assumption works well for deflection analysis. This may input directly into an elastic finite element model.
2. Clarify that the FEM for static stress analysis uses the full EI value, so as not to confuse the results from the deflection/drift analysis.
3. The deflection analysis is required to satisfy drift restrictions on masonry bearing wall buildings provided by the 2000 IBC, Table 1617.3.

4. IBC 2000 Deflection Amplification Factor

The 2000 IBC provides for stiffness degradation in a slightly different approach. Using the full gross stiffness properties, calculate the elastic defection and then multiply this value by C_d from Table 1617.6. The same requirement to satisfy Table 1617.3 still holds, but this is an equally conservative methodology.

Appendix C3: Deflection Analysis of Diaphragms

Diaphragm analysis facilitates the determination of the lateral drift contribution from diaphragm bending. The three fundamental classes of diaphragms are 1) Flexible, 2) Semi-rigid, and 3) Rigid. However, because there is no analysis protocol established for the semi-rigid category even through it does exist in the structural real world, the design professional is restricted to classifying diaphragms as either rigid or flexible. This appendix focuses on deflection analysis of the flexible diaphragm according to the 2000 IBC.

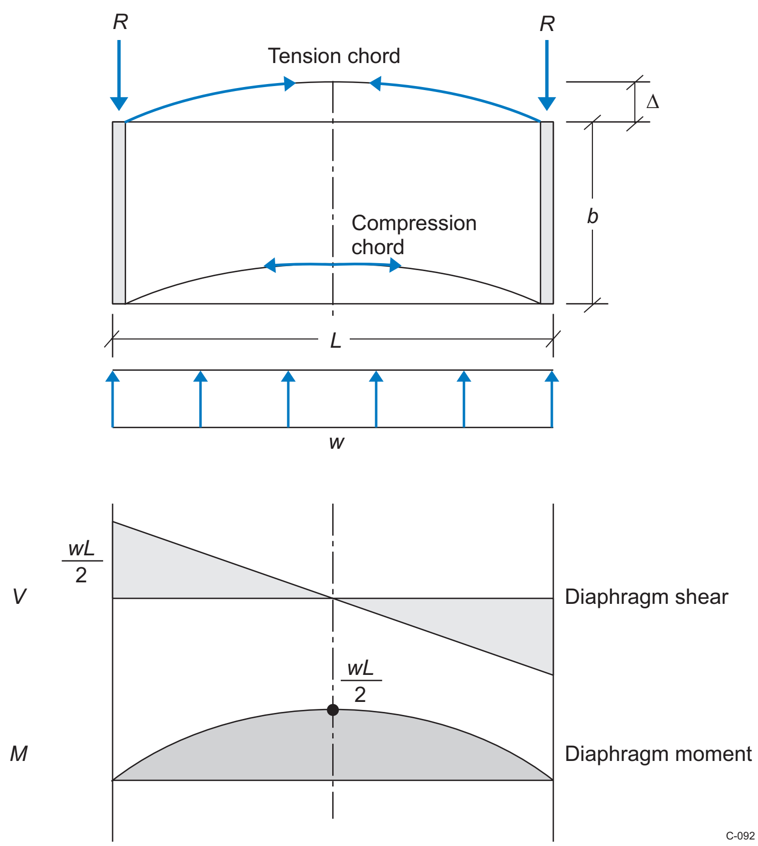
Rigid diaphragm analysis

The rigid diaphragm relies on the deflection response as a rigid plate section. Rigid diaphragms are usually constructed with reinforced concrete or steel frame systems with concrete decking. However, there are examples of wood frame diaphragms being classified as rigid; lightweight concrete is used in the deck material, which produces a rigid effect within the wood joists, or wood frame members are blocked and plywood sheathing with close nail spacing is used to create the effect of a rigid plate. Thus the deflection analysis is a direct function of the relative stiffness values of the shear walls. This requires a rigid diaphragm torsional analysis to compute rotation values and deflections. This calculation is performed by using a three-dimensional finite element model. Examples of this application are shown in Chapters 4 and 5.

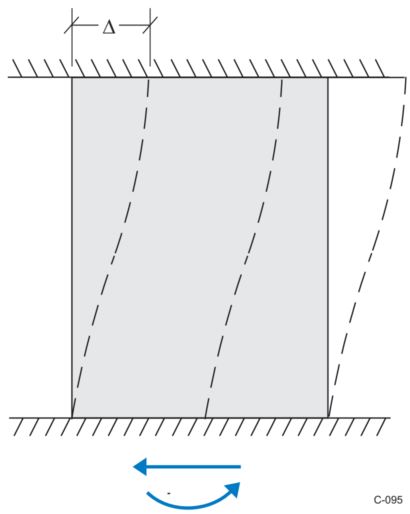
Because rigid diaphragms do not deflect in plane, the key issue is diaphragm rotation, which depends on the supporting shear wall elements.

Flexible diaphragm analysis

A flexble diaphragm behaves like a simple beam. The shear and moment diagrams are shown here.



Free-body diagram – moment resistance from force-couple system



$$M = \frac{wl^2}{8} = T_b$$

$$\therefore\ T = \frac{wl^2}{8b} = \text{Tension chord force}$$

Diaphragm deflection; 2000 IBC; 2305.2.2

$$\Delta = \frac{5vL^3}{8EAb} + \frac{vL}{4Gt} + 0.188Le_n + \frac{\Sigma(\Delta_c X)}{2b} \qquad \textbf{(EQ. 23-1)}$$

For SI: $$\Delta = \frac{0.052L^3}{EAb} + \frac{vL}{4Gt} + \frac{Le_n}{1627} + \frac{\Sigma(\Delta_c X)}{2b}$$

Where:

A = Area of chord cross section, in square inches (mm^2)

b = Diaphragm width, in feet (mm)

E = Elastic modulus of chords, in pounds per square inch (N/mm^2)

e_n = Nail deformation, in inches (mm)

G = Modulus of rigidity of wood structural panel, in pounds per square inch (N/mm^2)

L = Diaphragm length, in feet (mm)

t = Effective thickness of wood structural panel for shear, in inches (mm)

v = Maximum shear due to design loads in the direction under consideration, in pounds per lineal foot (N/mm)

Δ = The calculated deflection, in inches (mm)

$\Sigma(\Delta cX)$ = Sum of individual chord-splice values on both sides of the diaphragm, each multiplied by its distance to the nearest support

2305.2.3 Diaphragm aspect ratios. Size and shape of diagrams shall be limited as set forth in Table 2305.2.3.

Table 2305.2.3
MAMIMUM DIAPHRAGM DIMENSION RATIOS HORIZONTAL AND SLOPED DIAPHRAGM

TYPE	MAXIMUM LENGTH - WIDTH RATIO
Wood structural panel, nailed all edges	4:1
Wood structural panel, blocking omitted at intermediate joints	3:1
Diagnol sheathing, sizing	3:1
Diagnol sheathing, double	4:1

Example: wood diaphram

$E = 1800$ ksi for chords 4 x 6, D-F

$A_{\text{chord}} = 19.25 \text{ in}^2$

Calculate diaphram deflection:

1. $A_{\text{chord}} = 19.25 \text{ in}^2$
2. $b = 30$ ft (width) for force in the transverse direction
3. $E = 1{,}800{,}000$ psi
4. $e_n = 0.10$ inch (assumed)

   This may require a special calculation if the nail is shallow penetration with high seismic forces.
5. $\Sigma(\Delta c X) = 1 \text{ in } (15 \text{ ft}) + 1 \text{ in } (15 \text{ ft}) = 30$ ft-in

