CHAPTER 1

INTRODUCTION

1.1 BASIC CONCEPTS OF REINFORCED CONCRETE

Reinforced concrete is basically concrete in which steel bars of desirable magnitude are introduced in the casting stage so that the resulting composite material resist the stresses developed due to the external loads. In flexural members the steel reinforcement is generally provided near the tension face to resist the tensile stresses since the tensile strength of concrete is hardly one tenth of its compressive strength. In the case of compression members, the steel reinforcement is distributed uniformly in the cross section to resist the compressive stresses developed due to the external loads.

The revolutionary engineering concept of reinforcing the weak tensile zone of concrete by steel reinforcement was developed in mid-nineteenth century. The early 20th century witnessed significant improvements in the development and use of reinforced concrete mainly due to the production of good quality concrete with improved strength and improved quality of steel with surface characteristics suitable to develop good bond between concrete and steel.

The success of reinforced concrete as a revolutionary material for use in various types of structures is mainly due to the improved quality of concrete and steel over the years and also the improved bond characteristics between the two ingredients.

1.2 HISTORICAL DEVELOPMENT

The present state of development in the field of reinforced concrete is due to the continuous research done by scientists and engineers in this field during the last 150 years.

Isaac Johnson[1] first made the prototype of modern cement in 1845, by burning a mixture of clay and chalk until clinkering, so that reactions necessary for the formation of strong cementatious compounds are complete. Early 20th century witnessed the development of mass production of good quality cement. At present Ordinary Portland cement of various strengths designated as C-33, C-43, and C-53 are available for use in different types of structures. Different types of cements with specific properties have been developed for use in the construction of highways, marine structures, multistorey buildings and industrial structures.

Romans used iron-reinforced masonry during first century B.C. Lambot of France constructed a rowboat 3.3m long by plastering Roquefort cement on a skeleton network of iron and wire. Coignet[2] of France and his contemporaries in England filed the first patents for the use of reinforced concrete around 1855.

In 1855, Wilkinson secured a patent in England for a concrete arch floor reinforced with tie bars. Many scientists around this time obtained patents on reinforced concrete in different types of structures in various countries. In the Later part of 19th century, reinforced concrete passed through a period of patents held by several specialists.

Significant developments during the early part of 20th century resulted in improved quality of concrete and steel. Cement was mass-produced with quality control and improved method of proportioning concrete mixes resulted in concrete of desired compressive strength ranging from 15 N/mm^2 to 60 N/mm^2.

Early Investigators worked on the theoretical basis to explain the structural behavior of reinforced concrete as early as the end of the nineteenth century.

In 1877, Thaddeus Hyatt, an American lawyer established the basis of analysis of stress in reinforced concrete by explaining the concept of bond between steel rods and concrete.

Later Koenen of Germany developed the design rules of analysis of reinforced concrete sections in 1886. Coignet of France also published the principles of elastic design of reinforced concrete during the same period.

Early 20th century witnessed the end of patents in this field when the Prussian regulations comprising the complete set of design rules of reinforced concrete appeared in 1907. While the French commission on reinforced concrete had formulated the design rules in 1906, professional societies like the American Concrete Institute (ACI) and the American Society of Civil Engineers (ASCE) introduced the first joint code on reinforced concrete in 1909.

The first major application of reinforced concrete was in bridges mainly due to the economy in comparison with steel bridges. The elastic method of design was firmly established and widely used during this period. The rebuilding of bridges and buildings during the post war periods resulted in establishing reinforced concrete as an economical structural material for use in different types of structures.

However, the inadequacy of the elastic or working load design in predicting the ultimate loads of a structure paved the way for the ultimate load theories and design based on ultimate loads computed by applying load factors to the working loads.

Several Investigators[3] like Emperger (1936) Whitney (1937) Jenson (1943), Chambaud (1949) and Hognestad (1951) developed the ultimate load theory based on different types of stress blocks. Reinforced concrete structures designed solely on the basis of ultimate load theory resulted in slender structural elements and their serviceability characteristics (deflections and cracks) under working loads were not within the codified acceptable limits.

The ultimate load method of design ensures the safety of the structures against the collapse limit state only and as such does not give any information about the behaviour of the structure at service loads and the range between service and collapse loads. The inadequacy of the ultimate load method in not ensuring the serviceability of the structure resulted in the development of *limit State design.*

The philosophy of limit state design[4,5,6] was first incorporated in the Russian code in 1955. Basically, limit state design is a method of designing structures based on a statistical concept of safety and the associated statistical probability of failure. Limit state design is based on the concept of probability and comprises the application of the method of statistics to the variations that occur in practice in the loads acting on the structure and the strength of the materials.

The Limit state design overcomes the inadequacies of the working stress and ultimate load methods and ensures the safety of the structure against excessive deflections and cracking under service loads and also provides for the desirable load factor against failure. Hence, the British Code[7], American Code[8], Australian Code[9] and German Code[10] and the recently revised Indian Code[11] have adopted the limit State design concepts.

1.3 PHILOSOPHY OF STRUCTURAL DESIGN

The main objective of reinforced concrete structural design is to comply with the following essential requirements.

1) Structures designed should satisfy the criterion of desirable ultimate strength, in flexure, shear, compression, tension and torsion developed under a given system of loads and their combinations. In addition, the stresses developed in the structure under the given system of loads should be within the safe permissible limits under service loads.

2) The structure designed should satisfy the criterion of serviceability, which limits the deflections and cracking to be within acceptable limits. The structure should also have adequate durability and impermeability, resistance to acids, corrosion, frost etc.

3) The structure should have adequate stability against overturning, sliding, buckling, and vibration under the action of loads.

A satisfactory structural design should ensure the three basic criteria of strength, serviceability and stability. In addition, the structural designer should also consider aesthetics and economy. The structural designer and the architect should co ordinate so that the structure designed is not only aesthetically superior, but also strong enough to safely sustain the designed loads without any distress during the life time of the structure.

1.4 APPLICATIONS OF REINFORCED CONCRETE

Reinforced concrete is well established, as an important construction material often preferred to steel construction mainly due to its versatility, adaptability, and resistance to fire and corrosion resulting in negligible maintenance costs. Development of better quality cements during the last decade has resulted in stronger and more durable concrete for use in different types of structures.

Reinforced concrete is ideally suited for the construction of floor and roof slabs, columns and beams in residential and commercial structures.

The present trend is to adopt reinforced concrete for bridges of small, medium and long spans resulting in aesthetically superior and economical structures in comparison with steel bridges.

Typical use of reinforced concrete in earth retaining structures includes abutments for bridges and retaining walls for earthen embankments.

Reinforced concrete is ideally suited for water retaining structures like ground and overhead tanks and hydraulic structures like gravity and arch dams. The material is widely used for the construction of large domes for water tanks and sports stadiums and conference halls.

Reinforced concrete grid floors comprising beams and slabs are widely used for covering large areas like conference halls where column free space is an essential requirement.

For aircraft hangers, reinforcement concrete shells comprising of thin circular slabs and deep edge beams provide an economical solution.

Reinforced concrete folded plate construction has been used for industrial structures where large column free space is required under the roof.

In coastal areas where corrosion is imminent due to humid environment, reinforced concrete is ideally suited for the construction of marine structures like wharfs, quay walls, watchtowers, and lighthouses. For warehouses in coastal areas, reinforced concrete trusses are preferred to steel trusses.

Reinforced concrete poles have almost replaced steel poles for power transmissions. Tall towers for T.V.transmission are invariably constructed using reinforced concrete.

Multistorey reinforced concrete buildings are routinely adopted for both residential and office complexes. For heavy-duty floors in factories, reinforced concrete is ideally suited due to its resistance to wear and tear and improved durability.

In atomic structures, reinforced concrete is preferred to steel for pressure vessel construction due to the superior radiation absorption characteristics of high strength and high density concrete.

Reinforced concrete piles, both precast and cast in sites have been in use for foundations of structures of different types likes bridges and buildings.

Another novel application of reinforced concrete is in the construction of pavements for highways and airport runways.

The Twentieth century has witnessed reinforced concrete as a revolutionary material suitable for the construction of most simple to complex structures. With significant improvements in the quality of cement and steel, reinforced concrete will continue to find new applications and widespread use in the 21st century.

1.5 REINFORCED CONCRETE STRUCTURAL SYSTEMS

Any type of structure may be considered as an assemblage of various structural elements, which perform a predetermined function of resisting various types of forces. Basically a structure can be built up using structural and non-structural elements[12]. The structural elements (beams, slabs columns etc.) have the primary function of resisting the external loads, while the nonstructural elements (partitions, false ceiling, doors etc.) do not support the external loads.

Basically, the structural elements can be classified as one-dimensional elements (Ex: beams, columns, arches etc) or two-dimensional elements (Ex: slabs, plates, shells etc.) and three-dimensional elements (thick pipes, walls of nuclear reactor vessels, domes etc.)

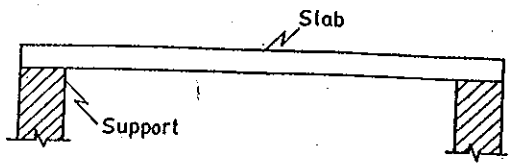
Circular girders generally used in water tanks are subjected combined flexure, shear and torsion while the corner columns in a multistorey framed structure is subjected to biaxial bending.

a) One Way Slab Systems

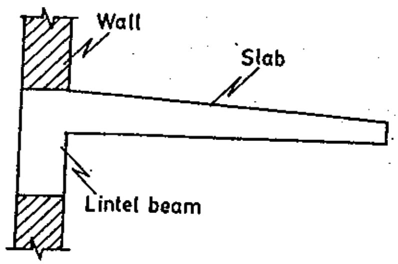
Fig. 1.1(a) shows the floor system comprising a one-way slab supported at the edges by walls or beams and supports dead and live loads. The slabs are subjected primarily to maximum flexure at centre of span along the shorter direction and maximum shear at supports under gravity loads.

Fig 1.1(b) shows a cantilever slab generally used in chajjas projecting from lintel beams.

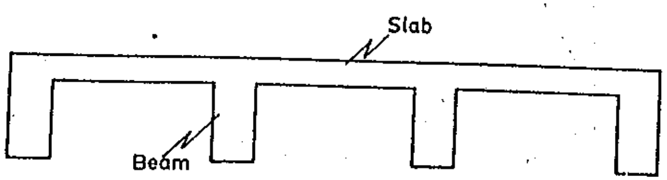
Fig 1.1(c) shows a continuous slab, supported on beams generally used in a building complex.



(a) One Way Slab



(b) Cantilever Slab



(c) Continuous Slab

Fig. 1.1 Types of Reinforced Concrete Slab System

b) Two Way Slab floor Systems

Fig 1.2. shows a typical two-way slab floor system commonly used in buildings. In this case the slab is supported at the edges and it is subjected to flexure in two principal directions while resisting gravity loads.

c) Beam and Slab floor systems

Fig 1.3 shows a typical beam and slab floor system generally used in residential and commercial building structures. In this case the gravity loads are resisted by flexure of slab and beams.

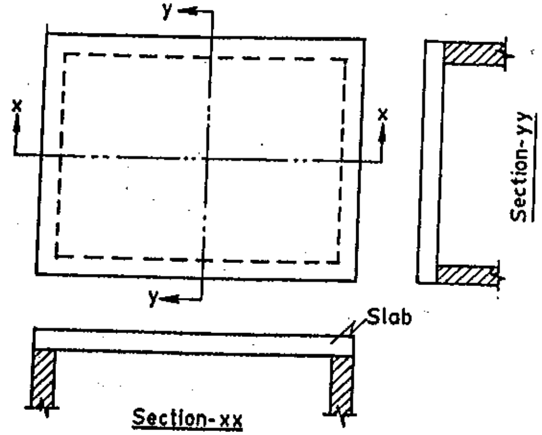


Fig. 1.2 Two Way Slab Floor System

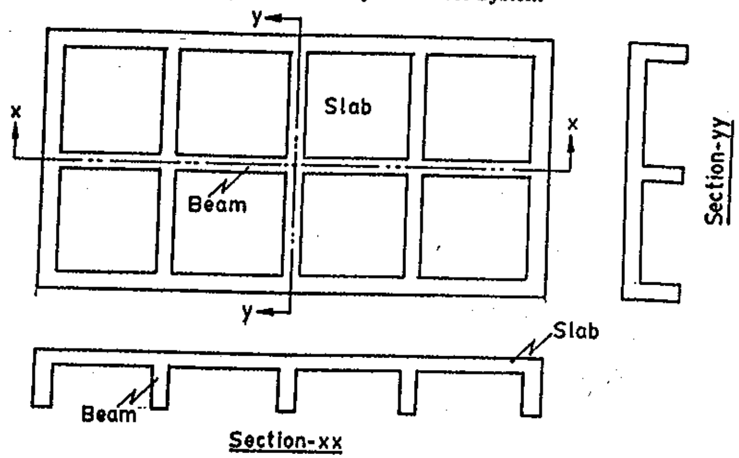


Fig. 1.3 Beam and Slab Floor System

d) Flat Slab floor System

Fig 1.4 shows a flat slab floor system in which the slab is supported on columns directly without any beams. This type of floor system is generally preferred for large span office complexes, commercial buildings and garages, where headroom is less.

e) Grid Floor system

Fig 1.5 shows a typical grid floor system comprising beams spaced at short intervals running in perpendicular directions and supports a thin slab. This

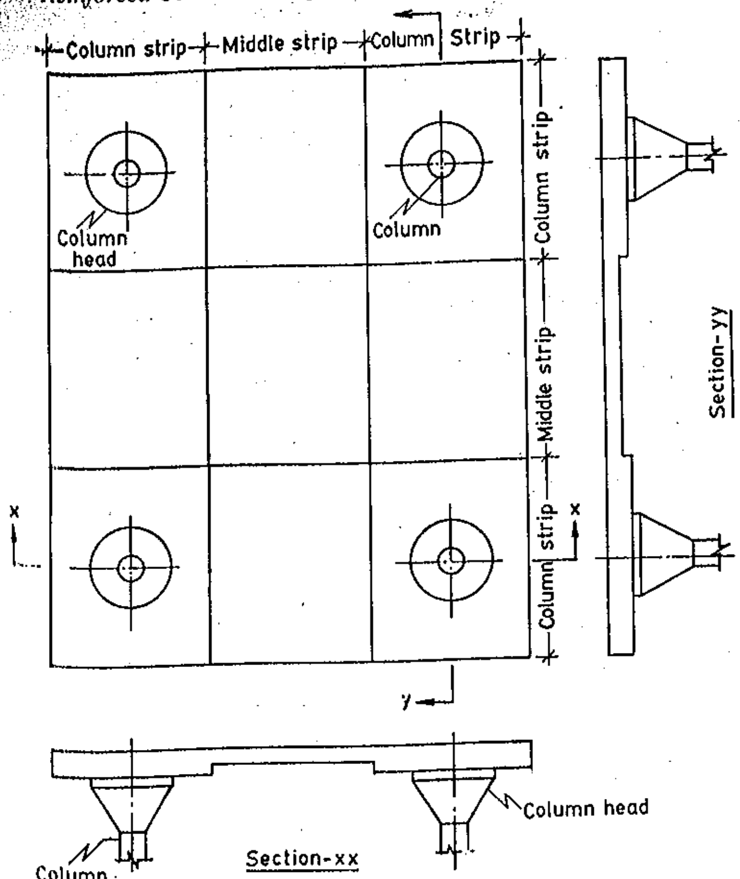


Fig. 1.4 Flat Slab Floor System

type of roof is generally used for large conference halls and commercial buildings requiring column free space. The grid floor is supported at the edges on solid walls or columns at regular intervals.

f) Multistorey Vertical Framing System

Fig 1.6 shows the multistorey vertical frame-comprising columns beams and slabs forming three-dimensional structure. The gravity loads are transmitted from slab to beams which in turn transfer the loads to columns and finally to the foundations. The rigid column and beam frame can resist lateral loads due to wind.

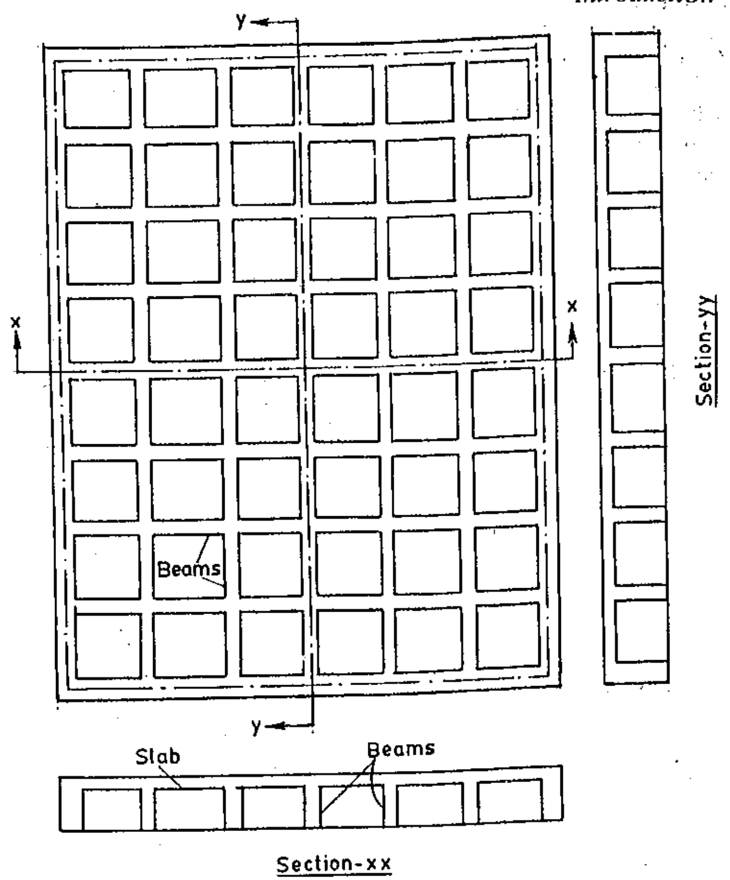


Fig. 1.5 Grid Floor System

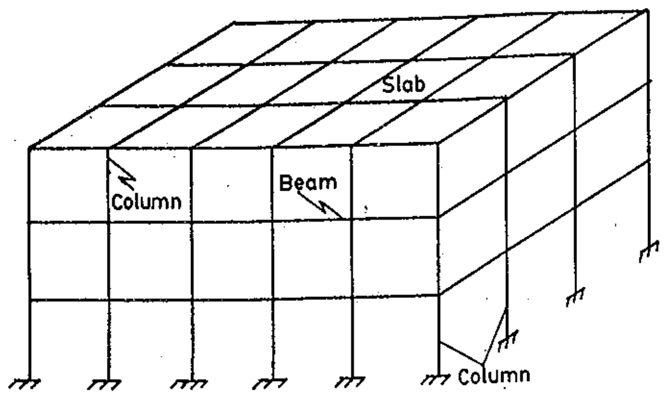


Fig. 1.6 Multistorey Vertical Framing System

g) Shear Wall System

This system consists of solid concrete walls covering the full height of the building. Generally the shear wall box is located at the lift/staircase regions. Sometimes the shear walls are located as exterior or interior walls placed along the transverse direction of the tall building to resist lateral loads due to wind. A typical shear wall provided at the core of a tall structure is shown in Fig 1.7.

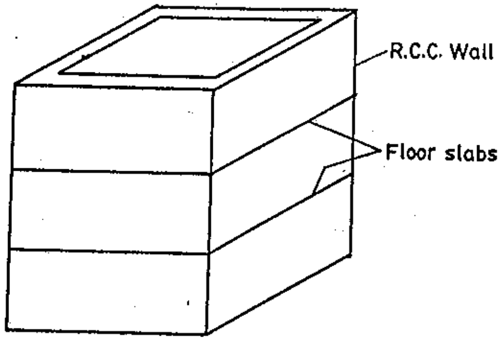


Fig. 1.7 Shear Wall System

1.6 DESIGN CODES AND HAND BOOKS

a) Objective of Codes

Based on extensive research and practical knowledge, various countries have evolved their national codes, which serve as guidelines for the design of structures. The main objectives of the codes are

1) To provide adequate structural safety by ensuring strength, serviceability and durability.
2) To specify simple design procedures, design tables and formulae for easy computations.
3) To provide legal validity and to protect structural engineers form any liability due to failures of structures caused by inadequate design and improper materials and lack of proper supervision during construction.
4) To provide a uniform set of design guide lines to be followed by various structural designers in the country.

National building codes are periodically revised to reflect the improvements in the quality of materials and design procedures evolved as a result of comprehensive research investigations conducted in the various institutions in the country and abroad.

b) Design Codes and Handbooks

All reinforced concrete structural design in our country should conform to the recently revised Indian Standard Code IS:456-2000 Code of practice for plain and reinforced concrete (Fourth Revision). The corresponding national codes of other countries, which are often referred to, are the American Concrete Institute Code ACI-318 and the British Code BS: 8110.

The design examples presented in this book conform to the Indian standard code. The Bureau of Indian standards have released over the years several handbooks to facilitate reinforced concrete structural designers to design routine structural elements quickly by referring to the various tables and graphs presented in the handbooks.

The following handbooks will serve as useful design aids for structural concrete designers.

1) SP: 16-1980[13]- Design Aids for Reinforced Concrete to IS: 456.
2) SP: 24-1983[14]- Explanatory Handbook on IS: 456.
3) SP: 34-1987[15]- Handbook on Concrete Reinforcement and Detailing.
4) SP: 23-1982[16]- Hand book on Concrete Mixes (Based on Indian Standards)

1.7 LOADING STANDARDS

Reinforced concrete structures are designed to resist the following types of loads: -

a) Dead Loads

These are loads that will not change with respect to time. The dead loads acting on the structure include the self-weight of the structural elements, partitions, and finishes, which depends upon the type of material used in the structure. The Indian standard code IS: 875(Part-1) - 1987[17] prescribes the unit weight of building materials and stored materials to be used in the design. Salient dead loads of most common materials used in structural elements are presented in Table 1.1

Table 1.1 Dead Loads of Materials

Material	Unit Weight (kN/m^3)
Brick Masonry	18.85 to 22
Plain Concrete	22.00 to 23.50
Reinforced concrete	22.75 to 26.50
Stone Masonry	21 to 27
Timber	6 to 10

(Contd...)

Table 1.1 Dead Loads of Materials *(Contd...)*

Cement Mortar	20.4
Lime Mortar	15.7 to 18.50
Steel	78.5
Floor Finishes	0.6 to 1.2
Roof Finishes	0.2 to 1.2
Steel work for Roofing	0.16 to 0.23
Cement plaster - 10mm thick	0.2
Concrete Tile flooring 25 mm thick	0.5
Terrazzo, 10mm thick	0.2
Brick Wall, 100mm thick	1.91

b) Live Loads

These are loads that change with respect to time. Live or imposed loads include the loads due to people occupying the floor and those due to materials stored or vehicles in garage floors. The imposed floor and roof loads for different occupancies are specified in IS 875 (Part-2) - 1987[18]. Some of the common live loads encountered in the design of buildings are compiled in Table 1.2

Table 1.2 Live or Imposed Loads

Loading Class	Types of Floors	Minimum Live Load kN/m²
2	Floors in dwelling houses, tenements, hospital wards, Bedrooms and private sitting rooms in hostels and dormitories.	2
2.5	Office floors other than entrance hall floors of light work rooms.	2.5 - 4.0
3.0	Floors of banking halls, office entrance halls and reading rooms	3.0
	ROOFS Types of Roof	Live Load in plan kN/m²
4.0	Shop floors used for display and sale of merchandise, floors of work rooms, floors of class rooms, restaurants, machinery halls power stations etc, where not occupied by plant or equipment.	4.0
5.0	Floors of warehouses, workshops, factories and other buildings or parts of building or similar category for light weight loads, office floors for storage and filling purposes. Assembly floor space without fixed seating, public rooms in hotels, dance halls and waiting halls.	5.0

(Contd.)

Table 1.2 Live or Imposed Loads *(Contd...)*

7.5	Floors of warehouses, workshops, factories and other buildings or parts of buildings of similar category for heavy weight loads, floors of bookstores and libraries.	7.5
10.0	Floors of ware houses, work shops, factories and other buildings or parts of buildings of similar category for heavy weight loads, floors of book stores and libraries	10.0
	Garages (light) Floors used for garages for vehicles not exceeding 25 kN gross weight. Slabs Beams	 4.0 2.5
	Garages (Heavy) Floors used for garages for vehicles not exceeding 40 kN gross weight.	7.5
	Staircases Stairs, landings and corridors for class 2 but not liable to over crowding.	3.0
	Balcony Balconies not liable to over-crowding for class 2 loading Loading for other classes Balconies liable to over crowding	 3.0 5.0 5.0
	Flat, Sloping or Curved roof with slopes up to and including to degrees. a) Access provided. b) Access not provided, except for maintenance. c) Sloping roof with slope greater than 10°:- 0.75 kN/m² less 0.001 kN/m² for every increase in slope over 10 degrees up to and including 20° and 0.002 kN/m² for every degree increase in slope over 20°	 1.5 0.75

c) Wind Loads

Wind loads have to be considered in the design of multistorey buildings, towers and poles. Wind loads depend upon the intensity of wind prevailing in the locality of the structure. IS: 875(Part-3) -1987[19] prescribes basic wind speeds in various zones by dividing the country into 6 zones. The design wind pressure is computed as

$$p_x = 0.6V_x^2$$

Where p_x = design wind pressure in N/mm² at a height Z and
V_x = design wind velocity in m/s at a height Z.

Wind Load '*F*' acting in a direction normal to the individual structural element or cladding unit is computed as,

$$F = (C_{pe} - C_{pi})\,A\,.p_d$$

Where C_{pe} = external pressure coefficient.
C_{pi} = internal pressure coefficient.
A = surface area of structural element or cladding unit and
p_d = design wind pressure.

The values of external and internal pressure coefficients depend upon the type of structure and are presented in a tabular form in IS 875 (Part-3) - 1987.

d) Snow Loads and Local Combinations

Structures subjected to snow loads have to be designed suitably by considering the snow loads prevailing in the region and also the various load combinations. These are specified in IS 875 (Part-4) and (Part-5) - 1987[20] respectively.

e) Earth Quake Loads

Seismic or earthquake forces have to be considered in the design of structures located in seismic zones according to IS:1893-84[21]. The horizontal seismic force (F_{eq}) is computed as,

$$F_{eq} = [\alpha\, \beta\, \lambda\, G]$$

Where α = Horizontal seismic coefficient depending on location with values of 0.08, 0.05, 0.04, 0.02, and 0.01 for Zones V, IV, III, II, and I.
β = A coefficient depending on soil-foundation system ranging from 1.0 to 1.5
λ = A coefficient depending upon the importance of the structure varying from 1.5 to 1.0
G = Dead load above the section considered.

Structures located in Zone V to III (Severe earthquake zone) should be designed for seismic forces.

CHAPTER 2

Materials For Reinforced Concrete

2.1 CONCRETE

Plain Concrete is a composite material composing of cement, aggregate and water, in suitable proportions. Cement reacts in the presence of water to produce complex compounds which gradually harden and bonds the aggregate comprising sand and coarse aggregate into a solid mass with time. Fresh concrete exhibits plasticity and flowability so that it can be placed into the moulds of required shape and compacted to form a dense mass. The compacted and hardened concrete is cured in the presence of water so that it gains most of its strength within four weeks, after which the external loads can be applied.

2.1.1 Cement

Various types of cements have been developed for use in different types of structures. For a detailed study of the type and properties of different types of cements, the reader may refer to the treatise on properties of concrete authored by Neville.

According to IS: 456-2000, the types of cements and their suitability for a specific situation are outlined in Table.2.1.

2.1.2 Aggregates

In concrete, aggregate volume is nearly 75 percent of the total volume. Hence, the structural behavior of concrete is significantly influenced by the type of aggregates used. Fine aggregate comprises of sand dug out from riverbeds and pits having particle sizes from 0.075 mm to 4.75 mm.

Crushed rock and gravel are generally used as coarse aggregates with maximum size of 10 mm, 20 and 40 mm. For reinforced concrete work 10 and 20 mm are commonly used. For mass concrete works like dams, larger sizes of aggregates upto 150 mm are used. The nominal maximum size of coarse aggregate should be as large as possible but it should be limited to one fourth of the minimum thickness of the member. Lightweight and heavy weight aggregates are also used in specific works. The various

Table 2.1 Types of Cements and their use

No	Type of Cement	IS:Code	Where used
1	Ordinary Portland Cement C-33 Grade C-43 Grade C-53 Grade	IS: 269 IS: 8112 IS: 12269	All General Concreting works. Multistorey structures. Bridges-Tall structures Prestressed concrete work.
2	Rapid Hardening Portland Cement	IS: 8041	Road works and Repairs.
3	Low Heat Portland Cement	IS: 12600	Mass Concrete Dams
4	Portland Slag Cement	IS: 455	Marine Structures.
5	Portland Pozzolana Cement	IS: 1489	Mass Concrete - Marine Structure and General building Works.
6	Sulphate Resisting Portland Cement	IS: 12330	Marine Structures foundations in Sulphate bearing soils.
7	Hydrophobic Cement	IS: 8043	Swimming Pools floors of food Processing plants.
8	High Alumina Cement	IS: 6452	Marine Structures.
9	Supersulphated Cement	IS: 6909	Marine Structures construction of sewers.

properties of aggregates like specific gravity, strength, toughness, hardness, soundness, particle size distribution (grading) should comply with the Indian Standard Code: IS: 383-1979[22].

Crushed rock and gravel aggregates with specific gravity in the range of 2.5 to 2.7 yields concrete with a density in the range of 23 to 24 kN/m^3. However special concretes like light weight and high density required for specific applications can be produced by using suitable aggregates.

Light weight aggregates[23] generally used to produce structural light weight concrete having a density in the range of 10 to 18 kN/m^3, widely used in U.K., U.S.A and Europe belong to the category of

a) Expanded shales, clays, and slates produced in a rotary kiln (Leca, Kermazite)
b) Expanded shales or clay produced on a sintering grate (Aglite, Agloporite)
c) Slags expanded mechanically or by water jet process (Foamed Slag)
d) Sintered pulverized fuel ash aggregate (Lytag)

Lightweight concrete is now a firmly established building material having extensive applications in most of the developing and developed countries. Light Weight concrete is used in the block making industry[24] and also in reinforced and prestressed concrete constructions.[25]

High density concrete with a density in the range of 30 to 40 kN/m^3, required for the construction of biological shields for atomic reactors is

made by using heavy aggregates like Magnetite, Hematite, Limonite and Barytes[26], Steel punchings[27] and shots have been successfully used for producing concrete with a density in the range of 50 to 60 kN/m^3.

2.1.3 Concrete Mix Proportions

The main objective of concrete mix design is to select the optimum proportion of the various ingredients of concrete, which will yield fresh concrete of desirable workability and hardened concrete possessing the specified characteristic compressive strength and durability. The mix proportions should also satisfy the additional requirement of the use of minimum possible cement content so that the maximum economy is achieved in the unit cost of concrete according to the author[28].

a) Nominal Mix Concrete

The Revised Indian Standard Code IS: 456-2000, prescribes the proportions of ingredients of concrete for nominal mixes of concrete grades lower than M-20 which are used for ordinary and small works, as shown in Table 2.2.

Table 2.2 Proportions for Nominal Mix Concrete (Table-9 of IS: 456-2000)

Grade of Concrete	Total Quantity of Dry Aggregates by mass per 50 kg of Cement, to be taken as the sum of the individual masses of Fine and Coarse Aggregates (kg) Max	Proportions of Fine Aggregate to coarse Aggregate (By Mass)	Quantity of water per 50 kg of Cement (Max) (litres)
1	2	3	4
M-5	800	Generally 1: 2 but subject to an upper Limit of 1:1½ and a lower limit of 1:2½	60
M-7.5	625		45
M-10	480		34
M15	330		32
M-20	250		30

b) Design Mix Concrete

For all-important works involving large quantities of concrete, it is preferable to use design mix, which results in considerable economy ensuring the required strength. The design mix uses the following parameters:-

1) Type of cement
2) Aggregate size and grading
3) Water / Cement ratio
4) Aggregate / Cement ratio

5) Workability of concrete
6) Relation between mean and maximum strength and standard deviation
7) Grade of concrete

Over the years, several mix design methods have been developed based on the above parameters. The most prominent, well-established and widely used methods are,

1) The American Concrete Institute Method[29].
2) The British Method, developed by Teychenne, Franklin and Erntroy[30]
3) The Indian Standard Method[31].

The above methods are based on extensive experimental investigations in their respective countries. A critical review of the Indian, British and American methods of concrete mix design has been reported by Krishna Reddy[28] and the author. The salient experimental observations being that the American and British methods resulted in concrete having compressive strength nearly equal to the desired characteristic strength while the Indian Standard method yields significantly higher compressive strength than the desired characteristic strength. Also the concrete mixes designed by the Indian Standard Code method utilized the highest cement content for unit volume of concrete in comparison with the American and British methods. For exhaustive information regarding the format of design of concrete mixes of various types and computer aided design of mixes, the reader may refer to the treatise on 'Design of Concrete Mixes' recently revised by the author[32].

c) Properties of Concrete

i) Compressive strength

The Characteristics strength is defined as the strength of material below which not more than 5 percent of the test results are expected to fall. The concrete mix should be designed for the target strength computed as,

Target Strength = (Characteristic Strength) + (1.65 time the Standard deviation)

The Indian Standard Code IS: 456-2000 specifies the characteristic compressive strength of 150 mm cubes at the age of 28days as Grades of concrete varying from 15 to 50 N/mm^2 designated as M-15 to M-50. For Reinforced concrete, the minimum grade of concrete to be used is M-20.

ii) Tensile Strength

The flexural strength of concrete generally referred to as *Tensile strength* is

required to compute the on set of visible cracks in a concrete structure under flexure. For computation of load factor against cracking, knowledge of the flexural strength is required. According to IS: 456-2000, the tensile strength of concrete can be computed from the compressive strength using the empirical relation given by

Flexural strength $f_{cr} = 0.7\sqrt{f_{ck}}$ N/mm^2

Where f_{ck} = Characteristic cube compressive strength of concrete (N/mm^2)

iii) Modulus of Elasticity

Modulus of elasticity of concrete which is significantly influenced by the type of the aggregates used, type of cement and mix proportions is an important property required for the computations of deflections of structural concrete members which forms an important limit state in the design of concrete members. In the absence of test data, the modulus of elasticity of concrete is normally related to the compressive strength and is computed by the empirical relation recommended by IS: 456-2000 code and is expressed as,

$$E_c = 5000\sqrt{f_{ck}}$$

Where E_c is the short-term static modulus of elasticity of concrete expressed in N/mm^2.
f_{ck} is the characteristic compressive strength of concrete expressed in N/mm^2.

iv) Shrinkage of Concrete

The ingredients of concrete and environmental conditions like temperature and humidity influence the total shrinkage of concrete. Water content in concrete significantly affects the shrinkage. The IS: Code 456-2000 recommends the total shrinkage strain as 0.0003 in the absence of test data. Drying shrinkage in plain concrete may result in surface cracks. Shrinkage of concrete also influences the deflections of reinforced concrete members.

v) Creep of concrete

The inelastic time dependent strain developed in a concrete member under sustained loading is referred to as *creep* of concrete. Creep of concrete is influenced by cement content, W/C ratio, A/C ratio, temperature and humidity, size of the structural element, type of loading and period of loading.

In the absence of reliable experimental data, the creep coefficient is expressed as the ratio of ultimate creep strain / elastic strain at various ages of loading as recommended by IS: 456-2000 are given in Table 2.3.

Table 2.3 Creep coefficient (IS: 456-2000)

Age at Loading	Creep coefficient
7 days	2.2
28 days	1.6
1 year	1.1

Creep of concrete significantly affects the deflections of reinforced concrete flexural members. Higher creep coefficient results in larger deflections. The value of creep coefficient is useful in the computation of time dependent deflections in reinforced concrete members.

vi) Coefficient of Thermal Expansion

The coefficient of thermal expansion of concrete, influenced mainly by the type of aggregate used in concrete is required for the design of structures like chimneys, water tanks, silos etc. The values recommended in IS: 456-2000 are compiled in Table 2.4.

Table 2.4 Coefficient of Thermal Expansion for Concrete

Type of Aggregate	Coefficient of Thermal expansion for concrete / °C
Quartzite	$1.2 \text{ to } 1.3 \times 10^{-5}$
Sand stone	$0.9 \text{ to } 1.2 \times 10^{-5}$
Granite	$0.7 \text{ to } .95 \times 10^{-5}$
Basalt	$0.8 \text{ to } 0.95 \times 10^{-5}$
Lime stone	$0.6 \text{ to } 0.9 \times 10^{-5}$

vii) Durability of Concrete

a) General Features

Concrete is durable if it performs satisfactorily without deterioration when exposed to different types of exposure conditions during its service life. The main factors influencing durability are, the type of environment, the type of quality of concrete, cement content, water/cement ratio, workmanship, cover to the embedded reinforcement, the shape and size of the structural member. IS: 456-2000 categorizes the exposure conditions into six types designated as a) mild b) moderate, c) severe d) very severe e) extreme f) abrasive.

The Indian code prescribes the minimum cement and maximum water-cement ratios to be used in concrete for different exposure conditions to

ensure the durability of concrete.

The values of cement content, water/cement ratio and minimum grade of concrete for normal weight aggregates of 20mm nominal maximum size are compiled in Table 2.5 and the adjustments required for cement content when other sizes of aggregates used are shown in Table 2.6.

Table 2.5 Minimum Cement Contents, Maximum W/C Ratio, and Minimum Grade of concrete for different exposure condition with normal weight aggregates of 20mm Nominal maximum size. (Table -5 of IS: 456-2000)

Exposure	Plain Concrete		Reinforced concrete		Minimum Grade of Concrete	
	Minimum cement content (kg/m³)	Maximum Free W/C Ratio	Minimum cement content (kg/m³)	Maximum Free W/C Ratio	P.C.C.	R.C.C
Mild	220	0.60	300	0.55		M-20
Moderate	240	0.60	300	0.50	M-15	M-25
Severe	250	0.50	320	0.45	M-20	M-30
Very severe	260	0.45	340	0.45	M-20	M-35
Extreme	280	0.40	360	0.40	M-25	M-40

Table 2.6 Adjustments to Minimum Cement contents for Aggregates other than 20 mm nominal maximum size. (Table -6 of IS: 456-2000)

Nominal Maximum Aggregate size (mm)	Adjustments to Minimum Cement Contents in Table 2.4 (kg/m³)
10	+ 40
20	0
40	- 30

b) Freezing and thawing

Under severe exposure conditions where concrete is subjected to freezing and thawing, it is preferable to use air-entrained concrete for grades less than M-50. Air entrained concrete obtained by using air entraining admixtures is ideally suited to resist the destructive effects of freezing and thawing conditions. The IS: 456-2000 code recommends the percentages of entrained air for nominal maximum size of aggregates of 20 and 40 mm as shown in Table 2.7.

Table 2.7 Air Entrained Concrete

Nominal Maximum size of Aggregate (mm)	Entrained air (percentage by volume)
20	5 ± 1%
40	4 ± 1%

Notes:

1) Minimum Cement Content prescribed in the Table is irrespective of grades of cement and it is inclusive of supplementary cementatious materials such as fly ash, ground granulated blast furnace slag or silica fume.
2) Minimum grade for PCC under mild exposure conditions not specified.

c) Exposure to sulphate attack

Concrete used in marine structures is subjected to extreme exposure conditions due to the sulphate bearing waters of the sea. Depending upon the concentration of sulphate expressed as SO_3 ,different types of cements are preferred to resist the destructive effects of sulphate bearing waters in marine environment.

IS: 456-2000 recommends different types of cements from ordinary Portland to sulphate resulting Portland depending upon the sulphate contents. The minimum cement content and the corresponding maximum free water/cement ratios are compiled in Table 2.8.

d) Fire resistance, Corrosion and Cover requirements for R.C.C. members

The alkaline environment of Portland cement concrete generally protects embedded steel reinforcement, against corrosion from various environmental agencies. However, the carbonation of hydrated cement gradually progresses from the surface to the interior of concrete, thus reducing the effective protection provided by the concrete against rusting of steel reinforcement. Many codes have provided for minimum cover requirements in this regard. It is important to note the thickness of clear cover and the density of concrete in the protection to steel against corrosion and fire resistance.

Notes:

1) Cement content given in Table 2.7 for ordinary Portland cement is irrespective of grades of cement.
2) Use of supersulphated cement is generally restricted where the prevailing temperature is above 40°C.
3) Supersulphated cement gives an acceptable life provided that the concrete is dense and prepared with a water/cement ratio of 0.4 or less. In mineral acids, down to pH 3.5.

Table 2.8 Requirements for Concrete Exposed to Sulphate Attack
(Table-4 of IS: 456-2000)

Class	Concentration of Sulphate expressed as SO_3			Type of cement	Requirements for dense fully compacted concrete made with Aggregates complying with IS: 383-1970	
	In Soil		In ground water (g/l)		Min. cement content (Kg/m^3)	Max. Free water / cement ratio
	Total SO_3 (percent)	SO_3 in 2:1 water soil extract (g/l)				
1	Traces Less than 0.2	Less than 1.0	Less than 0.3	Ordinary portland cement or portland slag cement or portland pozzolona cement.	280	0.55
2	0.2 to 0.5	1.0 to 1.9	0.3 to 1.2	Ordinary portland or portland slag or portland pozzolona cement.	330	0.50
				Supersulphated cement or sulphate resisting portland cement	310	0.50
3	0.5 to 1.0	1.9 to 3.1	1.2 to 2.5	Super sulphated cement or sulphate resisting portland cement.	330	0.50
				Portland pozzolona or portland slag cement	350	0.45
4	1.0 to 2.0	3.1 to 5.0	2.5 to 5.0	Super sulphated or sulphate resisting portland cement	370	0.45
5	More than 2.0	More than 5.0	More than 5.0	Sulphate resisting portland cement or super sulphated cement with protective coatings	400	0.40

4) The cement contents given in class 2 are the minimum recommended. For SO_3 contents near the upper limit of class 2, cement contents above these minimums are advised.
5) For severe conditions such as thin sections under hydrostatic pressure on one side only and sections partly immersed, considerations should be given to a further reduction of water/cement ratio.
6) Portland slag cement conforming to IS: 455-1989 with slag content more than 50 percent exhibits better sulphate resisting properties.
7) Where chloride is also encountered along with sulphate in soil or ground water, ordinary Portland cement with C_3A content from 5 to 8 percent shall be desirable to be used in concrete, instead of sulphate resisting cement. Alternatively, a blend of ordinary Portland cement and slag may also be used provided sufficient information is available on performance of such blended cements in these conditions.

The Indian Standard Code IS: 456-2000 provides for separate nominal cover requirements to meet durability and fire resistance requirements. The cover requirements varying from 20 to 75 mm for durability requirements depend upon the type of exposure conditions as outlined in Table 2.9. These covers may be used for reinforcements in beams and slabs. In the case of longitudinal reinforcements in columns, the code prescribes a minimum nominal cover of not less than 40mm or less than the diameter of the bars. In the case of columns having minimum dimension of 200mm or under and where reinforcing bars do not exceed 12 mm, a cover of 25mm may be used. For footings of columns where the footing slab is in contact with soil, the minimum cover shall be 50 mm.

The minimum nominal cover requirements to be provided to all reinforcement including links embedded in normal aggregate concrete to meet specified periods of fire resistance varying from 0.5 to 4 hours is compiled in Table 2.10. The cover requirements depend upon the type of Structural element such as, beam, floor, ribs and columns as well as the support conditions, which include simply supported or continuous members. These specifications are based on the British Code BS: 8110[33] recommendations for fire resistance.

Table 2.9 Nominal Cover to meet Durability Requirements (Table-16 of IS: 456-2000)

Exposure	Nominal Concrete cover in (mm) not less than
Mild	20
Moderate	30
Severe	45
Very Severe	50
Extreme	75

Notes:

1) For main reinforcement up to 12mm diameter bar, for mild exposure, the nominal cover may be reduced by 5mm.
2) Unless specified otherwise, actual concrete cover should not deviate from the required nominal cover by +10mm.
3) For exposure conditions 'severe' and 'very severe', reduction of 5mm may be made, where concrete grade is M-35 and above.

Table 2.10 Nominal covers to all Reinforcement to meet specified periods of Fire resistance (Table-16A of IS: 456-2000)

Fire Resistance	Nominal Cover						
	Beams		Floors		Ribs		Columns
	Simply Supported	Conti-nuous	Simply Supported	Conti-nuous	Simply Supported	Conti-nuous	
Hours	mm	mm	mm	mm	mm	mm	mm
0.5	20	20	20	20	20	20	40
1.0	20	20	20	20	20	20	40
1.5	20	20	25	20	35	20	40
2.0	40	30	35	25	45	35	40
3.0	60	40	45	35	55	45	40
4.0	70	50	55	45	65	55	40

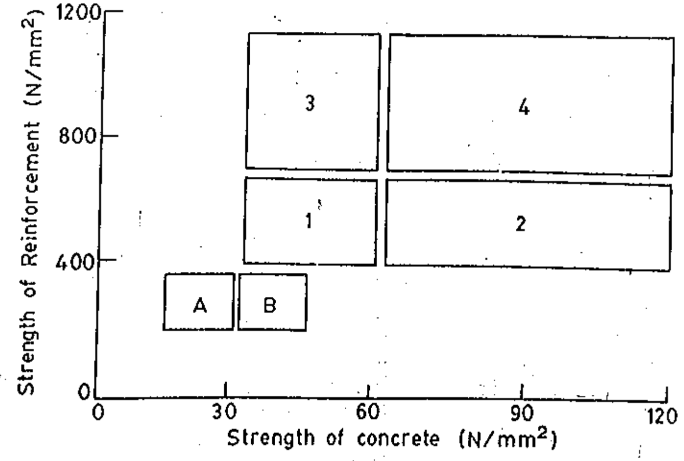
2.1.4 Progress in Concrete Strength and its use in Buildings

Over the years, phenomenal progress has been achieved to produce concrete of higher compressive strength through continuous research resulting in the production of cements of superior quality. In 1950, concrete grades of M-15 to M-40 were commonly used. The dawn of 21st century has witnessed concrete grades ranging from M-30 to M-100. Table 2.11 shows the progress in concrete strength from 1959 to 1990. Recent developments in the technology of cement production in Japan indicates that it is possible to achieve concrete grades exceeding M-100 by using Ultra High strength cements.

Fig 2.1 shows the various types of reinforced concrete buildings in which concrete of different grades and steel reinforcement of different strengths find extensive applications. 21st century will herald in a big way the use of high strength concrete and steel in the construction industry.

2.2 Steel Reinforcement

Steel bars are primarily used to reinforce concrete in the tension zone of flexural members to compensate for the low tensile strength of concrete



A - Low rise buildings in common
B - High rise buildings of last decade
1 - High strength concrete and reinforcement
2 - High strength concrete (ultra) and high strength reinforcement
3 - High strength concrete and ultra high strength reinforcement
4 - Ultra high strength concrete and reinforcement

Fig. 2.1 Types of R.C. Buildings and Materials

Table 2.11 Progress in Concrete Strength

Year	Place	Building	Height (m)	Concrete Grade (N/mm^2)
1959	Chicago	Executive House	150	34
1962	Chicago	Marine	200	34
1964	Montreal	Place Division	200	41
1964	Chicago	1000 Lake store Drive	212	
		Lake Point Tower	220	41
1968	Chicago	One Shell Plaza	230	51
1970	Houston	Watch Tower Palace	267	51
1975	Chicago	311 South Walker Drive		62
1989	Chicago	Tower	320	83
1990	Seattle	Pacific First Centre	–	96.5(124 in 56 days)

and in compression members to increase the load carrying capacity.

Steel reinforcement generally used comprises the following types of bars

a) Mild steel and Medium tensile steel bars conforming to IS: 432 (Part-I)[34]
b) High strength deformed steel bars conforming to IS: 1786[35].
c) Hard-drawn steel wire fabric conforming to IS: 1566[36].
d) Structural steel conforming to Grade A of IS: 2062[37] which covers various types of rolled steel sections.

The typical stress-strain curves of different grades of steel bars are shown in Fig. 2.2.

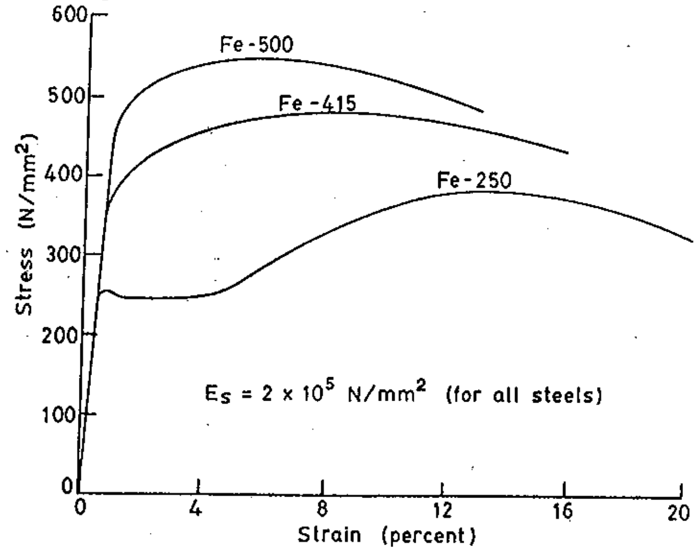


Fig. 2.2 Typical Stress-Strain Curves for Reinforcing Steels

Reinforcements used in reinforced concrete work should be free from loose mill scale, loose rust, oil, mud and any other substance, which reduces bond between steel and concrete which is vital for composite actions. The modulus of elasticity of steel of all grades is taken as 200 kN/mm^2. The characteristic yield strength of different types of steels shall be assumed as the minimum yield or 0.2 percent proof stress.

The nominal diameters presently available in India are 5, 6, 8, 10, 12, 16, 18, 20, 22, 25, 28, 32, 36, 40, 45 and 50mm. The most commonly used type of reinforcement is the high strength deformed bars with a specified yield strength of 415 N/mm^2, since the surface characteristics with protruding ribs result in increased bond between steel and concrete in comparison with mild steel bars with plain surface.

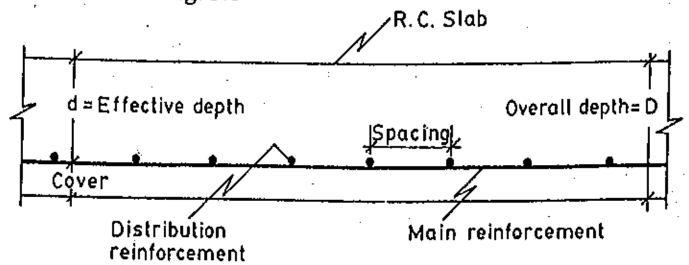
The stress-strain curve shown in Fig. 2.2 indicates that all steels exhibit increase in strength beyond the yield point due to strain hardening. However, for design purposes this increase in strength beyond yield point is generally neglected. In general, the design codes recommended the use of idealized elasto-plastic stress-strain curve with an initial linear elastic line up to yield followed by a line at constant stress, denoting post-yield behaviour.

CHAPTER 3

Reinforcement Specifications For Structural Concrete Members

3.1 REINFORCEMENTS IN SLABS

In concrete slabs, the minimum reinforcements to be provided in either direction together with details of reinforcement spacing and cover requirements are shown in Fig. 3.1



IS: 456-2000 Specifications

1) Minimum Reinforcement : $\nless$ 0·15 % of the total cross sectional area for mild steel and 0·12 % when HYSD bars are used
2) Spacing : a) Main Steel – $\ngtr$ 3d or 300 mm whichever is smaller
   b) Distribution Steel – $\ngtr$ 5d or 450 mm whichever is smaller
3) Maximum Diameter of Bars : $\ngtr$ 1/8 D
4) Cover: $\nless$ 20 mm nor < diameter of bar whichever is higher

Fig. 3.1 Reinforcement Specifications in R.C. Slabs (IS: 456-2000)

3.2 REINFORCEMENTS IN BEAMS

Generally, beams are provided with main reinforcement on the tension side for flexure and transverse reinforcement for shear and torsion.

a) Tension Reinforcement

The minimum area of tension reinforcement shall be not less than that given by the relation,

$$A_s = (0.85\, bd/f_y)$$

Where A_s = Minimum area of tension reinforcement.
b = breadth of beam or breadth of web of flanged sections.
d = effective depth and
f_y = characteristic strength of reinforcement expressed in N/mm^2.
D = overall depth of the member

The maximum area of tension reinforcement shall not exceed 0.04 bD

b) Compression Reinforcement

The maximum area of compression reinforcement shall not exceed 0.04 bD. The compression reinforcement in beams shall be enclosed by stirrups for effective lateral restraint as shown in Fig. 3.2.

c) Side Face Reinforcement

When the depth of web or rib in a beam exceeds 750 mm, side face reinforcement of cross sectional area not less than 0.1 percent of the web area is to be provided and distributed equally on two faces and the spacing of the bars not to exceed 300mm or web thickness whichever is smaller.

d) Transverse or Shear Reinforcement

Minimum or nominal area of shear reinforcement provided in the form of stirrups is computed by the relation,

$$A_{sv} \geq \frac{0.4\, bS_v}{0.87\, f_y}$$

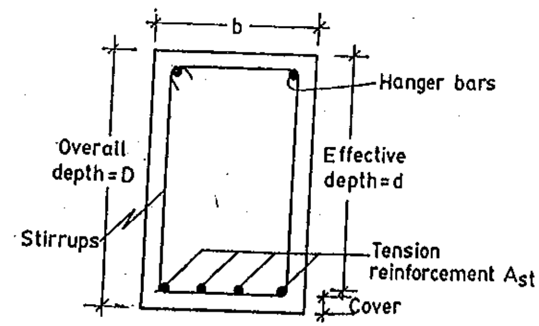
Where A_{sv} = total cross sectional area of stirrup legs in shear
S_v = spacing of stirrups along the length of the member
b = breadth of beam (or web in a flanged member)
f_y = characteristic strength of stirrup reinforcement in N/mm^2 which shall not exceed 415 N/mm^2.

The maximum spacing of shear reinforcement should not exceed '0.75d' for vertical stirrups and 'd' for inclined stirrups at 450 where 'd' is the effective depth. The maximum spacing is restricted to 300mm.

The specifications of reinforcements in beams are illustrated in Fig. 3.2.

3.3 REINFORCEMENTS IN COLUMNS

Reinforced concrete columns are generally of square, rectangular, or



IS: 456-2000 Specifications

1) Minimum Reinforcement: $A_{st} = (\frac{0.85\, db}{f_y})$ or
$A_{st} \nless 0.34\%$ for mild steel ($f_y = 250$ N/mm^2)
$\nless 0.20\%$ for HYSD bars ($f_y = 415$ N/mm^2)

2) Maximum Reinforcement: $\ngtr$ 0·04 bD for both tension and compression reinforcement

3) Spacing Between Bars: $\nless$ diameter of larger bar nor less than the maximum size of coarse aggregate + 5 mm, whichever is greater

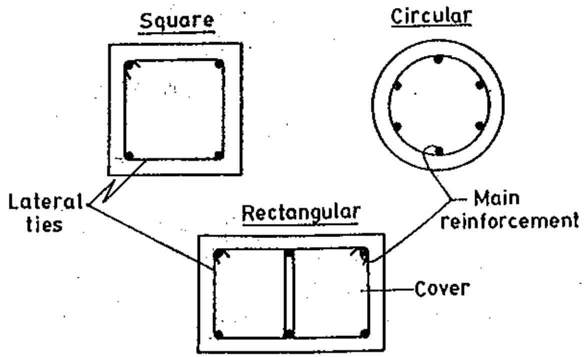
4) Cover: $\nless$ 25 mm nor less than the diameter of bar

5) Curtailment: Refer clause 26·2·3 of IS: 456-2000

Fig. 3.2 Reinforcement Specifications in R.C. Beams (IS: 456-2000)

circular cross section. Columns are provided with main longitudinal reinforcements and lateral ties to prevent buckling of the main bars. The minimum and maximum limits of reinforcements, minimum number of bars and their size, cover requirements and the diameter and spacing of lateral ties are illustrated in Fig. 3.3.

In R.C.Columns with helical ties, at least six main longitudinal reinforcements have to be provided within the helical reinforcement. The spacing of longitudinal bars measured along the periphery of the column shall not exceed 300 mm. The pitch of helical reinforcement is limited to a maximum value of 75 mm and a minimum of 25 mm. Helically reinforced columns have marginally higher load carrying capacity than those with ordinary lateral ties due to higher degree of confinement of concrete in the core.



IS: 456-2000 Specifications

1) Maximum Reinforcement: $\ngtr$ 6·0 %
2) Minimum Reinforcement: $\nless$ 0·8 %
3) Minimum Number of Bars: 4 in rectangular and 6 in circular columns
4) Diameter of Bars: $\nless$ 12 mm
5) Minimum Cover: 40 mm nor diameter of bar whichever is greater
6) Lateral Ties Diameter: $\nless$ 1/4 diameter of largest longitudinal bar nor less than 5 mm

Pitch Than: a) Least lateral dimension of member
b) 16 times the smaller diameter of longitudinal reinforcement
c) 48 times the diameter of transverse reinforcement
d) For effective arrangement of lateral ties refer Fig. 8, 9, 10 and 11 of IS: 456-2000

Fig. 3.3 Reinforcement Specifications in R.C. Columns (IS: 456-2000)

CHAPTER 4

Elastic Theory of Reinforced Concrete Sections in Flexure

4.1 ELASTIC THEORY OF REINFORCED CONCRETE SECTIONS

The working stress method of design of reinforced concrete structures developed during the beginning of 20th century is based on the elastic theory of reinforced concrete sections. The working stress method is based on the assumptions that the structural materials behave in a linear elastic manner and the required safety is ensured by restricting the stresses in the materials under service or working loads. The permissible stresses in concrete and steel are obtained by dividing the characteristic strength of the material by the factor of safety to restrict the working stress in the material under service loads to be well within the linear elastic phase of the materials.

4.2 NEUTRAL AXIS DEPTH AND MOMENT OF RESISTANCE OF SECTIONS

Consider a rectangular section shown in Fig. 4.1 subjected to a moment 'M' under working loads.

Let σ_{cbc} = compressive stress developed in Concrete.
σ_{st} = tensile stress developed in steel.
A_{st} = area of tension reinforcement.
d = effective depth.
b = width of member.
n = neutral axis depth.
k = neutral axis depth factor.
m = modular ratio. = $(280/3\,\sigma_{cbc})$
C = compressive force in concrete.
T = tensile force in Steel.
M = moment of Resistance of the section.

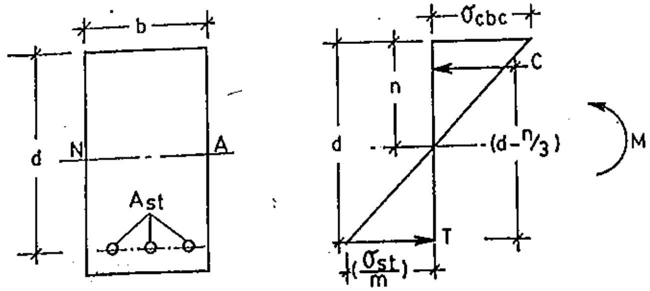


Fig. 4.1 Stress Distribution in Rectangular Section

In the cracked section, concrete below the neutral axis is neglected in computations. Below the neutral axis, the steel area is converted into an equivalent area of concrete by multiplying the steel area by modular ratio and this area contributes to the tensile force for equilibrium of the section.

From the stress distribution diagram shown in Fig. 4.1 we have the relation,

$$\left(\frac{\sigma_{cbc}}{\sigma_{st}/m}\right)=\left(\frac{n}{d-n}\right)=\left(\frac{kd}{d-kd}\right)$$

From the above relation we get

$$\sigma_{st}=m\left(\frac{d-n}{n}\right)\sigma_{cbc}$$

Further when the section is subjected to external loading, resisting moment is developed due to compression in concrete and tension in steel.

Moment of Resistance of the section is given by the relation.

$$M=C\,(d-n/3)=0.5\,\sigma_{cbc}\,.\,b.n(d-n/3)$$

$$M=0.5\,\sigma_{cbc}\,.\,b\,.kd\left(d-\frac{kd}{3}\right)=0.5\,\sigma_{cbc}\,.\,b.k.d^2\left(1-\frac{k}{3}\right)$$

The factor $\left(1-\frac{k}{3}\right)$ is termed as lever arm factor and is represented by 'j'

Hence, we have

$$M=0.5\,\sigma_{cbc}.b.k.j.d^2 \qquad (1)$$

For any given section with known values of b, d, σ_{st}, σ_{cbc}, and A_{st}, we can evaluate the neutral axis depth by equating the first moment of areas above and below the neutral axis.

$$0.5b.n^2=m.A_{st}.(d-n) \qquad (2)$$

By solving Eq. (2), the value of 'n' and 'k' can be determined

In equation (1) substituting, $Q=0.5\,\sigma_{cbc}\,k\,.\,j$

We have $M=Q.b.d^2$.

$$\therefore \quad d=\sqrt{M/Qb} \qquad (3)$$

Equation (3) is generally used to check the adequacy of the depth of section assumed to resist the given moment M.

The Moment of resistance of the section computed from the tension side is given by

$$M=A_{st}.\sigma_{st}\left(d-\frac{n}{3}\right)=A_{st}\cdot\sigma_{st}\left(d-\frac{kd}{3}\right)=A_{st}.\sigma_{st}.d\left(1-\frac{k}{3}\right)=(A_{st}.\sigma_{st}.d.j)$$

$$A_{st}=\left(\frac{M}{\sigma_{st}.j.d}\right) \qquad (4)$$

Equation (4) is generally used to compute the area of tension reinforcement in the section to resist the given moment.

Neutral axis depth factor

The neutral axis depth factor 'k' depends only on the permissible stresses in concrete and steel σ_{cbc} and σ_{st} and modular ratio 'm'. The value of 'k' can be evaluated by the following equations.

From Fig. 4.1, $\quad \dfrac{\sigma_{cbc}}{(\sigma_{st}/m)}=\left(\dfrac{kd}{d-kd}\right)$

Solving $\quad k=\left[\dfrac{1}{1+(\sigma_{st}/m\sigma_{cbc})}\right] \qquad (5)$

Also $\quad \left(\dfrac{m\sigma_{cbc}}{\sigma_{st}}\right)=\left(\dfrac{k}{1-k}\right)$

Substituting $(m.\ \sigma_{cbc})=(280/3)$
and Solving,

$$k=\left(\frac{280}{280+3(\sigma_{st})}\right) \qquad (6)$$

Equation (5) or (6) can be used to evaluate 'k'.

In the analysis of reinforced concrete sections, it is often necessary to evaluate the neutral axis depth factor k using Equation (5) or (6). Equation (3) and (4) is generally used in the design of reinforced concrete sections.

The values of the design coefficients k, j, and Q depend only on the permissible stresses σ_{cbc}, σ_{st} and the modular ratio m.

The permissible stresses in steel and concrete according to IS: 456-2000 are shown in Table 4.1 and 4.2 respectively. The values shown

Table 4.1 Permissible stresses in Steel Reinforcement (IS: 456-2000) (Table-22 of IS: 456-2000)

S.No.	Type of Stress in Steel Reinforcement	Permissible stresses in N/mm² Mild Steel Bars IS: 432	Medium Tensile bars IS: 432	HYSD bars IS:1786 Grade Fe-415
(1)	(2)	(3)	(4)	(5)
(i)	Tension (σ_{st} or σ_{sv}) a) Up to and including 20mm b) Over 20mm	 140 130	Half the guaranteed yield stress subject to a maximum of 190	 230 230
(ii)	Compression in Column Bars (σ_{sc})	130	130	190
(iii)	Compression in bars in beam or slab when the compressive resistance of the concrete is taken into account	The calculated compressive stress in the surrounding concrete multiplied by 1.5 times the modular ratio or σ_{sc} whichever is lower		
(iv)	Compression in bars in a beam or slab where the compressive resistance of the concrete is not taken into account. a) Up to and including 20mm b) Over 20mm	 140 130	Half the guaranteed yield stress subject to a maximum of 190	 190 190

Notes:

1) For high yield strength deformed bars of Grade Fe-500, the permissible stress in direct tension and flexural tension shall be 0.55 f_y. The permissible stresses for shear and compression reinforcement shall be as for Grade Fe-415.
2) For welded wire fabric conforming to IS:1566, the permissible value in tension is 230 N/mm².
3) For the purposes of this standard, the yield stress of steels for which there is no clearly defined yield point should be taken to be 0.2 percent proof stress.

4) When mild steel conforming to Grade II of IS: 432(Part-1) is used, the permissible stresses in col.3, or if the design details have already been worked out on the basis of mild steel conforming to Grade I of IS: 432 (Part-1), the area of reinforcement shall be increased by 10% of that required for Grade I steel.

Table 4.2 Permissible stresses in concrete (IS: 456-2000) (Table-21 of IS: 456-2000)

Grade of concrete	All values in N/mm² — Permissible stress in compression: Bending (σ_{cbc})	Direct (σ_{cc})	Permissible stress in bond (Avg.) for plain bars in tension. (σ_{bd})
M-10	3.0	2.5	
M-15	5.0	4.0	0.6
M-20	7.0	5.0	0.8
M-25	8.5	6.0	0.9
M-30	10.0	8.0	1.0
M-35	11.5	9.0	1.1
M-40	13.0	10.0	1.2

in Table 4.2 are obtained by applying a factor of safety of 3 to characteristic strength of concrete. Accordingly the permissible values of stresses in steel are obtained by applying a factor of safety of 1.78.

In the design of reinforced concrete members, the most commonly used grades of concrete are M-20 and M-25. The revised Indian standard code IS: 456-2000 prescribes M-20 as the minimum grade of concrete for reinforced concrete while M-15 and M-10 may be used for plain concrete constructions.

For Design office use, it is convenient to use the values of design coefficients 'j' and 'Q' to check the depth of the section and to compute the area of reinforcements required to resist the working moment 'M' using equations (3) and (4). The values of design coefficients are compiled in Table 4.3, for the most commonly used grades of concrete.

4.3 BALANCED, UNDER REINFORCED AND OVER REINFORCED SECTIONS

In reinforced concrete sections, the depth of neutral axis generally determines the type of section. The analysis of reinforced concrete sections

Table 4.3 Design Coefficients

σ_{cbc} (N/mm²)	M	σ_{st} (N/mm²)	k	j	Q
7	13.33	140 230 280	0.400 0.288 0.250	0.87 0.90 0.92	1.22 0.91 0.80
8.5	11	140 230 280	0.400 0.288 0.250	0.87 0.90 0.92	1.48 1.10 0.98
10	9.33	140 230 280	0.400 0.288 0.250	0.87 0.90 0.92	1.74 1.30 1.15

include the determination of 'critical neutral axis' which depends only on the permissible stresses in concrete and steel and modular ratio and the actual neutral axis, which is influenced by the sectional properties and the quantity of reinforcement used in the section.

Referring to the Fig. 4.2

Let b = width of section
d = effective depth.
n_c = critical neutral axis depth.
A_{st} = Area of tension reinforcement.
σ_{st} = Permissible Tensile stress in steel.
σ_{cbc} = Permissible Compressive stress in concrete.
m = modular ratio = $(280/3\sigma_{cbc})$

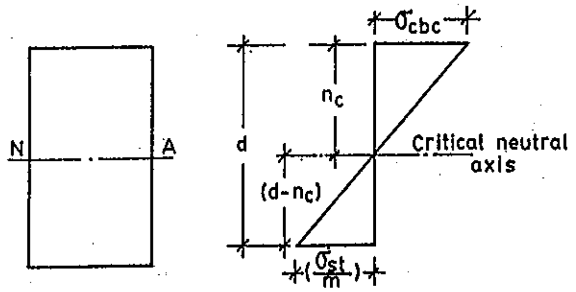


Fig. 4.2 Depth of Critical Neutral Axis

From the stress distribution diagram

$$\frac{\sigma_{cbc}}{(\sigma_{st}/m)} = \left[\frac{n_c}{d-n_c}\right]$$

Solving, the critical neutral axis depth is computed by the relation

$$n_c = \left[\frac{1}{1+(\sigma_{st}/m\sigma_{cbc})}\right]$$

Referring to the Fig. 4.3

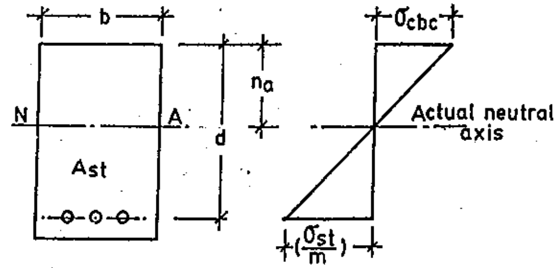


Fig. 4.3 Depth of Actual Neutral Axis

Let n_a = actual neutral axis depth. By equating the first moment of areas above and below the neutral axis, we have

$$0.5\, b n_a^2 = m\, A_{st}\,(d-n_a)$$

Solving this quadratic equation, the actual neutral axis depth can be determined.

Case-1 Under reinforced section

If $n_a < n_c$, the section is under reinforced. The moment of resistance is computed from tension side with steel reaching the maximum permissible stress σ_{st} and the moment of resistance is computed from Fig. 4.4.

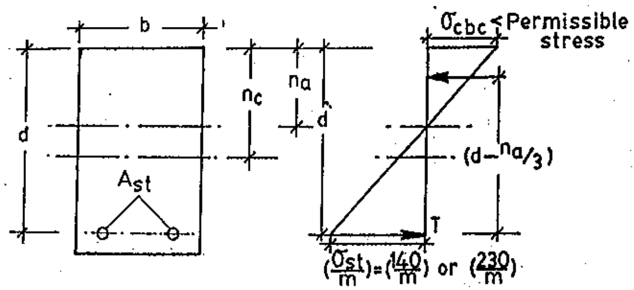


Fig. 4.4 Under Reinforced Section

If M_r = Moment of Resistance.

$$M_r = T.(d - n_a/3)$$

$$M_r = \sigma_{st} A_{st} (d - n_a/3)$$

Where σ_{st} = 140 N/mm^2 for Grade–I, Mild steel.
= 230 N/mm^2 for HYSD bars.

Case-2 Over reinforced Section

If $n_a > n_c$, the section is overreinforced (more reinforcement used) and the moment of resistance is computed from the compression side since the concrete in the extreme fibres reach the permissible stress σ_{cbc} first, hence the moment of resistance is computed from Fig. 4.5.

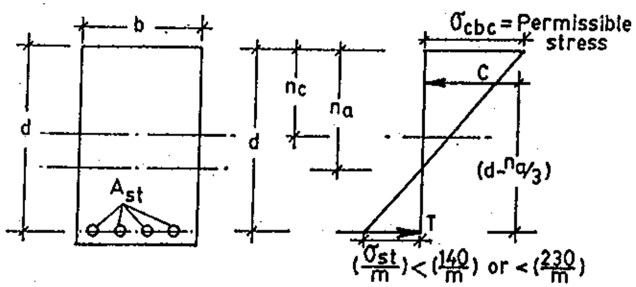


Fig. 4.5 Over Reinforced Section

If M_r = moment of resistance.

$$M_r = 0.5\ \sigma_{cbc} \cdot n_a \cdot b\ (d - n_a/3)$$

Where σ_{cbc} = 7 N/mm^2 for M-20 grade concrete.

Case-3 Balanced Section

If $n_a = n_c = n$, then the section is balanced. In this case, the steel and concrete reach their maximum permissible stresses simultaneously and the moment of resistance can be computed either from the compression or tension side.

From Fig. 4.6

$$M_{rb} = \sigma_{st} . A_{st}(d - n/3) = 0.5\ \sigma_{cbc} . n . b . (d - n/3)$$

Where M_{rb} = resisting moment of balanced section.

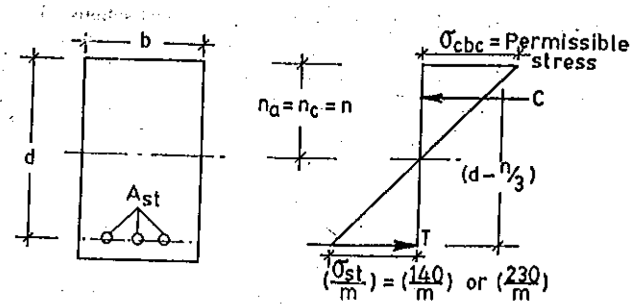


Fig. 4.6 Balanced Section

Also $C = 0.5\ \sigma_{cbc}\ k.d.b.$

$$T = \sigma_{st} \cdot A_{st}$$

Equating $C = T$

$$0.5\ \sigma_{cbc}\ k.d.b. = \sigma_{st} A_{st}$$

Hence percentage steel reinforcement in the balanced section is given by

$$p_{tb} = \left[\frac{100 A_{st}}{bd}\right] = 50\ k \left[\frac{\sigma_{cbc}}{\sigma_{st}}\right] \quad (1)$$

Also,

$$M_{rb} = 0.5\ \sigma_{cbc}\ .b.k.d\left(d - \frac{kd}{3}\right)$$

$$M_{rb} = 0.5\ \sigma_{cbc}\ .b.k.d^2\left(1 - \frac{k}{3}\right)$$

$$M_{rb} = 0.5\ \sigma_{cbc}\ .b.k.d^2.j \quad (2)$$

$$Q_b = \left(\frac{M_{rb}}{\sigma_{cbc}\ .b.d^2}\right) = 0.5\ k\left(1 - \frac{k}{3}\right) = 0.5\ k.j$$

Hence, Equations (1) and (2) can be conveniently used to compute the percentage reinforcement and moment of resistance of balanced sections in which steel and concrete reach the permissible stresses simultaneously and the section is economical since optimal utilization of materials is achieved in balanced sections. Typical values of the design constants p_{tb} and Q_b for different grades of concrete and steel most commonly used in structural concrete are compiled in Table 4.4.

Table 4.4 Design Constants for Balanced Sections

Grade of steel		Fe-415	Fe-500
k = Neutral Axis depth factor		0.288	0.253
Lever arm factor = j		0.904	0.916
Percentage Reinforcements p_b	M-20	0.439	0.322
	M-25	0.533	0.391
	M-30	0.627	0.460
$Q_b = (M_{rb} / \sigma_{cbc} b d^2)$		0.1304	0.1160

In practice, it is advisable to design R.C. sections as balanced or under reinforced since there will be clear warning of impending failure of the member in the form of larger deflections and well distributed cracks with smaller quantities of reinforcement. Over reinforced sections are not preferred since they require large quantities of reinforcement and the members under over loads fail suddenly with explosive failures and with negligible deflections and very few cracks.

Typical failure patterns of under reinforced and over reinforced beams are shown in Fig. 4.7 and 4.8 respectively.

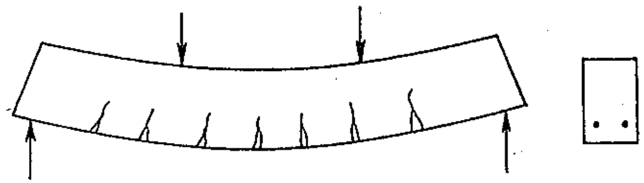


Fig. 4.7 Failure Pattern of Under Reinforced Beam

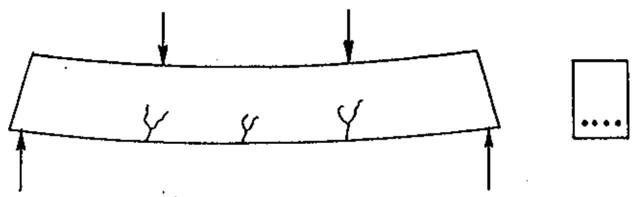


Fig. 4.8 Failure Pattern of Over Reinforced Beam

Table 4.5 shows the commonly used diameters of reinforcement along with the cross sectional areas and weight of bars. Table 4.6 shows the areas, perimeters and spacing of round bars.

Table 4.5 Areas of Given Number of Bars in Cm²

Number of Bars	Bar Diameter, mm 6	8	10	12	14	16	18	20	22	25	28	32	36
1	0.28	0.50	0.79	1.13	1.54	2.01	2.54	3.14	3.80	4.91	6.16	8.04	10.18
2	0.56	1.00	1.57	2.26	1.07	4.02	5.08	6.28	7.60	9.81	12.31	16.08	20.35
3	0.84	1.50	2.35	3.39	4.61	6.03	7.63	9.42	11.40	14.72	18.47	24.12	30.53
4	1.13	2.01	3.14	4.52	6.15	8.04	10.17	12.56	15.20	19.63	24.63	32.17	40.71
5	1.41	2.51	3.92	5.56	7.69	10.05	12.72	15.70	19.00	24.54	30.78	40.21	50.89
6	1.69	3.01	4.71	6.78	9.23	12.06	15.26	18.85	22.80	29.45	36.94	48.25	61.07
7	1.97	3.51	5.49	7.91	10.77	14.07	17.81	21.99	26.60	34.36	43.10	56.29	71.25
8	2.26	4.02	6.28	9.04	12.31	16.08	20.35	25.13	30.41	39.27	49.26	64.34	81.43
9	2.54	4.52	7.06	10.17	13.85	18.09	22.90	28.27	34.21	44.17	55.41	72.38	91.60
10	2.82	5.02	7.85	11.31	15.39	20.10	25.44	31.41	38.01	49.08	61.57	80.42	101.78
11	3.11	5.52	8.63	12.44	16.93	22.11	27.99	34.55	41.81	53.99	67.73	88.46	111.96
12	3.39	6.03	9.42	13.57	18.47	24.12	30.53	37.69	45.61	58.90	73.89	96.51	122.14
13	3.67	6.53	10.21	14.70	20.01	26.13	33.08	40.84	49.41	63.81	80.04	104.55	132.32
14	3.95	7.03	10.99	15.83	21.55	28.14	35.62	43.98	53.21	68.72	86.20	112.59	142.50
15	4.24	7.54	11.78	16.96	23.09	30.15	38.17	47.12	57.02	73.63	92.36	120.63	152.68
16	4.52	8.04	12.56	18.09	24.63	32.17	40.71	50.26	60.82	78.54	98.52	128.68	162.86
17	4.80	8.54	13.35	19.22	26.17	34.18	43.26	53.40	64.62	83.44	104.67	136.72	173.03
18	5.08	9.04	14.13	20.35	27.70	36.19	45.80	56.54	68.42	88.35	110.83	144.76	183.21
19	5.37	9.55	14.92	21.48	29.24	38.20	48.34	49.69	72.22	93.26	116.99	152.80	193.39
20	5.65	10.05	15.70	22.62	30.78	40.21	50.89	62.83	76.02	98.17	123.15	160.85	203.57

Table 4.6 Areas of Bars At Given Spacings Values in cm² per Meter Width

Spac.cm	Bar Diameter, mm										
	6	8	10	12	14	16	18	20	22	25	28	32
5	5.65	10.05	15.71	22.62	30.79	40.21	50.89	62.83	76.03	98.17	123.15	160.85
6	4.71	8.38	13.09	18.85	25.66	33.51	42.41	52.36	63.36	81.81	102.68	34.04
7	4.04	7.18	11.22	16.16	21.99	28.72	36.35	44.88	54.30	70.12	87.96	14.89
8	3.53	6.28	9.82	14.14	19.24	25.13	31.81	39.27	47.52	61.36	76.9	100.53
9	3.14	5.58	8.73	12.57	17.10	22.34	28.27	34.91	42.24	54.54	68.42	89.36
10	2.83	5.03	7.85	11.31	15.39	20.11	25.45	31.42	38.01	49.09	61.57	80.42
11	2.57	4.57	7.14	10.28	13.99	18.28	23.13	28.56	34.56	44.62	55.98	73.11
12	2.36	4.19	6.54	9.42	12.83	16.75	21.21	26.18	31.68	40.91	51.31	67.02
13	2.17	3.87	6.04	8.70	11.84	15.47	19.57	24.17	29.24	37.76	47.37	61.86
14	2.02	3.59	5.61	8.08	11.00	14.36	18.18	22.44	27.15	35.06	43.98	57.45
15	1.88	3.35	5.24	7.54	10.26	13.40	16.96	20.94	25.34	32.72	41.05	53.62
16	1.77	3.14	4.91	7.07	9.62	12.57	15.90	19.63	23.76	30.68	38.48	50.27
17	1.66	2.96	4.62	6.65	9.05	11.83	14.97	18.48	22.36	28.87	36.22	47.31
18	1.57	2.79	4.36	6.28	8.55	11.17	14.44	17.45	21.12	27.27	24.21	44.68
19	1.49	2.65	4.13	5.95	8.10	10.58	13.39	16.53	20.01	25.84	32.41	42.33
20	1.41	2.51	3.93	5.65	7.70	10.05	12.72	15.71	19.01	24.54	30.79	40.21
21	1.35	2.39	3.74	5.39	7.33	9.57	12.12	14.96	18.10	23.37	29.32	38.30
22	1.28	2.28	3.57	5.14	7.00	9.14	11.57	4.28	17.28	22.31	27.99	36.56
23	1.23	2.18	3.41	4.92	6.69	8.74	11.06	13.66	16.53	21.34	26.77	34.97
24	1.18	2.09	3.27	4.71	6.41	8.38	10.60	13.09	15.84	20.54	25.66	33.51

(Contd...)

Table 4.6 (Contd.)

Spac.cm	Bar Diameter, mm										
	6	8	10	12	14	16	18	20	22	25	28	32
25	1.13	2.01	3.14	4.52	6.16	8.07	10.18	12.57	15.20	19.63	24.63	32.17
26	1.09	1.93	3.02	4.35	5.92	7.73	9.79	12.08	14.62	18.88	23.68	30.93
27	1.05	1.86	2.91	4.19	5.70	7.45	9.42	11.64	14.08	18.18	22.81	29.79
28	1.01	1.79	2.80	4.04	5.50	7.18	9.09	11.22	13.58	17.53	21.99	28.76
29	0.97	1.73	2.71	3.90	5.31	6.93	8.77	10.83	13.11	16.93	21.23	27.73
30	0.94	1.68	2.62	3.77	5.13	6.70	8.48	10.47	12.67	16.36	20.52	26.81
32	0.88	1.57	2.45	3.53	4.18	6.28	7.95	9.82	11.88	15.34	19.24	25.13
34	0.83	1.48	2.31	3.33	4.53	5.91	7.48	9.24	11.18	14.44	18.11	23.65
36	0.78	1.40	2.18	3.14	4.28	5.58	7.07	8.73	10.56	13.63	17.10	22.34
38	0.74	1.32	2.07	2.98	4.05	5.29	6.70	8.27	10.00	12.92	16.20	21.16
40	0.71	1.26	1.96	2.83	3.85	5.03	6.36	7.85	9.50	12.27	15.39	20.11

These tables are very useful in design computations of reinforcements in structural concrete members like slabs, beams and columns.

4.4 ANALYSIS OF EXAMPLES OF R.C. SECTIONS

4.4.1 Example

A singly reinforced concrete beam with an effective span of 4m has a rectangular section with a width of 250 mm and an overall depth of 550 mm. The beam is reinforced with 3 bars of 10 mm diameter Fe-415 HYSD bars at an effective depth of 500 mm. The self-weight of beam together with the dead load is 4 kN/m. Calculate the maximum permissible live load on the beam. Assume M-20 grade concrete.

a) Data

Effective Span = 4 m
Width of beam = 250 mm
Effective depth = 500 mm
Overall depth = 550 mm
Tension steel (A_{st}) = (3 × 78.5) = 235.5 mm^2.
Materials: M-20 Grade concrete and Fe-415 HYSD bars.

b) Permissible Stresses

$\sigma_{cbc} = 7\ \text{N/mm}^2$ $\quad m = 13.33$

$\sigma_{st} = 230\ \text{N/mm}^2$ $\quad n_c = 0.288d$

c) Loads and Moment

Self-weight and dead load = g = 4 kN/m.

$\therefore\ M_g = (0.125 \times 4 \times 4^2) = 8$ kN.m

d) Actual Neutral Axis Depth

If n_a = depth of actual neutral axis.

$$0.5\, b\, n_a^2 = m.A_{st}\,(d - n_a)$$

$$(0.5 \times 250 \times n_a^2) = 13 \times 235.5\,(500 - n_a)$$

Solving n_a = 100.2 mm.

e) Critical Neutral Axis Depth

If n_c = Critical neutral axis depth.

$$n_c = \left[\frac{1}{1 + \left(\frac{\sigma_{st}}{m \cdot \sigma_{cbc}}\right)}\right] d = \left[\frac{1}{1 + \left(\frac{230}{13.33 \times 7}\right)}\right] 500 = 144.3 \text{ mm.}$$

Since $n_a < n_c$, the section is underreinforced.

f) Moment of Resistance

$$M_r = \sigma_{st} A_{st}\left(d - \frac{n_a}{3}\right)$$

$$= (230 \times 235.5)(500 - 100.2/3)$$

$$= (25.3 \times 10^6)\ \text{N.mm}$$

$$= 25.3\ \text{kN.m}$$

g) Permissible Live load

M_q = Live load moment = [25.3 − 8] = 17.3 kN.m
If q = live load on beam

$$q = \left(\frac{8M_q}{L^2}\right) = \left(\frac{8 \times 17.3}{4^2}\right) = 8.65\ \text{kN/m}$$

∴ (Permissible Live load on beam = 8.65 kN/m

4.4.2 Example

A reinforced concrete beam of rectangular section 300 mm wide by 650 mm over all depth is reinforced with 4 bars of 32 mm diameter at an effective depth of 600 mm. Using M-20 Grade concrete and Fe-415 HYSD bars, estimate the moment of resistance of the section.

a) Data

Width of beam = 300 mm
Effective depth = 600 mm

Area of tension steel = $(A_{st}) = (4 \times 804) = 3216\ \text{mm}^2$.
Materials: M-20 Grade concrete
Fe-415 HYSD bars.

b) Permissible Stresses

$\sigma_{cbc} = 7\ \text{N/mm}^2$ $\quad m = 13.33$
$\sigma_{st} = 230\ \text{N/mm}^2$ $\quad n_c = 0.288\ d$

c) Neutral Axis Depth

If n_a = depth of actual neutral axis.

$$(b\ n_a^2/2) = m.A_{st}\ (d - n_a)$$

$$(0.5 \times 300\ n_a^2) = [13.33 \times 32.16\ (600 - n_a)]$$

Solving $n_a = 295.16$ mm.

Critical neutral axis depth is

$$n_c = \left[\frac{1}{1+\left(\frac{\sigma_{st}}{m \cdot \sigma_{cbc}}\right)}\right] d = \left[\frac{1}{1+\left(\frac{230}{13.33 \times 7}\right)}\right] 600 = 172.8\ \text{mm}.$$

d) Moment of Resistance

Since $n_a > n_c$, the section is over reinforced.

$\therefore$ Moment of Resistance is computed as

$$M_r = 0.5.\sigma_{cbc}.b.n_a\ (d - n_a/3)$$

$$= [0.5 \times 7 \times 300 \times 295.16(600 - 295.16/3)]$$

$$= (155.46 \times 10^6)\ \text{N.mm}$$

$$= 155.46\ \text{kN.m.}$$

4.4.3 Example

The cross section of an R.C.C beam of rectangular section is to be designed to resist a bending moment of 65 kN.m. Assuming the width of beam as half the effective depth, determine the dimensions of the beam and the area of tension reinforcement for the balanced section. Adopt M-20 Grade concrete and Fe-415 Grade HYSD bars.

a) Data

Moment of Resistance = M_r = 65 kN.m
Width of beam = b
Effective depth = d.
Materials: M-20 Grade Concrete.
Fe-415 Grade HYSD bars.

b) Permissible Stresses

$\sigma_{cbc} = 7\ \text{N/mm}^2$ $\quad Q = 0.91.$
$\sigma_{st} = 230\ \text{N/mm}^2$ $\quad j = 0.90$
$m = 13.33$ $\quad b = 0.5\ d$

c) Cross Sectional Dimensions

$M_r = Q.b.d^2$
$(65 \times 10^6) = (0.91 \times 0.5 \times d \times d^2)$
Solving $d = 522.8$ mm
$\therefore\ b = (0.5 \times 522.8) = 261.4$ mm
Cover = 40 mm.
Adopt a section 265 mm by 570 mm,
Provided, $d = (570 - 40) = 530$ mm.

d) Reinforcements

Area of tension Reinforcement is

$$A_{st} = \left(\frac{M_r}{\sigma_{st}.j.d}\right) = \left(\frac{65 \times 10^6}{230 \times 0.9 \times 530}\right) = 593\ \text{mm}^2$$

4.4.4 Example

A reinforced concrete beam of rectangular section is required to resist a service moment of 120 kN.m. Design suitable dimensions and reinforcements for the balanced section of the beam assuming M-20 grade concrete and Fe-415 grade HYSD bars.

a) Data

Moment of Resistance of the balanced section = M_{rb} = 120 kN.m
Width of beam = b

Effective depth = d
Materials: M-20 Grade concrete and Fe-415 Grade HYSD bars.

b) Permissible stresses

$\sigma_{cbc} = 7\ \text{N/mm}^2$ Assume $b = (d/2)$
$\sigma_{st} = 230\ \text{N/mm}^2$
$m = 13.33$

c) Cross Sectional Dimensions

For the balanced section (Table 4.4)

$$\left(\frac{M_{rb}}{\sigma_{cbc}.b.d^2}\right) = Q_b = 0.1301$$

$$\left(\frac{120\times10^6}{7\times0.5d\times d^2}\right) = Q_b = 0.1301$$

Solving $d = 641$ mm.
∴ Effective depth = $d = 641$ mm
Cover = 39 mm.
Adopt overall depth, $D = 680$ mm,
Width of section $b = 0.5\ d = 321$ mm.

d) Reinforcements

Reinforcement in balanced section is computed from Table 4.4 as

$$p_{tb} = \left(\frac{100A_{st}}{bd}\right) = 0.438$$

$$\therefore \quad A_{st} = \left(\frac{0.438\ bd}{100}\right) = \left(\frac{0.438\times321\times641}{100}\right) = 901.23\ \text{mm}^2$$

Adopt 3 bars of 20mm ($A_{st} = 942\text{mm}^2$)

4.4.5 Example

Compute the moment of resistance of the reinforced concrete section shown in Fig. 4.9. The beam section is reinforced with tension and compression reinforcement. Adopt M-20 grade concrete and Fe-415 HYSD bars.

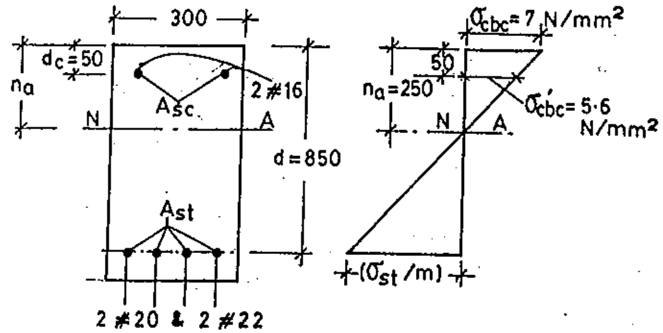


Fig. 4.9 Reinforced Concrete Section

a) Data

Area of tension steel (A_{st}) = 2 bars of 20 mm diameter and 2 bars of 22 mm diameter.
∴ $A_{st} = 1388\ \text{mm}^2$
Area of compression steel (A_{sc}) = 2 bars of 16 mm diameter = 402 mm²
Cover to compression steel = d_c = 50 mm
Width of beam = b = 300 m z
Effective depth = d = 850 mm
Materials: M-20 grade concrete and Fe-415 HYSD bars.

b) Permissible Stresses

$\sigma_{cbc} = 7\ \text{N/mm}^2$ $m = 13$ and $m_c = 1.5m$

$\sigma_{st} = 230\ \text{N/mm}^2$ $n_c = 0.288\ d$

c) Depth of Neutral Axis

Let n_a = actual depth of neutral axis

First moment of the areas above and below the neutral axis yields the following relation.

$$0.5b.n_a^2 + (1.5m-1)A_{sc}(n_a - d_c) = mA_{st}(d - n_a)$$

$$(0.5\times300n_a^2) + [(1.5\times13)-1]402(n_a - 50) = (13\times1388)(850 - n_a)$$

Solving, $n_a = 250$ mm
Critical neutral axis depth is given by the relation

$$n_c = \left[\frac{1}{1+\frac{\sigma_{st}}{(m.\sigma_{cb})}}\right]d = \left[\frac{1}{1+\left(\frac{230}{13\times7}\right)}\right]850 = (0.284\times850) = 241.4\ \text{mm}$$

Since $n_a > n_c$, the section is over reinforced.

∴ Moment of resistance of the section is computed from the compression side as

$$M_{rc} = 0.5\,\sigma_{cbc}.n_a.b(d - n_a/3) + (1.5m - 1)A_{sc}\sigma'_{cbc}(d - d_c)$$

$$= (0.5 \times 7 \times 250 \times 300)\,[850 - (250/3)] + [(1.5 \times 13) - 1]402 \times 5.6(850 - 50)$$

$$= (234.56 \times 10^6)\ \text{N.mm}$$

$$= 234.56\ \text{kN.m}$$

4.5 EXAMPLES FOR PRACTICE

1) A singly reinforced simply supported beam 200 mm wide by 550 mm overall depth is reinforced with 4 bars of 12 mm diameter at an effective depth of 500 mm. The self weight of the beam together with the dead load is 3.5 kN/m. Adopting M-20 grade concrete and Fe-415 HYSD bars estimate,the maximum permissible live load on the beam.
2) A reinforced concrete beam of rectangular section 300 mm wide by 650 mm deep is reinforced with 4 bars of 25 mm diameter at an effective depth of 600 mm. Calculate the neutral axis depth and estimate the safe moment of resistance of the section adopting M-25 grade concrete and Fe-415 HYSD bars.
3) A reinforced concrete beam of rectangular section 250 mm wide by 550 mm deep is reinforced with 4 bars of 32 mm diameter at an effective depth of 500 mm. Using M-20 grade concrete and Fe-415 HYSD bars, calculate the safe moment of resistance of the beam.
4) A reinforced concrete beam of rectangular section 350 mm wide by 750 mm overall depth is reinforced with 3 bars of 20 mm diameter at an effective depth of 700 mm. Adopting M-30 grade concrete and Fe-500 grade steel reinforcement, calculate the safe moment of resistance of the section. If the beam spans over 5 m, estimate the safe permissible live load on the beam.
5) A reinforced concrete beam of rectangular section is to be designed to resist a service load moment of 200 kN.m. Assuming the width of the beam as half the effective depth, calculate the dimensions of the beam adopting M-20 grade concrete and Fe-415 HYSD bars.
6) A reinforced concrete beam of rectangular section having a width of 400 mm and overall depth 850 mm is reinforced with 4 bars of 25 mm diameter both on the compression and tension sides at an effective cover of 50 mm. Using M-20 grade concrete and Fe-415 HYSD bars, compute a) the actual neutral axis, b) the critical neutral axis and c) the safe moment of resistance of the section.
7) A reinforced concrete beam of rectangular section 300 mm wide by 600 mm overall depth is reinforced with 3 bars of 36 mm diameter at an effective depth of 550 mm. The section is also reinforced with 2 bars of 25 mm diameter on the compression side at an effective cover of 50 mm. Adopting M-15 grade concrete and Fe-250 grade steel, calculate a) the stresses developed in concrete and steel corresponding to a service load moment of 175 kN.m. b) Determine the safe moment of resistance of the section.
8) A reinforced concrete rectangular section 300 mm wide by 600 mm overall depth is reinforced with 4 bars of 25 mm diameter at an effective cover of 50 mm on the tension side. Assuming M-20 grade concrete and Fe-415 HYSD bars, determine the allowable bending moment and the stresses in steel and concrete corresponding to this moment.

CHAPTER 5

Limit State Method of Design

5.1 PHILOSOPHY OF LIMIT STATE DESIGN

The inadequacies of the elastic and ultimate load methods of design paved the way for the limit state method of design with a semi-probabilistic approach. Limit state design is a method of designing structures based on a statistical concept of safety and the associated statistical probability of failure. Structures designed should satisfy the dual criterion of

a) Safety and
b) Serviceability.

Safety may be defined as an acceptable degree of security against complete collapse or failure, which in concrete structures can occur by various modes such as compression, tension, flexure, shear, torsion, fatigue or their combinations.

Serviceability requirement means that the member or structure should not in its intended lifetime deteriorate to such an extent that it fails to fulfil its function for which it is designed. In concrete structures, this state may be reached due to excessive deflection, cracking, vibration, corrosion of reinforcement etc.

Limit state design philosophy[38, 39, 40] uses the concept of probability and is based on the application of the method of statistics to the variations that occur in practice in the loads acting on the structure and the strengths of the materials. The evolution of limit State method of design over the years is presented in Fig. 5.1.

5.2 LIMIT STATE DESIGN AND CLASSICAL RELIABILITY THEORY

In Limit state design, probabilistic concepts are explicitly incorporated for the first time. Applications of classical reliability theory[41, 42] to structural design require comprehensive statistical data regarding loads and strengths and their exact shapes of normal distribution curves. At present the probabilities of failure that are socially acceptable must be kept very low (1 in a million). At such low levels, the probability of failure is very sensitive to the exact shape of the normal distribution curves. To determine exact shapes of normal distribution curves, we require very large numbers of

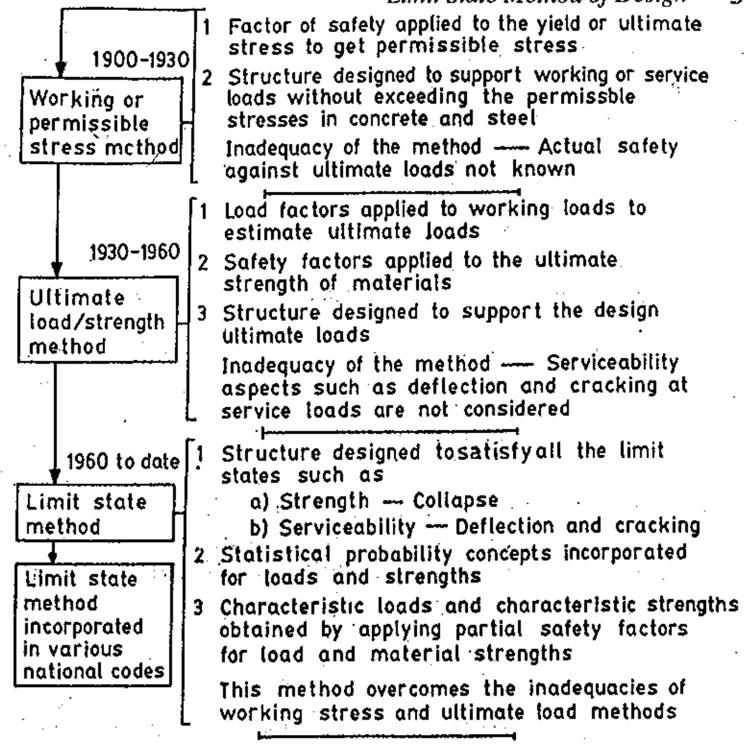


Fig. 5.1 Evolution of Limit State Design Method

statistical data and such comprehensive data is not yet available. In particular, sufficient numbers of extreme values of the strengths of complete structures (to define accurately the shapes of the tails of the normal distribution curves) may never be available.

In a simple example, only one type of load and one-strength variables are used. For a real structure, there will in general be many types of loads and many modes of failure, normally with complex correlations between them making it very difficult to calculate the probability of failure[43]. Hence in the limit state design, our engineering experience and judgement have been used to modify and to remedy the inadequacies of earlier design methods and partly use the probabilistic concepts. Hence, it is appropriate to designate the limit state design method currently practiced as **Semi probabilistic approach** to structural design.

5.3 LIMIT STATES

A structure may become unfit for its intended purpose in a number of ways in terms of either Safety or Serviceability. The prominent limit states are:

a) Ultimate or Failure Limit State

The structure may collapse due to rupture of one or more critical sections, as a result of static, sustained, pulsating or dynamic loading or loss of overall stability, disintegration due to fire or frost.

b) Serviceability Limit State

The structure may exhibit excessive deflections or displacements adversely affecting the finishes causing discomfort to the users. Also the structure may suffer excessive local damage resulting in cracking or spalling of concrete which impairs the efficiency or appearance of the structure.

5.4 SAFETY FACTORS

Safety is expressed in terms of the probability that the structure will not become unfit for its intended function during its useful life that is the structure will not reach a limit state. The initial idea of referring to a single failure criterion has been replaced by the comprehensive concept of multiple limit states. With this concept the local or the overall behaviour in all stages–elastic, cracked, inelastic and ultimate–are considered. In the limit state approach, a structure is considered as well designed if it could be shown that the probability of any limit state being attained is substantially constant for all the component members and for the structure as a whole and that consequently, the latter posses adequate and uniform structural safety. Due to the number of variables involved, a rational determination of the safety of a structure, based on probability theory is not yet practical in the design office. Partial safety factors are therefore introduced for each limit state and these consist of γ_m, reduction factor for characteristic strength of materials and γ_f, enhancement factors for characteristic loads on the structure.

5.5 CHARACTERISTIC AND DESIGN STRENGTHS AND PARTIAL SAFETY FACTORS

The variation in the properties of concrete and steel are expressed as characteristic values related to the mean values and standard variation.

Characteristic Strength (f) = [Mean strength – $k \times$ Standard Deviation]

Where 'k' is a factor chosen to ensure that the probability of the characteristic strength not being exceeded is small. Many of the national codes including the Indian standard code IS: 456 - 2000 have recommended a value of 1.65 for k so that only 5 percent of the test results could have a strength less than the characteristic strength.

In the absence of statistical data, the characteristic strength of concrete and steel may be taken as the works cube strength and minimum proof or yield strength respectively as recommended in the current codes.

Since the materials in the structure are likely to differ in quality from those tested, design strengths are obtained by dividing the characteristic strength by γ_m, the appropriate partial safety factor for the Limit State being considered. The proposed values for the partial safety factors are as given in Table 5.1.

In contrast the ACI Code[44] provides for these variations in material strengths and workmanship in the form of capacity reduction factors. Hence we have

$$\text{Design Strength} = \left(\frac{\text{Characteristic Strength}}{\text{Partial Safety Factor}}\right)$$

Table 5.1 Partial safety Factors for Material Strengths (γ_m) (IS: 456-2000)

Material	Limit State		
	Collapse	Deflection	Local Damage
Steel	1.15	1.00	1.00
Concrete	1.50	1.00	1.00 or 1.30

5.6 CHARACTERISTIC AND DESIGN LOADS AND PARTIAL SAFETY FACTORS

The Characteristic loads are expressed as

Characteristic Load (F) = [Mean Load + $k \times$ Standard Deviation]

Where k is a factor, which ensures that the probability of the characteristic load being exceeded is small. For the immediate future, the characteristic loads can not be assessed in this way due to lack of statistical information about the nature of loads and it is necessary to assume that the characteristic loads are equivalent to the values of loads currently recommended in the loading standards IS: 875[17-20]. The revised code IS: 456-2000 distinguishes between three types of loading in traditional use which are dead, imposed or live and wind load. In addition, loads resulting from the effects of creep, shrinkage or temperature are also considered if their effect is judged to be significant.

The characteristic loads do not allow for lack of precision in design calculation and inadequacies in the methods of analysis and construction. As such the design loads are obtained by enhancing the characteristic loads by suitable partial safety factors for the various limit states as given in Table 5.2. Hence we have,

Design Load = (*Characteristic load*) × (*Partial safety factor*)

Table 5.2 Partial Safety Factors for Loads (γ_f)
(Table-18 Of IS: 456-2000)

Load Combination	Limit State of Collapse			Limit state of Serviceability		
	DL	LL	WL	DL	LL	WL
DL + LL	1.5		–	1.0	1.0	–
DL + WL	1.5 or 0.9*	–	1.5	1.0	–	1.0
DL + LL + WL	1.2			1.0	0.8	0.8

Note

1) While considering earthquake effects, substitute EL for WL
2) For the Limit states of serviceability, the values of γ_f given in this Table are applicable for short-term effects. While assessing the Long-term effects due to creep, the dead load and that part of the Live Load likely to be permanent may only be considered.
   * This value is to be considered when stability against overturning or stress reversal is critical.

The interaction between load effects and strength is shown in Fig. 5.2 where the normal distribution curves for loads and material strength are superposed.

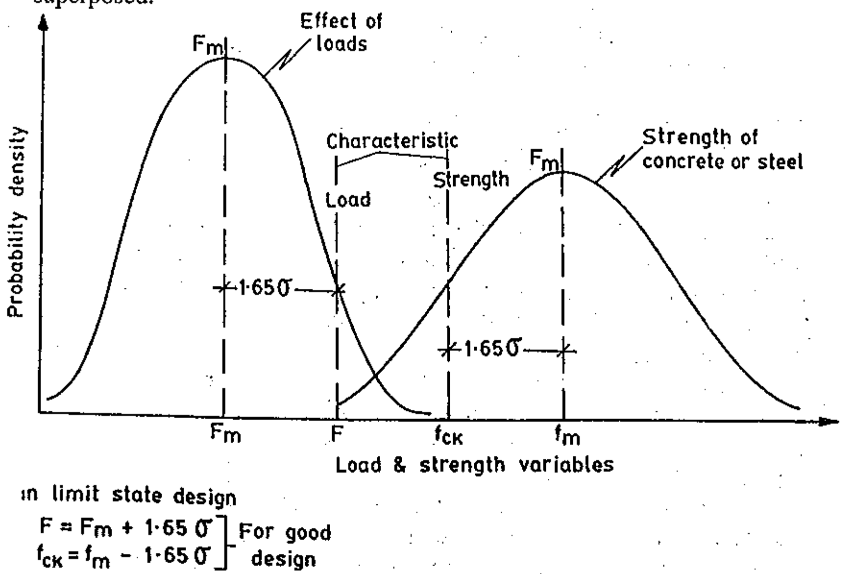


Fig. 5.2 Classical Reliability Model for Strength Design

The characteristic loads and strengths are expressed in terms of the standard deviation, mean strength, and the probability factor as,

$$F = F_m + 1.65\,\sigma$$

$$f_{ck} = f_m - 1.65\,\sigma$$

Where F = characteristic load
F_m = mean load
f_{ck} = characteristic strength
f_m = mean strength
σ = standard deviation.

CHAPTER 6

Ultimate Strength of Reinforced Concrete Sections

6.1 INTRODUCTION

The most common types of structural concrete elements comprise, slabs, beams and columns, which are used extensively in buildings of all types. Reinforced concrete slabs are primarily subjected to flexure and shear while beams have to be designed to resist flexure and shear and also torsion or a combination of these forces in some cases. Columns are primarily designed for compression but in some cases for compression and bending, which develops in edge or corner columns. The composite action of steel and concrete is mainly due to bond and anchorage.

This Chapter deals with the theoretical concepts involved in developing the strength computations of reinforced concrete sections under different states of stress and the basis of the various formulae given in IS: 456-2000, Section-5. Structural Design (Limit State Method) and Annexure-G containing various formulae for the computation of flexural strength of rectangular and flanged reinforced concrete sections.

Structural designers and students should be familiar with the derivations to have a better insight into the design process. However practical designers may use the formulae given in the codes or the tables and charts of "Design Aids to IS:456" published as special publication SP:16[45], by the Bureau of Indian Standards. Designers may also refer to the Manual for limit state design of Reinforced concrete members authored by Varyani and Radhaji[46] which contains exhaustive design tables and charts to facilitate faster design of Structural concrete members.

6.2 ULTIMATE FLEXURAL STRENGTH OF RECTANGULAR SECTIONS

6.2.1 Assumptions

The following assumptions are relevant in the computations of ultimate flexural strength of reinforced concrete sections as specified in IS: 456-2000 Clause 38.1,

1) Plane sections normal to the axis remain plane after bending.
2) The maximum strain in concrete at the extreme compression fibre is assumed as 0.003 in flexure.
3) The relationship between the compressive stress distribution in concrete and the strain in concrete may be assumed to be rectangle, trapezoid, parabola or any other shape which results in prediction of strength in substantial agreement with the test results. The recommended stress-strain curve is shown in Fig. 6.1, which shows the characteristic and design strength curves.

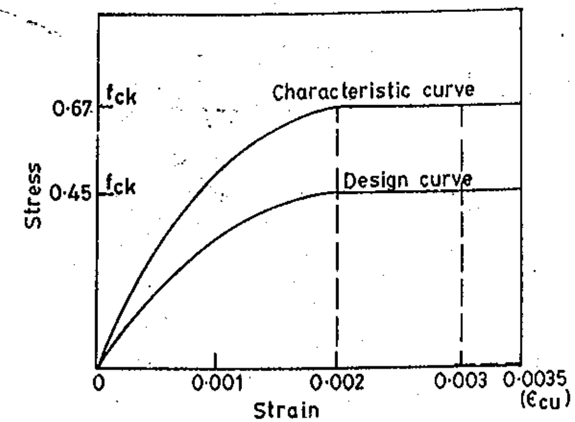


Fig. 6.1 Characteristics and Design Strength Curves for Concrete in Flexural Compression

Characteristic Strength $= f_{ck}$

Design Strength $= \left[\dfrac{0.67 f_{ck}}{\gamma_m}\right] = \left[\dfrac{0.67 f_{ck}}{1.5}\right] = 0.45 f_{ck}$

The stress block parameters are shown in Fig. 6.2.