   $2b = 60$ ft

   $$\therefore \frac{\Sigma(A_c X)}{2b} = \underline{0.5 \text{ in}}$$
6. $$v = \frac{wL}{2b} = \frac{200 \text{ \#/ft } (50')}{2(30')} = 167 \text{ \#/ft}$$
7. $$G = \frac{E}{2(1+v)} = 692 \text{ psi} \qquad (v = 0.3)$$

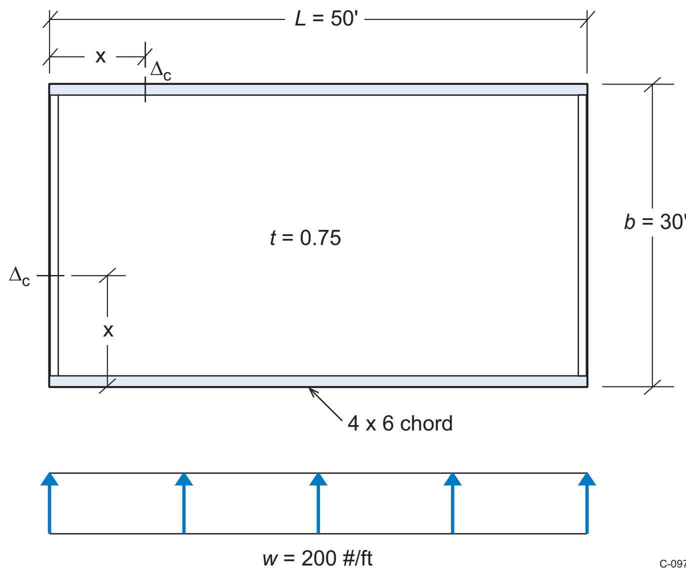
$$\therefore \Delta = \frac{5(167)(50)^3}{8(1{,}800{,}00)(19.25)(30)} + \frac{(167)(50)}{4(692\text{psi})(0.75)}$$

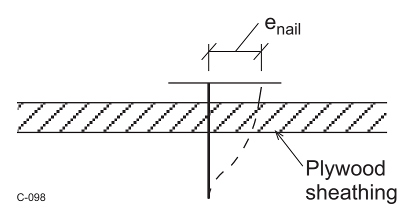
$$+ 50\ (0.10)(1.88) + 0.5 \text{ in}$$

$$\therefore \Delta = 0.013 + 4.0 + 0.94 + 0.5 = \underline{5.45 \text{ in}}$$

$$\frac{L}{\Delta} = \frac{50(12)}{5.45} = 110 < 180$$

(GOOD)





Evaluation of the Rigid versus the Flexible Diaphragm Analysis

Prevailing practice treats all wood frame diaphragms as flexible, but certain issues arise in California that make this practice questionable; whether the wood frame diaphragm is blocked or has lightweight concrete decking is of primary importance. For a thorough discussion involving this topic, consult the *Structural Engineers Association of Southern California Recommended Lateral Force Requirements* (i.e., Blue book), also the *Seismic Design Handbook* (volumes 1 and 2), published by ICBO and SEAOC, which contains specific commentary on this subject.

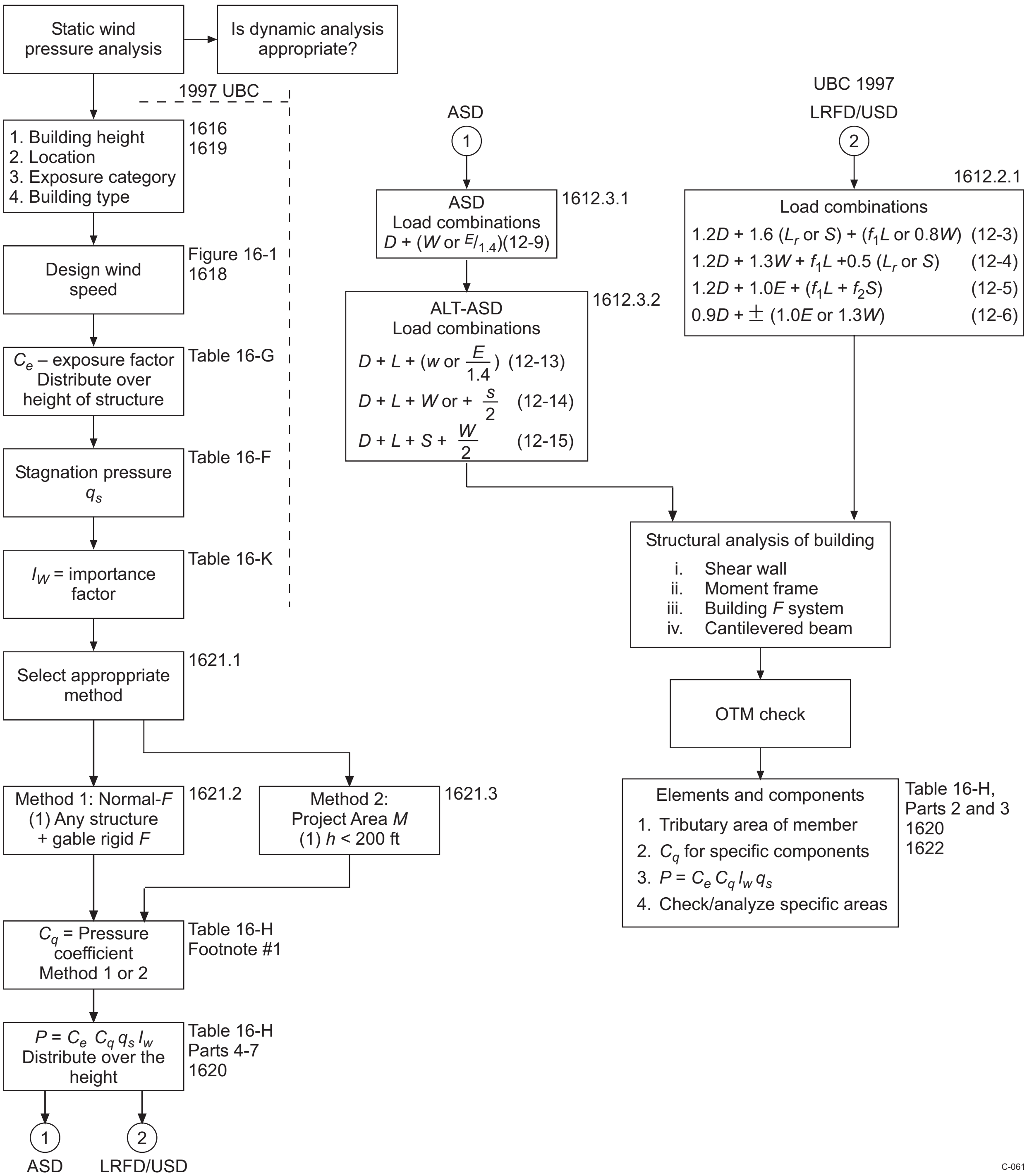
Appendix D

@Seismicisolation

Division III – Wind Load Design

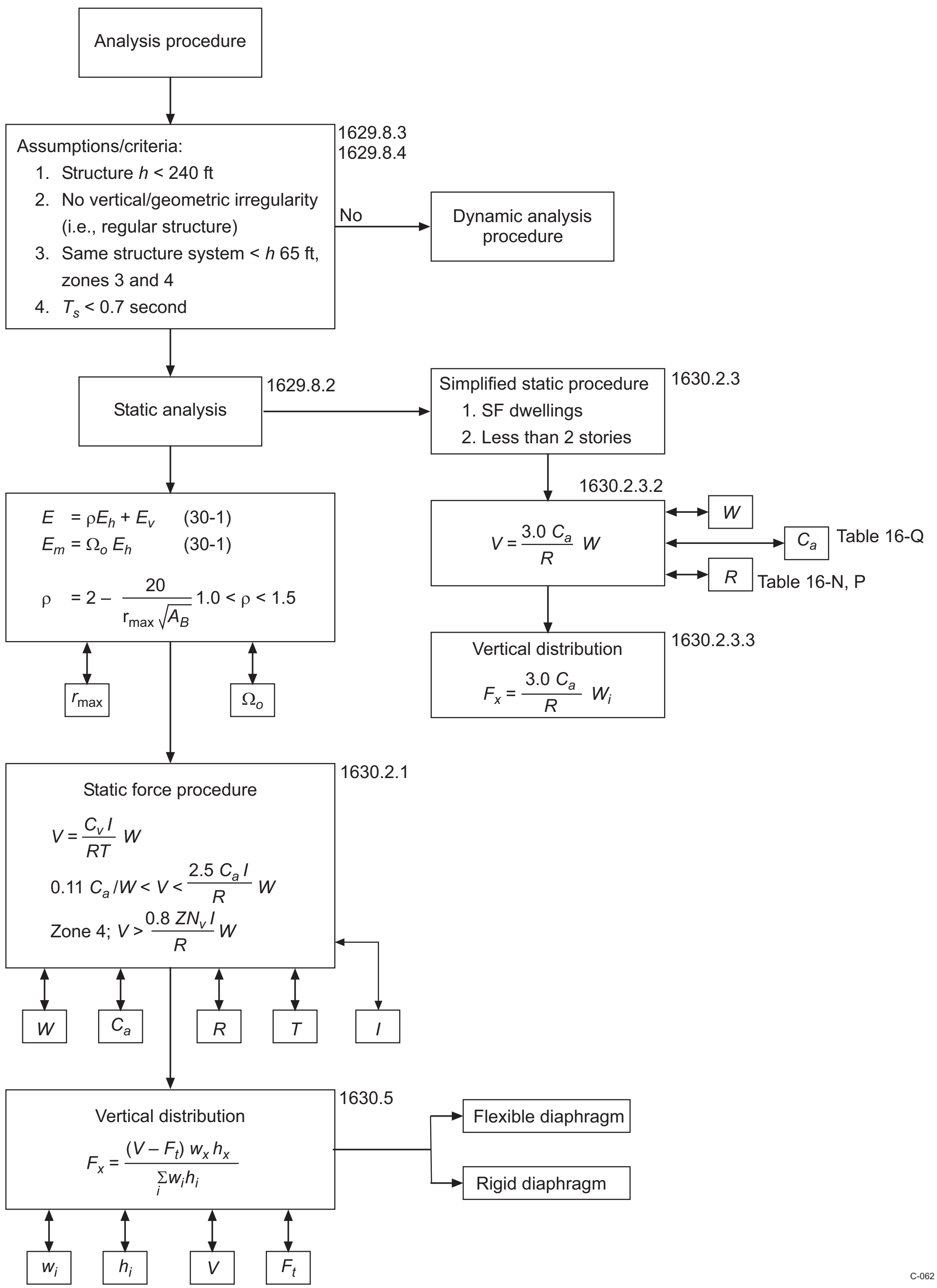
Design/analysis flowchart

$P = C_e\ C_q\ q_s\ I_w$



Appendix D2

Division IV – Earthquake Load Design



Appendix D3

Design Flowchart for Wind Loads
IBC 2000 (Based on ASCE 7-98)

1. Building/structure type

The first step is to select the correct method to calculate the wind forces

Based on the type of structure, and specific provisions for each calculation procedure in the IBC 2000.
There are three mehtods:
- Method 1: Simplified procedure
- Method 2: Analytical procedure
- Method 3: Wind tunnel procedure

→

2. Select method of calculation

If the building is less than 60 ft, it may classify for Method 1: Simplified procedure.

IBC 2000, 1609.1:
- Method 1: ASCE 7-98, 6.4
- Method 2: ASCE 7-98, 6.5
- Method 3: ASCE 7-98, 6.6

→

3C. Method 3: Wind tunnel testing

Complex wind tunnel testing that will require a prototype of the structure to be constructed. This method is for highly sophisticated structures entailing site-specific wind analysis using aerodynamic models.

Method 3 is not specifically covered in this flowchart because of its complexity.

3A. Method 1

Based on a simplified analytical procedure.

↓

Method 1

Step 1: Site location for *V*

Select wind speed *V*
2000 IBC, Figure 1609
ASCE 7-98, 6.5.4

↓

Step 2: Importance factor

Select *Iw*
2000 IBC, Table 1604.5
ASCE 7-98, 6.5.5

↓

Step 3: Exposure category

Select the correct exposure class for the building's site topography. 200 IBC, 1609.4
ASCE 7-98, 6.5.6

↓

Step 4: Enclosure classifiction

Is the structure open, closed, or partially enclosed?

2000 IBC, 1609.2
ASCE 7-98, 6.5.9

↓

Step 5: Select the loads for the MWFRS or cladding

Design for the Main Wind Force Resisting System (MWFRS) or components and cladding?

→ C

↓

MWFRS

↓

A

3B. Method 2

Analytical procedure that requires the calculation of site-specific parameters.

This procedure is detailed in ASCE 7-98 (adopted by reference in the 2000 IBC)

↓

Method 2

Step 1: Site location for *V* and wind directionality factor *kd*

Select wind speed *V*
ASCE7-98, 6.5.4
kd from Table 6-6

↓

Step 2: Importance factor

Select *Iw*
2000 IBC, Table 1604.5
ASCE 7-98, 6.5.5

↓

Step 3: Exposure category

Select the correct exposure class for the building's site topography.
2000 IBC, 1609.4
ASCE 7-98, 6.5.6

↓

Step 4: Topographic factor *kzt*

Topograph factor accounts for site physical geometry, such as hills, ridges, or abrupt changes in elevation. ASCE 7-98, 6.5.7
Figure 6-2