Area of stress block is the sum of rectangular and parabolic portion and is computed as

$$A = (0.45 f_{ck}\, 0.42\, x_u) + (2/3 \times 0.45 f_{ck} \times 0.58\, x_u) = 0.36 f_{ck}.x_u.$$

where x_u = depth of Neutral Axis.

f_{ck} = Characteristic Compressive Strength.

Position of centre of compression from extreme compression fibre

$$= 0.42\, x_u$$

4) The tensile strength of concrete is ignored.

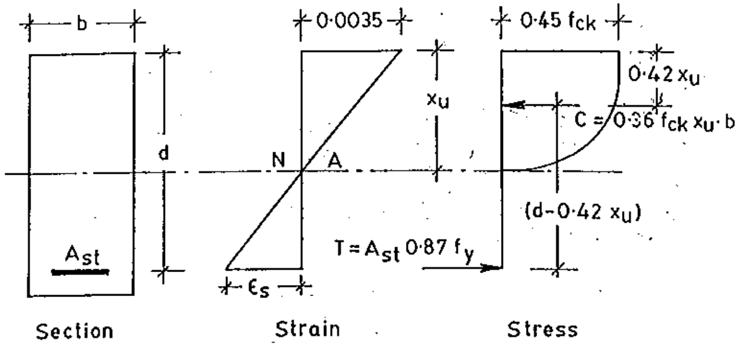


Fig. 6.2 Stress Block Parameters

5) The stresses in the reinforcemet are obtained from the stress-strain curves shown in Fig. 6.3. For design purposes the partial safety factor γ_m equal to 1.15 is applied to compute the design strength.

6) The maximum strain in the tension reinforcement in the section at the collapse limit state shall be not less than

$$\left[\frac{f_y}{1.15E_s}+0.002\right]=\left[\frac{0.87f_y}{E_s}+0.002\right]$$

Where f_y = characteristic strength of steel.
E_s = modulus of elasticity of steel.

6.2.2 Balanced, Under reinforced and Over reinforced Sections

Reinforced concrete sections in flexure reach the failure stage when the compressive strain in concrete reaches a value of 0.0035 as shown in Fig. 6.2. When the sections are reinforced in such a way that the tension steel reaches the yield strain of

$$\varepsilon_y=[(0.87f_y)/E_s+0.002]$$

and simultaneously the concrete strain is $\varepsilon_c = 0.0035$, then the section is termed as **Balanced**. In **Under reinforced** sections, the tension steel reaches yield strain at loads lower than the load at which concrete reaches the failure strain.

When the steel yields earlier than concrete, there will be excessive deflections and cracking with a clear indication of impending failure. Hence it is preferable to design beams as underreinforced since failure will take place after yielding of steel with clear warning signals like excessive deflections and cracking before the ultimate failure.

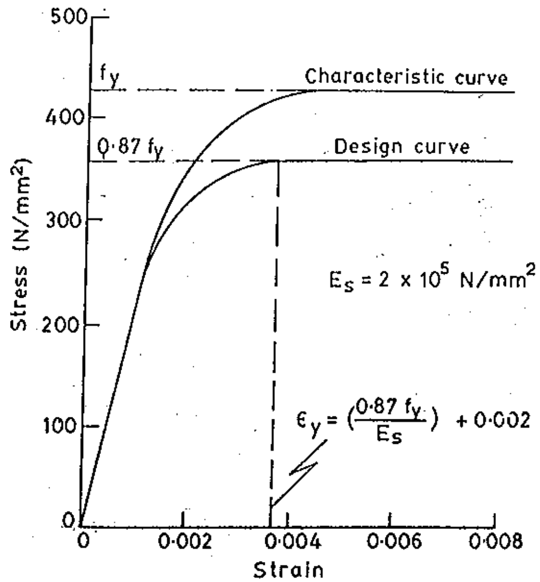


Fig. 6.3 Characteristic and Design Stress-Strain Curves for Fe 415 Grade Steel

Over reinforced sections are those in which concrete reaches the yield strain earlier than that of steel. Over reinforced beams fail by compression failure of concrete without much warning and with very few cracks and negligible deflections. Over reinforced concrete beams are not preferred since they require large quantities of steel and they fail suddenly with explosive failures without any warning.

6.2.3 Depth of Neutral Axis

Consider a rectangular beam section shown in Fig. 6.2.

Let b = width of section.
d = Effective depth.
A_{st} = Area of tension reinforcement.
x_u = depth of neutral Axis.

For equilibrium of forces at the limit state of collapse,

Total tension (T) = Total compression (C)

$$(A_{st}\cdot 0.87f_y)=0.36f_{ck}.b.x_u$$

$$\left(\frac{x_u}{d}\right)=\left[\frac{0.87f_yA_{st}}{0.36f_{ck}b.d}\right] \quad \ldots(6.1)$$

Limiting values of (x_u / d) to avoid brittle failure is determined from the condition that the steel strain ε_{su} at failure should be not less than the value given by

$$\varepsilon_{su} = \left[\frac{0.87 f_y}{E_s} + 0.002\right]$$

Assuming $E_s = 2 \times 10^5$ N/mm^2, the yield strain for design purposes for different grades of steel are given in Table 6.1.

From proportionality of strains, we have the relation,

$$\left(\frac{x_u}{d}\right) = \left[\frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{su}}\right] = \left[\frac{0.0035}{0.0035 + \varepsilon_{su}}\right] \qquad \ldots(6.2)$$

Substituting the various values of ε_{su} for different grades of steel, the maximum limiting values of (x_u/d) for different grades of steel are also shown in Table 6.1.

Table 6.1 Limiting values of $(x_{u,max} / d)$

Grade of Concrete	f_y	Yield strain (ε_{su})	$(x_{u,max} / d)$
Fe-250 Mild Steel	250	0.0031	0.53
HYSD bars Fe-415	415	0.0038	0.48
HYSD bars Fe-500	500	0.0042	0.46

6.2.4 Moment of Resistance of Reinforced Concrete Sections

The moment of resistance of rectangular reinforced concrete sections can be computed by using the stress diagram assumed at the limit state of collapse shown in Fig. 6.2. Taking moments about the centre of compression,

$$M_u = T\,(d - 0.42 x_u)$$

Substituting $x_u = \left[\frac{0.87 f_y A_{st}}{0.36 f_{ck} . b}\right]$ from Eq. (6.1)

and $T = (0.87 A_{st} . f_y)$

Hence $M_u = 0.87 A_{st} f_y \,[d - 0.42\,(0.87 f_y A_{st} / 0.36 f_{ck}\, b)]$

Simplifying and rearranging,

$$M_u = 0.87 A_{st} . f_y . d \left[1 - \frac{A_{st} . f_y}{b.d.f_{ck}}\right] \qquad \ldots(6.3)$$

Eq (6.3) can be used for estimating the flexural strength of sections in which (x_u / d) is less than the limiting value given in Table-6.1. This equation is specified in ANNEX-G of IS: 456-2000.

Expressing the area of steel as a percentage of the effective area, we have

$$p = \left[\frac{A_{st}}{bd}\right] \times 100$$

Where p is the percentage of steel. Substituting for (A_{st} / bd) from the above expression in Eq. (6.3), we get

$$M_u = 0.87 f_y \left(\frac{p}{100}\right)\left[1 - \frac{f_y . p}{f_{ck} . 100}\right] bd^2$$

$$\left[\frac{M_u}{bd^2}\right] = 0.87 f_y . \frac{p}{100}\left[1 - \left(\frac{f_y}{f_{ck}}\right)\left(\frac{p}{100}\right)\right] \qquad \ldots(6.4)$$

For a given value of $[M_u/bd^2]$, f_y and f_{ck}, the value of 'p' can be computed. This is presented in I.S.Publication SP: 16 as design tables. In these tables, the percentage of tension steel in the beam corresponds to the yield stress in steel when the beam fails by yielding of steel as in under reinforced sections. The design tables 1 to 4 in SP: 16 are very useful for structural designers to compute the percentage of tensile steel for known values of (M_u/bd^2) and different grades of steel and concrete. The moment of resistance of a concrete section can also be determined in terms of concrete strength by taking the moment of compression force about the tension force in steel, which yields the relation,

$$M_u = 0.36 f_{ck}\, b x_u (d - 0.42 x_u)$$

$$= 0.36 f_{ck} \left(\frac{x_u}{d}\right)\left[1 - 0.42\left(\frac{x_u}{d}\right)\right] bd^2$$

If $\left(\frac{x_u}{d}\right) = \left(\frac{x_{u,max}}{d}\right)$ which is the limiting value as given in Table-6.1,

then the limiting values of the moment of resistance of the section is given by,

$$M_{u,lim} = 0.36 f_{ck} \left(\frac{x_{u,max}}{d}\right)\left[1 - 0.42\left(\frac{x_{u,max}}{d}\right)\right] bd^2$$

$$= K.b.d^2 \qquad \ldots(6.5)$$

Where K = A constant.

For different grades of steel, the expression for M_u for different grades of steel are compiled in Table- 6.2.

Table 6.2 Moment of Resistance for Limiting Values of ($x_{u,max}/d$) for different Grades of Steel

Grade of Steel	$\left(\frac{x_{u,max}}{d}\right)$	Expression for M_u
Fe-250	0.53	$0.149 f_{ck} b d^2$
Fe- 415	0.48	$0.138 f_{ck} b d^2$
Fe- 500	0.46	$0.133 f_{ck} b d^2$

6.2.5 Expression for Steel Area for Balanced Singly Reinforced Section

Equating the compressive force in concrete and tensile force in steel (Fig. 6.2) we have.

$$0.87 f_y A_{st} = 0.36 f_{ck} b x_u$$

Rearranging the terms,

$$\left(\frac{A_{st}}{bd}\right) = \left(\frac{0.36 x_u}{0.87 d}\right)\left(\frac{f_{ck}}{f_y}\right) = (\text{Constant})\left(\frac{f_{ck}}{f_y}\right)$$

Since (x_u/d) is constant for a given value of f_y

If p_t = Limiting percentage of tension steel.

$$p_t = \left(\frac{100 A_{st}}{bd}\right)$$

The Reinforcement Index can be expressed as

$$\left(\frac{p_t f_y}{f_{ck}}\right) = \frac{100(0.36)}{0.87}\left(\frac{x_u}{d}\right) = 41.3\left(\frac{x_u}{d}\right) \qquad \ldots(6.6)$$

For different grades of steel, the reinforcement index and the limiting moment of resistance for singly reinforced rectangular sections are compiled in Table 6.3.

Table 6.3 Limiting moment of Resistance and Reinforcement Index for Singly Reinforced Rectangular Sections (Table-C of SP: 16)

f_y (N/mm²)	250	415	500
$\left(\frac{M_{u,lim}}{f_{ck} b d^2}\right)$	0.149	0.138	0.133
$\left(\frac{p_{t,lim} f_y}{f_{ck}}\right)$	21.97	19.82	18.87

Balanced percentage of steel, $p_{t\,lim}$, evaluated for different grades of concrete and steel are shown in Table 6.4.

Table 6.4 Balanced Percentage of Steel, $p_{t,lim}$ for Singly Reinforced Rectangular Sections (Table -E of SP: 16)

f_{ck} (N/mm²)	f_y (N/mm²)		
	250	415	500
15	1.32	0.72	0.57
20	1.76	0.96	0.76
25	2.20	1.19	0.94
30	2.64	1.43	1.13

6.2.6 Use of Design Charts and Tables of SP: 16 for Singly Reinforced beams and slabs

The Indian Standards Institution's special Publication SP: 16, Design Aids for Reinforced concrete to IS: 456(1978) or (2000) contains a number of charts and tables for design of reinforced concrete members. Based on Equations (6.3) and (6.6), the various charts and tables have been evolved.

The following are the data presented in SP: 16 for design and analysis of beams and slabs of singly reinforced concrete sections.

1) Tables 1 to 4 give the percentage steel required for various values of (M_u/bd^2) and f_y for concrete grades f_{ck} = 15, 20, 25, and 30. Three typical tables for f_{ck} = 20, 25 and 30 are presented in Tables 6.5, 6.6 and 6.7 in the text.
2) Tables 5 to 44 give the moment of resistance per metre width for various thicknesses of slabs (t = 10 to 25 cm) for different bar diameters and spacing for various values of f_y and f_{ck}.
3) Charts 1 to 18 present the moment of resistance per metre width for varying depths (5 to 80 cm) and varying percentage of steel and for two concrete grades of f_{ck} = 15 and 20 using steel grades of f_y = 250, 415 and 500.

The SP: 16 design tables and charts are very useful for structural designers, since the designs of beams and slabs can be quickly worked out and checked without using the detailed procedure of using the design equations.

6.2.7 Analysis Examples

1) A singly reinforced concrete beam having a width of 250 mm is reinforced with steel bars of area 3600 mm² at an effective depth of

Table 6.5 Flexure–Reinforcement Percentage, p_t For Singly Reinforced Sections
(Table–2 of SP: 16)

f_{ck} = 20 N/mm²

M_u/bd^2 N/mm²	f_y, N/mm² 240	250	415	480	500	M_u/bd^2 N/mm²	f_y, N/mm² 240	250	415	480	500
0.30	0.146	0.140	0.085	0.073	0.070	2.22	1.253	1.203	0.725	0.627	0.602
0.35	0.171	0.164	0.099	0.086	0.082	2.24	1.267	1.216	0.733	0.633	0.608
0.40	0.196	0.188	0.114	0.098	0.094	2.26	1.281	1.230	0.741	0.640	0.615
0.45	0.222	0.213	0.128	0.111	0.106	2.28	1.295	1.243	0.749	0.647	0.621
0.50	0.247	0.237	0.143	0.123	0.119	2.30	1.309	1.256	0.757	0.654	0.628
0.55	0.272	0.262	0.158	0.136	0.131	2.32	1.323	1.270	0.765	0.661	0.635
0.60	0.298	0.286	0.172	0.149	0.143	2.34	1.337	1.283	0.773	0.668	0.642
0.65	0.324	0.311	0.187	0.162	0.156	2.36	1.351	1.297	0.781	0.675	0.648
0.70	0.350	0.336	0.203	0.175	0.168	2.38	1.365	1.311	0.790	0.683	0.655
0.75	0.376	0.361	0.218	0.188	0.181	2.40	1.380	1.324	0.798	0.690	0.662
0.80	0.403	0.387	0.233	0.201	0.193	2.42	1.394	1.338	0.806	0.697	0.669
0.85	0.430	0.412	0.248	0.215	0.206	2.44	1.408	1.352	0.814	0.704	0.676
0.90	0.456	0.438	0.264	0.228	0.219	2.46	1.423	1.366	0.823	0.711	0.683
0.95	0.483	0.464	0.280	0.242	0.232	2.48	1.438	1.380	0.831	0.719	0.690
1.00	0.511	0.490	0.295	0.255	0.245	2.50	1.452	1.394	0.840	0.726	0.697

(Contd.)

Table 6.5 (Contd.)

f_{ck} = 20 N/mm²

M_u/bd^2 N/mm²	f_y, N/mm² 240	250	415	480	500	M_u/bd^2 N/mm²	f_y, N/mm² 240	250	415	480	500
1.05	0.538	0.517	0.311	0.269	0.258	2.52	1.467	1.408	0.848	0.734	0.704
1.10	0.566	0.543	0.327	0.283	0.272	2.54	1.482	1.423	0.857	0.741	0.711
1.15	0.594	0.570	0.343	0.297	0.285	2.56	1.497	1.437	0.866	0.748	0.719
1.20	0.622	0.597	0.359	0.311	0.298	2.58	1.512	1.451	0.874	0.756	0.726
1.25	0.650	0.624	0.376	0.325	0.312	2.60	1.527	1.466	0.883	0.764	0.733
1.30	0.678	0.651	0.392	0.339	0.326	2.62	1.542	1.481	0.892	0.771	0.740
1.35	0.707	0.679	0.409	0.354	0.339	2.64	1.558	1.495	0.901	0.779	0.748
1.40	0.736	0.707	0.426	0.368	0.353	2.66	1.573	1.510	0.910	0.786	0.755
1.45	0.765	0.735	0.443	0.383	0.367	2.68	1.588	1.525	0.919	0.794	
1.50	0.795	0.763	0.460	0.397	0.382	2.70	1.604	1.540	0,928		
1.55	0.825	0.792	0.477	0.412	0.396	2.72	1.620	1.555	0.937		
1.60	0.855	0.821	0.494	0.427	0.410	2.74	1.636	1.570	0.946		
1.65	0.885	0.850	0.512	0.443	0.425	2.76	1.651	1.585	0.955		
1.70	0.916	0.879	0.530	0.458	0.440	2.78	1.667	1.601			
1.75	0.947	0.909	0.547	0.473	0.454	2.80	1.683	1.616			

(Contd.)

Table 6.5 (Contd.)

f_{ck} = 20 N/mm²

M_u/bd^2 N/mm²	f_y, N/mm² 240	250	415	480	500	M_u/bd^2 N/mm²	f_y, N/mm² 240	250	415	480	500
1.80	0.978	0.939	0.565	0.489	0.469	2.82	1.700	1.632			
1.85	1.009	0.969	0.584	0.505	0.484	2.84	1.716	1.647			
1.90	1.041	1.000	0.602	0.521	0.500	2.86	1.732	1.663			
1.95	1.073	1.030	0.621	0.537	0.515	2.88	1.749	1.679			
2.00	1.106	1.062	0.640	0.553	0.531	2.90	1.766	1.695			
2.02	1.119	1.074	0.647	0.559	0.537	2.92	1.782	1.711			
2.04	1.132	1.087	0.655	0.566	0.543	2.94	1.799	1.727			
2.06	1.145	1.099	0.662	0.573	0.550	2.96	1.816	1.743			
2.08	1.159	1.112	0.670	0.579	0.556	2.98	1.833	1.760			
2.10	1.172	1.125	0.678	0.586	0.562						
2.12	1.185	1.138	0.685	0.593	0.569						
2.14	1.199	1.151	0.693	0.599	0.575						
2.16	1.212	1.164	0.701	0.606	0.582						
2.18	1.226	1.177	0.709	0.613	0.588						
2.20	1.239	1.190	0.717	0.620	0.595						

Note–Blanks indicate inadmissible reinforcement percentage (see Table E).

Table 6.6 Flexure–Reinforcement Percentage, p_t For Singly Reinforced Sections (Table-3 of SP: 16)

f_{ck} = 25 N/mm²

M_u/bd^2 N/mm²	f_y, N/mm² 240	250	415	480	500	M_u/bd^2 N/mm²	f_y, N/mm² 240	250	415	480	500
0.30	0.146	0.140	0.084	0.073	0.070	2.55	1.415	1.358	0.818	0.708	0.679
0.35	0.171	0.164	0.099	0.085	0.082	2.60	1.448	1.390	0.837	0.724	0.695
0.40	0.195	0.188	0.113	0.098	0.094	2.65	1.482	1.422	0.857	0.741	0.711
0.45	0.220	0.211	0.127	0.110	0.106	2.70	1.515	1.455	0.876	0.758	0.727
0.50	0.245	0.236	0.142	0.123	0.118	2.75	1.549	1.487	0.896	0.775	0.744
0.55	0.271	0.260	0.156	0.135	0.130	2.80	1.584	1.520	0.916	0.792	0.760
0.60	0.296	0.284	0.171	0.148	0.142	2.85	1.618	1.554	0.936	0.809	0.777
0.65	0.321	0.309	0.186	0.161	0.154	2.90	1.653	1.587	0.956	0.827	0.794
0.70	0.347	0.333	0.201	0.174	0.167	2.95	1.689	1.621	0.977	0.844	0.811
0.75	0.373	0.358	0.216	0.186	0.179	3.00	1.724	1.655	0.997	0.862	0.828
0.80	0.399	0.383	0.213	0.199	0.191	3.05	1.760	1.690	1.018	0.880	0.845
0.85	0.425	0.408	0.246	0.212	0.204	3.10	1.797	1.725	1.039	0.898	0.863
0.90	0.451	0.433	0.261	0.225	0.216	3.15	1.834	1.760	1.061	0.917	0.880
0.95	0.477	0.458	0.276	0.239	0.229	3.20	1.871	1.796	1.082	0.936	0.898
1.00	0.504	0.483	0.291	0.252	0.242	3.25	1.909	7.832	1.104	0.954	0.916

(Contd.)

Table 6.6 (Contd.)

$f_{ck} = 25$ N/mm²

M_u/bd^2 N/mm²	f_y, N/mm² 240	250	415	480	500	M_u/bd^2 N/mm²	f_y, N/mm² 240	250	415	480	500
1.05	0.530	0.509	0.307	0.265	0.255	3.30	1.947	1.869	1.126	0.973	0.935
1.10	0.557	0.535	0.322	0.279	0.267	3.32	1.962	1.884	1.135	0.981	0.942
1.15	0.584	0.561	0.338	0.292	0.280	3.34	1.978	1.899	1.144	0.989	
1.20	0.611	0.587	0.353	0.306	0.293	3.36	1.993	1.914	1.153		
1.25	0.638	0.613	0.369	0.319	0.306	3.38	2.009	1.929	1.162		
1.30	0.666	0.639	0.385	0.333	0.320	3.40	2.025	1.944	1.171		
1.35	0.693	0.666	0.401	0.347	0.333	3.42	2.040	1.959	1.180		
1.40	0.721	0.692	0.417	0.360	0.346	3.44	2.056	1.974	1.189		
1.45	0.749	0.719	0.433	0.374	0.359	3.46	2.072	1.989			
1.50	0.777	0.746	0.449	0.388	0.373	3.48	2.088	2.005			
1.55	0.805	0.773	0.466	0.403	0.387	3.50	2.104	2.020			
1.60	0.834	0.800	0.482	0.417	0.400	3.52	2.120	2.036			
1.65	0.862	0.828	0.499	0.431	0.414	3.54	2.137	2.051			
1.70	0.891	0.856	0.515	0.446	0.428	3.56	2.153	2.067			
1.75	0.920	0.883	0.532	0.460	0.442	3.58	2.170	2.083			

(Contd.)

Table 6.6 (Contd.)

$f_{ck} = 25$ N/mm²

M_u/bd^2 N/mm²	f_y, N/mm² 240	250	415	480	500	M_u/bd^2 N/mm²	f_y, N/mm² 240	250	415	480	500
1.80	0.949	0.911	0.549	0.475	0.456	3.60	2.186	2.099			
1.85	0.979	0.940	0.566	0.489	0.470	3.62	2.203	2.115			
1.90	1.009	0.968	0.583	0.504	0.484	3.64	2.219	2.131			
1.95	1.038	0.997	0.601	0.519	0.498	3.66	2.236	2.147			
2.00	1.068	1.026	0.618	0.534	0.513	3.68	2.253	2.163			
2.05	1.099	1.055	0.635	0.549	0.527	3.70	2.270	2.179			
2.10	1.129	1.084	0.653	0.565	0.542	3.72	2.287	2.196			
2.15	1.160	1.114	0.671	0.580	0.557	3.74	2.304				
2.20	1.191	1.143	0.689	0.596	0.572						
2.25	1.222	1.173	0.707	0.611	0.587						
2.30	1.254	1.204	0.725	0.627	0.602						
2.35	1.285	1.234	0.743	0.643	0.617						
2.40	1.317	1.265	0.762	0.649	0.632						
2.45	1.350	1.296	0.781	0.675	0.648						
2.50	1.382	1.327	0.799	0.691	0.663						

Note–Blanks indicate inadmissible reinforcement percentage (see Table E).

Table 6.7 Flexure–Reinforcement Percentage, p_t For Singly Reinforced Sections (Table–4 of SP: 16)

f_{ck} = 30 N/mm²

M_u/bd^2 N/mm²	f_y, N/mm² 240	250	415	480	500	M_u/bd^2 N/mm²	f_y, N/mm² 240	250	415	480	500
0.30	0.145	0.140	0.084	0.073	0.070	2.55	1.374	1.319	0.794	0.687	0.659
0.35	0.170	0.163	0.098	0.085	0.082	2.60	1.404	1.348	0.812	0.702	0.674
0.40	0.195	0.187	0.113	0.097	0.093	2.65	1.435	1.478	0.830	0.718	0.689
0.45	0.219	0.211	0.127	0.110	0.105	2.70	1.467	1.408	0.848	0.733	0.704
0.50	0.244	0.235	0.141	0.122	0.117	2.75	1.498	1.438	0.866	0.749	0.719
0.55	0.269	0.259	0.156	0.135	0.129	2.80	1.530	1.469	0.885	0.765	0.734
0.60	0.294	0.283	0.170	0.147	0.141	2.85	1.562	1.499	0.903	0.781	0.750
0.65	0.320	0.307	0.185	0.160	0.153	2.90	1.594	1.530	0.922	0.797	0.765
0.70	0.345	0.331	0.200	0.172	0.166	2.95	1.696	1.561	0.940	0.813	0.781
0.75	0.370	0.356	0.214	0.185	0.178	3.00	1.659	1.592	0.959	0.829	0.796
0.80	0.396	0.380	0.229	0.198	0.190	3.05	1.691	1.624	0.978	0.846	0.812
0.85	0.422	0.405	0.244	0.211	0.202	3.10	1.725	1.656	0.997	0.862	0.828
0.90	0.447	0.429	0.259	0.224	0.215	3.15	1.758	1.687	1.017	0.879	0.844
0.95	0.473	0.454	0.274	0.237	0.227	3.20	1.791	1.720	1.036	0.896	0.860
1.00	0.499	0.479	0.289	0.250	0.240	3.25	1.825	7.752	1.055	0.913	0.876

(Contd.)

Table 6.7 (Contd.)

f_{ck} = 30 N/mm²

M_u/bd^2 N/mm²	f_y, N/mm² 240	250	415	480	500	M_u/bd^2 N/mm²	f_y, N/mm² 240	250	415	480	500
1.05	0.525	0.504	0.304	0.263	0.252	3.30	1.859	1.785	1.075	0.930	0.892
1.10	0.552	0.529	0.319	0.276	0.265	3.35	1.893	1.818	1.095	0.947	0.909
1.15	0.578	0.555	0.334	0.289	0.277	3.40	1.928	1.851	1.115	0.964	0.925
1.20	0.604	0.580	0.350	0.302	0.290	3.45	1.963	1.884	1.135	0.981	0.942
1.25	0.631	0.606	0.365	0.315	0.303	3.50	1.998	1.918	1.156	0.999	0.959
1.30	0.658	0.631	0.380	0.329	0.316	3.55	2.034	1.952	1.176	1.017	0.976
1.35	0.685	0.657	0.396	0.342	0.329	3.60	2.069	1.986	1.197	1.035	0.993
1.40	0.712	0.683	0.411	0.356	0.342	3.65	2.105	2.021	1.218	1.053	1.001
1.45	0.739	0.709	0.427	0.369	0.355	3.70	2.142	2.056	1.239	1.071	1.028
1.50	0.766	0.735	0.443	0.383	0.368	3.75	2.178	2.091	1.260	1.089	1.046
1.55	0.793	0.762	0.459	0.397	0.381	3.80	2.215	2.127	1.281	1.108	1.063
1.60	0.821	0.788	0.475	0.410	0.394	3.85	2.253	2.163	1.303	1.126	1.081
1.65	0.849	0.815	0.491	0.424	0.407	3.90	2.291	2.199	1.325	1.145	1.099
1.70	0.876	0.841	0.507	0.438	0.421	3.95	2.329	2.236	1.347	1.164	1.118
1.75	0.904	0.868	0.523	0.452	0.434	4.00	2.367	2.273	1.369	1.184	

(Contd.)

Table 6.7 (Contd.)

$f_{ck} = 25$ N/mm²

M_u/bd^2 N/mm²	f_y, N/mm² 240	250	415	480	500	M_u/bd^2 N/mm²	f_y, N/mm² 240	250	415	480	500
1.80	0.932	0.895	0.539	0.466	0.448	4.05	2.406	2.310	1.391		
1.85	0.961	0.922	0.556	0.480	0.461	4.10	2.445	2.348	1.414		
1.90	0.989	0.950	0.572	0.495	0.475	4.15	2.485	2.386			
1.95	1.018	0.977	0.589	0.509	0.488	4.20	2.525	2.424			
2.00	1.046	1.005	0.605	0.523	0.502	4.25	2.566	2.463			
2.05	1.075	1.032	0.622	0.538	0.516	4.30	2.607	2.502			
2.10	1.104	1.060	0.639	0.552	0.530	4.35	2.648	2.542			
2.15	1.134	1.088	0.656	0.567	0.544	4.40	2.690	2.583			
2.20	1.163	1.116	0.673	0.581	0.558	4.45	2.733	2.623			
2.25	1.192	1.145	0.690	0.596	0.572						
2.30	1.222	1.173	0.707	0.611	0.587						
2.35	1.252	1.202	0.724	0.626	0.601						
2.40	1.282	1.231	0.742	0.641	0.615						
2.45	1.312	1.260	0.759	0.656	0.630						
2.50	1.343	1.289	0.777	0.671	0.645						

Note–Blanks indicate inadmissible reinforcement percentage (see Table E).

400 mm. If M-20 Grade Concrete and Fe-415 HYSD bars are used, compute the ultimate flexural strength of the section.

Method-1 (Using IS: 456-2000. Code Equations).

a) Data

$$b = 250 \text{ mm}$$
$$d = 400 \text{ mm}$$
$$A_{st} = 3600 \text{ mm}^2$$

b) Material Properties

$$f_{ck} = 20 \text{ N/mm}^2 \qquad f_y = 415 \text{ N/mm}^2$$

c) Depth of Neutral Axis:-

If x_u = depth of neutral Axis.

From Annexure G (IS: 456- 2000) Clause G- 1.1.

$$\left(\frac{x_u}{d}\right) = \left[\frac{0.87 f_y A_{st}}{0.36 f_{ck}\, b.d}\right] = \left[\frac{0.87 \times 415 \times 3600}{0.36 \times 20 \times 250 \times 400}\right] = 1.805$$

Limiting value of $\left(\frac{x_u}{d}\right)$ for Fe- 415-grade steel is 0.48

Since $\left(\frac{x_u}{d}\right) = 1.805 > 0.48$, section is over reinforced.

d) Moment of Resistance

Referring to Table 6.2, Limiting value of moment of resistance is computed as

$$M_{u,\,lim} = 0.138 f_{ck}.b.d^2.$$
$$= (0.138 \times 20 \times 250 \times 400^2)$$
$$= 110.4 \times 10^6 \text{ N.mm.}$$
$$= 110.4 \text{ kN.m}$$

Method–2 (Using SP: 16 Design Aids.)

$$\text{Percentage of steel} = p_t = \left(\frac{100\,A_{st}}{bd}\right) = \left(\frac{100\times 3600}{250\times 400}\right) = 3.6.$$

For Fe-415 grade steel and $f_{ck} = 20$ N/mm². Referring to Table 6.4 (Table E of SP: 16), maximum percentage of tensile reinforcement $p_{t,lim}$ for singly reinforced rectangular sections is 0.96.

Hence the section is over reinforced.

Hence,
$$M_u = M_{uc} = 0.138\,f_{ck}.b.d^2$$
$$= (0.138\times 20\times 250\times 400^2)$$
$$= 110.4\times 10^6 \text{ N.mm.}$$
$$= 110.4 \text{ kNm.}$$

2) A rectangular reinforced concrete section having a breadth of 350 mm is reinforced with 2 bars of 28 mm and 2 bars of 25 mm diameter at an effective depth of 700 mm. Adopting M-20 grade concrete and Fe-415 HYSD bars determine the ultimate moment of resistance of the section.

Method - 1(Using IS: 456 - 2000 Code Equations)

a) Data

b = 350 mm
d = 700 mm
A_{st} = 2 [491 + 616] = 2214 mm²

b) Material Properties

$$f_{ck} = 20 \text{ N/mm}^2,\quad f_y = 415 \text{ N/mm}^2.$$

c) Depth of Neutral Axis:-

Let x_u = Depth of neutral Axis.

$$\left(\frac{x_u}{d}\right) = \left[\frac{0.87\,f_y\,A_{st}}{0.36\,f_{ck}.b.d}\right] = \left[\frac{0.87\times 415\times 2214}{0.36\times 20\times 350\times 700}\right] = 0.453 < 0.48.$$

Hence, the section is under reinforced.

d) Moment of Resistance

$$M_u = 0.87\,f_y.A_{st}.d\left[1 - \frac{A_{st}.f_y}{b.d.f_{ck}}\right]$$
$$= 0.87\times 415\times 2214\times 700\left[1 - \left(\frac{2214\times 415}{350\times 700\times 20}\right)\right]$$
$$= (454.6\times 10^6) \text{ N.mm.}$$
$$= 454.6 \text{ kN.m}$$

Method–2 (Using SP: 16 Design Aids.)

Percentage reinforcement in the section.

$$= p_t = \left(\frac{100A_{st}}{b.d.}\right) = \left(\frac{100\times 2214}{350\times 700}\right) = 0.90$$

Refer Table 6.5 (Table-2 of SP: 16), and read out the value of (M_u/bd^2) corresponding to $f_y = 415$ N/mm² and $f_{ck} = 20$ N/mm²

$$\left(\frac{M_u}{b.d^2}\right) = 2.64$$

$$\therefore \quad M_u = (2.64\times 350\times 700^2)$$
$$= (452.7\times 10^6) \text{ N.mm}$$
$$= 452.7 \text{ kN.m}$$

3) A reinforced concrete slab 150mm thick is reinforced with 10mm diameter bars at 200mm centres, located at an effective depth of 125mm. If M- 20 grade concrete and Fe- 415 grade HYSD bars are used, estimate the ultimate moment of resistance of the section.

Method-1 (using IS: 456- 2000 code equations)

a) Data

b = 1000 mm
d = 125 mm

$$A_{st} = \left[\frac{1000(\pi\times 10^2/4)}{200}\right] = 393 \text{ mm}^2$$

$$f_{ck} = 20 \text{ N/mm}^2,\quad f_y = 415 \text{ N/mm}^2.$$

b) Depth of Neutral Axis

Let x_u = Depth of Neutral axis.

$$\left(\frac{x_u}{d}\right) = \left[\frac{0.87 f_y.A_{st}}{0.36\, f_{ck}.b.d}\right] = \left[\frac{0.87 \times 415 \times 393}{0.36 \times 20 \times 1000 \times 125}\right] = 0.157 < 0.48$$

Hence, the section is under reinforced.

c) Moment of Resistance

$$M_u = 0.87\, f_y.A_{st}.d\left[1 - \frac{A_{st}.f_y}{b.d.f_{ck}}\right]$$

$$= (0.87 \times 415 \times 393 \times 125)\left[1 - \left(\frac{393 \times 415}{100 \times 125 \times 20}\right)\right]$$

$$= (16.58 \times 10^6)\ \text{N.mm}$$

$$= 16.58\ \text{kN.m.}$$

Method–2 (Using SP: 16 Design Aids)

Percentage reinforcement in the Section

$$= p_t = \left(\frac{100\, A_{st}}{b.d}\right) = \left(\frac{100 \times 393}{1000 \times 125}\right) = 0.314$$

Refer Table-2 of SP: 16 and read out the value of (M_u/bd^2) corresponding to values of $p_t = 0.314$, $f_y = 415$ N/mm^2 and $f_{ck} = 20$ N/mm^2. Interpolating the value, we have

$$(M_u/bd^2) = 1.06$$

$\therefore \quad M_u = (1.06 \times 10^3 \times 125^2) = (16.56 \times 10^6)$ N.mm/m = 16.56 kN.m/m.

4) A reinforced concrete beam of rectangular section 300mm wide by 600mm deep is reinforced with 4 bars of 25mm diameter at an effective depth of 550mm. The effective span of the beam is 7m, If $f_y = 415$ N/mm^2 and $f_{ck} = 20$ N/mm^2, find the uniformly distributed ultimate load on the beam.

Method–1 (Using IS: 456-2000 Code equations)

a) Data

$b = 300$ mm
$D = 600$ mm

$d = 550$ mm
$A_{st} = (4 \times 491) = 1964$ mm^2
$f_{ck} = 20$ N/mm^2 and $f_y = 415$ N/mm^2.

b) Neutral Axis Depth

$$\left(\frac{x_u}{d}\right) = \left[\frac{0.87\, f_y.A_{st}}{0.36\, f_{ck}.b.d}\right] = \left[\frac{0.87 \times 415 \times 1964}{0.36 \times 20 \times 300 \times 550}\right] = 0.596 > 0.48$$

Hence, the Section is over reinforced.

c) Moment of Resistance

Referring to Table 6.2, Limiting value of moment of resistance is

$$M_{u,\,lim} = 0.138\, f_{ck}.b.d^2$$

$$= (0.138 \times 20 \times 300 \times 550^2)$$

$$= 250 \times 10^6\ \text{N.mm}$$

$$= 250\ \text{kN.m}$$

Method–2 (Using SP: 16 Design Aids)

Percentage of steel $= p_{t,\,lim} = \left(\frac{100 A_{st}}{b.d}\right) = \left(\frac{100 \times 1964}{300 \times 550}\right) = 1.19$

For Fe-415 grade steel and $f_{ck} = 20$ N/mm^2
Referring to Table 6.4 (Table-E of SP: 16)
Maximum percentage of tensile reinforcement $p_{t,\,lim}$ for singly reinforced rectangular sections is 0.96.

Hence the section is over reinforced

$$\therefore \quad M_u = M_{uc} = 0.138 f_{ck} bd^2 = (0.138 \times 20 \times 300 \times 550^2)$$

$$= (250 \times 10^6)\ \text{N.mm} = 250\ \text{kN.m}$$

If w_u = uniformly distributed ultimate load,

$$w_u = \left(\frac{8 M_u}{L^2}\right) = \left(\frac{8 \times 250}{7^2}\right) = 40.81\ \text{kN/m}$$

6.2.8 Design Examples

1) Determine the area of reinforcement required for a singly reinforced concrete section having a breadth of 675 mm to support a factored moment of 185 kNm. Adopt M- 20 grade concrete and Fe-415 Grade HYSD bars.

Method-1 (Using IS: 456 - 2000 Code Formulae)

a) Data

$$b = 300 \text{ mm} \qquad f_{ck} = 20 \text{ N/mm}^2$$
$$d = 675 \text{ mm} \qquad f_y = 415 \text{ N/mm}^2$$
$$M_u = 185 \text{ kNm}$$

b) Limiting Moment of Resistance

For Fe-415 HYSD bars,

$$M_{u,\,lim} = 0.138\, f_{ck}\, b.d^2$$
$$= (0.138 \times 20 \times 300 \times 675^2)10^{-6}$$
$$= 377 \text{ kN.m} > M_u = 185 \text{ kN.m}$$

Hence, the beam is under reinforced.

c) Area of Tensile Reinforcement:-

$$M_u = 0.87\, f_y A_{st} d\, (1 - A_{st} f_y / bd\, f_{ck})$$

$$(185 \times 10^6) = 0.87 \times 415 \times A_{st} \times 675 \left[1 - \left(\frac{A_{st}\,.\,415}{300 \times 675 \times 20}\right)\right]$$

Solving $A_{st} = 830 \text{ mm}^2$.

Method-2 (Using SP: 16 Design Tables)

a) Design Parameters

$$\left(\frac{M_u}{bd^2}\right) = \left(\frac{185 \times 10^6}{300 \times 675^2}\right) = 1.35$$

b) Area of Reinforcement

Refer Table-2 of SP: 16 (Table 6.5 of text) corresponding to f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2 read out the percentage reinforcement p_t for the required parameter.

$\left(\frac{M_u}{bd^2}\right) = 1.35$ and the corresponding value of $p_t = 0.409$ percent.

$$\therefore \qquad A_{st} = \left(\frac{p_t bd}{100}\right) = \left[\frac{0.409 \times 300 \times 675}{100}\right] = 828 \text{ mm}^2$$

The area of reinforcement obtained by both the methods are same.

2) Design the minimum effective depth required and the area of reinforcement for a rectangular beam having a width of 300mm to resist an ultimate moment of 200kNm, using M-20 grade concrete and Fe-415 HYSD bars.

Method–1 (Using IS: 456 - 2000 Code Formulae)

a) Data

$$b = 300 \text{ mm} \qquad f_{ck} = 20 \text{ N/mm}^2$$
$$M_u = 200 \text{ kNm} \qquad f_y = 415 \text{ N/mm}^2$$

b) Minimum Effective depth

For Fe–415 HYSD bars, limiting moment of resistance is given by

$$M_{u,\,lim} = 0.138\, f_{ck}.b.d^2$$

$$\therefore \qquad d = \sqrt{\frac{M_{u,\,lim}}{0.138\, f_{ck}.b}} = \sqrt{\frac{200 \times 10^6}{0.138 \times 20 \times 300}} = 492 \text{ mm.}$$

c) Area of Reinforcement

$$M_u = 0.87\, f_y.A_{st}.d\left[1 - \frac{A_{st}.f_y}{b.d.f_{ck}}\right]$$

$$200 \times 10^6 = 0.87 \times 415 \times A_{st} \times 492\left[1 - \left(\frac{A_{st}.415}{300 \times 492 \times 20}\right)\right]$$

Solving,

$$\therefore \qquad [A_{st}^2 - 7114\, A_{st} + 8 \times 10^6] = 0$$

$$\therefore \qquad A_{st} = 1400 \text{ mm}^2.$$

Method–2 (Using SP: 16 Design Tables)

Referring to Table–D of SP: 16 for $f_{ck} = 20$ N/mm² and $f_y = 415$ N/mm².

$$\left(\frac{M_{u,\,lim}}{b.d^2}\right) = 2.76$$

$$\therefore \qquad d = \sqrt{\frac{M_{u,\,lim}}{2.76b}} = \sqrt{\frac{200\times10^6}{2.76\times300}} = 492 \text{ mm}$$

Referring to Table–2 of SP: 16 (Table 6.5 of text), read out the value of percentage reinforcement p_t corresponding to the parameter $\left(\frac{M_u}{bd^2}\right) = 2.76$

$$\therefore \qquad p_t = 0.955$$

$$\therefore \qquad A_{st} = \left(\frac{p_t.b.d}{100}\right) = \left(\frac{0.955\times300\times492}{100}\right) = 1410 \text{ mm}^2.$$

6.3 ULTIMATE FLEXURAL STRENGTH OF FLANGED SECTIONS

The flexural strength of flanged beams (Tee and L-beams) depends upon the position of neutral axis. The Indian Standard Code IS: 456-2000 prescribes a method for computing the ultimate moment of resistance of flanged sections for different cases as detailed below: -

6.3.1 Neutral Axis within flange

Referring to Fig. 6.4

Let $\quad b_f$ = width of compression flange.
b_w = width of rib.
d = Effective depth.
D_f = Depth of flange.
A_{st} = Area of Tension Reinforcement.
x_u = Depth of Neutral Axis.

When neutral axis falls within the flange, the moment of resistance of the section can be calculated by the same procedure as that of rectangular sections.

Hence when $x_u < D_f$ the moment of Resistance of the section can be computed by the relation,

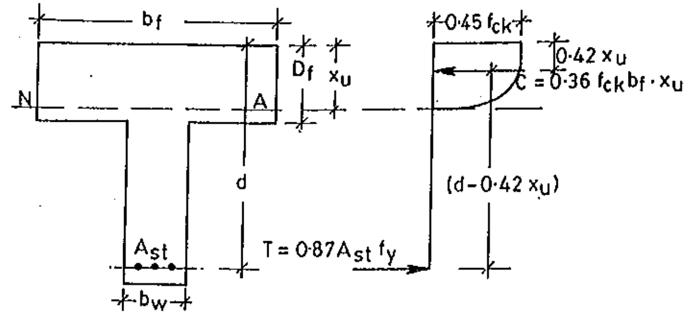


Fig. 6.4 Stress Block Parameters for Tee Beam ($x_u < D_f$)

$$M_u = 0.87\, f_y.A_{st}.d\left[1 - \frac{A_{st}.f_y}{b_f.d.f_{ck}}\right] \tag{6.7}$$

6.3.2 Neutral Axis falls outside the Flange ($D_f/d \not> 0.2$)

When the neutral axis falls outside the flange and the ratio $(D_f/d) \not> 0.2$, the moment of resistance can be computed using the stress block parameters shown in Fig. 6.5. The stress blocks are separately shown for the rectangular portion and the flange portion. The moment of resistance of the section of the Tee Section is computed by the relation,

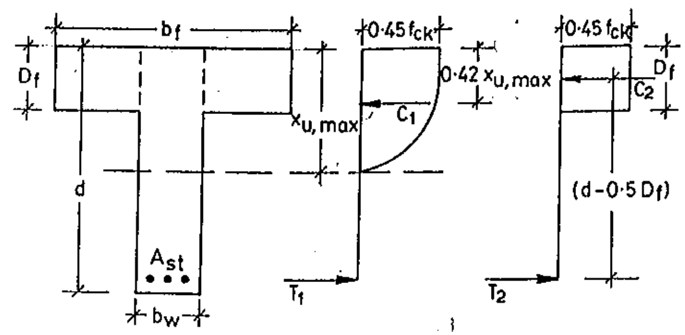


Fig. 6.5 Stress Block Parameters for Tee Beam ($x_u > D_f$)

$$M_u = C_1(d - 0.42x_{u,\,max}) + C_2(d - 0.5D_f)$$

Where $\qquad C_1 = 0.36\, f_{ck}.x_{u.max}.b_w$

$$C_2 = 0.45\, f_{ck}.D_f.(b_f - b_w)\,. \quad \text{For } (D_f/x_u) < 0.43$$

$$\therefore \quad M_u = 0.36\, f_{ck}.x_{u,\max}.b_w\,(d - 0.42x_{u,\max}) + 0.45\, f_{ck}\,(b_f - b_w)\, D_f(d - 0.5\, D_f)$$

This equation can be recast as that given IS: 456–2000 (ANNEX–G), in the form,

$$M_u = 0.36\left(\frac{x_{u,\max}}{d}\right)\left[1 - 0.42\left(\frac{x_{u,\max}}{d}\right)\right] f_{ck}.b_w.d^2$$

$$+\,0.45 f_{ck}(b_f - b_w)D_f(d - 0.5\, D_f) \qquad \ldots(6.8)$$

Note:

For $D_f / x_u > 0.43$, D_f to be replaced by y_f

6.3.3 Neutral Axis falls outside the Flange (D_f / d > 0.2)

When the neutral axis falls outside the flange and the ratio $(D_f / d) > 0.2$, we cannot assume that the flange is uniformly stressed as in case (2). Hence the expression for case (2) is modified by substituting y_f for D_f in Eq. (6.8) where

$y_f = (0.15\, x_u + 0.65\, D_f)$ but y_f should be not greater than D_f. Hence, the expression for moment of resistance is given by the relation,

$$M_u = 0.36\left(\frac{x_{u,\max}}{d}\right)\left[1 - 0.42\left(\frac{x_{u,\max}}{d}\right)\right] f_{ck}.b_w.d^2$$

$$+\,0.45 f_{ck}(b_f - b_w)y_f(d - 0.5\, y_f) \qquad \ldots(6.9)$$

Where $y_f = (0.15\, x_u + 0.65\, D_f)$ but not greater than D_f.

The equation $y_f = (0.15\, x_u + 0.65\, D_f)$ is based on the Whitney's stress block shown in Fig. 6.6.

Let x_u = Depth of Neutral Axis.
D_f = Depth of flange.

Let $$y_f = A\,x_u + B\, D_f \qquad (6.10)$$

The constants A and B are solved by specifying the following two conditions to be satisfied by this equation.

1) When $D_f = 0.43\, x_u$, $\quad y_f = 0.43\, x_u$
2) When $D_f = x_u$ $\quad y_f = 0.80\, x_u$

Substituting these conditions in Eq. (6.10) the constants A and B are evaluated as

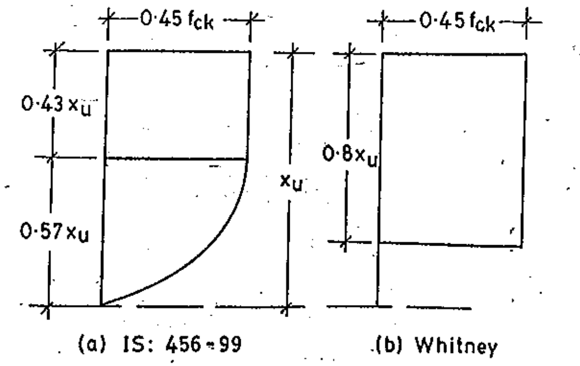


Fig. 6.6 Equivalent Stress Blocks

$$A = 0.15 \quad \text{and} \quad B = 0.65.$$

Hence, $\quad y_f = (0.15\, x_u + 0.65\, D_f)$

The Indian Standard code further stipulates that for $x_{u,\max} > x_u > D_f$, the moment of resistance may be calculated by the equations (6.8), when (D_f/x_u) does not exceed 0.43 and when (D_f/x_u) exceeds 0.43, the moment of resistance is computed by the equation (6.9) by substituting $x_{u,\max}$ by x_u.

6.3.4 Computation of Tension Reinforcement in Tee beam Sections

Case–1 $(x_u < D_f)$

In this case of Tee-Section the area of tension reinforcement can be computed by treating the section as rectangular and using the equation 6.7 and solving A_{st} for a given value of M_u expressed as

$$M_u = 0.87\, f_y.A_{st}.d\left[1 - \frac{A_{st}.f_y}{b_f.d.f_{ck}}\right]$$

Knowing the value of x_u,

$$A_{st} = \frac{0.36\, f_{ck}.x_u.b_f}{0.87\, f_y}$$

Case-2 $\left[x_u > D_f,\ \left(\frac{D_f}{d}\right) \not> 0.2 \text{ and } \left(\frac{D_f}{x_u}\right) \not> 0.43\right]$

For a given value of M_u, evaluate x_u by using Eq. 6.8 and replacing $x_{u,\max}$ by x_u. Referring to stress block parameters shown in Fig. 6.5, force equilibrium yields the following equations:-

$$T_1 = C_1$$

$$(A_{stw} 0.87 f_y) = 0.36 f_{ck} b_w x_u$$

$$\therefore \quad A_{stw} = \left[\frac{0.36 f_{ck} b_w x_u}{0.87 f_y}\right]$$

Also

$$T_2 = C_2$$

$$(A_{stf} 0.87 f_y) = 0.45 f_{ck} (b_f - b_w) D_f$$

$$A_{stf} = \left[\frac{0.45 f_{ck}(b_f - b_w) D_f}{0.87 f_y}\right]$$

Hence, the total tension reinforcement in the Tee Sections is given by

$$A_{st} = [A_{stw} + A_{stf}]$$

$$A_{st} = \left[\frac{0.36 f_{ck} b_w x_u}{0.87 f_y}\right] + \left[\frac{0.45 f_{ck} (b_f - b_w) D_f}{0.87 f_y}\right]$$

Case-3 $\left[x_u > D_f, \left(\frac{D_f}{d}\right) \ngtr 0.2 \quad \text{and} \quad \left(\frac{D_f}{x_u}\right) > 0.43\right]$

For a given value of M_u, evaluate x_u by using Eq.6.9 and replacing $x_{u,max}$ by x_u. Referring to stress block parameters shown in Fig. 6.6 in which the depth of stress block is

$y_f = (0.15\, x_u + 0.65\, D_f)$ but not greater than D_f.

Force Equilibrium yields the following Equations:-

$$T_1 = C_1$$

$$(A_{stw} 0.87 f_y) = 0.36 f_{ck} b_w x_u$$

$$\therefore \quad A_{stw} = \left[\frac{0.36 f_{ck} b_w x_u}{0.87 f_y}\right]$$

Also

$$T_2 = C_2$$

$$(A_{stf} 0.87 f_y) = 0.45 f_{ck} (b_f - b_w)\, y_f$$

$$\therefore \quad A_{stf} = \left[\frac{0.45 f_{ck}(b_f - b_w) y_f}{0.87 f_y}\right]$$

$$A_{st} = (A_{stw} + A_{stf})$$

Case-4 $\left[x_u > D_f \quad \text{and} \quad \left(\frac{D_f}{d}\right) > 0.2\right]$

Similar to case (3)

6.3.5 Use of Design Aids (SP: 16) for Design of Flanged Beams

In most cases of tee beams used in buildings, the neutral axis falls within the flange and the computation of steel area can be made as in the design of rectangular beams using Tables 1 to 4 of SP: 16. In the case of tee beams in which the neutral axis falls in the rib, design Tables provided in I.S. Special publication SP: 24[14] (Explanatory Hand Book on IS: 456) are very useful in computing the area of reinforcement for a given tee beam to resist a specified bending moment.

SP: 16 designs Tables 57 to 59 are also useful to compute the limiting moment of resistance factor ($M_{u,lim} / b_w \cdot d^2 \cdot f_{ck}$) for singly reinforced Tee beams. The tables cover different grades of steel (250, 415 and 500) and ratios of (D_f/d) varying from 0.06 to 0.45 and ratios of (b_f/b_w) varying from 1 to 10. These tables are presented as Tables 6.8, 6.9 covering Fe-415 and 500 grade steels in the text.

6.3.6 Analysis Examples

1) Determine the Ultimate flexural strength of the T-beam having the following section properties:-

Width of flange = 800 mm
Depth of flange = 150 mm
Width of rib = 300 mm
Effective depth = 420 mm
Area of steel = 1470 mm²

M-25 Grade concrete and Fe-415 Grade HYSD bars.

Method-1 (Using IS: 456-2000 formula)

a) Data

$b_f = 800$mm
$D_f = 150$mm
$d = 420$mm
$b_w = 300$mm

$A_{st} = 1470\ \text{mm}^2$
$f_{ck} = 25\ \text{N/mm}^2$
$f_y = 415\ \text{N/mm}^2$

Table 6.8 Flexure–Limiting Moment of Resistance Factor, $M_{u,lim}/b_w d^2 f_{ck}$, For Singly Reinforced T-Beams. N/mm²
(Table–58 of SP: 16)

$f_y = 415$ N/mm²

D_f/d	b_f/b_w 1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.00	10.0
0.06	0.138	0.164	0.190	0.216	0.242	0.268	0.294	0.320	0.346	0.372
0.07	0.138	0.168	0.198	0.228	0.259	0.289	0.319	0.349	0.379	0.409
0.08	0.138	0.172	0.207	0.241	0.275	0.309	0.344	0.378	0.412	0.446
0.09	0.138	0.176	0.215	0.253	0.291	0.330	0.368	0.406	0.445	0.483
0.10	0.138	0.180	0.223	0.265	0.308	0.350	0.392	0.435	0.477	0.519
0.11	0.138	0.184	0.231	0.277	0.324	0.370	0.416	0.463	0.509	0.555
0.12	0.138	0.188	0.239	0.289	0.339	0.390	0.440	0.490	0.541	0.591
0.13	0.138	0.192	0.247	0.301	0.355	0.409	0.463	0.518	0.572	0.626
0.14	0.138	0.196	0.254	0.312	0.370	0.428	0.487	0.545	0.603	0.661
0.15	0.138	0.200	0.262	0.324	0.386	0.428	0.509	0.571	0.633	0.695
0.16	0.138	0.204	0.269	0.335	0.401	0.466	0.532	0.598	0.663	0.729
0.17	0.138	0.207	0.277	0.346	0.416	0.485	0.554	0.624	0.693	0.762
0.18	0.138	0.211	0.284	0.357	0.430	0.503	0.576	0.649	0.723	0.796
0.19	0.138	0.215	0.291	0.368	0.445	0.522	0.598	0.675	0.752	0.828
0.20	0.138	0.218	0.299	0.379	0.459	0.540	0.620	0.700	0.780	0.861
0.21	0.138	0.221	0.305	0.388	0.471	0.554	0.638	0.721	0.804	0.887
0.22	0.138	0.224	0.309	0.395	0.480	0.566	0.651	0.737	0.822	0.908
0.23	0.138	0.226	0.314	0.402	0.489	0.577	0.665	0.753	0.841	0.928
0.24	0.138	0.228	0.318	0.408	0.498	0.588	0.678	0.768	0.859	0.949
0.25	0.138	0.230	0.323	0.415	0.507	0.600	0.692	0.784	0.876	0.969

(Contd.)

Table 6.8 (Contd.)

$f_y = 415$ N/mm²

D_f/d	b_f/b_w 1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.00	10.0
0.26	0.138	0.233	0.327	0.422	0.516	0.611	0.705	0.800	0.894	0.989
0.27	0.138	0.235	0.331	0.428	0.525	0.622	0.718	0.815	0.912	1.008
0.28	0.138	0.237	0.336	0.435	0.534	0.632	0.731	0.830	0.929	1.028
0.29	0.138	0.239	0.340	0.441	0.542	0.643	0.744	0.845	0.946	1.047
0.30	0.138	0.241	0.344	0.448	0.551	0.654	0.757	0.860	0.963	1.006
0.31	0.138	0.243	0.349	0.454	0.559	0.664	0.770	0.875	0.980	1.085
0.32	0.138	0.245	0.353	0.460	0.568	0.675	0.782	0.890	0.997	1.104
0.33	0.138	0.248	0.357	0.466	0.576	0.685	0.795	0.904	1.014	1.123
0.34	0.138	0.250	0.361	0.473	0.584	0.696	0.807	0.919	1.030	1.142
0.35	0.138	0.252	0.365	0.479	0.592	0.706	0.819	0.933	1.046	1.160
0.36	0.138	0.254	0.369	0.485	0.600	0.716	0.831	0.947	1.063	1.178
0.37	0.138	0.256	0.373	0.491	0.608	0.726	0.843	0.961	1.079	1.196
0.38	0.138	0.258	0.377	0.497	0.616	0.736	0.855	0.975	1.094	1.214
0.39	0.138	0.260	0.381	0.503	0.624	0.746	0.867	0.989	1.110	1.232
0.40	0.138	0.262	0.385	0.508	0.632	0.755	0.879	1.002	1.126	1.249
0.41	0.138	0.263	0.389	0.514	0.640	0.765	0.890	1.016	1.141	1.267
0.42	0.138	0.265	0.393	0.520	0.647	0.775	0.902	1.029	1.156	1.284
0.43	0.138	0.267	0.396	0.526	0.655	0.784	0.913	1.042	1.172	1.301
0.44	0.138	0.269	0.400	0.531	0.662	0.793	0.924	1.055	1.187	1.318
0.45	0.138	0.271	0.404	0.537	0.670	0.803	0.936	1.068	1.201	1.334

Table 6.9 Flexure–Limiting Moment of Resistance Factor, $M_{u,lim}/b_w d^2 f_{ck}$, For Singly Reinforced T-Beams. N/mm²
(Table–59 of SP: 16)

$f_y = 500$ N/mm²

D_f/d	b_f/b_w 1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0.06	0.133	0.159	0.185	0.211	0.237	0.263	0.289	0.315	0.341	0.367
0.07	0.133	0.163	0.193	0.223	0.254	0.284	0.314	0.344	0.374	0.404
0.08	0.133	0.167	0.202	0.236	0.270	0.304	0.339	0.373	0.407	0.441
0.09	0.133	0.171	0.210	0.248	0.286	0.325	0.363	0.401	0.440	0.478
0.10	0.133	0.175	0.218	0.260	0.303	0.345	0.387	0.430	0.472	0.514
0.11	0.133	0.179	0.226	0.272	0.318	0.365	0.411	0.458	0.504	0.550
0.12	0.133	0.183	0.234	0.284	0.334	0.385	0.435	0.485	0.536	0.586
0.13	0.133	0.187	0.241	0.296	0.350	0.404	0.458	0.513	0.567	0.621
0.14	0.138	0.191	0.249	0.307	0.365	0.423	0.481	0.540	0.598	0.656
0.15	0.133	0.195	0.257	0.319	0.381	0.442	0.504	0.566	0.628	0.690
0.16	0.133	0.199	0.264	0.330	0.396	0.461	0.527	0.593	0.658	0.724
0.17	0.133	0.202	0.272	0.341	0.411	0.480	0.549	0.619	0.688	0.757
0.18	0.133	0.206	0.279	0.352	0.425	0.498	0.571	0.644	0.717	0.791
0.19	0.133	0.210	0.286	0.363	0.440	0.516	0.593	0.670	0.747	0.823
0.20	0.133	0.213	0.292	0.372	0.452	0.532	0.611	0.691	0.771	0.850
0.21	0.133	0.215	0.297	0.379	0.461	0.543	0.625	0.707	0.789	0.871
0.22	0.133	0.217	0.302	0.386	0.470	0.555	0.639	0.723	0.808	0.892
0.23	0.133	0.220	0.306	0.393	0.479	0.566	0.653	0.739	0.826	0.912
0.24	0.133	0.222	0.311	0.400	0.488	0.577	0.666	0.755	0.844	0.933
0.25	0.133	0.224	0.315	0.406	0.497	0.589	0.680	0.771	0.862	0.953

(Contd.)

Table 6.9 (Contd.)

$f_y = 500$ N/mm²

D_f/d	b_f/b_w 1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0.26	0.133	0.226	0.320	0.413	0.506	0.600	0.693	0.786	0.880	0.973
0.27	0.133	0.229	0.324	0.420	0.515	0.611	0.706	0.802	0.897	0.993
0.28	0.133	0.231	0.328	0.426	0.524	0.622	0.719	0.817	0.915	1.012
0.29	0.133	0.233	0.333	0.433	0.532	0.632	0.732	0.832	0.932	1.032
0.30	0.133	0.235	0.337	0.439	0.541	0.643	0.745	0.847	0.949	1.051
0.31	0.133	0.237	0.341	0.445	0.550	0.654	0.758	0.862	0.966	1.070
0.32	0.133	0.239	0.346	0.452	0.558	0.664	0.770	0.877	0.983	1.039
0.33	0.133	0.241	0.350	0.458	0.566	0.675	0.783	0.891	1.000	1.108
0.34	0.133	0.243	0.354	0.464	0.575	0.685	0.795	0.906	1.016	1.127
0.35	0.133	0.245	0.358	0.470	0.583	0.695	0.808	0.920	1.033	1.145
0.36	0.133	0.248	0.362	0.476	0.591	0.705	0.820	0.934	1.049	1.163
0.37	0.133	0.250	0.366	0.483	0.599	0.716	0.832	0.949	1.065	1.181
0.38	0.133	0.252	0.370	0.489	0.607	0.725	0.844	0.962	1.081	1.199
0.39	0.133	0.254	0.374	0.494	0.615	0.735	0.856	0.976	1.097	1.217
0.40	0.133	0.255	0.378	0.500	0.623	0.745	0.868	0.990	1.112	1.235
0.41	0.133	0.257	0.382	0.506	0.630	0.755	0.879	1.004	1.128	1.252
0.42	0.133	0.259	0.386	0.512	0.638	0.764	0.891	1.017	1.143	1.270
0.43	0.133	0.261	0.389	0.518	0.646	0.774	0.902	1.030	1.158	1.287
0.44	0.133	0.263	0.393	0.523	0.653	0.783	0.913	1.043	1.174	1.304
0.45	0.133	0.265	0.397	0.529	0.661	0.793	0.925	1.056	1.188	1.320

b) Depth of Neutral Axis

$$\left(\frac{x_u}{d}\right)=\left(\frac{0.87\,f_y A_{st}}{0.36\,f_{ck}.b_f.d}\right)=\left(\frac{0.87\times415\times1470}{0.36\times25\times800\times420}\right)=0.175$$

$\therefore$ $x_u=(0.175\times420)=73.5\text{ mm}<D_f$

Neutral axis falls inside the flange. The section can be considered as a rectangular section with $b=b_f=800$mm. The section is under reinforced since $\left(\frac{x_u}{d}\right)<0.48$

Hence, the ultimate flexural strength is computed as,

$$M_u=0.87\,f_y A_{st}\,[d-0.42x_u]$$

$$=(0.87\times415\times1470)\,[0.42\times73.5)]$$

$$=(206.52\times10^6)\text{ N.mm}$$

$$=206.52\text{ kN.m}$$

Method–2 (Using SP: 16 Design Aids)

Percentage Reinforcement in the section (Assuming b_f = b for the rectangular section)

$$p_t=\left(\frac{100A_{st}}{bd}\right)=\left(\frac{100\times1470}{800\times420}\right)=0.44$$

Refer Table-3 of SP: 16 and read out the value of (M_u/bd^2) corresponding to

$f_y=415\text{ N/mm}^2$ and $f_{ck}=25\text{ N/mm}^2$

$$(M_u/bd^2)=1.47$$

$\therefore$ $$M_u=(1.47\times800\times420^2)10^{-6}$$

$$=207.4\text{ kN.m}$$

2) Calculate the Ultimate moment of resistance of a tee-beam having the following section properties.

Width of flange = 1300 mm
Thickness of flange = 100 mm

Width of rib = 325 mm
Effective depth = 600 mm
Area of steel = 4000 mm^2

M–20 Grade concrete and Fe–415 HYSD bars.

Method–1 (Using IS: 456-2000 code formula)

a) Data

b_f = 1300mm, f_{ck} = 20 N/mm^2
D_f = 100mm, f_y = 415 N/mm^2
b_w = 325mm, A_{st} = 4000 mm^2
d = 600mm.

b) Depth of Neutral Axis:-

Assuming Neutral Axis to fall within the flange, compute the depth of Neutral axis.

$$\left(\frac{x_u}{d}\right)=\left(\frac{0.87\,f_y A_{st}}{0.36\,f_{ck}.b.d}\right)=\left(\frac{0.87\times415\times4000}{0.36\times20\times1300\times600}\right)=0.257$$

$\therefore$ $x_u=(0.257\times600)=154.3\text{mm}>D_f$

Hence the assumption that $x_u<D_f$ is not correct. Neutral Axis falls outside the flange.

$$\left(\frac{D_f}{d}\right)=\left(\frac{100}{600}\right)=0.166<0.2$$

The neutral axis depth can be determined by referring to Fig. 6.5 and by compatibility of forces as shown below.

$$[C_1+C_2]=[T_1+T_2]=T$$

Assuming $\left(\frac{D_f}{x_u}\right)<0.43$

$$C_1=0.36\,f_{ck}b_w x_u.$$

$$=(0.36\times20\times325\times x_u)=2340\,x_u\text{ N}$$

$$C_2=0.45f_{ck}(b_f-b_w)D_f$$

$$=(0.45\times20)(1300-325)100=877500\text{ N.}$$

$$T = 0.87 f_y A_{st}$$
$$= (0.87 \times 415 \times 4000)$$
$$= 14442000 \text{ N.}$$

$\therefore$ $$2340\, x_u + 877500 = 1444200$$

$\therefore$ $$x_u = 242.18 \text{ mm.}$$

$$\left(\frac{D_f}{x_u}\right) = \left(\frac{100}{242.18}\right) = 0.413 < 0.43$$

$$x_{u,\,max} = 0.48\, d = (0.48 \times 600) = 288 \text{mm.}$$

Hence according to clause G-2.3 of I.S 456-2000, the moment of resistance is computed by replacing x_u by $x_{u,max}$ in Eqn.6.8.

$$M_u = 0.36\left(\frac{x_u}{d}\right)\left[1 - 0.42\left(\frac{x_u}{d}\right)\right] f_{ck} . b_w . d^2 + 0.45 f_{ck}(b_f - b_w) D_f (d - 0.5 D_f)$$

$$= 0.36 \times \left(\frac{242}{600}\right)\left[1 - 0.42\left(\frac{242}{600}\right)\right](20 \times 325 \times 600^2)$$

$$+ [0.45 \times 20(1300 - 325)100(600 - 0.5 \times 100)]$$

$$= (764.84 \times 10^6) \text{ N.mm} = 764.84 \text{ kN.m.}$$

Method–2 (Using SP: 16 Design Tables)

Tables 57, 58 and 59 of SP-16 give the values of $(M_{u,lim} / b_w\, d^2 f_{ck})$ for singly reinforced T-beams. These tables can be used when $x_u = x_{u,max}$ in which case the limiting moment of resistance can be computed. In the present example $x_u < x_{u,max}$ and hence the tables cannot be used for computing the moment of resistance as the section is under reinforced section.

3) A singly reinforced T- beams has a flange width of 950 mm,thickness of flange 80mm, width of rib = 250 mm. Effective depth 565mm. Area of tensile reinforcement 1256.6mm^2. If M-15 concrete and Fe-415 HYSD bars are used, estimate the ultimate flexural strength of the section using IS: 456-2000 code provisions.

Method–1 (Using IS: 456-2000 code provisions)

a) Data

b_f = 950 mm $\qquad f_{ck}$ = 15 N/mm^2.
D_f = 80 mm $\qquad f_y$ = 415 N/mm^2

d = 565 mm
b_w = 250 mm
A_{st} = 1256.6 mm^2

a) Depth of Neutral Axis

Assuming neutral axis to fall within the flange, the depth of neutral axis is computed.

$$\left(\frac{x_u}{d}\right) = \left(\frac{0.87 f_y A_{st}}{0.36 f_{ck} . b_f . d}\right) = \left(\frac{0.87 \times 415 \times 1256.6}{0.36 \times 15 \times 950 \times 565}\right) = 0.16$$

$\therefore$ $$x_u = (0.16 \times 565) = 90.4 \text{mm} > 80 \text{ mm}$$

Hence, the assumption that $x_u < D_f$ is not correct. The Neutral axis falls outside the flange.

$$\left(\frac{D_f}{d}\right) = \left(\frac{80}{565}\right) = 0.14 < 0.2$$

Neutral axis depth is determined by referring to Fig. 6.5 and by compatibility of forces as shown in figure we have

$$[C_1 + C_2] = [T_1 + T_2] = T.$$

Assuming $\left(\frac{D_f}{x_u}\right) < 0.43$

$$C_1 = 0.36 f_{ck}\, b_w\, x_u = (0.36 \times 15 \times 250 \times x_u) = 1350\, x_u.$$
$$C_2 = 0.45 f_{ck}\, (b_f - b_w)\, D_f$$
$$= (0.45 \times 15)\,(950\text{-}250)\, 80$$
$$= 378000 \text{ N.}$$
$$T = 0.87 f_y A_{st}$$
$$= (0.87 \times 415 \times 1256.6)$$
$$= 453695.43 \text{ N.}$$

Hence we have $(1350\, x_u + 378000) = 453695.43$

$\therefore$ $x_u = 56.07$ mm < 80 mm.

Therefore the assumption that

$\left(\frac{D_f}{x_u}\right) < 0.43$ is wrong and $\left(\frac{D_f}{x_u}\right)$ will be > 0.43. As such value of C_2 will change.

$$C_2 = 0.45 f_{ck} (b_f - b_w) y_f$$
$$y_f = (0.15 x_u + 0.65 D_f)$$
$$= (0.15 x_u + 0.65 \times 80)$$
$$= (0.15 x_u + 52)$$

$\therefore$
$$C_2 = 0.45 \times 15 (950-250) (0.15 xu + 52)$$
$$= (708.75 x_u + 245700)$$

$\therefore$
$$[1350 x_u + 708.75 x_u + 245700] = 453695.43.$$

$\therefore$
$$x_u = 101.03 \text{ mm} > 80 \text{ mm}.$$

$$\left(\frac{D_f}{x_u}\right) = \left(\frac{80}{101.03}\right) = 0.80 > 0.43$$

$$x_{u,\,max} = 0.48d = (0.48 \times 565) = 271.2 \text{ mm}.$$

$$x_u < x_{u,\,max}$$

Hence, according to Clause- G.2.3 of I.S.456-2000 the moment of resistance of Tee-section is computed by replacing $x_{u,max}$ by x_u in Eq.(6.9).

$$M_u = 0.36\left(\frac{x_{u,\,max}}{d}\right)\left[1 - 0.42\left(\frac{x_{u,\,max}}{d}\right)\right] f_{ck}.b_w.d^2 + 0.45 f_{ck}(b_f - b_w) y_f (d - 0.5 y_f)$$

$$\left(\frac{x_u}{d}\right) = \left(\frac{101.03}{565}\right) = 0.18 \quad \text{and} \quad y_f = [0.15 x_u + 0.65 D_f]$$

$$y_f = [(0.15 \times 101.03) + (0.65 \times 80)] = 67.15 \text{mm} < D_f$$

$\therefore$
$$M_u = 0.36\,(0.18)\;[1 - 0.42 \times 0.18]\,(15 \times 250 \times 565^2)$$
$$+ 0.45 \times 15\,(950-250)\,67.15\,(565 - 0.5 \times 67.15)$$
$$= (240.03 \times 10^6) \text{ N.mm}$$
$$= 240.03 \text{ kN.m}$$

Method 2 (using SP: 16 Design aids)

In the present example, $x_u < x_{u,max}$ and hence SP: 16 Design tables 57,58,59 cannot be used to compute the moment of resistance.

4) A singly reinforced T-beam has a flange width of 900mm, thickness of flange is 150mm width of rib = 300mm, Effective depth = 650mm. Area of tensile reinforcement = 4000 mm². M-20 grade concrete and Fe-415 HYSD bars are used. Estimate the ultimate flexural strength of the section using IS: 456-2000 code provisions.

Method–1 (Using IS: 456-2000 code provisions)

a) Data

$b_f = 900$ mm
$D_f = 150$ mm
$d = 650$ mm
$b_w = 300$ mm
$A_{st} = 4000 \text{ mm}^2$
$f_{ck} = 20 \text{ N/mm}^2$
$f_y = 415 \text{ N/mm}^2$

b) Depth of Neutral Axis

Assuming neutral axis to fall within the flange, the depth of neutral axis is computed.

$$\left(\frac{x_u}{d}\right) = \left(\frac{0.87 f_y A_{st}}{0.36 f_{ck}.b_f.d}\right) = \left(\frac{0.87 \times 415 \times 4000}{0.36 \times 20 \times 900 \times 650}\right) = 0.34$$

$\therefore$
$$x_u = (0.36 \times 650) = 221 \text{ mm} > D_f$$

Hence the assumption that $x_u < D_f$ is not correct. The neutral axis falls outside the flange.

$$\left(\frac{D_f}{d}\right) = \left(\frac{150}{650}\right) = 0.23 > 0.2$$

Neutral axis depth is determined by referring to Fig. 6.5 and by compatibility of forces as shown below.

$$[C_1 + C_2] = [T_1 + T_2] = T$$
$$C_1 = 0.36 f_{ck} b_w x_u$$
$$= (0.36 \times 20 \times 300 \times x_u) = 2160\, x_u$$
$$C_2 = 0.45 f_{ck}(b_f = b_w)\, y_f$$
$$y_f = 0.15\, x_u + 0.65\, D_f$$
$$= 0.15 x_u + (0.65 \times 150)$$
$$= (0.15 x_u + 97.5)$$

$\therefore$
$$C_2 = [(0.45 \times 20)(900 - 300)(0.15 x_u + 97.5)] = [810 x_u + 526500]$$
$$T = (0.87 f_y.A_{st}) = (0.87 \times 415 \times 4000) = 1444200 \text{ N}$$
$$(2160 x_u + 810 x_u + 526500) = 1444200$$

$$\therefore \quad x_u = 309 \text{ mm} > D_f$$

$$x_{u,max} = 0.48d = 312\text{mm}$$

$$x_u < x_{u,max}$$

Hence, according to Clause-G.2.3 of I.S: 456-2000, the moment of resistance of tee-section is computed by replacing $x_{u,max}$ by x_u in Eq. 6.9

$$M_u = 0.36\left(\frac{x_u}{d}\right)\left[1-0.42\left(\frac{x_u}{d}\right)\right] f_{ck}.b_w.d^2 + 0.45 f_{ck}(b_f - b_w)y_f(d - 0.5y_f)$$

$$\left(\frac{x_u}{d}\right) = \left(\frac{309}{650}\right) = 0.475 \quad \text{and} \quad y_f = (0.15x_u + 0.65D_f)$$

$$y_f = (0.15 \times 309) + (0.65 \times 150) = 143.85\text{mm} < D_f$$

$$M_u = [0.36 \times 0.475(1 - 0.42 \times 0.475)20 \times 300 \times 650^2]$$

$$+ [(0.45 \times 20)(900 - 300)143.85(650 - 0.5 \times 143.85)]$$

$$= (796.05 \times 10^6)\text{N.mm}$$

$$= 796.05 \text{ kN.m}$$

5) A tee-beam is singly reinforced and has the following sectional properties. Estimate the ultimate moment of resistance of the section using I.S code provisions.

Width of flange = 1200 mm
Thickness of Flange = 150 mm
Width of rib = 300 mm
Effective depth = 750 mm
Area of tension reinforcement = 5520 mm^2
M-20 Grade Concrete and Fe-415 HYSD bars

Method–1 (Using IS: 456-2000 Code Provisions)

a) Data

b_f = 1200 mm	A_{st} = 5520 mm^2
D_f = 150 mm	f_{ck} = 20 N/mm^2
b_w = 300 mm	f_y = 415 N/mm^2
d = 750 mm	

b) Depth of Neutral Axis

Assuming neutral axis to fall within the flange, we have,

$$\left(\frac{x_u}{d}\right) = \left(\frac{0.87f_y A_{st}}{0.36f_{ck} b_f d}\right) = \left(\frac{0.87 \times 415 \times 5520}{0.36 \times 20 \times 1200 \times 750}\right) = 0.307$$

$$\therefore \quad x_u = (0.307 \times 750) = 230.25 \text{ mm} > D_f$$

Hence, the assumption that $x_u < D_f$ is not correct. Compute the neutral axis depth by force compatibility of the section.

$$\left(\frac{D_f}{d}\right) = \left(\frac{150}{750}\right) = 0.2$$

Refer Fig. 6.5 and by equating the compressive and tensile forces, we have the relation

$$(C_1 + C_2) = (T_1 + T_2) = T$$

$$C_1 = 0.36 f_{ck} b_w . x_u$$

$$= (0.36 \times 20 \times 300 \times x_u) = 2160x_u$$

$$C_2 = 0.45 f_{ck}(b_f - b_w)D_f$$

$$= 0.45 \times 20(1200 - 300)150 = 1215000 \text{ N}$$

$$T = 0.87 f_y . A_{st}$$

$$T = (0.87 \times 415 \times 5520)$$

$$= 1992996 \text{ N}$$

$$[2160x_u + 1215000] = 1992996$$

$$\therefore \quad x_u = 360\text{mm} > D_f$$

For Fe-415 HYSD bars, $x_{u,lim} = 0.48\, d = (0.48 \times 750) = 360$ mm.

$$\therefore \quad x_u = x_{u,lim}$$

and
$$\left(\frac{D_f}{x_u}\right) = \left(\frac{150}{750}\right) = 0.2$$

$$\therefore \quad x_u = x_{u,max} > D_f$$

The moment of resistance is computed by using Eq. (6.8)

$$x_u = x_{u,max} = 360 \text{ mm} \ \& \left(\frac{x_u}{d}\right) = 0.48$$

$$M_u = 0.36\left(\frac{x_u}{d}\right)\left[1 - 0.42\left(\frac{x_u}{d}\right)\right] f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w)D_f(d - 0.5D_f)$$

$= 0.36 \times 0.48[1 - 0.42(0.48)]20 \times 300 \times 750^2$

$+0.45 \times 20(1200 - 300)150(750 - 0.5 \times 150)$

$= (1285 \times 10^6)$ N.mm

$= 1285$ kN.m

Method-2 (Using SP: 16 Design Tables)

Refer Table-58 of SP: 16 [Table 6.7 of text]
Corresponding to the ratios of

$$\left(\frac{D_f}{d}\right) = \left(\frac{150}{750}\right) = 0.2 \quad \text{And} \quad \left(\frac{b_f}{b_w}\right) = \left(\frac{1200}{300}\right) = 4$$

and Fe- 415 N/mm^2 and read out the value of ratio.

$$\left[\frac{M_{u,\,lim}}{b_w.d^2.f_{ck}}\right] = 0.379$$

$$M_{u,\,lim} = [0.379 \times 300 \times 760^2 \times 20] = [1279 \times 10^6] \text{ N.mm} = 1279 \text{ kN.m}$$

6) Determine the ultimate moment of resistance of the tee beam having the following section properties:-

Width of Flange = 900 mm
Thickness of Flange = 150 mm
Width of rib = 300 mm
Effective depth = 600 mm
Area of Tension Reinforcement = 3966 mm^2
M-20 Grade Concrete and Fe-415 HYSD bars.

Method–1 (Using IS 456-2000 Code Provisions)

a) Data

$b_f = 900$ mm	$A_{st} = 3966$ mm^2
$D_f = 150$ mm	$f_{ck} = 20$ N/mm^2
$b_w = 300$ mm	$f_y = 415$ N/mm^2
$d = 600$ mm	

b) Depth of Neutral Axis

Assuming neutral axis to fall within the flange thickness, we have,

$$\left(\frac{x_u}{d}\right) = \left(\frac{0.87 f_y A_{st}}{0.36 f_{ck}.b_f.d}\right) = \left(\frac{0.87 \times 415 \times 3966}{0.36 \times 20 \times 900 \times 600}\right) = 0.368$$

$\therefore$ $x_u = (0.368 \times 600) = 220.8 \text{ mm} > D_f$

Hence, the assumption that $x_u < D_f$ is not correct. Compute the neutral axis depth by force compatibility of the section.

$$\left(\frac{D_f}{d}\right) = \left(\frac{150}{600}\right) = 0.25$$

Refer Fig. 6.5 and by equating the compressive and tensile forces, we have the relation

$$[C_1 + C_2] = [T_1 + T_2] = T$$

$$C_1 = 0.36 f_{ck} b_w.x_u$$

$$= (0.36 \times 20 \times 300 \times x_u) = 2160 x_u$$

$$C_2 = 0.45 f_{ck}(b_f - b_w) D_f$$

$$= 0.45 \times 20(900 - 300)150$$

$$= 810000 \text{ N}$$

$$T = 0.87 f_y.A\text{st}$$

$$= (0.87 \times 415 \times 3966)$$

$$= 14,31,924 \text{ N}$$

$$(2160 x_u + 810,000) = 19,92,996$$

$\therefore$ $x_u = 288 \text{ mm} > D_f$

For Fe-415 HYSD bars, the limiting depth of the neutral axis is given by the expression, $x_{u,max} = 0.48\,d = (0.48 \times 600) = 288$ mm.

and $\left(\frac{D_f}{x_u}\right) = \left(\frac{150}{288}\right) = 0.52 > 0.43$

also $\left(\frac{D_f}{d}\right) = \left(\frac{150}{600}\right) = 0.25 > 0.2$

Hence according to Clause G.2.3 of IS: 456-2000, the moment of resistance is computed by the Eq. (6.9)

$$M_u = 0.36\left(\frac{x_{u,\,max}}{d}\right)\left[1 - 0.42\left(\frac{x_{u,\,max}}{d}\right)\right] f_{ck}.b_w.d^2 + 0.45 f_{ck}(b_f - b_w) y_f(d - 0.5 y_f)$$

$\therefore \quad y_f = (0.15x_u + 0.65D_f) = (0.15 \times 288) + (0.65 \times 150) = 140.7\text{mm} < D_f = 150\text{mm}$

$$M_u = (0.36 \times 0.48)[1-(0.42 \times 0.48)](20 \times 300 \times 600^2)$$
$$+(0.45 \times 20)(900-300)140.7[600-(0.5 \times 150)]$$
$$= (700 \times 10^6)\text{N.mm} = 700\text{kN.m}$$

Method-2 (Using SP: 16 Design Tables)

Refer Table-58 of SP: 16 [Table 6.7 of text]

$$\left(\frac{D_f}{d}\right) = \left(\frac{150}{600}\right) = 0.25 \quad \text{And} \quad \left(\frac{b_f}{b_w}\right) = \left(\frac{900}{300}\right) = 3$$

Read out the value of $\left[\dfrac{M_{u,\,lim}}{b_w.d^2.f_{ck}}\right] = 0.323$

$\therefore \quad M_{u,lim} = [0.323 \times 300 \times 600^2 \times 20] = (698 \times 10^6)\text{N.mm} = 698\text{ kN.m}$

6.3.7 Design Examples

1) Determine the area of tensile reinforcement required in a flanged beam having the following sectional dimensions to support a factored moment of 300 kN.m

Width of Flange (b_f) =750 mm
Width of rib (b_w) = 300 mm
Thickness of Flange (D_f) =120 mm
Effective Depth (d) = 600 mm
M-20 Grade Concrete and Fe-415 HYSD bars

Method–1 (Using IS: 456-2000 Code Provisions)

a) Data

b_f = 750 mm f_{ck} = 20 N/mm²
b_w = 300 mm f_y = 415 N/mm²
D_f = 120 mm M_u = 300 kNm
D = 600 m

b) Limiting moment of Resistance $\left(\dfrac{D_f}{d}\right) = \left(\dfrac{120}{600}\right) = 0.2$

$$\therefore \quad M_{u,lim} = [0.138 f_{ck} b_w.d^2 + 0.45 f_{ck}(b_f - b_w)D_f(d-0.5D_f)]$$
$$= [(0.138 \times 20 \times 300 \times 6002) + 0.45 \times 20(750-300)120(600-0.5 \times 120)]$$
$$= (554 \times 10^6)\text{N.mm}$$

Since $\quad M_u = (300 \times 10^6)\text{N.mm} < M_{u,\,lim}$

$x_u < 0.48d$

Assuming $x_u < D_f$, compute the value of x_u from moment equation

$$M_u = 0.36 f_{ck}.b_f.x_u(d-0.42x_u)$$
$$(300 \times 10^6) = (0.38 \times 20 \times 750 \times x_u)(600-0.42x_u)$$
$$[x_u^2 - 1428.5x_u + 1.32 \times 105] = 0$$

Solving x_u = 99 mm. Hence, $x_u < D_f$

c) Tensile Reinforcement

The section is considered as rectangular and under reinforced

$$M_u = 0.87 f_y A_{st}.d\left[1-\left(\frac{A_{st}.f_y}{b_f d.f_{ck}}\right)\right]$$
$$(300 \times 10^6) = (0.87 \times 415 \times A_{st} \times 600)\left[1-\left(\frac{A_{st} \times 415}{750 \times 600 \times 20}\right)\right]$$

Solving A_{st} = 1493 mm²

Method–2 (Using SP: 16 Design Aids)

Since neutral axis falls within flange, the section is considered as rectangular.

Referring to Table-2 of SP: 16 (Table 6.5 of Text)

$$\left(\frac{M_u}{bd^2}\right) = \left(\frac{300 \times 10^6}{750 \times 600^2}\right) = 1.11$$

Read out percentage reinforcement p_t corresponding to f_y = 415 N/mm² yielding $p_t = 0.33\%$

$$\therefore \quad A_{st} = \left(\frac{p_t.b.d}{100}\right) = \left(\frac{0.33 \times 750 \times 600}{100}\right) = 1485\text{ mm}^2$$

2) A tee-beam has the following dimensions:-

Effective width of flange (b_f) = 2000 mm
Thickness of Flange (D_f) = 150 mm
Width of rib (b_w) = 300 mm
Effective depth (d) = 1000 mm
M-20 Grade Concrete and Fe-415 HYSD bars

Calculate the limiting moment of capacity of the section and the corresponding area of tension reinforcement.

Method-1 (Using IS: 456-2000 Code Formulae)

a) Data

$b_f = 2000$ mm $f_{ck} = 20$ N/mm^2
$D_f = 150$ mm $f_y = 415$ N/mm^2
$b_w = 300$ mm
$d = 1000$ mm

b) Neutral axis Depth

For $f_y = 415$ N/mm^2, limiting depth of neutral axis is

$$x_u = 0.48\,d = (0.48 \times 1000) = 480 \text{ mm}$$

$$\left(\frac{D_f}{d}\right) = \left(\frac{150}{1000}\right) = 0.15 < 0.2$$

$$\left(\frac{D_f}{x_u}\right) = \left(\frac{150}{480}\right) = 0.31 < 0.43$$

b) Moment of resistance

Hence, Eq. (6.8) recommended in IS: 456-2000 yields,

$$M_u = 0.36\left(\frac{x_u}{d}\right)\left[1 - 0.42\left(\frac{x_u}{d}\right)\right] f_{ck}.b_w.d^2 + 0.45 f_{ck}(b_f - b_w)D_f(d - 0.5D_f)$$

This equation can be recast as

$$M_u = [0.138 f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w)D_f(d - 0.5D_f)]$$

$$= [(0.138 \times 20 \times 300 \times 1000^2) + 0.45 \times 20(1000 - 300)150(1000 - 0.5 \times 150)]$$

$$= (2980 \times 10^6) \text{ N.mm}$$

$$= 2980 \text{ kN.m}$$

c) Area of Tension Reinforcement

$$A_{st} = [A_{stw} + A_{stf}]$$

$$= \left[\left(\frac{0.36 f_{ck}\, b_w.x_u}{0.87 f_y}\right) + \left(\frac{0.45\, f_{ck}\,(b_f - b_w)D_f}{0.87\, f_y}\right)\right]$$

$$A_{st} = \left[\left(\frac{0.36 \times 20 \times 300 \times 480}{0.87 \times 415}\right) + \left(\frac{0.45 \times 20(2000 - 300)150}{0.87 \times 415}\right)\right] = 9227 \text{ mm}^2$$

Method–2 [Using SP: 16 Design Tables]

Table-58 of SP: 16(Table 6.8 of text) is used to compute the limiting moment of resistance. The parameters are

$$\left(\frac{D_f}{d}\right) = \left(\frac{150}{1000}\right) = 0.15 \quad \text{and} \quad \left(\frac{b_f}{b_w}\right) = \left(\frac{2000}{300}\right) = 6.66$$

Read out $\left(\dfrac{M_{u,\,lim}}{b_w.d^2.f_{ck}}\right)$ corresponding to the parameters

$$\left[\frac{M_{u,\,lim}}{b_w.d^2.f_{ck}}\right] = 0.490$$

$$M_{u,\,lim} = (0.490 \times 300 \times 1000^2 \times 20) = 2940 \times 10^6 \text{ N.mm} = 2940 \text{ kN.m}$$

3) A tee beam has a cross section as detailed below:-

Effective Width of Flange = 1500 mm
Thickness of Flange = 200 mm
Thickness of rib = 300 mm
Effective Depth = 750 mm

If $f_{ck} = 20$ N/mm^2 and $f_y = 415$ N/mm^2, design the tension reinforcement required to resist an ultimate design bending moment of 1600 kN.m

Method–1 [Using IS: 456-2000 Code Provisions]

a) Data

$b_f = 1500$ mm $f_{ck} = 20$ N/mm^2
$b_w = 300$ mm $f_y = 415$ N/mm^2
$d = 750$ mm $M_u = 1600$ kN.m
$D_f = 200$ mm

b) Limiting Moment of Resistance

$$\left(\frac{D_f}{d}\right) = \left(\frac{200}{750}\right) = 0.26 > 0.2$$

$$x_{u,\max} = 0.48d = (0.48 \times 750) = 360 \text{ mm}$$

Using IS: 456-2000 Code Equations given Under Clause G-2.2.1 for the ratio of $(D_f/d) > 0.2$,

$$M_u = 0.36\left(\frac{x_{u,\max}}{d}\right)\left[1 - 0.42\left(\frac{x_{u,\max}}{d}\right)\right] f_{ck}.b_w.d^2 + 0.45 f_{ck}(b_f - b_w) y_f (d - 0.5 y_f)$$

Where $y_f = (0.15 x_u + 0.65 D_f)$ but not greater than D_f.

$\therefore$ $y_f = [(0.15 \times 360) + (0.65 \times 200)]$

$= 184 \text{ mm} < D_f = 200 \text{ mm}$

$$\therefore \quad M_u = [0.138 f_{ck}.b_w.d^2 + 0.45 f_{ck}(b_f - b_w) y_f (d - 0.5 y_f)]$$

$$= [0.138 + 20 \times 300 \times 750^2] + 0.45 \times 20(1500 - 300)184(750 - 0.5 \times 184)]$$

$$= (1773 \times 10^6) \text{ N.mm}$$

$$= 1773 \text{ kN.m} > M_u = 1600 \text{ kN.m}$$

c) Determination of Neutral Axis:-

For the known value of M_u, compute x_u by replacing $x_{u,\max}$ by x_u, in the moment equation.

$$\therefore \quad (1600 \times 10^6) = 0.36\left(\frac{x_u}{750}\right)\left[1 - 0.42\left(\frac{x_u}{750}\right)\right](20 \times 300 \times 750^2)$$

$$+ (0.45 \times 20)(1500 - 300)[0.15 x_u + 130] \times [750 - 0.5(0.15 x_u + 130)]$$

Simplifying we have $(x_u^2 - 2500 x_u + 620700) = 0$

$$\therefore \quad x_u = 273 \text{ mm} \quad \text{and} \quad \therefore \left(\frac{D_f}{x_u}\right) = \left(\frac{200}{273}\right) = 0.73 > 0.43$$

d) Area of tensile reinforcement

$$A_{st} = [A_{stw} + A_{stf}]$$

$$A_{st} = \left[\frac{0.36 f_{ck} b_w \cdot x_u}{0.87 \times 415}\right] + \left[\frac{0.45 f_{ck}(b_f - b_w)(0.15 x_u + 0.65 D_f)}{0.87 f_y}\right]$$

$$A_t = \left[\frac{0.36 \times 20 \times 300 \times 273}{0.87 \times 415}\right] + \left[\frac{0.45 \times 20\,(1500 - 300)(0.15 \times 273 + 0.65 \times 200)}{0.87 \times 415}\right]$$

$$= 6746 \text{ mm}^2$$

Method–2 (Using SP: 16 Design tables)

SP: 16 Design tables cannot be used for the computation of area of tension reinforcement in tee-beams. Tables 58 and 59 of SP: 16 [Tables 6.8 and 6.9 of text] can be used only for determining the limiting moment of resistance ($M_{u,\lim}$) for known value of parameters (D_f/d) and (b_f/b_w) and f_y.

6.4 ULTIMATE FLEXURAL STRENGTH OF DOUBLY REINFORCED CONCRETE SECTIONS

6.4.1 Design principles

Reinforced concrete beams with compression reinforcement will be required in cases where the depth of the beam is restricted and the singly reinforced section is insufficient to resist the moment on the section.The behaviour of R.C.C. beams with compression steel for ultimate load design is sometimes referred to as the *Steel beam theory*. The final beam section with tension and compression steel is assumed to consist of two separate beams consisting of

a) A singly reinforced section which reaches its limiting value of moment of resistance expressed as $M_{u,\lim}$.
b) A steel beam without any concrete but reinforced with tension and compression steel.

The moment of resistance of the doubly reinforced section will be the sum of the moment of resistance of the two different sections specified in (a) and (b).

6.4.2 Design Equations

Consider the doubly reinforced concrete section split into two parts as shown in Fig. 6.7.

Let M_u = moment of resistance of the doubly reinforced section.

$M_{u1} = M_{u,\lim}$ = the limiting or the maximum moment capacity of the singly reinforced section [Eq. 6.5].

M_{u2} = moment capacity of the steel beam neglecting the effect of concrete

$= f_{sc} A_{sc} (d - d')$

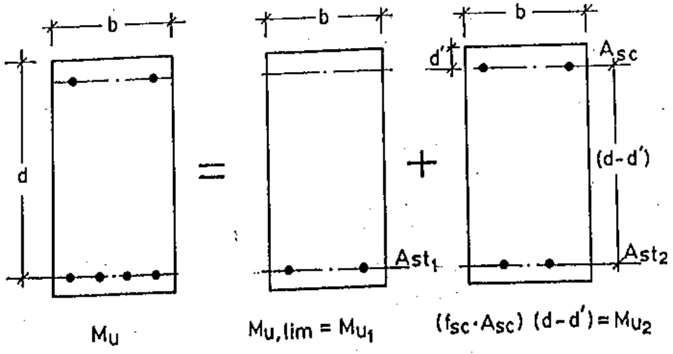


Fig. 6.7 Doubly Reinforced Section

where f_{sc} = the stress in the compression steel corresponding to the strain reached by it when the extreme concrete fibre reaches a strain of 0.0035.

Hence, we have the relation,

$$M_u = M_{u,lim} + f_{sc} A_{sc} (d-d')$$

Where, A_{sc} = area of compression reinforcement
d = effective depth to tension steel
d' = depth of compression reinforcement from compression face
A_{st1} = area of tensile reinforcement for a singly reinforced section.
A_{st2} = area of tensile reinforcement required to balance the compression reinforcement.

$$A_{st} = (A_{st1} + A_{st2}) = \text{total tensile steel.}$$

The value of stress in the compression reinforcement (f_{sc}) depends upon the ratio (d'/d) and the grade of steel as shown in Table 6.10 based on SP:16.

For values of (d'/d) up to 0.2, the stress in concrete in the extreme fibre is equal to 0.45 f_{ck} and for mild steel reinforcement f_{sc} would be equal to the design yield stress having a value of 0.87 f_y. When the reinforcement is cold worked bars, the design stress in compression reinforcement f_{sc} for different values of (d'/d) is shown in Table 6.10, based on SP: 16.

The reinforcements A_{st1}, A_{st2} and A_{sc} in the doubly reinforced section is computed using the following steps.

1) Equating tensile force in steel and compressive force in concrete, we have

Table 6.10 Stress in Compression Reinforcement (f_{sc}) in Doubly reinforced Beams with Cold Worked Bars(Table-F of SP:16)

f_y (N/mm²)	(d'/d)			
	0.05	0.10	0.15	0.20
415	355	353	342	329
500	424	412	395	370

$$T_1 = C_1$$

$$(A_{st1} \times 0.87 f_y) = (0.36 f_{ck} b x_{u,lim})$$

Hence, we have, $A_{st1} = [0.36 f_{ck}\, b\, x_{u,lim}]/(0.87 f_y)$

Where T_1 and C_1 are the tensile and compressive force capacity of balanced singly reinforced section.

2) $M_{u2} = (M_u - M_{u,lim}) = f_{sc} A_{sc} (d - d')$
Hence we have, $A_{sc} = [M_{u2}/f_{sc} (d - d')]$

3) $T_2 = C_2$. Hence, we have $(0.87 f_y A_{st2}) = (f_{sc} A_{sc})$
Therefore $A_{st2} = [(f_{sc} A_{sc}) / 0.87 f_y]$

Where T_2 and C_2 are the additional, tensile and compressive force carrying capacity of section.

6.4.3 Analysis of doubly Reinforced Sections

Method–1 (Using I.S. Code Formula)

1) As a first trial, assume $x_u = x_{u,lim}$, and calculate strain in concrete at the level of Compression steel (ε_{sc}) computed as

$$\varepsilon_{sc} = 0.0035\, [x_{u,lim} - d']/x_{u,lim}$$

2) For mild steel bars, if $\varepsilon_{sc} < 0.00125$, where 0.00125 is the yield strain of mild steel. We have the stress in mild steel as,

$$f_{sc} = (0.87 E_s\, \varepsilon_{sc}) = (0.87 \times 2 \times 10^5 \varepsilon_{sc})$$

If $\varepsilon_{sc} > 0.00125$, then $f_{sc} = 0.87 f_y$

For HYSD bars, obtain the value of f_{sc} from the stress-strain curve shown in Fig. 6.3. (Fig. 3 of SP: 16-1980).

3) Calculate the value of A_{st2} using the relation $A_{st2} = [(f_{sc} A_{sc})/ 0.87 f_y]$
4) Obtain the value of $A_{st1} = (A_{st} - A_{st2})$
5) Calculate the value of x_u using the relation,

$$x_u = [(0.87 f_y A_{st1})/ (0.36 f_{ck}\, b)]$$

6) Considering this value of x_u, repeat the steps 1 to 5 to obtain constant value of x_u
7) Comparing this value of x_u with $x_{u,lim}$, calculate the value of M_u
8) If $x_u < x_{u,lim}$, compute the value of the moment capacity of section as

$$M_u = [0.87 f_y A_{st1} + f_{sc} A_{sc}(d - d')]$$

9) If $x_u > x_{u,lim}$, compute the value of the moment capacity of section as

$$M_u = [0.36 f_{ck}\, x_{u,lim}\, b\,(d - 0.42\, x_{u,lim}) + f_{sc} A_{sc}(d - d')]$$

Method–2

Alternatively, strain compatibility method may be used to analyse the doubly reinforced sections as detailed below:

1) Select a trial value for x_u, the depth of neutral axis. Assuming that the extreme compression fibre in concrete fails at a strain of $\varepsilon_{cu} = 0.0035$, determine the strain in the tension reinforcement by the relation,

$$\varepsilon_{st} = \varepsilon_{cu}(d - d')/x_u$$

2) Compute the stress in steel ($f_{st} < f_y$) corresponding to ε_{st} by the stress-strain curve shown in Fig. 6.3.
3) Total tension in steel is computed as $T = f_{st} A_{st}$
4) Compute the strain in compression steel using the relation

$$\varepsilon_{sc} = [0.0035(x_u - d')/x_u]$$

5) Read out the compressive stress f_{sc} corresponding to the strain ε_{sc} from Fig. 6.3.
6) Compute the compressive force in steel as $C_s = f_{sc} A_{sc}$
7) Using the standard stress block for concrete on the compression side, compute the compressive force in concrete as

$$C_c = 0.36 f_{ck}\, b\, x_u$$

8) Determine the total compression as $C = (C_s + C_c)$
9) Check whether total tension T is equal to the compression C ($T = C$). If $T = C$, the assumed neutral axis depth x_u is satisfactory. Otherwise chose another trial value of x_u and repeat the steps from (1) to (8) until the value of $T = C$.
10) The ultimate moment of resistance is computed by taking the moments of forces C_s and C_c about tension steel yielding the relation,

$$M_u = [C_c(d - 0.42 x_u) + C_s(d - d')]$$

This method of computing the moment of resistance is referred to as the strain compatibility method or force equilibrium method and it gives a correct value of the moment of resistance of the section.

6.4.4 Use of Design Aids (SP: 16) for design of Doubly reinforced Sections

The tables and charts of SP: 16 are very useful in the analysis and design of doubly reinforced beams.

The moment of resistance of a doubly reinforced section can be expressed in the form,

$$M_u = [M_{u,lim} + p_{t2} bd(0.87 f_y)(d - d')]$$

$$(M_u/bd^2) = (M_{u,lim}/bd^2) + p_{t2}(0.87 f_y)[1 - (d'/d)]/100$$

Where p_{t2} = additional percentage of tensile reinforcement expressed as $(100 A_{st2}/b\,d)$
p_t = total percentage of tension reinforcement
$p_{t,lim}$ = percentage of tension reinforcement for the singly reinforced section = $(100 A_{st1}/b\,d)$

Hence, we have $p_t = (p_{t,lim} + p_{t2})$
p_c = percentage of compression reinforcement = $(100 A_{sc}/b\,d)$

SP: 16 design tables 45 to 56 present the percentage of tension and compression reinforcements (p_t and p_c) for different ratios of (d'/d) varying from 0.05 to 0.20 and for various grades of concrete (f_{ck} = 15 to 30 N/mm^2) and different grades of steel (f_y = 250, 415 and 500 N/mm^2) covering the moment of resistance factor (M_u/bd^2) varying from 2.24 to 8.30.

Some of the salient tables covering M-20, M-25 and M-30 grades of concrete and Fe-415 grade steel are reproduced in Tables 6.11, 6.12 and 6.13 of the text.

6.4.5 Analysis Examples

1) Determine the ultimate moment of resistance of a doubly reinforced beam of Rectangular section having a width of 300 mm and reinforced with 5 bars of 25 mm diameter at an effective depth of 600 mm. The compression steel is made up of 2 bars of 25 mm diameter at an effective cover of 60 mm. Adopt M-20 grade concrete and fe-415 HYSD bars.

Table 6.11 Flexure-Reinforcement Percentages For Doubly Reinforced Sections (Table-50 of SP: 16)

f_{ck} = 20 N/mm^2
f_y = 500 N/mm^2

M_u/bd^2 N/mm^2	$d'/d = 0.05$		$d'/d = 0.10$		$d'/d = 0.15$		$d'/d = 0.20$	
	P_t	P_c	P_t	P_c	P_t	P_c	P_t	P_c
2.77	0.958	0.002	0.958	0.002	0.959	0.003	0.959	0.003
2.80	0.967	0.011	0.968	0.012	0.968	0.013	0.969	0.015
2.90	0.996	0.042	0.998	0.045	1.001	0.049	1.004	0.054
3.00	1.025	0.072	1.029	0.077	1.034	0.084	1.038	0.093
3.10	1.055	0.103	1.060	0.109	1.066	0.119	1.073	0.132
3.20	1.084	0.133	1.091	0.142	1.099	0.154	1.108	0.171
3.30	1.113	0.164	1.122	0.174	1.131	0.190	1.142	0.210
3.40	1.142	0.194	1.152	0.207	1.164	0.225	1.177	0.249
3.50	1.171	0.224	1.183	0.239	1.197	0.260	1.212	0.288
3.60	1.200	0.255	1.214	0.271	1.229	0.295	1.246	0.327
3.70	1.230	0.285	1.245	0.304	1.262	0.331	1.281	0.366
3.80	1.259	0.316	1.276	0.336	1.294	0.366	1.315	0.405
3.90	1.288	0.346	1.306	0.369	1.327	0.401	1.350	0.444
4.00	1.317	0.376	1.337	0.401	1.360	0.437	1.385	0.483
4.10	1.346	0.407	1.368	0.433	1.392	0.472	1.419	0.522
4.20	1.375	0.437	1.399	0.466	1.425	0.507	1.454	0.561
4.30	1.405	0.468	1.429	0.498	1.457	0.542	1.489	0.600
4.40	1.434	0.498	1.460	0.530	1.490	0.578	1.523	0.640
4.50	1.463	0.528	1.491	0.563	1.523	0.613	1.558	0.679
4.60	1.492	0.559	1.522	0.595	1.555	0.648	1.593	0.718

(Contd.)

Table 6.11 (Contd.)

4.70	1.521	0.589	1.553	0.628	1.588	0.683	1.627	0.757
4.80	1.550	0.620	1.583	0.660	1.620	0.719	1.662	0.796
4.90	1.580	0.650	1.614	0.692	1.653	0.754	1.696	0.835
5.00	1.609	0.680	1.645	0.725	1.686	0.789	1.731	0.874
5.10	1.638	0.711	1.676	0.757	1.718	0.825	1.766	0.913
5.20	1.667	0.741	1.707	0.790	1.751	0.860	1.800	0.956
5.30	1.696	0.772	1.737	0.822	1.783	0.895	1.835	0.991
5.40	1.725	0.802	1.768	0.854	1.816	0.930	1.870	1.030
5.50	1.755	0.832	1.799	0.887	1.849	0.966	1.904	1.069
5.60	1.784	0.863	1.830	0.919	1.881	1.001	1.939	1.108
5.70	1.813	0.893	1.861	0.952	1.914	1.036	1.974	1.147
5.80	1.842	0.924	1.891	0.984	1.946	1.071	2.008	1.186
5.90	1.871	0.954	1.922	1.016	1.979	1.107	2.043	1.225
6.00	1.900	0.985	1.953	1.049	2.012	1.142	2.078	1.264
6.10	1.930	1.015	1.984	1.081	2.044	1.177	2.112	1.303
6.20	1.959	1.045	2.014	1.114	2.077	1.213	2.147	1.342
6.30	1.988	1.076	2.045	1.146	2.109	1.248	2.181	1.381
6.40	2.017	1.106	2.076	1.178	2.142	1.283	2.216	1.421
6.50	2.046	1.137	2.107	1.211	2.175	1.318	2.251	1.460
6.60	2.075	1.167	2.138	1.243	2.207	1.354	2.285	1.499
6.70	2.105	1.197	2.168	1.276	2.240	1.389	2.320	1.538
6.80	2.134	1.228	2.199	1.308	2.272	1.424	2.355	1.577
6.90	2.163	1.258	2.230	1.340	2.305	1.459	2.389	1.616
7.00	2.192	1.289	2.261	1.373	2.338	1.495	2.424	1.655
7.10	2.221	1.319	2.292	1.405	2.370	1.530	2.459	1.694

Table 6.12 Flexure–Reinforcement Percentages For Doubly Reinforced Sections (Table–51 of SP: 16)

f_{ck} = 20 N/mm²
f_y = 415 N/mm²

M_u/bd^2 N/mm²	$d'/d = 0.05$		$d'/d = 0.10$		$d'/d = 0.15$		$d'/d = 0.20$	
	P_t	P_c	P_t	P_c	P_t	P_c	P_t	P_c
3.46	1.197	0.002	1.197	0.002	1.197	0.003	1.197	0.003
3.50	1.209	0.014	1.210	0.015	1.210	0.017	1.211	0.019
3.60	1.238	0.045	1.240	0.048	1.243	0.052	1.246	0.58
3.70	1.267	0.076	1.271	0.081	1.276	0.088	1.281	0.097
3.80	1.296	0.106	1.302	0.113	1.308	0.123	1.315	0.137
3.90	1.325	0.137	1.33	0.146	1.341	0.159	1.350	0.176
4.00	1.355	0.167	1.363	1.178	1.373	0.194	1.385	0.215
4.10	1.384	0.198	1.394	0.211	1.406	0.230	1.419	0.254
4.20	1.413	0.229	1.425	0.244	1.439	0.265	1.454	0.294
4.30	1.442	0.259	1.456	0.276	1.471	0.301	1.488	0.333
4.40	1.471	0.290	1.487	0.309	1.504	0.336	1.523	0.372
4.50	1.500	0.320	1.517	0.341	1.536	0.372	1.558	0.412
4.60	1.530	0.351	1.548	0.374	1.569	0.407	1.592	0.451
4.70	1.559	0.382	1.579	0.407	1.602	0.443	1.627	0.490
4.80	1.588	0.412	1.610	0.439	1.634	0.478	1.662	0.530
4.90	1.617	0.443	1.641	0.472	1.67	0.514	1.696	0.569
5.00	1.646	0.474	1.671	0.504	1.699	0.549	1.731	0.608
5.10	1.675	0.504	1.702	0.537	1.732	0.585	1.766	0.648
5.20	1.705	0.535	1.733	0.570	1.765	0.620	1.800	0.687
5.30	1.734	0.565	1.764	0.602	1.797	0.656	1.835	0.726

(Contd.)