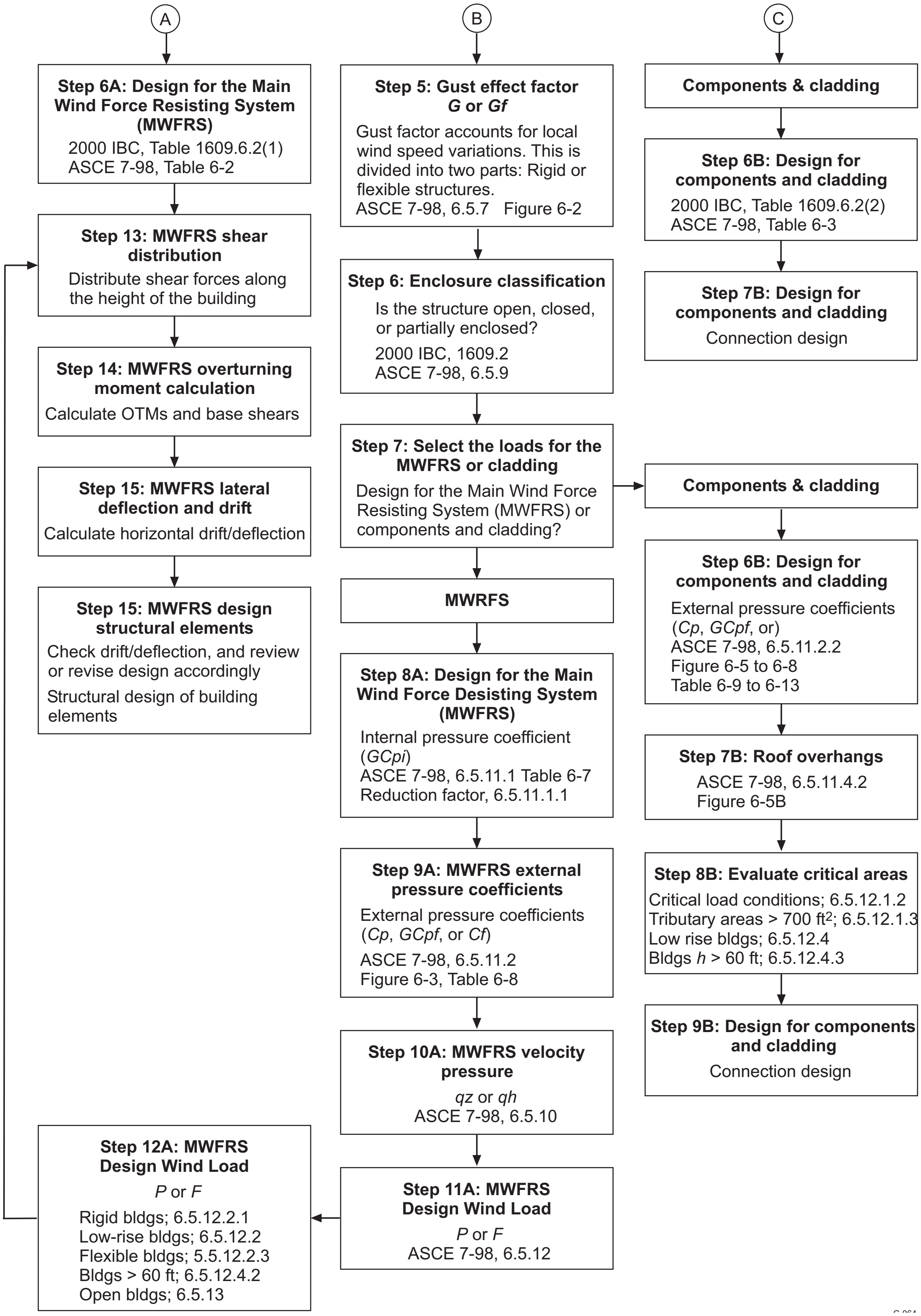
↓

B

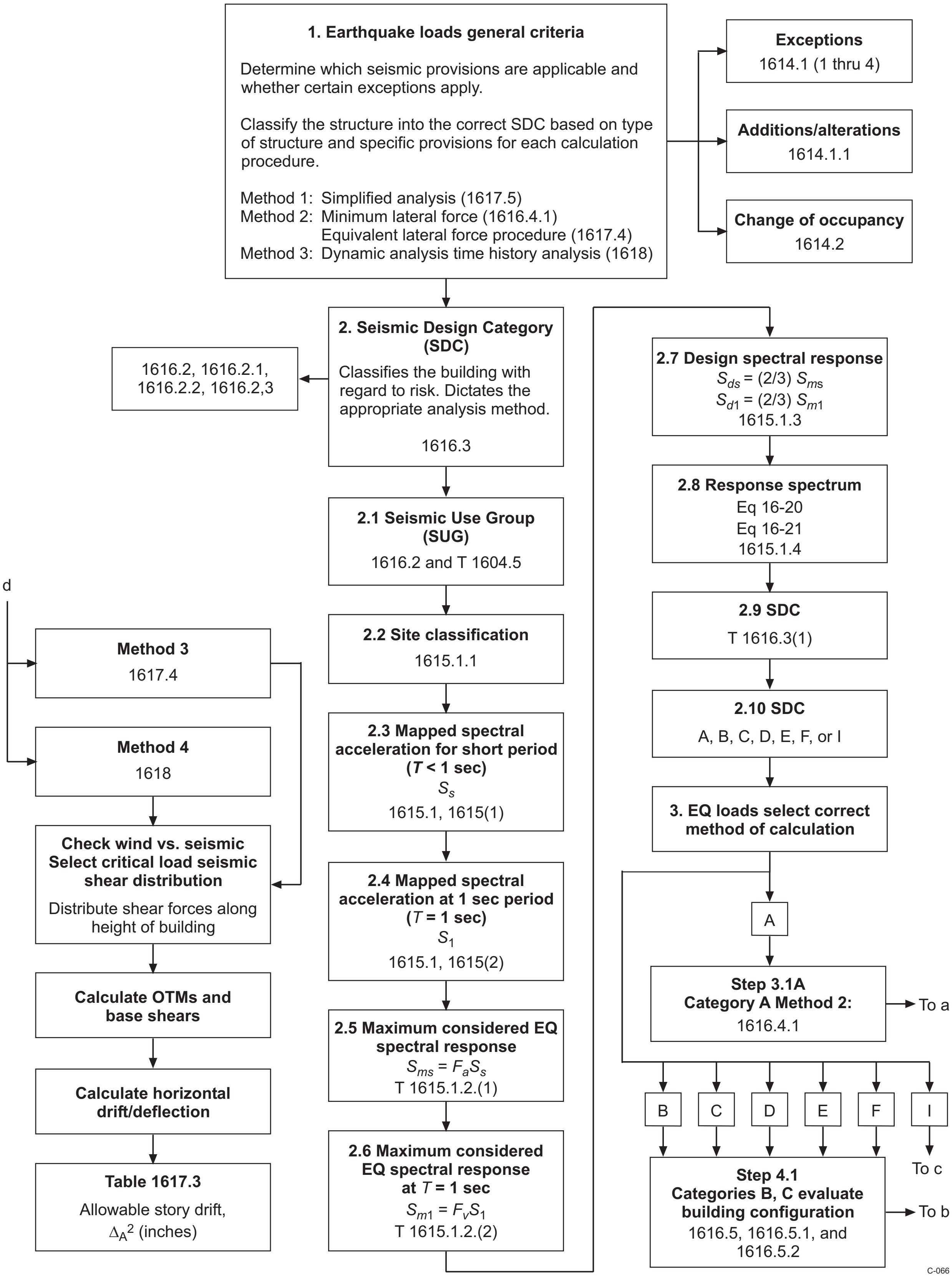
C-063

(continued)

Design Flowchart for Wind Loads
IBC 2000 (Based on ASCE 7-98) (cont'd)