Table 6.12 (Contd.)

5.40	1.763	0.596	1.795	0.635	1.830	0.691	1.869	0.766
5.50	1.792	0.627	1.825	0.667	1.862	0.727	1.904	0.805
5.60	1.821	0.657	1.856	0.700	1.895	0.762	10939	0.844
5.70	1.851	0.688	1.887	0.733	1.928	0.798	1.973	0.844
5.80	1.880	0.718	1.918	0.765	1.960	0.833	2.008	0.923
5.90	1.909	0.749	1.948	0.798	1.993	0.869	2.043	0.962
6.00	1.938	0.780	1.979	0.830	2.025	0.904	2.077	1.002
6.10	10967	0.810	2.010	0.863	2.058	0.940	2.112	1.041
6.20	1.996	0.841	2.041	0.896	2.091	0.975	2.147	1.080
6.30	2.026	0.871	2.072	0.928	2.123	1.011	2.181	1.120
6.40	2.055	0.902	2.102	0.961	2.156	1.046	2.216	1.159
6.50	2.084	0.933	2.133	0.993	2.188	1.082	2.251	1.198
6.60	2.113	0.963	2.164	1.026	2.221	1.118	2.285	1.238
6.70	2.142	0.994	2.195	1.059	2.254	1.153	2.320	1.277
6.80	2.171	1.024	2.226	1.091	2.286	1.189	2.354	1.316
6.90	2.201	1.055	2.256	1.124	2.319	1.224	2.389	1.356
7.00	2.230	1.086	2.287	1.157	2.351	1.260	2.424	1.395
7.10	2.259	1.116	2.318	1.189	2.384	1.295	2.458	1.434
7.20	2.288	1.147	2.349	1.222	2.417	1.331	2.493	1.513
7.30	2.317	1.177	2.380	1.254	2.449	1.336	2.528	
7.40	2.346	1.208	2.410	1.287	2.482	1.402	2.562	1.552
7.50	2.376	1.239	2.441	1.320	2.514	1.437	2.597	1.591
7.60	2.405	1.269	2.472	1.352	2.547	1.473	2.632	1.631
7.70	2.434	1.300	2.503	1.385	2.580	1.508	2.666	1.670
7.80	2.463	1.330	2.534	1.417	2.612	1.544	2.701	1.709

Table 6.13 Flexure–Reinforcement Percentages For Doubly Reinforced Sections
(Table–52 of SP: 16)

f_{ck} = 30 N/mm²
f_y = 415 N/mm²

M_u/bd^2 N/mm²	$d'/d = 0.05$		$d'/d = 0.10$		$d'/d = 0.15$		$d'/d = 0.20$	
	P_t	P_c	P_t	P_c	P_t	P	P_t	
4.15	1.436	0.002	1.436	0.002	1.436	0.002	1.436	0.003
4.20	1.451	0.017	1.451	0.019	1.452	0.020	1.454	0.022
4.30	1.480	0.048	1.482	0.051	1.485	0.056	1.488	0.062
4.40	1.509	0.079	1.513	0.084	1.518	0.092	1.523	0.102
4.50	1.538	0.110	1.544	0.117	1.550	0.127	1.558	0.141
4.60	1.567	0.141	1.575	0.150	1.583	0.163	1.592	0.181
4.70	1.596	0.171	1.605	0.183	1.615	0.199	1.627	0.220
4.80	1.626	0.202	1.636	0.215	1.648	0.235	1.661	0.260
4.90	1.655	0.233	1.667	0.248	1.681	0.270	1.696	0.300
5.00	1.684	0.264	1.698	0.281	1.713	0.306	1.731	0.339
5.10	1.713	0.295	1.729	0.314	1.746	0.324	1.765	0.379
5.20	1.742	0.325	1.759	0.347	1.778	0.378	1.800	0.418
5.30	1.771	0.356	1.790	0.380	1.811	0.413	1.835	0.458
5.40	1.801	0.387	1.821	0.412	1.844	0.449	1.869	0.498
5.50	1.830	0.418	1.852	0.445	1.846	0.485	1.904	0.537
5.60	1.859	0.449	1.883	0.478	1.909	0.521	1.939	0.577
5.70	1.888	0.479	1.913	0.511	1.941	0.556	1.973	0.616
5.80	1.917	0.510	1.944	1.544	1.974	0.592	2.008	0.656
5.90	1.946	0.541	1.975	0.576	2.007	0.628	2.042	0.696
6.00	1.976	0.572	2.006	0.609	2.039	0.664	2.077	0.735

(*Contd.*)

Table 6.13 (Contd.)

6.10	2.005	0.603	2.036	0.642	2.072	0.699	2.112	0.755
6.20	2.034	0.634	2.067	0.675	2.104	0.735	2.146	0.814
6.30	2.063	0.664	2.098	0.708	2.137	0.771	2.181	0.854
6.40	2.092	0.694	2.219	0.741	2.170	0.807	2.216	0.894
6.50	2.121	0.726	2.160	0.773	2.202	0.842	2.250	0.933
6.60	2.151	0.757	2.190	0.806	2.235	0.878	2.285	0.973
6.70	2.180	0.788	2.221	0.839	2.267	0.914	2.320	1.012
6.80	2.209	0.818	2.252	0.872	2.300	0.950	2.354	1.052
6.90	2.238	0.849	2.283	0.905	2.333	0.985	2.389	1.092
7.00	2.267	0.880	2.314	0.937	2.365	1.021	2.424	1.131
7.10	2.296	0.911	2.344	0.970	2.398	1.057	2.458	1.171
7.20	2.326	0.942	2.375	1.003	2.431	1.093	2.493	1.210
7.30	2.355	0.972	2.406	1.036	2.463	1.128	2.527	1.250
7.40	2.284	1.003	2.437	1.069	2.496	1.164	2.562	1.290
7.50	2.413	1.034	2.468	1.102	2.528	1.200	2.597	1.329
7.60	2.442	1.065	2.498	1.134	2.561	1.236	2.631	1.369
7.70	2.471	1.096	2.529	1.167	2.594	1.271	2.666	1.408
7.80	2.501	1.126	2.560	1.200	2.626	1.307	2.701	1.448
7.90	2.530	1.157	2.591	1.233	2.659	1.343	2.735	1.488
8.00	2.559	1.188	2.621	1.266	2.691	1.379	2.770	1.527
8.10	2.588	1.219	2.652	1.299	2.724	1.414	2.805	1.567
8.20	2.617	1.250	2.683	1.331	2.757	1.450	2.839	1.606
8.30	2.646	1.280	2.714	1.364	2.789	1.486	2.874	1.646
8.40	2.676	1.311	2.745	1.397	2.822	1.522	2.908	1.686
8.50	2.905	1.342	2.775	1.430	2.854	1.557	2.943	1.725

Method– (Strain compatibility)

a) Data

$b = 300$ mm $\quad f_{ck} = 20$ N/mm^2
$d = 600$ mm $\quad f_y = 415$ N/mm^2
$d' = 60$ mm
$A_{st} = (5 \times 491) = 2455$ mm^2
$A_{sc} = (2 \times 491) = 982$ mm^2

b) Neutral Axis Depth

As a first trial, assume the neutral axis depth as

$$x_u = x_{u,\,max} = 0.48d = (0.48 \times 600) = 288 \text{ mm}$$

c) Strain in tension reinforcement is computed as

$$\varepsilon_{st} = \left[\frac{\varepsilon_{cu}(d - x_u)}{x_u}\right] = \left[\frac{0.0035(600 - 288)}{288}\right] = 0.00379$$

From stress-strain curve [Fig. 6.3] read out, $f_{st} = 0.87 f_y$

d) Total tensile force

$$T = 0.87 f_y A_{st} = (0.87 \times 415 \times 2455 \times 10^{-3}) = 886.4 \text{ kN}$$

e) Strain in compression steel is computed by the equation

$$\varepsilon_{sc} = \left(\frac{0.0035(x_u - d')}{x_u}\right) = \left(\frac{0.0035(288 - 60)}{288}\right) = 0.00277$$

f) Read out the value of f_{sc} from Fig. 6.3 as $f_{sc} = 350$ N/mm^2

g) Force in compression steel = $C_s = f_{sc} A_{sc} = (350 \times 982 \times 10^{-3}) = 343.7$ kN

h) Compressive force in concrete = $C_c = (0.36 \times 20 \times 300 \times 288)\ 10^{-3}$ = 622 kN

i) Total compressive force = $[C_s + C_c] = [343.7 + 622] = 965.7$ kN

j) For equilibrium of forces at the section, $C = T$

Since $C = 965.7$ kN and $T = 886.4$ kN, $C > T$
Hence reduce the value of x_u and repeat the steps (b) to (j)

Second Trial

Assume $x_u = 250$ mm

$$\varepsilon_{sc} = [0.0035(250 - 60)/250] = 0.00266$$

From Fig. 6.3, read out the value of $f_{sc} = 345$ N/mm^2

$$C_s = (345 \times 982)10^{-3} = 338.8 \text{ kN}$$

$$C_c = (0.36 \times 20 \times 300 \times 250)10^{-3} = 540\text{kN}$$

Hence $\quad C = (C_s + C_c) = (338.8 + 540) = 878.8$ kN

$$\varepsilon_{st} = [0.0035(600 - 250)/250] = 0.0049$$

Tension steel yields and hence $f_{st} = 0.87 f_y$
Tensile force $T = (0.87 f_y A_{st}) = (0.87 \times 415 \times 2455 \times 10^{-3}) = 886.4$ kN.
Since C is nearly equal to T, compute the moment of resistance of the section by taking moments about tension steel.

$$M_u = C_c(d - 0.42x_u) + C_s(d - d')$$
$$= [540(600 - 0.42 \times 250) + 338.8(600 - 60)]\ 10^3$$
$$= (450.25 \times 10^6) \text{ N.mm}$$
$$= 450.25 \text{ kN.m}$$

Method–2 (Using IS: 456-2000 Code Formula)

First Trial

a) Assume $x_u = x_{u,lim} = (0.48 \times 600) = 288$ mm
b) $\varepsilon_{sc} = [0.0035\,(x_{u,lim} - d')] / x_{u,lim}$
$= [0.0035\,(288 - 60)] / 288 = 0.00277$
c) From the stress-strain curve (Fig. 6.3) read out $f_{sc} = 350$ N/mm^2
d) $A_{st2} = [(f_{sc} A_{sc}) / (0.87 f_y)] = [(350 \times 982) / (0.87 \times 415)] = 952$ mm^2
e) $A_{st1} = (A_{st} - A_{st2}) = (2455 - 952) = 1503$ mm^2
f) $x_u = [(0.87 f_y A_{st1}) / (0.36 f_{ck}\, b)]$
$= [(0.87 \times 415 \times 1503) / (0.36 \times 20 \times 300)] = 251.23$ mm < 288 mm

Second trial

a) Assume $x_u = 251.23$ mm
b) $\varepsilon_{sc} = [0.0035\,(251.23 - 60) / 251.23] = 0.00266$
c) From stress-strain curve read out $f_{sc} = 345$ N/mm^2
d) $A_{st2} = [(345 \times 982) / (0.87 \times 415)] = 938$ mm^2

e) $A_{st1} = (2455 - 938) = 1517 \text{ mm}^2$

f) $x_u = [(0.87 \times 415 \times 1517) / (0.36 \times 20 \times 300)]$
$= 253.6$ mm which is nearly equal to the assumed value
and $x_u < x_{u,lim}$ and hence the section is under reinforced

g) $M_u = [0.87 f_y A_{st1} (d - 0.42 x_u) + f_{sc} A_{sc} (d - d')]$
$M_u = [(0.87 \times 415 \times 1517)(600 - 0.42 \times 253.6)] 10^{-6}$
$+ [345 \times 982 \times (600 - 60)] 10^{-6}$
$= 453.24$ kN.m

Hence, the two methods yield nearly the same moment capacity of the section.

Method-3 (Using SP: 16 Design Tables)

a) Compute parameters for use of design tables.

$(d'/d) = (60/600) = 0.1$

$p_c = (100 A_{sc})/(bd) = (100 \times 982)/(300 \times 600) = 0.55$

$p_t = (100 A_{st})/(bd) = (100 \times 2455)/(300 \times 600) = 1.36$

a) Refer Table-50 of SP: 16 corresponding to $f_{ck} = 20$ N/mm^2 and $f_y = 415$ N/mm^2, read out the ratio (M_u / bd^2) for $(d'/d) = 0.1$

$(M_u / bd^2) = 4.45$

Hence $M_u = (4.45 \times 300 \times 600^2) 10^{-6} = 443$ kN.m

The moment of resistance computed is nearly equal to that computed by the rigorous methods of analysis.

2) Determine the ultimate moment capacity of a rectangular beam having a width of 280 mm and overall depth 550 mm. The tension and compression reinforcements provided at an effective cover of 50 mm are 2450 mm^2 and 400 mm^2 respectively. Assume m-30 grade concrete and Fe-500 HYSD bars.

Method–1 (Using IS: 456-2000 Code Formulae)

a) Data

$b = 280$ mm	$f_{ck} = 30$ N/mm^2
$d = 500$ mm	$f_y = 500$ N/mm^2
$d' = 50$ mm	$E_s = 2 \times 10^5$ N/mm^2
$A_{st} = 2450$ mm^2	$A_{sc} = 400$ mm^2

b) Neutral axis depth

$$x_u = x_{u,lim} = 0.46d = (0.46 \times 500) = 230 \text{ mm}$$

$$\varepsilon_{sc} = [0.0035(x_{u,lim} - d')/x_{u,lim}] = 0.0035(230 - 50)/230 = 0.00274$$

c) Stress in steel

Refer the stress-strain curve (Fig. 6.3) and read out $f_{sc} = 410$ N/mm^2

d) Check for neutral axis depth

$A_{st2} = (f_{sc} A_{sc})/(0.87 f_y) = (410 \times 400)/(0.87 \times 500) = 377 \text{ mm}^2$
$A_{st1} = (A_{st} - A_{st2}) = (2450 - 377) = 2073 \text{ mm}^2$
$x_u = (0.87 f_y A_{st1})/(0.36 f_{ck} b)$
$= [(0.87 \times 500 \times 2073) / (0.36 \times 30 \times 280)]$
$= 298.2 \text{ mm} > x_{u,lim} = 230$ mm

Hence, the section is over reinforced

e) Moment of resistance

$M_u = 0.36 f_{ck} x_{u,lim} b (d - 0.42 x_{u,lim}) + f_{sc} A_{sc} (d - d')$
$= [(0.36 \times 30 \times 230 \times 280)(500 - 0.42 \times 230) + 410 \times 400$
$(500 - 50)] 10^{-6}$
$= 354.37$ kN.m

Method–2 (Using SP: 16 Design Tables)

a) Compute parameters for use of design tables

$(d' / d) = (50 / 500) = 0.1$
$p_c = (100 \times 400) / (280 \times 500) = 0.285$
$p_t = (100 \times 2450) / (280 \times 500) = 1.75$

b) Refer Table-56 of SP: 16 for $f_{ck} = 30$ N/mm^2 and $f_y = 500$ N/mm^2

For $(d'/d) = 0.1$ read out the ratio (M_u/bd^2)
$(M_u/bd^2) = 5.00$, p_t required is 1.698
p_t provided is 1.75% which is only slightly higher than p_t required
Hence, $M_u = (5 \times 280 \times 500^2) 10^{-6} = 350$ kN.m

The ultimate moment capacity of the section computed using SP: 16 design tables is nearly the same as that computed by rigorous methods.

6.4.6 Design examples

1) A rectangular reinforced concrete beam of width 400 mm and effective depth 600 mm is to be designed to support an ultimate moment of 600 kN.m. Using M-20 grade concrete and Fe-415 HYSD bars design suitable reinforcements in the beam at an effective cover of 60 mm.

Method–1 (Using IS: 456-2000 Provisions)

a) Data

$b = 400$ mm $f_{ck} = 20$ N/mm^2
$d = 600$ mm $f_y = 415$ N/mm^2
$d' = 60$ mm $M_u = 600$ kN.m

b) Limiting Moment of Resistance

The limiting moment of resistance of singly reinforced section is

$$M_{u,lim} = (0.138 f_{ck}\, b\, d^2) = (0.138 \times 20 \times 400 \times 600^2)\, 10^{-6} = 397 \text{ kN.m}$$

c) Reinforcements

A_{st1} = area of tensile reinforcement for a singly reinforced section for $M_{u,lim}$

$$= [0.36 f_{ck}\, b\, (0.48\, d)\,] / (0.87 f_y)$$
$$= [0.36 \times 20 \times 400 \times 0.48 \times 600] / (0.87 \times 415)$$
$$= 2297 \text{ mm}^2$$

$[M_u - M_{u,lim}] = [600 - 397] = 203$ kN.m

But $[M_u - M_{u,lim}] = [f_{sc} A_{sc} (d - d')]$

$$\varepsilon_{sc} = [0.0035\, (x_{u,max} - d')] / x_{u,max}$$
$$= [0.0035\, \{(0.48 \times 600) - 60\}] / (0.48 \times 600)$$
$$= 0.00277$$

From stress-strain curve (Fig. 6.3) read out $f_{sc} = 350$ N/mm^2

Hence, $A_{sc} = [M_u - M_{u,lim}] / [f_{sc} (d - d')]$
$$= [203 \times 10^6] / [350 \times (600 - 60)]$$
$$= 1072 \text{ mm}^2$$

$$A_{st2} = (A_{sc} f_{sc}) / (0.87 f_y)$$
$$= [1072 \times 350] / (0.87 \times 415)$$
$$= 1039 \text{ mm}^2$$

Total tensile steel $= A_{st} = [A_{st1} + A_{st2}] = [2297 + 1039] = 3336$ mm^2
Compression steel $= A_{sc} = 1072$ mm^2

Method–2 (Using SP: 16 Design Tables)

a) Compute parameters to be used in SP: 16 design tables.

$[M_u / bd^2] = [600 \times 10^6]/[400 \times 600^2] = 4.16$

b) Refer Table-50 of SP: 16 corresponding to $f_{ck} = 20$ N/mm^2, $f_y = 415$ N/mm^2
And $(d'/d) = 0.10$
Interpolate the value of percentage of reinforcements p_t and p_c corresponding to the parameter 4.16.

$p_t = 1.386\%$ and $p_c = 0.452\%$
$A_{st} = [(p_t\, b\, d)/100] = [(1.386 \times 400 \times 600) / 100] = 3326$ mm^2
$A_{sc} = [(p_c\, b\, d)/100] = [(0.452 \times 400 \times 600)/100] = 1085$ mm^2

The results of reinforcements from SP:16 design tables are nearly the same as that obtained by using IS:456 code provisions.

2) Design the reinforcements for a doubly reinforced concrete beam section to support a factored moment of 1000 kN.m. Assume $b = 400$ mm, $d = 550$ mm, cover $d' = 50$ mm, $f_{ck} = 30$ N/mm^2 and $f_y = 415$ N/mm^2.

Method–1 (Using IS: 456-2000 code provisions)

a) Data

$b = 400$ mm $M_u = 1000$ kN.m
$d = 550$ mm $f_{ck} = 30$ N/mm^2
$d' = 50$ mm $f_y = 415$ N/mm^2

b) Limiting #bMoment of resistance

$$M_{u,lim} = 0.138 f_{ck}\, b\, d^2$$
$$= (0.138 \times 30 \times 400 \times 550^2)\, 10^{-6}$$
$$= 501 \text{ kN.m}$$

c) Reinforcements

$$A_{st1} = [0.36 f_{ck}\, b\, (0.48\, d)] / 0.87 f_y$$
$$= [0.36 \times 30 \times 400 \times 0.48 \times 550] / (0.87 \times 415)$$
$$= 3159 \text{ mm}^2$$
$$(M_u - M_{u,lim}) = (1000 - 501) = 499 \text{ kN.m}$$
$$\varepsilon_{sc} = [0.0035\, (x_{u,max} - d')/ x_{u,max}]$$
$$= [0.0035\, \{(0.48 \times 550) - 50\}/(0.87 \times 415)]$$
$$= 0.00207$$

From stress-strain curve (Fig. 6.3) read out the value of stress, f_{sc} = 330 N/mm^2

$$A_{sc} = [M_u - M_{u,lim}] / [f_{sc}(d - d')]$$
$$A_{sc} = [499 \times 10^6] / [330 (550 - 50)] = 3024 \text{ mm}^2$$
$$A_{st2} = [(A_{sc} f_{sc}) / (0.87 f_y)]$$
$$= [(3024 \times 330) / (0.87 \times 415)] = 2764 \text{ mm}^2$$
$$A_{st} = (A_{st1} + A_{st2}) = (3159 + 2764) = 5923 \text{ mm}^2$$

Method–2 (Using SP: 16 design Tables)

Compute the parameters to use SP: 16 design tables.

$[M_u/bd^2] = [(1000 \times 10^6) / (400 \times 550^2)] = 8.26$

Refer Table-52 of SP: 16 for f_{ck} = 30 N/mm^2, f_y = 415 N/mm^2 and (d'/d) = 0.1 and interpolate the values of p_t and p_c.

$$p_t = 2.70\% \quad \text{and} \quad p_c = 1.35\%$$
$$A_{st} = [(2.7 \times 400 \times 550) / 100] = 5940 \text{ mm}^2$$
$$A_{sc} = [(1.35 \times 400 \times 550) / 100] = 2970 \text{ mm}^2$$

The reinforcement values are nearly the same as those obtained by method-1.

6.5 ULTIMATE SHEAR STRENGTH OF REINFORCED CONCRETE SECTIONS

6.5.1 Introduction

Investigations over the years have shown that there are two major modes of shear cracking in structural concrete beams[47,48]. Near supports of reinforced concrete members, the shear stresses developed are accompanied by diagonal tension as shown in Fig. 6.8. As concrete is weak in tension, if the tensile stresses developed exceed the low tensile strength of concrete, diagonal tension cracks develop near supports as shown in Fig. 6.8. Hence, beams are invariably checked for safety against 'shear failure'. If the nominal shear stress is excessive, steel in the form of vertical stirrups or bent up bars should be designed to resist the large shear forces. Limit state design of reinforced concrete beams comprises of the design for flexure at centre of span and the design for shear in the vicinity of supports.

6.5.2 Shear Failure Mechanisms

The major types of shear failure modes encountered in reinforced concrete beams are identified under the following groups: -

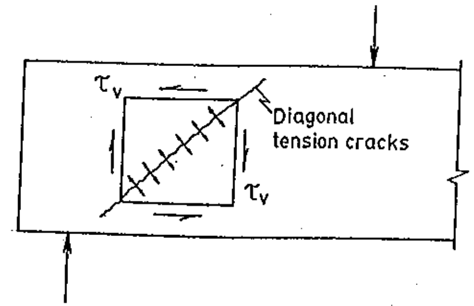


Fig. 6.8 Diagonal Tension in Beams

1) Shear-Tension or Diagonal Tension
2) Flexure-Shear
3) Shear-Compression
4) Shear-Bond.

The types of cracks developed and the failure modes are shown in Fig. 6.9 (a to e). The four different types of shear failure modes depend upon the ratio of shear span to effective depth. The transverse shear force in a flexure-shear mode is generally resisted by the following major mechanisms as outlined in Fig. 6.10.

1) Shear resistance V_c of the uncracked portion of concrete.
2) Vertical component of the interface shear (aggregate interlock) force V_a
3) The dowel force V_d developed due to the tension reinforcement.
4) The shear resistance V_s developed in the shear reinforcement.

In the case of very deep beams having shear span/depth ratio (a/d) <1 without web reinforcement, inclined cracks from supports develop transforming the beam into a tied arch which may fail by yielding of longitudinal reinforcement or due to crushing of concrete of the compression chord.

Normally short beams have the shear span/depth ratio (a/d) greater than 1 but less than 2.5. In such beams the failure may be due to

a) Crushing of reduced concrete section above the progressing diagonal tension crack under combined shear and compression.
b) Secondary cracking along the tension reinforcement termed as shear-bond failure.
c) Failure may also be initiated by flexure-shear cracking mode.

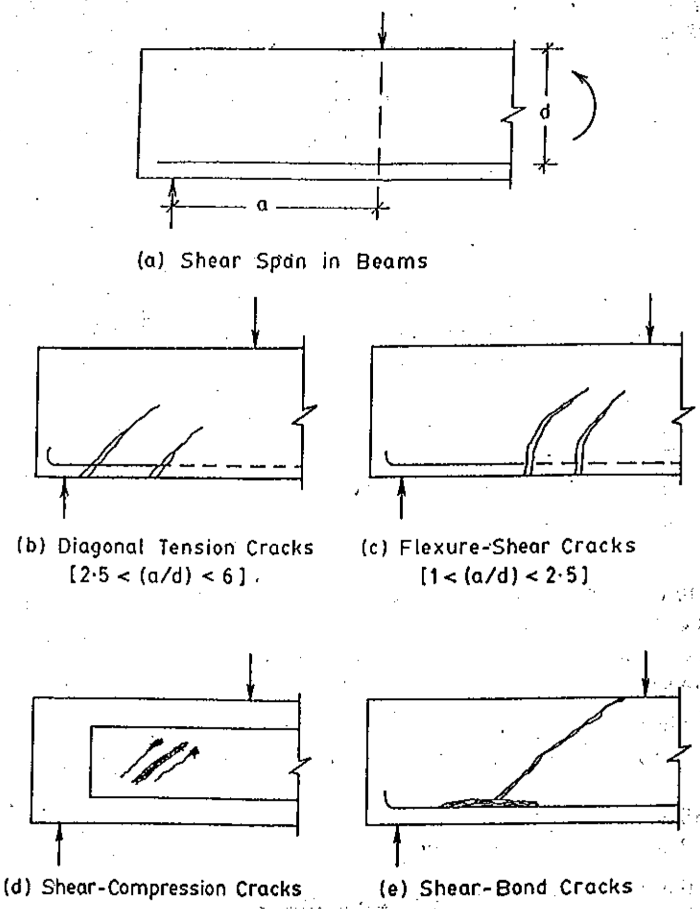


Fig. 6.9 Types of Shear Failures

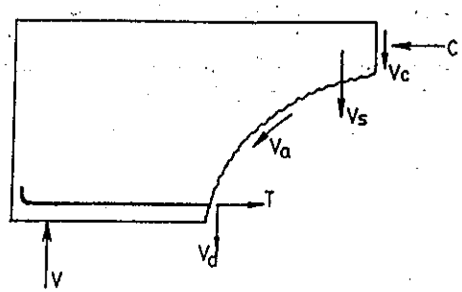


Fig. 6.10 Shear Transfer Mechanism at a Flexure Shear Crack

In most of the normal cases of beams, the shear span /depth ratio (*a*/*d*) will be greater than 2.5 but less than 6. The limiting (*a*/*d*) ratio above which flexural failure is certain is dependent upon the area of tension reinforcement and the compressive strength of concrete and it is generally common when the (*a*/*d*) ratio is in the neighbourhood of 6.

In beams with (*a*/*d*) ratio between 2.5 and 6, flexural cracks develop first and the failure is due to propagation of flexure cracks as flexure-shear cracks. If web reinforcement is provided, the shear strength of such beams can be considerably enhanced.

In the case of I-beams with thin webs, failure due to web crushing of concrete is common and this type of failure can be prevented by proper designing of web reinforcement and using high strength concrete.

6.5.3 Nominal Shear stress

The theoretical shear stress distribution in a rectangular reinforced concrete section in the elastic and ultimate stages vary parabolically in the compression zone assuming that concrete below neutral axis is ineffective due to cracking and neglecting the tension in concrete. For the sake of simplicity the nominal shear stress across the cross section of a beams is computed as the average shear stress on the section and evaluated as follows: -

$$\tau_v = \left(\frac{V_u}{bd}\right)$$

Where V_u = ultimate shear force at the section
τ_v = nominal shear stress
b = breadth (width of rib for flanged beams)
d = effective depth

In case of members with varying depth,

$$\tau_v = \left[\frac{V_u \pm (M_u/d)\tan\beta}{bd}\right]$$

Where,
β = inclination of flexural tensile force to the horizontal.
M_u = factored bending moment at the section.

Negative sign to be used when M_u increases in the same direction as the depth and positive sign when M_u decreases in this direction.

6.5.4 Design shear strength of concrete

Experimental investigations have shown that the resistance of reinforced concrete beams to diagonal tension failure depends upon,

a) The grade of concrete (f_{ck})
b) The percentage of tension reinforcement (A_{st}) in the beam.

Table-19 of IS: 456-2000 and Table- 61 of SP: 16 and Table 6.14 of the text give the ultimate allowable shear stress τ_c, termed as design shear strength of concrete in beams as a function of concrete grade and percentage of tension reinforcement.

The IS: Code Table-19 is based on the research investigations of the study group of the Institution of Structural Engineers London, (U.K) in 1969.

Table 6.14 Design Shear Strength of Concrete (τ_c, N/mm²)
(Table-19 of IS: 456-2000)

(100 A_s/bd)	Concrete Grade					
	M-15	M-20	M-25	M-30	M-35	M-40 and above
(1)	(2)	(3)	(4)	(5)	(6)	(7)
≤ 0.15	0.28	0.28	0.29	0.29	0.29	0.30
0.25	0.35	0.36	0.36	0.37	0.37	0.38
0.50	0.46	0.48	0.49	0.50	0.50	0.51
0.75	0.54	0.56	0.57	0.59	0.59	0.60
1.00	0.60	0.62	0.64	0.66	0.67	0.68
1.25	0.64	0.67	0.70	0.71	0.73	0.74
1.50	0.68	0.72	0.74	0.76	0.78	0.79
1.75	0.71	0.75	0.78	0.80	0.82	0.84
2.00	0.71	0.79	0.82	0.84	0.86	0.88
2.25	0.71	0.81	0.85	0.88	0.90	0.92
2.50	0.71	0.82	0.88	0.91	0.93	0.95
2.75	0.71	0.82	0.90	0.94	0.96	0.98
3.00 and above	0.71	0.82	0.92	0.96	0.99	1.01

Note: The term A_s is the area of longitudinal tension reinforcement which continues at least one effective depth beyond the section being considered except at support where the full area of tension reinforcement may be used provided the detailing conforms to Clauses 26.2.2 and 26.2.3

The study group recommended an empirical equation for the computation of shear strength, concrete as

$$\tau_c = \left[\frac{0.85\sqrt{(0.8 f_{ck})}\,\sqrt{(1+5\beta)}-1}{6\beta}\right] \quad \ldots(6.11)$$

Where $\beta = \left[\dfrac{0.8 f_{ck}}{6.89\, p_t}\right]$ but not less than 1.

$$p_t = \left[\frac{100 A_{st}}{bd}\right]$$

$0.8\ f_{ck}$ = Cylinder Strength in terms of cube Strength
0.85 = Reduction factor similar to $(1/\gamma_m)$

Table-19 of IS: 456-2000 is applicable for sections with effective depth 300 mm or more. For smaller depth (R.C.Slabs), the strength is larger as given in clause 40.2.1.1 of the IS: 456 code which suggests the shear strength as '$k\tau_c$' where k is multiplying factor depending upon the overall depth of slab as given in Table 6.15.

Table 6.15 Multiplying Factor (k) for slabs

Overall depth of slab (mm)	300 or more	275	250	225	200	175	150 or Less
k	1.00	1.05	1.10	1.15	1.20	1.25	1.30

It is important to note that there is a limit to the maximum nominal shear stress value for which the beam can be strengthened by shear reinforcements. Beyond these values, diagonal compression is prevalent even if the diagonal tension is resisted by shear reinforcements. Hence the maximum nominal shear stress computed should not exceed the maximum shear stress in concrete ($\tau_{c,max}$) given in Table.20 of IS: 456-2000 and Table 6.16 of the text. According to SP: 24 these values are computed by the empirical relation,

Table 6.16 Maximum Shear Stress in Concrete ($\tau_{c,max}$)

Concrete Grade	M-15	M-20	M-25	M-30	M-35	M-40 and above
$\tau_{c,max}$ (N/mm²)	2.5	2.8	3.1	3.5	3.7	4.0

$$\tau_{c,\,max} = 0.83\sqrt{f_c}\ \text{N/mm}^2 \quad \ldots(6.12)$$

Where f_c = cylinder strength of concrete by applying a reduction factor of 0.85 to convert the cube to cylinder strength and a partial safety factor for material strength, γ_m = 1.25. The empirical equation expressed in terms of cube strength is given as,

$$\tau_{c,max} = 0.62\sqrt{f_{ck}} \quad \ldots(6.13)$$

If the value of the nominal shear stress $\tau_v = (V/bd)$ exceeds the values of

$\tau_{c,max}$, the section should be redesigned by enhancing the cross sectional dimensions.

6.5.5 Design of Shear Reinforcements

The various types of reinforcements used to resist shear can be classified under the following two groups:-

1) Vertical Stirrups.
2) Main tension reinforcement bent up near supports.

The typical arrangement of these types is shown in Fig. 6.11 (a), (b) Inclined stirrups are also used but they are not preferred due to practical difficulties.

At the Limit state of collapse in shear, the forces are resisted by the combined action of concrete and steel.

If V_u = total Shear Force.
V_c = shear resisted by concrete
V_{us} = shear resisted by reinforcements (Links or bent up bars)

Then
$$V_{us} = (V_u - V_c) = (\tau_v - \tau_c)\, bd \quad \ldots(6.14)$$

Where τ_v = nominal shear stress.
τ_c = design shear stress of concrete (Table 6.14)
Let A_{sv} = total area of the legs of shear reinforcement.
S_v = spacing of the links.
d = effective depth of section.

The number of stirrups links cut by a 45° crack line is given by

$$N = (d/S_v)$$

Hence, the total shear resistance of the vertical stirrup system across the section is expressed as

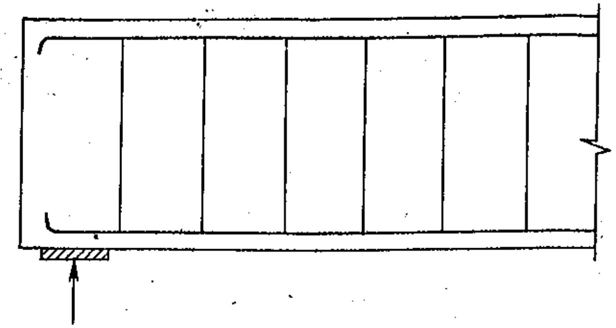
$$V_{us} = 0.87 f_y A_{sv} (d/S_v)$$

Hence, the spacing of vertical stirrups is given by the relation,

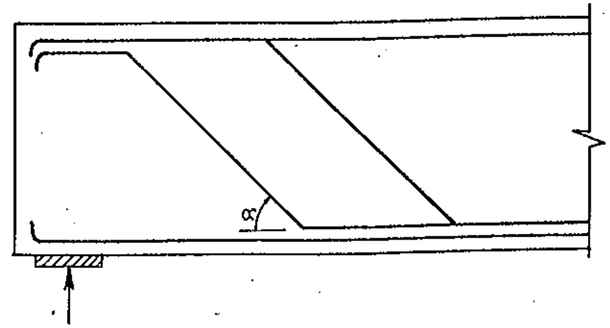
$$S_v = \left[\frac{0.87 f_y . A_{sv} . d}{V_{us}}\right] \quad \ldots(6.15)$$

The shear force resisted by the bent up bar inclined at an angle 'α' to the horizontal is expressed as,

$$V_{us} = 0.87 f_y . A_{sv} . \sin\alpha \quad \ldots(6.16)$$



(a) Vertical Stirrups



(b) Bent up Bars

Fig. 6.11 Types of Shear Reinforcements

Equations (6.15) and (6.16) are recommended for the design of shear reinforcements in IS: 456-2000 clause 40.4.

Eq. (6.15) is recast in the form.

$$\left(\frac{V_{us}}{d}\right) = \left(\frac{A_{sv}}{S_v}(0.87 f_y)\right) = \left[\frac{\text{Shear carried by steel}}{\text{Depth in cm.}}\right] \quad (6.17)$$

Table-62 of SP: 16 adopts this equation facilitating the direct design of two legged stirrups for known values of (V_{us}/d) expressed in (kN/cm). This table is reproduced as Table 6.17 in this text. The design table covers two grades of steel Fe-250 and Fe-415 and diameter of stirrups of 6,8,10 and 12mm. The spacing of stirrups can be directly read out for a given shear/depth ratio.

Table-63 of SP: 16 gives the shear resistance of single bars of diameter ranging from 10 to 36mm which are bent up at $\alpha = 45°$ or 60° and covering steels of Fe-250 and Fe-415 grades. This table which is useful in designing

Table 6.17 Shear—Vertical Stirrups
(Table–62 of SP: 16)

Values of V_{us}/d for two legged stirrups, kN/cm.

Stirrup Spacing, cm	$f_y = 250 N/mm^2$ Diameter, mm				$f_y = 415 N/mm^2$ Diameter, mm			
	6	8	10	12	6	8	10	12
5	2.460	4.373	6.833	9.839	4.083	7.259	1.342	16.334
6	2.050	3.644	5.694	8.200	3.403	6.049	9.452	13.611
7	1.757	3.124	4.881	7.028	2.917	5.185	8.102	11.667
8	1.537	2.733	4.271	6.150	2.552	4.537	7.089	10.208
9	1.367	2.429	3.796	5.466	2.269	4.033	6.302	9.074
10	1.230	2.186	3.416	4.920	2.042	3.630	5.671	8.167
11	1.118	1.988	3.106	4.472	1.856	3.299	5.1256	7.424
12	1.025	1.822	2.847	4.100	1.701	3.025	4.726	6.806
13	0.946	1.682	2.628	3.784	1.571	2.792	4.363	6.286
14	0.879	1.562	2.440	3.514	1.458	2.593	4.051	5.833
15	0.820	1.458	2.278	3.280	1.361	2.420	3.781	5.445
16	0.769	1.366	2.135	3.075	1.276	2.269	3.545	5.104
17	0.723	1.286	2.010	2.894	1.201	2.135	3.336	4.804
18	0.683	1.215	1.898	2.733	1.134	2.016	3.151	4.537
19	0.647	1.151	1.798	2.589	1.075	1.910	2.985	4.083
20	0.615	1.093	1.708	2.460	1.020	1.815	2.836	4.083
25	0.492	0.875	1.367	1.968	0.817	1.452	2.269	3.267
30	0.410	0.729	1.139	1.640	0.681	1.210	1.890	2.722
35	0.351	0.625	0.976	1.406	0.583	1.037	1.620	2.333
40	0.307	0.547	0.854	1.230	0.510	0.907	1.418	2.042
45	0.273	0.486	0.759	1.093	0.454	0.807	1.260	1.815

Table 6.18 Shear—Bent-up Bars
(Table–63 of SP: 16)

Values of V_{us} for singal bar, kN

Bar Diameter, mm	$f_y = 250 N/mm^2$		$f_y = 415 N/mm^2$	
	$\alpha = 45^\circ$	$\alpha = 60^\circ$	$\alpha = 45^\circ$	$\alpha = 60^\circ$
10	12.03	14.79	20.05	24.56
12	17.39	21.30	28.87	35.36
16	30.92	37.87	51.33	62.87
18	39.14	47.93	64.97	79.57
20	48.32	59.18	80.21	98.23
22	58.46	71.60	97.05	118.86
25	75.49	92.46	125.32	153.48
28	94.70	115.98	157.20	192.53
32	123.69	151.49	205.32	251.47
36	156.54	191.73	259.86	318.27

Note–α is the angle between the bent-up bar and the axis of the member.

the number of bent up bars to resist a known magnitude of shear force is reproduced in Table 6.18 of this text.

6.5.6 Minimum Shear Reinforcements

IS: 456-2000 clause 26.5.1.6 stipulates that all reinforced concrete beams should be provided with at least some minimum shear reinforcements even if computation does not require them.

The minimum shear reinforcements are required to prevent the following types of failures:-

1) Brittle shear failure cracks which can occur without shear reinforcements.
2) Sudden failure due to bursting of concrete cover and bond to the tension reinforcements.
3) The shear reinforcements help to hold the main reinforcements while concreting, forming an effective cage.
4) Formation of cracks due to the thermal and shrinkage stresses are minimized.
5) Shear reinforcements act as effective ties for the compression steel and make them effective.

The minimum shear reinforcements to be provided in all the beams is computed by the relation,

$$\left(\frac{A_{sv}}{bS_v}\right) \geq \left(\frac{0.4}{0.87 f_y}\right) \qquad \ldots(6.18)$$

Where A_{sv} = total cross sectional area of stirrup legs effective in shear.
S_v = spacing of stirrups.
b = breadth of the beam or breadth of web of flanged beams.
f_y = characteristic strength of stirrup reinforcement in N/mm^2 which shall not be taken greater than 415 N/mm^2.

The spacing of the stirrups can also be expressed as;

$$\begin{Bmatrix} S_v = 543\left[\dfrac{A_{sv}}{b}\right] \text{ for Fe-250} \\ S_v = 902\left[\dfrac{A_{sv}}{b}\right] \text{ for Fe-415} \end{Bmatrix} \qquad \ldots(6.19)$$

Also, the spacing of the links should not exceed 0.75d or 300mm whichever is less. The IS: 456-2000 code also specifies that in cases where the maximum shear stress ($\tau_{c,max}$) computed is less than half the permissible

value and in members of minor structural importance such as lintels, the provision of minimum reinforcement may be waived.

This equation (6.18) can also be written as

$$\left(\frac{A_{sv}}{S_v}\right) = \left(\frac{0.4bd}{0.87 f_y d}\right) \qquad \ldots(6.20)$$

Comparing this equation with Eq. (6.15) it can be seen that providing for nominal reinforcements is equivalent to designing the shear reinforcement for a shear stress of

$$(\tau_v - \tau_c) = 0.4 \text{ N/mm}^2$$

and $$V_{us} = (\tau_v - \tau_c)\, bd \qquad \ldots(6.21)$$

This concept is useful while designing nominal shear reinforcements using SP: 16 design tables.

6.5.7 Enhanced Shear Near Supports

Investigations on shear failures of beams and cantilevers without shear reinforcement indicate that shear cracks develop on planes inclined at an angle of 30^0 as shown in Fig. 6.12(a). Hence if a section is considered near the support, it is customary to enhance the shear strength capacity, the common examples being the design of brackets, corbels etc. Hence, the design shear strength is different when beams are supported on members, which are in compression as shown in Fig 6.12(b) and when supported on members which are in tension such as that shown in Fig 6.12(c).

The following specifications of IS: 456 Code are useful in the design of shear reinforcements in the vicinity of supports.

1) In the simplified approach, the IS: 456 code specifies that the critical section for shear is taken at a distance equal to the effective depth 'd' from the face of support when the beam supports uniformly distributed load or a concentrated load farther than 2d from the face of support. The value of τ_c is calculated in accordance with Table-19 and appropriate shear reinforcements are designed at sections closer to the support without any further check for shear at sections closer to the support.
2) The enhancement of shear strength may be taken into account while designing sections near a support. The value of τ_c is enhanced by a factor and is given by the equation,

$$\text{Enhanced shear strength} = [(\tau_c.2d) / a_v] \qquad \ldots(6.22)$$

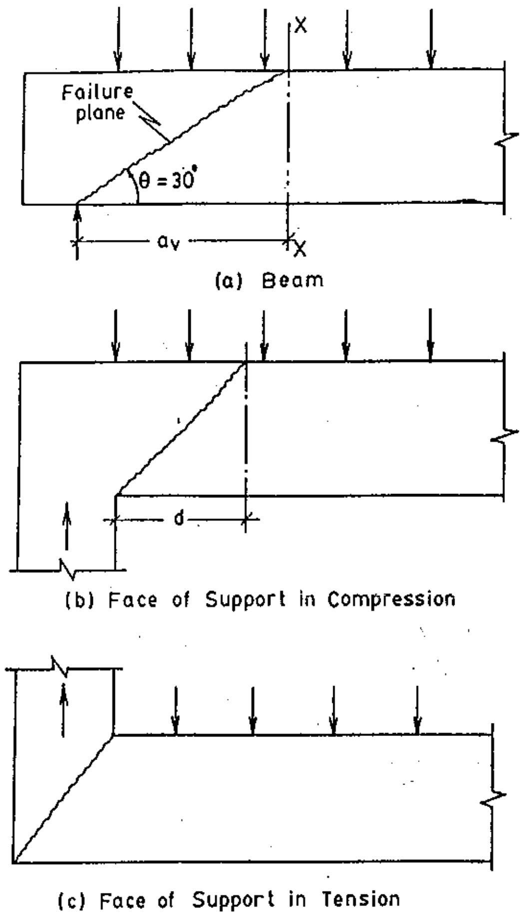


Fig. 6.12 Critical Sections for Shear

Where a_v = length of that part of a member traversed by a shear plane called the shear span as shown in Fig 6.12(a).

If shear reinforcement is required, the total area of this reinforcement is given by

$$\left\{ \begin{aligned} A_{sv} &= a_v . b\left(\tau_v - \frac{2d.\tau_c}{a_v}\right) / \, 0.87 f_y \\ &\geq (0.4 a_v . b)/0.87 f_y \end{aligned} \right\} \qquad \ldots(6.23)$$

This area is provided within a distance of 0.75 a_v. If a_v is less than effective depth, horizontal shear reinforcement will be more effective than vertical steel since the action is similar to that of deep beams.

6.5.8 Influence of Axial Force on Design Shear Strength

In general, the actual shear strength of concrete is improved in the presence of uniaxial compression and weakened in the presence of uniaxial tension. The design shear strength of concrete is based on a safe estimate of the limiting nominal stress at which the first inclined crack develops.

The presence of tension accelerates the process of cracking and also increases the angle of inclination of the shear cracks while the presence of uniaxial compression has the opposite effect which is generally prevalent in prestressed concrete beams[49].

Hence the IS: 456-2000 code (Clause 40.2.2) specifies that the design shear strength in the presence of axial compression should be taken as $\delta\tau_c$, the multiplying factor δ is defined as

$$\delta = \left[1 + \frac{3P_u}{A_g f_{ck}}\right] \qquad \ldots(6.24)$$

Or 1.5, whichever is less

Where P_u = factored compressive force (Newtons)
A_g = gross area of concrete section (mm^2)
f_{ck} = characteristic strength of concrete(N/mm^2)

The Indian Standard Code does not mention the case of axial tension which evidently reduces the design shear strength. However the American Code ACI: 318-89[50] specifies the multiplying factor as

$$\delta = \left[1 + \frac{P_u}{3.45 A_g}\right] \text{ for } P_u < 0 \qquad \ldots(6.25)$$

Where P_u = factored axial tension (N) with a negative sign.

6.5.9 Analysis Examples

1) A reinforced concrete beam has a support section with a width of 300mm and effective depth of 600mm. the support section is reinforced with 3 bars of 20mm diameter at an effective depth of 600mm. 8mm diameter 2 legged stirrups at a spacing of 200mm is provided as shear reinforcement near supports using M-20 Grade concrete and Fe-415 HYSD bars, estimate the shear strength of the support section.

Method–1(Using IS: 456-2000 Code formulae)

a) Data

$b = 300$ mm, $f_{ck} = 20$ N/mm^2
$d = 600$ mm, $f_y = 415$ N/mm^2
$A_{st} = (3 \times 314) = 942$ mm^2, $A_{sv} = (2 \times 50) = 100$ mm^2
$S_v = 200$ mm

b) Percentage Reinforcement

$$p_t = \left(\frac{100 A_{st}}{bd}\right) = \left(\frac{100 \times 942}{300 \times 600}\right) = 0.52$$

Refer Table–19 of IS: 456 (Table 6.17 of text) and read out the design shear strength of concrete τ_c corresponding to $f_{ck} = 20$ N/mm^2.

$\tau_c = 0.48$ N/mm^2

c) Shear Resisted by Concrete

$$V_{uc} = (\tau_c\, b\, d) = (0.48 \times 300 \times 600)\, 10^{-3} = 86.4 \text{ kN}$$

d) Shear resisted by Stirrups:-

$$V_{us} = \left[\frac{A_{sv}(0.87 f_y)\, d}{S_v}\right] = \left[\frac{100 \times 0.87 \times 415 \times 600}{200}\right] 10^{-3} = 108.3 \text{ kN.}$$

e) Total shear resistance of support section:-

$$V_u = [V_{uc} + V_{us}] = [86.4 + 108.3] = 194.7 \text{ kN}$$

Method–2 (Using SP: 16 Design Tables)

a) Shear resistance of concrete

Refer Table–61 of SP: 16 (Table 6.14 of text) and read out the design strength of concrete as

$$\tau_c = 0.48 \text{ N/mm}^2 \quad \text{for} \quad p_t = 0.52$$

$$V_{uc} = (\tau_c . b . d) = (0.48 \times 300 \times 600) 10^{-3} = 86.4 \text{ kN}$$

b) Shear resistance of two legged vertical stirrups

Refer Table 62 of SP: 16 (Table 6.17 of text) and read out the ratio (V_{us}/d) corresponding to $f_y = 415$ N/mm^2 and diameter of stirrups as 8 mm and spacing $S_v = 200$ mm (20 cm).

$$\left(\frac{V_{us}}{d}\right) = 1.815 \text{ kN/cm}$$

$$\therefore \quad V_{us} = (1.815 \times 60) = 108.9 \text{ kN.}$$

c) Total Shear Resistance

$$V_u = (V_{uc} + V_{us}) = (86.4 + 108.9) = 195.3 \text{ kN.}$$

2) A reinforced concrete beam of rectangular section has a width of 250mm and an effective depth of 500mm. The beam is reinforced with 4 bars of 25mm diameter on the tension side. Two of the tension bars are bent up at 45^0 near the support section. In addition the beam is provided with two legged stirrups of 8mm diameter at 150mm centres near the supports. If $f_{ck} = 25$ N/mm^2 and $f_y = 415$ N/mm^2, estimate the ultimate shear strength of the support section.

Method–1 (Using IS: 456-2000 Code Formula)

a) Data

$b = 250$ mm, $f_{ck} = 25$ N/mm^2
$d = 500$ mm, $f_y = 415$ N/mm^2
$A_{st} = (2 \times 491) = 982$ mm^2, $S_v = 150$ mm
$A_{sv} = (2 \times 50) = 100$ mm^2

b) Percentage Reinforcement

$$p_t = \left(\frac{100\, A_s}{bd}\right) = \left(\frac{100 \times 982}{250 \times 500}\right) = 0.78$$

Refer Table–19 of IS: 456 (Table 6.11 of text) and read out τ_c corresponding to $f_{ck} = 25$ N/mm^2.

$$\tau_c = 0.584 \text{ N/mm}^2$$

c) Shear Resisted by Concrete

$$V_{uc} = (\tau_c . b . d) = (0.584 \times 250 \times 500) 10^{-3} = 73.0 \text{ kN}$$

d) Shear resisted by Stirrups

$$V_{us} = \left[\frac{A_{sv}(0.87 f_y)d}{S_v}\right] = \left[\frac{100 \times 0.87 \times 415 \times 500}{150}\right] 10^{-3} = 120.3 \text{ kN}$$

e) Shear resisted by bent up bars

$$V_{us\alpha} = A_s(0.87 f_y)\sin\alpha$$

$$= [982 \times 0.87 \times 415 \times \sin 45^0] 10^{-3}$$

$$= 250.7 \text{ kN}.$$

f) Total Shear resistance of Support Section

$$V_u = [V_{uc} + V_{us} + V_{us\alpha}] = [73.0 + 120.3 + 250.7] = 444 \text{ kN}.$$

Method–2 (Using SP: 16 Design Tables)

a) Refer Table 61 of SP: 16 (Table 6.14 of text) and read out $\tau_c = 0.58$ N/mm^2 for $p_t = 0.78$ and $f_{ck} = 25$ N/mm^2

$\therefore$ $$V_{uc} = (\tau_c bd) = (0.584 \times 250 \times 500)10^{-3} = 73.0 \text{ kN}$$

b) Refer Table 62 of SP: 16 (Table 6.17 of text) and readout $(V_{us}/d) = 2.420$,

$$V_{us} = (2.42 \times 50) = 121 \text{ kN}.$$

c) Refer Table 63 of SP: 16 (Table 6.18 of text) and readout $V_{us\alpha}$ for single bar in kN as

$$V_{us\alpha} = (2 \times 125.32) = 250.64 \text{ kN}.$$

d) Total Shear Strength = $V_u = [V_{uc} + V_{us} + V_{us\alpha}] = [73.0 + 121 + 250.64] = 444.64$ kN

6.5.10 Design Examples

1) A reinforced Concrete beam of rectangular section 300mm wide is reinforced with four bars of 25mm diameter at an effective depth of 600mm. the beam has to resist a factored shear force of 400 kN at support section. Assuming $f_{ck} = 25$ N/mm^2 and $f_y = 415$ N/mm^2, design vertical stirrups for the section.

Method–1 (Using IS: 456-2000 code formulae)

a) Data

$b = 300$ mm $\quad f_{ck} = 25$ N/mm^2

$d = 600$ mm $\quad f_y = 415$ N/mm^2

$A_s = (4 \times 491) = 1964$ mm^2 $\quad V_u = 400$ kN.

b) Nominal Shear Stress

$$V_u = 400 \text{ kN},$$

$$\tau_v = \left(\frac{V_u}{bd}\right) = \left(\frac{400 \times 10^3}{300 \times 600}\right) = 2.22 \text{ N/mm}^2 < \tau_{c,max} = 3.1 \text{ N/mm}^2$$

c) Shear resisted by concrete:-

$$p_t = \left(\frac{100 A_s}{bd}\right) = \left(\frac{100 \times 1964}{300 \times 600}\right) = 1.09$$

Refer Table-19 of IS: 456–2000 and read out τ_c, corresponding to p_t and f_{ck}.

$\therefore$ $\tau_c = 0.658$ N/mm$^2 < \tau_v$

Hence, stirrups are to be designed.

$$V_{uc} = (\tau_c bd) = (0.658 \times 300 \times 600)10^{-3} = 118.4 \text{ kN}$$

$\therefore$ Balance shear is given by

$$V_{us} = [V_u - V_{uc}] = [400 - 118.4] = 281.6 \text{ kN}.$$

d) Design of Vertical Stirrups

Using 10mm diameter 2 legged vertical stirrups, spacing is given by

$$S_v = \left[\frac{0.87 f_y A_{sv} d}{V_{us}}\right] = \left[\frac{0.87 \times 415 \times 2 \times 78.5 \times 600}{281.6 \times 10^3}\right] = 120.77 \text{ mm}$$

$$S_{v,max} = 0.75\, d = (0.75 \times 600) = 450 \text{ mm}$$

Also $S_v \leq 300$ mm

Provide 10mm diameter 2 legged vertical stirrups at 120 mm centers at support section.

Method–2 (Using SP: 16 design Tables)

Compute the ratio $\left(\dfrac{V_{us}}{d}\right)$ kN/cm $= \left(\dfrac{281.6}{60}\right) = 4.69$ kN/cm

Refer Table-62 of SP: 16 (Table 6.14 of text) and read out the spacing corresponding to $f_y = 415$ N/mm^2, 10mm diameter stirrups and $(V_{us}/d) = 4.69$

$\therefore \quad S_v = 12$ cm $= 120$ mm

Provide 10 mm diameter 2 legged stirrups at 120 mm centres.

2) Design the shear reinforcements in a beam of rectangular section having a width of 300 mm and effective depth 600 mm. the ultimate shear at the section is 100 kN. Use $f_{ck} = 20$ N/mm^2 and $f_y = 415$ N/mm^2. The beam is reinforced with 4 bars of 25 mm diameter in the tensile zone.

Method–1 (Using IS: 456-2000 Code Formulae)

a) Data

$b = 300$ mm, $d = 600$ mm, $A_s = (1964\ \text{mm}^2)$

$f_{ck} = 20$ N/mm^2, $f_y = 415$ N/mm^2, $V_u = 100$ kN

b) Nominal Shear Stress

$V_u = 100$ kN

$$\tau_v = \left(\frac{V_u}{bd}\right) = \left(\frac{100 \times 10^3}{300 \times 600}\right) = 0.55 \text{ N/mm}^2$$

c) Shear Strength of Concrete

$$p_t = \left(\frac{100A_s}{bd}\right) = \left(\frac{100 \times 1964}{300 \times 600}\right) = 1.09$$

Refer Table-19 of IS: 456-2000 and read out τ_c, corresponding to p_t and f_{ck}.

$\therefore \quad \tau_c = 0.658$ N/mm$^2 > \tau_v$

Hence, nominal shear reinforcements are to be designed.

d) Nominal Shear Reinforcements

Using 8mm diameter two legged stirrups the spacing is computed by Eq.(6.18)

$$S_v = \left(\frac{A_{sv} 0.87 f_y}{0.4b}\right) = \left(\frac{2 \times 50 \times 0.87 \times 415}{0.4 \times 300}\right)$$

$$= 300.8 \text{ mm} < (0.75d) = (0.75 \times 600) = 450 \text{ mm.}$$

Use 8 mm diameter 2 legged stirrups at 300 mm centres.

Method–2 (Using SP: 16 Design Tables)

SP: 16 requires the computation of the parameter $\left(\dfrac{V_{us}}{d}\right)$ kN/cm.

Design for nominal steel is equivalent to designing for a shear stress of 0.4 N/mm^2 (Refer Eq.6.18)

$$\left(\frac{V_{us}}{d}\right) = \left(\frac{0.4 \times 300 \times 600}{10^3 \times 60}\right) = 1.2$$

Where V_{us} is expressed in kN and 'd' is expressed in cm.
Refer Table–62 of SP: 16 (Table–6.17 of text) and read out the spacing corresponding to $f_y = 415$ N/mm^2 and diameter = 8mm and $(V_{us}/d) = 1.20$

$S_v = 30$ cm $= 300$ mm

Adopt 8mm diameter 2 legged vertical stirrups at 300 mm centers.

3) A reinforced concrete beam of rectangular section 350 mm wide is reinforced with 4 bars of 20 mm diameter at an effective depth of 550 mm out of which 2 bars are bent up near the support section where a factored shear force of 400 kN is acting. Using M-20 grade concrete and Fe-415 Grade HYSD bars design suitable shear reinforcements at the support section.

Method–1 (Using IS: 456-2000 Code Formulae)

a) Data

$b = 350$ mm, $d = 550$ mm, $A_s = 628$ mm^2

$f_{ck} = 20$ N/mm^2, $f_y = 415$ N/mm^2, $V_u = 400$ kN

b) Nominal Shear Stress

$V_u = 400$kN

$$\tau_v = \left(\frac{V_u}{bd}\right) = \left(\frac{400 \times 10^3}{350 \times 550}\right) = 2.07 \text{ N/mm}^2$$

$$p_t = \left(\frac{100A_s}{bd}\right) = \left(\frac{100 \times 628}{350 \times 550}\right) = 0.32$$

Refer Table–19 of IS: 456-2000 and read out τ_c corresponding to p_t and f_{ck}

$\therefore \quad \tau_c = 0.40\ \text{N/mm}^2 < \tau_v < \tau_{c,max} = 2.8\text{N/mm}^2$

Hence, shear reinforcements have to be designed.

c) Shear resisted by Concrete

$$V_{uc} = (\tau_c bd) = (0.40 \times 350 \times 550)\ 10^{-3} = 77\ \text{kN}.$$

$\therefore$ Shear to be carried by steel is $V_{us} = [V_u - V_{uc}] = [400-77] = 323$ kN.

d) Shear carried by bent up bars

$$V_{us\alpha} = A_s(0.87f_y)\sin\alpha$$

$$= (628 \times 0.87 \times 415 \times \sin 45°)10^{-3}$$

$$= 160.3\ \text{kN}.$$

$\therefore$ Shear to be carried by Vertical Stirrups is = [323–160.3] = 162.7 kN.

e) Design of Vertical Stirrups

Using 10 mm diameter 2 legged stirrups, the spacing S_v is calculated as

$$S_v = \left[\frac{0.87f_y.A_{sv}.d}{V_s}\right] = \left[\frac{0.87 \times 415 \times 2 \times 78.5 \times 550}{162.7 \times 10^3}\right] = 192.8\ \text{mm}$$

$S_v < 0.75\ d$ and $S_v < 300$ mm. Use 10mm diameter 2 legged stirrups at 190 mm centres.

Method–2 (Using SP: 16 Design Tables)

The shear to be carried by bent up bars and vertical stirrups = 323 kN.

a) Shear resisted by bent up bars

2 bars of 20mm diameter are bent up at an angle $\alpha = 45°$ near support section. Compute the shear taken by bent bars using Table-63 of SP: 16 corresponding to

$f_y = 415\ \text{N/mm}^2$, $\phi = 20$ mm and $\alpha = 45°$, $\quad V_{us}\alpha = (2 \times 80.21) = 160.4$ kN.

a) Design of Vertical Stirrups:-

Shear force to be resisted by vertical stirrups is V_{us} = [323–160] =162.6 kN.

Compute the ratio $\left(\frac{V_{us}}{d}\right)$ kN/cm.

$$\left(\frac{V_{us}}{d}\right) = \left(\frac{162.6}{55}\right) = 2.96\ \text{kN/cm}$$

Using 10mm diameter 2 legged stirrups, Refer Table - 62 of SP: 16 and read out the spacing S_v, corresponding to ϕ = 10 mm and f_y = 415 N/mm^2 and the ratio of (V_{us}/d)

$$\therefore \qquad S_v \cong 19\text{cm} = 190\text{mm}$$

$\therefore$ Adopt 10mm diameter 2 legged vertical stirrups at 190 mm centres.

6.6 TORSIONAL STRENGTH OF REINFORCED CONCRETE SECTIONS

6.6.1 Introduction

In reinforced concrete members, torsion generally occurs in combination with flexure and transverse shear. Pure torsion (metallic shafts) rarely develops in reinforced concrete. Several investigations have revealed the complex behaviour of interaction between flexure, shear and torsion. Based on various experimental investigations, the national codes prescribe simplified procedures combining the significant aspects of theoretical considerations and experimental results.

Several types of loading produce torsion in reinforced concrete structural elements. The resultant torsion in classified under two main groups[51].

a) Primary or equilibrium torsion
b) Secondary or compatibility torsion

6.6.2 Primary Torsion

Primary or equilibrium torsion is induced by eccentric loading, the common examples being,

a) Cantilever Beam with slab
b) Bow Girder
c) L-Beams

The loading on these elements developing torsion is shown in Fig. 6.13. In all these cases the loading is eccentric to the line of reaction at supports. The total torsion is equally distributed to the support sections equilibrium or primary torsion is induced by eccentric loading and equilibrium conditions are sufficient to estimate the torsional moments.

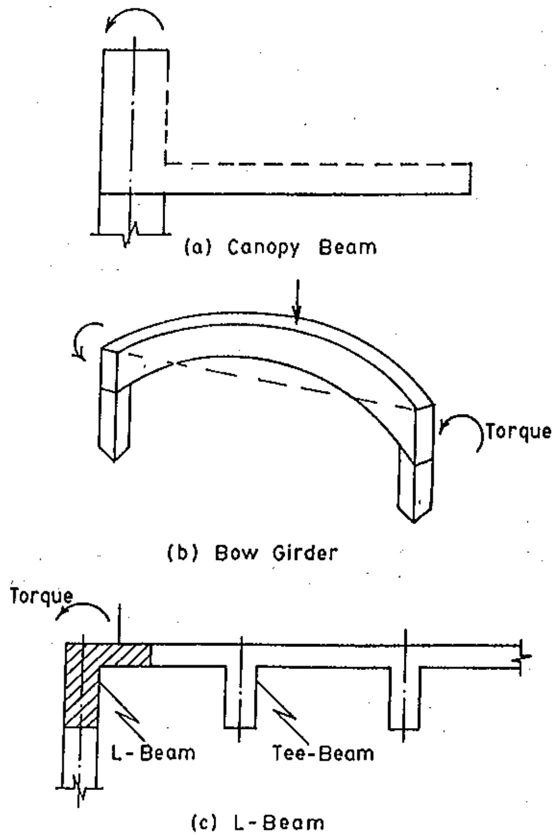


Fig. 6.13 Examples of Equilibrium Torsion

6.6.3 Secondary or Compatibility Torsion

In compatibility torsion, the torsion is induced by the application of an angle of twist such as the rotation of a member and the resulting torsional moment depends upon the torsional stiffness of the member. A typical

example of compatibility torsion is shown in Fig. 6.14 in which the secondary beam AB is monolithically connected to the main beam CD at A and B. The beam AB rotates at the junction due to loading on AB. Corresponding to the angle θ_A, a torsional moment will develop at A in beam CD and a bending moment will develop at A in beam AB.

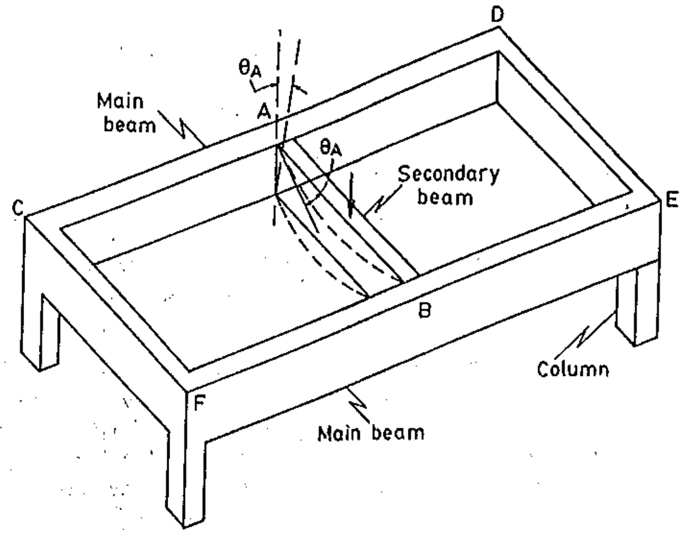


Fig. 6.14 Typical Example of Compatibility Torsion

The bending moment will be equal to and act in a direction opposite to the torsional moment to maintain static equilibrium. The magnitude of rotation θ_A and the torsional and bending moments at A depends upon the torsional stiffness of beam CD and the flexural stiffness of beam AB.

6.6.4 Torsional shear stress

The theory of torsion[52, 53] of prismatic homogeneous members of different types of cross section is well established and described in detail in books of mechanics of materials.[54, 55] The effect of torsion is to induce shear stresses and causes warping of non-circular sections. The failure of a plain concrete member in torsion is caused by torsional cracking due to the diagonal tensile stresses. Due to torsion, a plain concrete rectangular member develops diagonal tension cracks in the inter fibres as shown in Fig. 6.15 leading to a sudden failure of the entire section due to low strength of concrete in tension. To improve the torsional strength of rectangular concrete sections torsional reinforcements are generally provided in the form of longitudinal and transverse steel, the former in the form of bars distributed around the

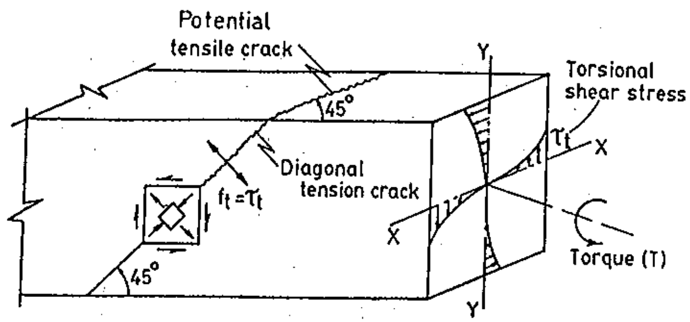


Fig. 6.15 Torsional Shear Stresses and Cracks in a Beam of Rectangular Section

cross sections close to the periphery and the latter in the form of closed rectangular stirrups, placed perpendicular to the axis of the beam, the longitudinal reinforcements resists the tension and the transverse reinforcement is required to resist shear.

The torque-twist characteristics of torsionally reinforced concrete members is similar to that of plain concrete until the formation of the first torsional cracks as shown is Fig. 6.16 (a) and (b). The value of cracking torque T_{cr} is practically the same for both plain and reinforced concrete members. When cracking develops, there is a large increase in twist under constant torque due to drastic loss of torsional stiffness[56]. Post cracking behaviour is influenced by the magnitude of torsional reinforcement in the member as shown in Fig. 6.16 (b). Increase of torsional reinforcement will also increase the ultimate torsional strength and the ductile failure is preceded by yielding of steel, which can be realised only at very large angles of twist. However, the increase in strength is limited since failure due to crushing of concrete may take place prior to the yielding of reinforcement in tension.

For rectangular sections subjected to a torque T_u, using the sand heap analogy,[57,58] we can express the torque in terms of the torsional shear stress τ_t and the cross sectional dimensions 'b', 'D' and 'd' as,

$$\tau_t = \frac{2T_u}{b^2 d}\left(\frac{1}{\left(\frac{D}{d}\right)-\left(\frac{b}{3d}\right)}\right) \qquad \ldots(6.26)$$

$$= \frac{2T_u}{b^2 d}\left(\frac{1}{\text{Constant}}\right)$$

However, the constant $[(D/d) - (b/3d)]$ has a value in the range of 0.8 to 1.15 for most of the rectangular sections in practice. Selecting an average value for the constant and applying a correction factor for the assumption of full plastification of the section the final expression reduces to

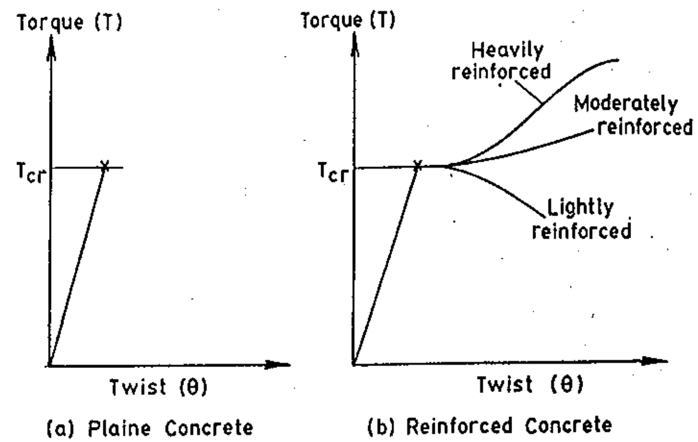


Fig. 6.16 Torque-Twist Characteristics of Plain and Reinforced Concrete Members

$$\tau_t = \left[\frac{1.6\,(T_u/b)}{b.d}\right] \qquad \ldots(6.27)$$

If V_t = shear due to torsion, the expression specified in IS: 456-2000 code is given by

$$V_t = (\tau_t.b.d) = 1.6\left(\frac{T_u}{b}\right) \qquad \ldots(6.28)$$

This is similar to the expression for flexural shear stress, $\tau_v = (V_u/bd)$

Hence for a section of overall dimensions b and D subjected to shear V_u and torsion T_u the equivalent shear V_e is calculated from the relation as specified in IS: 456 code as

$$V_e = V_u + 1.6(T_u/b) \qquad \ldots(6.29)$$

6.6.5 Reinforcement Design for shear and torsion

Reinforced concrete members, when subjected torsion and shear have to be suitably reinforced so that the equivalent nominal shear stress τ_{ve} expressed as

$$\tau_{ve} = \left[\frac{V_u + 1.6(T_u/b)}{bd}\right] \qquad \ldots(6.30)$$

lies between τ_c, the permissible shear stress given in table 6.14 and the maximum shear stress $\tau_{c,max}$ compiled in Table 6.16.

If the shear tve exceeds the value of $\tau_{c,max}$, the section has to be suitably redesigned by increasing the cross sectional area and /or increasing the grade of concrete. If τ_{ve} is less than the design shear strength τ_c, minimum shear reinforcements has to be provided conforming to the equation (6.18) explained in section 6.5.6.

6.6.6 Torsion-Shear Interaction

The design of reinforcements for torsion and shear is based on the interaction curve for concrete with web steel under shear and torsion as investigated by Hsu[59], Collins et al[60], who formulated the space-truss analogy and skew bending theory. The interaction curve for concrete with and without web steel under combined torsion and shear is shown in Fig. 6.17 (a) and (b)[57].

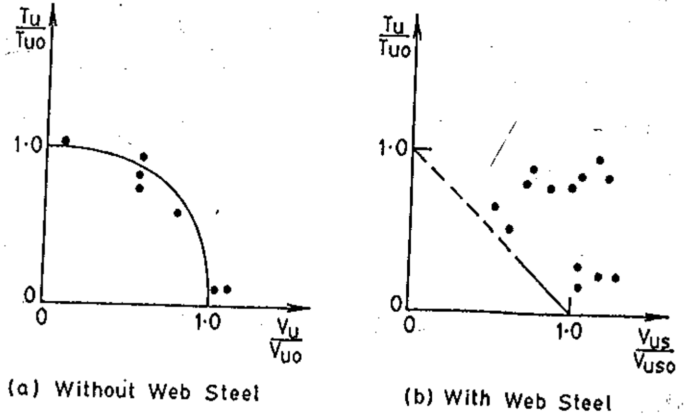


Fig. 6.17 Interaction Curves for Torsion and Shear

The interaction relation shown is Fig. 6.17 (b) for beams with web reinforcement can be assumed to follow the conservative linear relation given by the equation.

$$\left[\left(\frac{T_u}{T_{uo}}\right)+\left(\frac{V_{us}}{V_{uso}}\right)\right]=1 \qquad \ldots(6.31)$$

Where

T_u = torsional moment in the section

V_{us} = shear force shared by web steel out of the total shear force in the section (V_u)

V_{uso} = shear strength of reinforcement assuming no torsion is present.

T_{uo} = Torsional strength of reinforcement assuming no shear is present.

Based on experimental investigations, it can be safely assumed that the shear carried by the web reinforcement (V_{us}) is about 40 percent of the

ultimate shear (V_u).

Therefore $V_{us} = 0.4\ V_u$.

Using this relation, Eq (6.30) reduces to

$$\left[\left(\frac{T_u}{T_{uo}}\right)+\left(\frac{V_{us}}{2.5V_{uso}}\right)\right]=1 \qquad \ldots(6.32)$$

If S_v = spacing of stirrups (Fig. 6.18)

A_{sv} = cross sectional area of web reinforcement

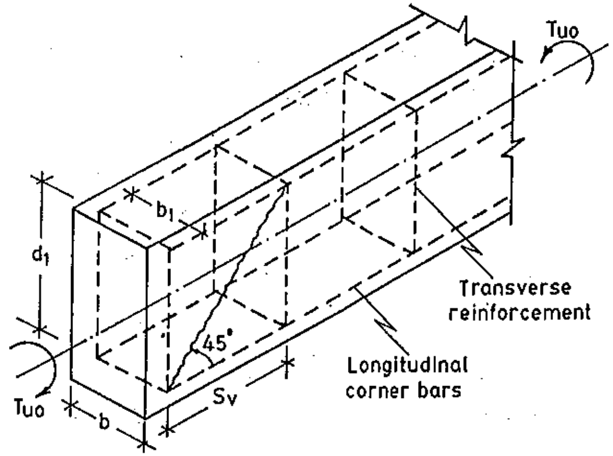


Fig. 6.18 Space Truss Model for Torsion in R.C.C. Beams

Then, $V_{uso} = 0.87\, f_y\, A_{sv}\, (d/S_v)$ $\qquad \ldots(6.33)$

Where d = effective depth

Using the thin walled tube model shown in Fig. 6.19, the shear flow 'q' (force per unit length) across the thickness of the tube (Ref. 58) is given by

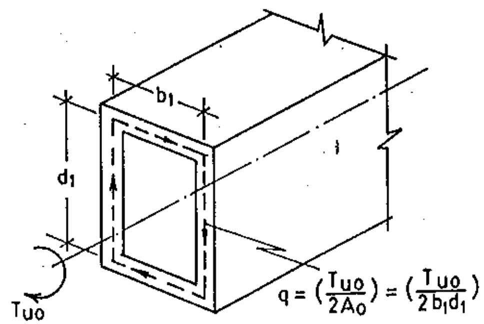


Fig. 6.19 Shear Flow in Thin Walled Tube

$$q = [T_{u} / 2A_o] \quad \text{...(6.34)}$$

Where A_o = area enclosed by the centre line of the thickness.

But $A_o = b_1 d_1$

Where b_1 and d_1 denote the centre-to-centre distances between the corner bars in the directions of width and depth respectively.

Accordingly, we have

$$q = (T_{uo}/2b_1 d_1) \quad \text{...(6.35)}$$

Assuming torsional cracks (pure torsion) at 45° to the longitudinal axis of the beam (Fig. 6.18) and considering the equilibrium of forces normal to the section (Fig. 6.19),

$$q.S_v = A_t(0.87f_y) \quad \text{...(6.36)}$$

Where A_t = cross sectional area of the stirrups = $(A_{sv}/2)$ for two legged stirrups. Substituting in Eq (6.34), we have

$$T_{uo} = \left(\frac{2A_t.b_1.d_1(0.87f_y)}{S_v}\right)$$

or

$$T_{uo} = \left(\frac{0.87f_y A_{sv}.b_1.d_1}{S_v}\right) \quad \text{...(6.37)}$$

Substituting Eq. (6.33) and (6.37) in Eq. (6.31) we have the final equation for the web reinforcement given by the relation,

$$A_{sv} = \left[\left(\frac{T_u.S_v}{b_1.d_1 0.87 f_y}\right) + \left(\frac{V_u.S_v}{2.5d_1 0.87f_y}\right)\right] \quad \text{...(6.38)}$$

Which is the same equation specified in IS: 456-2000 for the computation of transverse reinforcement for combined torsion and shear in limit state design.

Also clause 41.4.3 also specifies that the transverse steel should naturally be not less than that required to with stand the full equivalent shear V_e given by the relation,

$$A_{sv} = \left[\frac{(\tau_{ve} - \tau_c)b.S_v}{0.87 f_y}\right] \quad \text{...(6.39)}$$

It is more convenient to recast the equations (6.38) and (6.39) in terms of the spacing S_v since the diameter and hence the cross sectional area of the web steel (A_{sv}), is generally assumed and the spacing is computed. Hence, we have the equations for the spacing S_v as

$$S_v = \left[\left(\frac{A_{sv} b_1 d_1(0.87f_y)}{T_u}\right) + \left(\frac{A_{sv}.2.5d_1(0.87 f_y)}{V_u}\right)\right] \quad \text{...(6.40)}$$

And

$$S_v = \left[\frac{A_{sv}(0.87f_y)}{(\tau_{ve} - \tau_c)b}\right] \quad \text{...(6.41)}$$

In practical examples, it has been found that equation (6.40) results in very large values of spacing and hence Eq (6.41) is generally used for designing the transverse reinforcements for beams subjected to combined torsion and shear.

6.6.7 Design strength in Torsion combined with Flexure

When a section is subjected to torsion (T_u) and flexure (M_u), experimental investigations by Iyengar et al[61] have shown that the interaction behavior of rectangular sections under torsion and flexure following a parabolic relation is influenced by the magnitude of longitudinal reinforcements provided in the flexural tension and compression zones. Warner and Rangan[62] have investigated the different modes of failure of rectangular reinforced concrete members under combined flexure and torsion. They have identified three different modes of failure depending upon the various variables and their combinations as shown in Figs. 6.20 (a), (b) and (c) and (d).[63]

a) Mode–1 Type Failure

The most common type of failure encountered in beams is the mode-1 type failure which occurs when flexure is predominant over torsion. The member fails by skew compression at top as shown in Fig. 6.20 (b). This type of failure is often referred to as modified flexural failure.

b) Mode–2 Type Failure

In the case of beams having a narrow section with depth exceeding the width and when torsion is greater than flexure, mode-2 type failure characterized by the compression zone skewed to the side of the member as shown in Fig. 6.20 (c) generally occurs and this type of failure is also referred to as lateral flexural failure.

c) Mode–3 Type Failure

In rectangular beams having longitudinal top reinforcements much less than that of the bottom reinforcements, mode-3 type failure, also termed as negative flexural failure develops with the compression zone occurring towards the soffit of the member. This type of failure is shown in Fig. 6.20(d).

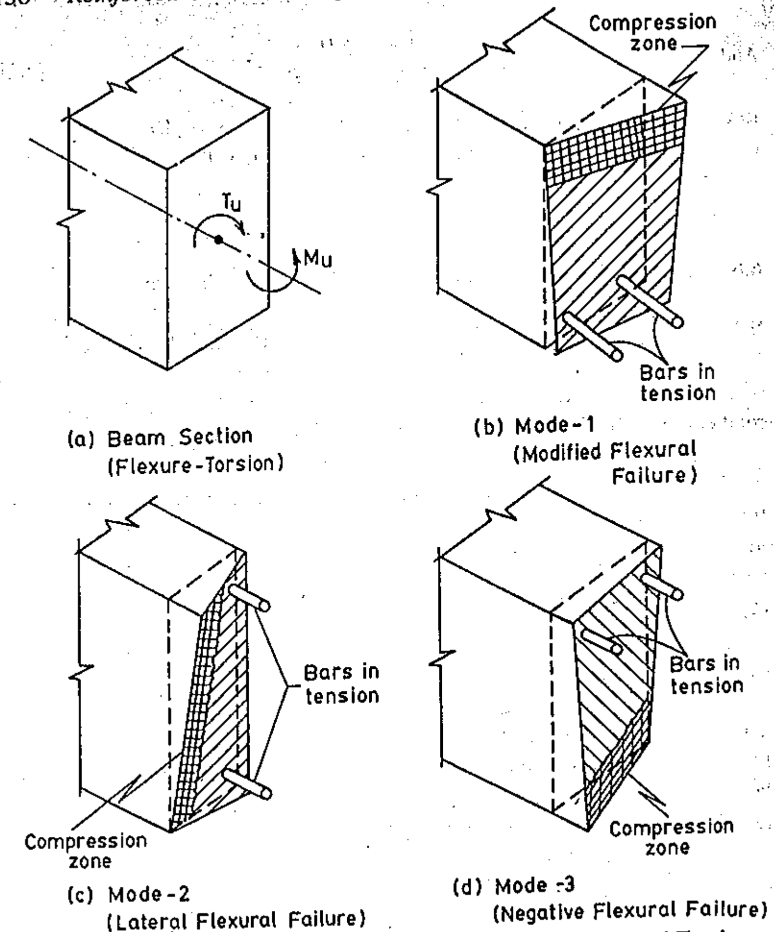


Fig. 6.20 Failure Models of R.C. Beams Under Combined Flexure and Torsion

The ultimate strength of rectangular reinforced concrete section subjected to combined flexure (M_u) and torsion (T_u) is generally described by the interaction diagram shown in Fig. 6.21.

Let T_{ur} = pure torsional strength

M_{ur} = pure Flexural strength

Based on experimental investigations, Iyengar et al[61] have suggested the following interaction formulas:

For mode-1 Failure,

$$\left(\frac{A_{sc}}{A_{st}}\right)\left(\frac{T_u}{T_{ur}}\right)^2+\left(\frac{M_u}{M_{ur}}\right)^2 \le 1 \quad \ldots(6.42)$$

For mode-3 Failure,

$$\left(\frac{T_u}{T_{ur}}\right)^2-\left(\frac{A_{st}}{A_{sc}}\right)\left(\frac{M_u}{M_{ur}}\right)^2 \le 1 \quad \ldots(6.43)$$

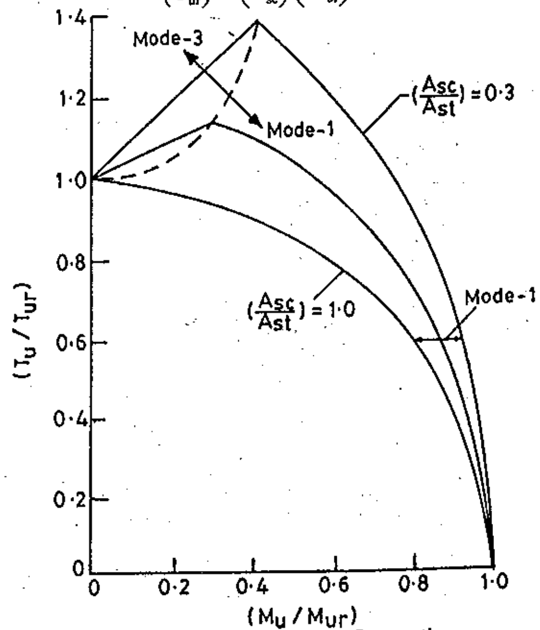


Fig. 6.21 Torsion-Flexure Interaction

Where A_{st} = Area of longitudinal steel in the flexural tension zone.

A_{sc} = Area of longitudinal steel in the flexural Compression zone

The flexure-torsion interaction curves based on the above relations are shown in Fig. 6.21 for the ratio of $\left(\frac{A_{sc}}{A_{st}}\right)$ varying from 0.3 to 1.0.

For low values of the ratio $\left(\frac{M_u}{M_{ur}}\right)$, the torsional strength is marginally higher. When $\left(\frac{A_{sc}}{A_{st}}\right)$ = 1, mode-1 type failure generally occurs with the yielding of tension reinforcement. In general, presence of torsion reduces the flexural strength of the member.

6.6.8 Reinforcement design for Flexure and Torsion

The revised Indian standard code clause 41.4.2 IS: 456-2000 recommendations are based on the skew bending approach[61] in which the torsional moment T_u is converted into an effective bending moment M_t defined by the relation,

$$M_t = \left\{\frac{T_u(1+D/b)}{1.7}\right\} \quad \ldots(6.44)$$

Where D = overall depth of the beam.
The longitudinal reinforcement is designed to resist an equivalent bending moments M_{e1} and M_{e2} expressed as,

$$M_{e1} = (M_u + M_t) \quad \ldots(6.45)$$
$$M_{e2} = (M_t - M_u) \quad \ldots(6.46)$$

For values of $M_t \le M_u$, the longitudinal reinforcement is designed to resist the equivalent moment M_{e1} with the steel located in the flexural tension zone. For values of $M_t > M_u$, the reinforcement is designed to resist M_{e2} with the steel located in the flexural compression zone.

In the case of pure torsion ($M_u = 0$), it follows that equal longitudinal steel is required at the top and bottom of the section.

6.6.9 Design of Reinforcements for Flexure, Shear and Torsion according to IS: 456 - 2000 Code Specifications

The procedure outlined in the following steps may be followed to design sections subjected to combined bending, shear and torsion.

Design Steps

1) Compute design moments M_u and T_u and design shear force V_u
2) Calculate equivalent moment

$$M_{e1} = (M_u + M_t) = \left\{M_u + \frac{T_u(1+D/b)}{1.7}\right\}$$

3) Design tension steel for M_{e1}
4) If $M_t > M_u$ then compute $M_{e2} = (M_t - M_u)$
5) Design steel on compression face for the moment M_{e2}
6) Compute the equivalent shear force as

$$V_e = V_u + 1.6\,(T_u / b)$$

7) Determine shear stress $\tau_{ve} = (V_e / bd)$. Check that the shear stress is less than the maximum shear stress $\tau_{c,max}$

8) If $\tau_{ve} > \tau_c$ given in Table-19 of IS: 456 and $< \tau_{c,max}$ given in Table-20 of IS:456, then assume diameter of shear stirrups and compute their spacing using the equations:

a) $$S_v = \left(\frac{A_{sv}.0.87 f_y}{(\tau_{ve} - \tau_c)b}\right)$$

b) $$S_v = \left[\left(\frac{A_{sv}(0.87 f_y) b_1 d_1}{T_u}\right) + \left(\frac{A_{sv}(0.87 f_y) 2.5 d_1}{V_u}\right)\right]$$

9) The spacing S_v selected should be the smaller of the two values from (a) and (b). Also S_v should not exceed b_1 or $[(b_1 + d_1)/4]$ or 300 mm which ever is less.

6.6.10 Design procedure using SP: 16 design aids

The computation in steps 7 and 8 are modified to use the SP: 16 Table-62 (Table-6.17 of the text).
Compute the parameter

$$\left(\frac{V_{us}}{d}\right) = (\tau_{ve} - \tau_c)b = \left(\frac{A_{sv}(0.87 f_y)}{S_v}\right)$$

Also compute $$\left(\frac{A_{sv}(0.87 f_y)}{S_v}\right) = \left[\left(\frac{T_u}{b_1 . d_1}\right) + \left(\frac{V_u}{2.5 d_1}\right)\right]$$

For known values of the ratios, read out the diameter of stirrups and their spacing from Table-62 of SP: 16 or Table 6.17 of the text.

6.6.11 Analysis examples

1) A cantilever canopy is made up of monolithically constructed beam of 8m span with a cantilever slab of 4m. The beam section is 400mm wide by 1200 mm deep. The live load on slab is 1.5 kN/m^2. Determine the torsion and shear due to live load for which the beam should be designed.

a) Data

Span of beam = L = 8m
Cantilever slab span = 4m
Cross section of beam = 400 mm wide by 1200 mm deep
Live load = 1.5 kN/m^2

b) Loads on beam

Live load on beam = $(1.5 \times 4) = 6$ kN/m
Maximum shear at support = $V = (6 \times 0.5 \times 8) = 24$ kN

c) Design Torsion in beam

Torsion per metre length of beam = $(1.5 \times 4)(0.5 \times 4) = 12$ kNm/m

Torsion is zero at centre of span and maximum at the two fixed ends of beam.

Design torsion at support section is

$$T = (12 \times 0.5 \times 8) = 48 \text{ kN.m}$$

d) Design shear

$$V_e = V + 1.6\left(\frac{T}{b}\right) = \left[24 + 1.6\left(\frac{48}{0.4}\right)\right] = 216 \text{ kN}$$

2) A circular R.C.C. girder has a rectangular section with a width of 500 mm and over all depth of 1000 mm. At a particular section, the factored values of bending and torsional moments are 150 and 30 kN.m respectively. The ultimate shear force at the section is 150 kN. Analyse the design moment and shear for which the beam has to be designed.

a) Data

$M_u = 150$ kN.m	$b = 500$ mm
$T_u = 30$ kN.m	$D = 1000$ mm
$V_u = 150$ kN	

b) Equivalent bending moment

$$M_{e1} = (M_u + M_t)$$

Where $$M_t = T_u\left[\frac{(1 + D/b)}{1.7}\right] = 30\left[\frac{(1 + 1000/500)}{1.7}\right] = 53 \text{ kN.m}$$

$$\therefore \quad M_{e1} = (150 + 53) = 203 \text{ kN.m.}$$

c) Equivalent Shear Force

$$V_e = V_u + 1.6\left(\frac{T}{b}\right) = \left[150 + 1.6\left(\frac{30}{0.5}\right)\right] = 246 \text{ kN}$$

The cross-section has to be designed for an equivalent bending moment of 203 kN.m and an equivalent shear force of 246 kN.

3) Determine the design torsional resistance of a reinforced concrete beam of rectangular section using the following data. The beam is reinforced using Fe-415 HYSD bars. Adopt IS: 456-2000 code provisions.

a) Data

$b = 350$ mm	$b_1 = 300$ mm
$D = 900$ mm	$d_1 = 800$ mm
$A_{sv} = (2 \times 79)$	$S_v = 150$ mm
$= 158$ mm^2	

b) Design torsional resistance

According to IS: 456-2000 code, considering the shear-torsion interaction [Eq.6.38] with $V_u = 0$, which corresponds to the space truss analogy, considering the contribution of the transverse reinforcement only (Eq.6.40] the ultimate torsional resistance is computed as.

$$T_u = \left(\frac{0.87 f_y A_{sv} . b_1 . d_1}{S_v}\right) = \left(\frac{0.87 \times 415 \times 158 \times 300 \times 800}{150}\right) = (86.65 \times 10^6) = 86.65 \text{ kN.m}$$

4) A reinforced concrete beam of rectangular section has a width of 350 mm and overall depth of 700 mm. The beam is reinforced with 2 bars of 25mm diameter both on the tension and compression faces at an effective cover of 50mm. The side covers are 25mm. 10 mm diameter two legged stirrups are provided at 100mm centres. The section is subjected to a factored shear force of 200 kN. If Fe-415 HYSD bars are used, estimate the torsional resistance of the beam using I.S. code provisions.

a) Data

$b = 350$ mm	$b_1 = 300$ mm
$D = 700$ mm	$d_1 = 600$ mm
$V_u = 200$ kN	$A_{sv} = (2 \times 79) = 158$ mm^2
$S_v = 100$ mm	

b) Torsional Strength considering $V_u = 0$

$$T_u = \left(\frac{0.87 f_y A_{sv} b_1 d_1}{S_v}\right) = \left(\frac{0.87 \times 415 \times 158 \times 300 \times 600}{100}\right) = (102.6 \times 10^6)\ \text{N.mm}$$

$= 102.6$ kN.m

c) Torsional strength considering $V_u = 200$ kN

$$A_{sv} = \left[\left(\frac{T_u S_v}{b_1 d_1 0.87 f_y}\right) + \left(\frac{V_u S_v}{2.5 d_1 0.87 f_y}\right)\right]$$

$$158 = \left[\left(\frac{T_u \times 100}{300 \times 600 \times 0.87 \times 415}\right) + \left(\frac{200 \times 10^3 \times 100}{2.5 \times 600 \times 0.87 \times 415}\right)\right]$$

Solving, $T_u = (78.6 \times 10^6)$ N.mm $= 78.6$ kN.m. Hence, torsional strength is smaller of the two values.

5) A reinforced concrete rectangular beam has a breadth of 400 mm and effective depth of 800 mm. It has a factored shear force of 120 kN at a particular section. Assuming that $f_{ck} = 25$ N/mm^2 and $f_y = 415$ N/mm^2 and percentage of tensile steel at that section as 0.5 percent, determine the torsional moment the section can resist for the following cases.

Case- 1: If no additional reinforcement for torsion is provided.
Case- 2: If the maximum steel for torsion is provided in the section.
Adopt IS: 456 code provisions for the analysis.

a) Data

$b = 400$ mm $\quad f_{ck} = 25$ N/mm^2
$d = 800$ mm $\quad f_y = 415$ N/mm^2

b) Permissible shear stress

For $p_t = 0.5\%$ and $f_{ck} = 25$ N/mm^2 referring to Table–19 of IS:456 code

$$\tau_{c(min)} = 0.49\ \text{N/mm}^2$$

c) Allowable torsion

$$V_e = V_u + 1.6(T/b) \qquad \therefore \left(\frac{V_e}{bd}\right) = 0.49$$

$$\left[\frac{V_u + 1.6\,(T/b)}{bd}\right] = 0.49$$

$$\left[\frac{(120 \times 10^3) + 1.6(T/400)}{400 \times 800}\right] = 0.49$$

Solving $T = (9.2 \times 10^6)$ N.mm $= 9.2$ kN.m

d) Maximum torsional capacity of the section (Torsion + shear)

Refer Table-20 of IS: 456 and read out the value of $\tau_{c,max}$ for $f_{ck} = 25$ N/mm^2

$$\therefore \qquad \tau_{c,\,max} = 3.1\ \text{N/mm}^2$$

It T = Maximum allowable torsion, then

$$\left[\frac{V_u + 1.6(T/b)}{bd}\right] = 3.1$$

$$\left[\frac{(120 \times 10^3) + 1.6(T/400)}{400 \times 800}\right] = 3.1$$

Solving $T = (218 \times 10^6)$ N.mm $= 218$ kN.m

6) A reinforced concrete beam of rectangular section with a breadth of 350mm and overall depth of 800mm is reinforced with 4 bars of 20mm diameter on the tension side at an effective depth of 750mm. The section is subjected to an ultimate moment of 215 kN.m. If $f_{ck} = 30$ N/mm^2 and $f_y = 415$ N/mm^2, estimate the ultimate torsional moment that can be allowed on the section.

a) Data

$b = 350$ mm $\quad f_{ck} = 30$ N/mm^2
$d = 750$ mm $\quad f_y = 415$ N/mm^2
$D = 800$ mm $\quad A_{st} = (4 \times 314) = 1256$ mm^2

b) Neutral axis depth

$$x_{u,\,max} = (0.48d) = (0.48 \times 750) = 360\ \text{mm}$$

But $$x_u = \left[\frac{0.87 f_y A_{st}}{0.36 f_{ck} b}\right] = \left[\frac{0.87 \times 415 \times 1256}{0.36 \times 30 \times 350}\right] = 120\ \text{mm} < x_{u,\,max}$$

Hence, the beam is under reinforced

c) Equivalent Ultimate moment capacity of section

$$M_e = 0.87 f_y A_{st} d\left[1 - \left(\frac{A_{st} f_y}{b d f_{ck}}\right)\right]$$

$$M_e = 0.87\times415\times1256\times750\left[1-\left(\frac{1256\times415}{350\times750\times30}\right)\right] = (317\times10^6)\ \text{N.mm}$$

$$= 317\ \text{kN.m.}$$

d) Allowable torsional moment

$$M_e = (M_u + M_t) = M_u + T_u\left(\frac{1+(D/b)}{1.7}\right)$$

$$317 = 215 + T_u\left(\frac{1+(800/350)}{1.7}\right)$$

Solving $T_u = 62.2$ kN.m

6.6.12 Design examples

1) An R.C.C. Section 200 ×400 mm is subjected to a characteristic torsional moment of 2.5 kN.m and a transverse shear of 60 kN. Assuming the use of M-25 grade concrete and Fe-415 HYSD bars, determine the reinforcements required according to the IS: 456 code provisions, using the following data.

Method–1 (Using IS: 456 Code Formulae)

a) Data

$b = 200$ mm	$f_{ck} = 25$ N/mm^2
$d = 350$ mm	$f_y = 415$ N/mm^2
$D = 400$ mm	$b_1 = 150$ mm
$T_u = 2.5$ kNm	$d_1 = 300$ mm
$V_u = 60$ kN	

b) Equivalent shear force

$$V_e = V_u + 1.6\left(\frac{T_u}{b}\right) = 60 + 1.6\left(\frac{2.5}{0.2}\right) = 80\ \text{kN}$$

c) Equivalent bending moment

$$M_e = (M_u + M_t) = M_u + \frac{T_u(1+D/b)}{1.7} = 0 + \frac{2.5(1+400/200)}{1.7} = 4.41\ \text{kN.m}$$

d) Longitudinal reinforcements

Since the equivalent bending moment $M_e = 4.41$ kN.m, design the longitudinal reinforcement for this moment.

The section is under reinforced since the steel requirement to resist the small moment will be less than the minimum.

$$M_e = 0.87 f_y A_{st} d\left[1-\left(\frac{A_{st}f_y}{bdf_{ck}}\right)\right]$$

$$M_e = 0.87\times415\times A_{st}\times350\left[1-\left(\frac{415A_{st}}{200\times350\times25}\right)\right]$$

$$[A_{st}^2 - 4218\ A_{st} + (1.47\times10^5)] = 0$$

$$\therefore \quad A_{st} = 36\ \text{mm}^2$$

Providing minimum reinforcement of

$$A_s = \left(\frac{0.85bd}{f_y}\right) = \left(\frac{0.85\times200\times350}{415}\right) = 143\ \text{mm}^2$$

Provide 2 bars of 10mm diameter as tension reinforcement and 2 hanger bars of 10mm diameter on compression side at an effective cover of 50mm($A_{st} = 158$ mm^2).

e) Permissible shear stress

$$\tau_{ve} = \left(\frac{V_e}{bd}\right) = \left(\frac{80\times10^3}{200\times350}\right) = 1.14\ \text{N/mm}^2$$

$$p_t = \left(\frac{100A_s}{bd}\right) = \left(\frac{100\times158}{200\times350}\right) = 0.225$$

Refer Table–19 of IS: 456 and read out τ_c corresponding to $f_{ck} = 25$ N/mm^2,

$$\tau_c = 0.34\ \text{N/mm}^2$$

$$\tau_{ve} > \tau_c \quad \text{and} \quad \tau_{ve} < \tau_{c,max} = 3.1\ \text{Nmm}^2 \quad \text{(Table-20)}$$

∴ Design transverse reinforcements using the IS: code recommendations.

f) Transverse reinforcement

Using 8mm diameter 2 legged stirrups with side covers of 25mm, the spacing is given by

$$S_v = \left(\frac{A_{sv}.0.87 f_y}{(\tau_{ve}-\tau_c)b}\right) = \left(\frac{0.87\times415\times2\times50}{(1.14-0.34)200}\right) = 225\text{mm}$$

Also,
$$A_{sv} = \left[\left(\frac{T_u S_v}{b_1 d_1 0.87 f_y}\right) + \left(\frac{V_u S_v}{2.5 d_1 0.87 f_y}\right)\right] = 100$$

$$100 = \left[\left(\frac{2.5\times10^6}{150\times300\times0.87\times415}\right) + \left(\frac{60\times10^3}{2.5\times300\times0.87\times415}\right)\right] S_v$$

Solving $S_v = 266\text{mm}$

Adopting the smaller of the two values, use 8mm diameter 2 legged stirrups at a spacing of 225mm.

Method–2 (using SP: 16 Design Tables)

a) Longitudinal reinforcements

Equivalent bending moment $= M_e = M_u = 4.41$ kN.m

$$\left(\frac{M_u}{bd^2}\right) = \left(\frac{4.41\times10^6}{200\times350^2}\right) = 0.18$$

Referring to Table-3 of SP: 16 ($f_{ck} = 25$ N/mm^2) the minimum value of the parameter (M_u/bd^2) listed as 0.30. Hence, the table cannot be used.

Provide minimum longitudinal reinforcement of $p_t = 0.20$ percent for $f_y = 415$ N/mm^2, as worked out in method-1.

b) Transverse Reinforcements

Compute the parameter

$$\left(\frac{A_{sv}(0.87 f_y)}{S_v}\right) = \left[\left(\frac{T_u}{b_1 d_1}\right) + \left(\frac{V_u}{2.5 d_1}\right)\right] = \left[\left(\frac{2.5\times10^6}{150\times300}\right) + \left(\frac{60\times10^3}{2.5\times300}\right)\right] = 135.5 \text{ N/mm}$$

Refer Table-62 (SP : 16) and read out spacing of 8mm diameter two legged stirrups at 27cm = 270 mm.

Also
$$\left(\frac{A_{sv}(0.87 f_y)}{S_v}\right) = (\tau_{ve}-\tau_c)\,b = (1.14-0.34)200 = 1600 \text{ N/mm} = 1.6 \text{ kN/cm}$$

Refer Table-62 and read out spacing of 8mm diameter two legged stirrups as 22.5 cm = 225 mm. Provide the smaller spacing of the two values which is 225 mm.

2) A reinforced concrete beam of rectangular section with a width of 350mm and over all depth 700mm is subjected to an ultimate torsional moment of 100 kNm together with an ultimate bending moment of 200 kN.m. Adopting M-20 Grade concrete and Fe-415 HYSD bars and assuming top and bottom covers of 50 mm and side covers of 25 mm, design suitable longitudinal and transverse reinforcements for the section.

Method–1(using IS: 456 code formulae)

a) Data

b = 350 mm	b_1 = 300 mm
d = 650 mm	d_1 = 600 mm
D = 700 mm	f_{ck} = 20 N/mm^2
M_u = 200 kN.m	f_y = 415 N/mm^2
T_u = 100 kN.m	

b) Equivalent bending moment and shear forces

$$M_e = M_u + T_u\left[\frac{1+D/b}{1.7}\right] = 200 + 100\left[\frac{1+700/350}{1.7}\right] = 376 \text{ kN.m}$$

$$V_e = V_u + 1.6\left(\frac{T_u}{b}\right) = 0 + 1.6\left(\frac{100}{0.35}\right) = 457 \text{ kN}$$

The longitudinal reinforcement is designed for M_e and transverse reinforcement for V_e

c) Longitudinal reinforcements

$$M_u = 0.138 f_{ck}.b.d^2 = (0.138\times20\times350\times650^2) = 408\times10^6 \text{ N.mm} > M_e$$

Hence, section is under reinforced

$$M_e = 0.87 f_y A_{st}.d\left[1-\left(\frac{A_{st} f_y}{b\,d\,f_{ck}}\right)\right]$$

$$(376\times10^6) = (0.87\times415\times A_{st}\times650)\left[1-\left(\frac{415 A_{st}}{350\times650\times20}\right)\right]$$

Solving $A_{st} = 1940$ mm^2

Use 4 bars of 25mm diameter (A_{st} = 1964 mm^2) on the tension side and 2 hanger bars of 16 mm diameter on the compression side with effective covers of 50 mm.

d) Transverse Reinforcement

$$\tau_{ve} = \left(\frac{V_e}{bd}\right) = \left(\frac{457\times10^3}{350\times650}\right) = 2\ \text{N/mm}^2$$

$$p_t = \left(\frac{100A_s}{bd}\right) = \left(\frac{100\times1964}{350\times650}\right) = 0.86$$

Refer Table-19 of IS: 456 and read out permissible shear shear stress as τ_c = 0.59 N/mm^2 $\cdot$ $\tau_{ve} > \tau_c$ but less than $\tau_{c,max}$ = 2.8 N/mm^2. Hence, shear reinforcements are required. Assuming 10mm diameter 2 legged stirrups, the area of shear reinforcement is computed by using the equation specified in IS: 456 code, clause 41.4.3.

$$A_{sv} = \left[\left(\frac{T_u S_v}{b_1 d_1 0.87 f_y}\right) + \left(\frac{V_u S_v}{2.5 d_1 0.87 f_y}\right)\right]$$

$$158 = \left[\left(\frac{100\times10^6}{300}\right) + (0)\right]\frac{S_v}{600\times0.87\times415}$$

Solving $S_v = 102.7$ mm

Also,

$$S_v \not> \left(\frac{A_{sv}0.87 f_y}{(\tau_{ve}-\tau_c)b}\right)$$

$$S_v \not> \left(\frac{158\times0.87\times415}{(2-0.59)350}\right) \not> 115.6\text{mm}$$

Hence, adopt 10mm diameter two legged stirrups at a spacing given by smaller of the above two equations which is 100 mm.

Method–2 (using SP: 16 Design Tables)

a) Longitudinal Reinforcements

$M_e = M_u = 376$ kN.m	$b = 350$mm
$f_{ck} = 20$ N/mm^2	$b_1 = 300$mm
$f_y = 415$ N/mm^2	$d_1 = 600$mm
$d = 650$ mm	$V_u = 0$

Compute the parameter $\left(\frac{M_u}{bd^2}\right) = \left(\frac{376\times10^6}{350\times650^2}\right) = 2.54$

Refer Table–2 (SP: 16) and read out the percentage reinforcement p_t for f_{ck} = 20 and f_y = 415 N/mm^2.

$$p_t = 0.857 \quad \therefore \left(\frac{100A_{st}}{bd}\right) = 0.857$$

$$\therefore \quad A_{st} = \left(\frac{0.857\times350\times650}{100}\right) = 1950\ \text{mm}^2$$

b) Transverse Reinforcement

Compute the parameters given by

$$\left(\frac{A_{sv}(0.87 f_y)}{S_v}\right) = \left[\left(\frac{T_u}{b_1 d_1}\right) + \left(\frac{V_u}{2.5 d_1}\right)\right] = \left[\left(\frac{100\times10^6}{300\times600}\right) + (0)\right]$$

$$= 555.5\ \text{N/mm} \cong 5.55\text{kN/cm}$$

Refer Table–62 (SP: 16) and read out spacing of 10mm diameter 2 legged stirrups as S_v =10.2 cm = 102 mm.

Also

$$\left(\frac{A_{sv}(0.87 f_y)}{S_v}\right) = (\tau_{ve}-\tau_c)\,b = (2-0.59)350 = 493.5\ \text{N/mm} = 4.93\ \text{kN/cm}$$

Refer Table-62 (SP: 16) and read out spacing of 10mm diameter two legged stirrups as

S_v = 11.5 cm =115 mm. Provide the smaller of the two spacing, S_v = 100mm.