Design Flowchart for Seismic Loads, 2000 IBC



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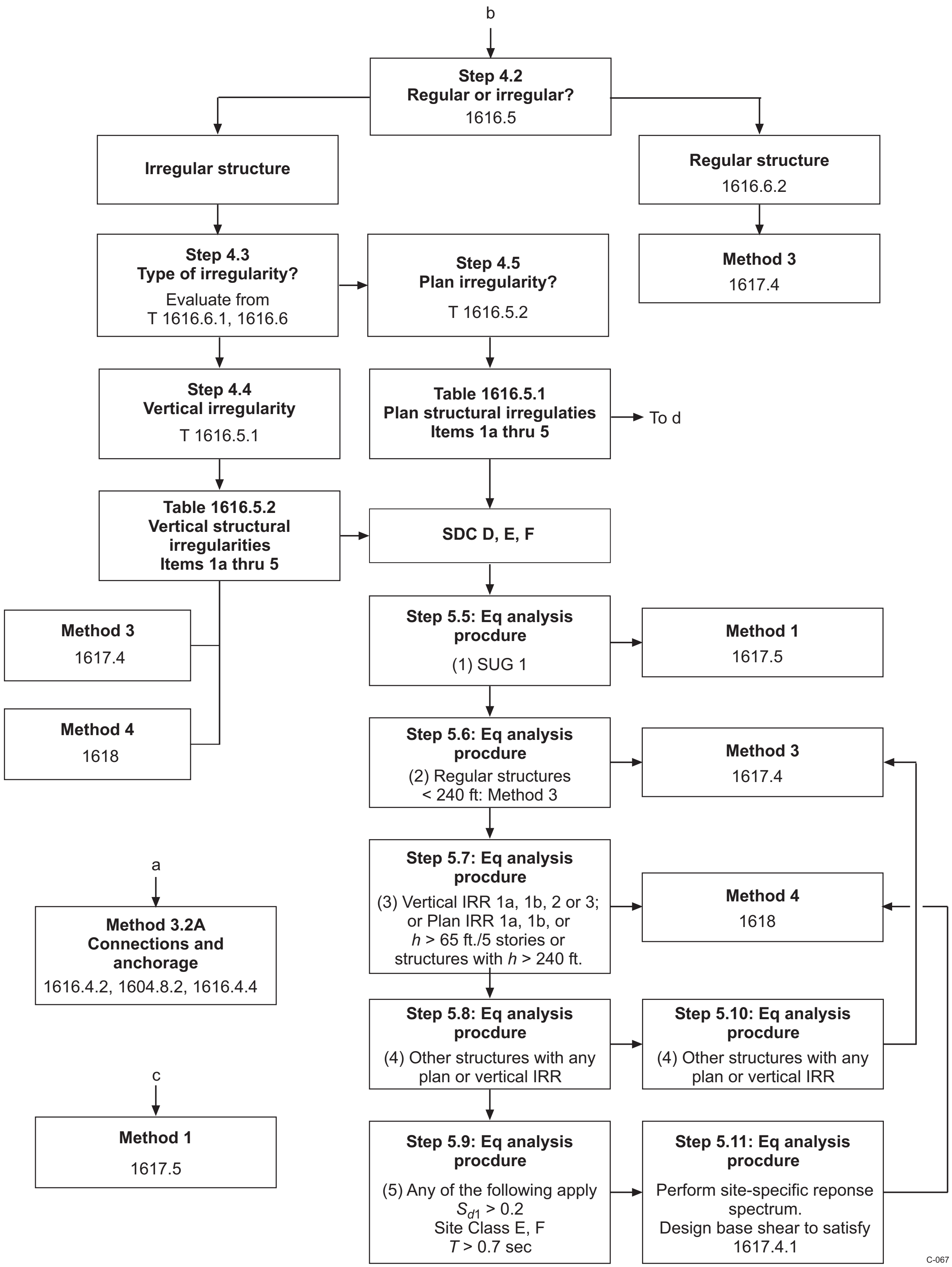
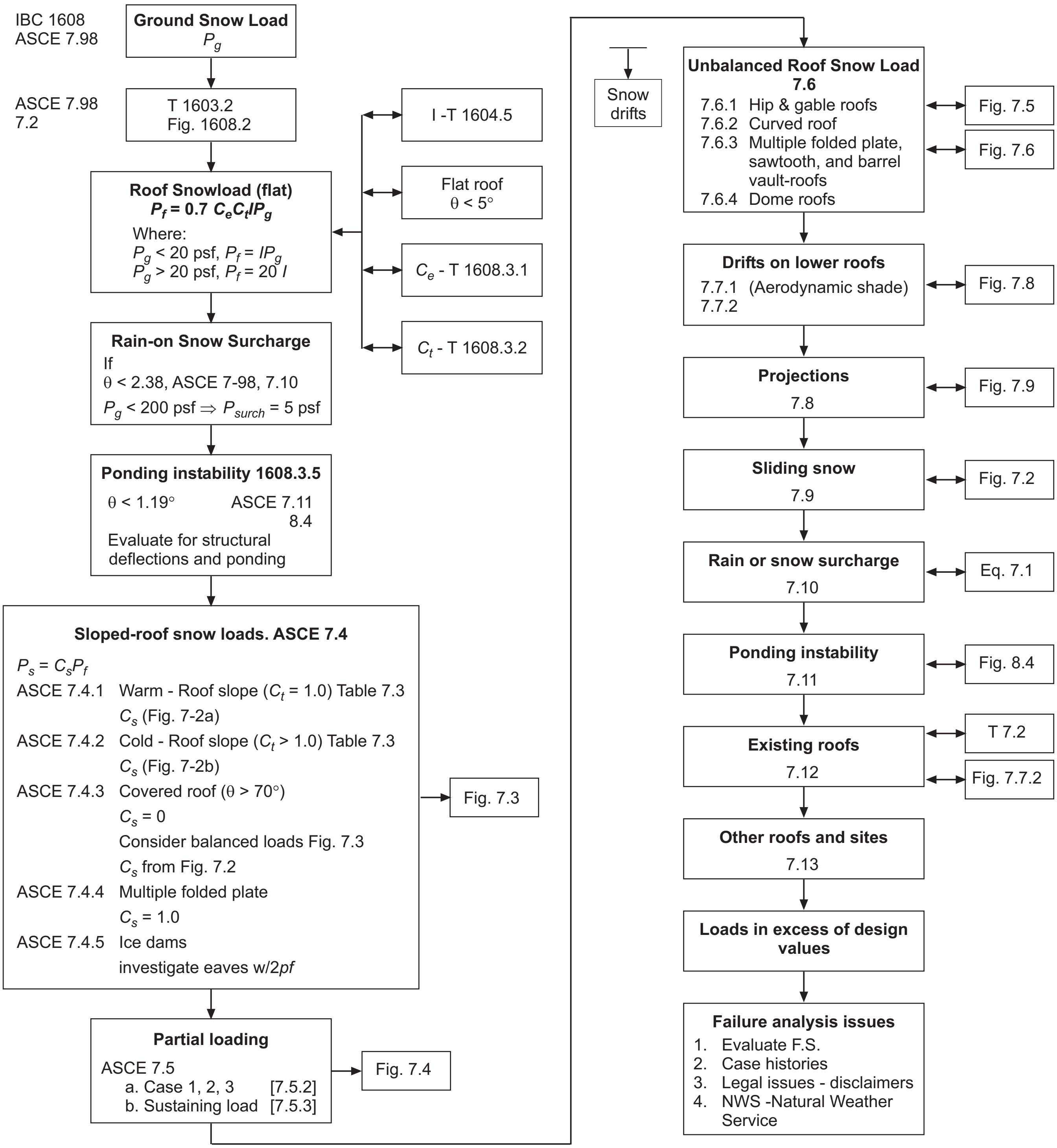
Design Flowchart for Seismic Loads, 2000 IBC (Cont'd)



Snow Load Design
IBC-2000



All referenced text pertains to 2000 IBC, 1608
All referenced figures pertain to ASCE 7-98

C-068

(Continued)

Appendix D6

Working Stress Design Equations

Type of stress	Allowable stress design	Code ref. UBC '97	Code ref. IBC 2000
Tensile stress in steel reinforcment	Deformed bars, $F_s = 0.5F_y$, 24 ksi max. Wire reinforcement, $F_s = 0.5F_y$, 24 ksi max. Ties, anchors, and smooth bars $F_s = 0.4F_y$, 20 ksi max.	2107.2.11	
Compression stress in steel reinforcing	Deformed bars in column, $F_{sc} = 0.4F_y$, 24 ksi max. Deformed bars in flexural members, $F_s = 0.5F_y$, 24 ksi max. Confined, deformed bars in shear walls, $F_{sc} = 0.4F_y$, 24 ksi max.	2107.2.11	
Modulus of elasticity	Reinforcing steel, $E_s = 29{,}000{,}000$ psi. Masonry (clay), $E_m = 700f'_m$, 3,000,000 psi max. Grout, $E_g = 500f_g$ $E_m = 900f'_m$, for concrete masonry.	2106.2.12.1 $E_m = 700f'_m$	
Shear modulus	$G = 0.4\,E_m$ (UBC) $E_v = 0.4\,E_m$ (ACI/ASCE)	2106.2.13	
Axial compressive stress	UBC: Members other than columns ACI/ASCE: walls or columns, when $h'/r \le 99$ $F_a = 0.25f'_m\left[1-\left(\frac{h'}{140r}\right)^2\right]$ when $h'/r > 99$, $F_a = 0.25f'_m\left(\frac{70r}{h'}\right)^2$ UBC: For reinforced masonry columns, when $h'/r \le 99$ $P_a = (0.25f'_m A_e + 0.65A_s F_{se})\left(\frac{70r}{h'}\right)^2$	2107.25	
Flexural compressive stress	$F_a = 0.33f'_m$ (UBC limits F_b to a maximum of 2000 psi)	2107.2.6	
Combined compressive stresses (Unity equation)	$\frac{f_a}{F_a} + \frac{f_b}{F_b} \le 1$	2107.2.8	
Shear stress	Flexural members without shear reinforcing, $F_v = 1.0\sqrt{f'_m}$ $F_{v(\max)} = 50$ psi Flexural members will reinforce steel carrying shear forces, $F_v = 3.0\sqrt{f'_m}$ $F_{v(\max)} = 150$ psi	2107.2.8	