3) A reinforced concrete beam of rectangular section with a width of 350 mm and overall depth of 800 mm is subjected to a factored bending moment of 215 kNm, ultimate torsional moment of 105 kNm and ultimate shear forcec of 150 kN. Using M-20 grade concrete and Fe-415 HYSD bars and side, top and bottom covers of 50 mm, design suitable reinforcement in the section.

a) Data

b = 350 mm	b_1 = 250 mm
d = 750 mm	d_1 = 700 mm
D = 800 mm	f_{ck} = 20 N/mm^2
M_u = 215 kN.m	f_y = 415 N/mm^2
T_u = 105 kN.m	V_u = 150 kN

b) Equivalent bending moment and shear force

$$M_e = M_u + M_t$$

$$M_e = M_u + T_u\left[\frac{1+D/b}{1.7}\right] = 215 + 105\left[\frac{1+800/350}{1.7}\right] = (215+203) = 418 \text{ kN.m}$$

$$V_e = V_u + 1.6\left(\frac{T_u}{b}\right) = 150 + 1.6\left(\frac{105}{0.35}\right) = 630 \text{ kN}$$

Longitudinal reinforcements are designed for the equivalent bending moment M_e.

c) Longitudinal reinforcements

Since $M_t > M_u$, design reinforcements for M_e only.

$$\begin{aligned} M_{u,lim} &= 0.138 f_{ck}\, b.d^2 \\ &= (0.138 \times 20 \times 350 \times 750^2) \\ &= (543.3 \times 10^6) \text{ N.mm} \\ &= 543.3 \text{ kN.m} > M_e \end{aligned}$$

Hence, section is under reinforced

$$M_e = 0.87 f_y A_{st}.d\left[1 - \left(\frac{A_{st} f_y}{b d f_{ck}}\right)\right]$$

$$(418 \times 10^6) = 0.87 \times 415 \times A_{st} \times 750\left[1 - \left(\frac{415 A_{st}}{350 \times 750 \times 20}\right)\right]$$

Solving $A_{st} = 1802 \text{ mm}^2$

Provide 4 bars of 25 mm diameter ($A_{st} = 1964 \text{mm}^2$)

d) Transverse Reinforcements

$$\tau_{ve} = \left(\frac{V_e}{bd}\right) = \left(\frac{630 \times 10^3}{350 \times 750}\right) = 2.4 \text{ N/mm}^2$$

$$p_t = \left(\frac{100 A_{st}}{bd}\right) = \left(\frac{100 \times 1964}{350 \times 750}\right) = 0.75$$

Refer Table–19 (IS: 456-2000) and read out τ_c for $f_{ck} = 20 \text{ N/mm}^2$ as

$$\tau_c = 0.56 \text{ Nmm}^2 < \tau_{ve}$$

and

$$\tau_{ve} < \tau_{c,max} = 2.8 \text{ N/mm}^2$$

Hence, transverse reinforcements are required.

Using 10mm diameter 2 legged stirrups spacing is computed as

$$A_{sv} = \left[\left(\frac{T_u S_v}{b_1 d_1 0.87 f_y}\right) + \left(\frac{V_u S_v}{2.5 d_1 0.87 f_y}\right)\right]$$

$$(2 \times 79) = \left[\left(\frac{105 \times 10^6}{250}\right) + \left(\frac{150 \times 10^3}{2.5}\right)\right]\frac{S_v}{0.87 \times 415 \times 700}$$

Solving $S_v = 83.2$ mm

Also the spacing should conform to the equation,

$$S_v \ngtr \left(\frac{A_{sv} 0.87 f_y}{(\tau_{ve} - \tau_c) b}\right) \ngtr \left(\frac{158 \times 0.87 \times 415}{(2.4 - 0.56)350}\right) \ngtr 88.5 \text{ mm}$$

Hence, provide 10mm diameter 2 legged stirrups at a spacing of 80mm centers.

Method–2 (Using SP: 16 Design Charts)

a) Longitudinal Reinforcements

$$\begin{aligned} M_e &= M_u = 418 \text{ kN.m} & b &= 350 \\ f_{ck} &= 20 \text{ N/mm}^2 & d &= 750 \text{ mm} \\ f_y &= 415 \text{ N/mm}^2 & b_1 &= 250 \text{ mm} \\ & & d_1 &= 700 \text{ mm} \end{aligned}$$

Compute the parameter

$$\left(\frac{M_u}{bd^2}\right) = \left(\frac{418 \times 10^6}{350 \times 750^2}\right) = 2.12$$

Refer Table–2 (SP: 16) and read out the percentage reinforcement p_t for

$f_y = 415 \text{ N/mm}^2$, as $p_t = 0.685$

$$\therefore \quad A_{st} = \left(\frac{0.685 \times 350 \times 750}{100}\right) = 1798 \text{ mm}^2$$

b) Transverse reinforcements

Compute the parameter given by

$$\left(\frac{A_{sv}(0.87 f_y)}{S_v}\right) = \left[\left(\frac{T_u}{b_1 d_1}\right) + \left(\frac{V_u}{2.5 d_1}\right)\right] = \left[\left(\frac{105 \times 10^6}{250 \times 700}\right) + \left(\frac{150 \times 10^3}{2.5 \times 700}\right)\right] = 685.7 \text{ N/mm}$$

Refer Table-62 (SP: 16) and using 10mm diameter two legged stirrups read out spacing $S_v = 8.8\text{cm} = 88\text{mm}$.

Also

$$\left(\frac{A_{sv}(0.87 f_y)}{S_v}\right) = (\tau_{ve} - \tau_c)b = (2.4 - 0.56)350 = 644 \text{N/mm} = 6.44 \text{kN/cm}$$

Refer Table-62 (SP: 16) and using 10mm diameter two legged stirrups, read out the spacing as

$$S_v = 8.5\text{cm} = 85 \text{ mm}$$

Adopt smaller of the two spacings , ($S_v = 85$ mm).

6.7 BOND and ANCHORAGE IN REINFORCED CONCRETE MEMBERS

6.7.1 Introduction

The composite action of a reinforced concrete member is mainly due to the bond or the adhesion between the reinforcing steel and the surrounding concrete. Bond between concrete and steel facilitates the transfer of axial force form a reinforcing bar to the surrounding concrete and bond ensures strain compatibility and composite action of the composite material. The assumption of plane sections remains plane even after bending in the simple bending theory is valid only when there is effective bond between concrete and steel. The stress in a reinforcing bar can vary form point to point along its length mainly due to the bond resistance. If there is no bond, the stress in the bar will be constant along its length as in the case of a straight cable used in a prestressed concrete member.

6.7.2 Bond Mechanism

Bond between concrete and steel develops due to the following three mechanisms: -

a) Chemical adhesion is the grip developed due to the gum like property of the hydration products of cement in concrete.

b) Frictional Resistance developed due to the relative movement between concrete and steel bars depending upon the surface characteristics of the bar and the grip developed due to shrinkage of concrete.

c) Shearing resistance or dilatancy due to mechanical interlock developed as a consequence of surface protrusions or ribs provided in deformed bars.

Plain bars cannot develop the bond resistance due to mechanical interlock and the development and widespread use of deformed bars is attributed to their superior bond resistance. The present trend is to prefer deformed bars for main reinforcements and plain bars for ties and stirrups.

6.7.3 Bond Stresses

The tangential or shear stress developed along the contact surface of the reinforcing bar and the surrounding concrete if generally termed as '*Bond stress*' and is expressed in terms of the tangential force per unit nominal surface area of the reinforcing bar.

Depending on the type of Load situations the following types of bond stresses develop in structural concrete members.

1) Flexural Bond stress

Flexural bond stresses develop in a member under flexure due to variation of bending moment or shear at a section. Referring to Fig. 6.22, the differential moment dM from section X to Y causes the additional tension dT expressed as

$$dT = (dM / jd)$$

Where jd = lever arm

This unbalanced bar force is transferred to the surrounding concrete by means of flexural bond developed along the interface.

If τ_b = flexural bond stress, then the equilibrium of forces yields the relation

$$\tau_b(\Sigma O)dx = dT$$

Where ΣO = total perimeter of the bars

$$\therefore \quad \tau_b = \left(\frac{(dm/dx)}{(\Sigma O).jd}\right)$$

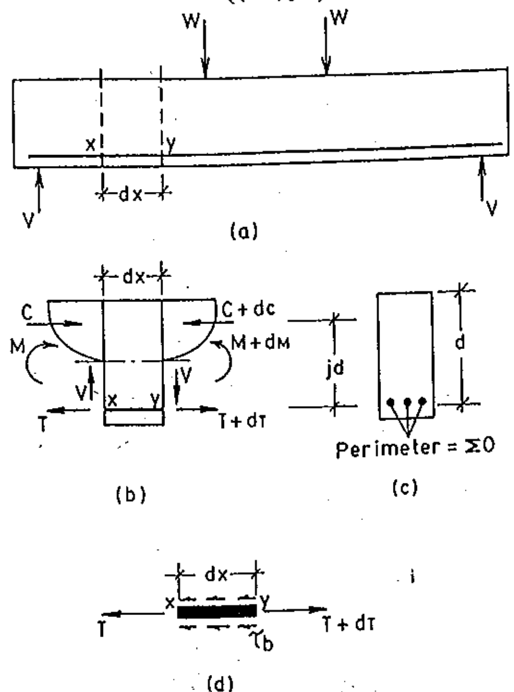


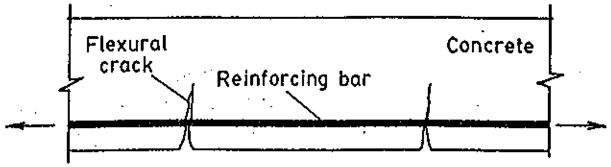
Fig. 6.22 Flexural Bond Stress

But $V = (dm / dx)$, Hence we have

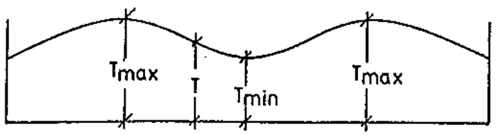
$$\tau_b = \left(\frac{V}{(\Sigma O).jd}\right) \qquad \ldots(6.47)$$

The flexural bond stress is higher at locations of high shear force but it can be reduced by providing an increased number of bars of smaller diameter yielding the same equivalent area of reinforcement.

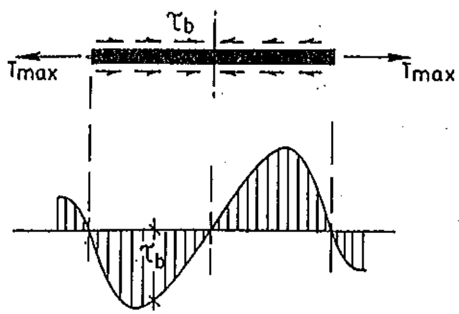
The actual bond stress is influenced by flexural cracking, local slip and other secondary effects which are not taken into account in Eq.[6.47]



(a) Constant Moment Region Between Flexural Cracks



(b) Variation of Tension in Reinforcing Bar



(c) Variation of Flexural (Local) Bond Stress

Fig. 6.23 Variation of Flexural Bond Stress Between Cracks

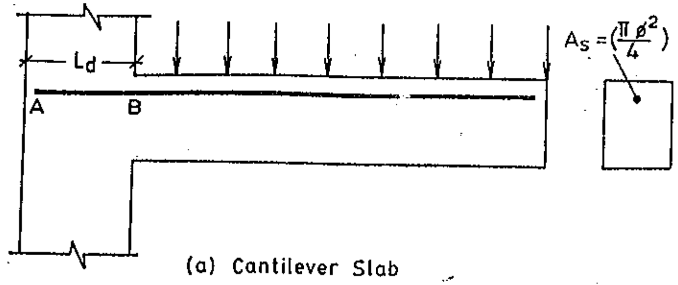
The effect of flexural cracks on flexural bond stresses in the constant moment region is shown in Fig. 6.23(a).The variation of tension in the reinforcing bar is shown in Fig. 6.23(b) and the bond stress variation in Fig. 6.23(c) Experimental investigations have shown that splitting cracks develop in the vicinity of the flexural cracks where the local bond stresses

are high. Hence, it is preferable to limit the magnitude of local bond stress by using a larger number of smaller diameter bars than using a few large diameter bars. However, with the development and wider use of deformed or high bond bars, more emphasis is laid on anchorage or development length requirements than the local bond stress. Hence, the Indian standard code IS: 456-2000 does not specify any permissible values for the flexural or local bond stress.

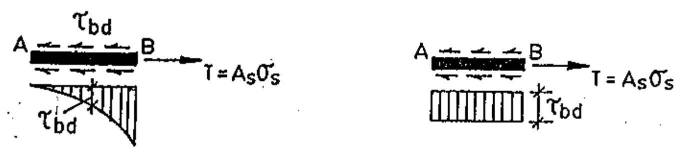
2) Anchorage (Development) bond stress

The anchorage bond stress is the stress developed in the vicinity of the extreme end (or cut off point) of bars in tension or compression.

Fig. 6.24(a) shows a cantilever slab in which the tensile stress in the bar varies from a maximum (σ_s) at the continuous end B to a value of zero at the end A. Since the moment is maximum at B, the tensile stress is also maximum at B and to develop the maximum stress (σ_s the bar requires a certain length (AB), which is termed as *anchorage*, or *Development Length.* Also, the stress is zero at the discontinuous end A.



(a) Cantilever Slab



(b) Variation of Bond Stress (c) Average Bond Stress

Fig. 6.24 Anchorage Bond Stress

The variation of bond stress from A to B is shown in Fig. 6.24 (b). A similar situation exists in the bars terminated at the supports of a simply supported beam. For design purposes an average bond stress assuming uniform distribution over the length AB [Fig. 6.24(c)] is specified in the Indian Standard Code IS: 456-2000. The average bond stress τ_{bd} can be expressed in terms of the diameter of the bar ϕ, the stress in steel σ_s and the anchorage length L_d, by considering the equilibrium of forces shown in Fig. 6.24(c),

$$(\pi\phi.L_d)\tau_{bd} = (\pi\phi^2/4)\sigma_s$$

$$\therefore \quad \tau_{bd} = \left(\frac{\phi\sigma_s}{4L_d}\right) \quad \ldots(6.48)$$

The development length 'L_d' required to develop the design stress σ_s in the bar is expressed as,

$$L_d = \left(\frac{\phi\sigma_s}{4\tau_{bd}}\right) \quad \ldots(6.49)$$

The values of τ_{bd} depend upon the grade of concrete and in the limit state method for plain bars, the values of design bond stress specified in clause 26.2.1.1 of IS: 456-2000 are compiled in Table 6.19.

Table 6.19 Design Bond stress in Limit state method for Plain bars in Tension

Grade of Concrete	M-15	M-20	M-25	M-30	M-35	M-40
Design Bond stress τ_{bd} (N/mm^2)	1.0	1.2	1.4	1.5	1.7	1.9

a) For deformed bars conforming to IS: 1786, these values have to be increased by 60 percent.

b) For bars in compression, the value of bond stress in tension is increased by 25 percent.

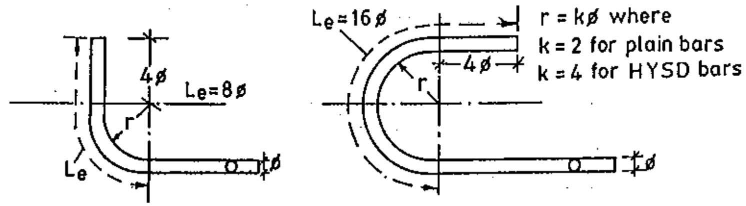
6.7.4 Code Requirements for Bond

The design for safety aginst bond failures requires the consideration of both flexural or local bond stress and the Anchorage or development bond stress. Due to non uniform distribution of actual bond stress and several factors influencing bond strength and despite checks provided by the computations, localized bond failures which occur do not significantly affect the ultimate strength of the member provided the reinforcement are properly anchored at their ends. In addition the wide spread use of deformed bars in place of plain bars, the design emphasis is centered around the anchorage or development bond stress rather than the flexural or local bond stress.

Hence, the Indian Standard Code IS: 456-2000 prescribes a check on

anchorage in terms of development length given by Eq.6.49.

The code prescribes that deformed bars may be used without end anchorage provided, the development Length requirements is satisfied. Hooks are normally provided for plain bars in tension. Bends and hooks shown in Fig. 6.25 should conform to the specifications of IS: 2502-1963[64] and SP: 34(1987).



(a) Standard 90° Bend (b) Standard U-Type Hook

Fig. 6.25 Anchorage Lengths of Standard Bends and Hooks (SP: 34)

6.7.5 Reinforcement Splicing

Splicing of reinforcement is required when the bars are to be extended beyond their available length as in the case of column bars in multistoried buildings. It is recommended that splices in flexural members should be kept away from sections with high bending stresses and shear stresses. Also the splices should be staggered in the individual bars of a group. The I.S. Code recommends that "Splices in flexural members should not be at sections where the bending moment is more than 50 percent of the moment of resistance, and not more than half the bars shall be spliced at a section". When splicing in such situations becomes unavoidable, special precautions such as a) increasing lap length and b) using spirals or closely spaced stirrups around the length of splice, should be adopted.

The various types of splicing of reinforcement are:

1) Lapping of bars (Lap splice)
2) Stirrups at splice locations.
3) Staggered splicing.
4) Mechanical connections.
5) Butt welding of bars.
6) Lap welding of bars.

The different types are shown in Fig. 6.26(a to f)

In the case of beams, particular care has to be taken in providing sufficient anchorage or development length near supports to limit the magnitude of bond stress. If the bond stresses are excessive, horizontal cracks at the

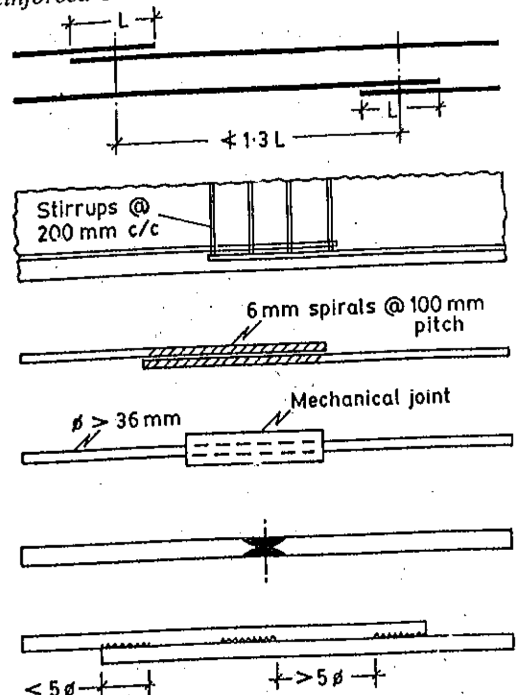


Fig. 6.26 Splicing of Reinforcement Bars
(a) Splicing of Bars, (b) Stirrups at Splice Points,
(c) Splicing Using Spirals, ø > 36 mm, (d) Mechanical Joint,
(e) Butt Welding, (f) Lap Welding of Bars.

level of reinforcement are formed at such locations. Typical horizontal cracks developed due to the failure of bond between concrete and steel bars in beams are shown in Figs. 6.27 and 6.28.

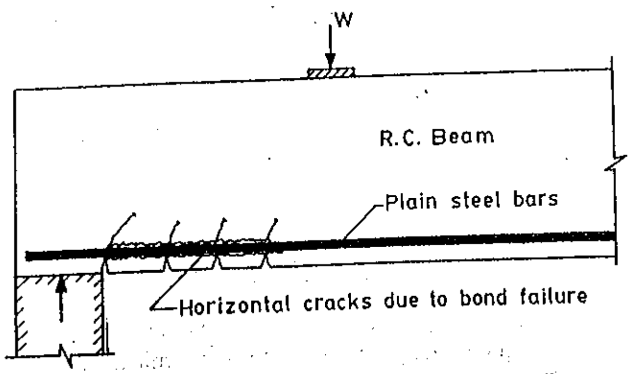


Fig. 6.27 Typical Horizontal Cracks due to Bond Failure

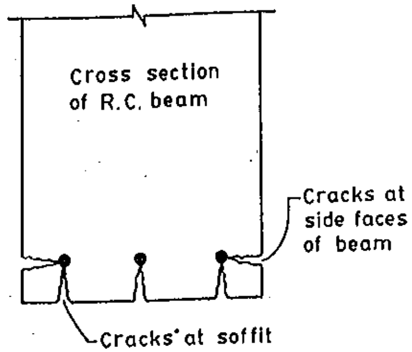


Fig. 6.28 Typical Crack Pattern due to Failure of Bond Between Concrete and Steel

6.7.6 Use of SP: 16 for checking Development Length

Development Length can be easily checked by using the Tables of SP: 16. The Tables 64, 65 and 66 of SP: 16 (Tables 6.20, 6.21, 6.22, of text) cover plain and deformed bars and different grades of concrete from M-15 to M-30. For a given bar diameter varying from 6 mm to 36mm the development length required for bars fully stressed to design strength of $0.87 f_y$ in tension or compression can be directly read out from the appropriate tables.

In general

$$L_d\,(Compression) = \left(\frac{L_d(Tension)}{1.25}\right)$$

For any other design stress level less than $0.87 f_y$, the development length L'_d required is computed using the relation.

$$L'_d = \left(\frac{\sigma_s}{0.87 f_y}\right) L_d \qquad \ldots(6.50)$$

Where σ_s = design stress in the bars.

Table 67 of SP-16 (Table 6.23 of text) gives the anchorage value of hooks and bends for tension reinforcement. The effect of hooks and bends, if provided can also be considered as development length. However, in bars under compression, only the projected length of hooks or bends are considered as effective towards development length.

Table 6.20 Devlopment Length For Fully Stressed Plain Bars

f_y = 250 N/mm² for bars up to 20 mm diameter.
= 240 N/mm² for bars over 20 mm diameter.
(Table–64 of SP:16)
Tabulated values are in centimetres.

Bar Diameter mm	Tension Bars Grade of Concrete				Compression Bars Grade of Concrete			
	M15	M20	M25	M30	M15	M20	M25	M20
6	32.6	27.2	23.3	21.8	26.1	21.8	18.6	17.4
8	43.5	36.3	31.1	29.0	34.8	29.0	24.9	23.2
10	54.4	45.3	38.8	36.3	43.5	36.3	31.1	29.0
12	65.3	54.4	46.6	43.5	52.2	43.5	37.3	34.8
16	87.0	72.5	62.1	58.0	69.6	58.0	49.7	46.4
18	97.9	81.6	69.9	65.3	78.3	65.3	55.9	52.2
20	108.8	90.6	77.7	72.5	87.0	72.5	62.1	58.0
22	114.8	95.7	82.0	76.6	91.9	76.6	65.6	61.2
25	130.5	108.8	93.2	87.0	104.4	87.0	74.6	69.6
28	146.2	121.8	104.4	97.4	116.9	97.4	83.5	78.0
32	167.0	139.2	119.3	111.4	133.6	111.4	95.5	89.1
36	187.9	156.6	134.2	125.3	150.3	125.3	107.4	100.2

Note–The develpment lengths given above are for a stress of $0.87 f_y$ in the bar.

Table 6.21 Development Length For Fully Stressed Plain Bars

(Table–65 of SP:16)

f_y = 415 N/mm²

Tabulated values are in centimetres.

Bar Diameter mm	Tension Bars Grade of Concrete				Compression Bars Grade of Concrete			
	M15	M20	M25	M30	M15	M20	M25	M20
6	33.8	28.2	24.2	22.6	27.1	22.6	19.3	18.1
8	45.1	37.6	32.2	30.1	36.1	30.1	25.8	24.1
10	56.4	47.0	40.3	37.6	45.1	37.6	32.2	30.1
12	67.7	56.4	48.4	45.1	54.2	45.1	38.7	36.1
16	90.0	75.2	64.5	60.2	72.2	60.2	51.6	48.1
18	101.5	84.6	72.5	67.7	81.2	67.7	58.0	54.2
20	112.8	94.0	80.6	75.2	90.3	75.2	64.5	60.2
22	124.1	103.4	88.7	82.7	99.3	82.7	70.9	66.2
25	141.0	117.5	100.7	94.0	112.8	94.0	80.6	75.2
28	158.2	131.6	112.8	105.3	126.4	105.3	90.3	84.2
32	180.5	150.4	128.9	120.3	144.4	120.3	103.2	96.3
36	203.1	169.3	145.0	135.4	162.5	135.4	116.1	108.3

Note–The develpment lengths given above are for a stress of $0.87 f_y$ in the bar.

Table 6.22 Development Length For Fully Stressed Deformed Bars

(Table–66 of SP:16)

$f_y = 500$ N/mm^2

Tabulated values are in centimetres.

Bar Diameter mm	Tension Bars Grade of Concrete				Compression Bars Grade of Concrete			
	M15	M20	M25	M30	M15	M20	M25	M20
6	40.8	34.0	29.1	27.2	32.6	27.2	23.3	21.8
8	54.4	45.3	38.8	36.3	43.5	36.3	31.1	29.0
10	68.0	56.6	48.5	45.3	54.4	45.3	38.8	36.3
12	81.6	68.0	58.3	54.4	65.3	54.4	46.6	43.5
16	108.8	90.6	77.7	72.5	87.0	72.5	62.1	58.0
18	122.3	102.0	87.4	81.6	97.9	81.6	69.9	65.3
20	135.9	113.3	97.1	90.6	108.8	90.6	77.7	72.5
22	149.5	124.6	106.8	99.7	119.6	99.7	85.4	79.8
25	169.9	141.6	121.4	113.3	135.9	113.3	97.1	90.6
28	190.3	158.6	135.9	126.9	152.3	126.9	108.8	101.5
32	217.5	181.3	155.4	145.0	174.0	145.0	124.3	116.0
36	244.7	203.9	174.8	163.1	195.8	163.1	139.8	130.5

Note–The develpment lengths given above are for a stress of $0.87\,f_y$ in the bar.

6.7.7 Analysis Examples

1) A cantilever beam having a width of 200mm and effective depth 300mm supports a uniformly distributed load and is reinforced with four bars of 16mm diameter. If the factored total load is 80 kN, calculate

a) The maximum local bond stress.
b) The anchorage length required.
c) If the anchorage length provided is 900 mm, the average bond stress.

Assume M-20 Grade concrete and Fe-415 HYSD bars.

Method-1 (Using IS: 456-2000 Code Formulae)

a) Data

$b = 200$ mm $f_{ck} = 20$ N/mm^2
$d = 300$ mm $f_y = 415$ N/mm^2
$A_{st} = 4$ bars of 16mm diameter. $\tau_{bd} = (1.6 \times 1.2) = 1.92$ N/mm^2
$\Sigma O = 4\,(\pi \times 16) = 201$ mm $L_d = 900$ mm

b) Maximum Local Bond Stress

$$\tau_b = \left[\frac{V}{\Sigma O.d}\right] = \left[\frac{80 \times 10^3}{201 \times 300}\right] = 1.32 \text{ N/mm}^2$$

c) Anchorage Length

$$L_d = \left(\frac{\phi\sigma_s}{4\tau_{bd}}\right) = \left(\frac{16 \times 0.87 \times 415}{4 \times 1.92}\right) = 752 \text{ mm}$$

d) Average Bond Stress

$$\tau_{bd} = \left(\frac{\phi\sigma_s}{4L_{bd}}\right) = \left(\frac{16 \times 0.87 \times 415}{4 \times 900}\right) = 1.6 \text{ N/mm}^2$$

Method–2 (using SP: 16 Design tables)

Refer Table 65 of SP: 16 (Table 6.21 of text) and read out L_d for 16mm bars (M-20 concrete) as $L_d = 752$mm.

2) A reinforced concrete beam of 6m span is uniformly loaded and is

reinforced with 5 bars of 20mm diameter on the tension side at an effective depth of 400mm. Find the distance from the center of the beam where one of the bars can be curtailed. Adopt M-20 grade concrete and Fe-415 HYSD bars.

a) Data

$L = 6$ m $f_{ck} = 20$ N/mm^2
$d = 400$ mm $f_y = 415$ N/mm^2
$\phi = 20$ mm $\tau_{bd} = (1.6 \times 1.2) = 1.92$ N/mm^2

b) Theoretical cut of point from Bending Moment considerations

Let x = Distance of cut of point measured from center of span.

Then $$\frac{(wL^2/8)-(wx^2/2)}{(wL^2/8)} = \left(\frac{4}{5}\right)$$

$\therefore$ Solving $x = 1.34$ m $= 1340$ mm

c) Development Length for Maximum Tension at Centre in Fe-415 grade steel

$$L_d = \left(\frac{\phi\sigma_s}{4\tau_{bd}}\right) = \left(\frac{20(0.87 \times 415)}{4 \times 1.92}\right) = 940 \text{ mm}$$

d) Physical cut of point (PCP)

The theoretical cut off point (TCP) is larger of (b) or (c).

$$\begin{aligned} PCP &= T_{CP} + d \text{ or } 12\phi \\ &= 1340 + 400 \text{ or } (12 \times 20) \text{ which ever is higher} \\ &= 1740 \text{ mm.} \end{aligned}$$

Hence, one bar can be curtailed at 1.74 m from center from span.

3) A reinforced concrete column of a multistoreyed building is reinforced with 36mm diameter longitudinal bars and with ties at regular intervals. Assuming M-25 Grade concrete and Fe-415 HYSD bars calculate (i) the lap length required and (ii) specify the method of reducing the lap length to reduce the quantity of steel.

a) Data

Diameter of bars $\phi = 36$ mm.
$f_{ck} = 25$ N/mm^2
$f_y = 415$ N/mm^2

b) Computation of Lap Length (L)

L = Development length of bars (only projected length of hooks and bends is considered)

Average bond stress τ_{bd} in compression (Table 6.19 of text)

$$\begin{aligned} &= (\tau_{bd} \text{ in Tension}) \times 1.25 \\ &= (1.6 \times 1.4) \times 1.25 = 2.8 \text{ N/mm}^2 \end{aligned}$$

$$L_d = \left(\frac{\phi\sigma_s}{4\tau_{bd}}\right) = \left(\frac{\phi \times 0.87 \times 415}{4 \times 2.8}\right) = 32.2\,\phi = (32.2)36 = 1160 \text{ mm.}$$

From Table 65 (SP: 16) Read out $L_d = 1161$ mm.

c) Method of Reducing Lap Length

Shorter lap length can be used with welding. Using a lap length of 15ϕ together with lap welding at 5ϕ intervals, the welds are designed to resist the equivalent force (F) for a lap of $[32.2 - 15]\,\phi = 17.2\phi$

$$F = (0.87 \times 415)\left(\frac{\pi \times 36^2}{4}\right)\left(\frac{17.2}{32.2}\right) \text{N} = 196306 \text{ N} = 196.306 \text{ kN}$$

4) A simply supported beam of 8 m span is reinforced with 6 bars of 25mm diameter at center of span and 50 percent of the bars are continued into the supports. Check the development length at supports assuming M-20Grade concrete and Fe-415 HYSD bars. The beam supports a characteristics total load of 50 kN/m.

a) Data

$L = 8$ m. $f_{ck} = 20$ N/mm^2
$w = 50$ kN/m $f_y = 415$ N/mm^2
No. of bars at center of span = 6. $\tau_{bd} = (1.6 \times 1.2) = 1.92$ N/mm^2
No. of bars at supports = 3
$\phi = 25$mm

b) Bending Moment and Shear Force

Design Load $= w_u = (1.5 \times 50) = 75$ kN/m.
$M_{max} = (0.125\, w_u.L^2) = (0.125 \times 75 \times 82) = 600$ kN.m
$V_{max} = (0.5\, w_u.L) = (0.5 \times 75 \times 8) = 300$ kN.

c) Moment of Resistance of bars continued into supports

$$M_1 = \frac{3}{6}(600) = 300 \text{ kN.m.}$$

d) Development Length of 25 mm diameter bars

Using M-20 Concrete and Fe-415 grade HYSD bars.

$$L_d = \left(\frac{\phi\sigma_s}{4\tau_{bd}}\right) = \left(\frac{25\times0.87\times415}{4\times1.92}\right) = 1175\text{mm}$$

From Table-65 (SP: 16), read out $L_d = 1175$mm.

e) Check for Development Length at Supports

According to clause 26.2.3.3. (IS: 456) assuming 30% increase in development length computed as (M_1/V) we have

$$\left(\frac{1.3M_1}{V}\right) = \left(\frac{1.3\times300\times10^6}{300\times10^3}\right) = 1300 \text{ mm}$$

Condition to be satisfied is given by

$$\left[L_O + \left(\frac{1.3M_1}{V}\right)\right] > L_d \text{ where } L_O = \text{anchorage beyond support line.}$$

$$[L_O + 1300] > 1175$$

Hence, Development length is satisfied without any anchorage value.

6.7.8 Design Examples

1) A reinforced concrete cantilever beam of rectangular section 300mm wide by 600mm deep is built into a column 500mm wide as shown in Fig. 6.29.The cantilever beam is subjected to a hogging moment of 200 kNm at the function of beam and column. Design suitable reinforcements in the beam and check for the required anchorage length. Adopt M-20 grade concrete and Fe-415 HYSD bars.

Method–1 (Using IS: 456 code formulae)

a) Data

$b = 300$ $\quad f_{ck} = 20 \text{ N/mm}^2$

$d = 550$ $\quad f_y = 415 \text{ N/mm}^2$

$M_u = 200$ kN.m

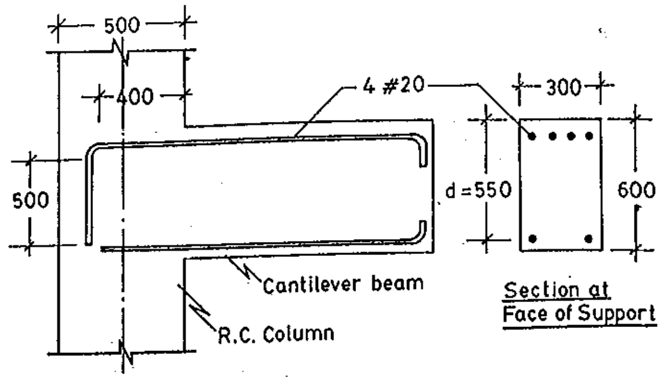


Fig. 6.29 Cantilever Beam

b) Limiting moment of resistance

$$M_{u,lim} = 0.138 f_{ck}.b.d^2 = (0.138\times20\times300\times5502) = (250\times10^6) \text{ N.mm} = 250 \text{ kN.m}$$

Since $M_u < M_{u,lim}$, section under reinforced

c) Reinforcements

$$M_u = (0.87 f_y A_{st}.d)\left[1 - \left(\frac{A_{st}.f_y}{bd.f_{ck}}\right)\right]$$

$$(200\times10^6) = (0.87\times415\times A_{st}\times550)\left[1 - \left(\frac{415A_{st}}{300\times550\times20}\right)\right]$$

Solving $A_{st} = 1181 \text{ mm}^2$

$\therefore$ (Provide 4 bars of 20mm diameter ($A_{st} = 1256 \text{ mm}^2$)

d) Anchorage Length

$$L_d = \left[\frac{0.87f_y}{4\tau_{bd}}\right]\phi$$

Using 20 mm diameter bars, $\phi = 20$ mm

For bars in tension, $\tau_{bd} = (1.6\times1.2) = 1.92 \text{ N/mm}^2$ and M-20 Grade Concrete

$$\therefore \quad L_d = \left[\frac{0.87\times415}{4\times1.92}\right]\phi = 47\phi = (47\times20) = 940\text{mm}$$

The bars are extended into the column to a length of 400 mm with a 90° bend and 500mm length as shown in Fig. 6.29.

Anchorage Length provided = [400 + (8 × 20) + 500] = 1060 mm > 940 mm

Method–2 (Using SP: 16 Design Tables)

Refer Table-2 (SP: 16) and read out the percentage steel corresponding to the ratios,

$$\left(\frac{M_u}{bd^2}\right) = \left(\frac{200 \times 10^6}{300 \times 550^2}\right) = 2.2$$

Hence $$p_t = \left(\frac{100 A_{st}}{bd}\right) = 0.717$$

$$\therefore \quad A_{st} = \left[\frac{0.717 \times 300 \times 550}{100}\right] = 1183 \text{mm}^2 (\text{Adopt 4 bars of } 20\phi)$$

From Table-65 (SP: 16) for f_y = 415 N/mm² and 20mm diameter, the required anchorage length is L_d = 94 cm = 940 mm

Hence, the bars are built into the column and bent at 90° for the required total anchorage length.

2) A reinforced concrete column subjected to compression combined with flexure is shown in Fig. 6.30. It is required to reduce the longitudinal reinforcement diameter from 25mm in the ground floor to 20mm in the first floor. Design a suitable lap slice. Assume M-20 grade concrete and Fe-415 HYSD bars.

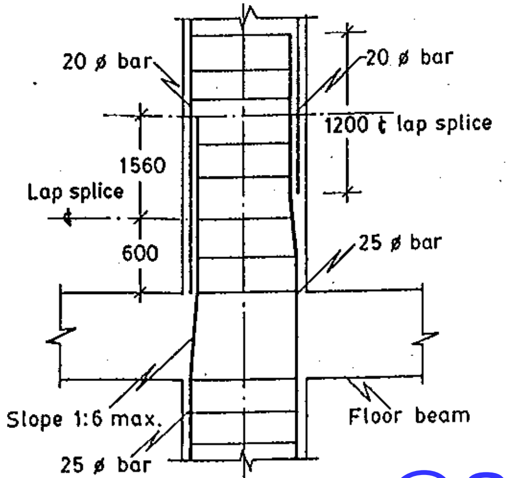


Fig. 6.30 Typical Lap Splice

a) Data

Diameter of bars: 25 ϕ at G.F f_{ck} = 20 N/mm²
20 ϕ at F.F f_y = 415 N/mm²
$\tau_{bd} = (1.2 \times 1.25) = 1.5$ N/mm²

b) Lap Length

At location of splice, smaller diameter bars (20mm diameter) are adequate in providing the desired strength. The lap length is based on smaller diameter bars.

$$L = L_d \quad \text{or} \quad 30\,\phi \quad \text{whichever is greater.}$$

$$L_d = \left[\frac{0.87 f_y}{4\tau_{bd}}\right]\phi = \left[\frac{0.87 \times 415}{4 \times 1.5}\right]\phi = 60\,\phi$$

$$\therefore \quad L = L_d = (60 \times 20) = 1200\text{mm}$$

c) Staggered Splicing

According to IS: 456 Code (Clause 26.2.5.1) the splicing of bars should be ideally staggered with minimum centre-to-centre spacing of

$$1.3L = (1.3 \times 1200) = 1560\text{mm}$$

The lap length and staggering of spacing is shown in detail in Fig. 6.30

3) A doubly reinforced beam of width 300 mm and overall depth 500mm is built into a column having a width of 600 mm as shown in Fig. 6.31. The section of the beam at supports is reinforced with 3 bars of 16mm diameter to resist the hoging moment and 2 bars of 12 mm diameter on the compression side. Using f_{ck}= 20 N/mm². and f_y = 415 N/mm², design and detail the anchorage length required at the junction of column and beam.

a) Data

b = 300 mm	f_{ck} = 20 N/mm²
d = 500 mm	f_y = 415 N/mm²
A_{st} = 3 bars of 16 ϕ	τ_{bd} = 1.2 N/mm²
A_{sc} = 2 bars of 12 ϕ	

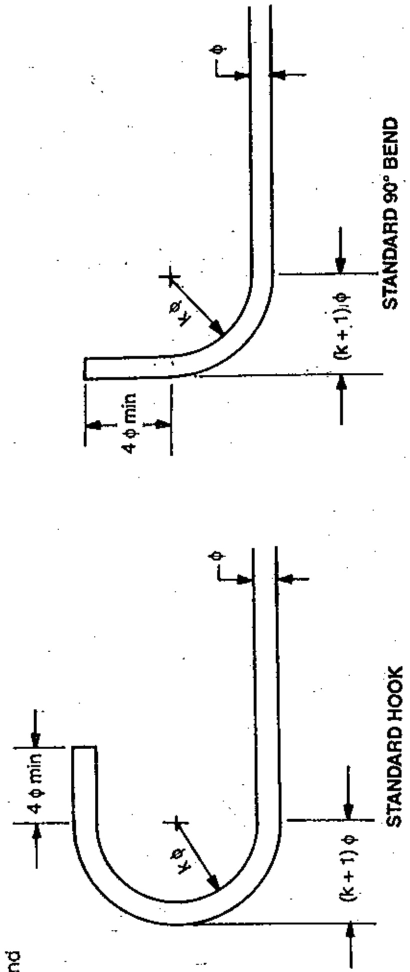
b) Anchorage Length for Tension bars (top)

$$L_d = \left(\frac{0.87 f_y}{4\tau_{bd}}\right)\phi = \left(\frac{0.87 \times 415}{4 \times 1.2 \times 1.6}\right)\phi = 47\phi = (47 \times 16) = 752\text{mm}$$

Table 6.23 Anchorage Value of Hooks and Bends

Tabulated values are in centimeters.
(Table–67 of SP:16)

Bar Diameter mm	6	8	10	12	16	18	20	22	25	28	32	36
Anchorage Value of hook	9.6	12.8	16.0	19.2	25.6	28.8	32.0	35.2	40.0	44.8	51.2	57.6
Anchorage Value of 90° bend	4.8	6.4	8.0	9.6	12.8	14.4	16.0	17.6	20.0	22.4	25.6	28.8



STANDARD HOOK AND BEND

Type of Steel	*Min. Value of k*
Mild steel	2
Cold worked steel	4

NOTE 1 — Table is applicable to all grades of reinforcement bars.
NOTE 2 — Hooks and bends shall conform to the details given above.

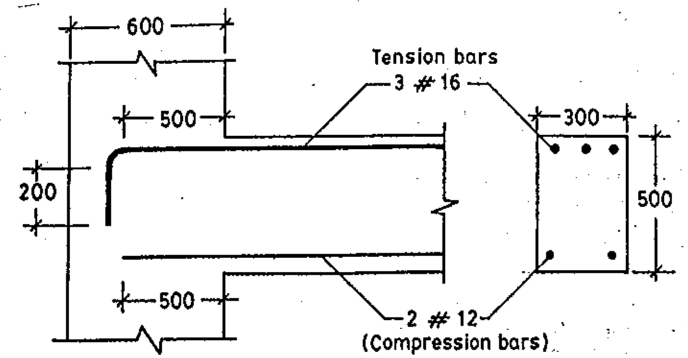


Fig. 6.31 Anchorage Details in Doubly Reinforced Beam

Anchorage Length Provided = [500 + (8 × 16) + 200] = 764 mm

c) Anchorage Length for Compression bars (Soffit)

τ_{bd} can be increased by 25%

$$\therefore \quad L_d = (47\phi)0.8 = 37.6\,\phi = (37.6 \times 12) = 451\text{mm}$$

Provide an Anchorage length L_d = 500 mm as shown in Fig. 6.31.

6.8 EXAMPLES FOR PRACTICE

1) A rectangular R.C.C beam has a width of 200 mm and is reinforced with 2 bars of 20 mm diameter at an effective depth of 400 mm. If M-20 grade concrete and Fe-415 HYSD bars are used, estimate the ultimate moment of resistance of the section.

2) A reinforced concrete beam of rectangular section 200 mm wide by 550 mm deep is reinforced with 4 bars of 25 mm diameter at an effective depth of 500 mm. Using M-20 grade concrete and Fe415 HYSD bars, calculate the safe moment of resistance on the section.

3) A reinforced concrete beam 300 mm wide is reinforces 1436 mm^2 of Fe-415 HYSD bars at an effective depth of 500 mm. If M-20 grade concrete is used, estimate the flexural strength of the section.

4) Determine the area of reinforcement required for a singly reinforced concrete section having a breadth of 300 mm and an effective depth of 600 mm to support a fractured moment of 200 kN.m

5) Determine the minimum effective depth required and the corresponding area of tension reinforcement for a rectangular beam having a width of 200 mm to resist an ultimate moment of 200 kN.m, Using M-20 Grade concrete and Fe-415 HYSD bars

6) Determine the moment of resistance of a tee-beam having the following section properties:
Effective width of flange = 2500 mm
Depth of flange = 150 mm
Width of rib = 300 mm
Effective depth = 800 mm
Area of steel: 6 bars of 25 mm diameter
Materials: M-20 grade Concrete
Fe-415 HYSD bars.

7) Calculate the ultimate flexural strength of the tee-beam section having the following section properties.
Width of flange =1200 mm
Depth of flange = 120 mm
Width of rib = 300 mm
Effective depth = 600 mm
Area of tension steel = 4000 mm^2
Materials: M-20 grade Concrete
Fe-415 HYSD bars.

8) A tee beam has an effective flange width of 2500 mm and depth of 150 mm. Width of rib = 300mm. Effective depth = 800mm.Using M-20 grade Concrete and Fe-415 HYSD bars, estimate the area of tension steel required if the section has to resist a factored moment of 1000 kN.m

9) A tee beam has the following cross sectional details: -
Effective width of flange = 2000 mm
Thickness of flange = 150 mm
Width of rib = 300 mm
Effective depth = 1000 mm

Calculate the limiting moment capacity of the section and the corresponding area of tension reinforcement. Adopt M-20 grade concrete and Fe-415 HYSD bars.

10) A doubly reinforced concrete beam having a rectangular section 250 mm wide and 540 mm overall depth is reinforced with 2 bars of 12 mm diameter on the compression face and 4 bars of 20 mm diameter on the tension side. The effective cover to the bars is 40 mm. Using M-20 grade concrete and Fe-415 HYSD bars, estimate the flexural strength of the section using IS: 456-2000 code specifications.

11) A doubly reinforced concrete section has a width of 300 mm and is reinforced with tension reinforcement of area 2455 mm^2 at an effective depth of 600 mm. Compression Steel of area 982 mm^2 is provided at an effective cover of 60 mm. Using M-20 Grade Concrete and Fe-415 HYSD bars, estimate the ultimate moment capacity of the section.

12) A reinforced concrete beam has a support section with a width of 250 mm and effective depth of 500 mm. The support section is reinforced with 3 bars of 20 mm diameter on the tension side.8mm diameter two legged stirrups are provided at a spacing of 200 mm centers. Using M-20 grade concrete and Fe-415 HYSD bars, estimate the shear strength of the support section.

13) A reinforced concrete beam of rectangular section with a width of 300 mm and an effective depth of 600 mm is reinforced with 4 bars of 25 mm diameter as tension reinforcement. Two of the tension bars are bent up at 45^0 near the support section. The beam is also provided with two-legged vertical links of 8 mm diameter at 150 mm centers near supports. Using M-25 grade concrete and Fe-415 HYSD bars, compute the ultimate shear strength of the support section. Using IS: 456-2000 code specifications.

14) A reinforced concrete beam of rectangular section having width of 300 mm and overall depth 600 mm is reinforced with 4 bars of 25 mm diameter, distributed at each of the corners at an effective cover of 50 mm in the depth direction and side covers of 25 mm in the width direction. 8 mm diameter 2 legged stirrups are provided at 100mm centers
Estimate the torsional strength of the section adopting Fe-415 HYSD bars for the following cases: -

a) Factored shear force is zero
b) Factored shear force is 100 kN

15) A reinforced concrete beam of rectangular section with a breadth of 300 mm and overall depth 850 mm is reinforced with 4 bars of 20 mm diameter on the tension side at an effective depth of 800 mm. The section is subjected to a factored bending moment 200 kNm. If fck = 415 N/mm^2, calculate the ultimate torsional resistance of the section.

16) A beam of rectangular section in a multistorey frame is 250 mm wide by 500 mm deep. The section is subjected to a factored bending

moment of 55 kN.m and torsional moment of 30 kN.m together with an ultimate shear force of 40 kN. Using M-20 grade concrete and Fe-415 HYSD bars design suitable reinforcements in the section assuming an effective cover of 50 mm in the depth and width directions and using IS: 456-2000 code specifications.

17) A cantilever beam having a width of 200 mm and overall depth 400 mm is reinforced with 3 bars of 20 mm diameter as tension steel and 3 bars of 12 mm diameter as compression reinforcement. The beam supports a total uniformly distributed factored load of 100 kN over a span of 2 m. Using M-20 grade concrete and Fe-415 HYSD bars, compute the anchorage length required for the reinforcements.

CHAPTER 7

Serviceability Requirements of Reinforced Concrete Members

7.1 INTRODUCTION

Reinforced concrete members should be designed to conform to the limit state of strength and serviceability. In addition to the limit state of strength outlined in Chapter-6, the members should also satisfy the serviceability conditions under the action of dead and live loads. The primary serviceability conditions are:-

1) The member should not undergo excessive deformation under service loads. This limit state is generally referred to as the *'limit state of deflection'*.
2) The width of cracks developed on the surface of reinforced concrete members under service loads should be limited to the values prescribed in the codes of practice. This limit state is referred to as *'limit state of cracking'*.

Depending upon the environmental conditions and type of structure, other limit states such as durability and vibration should also be considered. These limit states are also important for structures like bridges located in marine environment.

IS: 456-2000 code has specified the partial safety factors for load combinations under which the deflection and cracking are to be checked.

Table-18 of IS: 456-2000 code (Table-5.2 of text) outlines the combinations of loads for serviceability conditions. The largest value should be used for the computations. The load combinations are as follows:

1) 1.0 D.L + 1.0 L.L
2) 1.0 D.L + 1.0 W.L
3) 1.0 D.L + 0.8 L.L + 0.8 W.L (E.L)

Generally the codes specify the following two methods for control of deflection.

a) The empirical method in which the span/effective depth ratio of the structural members are limited to specified values in the codes.

b) The theoretical method in which the actual deflection is computed and checked with the codified permissible deflections.

For control of crack widths also, two methods are specified.

a) The empirical method which requires the detailing of reinforcements according to the codified provisions, such as minimum percentage of steel in the section, spacing of bars, curtailment and anchorage bars, lapping bars etc.

b) The theoretical method of computing the actual width of cracks and checking them with the codified requirements for the specified environmental conditions.

The widespread use of high grade steels like Fe-415 and Fe-500 with higher allowable stresses replacing the Fe-250 grade during the last few decades coupled with the use of high performance concrete generally results in slender structural elements necessitating greater attention to deflections and crack control in the modern methods of design of reinforced concrete structures.

7.2 CODIFIED DEFLECTION LIMITS

Deflections of flexural members like beams and slabs if excessive causes distress to users of the structure and also likely to cause cracking of partitions. As given IS: 456 code Clause 23.2, the accepted limits to permissible deflections are given as,

a) The final deflection including the effects of all loads, temperature, creep and shrinkage of horizontal structural members should not exceed the value of span/250.

b) The deflection including the effects of temperature, creep and shrinkage occurring after the erection of partitions and the application of finishes should not exceed span/350 or 20 mm whichever is less.

7.3 DEFLECTION CONTROL IN BEAMS AND SLABS (EMPIRICAL METHOD)

7.3.1 Theoretical Basis of Empirical Method

The empirical method is based on the principle of expressing deflection span ratio of beams in terms of span/depth ratio and assuming constant values of material properties. The deflection of a beam or slab supporting uniformly distributed load can be expressed as

$$\delta = \left(\frac{5}{384}\right)\left(\frac{WL^4}{EI}\right) = \left(\frac{5}{48}\right)\left(\frac{WL^2}{8}\right)\left(\frac{L^2}{EI}\right)$$

$$= \left(\frac{5}{43}\right)\left(\frac{M_{max}}{8}\right)\left(\frac{L^2}{E}\right)$$

Substituting $\left(\frac{M}{I}\right) = \left(\frac{\sigma}{y}\right)$ and $y = \left(\frac{d}{2}\right)$

We have $\left(\frac{\delta}{L}\right) = \left(\frac{5}{48}\right)\left(\frac{2\sigma}{d}\right)\left(\frac{L}{E}\right) = \left(\frac{5}{24}\right)\left(\frac{\sigma}{E}\right)\left(\frac{L}{d}\right)$

In this relation, it can be seen that the ratio of deflection/span is a function of span/depth ratio and the values of σ and E can be suitably assumed.

For example, assigning the values for $\sigma = 5$ N/mm², $E = 10$ kN/mm² and $(\sigma/L) = (1/350)$

The span/depth ratio is obtained as

$$\left(\frac{L}{d}\right) = \left[\frac{24(10 \times 10^3)}{350 \times 5 \times 5}\right] = 27$$

This relation indicates that assuming certain terms as constants, the permissible deflection/span ratio can be controlled by span/depth ratio. The IS: 456 code recommendations are based on this principle.

7.3.2 Basic Span/Depth ratios

The following factors are considered in the specifications of basic span/depth ratios recommended in IS: 456 codes.

a) The span/effective depth ratios
b) Percentage tension and compression reinforcement in the section
c) Type of beam (Rectangular or flanged)
d) Type of supports (simply supported, fixed or continuous)

Table-7.1 gives the basic span to effective depth ratios to be used for beams and slabs with spans up to 10 m. For spans greater than 10 m, the ratios have to be multiplied by a factor F = (10/span) in metres. A graphical representation of the basic span/depth ratios is shown in Fig. 7.1.

7.3.3 Modification Factors for Basic Span/Depth Ratios

Modification factors have to be applied to the basic span/depth ratio to account for the percentage of tension and compression reinforcements in the section and also the type of section such as rectangular or flanged. The final expression of the span/effective depth ratio can be expressed as

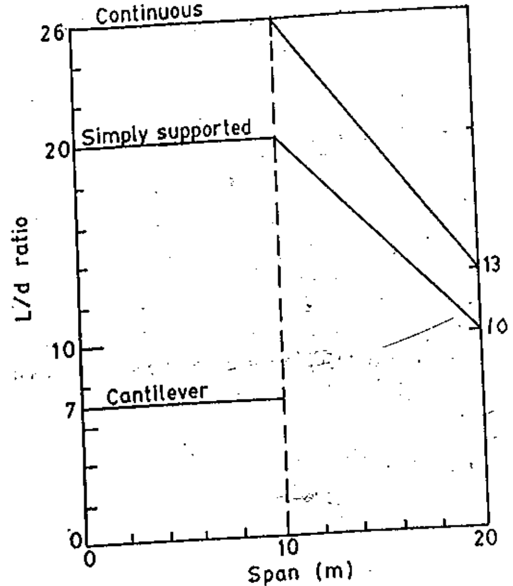


Fig. 7.1 Basic Span-Depth Ratios for Beams and Slabs

Table 7.1 Basic Span-Effective depth Ratios for Beams and Slabs.
(Clause 23.2.1 of IS: 456-2000)

Type of Support	Rectangular Sections	Flanged Sections
Cantilever	7	Multiply Values for rectangular sections by factor K_f (Refer Fig. 7.4)
Simply supported	20	
Continuous	26	

$$(L/d) = [(L/d)_{basic} \times K_t \times K_c \times K_f]$$

Where $(L/d)_{basic}$ is as given in Table-7.1

K_t = modification factor for tension reinforcement (Refer Fig. 7.2)

K_c = modification factor for compression reinforcement (Refer Fig. 7.3)

K_f = modification or reduction factor for flanged sections (Refer Fig. 7.4)

In general, higher percentages of tension reinforcement are associated with lower values of K_t and higher values of K_c. For Fe-415 grade HYSD bars, the value of K_t is unity corresponding to the unit percentage of tension reinforcement.

For spans above 10 m, the $(L/d)_{basic}$ values have to be multiplied by

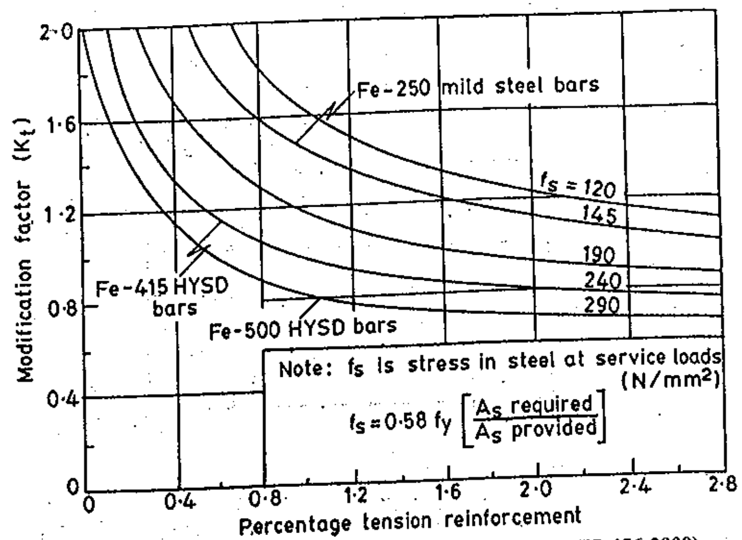


Fig. 7.2 Modification Factor for Tension Reinforcement (IS:456-2000)

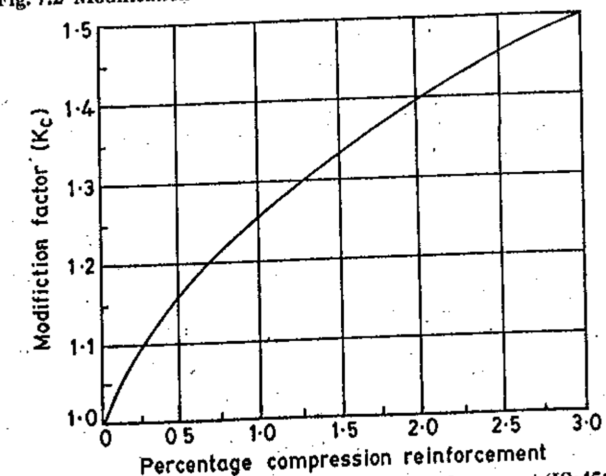


Fig. 7.3 Modification Factor for Compression Reinforcement (IS:456-2000)

(10/span) in metres, except for cantilevers in which case deflection computations are required to satisfy the limit state of deflection.

In the case of flanged beams (*T* and *L* beams), the modification factors K_t and K_c should be based on an area of section $b_f\, d$ (flange width × effective depth) and the calculated (L/d) ratio is further modified by a reduction factor which depends on the ratio (b_w/b_f) as shown in Fig. 7.4 (Fig.6 of IS: 456 Code).

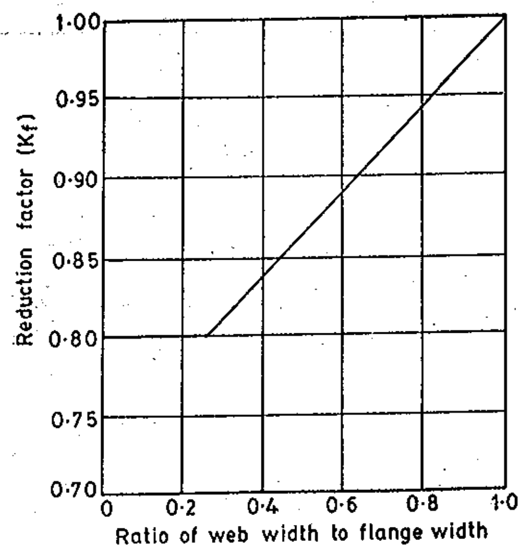


Fig. 7.4 Reduction Factors for Ratios of Span to Effective Depth for Flanged Beams (IS: 456-2000)

The codal procedure yields anomalous results in the case of flanged beams as outlined in the explanatory handbook to the code[14]. Hence it is preferable to consider the width of the web b_w in place of b_f in computations and this procedure yields conservative results.

The empirical procedure recommended for control of deflection in slabs is the same as in beams, i.e. to limit the span/depth ratios and the use of same modification factors.

For preliminary proportioning the thickness of slabs using Fe-415 HYSD bars, it is recommended to assume 0.4 percent value for p_t which gives a value of K_t of about 1.25 and the corresponding (L/d) ratio being 25.

Table 7.2 Span / Depth Ratios for Two Way Slabs (IS: 456-2000 Clause 24.1)

Support Conditions	Span/Overall Depth Ratio	
	Fe-250 Grade Steel	Fe-415 Grade HYSD bars
Simply Supported Slabs	35	28
Continuous Slabs	40	32

In the design of beams which carry heavy loading, it is preferable to assume span/effective depth ratio in the range of 10 to 12 from practical

considerations. For two way slabs of spans not exceeding 3.5 m and for loading class not exceeding 3 kN/m^2, the span to overall depth ratios recommended by IS: 456-2000 code are compiled in Table-7.2

7.4 DEFLECTION COMPUTATIONS (THEORETICAL METHOD)

The deflections of reinforced concrete members are influenced by the following salient factors:

1) Self weight and imposed loads
2) Strength and modulus of elasticity of concrete
3) Reinforcement percentage
4) Span of the structural member
5) Type of supports (simply supported, fixed or continuous)
6) Flexural rigidity
7) Creep and shrinkage of concrete

The computation of deflections are generally considered in two parts. Instantaneous or short term deflections occurring on application of load. Long term deflection resulting from differential shrinkage and creep under sustained loading. Annexure-C of IS: 456-2000 code presents a method of computing short and long term deflections as outlined below:

a) Short term deflection

The short-term deflection is computed using the elastic theory and short-term modulus of elasticity E_c and an effective moment of inertia I_{eff} which is expressed as

$$I_{eff} = \left[\frac{I_r}{1.2 - \left(\frac{M_r}{M}\right)\left(\frac{z}{d}\right)\left(1 - \frac{x}{d}\right)\left(\frac{b_w}{b}\right)}\right]$$

But

$$I_r \leq I_{eff} \leq I_{gr}$$

Where
- I_r = moment of inertia of the cracked section
- M_r = cracking moment, equal to $[(f_{cr} I_{gr})/y_t]$
- f_{cr} = modulus of rupture of concrete
- I_{gr} = moment of inertia of gross section about centroidal axis, neglecting the reinforcement
- y_t = distance of extreme fibre in tension from centroidal axis.
- M = maximum moment under service loads
- z = lever arm
- x = depth of neutral axis
- d = effective depth
- b_w = breadth of web
- b = breadth of compression flange

The expression for I_{eff} given in IS: 456 is based on the earlier version of the British code BS: 8110-1985[7].

Hence the short-term deflection is expressed as

$$a_{sd} = K_w\left(\frac{WL^3}{E_c I_{eff}}\right)$$

where K_w = Constant depending upon the type of load and support conditions

E_c = $5000\sqrt{f_{ck}}$

W = total load on the beam

L = span of the beam

b) Shrinkage Deflection

The deflection due to shrinkage (Clause C-3 of IS: 456-2000) is expressed as

$$a_{cs} = k_3 \psi_{cs} L^2$$

Where k_3 = a constant depending upon the support conditions
= 0.5 for cantilevers
= 0.125 for simply supported members
= 0.086 for members continuous at one end and
= 0.063 for fully continuous members

ψ_{cs} = shrinkage curvature $= k_4(\varepsilon_{cs}/D)$

where ε_{cs} = ultimate shrinkage strain of concrete (Clause 6.2.4.1 of IS:456-2000)

D = total depth of section

$$k_4 = 0.72\left[\frac{P_t - P_c}{\sqrt{P_t}}\right] \le 1.0 \quad \text{for} \quad 0.25 \le (P_t - P_c) < 1.0$$

$$= 0.65\left[\frac{P_t - P_c}{\sqrt{P_t}}\right] \le 1.0 \quad \text{for} \quad (P_t - P_c) \ge 1.0$$

where $P_t = (100A_{st}/bd)$ and $P_c = (100A_{sc}/bd)$

and L = span of the member

The expression for shrinkage deflection involving shrinkage curvature ψ_{cs} is based on empirical fits with the test data[14].

c) Creep Deflection

The creep deflection due to permanent loads $a_{cc(perm)}$ may be expressed as

$a_{cc(perm)}$ = initial plus creep deflection due to permanent loads obtained using an elastic modulus of elasticity $E_{ce} = [E_c/(1+\theta)]$ &

θ = creep coefficient

$a_{i(perm)}$ = short term deflection due to permanent load using E_c

The deflection due to creep depends upon the effective modulus of elasticity and creep coefficient. It is based on the assumption that the total strain in concrete (i.e. initial elastic strain plus creep strain) is directly proportional to the stress induced by the permanent loads[14].

7.5 CONTROL OF CRACKING IN R.C.MEMBERS

7.5.1 Cracks in Reinforced Concrete Members

Cracks in Reinforced Concrete Members develop due to various reasons[65]. The main causes of cracking are due to

a) Excessive flexural tensile stresses due to bending under applied loads since the tensile strength of concrete is only a tenth of its compressive strength.
b) Differential Shrinkage, Creep, thermal & aggressive environmental effects.
c) Settlement of supports & excessive curvature due to continuity effects.
d) Shear & diagonal tension cracks
e) Splitting cracks along with reinforcement due to bond and anchorage failure.

The various specifications prescribed in the codes regarding detailing of reinforcements are generally meant to reduce the width of cracks to allowable limits. According to the specifications of IS: 456 Code Clause 35.3.2 cracks in concrete should not adversely affect the appearance or durability of the structure.

7.5.2 Codal Crack Width Limits

The IS: 456 Code recommends a limit of 0.3mm for the surface crack widths of reinforced concrete members. This limit is adequate for the purpose of durability when the structural member is completely protected against aggressive environmental conditions[14]. For particularly aggressive environments, (Refer Table 2.4 of text or Table-5 of IS: 456) a more stringent limiting crack width of 0.004 times the nominal cover is prescribed. Using a nominal cover of 25mm, the limiting crack width works out to 0.1 mm. This limiting crack width is generally prescribed in most of the national codes where water tightness is required[14]. For structures exposed to moderate environmental conditions, the limiting crack width maybe taken as 0.2mm.

IS: 456 code clause-43.1 does not require explicit calculations of crack widths in the case of normal flexural members in which the spacing and cover requirements of reinforcements specified in section 26 of the code are adopted. In special structures and in aggressive environments, crack widths should be calculated by the method specified in Annexure-F of the code.

7.5.3 Empirical Method of Crack control

The empirical method of crack control renders the structures with cracks well within the permissible limits by detailing the reinforcements according to the specifications of the code of practice. The following factors are considered in this method.

1) Maximum and Minimum spacing of reinforcements

The horizontal & vertical spacing of bars are specified in Clause26.3.2 & 26.3.3 of IS: 456 Code. The minimum spacing requirements are detailed in Fig. 7.5.

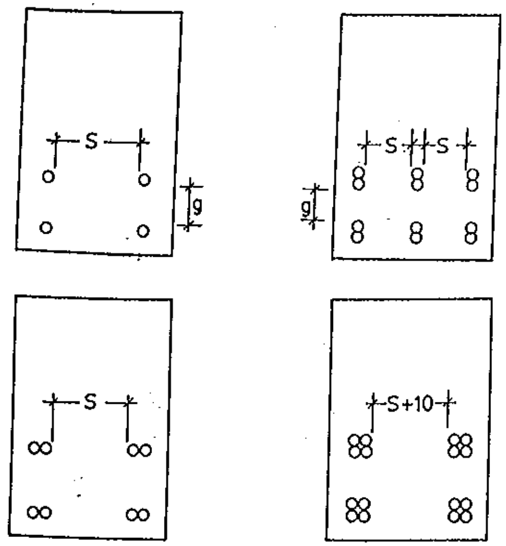
The maximum spacing requirements in the tension zone is a function of the stress level in the steel and the redistribution of the moments to and from that section. It should not be more than the values specified in Table-15 in IS: 456 Code. (Table 7.3 of Text). The spacing depends upon the grade of steel and percentage redistribution of moments. In the case of negative redistribution (the moment of the section is distributed to another section) the actual working stress in steel is higher than that with no redistribution and hence the difference in spacings shown in Table 7.3.

Table 7.3 Clear Distance Between Bars (mm)
(Clause 26.3.3 & Table-15 of IS: 456)

f_y (N/mm^2)	Percentage Redistribution to or from the section considered Clear distance between bars (mm)				
	–30	–15	0	+15	+30
250	215	260	300	300	300
415	125	155	180	210	235
500	105	130	150	175	195

Note: The spacings given in the table are not applicable to members subjected to particularly aggressive environments unless in the calculation of the moment of resistance, f_y has been limited to 300 N/mm^2 in the limit state design and σ_{st} limited to 165 N/mm^2 in working stress design.

In the case of slabs, the horizontal distance between parallel main reinforcement shall not be more than three times the effective depth of solid slab or 300 mm whichever is smaller. For bars provided against shrinkage



S $\nless$ Diameter of largest bar
$\nless$ (maximum aggregate size + 5mm)
g $\nless$ 15 mm
$\nless$ Diameter of largest bar
$\nless$ 2/3 the maximum size of aggregate

Fig. 7.5 Maximum Spacing Between Group of Bars

and temperature (distribution bars), the horizontal distance is limited to five times the effective depth of the solid slab or 450 mm whichever is smaller (Refer Fig. 3.1 for details).

2) Side Face Reinforcements

According to Clause 26.5.1.3 of IS: 456 code, if the total depth of beam is greater than 750 mm, side face reinforcements of area not less than 0.1 percent of web area should be distributed equally on two faces at a spacing not exceeding 300 mm or web thickness whichever is smaller as shown in Fig. 7.6.

The British code BS: 8110[7], limits the spacing of side face reinforcements to 250 mm and the diameter of the side face reinforcements (ϕ) should be not less than the value in mm given by the expression

$$\phi = \sqrt{\frac{S_b . b}{f_y}} \qquad ...(7.1)$$

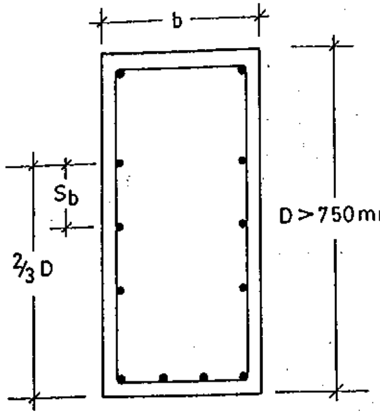


Fig. 7.6 Spacing of Side Face Reinforcement

Where S_b = vertical spacing (mm)
b = breadth of beam (mm)
f_y = characteristic strength (N/mm^2)

This provision is made to guard against the bar yielding locally at a crack. The side face reinforcement should invariably be provided and is mandatory on the tension zone of deep beams below the neutral axis as shown in Fig. 7.6.

3) Minimum percentage of steel

The minimum percentage of steel prescribed in the IS: 456 code is derived by considering the probability of cracking of the beams on the tension side when the tension in concrete reaches a stress equal to the modulus of rupture. Hence the main criterion for minimum percentage of steel is expressed as

[Strength of reinforced concrete beam] > [Strength as plain concrete beam]

According to IS: 456 code, clause 6.2.2 , the modulus of rupture of concrete is computed by the empirical relation,

$$f_{cr} = 0.7\sqrt{f_{ck}} \quad \text{and} \quad M_{cr} = f_{cr} Z$$

For a rectangular section having width 'b' and overall depth 'd', the value of section modulus, $Z = (b\, d^2 / 6)$.

Hence $$M_{cr} = 0.7\sqrt{f_{ck}}(bd^2/6) \qquad \ldots(7.2)$$

Assuming the lever arm, $a = 0.7\, d$, and in rectangular beams, the ratio of total depth to effective depth is very nearly unity. Hence replacing effective depth as total depth, the moment of resistance is expressed as

$$M_u = (A_s 0.87 f_y\, a) \qquad \ldots(7.3)$$

Equating $$M_u = M_{cr}$$

$$A_s(0.87 f_y)(0.7d) = 0.7\sqrt{f_{ck}}(bd^2/6)$$

$$\left(\frac{A_s}{bd}\right) = \left(\frac{0.19\sqrt{f_{ck}}}{f_y}\right)$$

For $f_{ck} = 20$ N/mm^2

$$\left(\frac{A_s}{bd}\right) = \left(\frac{0.85}{f_y}\right)$$

$$A_s = \left(\frac{0.85\, bd}{f_y}\right) \qquad \ldots(7.4)$$

This value of minimum tension reinforcement is prescribed in IS: 456 code clause 26.5.1.1. Provision of this magnitude of reinforcement will prevent the failure in tension zone when the first cracks develops in concrete.

In the case of slabs the ratio of total depth to effective depth , compared to beams is as large is 1.25 so that the coefficient worked out in the equation of the beams will be much smaller. In addition, the loads will be better distributed laterally in slabs than in beams. Hence the minimum reinforcement (0.15 percent for mild steel and 0.12 percent for HYSD bars) specified in the IS: 456 code is based on shrinkage and temperature effects rather than strength considerations.

7.5.4 Calculation of crack width

The empirical method of controlling crack widths to permissible limits is sufficient for most of the reinforced concrete structures. However in special structures and in aggressive environments, crack widths should be calculated by appropriate methods. The Indian Standard code IS:456-2000 prescribes an analytical method for estimation of design surface crack width which is the same as that specified in British code BS: 8110-1985[7].

The formula recommended by the British code BS: 8110-1985 for the estimation of surface crack width W_{cr} is based on the research investigations of Beeby[66] and the same has been used by author[67] in the computation of crack widths in Prestressed concrete members.

The empirical formula for the design surface crack width W_{cr} is expressed as

$$W_{cr} = \left[\frac{3a_{cr}\,\varepsilon_m}{1+2\left\{\frac{a_{cr}-C_{min}}{h-x}\right\}} \right] \quad \ldots(7.5)$$

Where a_{cr} = distance from the point considered to the surface of the nearest longitudinal bar

Referring to Fig. 7.7

$$a_{cr} = [(0.5\,S)^2 + C_{min}^2]^{0.5}$$

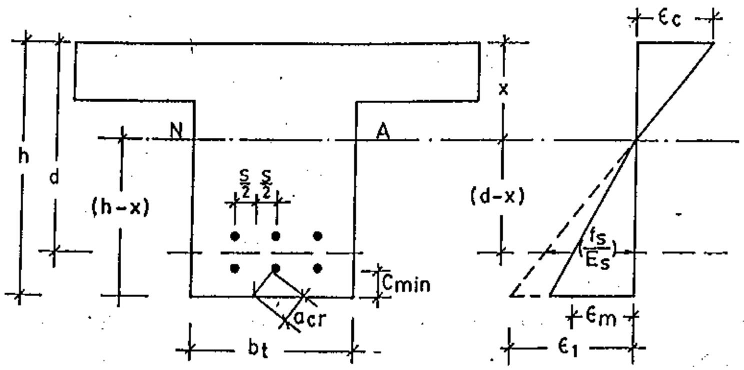


Fig. 7.7 Parameters for Crack Width Computation

Where S = spacing between bars

C_{min} = minimum cover to the longitudinal bars

x = depth of neutral axis

h = Overall depth of the member

ε_m = average strain at the level of steel where cracking is being considered and calculated by allowing for the stiffening effect of concrete in the tension zone and obtained from Eq.(7.6).

$$\varepsilon_m = \varepsilon_1 - \left\{ \frac{b_t(h-x)(a'-x)}{3E_s A_s(d-x)} \right\} \quad \ldots(7.6)$$

where ε_1 = strain at the level considered, considering a cracked section

b_t = width of section at the centroid of tension steel

a' = distance from the compression face to the point at which crack width is being measured.

E_s = modulus of elasticity of steel (N/mm^2)

A_s = area of tension reinforcement

It is important to note that Eq.(7.6) is empirical, based on test data and dimensional homogeneity. The constant 3 in the denominator has the inverse unit of stress.

If the crack width is measured at the soffit of the beam, then Eq.(7.6) can be modified as follows:

$$a' = h \quad \text{and} \quad \varepsilon_1 = \frac{f_s}{E_s}\left[\frac{h-x}{d-x}\right] \quad \text{and}$$

$$\varepsilon_m = \frac{1}{E_s}\left[\frac{h-x}{d-x}\right]\left[f_s - \frac{b_t(h-x)}{3A_s}\right] \quad \ldots(7.7)$$

where f_s = stress at the centroid of the tension reinforcement expressed in N/mm^2 units.

In flexural members, the maximum widths of cracks are generally encountered at the soffit of the beam and at sections where the moment is maximum and at points mid way at soffit between the reinforcements and at corners.

7.6 Examples

1) A simply supported rectangular beam of 12 m span has an effective depth of 800 mm. The area of tension reinforcement required to support the loads is designed as 1.6 percent. Check the deflection control of the beam by empirical method if a) Fe-415 grade HYSD bars are used and b) Fe-500 grade bars are used.

a) Data

L = 12 m $f_y = 415$ N/mm^2

d = 800 mm $f_y = 500$ N/mm^2

A_{st} = 1.6 percent

b) Actual span/depth ratio

$$(L/d) = (12/0.8) = 15$$

c) Allowable span/depth ratio

$$(L/d) = [(L/d)_{basic} \times K_t \times K_c \times K_f]$$

Since $A_{sc} = 0$, $K_c = 1$

Since the beam is rectangular, $K_f = 1$

From Table-7.1, $(L/d)_{basic} = 20$
From Fig. 7.2, $K_t = 0.9$ for Fe-415 steel
$= 0.8$ for Fe-500 steel

Case (a): $(L/d) = [20 \times 0.9 \times 1 \times 1] = 18$
Case (b): $(L/d) = [20 \times 0.8 \times 1 \times 1] = 16$

Since permissible span/depth ratio is greater than the actual, the deflection control is satisfactory.

2) A doubly reinforced beam of rectangular section 250 mm wide by 550 mm overall depth is reinforced with 4 bars of 22 mm diameter on the tension face and 2 bars of 16 mm diameter at the compression face. The effective cover is 50 mm. The beam spans over 8 m. If Fe-415 HYSD bars are used, check for the deflection control using the empirical method.

a) Data

$b = 250\text{mm}$ $\quad A_{st} = (4 \times 380) = 1520\ \text{mm}^2$
$d = 500\ \text{mm}$ $\quad A_{sc} = (2 \times 201) = 402\ \text{mm}^2$
$L = 8\text{m}$ $\quad$ Fe-415 HYSD bars

b) Actual Span/Depth ratio

$$\left(\frac{L}{d}\right)_{actual} = \left(\frac{8000}{500}\right) = 16$$

c) Allowable span/depth ratio

$$\left(\frac{L}{d}\right)_{max} = \left(\frac{L}{d}\right)_{basic} \times K_t \times K_c \times K_f$$

$$\left(\frac{L}{d}\right)_{basic} = 20 \text{ from the table 7.1}$$

$$p_t = \left(\frac{100A_{st}}{bd}\right) = \left(\frac{100 \times 1520}{250 \times 500} = 1.21\%\right)$$

$$p_c = \left(\frac{100A_{sc}}{bd}\right) = \left(\frac{100 \times 402}{250 \times 500} = 0.32\%\right)$$

From $\quad$ Fig. 7.2, $K_t = 0.95$
Fig. 7.3, $K_c = 1.1$
Fig. 7.4, $K_f = 1.0$

$$\therefore \quad \left(\frac{L}{d}\right)_{max} = (20 \times 0.95 \times 1.1 \times 1.0) = 20.9 > 16$$

Hence, the beam is safe with regard to serviceability limit state of deflection.

3) Check the deflection requirement for the following tee-beam continuous over 10m spans and having flange width of 1200 mm, web width of 250 mm and effective depth = 400 mm, Area of tension reinforcement =1500 mm², Area of compression reinforcement = 960 mm². Adopt Fe-415 grade HYSD bars.

a) Data

$L = 10\ \text{m}$ (Continuous beam)
$b_f = 1200\ \text{mm}$
$d = 400\ \text{mm}$
$b_w = 250\ \text{mm}$ $\quad \left(\frac{b_w}{b_f}\right) = \left(\frac{250}{1200}\right)$
$A_{st} = 1500\ \text{mm}^2$
$A_{sc} = 960\ \text{mm}^2$

b) Percentage reinforcement

$$p_t = \left[\frac{100A_{st}}{b_f d}\right] = \left[\frac{100 \times 1500}{1200 \times 400}\right] = 0.31$$

$$p_c = \left[\frac{100 \times A_{sc}}{b_f d}\right] = \left[\frac{100 \times 960}{1200 \times 400}\right] = 0.20$$

c) Actual span/depth ratio

$$\left[\frac{L}{d}\right]_{actual} = \left[\frac{10000}{400}\right] = 25$$

d) Modification factors

From $\quad$ Fig. 7.2, $K_t = 1.50$
From $\quad$ Fig. 7.3, $K_c = 1.00$
From $\quad$ Fig. 7.4, $K_f = 0.80$

e) Permissible span/depth ratio

$$\left[\frac{L}{d}\right]_{max} = \left[\frac{L}{d}\right]_{basic} \times K_t \times K_c \times K_f$$

$$= [26 \times 1.50 \times 1.00 \times 0.8]$$

$$= 31.2 > 25$$

Hence, the tee beam is safe with regard to the limit state of deflection.

4) A rectangular section beam 200 mm wide by 450 mm overall depth is reinforced with 3 bars of 16 mm diameter at an effective depth of 420 mm. Two hanger bars of 12 mm diameter are provided at the compression face. The effective span of the beam is 5 m. The beam supports a service live load of 10 kN/m. If f_{ck} = 20 N/mm² and f_y = 415 N/mm², compute a) the short-term deflection b) the long term deflection according to IS: 456 code specifications.

a) Data

b = 200 mm $\quad f_{ck}$ = 20 N/mm²

D = 450 mm $\quad f_y$ = 415 N/mm²

d = 420 mm $\quad E_c = 5000\sqrt{f_{ck}}$

L = 5 m $\quad = 5000\sqrt{20} = 22360$ N/mm²

q = 10 kN/m

A_{st} = 603 mm² $\quad f_{cr} = 0.7\sqrt{f_{ck}}$

A_{sc} = 226 mm² $\quad = 0.7\sqrt{20} = 3.13$ N/mm²

m = 13

b) Service loads on beam

Self weight = $g = (0.2 \times 0.45 \times 25) = 2.25$ kN/m
Live load = q = 10 kN/m
Service load = $w = (g + q) = (2.25 + 10) = 12.25$ kN/m = 12.25 N/mm

c) Maximum Short term Deflection

$$(a_{sd})_{max} = \left[\frac{5WL^4}{384E_cI_{eff}}\right]$$

where
$$I_{eff} = \left[\frac{I_r}{1.2 - \left(\frac{M_r}{M}\right)\left(\frac{z}{d}\right)\left(1 - \frac{x}{d}\right)\frac{b_w}{b}}\right] \quad \text{but} \quad I_r \le I_{eff} \le I_{gr}$$

I_r = Moment of inertia of cracked section

Let x = depth of neutral axis, then

$$0.5\,b\,x^2 = m\,A_{st}(d - x)$$

$$(0.5 \times 200 \times x^2) = 13 \times 600(420 - x)$$

Solving $x = 146$ mm

Distance of centroid of steel from neutral axis is obtained as

$$r = (d - x) = (420 - 146) = 274\text{mm}$$

$$\therefore \quad I_r = (b\,x^3/3) + m\,A_{st}\,r^2$$

$$= \left(\frac{200 \times 146^3}{3}\right) + (13 \times 603 \times 274^2)$$

$$= (7.95 \times 10^8)\text{ mm}^4$$

$$I_{gr} = \left(\frac{bD^3}{12}\right) = \left(\frac{200 \times 450^3}{12}\right) = (15.18 \times 10^8)\text{ mm}^4$$

$$M_r = \left(\frac{f_{cr}I_{gr}}{y_t}\right) = \left(\frac{3.13 \times 15.18 \times 10^8}{0.5 \times 450}\right) = (0.211 \times 10^8)\text{ N.mm}$$

$$z = \text{lever arm} = [d - (x/3)] = [420 - (146/3)] = 371.34\text{mm}$$

$$M = (0.125w\,L^2) = (0.125 \times 12.25 \times 5^2) = 38.3\text{ kN.m} = (0.383 \times 10^8)\text{ N.mm}$$

$$I_{eff} = \left[\frac{7.95 \times 10^8}{1.2 - \left(\frac{0.211 \times 10^8}{0.38 \times 10^8}\right)\left(\frac{371.34}{420}\right)\left\{1 - \left(\frac{146}{420}\right)\right\}}\right] = (9.037 \times 10^8)\text{ mm}^4$$

$$I_r \le I_{eff} \le I_{gr}$$

$$(7.85 \times 10^8) \le (9.037 \times 10^8) \le (15.18 \times 10^8)$$

Maximum short-term deflection is computed as

$$a_{1(perm)} = \left[\frac{5wL^4}{384E_cI_{eff}}\right] = \left[\frac{5 \times 12.25 \times 5000^4}{384 \times 22360 \times 9.02 \times 10^8}\right] = 4.94\text{ mm}$$

d) Long term Deflection

a_{Ld} = (Short term deflection) + (Shrinkage deflection) + (Creep deflection)

$a_{Ld} = [a_{1(perm)} + a_{cs} + a_{cc(perm)}]$

Shrinkage Deflection

$a_{cs} = k_3\,\psi_{cs}\,L^2$

where k_3 = a constant = 0.125 for simply supported beams

ψ_{cs} = shrinkage curvature = $k_4\,(\varepsilon_{cs}/D)$

where ε_{cs} = ultimate shrinkage strain of concrete = 0.0003

$$k_4 = \left[\frac{0.72(p_t - p_c)}{\sqrt{p_t}}\right] \le 1.0 \quad \text{for} \quad 0.25 \le (p_t - p_c) < 1.0$$

$$p_t = \left(\frac{100 \times 603}{200 \times 420}\right) = 0.71 \quad \text{and} \quad p_c = \left(\frac{100 \times 226}{200 \times 420}\right) = 0.269$$

$\therefore$ $$(p_t - p_c) = (0.71 - 0.269) = 0.441 > 0.25 \quad \text{and} \quad < 1.00$$

$\therefore$ $$k_4 = \left[\frac{0.72 \times 0.441}{\sqrt{0.71}}\right] = 0.377$$

$\therefore$ $$\psi_{cs} = k_4\,[\varepsilon_{cs}/D] = 0.377[0.0003/450] = (2.51 \times 10^{-7})$$

$\therefore$ $a_{cs} = [k_3\,\psi_{cs}\,L^2] = [0.125 \times 2.51 \times 10^{-7} \times 5000^2] = 0.784$ mm

Creep Deflection

$a_{1,cc(perm)}$ = Initial + creep deflection due to permanent loads obtained by using the effective modulus of elasticity as $E_{ce} = [E_c/(1+\theta)]$

where θ = creep coefficient = 1.6 (at 28 days loading)

$$a_{1,cc(perm)} = \left[\frac{5wL^4}{384\,E_{ce}I_{eff.}}\right] \quad \text{and} \quad E_{ce} = \left[\frac{E_c}{1+1.6}\right] = \left[\frac{E_c}{2.6}\right]$$

$\therefore$ $a_{1,cc(perm)}$ = 2.6 (short term deflection) = (2.6 × 4.94) = 12.84 mm

Hence the creep deflection due to permanent loads may be obtained from the relation,

$$a_{cc\,(perm)} = [a_{1,cc\,(perm)} - a_{1\,(perm)}] = (12.84 - 4.94) = 7.9 \text{ mm}$$

Total long-term deflection is given by the relation,

a_{Ld} = (short term deflection)+(shrinkage deflection)+(creep deflection)

$= a_{1,\,(perm)} + a_{cs} + a_{cc,\,(perm)}$

$= (4.94 + 0.784 + 7.9) = 13.664$ mm

According to IS: 456-2000 code, the maximum permissible long-term deflection should not exceed the value of (span/250).

$\therefore$ Permissible deflection = (span / 250) = (5000 / 250) = 20 mm

Actual deflection = 13.664 mm

Hence, the beam satisfies the limit state of deflection.

5) A simply supported beam spanning over 8 m is of rectangular section with a width of 300 mm and overall depth 600 mm. The beam is reinforced with 4 bars of 25 mm diameter on the tension side at an effective depth of 550 mm. Two nominal hanger bars of 12 mm diameter are provided on the compression side. The beam is subjected to a service load moment of 140 kN.m at the centre of span section. Assuming M-20 grade concrete and Fe-415 HYSD bars, check the beam for the serviceability limit states of deflection and cracking using the following methods:

1) Deflection control (empirical method)
2) Deflection control (Theoretical method)
3) Maximum width of cracks (Theoretical method)

Data

b = 300 mm		f_{ck} = 20 N/mm²	
D = h = 600 mm		f_y = 415 N/mm²	
d = 550 mm		m = 13	
M = 140 kN.m		E_c = $5000\sqrt{20} = 22360$ N/mm²	
A_{st} = 1963 mm²		f_{cr} = $0.7\sqrt{20} = 3.13$ N/mm²	
A_{sc} = 226 mm²		E_s = (2×10^5) N/mm²	
L = 8 m			

1) Deflection Control (Empirical Method)

$$p_t = \left(\frac{100A_{st}}{bd}\right) = \left(\frac{100 \times 1963}{300 \times 550}\right) = 1.19$$

$$p_c = \left(\frac{100A_{sc}}{bd}\right) = \left(\frac{100 \times 226}{300 \times 550}\right) = 0.14$$

a) Modification Factors

From Fig. 7.2, $K_t = 0.95$
From Fig. 7.3, $K_c = 1.05$
From Fig. 7.4, $K_f = 1.00$

b) Span/depth Ratio

Allowable span depth ratio is expressed as

$$(L/d) = [(L/d)_{basic} \times K_t \times K_c \times K_f]$$
$$= (20 \times 0.95 \times 1.05 \times 1.00)$$
$$= 19.95$$

Actual span/depth ratio = $(L/d) = (8000/550) = 14.54 < 19.95$
Hence the deflection control is satisfactory.

2) Deflection Computations (Theoretical Method)

a) Neutral Axis depth

Let x = depth of neutral axis
$r = (d - x)$, neglecting the compression steel in computations,

$$0.5bx^2 = mA_{st}(d-x)$$

$$(0.5 \times 300 \times x^2) = [13 \times 1963(550 - x)$$

Solving $x = 233$ mm and $r = (550 - 233) = 317$ mm
Lever arm = $z = [d - (x/3)] = [550 - (233/3)] = 472.34$ mm

b) Moment of Inertia of Cracked Section

$$I_r = (bx^3/3) + mA_{st}r^2$$

$$= [(300 \times 233^3)/3] + [13 \times 1963 \times 317^2] = (38.2 \times 10^8)\text{mm}^4$$

c) Moment of Inertia of Gross Section

$$I_{gr} = (b\,h^3/12) = [(300 \times 600^3)/12] = (54 \times 108)\ \text{mm}^4$$

d) Effective Moment of Inertia

$$M_r = [f_{cr}I_{gr})/y_t] = [(3.13 \times 54 \times 10^8)/300] = (0.563 \times 10^8)\ \text{N.mm}$$

$$I_{eff} = \left[\frac{I_r}{1.2 - \left(\frac{M_r}{M}\right)\left(\frac{z}{d}\right)\left(1 - \left(\frac{x}{d}\right)\right)\left(\frac{b_w}{b}\right)}\right]$$

$$= \left[\frac{38.2 \times 10^8}{1.2 - \left(\frac{0.563 \times 10^8}{1.4 \times 10^8}\right)\left(\frac{472.34}{550}\right)\left\{1 - \left(\frac{233}{550}\right)\right\}}\right]$$

$$= (38.2 \times 10^8)\ \text{mm}^4$$

e) Short term Deflection

$$a_{sd} = \left[\frac{5ML^2}{48E_cI_{eff}}\right] = \left[\frac{5 \times 1.4 \times 10^8 \times 8000^2}{48 \times 22360 \times 38.2 \times 10^8}\right] = 10.92\ \text{mm}$$

f) Shrinkage deflection

$$\psi_{cs} = k_4\left(\frac{\varepsilon_{cs}}{D}\right)$$

$$(p_t - p_c) = (1.19 - 0.14) = 1.05 > 1$$

$$\therefore\ k_4 = 0.65\left[\frac{p_t - p_c}{\sqrt{p_t}}\right] = 0.65\left[\frac{1.05}{\sqrt{1.19}}\right] = 0.626$$

For simply supported beams

$$\psi_{cs} = 0.626(0.0003/600) = (3.13 \times 10^{-7})$$

$$k_3 = 0.125$$

$$a_{cs} = [k_3\psi_{cs}L^2] = [0.125 \times 3.13 \times 10^{-7} \times 8000^2] = 2.5\ \text{mm}$$

g) Creep deflection

$$E_{ce} = \left(\frac{E_c}{1+\theta}\right) = \left(\frac{E_c}{2.6}\right)$$

$$\therefore\ a_{1,cc(perm)} = (1+\theta)(a_{sd}) = (2.6 \times 10.92) = 28.4\text{mm}$$

Deflection due to creep only is

$a_{cc,(perm)} = [a_{1,cc(perm)} - a_{1(perm)}] = (28.4 - 10.92) = 17.48$ mm

h) Long term Deflection

$a_{Ld} = [a_{sd} + a_{cs} + a_{cc,(perm)}] = (10.92 + 2.5 + 17.48) = 30.9$ mm

i) Check for Deflection Control

Maximum permissible long-term deflection is given by the expression

$$a_{limiting} = \left(\frac{\text{span}}{250}\right) = \left(\frac{8000}{250}\right) = 32\text{mm}$$

Actual deflection is = $a_{Ld} = 30.9$ mm < 32 mm
Hence the deflection is within safe permissible limits.

3) Maximum Width of Cracks

Assuming the spacing of tension steel (25 ϕ) at 60 mm centers, $S = 60$ mm
Cover = $C_{min} = (50 - 12.5) = 37.5$ mm

$a_{cr} = [(0.5S)^2 + C_{min}]^{0.5} = [(0.5 \times 60)^2 + 37.5^2]^{0.5} = 48$ mm

Crack width will be maximum at the soffit of the beam.

$$a' = h = 600\text{mm}$$

$$\therefore \quad \varepsilon_1 = \frac{f_s}{E_s}\left[\frac{h-x}{d-x}\right]$$

$$f_s = m\left[\frac{My}{I_r}\right] \quad \text{where} \quad y = r = 317 \text{ mm}$$

$$= 13\left[\frac{140 \times 10^6 \times 317}{38.2 \times 10^8}\right] = 151 \text{ N/mm}^2$$

$$\therefore \quad \varepsilon_1 = \left(\frac{151.3}{2 \times 10^5}\right)\left[\frac{600-233}{550-233}\right] = (8.74 \times 10^{-4})$$

$$\varepsilon_m = \varepsilon_1 - \left[\frac{b_t(h-x)(a'-x)}{3E_sA_s(d-x)}\right]$$

$$= (8.74 \times 10^{-4}) - \left[\frac{300(600-233)(600-233)}{3 \times 2 \times 10^5 1963(550-233)}\right] = (7.66 \times 10^{-4})$$

$$\therefore \quad W_{cr} = \left[\frac{3a_{cr}\varepsilon_m}{1+2\left(\frac{a_{cr}-C_{min}}{h-x}\right)}\right] = \left[\frac{3 \times 48 \times 7.66 \times 10^{-4}}{1+2\left(\frac{48-37.5}{600-233}\right)}\right] = 0.104 \text{ mm}$$

According to IS: 456 code clause 35.3.2, under general (normal) conditions,

Maximum width of crack $\ngtr (0.004 \times C_{min})$
$\ngtr (0.004 \times 37.5)$
$\ngtr 0.15$ mm

Since $W_{cr} = 0.104$ mm < 0.15 mm, the serviceability limit state of cracking is satisfied.

7.7 EXAMPLES FOR PRACTICE

1) A simply supported beam of 8 m span has an effective depth of 800 mm. The beam is reinforced with tension steel of 1.5 percent. Check the deflection control of the beam by empirical method using
   a) Fe-415 HYSD bars and
   b) Fe-500 grade steel bars

2) A doubly reinforced concrete beam of rectangular section 250 mm wide by 500 mm effective depth is reinforced with 3 bars of 20 mm diameter on the tension face and 2 bars of 16 mm diameter at the compression face at an effective cover of 50 mm. The beam spans over 6 m. Using Fe-415 grade HYSD bars check for the serviceability limit state of deflection using the empirical method.

3) A tee beam continuous over 8 m spans and having a flange width of 1200 mm and web width 300 mm, effective depth of 500 mm is reinforced with tension reinforcement of area 1600 mm². Adopting Fe-415 grade HYSD bars, check for the limit state of deflection using IS: 456-2000 code empirical method.

4) A simply supported reinforced concrete beam of rectangular section 250 mm wide by 450mm overall depth is used over an effective span of 4 m. The beam is reinforced with 3 bars of 20 mm diameter Fe-415 HYSD grade steel at an effective depth of 400 mm. Two hanger bars of 10 mm diameter are provided. The self weight together with the dead load on the beam is 4 kN/m
   Service live load is 10 kN/m. Using M-20 grade concrete, compute
   a) The short term deflection
   b) The long-term deflection according to the provisions of the IS: 456-2000 code.

5) A simply supported beam of rectangular section spanning over 6 m has a width of 300 mm and overall depth of 600 mm. The beam is reinforced with 4 bars of 25 mm diameter on the tension side at an effective depth of 550 mm spaced at 50 mm centers. The beam is subjected to a working moment of 160 kN.m at the centre of span section. Using Fe-415 HYSD bars and M-25 grade concrete, check the beam for the serviceability limit state of cracking according to the provisions of the Indian standard code IS: 456-2000.

6) A simply supported beam of rectangular section 300 mm wide by 700 mm overall depth has an effective span of 8 m. The beam is reinforced with 4 bars of 25 mm diameter spaced 50 mm apart on tension side at an effective depth of 650 mm. Two nominal hanger bars of 12 mm diameter are provided on the compression side at a cover of 50 mm. The beam is subjected to a service load moment of 140 kN.m at centre of span section. Using M-25 grade concrete and Fe-415 HYSD bars, check the beam for the limit states of deflection and cracking using the following methods:
   a) Deflection control using empirical method
   b) Deflection computations using the theoretical method.
   c) Maximum width of cracks using the theoretical method.

CHAPTER 8

Limit State Design of Beams

8.1 INTRODUCTION

The design of a reinforced concrete beam element to resist a given system of external loads involves the material properties and the skeletal dimensions such as width and depth are assumed based on specific guidelines. The cross sectional dimensions generally assumed to satisfy the serviceability criteria and housing of reinforcements with suitable spacing and cover is required to estimate the dead loads and moments. In designing the reinforcements for flexure and shear, the bending moments and shear forces along the length of beam must be obtained from structural analysis. The designed beam should satisfy the limit states of safety and serviceability discussed in earlier chapters.

In contrast to the analysis problem, the design problem does not have a unique solution since the flexural strength of a section is governed by its cross sectional dimensions, material properties and magnitude of reinforcements in the section. The desired strength can be obtained by several combinations of these variables. It is possible that different designers may produce different solutions all of which may satisfy the design criteria.

A comprehensive design of a beam requires the considerations of safety under the ultimate limit states of flexure, shear, torsion and bond together with the limit states of serviceability criteria by empirical methods.

8.2 GUIDE LINES FOR SELECTION OF CROSS SECTIONAL DIMENSIONS AND DETAILING OF REINFORCEMENTS

The following guidelines may be used to select the cross sectional dimensions of reinforced concrete beams.

1) The depth of the beam is fixed based on span/depth ratios to satisfy the deflection requirements. The ratio of overall depth to width should be between 1.5 and 2.0

Table 8.1 shows the trial section(span/depth) ratios to be assumed as a function of span and loading.

Table 8.1 Span/Depth ratios for Trial Section

Sl. No.	Span Range	Loading	Span/Depth ratio (L/d)
1	3 to 4m	Light	15 to 20
2	5 to 10	Medium to Heavy	12 to 15
3	> 10m	Heavy	12

2) The minimum percentage of tension steel should be around 0.3 percent. Generally the depth of the beam should be such that the percentage of steel required is about 75 percent of that required for balanced section.
3) The minimum number of bars used as tension reinforcement should be atlest two and not more than six bars should be used in one layer in the beam.
4) The width of the section should accommodate the required number of bars with sufficient spacings between them and a minimum side cover of 20mm to the stirrups. The minimum spacing between groups of bars is shown in Fig. 7.5
5) The diameter of hanger bars should be not less than of 10mm and that of main bars 12mm. The normal diameter of bars used are 08,10,12,16,20,22,25 and 32mm. If different sizes of bars are used in one layer, the hanger bars should be placed near the faces of the beam.
6) In flanged Tee-beams, the depth of the slab is usually taken as 20percent of the overall depth.
7) The general widths of beams used are 150, 200, 230, 250 and 300mm. Also the widths of beam should be equal to or less than the dimension of the columns supporting the beam.

8.3 DESIGN OF SINGLY REINFORCED RECTANGULAR BEAMS

1) Design a singly reinforced concrete beam to suit the following data:

Data:

Clear span = 3m
Width of supports = 200mm
Working Live load = 6 kN/m
M-20 grade concrete
Fe-415 grade HYSD bars

Method-1 (Using IS: 456 Code provisions)

a) Stresses

$f_{ck} = 20\ \text{N/mm}^2$
$f_y = 415\ \text{N/mmm}^2$
Load Factor = 1.5 for dead and Live Loads.

b) Cross Sectional Dimension

Refer Table-8.1 and adopt a span/depth ratio of 20 for the given span and range of loading.

$$\text{Effective depth} = d = \left(\frac{\text{span}}{20}\right) = \left(\frac{3000}{20}\right) = 150\ \text{mm}$$

Adopt $d = 160\text{mm}$
$D = 200\text{mm}$
$b = 200\text{mm}$

$\therefore$ Effective Span = [Clear span + effective depth] = (3 + 0.16) = 3.16 m

Center to center of supports = (3 + 0.2) = 3.2 m

Hence $L = 3.16$ m.

c) Loads

Self weight = g = (0.2 × 0.2 × 25) = 1.00 kN/m
Live Load = q = 6.00 kN/m
Total Load = w = 7.00 kN/m
Design Ultimate Load = w_u = (1.5 × 7) = 10.5 kN/m

d) Ultimate Moments and Shear Forces

$M_u = (0.125\ w_u\ L^2) = (0.125 \times 10.5 \times 3.162) = 13.1$ kNm.
$V_u = (0.5\ w_u\ L) = (0.5 \times 10.5 \times 3.16) = 16.59$ kN.

e) Tension Reinforcements

$M_{u,lim} = 0.138\, f_{ck}.b.d^2$
$= (0.138 \times 20 \times 200 \times 160^2)\ 10^{-6}$
$= 14.13$ kNm.

Since $M_u < M_{u,lim}$, section is under reinforced.

$$\therefore \quad M_u = 0.87\, f_y.A_{st}.d\left[1 - \frac{A_{st}.f_y}{b.d.f_{ck}}\right]$$

$$(13.1 \times 10^6) = (0.87 \times 415 \times A_{st} \times 160)\left[1 - \frac{A_{st} \times 415}{200 \times 160 \times 20}\right]$$

Solving $A_{st} = 275\ \text{mm}^2 > A_{st,min} = [(0.85\ bd)/f_y]$
Provide 3 bars of 12mm diameter ($A_{st} = 339\ \text{mm}^2$) and 2 hanger bars of 10mm diameter on compression side.

f) Check for Shear stress

$$\tau_v = \left(\frac{V_u}{bd}\right) = \left(\frac{16.59 \times 10^3}{200 \times 160}\right) = 0.51\ \text{N/mm}^2$$

$$p_t = \left(\frac{100A_{st}}{bd}\right) = \left(\frac{100 \times 339}{200 \times 160}\right) = 1.05$$

Refer Table-19 of IS:456-2000 (Table-6.11 of text) and read out the design shear strength of concrete as,

$$\tau_c = 0.62\ \text{N/mm}^2 > \tau_v.$$

$\therefore$ Nominal shear reinforcements are provided. Using 6 mm diameter two legged stirrups,

$$S_v = \left(\frac{A_{sv}0.87\, f_y}{0.4\, b}\right) = \left(\frac{2 \times 28 \times 0.87 \times 250}{0.4 \times 200}\right) = 152\ \text{mm}$$

But $S_v \not> 0.75\, d = (0.75 \times 160) = 120$ mm and $S_v \not> 300$ mm.
Adopt spacing of stirrups as 120 mm.

g) Check for Deflection Control

$p_t = 1.05$. From Fig.7.2 read out the modification factor $K_t = 0.95$.

$$\therefore \quad \left(\frac{L}{d}\right)_{max} = \left(\frac{L}{d}\right)_{basic} \times K_t \times K_c \times K_f$$

$$= (20 \times 1.1 \times 1 \times 1) = 22$$

$$\left(\frac{L}{d}\right)_{provided} = \left(\frac{3.16}{0.16}\right) = 19.75 < 22.$$

Hence, deflection control is satisfactory.

Method-2 (using SP: 16 Design Charts)

a) Compute $\left(\frac{M_u}{bd^2}\right) = \left(\frac{13.1 \times 10^6}{200 \times 160^2}\right) = 2.55$

b) Refer Table-2 (SP: 16) or Table-6.5 of text and read out $p_t = 0.861$

$\therefore \quad A_{st} = \left(\frac{0.861 \times 200 \times 160}{100}\right) = 276\ \text{mm}^2$(Provide 3 bars of 12 mm diameter)

c) Compute parameter $\left(\frac{V_{us}}{d}\right)$ kN/cm

Where $\quad V_{us} = (0.4b\ d) = (0.4 \times 200 \times 600)10^{-3} = 12.8\text{kN}$

$$\therefore \quad \left(\frac{V_{us}}{d}\right) = \left(\frac{12.8}{16}\right) = 0.8$$

Refer Table-62 (SP: 16) and read out the spacing of 6 mm two legged stirrups

As $S_v = 25$ cm $= 250$ mm. But $S_v \not> 0.75\ \text{d} = (0.75 \times 160) = 120$ mm

d) The detailing of reinforcements are shown in Fig. 8.1.

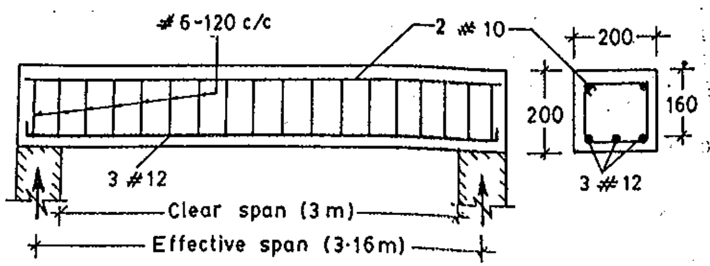


Fig. 8.1 Reinforcement Details in Singly Reinforced Rectangular Beam

2) Design a singly reinforced concrete beam of clear span 5m to support a design working live load of 10 kN/m. Adopt M-20 grade concrete and Fe-415HYSD bars.

Data:

Clear span = 5m
Working live load = 10 kN/m
M-20 grade concrete and Fe-415 HYSD bars.

Method-1 (using IS: 456 Code Formulae)

a) Stresses

$f_{ck} = 20\ \text{N/mm}^2,\ f_y = 415\ \text{N/mm}^2$
Load factor = 1.5 for dead and live loads.

b) Cross sectional dimensions

Refer Table-8.1 and adopt a span/depth ratio of 15 for the given span and range of loading.

Effective depth = (span/15) = (5000/15) = 333 mm.
Adopt effective depth = d = 350 mm
Overall depth = D = 400 mm
Width of beam = b = 200 mm
$\therefore$ Effective span = [Clear span + effective depth]
= [5 + 0.35] = L = 5.35 m

c) Loads

Self weight of beam = $g = (0.2 \times 0.4 \times 25) = 2.00$ kN/m
Live load = q = 10.00 kN/m
Total working load = w = 12.00 kN/m
Design ultimate load = $w_u = (1.5 \times 12) = 18.00$ kN/m

d) Ultimate Moments and Shear Forces

$M_u = (0.125\ w_u\ L^2) = (0.125 \times 18 \times 5.35^2) = 64.4$ kN
$V_u = (0.5\ w_u\ L^2) = (0.5 \times 18 \times 5.35) = 48.2$ kN

e) Reinforcements

Limiting moment of the section is
$M_u = 0.138 f_{ck}\ b\ d^2 = (0.138 \times 20 \times 200 \times 350^2)\ 10^{-6} = 68$ kN.m
Since $M_u < M_{u,lim}$, the section is underreinforced

$$M_u = (0.87 f_y A_{st} d)\left[1 - \left(\frac{A_{st} f_y}{b\,d\,f_{ck}}\right)\right]$$

$$(64.4 \times 10^6) = (0.87 \times 415 \times A_{st} \times 350)\left[1 - \left(\frac{415 A_{st}}{200 \times 350 \times 20}\right)\right]$$

Solving $A_{st} = 624 \text{ mm}^2$

Provide 2 bars of 20 mm diameter ($A_{st} = 628 \text{ mm}^2$) as tension reinforcement and 2 bars of 10 mm diameter as hanger bars on compression side.

f) Check for Shear Stress

$V_u = 48.2 \text{ kN}$

$$\tau_v = \left(\frac{V_u}{bd}\right) = (48.2 \times 10^3 / 200 \times 350) = 0.68 \text{ N/mm}^2.$$

$$p_t = \left(\frac{100A_{st}}{bd}\right) = \left(\frac{100 \times 628}{200 \times 350}\right) = 0.89$$

Refer Table-19 of IS:456 (Table-6.11 of text) and read out the design shear strength of concrete as $\tau_c = 0.59 \text{ N/mm}^2$.

Since $\tau_v > \tau_c$, shear reinforcements are required.

Balance shear, $V_{us} = [V_u - (\tau_c\, b\, d)]$

$= [48.2 - (0.59 \times 200 \times 350)\, 10^{-3}] = 7 \text{ kN}$

Using 6 mm diameter 2 legged mild steel stirrups,

$$S_v = \left[\frac{0.87 f_y A_{sv} d}{V_{us}}\right] = \left[\frac{0.87 \times 250 \times 2 \times 28 \times 350}{7 \times 1000}\right] = 609 \text{ mm}$$

$S_{v,max} = 0.75\, d = (0.75 \times 350) = 262.5$ mm and also $S_v \not> 300$ mm

Provide 6 mm diameter 2 legged stirrups at a spacing of 250 mm centres.

g) Check for deflection Control

$p_t = 0.89$ From Fig. 7.2 read out the modification factor $K_t = 0.99$

$$\therefore \quad \left(\frac{L}{d}\right)_{max} = \left(\frac{L}{d}\right)_{basic} \times K_t \times K_c \times K_f$$

$$= (20 \times 0.99 \times 1 \times 1) = 19.8$$

$$\left(\frac{L}{d}\right)_{provided} = \left(\frac{5.35}{0.35}\right) = 15.3 < 19.8.$$

Hence deflection control is satisfied.

Method-2 (Using SP: 16 Design Tables)

a) Compute the parameter, $\left(\frac{M_u}{bd^2}\right) = \left(\frac{64.4 \times 10^6}{200 \times 350^2}\right) = 2.62$

b) Refer Table-2 of SP: 16 (Table-6.5 of text) and read out p_t corresponding to the parameter $(M_u/b\,d^2)$ and $f_y = 415 \text{ N/mm}^2$.

$$p_t = 0.892 = (100A_{st}/bd)$$

$$\therefore \quad A_{st} = [(0.892 \times 200 \times 350)/(100)] = 625 \text{mm}^2$$

c) Compute parameter (V_{us}/d) kN/cm

$$(V_{us}/d) = (7/35) = 0.2$$

Refer Table-62 of SP: 16 and read out the spacing of 6 mm diameter two legged mild steel stirrups, $S_v = 300$ mm.

But $S_{v,max} > 0.75\, d = (0.75 \times 350) = 262.5$ mm

Adopt a spacing of 250 mm

d) The detailing of reinforcements are shown in Fig. 8.2.

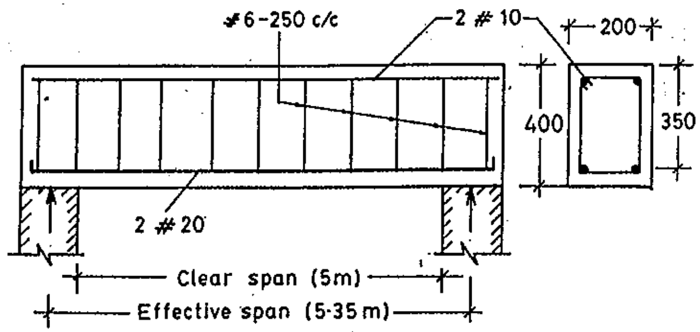


Fig. 8.2 Reinforcement Details in Singly Reinforced Rectangular Beam

8.4 DESIGN OF DOUBLY REINFORCED RECTANGULAR BEAMS

1) Design a reinforced concrete beam of rectangular section using the following data:

Data:

Effective span = 8 m

Working live load = 30 kN/m

Overall depth restricted to 650 mm
M-20 grade concrete and Fe-415 HYSD bars
Assuming width = b = 300 mm
Effective cover = d' = 50 mm
Effective depth = d = 600 mm

Method -1 (Using IS: 456 Code provisions)

a) Stresses

$f_{ck} = 20$ N/mm^2
$f_y = 415$ N/mm^2
Load factor = 1.5 for dead and live loads.

b) Loads

Self weight = $g = (0.3 \times 0.65 \times 25) = 4.875$ kN/m
Live load = $q = 30.000$ kN/m
Total working load = $w = 34.875$ kN/m
$\therefore$ Ultimate Design load = $w_u = (1.5 \times 34.875) = 52.3$ kN/m

c) Moments and Shear Forces

$M_u = (0.125 \times 52.3 \times 8^2) = 418$ kN.m
$V_u = (0.5 \times 52.3 \times 8) = 209$ kN

d) Limiting Moment of Resistance

$$M_{u,lim} = 0.138 f_{ck}\, b\, d^2 = (0.138 \times 20 \times 300 \times 600^2)\, 10^{-6} = 298 \text{ kN.m} < M_u$$

Hence design as doubly reinforced section.

e) Main Reinforcements

$$(M_u - M_{u,lim}) = (418 - 298) = 120 \text{ kN.m}$$

$$f_{sc} = \left[\frac{0.0035\,(x_{u,max} - d')}{x_{u,max}}\right] E_s$$

$$= \left[\frac{0.0035[(0.48 \times 600) - 50]}{(0.48 \times 600)}\right] (2 \times 10^5) = 578 \text{ N/mm}^2$$

But $f_{sc} \not> 0.87 f_y = (0.87 \times 415) = 361$ N/mm^2

$$\therefore \quad A_{sc} = \left[\frac{(M_u - M_{u,lim})}{f_{sc}\,(d - d')}\right] = \left[\frac{120 \times 10^6}{361 \times 550}\right] = 604 \text{ mm}^2$$

Provide 2 bars of 20 mm diameter (A_{sc} = 628 mm^2)

$$A_{st2} = \left[\frac{A_{sc} f_{sc}}{0.87 f_y}\right] = \left[\frac{604 \times 361}{0.87 \times 415}\right] = 604 \text{ mm}^2$$

$$A_{st1} = \left[\frac{0.36 f_{ck}\, b(x_{u,lim})}{0.87 f_y}\right] = \left[\frac{0.36 \times 20 \times 300 \times 0.48 \times 600}{0.87 \times 415}\right] = 1723 \text{ mm}^2$$

$$\therefore \quad A_{st} = (A_{st1} + A_{st2}) = (1723 + 604) = 2327 \text{ mm}^2$$

Provide 5 bars of 25 mm diameter [A_{st}= (5 × 491) = 2455 mm^2]

f) Shear Reinforcements

$$\tau_v = \left(\frac{V_u}{bd}\right) = \left(\frac{209 \times 10^3}{300 \times 600}\right) = 1.16 \text{ N/mm}^2$$

$$p_t = \left(\frac{100 A_{st}}{bd}\right) = \left(\frac{100 \times 2455}{300 \times 600}\right) = 1.36$$

Refer Table-19 (IS: 456) and read out $\tau_c = 0.70$ N/mm^2

Since $\tau_v > \tau_c$, shear reinforcements are required.

$$V_{us} = [V_u - (\tau_c.b.d)]$$
$$= [209 - (0.70 \times 300 \times 600)\, 10^{-3}]$$
$$= 83 \text{ kN}$$

Using 8mm diameter 2 legged stirrups

$$S_v = \left[\frac{0.87 f_y A_{sv}\, d}{V_{us}}\right] = \left[\frac{0.87 \times 415 \times 2 \times 50 \times 600}{83 \times 10^3}\right] = 261 \text{ mm}$$

$S_v \not> 0.75d = (0.75 \times 600) = 450$ mm

Also $S_v \not> 300$ mm
Adopt spacing S_v = 260 mm near the supports and gradually increasing to 300mm towards the centre of the span.

g) Check for Deflection Control

$$\left(\frac{L}{d}\right)_{actual} = \left(\frac{8000}{600}\right) = 13.33$$

$$\left(\frac{L}{d}\right)_{max} = \left(\frac{L}{d}\right)_{basic} \times K_t \times K_c \times K_f$$

$$p_t = 1.36 \quad \text{and} \quad p_c = \left(\frac{100 \times 628}{300 \times 600}\right) = 0.34$$

Refer Fig. 7.2, $K_t = 0.9$
Fig. 7.3, $K_c = 1.1$
Fig. 7.4, $K_f = 1.0$

$$\therefore \left(\frac{L}{d}\right)_{max} = [20 \times 0.9 \times 1.1 \times 1.0] = 19.8 > 13.33$$

Hence, deflection criteria is satisfied

Method-2 (Using SP: 16 Design Tables)

a) Main Reinforcements

$$[M_u / bd^2] = [(418 \times 10^6)/(300 \times 600^2)] = 3.87 \quad \text{and} \quad (d'/d) = 0.1$$

Refer Table-50 (SP: 16) and read out the values of p_t and p_c.

$$p_t = 1.297 \quad \text{and} \quad p_c = 0.359$$

$$A_{st} = \left[\frac{1.297 \times 300 \times 600}{100}\right] = 2335 \text{ mm}^2$$

$$A_{sc} = \left[\frac{0.34 \times 0.359 \times 600}{100}\right] = 646 \text{ mm}^2$$

The tension and compression reinforcements are similar to that obtained from method-1.

b) Shear reinforcements

$$(V_{us}/d) = (83/60) = 1.38 \text{ kN/cm}$$

Refer Table-62 (SP: 16). Using 8 mm diameter two legged stirrups, spacing is nearly 260 mm.

c) The reinforcement details in the beam is shown in Fig. 8.3.

2) A rectangular beam of span 7m (centre to centre of supports), 250mm wide by 550 mm deep is to carry a uniformly distributed load (excluding self weight) of 15 kN/m and a live load of 20 kN/m. Using M-20 grade concrete and Fe-415 HYSD bars, design the beam section at mid span. Check the adequacy of the section for shear and perform a check for deflection control.

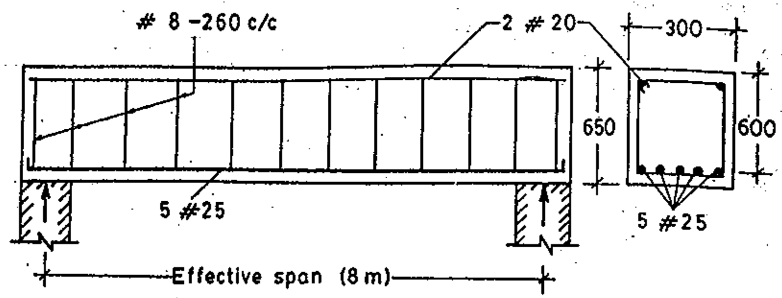


Fig. 8.3 Reinforcement Details in Doubly Reinforced Beam

a) Data:

b	= 250 mm	f_{ck}	= 20 N/mm^2
D	= 550 mm	f_y	= 415 N/mm^2
d	= 500 mm	q	= 20 kN/m
d'	= 50 mm	L	= 7 m
g_2	= 15 kN/m	L.F	= 1.5 for dead and live loads

b) Loads

Self weight Of beam	$= g_1 = (0.25 \times 0.55 \times 25)$	= 3.44 kN/m
Dead load	$= g_2 =$	= 15.00 kN/m
Live load	$= q$	= 20.00 kN/m
Finishes etc		= 1.56 kN/m
Total service load		= 40.00 kN/m

Ultimate design load $= w_u = (1.5 \times 40) = 60$ kN/m

c) Ultimate Moments and Shear Forces

$$M_u = (0.125 \times 60 \times 7^2) = 367.5 \text{ kN.m}$$
$$V_u = (0.5 \times 60 \times 7) = 210 \text{ kN}$$

d) Limiting moment of resistance

$$M_{u,lim} = 0.138 f_{ck}\, b\, d^2 = (0.138 \times 20 \times 250 \times 500^2)\, 10^{-6} = 172.5 \text{ kN.m} < M_u$$

Hence the beam is designed as doubly reinforced.

e) Tension and Compression Reinforcements

$$(M_u - M_{u,lim}) = (367.5 - 172.5) = 195 \text{ kN.m}$$

$$f_{sc} = \left[\frac{0.0035(x_{u,max} - d')}{x_{u,max}}\right] E_s$$

$$f_{sc} = \left[\frac{0.0035\{(0.48 \times 500) - 50\}}{(0.48 \times 500)}\right](2 \times 10^5)$$

$$= 554 \text{ N/mm}^2 \not> (0.87 \times 415) = 361 \text{ N/mm}^2$$

$$A_{sc} = \left[\frac{(M_u - M_{u,lim})}{f_{sc}(d - d')}\right] = \left[\frac{195 \times 10^6}{361 \times 450}\right] = 1200 \text{ mm}^2$$

Provide 4 bars of 20 mm diameter ($A_{sc} = 1256$ mm^2)

$$A_{st2} = \left[\frac{A_{sc} f_{sc}}{0.87 f_y}\right] = \left[\frac{1200 \times 361}{0.87 \times 415}\right] = 1200 \text{ mm}^2$$

$$A_{st1} = \left[\frac{0.36 f_{ck}\, b(0.48d)}{0.87 f_y}\right] = \left[\frac{0.36 \times 20 \times 250 \times 0.48 \times 500}{0.87 \times 415}\right] = 1197 \text{ mm}^2$$

$$\therefore \quad A_{st} = (A_{st1} + A_{st2}) = (1197 + 1200) = 2397 \text{mm}^2$$

Provide 4 bars of 28 mm diameter ($A_{st} = 2464$ mm^2)

$$p_t = \left(\frac{100A_{st}}{bd}\right) = \left(\frac{100 \times 2464}{250 \times 500}\right) = 1.97$$

$$p_c = \left(\frac{100A_{sc}}{bd}\right) = \left(\frac{100 \times 1256}{250 \times 500}\right) = 1.00$$

f) Shear reinforcements

$$\tau_v = \left(\frac{V_u}{bd}\right) = \left(\frac{210 \times 10^3}{250 \times 500}\right) = 1.68 \text{ N/mm}^2$$

For $p_t = 1.97$, read out from Table-19(IS: 456), $\tau_c = 0.79$ N/mm^2

$$V_{us} = [V_u - (\tau_c.b.d)] = [210 - (0.79 \times 250 \times 500)10^{-3}] = 111 \text{ kN}$$

Using 10 mm diameter two legged stirrups

$$S_v = \left[\frac{0.87 f_y A_{sv} d}{V_{us}}\right] = \left[\frac{0.87 \times 415 \times 2 \times 79 \times 500}{111 \times 10^3}\right] = 256 \text{ mm}$$

$$S_v \not> (0.75d) = (0.75 \times 500) = 375 \text{ mm and } S_v \not> 300 \text{ mm}$$

Provide spacing of stirrups = 250 mm.

g) Check for Deflection Control

$$\left(\frac{L}{d}\right)_{max} = \left(\frac{L}{d}\right)_{basic} \times K_t \times K_c \times K_f$$

For $p_t = 1.97$ and $p_c = 1.00$

From Fig. 7.2, $K_t = 0.82$

Fig. 7.3, $K_c = 1.24$

Fig. 7.4, $K_f = 1.00$

$$\therefore \quad \left(\frac{L}{d}\right)_{max} = (20 \times 0.82 \times 1.24 \times 1.00) = 20.33$$

$$\left(\frac{L}{d}\right)_{provided} = \left(\frac{7000}{5000}\right) = 14 < 20.33$$

Hence, deflection control check is satisfactory

h) The reinforcement details in the beam are shown in Fig. 8.4.

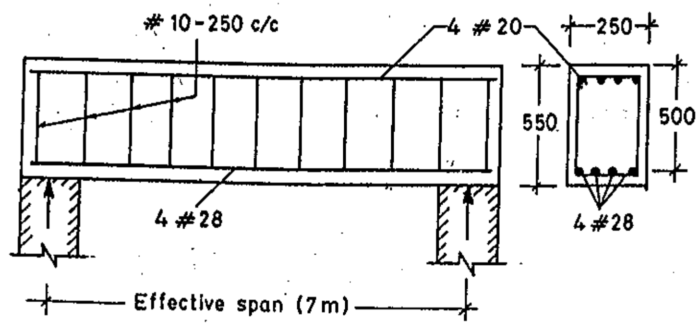


Fig. 8.4 Reinforcement Details in Doubly Reinforced Beam

8.5 DESIGN OF FLANGED BEAMS

8.5.1 Introduction

When a reinforced concrete slab is cast monolithically with the beam as in the case of beam supported floor slab system, the beams can be considered as flanged beams with slab acting as an effective flange on the compression side. It is important to note that continuous T or L beams act as flanged beams only between the supports where the bending moments are negative (sagging) and the slabs are on the compression side of the beam. In the vicinity of the supports where the bending moments are negative(hogging),the slab is on the tension side and hence the beam acts as a rectangular beam with the tension steel located in the slab portion of the beam . Hence at the locations of negative moments, the beams have to be designed as singly or doubly reinforced rectangular beams

8.5.2 Design Parameters

Fig. 8.5 shows the salient design parameters of flanged beams using the notations used in IS: 456 code.

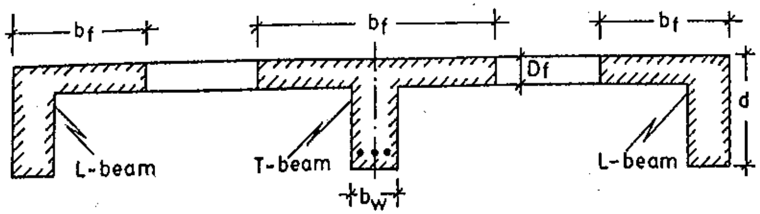


Fig. 8.5 Parameters of Flanged Beams

a) Effective width of Flange (b_f)

The effective width of flange should in no case be greater than the breadth of the web plus half the sum of the clear distances to the adjacent beams on either side.

i) For T-beams, $b_f = [(L_o/6) + b_w + 6\,D_f]$

ii) For L-beams, $b_f = [(L_o/12) + b_w + 3\,D_f]$

iii) For isolated beams, the effective flange width is computed as,

T-beam, $$b_f = \left\{\frac{L_o}{(L_o/b)+4} + b_w\right\}$$

T-beam, $$b_f = \left\{\frac{0.5L_o}{(L_o/b)+4} + b_w\right\}$$

Where

b_f = effective width of flange
L_o = distance between points of zero moments
b_w = breath of the web
D_f = thickness of flange and
b = actual width of flange

The effective width of flange is the assumed equivalent width of slab with uniform stress distribution shown in Fig. 8.6 to replace the actual

stress distribution, which is not suitable for computations.

b) Effective depth (d)

The basic span/effective depth ratios specified for beams and slabs in IS:456 code (Clause 23.2.1 or Table.7.1 of text) can as well be used for flanged beams along with the modification or reduction factor K_f (Refer Fig. 7.4 of text). However for purposes of design, the span/depth ratio of the trial section may be assumed in the range of 12 to 20 depending upon the span range and type of loading as given in Table-8.1.

c) Width of web (b_w)

The width of web generally depends upon the width of column on which the beam is supported. The normal range of values being 150 mm to 400 mm.

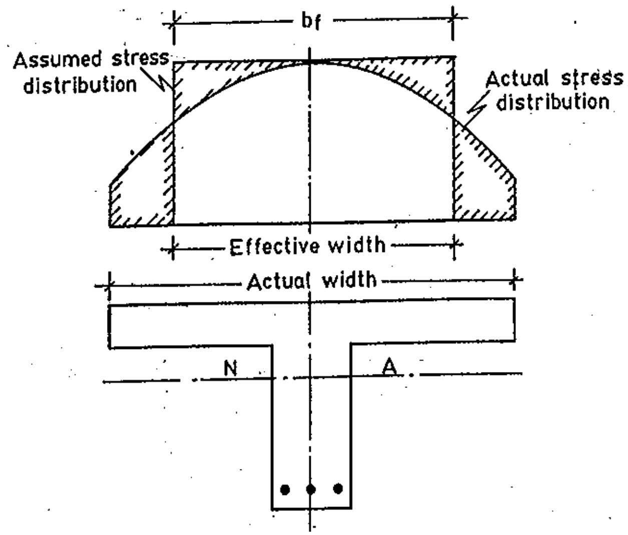


Fig. 8.6 Compressive Stress Distribution in Flange of Tee Beams

d) Thickness of flange (D_f)

The thickness of flange is governed by the thickness of slab which is continuous over T-beams. The slab thickness is influenced by the spacings of T-beams and the type of loading and generally governed by the basic span/depth ratios specified in the IS: 456 code.The thickness of flange generally varies from 100 to 200 mm.

e) Minimum and Maximum Reinforcement in Flanged Beams

The minimum percentage of reinforcement to be provided in a flanged beam as per IS: 456 code clause 26.5.1.1 is to be computed using the width of web and effective depth. The code recommendation gives the minimum reinforcement as

$$\left(\frac{A_s}{b_w d}\right) = \left(\frac{0.85}{f_y}\right)$$

Using Fe-415 grade HYSD bars, the minimum percentage works out to about 0.2 percent. Also, the maximum percentage of tension reinforcement in T-beams (based on web width) is limited to 4 percent.

8.5.3 Expressions for M_u and A_{st} for Preliminary Design

The moment of resistance of a T-beam section can be expressed by a simple equation by assuming the neutral axis to coincide with the bottom of the flange and the lever arm length is (d – 0.5 D_f). Accordingly we have the relation,

$$M_u = 0.36 f_{ck}\, b_f\, D_f\, (d - 0.5\, D_f)$$

The area of flange width required for the neutral axis to be at the soffit of the flange can be expressed as

$$b_f = \left[\frac{2M_u}{0.36 f_{ck} D_f (2d - D_f)}\right] \quad \text{or} \quad b_f = \left[\frac{5.56\, M_u}{f_{ck} D_f (2d - D_f)}\right]$$

This required width is compared with the effective width of T-beam to ascertain whether the neutral axis will be within the flange or below the flange. An approximate estimate of tension reinforcement required to resist the factored moment M_u can be evaluated using the simple expression,

$$A_{st} = \left[\frac{M_u}{0.87 f_y (d - 0.5 D_f)}\right]$$

8.5.4 Transverse Reinforcement

The compression flange of a flanged beam should be adequately reinforced for effective T-beam action. According to IS: 456 code clause 23.1.1, the transverse reinforcement in the slab should be not less than 60 percent of the main reinforcement at mid span of the slab and the reinforcement should extend to a length of at least one fourth of the span of the slab on either side of the beam.

For example, the transverse reinforcement in a T-beam with D_f = 150mm and with the area of main reinforcement of the slab(parallel to the beam) at its middle is 500 mm²/m, the transverse reinforcement required according to the provisions of the IS:456 code will be equal to (0.6 × 500) = 300 mm²/m.

8.5.5 Design of T-beams Using SP: 16 Design Tables

The limiting moment capacity of a T-beam section (by failure of concrete in compression) can be computed by the design equations presented in section 6.3. The moment of resistance factor based on the IS: 456 code provisions have been tabulated in SP: 16 using the parameter $[(M_{u,lim}) / (f_{ck}\, b_w\, d^2)]$ for different values of the ratios (b_f/b_w) and (D_f/d) in Tables-58 and 59 of SP: 16 for the two different grades of steel Fe-415 and Fe-500 respectively.(Tables 6.8 and 6.9 of text.)

The amount of reinforcement required to resist the limiting moment is not given in these tables. However the tables are useful to estimate the maximum moment the T-beam can resist as singly reinforced beams with failure of concrete in compression. In most cases of practical design examples, the moment capacity of the T-beam will be considerably greater than the applied moment and the steel requirement will be much lower than that required for the limiting moment. Hence the tables are useful only to check the capacity of the concrete section. In cases where the neutral axis falls within the flange, the section can be treated as rectangular beam and the percentage of reinforcement can be readily obtained from Tables-1 to 4 of SP: 16 for different grades of concrete and steel.

8.5.6 Design Procedure for L-beams

In the case of beam and slab floor systems, beams are spaced at regular intervals. The interior beams which are cast monolithic with slabs on both sides of the rib behave as T-beams. The edge beams which are cast monolithic with slabs on one side of the rib only are termed as L-beams. Due to eccentricity of the load coming on the flange as shown in Fig. 8.7, torsional moments are induced in the L-beams in addition to the bending moments and shear forces.

Torsional and hogging bending moments are maximum in the vicinity of the column supports where the L-beam is built into the column while the sagging moments are maximum at the centre of span. The support section of the L-beam is the most critical section which is designed for combined torsion, bending and shear forces according to the IS: 456 code provisions.

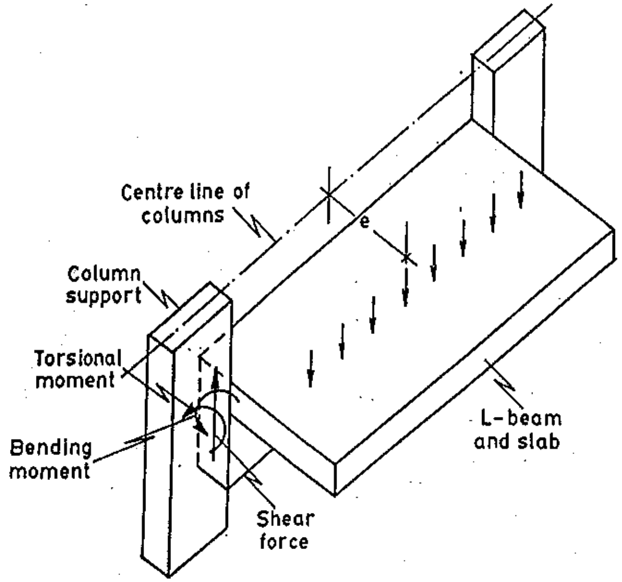


Fig. 8.7 Moments and Forces at Support Section of L-Beam

8.5.7 Design Examples

1) A T-beam slab floor of reinforced concrete has a slab 150 mm thick spanning between the T-beams which are spaced 3 m apart. The beams have a clear span of 10 m and the end bearings are 450 mm thick walls. The live load on the floor is 4 kN/m^2. Using M-20 grade concrete and Fe-415 HYSD bars, design one of the intermediate T-beams.

Method-1 (using IS: 456 code equations)

a) Data:

Clear span = 10 m
Bearing thickness = 450 mm
Working live load = q = 4 kN/m^2
Spacing of T-beams = 3 m

f_{ck} = 20 N/mm^2
f_y = 415 N/mm^2
D_f = 150 mm

b) Cross sectional Dimensions

Assuming effective depth $= \left(\dfrac{\text{span}}{15}\right) = \left(\dfrac{10 \times 10^3}{15}\right) = 666$ mm

Adopt $d = 700$ mm, $D = 750$ mm and $b_w = 300$ mm

c) Effective span

The least value of
i) Centre to centre of bearings = (10 + 0.45) = 10.45 m
ii) Clear span + effective depth = (10 + 0.70) = 10.70 m
∴ Effective span = L = 10.45 m

d) Loads

Self weight of slab = (0.15 × 25 × 3) = 11.25 kN/m
Floor finish = (0.6 × 3) = 1.80 kN/m
Self weight of rib = (0.3 × 0.6 × 25)= 4.50 kN/m
Plaster finishes = 0.45 kN/m
Total dead load = g = 18.00 kN/m
Live load = q = (4 × 3) = 12.00 kN/m
Design Ultimate load = w_u = 1.5 (18 + 12) = 45 kN/m

e) Ultimate Moments and Shear Forces

$$M_u = (0.125 \times 45 \times 10.45^2) = 614 \text{ kN.m}$$
$$V_u = (0.5 \times 45 \times 10.45^2) = 235 \text{ kN}$$

f) Effective width of Flange (b_f)

$$b_f = [(L_o/6) + b_w + 6\,D_f\,]$$
$$= [(10.45/6) + 0.3 + (6 \times 0.15)] = 2.94 \text{ m} < 3.00 \text{ m}$$
$$\therefore\ b_f = 2940 \text{ mm}$$

g) Moment capacity of Flange section (M_{uf})

$$M_{uf} = 0.36 f_{ck}\, b_f\, D_f\,(d - 0.42\, D_f\,)$$
$$= [0.36 \times 20 \times 2940 \times 150)\,(700 - 0.42 \times 150)]$$
$$= (2022 \times 10^6) \text{ N.mm}$$
$$= 2022 \text{ kN.m}$$

Since $M_u < M_{uf}$, $x_u < D_f$
Hence the section is treated as rectangular with $b = b_f$ for designing reinforcements.

h) Tension Reinforcements

$$M_u = (0.87\, f_y\, A_{st}\, d)\left[1 - \left(\frac{A_{st}\, f_y}{b\; d\; f_{ck}}\right)\right]$$

$$(614 \times 10^6) = (0.87 \times 415 \times A_{st} \times 700)\left[1 - \left(\frac{415 A_{st}}{2940 \times 700 \times 20}\right)\right]$$

Solving $A_{st} = 2492\ \text{mm}^2$
Provide 2 bars of 32 mm diameter and 2 bars of 25 mm diameter
Total $A_{st} = 2590\ \text{mm}^2$.

i) Shear reinforcements

$$\tau_v = \left(\frac{V_u}{b_w\, d}\right) = \left(\frac{235 \times 10^3}{300 \times 700}\right) = 1.19\ \text{N/mm}^2$$

$$p_t = \left(\frac{100\, A_{st}}{b_w\, d}\right) = \left(\frac{100 \times 2590}{300 \times 700}\right) = 1.23$$

Refer Table-19 (IS: 456) and read out the value of $\tau_c = 0.67\ \text{N/mm}^2$.

Balance Shear $= V_{us} = [V_u - (\tau_c\, b_w\, d)]$
$= [235 - (0.67 \times 300 \times 700)]\ 10^{-3} = 95\ \text{kN}$

Using 8 mm diameter 2 legged stirrups, spacing is given by

$$S_v = \left[\frac{0.87 \times 415 \times 2 \times 50 \times 700}{95 \times 10^3}\right] = 266\ \text{mm}$$

Provide a spacing of 250 mm at supports and gradually increasing to 300 mm at centre of span.

j) Check for Deflection Control

$$p_t = \left(\frac{100\, A_{st}}{b_f\, d}\right) = \left(\frac{100 \times 2590}{2940 \times 700}\right) = 0.126$$

$$(b_w/b_f) = (300/2940) = 0.102$$

Refer Fig. 7.3 and read out $K_t = 2.0$
Fig. 7.3 and read out $K_c = 1.0$
Fig. 7.4 and read out $K_f = 0.8$

$$\left(\frac{L}{d}\right)_{max} = \left(\frac{L}{d}\right)_{basic} \times K_t \times K_c \times K_f$$

$$= (20 \times 2.0 \times 1.0 \times 0.8) = 32$$

$$\left(\frac{L}{d}\right)_{provided} = \left(\frac{10450}{700}\right) = 14.92 < 32$$

Hence, check for deflection control is satisfactory.

k) Details of reinforcements

The reinforcement details in the T-beam are shown in Fig. 8.8.

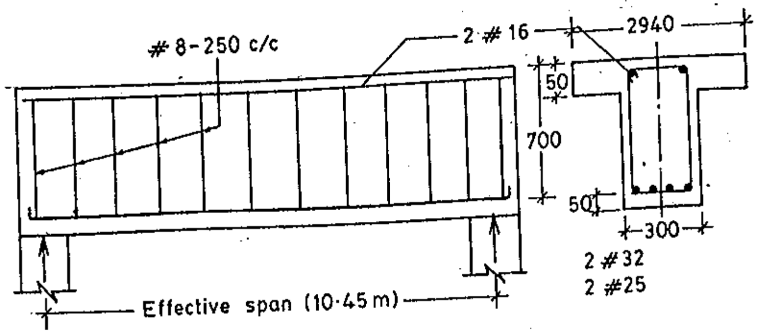


Fig. 8.8 Reinforcement Details in Tee-Beam

Method-2 (Using SP: 16 Design Charts)

a) Tension Reinforcement

Since $x_u < D_f$, the section can be treated as rectangular and Table-2 of SP:16 can be used.

$$\left(\frac{M_u}{b_f\, d^2}\right) = \left(\frac{614 \times 10^6}{2940 \times 700^2}\right) = 0.42 \quad \text{Also} \quad p_t = 0.120 = \left(\frac{100 A_{st}}{b_f d}\right)$$

$$\therefore \quad A_{st} = \left(\frac{0.120 \times 2940 \times 700}{100}\right) = 2470\ \text{mm}^2$$

b) Shear reinforcements

$$\left(\frac{V_{us}}{d}\right) = \left(\frac{95}{70}\right) = 1.35\ \text{kN/cm}$$

Using 8 mm diameter 2 legged stirrups and referring to Table-62 (SP:16)
Spacing $S_v = 26\ \text{cm} = 260\ \text{mm}$.
The values of tension and shear reinforcements are almost the same as that obtained by method-1.

3) Design a L-beam for an office room floor to suit the following data.

a) **Data**

Clear span = 6 m
Centre to centre of supports = 6.3 m
The L-beams are monolithic with R.C. columns
Spacing of beams = 2.75 m c/c
Loading (Office floor) = 4 kN/m^2
Thickness of slab = 100 mm
Width of column = 300 mm
Materials: M-20 grade concrete and Fe-415 HYSD bars.

b) **Cross sectional Dimensions**

Since L-beam is subjected to flexure, torsion and shear forces, assume a trial section having span/depth ratio of 12.

$$\therefore \quad d = \left(\frac{6300}{12}\right) = 525 \text{ mm}$$

Hence adopt $d = 550$ mm, $D = 600$ mm and $b_w = 300$ mm.

c) **Effective span**

Effective span is the least of

i) Centre to centre of supports = 6.3 m
ii) Clear span + effective depth = (6 + 0.55) = 6.55 m. Hence, $L = 6.3$ m

d) **Loads**

Dead load of slab = $(0.1 \times 25 \times 0.5 \times 2.75) = 3.43$ kN/m
Floor finish = $(0.6 \times 0.5 \times 2.75) = 0.83$ kN/m
Self weight of rib = $(0.5 \times 0.3 \times 25) = 3.75$ kN/m
Live load = $(4 \times 0.5 \times 2.75) = 5.50$ kN/m
Plaster finishes etc = 0.49 kN/m
Total working load = w = 14.00 kN/m

e) **Effective Flange width**

Effective flange width (b_f) is the least of the following values.

i) $b_f = [(L_o/12) + b_w + 3\,D_f]$
$= [(6300/12) + 300 + (3 \times 100)] = 1125$ mm
ii) $b_f = b_w + 0.5$ (spacing between the ribs)
$= 300 + (0.5 \times 2450) = 1525$ mm
$\therefore\ b_f = 1125$ mm

f) **Ultimate bending moment and shear forces**

At support section,

$M_u = 1.5\,(w\,L^2/12) = 1.5\,(14 \times 6.3^2)/12 = 70$ kN.m
$V_u = 1.5\,(0.5\,wL) = 1.5\,(0.5 \times 14 \times 6.3) = 66$ kN

At centre of span section,

$M_u = 1.5\,(w\,L^2/24) = 1.5\,(14 \times 6.3^2)/24 = 35$ kN.m

g) **Torsional moments at support section**

Torsional moment is produced due to dead load of slab and live load on it.

(Working load/m–rib self weight) = (14–3.75) = 10.25 kN/m
$\therefore$ Total ultimate load on slab = $1.5\,(10.25 \times 6.3) = 97$ kN
Total ultimate shear force = $(0.5 \times 97) = 48.5$ kN
Distance of centroid of shear force from the centre line of beam (Refer Fig. 8.9) = 0.4125 m
Ultimate torsional moment is computed as

$$T_u = (48.5 \times 0.4125) = 20 \text{ kN.m}$$

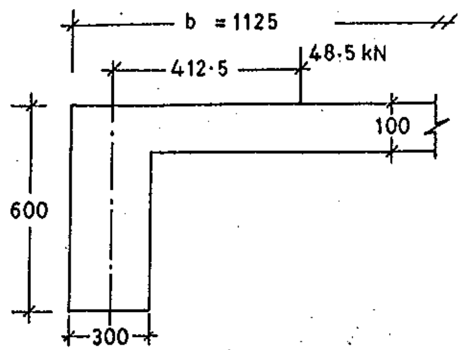


Fig. 8.9 L-Beam Loads (Support section)

h) Equivalent Bending moment and Shear force

According to IS: 456-2000 code clause 41.4.2, at the support section, the equivalent bending moment is computed as

$$M_{e1} = (M_u + M_t)$$

Where $M_t = T_u\left[\frac{1+(D/b)}{1.7}\right] = 20\left[\frac{1+(600/300)}{1.7}\right] = 35.3 \text{ kN/m} < M_u$

$\therefore \quad M_{e1} = (70 + 35.3) = 105.3 \text{ kN.m}$

Equivalent shear force (Clause 41.3.1 of IS:456) is computed as

$$V_e = V_u + 1.6(T_u/b) = [66 + 1.6(20/0.3)] = 173 \text{ kN}$$

i) Main Longitudinal reinforcements

Support section is designed as rectangular section to resist the hogging equivalent moment $M_{e1} = 105.3$ kN.m

$$M_{u,lim} = (0.138 f_{ck}\, b.d^2) = (0.138 \times 20 \times 300 \times 550^2)10^{-6} = 250 \text{ kN.m}$$

Since $M_{e1} < M_{u,lim}$, the section is underreinforced and $x_u < x_{u,lim}$.

$$\therefore \quad M_{e1} = 0.87 f_y A_{st} d\left[1 - \left(\frac{A_{st} f_y}{bd\, f_{ck}}\right)\right]$$

$$(105.3 \times 10^6) = (0.87 \times 415 \times A_{st} \times 550)\ 1 - \left(\frac{415 A_{st}}{(300 \times 550 \times 20)}\right)$$

Solving $A_{st} = 572 \text{ mm}^2 > A_{st(min)}$

Provide 2 bars of 20 mm diameter on the tension face (top) ($A_{st} = 628$ mm^2) Area of steel required at centre of span section to resist a moment M_u = 35 kN.m will be less than the minimum given by

$$A_{st(min)} = \left(\frac{0.85 b_w d}{f_y}\right) = \left(\frac{0.85 \times 300 \times 550}{415}\right) = 338 \text{ mm}^2$$

Provide 2 bars of 16 mm diameter at the bottom face ($A_{st} = 402$ mm^2).

j) Side face reinforcement

According to clause 26.5.1.7 of IS: 456 code, side face reinforcement of 0.1 percent of web area is to be provided for members subjected to torsion, when the depth exceeds 450 mm.

$\therefore$ Area of reinforcement $= (0.1 \times 300 \times 600)/100 = 180 \text{ mm}^2$

Provide 8 mm diameter bars (4 numbers) two on each face as horizontal reinforcement spaced 200 mm centres.

k) Shear Reinforcement

$$\tau_{ve} = \left(\frac{V_e}{b_w d}\right) = \left(\frac{173 \times 10^3}{300 \times 550}\right) = 1.05 \text{ N/mm}^2$$

$$p_t = \left(\frac{100 A_{st}}{b_w d}\right) = \left(\frac{100 \times 628}{300 \times 550}\right) = 0.38$$

From Table-19 (IS: 456 Code) read out $\tau_c = 0.42 \text{ N/mm}^2 < \tau_{ve}$

Hence shear reinforcements are required. Using 10 mm diameter two legged stirrups with side covers of 25 mm and top and bottom covers of 50 mm, we have

$$b_1 = 250 \text{ mm}, \quad d_1 = 500 \text{ mm}, \quad A_{sv} = (2 \times 78.5) = 157 \text{ mm}^2$$

The spacing S_v is computed using the equations specified in clause 41.4.3 of IS: 456 code.

$$S_v = \left\{\frac{0.87 f_y A_{sv} d_1}{\left(\frac{T_u}{b_1}\right) + \left(\frac{V_u}{2.5}\right)}\right\} \quad \text{or} \quad S_v = \left\{\frac{A_{sv} 0.87 f_y}{(\tau_{ve} - \tau_c) b}\right\}$$

$$S_v = \left\{\frac{(0.87 \times 415 \times 157 \times 500)}{\left(\frac{20 \times 10^6}{250}\right) + \left(\frac{66 \times 10^3}{2.5}\right)}\right\} = 266 \text{ mm}$$

Or $$S_v = \left\{\frac{157 \times 0.87 \times 415}{(1.05 - 0.42)300}\right\} = 300 \text{ mm}$$

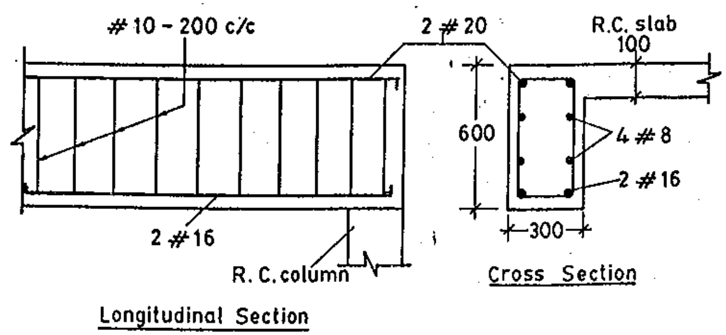


Fig. 8.10 Reinforcement Details in L-Beam

Provide 10 mm diameter two legged stirrups at a minimum spacing given by the clause 26.5.1.7 of IS: 456 code.

i) $x_1 =$ [illegible] $+ 10) = 280$ mm
ii) $[(x_1 + y_1)/4] = [(280 + 530)/4] = 202$ mm
iii) 300 mm

Adopt minimum spacing $S_v = 200$ mm.
Details of reinforcements are shown in Fig. 8.10.

l) Check for Deflection Control

$$p_t = 1.38 \quad \text{and} \quad p_c = [(100\,A_{sc}/b_w d)] = [(100 \times 402)/(300 \times 550)] = 0.24$$

$$(b_w/b_f) = (300/1125) = 0.266$$

Refer Fig. 7.2, $K_t = 1.30$
Fig. 7.3, $K_c = 1.07$
Fig. 7.4, $K_f = 0.80$

$$\left(\frac{L}{d}\right)_{max} = \left(\frac{L}{d}\right)_{basic} \times K_t \times K_c \times K_f$$

$$= (20 \times 1.30 \times 1.07 \times 0.8) = 22.2$$

$$\left(\frac{L}{d}\right)_{provided} = \left(\frac{6300}{550}\right) = 11.45 < 22.2$$

Hence, the check for deflection control is satisfactory.

8.6 DESIGN OF CANTILEVER BEAMS

Design a cantilever beam to suit the following data:

a) Data

Clear span = 2.5 m
Working live load = 20 kN/m
Cantilever beam is monolithic with R.C. column 300 mm wide and 450 mm deep
$f_{ck} = 20$ N/mm^2 and $f_y = 415$ N/mm^2

b) Cross sectional dimensions

For cantilever beams, the trial section is based on the (span/depth) ratio of 7.

$\therefore$ depth = (span/7) = (2500/7) = 357 mm.
Adopt effective depth = $d = 400$ mm
Overall depth = $D = 450$ mm
Width = $b = 300$ mm

c) Loads

Self weight of beam = $(0.3 \times 0.45 \times 25) = 3.375$ kN/m
Live load = 20.000 kN/m
Finishes = 0.625 kN/m
Total working load = w = 24.000 kN/m

d) Ultimate moments and shear forces

$$M_u = 1.5[0.5wL^2] = 1.5[0.5 \times 24 \times 2.5^2] = 112.5 \text{ kN.m}$$

$$V_u = 1.5[w\,L] = 1.5\,[24 \times 2.5] = 90 \text{ kN}$$

e) Main Reinforcements

$$M_{u,\,lim} = (0.138\,f_{ck}\,b\,d^2)$$

$$= (0.138 \times 20 \times 300 \times 400^2)10^{-6}$$

$$= 132 \text{ kN.m}$$

Since $M_u < M_{u,lim}$, the section is under reinforced.

$$\therefore \quad M_u = 0.87\,f_y\,A_{st}\,d\left[1 - \left(\frac{A_{st}\,f_y}{b\,d\,f_{ck}}\right)\right]$$

$$= (0.87 \times 415\,A_{st} \times 400)\left[1 - \left(\frac{415 A_{st}}{300 \times 400 \times 20}\right)\right]$$

Solving $A_{st} = 928$ mm^2

Provide 3 bars of 20 mm diameter ($A_{st} = 942$ mm^2) on the tension face (top) and 2 bars of 10 mm diameter as hanger bars on the compression face.

f) Shear Reinforcements

$$\tau_v = \left(\frac{V_u}{bd}\right) = \left(\frac{90 \times 10^3}{300 \times 400}\right) = 0.75 \text{ N/mm}^2$$

$$p_t = \left(\frac{100 A_{st}}{bd}\right) = \left(\frac{100 \times 942}{300 \times 400}\right) = 0.785$$

Refer Table-19 (IS: 456) and read out $\tau_c = 0.56\ \text{N/mm}^2 < \tau_v$
Hence, shear reinforcements are required.

Balance shear $= V_{us} = [V_u - \tau_c\, b\, d] = [90 - (0.56 \times 300 \times 400)]10^{-3} = 23\ \text{kN}$

Using 8 mm diameter 2 legged stirrups, the spacing S_v of the stirrups is given by

$$S_v = \left[\frac{0.87 f_y A_{sv} d}{V_{us}}\right] = \left[\frac{0.87 \times 415 \times 2 \times 50 \times 400}{23 \times 10^3}\right] = 631\text{mm}$$

But $S_v \ngtr (0.75\,d) = (0.75 \times 400) = 300$ mm
Adopt 8 mm diameter 2 legged stirrups at 300 mm centres.

g) Anchorage length at supports

Anchorage length required is given by

$$L_d = \left(\frac{0.87 f_y \phi}{4\,\tau_{bd}}\right) = \left(\frac{0.87 \times 415 \times 20}{4 \times 1.2 \times 1.6}\right) = 940\text{mm}$$

The main tension bars are extended into the column to a length of 400 mm and bent at 90° and extended up to 500 mm as shown in Fig. 8.11.

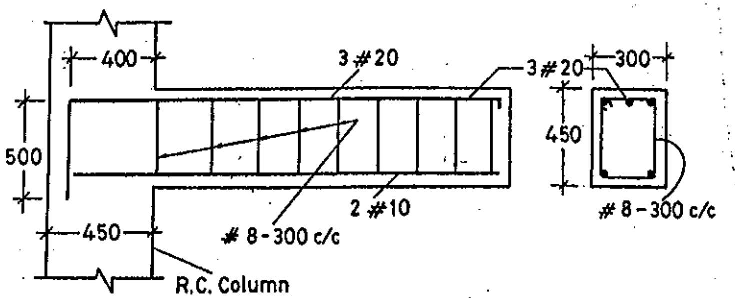


Fig. 8.11 Reinforcement Details in Cantilever Beam

h) Check for Deflection Control

$p_t = 0.785$, From Figs. 7.2, 7.3 and 7.4 read out the modification factors

$K_t = 1.025$, $K_c = 1.0$ and $K_f = 1.0$

$$\left(\frac{L}{d}\right)_{\max} = \left(\frac{L}{d}\right)_{\text{basic}} \times K_t \times K_c \times K_f = (7 \times 1.025 \times 1.0 \times 1.0) = 7.715$$

$$\left(\frac{L}{d}\right)_{\text{provided}} = \left(\frac{2500}{400}\right) = 6.25 < 7.175$$

Hence, the deflection criteria is satisfied.

i) The reinforcement details in the cantilever beam are shown in Fig. 8.11.

8.7 DESIGN OF CONTINUOUS BEAMS

8.7.1 Bending moment and Shear Force Coefficients

In the case of multistoried reinforced concrete framed structures, the floor slabs are cast monolithic with secondary and main beams continuous over several spans supported on columns at regular intervals. The continuous beams framing into the columns are designed for maximum bending moments and shear forces developed due to dead and superimposed loads.

Rigorous analysis of moments and shear forces in continuous beams is generally made by using the classical methods such as moment distribution method, Kani's rotation contribution method, stiffness or flexibility matrix methods which involve lengthy computations. However the IS: 456-2000 code permits the use of moment and shear force coefficients shown in Tables 8.2 and 8.3 of the text (Tables 12 and 13 of IS: 456 code) for computing the design bending moments and shear forces in continuous beams supporting substantially uniformly distributed loads over three or more spans which do not differ by more than 15 percent of the longest span. However redistribution of moments are not permitted when using these coefficients.

8.7.2 Effective Span

According to IS: 456-2000 code clause 22.2, for a continuous beam having a support width less than 1/12 clear span, the effective span shall be as per freely supported beams, i.e. clear span plus the effective depth or centre to centre of supports whichever is less. If the supports are wider than 1/12 the clear span or 600 mm whichever is less, the effective span is computed using the following specifications.

a) For end span with one end fixed and the other continuous or for intermediate spans, the effective span shall be the clear span between the supports.

b) For end span with one end free and the other end continuous, the effective span shall be equal to the clear span plus half the effective

Table 8.2 Bending Moment Coefficients
(Clause 22.5.1 of IS:456-2000)

Type of Load	Span Moments		Support Moments	
	Near middle of end span	At middle of interior span	At support next to end support	At other interior supports
Dead load and Imposed load (fixed)	$+\frac{1}{12}$	$+\frac{1}{16}$	$-\frac{1}{10}$	$-\frac{1}{12}$
Imposed load (not fixed)	$+\frac{1}{10}$	$+\frac{1}{12}$	$-\frac{1}{9}$	$-\frac{1}{9}$
Note:- For obtaining the bending moment, the coefficient shall be multiplied by the total design load and effective span				

Table 8.3 Shear Force Coefficients

Type of Load	At End Support	At support next to the end support		At all other interior supports
		Outer side	Inner Side	
Dead load and imposed load (fixed)	0.4	0.6	0.55	0.5
Imposed load (not fixed)	0.45	0.6	0.6	0.6
Note:- For obtaining the shear force, the coefficient shall be multiplied by the total design load				

depth of beam or the clear span plus half the width of the discontinuous support whichever is less.

c) In the case of spans with roller and rocker bearings, the effective span shall always be the distance between the centers of bearings.

d) In the case of continuous monolithic frames, the effective span of continuous beams are taken as the centre line distance between the members

8.7.3 Span/Depth Ratio

The span to effective depth ratios has an important influence on the deflection characteristics of the beams. The vertical deflection limit of span/250 specified in IS: 456 code (Clause 23.2) may generally be assumed to be satisfied if the basic span/depth ratio of continuous members are not greater than 26. For spans greater than 10 m, this value is multiplied by the ratio of 10/span in metres.

In general, continuous beams carry heavy dead and superimposed loads and consequently the span/depth ratios recommended in practical designs are normally between 10 to 15. The use of upper limit of

span/depth ratio of 26, results in shallow depths requiring high percentages of tensile and compressive reinforcements tending towards over reinforced sections. The span/depth ratios are modified using the modification factors explained in section 7.3.3.

8.7.4 Design Example

1) Design a continuous reinforced concrete beam of rectangular section to support a dead load of 10 kN/m and live load of 12 kN/m over 3 spans of 6 m each. The ends are simply supported. Adopt M-20 grade concrete and Fe-415 HYSD bars. Sketch the details of reinforcements in the beam.

a) Data

Effective span = 6 m $\qquad f_{ck} = 20\ \text{N/mm}^2$
Dead load = 10 kN.m $\qquad f_y = 415\ \text{N/mm}^2$
Live load= 12 kN/m
Concrete : M-20 Grade
Steel: Fe-415 HYSD bars

b) Cross sectional Dimensions

As the continuous beam supports heavy loads, span/depth ratio is assumed as 10

$\therefore$ Effective depth $= d = \left(\frac{\text{span}}{10}\right) = \left(\frac{6000}{10}\right) = 600$ mm

$\therefore$ Adopt $d = 600$ mm
$D = 650$ mm
$b = 300$ mm

Cover to tension steel = 50 mm

c) Loads

Self weight of beam $= (0.3 \times 0.65 \times 25) = 4.875$ kN/m
Dead load = 10.000 kN/m
Finishes = 0.125 kN/m
Total Dead load $= g = 15.000$ kN/m
Live load $= q = 12$ kN/m

d) Bending Moments and Shear Forces

Referring to the bending moment and shear force coefficients (Tables-8.2 and 8.3)

Negative B.M at interior support is computed as,

$$M_u(-ve) = 1.5\left[\frac{gL^2}{10} + \frac{qL^2}{9}\right] = 1.5\left[\frac{15\times 6^2}{10} + \frac{12\times 6^2}{9}\right] = 153 \text{ kN.m}$$

Positive B.M at centre of span is computed as,

$$M_u(+ve) = 1.5\left[\frac{gL^2}{12} + \frac{qL^2}{10}\right] = 1.5\left[\frac{15\times 6^2}{12} + \frac{12\times 6^2}{10}\right] = 132.3 \text{ kN.m}$$

Maximum Shear force at the support section is given by

$$V_u = 0.6\,L\,(g+q)\,1.5 = (0.6\times 6)\,(15+12)\,1.5 = 145.8 \text{ kN}$$

e) Limiting Moment of Resistance

$$M_{u,lim} = (0.138 f_{ck}\, b\, d^2) = (0.138\times 20\times 300\times 600^2)\,10^{-6} = 298 \text{ kN.m}$$

Since $M_u < M_{u,lim}$, the section is under reinforced.

f) Main Reinforcements

$$M_u = (0.87 f_y A_{st}\, d)\left[1 - \left(\frac{A_{st} f_y}{bd\, f_{ck}}\right)\right]$$

$$(153\times 10^6) = (0.87\times 415\, A_{st}\times 600)\left[1 - \left(\frac{415 A_{st}}{300\times 600\times 20}\right)\right]$$

Solving $A_{st} = 780 \text{ mm}^2$

Using 2 bars of 25 mm diameter on the tension side ($A_{st} = 982 \text{ mm}^2$) at supports. For positive bending moment the area of steel required is 675 mm². Hence provide 2 bars of 22 mm diameter on the tension face at mid span.

g) Shear Reinforcements

$$\tau_v = \left(\frac{V_u}{bd}\right) = \left(\frac{145.8\times 10^3}{300\times 600}\right) = 0.81 \text{ N/mm}^2$$

$$p_t = \left(\frac{100\, A_{st}}{bd}\right) = \left(\frac{100\times 982}{300\times 600}\right) = 0.54$$

Refer Table-19 (IS: 456) and read out the permissible shear stress in concrete as $\tau_c = 0.49 \text{ N/mm}^2$. Since $\tau_v > \tau_c$, shear reinforcements are to be designed to resist the balance shear computed as

$$V_{us} = [145.8 - (0.49\times 300\times 600)10^{-3}] = 57.6 \text{ kN}$$

Using 8 mm diameter 2 legged stirrups, the spacing near supports is

$$S_v = \left[\frac{0.87 f_y A_{sv} d}{V_{us}}\right] = \left[\frac{0.87\times 415\times 2\times 50\times 600}{57.6\times 10^3}\right] = 376 \text{ mm}$$

Adopt 8 mm diameter 2 legged stirrups at 300 mm centres throughout the beam.

h) Check for Deflection Control

$p_t = 0.54$. From Fig. 7.2, read out the modification factor $K_t = 1.2$

Neglecting hanger bars, $K_c = 1.0$ and $K_f = 1.0$

$$\therefore \quad \left(\frac{L}{d}\right)_{max} = \left(\frac{L}{d}\right)_{basic} \times K_t \times K_c \times K_f$$

$$= (26\times 1.2\times 1.0\times 1.0) = 31$$

$$\left(\frac{L}{d}\right)_{actual} = \left(\frac{6000}{600}\right) = 10 < 31$$

i) The reinforcement details in the continuous beam are shown in Fig 8.12.

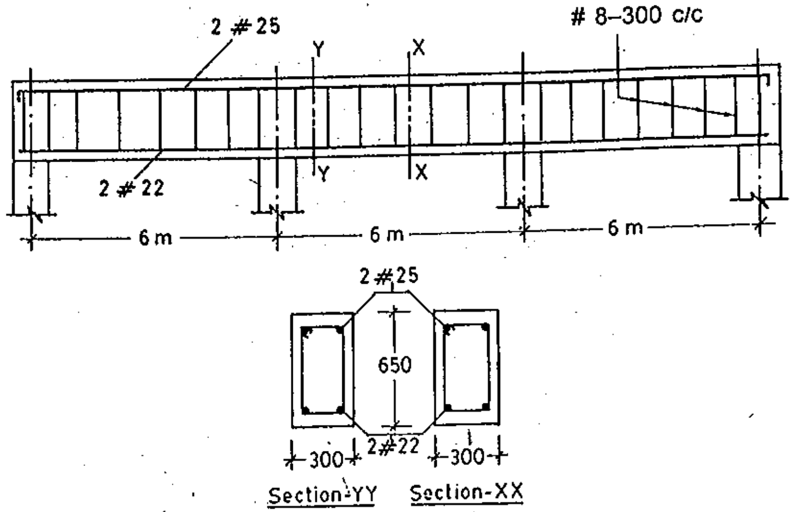


Fig. 8.12 Reinforcement Details in Continuous Beam

8.8 EXAMPLES FOR PRACTICE

1) Design a singly reinforced concrete beam to suit the following data.
Clear span = 4 m
Width of supports = 300 mm brick walls
Service live load = 5 kN/m
Materials: M-20 grade concrete
Fe-415 HYSD bars

2) A reinforced concrete beam is to be designed over an effective span of 5 m to support a design service live load of 8 kN/m. Adopt M-20 grade concrete and Fe-415 HYSD bars and design the beam to satisfy the limit states of collapse and Serviceability.

3) Design a reinforced concrete beam of rectangular section using the following data:-
Effective span = 5 m
Width of beam = 250 mm
Overall depth = 500 mm
Service dead and live loads including self weight = 40 kN/m
Effective cover = 50 mm
Materials: M-20 grade concrete
Fe-415 HYSD bars
Check the beam for deflection control and sketch the details of reinforcements.

4) A tee beam slab floor of an office comprises of a slab 150 mm thick spanning between ribs spaced 3 m centres. The effective span of the beam is 8 m. Service live load on floor = 4 kN/m^2. Using M-20 grade concrete and Fe-415 HYSD bars, design one of the intermediate tee beams and sketch the details of reinforcements.

5) The floor of a school building is made up of tee beams and slab with the following data:-
Clear span of tee beam = 7 m
Distance between c/c of supports = 7.45 m
Spacings of tee beams = 2.75 m c/c
Width of rib = 250 mm
Thickness of slab = 100 mm
Service live load on floor = 4 kN/m^2
Floor finish = 0.6 kN/m^2
Materials: M-20 grade concrete
Fe-415 HYSD bars
Design an intermediate tee beam and sketch the details of reinforcements.

6) Design a L-beam for an office floor using the following data:-
Clear span = 8 m
Thickness of flange = 150 mm
Spacings of beams = 3 m centres
Live load = 4 kN/m^2
Adopt M-20 grade concrete and Fe-415 HYSD bars. The L-beams are monolithic with columns 300 mm wide. Design the beam and sketch the details of steel reinforcements in the beam.

7) Design a cantilever beam to suit the following data.
Clear span = 1.75 m
Service load live load = 18 kN/m
Cantilever beam is monolithic with reinforced concrete column 250 mm wide by 400 mm deep. Adopt M-20 grade concrete and Fe-415 HYSD bars.

8) A canopy of the entrance of a building comprises of cantilever beams supporting a reinforced concrete slab. The cantilever beams are spaced at 3 m intervals and have a span of 3 m. The thickness of the slab is 120 mm.The live load on the slab may be assumed as 1.5 kN/m^2. Using M-25 grade concrete and Fe-500 grade reinforcements, design a typical cantilever beam and sketch the details of steel reinforcements in the beam.

9) Design a reinforced concrete continuous beam of rectangular section to support a dead load of 8 kN/m and service live load of 15 kN/m over 4 spans of 8 m each. Assume the ends as simply supported. Adopt M-20 grade concrete and Fe-415 HYSD bars. Sketch the details of reinforcements in the continuous beam.

10) A three span continuous beam is to be designed to support an imposed dead load 15 kN/m and a service live load of 15 kN/m. The three spans are 8 m each. Adopt suitable load factors as specified in IS: 456-2000 and design the beam, using M-20 grade concrete and Fe-415 HYSD bars.

CHAPTER 9

Limit State Design of Slabs

9.1 INTRODUCTION

The most common type of structural element used to cover floors and roofs of buildings are reinforced concrete slabs of different types. One-way slabs are those supported on the two opposite sides so that the loads are carried along one direction only. A common example of one way slab is the verandah slab spanning in the shorter direction with main reinforcements and distribution reinforcements in the transverse direction.

Two- way slabs are supported on all the four sides with such dimensions such that the loads are carried to the supports along both directions. Two-way slabs are common in the floors of multistorey buildings. Cantilevered slabs are generally used for chajjas over windows & in balconies projecting from the buildings. In Tee beam-slab floors, the slab is continuous over tee beams and designed as a continuous slab with positive moments at mid span and negative moments over supports.

Flat slabs are generally multispan slabs, which are directly supported on columns at regular intervals without beams. In the case of basements where headroom available is limited, flat slabs can be conveniently adopted. Flat slabs are commonly used for garages where limited headroom is available.

9.2 DESIGN OF ONE-WAY SLABS

9.2.1 Design Principles

Reinforced concrete slabs supported on two opposite sides with their longer dimension exceeding two times the shorter dimension are referred to as one-way slabs.

One way reinforced concrete slabs supporting floor or roof loads are generally designed as beams of unit width. For a given type of support condition, the span/depth ratio applicable for beams in IS: 456 is also valid for slabs. Since the percentage of reinforcements in slabs is generally low in the range of 0.3 to 0.5 per cent, a span/depth ratio of 25 to 30 is more appropriate by considering the modification factor K_t (1.2 to 1.4 for Fe-415 steel) Normally the thickness of slabs is so chosen that the shear can be

resisted by concrete alone without any extra shear reinforcements. The shear enhancement factor (k) specified in clause 40.2.1.1 of IS: 456 code varying from 1 to 1.3 depending upon the thickness of slab will considerably increase the permissible shear stress in slabs when multiplied with the values of shear stress given in Table-19 of IS: 456- 2000.

In slabs, shear reinforcements may be allowed if the thickness is 200 mm or more but in no case the maximum shear stress in slabs due to ultimate load exceed one half of that given in Table-20 of IS: 456-2000.

In the case of slabs the depth selected is usually greater than the minimum depth for the balanced section and hence the steel required may be calculated by the formula given in IS: 456 or by use of SP-16 charts and Tables. The designed slab should be checked for shear stress and deflection control.

9.2.2 Design Example

Design a simply supported R.C.C. slab for an office floor having clear dimensions of 4m by 10 m with 230 walls all-round. Adopt M - 20 grade concrete & Fe-415 grade HYSD bars.

a) Data

Clear span = 4 m
Wall thickness= 230 mm
Live load = 4 kN/m^2
Floor finish = 0.6 kN/m^2
$f_{ck} = 20$ N/mm^2
$f_y = 415$ N/mm^2

b) Thickness of Slab

$$\text{Assume effective depth} = d = \left(\frac{\text{span}}{25}\right) = \left(\frac{4000}{25}\right) = 160 \text{ mm}$$

Adopting a clear cover of 20 mm and using 10 mm diameter bars the total depth is computed as $D = 185$ mm.

c) Effective span

The least of

i) (Clear span + Effective depth) = (4 + 0.16) = 4.16 m
ii) (centre to centre of supports) = (4 + 0.23) = 4.23 m

$$\therefore \quad L = 4.16 \text{ m}$$

d) Loads

Self weight of slab = (0.185 × 25) = 4.625 kN/m^2
Finishes = 1.500
Live load = 4.000
Total service load = 10.125 kN/m^2
∴ Ultimate load = 1.5 (10.125) = 15.19 kN/m^2

e) Ultimate Moments and Shear Forces

$$M_u = (0.125 w_u L^2) = (0.125 \times 15.19 \times 4.16^2) = 32.86 \text{ kN.m}$$

$$V_u = (0.5 w_u L) = (0.5 \times 15.19 \times 4.16) = 31.60 \text{ kN}$$

f) Limiting Moment of Resistance

$$M_{u,lim} = 0.138 f_{ck}\, b.d^2 = (0.138 \times 20 \times 10^3 \times 160^2)\, 10^{-6} = 70.65 \text{ kN.m}$$

Since Mu < Mu,lim, section is under reinforced

g) Tension Reinforcements

$$M_u = (0.87\, A_{st} f_y\, d)\left[1 - \left(\frac{A_{st} f_y}{b\, d\, f_{ck}}\right)\right]$$

$$(32.86 \times 10^6) = (0.87 \times 415 A_{st} \times 160)\left[1 - \left(\frac{415 A_{st}}{10^3 \times 160 \times 20}\right)\right]$$

Solving, $A_{st} = 531$ mm^2 > A_{st}(min) = 216 mm^2
Using 10 mm diameter bars, the spacing of the bars is computed as

$$S = \left(\frac{1000 a_{st}}{A_{st}}\right) = \left(\frac{1000 \times 78.5}{531}\right) = 147 \text{ mm}$$

Adopt a spacing of 140 mm. Alternate bars are bent up at supports.

h) Distribution Bars

A_{st} = 0.12 percent = (0.0012 × 1000 × 185) = 220 mm^2

Provide 8 mm diameter bars at 230 mm centres (Ast = 217 mm^2).

i) Check for Shear stress

$$\tau_v = \left(\frac{V_u}{bd}\right) = \left(\frac{31.6 \times 10^3}{1000 \times 160}\right) = 0.198 \text{ N/mm}^2$$

$$p_t = \left(\frac{100 A_{st}}{bd}\right) = \left(\frac{100 \times 531 \times 0.5}{1000 \times 160}\right) = 0.166$$

Permissible shear stress in slab (Refer Table-19 of IS: 456) is computed as

$$k\tau_c = (1.23 \times 0.293) = 0.36 \text{ N/mm}^2 > \tau_v$$

Hence the slab is safe in shear.

j) Check for deflection control

$$\left(\frac{L}{d}\right)_{max} = \left(\frac{L}{d}\right)_{basic} \times K_t \times K_c \times K_f$$

Refer Fig. 7.2, $K_t = 1.4$ for p_t = [(100 × 531)/(1000 × 160)] = 0.33 percent
Fig. 7.3, $K_c = 1.0$
Fig. 7.4, $K_f = 1.0$

$$\left(\frac{L}{d}\right)_{max} = (20 \times 1.4 \times 1 \times 1) = 29$$

$$\left(\frac{L}{d}\right)_{provided} = \left(\frac{4160}{160}\right) = 26 < 29.$$

Hence, the deflection criterion is satisfied.

k) Design using SP: 16 Charts

Referring to Chart-13 of SP: 16, for $d = 160$ mm and $M_u = 32.86$ kN.m
$p_t = 0.3625$ and $A_{st} = 580$ mm^2

Spacing of 10 mm diameter bars is [(1000 a_{st})/A_{st}] = [(1000 × 78.5)/580] = 135 mm

Which is almost the same as that of analytical computations.

l) The reinforcement details in the slab are shown in Fig. 9.1.

9.3 DESIGN OF TWO WAY SLABS

9.3.1 General Features

Reinforced concrete slabs supported on all the four sides with their effective span in the longer direction not exceeding two times the effective span in the shorter direction are designed as two-way slabs. Two-way slabs

Where M_x and M_y are the design moments in the x and y directions.
w = uniformly distributed load on slab.
L_x and L_y are the short and long span dimensions of the simply supported slab.

The values of coefficients α_x and α_y are compiled in Table-9.1. (Table-27 of IS: 456)

These coefficients are due to Rankine-Groshoff theory[68] in which the slab is divided into a series of orthogonal beam strips and the load is apportioned to the short and long strips such that there is compatibility of deflection at the junction of strips.

Table 9.1 Bending Moment Coefficients for Slabs spanning in two directions at right angles, simply supported on four sides (Table-27 of IS: 456-2000)

$\left(\frac{L_y}{L_x}\right)$	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0	2.5	3.0
α_x	0.062	0.074	0.084	0.093	0.099	0.104	0.113	0.118	0.122	0.124
α_y	0.062	0.061	0.059	0.055	0.051	0.046	0.037	0.029	0.020	0.014

Clause D.2.1.1 of the IS: 456 codes specifies that at least 50 percent of the tension reinforcement provided at mid span should extend to the supports. The remaining 50 percent should extend to within 0.1 L_x or 0.1 L_y of the support, as appropriate.

9.3.3 Two Way Restrained Slabs with Corners Held Down

Restrained slabs are referred to as slabs whose corners are prevented from lifting. They may be supported on continuous or discontinuous edges. All the four edges of the two-way slab are assumed to be supported rigidly against vertical translation. The design moments in restrained slabs are easily evaluated using the moment coefficients recommended in IS: 456-2000 code and as shown in Table-9.2 (Table-26 of IS: 456 code).

These moment coefficients are based on inelastic analysis or yield line theory[69,70,71] with the following assumptions:

1) The reinforcement for positive moment is uniformly distributed over the middle strip extending over 75 percent of the span.
2) Edge strips cover a width equal to (L_x/8) or (L_y/8) as shown in Fig. 9.4.
3) Minimum reinforcements prescribed for slabs should be provided in edge strips.

Table 9.2 Bending Moment Coefficients for Rectangular Panels Supported on Four sides with Provision for Torsion at Corners (Table-26 of IS:456-2000)

No.	Type of panel and moments considered		Coefficients at point: Values of L_y/L_x 1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0 (or more)	B.M. coefficient α_y for long span for all values of (L_y/L_x)
1	Interior panels	X	0.032	0.037	0.043	0.047	0.051	0.053	0.060	0.065	0.032
		Y	0.024	0.028	0.032	0.036	0.039	0.041	0.045	0.049	0.024
2	One short edge discontinuous	X	0.037	0.043	0.048	0.051	0.055	0.057	0.064	0.068	0.037
		Y	0.028	0.032	0.036	0.039	0.041	0.044	0.048	0.052	0.028
3	One long edge discontinuous	X	0.037	0.044	0.052	0.057	0.063	0.067	0.077	0.085	0.037
		Y	0.028	0.033	0.039	0.044	0.047	0.051	0.059	0.065	0.028
4	Two edjacent edges discontinuous	X	0.047	0.053	0.060	0.066	0.071	0.075	0.084	0.091	0.042
		Y	0.035	0.040	0.045	0.049	0.053	0.056	0.069	0.069	0.035
5	Two short edges discontinuous	X	0.045	0.049	0.052	0.056	0.059	0.060	0.065	0.069	–
		Y	0.035	0.037	0.040	0.043	0.044	0.045	0.049	0.052	0.035
6	Two long edges discontinuous	X	–	–	–	–	–	–	–	–	0.045
		Y	0.035	0.043	0.051	0.057	0.069	0.068	0.080	0.88	0.035

(*Contd.*)

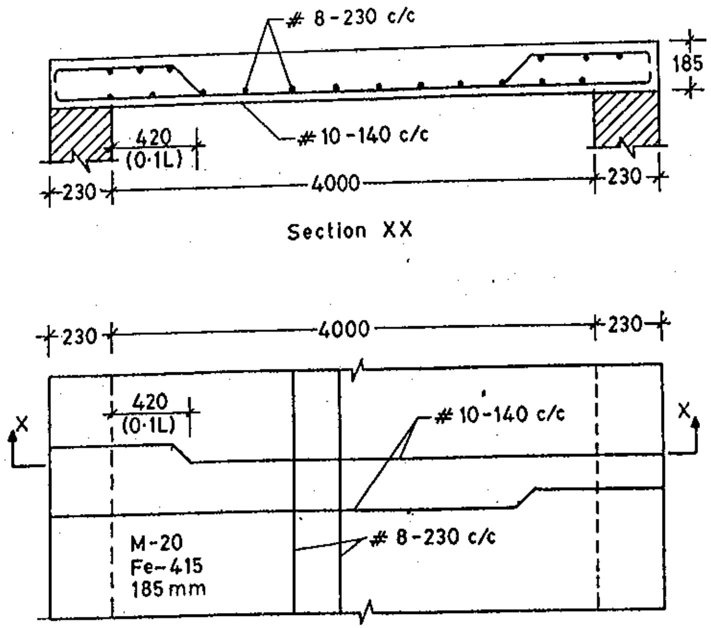


Fig. 9.1 Details of Reinforcement in One Way Slab

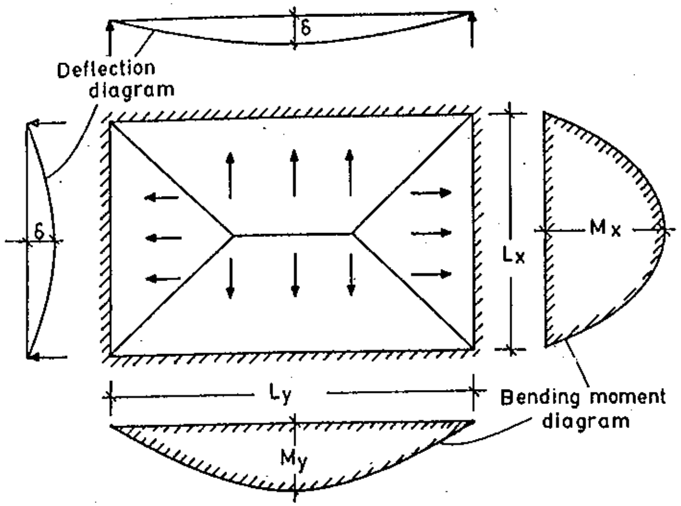


Fig. 9.2 Two Way Slab Action With Moment & Deflection Diagrams

deform with significant curvatures in two orthogonal directions with moments developed in the principal directions as shown in Fig. 9.2. The

bending moments are maximum at the centre of the slab and the larger moment invariably develops along the short span.

The bending moment in the slab depend upon the following parameters.

i) The short and long span (L_x and L_y)

ii) Edge conditions at the support (Fixed, free, continuous etc)

iii) Magnitude and type of load on the slab (Uniformly distributed, concentrated etc).

The reinforcements are generally placed along the transverse and longitudinal directions of the slab.

9.3.2 Simply Supported Slabs

When a slab simply supported on all the four sides is subjected to transverse loads, the bending of the slab in the two principal directions causes the corners to curl and lift up as shown in Fig. 9.3 due to non-uniform variation of load transmitted to the supports. Simply supported slabs which do not have adequate provision to resist torsion at corners and to prevent the corners from lifting, the maximum moments per unit width are specified in the IS: 456-2000 code and computed by the following equations.

$$M_x = \alpha_x \, w \, L_x^2$$

$$M_y = \alpha_y \, w \, L_x^2$$

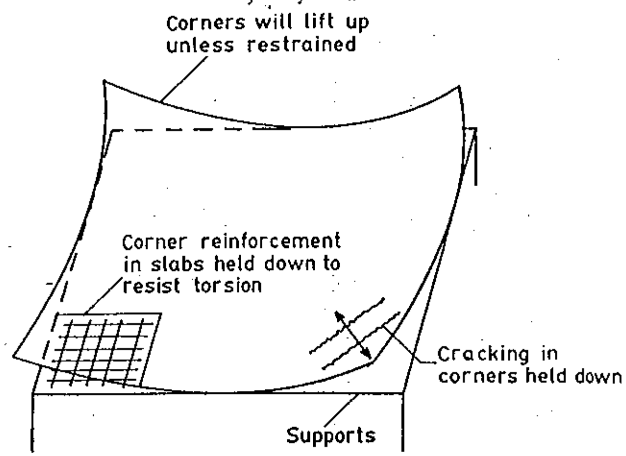


Fig. 9.3 Torsion Effects in Two Way Slab

Table 9.2 (Contd.)

No.	Type of panel and moments considered		Coefficients at point — Values of L_y/L_x: 1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0 (or more)	B.M. coefficient α_y for long span for all values of (L_y/L_x)
7	Three edges discontinuous	X	0.057	0.064	0.071	0.076	0.080	0.084	0.091	0.097	–
		Y	0.043	0.048	0.053	0.057	0.060	0.064	0.069	0.073	0.043
8	Three edges dicontinuous	X	–	–	–	–	–	–	–	–	0.057
		Y	0.043	0.051	0.059	0.065	0.071	0.076	0.087	0.096	0.043
9	Four edges discontinuous	X	0.056	0.056	0.072	0.079	0.085	0.089	0.100	0.107	0.056

X-Negative moment at continuous edge. Y-Positive moment at mid span.

4) Torsion reinforcement is provided at corners where the slab is simply supported on both edges meeting at that corner. The reinforcement comprising three quarters of the area required for the maximum mid span moment in the slab is provided in each of the four layers in the form of a mesh extending to a minimum distance of one-fifth of the shorter span. As shown in Fig. 9.4, full torsional steel is provided at corner A where the slab is discontinuous on both edges meeting at that corner. At corner B where the slab is discontinuous on only one edge meeting at that corner, 50 percent of full torsional steel is provided. At corner C, as the slab is continuous on both edges meeting at the corner, torsional steel is not required (Refer Fig. 9.4a).

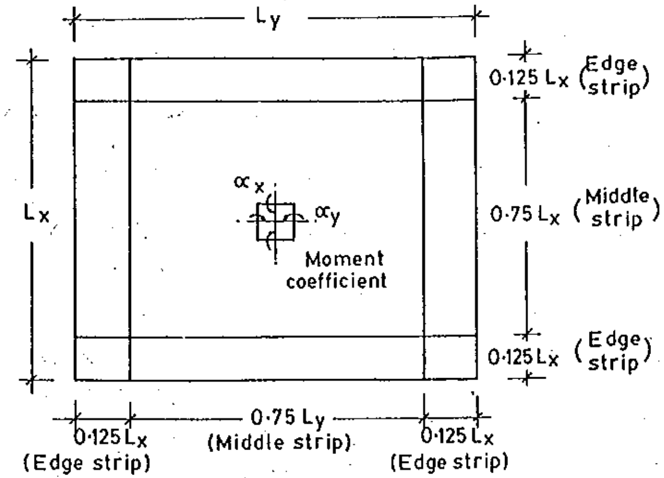


Fig. 9.4 Middle & Edge Strips in Two Way Slabs

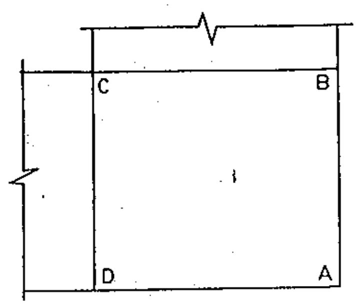


Fig. 9.4a Provison of Torsional Steel in Slabs

9.3.4 Span/Depth Ratio

In the case of two-way slabs, the magnitude of moments will be smaller than one-way slabs since the load is distributed in two principal directions. Consequently the percentage reinforcement being small, the modification factor for tension steel K_t is higher resulting in higher values of maximum permissible span/depth ratios. Hence the following span/overall depth ratios have been recommended in IS: 456 code clause 24.1 for two way slabs with shorter spans up to 3.5 m, using Fe-425 HYSD bars.

a) Simply supported slabs = 28
b) Continuous slabs = 32

9.3.5 Deflection and Crack control

The deflection of two-way slabs is controlled by span/depth ratio similar to the case of one-way slabs and beams. In two way slabs the shorter span and the percentage of steel in that direction have to be considered for computations of modification factors.

Crack control in two way slabs may be assumed to be satisfactory if the empirical rules for detailing of reinforcements outlined in section 7.5.3 are followed.

9.3.6 Design Example

Design a two way slab for a room of size 4m by 5m with discontinuous and simply supported edges on all the sides with corners prevented from lifting to support a live load of 4 kN/m² Adopt M-20 grade concrete & Fe-415 HYSD bars.

a) Data

$L_x = 4$ m $\qquad (L_y/L_x) = 1.25$
$L_y = 5$ m
$f_{ck} = 20$ N/mm²
$f_y = 415$ N/mm³

Two way slab simply supported on all the sides with provision for torsion at corners.

b) Depth of slab

As the span is more than 3.5m, adopt a span/depth ratio of 25
∴ Depth = (span/25) = (4000 / 25) = 160 mm

Adopt effective depth = d = 145 mm
Overall depth = D = 170 mm

c) Effective span

$$\text{Effective span} = (\text{Clear span} + \text{effective depth}) = (4 + 0.145) = 4.145 \text{ m}$$

d) Loads

Self weight of slab = (0.17 × 25) = 4.25 kN/m²
Live load on slab = 4.00
Floor finish = 0.60
Total working load = w = 8.85 kN/m²
∴ Design ultimate load = w_u = (1.5 × 8.85) = 13.275 kN/m²

e) Ultimate Design Moments and Shear Forces

Refer Table-9.2 and read out the moment coefficients for $(L_y/L_x) = 1.25$

$a_x = 0.076$ and $a_y = 0.056$

$M_{ux} = (a_x\, w_u\, L_x^2) = (0.076 \times 13.275 \times 4.1452) = 17.37$ kN.m

$M_{uy} = (a_y\, w_u\, L_y^2) = (0.056 \times 13.275 \times 4.1452) = 12.80$ kN.m

$V_{ux} = (0.5\, w_u\, L_x) = (0.5 \times 13.275 \times 4.145) = 27.6$ kN

f) Check for Depth

$$M_{max} = 0.138\, f_{ck}\, b\, d^2$$

$$\therefore \quad d = \sqrt{\frac{17.37 \times 10^6}{0.138 \times 20 \times 10^3}} = 79.33 \text{ mm} < 145 \text{ mm}$$

Hence, the effective depth selected is sufficient to resist the design ultimate moment

$$(A_{st})_{min} = (0.0012 \times 1000 \times 170) = 204 \text{ mm}^2$$

g) Reinforcements (Short and Long span)

$$M_u = 0.87\, A_{st}\, f_y\, d\left[1 - \frac{A_{st} f_y}{b\, d\, f_{ck}}\right]$$

$$(17.37 \times 10^6) = (0.87 \times 415\, A_{st} \times 145)\left[1 - \frac{415 A_{st}}{(10^3 \times 145 \times 20)}\right]$$

Solving $A_{st} = 302\ mm^2$

Adopt 10 mm diameter bars at 255 mm centres in short span direction.

Using 10 mm diameter bars in the long span direction,

Effective depth = (145−10) = 135 mm. Hence reinforcements in the long span direction is computed using the relation,

$$(12.8 \times 10^6) = (0.87 \times 415 A_{st} \times 135)\left[1 - \frac{415 A_{st}}{10^3 \times 135 \times 20}\right]$$

Solving $A_{st} = 237\ mm^2$. Hence provide 10 mm diameter bars at 300 mm centres in the long span direction ($A_{st} = 262\ mm^2$)

h) Check for Shear Stress

Considering the short span L_x and unit width of slab, the shear stress is given by

$$\tau_v = \left(\frac{V_u}{b\,d}\right) = \left(\frac{27.6 \times 10^3}{10^3 \times 145}\right) = 0.18$$

$$p_t = \left(\frac{100 A_{st}}{b\,d}\right) = \left(\frac{100 \times 302}{10^3 \times 145}\right) = 0.20\ N/mm^2$$

Refer Table-19 (IS: 456) and read out the permissible shear stress as

$$k\tau_c = (1.26 \times 0.32) = 0.40\ N/mm^2 > \tau_v$$

Hence the slab is safe against shear forces.

i) Check for Deflection

Considering unit width of slab in the short span direction L_x

$$\left(\frac{L}{d}\right)_{basic} = 20 \text{ and for } p_t = 0.20, \quad \text{From Fig. 7.2, read out } K_t = 1.7$$

$$\left(\frac{L}{d}\right)_{max} = (20 \times 1.7) = 34$$

$$\therefore \left(\frac{L}{d}\right)_{provided} = \left(\frac{4150}{145}\right) = 28.6 < 34$$

Hence deflection control is satisfied

j) Check for Cracking

i) Steel provided is more than the minimum percentage of 0.12 percent.

ii) Spacing of main steel $< 3d < (3 \times 145) = 435$ mm

iii) Diameter of reinforcement $< (D/8) = (175/8) = 21.8$ mm

Hence cracks will be within permissible limits as per the specifications of IS:456 code.

k) Torsion Reinforcement at Corners

Area of reinforcement in each of the four layers = (0.75 × 302) = 226.5 mm^2.

Distance over which torsion reinforcement is provided = (1/5 short span) = (0.2 × 4000) = 800 mm. Provide 6 mm diameter bars at 100 mm centres for a length of 800 mm at all four corners in 4 layers.

l) Reinforcement in Edge Strips

A_{st} = 0.12 percent of cross sectional area = $(0.0012 \times 10^3 \times 170) = 204\ mm^2/m$

Provide 10 mm diameter bars at 300 mm centres ($A_{st} = 262\ mm^2$) in all edge strips.

m) Design using SP: 16 design Tables

$M_{ux} = 17.37$ kN.m and $M_{uy} = 12.80$ kN.m, overall depth $= D = 170$ mm

Referring to Table-41 of SP: 16, read out the reinforcement i along short & long spans as

a) 10 mm diameter bars at 240 mm centres along the short span.

b) 10 mm diameter bars at 300 mm centres along the long span.

n) The details of reinforcements in the two-way slab are shown in Fig. 9.5.

9.3.7 Design Example

Design a two-way slab for an office floor to suit the following data.

a) Data

Size of office floor = 4 m by 6 m

Edge conditions = Two adjacent edges discontinous

Materials: M-20 grade concrete and Fe-415 HYSD bars.

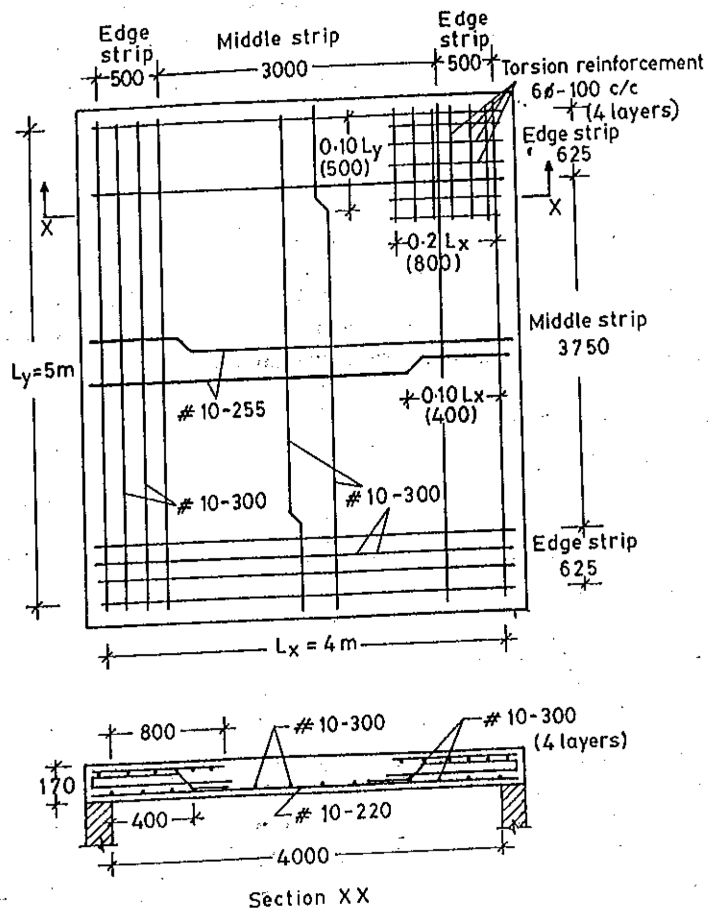


Fig. 9.5 Reinforcement Details in Two Way Slabs
(with provision for torsion at corners)

b) Depth of slab

As the span is more than 3.5 m, adopt a span/depth ratio of 25.

∴ Overall depth = (span/25) = (4000/25) = 160 mm

Adopt effective depth = 145 mm

And Overall depth = 170 mm

c) Effective Span

Effective span in along the short and long span directions are computed as

L_{ex} = (clear span + effective depth) = (4.000 + 0.145) = 4.145 m

L_{ey} = (clear span + effective depth) = (6.000 + 0.145) = 6.145 m

d) Loads

Self weight of slab = (0.17 * 25) = 4.25 kN/m²

Live load = 4.00

Finishes = 1.50

Total working load = w = 9.75 kN/m²

∴ Design Ultimate load = $w_u = (1.5 \times 9.75) = 14.625$ kN

e) Ultimate Design Moments

Refer Table-9.2 and read out the moment coefficients for $(L_y/L_x) = (6/4) = 1.5$

Short span moment coefficients:

a) – ve moment coefficient = $\alpha_x = 0.075$

b) + ve moment coefficient = $\alpha_x = 0.056$

Long span moment coefficients:

a) – ve moment coefficient = $\alpha_y = 0.047$

b) + ve moment coefficient = $\alpha_y = 0.035$

$M_{ux}(-ve) = (\alpha_x\, w_u\, L_{ex}^2) = (0.0075 \times 14.625 \times 4.145^2) = 18.85$ kN.m

$M_{ux}(+ve) = (\alpha_x\, w_u\, L_{ex}^2) = (0.056 \times 14.625 \times 4.145^2) = 14.07$ kN.m

$M_{uy}(-ve) = (\alpha^y\, w_u\, L_{ex}^2) = (0.047 \times 14.625 \times 4.145^2) = 11.81$ kN.m

$M_{uy}(+ve) = (\alpha_y\, w_u\, L_{ex}^2) = (0.035 \times 14.625 \times 4.145^2) = 8.80$ kN.m

f) Check for depth

$$M_{u,lim} = 0.138 f_{ck}\, b\, d^2$$

$$d = \sqrt{\frac{18.85 \times 10^6}{0.138 \times 20 \times 1000}} = 82.64\text{mm} < 145 \text{ mm}$$

Hence, the effective depth selected is sufficient to resist the design ultimate moment.

$$A_{st,min} = (0.0012 \times 1000 \times 170) = 204 \text{ mm}^2$$

g) Reinforcements along short and long span directions

The area of reinforcement is calculated using the relation,

$$M_u = 0.87\, f_y A_{st}\, d\left[1 - \frac{A_{st} f_y}{bd\, f_{ck}}\right]$$

Spacing of the selected bars are computed using the relation,

$$\text{Spacing} = S = \left(\frac{\text{Area of one bar}}{\text{Total Area}}\right) \times 1000 \text{ such that } A_{st}\text{ (provided)} \geq A_{st}\text{ (minimum)}$$

In addition, the spacing should be the least of three times the effective depth or 300 mm.

Using 10 mm diameter bars for long span, d = 145 mm & for short span, d = 135 mm.

The details of reinforcements provided in the two-way slab is compiled in Table-9.3.

Table 9.3 Reinforcement details in Two way slab

Location	A_{st} (Required)	Spacing of 10 mm ϕ bars
1) Short span		
a) –ve B.M(top of supports)	328.7 mm^2	235 mm c/c
b) +ve B.M (centre of span)	242.2 mm^2	300 mm c/c
2) Long span		
a) –ve B.M (top of supports)	218.10 mm^2	300 mm c/c
b) +ve B.M (Centre of span)	204 mm^2 ($A_{st,min}$)	300 mm c/c

h) Torsion Reinforcement at corners

Referring to Fig. 9.4 (a),

Area of torsional steel in each of 4 layers at – A = (0.75 × 242.2) = 181.65 mm^2

Provide 4 layers of reinforcement at A with 4 bars of 8 mm diameter in each layer (two layers at top level and two layers at bottom level) over a length of 800 mm in each direction from the corner.

At 'B' 50% of total torsional steel is 2 bars of 8 mm dimeter in each of form layers.

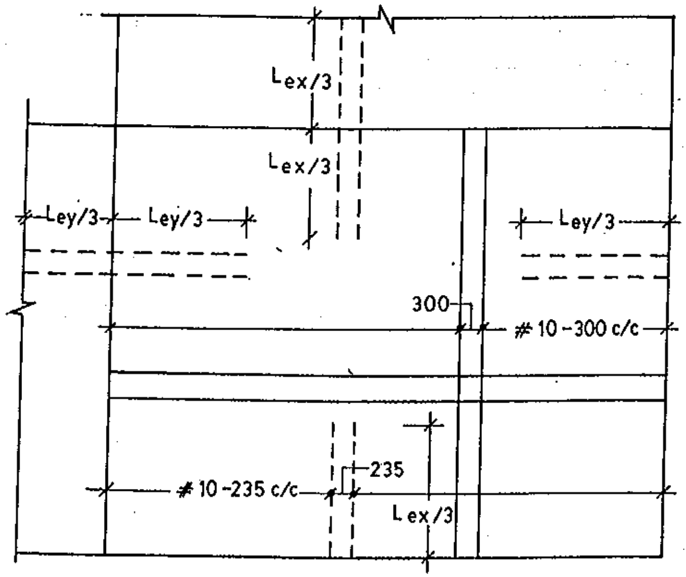
At 'C' torsional steel is not required.

i) Details of reinforcements are shown in Fig. 9.6.

9.4 DESIGN OF CANTILEVER SLABS

9.4.1 General features

Cantilever Slabs are commonly used for chajjas and balconies projecting



Long span: # 10 at 300 c/c
Slab thickness : 170 mm

Fig. 9.6 Reinforcement Details in Two Way Slabs

from the wall face from lintel beams or floor slabs. The slabs are generally designed as one-way slabs as a cantilever fixed or continuous at the supports. The trial depth is selected based on span/depth ratio of 7 recommended in IS: 456 codes. The reinforcements provided in the slab at the tension face should be checked for the anchorage length near the supports.

The thickness of the cantilever slab is generally varied from a maximum at the fixed end to a minimum of 100 to 150 mm at the free end. Distribution steel is provided in the transverse direction.

Proper selection of depth and detailing of reinforcements will safegaurd against excessive deflections and cracking of the cantilever slabs. Cantilever structural elements should be checked for safety against overturning.

9.4.2 Design Example

Design a cantilever slab projecting 2.1 m from the support using M-20 concrete and Fe-415 grade steel:

a) Data

Cantilever Projection = L = 2.1m

Materials: M-20 Concrete
Fe-415 Grade Steel
$f_{ck} = 20$ N/mm^2 and $f_y = 415$ N/mm^2

b) Depth of slab

Effective Depth = (span/10) = (2100/10) = 21
Provide d = 240 mm
D = 215 mm
Maximum depth of 240 mm at support is gradually reduced to 120 mm at free end.

c) Load

Self-weight of slab = 0.5(0.24 + 0.12) 25 = 4.5 kN/m^2
L.L. (Assuming Residential Building) = 2.0
Finishes = 1.5
Total working load = w = 8.0 kN/m^2
∴ Ultimate load $w_u = (1.5 \times 8) = 12.0$ kN/m^2

d) Ultimate Moments

$M_u = 0.5\, w_u\, L^2 = (0.5 \times 12 \times 2.1^2) = 26.46$ kN.m

e) Check for depth

$$M_u = 0.138\, f_{ck}\, b\, d^2$$

$$d = \sqrt{\frac{26.46 \times 10^6}{0.138 \times 20 \times 1000}} = 98.9 \text{ mm} < 215 \text{ mm}$$

Hence the effective depth selected is sufficient to resist the design ultimate moment .

$$A_{st,min} = (0.0012 \times 1000 \times 240) = 288 \text{ mm}^2$$

f) Reinforcement details

$$M_u = 0.87\, A_{st} f_y\, d \left[1 - \frac{A_{st} f_y}{bd\, f_{ck}}\right]$$

$$(26.46 \times 10^6) = (0.87 \times 415\, A_{st} \times 215)\left[1 - \frac{415 A_{st}}{1000 \times 215 \times 20}\right]$$

Solving $A_{st} = 305.6$ mm^2
Provided 10 mm diameter bars at 255 centres at top of slab.

g) Distribution steel

$A_{st} = 288$ mm^2. Provide 10 mm dimeter bars at 270 mm centres.

h) Anchorage Length

$$L_d = \left(\frac{0.87\, f_y}{4\tau_{bd}}\right)\phi = \left(\frac{0.87 \times 415}{4 \times 1.2 \times 1.6}\right) 10 = 470 \text{ mm}$$

Main tension bars are extended into the support to a minimum length of 470 mm including anchorage value of hooks and 90o bends. Further safety against overturning has to be satisfied by providing sufficient balancing moment.

i) Check for Deflection Control

$$\left(\frac{L}{d}\right)_{max} = \left(\frac{L}{d}\right)_{basic} \times K_t \times K_c \times K_f$$

$$p_t = \left(\frac{100 A_{st}}{b\, d}\right) = \left(\frac{100 \times 305.6}{100 \times 215}\right) = 0.142$$

From Fig. 7.2 read out $K_t = 2$, $K_c = 1$ and $K_f = 1$

∴ $$\left(\frac{L}{d}\right)_{max} = (7 \times 2 \times 1 \times 1) = 14$$

$$\left(\frac{L}{d}\right)_{provided} = \left(\frac{2100}{215}\right) = 9.76 < 14$$

Hence the cantilever slab satisfies the deflection limits prescribed in the code.

j) Reinforcement details in the cantilever slab are shown in Fig. 9.7.

9.5 DESIGN OF CONTINUOUS SLABS

9.5.1 Introduction

In the case of tee beam and slab floors, the slab is continuous over tee

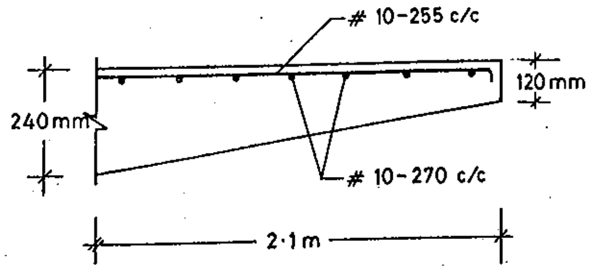


Fig. 9.7 Reinforcement Details in Cantilever Slab

beams spaced at regular intervals of 2.5 to 3.5m. Continuous slabs are designed similar to that of continuous beams using moment and shear coefficients recommended in IS:456 - 2000. The depth of the slab is based on the basic span depth ratio of 26 recommended in the IS. Code with suitable modification factors applied for tension reinforcement.

The limitations regarding variations in spans and redistribution of moments discussed in continuous beams, also apply for the design of continuous slabs when the moment and shear co-efficients specified in Tables -12 and 13 of the IS : 456-2000 code, are used in design.

9.5.2 Design example

Design a continuous one- way slab for an office floor. The slab is continuous over tee beams spaced at 4m intervals. Assume live load of 4 kN/m² and adopt M-20 grade concrete and Fe-415 HYSD bars.

a) Data

Live Load = $q = 4\ \text{kN/m}^2$, $f_{ck} = 20\ \text{N/m}^2$ and $f_y = 415\ \text{N/mm}^2$

b) Depth of slab

Since the slab is continuous and the percentage of reinforcement is small, the span/depth ratio may be assumed as 30

$$\therefore \quad \text{Depth} = \left(\frac{\text{span}}{30}\right) = \left(\frac{4000}{30}\right) = 134\ \text{mm}$$

Adopt effective depth = d = 140 mm and Overall depth = D = 160 mm

c) Loads

Self weight of slab = $(0.16 \times 25) = 4.00\ \text{kN/m}^2$
Finishes = 1.00
Total dead Load = $g = 5.00\ \text{kN/m}^2$
Live load = $q = 4.00\ \text{kN/m}^2$

d) Effective span = centre to centre of supports = $L = 4$ m

e) Moments and shear forces

Referring to Table-12 and 13 of IS: 456 - 2000 code,
Maximum negative moment at support next to the end support is

$$M_u(-ve) = 1.5\left[\left(\frac{gL^2}{10}\right)+\left(\frac{qL^2}{9}\right)\right] = 1.5\left[\left(\frac{5\times4^2}{10}\right)+\left(\frac{4\times4^2}{9}\right)\right] 22.66\ \text{kN.m}$$

$$M_u(+ve) = 1.5\left[\left(\frac{gL^2}{12}\right)+\left(\frac{qL^2}{10}\right)\right] = 1.5\left[\left(\frac{5\times4^2}{12}\right)+\left(\frac{4\times4^2}{10}\right)\right] = 20.65\ \text{kN.m}$$

Maximum shear force is computed as

$$V_u = (1.5\times0.6)\,(g+q)L$$

$$= (1.5\times0.6)(5+4)4 = 32.4\ \text{kN}$$

e) Check for depth

$$M_{u,lim} = 0.138\, f_{ck}\, b\, d^2$$

$$\therefore \quad d = \sqrt{\frac{22.66\times10^6}{0.138\times20\times10^3}} = 90.6\ \text{mm} < 140\ \text{mm}.$$

Hence the provided depth is safe.

f) Reinforcements

$$M_u = (0.87\, f_y A_{st} d)\left[1-\frac{A_{st} f_y}{b d f_{ck}}\right]$$

$$(22.66\times10^6) = (0.87\times415\times A_{st}\times140)\left[1-\frac{415 A_{st}}{10^3\times140\times20}\right]$$

Solving, $A_{st} = 485\ \text{mm}^2$

Provide 10 mm diameter bars at 150 mm centres at supports (A_{st} = 524 mm²)

The same reinforcement is provided for positive moment at mid span.
Distribution reinforcement = $(0.0012 \times 1000 \times 160) = 192\ mm^2$
Provide 10 mm diameter bars at 300 mm centres.

g) Check for Shear

$$\tau_v = \left(\frac{V_u}{bd}\right) = \left(\frac{32.4 \times 10^3}{10^3 \times 140}\right) = 0.23\ N/mm^2$$

Refer Table-19 of IS: 456 and read out the permissible shear stress as

$$\tau_c = (1.25 \times 0.36) = 0.45\ N/mm^2 > \tau_v$$

Hence, the slab is safe against shear failures.

h) Check for Deflection Control

$$\left(\frac{L}{d}\right)_{max} = \left(\frac{L}{d}\right)_{basic} \times K_t \quad \text{and} \quad p_t = \left(\frac{100 \times 524}{1000 \times 140}\right) = 0.37. \text{ From Fig. 7.2, } K_t = 1.35$$

$$\left(\frac{L}{d}\right)_{max} = \left(\frac{20+26}{2}\right) 1.35 = 29.9$$

$$\left(\frac{L}{d}\right)_{provided} = \left(\frac{4000}{140}\right) = 28.5 < 29.9$$

Hence, the slab is safe against excessive deflections.

i) The details of reinforcements in the continuous slab are shown in Fig. 9.8.

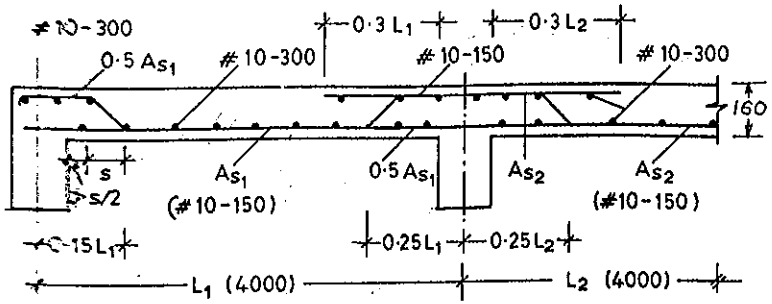


Fig. 9.8 Reinforcement Details in One Way Continuous Slab

9.6 DESIGN OF FLAT SLABS

9.6.1 Introduction

A flat slab is a reinforced concrete slab supported directly over columns without beams generally used when headroom is limited such as in cellars and warehouses.

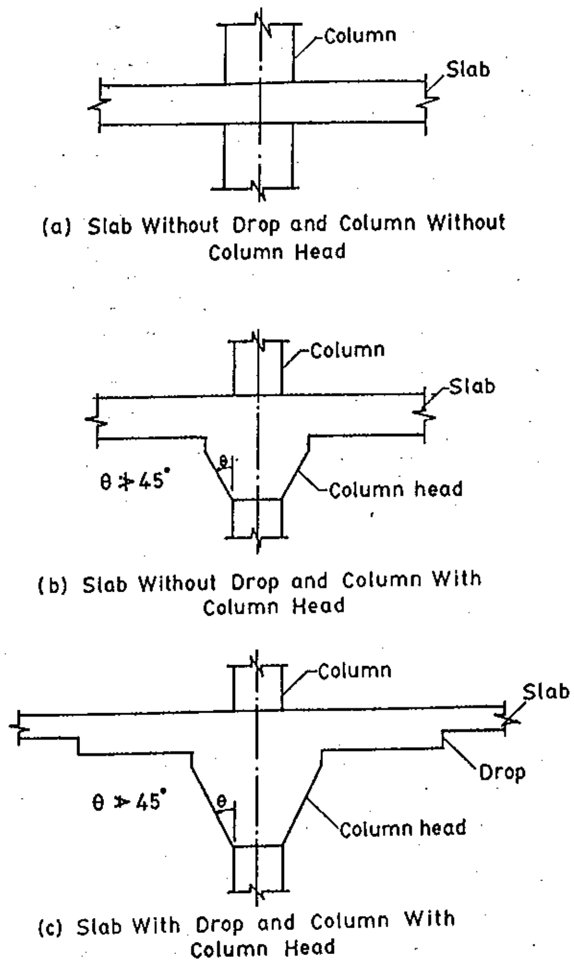


Fig. 9.9 Different Types of Flat Slabs

The different types of flat slabs shown in Fig. 9.9 are referred to as

(i) Slabs without drops and column heads

(ii) Slabs without drops
iii) Slabs with drops and column with column head

9.6.2 Panel Divisions

The flat slab panel is generally divided into column strip and middle strip.

a) **Panel:** Panel is that part of the slab bounded on each of its form sides by the centre line of columns or centre lines of adjacent spans.

b) **Column Strip:** Column strip is a design strip having a width of 0.25 L_2 but not greater than 0.25 L_2, on each side of the column centre line where L_1 is the span in the direction, moments are being determined measured centre to centre of supports and L_2 is the span transverse to L_1 measured centre to centre of supports.

c) **Middle Strip:** Middle strip is a design strip bounded on each of its opposite sides by the column strip.

Fig. 9.10 shows the division of flat slab into column and middle strips.

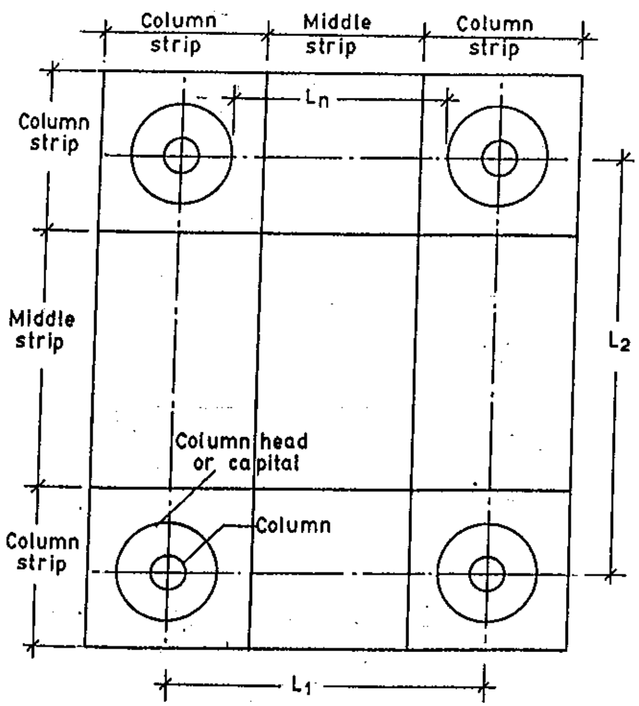


Fig. 9.10 Division of Flat Slab into column and Middle Strips

9.6.3 Proportioning of Slab thickness, Drop panel and Column head

a) Thickness of Flat slab

The thickness of flat slab depends upon the span / effective depth ratio which is specified as 40 for two-way slabs. However the IS: 456 code permits a reduction factor of 0.9 resulting in a span / effective depth ratio of 36 for flat slabs. However the longer span should be considered in the computations. The minimum thickness of a flat slab is 125 mm.

b) Drops

The drop panel is formed by increasing the thickness of slab in the vicinity of the supporting column. The main purpose of providing drops is to reduce the shear stress around the column supports. Since the moments in the column strip are higher than in middle strips, drops help to reduce the steel requirement to resist the negative moments at the column supports.

The code clause (C1.31.2.2) prescribes that drops should be rectangular in plan, and have a length in each direction not less then one third of the panel length in that direction. For exterior panels, the length measured perpendicular to the discontinuous edge from the column centre line should be taken as one half of the corresponding width of drop for the interior panel (Fig. 9.12). Although the code does not specify the thickness of the drop, it is recommended by the ACI code[72], that the thickness of the drop should be not less than one fourth the slab thickness and preferably not less than 100 mm.

c) Column Head

The column head or capital located by flaring of the column at the top is primarily intended to increase the punching shear strength of the slab. The IS: 456 Code clause (C1.31.2.3) specifies the useful portion of the column capital as that which lies within the largest circular cone or pyramid that has a vertex angle of 90° and can be included entirely within the outlines of the column and column head.

9.6.4 Direct Design Method

The direction design method facilitates the computation of positive and negative design moments under design loads at critical sections in the slab using empirical moment coefficients. However, the code (C1.21.4.1) specifies that the following conditions must be satisfied by the flat slab system for the application of the direct design method.

a) There must be at least three continuous spans in each direction.

b) The panels should be rectangular and the ratio of the longer span to the shorter span within a panel should not exceed 2
c) The columns must not be offset by more than 10 percent of the span from either axis between centre lines of successive columns.
d) The successive span lengths in each direction must not differ by more than one third of the longer span.
e) The design live load must not exceed three times the design dead load.

9.6.5 Total Design Moment for a span

In the direct design method, the total design moment for a span bounded laterally by the centre lines of the panel on each side of the centre line of supports is expressed as (C1.31.4.2.2)

$$M_o = \left(\frac{WL_n}{8}\right)$$

Where M_o = absolute sum of the positive and average negative bending moment in each direction.
W = total design load covered on an area $L_2\,L_n$
L_n = clear span extending from face to face of columns,capitals brackets or walls, but not less than 0.65 L_1 (Refer Fig. 9.12)
L_1 = length of span in the direction of M_o
L_2 = span length transverse to L_1

The expressions for M_o is computed as the maximum mid span static moment in an equivalent simply supported span L_n, subjected to a uniformly distributed total load $W = w\ (L_2\ L_n)$ where $L_2\ L_n$ is the effective panel area on which the unit load 'w' acts.

According to IS: 456-2000 Clauses 31.4.3.2, the total moment M_o in the panel is distributed to the column and middle strips in the following proportions.

a) Moments in Interior Panel

	Bending Moment Distribution (Percent of M_o)	
Type of Moment	Column Strip	Middle Strip
Negative Moment	$(0.65 \times 0.75) = 49\%$	15%
Positive Moment	$(0.35 \times 0.60) = 21\%$	15%

b) Moments in Exterior Panel

The moments in the exterior panel are influenced by the flexural stiffness of columns and slab.