(Continued)

Working Stress Design Equation (Cont'd)

Type of stress	Allowable stress design	Code ref. UBC '97	Code ref. IBC 2000
Shear stress − shear walls	Shear walls with masonry designed to carry all the shear force, When $\frac{M}{Vd} <, F_v = {}^1/_3\left[4 - \left(\frac{M}{Vd}\right)\right]\sqrt{f'_m}$ $F_{v(\max)} = 80 - 45\left(\frac{M}{Vd}\right)$ psi When $\frac{M}{Vd} \geq 1, F_v = 1.0\sqrt{f'_m}$ $F_{v(\max)} = 35$ psi Shear walls with reinforcing steel designed to carry all the shear force, When $\frac{M}{Vd} < 1, F_v = {}^1/_2\left[4 - \left(\frac{M}{Vd}\right)\right]\sqrt{f'_m}$ $F_{v(\max)} = 120 - 45\left(\frac{M}{Vd}\right)$ psi When $\frac{M}{Vd} \geq 1, F_v = 1.5\sqrt{f'_m}$ $F_{v(\max)} = 75$ psi	2107.2.9	
Bearing stress	One full area, $F_{br} = 0.26\sqrt{f'_m}$ On one-third area or less, $F_{br} = 0.38\sqrt{f'_m}$ ForACI/ASCE Code, $F_{br} = 0.25\sqrt{f'_m}$	2107.2.10	
Tension on embedded anchor bolts	The lesser of, $B_t = 0.5A_p\sqrt{f'_m}$ $B_t = 0.2A_b f_y$ (Note ACI/ASCE uses B_a to denote the allowable bolt tensile capacity)	2107.1.5.1	2108.6.3 $B_v = 4\phi A_{pr}\sqrt{f'_m}$ (21-15)
Shear on embedded anchor bolts	The lesser of, $B_v = 350A_p\sqrt[4]{f'_m A_b}$ $B_v = 0.12A_b f_y$	2107.5.3	
Combined shear and tension on anchor bolts	$\frac{b_t}{B_t} + \frac{b_y}{B_v} \leq 1.0$ (Note ACI/ASCE uses B_a to denote the allowable bolt tensile capacity)	2107.1.5.4	2108.6.4

Appendix D7

Working Stress Analysis Equations

Item	Design Formula
Modular ratio, n	$n = \frac{E_s}{E_m}$
Tension steel reinforcing ratio, p	$p = \frac{A_s}{bd} = \frac{K}{f_s j}$
Area of tension steel, A_s	$A_s = pbd = \frac{M}{f_s jd} = \frac{T}{f_s}$
Compression steel reinforcing ratio, p'	$p' = \frac{K - K_b}{(2n-1)\left(\frac{k - \frac{d'}{d}}{k}\right)\left(1 - \frac{d'}{d}\right)} = \frac{A'_s}{bd}$
Area of compression steel, A'_s	$A'_s = \frac{M - kF}{cd}$
Perimeter of circular reinforcing, bar Σ_o	$\Sigma_o = \pi d$
Moment capacity of the masonry, M_m	$M_m = F_b kjbd^2 = kbd^2$
Moment capacity of the tension steel	$M_s = F_s A_s jd = kbd^2$
Flexural coefficient, K	$K = \frac{1}{2} F_b kj = \frac{M}{bd^2} = f_s pj$
Coefficient, k	For members with tension steel only, $k = \sqrt{(np)^2 + 2np} - np$ $k = \frac{1}{1 + \frac{f_s}{nf_b}}$ Member with tension and compression reinforcement, $k = \sqrt{[np + (2n-1)]^2 + 2(2n-1)p'\frac{d'}{d}}$ $-[np + (2n-1)p']$
Coefficient, j	Members with tension steel only. $j = 1 - \frac{k}{3}$ members with tension and compression steel, $j = 1 - \frac{z}{3}$
Coefficient, z	$z = \frac{\frac{1}{6} + \frac{(2n-1)A'_s}{kbd} \times \left(1 - \frac{d'}{kd}\right)}{\frac{1}{2} + \frac{(2n-1)A'_s}{kbd} \times \left(1 - \frac{d'}{kd}\right)}$

(Continued)

(Cont'd)

Item	Design Formula
Resultaant compression force, C	$C = \frac{1}{2} f_b kdb$
Resultant tension force, T	$T = A_s f_s$
Tension steel stress, f_s	$f_s = \frac{M}{A_s jd}$
Compression steel stress, f_{sc}, f'	$f_{sc} = 2nf_b\left(\frac{kd - d'}{kd}\right)$
Masonry stress, f_b	$f_b = \frac{2M}{bd^2 jk} = \frac{2}{jk}K$
Shear stress, f_v or v, for beams and shear walls	$f_v = \frac{V}{bjd} = \frac{V}{bd}$ or $\frac{V}{bl}$
Spacing of shear steels, s	$s = \frac{A_v F_s d}{V}$
Shear strength provided by the reinforcing steel, F_v	$F_v = \frac{A_v F_s}{bjs}$ or conservatively, $F_v = \frac{A_v F_s}{bs}$
Area of shear steel, A_v	$A_v = \frac{Vs}{F_s d}$
Bond stress, u	$u = \frac{V}{\Sigma_o jd}$
Effective height to thickness reduction factor, R	$R = \left[1 - \left(\frac{h'}{140r}\right)^2\right], \frac{h'}{r} \leq 99; \left(\frac{70r}{h'}\right), \frac{h'}{r} > 99$
Interaction of load and moment	$\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.00$ or 1.33 $f_v = \left(1 - \frac{f_a}{F_a}\right)F_b$ $f_b = 0.33f'_m$ $f_a = \frac{P}{Ae} = \frac{P}{bd}$ $f_m = f_a + f_b$ $k_d = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $a = \frac{1}{6}tfm$ $b = \frac{1}{2}tfmd$ $c = p\left(\frac{1}{2} - d_1\right) + M$