The total design moment M_o is distributed in the following proportions.

$$\text{Interior Negative Design Moment} = 0.75 - \left[\frac{0.10}{1+(1/\alpha_c)}\right]$$

$$\text{Exterior Negative Design Moment} = \left[\frac{0.65}{1+(1/\alpha_c)}\right]$$

$$\text{Positive Design Moment} = 0.63 - \left[\frac{0.28}{1+(1/\alpha_c)}\right]$$

Where α_c = Ratio of flexural stiffness of exterior columns to the flexural stiffness of the slab at a joint taken in the direction, moments are being determined and is given by
$\alpha_c = [\sum K_c / K_s]$
Where $\sum K_c$ = Sum of the flexural stiffness of the columns meeting at the joint and
K_s = Flexural stiffness of the slab, expressed as moment per unit rotation.

At an exterior support, the column strip must be designed to resist the total negative moment in the panel at that support.

9.6.6 Equivalent Frame Method

The structure is analysed as a continuous frame with the following assumptions.

a) The structure is considered to be made up of equivalent frames longitudinally and transversely consisting of row of columns and strip of slab with a width equal to the distance between the centre lines of the panel on each side of the row of columns.
b) Each frame is analysed by any established method like moment distribution or any other suitable method. Each strip of floor and roof may be analysed as a separate frame with the columns above and below assumed fixed at their extremities.
c) The relative stiffness is computed by assuming gross cross section of the concrete alone in the calculation of the moment of inertia.
d) Any variation of moment of inertia along the axis of the slab on account of provision of drops should be considered. In the case of recessed or coffered slab which is made solid in the region of the columns, the stiffening effect may be ignored provided the solid part of the slab does not extend more than 0.15 Lef into the span measured from the centre line of the columns. The stiffening effect of flared column heads may be ignored.

9.6.7 Shear in Flat Slab

In the case of flat slabs, the critical section for shear is at a distance ($d/2$) from the periphery of the column / capital / drop panel, perpendicular to the plane of the slab where 'd' is the effective depth of the section. The shape in plan is geometrically similar to the support immediately below the slab.

The nominal shear stress in flat slabs is computed as ($V/b_o.d$) where V is the shear force due to design load and b_o is the periphery of the critical section and d is the effective depth.

When shear reinforcement is not provided, the calculated shear stress at the critical section shall not exceed $k_s.\tau_c$ where

k_s = $(0.5 + \beta_c)$ but not greater than 1.
β_c = Ratio of short side to long side of the column / capital and
τ_c = $0.25\sqrt{f_{ck}}$ in limit state method of design and $0.16\sqrt{f_{ck}}$ in working stress method of design.

When the shear stress exceeds this value, suitable shear reinforcements according to the provisions of the code should be provided.

In practice it is preferable to increase the thickness of the slab near the column head to reduce the shear stresses rather than providing shear reinforcements.

9.6.8 Design Example

Design the interior panel of a flat slab for a ware house to suit the following data:

a) Data

Size of ware house 24 m by 24 m divided into panels of 6 m by 6 m
Loading class–5 kN/m^2
Materials : M–20 Grade concrete
Fe–415 grade HYSD bars.

b) Interior Panel - Proportions

Thickness of slab = (Span/40) = (6000/40) = 150 mm
Thickness of slab at drops = (150 + 50) = 200mm
Column head diameter is computed as
$D > 0.25\ L = (0.25 \times 6) = 1.5$ m
Adopt diameter of column head = D = 1.5 m

Length of drop < ($L/3$) in either direction < (6/3) = 2 m
Adopt drop width = 3m
∴ Column strip = drop width = 3m
Middle strip = 3m
Span of flat slab = $L_1 = L_2$ = 6m

c) Loads

Self weight of slab = $(0.15 \times 25) = 3.75$ kN/m^2
Live Load = 5.00
Dead Load due to extra depth of slab at drops = (0.05×25). = 1.25
Total working load = w = 10.00 kN/m^2
∴ Ultimate load = $w_u = (1.5 \times 10) = 15$ kN/m^2

d) Ultimate Bending Moments

$M_o = (W_{Ln} / 8)$
$L_n = (6 - 1.5) = 4.5\text{ m} > 0.65\ L_1 > (0.65 \times 6) = 3.9$ m
and $L_1 = L_2$ = 6m
∴ $W = (w_u.L_2.L_n) = (15 \times 6 \times 4.5) = 405$ kN
∴ $M_o = [(405 \times 4.5) / 8] = 230$ kN.m.

For Interior panel with drops:
Columns strip moments

Negative B.M= 49% $M_o = (0.49 \times 230) = 113$ kN.m
Positive B.M.= 21% $M_o = (0.21 \times 230) = 48$ kN.m

Middle Strip Moments

Negative B.M.= 15% $M_o = (0.15 \times 230) = 35$ kNm
Positive B.M.= 15% $M_o = (0.15 \times 230) = 35$ kNm

e) Check for thickness of slab

i) Thickness of slab required near drops

$$d = \sqrt{\frac{M_u}{0.138\, f_{ck}\, b}} \quad \text{where} \quad b = 3000 \text{ mm}$$

$$= \sqrt{\frac{113 \times 10^6}{0.138 \times 20 \times 3000}} = 117 \text{ mm}$$

Effective depth provided = d = 170 mm
Overall depth = 200mm

ii) Thickness of slab required in middle strips

$$d = \sqrt{\frac{49 \times 10^6}{0.138 \times 20 \times 3000}} = 77 \text{ mm}$$

Provide effective depth = d = 120 mm and overall depth =150 mm

f) Check for shear stress

Shear stress is checked near the column head at section $(D + d)$.
Total load on the circular area with $(D + d)$ as diameter is given by

$$W_1 = (\pi/4)(D + d)^2 w_u$$

$$= (\pi/4)(1.5 \times 0.17)^2\ 15$$

$$= 33 \text{ kN}$$

$$\begin{aligned} \text{Shear force} &= [(\text{Total Load}) - (\text{Load on circular area})] \\ &= [(15 \times 6 \times 6) - (33)] \\ &= 507 \text{ kN} \end{aligned}$$

Shear force / meter of perimeter

$$V_u = \left[\frac{507}{\pi(D + d)}\right] = \left[\frac{507}{\pi(1.5 + 0.17)}\right] = 97 \text{ kN/m}$$

$$\therefore \quad \text{Shear Stress} = \tau_v = \left(\frac{V_u}{bd}\right) = \left(\frac{97 \times 10^3}{10^3 \times 170}\right) = 0.57 \text{ N/mm}^2$$

According to clause 31.6.3.1 of IS: 456,
Permissible shear stress = $k_s.\tau c$

Where $k_s = (0.5 + \beta_c)$ where $\beta_c = (L_1/L_2) = (6/6) = 1$
$= (0.5 + 1) = 1.5 > 1.0 \ \therefore\ k_s = 1.0$

and $\tau_c = 0.25\sqrt{f_{ck}} = 0.25\ \sqrt{20} = 1.12$ N/mm^2

$\therefore \quad k_s.\tau_c = (1.0 \times 1.12) = 1.12$ N/mm^2

The actual shear stress of 0.57 N/mm^2 is within safe permissible limits.

g) Reinforcements in Column and Middle Strips

i) Column Strip

A_{st} (for $-$ve Moment) is computed as

$$M_u = 0.87 f_y A_{st} d \left[1 - \frac{A_{st} f_y}{b\, d f_{ck}}\right]$$

$$(113 \times 10^6) = (0.87 \times 415\ A_{st} \times 170)\left[1 - \frac{415 A_{st}}{(3000 \times 170 \times 20)}\right]$$

Solving A_{st} = 2000 mm^2

$\therefore$ A_{st}/metre = (2000/3) = 667 mm^2

Adopt 16 mm diameter bars at 300 mm centres (A_{st} = 670 mm^2)

A_{st} (for +ve moment) is given by

$$(49 \times 10^6) = (0.87 \times 415 A_{st} \times 120)\left[1 - \frac{415 A_{st}}{(3000 \times 120 \times 20)}\right]$$

Solving A_{st} = 1215 mm^2

$\therefore$ A_{st}/metre = (1215/3) = 405 mm^2/m

Provide 12 mm diameter bars at 250 mm centres (A_{st} = 452 mm^2)

ii) Middle Strip

A_{st} for +ve and $-$ve B.M. is computed as

$$(35 \times 10^6) = (0.87 \times 415 A_{st} \times 120)\left[1 - \frac{415 A_{st}}{(3000 \times 120 \times 20)}\right]$$

Solving A_{st} = 850 mm^2
$\therefore$ A_{st}/metre = (850/3) = 284 mm^2/m

Provide 10mm diameter bars at 270 mm centres (A_{st} = 290 mm^2)

h) Reinforcement Details

The details of reinforcements in the flat slab are shown in Fig. 9.11.

9.7 YIELD LINE ANALYSIS OF SLABS

9.7.1 Introduction

The failure of reinforced concrete slabs of different shapes such as square, rectangular, circular with different types of edge conditions is preceded by a characteristic pattern of cracks which are generally referred to as yield lines which are characteristic of the shape of slab, type of loading and edge conditions. The yield line theory was innovated by a Danish engineer

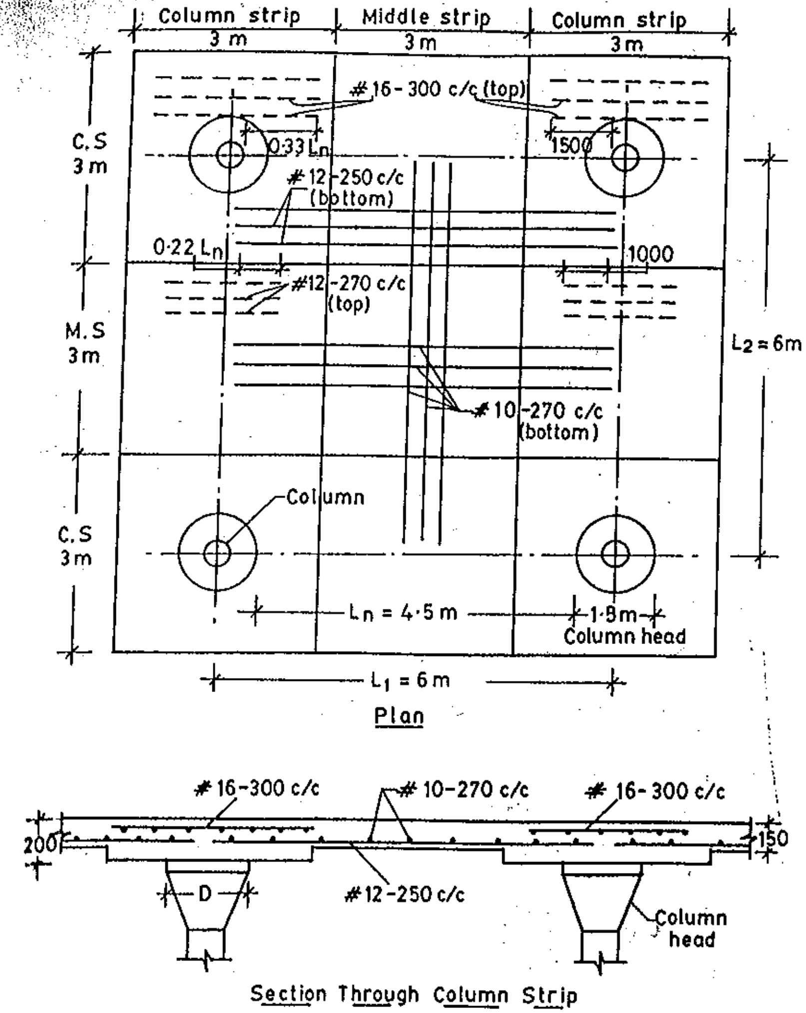


Fig. 9.11 Reinforcement Details in Flat Slab

Ingerslav[73], and was considerably improved and advanced by Johanssen[74, 75]. In the case of slabs the computation of ultimate loads is really complicated and is a challenge to the research workers and designers.

The determination of ultimate loads on slabs based on yield line theory has been further extended by Wood[76, 77] and Jones[78] of Great Britain. Shukla's[79] hand book published by SERC is also a useful reference for the

design of slabs using the yield line theory. The Indian standard code IS: 456-2000 specifies that two way slabs carrying uniformly distributed loads may be designed by any acceptable theory. The most generally used elastic methods are based on Poteau's or Westergaard's theory[80, 81] and the ultimate load methods are based on Johanssen's yield line theory and Hillerborg's[82] strip method of design. The ultimate load methods have been used by the author[83, 84] for the design of different types of slabs.

9.7.2 Characteristic Features of yield lines

The typical crack pattern (yield lines) developed in an isotropically reinforced square slab is shown in Fig. 9.12. As the load is gradually increased on the slab, the region of highest moment will yield first and the yield lines are propagated until they reach the boundaries of the slab. The final failure will take place by the rotation of the slab elements about the axes of rotation which are usually the supporting edges of the slab.

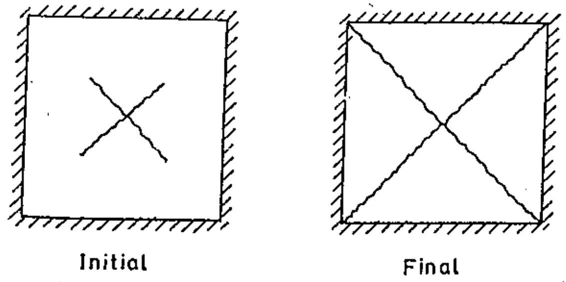


Fig. 9.12 Yield Line Pattern in a Simply Supported Slab

It is important to note that for the complete yield line pattern to develop, the slab must be under reinforced so that sufficient rotation capacity is available for the initiation and propagation of the yield lines.

The following characteristic features of yield lines are helpful in selecting a possible yield line mechanism in a typical slab.

a) Yield lines end at the supporting edges of the slab
b) Yield lines are straight
c) A yield line or yield line produced, passes through the intersection of the axes of rotation of adjacent slab elements.
d) Axes of rotation generally lie along lines of supports and pass over any columns.

Fig. 9.13 shows the notations used to represent the yield line and supports. The yield line patterns developed in slabs of different shapes and

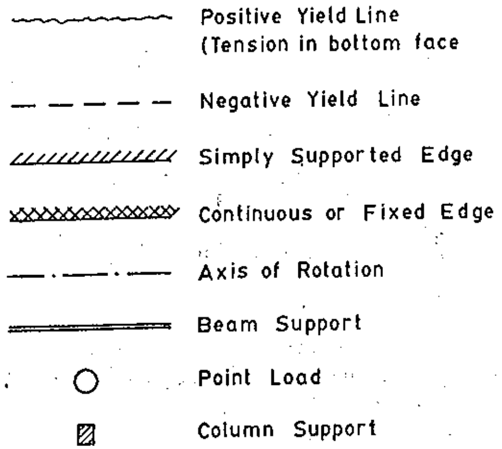


Fig. 9.13 Notations For Yield Lines and Supports

with different edge conditions are compiled in Fig. 9.14. Negative yield lines form near the supports in the case of slabs fixed or continuous at the edges.

9.7.3 Yield Moments

When the yield lines form at right angles to the direction of the reinforcement as shown in Fig. 9.15 (a).The yield or ultimate moments is computed by considering the slab section as under reinforced.

According to IS: 456–2000, the yield or ultimate moment is expressed as

$$m = M_u = 0.87\, f_y . A_{st} . d\left[\frac{1 - A_{st} f_y}{b d f_{ck}}\right]$$

Referring to Fig. 9.15 (b), if an yield line ab has an ultimate moment 'm' per unit length and the yield line ab makes an angle 'a' with the yield line cd which is at right angles to the reinforcement, the yield moment 'm' is calculated as follows:

$$m_a . \text{ab} = (m . \cos\alpha \times \text{cd})$$

$$m_a = m . \cos a (\text{cd/ab}) = m . \cos^2\alpha$$

If there is more than one mesh reinforcement

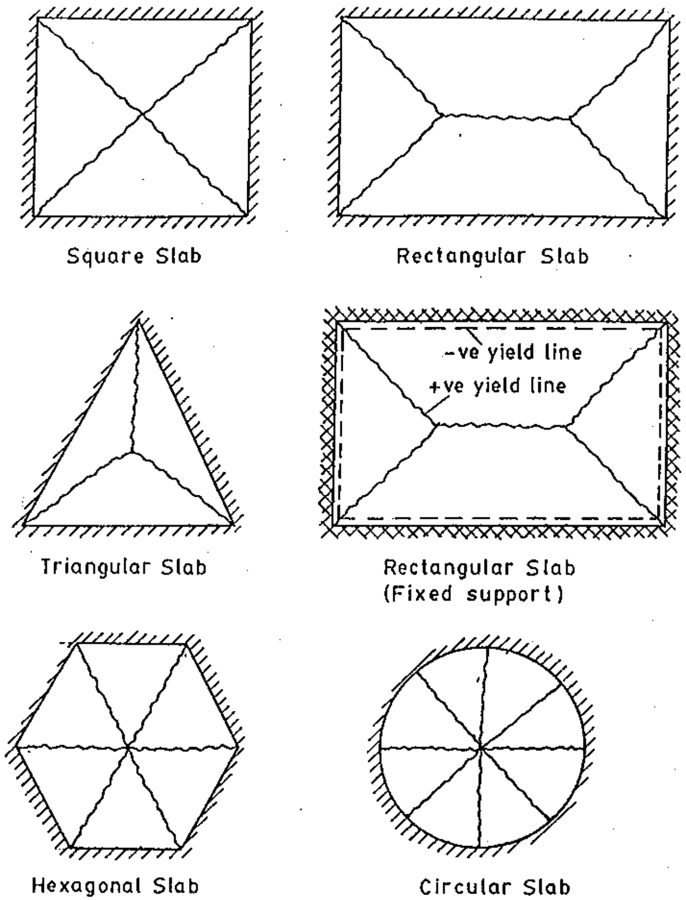


Fig. 9.14 Typical Yield Line Patterns in Reinforced Concrete Slabs

$$m_a = \Sigma m . \cos^2\alpha$$

In square slabs, isotropically reinforced, equal steel is provided in perpendicular directions. If 'm' is the ultimate moment of yield lines at right angles to the direction of the reinforcement, then the ultimate moment of any yield line at an angle a to the horizontal is given by

$$\begin{aligned} m_a &= m . \cos^2\alpha + \mu m . \cos^2(90 - \alpha) \\ &= m . \cos^2\alpha + \mu m . \sin^2\alpha \end{aligned}$$

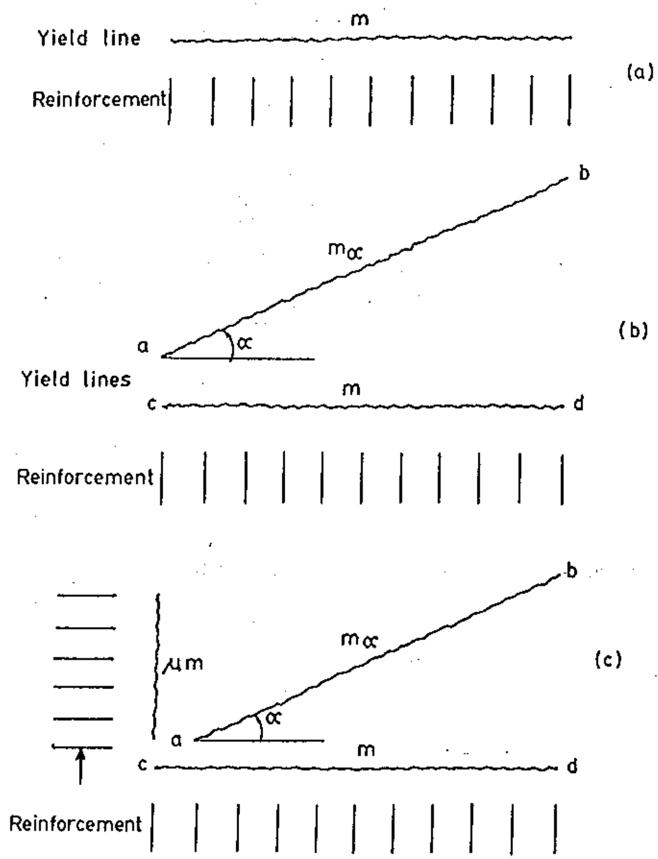


Fig. 9.15 Yield Moments

$$= m\,(\cos^2\alpha + \sin^2\alpha)$$
$$= m$$

This criterion indicates that in an isotropically reinforced slab, the yield moment is the same in all directions. Referring to Fig. 9.15 (c), if the reinforcement is arranged in two directions at right angles but with unequal magnitude, the slab is said to be orthotropically reinforced. This type of arrangement of different steel in perpendicular directions is very common in rectangular slabs. In such cases the yield moment along a line inclined at an angle 'α' to the horizontal is computed as

$$m\alpha = m\,.\,\cos^2\alpha + \mu\, m\,.\,\cos^2(90 - \alpha)$$
$$= m\,.\,\cos^2\alpha + \mu\, m\,.\,\sin^2\alpha$$

9.7.4 Ultimate loads on slabs

There are two methods of determining the ultimate load capacity of slabs. They are based on the principles of (a) Virtual work (b) Equilibrium.

The virtual work method is based on the principle that the applied loads causing a small virtual displacement is equal to the internal work done or energy dissipated in rotation along the yield lines. It is generally assumed that the elastic deformations in the slab are negligible and all the plastic deformation takes place at the yield lines.

In the equilibrium method, the equilibrium of the individual segments of slab formed by the yield lines under the action of the applied loads and moments and forces acting on the edges of the segments are considered.

Both the virtual work and equilibrium methods give an upper bound to the collapse load on the slab. Hence it is essential that all possible yield line patterns have to be investigated to find the lowest value of the ultimate load.

If a correct yield line pattern has been assumed, the lower bound solution will coincide with the upper bound solution but lower bound solutions are not available except for a few simple cases of slabs. Test results have shown that the actual failure loads of slabs is greater than the predicated values by yield line analysis because of membrane action. Hence the upper bound solutions resulting from yield line analysis can be used with a reasonable degree of safety.

9.7.5 Yield line analysis by virtual work method

(1) Isotropically reinforced square slab simply supported and supporting uniformly distributed load

The principle of the virtual work method is to equate the internal work done due to rotation of yield lines to the external work done due to the loads having a virtual displacement.

External work done = $(W.\,\delta)$
Where W = Loads
δ = Virtual displacement
Internal work done = $(M\,\theta) = \Sigma\,(m.L.\theta)$

Where m = ultimate moment per unit length of yield line
L = length of yield line

Referring to Fig. 9.16,

The square slab is isotropically reinforced. The ultimate moment along the yield line is also 'm' and the total work done in yield line ac is given by

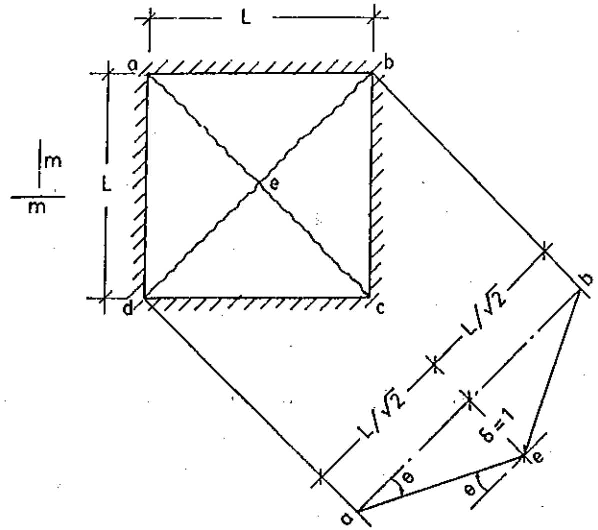


Fig. 9.16 Yield Line Pattern in a Square Slab (Simply Supported)

$$\Sigma\,(M\,\theta)_{ac} = \Sigma(m\,L\,\theta) = [m\,\sqrt{2}L\,(2\sqrt{2})/L] = 4\,m$$

The work done in yield line bd is the same as in ac.
Total internal work done = $\Sigma(M\theta) = 8$ m
For a virtual displacement of $\delta = 1$ at e, the centre of gravity of each of the triangular elements deflects by 1/3

$\therefore$
$$\Sigma\,(W\delta) = (1/3)\,w.L^2$$

Where w = uniformly distributed load on slab, By equating

$$\Sigma\,(M.\theta) = \Sigma(W.\delta)$$

We have,
$$m = \left(\frac{wL^2}{24}\right)$$

(2) Isotropically reinforced square slab fixed on all edges and subjected to a uniformly distributed load

Referring to Fig. 9.17, since the edges are fixed negative yield lines will form along the edges.

Internal work done along the positive yield lines ac and db is given by

$\Sigma(M.\theta) = 8$ m (Refer previous problem).

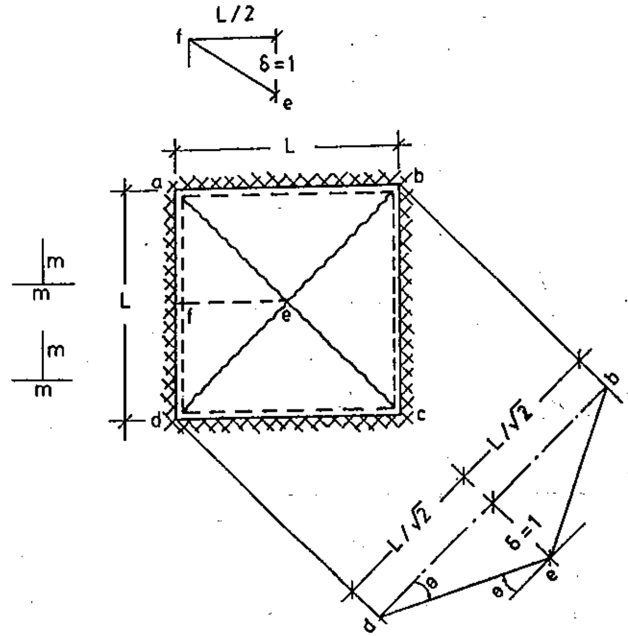


Fig. 9.17 Yield Line Pattern in a Square Slab (Fixed)

Internal work done along the negative yield lines ab, bc, cd, and de is given by

$$\Sigma(M.\theta) = 4\,[m.L\,(2/L)] = 8\text{ m}$$

$\therefore$ Total internal work done = $\Sigma\,(M\,.\,\theta) = 16$ m
External work done = $\Sigma\,(W\,.\,\delta) = (1/3)\,w.L^3$
Equating internal to external work done

$$\Sigma(M.\theta) = \Sigma(W.\delta)$$

$$16\text{m} = (1/3)\text{ wL}^2$$

$$m = \left(\frac{wL^2}{48}\right)$$

(3) Triangular slab simply supported on adjacent edges and subjected to uniformly distributed load and isotropically reinforced

Referring to Fig. 9.18, the triangular slab acb is simply supported at ac and cb. The yield line formed is cd. Unit displacement is given for point d. Since slab is isotropically reinforced $m_x = m_y = m$

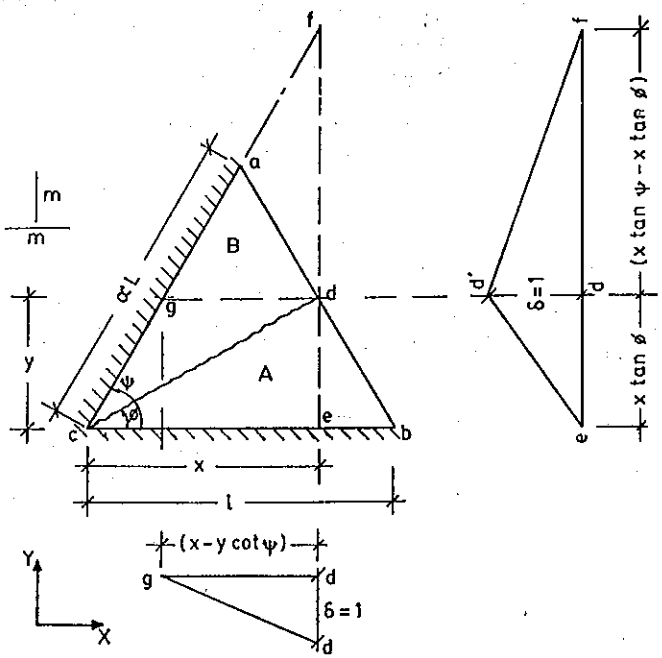


Fig. 9.18 Triangular Slab Simply Supported on Adjacent Edges

For Element A, $\theta_{Ax} = (1/de) = (1/x.\tan\phi)$ and $\theta_{Ay} = 0$

$\therefore$ $$(M_x.\theta_x + M_y.\theta_y)_A = (x.m.\theta_x) = m\cot\phi$$

For element B, $\theta_{Bx} = (1/df) = 1/(x\tan\Psi - x\tan\phi)$

and $\theta_{By} = (1/gd) = 1/(x - y\cot\psi)$

$\therefore$ $$(M_x.\theta_x + M_y.\theta_y)_B = m\left[\frac{1}{\tan\psi - \tan\phi} + \frac{1}{\cot\phi - \cot\psi}\right]$$

$$= m\left[\frac{1 + \tan\psi\tan\phi}{\tan\psi - \tan\phi}\right] = m\cot(\psi - \phi)$$

Thus $\Sigma(M.\theta) = m[\cot(\Psi - \phi) + \cot\phi]$

And $\Sigma(W.\delta) = (1/6)\, w\alpha L^2 .\sin\Psi$

Equating $\Sigma(M.\theta) = \Sigma(W.\delta)$

We have

$$m = \frac{w\alpha L^2 \sin\psi}{6[\cot(\psi - \phi) + \cot\phi]}$$

$$m = (1/6) w\alpha L^2 .\sin\phi.\sin(\Psi - \phi)$$

For a maximum value of m, $(dm/d\phi) = 0$

$\therefore$ $$\cos\phi.\sin(\Psi - \phi) = \sin\phi\cos(\Psi - \phi)$$

$$\tan\phi = \tan(\Psi - \phi)$$

$$\phi = (1/2)\Psi$$

Hence the yield line bisects the angle opposite the free edge.

Substituting the value of ϕ we have the final value given by

$$m = \{(1/6)\, w\,\alpha\, L^2 .\sin^2(\Psi/2)\}$$

In a right angled triangle $\Psi = 90°$. Then $m = (w.\alpha.L^2)/6$

4) Orthotropically Reinforced Rectangular slab, Simply supported along its edges and subjected to a Uniformly distributed load of w/unit area

Referring to Fig. 9.19, the rectangular slab abcd is simply supported at the edges. The yield line pattern assumed is given by ae, de, bf, cf and ef. M and μm are the yield moments along the x and y-axis respectively. In the yield line pattern shown 'βL' is an unknown dimension. The yield line ef is given a virtual displacement of unity.

For Element A, $\theta_x = (2/\alpha.L), \theta_y = 0, M_x = mL$

$\therefore$ $$(M_x.\theta_x + M_y.\theta_y)_A = (2m/\alpha.)$$

For element D, $\theta_X = 0, \theta_y = (1/\beta), M_y = (\alpha.L\mu m)$

$\therefore$ $$(M_x.\theta_X + M_y.\theta_y)_D = (\alpha.\mu m/\beta)$$

Since elements A and C and B and D are similar

$$\Sigma(M.\theta) = 2\left[\frac{2m}{\alpha} + \frac{\alpha\mu m}{\beta}\right]$$

The external work done is given by

$$\Sigma(W.\delta) = w.L^2\left[\frac{2\beta\alpha}{3} + \frac{\alpha(1 - 2\beta)}{2}\right]$$

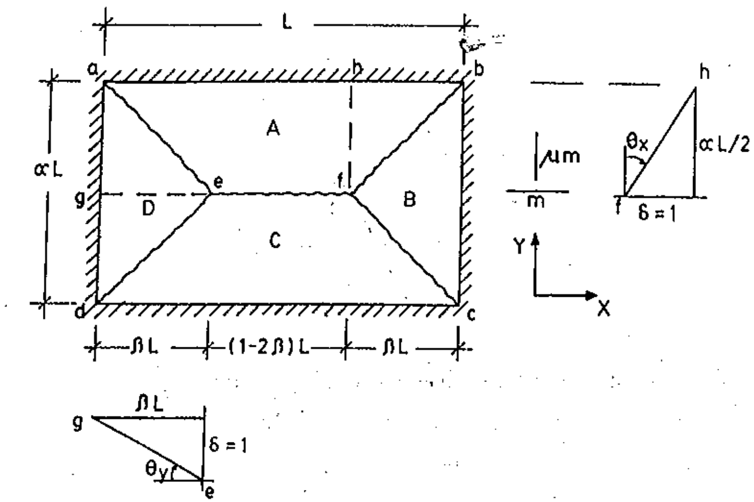


Fig. 9.19 Rectangular Slab Simply Supported at the Edges

Equating $\Sigma(M.\theta) = \Sigma(W.\delta)$ we get

$$m = \left(\frac{1}{12}\right)\alpha^2 L^2 \left[\frac{3\beta - 2\beta^2}{2\beta + \mu\alpha^2}\right]$$

If the work equation is of the form

$$m = W\left[\frac{f_1(x_1x_2)}{f_2(x_1x_2)}\right]$$

For a maximum value of m,

$$\left(\frac{\delta m}{\delta x}\right) = 0$$

This is obtained for the condition

$$\frac{f_1(x_1,x_2)}{f_2(x_1,x_2)} = \frac{\left(\frac{\delta}{\delta x}\right)[f_1(x_1,x_2)]}{\left(\frac{\delta}{\delta x}\right)[f_2(x_1,x_2)]}$$

Using this criterion, for a maximum value of m,

$$\left(\frac{\delta m}{\delta \beta}\right) = 0$$

Hence we have

$$\left(\frac{3\beta - 2\beta^2}{2\beta + \mu\alpha^2}\right) = \left(\frac{3 - 4\beta}{2}\right)$$

Cross multiplying, we get the quadratic as

$$[4\beta^2 + 4\mu\alpha^2\beta - 3\mu\alpha^2] = 0$$

The positive root of this quadratic as

$$\beta = {}^1/_2\,[\sqrt{(3\mu\alpha^2 + \mu^2\alpha^4)} - 3\alpha\sqrt{\mu}$$

Substituting the value of β in the equation for m, we have

$$m = \left(\frac{w\alpha^2L^2}{24}\right)[\sqrt{(3 + \mu\alpha^2)} - \mu\alpha^2]^2$$

5) Isotropically reinforced circular slab, simply supported all round and uniformly loaded

Referring to Fig. 9.20, a circular slab of radius 'r' is simply supported at the edges and supports a uniformly distributed load of w/unit area. In the circular slabs, the failure will take place by the formation of an infinite number of positive yield lines running radially from the centre to the circumference, resulting in the formation of a flat cone at collapse.

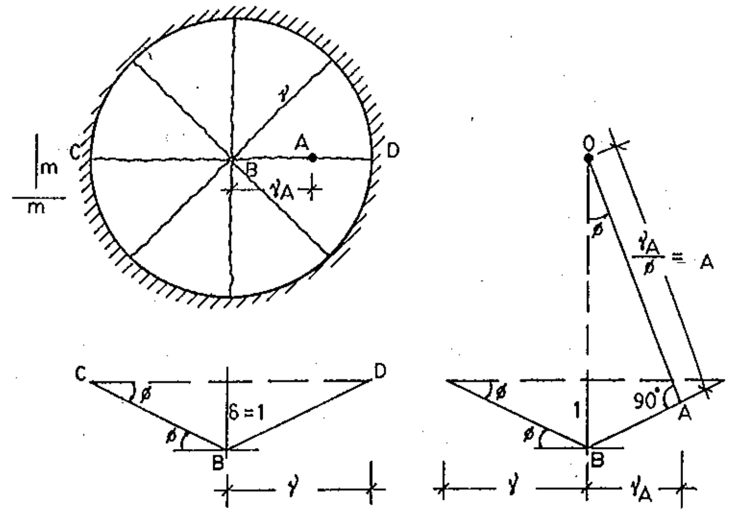


Fig. 9.20 Circular Slab Simply Supported at the Edges

For unit displacement at the centre of slab

External work done = $\Sigma(W\,.\,\delta) = (\pi.r^2.w/3)$

For a central displacement of unity

$$\angle AOB = \phi = (1/r)$$

$\therefore$ Length OA $= (r_A/\phi) = r_A.r$

Change of slope of the slab in the tangential direction at A, per unit length of are is equal to the angle between the two normal unit length of arc apart at A and is given by $(1/r_A.r)$. Total change of slope in one complete revolution is give by

$$\Sigma\theta = (2\mu r_A \times 1/r_A.r) = (2\pi/r)$$

Internal work done in rotation at yield line = $\Sigma(mL\theta)$ since all the yield lines are of equal length.

Work done $= mL\Sigma\theta = m\ r\ (2\pi/r) = 2\pi m$

Equating internal work to external work done we have

$$(1/3).\pi m.r^2 w = 2\pi\, m$$

$$m = \left(\frac{wr^2}{6}\right)$$

9.7.6 Yield Line Analysis By Equilibrium Method

1) Square slab, isotropically Reinforced and Subjected to a Uniformly Distributed Load

The assumed yield line pattern is shown in Fig. 9.21. Considering the equilibrium of the triangular element C, we have by taking moments about the edge ab

$$m.L = (1/2)\, L.(L/2)\,(w)\,(L/6)$$

$$m = \left(\frac{wL^2}{24}\right)$$

2) Rectangular slab Orthotropically Reinforced and subjected to a Uniformly distributed load

The assumed yield line pattern is shown in Fig. 9.22. Considering the equilibrium of the trapezodial element A.

$$m.L = w\left[(1-2\beta)L.\left(\frac{\alpha L}{2}\right)\left(\frac{\alpha L}{4}\right) + 2\left(\frac{\beta L}{2}\right)\left(\frac{\alpha L}{2}\right)\left(\frac{\alpha L}{6}\right)\right]$$

$$m = \left[\left(\frac{w\alpha^2.L^2}{24}\right)(3-4\beta)\right]$$

Taking moments about be for element B

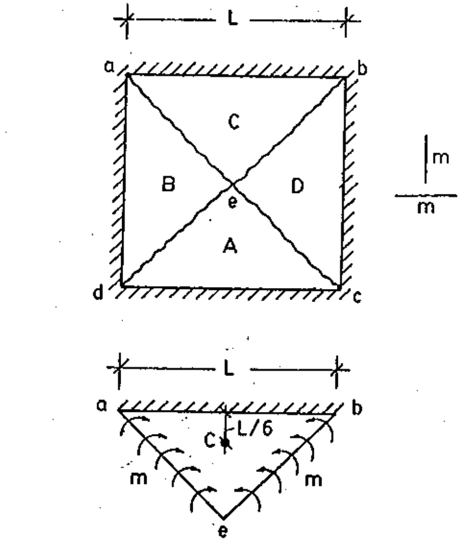


Fig. 9.21 Equilibrium of Element C

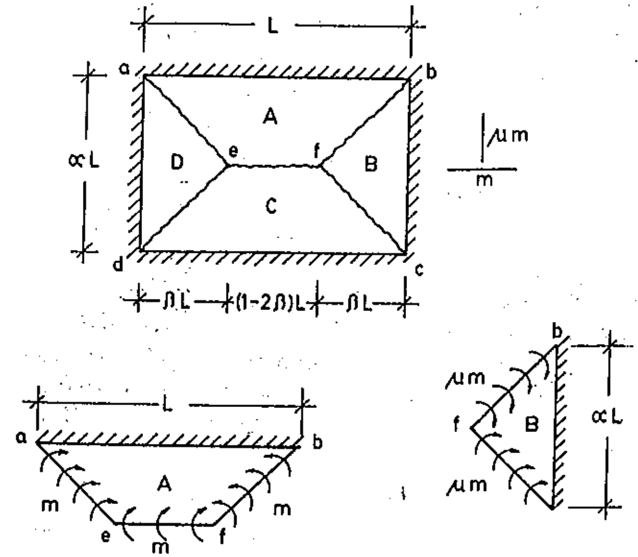


Fig. 9.22 Equilibrium of Element in a Rectangular Slab

$$\mu.m.L = (1/6)\, w.\alpha.\beta^2.L^3$$

$$m = \left(\frac{w\beta^2L^2}{6\mu}\right)$$

Equating the two equilibrium equations we have,

$$\left[\frac{\alpha^2(3-4\beta)}{24}\right] = \left(\frac{\beta^2}{6\mu}\right)$$

or

$$[4\beta^2 + 4\mu\alpha^2\beta - 3\mu\alpha^2] = 0$$

the positive root of this quadratic in β is

$$\beta = {}^1/_2\, [\sqrt{(3\mu\alpha^2 + \mu^2.\alpha^4)} - \mu\alpha^2]$$

Substituting the value of β in the equilibrium equation we have

$$m = \left(\frac{w\alpha^2L^2}{24}\right)[\sqrt{(3+\mu\alpha^2)} - \alpha\sqrt{\mu}]^2$$

3) Hexagonal Slab isotropically reinforced and subjected to uniformly distributed load

The isotropically reinforced hexagonal slab is shown in Fig. 9.23. Considering the equilibrium of element A we have,

$$m.L = \left[\frac{1}{2}.L.\frac{\sqrt{3}}{2}.L.\frac{1}{3}.\frac{\sqrt{3}}{2}.L\right]w$$

$$m = \left(\frac{w.L^2}{8}\right)$$

9.7.7 Design Example

Design a simply supported square slab of 5 m side length to support a service live load of 4 kN/m². Adopt M-20 Grade concrete and Fe-415 grade HYSD bars. Assume load factors according to the IS: 456 - 2000 standard code.

a) Data

Square slab, simply supported at edges
Side length = L = 5 m
Live Load = q = 4 kN/m²
f_{ck} = 20 N/mm², f_y = 415 N/mm²

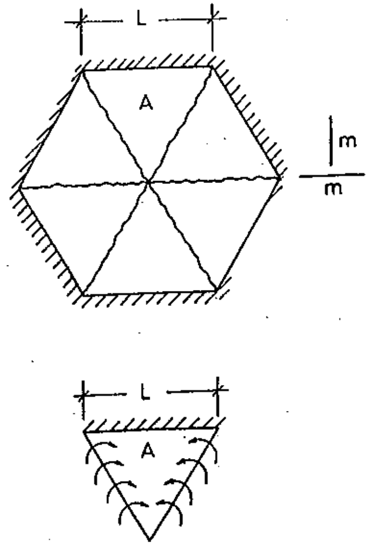


Fig. 9.23 Equilibrium of Elements in a Hexagonal Slab

b) Depth of Slab

For simply supported slabs using Fe-415 HYSD bars, according to IS: 456-2000 code (span / overall depth) ratio = (35 × 0.8) = 28

$$\therefore \quad \text{Overall depth} = D = \left(\frac{\text{span}}{28}\right) = \left(\frac{5000}{28}\right) = 178.5 \text{ mm}$$

Hence adopt overall depth = 180 mm
And effective depth = d = 160 mm

c) Ultimate loads

Self weight of slab = (18 × 25) = 4.5 kN/m²
Live load = 4.0
Floor Finishes = 1.5
Total Load = w = 10.0 kN/m²
(Ultimate load = w_u = (1.5 × 10) = 15 kN/m²

d) Ultimate moments and shear forces

The yield or ultimate moment capacity of a simply supported square slab is given by

$$m = M_u = (w_u.L^2/24) = (15 \times 5^2)/24 = 15.625 \text{ kN.m/m}$$

Ultimate shear $= V_u = (0.5\, w_u.L) = (0.5 \times 15 \times 5) = 37.5$ kN/m

e) Limiting Moment capacity of slab section

$$M_u = 0.138\, f_{ck}.b.d^2$$
$$= (0.138 \times 20 \times 10^3 \times 160^2)\, 10^{-6}\text{kN.m}$$
$$= 70.65 \text{ kN.m}$$

Since $M_u < M_{u,lim}$ the section is under reinforced.

f) Reinforcements in slab

$$M_u = m = 0.87\, f_y A_{st} d \left[1 - \frac{A_{st} f_y}{b\, d\, f_{ck}}\right]$$

$$(15.625 \times 10^6) = (0.87 \times 415 A_{st} \times 160)\left[1 - \frac{415 A_{st}}{(1000 \times 160 \times 20)}\right]$$

Solving $A_{st} = 280 \text{ mm}^2$

Adopt 10 mm diameter bars at 280 mm centres ($A_{st} = 280 \text{ mm}^2$)

g) Check for shear stress

$$\tau_v = \left(\frac{V_u}{bd}\right) = \left(\frac{37.5 \times 10^3}{1000 \times 160}\right) = 0.23 \text{ N/mm}^2$$

$$\left(\frac{100 A_{st}}{bd}\right) = \left(\frac{100 \times 280}{1000 \times 160}\right) = 0.175$$

Permissible shear stress $= k_s \tau_c = (1.25 \times 0.36) = 0.45 \text{ N/mm}^2$
Since $k_s \tau_c > \tau_v$, shear stresses are within safe permissible limits.

9.7.8 Design Example

Design a rectangular slab 6m by 4m in size and simply supported at the edges for a service live load of 4 kN/m². Assume co-efficient of orthotropy (μ) as 0.7, M-20 grade concrete and Fe-415 HYSD Bars.

a) Data

L	= 6m	μ	= 0.7
αL	= 4m	f_{ck}	= 20 N/mm²
α	= 0.666	f_y	= 415 N/ mm²

b) Depth of Slab

$$\text{Overall depth} = D = \left(\frac{\text{span}}{28}\right) = \left(\frac{4000}{28}\right) = 143 \text{ mm}$$

Adopt overall depth = 150 mm
Effective depth = 130 mm

c) Ultimate loads

Self weight of slab = (0.15 × 25)	= 3.75 kN/m²
Live load	= 4.00
Floor Finishes	= 1.25
Total service load	= 9.00 kN/m2

Therefore, ultimate load $= w_u = (1.5 \times 9) = 13.5 \text{ kN/m}^2$

d) Ultimate Moments and Shear Forces

$$M_u = m = \left(\frac{w_u \alpha^2 L^2}{24}\right)\left[\sqrt{(3+\mu\alpha^2)} - \alpha\sqrt{\mu}\right]^2$$
$$= \left(\frac{13.5 \times 16}{24}\right)\left[\sqrt{(3 + 0.7 \times 0.44)} - 0.666\sqrt{0.7}\right]^2$$
$$= 14.3 \text{ kN.m/m}$$

$$V_u = (0.5 w_u L) = (0.5 \times 13.5 \times 4) = 27 \text{ kN/m}$$

e) Limiting Moment capacity of the Slab

$$M_{u,lim} = (0.138\, f_{ck}\, b\, d^2) = (0.138 \times 20 \times 1000 \times 130^2)\, 10^{-6}$$
$$= 46.64 \text{ kN.m/m} < M_u$$

Hence, the section is underreinforced.

f) Reinforcements

$$M_u \text{ (short span)} = 0.87\, f_y A_{st}\, d\left[1 - \frac{A_{st} f_y}{b\, d\, f_{ck}}\right]$$

$$(14.3 \times 10^6) = (0.87 \times 415 A_{st} \times 130)\left[1 - \frac{415 A_{st}}{(1000 \times 130 \times 20)}\right]$$

Solving $A_{st} = 323\ \text{mm}^2/\text{m}$

Adopt 10 mm diameter bars at 240 mm centres ($A_{st} = 327\ \text{mm}^2$) in the short span direction.

$$A_{st}\ (\text{Long span}) = \mu\ (A_{st}) = (0.7 \times 323) = 226\ \text{mm}^2 > A_{st},\ \text{min}$$

Provide 10mm diameter bars at 340 mm centers along the long span (A_{st} = 231 mm^2)

g) Check for shear Stress

$$\tau_{nu} = \left(\frac{V_u}{b\,d}\right) = \left(\frac{27 \times 1000}{1000 \times 130}\right) = 0.20\ \text{N/mm}^2$$

$$\left(\frac{100A_{st}}{b\,d}\right) = \left(\frac{100 \times 327}{1000 \times 130}\right) = 0.25$$

Permissible shear stress (Table-19 of IS:456) = $k_s.\tau_c$= (1.3 × 0.36) = 0.468 N/mm^2 which is greater than τ_v. Hence shear stresses are within safe permissible limits.

9.7.9 Analysis Example

A right angled triangular slab is simply supported at the adjacent edges AB and BC. The side AB = BC = 4m and CA = 6m. The slab is isotropically reinforced with 10mm diameter bars at 100mm centres, both ways at an average effective depth of 120mm. The overall depth of the slab is 150mm. If f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2. Estimate the safe permissible service live load on the slab.

a) Data

Triangular Slab ABC right angled at B

AB = BC = 4m

$L = 4\text{m},\ \alpha L = 4\text{m}\ \therefore\ \alpha = 1$

Reinforcement provided (10mm diameter) at 100 mm centres both ways.

$$A_{st} = \left(\frac{1000 \times 78.5}{100}\right) = 785\ \text{mm}^2/\text{m}$$

$$f_{ck} = 20\ \text{N/mm}^2 \quad \text{and} \quad f_y = 415\ \text{N/mm}^2.$$

b) Yield or ultimate moment

$$m = M_u = 0.87\,f_y\,A_{st}.d\left[1 - \frac{A_{st}f_y}{b d\,f_{ck}}\right]$$

$$= (0.87 \times 415 \times 785 \times 120)\left[1 - \frac{(785 \times 415)}{(1000 \times 120 \times 20)}\right]$$

$$= (29.4 \times 10^6)\ \text{N.mm}$$

$$= 29.4\ \text{kN.m}$$

c) Ultimate load on Slab

$$w_u = \left(\frac{6m}{\alpha L^2}\right) = \left(\frac{6 \times 29.4}{1 \times 16}\right) = 11.025\ \text{kN/m}^2$$

d) Service Live Load

Total Service Load = (11.025/1.5) = 7.35 kN/m^2
Dead load of slab = (0.15 × 25) = 3.75 kN/m^2
Therefore, service live load = (7.35 − 3.75) = 3.6 kN/m^2

9.7.10 Analysis Example

A hexagonal slab of side length 3m is simply supported at the edges and it is isotropically reinforced with 12mm diameter bars at 150mm centres, both ways at an average effective depth of 118 mm. The overall depth of the slab is 150 mm. Calculate the ultimate load capacity of the slab and also the safe permissible service live load if f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2.

a) Data

Hexagonal slab, simply supported at edges
Side length = L = 3m
12 mm diameter bars provided at 150mm centres

$$\therefore \quad A_{st} = \left(\frac{1000 \times 113}{150}\right) = 753\ \text{mm}^2/\text{m}$$

b) Yield or ultimate moment

$$m = M_u = 0.87\,f_y\,A_{st}d\left[1 - \frac{A_{st}f_y}{b d\,f_{ck}}\right]$$

$$= (0.87 \times 415 \times 753 \times 118)\left[1 - \frac{(753 \times 415)}{(1000 \times 118 \times 20)}\right]$$

$$= 27.84 \times 10^6 \text{ N.mm}$$

$$= 27.84 \text{ kN.m}$$

c) Ultimate load on Slab

$$w_u = \left(\frac{8m}{L^2}\right) = \left(\frac{8 \times 27.84}{9}\right) = 24.75 \text{ kN/m}^2$$

d) Service live load

$$\text{Total service load} = \left(\frac{24.75}{1.5}\right) = 16.5 \text{ kN/m}^2$$

Dead load of slab = $(0.15 \times 25) = 3.75$ kN/m^2

$\therefore$ Safe permissible live load = $(16.5 - 3.75) = 12.75$ kN/m^2

9.7.11 Design Example

Design a circular slab of diameter 5m which is simply supported at the edges. Live load = 4 kN/m^2. Assume M-20 grade concrete and Fe-415 HYSD bars. Assume load factors according to IS:456-2000.

a) Data

Circular Slab, simply supported at edges
Diameter of slab = 5 m, radius = r = 2.5 m
Live load = 4 kN/m^2
$f_{ck} = 20$ N/mm^2, $f_y = 415$ N/mm^2

b) Depth of Slab

$$\text{Overall depth of slab} = D = \left(\frac{\text{span}}{28}\right) = \left(\frac{5000}{28}\right) = 178.5 \text{ mm}$$

Adopt overall depth = 180 mm
Effective depth = d = 150 mm

c) Ultimate Loads

Self weight of slab = (0.18×25)	=	4.5 kN/m^2
Live Load	=	4.0
Floor Finishes	=	1.5
Total Service Load	=	10.0 kN/m2

Ultimate design load = $w_u = (1.5 \times 10) = 15$ kN/m^2

d) Ultimate moments and shear forces

The yield moment or ultimate moment capacity of a simply supported circular slab is given by the relation.

$$m = \left(\frac{w_u r^2}{6}\right) = \left(\frac{15 \times 2.5^2}{6}\right) = 15.625 \text{ kN.m/m}$$

e) Limiting or balanced moment capacity of slab

$$\begin{aligned} M_{u,lim} &= 0.138 f_{ck}.b.d^2 \\ &= (0.138 \times 20 \times 10^3 \times 150^2)\,10^{-6} \\ &= 62.1 \text{ kN.m} \end{aligned}$$

Since $m < M_{0,lim}$, section is under reinforced.

f) Reinforcements

$$m = M_u = 0.87 f_y A_{st}.d\left[1 - \frac{A_{st} f_y}{b d f_{ck}}\right]$$

$$(15.625 \times 10^6) = (0.87 \times 415 \times A_{st} \times 150)\left[1 - \frac{415 A_{st}}{(1000 \times 150 \times 20)}\right]$$

Solving $A_{st} = 300$ mm^2/m

Adopt 10mm diameter bars at 250mm centres ($A_{st} = 314$ mm^2)

g) Check for shear stress

$$V_u = (0.5 w_u.L) = (0.5 \times 15 \times 5) = 37.5 \text{ kN}$$

$$\tau_v = \left(\frac{V_u}{bd}\right) = \left(\frac{37.5 \times 10^3}{1000 \times 150}\right) = 0.25 \text{ N/mm}^2$$

$$\left(\frac{100 A_{st}}{bd}\right) = \left(\frac{100 \times 314}{1000 \times 150}\right) = 0.209$$

From Table-19 of IS:456 code, $k_s \tau_c = (1.25 \times 0.36) = 0.45$ N/mm^2

Since $k_s \tau_c > \tau_v$, shear stresses are within safe permissible limits.

9.7.12 Analysis Example

A rectangular slab 4.5m by 6.5m is simply supported at the edges. The coefficient of orthotrophy. μ = 0.75. If the ultimate design load is 12 kN/m^2, estimate the ultimate moment capacity of the slab in the short span direction by deriving the expression.

$$w = (24m/L_y^2)\,(\mu/\tan^2\phi)$$

from first principles where

w = ultimate design load
m = ultimate moment capacity of the slab in the short span direction
μ = coefficient of orthotropy
L_y = short span length
ϕ = angle made by the shorter yield line with the side L_y

a) Data

Short span length = L_y = 4.5m
Long span length = $\alpha L_y = L_x$ = 6.5m
Ultimate design load = w = 12 kN/m^2
Coefficient of orthotrophy = μ = 0.75
Ultimate moment capacity of slab in the short span direction = m kN.m/m
Ultimate moment capacity of slab in the long span direction = $\mu\, m$ kN.m/m

b) Derivation of relation between ultimate load and ultimate moment

Referring to Fig. 9.24
External work done = $\Sigma\,(W\,.\,\delta)$
Yield line of 'ef' is given unit deflection, δ = 1
Work done by the elements 1, 2, 3 and 4 are computed as detailed below
For element – 1,

$$\begin{aligned}\text{Work done} &= [L_y\,(\alpha - \tan\phi)\,(L_y/2).(1/2)] \\ &\quad + [(2\times {}^1/_2)\times(L_y/2)\tan\phi)\times(L_y/2)\times 1/3)] \\ &= [(1/4)\,L_y^2\,(\alpha - \tan\phi) + (1/12)\,L_y^2\tan\phi]\end{aligned}$$

For element 2,

$$\begin{aligned}\text{Work done} &= [(1/2).L_y\,(L_y/2)\tan\phi\,(1/3)] \\ &= (1/12)L_y^2\tan\phi\end{aligned}$$

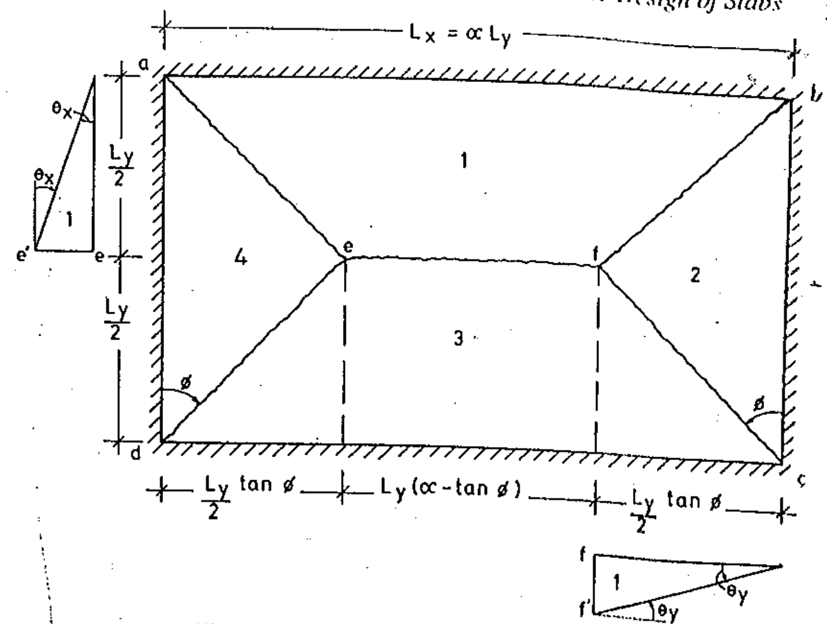


Fig. 9.24 Rectangular Slab With Simply Supported Edges

Therefore,
Total external work done in elements 1, 2, 3 and 4 is obtained as

$\Sigma(W.\delta)$ = 2 (work done in element 1 + work done in element 2)

$\Sigma(W.\delta) = (1/2).w.L_y^2\,(\alpha - 1/3.\tan\phi)$

Internal Work done on yield lines ae, bf, cf and de by rotation of elements 1, 2, 3 and 4 is obtained as follows:

For element – 1,

$$\theta_x = (2/L_y),\ \theta_x = 0,\ M_x = m.L_x = m.\alpha L_y$$

$$\therefore \quad (M_x.\theta_x + M_y.\theta_y) = 2m.\alpha)$$

For Element – 2,

$$\theta_y = (2/L_y.\tan\phi),\ \theta_x = 0 \quad \text{and} \quad M_y = \mu.m.L_y$$

$$\therefore \quad (M_x.\theta_x + M_y.\theta_y) = 2\mu m/\tan\phi)$$

(Total internal work done in all the yield lines is

$$\Sigma(M.\theta) = 2\,[2m.\alpha + (2\mu m/\tan\phi)]$$

$$= 4m[\alpha + (\mu/\tan\phi)]$$

For equilibrium we must equate

$\Sigma(W.\delta) = \Sigma(M.\theta)$

$(1/2).w.L_y^2(a - 1/3 \tan\phi) = 4m(\alpha + \mu/\tan\phi)$

$(w\, L_y^2/24m) = (\alpha + \mu/\tan\phi)(3\alpha - \tan\phi)$ ...(3)

For a maximum value of 'm', we have

$[dm/d\,(\tan\phi)] = 0$

Differentiating the R.H.S. of equation – 3 we have

$$(\alpha + \mu/\tan\phi)(3\alpha - \tan\phi) = (-\mu/\tan^2\phi)/(-1) = (\mu/\tan^2\phi)$$

$$[\alpha.\tan^2\phi + 2\mu.\tan\phi - 3\alpha.\mu] = 0$$

The positive root of the quadratic is

$$\tan\phi = \sqrt{(3\mu + \mu^2/\alpha^2)} - (\mu/\alpha)$$

Which gives the values tan ϕ for minimum collapse load.
Substituting this in equation 3, the collapse load is expressed as

$$w = (24.m/L_y^2)(\mu/\tan^2\phi) \quad ...(4)$$

c) Example

In the given example:

$$w = 12\ \text{kN/m}^2$$
$$\mu = 0.75$$
$$L_x = 6.5\ \text{m}$$
$$L_y = 4.5\ \text{m}$$

$\therefore \quad \alpha = (6.5 / 4.5) = 1.44$

$$\tan\phi = \sqrt{(3\mu + \mu^2/\alpha^2)} - (\mu/\alpha)$$

$$= \sqrt{(2 \times 0.75) + (0.75^2/1.44^2)} - (0.75/1.44)$$

$$= 1.06$$

$$m = (wL_y^2/24)(\tan^2\phi/\mu)$$

$$= [(12 \times 4.5^2)/(24)]\,[(1.06^2/0.75)]$$

$$= 15.16\ \text{kN.m/m}$$

If the slab is fixed on all four sides and m' is the moment capacity of the negative reinforcement, then the relation between the ultimate moment and ultimate load on the slab is expressed by the relation,

$$w = [24(m + m')/L_y^2]\,[\mu/\tan^2\phi]$$

If $\quad m = m'$, then

$$w = (48m/L_y^2)(\mu/\tan^2\phi)$$

9.7.13 Analysis Example

A two- way R.C.C. slab is rectangular having a size 4m by 5 m with two longer edges fixed in position and the two shorter edges are simply supported. Derive the relation between moment capacity of slab and ultimate load by first principles and hence design the slab for a working live load of 3 kN/m^2 by yield line theory. Assume $\mu = 0.8$ Adopt M–15 grade concrete and HYSD bars.

a) Data

Short span length = $L_y = 4$ m
Long span length = $L_x = 5$ m
Coefficient of orthotropy = $\mu = 0.8$
Working live load = $q = 3$ kN/m^2
Longer edges are fixed and shorter edges are simply supported.
Concrete : M-15 grade
Steel : Fe-415 grade HYSD bars

b) Stresses

$$f_{ck} = 15\ \text{N/mm}^2, f_y = 415\ \text{N/mm}^2$$

c) Derivation of Relation

Referring to Fig. 9.25

External work done is obtained as

$$\Sigma(W.\delta) = w.(1/2)L_y^2(\alpha - 1/3\tan\phi)$$

Internal work done by rotation of

(a) Positive yield lines = $4m\,(\alpha + \mu/\tan\phi)$
(b) Negative yield lines = $4\,\alpha.m'$

(Total internal work done $(M\theta)$ is expressed as

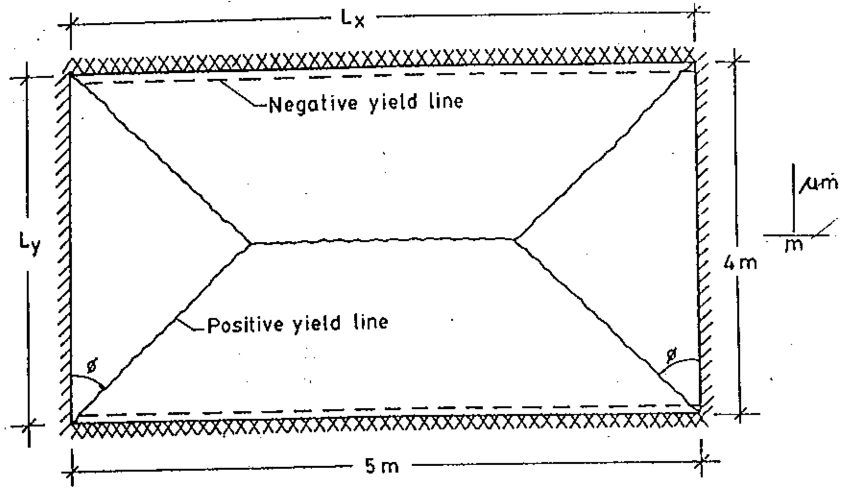


Fig. 9.25 Rectangular Slab With Fixed and Simply Supported Edges

$$\Sigma(M\theta) = 4\alpha(m+m') + (4m\mu/\tan\phi)$$

Equating $\Sigma(W.\delta) = \Sigma(M.\theta)$

$$w.(1/2)L_y^2(\alpha - 1/3\tan\phi) = 4\alpha(m+m') + (4m\mu/\tan\phi)$$

Assuming $m = m'$

$$w.(1/2)L_y^2(\alpha - 1/3\tan\phi) = 4m(2\alpha + \mu/\tan\phi)$$

$$(w.L_y^2)(24m) = (2\alpha + \mu/\tan\phi)/(3\alpha - \tan\phi)$$

For a maximum value 'm' differentiating the right hand side of the equation we have

$$2\alpha + (\mu/\tan\phi)/(3\alpha - \tan\phi) = (-\mu/\tan^2\phi)(-1) = (\mu/\tan^2\phi)$$

$$[2\alpha\tan^2\phi + 2\mu\tan\phi - 3\alpha.\mu] = 0$$

The positive root of this quadratic is

$$\tan\phi = (1.5\mu + \mu^2/4\alpha^2) - (\mu/2\alpha)$$

and $$m = (wL_y^2/24)(\tan^2\phi/\mu)$$

d) Example

$\alpha = (L_x / L_y) = (5/4) = 1.25$
$L_x = 5\ m,\ L_y = 4m,\ \mu = 0.8$

e) Thickness of Slab

For a two way slab (IS: 456)
Effective depth $= d =$ (span/35) $= (4000 / 35) = 114$ mm.
Adopt $d = 120$ mm and overall depth $D = 150$

f) Loads

Self weight of slab $= (0.15 \times 24) = 3.6$ kN/m^2
Finishes $= 0.6$
Live load $= 3.0$
Total working load $= 7.2$ kN/m^2
(Ultimate load $= w = (1.5 \times 7.2) = 10.8$ kN/m^2

g) Moment of Resistance

$$\tan\phi = \sqrt{(1.5\mu + \mu^2/4\alpha^2)} - (\mu/2\alpha)$$

$$= \sqrt{(1.5\times0.8) + (0.8^2/4\times1.25^2)} - (0.8/2\times1.25)$$

$$= 0.82$$

$$\therefore \quad m = (wL_y^2/24)(\tan^2\phi/\mu)$$

$$= (10.8\times4^2/24)(0.82^2/0.8)$$

$$= 6.05 \text{ kNm/m}$$

For the R.C.C. slab with $d = 120$ mm

$$M_{u,lim} = 0.48 f_{ck}.b.d^2$$

$$= (0.148\times15\times1000\times120^2)/10^6$$

$$= 331.968 \text{ kN m/m}$$

Since $m < M_{u,lim}$, the slab is under reinforced

$$\therefore \quad (6.05\times10^6) = 0.87 f_y.A_{st}d\ [1-(A_{st}f_y)/(b.d.f_{ck})]$$

$$= (0.87\times415\times A_{st}\times120)[1-(A_{st}.415)/(1000\times120\times15)]$$

Solving, $A_{st} = 150\ \text{mm}^2$

But minimum quantity of steel $= 0.12\%$

$$= (0.12 \times 1000 \times 150/100) = 180\ \text{mm}^2/\text{m}$$

Adopt 6 mm diameter bars at 150 mm centres both ways and also over the fixed edges as negative reinforcement.

9.7.14 Analysis Example

A uniformly loaded isotropically reinforced concrete square slab is simply supported on three sides and unsupported on the fourth. If w = load per unit area at collapse of slab and m = positive plastic moment per unit width, show that for the yield line pattern shown in Fig 9.26, the minimum upper bound solution is given by the relation,

$$w = \left(\frac{14.2m}{L^2}\right) \quad \text{Where } \tan\phi = 1.3$$

Where ϕ = Angle made by the inclined yield line with the edge.

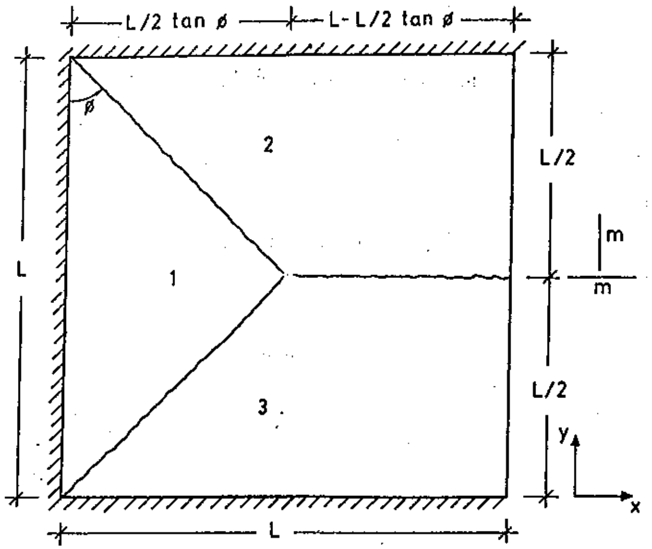


Fig. 9.26 R.C. Slab With Three Edges Simply Supported and One Edge Free

a) Data

Side length of slab $= L_x = L_y = L$

Moment capacity of slab $= m$

Ultimate load $= w$

Slab is isotropically reinforced.

Three adjacent edges are simply supported and the remaining edge is unsupported.

b) Derivation of Relation

Referring to Fig. 9.26, the external work done is computed for elements 1, 2 and 3.

For element 1, we have

$$(W.\delta) = [(0.5 \times 0.5L \tan\phi \times (1/3)]w = [w.L^2.\tan\phi/12]$$

For element 2, we have

$$(W.\delta) = [0.5w \times 0.5L(L - 0.5L\tan\phi)] + [(0.5w \times 0.5L\tan\phi \times 0.5L(1/3)]$$

$$= \left(\frac{wL^2}{4}\right)\left(1 - \frac{\tan\phi}{3}\right)$$

(Total external work done for elements 1, 2 and 3 is given by

$$(W.\delta) = [(wL^2.\tan\phi)/12] + [2(w.L^2/4)(1 - \tan\phi/3)]$$

$$= [(w.L^2)/2]\ [1 - (\tan(/6)]$$

Internal work done by rotation of yield lines is computed for the elements 1, 2, and 3.

For element 1, we have

$$\theta_y = (2/L\ .\ \tan\theta),\ \theta_y = 0, \text{ and } M_x = m.L$$

$$(M_x\,\theta_x + My\,\theta_y) = (2/L.\tan\theta)\ (m.L) = (2m/\tan\theta)$$

For elements 2 and 3 we have

$$\theta_y = (2/L),\ \theta_x = 0, \text{ and } M_y = m.L$$

$$(M_x\,\theta_x + My\,\theta_y) = (2/L)\ (m.L) = 4\,m$$

$\therefore$ Total internal work done is given by the equation

$$\Sigma(M\theta) = [4m + (2m/\tan\phi)]$$

Equating $\Sigma(W.\delta) = \Sigma(M\theta)$

$$(wL^2)\left[1 - \frac{\tan\phi}{6}\right] = \left[4m + \frac{2m}{\tan\phi}\right]$$

$$\left(\frac{w.L^2}{24}m\right) = \left[\frac{2 + (1/\tan\phi)}{(6 - \tan\phi)}\right]$$

For a maximum value of 'm', differentiating the right hand side of the equation, we have the relation,

$$\left[\frac{(2+(1/\tan\phi))}{(6-\tan\phi)}\right]=\left[\frac{-(1/\tan^2\phi)}{-1}\right]=\left[\frac{1}{\tan^2\phi}\right]$$

Cross multiplying, the quadratic equation is obtained as

$$(\tan^2\phi+\tan\phi-3)=0$$

The positive root of this quadratic equation is

$$\tan\phi=\left[\frac{-1+\sqrt{1+12}}{2}\right]=1.3$$

Substituting this value of tan ϕ in equation (1), we have the final relation between collapse load and ultimate moment capacity of the slab as

$$\left(\frac{w.L^2}{24m}\right)=\left[\frac{1}{(1.3)^2}\right]$$

$$\therefore \qquad w=\left(\frac{14.2m}{L^2}\right)$$

9.8 EXAMPLES FOR PRACTICE

1) A simply supported slab has a clear span of 2.1 m and is supported on walls 400mm thick along the edges. If the live load on the slab is 4 kN/m^2, and the floor finish weighs 0.6 kN/m^2,design the slab using M-20 grade concrete and Fe-415 HYSD bars.

2) Design a two- way slab for a residential roof to suit the following data:
Size of roof = 4.5 m by 6 m
Edge conditions: simply supported on all the sides on load bearing masonry walls 300 mm thick without any provision for torsion at corners.
Materials: M-20 grade concrete and Fe-415 HYSD bars.

3) Design a two- way slab 4 m by 6 m continuous on all the edges and supported on 300 mm wide beams to serve as an office floor. Adopt M-25 grade concrete and Fe-500 HYSD bars. Sketch the details of reinforcements in the slab.

4) A cycle stand shade consists of a R.C slab which cantilevers 3 m on each side of a central R.C. beam and is monolithic with the beam. Design the cantilever slab for a superimposed load of 1.5 kN/m^2. Adopt M-20 grade concrete and Fe-415 HYSD bars.

5) Design the interior span of a continuous slab for an office floor to suit the following data:
Slab is continuous over tee-beams spaced at 4 m intervals. Width of rib = 250 mm
Superimposed load on office floor = 4 kN/m^2.
Materials: M-20 grade concrete and Fe415 HYSD bars.

6) A flat slab floor with drops is proposed for a ware house 20 m by 30 m in size. Using a column grid of 5 m by 5 m design an interior panel of the flat slab to support a live load of 7.5 kN/m^2. Adopt M-20 grade concrete and Fe-415 HYSD bars.

7) Design the exterior panel of a flat slab using the following data:
Size of panel = 6m by 6 m, Loading class = 5 kN/m^2, Column size = 400 mm diameter. Height between floors = 4 m. Thickness of slab in column strip = 250 mm and thickness of slab in middle strips = 200 mm. Adopt M-20 grade concrete and Fe-415 HYSD bars. Sketch the details of reinforcements in the slab.

8) A square slab of 4 m side length is simply supported along the edges. The slab is required to support a uniformly distributed load of 4 kN/m^2. Using the yield line theory, design the slab using M-20 grade concrete and Fe-415 HYSD bars.

9) A rectangular slab 6.5 m by 4.5 m, simply supported along its edges is to be designed as an isotropically reinforced slab to support an uniformly distributed working live load of 4 kN/m^2. Design the slab using yield line theory and adopting M-20 grade concrete and Fe-415 HYSD bars.

10) A triangular reinforced concrete slab has equal sides of length 5 m. The isotropically reinforced slab is simply supported on two sides and carries a uniformly distributed load. If the moment of resistance of the section of the slab is 30 kN.m/m, estimate the ultimate collapse load carried by the slab from first principles.

11) A hexagonal slab, simply supported on all the edges has a side length of 4 m. Find the uniformly distributed load which would cause collapse of the isotropically reinforced slab if the ultimate moment of resistance of the slab is 6 kN.m/m.

12) A rectangular slab 6 m by 4 m is isotropically reinforced and is continuous over all the edges. The slab is reinforced with similar reinforcements to resist both positive and negative moments. Show that the ultimate moment capacity of the slab can be expressed as

$$m=(w.L_y^2.\tan^2\varphi)/(48)$$

where m = ultimate moment of resistance of the slab per unit length
L_y = short span length
ϕ = angle made by the positive yield line with shorter edge

Also, design the slab for a service design live load of 6 kN/m^2 using yield line theory. Adopt M-20 grade concrete and Fe-415 HYSD bars.

13) A rectangular slab 6m by 4.5 m is simply supported at the edges. The coefficient of orthotropy μ = 0.7. If the ultimate design load is 16 kN/m^2, estimate the ultimate moment capacity of the slab in the short span direction using yield line theory.

14) A two way reinforced concrete slab 6 m by 4 m has two longer edges fixed in position and the two shorter edges are simply supported. Derive the relation between the moment of resistance of the slab and the ultimate load using yield line principles. Also design the slab for a service load of 4 kN/m^2 using yield line theory. Assume the coefficient of orthotropy as 0.8. Adopt M-20 grade concrete and Fe-415 HYSD bars.

15) An isotropically reinforced square slab of side length 5 m is simply supported on three sides and unsupported on the fourth side. Derive the relation between the moment of resistance and the ultimate load carrying capacity of the slab using yield line theory. Also, design the slab to support a working live load of 4 kN/m^2. Adopt M-25 grade concrete and Fe-500 HYSD bars.

16) A square slab of 4 m side length is simply supported along the edges and supports a uniformly distributed load of 20 kN/m^2, including its own self- weight. If the slab is reinforced isotropically to give an ultimate moment of resistance of 20 kN.m/m, calculate the magnitude of the additional central point load required to cause collapse of the slab. Assume a pattern of simple diagonal yield lines.

CHAPTER 10

Limit State Design of Columns and Footings

10.1 INTRODUCTION

Structural concrete members in compression are generally referred to as columns and struts. The term 'Column' is associated with members transferring loads to the ground and the term 'strut' is applied to compression members is any direction such as those in a truss.The IS:456-2000 code clause 25.1.1 defines the column as a 'compression member' the effective length of which exceeds three times the least lateral dimension.The term 'pedestal' is used to describe a vertical compression member whose effective length is less than three times to least lateral dimension.

Axially loaded columns may fail in any of the following three modes :

1) Pure compression failure
2) Combined compression and bending failure
3) Failure by elastic instability.

The failure modes depend primarily on the slenderness ratio of the member which is turn depends on the cross sectional dimensions, effective length, and support conditions of the member.

10.2 Classification of Columns

a) Based on Type of Reinforcement

Depending on the type of reinforcement used, reinforced concrete columns are classified into the following three groups.

1) 'Tied Columns' in which the main longitudinal bars are confined within closely spaced lateral ties [Fig. 10.1 (a)]
2) 'Spiral Columns' having main longitudinal reinforcements enclosed within closely spaced and continuously wound spiral reinforcement [Fig. 10.1 (b)]
3) 'Composite Columns' in which the longitudinal reinforcement is in the form of structural steel section or pipes with or without longitudinal bars [Fig. 10.1 (c)]

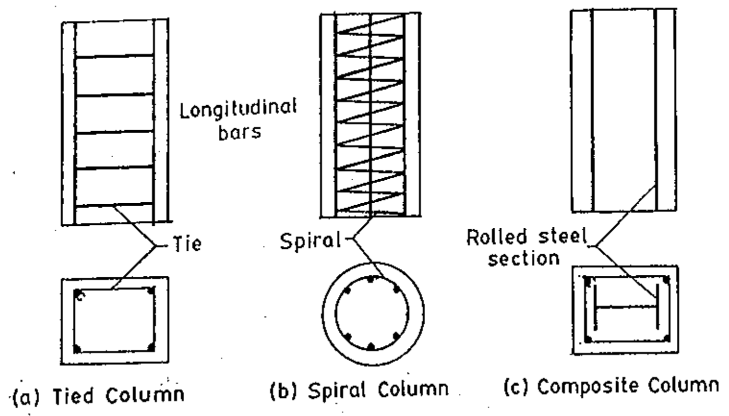


Fig. 10.1 Types of Columns

In general tied columns are the most commonly used having different shapes (Square, rectangular, T, L, circular etc).

Spiral columns are adopted with circular cross sections and also for square and octagonal sections.

b) Based on type of loading

Depending upon the type of loading columns may be classified into the following three types.

i) Axially loaded columns supporting concrete loads are relatively rare. Interior columns of multistoried buildings with symmetrical loads from floor slabs from all sides are common examples of this type [Fig. 10.2 (a)].

ii) Column with uniaxial eccentric loading are generally encountered in the case of columns rigidly connected to beams from one side only such as the edge columns [Fig. 10.2 (b)].

iii) Columns with biaxial eccentric loading is common in corner columns with beams rigidly connected at right angles on the top of the column [Fig. 10.2 (c)].

Eccentrically loaded columns have to be designed for combined axial force and bending moments.

c) Based on Slenderness Ratio

Depending on the slenderness ratio, (Effective length/Least lateral dimension)

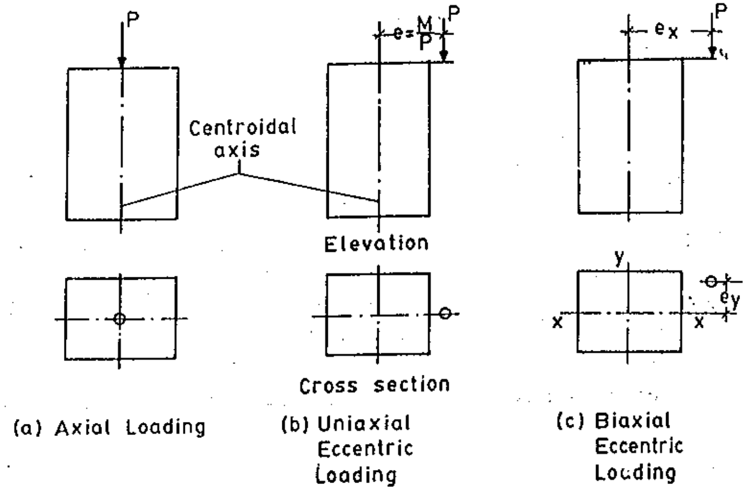


Fig. 10.2 Types of Loading on Columns

Columns may be classified as.

i) Short Columns
ii) Slender or Long columns

IS: 456-2000 code clause 25.1.2 classifies a rectangular compression member as short when both the slenderness ratio's (L_{ex}/D) and (L_{ey}/b) are less than 12,

Where L_{ex} = effective length in respect of major axis
D = depth in respect of major axis
L_{ey} = Effective length in respect of minor axis and
b = width of the member.

If any of these ratios is equal to or more than 12, then it is termed as slender or long column. This definition is not suitable for non-rectangular and non-circular sections where the slenderness ratio is better defined in terms of the radius of gyration rather than the lateral dimensions.

10.3 EFFECTIVE LENGTH OF COLUMNS

10.3.1 Computation of Effective Length

The effective length of a column depends upon the unsupported length (distance between lateral connections) and the boundary conditions at the ends of column due to the conditions of the framing beams and other members.

The effective length 'L_{ef}' can be expressed in the form.

$$L_{ef} = kL$$

Where L = Unsupported length or clear height of columns

k = Effective length ratio or a constant depending upon the degrees of rotational and translational restraints at the ends of column.

The effective length of compression members depends upon the bracing and end conditions. For braced (Laterally restrained at ends) columns, the effective length is less than the clear height between the restrains, whereas for unbraced and partially braced columns, the effective length is greater than the clear length between the restraints.

For design purposes, assuming idealized conditions, the effective length Le may be assessed for different types of end conditions using the Table-10.1 (Table 28 of IS: 456-2000).

Table 10.1 Effective Length of Compression members (Table-28 of IS: 456-2000)

Degree of End Restraint of Compression Member	Theoretical Value of Effective Length	Recommended Value of Effective length
1	2	3
Effectively held in position and restrained against rotation at both ends	0.5 L	0.65 L
Effectively held in position at both ends, restrained, against rotation at one end	0.7 L	0.80 L
Effectively held in position at both ends, but no restrained rotation	1.00 L	1.00 L
Effectively held in position and restrained against rotation at one end, and at the other restrained against rotation but not held in position	1.00 L	1.20 L
Effectively held in position and restrained against rotation at one end, and at the other partially restrained against rotation but not held in position	-	1.50 L
Effectively held in position at one end but not restrained against rotation, at the other end restrained against rotation but not held in position	2.00 L	2.00 L
Effectively held in position and restrained against rotation at one end but not held in position nor restrained against rotation at the other end	2.00 L	2.00 L

10.3.2 Slenderness Limits

The columns dimensions should be selected in such a way that it fails by material failure only and not by buckling. To ensure this criterion, the code

recommends that the clear distance between restraints (un supported length) should never exceed 60 times the least lateral dimensions of the column (clause 25.3.1). For unbraced columns, it is recommended that this value is limited to 30. In cantilever columns, in addition to the above restriction ($L \leq 60b$), the clear height should also not exceed the value of $L = (100\, b^2/D)$, where D is the depth of cross section measured in the plane under consideration and 'b' is the width of cross section (clause 25.3.2)

10.3.3 Minimum Eccentricities

All columns should be designed for minimum eccentricity (Clause 25.4), which may arise due to imperfections in constructions and inaccuracy in loading given by the relation,

$$e_{min} = \left[\frac{L}{500} + \frac{D}{30}\right] \quad \ldots(10.1)$$

but not less than 20 mm

Where L = Unsupported length

D = Lateral dimensions in the plane of bending

For non-rectangular and non-circular cross sectional shapes, SP: 24[14] recommends the minimum eccentricity as

$$e_{min} = (L_e/300) \text{ or } 20 \text{ mm (whichever is greater)}$$

10.3.4 Braced and Unbraced Columns

In a framed structure, an approximate method of deciding whether a column is 'braced' or 'unbraced' is specified in the ACI code commentary[85] and is reproduced in the revised IS: 456-2000 code. For this purpose, the 'stability index' (Q) of a storey in a framed multistorey structure is defined as

$$Q = \left[\frac{\Sigma P_u}{h_s} \times \frac{\Delta_u}{H_u}\right] \quad \ldots(10.2)$$

Where ΣP_u = sum of axial loads on all columns in the storey

h_s = height of the storey

Δ_u = elastically computed first order lateral deflection of the storey

H_u = total lateral force acting within the storey.

In the absence of bracing elements, Taranath[86] has shown that the lateral flexibility measure of the storey (Δ_u/H_u) (storey drift per unit storey shear) can be expressed by the relation.

$$\left(\frac{\Delta_u}{H_u}\right)=\left[\frac{h_s^2}{12E_{c,col}\Sigma(I_c/h_s)}+\frac{h_s^2}{12E_{c,beam}\Sigma(I_b/L_b)}\right] \qquad \ldots(10.3)$$

Where ΣI_c = sum of second moment of areas of all columns in the storey in the plane under consideration.

$\Sigma(I_b/L_b)$= sum of the ratios of second moment of area to span of all floor members in the storey in the plane under consideration.

E_c = modulus of elasticity of concrete

The equation for the stability index 'Q' is based on the assumption that the points of contra flexure occurs at the mid heights of all columns and mid span points of all beams and by applying unit load method to an isolated store[86]. If Bracing elements such as trusses, shear walls and infill walls are used then their beneficial effect will be to reduce the ratio (Δ_u/H_u) significantly.

If the value of $Q \le 0.04$, then the column may be considered as no sway column (braced), otherwise the column may be treated as sway column (unbraced).

IS: 456-2000 codal charts (Fig. 10.3 & 10.4) are very useful in determining the effective length ratios of braced and unbraced columns respectively; in terms of β_1 & β_2 which represent the degree of rotational freedom at the top and bottom ends of the column. The values of β_1 and β_2 for braced and unbraced columns are given by the relations,

$$\beta_1=\left[\frac{\Sigma I_c/h_s}{\Sigma I_c/h_s+\Sigma 0.5(I_b/L_b)}\right] \quad \text{(For braced columns)} \qquad \ldots(10.4)$$

$$\beta_2=\left[\frac{\Sigma I_c/h_s}{\Sigma I_c/h_s+\Sigma 1.5(I_b/L_b)}\right] \quad \text{(For unbraced columns)} \qquad \ldots(10.5)$$

The limiting values $\beta = 0$ and $\beta = 1$, represent the 'fully fixed' 'fully hinged' conditions respectively.

The following example illustrates the checking of braced and unbraced columns and the computation of effective length.

10.3.5 Example

A multistoreyed building plan shown in Fig. 10.5 (a) has 16 columns of size 300 × 300 mm interconnected by floor beams of size 250 mm by 500 mm in the longitudinal & transverse directions. The storey height is 3.5 m.

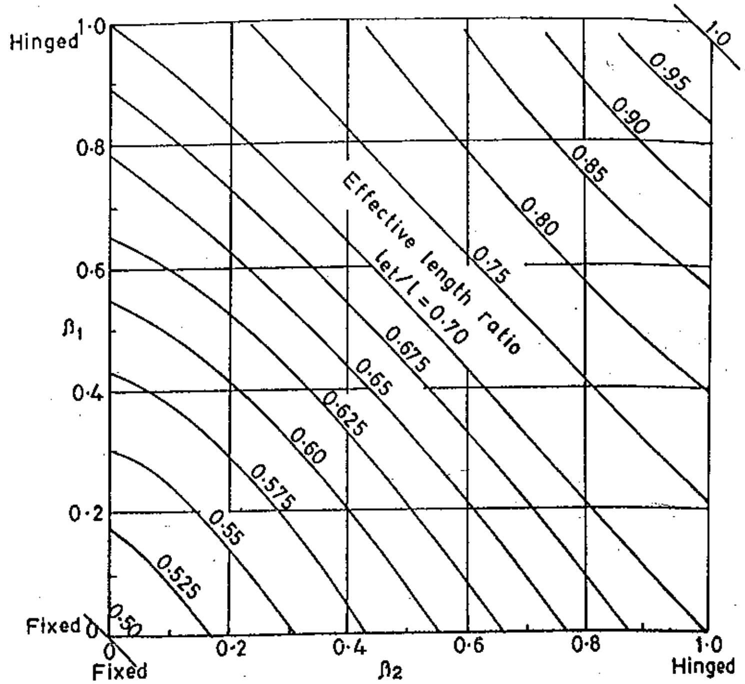


Fig. 10.3 Effective Length Ratios For a Column in a Frame With no Sway (Braced Columns) (IS: 456:2000 Fig. 26)

Calculate the effective length of the typical lower storey columns assuming a total distributed load 30 kN/m² from all the floors above & the grade of concrete as M-20. Adopt IS: 456-2000 codal method for computations.

a) Data

Size of columns = 300 × 300 mm
Height of storey = h_s = 3.5 m
Width of beam = 250mm
Depth of beam = 500 mm
Length of beam = 4 m
Total distributed load = 30 kN/m²
No. of Columns = 16
No. of Beams in XX or YY–directions = 12
Grade of concrete= M-20

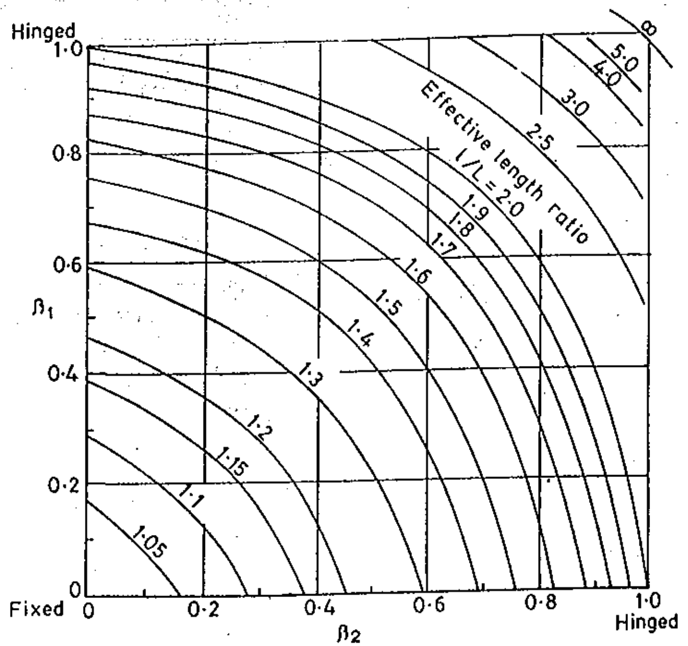


Fig. 10.4 Effective Length Ratios For a Column in a Frame Without Restraint Against Sway (Unbraced Columns) (IS: 456:200 Fig. 27)

b) Relative stiffness of Columns and Beams

Referring to Fig. 10.5(b)

Un-supported length of column = L = (3500–500) = 300 mm

i) Columns : 16 Nos, (300 × 300mm) and h_s = 3500 m

$$\Sigma\left(\frac{I_c}{h_s}\right) = \left[\frac{16\times(300)^4/12}{3500}\right] = (3086\times10^3)\ \text{mm}^3$$

ii) Beams in each direction XX or YY

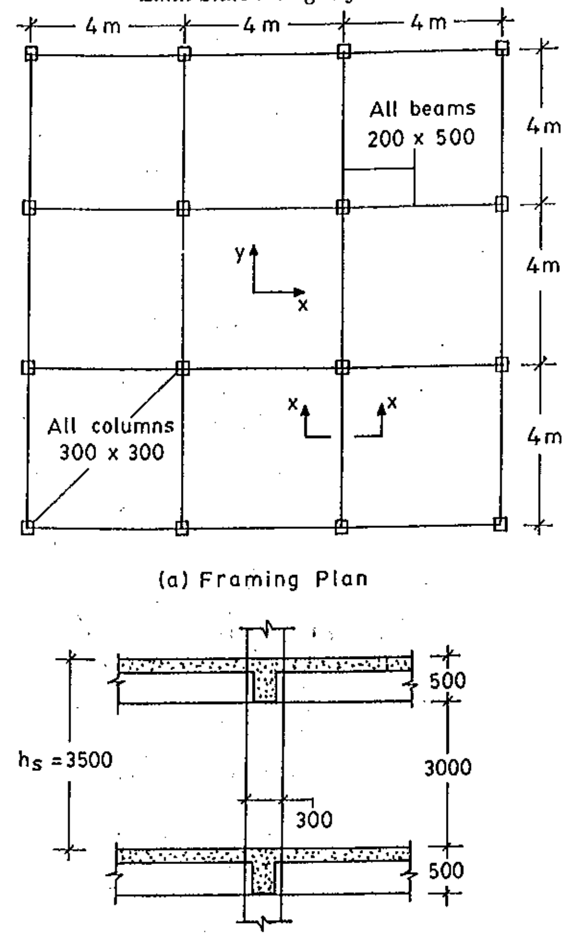
$$\Sigma\left(\frac{I_b}{L_b}\right) = \left[\frac{12\times250\times(500)^3/12}{4000}\right] = (7812\times10^3)\ \text{mm}^3$$

c) Check for braced or Unbraced Columns

$$\left(\frac{\Delta_u}{H_u}\right) = \frac{h_s^2}{12E_c}\left[\frac{1}{\Sigma(I_c/h_s)} + \frac{1}{\Sigma(I_b/L_b)}\right]$$

$E_c = 5000\sqrt[4]{f_{ck}}$ according to clause 6.2.3.1 of IS: 456 - 2000

Therefore $E_c = 5000\sqrt{20} = 22360$ N/mm^2



(b) Section XX

Fig. 10.5 Multi-Storey Building Frame

$$\left(\frac{\Delta_u}{H_u}\right) = \frac{3500^2}{(12\times22360)}\left[\frac{1}{(3086\times10^3)} + \frac{1}{(7812\times10^3)}\right] = (5.991\times10^{-6})\ \text{mm/N}$$

Total axial Load on all columns = (12 × 12 × 30) = P_u = 4320 kN

$$\text{Stability Index} = Q = \left[\frac{P_u}{h_s}\times\frac{\Delta_u}{H_u}\right] = \left[\frac{4320\times10^3}{3500}(5.991\times10^{-6})\right] = 0.00739 < 0.04$$

Hence, the columns in the storey can be considered as braced in XX and YY directions.

d) Effective length of columns using IS: 456 code charts

$$\beta_1 = \beta_2 = \frac{\Sigma(I_c/h_s)}{\left[\Sigma(I_c/h_s) + \Sigma 0.5(I_b/L_b)\right]}$$

$$\Sigma(I_c/h_s) = \left[\frac{(300)^4/12}{3500} \times 2\right] = (385 \times 10^3)\ \text{mm}^3$$

$$\Sigma(I_b/L_b) = \left[\frac{250 \times 500^3/12}{4000} \times 2\right] = (1302 \times 10^3)\ \text{mm}^3$$

$$\therefore \quad \beta_1 = \beta_2 = \left[\frac{385 \times 10^3}{(385 \times 10^3) + (0.5 \times 1302 \times 10^3)}\right] = 0.371$$

Referring to Fig. 10.3 [Fig. 26 of IS: 456-2000] and interpolating the effective length ratio as,

$$k = \left(\frac{L_e}{L}\right) = 0.630$$

$$\therefore \quad L_e = (0.630 \times 3000) = 1890\ \text{mm}$$

Slenderness ratio of the column is $= \left(\frac{L_e}{D}\right) = \left(\frac{1890}{300}\right) = 6.3 < 12$

Hence, the column should be designed as short column.

10.4 DESIGN OF SHORT COLUMNS UNDER AXIAL COMPRESSION

10.4.1 Assumptions

The main assumptions made for limit state design of columns failing under pure compression as specified in clause 39.1 are as follows:

a) The maximum compressive strain in concrete in axial compression is 0.002.
b) Plane sections remain plane in compression
c) The design stress-strain curve of steel in compression is taken to be the same as in tension

The design stress in steel is $0.87 f_y$ in Fe-250, 415 and Fe-500 grade steels.

Accordingly, under pure axial loading conditions the design strength of short columns is expressed as

$$P_u = [0.45 f_{ck} \cdot A_c + f_{sc} \cdot A_{sc}] \qquad \ldots(10.6)$$

Where $\qquad f_{sc} = 0.87 f_y$

The IS: 456-2000 code requires that all columns are to be designed for minimum eccentricity of 0.05 times the lateral dimension. Hence the final expression for the ultimate load is obtained by reducing the value of P_u by 10 percent in the equation (10.6) specified above as

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y \cdot A_{sc} \qquad \ldots(10.7)$$

$$= [0.4 f_{ck} \cdot A_g + (0.67 f_y - 0.4 f_{ck}) A_{sc}] \qquad \ldots(10.8)$$

Where P_u = axial ultimate load on the member
f_{ck} = characteristic compressive strength of concrete
A_c = area of concrete
f_y = characteristic strength of the compression reinforcement
A_{sc} = area of longitudinal reinforcement.

Short columns with helical reinforcement (spiral columns) have increased ductility prior to collapse and hence the code permits 5 percent increase in the load carrying capacity of spiral columns. However the ratio of the volume of helical reinforcement to the volume of the core shall be not less than,

$$0.36\left(\frac{A_g}{A_c} - 1\right)\left(\frac{f_{ck}}{f_y}\right)$$

according to clause 39.4.1 of IS:456-2000.

10.4.2 Design Example

Design the reinforcements in a column of size 400 mm by 600 mm subjected to an axial working load of 2000 kN. The column has an unsupported length of 3m and is braced against side sway in both directions. Adopt M-20 grade concrete and Fe-415 HYSD bars.

a) Data

Column Dimensions 400 mm by 600mm
Axial service load = 2000 kN
Un supported length $L = 3$ m
Column Braced against side sway
$f_{ck} = 20\ \text{N/mm}^2$ and $f_y = 415\ \text{N/mm}^2$
D_y = 400 mm and $D_x = 600$ mm

b) Slenderness Ratio

$$k_x = \left(\frac{L_{ex}}{L}\right) \quad \text{and} \quad k_y = \left(\frac{L_{ey}}{L}\right)$$

As the column is braced against side sway in both directions, effective length ratio k_x and k_y are both less than unity.

And $$\left(\frac{L}{D_y}\right) = \left(\frac{3000}{400}\right) = 7.5 < 12$$

Hence, the columns is designed as a short column.

c) Minimum Eccentricity

$$e_{x,\min} = \left[\frac{3000}{500} + \frac{600}{30}\right] = 26 > 20 \text{ mm}$$

$$e_{y,\min} = \left[\frac{3000}{500} + \frac{400}{30}\right] = 19.33 < 20 \text{ mm}$$

Also $0.05\,D_x = (0.05 \times 600) = 30 > e_{x,\min}$
$0.05\,D_y = (0.05 \times 400) = 20 > e_{y,\min}$

Hence, the codal formula (Eq:10.6) for short columns is applicable

d) Factored (Ultimate) Load

$$P_u = (1.5 \times 2000) = 3000 \text{ kN}$$

a) Longitudinal Reinforcements

$$P_u = [0.4 f_{ck} \cdot A_g + (0.67 f_y - 0.4 f_{ck}) A_{sc}]$$

$$(3000 \times 10^3) = (0.4 \times 20 \times 400 \times 600) + [(0.67 \times 415) - (0.4 \times 20)] A_{sc}$$

Solving $A_{sc} = 4000 \text{ mm}^2$
Provide 6-25 mm diameter bars: $(6 \times 491) = 2946 \text{ mm}^2$
4-20 mm diameter bars: $(4 \times 314) = 1256 \text{ mm}^2$
Total $A_{sc} = 4202 \text{ mm}^2 > 4000 \text{ mm}^2$

The area of reinforcement provided is greater than the minimum steel requirement of 0.8 percent = $(0.008 \times 400 \times 600) = 1920 \text{ mm}^2$

f) Lateral Ties

Tie diameter: $< (1/4)(25) = 6.25$ mm
> 16 mm

Hence, provide 8 mm diameter ties

Tie spacing: > 400 mm
$> (16 \times 20) = 320$ mm

∴ Provide 8 mm diameter ties at 300 mm c/c

g) The detailing of reinforcements in the column section is shown in Fig. 10.6.

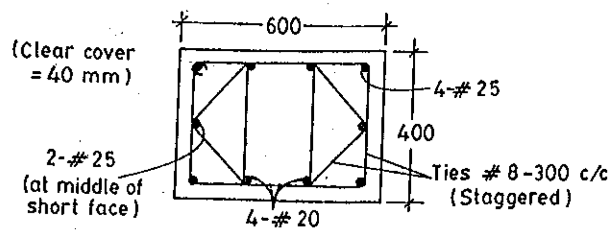


Fig. 10.6 Reinforcements in Short Column

10.4.3 Design Example

Design the reinforcements in a circular column of diameter 300 mm with helical reinforcement to support a factored load of 1500 KN. The columns has an unsupported length of 3 m and is braced against sidesway. Adopt M-20 grade concrete and Fe-415 HYSD bars.

a) Data

Diameter of column $= D = 300$ mm
Unsupported length $= L = 3000$ mm
Column braced against sidesway.
Factored Load $= P_u = 1500$ kN
$f_{ck} = 20 \text{ N/mm}^2$
$f_y = 415 \text{ N/mm}^2$

b) Slenderness Ratio

$$(L_e/D) = (3000/300) = 10 < 12$$

Hence, the column designed as short column.

c) Minimum Eccentricity

$$e_{min} = \left[\frac{L}{500} + \frac{D}{30}\right] = \left[\frac{3000}{500} + \frac{300}{30}\right] = 16 \text{ mm}(20 \text{ mm})$$

Also $0.05D = (0.05 \times 300) = 15 \text{ mm} < 20 \text{ mm}$

Hence, the codal formula for axially compressed column can be used.

d) Longitudinal Reinforcements

According to IS: 456-code clause 39.4

$$P_u = 1.05[0.4\, f_{ck}\, A_g + (0.67\, f_y - 0.4\, f_{ck})\, A_{sc}]$$

$$\left(\frac{1500 \times 10^3}{1.05}\right) = \left[\frac{0.4 \times 20 \times \pi \times 300^2}{4} + \{(0.67 \times 415) - (0.4 \times 20)\}A_{sc}\right]$$

Solving $A_{sc} = 3197 \text{ mm}^2$

$A_{sc,min} = 0.8\%$ of gross cross section $= (0.008 \times \pi \times 300^2/4) = 565 \text{ mm}^2$

Provide 6 bars of 28 mm diameter ($A_{sc} = 3696 \text{ mm}^2$)

e) Helical Reinforcement (spirals)

Assuming clear cover of 40 mm over spirals

Core diameter $= [300-(2 \times 40)] = 220$ mm

$$\text{Area of core} = A_c = \left[\left(\frac{\pi \times 220^2}{4}\right) - 3696\right] = 34317 \text{ mm}^2$$

$$\text{Volume of core/m} = V_c = (34317 \times 10^3) \text{ mm}^3$$

$$\text{Gross Area of section} = A_g = \left(\frac{\pi \times 300^2}{4}\right) = 70685 \text{ mm}^2$$

Using 8mm diameter helical spirals at a pitch 'p' mm, the volume of helical spiral per metre length is given by

$$V_{ns} = \pi(300 - 80 - 8)50 \times (1000/p) \text{ mm}^3/\text{m}$$

$$= (33301 \times 10^3)/p \text{ mm}^3/\text{m}$$

According to code clause 39.4.1 (IS:456)

$$\left(\frac{V_{ns}}{V_c}\right) < 0.36[(A_g/A_c) - 1]\,(f_{ck}/f_y)$$

$$\left(\frac{33301 \times 10^3}{p(34317 \times 10^3)}\right) < 0.36\left[\left(\frac{70685}{34317}\right) - 1\right]\left(\frac{20}{415}\right)$$

Solving pitch 'p' 52.78 mm

Codal restriction on pitch [Clause 26.5.3.2 (d)]

p < 75 mm or (core diameter/6) = (220/6) = 36.6 mm

p > 25 mm or (3 times the diameter of helix) = (3 × 8) = 24 mm

Hence, provide 8 mm diameter spirals at a pitch of 36 mm.

f) Reinforcement Details

The details of reinforcements in the helically reinforced column are shown in Fig. 10.7.

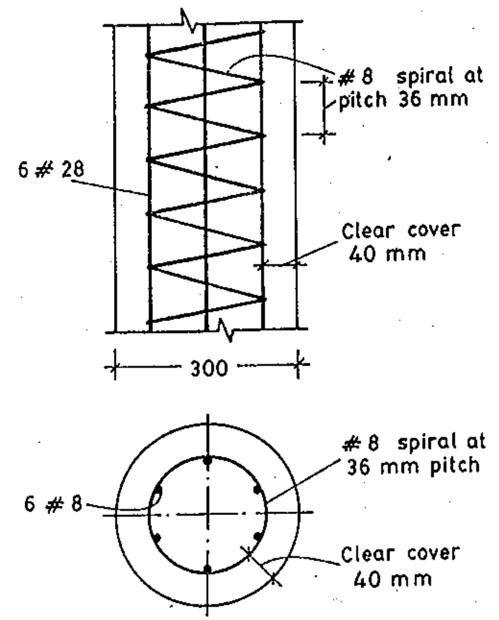


Fig. 10.7 Reinforcement in Helically Reinforced Column

10.5 DESIGN OF SHORT COLUMNS UNDER COMPRESSION WITH UNIAXIAL BENDING

10.5.1 Introduction

The external columns of multistoreyed buildings and columns supporting crane loads through corbels are subjected to direct loads and bending moments. The compression members should be designed for axial load and bending moment based on the assumptions prescribed in IS:456-2000 code clauses 39.1 and 39.2

The analytical design of members subjected to combined axial load and uniaxial bending involves lengthy calculation by trial and error and the method uses equilibrium equal to determine the area of reinforcement required to resist direct loads and uniaxial moment. In order to overcome these difficulties, I.S code recommends the use of interaction diagrams involving non-dimensional parameters presented in SP: 16 design aids for reinforced concrete.

10.5.2 Interaction Diagrams

The interaction diagram represents the design strength of eccentrically loaded column of known section properties. The salient points on the interaction curve corresponds to the design strength values of axial load P_u and the moment M_u associated with an eccentricity 'e'. Fig. 10.8 shows a typical interaction curve with P_u on Y-axis and M_u on X-axis along with strain profiles.

The interaction curve defines the different load-moment (P_u & M_u) combinations for all possible eccentricities of loading. For design purposes, the calculations of M_u and P_u are based on the design stress-strain curves (including partial safety factors).The design interaction curve represents the failure envelope and the point given by the co-ordinates (M_u and P_u) falling within the interaction curve indicates the safe values of the combination of load and moments.

The salient points on the interaction curve are note worthy.

1) Point-1, on the load axis corresponds to the axial loading with zero moment (P_{uo}) and $e = 0$.
2) Point-1' corresponds to the condition of axial load with the minimum eccentricity prescribed in IS:456 code clause-25.4. The corresponding ultimate load is represented as P'_{uo}.
3) As the eccentricity increases, the moment, increases with the neutral axis x_u moving from outside towards the extreme fibre.

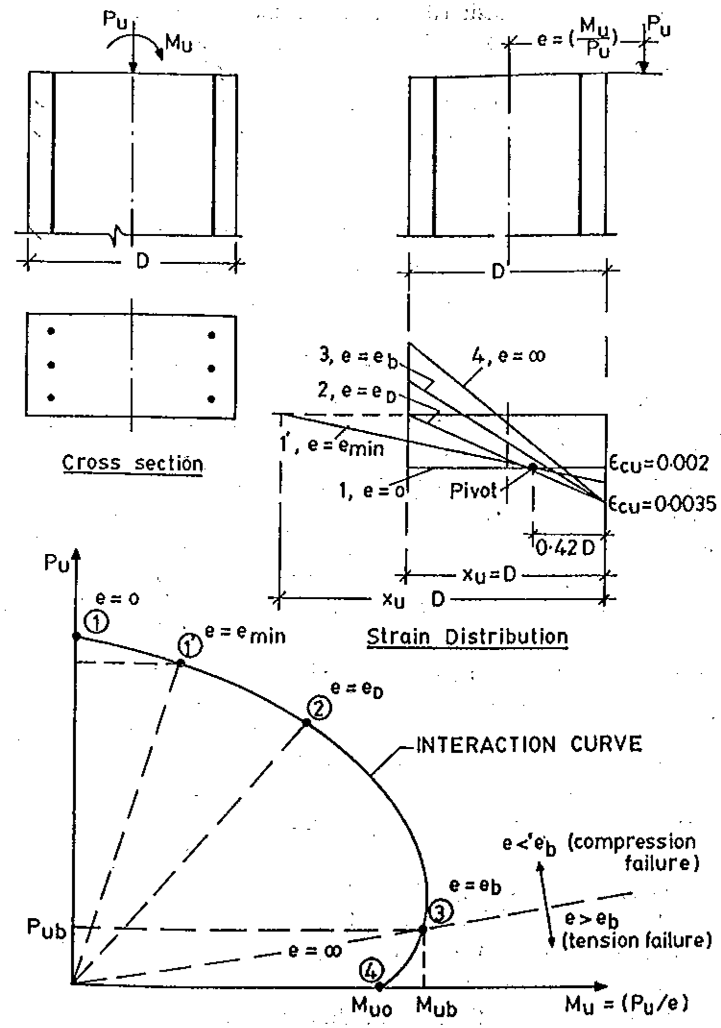


Fig. 10.8 Column Under Compression With Uniaxial Bending

Point-2 corresponds to the condition,
$x_u = D$ and $e = e_D$. For $e < e_D$, the entire cross section is under compression and the neutral axis is located outside the section ($x_u > D$) and the extreme fibre strain in concrete lies between 0.002 & 0.0035. With a further increase in the moment and eccentricity ($e > e_D$), the neutral axis lies within the section (xu < D) and the extreme concrete fibre strain ecu = 0.0035.