Appendix D8

Strength Design Equations

Type of stress	Design formula	UBC 1997	IBC 2000
Limiting vertical stress for slender wall design	$\frac{P_w + P_f}{A_g} \le 0.04f'_m$	2108.2.4.4 (8-19)	2108.9.4.4 (21-32)
Maximum reinforcement ratio	$\rho_{max} = 0.5\bar{\rho}_b$ For IBC 2000, Refer to Chapter 5	2108.2.4.2 N/A	N/A 2108.9.2.13 Method A Method B
Total factored loads	$1.4D$ $1.2D + 1.6L + 0.5\ (L_r \text{ or } S \text{ or } R)$ $1.2D + 1.6\ (L_r \text{ or } S) + (f_1L \text{ or } 0.8W)$ $1.2D + 1.3W + f_1L + 0.5\ (L_r \text{ or } S)$ $1.2D + 1.0E + (f_1L + f_2S)$ $0.9D \pm (1.0E \text{ or } 1.3W)$ Where: f_1 = 1.0 for floors in places of public assembly, for live loads in excess of 100 psf (4.9 kN/m^2), and for garage live load. = 0.5 for other live loads. f_2 = 0.7 for roof configurations (such as saw tooth) that do not shed snow off the structure. = 0.2 for other roof configurations. **Exceptions:** 1. Factored load combinations for concrete per Section 1909.2. where load combinations do not include seismic forces. 2. Factored load combinations of this section multiplied by 1.1 for concrete and masonry where load combinations include seismic forces. 3. Where other factored load combinations are specifically required by the provisions of this code. **1612.2.2 Other loads.** Where *F*, *H*, *P* or *T* are to be considered in design, each applicable load shall be added to the above combinations factored as follows: 1.3*F*, 1.6*H*, 1.2*P* and 1.2*T*.	1612.2.1 (12-1) (12-2) (12-3) (12-4) (12-5) (12-6)	1605.2.1 (16-1) (16-2) (16-3) (16-4) (16-5) (16-6)
E_m = modulus of elasticity	$E_m = 700f'_m$ for clay masonry $E_m = 900f'_m$ for concrete masonry	2106.2.1.12.1 (6-30 (6-4)	2108.7.2
E_{mu} = maximum masonry strain	$E_{mu} = 0.025$ for concrete masonry $E_{mu} = 0.035$ for clay masonry		2108.9

$\rightarrow \frac{P_{uw} + P_{uf}}{A_g} \le 0.05f'_m$

(Continued)

Strength Design Equations (Cont'd)

Type of stress	Design formula	UBC 1997	IBC 2000
M_u = factored moment at midheight of slender wall	$M_u = \frac{Wuh^2}{8} + P_{uf}\left(\frac{e}{2}\right) + P_u\Delta_u$	2108.2.4.4 (8-21)	2108.9.4.4 (21-34)
P_u = factored axial load at midheight of slender wall	$P_u = P_{uw} + P_{uf}$	2108.2.4.4 (8-21)	2108.9.4.4 (21-34)
Limiting moment equation	$M_u = \phi M_n$	(8-22)	(21-35)
M_u = nominal moment strength	$M_n = A_{se}F_f\left(d - \frac{a}{2}\right)$ where $A_{se} = (A_sf_y + P_u)/f_y$	(8-23)	(21-36)
a = depth of stress block	$a = \frac{P_u + A_sF_y}{0.85f'_mb}$	(8-25)	(21-38)
$(A_s)_{\max}$ = midheight deflection limit for slender walls	$A_s = 0.007h$ maximum	2108.2.4.6 (8-27)	2108.9.4.6 (21-39)
Δ_s = calculate midheight deflection	$M_{ser} \le M_{cr}$, $\Delta_s = \frac{5M_sh^2}{48E_mI_g}$ $M_{cr} < M_{ser} < M_n$, $\Delta_s = \frac{5M_sh^2}{48E_mI_g} + \frac{5(M_{ser} - M_{cr})h^2}{48E_mI_{cr}}$	(8-28) (8-29)	(21-40) (21-41)
Cracking moment strength M_{cr}	$M_{cr} = Sf_r$	(8-30)	(21-42)
Modulus of rupture, f_r	Fully grouted hollow unit masonry, $f_r = 4.0\sqrt{f'_m}$, 235 psi max Partially grouted hollow unit masonry, $f_r = 2.5\sqrt{f'_m}$, 125 psi max Fully grouted two-wythe brick masonry, $f_r = 2.0\sqrt{f'_m}$, 125 psi max	(8-31) (8-32) (8-33)	2108.7.5 (21-21) (21-20) (21-19)
Nominal balanced axial strength, P_b	$P_b = 0.85f'_m\,ba_b$	(8-2)	(21-4)
Balanced depth stress block, a_b	$a_b = 0.85\left[\frac{e_{mu}}{e_{mu} + \frac{f_y}{E_s}}\right]d$	(8-3)	(21-5)
Nominal axial strength of a shear wall supporting only axial loads, P_o	$P_o = 0.85f'_m\,(A_e - A_s) + f_sA_s$	2108.2.5.4 (8-34)	2108.9.5.4 (21-43)
Nominal shear strength of a shear wall, V_n	$V_n = V_m + V_s$ or $V_m = A_{mv}\rho_nf_y$	2108.2.5.5 (8-36)	2108.9.3.5.2 (21-27)

Appendix D9

Strength Design Equations

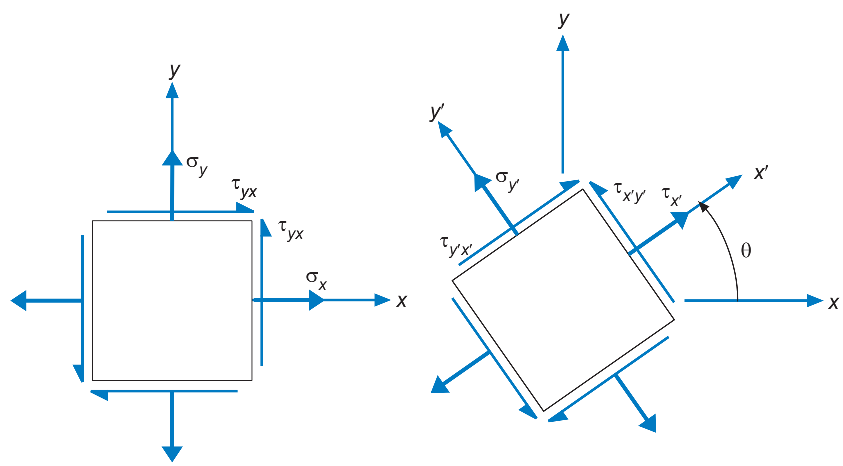
Type of stress	Design formula	UBC 1997	IBC 2000
Nominal shear strength provided by the masonry, V_m	$V_m = C_d A_{mv} (f'_m)^{\frac{1}{2}}$	2108.2.5.5 (8-37)	2108.9.3.5.2.1 (21-30)
Nominal shear strength provided by the shear reinforcement, V_s	$V_s = A_{mv} \rho_n f_y$	2108.2.5.5 (8-39)	2108.9.3.5.2.2 (21-31)
Reinforcing steel raatio, p	$\rho = \frac{A_s}{bd}$		
Area of tension steel, A_s	$a = \frac{pbdf_{uy}}{0.85 f'_m} = 0.85c$		
Coefficient, a			
Coefficient, a_b	$A_b = 0.85 c_b$		

$$\rightarrow V_m = \left[4.0 - 1.75\left(\frac{M}{vd_v}\right)\right] x A_n \sqrt{f'_m} + 0.25P$$

$$\rightarrow V_s = 0.5\left(\frac{A_v}{s}\right) f_r d_r$$

Plane stress and Mohr's circle

Plane stress formulation



(a)
Initial element

(b)
Transformed element

$$\sigma_z \quad = \tau_{yz} = \tau_{zx} = 0$$

From Budynas and Timoshenko, and using two-dimensional stress-strain transformations:

$$\sigma_{x'} \quad = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\cos\theta\sin\theta \qquad \textbf{Eq (1)}$$

$$\sigma_{y'} \quad = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy}\sin\theta\cos\theta \qquad \textbf{Eq (2)}$$

$$\tau_{x'y'} \quad = -\left(\sigma_x - \tau_y\right)(\sin\theta)(\cos\theta) + \tau_{xy}\left(\cos^2\theta - \sin^2\theta\right) \qquad \textbf{Eq (3)}$$