4) Point-3 on the interaction diagram represents the balanced failure state with $e = e_b$ and $x_u = x_{u,max}$. The design strength values for the balanced failure condition are denoted as Pub and Mub. For values of $e > e_t$. $P_{ult} < P_{ub}$ and the failure mode is termed as tension failure, similar to that of beams. It is important to note that M_{ub} is only marginally less than the ultimate moment of resistance of the section M_{uo} under pure flexural condition.
5) Point-4 on the interaction curve refers to the pure flexural state ($e = \infty$ and $P_{uR} = 0$) with the ultimate moment of resistance M_{uo} associated with the minimum neutral axis depth $x_{u,min}$.

10.5.3 Design charts (Uniaxial eccentric compression) in SP: 16

The design of structural concrete members subjected to combined axial load and uniaxial bending moment involves lengthy theoretical computations by trial and error procedure. To overcome these difficulties, interaction diagrams involving non dimensional parameters are useful in the rapid design of reinforcements in eccentrically loaded columns. SP:16 presents the design charts covering the following three different cases of symmetrically reinforced column sections, covering rectangular and circular cross sections.

The non dimensional parameters used for the construction of design charts are $(P_u/b.d.f_{ck})$ and $(M_u/b.d^2f_{ck})$ plotted along the Y and X-axis respectively. These parameters are plotted for different values of the ratio (p/f_{ck}) where 'p' is the percentage reinforcement in the section.

The following cases are covered in the SP: 16 Design charts: -

1) Rectangular section reinforced with equal number of bars on opposite sides parallel to that axis of bending (Charts 27 to 38)
2) Rectangular sections reinforced with equal number of bars on all the four sides (Charts 39 to 50)
3) Circular sections reinforced with 8 bars symmetrically spaced (charts 51 to 62) and these charts can also be used for bars not less than 6.

The charts for each of these types have been given for three grades of steel (Fe-250, Fe-415 and Fe-500) and four values of the ratio (d'/D).

The dotted lines in these charts indicate the stress in the bars nearest to the tension face of the member. It is pertinent to note that all these stress values are at the failure condition corresponding to the limit state of collapse and not at working loads.

The construction of these design charts are based on the equilibrium equations at the limit state collapse as outlined in SP:16.

Typical design charts covering the parameters f_y = 415 N/mm^2 and $(d'/D) = 0.10$ are reproduced in Figs. 10.9, 10.10 & 10.11 for the three different arrangements of reinforcements in the cross section.

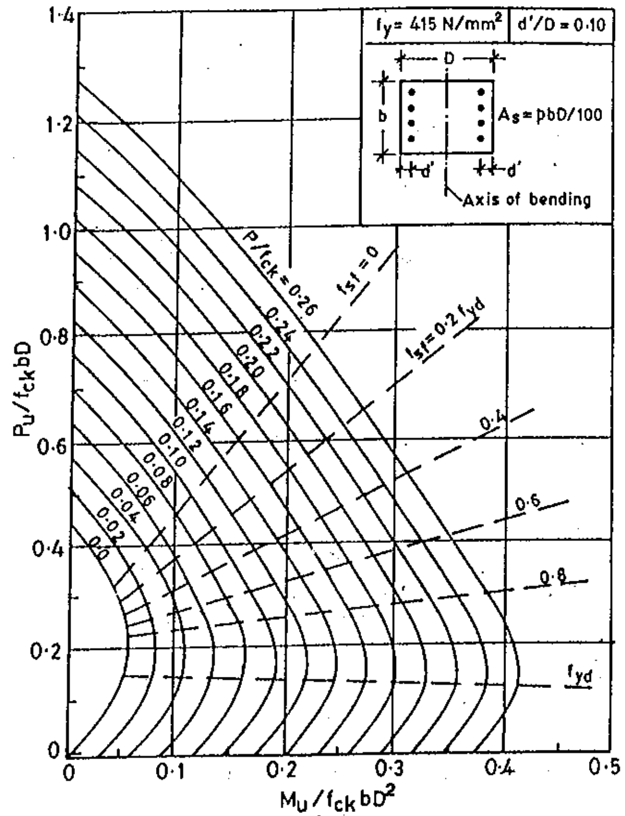


Fig. 10.9 Compression With Bending-Rectangular Section-Reinforcement Distributed Equally on Two Sides (SP: 16 Chart 32)

The following examples demonstrate the use of the design charts to design reinforcements in the columns subjected to combined axial load and uniaxial bending moment.

10.5.4 Design Example

Design the longitudinal and lateral reinforcement in a rectangular reinforced concrete column of size 300mm by 400 mm subjected to a design

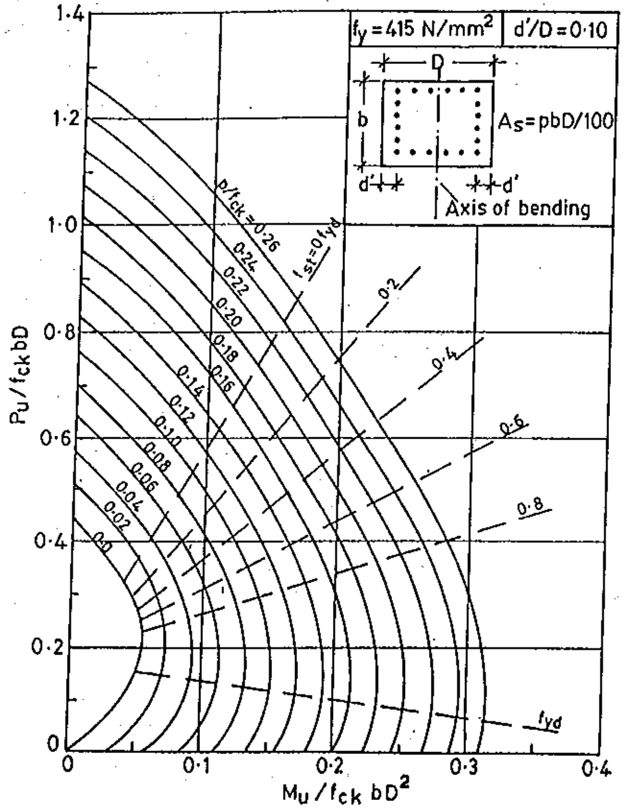


Fig. 10.10 Compression With Bending-Rectangular Section-Reinforcement Distributed Equally on Four Sides (SP: 16 Chart-44)

ultimate load of 1200 kN and an ultimate moment of 200 kN.m with respect to the major axis. Adopt M-20 grade concrete and Fe-415 grade HYSD bars.

a) Data

b = 300 mm	f_{ck} = 20 N/mm^2
D = 400 mm	f_y = 415 N/mm^2
P_u = 1200 kN	
M_u = 200 kN.m	

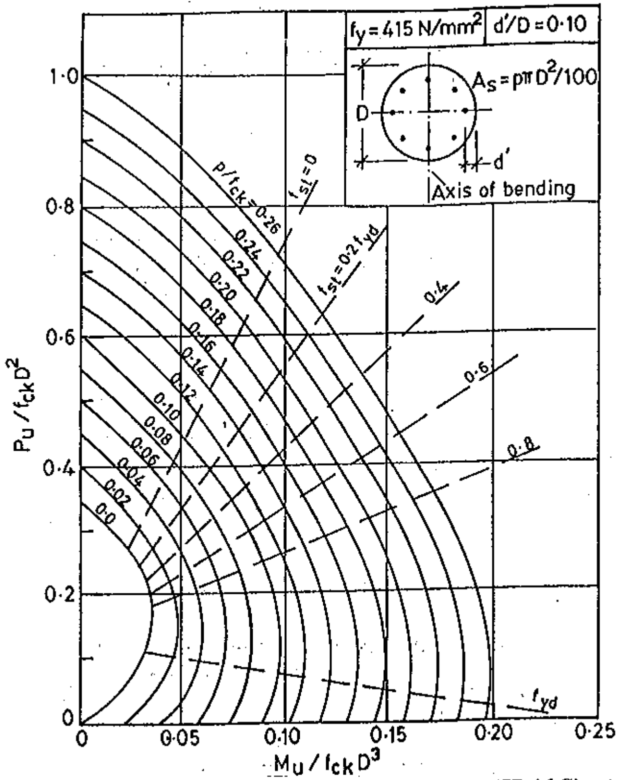


Fig. 10.11 Compression With Bending-Circular Section (SP:16 Chart-56)

b) Non Dimensional Parameters

$$\left(\frac{P_u}{f_{ck}b\,D}\right) = \left(\frac{1200\times10^3}{20\times300\times400}\right) = 0.5$$

$$\left(\frac{M_u}{f_{ck}bD^2}\right) = \left(\frac{200\times10^6}{20\times300\times400^2}\right) = 0.208$$

c) Longitudinal Reinforcement

Adopting an effective cover of 50 mm = d'

(d'/D) = (50/400) = 0.124 nearly equal to 0.15

Refer Chart 33 of SP: 16 and read out the ratio $(p/f_{ck}) = 0.20$

$$\therefore \quad p = (20\times0.20) = 4$$

$= \left(\frac{\ldots 300 \times 400}{100}\right) = 4800 \text{ mm}^2$

... mm²

... of the short faces at an effective cover of 50 mm.

... clause 26.5.3.1, the spacing of longitudinal bars ... of the column shall not exceed 300 mm.

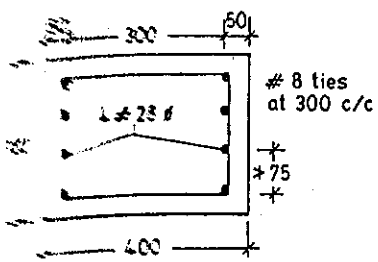
... centre lines of reinforcement = [400 − (2 × 50)] =

... mm and not greater than 16 mm

... and $\ngtr (16 \times 28) = 448$ mm

... reinforcements

... reinforcement is shown in Fig. 10.12



... Reinforcements in Columns With Uniaxial Bending

... concrete rectangular column of size 300 × 500 ... compressive factored load of 200 kN and a ... about the major axis. Adopting M-25 grade ... determine the reinforcement in the col-

$f_{ck} = 25 \text{ N/mm}^2$
$f_y = 415 \text{ N/mm}^2$

$P_u = 1000$ kN Adopt $d' = 50$ mm
$M_u = 250$ kN.m Ratio of $(d'/D) = 0.1$

b) Non Dimensional Parameters

$$\left(\frac{P_u}{f_{ck} b D}\right) = \left(\frac{1000 \times 10^3}{25 \times 300 \times 500}\right) = 0.266$$

$$\left(\frac{M_u}{f_{ck} b D^2}\right) = \left(\frac{250 \times 10^6}{25 \times 300 \times 500^2}\right) = 0.133$$

c) Longitudinal Reinforcements

Refer chart - 44 (SP:16) with equal steel on all the sides and read out

$$\therefore \quad \left(\frac{P}{f_{ck}}\right) = 0.09 \quad \therefore \quad p = (0.09 \times 25) = 2.25$$

$$\therefore \quad A_s = \left(\frac{pbd}{100}\right) = \left(\frac{2.25 \times 300 \times 500}{100}\right) = 3375 \text{ mm}^2$$

Provide 8 bars of 25 mm diameter ($A_{sc} = 3927$ mm²).
The bars are arranged equally on all the four sides (3 bars on each face)

d) Ties

Tie diameter $\nless (25/4) = 6.25$ mm. Hence, provide 8 mm ties.
$\nless 6$ mm
Tie spacing $\ngtr$ 300 mm and $\ngtr (16 \times 25) = 400$ mm
Hence provide 8 mm diameter ties at 300 mm c/c (staggered)

e) Details of Reinforcements

Fig. 10.13 shows the detailing of reinforcements in the column section

10.5.6 Design Example

Design a short circular column of diameter 400 mm to support a factored axial load of 900 kN, together with a factored moment of 100 kN.m. Adopt M-20 grade concrete and Fe-415 grade reinforcements.

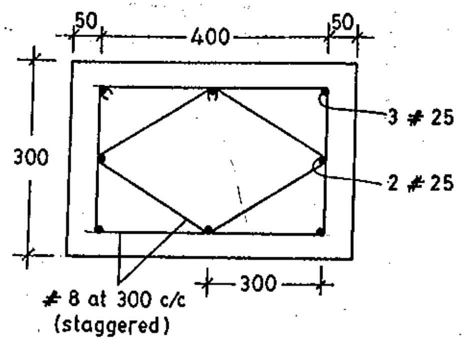


Fig. 10.13 Reinforcements in Columns With Uniaxial Bending

a) Data

$D = b = 400$ mm   Assume $d' = 40$ mm
$P_u = 900$ kN $\therefore (d'/D) = 0.10$
$M_u = 100$ kN.m
$f_{ck} = 20$ N/mm^2
$f_y = 415$ N/mm^2

b) Non Dimensional Parameters

$$\left(\frac{P_u}{f_{ck}D}\right) = \left(\frac{900 \times 10^3}{20 \times 400^2}\right) = 0.28$$

$$\left(\frac{M_u}{f_{ck}D^3}\right) = \left(\frac{100 \times 10^6}{20 \times 400^3}\right) = 0.078$$

c) Longitudinal Reinforcements

Refer Chart-56 of SP:16 and read out the values of the parameter

$$\therefore \quad \left(\frac{p}{f_{ck}}\right) = 0.10 \qquad \therefore \quad p = (20 \times 0.10) = 2$$

$$\therefore \quad A_s = \left(\frac{p\pi D^2}{400}\right) = \left(\frac{2 \times \pi \times 400^2}{400}\right) = 2512 \text{ mm}^2$$

Provide 6 bars of 25 mm diameter ($A_s = 2945$ mm^2)

d) Lateral ties

Tie diameter $\not<$ (25/4) = 6.25 mm
$\not>$ 16 mm (Hence select 8 mm diameter ties)
Tie spacing $\not>$ 400 mm
$\not>$ (16 × 25) = 400 mm
$\not>$ 300 mm

Provide 8 mm diameter ties at 300 mm centers.

e) Reinforcements

Fig. 10.14 shows the details of reinforcements in the column section.

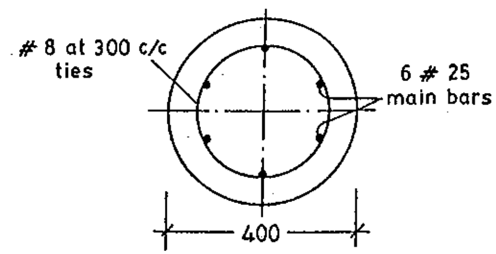


Fig. 10.14 Reinforcements in Circular Column

10.6 DESIGN OF SHORT COLUMNS UNDER COMPRESSION AND BIAXIAL BENDING

10.6.1 Introduction

Columns located at the corners of a multistoreyed building with rigidly connected beams at right angles, develop biaxial moments together with the axial compressive load transmitted from beams. Fig. 10.15(a) shows the column section subjected to the axial compressive load P_u and the moments M_{ux} and M_{uy} about the major and minor axis respectively. Fig. 10.15(b) shows the axis of bending and the resultant moment M_u acts about this axis inclined to the two principal axes. The resultant eccentricity is computed ass $e = (M_u/P_u)$ and this can also be expressed as,

$$e = \sqrt{e_x^2 + e_y^2} \quad \text{where} \quad e_x = (M_{ux}/P_u) \quad \text{and} \quad e_y = (M_{uy}/P_u)$$

The possible neutral axis lies in the X-Y plane as shown in Fig. 10.15(c).

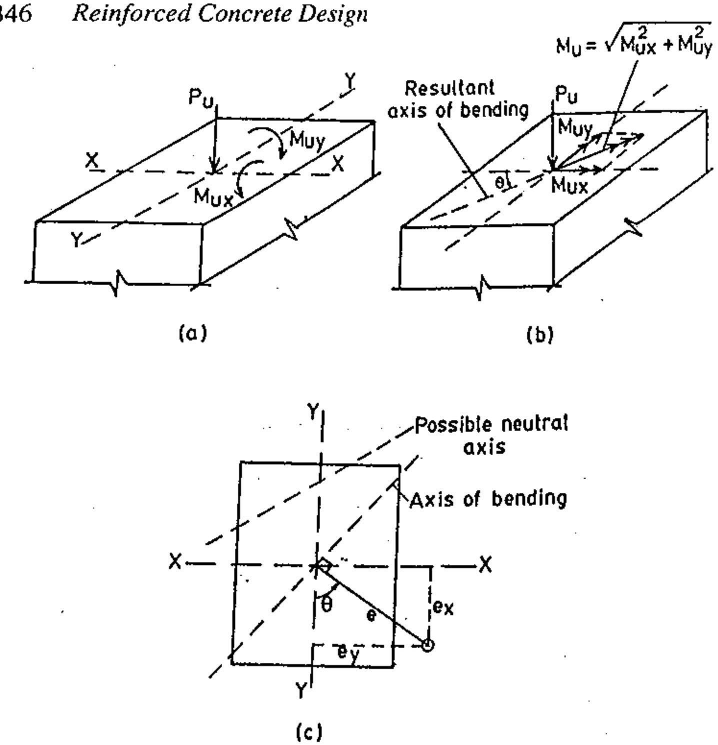


Fig. 10.15 Biaxial Bending of Short Columns

By choosing the neutral-axis which is in the X-Y plane, calculations are made from fundamentals to satisfy the equilibrium of load and moments about both the axes. This procedure is tedious and is not generally recommended for routine design.

To overcome the difficulties of trial and error procedure in the design of columns subjected to biaxial moments, The Indian standardcode IS:456-2000 recommends a simplified procedure based on Bresler's[87] formulation which facilitates faster design of reinforcements in the columns. This method is outlined in the following section.

10.6.2 Codal Method for Design of Compression members subject to Biaxial Bending

The simplified procedure adopted by the code (clause 39.6) based on Bresler's empirical formulation is expressed by the relation,

$$\left(\frac{M_{ux}}{M_{ux1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy1}}\right)^{\alpha_n} \leq 1.0$$

Where M_{ux}, M_{uy} are the moments about X and Y axes respectively due to design loads. M_{ux1} and M_{uy1} are the maximum uniaxial moment capacities with an axial load P_u, bending about X and Y axes respectively.
α_n is an exponent whose value depends on the ratio (P_u/P_{uz}) where

$$P_{uz} = [0.45\, f_{ck} \cdot A_{sc} + 0.75 f_y \cdot A_{sc}] \quad \text{i.e., Value of } P_u \text{ when } M = 0$$

The range of values of the ratio (P_u/P_{uz}) and the corresponding value of an are shown in Table-10.2 as well as in Fig. 10.16.

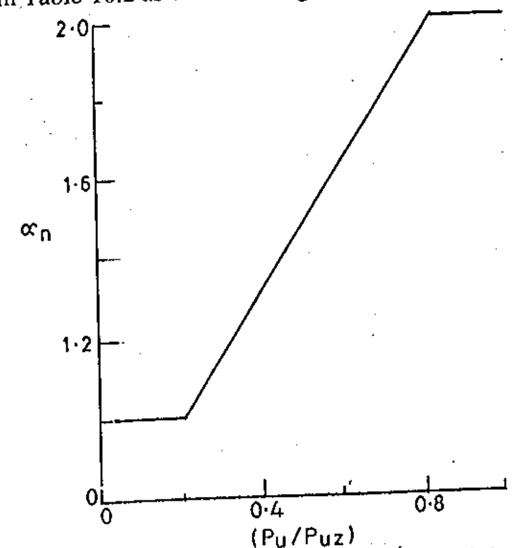


Fig. 10.16 Coefficient α_n For Biaxial Bending of Columns

For intermediate values, linear interpolation may be done. Chart-63 of SP-16 can be used for evaluating P_{uz} for different grades of concrete and steel and the percentage of reinforcement in the section.

Table 10.2 Values of α_n

(P_u/P_{uz})	α_n
≤ 0.2	1.0
≥ 0.8	2.0

Chart-64 of SP: 16 shows the relation,

$$\left(\frac{M_{ux}}{M_{ux1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy1}}\right)^{\alpha_n} = 1.0 \text{ for different values of } (P_u/P_{uz})$$

These curves are very useful in the design of columns subjected to biaxial bending.

The use of SP:16 charts for the design of columns subjected to axial compression and biaxial bending is illustrated in the example 10.6.4.

10.6.3 Selection of Trial Section and reinforcements

In practice, the cross sectional dimensions of the column are selected before the structural analysis is performed and the biaxial moments are derived from the frame analysis. Hence, only reinforcements need to be suitably assumed for the design. However, Devdas Menon[88] has suggested a simpler approach for the selection of reinforcements based on the resultant moment given by the relation,

$$M_u = 1.15\sqrt{M_{ux}^2 + M_{uy}^2}$$

This bending moment is considered to act in association with the axial compressive load Pu and using the design charts, the reinforcement percentage in the cross section is determined. Thereafter the procedure is the same as specified in section 10.6.2 for checking the adequacy of the designed section.

10.6.4 Design Example

Design the reinforcements in a short column 400 mm by 600 mm subjected to an ultimate axial load of 1600 kN together with ultimate moments of 120 kN.m and 90 kN.m about the major and minor axis respectively. Adopt M-20 grade concrete and Fe-415 HYSD bars.

a) Data

b	= 400 mm	f_{ck}	= 20 N/mm^2
D	= 600 mm	f_y	= 415 N/mm^2
P_u	= 1600 kN	d'	= 60 mm
M_{ux}	= 120 kN.m	(d'/D)	= 0.1
M_{uy}	= 90 kN.m		

b) Reinforcements

Reinforcements are distributed equally on all the four sides.
As a first trial, adopt percentage of reinforcement in the cross section as p = 1 percent

$$\therefore \quad A_s = \left(\frac{p\,b\,D}{100}\right) = \left(\frac{1\times400\times600}{100}\right) = 2400 \text{ mm}^2$$

Use 8 bars of 20 mm diameter distributed 3 on each face (A_s = 2512 mm^2)

$$\therefore \quad p = \left(\frac{100\times2512}{400\times600}\right) = 1.04 \quad \text{and} \quad \left(\frac{p}{f_{ck}}\right) = \left(\frac{1.04}{20}\right) = 0.052$$

$$\left(\frac{P_u}{f_{ck} b\,D}\right) = \left(\frac{1600\times10^3}{20\times400\times600}\right) = 0.333$$

Refer Chart-44 of SP:16 and read out the ratio $[M_{ux1}/f_{ck}\,b\,D^2]$ corresponding to the ratio $[P_u/f_{ck}\,b\,D] = 0.333$ and $(d'/D) = 0.10$ and $(p/f_{ck}) = 0.052$.

$$\therefore \quad \left(\frac{M_{ux1}}{f_{ck} b D^2}\right) = 0.085$$

$$\therefore \quad M_{ux1} = (0.085\times20\times400\times600^2)10^{-6} = 245 \text{ kN.m}$$

For moments about the minor axis YY, b = 600 mm, D = 400 mm and d' = 60 mm

$$\therefore \quad \left(\frac{d'}{D}\right) = \left(\frac{60}{400}\right) = 0.15$$

Refer chart-45 of SP:16 and read out the ratio $[M_{uy1}/f_{ck}\,b\,D^2]$ corresponding to the ratio $[P_u/f_{ck}\,b.D] = 0.333$ and $(p/f_{ck}) = 0.052$

$$\therefore \quad \left(\frac{M_{uy1}}{f_{ck} b D^2}\right) = 0.08$$

$$\therefore \quad M_{uy1} = (0.08\times20\times600\times400^2)10^{-6} = 153 \text{ kN.m}$$

$$P_{uz} = [0.45\,f_{ck}A_c + 0.75\,f_y A_s]$$

$$= [0.45\times20\{(600\times400) - 2512\} + (0.75\times415\times2512)]10^{-3} \text{ kN} = 2919 \text{ kN}$$

$$\therefore \quad \text{Ratio} \quad \left(\frac{P_u}{P_{uz}}\right) = \left(\frac{1600}{2929}\right) = 0.548$$

Refer Fig. 10.16 and read out the coefficient an corresponding to the ratio $(P_u/P_{uz}) = 0.548$. The value of $\alpha_n = 1.58$.

$$\therefore \quad \left(\frac{M_{ux}}{M_{ux1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy1}}\right)^{\alpha_n} \le 1$$

$$\left(\frac{120}{245}\right)^{1.58}+\left(\frac{90}{153}\right)^{1.58}=0.756<1$$

Hence, the design is safe. Provide suitable lateral ties as per codal provisions.

Provide 8 mm diameter lateral ties at 300 mm centers will conform to the codal requirements.

The above problem can be solved by using Charts-63 and 64 of SP:16 as shown below:

From Chart-63, for $p = 1$ percent, $f_{ck} = 20$ N/mm^2 and $f_y = 415$ N/mm^2, read out the

Corresponding ratio,

$$\left(\frac{P_{uz}}{A_g}\right)=12 \text{ and hence } P_{uz}=(12\times 600\times 400)10^{-3}=2880 \text{ kN}$$

Also $$\left(\frac{M_{ux}}{M_{ux1}}\right)=\left(\frac{120}{245}\right)=0.49$$

And $$\left(\frac{M_{uy}}{M_{uy1}}\right)=\left(\frac{90}{153}\right)=0.59$$

Ratio $$\left(\frac{P_u}{P_{uz}}\right)=\left(\frac{1600}{2880}\right)=0.55$$

From Chart-64 of SP: 16 for $[M_{ux}/M_{ux1}] = 0.49$ and $[P_u/P_{uz}] = 0.55$, read out the ratio

$$\left(\frac{M_{uy}}{M_{uy1}}\right)=0.8> \text{ calculated value of } 0.59$$

Hence, the design is safe. However for economical design, a second trial is made with lower value of reinforcement and the various steps repeated such that the ratio of $[M_{uy}/M_{uy1}]$ obtained from Chart-64 is slightly greater than the calculated value.

10.6.5 Design Example

A short column located at the corner of a storied building is subjected to an axial factored load of 2000 kN together with factored moments of 75 and 60 kN.m acting in perpendicular planes. The size of the column is fixed as 450 by 450 mm. Adopting concrete of M-20 grade and Fe-415 HYSD bars, design suitable reinforcements in the column section.

a) Data

b	= 450 mm	f_{ck}	= 20 N/mm^2
D	= 450 mm	f_y	= 415 N/mm^2
M_{ux}	= 75 kN.m	d'	= 50 mm
M_{uy}	= 60 kN.m	(d'/D)	= 0.10

b) Equivalent Moment

The reinforcement in section is designed for the axial compressive load P_u and the equivalent moment given by the relation,

$$M_u = 1.15\sqrt{M_{ux}^2+M_{uy}^2}$$
$$= 1.15\sqrt{75^2+60^2} = 110 \text{ kN.m}$$

c) Non Dimensional Parameters

$$\left(\frac{P_u}{f_{ck}b\,d}\right)=\left(\frac{2000\times 10^3}{20\times 4505450}\right)=0.49$$

$$\left(\frac{M_u}{f_{ck}b\,D^2}\right)=\left(\frac{110\times 10^6}{20\times 450\times 450^2}\right)=0.06$$

d) Reinforcements

Refer chart-44 of SP: 16 (equal reinforcement on all faces) with $(d'/D) =$ 0.10 and read out the value of $(p/f_{ck}) = 0.06$.

$$\therefore \quad p=(20\times 0.06)=1.2$$

$$\therefore \quad A_s=\left(\frac{pbD}{100}\right)=\left(\frac{1.2\times 450\times 450}{100}\right)=2430 \text{ mm}^2$$

Provide 8 bars of 20 mm diameter (A_s = 2512 mm^2) with 3 bars in each face.

$$p=\left(\frac{100\times 2512}{450\times 450}\right)=1.24 \text{ and the ratio}\left(\frac{p}{f_{ck}}\right)=\left(\frac{1.24}{20}\right)=0.062$$

Refer Chart-44 (SP:16) and read out the value of the ratio $[M_{ux1}/(f_{ck}\,b\,D^2)]$, corresponding to the value of ratio $[P_u/f_{ck}\,b\,D] = 0.49$ and $(p/f_{ck}) = 0.062$.

$$\left(\frac{M_{ux1}}{f_{ck}bD^2}\right) = 0.06$$

$\therefore \quad M_{ux1} = (0.06 \times 20 \times 450 \times 450^2)10^{-6} = 109$ kN.m

Due to symmetry, $M_{ux1} = M_{uy1} = 109$ kN.m

$$P_{uz} = [0.45\, f_{ck}.A_c + 0.75\, f_y A_s]$$

$$= (0.45 \times 20)[(450 \times 450) - 2512] + (0.75 \times 415 \times 2512)$$

$$= (2581 \times 10^3)\text{N}$$

$$= 2581 \text{ kN}$$

$$\therefore \quad \left(\frac{P_u}{P_{uz}}\right) = \left(\frac{2000}{2581}\right) = 0.77$$

Refer Fig. 10.16 and read out the coefficient an = 1.95

e) Check for Safety under Biaxial Loading

$$\therefore \quad \left(\frac{M_{ux}}{M_{ux1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy1}}\right)^{\alpha_n} \le 1$$

$$\left(\frac{75}{109}\right)^{1.95} + \left(\frac{60}{109}\right)^{1.95} = 0.79 \le 1$$

Hence, the section is safe under specified loading.

f) Reinforcements

Provide 8 bars of 20 mm diameter as main reinforcement and 8 mm lateral ties at 300 mm centres.

10.7 DESIGN OF SLENDER COLUMNS

10.7.1 Introduction

Compression members having the ratio of effective length to its least lateral dimension (slenderness ratio) exceeding 12 are categorized as slender or long columns according to IS:456-2000 code. The deformation characteristics of slender columns are significantly different from that of short columns. When slender columns are loaded even with axial loads, the

lateral deflection is significantly greater in comparison with short columns as shown in Fig. 10.17. Consequently, in slender columns, the moment produced by the deflection is large and should be considered in design.

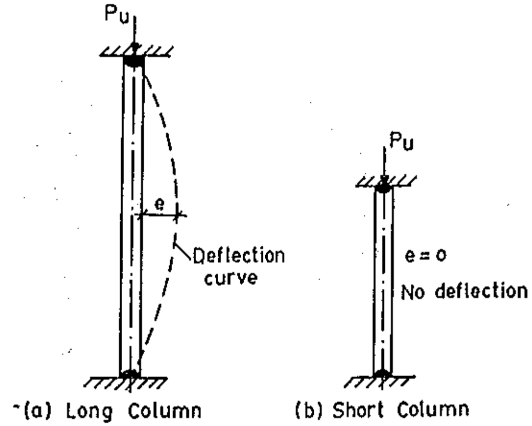


Fig. 10.17 Behaviour of Long And Short Columns

In the case of eccentrically loaded long columns, the effect of secondary moments developed due to the lateral deflection together with the primary moments significantly influences the load carrying capacity of the compression member.

10.7.2 Behaviour of Slender Columns

The structural behaviour of slender columns is significantly different from that of short columns with increasing slenderness ratios. Consider a column hinged at supports subjected to an eccentric load 'P' at an eccentricity 'e' as shown in Fig. 10.18(a).

As the load is increased, the lateral deflection of the column increases.

If $\quad \Delta$ = lateral deflection of the longitudinal axis,
$(e + \Delta)$ = total eccentricity

The moment at any section is expressed as,

$$M = P\,(e + \Delta)$$
$$M = M_{pr} + \Delta$$

Where M_{pr} = primary moment due to eccentricity of the load.
$P.\Delta$ = secondary moment which varies along the length of the column.

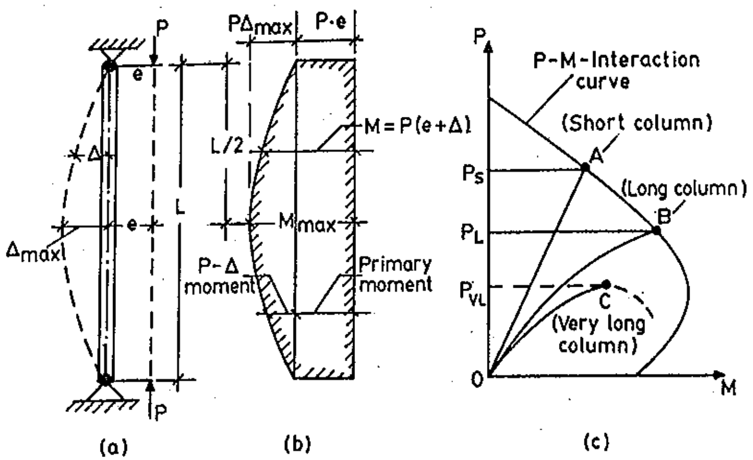


Fig. 10.18 Behaviour of Slender Columns

The maximum moment occurs at the mid height of the column and is expressed as

$$M_{max} = P\,(e + \Delta_{max})$$

The variation of maximum moment is non linear with the flexural stiffness reducing with increasing values of the load P [Refer Fig. 10.18(b)].

In the case of very short column, the flexural stiffness being very high, the lateral deflection ? is very small and the primary moment controls the behaviour of the column.

In the case of very slender column, it is possible that the flexural stiffness is effectively reduced to zero resulting in buckling or instability failure.

Fig. 10.18(c) shows the load-moment interaction diagram at the limit state of collapse representing the strength of the column with varying slenderness ratios.

In the case of short column, $\Delta_{max} = 0$ and hence the failure is due to the primary moment and axial load. Point A represents the behaviour of short column with material failure. Point B indicates the long column behaviour with primary and secondary moments with material failure.

In the case of very long columns, the failure is due to buckling or instability. The curve OC represents the behaviour of very long columns.

In the case of braced slender columns which is not subjected to sidesway, there is no significant relative lateral displacement between the top and bottom ends of the column. The ends of a braced column are partially

restrained against rotation due to the floor level beams and moments M_1 and M_2 may develop at the ends. The column may be bent in single or double curvature, depending upon the nature of moments. The effect of these moments are taken into account in the design of such columns.

Unbraced slender columns are subjected to sidesway or lateral drift due to the action of lateral loads or gravity loads inducing additional moments at the supports. The moment amplification due to the lateral drift effect which is significantly greater than that of braced columns should be considered in the design of such columns.

The design of slender columns is similar to that of columns subjected to a given factored axial compression P_u and factored moments M_{ux} and M_{uy}, the only difference being that the moments should include the secondary moment components in slender column design, where as these are ignored being negligible in short column design.

10.7.3 Codal method for design of Slender columns

The IS:456-2000 code (clause 39.7) prescribes that the design of slender compression members should include the forces and moments determined from structural analysis and also the effects of deflections on moments and forces. The second order analysis involving deflections and their effect on moments and forces being computationally difficult and laborious, the code recommends simplified procedures for the design of slender columns, which involves the process of increasing the moments or reducing the strength to take care of slenderness effects.

The IS:456 code clause 39.7.1 recommends additional moments M_{ux} and M_{uy} expressed in terms of the factored axial load P_u, overall depth of the member (D) and the slenderness ratios (L_{ex}/D) and (L_{ey}/D) derived from the deformation characteristics of a pin ended braced slender column shown in Fig. 10.19.

The additional eccentricity Δ_{max} is a function of curvature. Denoting the maximum curvature at mid height as ϕ_{max}, it can be shown that Δ_{max} lies between (ϕ_{max}, $L^2/12$) and (ϕ_{max}, $L^2/8$). From Fig. 10.19, case(a) & (b), considering an average value for eccentricity as,

$$e_a = \Delta_{max} = (\phi_{max} L^2/10)$$

Referring to Fig. 10.20 showing the relation between curvature and failure strain profile and assuming,

$\varepsilon_{cu} = 0.0035$ and $\varepsilon_{st} = 0.002$, $d' = 0.1D$ and $(D - d') = 0.9D$

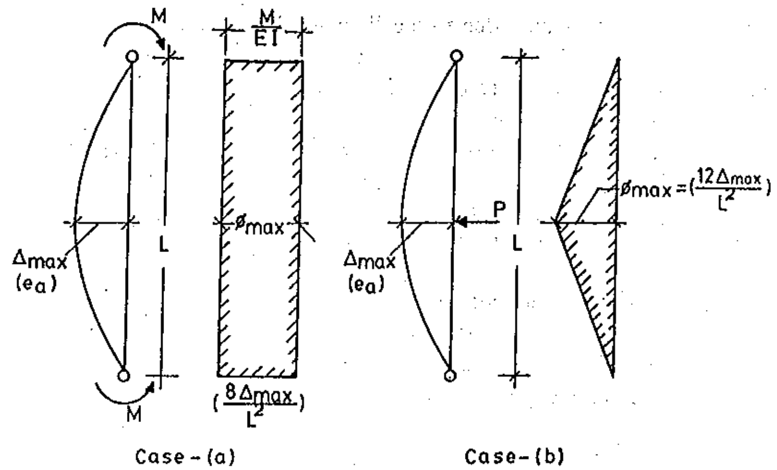


Fig. 10.19 Relation Between Deflection And Curvature in Pin Ended Slender Column

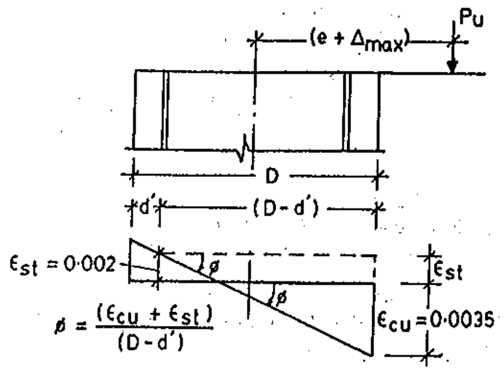


Fig. 10.20 Curvature-Strain Relationship

the additional moment comprises about 80 percent of the total moment. We can express the maximum curvature as,

$$\phi_{max} = \left[\frac{(0.0035 + 0.002)}{0.9D} \times 0.8\right] = \left(\frac{1}{200D}\right)$$

Substituting the value of ϕ_{max} in the expression for deflection e_a or Δ_{max} we have

$$\left(\frac{e_a}{D}\right) = \left(\frac{(L/D)^2}{2000}\right)$$

Hence, the expressions recommended in IS: 456-2000 code for additional moments are

$$M_{ax} = P_u e_{ax} = \frac{P_u D_x}{2000}\left[\frac{L_{ex}}{D_x}\right]^2$$

$$M_{ay} = P_u e_{ay} = \frac{P_u b}{2000}\left[\frac{L_{ey}}{b}\right]^2$$

Where

P_u = axial load on the member
L_{ex} = effective length in respect of major axis
L_{ey} = effective length in respect of minor axis
D = depth of cross section at right angles to the major axis
b = width of member
e_{ax} and e_{ay} are additional eccentricities (Refer Table-I of SP:16).

It is important to note that the additional moments to be considered are in addition to the factored primary moments M_{ux} and M_{uy} in the design of columns. The additional moments specified in the code are derived on the assumption that the column is braced and bent symmetrically in single curvature. Also the axial load corresponds nearly to the balanced failure condition ie $P_u = P_b$ .If these conditions are not satisfied, the code recommends the following modifications.

For $P_u > P_b$, the additional moments may be reduced by the multiplying factor 'k' given by the relation,

$$k = \left[\frac{P_{uz} - P_u}{P_{uz} - P_b}\right] \leq 1$$

Where $P_{uz} = [0.45 f_{ck} A_c + 0.75 f_y A_s]$ and this value can be read out from chart-63 of SP:16 and P_b is the axial load corresponding to the condition of maximum compressive strain of 0.0035 in concrete and tensile strain of 0.002 in the outer most layer of tension steel.

The modification suggested in the code is optional and it should always be taken advantage of since the value of 'k' could be substantially less than unity.

The value of P_b depends on the arrangement of reinforcement and the cover ratio (d'/D) and the grades of concrete and steel. The values of P_b can be computed for rectangular and circular sections using the constants k_1 and k_2 given in Table-60 of SP: 16 and the relation expressed as,

$$\left(\frac{P_b}{f_{ck} b\, d}\right) = k_1 + k_2\left(\frac{p}{f_{ck}}\right)$$

The value of the reduction factor 'k' can be read out from Chart-65 of SP: 16 after evaluating the ratios (P_u/P_{uz}) and (P_b/P_{uz})

For braced columns subjected to unequal primary moments M_1 and M_2 at the two ends, the value of M_u to be considered in computations of the total moment may be taken as (clause 39.7.1),

$$M_u = (0.4\,M_1 + 0.6\,M_2) \geq 0.4\,M_2$$

For un-braced columns, the lateral drift effect has to be included. Hence, an approximate method of including this effect is to assume the additional moment M_a to act at the column end where the maximum primary moment M_1 is operational. For design purposes, the total moment is computed as,

$$M_u = (M_a + M_1)$$

The use of these design principles is illustrated in the following example.

10.7.4 Design Example

Design the reinforcements required for a column which is restrained against sway using the following data:

a) Data

Size of column = 530 mm by 450 mm
Effective length = 6.6 m
Un supported length = 7.7 m
Factored load = 1600 kN
Factored moment about major axis = 45 kN.m at top and 30 kN.m at bottom
Factored moment about minor axis = 35 kN.m at top and 20 kN.m at bottom.
Concrete grade = M-25
Steel grade = Fe-500 HYSD bars

Column is bent in double curvature and reinforcement is distributed equally on all the four sides of the section.

b) Slenderness ratio

$$\left(\frac{L_e}{D}\right) = \left(\frac{6600}{530}\right) = 12.45 > 12$$

$$\left(\frac{L_e}{b}\right) = \left(\frac{6600}{450}\right) = 14.67 > 12$$

Hence, the column is slender about both axes

c) Additional Eccentricities

From Table-I of SP: 16, for $(L_e/D) = 12.45$, $(e_{ax}/D) = 0.078$
For $(L_e/b) = 14.67$, $(e_{ay}/b) = 0.108$

$\therefore$ $e_{ax} = (0.078 \times 530) = 41.34$ mm
$e_{ay} = (0.108 \times 450) = 48.60$ mm

d) Additional Moments

$M_{ax} = [1600\,(41.34/1000)] = 66.14$ kN.m
$M_{ay} = [1600\,(48.60/1000)] = 77.76$ kN.m

The above moments have to be multiplied by modification factor (k) as per clause 39.7.1.1 of IS:456-2000.

$$k = \left[\frac{P_{uz} - P_u}{P_{uz} - P_b}\right] \leq 1$$

Assuming 3.28 percent reinforcement for the first trial, the ratio,

$$(p/f_{ck}) = (3.28/25) = 0.131$$

From Chart-63 of SP:16, read out the ratio of $[P_{uz}/A_g] = 21$

$$\therefore \quad P_{uz} = \left[\frac{21 \times 530 \times 450}{100}\right] = 5008 \text{ kN}$$

Assuming 25 mm diameter bars with 50 mm cover,

$(d'/D) = (50/530) = 0.1$ and $(d'/b) = (50/450) = 0.1$

From Table-60 of SP: 16, read out the values of k_1 and k_2 as

$k_1 = 0.207$ and $k_2 = 0.425$

$$P_{bx} = P_{by} = [k_1 + k_2(p/f_{ck})]f_{ck}.b.D$$

$$= [0.207 + 0.425(0.131)]\,[(25 \times 450 \times 530)/1000] = 1566 \text{ kN}$$

$$k_x = k_y = [(5008 - 1600)/(5008 - 1566)] = 0.99$$

Additional moments are modified as,

$M_{ax} = (66.14 \times 0.99) = 65.48$ kN.m
$M_{ay} = (77.16 \times 0.99) = 76.39$ kN.m

As per clause 39.7.1 of IS: 456-2000 code, the initial moment acting on the column should be modified as follows:

$$M_{ux} = [(0.6 \times 45) - (0.4 \times 30)] = 15 < (0.4 \times 45) = 18$$
$$M_{uy} = [(0.6 \times 35) - (0.4 \times 20)] = 13 < (0.4 \times 35) = 14$$

As the above values are less than 0.4 times the larger end moment, we have to consider for design the modified initial moments as,

$$M_{ux} = 18 \text{ kN.m} \quad \text{and} \quad M_{uy} = 14 \text{ kN.m}$$

These moments are to be compared with the moment due to minimum eccentricity and greater of the two values is to be taken as the initial moment.

From clause 25.4 of IS: 456-2000, the minimum eccentricities are computed as,

$$e_x = \left[\frac{7700}{500} + \frac{530}{30}\right] = 33.07 \text{ mm} > 20 \text{ mm}$$

$$e_y = \left[\frac{7700}{500} + \frac{450}{30}\right] = 30.4 \text{ mm} > 20 \text{ mm}$$

$$M_{ux,min} = 1600\,(33.07/1000) = 52.92 \text{ kN.m} > 18.0 \text{ kN.m}$$
$$M_{uy,min} = 1600\,(30.4/1000) = 48.64 \text{ kN.m} > 14.0 \text{ kN.m}$$

Therefore the total moment for which the column is to be designed are,

$$M_{ux} = (52.92 + 65.48) = 118.40 \text{ kN.m}$$
$$M_{uy} = (48.64 + 76.39 = 125.03 \text{ kN.m}$$

$$\left(\frac{P_u}{f_{ck} b d}\right) = \left(\frac{1600 \times 10^3}{25 \times 530 \times 450}\right) = 0.27$$

From Chart-48 of SP:16, for the ratio (p/f_{ck}) read out the moments as,

$$M_{ux1} = (0.19 f_{ck}\, b\, d^2) = (0.19 \times 25 \times 450 \times 530^2)\, 10^{-6} = 600 \text{ kN.m}$$
$$M_{uy1} = (0.19 f_{ck}\, d\, b^2) = (0.19 \times 25 \times 530 \times 450^2)\, 10^{-6} = 510 \text{ kN.m}$$

$$\left(\frac{M_{ux}}{M_{ux1}}\right) = \left(\frac{118.40}{600}\right) = 0.20$$

$$\left(\frac{M_{uy}}{M_{uy1}}\right) = \left(\frac{125.03}{510}\right) = 0.25$$

$$\left(\frac{P_u}{P_{uz}}\right) = \left(\frac{1600}{5008.5}\right) = 0.32$$

From Chart-64, for $(P_u/P_{uz}) = 0.32$ and $(M_{ux}/M_{ux1}) = 0.2$ read out the value of $(M_{uy}/M_{uy1}) = 0.92 > 0.25$.

Hence, the section is safe but not economical. In the second trial, the area of reinforcement may be reduced in the section and the various design steps are repeated until an economical section is obtained.

Provide 12 bars of 25 mm diameter equally spaced on each face and lateral ties as per codal specifications.

10.8 DESIGN OF FOOTINGS

10.8.1 Introduction

Reinforced concrete columns are generally supported by the footings which are located below the ground level and is referred to as the foundation structure. The main purpose of the footing is to effectively support the super structure like columns by transmitting the applied loads, moments and other forces to the soil without exceeding the safe bearing capacity and also the settlement of the structure should be within tolerable limits and as nearly uniform as possible.

The footings are generally designed to resist the bending moments and shear forces developed due to soil reaction as specified in the Indian standard code IS:456-2000. This chapter deals with the design principles of different types of footings outlined in the following section.

10.8.2 Types of Footings

Footings are grouped under shallow foundations (in contrast to deep foundations like piles and caissons) which are adopted when the soil of adequate bearing capacity is available at a relatively short depth below the ground level. Column footing has a large plan area in comparison with the cross sectional area of the column. The loads on the columns are resisted by concrete and steel and these load effects are transmitted by the footing to the relatively weak supporting soil by bearing pressure.

Generally, the safe bearing capacity of the soil is very low in the range of 100 to 400 kN/m^2, whereas the permissible compressive stress in concrete is around 5 to 15 N/mm^2 and in steel, it is in the range of 130 to 190 N/mm^2 in reinforced concrete columns under working loads.

a) Isolated Column Footing

In the case of framed buildings with columns located on reasonably firm soil, it is generally sufficient to provide separate independent footings for each of the columns. Such a footing is referred to as isolated footing which is square, rectangular or circular in shape depending upon the shape of column cross section. Isolated footings comprise of a thick slab which may be flat or stepped or sloped as shown in Fig. 10.21(a). The footings are generally reinforced by a steel mesh located at the bottom of the slab to resist the bending moment and shear forces developed due to the soil pressure.

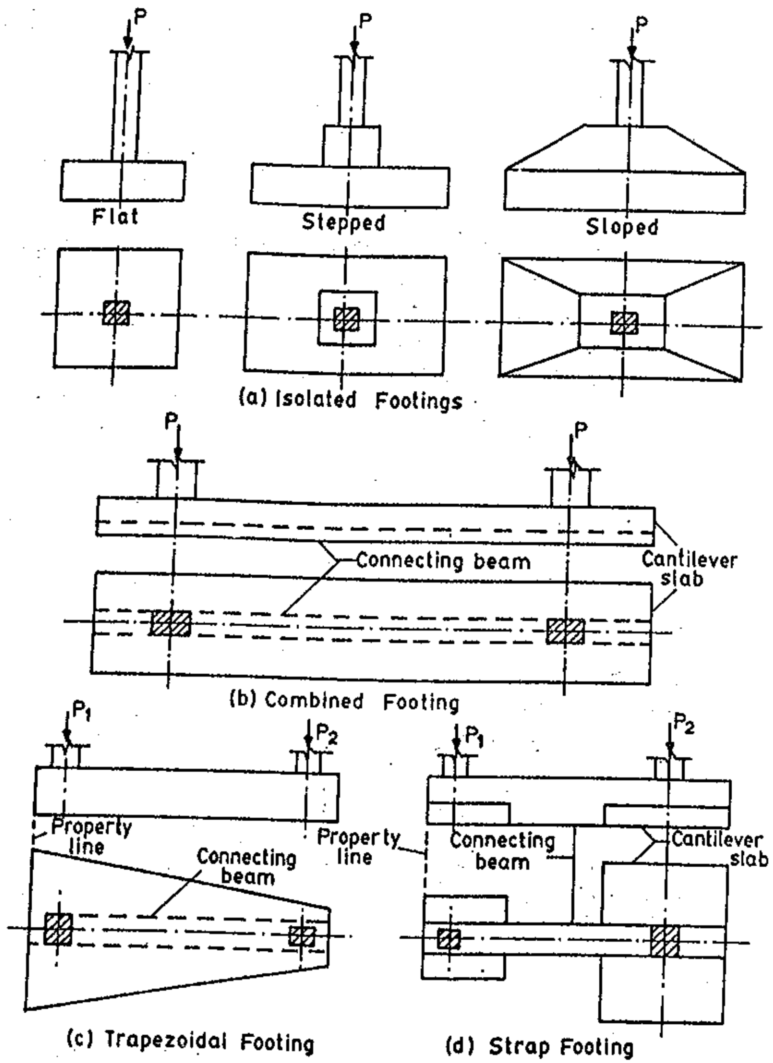


Fig. 10.21 Types of Footings

b) Combined Footings

When two are more heavily loaded columns are located close to each other resting on soil with low bearing capacity, the area of isolated footings overlap on each other and hence it is advantageous to provide a single combined footing contributing to the improved integral behaviour of the columns with the footing. Typical combined footing having rectangular shape is shown in Fig. 10.21(b). The combined footings comprise of a connecting beam between the columns integrally cast with a slab on either side of the connecting beam.

In the case of columns located close to the property line, footings can not be extended on one side. To overcome this problem of non availability of space near the exterior column, the footings of the exterior and interior columns are combined by using a connecting beam and trapezoidal shaped slab as shown in Fig. 10.21(c). Due to the soil pressure, the slab bends transversely while the connecting beam bends longitudinally between the columns. Strap footing shown in Fig. 10.21(d) is an alternative method of providing combined foundation connecting column located on property line and the interior of the building. In the case of strap footing, independent slabs are provided below the columns, connected by a strap beam.

10.8.3 Design principles and Codal requirements

The structural design of the footing, which includes the design of the depth and reinforcements, is done for factored loads using the relevant safety factors applicable for the limit state of collapse. The computation of factored moments and shear forces acting at the critical sections of the footing, is based on the fictitious factored soil pressure corresponding to the factored loads on the column.

The soil pressure developed due to self-weight of the footing does not induce any moments and shear forces and hence neglected in computations. The loads acting on the column and the soil pressure developed due to the service loads and the factored soil pressure to be used in design is shown in Fig. 10.22. The following design principles are relevant in the design of footings.

a) General design Features

Footings are designed for flexure and shear (both one way and two way action), bearing and bond, mainly due to the soil pressure from the soffit of the slab. The design is more or less similar to that of beams and two way slabs supported on columns. Additional design considerations being the

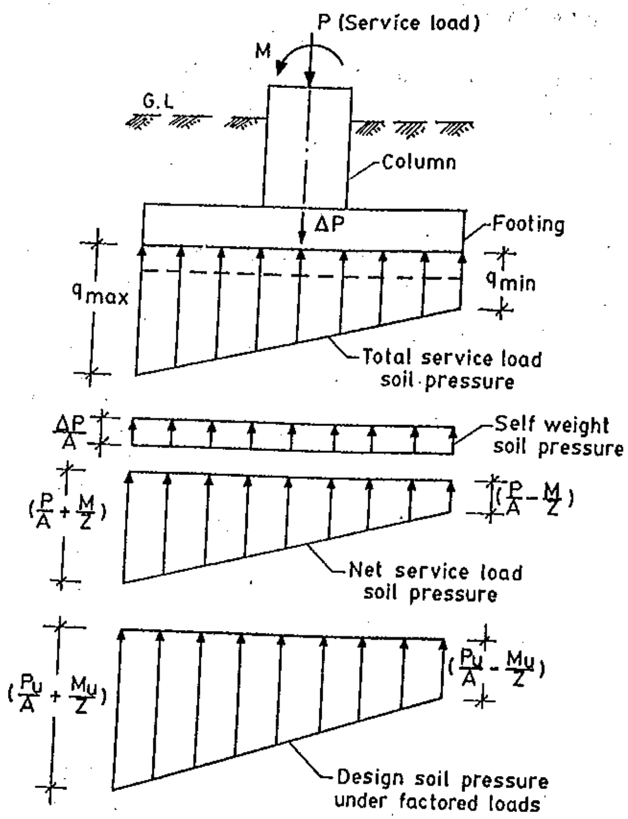


Fig. 10.22 Soil Pressure Under Column Footing

transfer of force from the column to the footing and also safety against sliding and overturning when horizontal forces are acting on the structure. Since footings are buried below the ground level, deflection control is not important but crack widths should be limited to 0.3 mm, with general detailing requirements and it is desirable to provide a clear cover of 75 mm for protection of main reinforcements especially under aggressive environments.

b) Depth of Foundation

The minimum depth of foundation according to Rankine's theory[89] is given by

$$h = \frac{p}{w}\left[\frac{1-\sin\phi}{1+\sin\phi}\right]^2$$

Where

h = depth of foundation
w = unit weight of soil
p = safe bearing capacity of soil under the footing
ϕ = angle of repose

c) Thickness of Footing

The thickness of footing is generally designed based on the considerations of shear and flexure which are critical in the vicinity of the column and footing junction.

Shear forces being more critical, the thickness is generally based on shear criteria. It is generally economical to vary the thickness of the slab from a minimum of 150 mm at the edges to a maximum near the face of the column depending upon the variations in bending moment and shear force. A leveling course of lean concrete of 100 mm thickness is generally provided below the footing.

d) Design for Shear

The overall depth of the footing is mainly dictated by the shear stress considerations and generally preceeds the design for flexure. To check for shear stress, the tension reinforcement in the slab is assumed as 0.25 to 0.3 percent and the procedure used for slabs is adopted. One way shear is checked at a critical section distant 'd' from the column face as shown in Fig. 10.23(a). The behaviour of footings in two way (punching) shear is similar to that of flat slab supported on columns. The critical section for two way shear is considered at a distance ($d/2$) from the periphery of the column as shown in Fig. 10.23(b).

Shear reinforcements are generally avoided in footing slabs and the required depth is designed by one way and two way shear considerations. The design shear strength tc of concrete is computed by assuming a normal percentage of flexural reinforcement of 0.25 percent in preliminary calculations. The design ultimate shear force V_u is limited to the shear resistance of concrete V_{uc} by providing the necessary depth. If $V_u > V_{uc}$, suitable shear reinforcements should be designed to resist the balance shear force of ($V_u - V_{uc}$) in a way similar to that of beams.

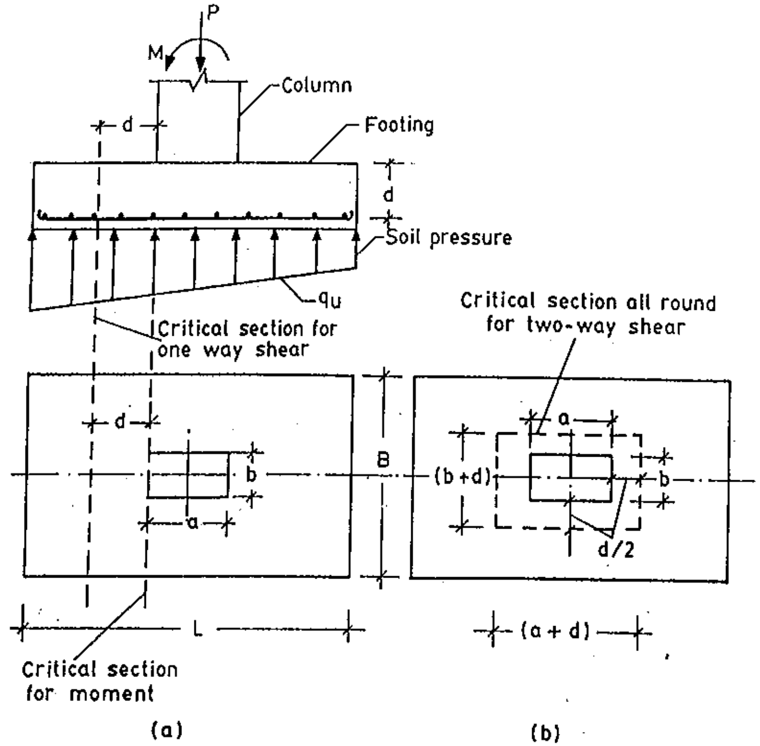


Fig. 10.23 Critical Sections For Moment & Shear in Column Footing

e) Design for Flexure

The critical section for moment is at the face of the column as shown in Fig. 10.23. The reinforcement is designed to resist the factored moment at the critical section. In two-way reinforced rectangular footings, the reinforcement in the long direction is uniformly spaced across the full width of the footing. In the shorter direction where the moments are less, the code clause (34.3.1.c) specifies a larger concentration of reinforcement to be provided within a central band width equal to the width(shorter dimension) of the footing given by the relation,

[Reinforcement in central band width] = $(A_{st,short})\,[2/(\beta + 1)]$

Where $A_{st,short}$ = total reinforcement in short direction
β = ratio of long side to the short side of the footing.

The remainder of the reinforcement is uniformly distributed in the outer portions of the footing as shown in Fig. 10.24.

f) Force transfer at Column Base

The axial force and moment acting at the base of the column must be transferred to the footing either by compression in concrete or by compression/tension in reinforcements. The bearing resistance or compressive stress developed at the junction of column and footing is limited to a value given by the IS:456 code clause 34.4 as

$$f_{br,max} = 0.45\, f_{ck} \sqrt{(A_1/A_2)}$$

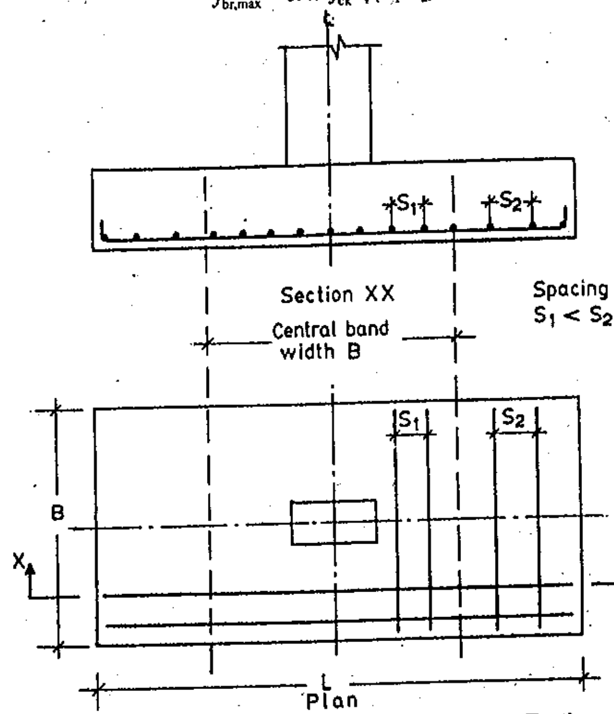
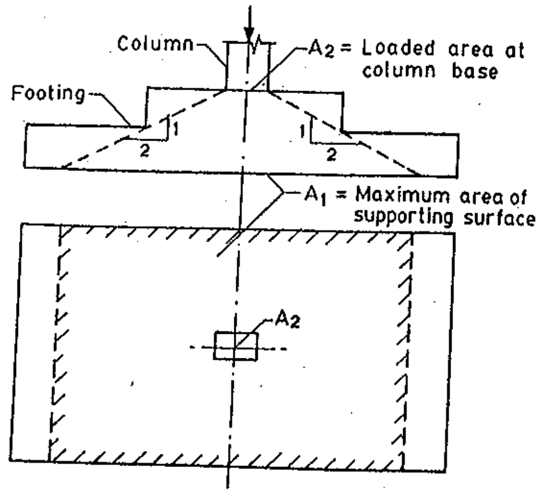


Fig. 10.24 Reinforcement Details in Rectangular Footings

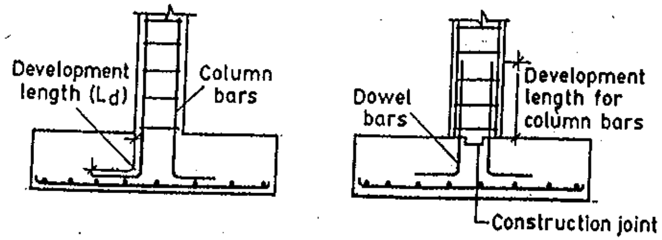
Where A_1 = supporting area for bearing of footing which in sloped or stepped footing may be taken as the area of the lower base of the largest frustrum of a pyramid or cone contained wholly within the footing and having for its upper base, the area actually loaded and having side slope of one vertical to two horizontal.

A_2 = loaded area at the base of the column.

The factor $\sqrt{(A_1/A_2)}$ accounts for the increase in concrete strength in the bearing area due to confinement of surrounding concrete. However, this factor is limited to 2 since very high compressive stresses result in transverse tensile strains leading to spalling, lateral splitting or bursting of concrete. The area A_1 geometrically similar to A_2 is shown in Fig. 10.25(a). If the actual compressive stress exceeds $f_{br,max}$, then the excess force is transferred by reinforcement, dowels or mechanical connectors.



(a) Loaded and Supporting Areas



(b) Reinforcement at Column-Footing Interface

Fig. 10.25 Force Transfer at Base of Column

The detailing of reinforcement across column-footing interface is shown in Fig. 10.25(b). according to the code clause 34.4.3, the diameter of dowels should not exceed the diameter of the column bars by 3 mm and the reinforcement at the interface must comprise of at least four bars with a total area not less than 0.5 percent of the cross sectional area of the supported column or pedestal. In addition, all reinforcement provided across the interface must have the necessary development length in compression

or tension on both sides of the interface.

The design of a typical column footing is illustrated by the following example.

10.8.4 Design Example

A reinforced concrete column 400 mm by 400 mm supports an axial service load of 1000 kN. The safe bearing capacity of the soil at site is 200 kN/m². Adopting M-20 grade concrete and Fe-415 HYSD bars design a suitable footing for the column and sketch the details of reinforcements.

a) Data

Axial service load = P = 1000 kN
Size of column = 400 mm by 400 mm
S.B.C. of soil = 200 kN/m²
M-20 grade concrete and Fe-415 HYSD bars.

b) Size of Footing

Load on column = 1000 kN
Weight of footing and back fill at 10% = 100 kN
Total load = 1100 kN

$$\text{Area of footing} = \left(\frac{1100}{200}\right) = 5.5 \text{ m}^2$$

Size of footing = $L = B = \sqrt{5.5} = 2.345$ m
Adopt 2.4 m by 2.4 m square footing
Net soil pressure at ultimate loads with a load factor of 1.5 is given by

$$q_u = \left(\frac{1000 \times 1.5}{2.4 \times 2.4}\right) = 260 \text{ kN/m}^2 = 0.26 \text{ N/mm}^2$$

c) One way Shear

The critical section is at a distance 'd' from the column face (Refer Fig. 10.26)

Factored shear force = $V_{u1} = (0.26 \times 2400)(1000 - d) = 624(1000 - d)$
Assuming percentage of reinforcement in the footing p_t = 0.25 percent, for M-20 grade concrete, read out from Table-19 of IS:456 code the permissible shear stress as,

$$\tau_c = 0.36 \text{ N/mm}^2$$

One way shear resistance = $V_{c1} = (0.36 \times 2400 \times d) = (864\ d)$ N

$$V_{u1} \leq V_{c1}$$

$$624\ (1000 - d) \leq 864\ d$$

$\therefore$

$$d \geq 722 \text{ mm}$$

d) Two way Shear

Assuming the effective depth of slab = d = 722 mm and computing the two way shear resistance at a critical section (d/2) from the face of the column, we have the relation,

$V_{u2} = 0.26\ [2400^2 - (400 + d)^2] = 0.26\ [2400^2 - (400 + 722)^2] = 1170290$ N

Two- way shear resistance V_{c2} is computed as,

$V_{c2} = k_s\ \tau_c\ [4(400 + d)\ d]$ where $k_s = 1.0$ and $\tau_c = 0.25 = 1.118$ N/mm^2

$V_{c2} = (1 \times 1.118)\ [4(400 + d)d] = 1788.8\ d + 4.472\ d^2$

$\therefore\ V_{u2} \leq V_{c2}$

$1170290 \leq (1788.8 + 4.472\ d^2)$

Solving, d = 349 mm
Hence, one-way shear is more critical.
Adopt effective depth = d = 725 mm and Overall depth = 800 mm

e) Design of Reinforcements

Ultimate moment at column face (refer Fig. 10.26) is computed as,

$$M_u = (260 \times 1 \times 0.50) = 130 \text{ kN.m/m}$$

$$\therefore \quad \left(\frac{M_u}{bd^2}\right) = \left(\frac{130 \times 10^6}{10^3 \times 725^2}\right) = 0.247$$

Refer Table-2 of SP: 16 and interpolate the percentage reinforcement as, p_t = 0.080 which is less than 0.25 percent assumed for one-way shear.

$$\therefore \quad A_{st} = \left(\frac{p_t b\ d}{100}\right) = \left(\frac{0.25 \times 10^3 \times 725}{100}\right) = 1813 \text{ mm}^2\text{/m}$$

Using 20 mm diameter bars,

Spacing of the bars is $S = \left(\dfrac{1000 \times 314}{1813}\right) = 173$ mm c/c

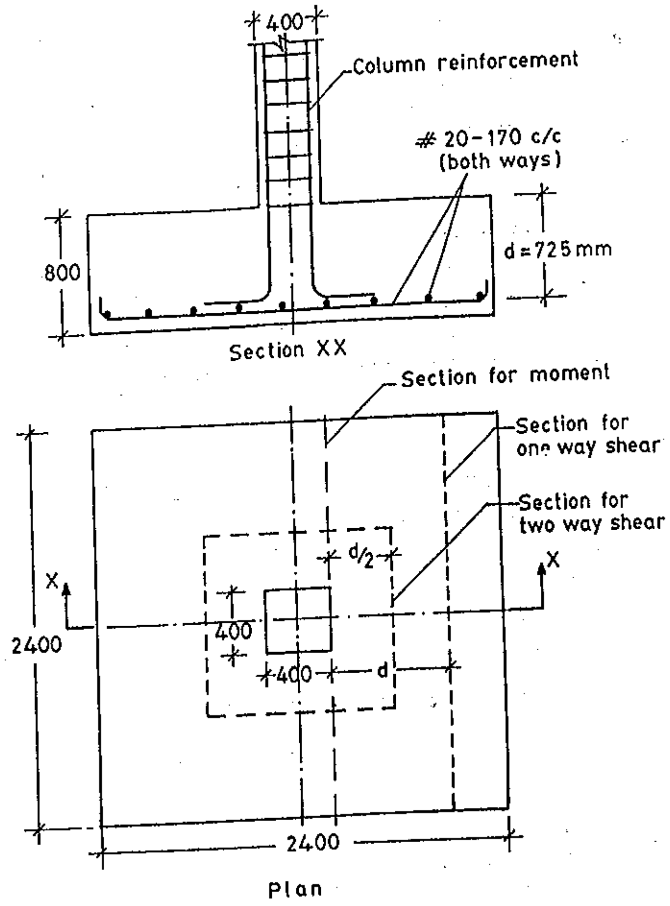


Fig. 10.26 Design of Column Footing

Adopt 20 mm diameter bars at 170 mm centres in both directions.

f) Transfer of Force at column Base

Ultimate compressive force at column base = $P_u = (1.5 \times 1000) = 1500$ kN
Limiting bearing stress at column footing interface is expressed as,

$$f_{br,max} = 0.45\ f_{ck}\ \sqrt{A_1/A_2}$$

i) For column face, $f_{ck} = 20\ \text{N/mm}^2$

$$A_1 = A_2 = 400^2\ \text{mm}^2$$

$\therefore$ $f_{br,max}$ (column) $= (0.45 \times 20 \times 1) = 9\ \text{N/mm}^2$

ii) For footing face, $f_{ck} = 20\ \text{N/mm}^2$, $A_1 = 2400^2\ \text{mm}^2$ and $A_2 = 400^2\ \text{mm}^2$

$$\therefore \quad \sqrt{A_1/A_2} = \sqrt{(2400^2)/(400^2)} = 6 \text{ limited to } 2.0$$

$$f_{br,max}\text{(footing)} = (0.45 \times 20 \times 2) = 18\ \text{N/mm}^2 > 9\ \text{N/mm}^2$$

Hence the column face governs the design and $f_{br,max} = 9\ \text{N/mm}^2$

$\therefore$ Limiting bearing resistance is computed as,

$$F_{br} = [(9 \times 400^2)/1000] = 1440\ \text{kN} < P_u = 1500\ \text{kN}$$

$\therefore$ Excess force (to be transferred by reinforcement) is

$$\Delta P_u = (1500 - 1440) = 60\ \text{kN}$$

The required development length for transferring the force of 60 kN can be provided by extending the column reinforcement into the footing and bent at 90° (standard bend) resting directly on top of reinforcement mesh as shown in Fig. 10.26.

10.8.5 Design Example

Design a reinforced concrete circular footing for a circular column of 300 mm diameter supporting a design ultimate load of 750 kN. The safe bearing capacity of the soil at site is 200 kN/m². Adopt M-20 grade concrete and Fe-415 HYSD bars.

a) Data

$P_u = 750\ \text{kN}$	$f_{ck} = 20\ \text{N/mm}^2$
$D = 300\ \text{mm}$	$f_y = 415\ \text{N/mm}^2$
$p = 200\ \text{kN/m}^2$	$p_u = (1.5 \times 200) = 300\ \text{kN/m}^2$

b) Dimensions of Footing

Load on column = 750 kN

Self weight of footing (10%) = 75 kN

Total load on soil = w_u = 825 kN

Let D_f = diameter of the circular footing

A_f = area of the footing

$$A_f = \left(\frac{\pi D_f^2}{4}\right) = \left(\frac{w_u}{p_u}\right) = \left(\frac{825}{300}\right) = 2.74\ \text{m}^2$$

$$D_f = \sqrt{\frac{(4 \times 2.75)}{\pi}} = 1.87\ \text{m}$$

Adopt diameter of footing $= D_f = 2$ m

$$\text{Upward soil pressure} = p_u = \left(\frac{750 \times 4}{\pi \times 2^2}\right) = 238.8\ \text{kN/m}^2 < 300\ \text{kN/m}^2$$

Hence, the diameter of the footing is adequate to resist the loads. Referring to Fig. 10.27, centre of gravity of quadrant of footing (bb'c'c) from 'O' is R_x and is computed as,

$$R_x = 0.6\left[\frac{R^2 + r^2 + Rr}{R + r}\right] = 0.6\left[\frac{1000^2 + 150^2 + (1000 \times 150)}{(1000 + 150)}\right] = 610\ \text{mm}$$

Upward load on area (b b'c'c) is expressed as W_q and computed as,

$$W_q = \left[\frac{\pi(1 - 0.15^2)238.8}{4}\right] = 183\ \text{kN}$$

c) Bending Moment

Maximum bending moment at the face of the column quadrant is computed as,

$$M_u = 183\,(0.61 - 0.15) = 84.2\ \text{kN.m}$$

Breadth of footing at column face (for one quadrant c'b') =[(p ×300/4)] = 235 mm

$$\text{Depth of footing} = d = \sqrt{\frac{M_u}{0.138 f_{ck} b}} = \sqrt{\frac{84.2 \times 10^6}{(0.138 \times 20 \times 235)}} = 360\ \text{mm}$$

Depth required from shear considerations will nearly 1.5 times that for moment computations.

Hence adopt effective depth = d = 525 mm and overall depth = D = 600 mm

d) Reinforcements

$$M_u = (0.87 f_y A_{st} d)\left[1 - \frac{A_{st} f_y}{b d f_{ck}}\right]$$

$$(84.2 \times 10^6) = (0.87 \times 415 A_{st} \times 525)\left[1 - \frac{415 A_{st}}{(235 \times 525 \times 20)}\right]$$

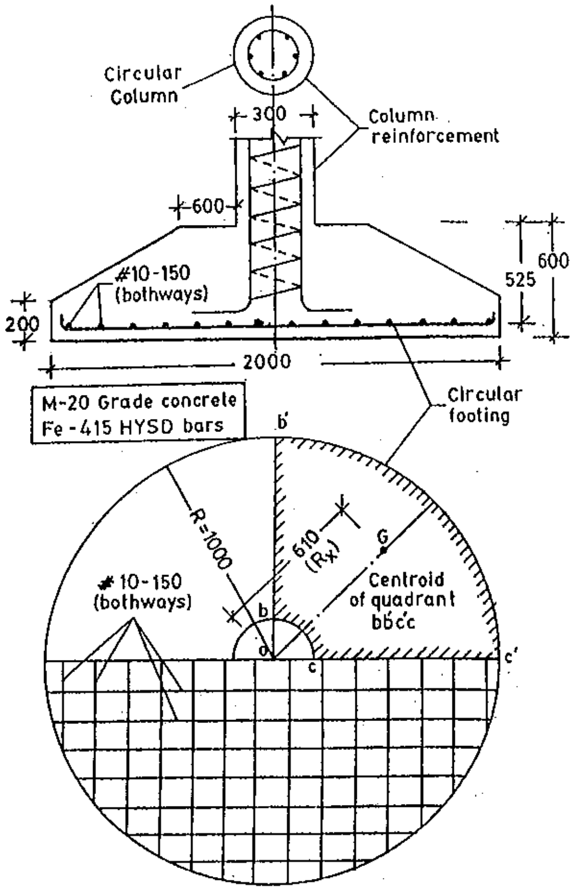


Fig. 10.27 Reinforcement Details in Circular Footing

Solving $A_{st} = 484\ \text{mm}^2$
$A_{st,min} = (0.0012 \times 235 \times 600) = 169\ \text{mm}^2$
Provide 10 mm diameter bars at 150 mm centres (A_{st} = 524 mm^2) both ways.

d) Check for Shear stress

Ultimate Shear force at a distance of 0.525 m from the face of column is given by,

$$V_u = 238.8\,(2^2 - 1.35^2)\,(\pi/4) = 408\ \text{kN}$$

$$\text{Shear per metre width of perimeter} = \left(\frac{408}{\pi \times 1.35}\right) = 96\ \text{kN}$$

$$\tau_v = \left(\frac{V_u}{bd}\right) = \left(\frac{96 \times 10^3}{10^3 \times 525}\right) = 0.18\ \text{N/mm}^2$$

$$\left(\frac{100A_{st}}{bd}\right) = \left(\frac{100 \times 754}{10^3 \times 525}\right) = 0.143$$

Refer Table-19 of IS: 456-2000 and read out the permissible shear stress in concrete

$$(k_s\,\tau_c) = (1 \times 0.28) = 0.28\ \text{N/mm}^2 > 0.18\ \text{N/mm}^2$$

Hence, the shear stresses are within safe permissible limits.

e) Reinforcement Details

The reinforcement details in the circular footing are shown in Fig. 10.27.

10.8.6 Design Example

Design a combined column footing with a strap beam for two reinforced concrete columns of size 300 mm by 300 mm spaced 4m c/c and each supporting a service axial load of 500 kN. The safe bearing capacity of soil at site is 150 kN/m^2. Adopt M-20 grade concrete and Fe-415 HYSD bars.

a) Data

Size of columns = 300 mm by 300 mm
Spacing of columns = 4 m
Working load on each column = 500 kN
Safe bearing capacity of soil = 150 kN/m^2
M-20 grade concrete and Fe-415 HYSD bars.

b) Design loads and Stresses

Design ultimate load on each column = (1.5 × 500) = 750 kN
Ultimate bearing capacity of soil = (1.5 × 150) = 225 kN/m^2
$f_{ck} = 20\ \text{N/mm}^2$, $f_y = 415\ \text{N/mm}^2$

c) Loads on Footing

Total load on both the columns = $(2 \times 750) = 1500$ kN
Self weight of footing (10%) = 150 kN
Total ultimate load = P_u = 1650 kN

d) Size of Footing

Area of footing = (1650/225) = 7.33 m^2
Adopt a footing of size 6m long by 1.5 m wide.
Adopt width of strap beam = b = 400 mm

e) Design of Footing

Soil pressure = $p_u = \left(\dfrac{1500}{6 \times 1.5}\right) = 166.6 \text{ kN/m}^2 < 225 \text{ kN/m}^2$

Cantilever projection of footing = 0.5 (1.5–0.4) = 0.55 m
Ultimate design moment = $M_u = (0.5\, p_u\, L^2) = (0.5 \times 166.6 \times 0.55^2) = 25.2$ kN.m

Effective depth of footing = $d = \sqrt{\dfrac{M_u}{0.138 f_{ck} b}} = \sqrt{\dfrac{25.2 \times 10^6}{0.138 \times 20 \times 10^3}} = 96$ mm

But the depth based on shear considerations is nearly double than that due to moment considerations.
Hence adopt effective depth = d = 250 mm and overall depth = D = 300 mm

$$M_u = (0.87\, f_y A_{st} d)\left[1 - \frac{A_{st} f_y}{bd\, f_{ck}}\right]$$

$$(25.2 \times 10^6) = (0.87 \times 415 A_{st} \times 250)\left[1 - \frac{415 A_{st}}{(10^3 \times 250 \times 20)}\right]$$

Solving A_{st} = 287 mm^2
But $A_{st,min} = (0.0012 \times 1000 \times 300) = 360$ mm^2
Adopt 10 mm diameter bars at 200 mm centres (A_{st} = 393 mm^2) as main and distribution reinforcement.

f) Check for Shear stress

Design shear force = $V_u = (0.55 - 0.25)\, 166.6 = 50$ kN

$$\tau_v = \left(\frac{V_u}{bd}\right) = \left(\frac{50 \times 10^3}{10^3 \times 250}\right) = 0.2 \text{ N/mm}^2$$

$$\left(\frac{100 A_{st}}{bd}\right) = \left(\frac{100 \times 393}{10^3 \times 250}\right) = 0.157$$

Refer table-19 (IS: 456-2000) and read out the shear strength of concrete as,

$$(k_s\, \tau_c) = (1 \times 0.28) = 0.28 \text{ N/mm}^2 > 0.2 \text{ N/mm}^2$$

Hence, shear stresses are within safe permissible limits.

g) Design of Strap Beam

Design ultimate load on beam = $w_u = (1.5 \times 166.6) = 250$ kN/m
Neglecting the small cantilever portion of the beam,

$$M_u = 0.125\, w_u L^2 = (0.125 \times 250 \times 4^2) = 500 \text{ kN.m}$$
$$V_u = 0.5\, w_u L = (0.5 \times 250 \times 4) = 500 \text{ kN}$$

Depth of strap beam computed based on shear will be greater than that based on moment.
Assuming $\tau_c = 1.2$ N/mm^2

$$d = \left(\frac{V_u}{b \tau_c}\right) = \left(\frac{500 \times 10^3}{400 \times 1.2}\right) = 1042 \text{ mm}$$

Adopt effective depth = d = 1150 mm and overall depth = D =1200 mm

$$M_u = (0.87\, f_y A_{st} d)\left[1 - \frac{A_{st} f_y}{bd\, f_{ck}}\right]$$

$$(500 \times 10^6) = (0.87 \times 415 A_{st} \times 1150)\left[1 - \frac{415 A_{st}}{(400 \times 1150 \times 20)}\right]$$

Solving A_{st} = 1290 mm^2
Provide 4 bars of 22 mm diameter (A_{st} = 1520 mm^2)
Shear stress = N/mm^2

$$\tau_v = \left(\frac{V_u}{bd}\right) = \left(\frac{500 \times 10^3}{400 \times 1150}\right) = 1.09 \text{ N/mm}^2$$

$$\left(\frac{100 A_{st}}{bd}\right) = \left(\frac{100 \times 1520}{400 \times 1150}\right) = 0.33$$

Refer table-19 of IS:456-2000 and read out the permissible shear stress as

$$\tau_c = 0.40 \text{ N/mm}^2 < \tau_v$$

Hence, shear reinforcements are required to resist the balance shear force computed as

$$V_{us} = [500 - (0.4 \times 400 \times 1150)\, 10^{-3}] = 316 \text{ kN}$$

Using 8 mm diameter 4 legged stirrups, the spacing is

$$S_v = \left(\frac{0.87 \times 415 \times 4 \times 50 \times 1150}{316 \times 10^3}\right) = 262 \text{ mm}$$

Adopt 8 mm diameter 4 legged stirrups at 250 mm centres in the strap beam.
Side face reinforcement of 0.1 percent of web area as specified in IS:456 code is provided.

h) Reinforcement Details

The details of reinforcements in the combined footing and strap beam are shown in Fig. 10.28.

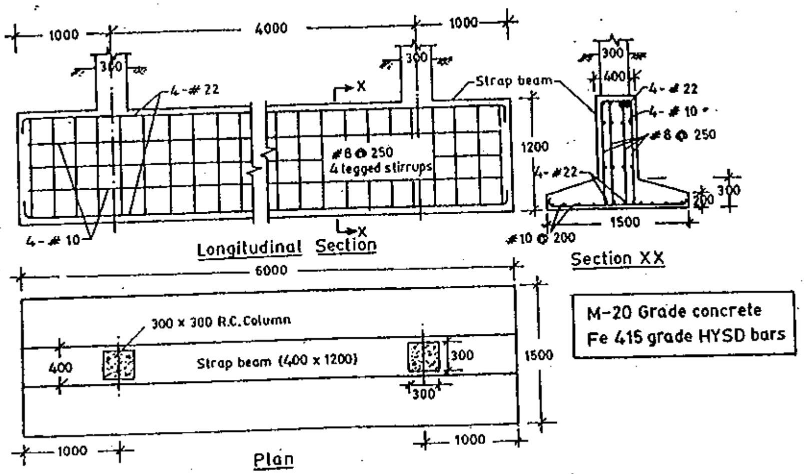


Fig. 10.28 Reinforcement Details in Combined Footing

10.8.7 Design Example

The column section of a reinforced concrete portal frame is 450 mm wide by 600 mm deep at the base. The column section transmits an axial service load of 160 kN together with an uniaxial moment of 52 kN.m about the major axis to the foundation . Design a suitable footing for the column of the portal frame assuming the safe bearing capacity of the soil as 200 kN/m². Adopt M-20 grade concrete and Fe-415 HYSD bars.

a) Data

Axial service load = 160 kN
Uniaxial moment = M = 52 kN.m
Size of column = 450 mm by 600 mm
S.B.C. of the soil = 200 kN/m²
M-20 grade concrete and Fe-415 HYSD bars

b) Size of Footing

Load on column = 160 kN
Self weight of footing(10%) = 20 kN
Total load on soil = P = 180 kN
Service load moment = M = 52 kN.m
Eccentricity = $e = (M/P) = [(52 \times 10^6)/(180 \times 10^3)] = 290$ mm
To avoid tension in the foundation the total breadth of foundation footing is expressed as,

$$b = 6e = (6 \times 290) = 1740 \text{ mm}$$

Hence, provide a foundation of size 1m by 2 m

c) Pressure Distribution at Base

Intensity of maximum pressure =

$$p = \left(\frac{2P}{A}\right) = \left(\frac{2 \times 180}{1 \times 2}\right) = 180 \text{ kN/m}^2 < 200 \text{ kN/m}^2$$

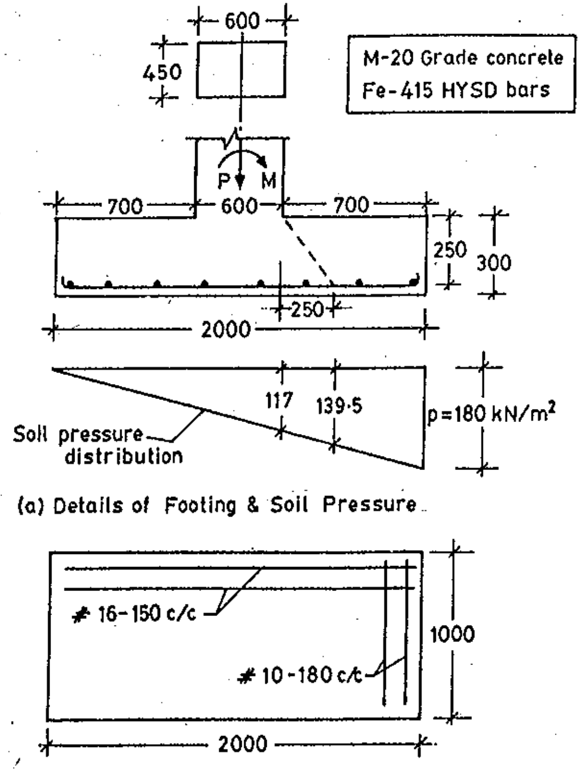
Hence, the soil pressure is within the safe permissible limits. The distribution of soil pressure below the footing is shown in Fig. 10.29(a).
If p' = soil pressure below the footing at the face of the column,

$$p' = \left(\frac{1.3 \times 180}{2}\right) = 117 \text{ kN/m}^2$$

Total pressure on the cantilever portion of the footing is expressed as,

$$P_s = \left(\frac{180+117}{2}\right)0.7 = 104 \text{ kN}$$

acting at a distance of 0.4 m from column face.
Hence, the bending moment at column face $= M = (104 \times 0.4) = 42$ kN.m
Factored bending moment $= M_u = (1.5 \times 42) = 63$ kN.m



(a) Details of Footing & Soil Pressure

(b) Reinforcements in Footing

Fig. 10.29 Footing Subjected to Axil Load And Moment

d) Thickness of Footing slab

Effective depth required for balanced section is computed as,

$$d = \sqrt{\frac{M_u}{0.138 f_{ck} b}} = \sqrt{\frac{63\times10^6}{(0.138\times20\times10^3)}} = 151 \text{ mm}$$

Depth required from shear considerations will be more than that required from moment considerations.
Hence, adopt effective depth $= d = 250$ mm
And overall depth $= D = 300$ mm

e) Reinforcements

$$M_u = 0.87 f_y A_{st} d\left[1 - \frac{A_{st} f_y}{b d f_{ck}}\right] = (0.87\times415 A_{st}\times250)\left[1-\frac{415A_{st}}{(10^3\times250\times20)}\right]$$

Solving $A_{st} = 750$ mm²
Provide 16 mm diameter bars at 150 mm centres ($A_{st} = 1341$ mm²)
Distribution reinforcement $= 0.12$ percent $= (0.0012 \times 300 \times 10^3) = 360$ mm²
Provide 10 mm diameter bars at 180 mm centres ($A_{st} = 436$ mm²)

f) Check for Shear stress

Factored shear force acting at a distance of 250 mm from the face of the column is given by [Refer Fig. 10.29(a)]

$$V_u = 1.5\left[\frac{180+139.5}{2}\right]0.45 = 108 \text{ kN}$$

$$\therefore \quad \tau_v = \left(\frac{V_u}{bd}\right) = \left(\frac{108\times10^3}{10^3\times250}\right) = 0.43 \text{ N/mm}^2$$

$$\left(\frac{100A_{st}}{bd}\right) = \left(\frac{100\times1341}{1000\times250}\right) = 0.53$$

Refer Table-19 (IS:456-2000) and read out the permissible shear stress as,

$$(k_s\,\tau_c) = (1\times0.48) = 0.48 \text{ N/mm}^2 > \tau_v$$

Hence, shear stresses are within safe permissible limits.

g) Reinforcement details

The details of reinforcements in the footing slab are shown in Fig. 10.29(b).

10.9 EXAMPLES FOR PRACTICE

1) Design the longitudinal and lateral reinforcements in a rectangular reinforced concrete column of size 300 mm by 600 mm to support a factored axial load of 1400 kN. The column has an unsupported length of 3 m and is braced against side sway in both directions. Adopt M-20 grade concrete and Fe-415 HYSD bars.

2) Design the reinforcements in a circular column of diameter 350 mm with helical ties to support a factored load of 1600 kN. The column has an unsupported length of 3.5 m and is braced against side sway. Adopt M-25 grade concrete and Fe-500 grade reinforcements.

3) Design a suitable reinforced concrete column of square section to support an axial service load of 1000 kN. The size of the column is 400 mm by 400 mm. Design a suitable footing for the column. The safe bearing capacity of the soil at site is 200 kN/m^2. Adopt M-20 grade concrete and Fe-415 HYSD bars. Sketch the details of reinforcements in the column and footing.

4) Design the longitudinal reinforcements in a rectangular reinforced concrete column of size 300 mm by 600 mm subjected to a factored load of 1500 kN and a factored moment of 300 kN.m with respect to the major axis. Adopt M-20 grade concrete and Fe-415 HYSD bars.

5) A multi-storeyed building with a floor-to-floor height of 4 m and a plan area of 18 m by 30 m has the columns spaced at 6 m intervals in both directions. The columns have a size of 400 mm by 400 mm with M-30 grade concrete and all the primary beams are of size 300 mm wide by 600 mm deep with M-25 grade concrete. Calculate the effective length of the typical lower storey column assuming a total distributed load of 50 kN/m^2 from all the floors above the ground floor.

6) Design a short circular column of diameter 350 mm to support a factored axial load of 1000 kN, together with a factored moment of 100 kN.m. Adopt M-20 grade concrete and Fe-415 HYSD bars.

7) Design the longitudinal and lateral reinforcements in a short column of size 300 mm by 500 mm subjected to an ultimate axial load of 1200 kN with ultimate moments of 80 and 60 kN.m about the major and minor axis respectively. Adopt M-25 grade concrete and Fe-415 HYSD bars.

8) A short reinforced concrete column located at the corner of a multi-storeyed building is subjected to an axial factored load of 1600 kN together with factored moments of 60 and 40 kN.m acting in

perpendicular planes. The size of the column is fixed as 400 mm by 400 mm. Adopting M-20 grade concrete and Fe-415 HYSD bars design suitable reinforcements in the corner column.

9) A reinforced concrete braced column of size 300 mm by 400 mm is to be designed to support a factored axial load of 1500 kN together with factored moments of 60 and 40 kN.m with respect to the major and minor axis respectively at the top end. Assume that the column is bent in double curvature in both directions and are subjected to moments at the bottom end equal to 50 percent of the corresponding moments at top.Also assume that the unsupported length of the column as 7 m and an effective length ratio of 0.85 in both directions. Adopt M-30 grade concrete and Fe-415 HYSD bars. Design suitable reinforcements in the column.

10) Design a suitable footing for a reinforced concrete column of size 300 mm by 500 mm supporting a factored axial load of 1500 kN. Assume the safe bearing capacity of the soil as 200 kN/m^2. Adopt M-20 grade concrete and Fe-415 HYSD bars. Sketch the details of reinforcements in the footing.

11) Design a combined footing for the two columns of a multistorey building. The columns of size 400 mm by 400 mm transmit a working load of 800 kN each and they are spaced at 5 m centres. The safe bearing capacity of soil at site is 200 kN/m^2. Adopt M-20 grade concrete and Fe-415 grade reinforcement. Sketch the details of reinforcements in the combined footing.

12) Design a trapezoidal footing for the two columns A and B transmitting service loads of 800 kN and 1600 kN respectively. The column A is 400 mm square and column B is 600 mm square in size and they are spaced at 5 m centres. The property line is 300 mm beyond the face of column A. The safe bearing capacity of soil at site is 150 kN/m^2. Adopt M-20 grade concrete and Fe-415 HYSD bars.

13) Design a strap footing combined foundation for two columns C and D spaced 6 m apart between their centres. Column C is 400 mm square and supports a service load of 500 kN. Column D is 500 mm square and supports a service load of 1200 kN. The safe bearing capacity of the soil at site is 200 kN/m^2. Adopt M-20 Grade concrete and Fe-415 grade HYSD bars.

14) Design an isolated footing for a column 350 mm by 600 mm reinforced with 6 bars of 25 mm diameter and is subjected to a service load of 600 kN and a service moment of 80 kN.m with respect to the major axis. At the column base. The safe bearing capacity of soil is 200 kN/m^2. Adopt M-20 grade concrete and Fe-415 HYSD bars.

CHAPTER 11

Limit State Design of Retaining Walls

11.1 INTRODUCTION

Retaining walls are generally used to retain earth or such materials to maintain unequal levels on its two faces. The soil on the back face is at a higher level and is called the backfill. Retaining walls are extensively used in the construction of basements below ground level, wing walls of bridge and to retain slopes in hilly terrian roads. The retaining wall prevents the retained earth to exert a lateral pressure on the wall tending to bend, overturn and slide the retaining wall. Retaining walls should be designed to resist the lateral earth pressure from the sides and the soil pressure acting vertically on the footing slab integrally built with the vertical slab.

Gravity walls of stone masonry were generally used in the earlier days to retain earthen embankments. The thickness of the masonry walls increased with the height of the earth fill. The advent of reinforced concrete has resulted in thinner retaining walls of different types resulting in considerable reduction of costs coupled with improved aesthetics.

11.2 TYPES OF RETAINING WALLS

a) Cantilever retaining wall

The most common and widely used retaining wall is of the cantilever type comprising the following structural parts (Fig. 11.1)

i) Vertical stem resisting earth pressure from one side and the slab bends like a cantilever. The thickness of the slab is larger at the bottom and gradually decreases towards the top in proportion to the variation in soil pressure.

ii) The base slab forming the foundation comprises the heel slab and the toe slab. The heel slab acts as a horizontal cantilever under the combined action of the weight of retained earth from the top and the soil pressure acting from the soffit. The toe slab also acts as a cantilever under the action of the resulting soil pressure acting upward. The stability of the wall is maintained by the weight of the earth fill on the

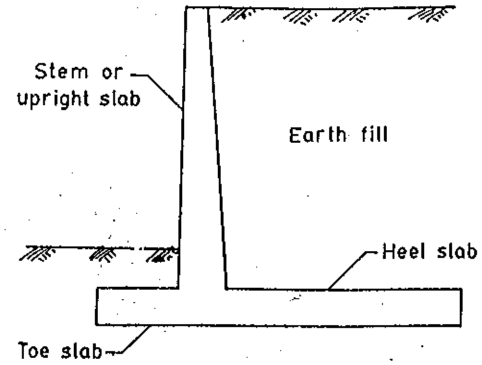


Fig. 11.1 Cantilever Retaining Wall

heel slab together with the self weight of the structural elements of the retaining wall. Cantilever type retaining walls are adopted for small to medium heights up to 5m.

b) Counterfort retaining wall

For larger heights exceeding 5 m of earth fill, the bending moment developed in the stem, heel and toe slabs are very large resulting in larger thickness of the structural elements tending to be uneconomical. Hence, counterfort type retaining walls are adopted for larger heights. Fig. 11.2 shows a typical counterfort type retaining wall consisting of a stem or upright slab, toe slab, heel slab and the counterforts which subdivide the vertical slab and they behave as vertical cantilever beams of tee-section with varying width. The stem and heel slab are effectively fixed to the counterforts so that the stem bends horizontally between the counerforts due to lateral earth pressure. Consequently the thickness of the stem and the heel slab is considerably reduced due to the reduction of moment due to the fixity of these slabs between the counterforts.

11.3 FORCES ACTING ON RETAINING WALLS

The various forces acting on retaining wall are shown in Fig. 11.3 and detailed as follows:

a) Lateral earth Pressure

The lateral forces due to earth pressure is the major force acting on the

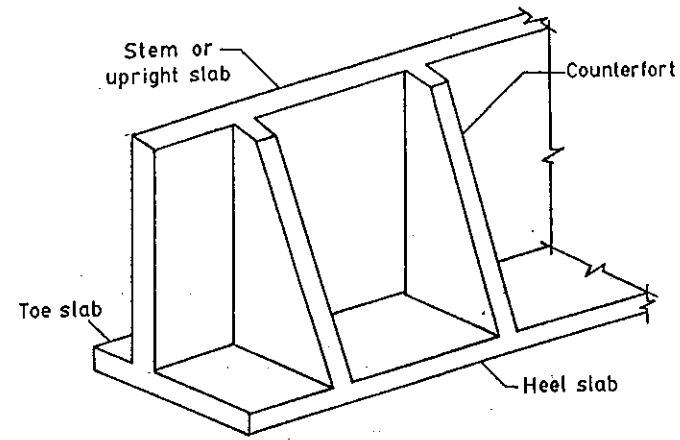


Fig. 11.2 Counterfort Retaining Wall

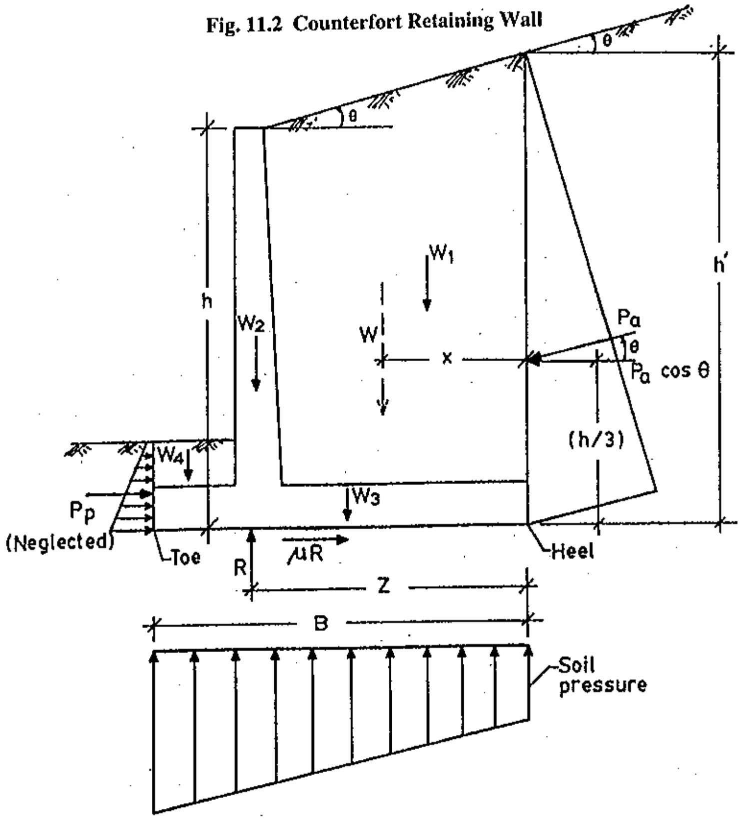


Fig. 11.3 Forces Acting on Retaining Wall

retaining wall. The magnitude of the force is expressed by the relation.

$$P_a = C_a \gamma_e (h')^2/2$$

Where C_a = Coefficient of active earth pressure
γ_e = Density of Soil
h' = height of the back fill measured vertically above the heel (Fig 11.3)

The coefficient of earth pressure C_a depends upon the angle of shearing resistance (angle of repose) 'ϕ' and the inclination or slope of the back fill to the horizontal expressed as 'θ'.

The general relation for the coefficient of active earth pressure based on Rankine's theory is given by the relation.

$$C_a = \left[\frac{\cos\theta - \sqrt{\cos^2\theta - \cos^2\phi}}{\cos\theta + \sqrt{\cos^2\theta - \cos^2\phi}}\right]\cos\theta$$

For the case of a level backfill, $\theta = 0$ and $h' = h$

Hence, $$C_a = \left\{\frac{1 - \sin\phi}{1 + \sin\phi}\right\}$$

The coefficient of passive earth pressure is given by the relation,

$$C_p = \left\{\frac{1 + \sin\phi}{1 - \sin\phi}\right\}$$

The magnitude of the earth pressure P_a acts at one- third the height of the back fill as shown in Fig. 11.3. The force P_p developed due to the passive pressure acts on the toe side of the retaining wall and its magnitude being very small (due to the small height of earth fill on toe slab) is generally neglected in the design computations.

Due to the construction of buildings on a level back fill or due to the movement of vehicles near the top of the retaining wall, gravity loads acting can be considered as uniformly distributed load. This additional load of w_s kN/m^2 can be treated as statically equivalent to an additional (fictitious) height of soil $h_s = (w_s / \gamma_e)$ acting over the level surface. The force developed due to the effect of surcharge on a level back fill together with the other forces are shown in Fig. 11.4.

The total force due to active earth pressure is expressed as,

$$P_a = P_{a1} + P_{a2}$$

Where $P_{a1} = C_a . w_s . h = C_a . h_s . \gamma_e\, h$ and $P_{a2} = (C_a . \gamma_e . h^2)/2$

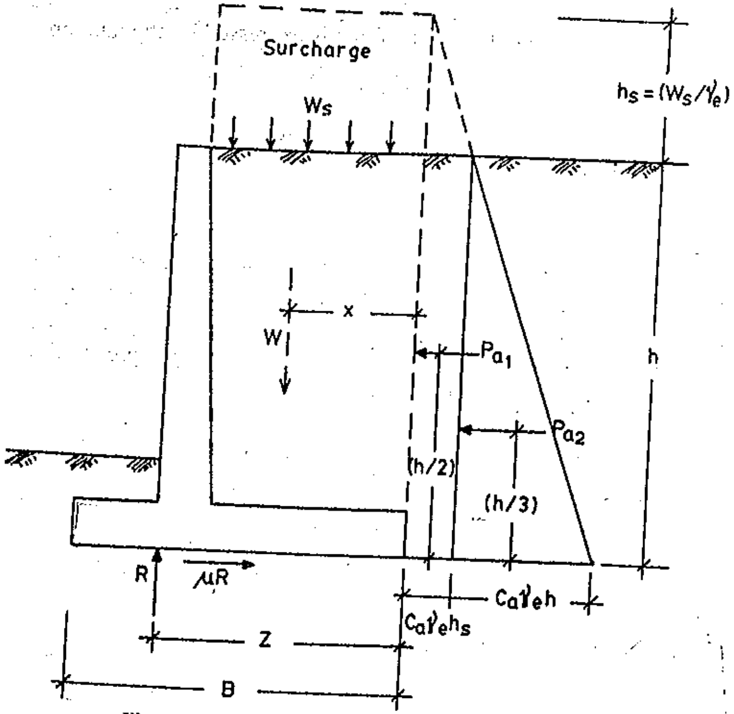


Fig. 11.4 Effective Surcharge on a Level Backfill

The forces P_{a1} and P_{a2} act at a height of $h/2$ and $h/3$ respectively above the heel.

b) The vertical forces include the weight of soil, weight of stem, heel, toe slab and the soil fill above toe slab.

c) The soil pressure developed to resist the earth pressure and other vertical forces acting upwards from heel to toe. The pressure distribution at base is obtained by stability calculations comprising the equilibrium condition of vertical forces and moments.

11.4 STABILITY REQUIREMENTS

The design of retaining walls should conform to the stability requirements specified in clause-20 of IS: 456 which includes overturning and sliding. The factor of safety against overturning and sliding should be not less than 1.4 since the stabilizing forces are due to dead loads. The code specifies

that these stabilizing forces should be factored by a value of 0.9 in calculating the factor of safety.

Hence, the factor of safety can be expressed by the relation,

$$\text{F.S.} = \left[\frac{0.9(\text{Stabilising Force or Moment})}{\text{Destabilising Force or Moment}}\right] \geq 1.4$$

a) Overturning

The retaining wall overturns with the toe as the centre of rotation. When the structure overturns, the upward reaction R will not act and the expressions for the overturning moment M_o and the stabilizing moment M_s depend only on the lateral earth pressure and the geometry of the retaining wall.

Considering the retaining wall with sloping back fill (Fig. 11.3), the expressions for the overturning and stabilizing moment are,

$$M_o = (P_a \cos\theta)(h'/3) = [C_a \gamma_e (h')^3/6] \cos\theta$$

$$M_s = W(B - x) + (P_a \sin\theta)B$$

Where $W = W_1 + W_2 + W_3 + W_4$

And W_1 = weight of earth fill

W_2 = weight of stem

W_3 = weight of heel and toe slab

W_4 = weight of earth fill over toe slab

And x = distance of W from the heel

B = Base width of slab

The factor of safety against overturning is expressed as

$$(\text{F.S})_{\text{overturning}} = \left(\frac{0.9 M_s}{M_o}\right) \geq 1.4$$

b) Sliding

The resistance developed against sliding of the retaining wall is mainly due to the frictional forces generated between the base slab and the supporting soil expressed as

$$F = \mu R$$

Where $R = W$ = Resultant soil pressure acting on the base slab and

μ = Coefficient of friction between concrete and soil (Value of μ varies in the range of 0.35 for silt to about 0.60 for rough rock)

Hence, the factor of safety against sliding is computed by the relation

$$(F.S)_{sliding} = \left[\frac{0.9\mu W}{P_a \cdot \cos\theta}\right] \geq 1.4$$

c) Shear key

In the case of back fills with surcharge, the active pressures are relatively high and consequently the required factor of safety against sliding by the frictional forces above will not be sufficient. In such cases, it is advantageous to provide a shear key projecting below the base slab as shown in Fig. 11.5.

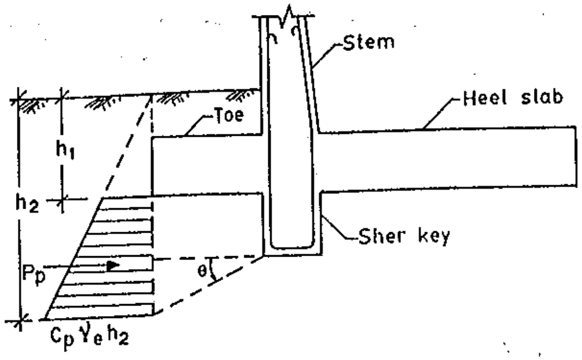


Fig. 11.5 Passive Pressure Due to Shear Key

The passive resistance developed against sliding is computed as

$$P_p = C_p \gamma_e (h_2^2 - h_1^2)/2$$

It is advantageous to provide a shear key just below the stem so that the reinforcements can be extended into the shear key.

The enhanced factor of safety against sliding by the use of the shear key can be expressed as.

$$(F.S)_{sliding} = \left[\frac{0.9\mu W + P_p}{P_a \cdot \cos\theta}\right] \geq 1.4$$

11.5 PROPORTIONING AND DESIGN OF RETAINING WALLS

11.5.1 Preliminary dimensioning of Stem and Base slab

a) Width of Base slab

An economical design of the retaining wall can be obtained by proportioning the base slab so as to align the vertical soil reaction R at the base with

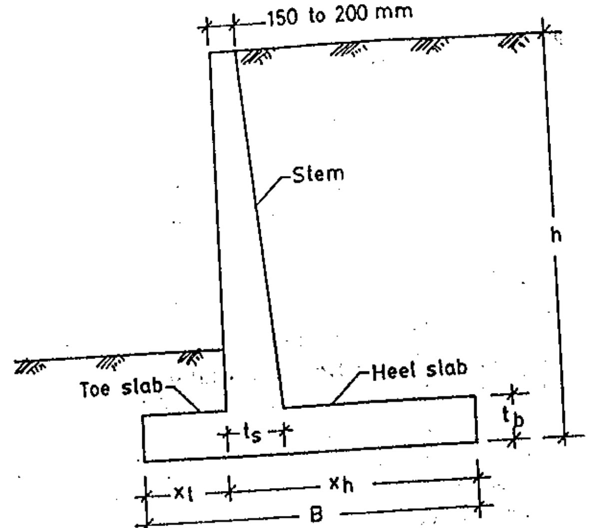


Fig. 11.6 Dimensioning of Retaining Wall Ellements

the front face of stem or up right slab. Referring to Fig. 11.6,

Let h = height of earth fill from the soffit of base slab
B = width of base slab
x_h = width of heel slab
x_t = width of toe slab
C_a = coefficient of active earth pressure.