Let $\sigma = \sigma_{x'}$ = normal stress along the $_{x'}$ face

$\tau = \tau_{x'y'}$ = shear stress along the $_{x'}$ face

Using the trigonometric identities

$$\cos^2\theta = -\frac{1+\cos^2\theta}{2},\ \sin^2\theta = \frac{1-\cos^2\theta}{2}$$

$$\sin\theta\cos\theta = \frac{1}{2}\sin^2\theta,\ \cos^2\theta - \sin^2\theta = \cos^2\theta$$

Equations (1) and (3) are formulated as

$$\sigma \quad = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \qquad \textbf{Eq (4)}$$

$$\tau \quad = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \qquad \textbf{Eq (5)}$$

Equations (4) and (5) may be combined as follows

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$$\left[\sigma - \left(\frac{\sigma_x + \sigma_y}{2}\right)\right]^2 = \left[\frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \sigma_{xy}\sin 2\theta\right]^2$$

$$\tau^2 = \left[\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \sigma_{xy}\cos 2\theta\right]^2$$

Add

$$\left[\sigma - \frac{\sigma_x + \sigma_y}{2}\right]^2 + \tau^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2 \qquad \textbf{Eq (6)}$$

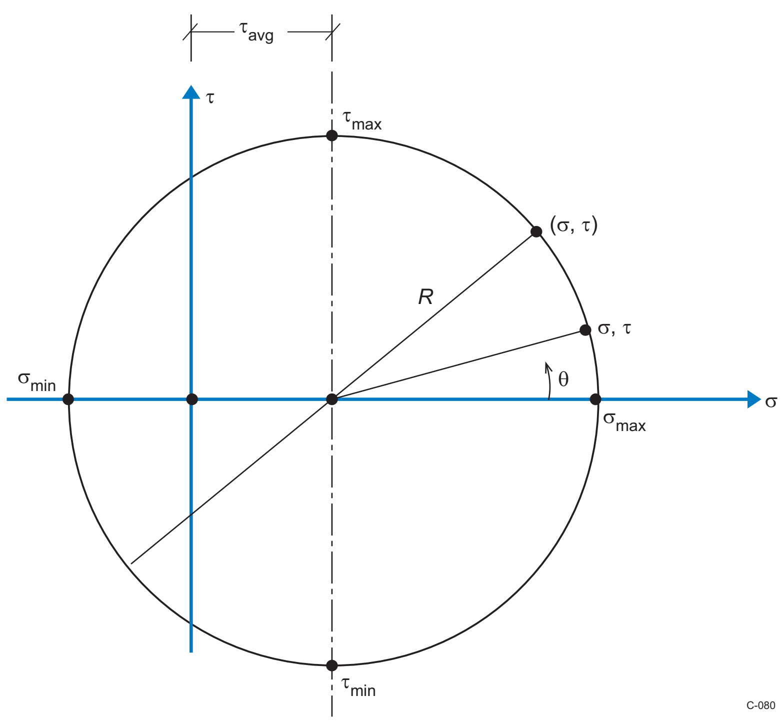
Equation (6) represents the equation of a circle in the σ, τ space

$$R = \text{radius of the circle} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad \textbf{Eq (7)}$$

$$\sigma_{avg} = \text{location of center} = \frac{\sigma_x + \sigma_y}{2} \qquad \textbf{Eq (8)}$$

Therefore (6) is rewritten

$$(\sigma - \sigma_{avg})^2 + \tau^2 = R^2 \qquad \textbf{Eq (9)}$$



Principal stress

From Budynas and Timoshenko, the definition of principal stress is

$$\sigma_p = \frac{\sigma_x - \sigma_y}{2} \pm \sqrt{\frac{\sigma_x - \sigma_y}{2} + \tau_{xy}^2} \qquad \textbf{Eq (10)}$$

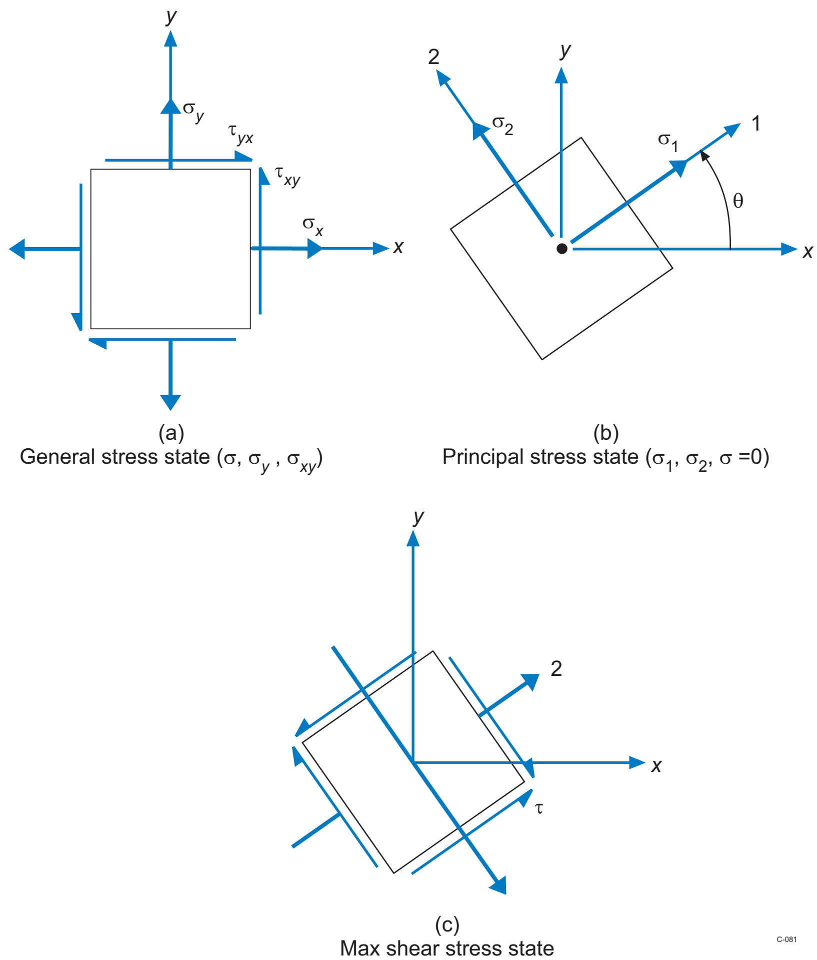
Which comes from Mohr's circle as,

$$\sigma_p = \sigma_{avg} \pm R \qquad \textbf{Eq (11)}$$

To calculate angle of principal stress state (i.e., zero shear sress), we start from Equation (4) and differentiate with respect to θ

$$\frac{d\sigma}{d\theta} = -\,(\sigma_x - \tau_y)\sin^2\theta_p + 2\tau_{xy}\cos^2\theta_p = 0 \qquad \textbf{Eq (12)}$$

where θ_p = angle of principal stress state



(a)
General stress state (σ, σ_y , σ_{xy})

(b)
Principal stress state (σ_1, σ_2, σ =0)

(c)
Max shear stress state

Solve Equation (12) for θ_p,

$$\theta_p = \frac{1}{2} \tan{-1} \left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) \qquad \text{Eq (13)}$$

Maximum in-plane shear stress

From Equation (3) for $\tau_{x'y'}$, differentiate with respect to θ

$$\frac{d\tau_{x'y'}}{d\theta} = -(\sigma_x - \tau_y) \cos^2 \theta_s + 2\tau_{xy} \sin^2 \theta_s = 0 \qquad \text{Eq (14)}$$

θ_s = angle at which $\tau_{x'y'}$ is maximum

Solve for θ_s

$$\therefore \theta_p = \frac{1}{2} \tan{-1} \left(\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right) \qquad \text{Eq (15)}$$

This defines the angle of maximum shear stress.

Examine the relationship between equations (15) and (13) and the arguments are negative reciprocals, which implies that θ_p and θ_s are 90° apart.

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