Assuming the soil pressure distribution as triangular with maximum pressure at the toe and zero at the heel, the resultant vertical pressure will pass through the middle third point. For economical design, the soil pressure resultant should line up with the front face of the stem.

Adopting this principle, Unnikrishna Pillai and Devadas Menon[88] have developed an expression for the minimum width of heel slab as,

$$x_h = h\sqrt{C_a/3}$$

The effect of surcharge or sloping back fill can be included by replacing 'h' with $h+h_s$ or h' respectively. With known values of 'h' and C_a, x_h can be computed.

Hence, base width = B = 1.5 x_h

And x_t = (1/3) B so that x_h = (2//3) B

b) Thickness of base slab and stem

The preliminary computations of the thickness of base slab is expressed as,

$$t_b = (h/12) \quad \text{or} \quad 0.08h$$

But not less than 300mm
Thickness of stem at bottom is assumed as,

$$t_s = t_b$$

The stem thickness is gradually decreased to a minimum value of 150 to 200mm at top. The front face of the stem is maintained vertical.

c) Design of stem, heel, and toe slabs

The stem, heel and toe slabs, structurally behave as cantilever slabs and deform as shown in Fig. 11.7. Hence, the critical sections XX, YY and ZZ shown in figure have to be designed to resist the factored moment and shear forces with a load factor of 1.5. Usually shear is not a critical design factor and the flexural reinforcement is provided near the tension face in the slabs with a clear cover of 50 mm. The reinforcements in the stem may be curtailed in stages for economy. Temperature and shrinkage reinforcement of 0.12 percent of the gross cross section should be provided transverse to the main reinforcement. Normal vertical and horizontal reinforcement should be provided near the front face of stem and also at the bottom face of heel slab and top face of toe slab.

d) Design of Counterforts

The counter forts should be integrally built with proper ties with stem and heel slab so that the horizontal forces due to earth fill is resisted by the tension steel provided in the counterforts. In a similar way the vertical forces on base slab are resisted by the vertical ties in the counterfort.

The counterfort is designed as a vertical cantilever, fixed at base. Since the stem acts integrally with the counterfort, the effective section resisting the cantilever moment is a flanged section, with the flange under compression. The counterforts are designed as tee beams with the depth of the section varying linearly from the top to the bottom where the section is maximum to resist the maximum moments.

The stem is designed as a continuous slab spanning between the counterforts with negative and positive moments at supports and mid span respectively. The heel and toe slabs are designed for soil pressure as continuous and cantilever slabs receptively.

11.6 DESIGN EXAMPLES

11.6.1 Design a cantilever retaining wall to retain an earth embankment

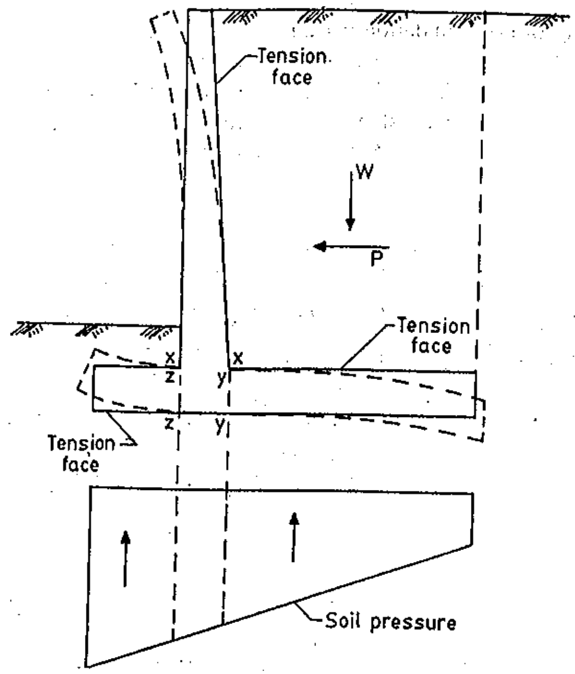


Fig. 11.7 Deformation Characteristics of Retaining Wall

4m high above ground level. The density of earth is 18 kN/m^3 and its angle of repose is 30°. The embankment is horizontal at top. The safe bearing capacity of the soil may be taken as 200 kN/m^2 and the coefficient of friction between soil and concrete is 0.5. Adopt M-20 grade concrete and Fe-415 HYSD bars.

a) Data

Height of embankment above ground level = 4m
Density of soil = 18 kN/m^3
Angle of repose = 30°
S.B.C. of soil= 200 kN/m^2
Coefficient of friction = 0.5
Materials : M-20 grade concrete and Fe-415 HYSD bars

b) Dimensions of the Retaining wall

Minimum depth of Foundation $= \left(\frac{p}{\gamma_e}\right)\left[\frac{1-\sin\phi}{1+\sin\phi}\right]^2 = \left(\frac{200}{18}\right)\left(\frac{1}{3}\right)^2 = 1.2$ m

Provide depth of foundation = 1.2 m
Over all depth of wall (h) = (4 + 1.2) = 5.2 m
Thickness of base slab = (h/12) or 0.08h = (5200/12) = 433 mm
Adopt thickness of base slab = t_b = 450 mm
∴ Height of stem = h_s = (5.2 − 0.45) = 4.75 m

Coefficient of active earth pressure is computed as,

$$C_a = \left\{\frac{1-\sin\phi}{1+\sin\phi}\right\} = \left\{\frac{1-\sin 30°}{1+\sin 30°}\right\} = 0.333$$

Width of heel slab = $x_h = h\sqrt{C_a/3} = 5.2\sqrt{(0.333/3)} = 1.73$ m
Width of base slab = $B = 1.5\,x_h = (1.5 \times 1.73) = 2.6$ m
Adopt B = 3 m
Width of toe slab = 1m and Width of heel slab = 2 m (Refer Fig.11.8)

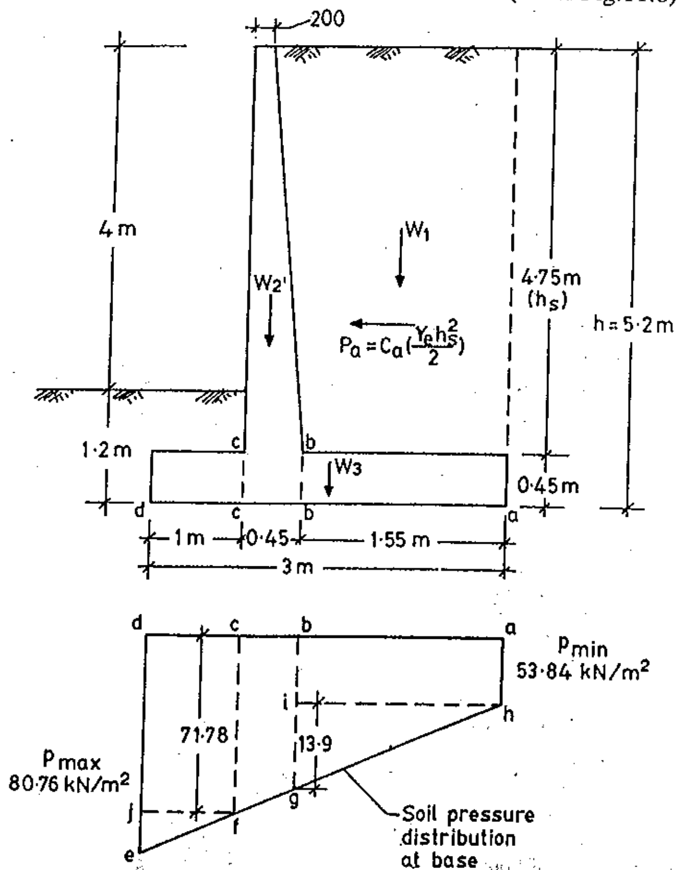


Fig. 11.8 Forces Acting on Retaining Wall

c) Design of stem

Height of stem = h_s = 4.75 m
Thickness of stem = thickness of base slab = 450 mm at bottom tapering to 200 mm at top.

$$M_u = 1.5C_a\left(\frac{\gamma_e h_s^3}{6}\right) = (1.5 \times 0.333 \times 18 \times 4.75^3)/6 = 161 \text{ kN.m}$$

Providing a cover of 50 mm, d = 400mm

$$\left(\frac{M_u}{bd^2}\right) = \left(\frac{161 \times 10^6}{10^3 \times 400^2}\right) = 1.006$$

Refer Table -2 (SP: 16) and read out the percentage of reinforcement as

$$p_t = 0.30 = \left(\frac{100A_{st}}{bd}\right) \quad \text{and} \quad \therefore \quad A_{st} = \left(\frac{0.30 \times 10^3 \times 400}{100}\right) = 1200 \text{ mm}^2\text{/m}$$

Provide 16mm diameter bars at 160mm centres at the bottom for a height of 1.75m and gradually increased spacing of 200mm for the next one metre and as 300mm centres for the remaining height.

Distribution Reinforcement = 0.12 percent = $(0.0012 \times 10^3 \times 450) = 540$ mm^2/m.

Provide 8mm diameter bars at 180mm centres in the horizontal direction at both front and back faces. Gradually the spacing is increased to 250 and 300 mm centres towards the top of the stem.

d) Stability computations (Pressure distribution at base)

The overall dimensions of the retaining wall is shown in Fig. 11.8 the stability calculations are shown in Table 11.1.

Table 11.1 Stability calculations for one metre run of wall

Loads	Magnitude of Load (kN)	Distance from 'a' (m)	Moment (kN.m)
$W_1 = (1.55 \times 4.75 \times 18)$	132.50	0.78	103.35
$W_2 = (0.2 \times 4.75 \times 25)$	22.80	1.65	37.62
$+ (0.5 \times 0.25 \times 4.75 \times 25)$	14.25	1.83	26.07
$W_3 = (3 \times 0.45 \times 25)$	32.40	1.50	48.60
Moment due to earth pressure $M = C_a\,(\gamma_e.h_s^3)/6$ $= 0.333(18 \times 4.75^3)/6$			
Total. $\Sigma W =$	201.95	$\Sigma M =$	322.81

Distance of point of application of resultant from point '*a*' = $Z = (\Sigma M/\Sigma W)$

$Z = [(322.81)/(201.95)] = 1.6$ m

Eccentricity = $e = (Z - 0.5B) = (1.6 - 0.5 \times 3) = 0.1$ m

$(B/6) = (3/6) = 0.5$ m $\quad \therefore \; e = 0.1 \text{ m} < (B/6)$

Hence, the soil pressure is compressive from *a* to *d*

Maximum and minimum soil pressure at '*d*' and '*a*' respectively are computed as

$$p_{(max,\,min)} = \left(\frac{W}{B}\right)\left[1 \pm \frac{6e}{B}\right] = \left(\frac{201.95}{3}\right)\left[1 \pm \frac{(6\times 0.1)}{3}\right]$$

$\therefore$ $\quad p_{max} = 80.76$ kN/m² (at *d*).... toe

$\quad p_{min} = 53.84$ kN/m² (at *a*).... heel

Maximum soil pressure at toe is less than the safe bearing capacity of soil given by (200 kN/m²). Hence, soil pressure is within safe permissible limits.

e) Design of heel slab

Maximum bending moment in the heel slab is computed at section '*bb*' using the moment computations shown in Table 11.2 (Refer Fig 11.8).

Table 11.2 Moment calculations for 1m length of heel slab

Loads	Magnitude of Load (kN)	Distance from 'b' (m)	Moment (kN.m)
$W_1 = (1.55 \times 4.75 \times 18)$	132.50	0.775	102.68
Self weight of heel slab $(1.55 \times 0.45 \times 25)$	16.70	0.775	12.94
Total			115.62
Deduct for upward pressure (abih) $= (53.84 \times 1.55)$	83.45	0.775	64.67
Up ward pressure (ghi) $(0.5 \times 1.55 \times 13.9)$	10.77	0.516	5.55
Total Deduction			70.22

$\therefore$ Maximum B.M. in heel slab = $M_w = (115.62 - 70.22) = 45.40$ kN.m

Maximum ultimate moment $M_u = (1.5 \times 45.40) = 68.10$ kN.m

Maximum Shear Force $V_u = 1.5\ V_w = 1.5\ [132.50 + 16.70 - 83.45 - 10.77] = 82.5$ kN

$$\tau_v = \left(\frac{V_u}{bd}\right) = \left(\frac{82.5\times 10^3}{10^3 \times 400}\right) = 0.20 \text{ N/mm}^2$$

Refer Table-19 (IS: 456) and for M-20 grade concrete, the percentage reinforcement required for $\tau_c = 0.20$ N/mm² is read out as,

$$\left(\frac{100A_{st}}{bd}\right) = 0.15$$

$$\therefore \quad A_{st} = \left(\frac{0.15 \times 1000 \times 400}{100}\right) = 600 \text{ mm}^2/\text{m}$$

$$\left(\frac{M_u}{bd^2}\right) = \left(\frac{68.10 \times 10^6}{1000 \times 400^2}\right) = 0.425$$

Refer Table-2 (SP:16) and read out the percentage reinforcement as

$$p_t = 0.121 < 0.15$$

Hence, provide 12 mm diameter bars at 180 mm centres ($A_{st} = 628$ mm²).

Provide 8 mm diameter bars at 180 mm centres in the transverse direction as distribution reinforcement at both faces of the heel slab.

f) Design of toe slab

Maximum bending moment in the toe slab is computed at the section CC as shown in Table-11.3 (Refer Fig 11.8)

Table 11.3 Moment calculations for 1m length of Toe slab

Loads	Magnitude of Load (kN)	Distance from 'c' (m)	Moment (kN.m)
Upward pressure cdjf $= (71.78 \times 1)$	71.78	0.50	35.89
Upward pressure jfe $= (0.5 \times 1 \times 8.98)$	4.49	0.67	3.00
Total			38.89
Deduct self weight of toe slab $(1 \times 1 \times 0.45 \times 25)$	10.8	0.50	5.40
Neglect self weight of soil above toe slab			
Maximum moment in toe slab			33.49

Maximum ultimate moment in toe slab at CC = M_u = 1.5 (33.49) = 50.23 kN.m

Maximum shear force = $V_u = 1.5\ [71.78 + 4.49 - 10.8] = 98.2$ kN

$$\tau_v = \left(\frac{V_u}{bd}\right) = \left(\frac{98.2 \times 1000}{1000 \times 400}\right) = 0.245 \text{ N/mm}^2$$

Refer Table-19 (IS : 456) and read out the percentage reinforcement required for
$\tau_c = 0.245$ N/mm^2 using M-20 grade concrete as,

$$\left(\frac{100A_{st}}{bd}\right) = 0.15$$

$$\therefore \quad A_{st} = \left[\frac{0.15 \times 1000 \times 400}{100}\right] = 600 \text{ mm}^2/\text{m}$$

The ultimate moment being small, the reinforcement required will be less than that from shear considerations. Hence, provide 12mm diameter bars at 180 mm centres ($A_{st} = 628$ mm^2)

Provide 8mm diameter bars at 180 mm centres in the transverse direction and in both directions at the top of the slab.

g) Stability against overturning

Stability against overturning is checked by computing the overturning and stabilizing moment about toe.

$$\text{Overturning moment } = M_o = \left(\frac{C_a \gamma_e h_s^3}{6}\right) = \left(\frac{0.33 \times 18 \times 4.75^3}{6}\right) = 106.1 \text{ kN.m}$$

Stabilizing moment due to vertical forces about the toe is computed as,

$$M_s = \Sigma W (B - Z) = 201.95\,(3 - 1.6) = 282.73 \text{ kN.m}$$

Factor of safety against overturning is expressed as,

$$\text{F.S.} = \left(\frac{0.9\, M_s}{M_o}\right) = \left(\frac{0.9 \times 282.73}{106.1}\right) = 2.39 > 1.40 \text{ (Hence safe)}$$

h) Stability against sliding

Sliding Force $= P_a = (0.5\, C_a\, \gamma_e\, h^2) = (0.5 \times 0.33 \times 18 \times 5.2^2) = 81.12$ kN

Resisting force (ignoring passive pressure) is expressed as

$F = \mu R = \mu W = (0.5 \times 201.5) = 100.975$ kN.

Factor of safety against sliding $= [(0.9 \times 100.975)]/(81.12) = 1.12 < 1.4$

Hence, the wall is unsafe against sliding. Therefore a shear key has to be designed below the stem.

i) Design of shear key

Let P_p = Passive force resisting the sliding of retaining wall

p_p = Intensity of passive pressure developed just in front of shear key at C Refer (Fig.11.8)

$$\text{Then,} \quad P_p = C_p . p = \left\{\frac{1 + \sin\phi}{1 - \sin\phi}\right\} 71.78 = (3 \times 71.78) = 215.34 \text{ kN/m}^2$$

If a = depth of shear key = 450 mm

Total passive force $P_p = (p_p \,.\, a) = (215.34 \times 0.45) = 96.9$ kN

Factor of safety against sliding

$$= \left[\frac{\mu W + P_p}{\Sigma P_a}\right] = \left[\frac{100.975 + 96.9}{81.12}\right] = 2.43 > 1.40$$

Provide minimum reinforcement of 8 mm diameter bars at 180 mm centers. The reinforcement details in the retaining wall is shown in Fig. 11.9.

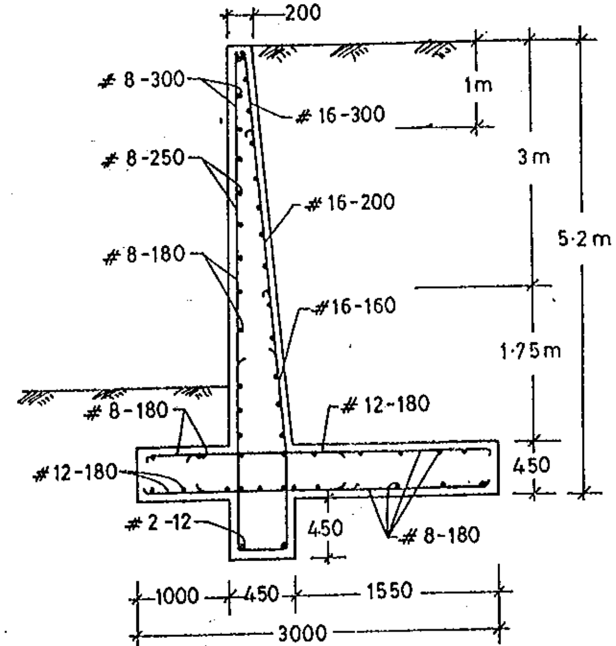


Fig. 11.9 Reinforcement Details in Cantilever Retaining Wall

11.6.2 Design Example (Counterfort Retaining Wall)

Design a counterfort type retaining wall to suit the following data.

Height of wall above ground level = 6 m

Safe bearing capacity of soil at site = 160 kN/m^2

Angle of internal friction = 33 degrees

Density of soil = 16 kN/m^3

Spacings of counterforts = 3 m c/c
Materials: M-20 grade concrete and Fe-415 HYSD bars.
Sketch the details of reinforcements in the retaining wall.

a) Dimensions of retaining wall

$$\text{Minimum depth of foundation } = \frac{p}{\gamma_e}\left[\frac{1-\sin\phi}{1+\sin\phi}\right]^2 = \frac{160}{16}\left(\frac{1}{3}\right)^2 = 1.11 \text{ m}$$

Provide depth of foundation = 1.2 m
∴ Overall height of wall = $H = (6 + 1.2) = 7.2$ m
Thickness of base slab = $2.L.H$ cm = $(2 \times 3 \times 7.2) = 43.2$ cm
Provide 450 mm thick base slab.
Base width = 0.6 H to 0.7 H
$(0.6 \times 7.2) = 4.32$ m and $(0.7 \times 7.2) = 5.04$ m
Hence, adopt base width = 4.5 m
Toe projection = (1/4) 4.5 = 1.1 m

b) Design of stem

$$\text{Pressure intensity at base} = \gamma_e h\left(\frac{1-\sin\phi}{1+\sin\phi}\right)$$

where $h = (7.2 - 0.45) = 6.75$ m
∴ Pressure intensity = $(16 \times 6.75)(1/3) = 36$ kN/m²

$$\text{Maximum working moment } = M_w = \left(\frac{36 \times 3^2}{12}\right) = 27 \text{ kN.m}$$

Factored moment = $M_u = (1.5 \times 27) = 40.5$ kN.m
Effective depth required for balanced section is computed as,

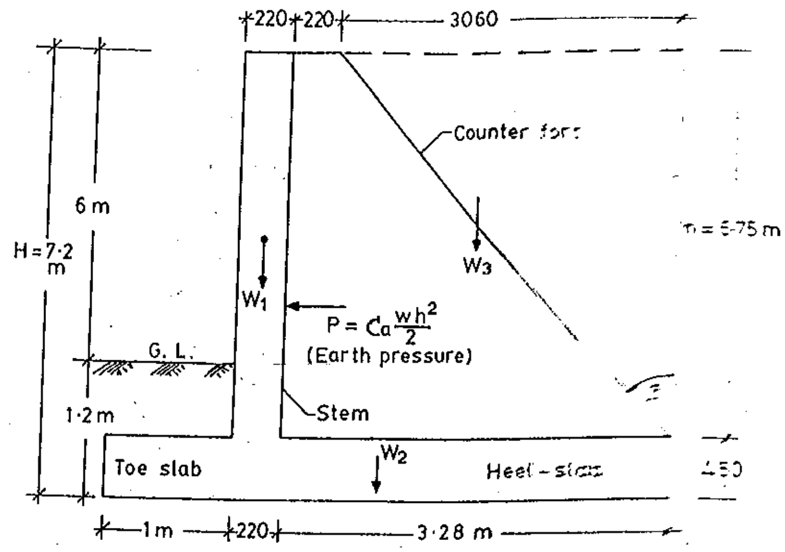
$$d = \sqrt{\frac{M_u}{(0.138 f_{ck} b)}} = \sqrt{\frac{405.\times 10^6}{(0.138 \times 20 \times 10^3)}} = 121 \text{ mm}$$

Assuming an un-reinforced section and to provide a suitable thickness to resist the design shear at base of stem, adopt an overall thickness of 220 mm constant up to the top of the retaining wall.
Effective depth = $d = 175$ mm
The reinforcements in the stem is computed using the relation,

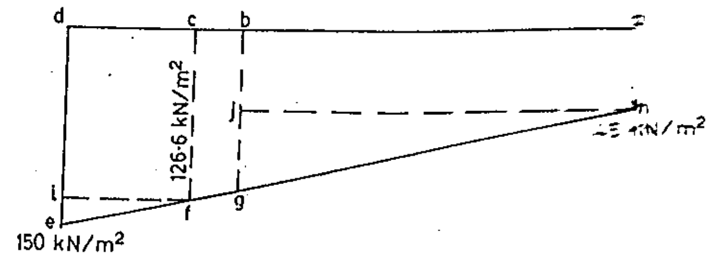
$$(40.5 \times 10^6) = (0.87 \times 415\, A_{st} \times 175)\left[1 - \frac{415 A_{st}}{(10^3 \times 175 \times 20)}\right]$$

Solving, $A_{st} = 700$ mm²

Provide 12 mm diameter bars at 150 mm centers (A_{st} = 754 mm²)
Distribution reinforcement = 0.12% = $(0.0012 \times 220 \times 1000)$ = 264 mm²/m
Adopt 6 mm diameter bars at 200 mm centers (A_{st} = 283 mm²)
The dimensions of the various structural elements of the counterfort retaining wall are shown in Fig.11.10(a).



(a) Counterfort Retaining Wall (overall dimensions)



(b) Pressure Distribution

Fig. 11.10 Counterfort Retaining Wall

c) Stability computations

The pressure distribution at base is computed by calculating the various forces acting and taking moments of all the forces about the heel. The various forces acting and their moments about the heel point a are shown in Table 11.4.

Table 11.4 Stability Computations

Loads	Magnitude of Load (kN)	Distance from 'a' (m)	Moment about 'a' (kN.m)
$W_1 = (0.22 \times 6.75 \times 25)$	35.64	3.39	120.80
$W_2 = (0.45 \times 4.5 \times 25)$	48.60	2.25	109.35
$W_3 = (3.28 \times 6.75 \times 16)$	354.24	1.64	580.95
Moment due to earth pressure $= C_a(\gamma_e h^3/6)$ $= (1/3)(16 \times 7.2^3)/6$			331.77
Total	$\Sigma W = 438.48$		$\Sigma M = 1142.87$

Distance of the point of application of the resultant from point 'a' is

$$Z = \left(\frac{\Sigma M}{\Sigma W}\right) = \left(\frac{1142.87}{438.49}\right) = 2.66 \text{ m}$$

$\therefore$ Eccentricity $= e = (Z - b/2) = (2.66 - 4.5/2) = 0.41$ m

But $(b/6) = (4.5/6) = 0.75$ m. Hence, $e < (b/6)$

$\therefore$ Maximum and minimum pressures at the base are given by

$$p_{max} = \frac{438.49}{4.5}\left(1 + \frac{6 \times 0.41}{4.5}\right) = 150 \text{ kN/m}^2$$

$$p_{min} = \frac{438.49}{4.5}\left(1 - \frac{6 \times 0.41}{4.5}\right) = 45 \text{ kN/m}^2$$

The maximum intensity of pressure does not exceed the permissible value of 160 kN/m^2.

The pressure distribution at the base of the retaining wall is shown in Fig.11.10(b).

d) Design of toe slab

The maximum bending moment acting on the toe slab is calculated by considering moments of all forces about the point 'c'. The computations are compiled in Table-11.5.

Maximum working moment in toe slab is obtained as,

$$M_w = (71.14 - 11.4) = 59.74 \text{ kN.m}$$

Design Ultimate moment $= M_u = (1.5 \times 59.74) = 89.61$ kN.m

Effective depth of toe slab = 400 mm

Reinforcements in toe slab is computed using the relation,

Table 11.5 Moments of Toe Slab

Loads	Magnitude of Load (kN)	Distance from 'c' (m)	Moment about 'c' (kN.m)
Upward pressure 'cdjf' $= (126.6 \times 1)$	26.6	0.50	63.30
Upward pressure 'efi' $= (0.5 \times 1 \times 23.4)$	11.7	0.67	7.84
Total			71.14
Deduct self weight of toe slab $(1 \times 0.45 \times 25)$	10.8	0.5	5.40
Deduct weight of soil above toe slab $(0.75 \times 1 \times 16)$	12.0	0.5	6.00
Total deduction			11.40

$$(89.61 \times 10^6) = (0.87 \times 415 A_{st} \times 400)\left[1 - \frac{415 A_{st}}{(10^3 \times 400 \times 20)}\right]$$

Solving $A_{st} = 644$ mm^2

Provide 12 mm diameter bars at 150 mm centres ($A_{st} = 754$ mm^2)

Distribution bars $= 0.12\% = (0.0012 \times 1000 \times 450) = 540$ mm^2

Provide 10 mm diameter bars at 280 mm centres on both faces ($A_{st} = 561$ mm^2)

e) Design of Heel Slab

Considering 1 m wide strip of heel slab near heel end 'a',

Upward soil pressure = 45 kN/m^2

Weight of soil on strip $= (16 \times 6.75)$ = 108.00 kN/m^2

Self weight of strip $= (1 \times 0.45 \times 25)$ = 0.80 kN/m^2

Total load = 118.80 kN/m^2

Deduct downward pressure = – 45.00 kN/m^2

Net downward pressure = 73.80 kN/m^2

Spacings of counterforts = 3 m

$\therefore$ Maximum negative working moment at counterfort is given by,

$$M_w = \left(\frac{73.80 \times 3^2}{12}\right) = 55.35 \text{ kN.m}$$

Design Ultimate moment $= M_u = (1.5 \times 55.35) = 83$ kN.m

$$(83 \times 10^6) = 0.87 \times 415 A_{st} \times 400)\left[1 - \frac{415 A_{st}}{1000 \times 400 \times 20)}\right]$$

Solving $A_{st} = 600$ mm^2

Provide 12 mm diameter bars at 150 mm centres (A_{st} = 754 mm^2)
Distribution bars = 0.12% = (0.0012 × 1000 × 450) = 540 mm^2
Provide 10 mm diameter bars at 280 mm centres on both faces(A_{st} = 561 mm^2)

f) Design of counterforts

Thickness provided at the top = (220 + 220) = 440 mm
Thickness of counterfort = 440 mm
Maximum working moment in counterfort is

$$M_w = \left(C_a \frac{\gamma_e . h^3}{6} L\right) = \left(\frac{1}{3} \times \frac{16 \times 6.75^3}{6} \times 3\right) = 820.12 \text{ kN.m}$$

Factored Design moment = M_u = (1.5 × 820.12) = 1230 kN.m
Reinforcement at the bottom of counterfort is computed by using the relation,

$$(1230 \times 10^6) = (0.87 \times 415 A_{st} \times 4400)\left[1 - \frac{415 A_{st}}{(440 \times 4400 \times 20)}\right]$$

Solving A_{st} = 800 mm^2
But minimum reinforcement as per IS:456-2000 code is stipulated as

$$A_s = \left(\frac{0.85\, bd}{f_y}\right) = \left[\frac{(0.85 \times 440 \times 4400)}{415}\right] = 3965 \text{ mm}^2$$

Provide 5 bars of 32 mm diameter (A_{st} = 4020 mm^2)

g) Curtailment of bars

Let h_1 = depth at which 1 bar can be curtailed

Then $\left(\frac{5-1}{5}\right) = \left(\frac{h_1}{6.75^2}\right)$ ∴ $h_1 = 6\text{m}$ from top

Let h_2 = depth at which 2 bars can be curtailed

Then $\left(\frac{5-2}{5}\right) = \left(\frac{h_2}{6.75^2}\right)$ ∴ $h_2 = 5.2\text{m}$ from top

Let h_3 = depth at which 3 bars can be curtailed

Then $\left(\frac{5-3}{5}\right) = \left(\frac{h_3}{6.75^2}\right)$ ∴ $h_3 = 5.2\text{m}$ from top

The remaining two bars are continued right up to the top.

h) Connection between counterfort and upright slab

Consider the bottom 1 m height of upright slab.
Pressure on this strip = 36 kN/m^2
Total lateral pressure transferred to the counterfort for 1m height is = 36 (3 − 0.44) = 91.8 kN
Factored force = (1.5 × 91.8) = 137.7 kN

$$\text{Reinforcement required per metre height} = \left(\frac{137.7 \times 10^3}{0.87 \times 415}\right) = 381 \text{ mm}^2$$

Provide minimum reinforcement of 10 mm diameter bars in the form of horizontal links at 280 mm centres.

i) Connection between counterfort and heel slab

Tension transferred in 1 m width of counterfort near the heel end is = 73.8(3 − 0.44) = 189 kN
Factored force = (1.5 × 189) = 283.5 kN

$$\text{Reinforcement required for 1 m height} = \left(\frac{283.5 \times 10^3}{0.87 \times 415}\right) = 785 \text{ mm}^2$$

Spacing of 10 mm diameter bars provided in the form of two legged vertical links is computed as $S_v = \left(\frac{2 \times 78.5 \times 10^3}{785}\right) = 200$ mm

Provide 10 mm diameter two legged vertical links at 200 mm centres.

j) Reinforcement Details

The detail of reinforcements in the counterfort retaining wall is shown in Fig.11.11.

11.7 EXAMPLES FOR PRACTICE

1) Design a reinforced concrete cantilever retaining wall to retain earth level with the top of the wall to a height of 5.5 m above ground level. The density of soil at site is 17 kN/m^3 with a safe bearing capacity of 120 kN/m^2. Assume the angle of shearing resistance of the soil as 35 degrees. Further assume a coefficient of friction beween soil and concrete as 0.55. Adopt M-20 grade concrete and Fe-415 HYSD bars.

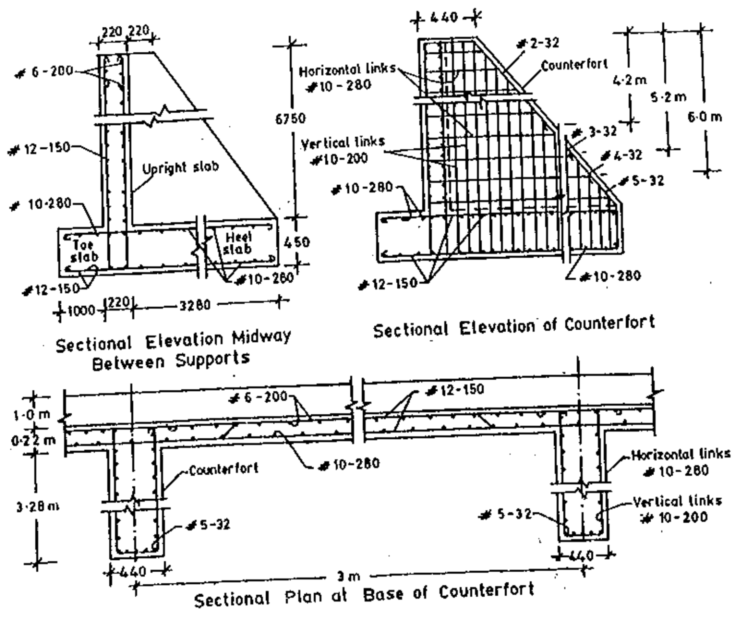


Fig. 11.11 Reinforcement Details in Counterfort Retaining Wall

2) A cantilever type retaining wall is to be designed to support a bank of earth 4 m above ground level on the toe side of the wall. The backfill surface is inclined at an angle of 15 degrees with the horizontal. Assume that good soil is available for foundations at a depth of 1.25 m below ground level with a safe bearing capacity of 160 kN/m^2 and an angle of shearing resistance of 30 degrees. Assume coefficient of friction between soil and concrete as 0.5. Adopt M-20 grade concrete and Fe-415 HYSD reinforcement. Assume the unit weight of soil as 16 kN/m^3.

3) Design a counterfort type retaining wall to support an earth fill of 7.5 m above ground level. The foundation depth may be taken as 1.5 m below ground level. The safe bearing capacity of soil at site is 150 kN/m^2. Unit weight of soil may be taken as 16 kN/m^3 and an angle of shearing resistance of 30°. Assume the value of coefficient of friction as 0.55. Adopt M-20 grade concrete and Fe-415 HYSD bars. Sketch the details of reinforcements in the retaining wall.

4) Design a cantilever retaining wall to retain earth with a backfill sloped 20° to the horizontal. The top of the wall is 5.5 m above the ground level. Assume the depth of foundation as 1.2 m below ground level with a safe bearing capacity of 120 kN/m^2. The unit weight of backfill is 18 kN/m^3 and an angle of of shearing resistance of 35°. Also assume the coefficient of friction between soil and concrete as 0.55. Adopt M-20 grade concrete and Fe-415 HYSD steel bars.

CHAPTER 12

Design of Staircases

12.1 INTRODUCTION

Staircase flights are generally designed as slabs spanning between wall supports or landing beams or as cantilevers from a longitudinal inclined beam. The staircase fulfills the function of access between the various floors in the building. Generally, the flight of steps consists of one or more landings provided between the floor levels. [Fig. 12.1]

The structural components of a flight of stairs comprises of the following elements.

a) Tread

The horizontal portion of a step where the foot rests is referred to as tread. 250 to 300 mm is the typical dimensions of a tread.

b) Riser

Riser is the vertical distance between the adjacent treads or the vertical projection of the step with value of 150 to 190 mm depending upon the type of building. The width of stairs is generally 1 to 1.5 m and in any case not less than 850 mm. Public buildings should be provided with larger widths to facilitate free passage to users and prevent over crowding.

c) Going

Going is the horizontal projection (plan) of an inclined flight of steps between the first and last riser. A typical flight comprises two landings and one going as shown in Fig. 12.1 (e).

To break the monotony of climbing, the number of steps in a flight should not generally exceed 10 to 12.

The tread-riser combination can be provided in conjunction with

i) Waist slab [Fig. 12.1(a)]

ii) Tread - Riser type (continuous folded plate) [Fig. 12.1 (b)]

iii) Isolated cantilever tread slab [Fig. 12.1 (c)]

iv) Double cantilever precast tread slab with a central inclined beam [Fig. 12.1(d)]

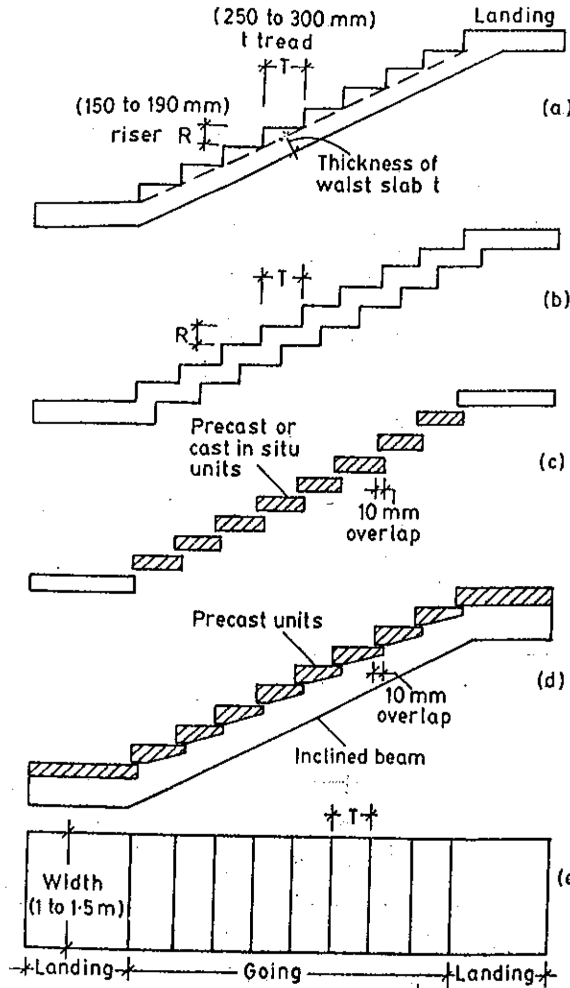


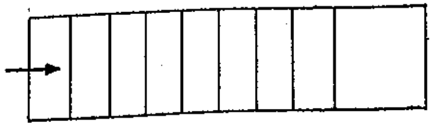
Fig. 12.1 Typical Flight in a staircase

12.2 TYPES OF STAIRCASES

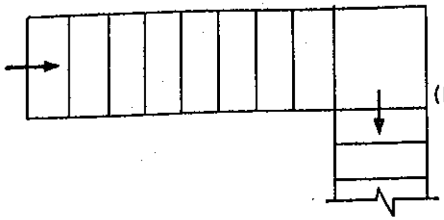
The various types of staircases adopted in different types of buildings can be grouped under geometrical and structural classifications depending upon their shape and plan pattern and their structural behavior under loads.

12.2.1 Geometrical classification

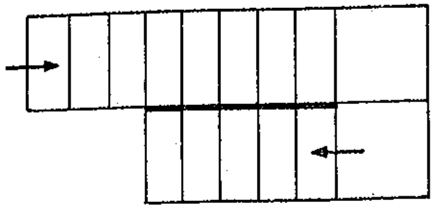
Aesthetic considerations have evolved a wide variety of staircases over the years. Some of the common geometrical configurations used are compiled in Fig. 12.2.



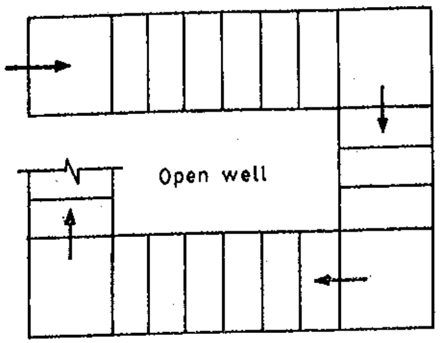
(a) Single Flight Staircase



(b) Quarter Turn Staircase



(c) Dog Legged Staircase



(d) Open Well Staircase

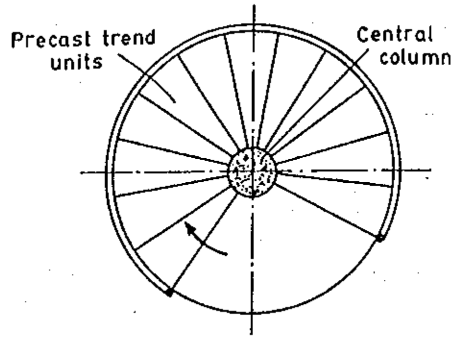
Fig. 12.2 Various Types of staircases (Contd.)

a) Single Flight Staircase

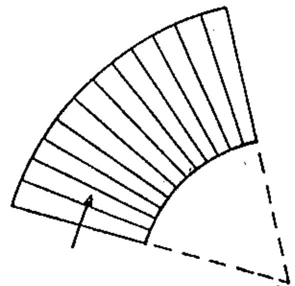
A typical single flight stairs is shown in Fig. 12.2 (a). This type is used in cellars or attics where the height between floors is small and the frequency of its use is less.

b) Single or quarter turn staircase

Fig. 12.2 (b) shows the plan arrangement of single right angled turn stair-



(e) Spiral Staircase



(f) Helicoidal Staircase

Fig. 12.2

case. The staircase flight generally runs adjoining the walls and provides uninterrupted space at the centre of the room. Generally used in domestic houses where floor heights are limited to 3 m.

c) Dog legged staircase

The most common type of stairs arranged with two adjacent flights running parallel with a mid landing as shown in Fig. 12.2 (c). Where space is at a premium, dog legged staircase is generally adopted resulting in economical utilisation of available space.

d) Open well staircase

In public buildings where large spaces are available, open well staircase shown in Fig.12.2 (d) is generally preferred due to its better accessibility, comfort and ventilation due to its smaller flights with an open well at the centre.

e) Spiral staircase

In congested locations, where space available is small, spiral stairs are ideally suited. A typical spiral staircase shown in Fig. 12.2(e) comprises a central post with precast slab treads anchored to the central column. It is not user friendly due to the reduced tread width near the port and is suitable only for single person to use the staircase at a time.

f) Helocoidal staircase

Helicoidal staircase shown Fig.12.2.(f) is aesthetically superior compared to other types and is generally used in the entrance foyer of cinema theatres and shopping malls to connect the ground and first floors. Helicoidal stair which is built as a ramp following the helicoidal curve with supports at ground and first floor or with intermediate supports involves rigorous structural computations for the determination of design moments and shear forces. The reader may refer to the publications of Bergman[90] and Scordelis[91] for the design aspects of helicoidal stairs.

g) Free standing staircase

Free-standing staircase is built out of a large fixed base with the stair flight cantilevering out of the base. It is essentially a cantilever in space and is aesthetically superior to other types. The reader may refer to the works of Gould92 and Solanki93 for the analysis and design aspects of free standing staircase.

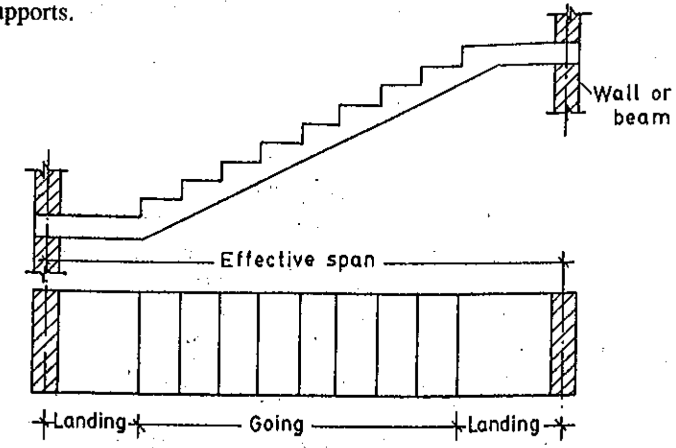
12.2.2 Structural Behavior of Staircases

Staircases can be grouped depending upon the support conditions and the direction of major bending of the slab component under the following categories.

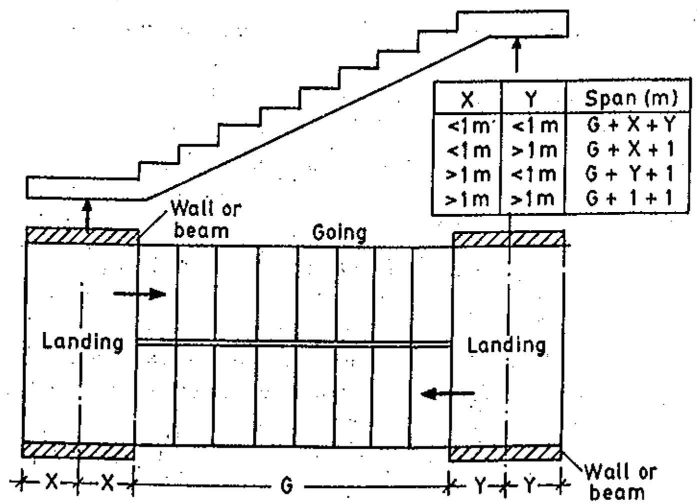
a) Staircase slab spanning longitudinally (along the sloping line)
b) Staircase slab spanning transversely (slab width wise with central or side supports)

a) Staircase Slab spanning in the longitudinal Direction

In this type, the inclined stair flight together with the landings are supported on walls or beams as shown in Fig. 12.3 (a). The effective span to be considered in design computations is between the centre to centre of supports.



(a) Simply Supported Staircase



X	Y	Span (m)
<1m	<1m	G + X + Y
<1m	>1m	G + X + 1
>1m	<1m	G + Y + 1
>1m	>1m	G + 1 + 1

(b) Transverse Spanning of Landings

Fig. 12.3 Staircase Flight (longitudinal spanning)

The slab arrangement may be of the conventional waist slab or the tread-riser type between the supports. The slab thickness depends upon the span and its thickness can be reduced by providing intermediate supports at the junctions of inclined waist slab and horizontal landing slabs.

Alternatively it is possible to provide supports to the landing slabs in the transverse direction as shown in Fig. 12.3 (b).

In such cases the effective span to be considered according to the IS: 456-code clause 33.1 (b) is given by

$$L = (G + X + Y)$$

Where G = going

X or Y = Half the width of landings

The values to be considered for X and Y are shown in Fig. 12.3 (b).

In the case stairs with open wells, where spans partly criss cross at right angles, the load on areas common to any two such spans may be distributed as one-half in each direction as shown in the Fig. 12.4.

The IS:456 code also specifies that when flights are landings are embedded into walls for a length of not less than 110 mm and are designed to span in the direction of flight, a length of 150 mm strip may be deducted from the loaded area and the effective breadth of the section increased by 75 mm for purposes of design.

b) Staircase Slabs spanning in the Transverse Direction

The most common examples of staircase slabs spanning in the transverse direction are grouped under the following :-

i) Slab supported between two edge beams are walls [Fig. 12.5 (a)]
ii) Slab cantilevering on either side of a central beam [Fig. 12.5 (b)]
iii) A Cantilevered slab from a wall or a spandrel beam [Fig. 12.5 (c)]

In these types of slabs, the width of flight being small (1 to 1.5 m), the designed thickness will be very small from structural computations. However, from practical considerations a minimum thickness of 75 to 80 mm should be provided and suitable reinforcements to resist the maximum bending moments or the minimum percentage reinforcements (whichever is higher) should be provided in the slab.

12.3 LOADS ON STAIRCASES

The loads to be considered in the design of staircases comprise the following types.

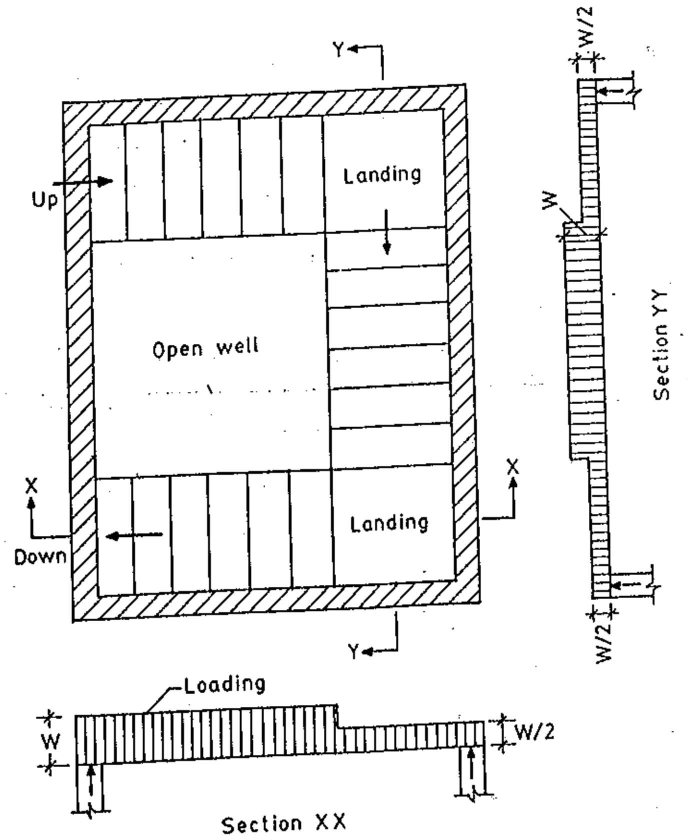


Fig. 12.4 Loading on Stairs With Open Wells

a) Dead Loads

The various dead loads are

i) Self weight of stair slab concrete which includes the waist slab, tread-riser etc.
ii) Self weight of finishes (0.5 to 1 kN/m²)

b) Live Loads

The IS: 875-1987 (part II) code specifies the live loads to be considered as uniformly distributed load of intensity 5 kN/m^2 for public buildings and 3 kN/m^2 for residential buildings where the specified floor loads do not exceed 2 kN/m^2, and the staircases are not liable for over crowding.

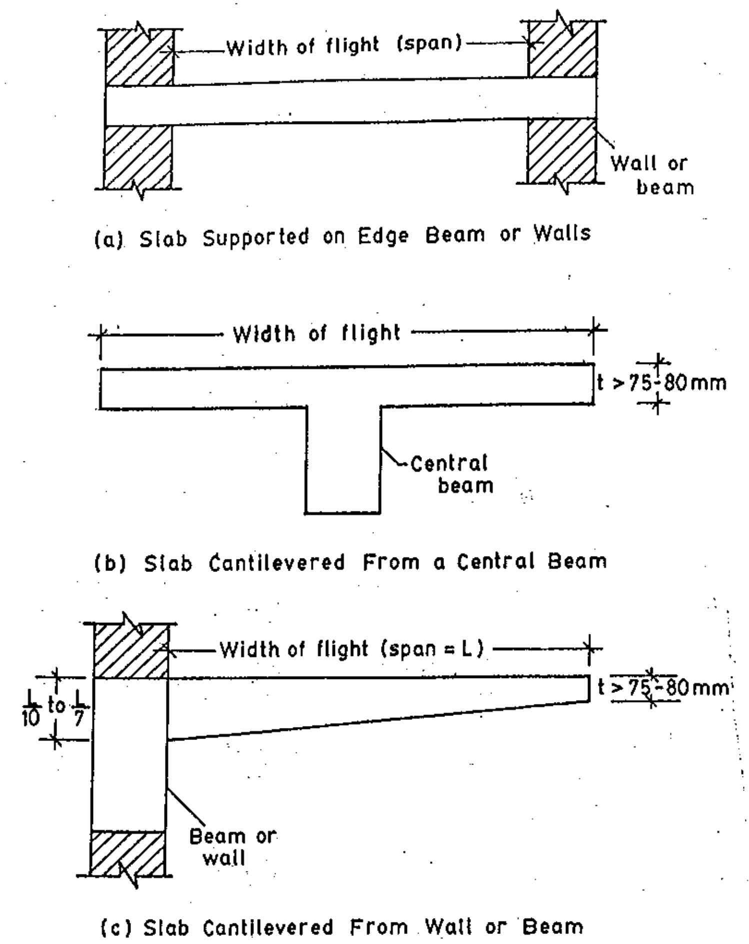


Fig. 12.5 Staircase Slabs Spanning in the Transverse Direction

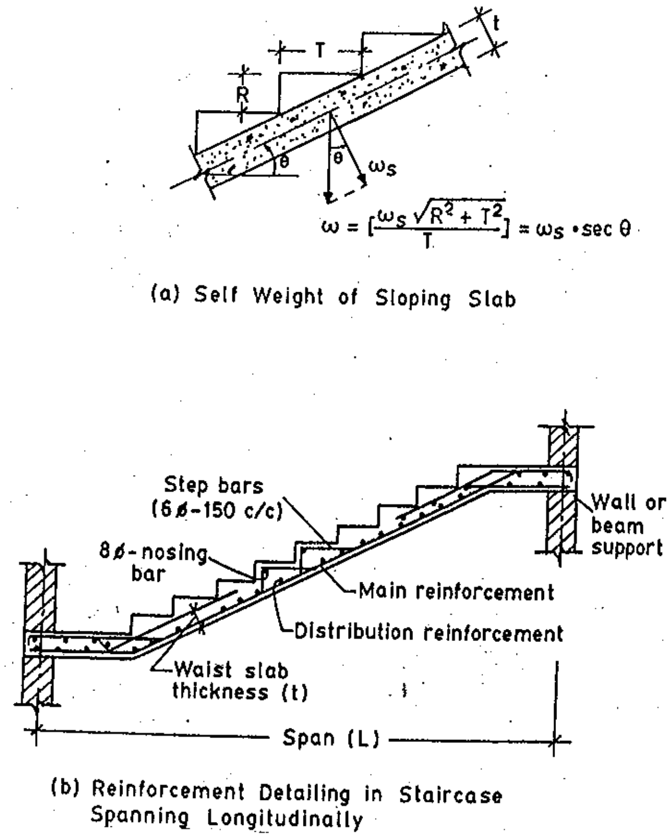
In the case of structurally independent cantilever steps, the code prescribes the tread slab to be designed to resist a concentrated live load of 1.3 kN applied at the free end of the cantilevered tread.

The loads specified in the IS:875 code being characteristic loads, a load factor of 1.5 has to be applied to arrive at the design loads for limit state design.

c) Load Effects on Waist slab spanning in the longitudinal direction

The thickness of the waist slab (t) normal to the slope may be assumed as approximately ($L/20$) for simply supported and ($L/25$) for continuous slabs. The steps are usually treated as non-structural elements and only the thickness of the waist slab is designed to resist the loads. Nominal reinforcement is generally provided in the step to protect the nosing from cracking comprising of 6 mm diameter bars at 150 mm centres.

The waist slab (sloping slab) is inclined at an angle 'θ' to the horizontal in a flight of stairs spanning in the longitudinal direction as shown in Fig. 12.6(a).



$$\omega = \left[\frac{\omega_s\sqrt{R^2+T^2}}{T}\right] = \omega_s \cdot \sec\theta$$

Fig. 12.6 Load Effects and Detailing in Waist Slabs

Let R = Rise
T = Tread
w_s = Self weight of slab (on slope) per metre
w = Self weight of slab on horizontal span
θ = Angle between the sloping slab and horizontal

Then dead load of slab on horizontal span is expressed as

$$w = \left(\frac{w_s\sqrt{R^2+T^2}}{T}\right) = w_s.\sec\theta$$

The self weight of steps (triangular shape) are computed and their weight per metre length of horizontal span is added to the dead weight of slab along with the specified live load. The factored load is used to compute the design moments and the reinforcements are designed to resist the ultimate moment and the standard detailing is shown in Fig. 12.6 (b)

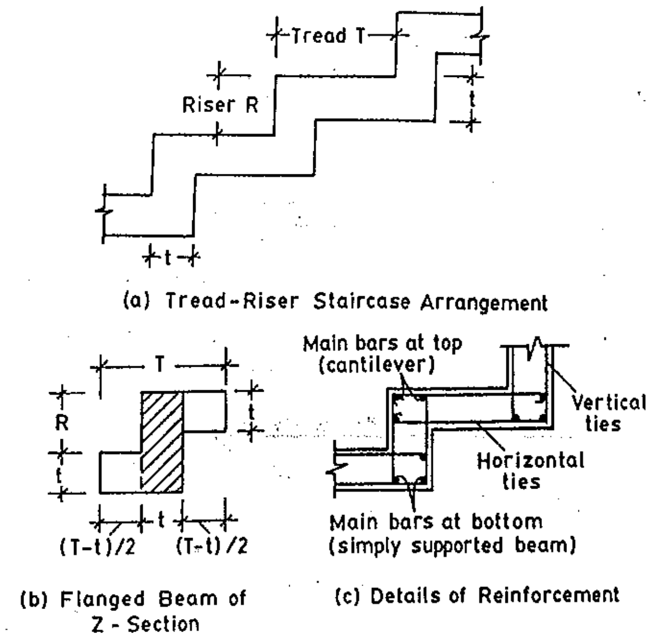
d) Load effects on Waist slab spanning Transversely

The load acts normal to the waist slab which bends in transverse planes normal to the sloping surface of the slab. The main bars are designed to resist the maximum bending moment and are provided transversely either at the bottom or top, depending upon whether the slab is simply supported or cantilevered from the wall support. As the span is very small (1 to 2m), the thickness of waist slab required to resist the bending moment will be small but from practical consideration, a minimum thickness of 75-80 mm is provided with minimum reinforcement as per IS: 456 code.

e) Load Effects on Tread-Riser Stairs spanning Transversely

The tread-riser unit comprising the riser and tread slab shown in Fig. 12.7 (a) behaves as beams spanning in the transverse direction. For purposes of analysis it is assumed that the riser slab and one half of the tread slab on either side can be assumed to behave as a Z-section.

The tread-riser unit structurally behaves as a flanged beam which is transversely loaded with an overall depth of $(R + t)$ as shown in Fig. 12.7 (b). For design of reinforcements, the contribution of flanges can be ignored and the rectangular section comprising the thickness of the riser as width (b) and the depth of beam as $(R + t)$ will yield conservative results. The reinforcements are designed for the maximum bending moment developed at the centre if simply supported or at support if it is a cantilever. The detailing of reinforcements is generally with horizontal bars and ties as shown in Fig. 12.7(c).



(a) Tread-Riser Staircase Arrangement

(b) Flanged Beam of Z - Section

(c) Details of Reinforcement

Fig. 12.7 Tread-Riser Stairs Spanning Trasversely

f) Load Effects on Tread-Riser Stairs Spanning Longitudinally

A typical tread-riser type staircase flight spanning longitudinally is shown in Fig. 12.8 (a) The structural behavior of this type of stairs is similar to that of waist slabs spanning longitudinally. The bending moments developed in various tread slabs which increase gradually towards the centre of span is shown in Fig. 12.8 (b). The bending moment in risers is constant for each riser. The variation of shear force in the treads is also shown in Fig. 12.8 (c). For practical purposes it is sufficient to design both tread and riser slabs for flexure only as the shear stresses in tread slabs and axial stresses in riser slabs are relatively of low magnitude.

The thickness of riser and tread slabs is generally kept the same with values of span / 25 for simply supported and span /30 for continuous staircases. However, the minimum thickness of 80 mm should be provided from practical considerations.

The reinforcement details are shown in Fig. 12.8 (d). The main bars comprise of closed ties in the longitudinal direction while the distribution bars are provided in the transverse direction. The top leg of the ties resists the negative moments developed near the supports. The close loop system of ties ensures both flexure and shear resisting capacity together with the ductility of the tread riser slabs. In the case of longer flights, the spacing and diameter of the main reinforcement in the tread-riser units is suitably varied along the span in conformity with the bending moment diagram, resulting in an economical design.

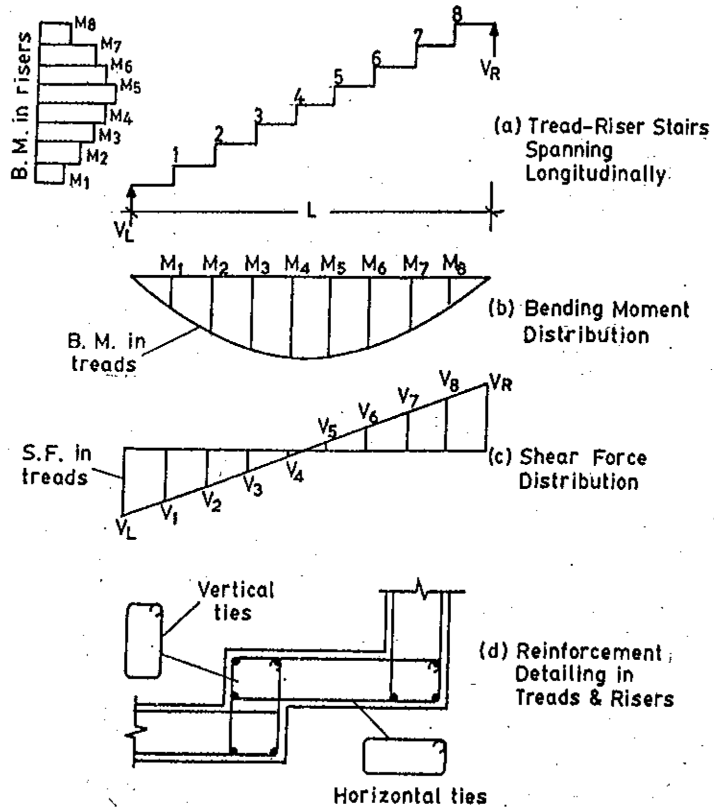


Fig. 12.8 Tread-Riser staircase Spanning Longitudinally

12.4 DESIGN EXAMPLES

12.4.1 Design one of the flights of stairs of a school building spanning between landing beams to suit the following data.

a) Data

Type of staircase:- Waist slab type
Number of steps in flight = 12
Tread T = 300 mm

Riser R = 160 mm
Width of landing beams = 400 mm
Materials: M-20 Concrete and Fe-415 HYSD bars

b) Effective span

Effective span = $L = \{(12 \times 300) + 400\} = 4000$ mm

Thickness of waist slab $= \left(\frac{\text{span}}{20}\right) = \left(\frac{4000}{20}\right) = 200$ mm

c) Loads

Dead load of slab (on slope) = $w_s = (0.2 \times 1 \times 25) = 5$ kN/m
Dead load of slab on horizontal span is expressed as,

$$w = \left(\frac{w_s\sqrt{R^2+T^2}}{T}\right) = \left(\frac{5\sqrt{160^2+300^2}}{300}\right) = 5.66 \text{ kN/m}$$

Dead load of one step = $(0.5 \times 0.16 \times 0.3 \times 25) = 0.6$ kN/m
Load of steps/m length = $[0.6 \times (1000/300)] = 2$ kN/m
Finishes = 0.6 kN/m
∴ Total dead load = $(5.66 + 2 + 0.6) = 8.26$ kN/m
Live load (liable of over crowding) = 5 kN/m
∴ Total service load = 13.26 kN/m
∴ Total Ultimate load = $w_u = (1.5 \times 13.26) = 19.89$ kN/m

d) Bending Moments

Maximum B.M at Centre of span is given by

$$M_u = 0.125\, w_u.L^2 = (0.125 \times 19.89 \times 4^2) = 39.78 \text{ kN.m}$$

e) Check for depth of waist slab

$$d = \sqrt{\frac{M_u}{0.138 f_{ck} b}} = \sqrt{\frac{39.78 \times 10^6}{0.138 \times 20 \times 10^3}} = 120\text{mm}$$

Assuming a clear cover of 20 mm and using 12 mm diameter bars,
Effective depth = $d = [200 - 20 - 6] = 174$ mm
Hence, the effective depth provided is greater than the required depth.

f) Reinforcements

Reinforcements are computed using the relation,

$$M_u = 0.87\, f_y\, A_{st}\, d\left[1 - \frac{A_{st} f_y}{b\, d\, f_{ck}}\right]$$

Where f_y = 415 N/mm^2
f_{ck} = 20 N/mm^2 and $M_u = (39.78 \times 10^6)$ N.mm
d = 174 mm
b = 1000 mm

Or by using Table-2 of SP:16, read out the percentage reinforcement corresponding to

$$\left(\frac{M_u}{bd^2}\right) = \left(\frac{39.78 \times 10^6}{10^3 \times 174^2}\right) = 1.31$$

$$\therefore \quad p_t = 0.40 = \left(\frac{100 A_{st}}{bd}\right)$$

$$\therefore \quad A_{st} = \left(\frac{0.40 \times 10^3 \times 174}{100}\right) = 696 \text{ mm}^2$$

Provide 12mm diameter bars at 160mm centres (A_{st} = 707 mm^2) as main reinforcement.
Distribution reinforcement = $(0.0012 \times 10^3 \times 200)$ = 240 mm^2/m
Provide 8 mm diameter bars at 200 mm centres (A_{st} = 251 mm^2)
The details of reinforcements in the staircase flight is shown in Fig. 12.9.

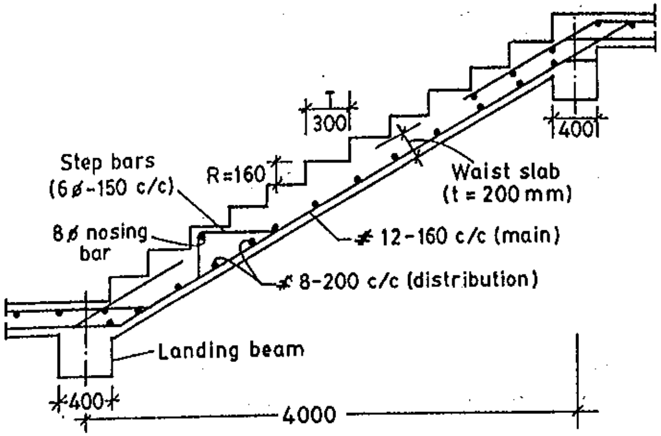


Fig. 12.9 Reinforcement in staircase Flight

12.4.2 A staircase flight comprises of independent tread slabs, cantilevered from a reinforced concrete wall. Assuming the riser is 150mm and tread length as 300 mm, width of flight = 1.75 m, design the cantilevered slab using M-20 grade concrete and Fe-415 HYSD bars to suit the loading requirements of IS:875 and IS:456 codes.

a) Data

Rise = R = 150 mm
Tread = T =300 mm
Span Length = L = 1.75 m
M-20 grade Concrete, f_{ck} = 20 N/mm^2
Fe-415 HYSD bars, f_y = 415N/mm^2
Width of Tread = (300 + 10 mm overlap)
B = 310 mm

b) Thickness of Slab

$$t = \left(\frac{\text{span}}{10}\right) = \left(\frac{1750}{10}\right) = 175 \text{ mm}$$

c) Dead Loads

i) Self weight of tread slab = $(0.175 \times 0.31 \times 25)$ = 1.356 kN/m
ii) Finishes = (0.6×0.31) = 0.186 kN/m
$\therefore$ Total Dead Load = 1.542 kN/m
Total Ultimate Dead Load = $g_u = (1.5 \times 1.542)$ = 2.313 kN/m

d) Live Loads

Case–1, $q_u = (5 \times 0.3) \times 1.5$ = 2.25 kN/m
Case–2, $q_u = (1.3 \times 1.5)$ = 1.95 kN (At free end)

e) Design Moments

At fixed end, M_u (Dead load) = $(0.5 \times 2.313 \times 1.75^2)$ = 3.54 kN.m

M_u (Live load) = $(0.5 \times 2.25 \times 1.752)$ = 3.44 kN.m
And (1.95×1.75) = 3.41 kN.m

Considering the critical values,
Total Design Moment = M_u = (3.54 + 3.44) = 6.98 kN.m

f) Reinforcements

As the moment is small, check for thickness is not required.
Using a cover of 25 mm, effective depth = d = 150 mm.

$$\left(\frac{M_u}{bd^2}\right) = \left(\frac{6.98\times10^6}{310\times150^2}\right) = 1.0007$$

Refer Table-2 of SP:16 and read out the percentage of reinforcement as,

$$p_t = \left(\frac{100A_{st}}{bd}\right) = 0.3$$

$$\therefore \quad A_{st} = \left(\frac{0.3\times310\times150}{100}\right) = 139.5 \text{ mm}^2$$

Provide 3 bars of 8 mm diameter (A_{st} = 150 mm^2) on the tension side.
Anchorage length required is

$$L_d = \left(\frac{0.87\times415\times8}{4\times1.2\times1.6}\right) = 376 \text{ mm}$$

In addition to L_d, we have to provide sufficient balancing moment to prevent overturning.
Distribution reinforcement = $(0.0012\times10^3\times175) = 210$ mm^2/m

$$\text{Spacing of 6 mm diameter bars} = \left(\frac{1000a_{st}}{A_{st}}\right) = \left(\frac{1000\times28}{210}\right) = 133 \text{ mm}$$

Provide 6 mm diameter bars at 130 mm centres.

g) Check for shear at support section

$$V_u = (g_u + q_u)L = (2.313 + 2.25)1.75 = 8.00 \text{ kN}$$

$$\therefore \quad \tau_v = \left(\frac{V_u}{bd}\right) = \left(\frac{8\times10^3}{310\times150}\right) = 0.172 \text{ N/mm}^2$$

Refer Table-19 of IS: 456 and read out the permissible shear stress for p_t = 0.30,

$$(k_s\tau_c) = (1.25\times0.39) = 0.48 \text{ N/mm}^2 > \tau_v$$

Hence, shear stresses are within safe permissible limits.

h) The reinforcement details in the Cantilever slab are shown in Fig. 12.10.

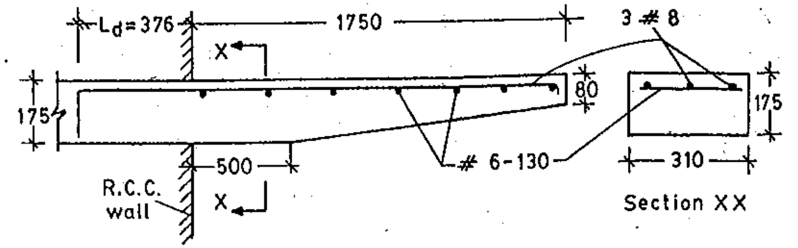
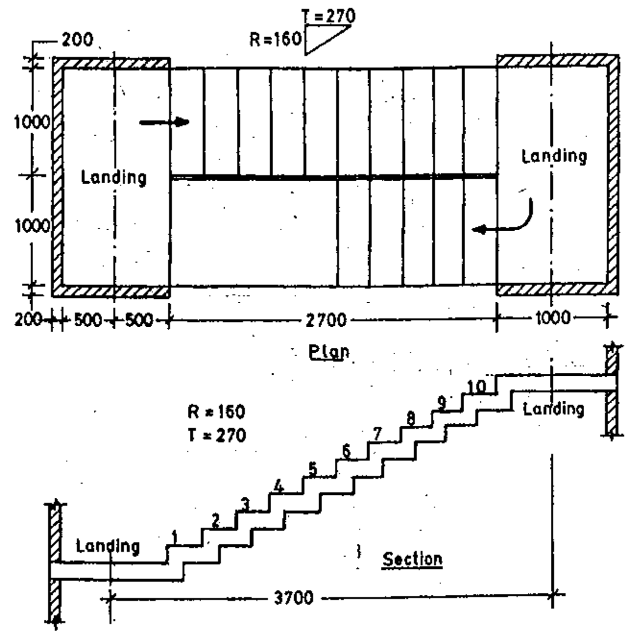


Fig. 12.10 Cantilever staircase

12.4.3. Design a tread-riser type staircase flight between the landings shown in Fig. 12.11(a). The landing slabs are supported on both the adjacent edges. Assume the following additional data:-

a) Data

Height between landings = 1.76m
Riser = R = 160 mm



(a) Tread-Riser Flight (Plan and Section)

Fig. 12.11 Tread-Riser Type staircase

Tread = T = 270 mm
Width of Flight = landing width = 1.0 m
Materials: M-20 grade concrete (f_{ck} = 20 N/mm²)
Fe-415 HYSD bars (f_y = 415 N/mm²)

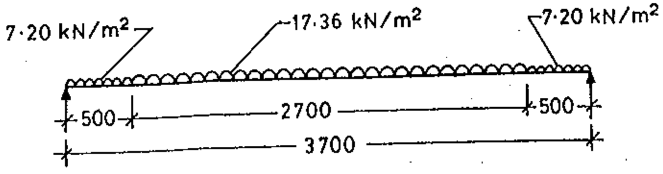
b) Effective span and thickness of slab

L = [2.70 + 1.00] = 3.70 m

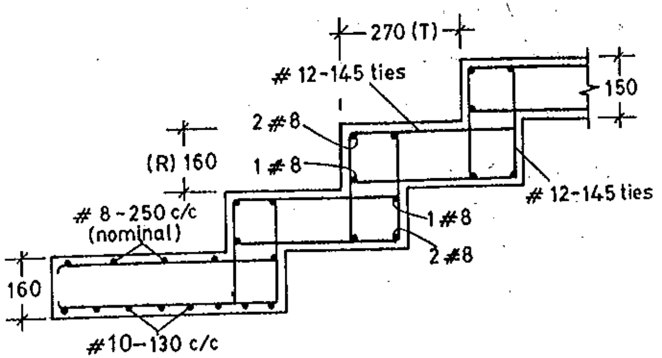
Assume thickness of riser slab = thickness of Tread

$$t = \left(\frac{\text{span}}{25}\right) = \left(\frac{L}{25}\right) = \left(\frac{3700}{25}\right) = 148 \text{ mm}$$

Adopt Effective depth, d = 125 mm and Overall depth h = 150 mm



(b) Loading on Staircase Flight



(c) Reinforcement Details in Tread-Risers and Landing Slab

Fig. 12.11 (Contd.)

c) Loads on Staircase flight

i) Loads on going (on projected plan area)
Self weight of tread and Riser slab/step = [(0.16+0.27) 0.15×25]
= 1.61 kN

Dead load of steps/m length = [(1.61 × 1000/270)] = 5.97 kN/m²
Weight of Finishes = 0.60 kN/m²
Live Load = 5.00 kN/m²
∴ Total Load = 11.57 kN/m²
∴ Factored Load = (11.57 × 1.50) = 17.36 kN/m²

ii) Loads on landing slab (Assuming 160 mm thick)
Self weight of slab = (0.16 × 25) = 4.00 kN/m²
Finishes = 0.60 kN/m²
Live load = 5.00 kN/m²
∴ Total load = 9.60 kN/m²
∴ (Factored Load = (9.60 × 1.5) = 14.4 kN/m²
50 percent of this load is assumed to be acting longitudinally in the direction of span.
∴ Load on Landing slab = (0.5 × 14.40) = 7.20 kN/m²

d) Design of Tread-Riser portion

Referring to Fig. 12.11(b) showing the loading on the horizontal span L = 3.70m, the reactions and bending moments are computed.
Reaction on Landing is computed as,

$$V = (7.2 \times 0.5) + 0.5\,(17.36 \times 2.70) = 27.03 \text{ kN}$$

Maximum moment at mid span

M_u = (27.03 × 1.85)−(7.2 × 0.5) 1.60−(17.36 × 1.35 × 0.675) = 28.42 kN·m

Effective depth provided = d = 125 mm.

$$\left(\frac{M_u}{bd^2}\right) = \left(\frac{28.42 \times 10^6}{10^3 \times 125^2}\right) = 1.82$$

Referring & Table-2 of SP: 16, read out, $p_t = \left(\frac{100A_{st}}{bd}\right) = 0.574$

$$A_{st} = \left(\frac{0.574 \times 10^3 \times 125}{100}\right) = 717.5 \text{ mm}^2$$

Provide 12 mm diameter bars at 150 mm centres in the form of stirrups as shown in Fig. 12.11 (c)

Distribution bars of 8 mm diameter at each bend as shown in Fig. 12.11

e) Design of Landing Slabs

Factored load on landing slab = 14.4 kN/m

Tread = T = 270 mm
Width of Flight = landing width = 1.0 m
Materials: M-20 grade concrete (f_{ck} = 20 N/mm²)
Fe-415 HYSD bars (f_y = 415 N/mm²)

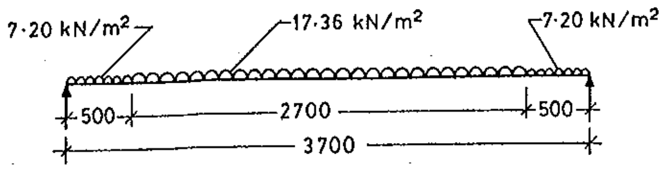
b) Effective span and thickness of slab

$L = [2.70 + 1.00] = 3.70$ m

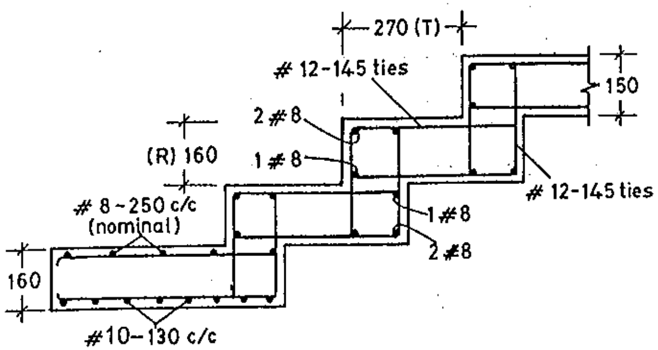
Assume thickness of riser slab = thickness of Tread

$$t = \left(\frac{\text{span}}{25}\right) = \left(\frac{L}{25}\right) = \left(\frac{3700}{25}\right) = 148 \text{ mm}$$

Adopt Effective depth, d = 125 mm and Overall depth h = 150 mm



(b) Loading on Staircase Flight



(c) Reinforcement Details in Tread-Risers and Landing Slab

Fig. 12.11 (Contd.)

c) Loads on Staircase flight

i) Loads on going (on projected plan area)
Self weight of tread and Riser slab/step = [(0.16+0.27) 0.15×25]
= 1.61 kN

Dead load of steps/m length = [(1.61 × 1000)/270] = 5.97 kN/m²
Weight of Finishes = 0.60 kN/m²
Live Load = 5.00 kN/m²
∴ Total Load = 11.57 kN/m²
∴ Factored Load = (11.57 × 1.50) = 17.36 kN/m²

ii) Loads on landing slab (Assuming 160 mm thick)
Self weight of slab = (0.16 × 25) = 4.00 kN/m²
Finishes = 0.60 kN/m²
Live load = 5.00 kN/m²
∴ Total load = 9.60 kN/m²
∴ (Factored Load = (9.60 × 1.5) = 14.4 kN/m²

50 percent of this load is assumed to be acting longitudinally in the direction of span.
∴ Load on Landing slab = (0.5 × 14.40) = 7.20 kN/m²

d) Design of Tread-Riser portion

Referring to Fig. 12.11(b) showing the loading on the horizontal span of L = 3.70m, the reactions and bending moments are computed.
Reaction on Landing is computed as,

$$V = (7.2 \times 0.5) + 0.5\,(17.36 \times 2.70) = 27.03 \text{ kN}$$

Maximum moment at mid span

$M_u = (27.03 \times 1.85) - (7.2 \times 0.5)\,1.60 - (17.36 \times 1.35 \times 0.675) = 28.42$ kN.m

Effective depth provided = d = 125 mm.

$$\left(\frac{M_u}{bd^2}\right) = \left(\frac{28.42 \times 10^6}{10^3 \times 125^2}\right) = 1.82$$

Referring & Table-2 of SP: 16, read out, $p_t = \left(\frac{100A_{st}}{bd}\right) = 0.574$

$$\therefore \quad A_{st} = \left(\frac{0.574 \times 10^3 \times 125}{100}\right) = 717.5 \text{ mm}^2$$

Provide 12 mm diameter bars at 150 mm centres in the form of closed ties as shown in Fig. 12.11 (c)

Distribution bars of 8 mm diameter at each bend as shown in Fig. 12.11(c)

e) Design of Landing Slabs

Factored load on landing slab = 14.4 kN/m

Load from going = $(0.5 \times 17.36 \times 3.7) = 32.1$ kN/m
Total load = 46.5 kN/m

Effective depth = $(160 - 25) = 135$ mm
Effective span = 2.135 m
$M_u = (0.125 \times 46.5 \times 2.135^2) = 26.5$ kN.m

$$\left(\frac{M_u}{bd^2}\right) = \left(\frac{26.5 \times 10^6}{10^3 \times 135^2}\right) = 1.45$$

Referring to Table-2 of SP: 16, $p_t = \left(\frac{100A_{st}}{bd}\right) = 0.443$

$$A_{st} = \left(\frac{0.433 \times 1000 \times 135}{100}\right) = 598 \text{ mm}^2$$

Provide 10 mm diameter bars at 130 centres parallel to risers at bottom and provide nominal reinforcements at top.

12.5 EXAMPLES FOR PRACTICE

1) Design a Staircase flight for an office type building to suit the following data:
Height between floors = 4
Mid landing is cantilevered out and the width is 1.5 m
Tread = 300 mm and rise = 150 mm
Adopt M-20 grade concrete and Fe-415 HYSD bars.
Sketch the details of reinforcements in the stair flight.

2) A Staircase flight comprises of independent cantilevered slabs from a reinforced concrete wall. Assuming the risers of 150 mm and treads of 300 mm, width of flight as 1.7 m, design a typical tread slab. Assume the live loads specified in IS:875 code loading standards for an office building. Use M-20 grade concrete and Fe-415 grade reinforcements.

3) Design the waist slab type staircase consisting of a straight flight of stairs resting on two stringer beams along the two sides. Assume the span of the slab as 2m with risers of 160 mm and treads of 270 mm. Live load = 3 kN/m^2. Adopt M-20 grade concrete and Fe-250 grade steel.

4) Design a suitable open well type staircase for a multistoried building complex using the following data: -
Interior dimensions of staircase room is 3.25 m by 3.25 m
Staircase flights are supported on 200 mm thick masonry walls on all sides.
Height between floors = 3 m

Risers are 150 mm and treads are 250 mm
Live load = 4 kN/m^2
Materials: M-20 grade concrete and Fe-415 HYSD bars.

5) Design a dog legged staircase in a public building to be located in a staircase room 6 m long and 3 m wide.
Height between floors = 3.6 m
Live load = 4 kN/m^2
The stairs are supported on beams over walls and sides of steps are built into the Wall by 120 mm. Adopt M-20 grade concrete and Fe-415 reinforcements.

6) Design a tread-riser type staircase flight between the landings 1.5 m long in the direction of span. Adopt 10 treads of 300 mm and risers of 150 mm in the flight. The landings are built into the reinforced concrete walls. Adopt a live load of 5 kN/m^2. Use M-20 grade concrete and Fe-415 Grade HYSD bars. Sketch the details of reinforcements in the staircase.

7) Design a waist slab type dog legged staircase for an office building using the following data:
Height between floors = 3.2 m
Tread = 270 mm and riser = 160 mm
Width of flight = landing width = 1.25 m
The stairs are supported on 300 mm load bearing masonry walls at the outer edges of the landing, parallel to the risers.
Materials: M-20 grade concrete and Fe-415 HYSD bars.

CHAPTER 13

Design of Corbels (Brackets) and Nibs

13.1 INTRODUCTION

Corbels are short brackets projecting from the columns, generally provided to support rails, which transmit heavy loads from moving cranes in heavy-duty factory workshops. A typical Corbel subjected to loads at a short distance from the face of the column is shown in Fig. 13.1 (a) Corbels are also provided at the Cantilever end of girders in double cantilever balanced reinforced concrete bridges to supports the end spans of the bridge[94].

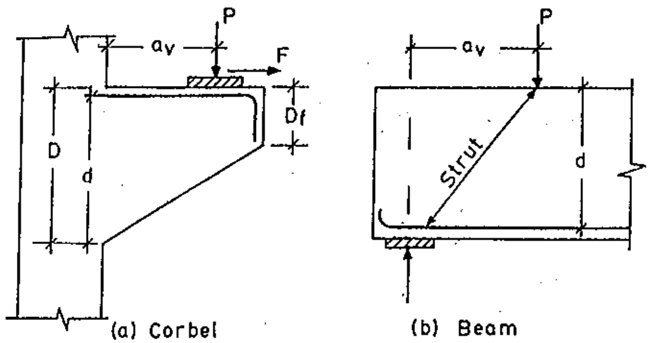


Fig. 13.1 Shear Span/Depth Ratio in Carbels and Beams

Corbels are short cantilevers whose shear span/depth (a_v/d) ratio is less than 1.0 and the depth (D_f) at the end face is not less than one half of the depth D_s at support. In the case of corbels, the load transfer at support is mainly by 'Strut' action than by simple flexure as shown in Fig. 13.1 (b). The revised IS:456-2000 code does not specify any method for the design of corbels except prescribing the enhanced shear strength of concrete near the supports.

However the British Code BS:8110[95], based on several research investigations has recommended some design principles which are outlined in the subsequent sections.

13.2 SHEAR SPAN/DEPTH RATIO AND SHEAR RESISTANCE

In the case of corbels, heavy loads are transmitted very near to the supporting column and the shear resistance of reinforced concrete members is different from that in which the loads are applied for away from the supports. The shear resistance of concrete depends upon the shear span/depth (a_v/d) ratio and it varies as shown in Fig. 13.2. As the shear span/depth ratio increases from 0.5 to 2, the shear strength of concrete decreases rapidly as indicated by the ratio of enhanced shear strength to the normal shear strength (τ_m/τ_c) which decreases from 4 to 1.

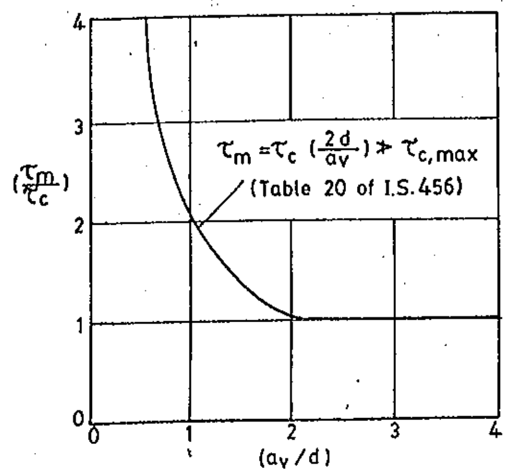


Fig. 13.2 Influence of Shear Span/Depth Ratio of Enhanced Shear/Strength

The IS: 456-2000 Code considers this enhancement of shear strength in clause 40.5.1 and the reinforcements from sections close to the supports are designed from the balance shear as per clause 40.5.2. The maximum shear strength τ_m is limited to the values specified in Table -20 of IS: 456 code. However, the British code[95] restricts the value of the maximum shear strength of concrete (τ_m) to 0.8 $\sqrt{f_{ck}}$ subject to a maximum value of 5 N/mm^2.

13.3 DIMENSIONING OF CORBELS

The initial dimensions of the corbel is based on the permissible bearing stresses in concrete which are compiled by Varghese[96] based on the British and Indian standard code recommendations. The bearing stress is limited to the following values.

1) Bearing stress with no packing material not to exceed $0.4 f_{ck}$
2) Bearing stress on cement mortar packing not to exceed $0.6 f_{ck}$
3) Bearing stress on steel plate cast into concrete not to exceed $0.8 f_{ck}$ (British Code)

   The width of the corbel is determined by considering the permissible bearing stress. If a plate is used, the size of the bearing plate is to be calculated by limiting the bearing stress to $0.8 f_{ck}$ (Provided the horizontal force acting is less than 10 percent of the vertical force).
4) The shear span/depth ratio(a_v/d) should preferably restricted to 0.6 and it should in no case exceed 1.0.
5) The depth of corbel at the free end should be not less than one half of the depth at the support.

13.4 ANALYSIS OF FORCES IN A CORBEL

The design of Corbels is based on the assumption of strut action recommended by both the British code BS: 8110 and the American Concrete Institute Code ACI–318[97].

The forces acting on a corbel is shown in Fig. 13.3. The vertical force F_v is in equilibrium under the action of the horizontal tensile force F_t in steel reinforcement and the inclined compressive force F_c developed in concrete simulating strut action.

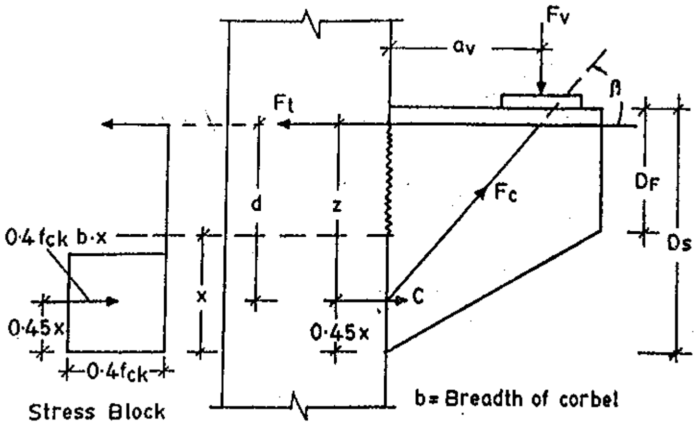


Fig. 13.3 Forces in a Corbel

The following notations are used for the analysis of forces in corbels.

a_v = distance of vertical force from the face of the support.
b = breadth of the corbel

d = effective depth of Corbel at Support
F_v = applied Vertical Load
F_t = tension in the horizontal direction
F_c = compression developed in concrete due to strut action
β = angle of inclination of force Fc to the horizontal
D_F = depth of Corbel at free end
D_S = depth of Corbel at Support

From Triangle of forces, we have

$$F_t = F_v(a_v/z)$$

$$F_c = \left[\frac{F_v\sqrt{a^2+z^2}}{z}\right] \quad \ldots(13.1)$$

Let x = height of compression concrete at the support section

Using the stress block recommended in BS: 8110 (British code),

$$z = (d - 0.45\,x)$$

Hence $$x = 2.2(d - z)$$

The force F_c acts over an area normal to direction of action given by $x.\cos\beta$

Hence $$F_c = 0.4\, f_{ck}\, b\, (x.\cos\beta) \quad \ldots(13.2)$$

And $$\cos\beta = \left\{\frac{a_v}{\sqrt{a_v^2+z^2}}\right\}$$

Substituting for x and $\cos\beta$ in Eq. (13.2)

We have the relation,

$$F_c = \left\{[0.88\, f_{ck}\, b(d-z)]\frac{a_v}{\sqrt{a_v^2+z^2}}\right\} \quad \ldots(13.3)$$

Equating Eqs (13.1) and (13.3) we have the relation,

$$F_v\,(a^2 + z^2) = 0.88\, f_{ck}.b.d\,[1 - (z/d)]\,a.z$$

Substituting, $$\left(\frac{F_v}{0.88\, f_{ck}\, bd}\right) = k \quad \text{and} \quad (a/d) = \alpha \quad \ldots(13.4)$$

Combining Eqs. (13.1) and (13.4), the resulting equation is expressed as

$$\left(\frac{z}{d}\right)^2 - \left(\frac{\alpha}{\alpha+k}\right)\left(\frac{z}{d}\right) + \left(\frac{k}{k+\alpha}\right)\left(\frac{a_v}{d}\right)^2 = 0 \quad \ldots(13.5)$$

For any given value of $\left(\frac{a_v}{d}\right)$ and $\left(\frac{F_v}{f_{ck}bd}\right)$ or $\left(\frac{\tau}{f_{ck}}\right)$ the values of the ratio

(z/d) can be computed using the equation (13.5). The values can directly be read out using the graphical chart recommended by Varghese[96], shown in Fig. 13.4.

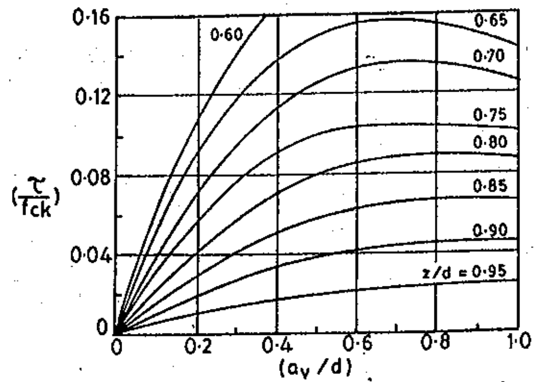


Fig. 13.4 Chart for (z/d) Values for Design of Corbels

For known values of (a_v/d) and $\left(\frac{F_v}{f_{ck}bd}\right)=\left(\frac{\tau}{f_{ck}}\right)$ the ratio of (z/d) can be directly obtained from the chart.

Knowing the value of z, the values of x, F_t and the strain ε_s can be easily computed facilitating the design of reinforcement in the corbel.

13.5 DESIGN PROCEDURE OF CORBELS

The following stepwise procedure is recommended for the design of corbels.

1) Breadth of bearing plate

Based on design bearing pressure of $0.8 f_{ck}$, and the length of bearing plate being equal to the width of column, the width of bearing plate is determined.

2) Corbel depth at support

Due to enhanced shear strength near supports permitted by the IS: 456 code, assume a suitable value for τ_c, nearer to $\tau_{c,max}$ [Table-20 of IS: 456] but not exceeding this value and compute the effective depth 'd' at the support section using the relation,

$$d=\left[\frac{F_v}{\tau_c . b}\right]$$

Hence overall depth = $D_s = (d + \text{effective cover})$

3) Check for Corbel dimensions

The value of (a_v/d) should preferably be less than 0.6 but not greater than 1.0.

4) Determination of Lever arm depth (z)

Compute the value of 'z' from Eq. (13.5) or by using the graphical chart of Fig. 13.4.

Also compute $x = 2.22d\,(1 - z/d)$ and Check for the ratio of (x/d).

The Limiting value of $(x/d) = 0.53$ for Fe-250 steel and $(x/d) = 0.48$ from Fe-415 HYSD bars.

If the value of (x/d) is greater than the limiting value, adequate steel should be provided in compression also. The support steel for main reinforcement and shear reinforcements will satisfy this condition.

5) Computation of Force (F_t)

$$F_t = F_v\,(a_v/z)$$

Also according to the British Code BS: 8110, F_t should be at least equal to $(F_v/2)$.

6) Area of Main Reinforcement

The stress in steel f_s Corresponds to the strain ε_s.
Using the value of x obtained in step- 4, compute

$$\varepsilon_s = \varepsilon_c\left(\frac{d-x}{x}\right)$$

Where $\varepsilon_c = 0.0035$ & compute f_s and $A_{st} = \left(\frac{F_t}{f_s}\right)$

If there is any horizontal force F_h, then the area of steel is computed as,

$$A_{st} = \left(\frac{F_t + F_h}{f_s}\right)$$

7) Check for Minimum and Maximum percentages of steel

The area of steel A_{st} should not exceed 1.3 percent and not less than 0.4 percent of the value of 'bd'. If it exceeds the maximum limit, increase the depth (D_s) and redesign.

8) Area of horizontal shear reinforcement (A_{sh})

$$A_{sh} = (A_{st}/2)$$

The shear reinforcements are provided as closed loops in the upper two-third portion of the total depth of corbel at support.

9) Check for shear

Knowing the percentage of steel ($100\,A_{st}/bd$), the exact value of the allowable shear stress is given by

$$\tau_m = \tau_c\,(2d/a_v)$$

The support section is checked for safety against shear.

10) Reinforcement detailing

The detailing of reinforcements in the corbel should conform to clause 7.7 and Fig. 7.18 and 7.19 of SP: 34[98] which are reproduced as Figs. 13.5 and 13.6 in the text.

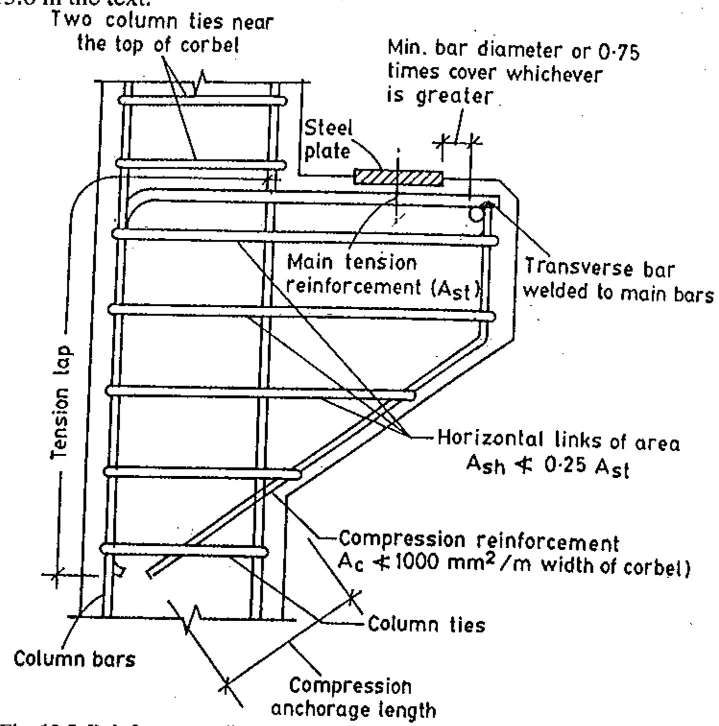


Fig. 13.5 Reinforcement Details in Corbel With Main Reinforcement of 18 mm Diameter or More (SP:34)

13.6 DESIGN OF NIBS (BEAM SHELVES)

In prefabricated structural systems, the reinforced concrete walls or

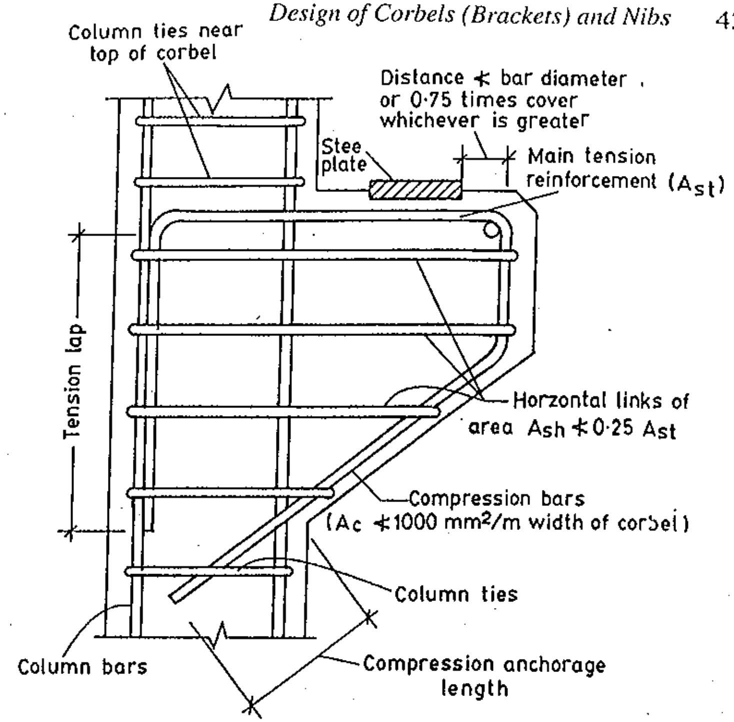


Fig. 13.6 Reinforcement Details in Corbel With Main Reinforcement of 16 mm Diameter and Less (SP:34)

columns are provided with nibs or beam shelves to support floor units comprising slabs and beams. Continuous nibs less than 300mm in depth are designed as cantilever slabs with suitable reinforcements provided in the form of horizontal loops to resist the shear forces applied close to the supports similar to corbels. The Following guide lines recommended by the Cement and Concrete association, U.K is useful in the design of nibs.

1) The bending moment and enhanced shear strength is computed by considering the distance a_v representing the line of action of the load as the distance from the centre line of the nearest vertical leg of the stirrup in the beam to the outer face of the main horizontal reinforcement of the nib as shown in Fig. 13.7.
2) Additional ties or links are provided as hangers in the beam connected to the nib. The load on the nib has to be resisted by the compression zone of the supporting beam. Hanger bars are used to resist not only shear in the beam but also to transfer the load from the nib to the compression side of the beam.

The additional reinforcement area (in addition to the area necessary to support the shear force) for the hangers is computed using the relation,

$$A_s = \left(\frac{F_v}{0.87 f_y}\right)$$

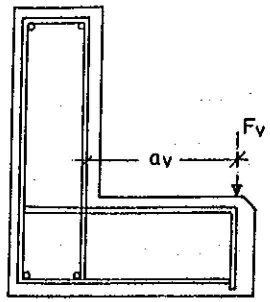


Fig. 13.7 Distance a_v for Bending Moment and Shear Force

3) According to the recommendations of the Cement and concrete Association, U.K, the cantilever portion of the nibs should be reinforced with both horizontal and vertical systems of reinforcements as shown in Fig.13.8. The area of horizontal nib steel is given by

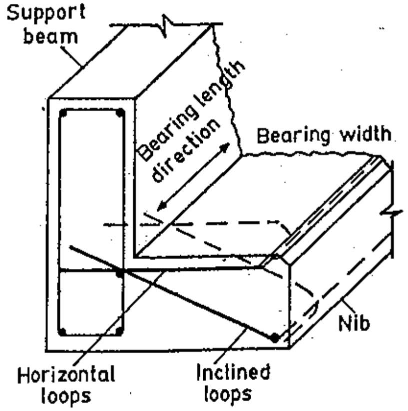


Fig. 13.8 Reinforcements in Nibs With Large Loads

$$A_{sh} = \left(\frac{F_v a_v}{0.87 f_y z}\right)$$

Where a_v = Distance of the Load F_v from the nearest hanger bar.
z = Lever arm

If inclined loops are used the area of the inclined nib reinforcement is given by

$$A_{si} = \left(\frac{F_v}{0.87 f_y \sin\theta}\right)$$

Where θ = angle of the inclined loop with the horizontal (Fig.13.8)

These reinforcements are to be held securely in position by using additional fixing bars running parallel to the nib as shown in Fig.13.8.

4) In the case of nibs supporting light loads the horizontal reinforcement may be bent as a loop as shown in Fig. 13.9.

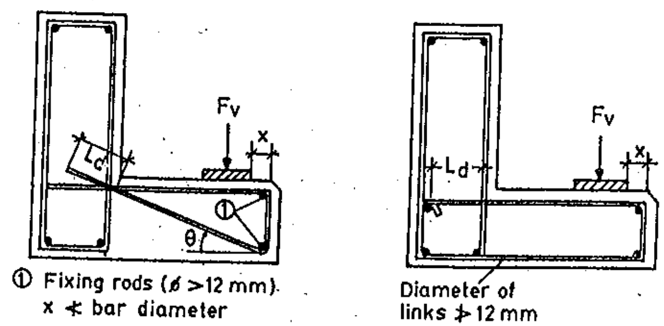


Fig. 13.9 Reinforcements in Nibs With Light Loads

13.7 DESIGN EXAMPLES

1) Design a corbel to support a factored load of 400 kN at a distance of 200 mm from the face of a column 300 mm by 400 mm. Adopt M-25 grade concrete and Fe-415 grade HYSD bars. Sketch the details of reinforcements in the Corbel.

a) Data

Factored load = F_v = 400 kN
Width of column = Length of Corbel = 300 mm
Shear span = a_v = 200 mm
Materials: M-25 grade Concrete (f_{ck} = 25 N/mm^2)
Fe–415 HYSD bars (f_y = 415 N/mm^2)

b) Dimensions of Corbel

Bearing Length = Width of Column = 300 mm
Using a bearing plate of length = 300 mm,
Bearing pressure = $0.8 f_{ck} = (0.8 \times 25) = 20$ N/mm^2

$$\text{Width of plate} = \left[\frac{400 \times 10^3}{300 \times 20}\right] = 66.66 \text{ mm}$$

Provide a minimum width of 100mm and adopt a bearing plate 100 × 300 mm

c) Estimation of depth (d)

From Table-19 of IS: 456, For M-25 grade concrete $\tau'_{c,max} = 3.1$ N/mm^2

$$\therefore \quad d = \left(\frac{F_v}{\tau.b}\right) = \left(\frac{400 \times 10^3}{3.1 \times 300}\right) = 430 \text{ mm}$$

Adopt effective depth = d = 450 mm
Total depth at support is

$$\begin{aligned} D_s &= (d + \text{cover} + {}^1/_2 \text{ diameter of bar}) \\ &= (450 + 40 + 10) \\ &= 500 \text{ mm} \end{aligned}$$

Depth at face = $D_f = (0.5\, D_s) = (0.5 \times 500) = 250$ mm

d) Check for strut action

$$\text{Ratio} \left(\frac{a_v}{d}\right) = \left(\frac{200}{450}\right) = 0.44 < 0.6, \text{ Hence acts as a corbel.}$$

e) Determination of Lever arm (z)

Using the Eq. (13.5),

$$\left(\frac{z}{d}\right)^2 - \left(\frac{\alpha}{\alpha + k}\right)\left(\frac{z}{d}\right) + \left(\frac{k}{\alpha + k}\right)\left(\frac{a_v}{d}\right)^2 = 0$$

$$\text{Where} \quad k = \left(\frac{F_v}{0.88 f_{ck}\, bd}\right) = \left(\frac{400 \times 10^3}{0.88 \times 25 \times 300 \times 450}\right) = 0.134$$

$$\alpha = \left(\frac{a_v}{d}\right) = \left(\frac{200}{400}\right) = 0.444$$

$$\therefore \quad \left(\frac{\alpha}{\alpha + k}\right) = \left(\frac{0.444}{0.444 + 0.134}\right) = 0.77$$

$$\text{and} \quad \left(\frac{k}{\alpha + k}\right) = \left(\frac{0.134}{0.44 + 0.134}\right) = 0.23$$

Substituting, we have

$$\left(\frac{z}{d}\right)^2 - 0.77\left(\frac{z}{d}\right) + 0.23(0.444)^2 = 0$$

$$\text{Solving,} \quad \left(\frac{z}{d}\right) = 0.7$$

f) Check for 'z' using chart

$$\left(\frac{\tau}{f_{ck}}\right) = \left(\frac{F_v}{f_{ck} bd}\right) = \left(\frac{400 \times 10^3}{25 \times 300 \times 450}\right) = 0.118$$

From Fig. 13.4, for $(a_v/d) = 0.465$ and $(\tau/f_{ck}) = 0.118$, read out $(z/d) = 0.7$
Therefore, $z = (0.7 \times 450) = 315$ mm
$(d - z) = 0.45\, x$ (Refer Fig. 13.3)
$\therefore \ (450 - 315) = 0.45\, x$
$\therefore \ x = 300$ mm

$$\left(\frac{x}{d}\right) = \left(\frac{300}{450}\right) = 0.66 > \text{the limiting value of 0.48 for Fe-415 grade bars.}$$

Hence, adequate steel should be used in compression also. The support reinforcement for main steel and horizontal links used as shear reinforcement will satisfy this condition.

g) Resolution of Forces

$$F_t = \left(\frac{F_v \times a_v}{z}\right) = \left(\frac{400 \times 200}{315}\right) = 254 \text{ kN}$$

$$F_t \text{ not less than } 0.5\, F_v = \left(\frac{400}{2}\right) = 200 \text{ kN}$$

h) Area of Tension Reinforcement

$$A_{st} = \left(\frac{F_t + F_h}{f_s}\right) \quad \text{But } F_h = 0$$

$$\varepsilon_s = \left[\frac{0.0035\,(d-x)}{x}\right] = \left[\frac{0.0035\,(450-300)}{300}\right] = 0.00175$$

From Fig.3 of SP: 16 read out the stress in steel (f_s) corresponding to strain $\varepsilon_s = 0.00175$

$$f_s = 320 \text{ N/mm}^2$$

$$\therefore \quad A_{st} = \left(\frac{254\times 10^3}{320}\right) = 794 \text{ mm}^2$$

Use 4 bars of 16 mm diameter ($A_{st} = 804$ mm²)

i) Check for minimum and maximum reinforcement

$$\left(\frac{100A_{st}}{bd}\right) = \left(\frac{100\times 804}{320}\right) = 0.595 > 0.4 \text{ but} < 1.3 \text{ percent}$$

Hence satisfactory

j) Area of shear reinforcement

$A_{sv\,(min)} = (^1/_2\, A_{st}) = (804/2) = 402$ mm²

Provide 4 numbers of 10 mm diameter 2 legged horizontal links in the upper two third depth ($A_{sv} = 628$ mm²).

$$\text{Spacing of links} = S_v = \left(\frac{2\times 450}{3\times 4}\right) = 75 \text{ mm}$$

k) Shear Capacity of Section

Using Table -19 of IS: 456, for M-25 grade concrete and 0.595 percent steel,

$$\tau_c = 0.53 \text{ N/mm}^2 \quad \text{and} \quad \left(\frac{a_v}{d}\right) = 0.444$$

$$\therefore \quad \text{Enhanced shear strength} = \left(\frac{2\times 0.53}{0.44}\right) = 2.4 \text{ N/mm}^2$$

Hence, shear capacity of concrete is computed as,

$$V_c = \left(\frac{2.4\times 300\times 450}{1000}\right) = 324 \text{ kN}$$

$$\text{Shear capacity of steel} = \left(\frac{0.87\,f_y A_s d}{S_v}\right) = \left(\frac{0.87\times 415\times 157\times 450}{75\times 1000}\right) = 340 \text{ kN}$$

∴ Total shear capacity = (324 + 340) = 664 kN > 400 kN
Hence, design is safe

l) Reinforcement Details

The details of reinforcements in the corbel in shown in Fig. 13.10.

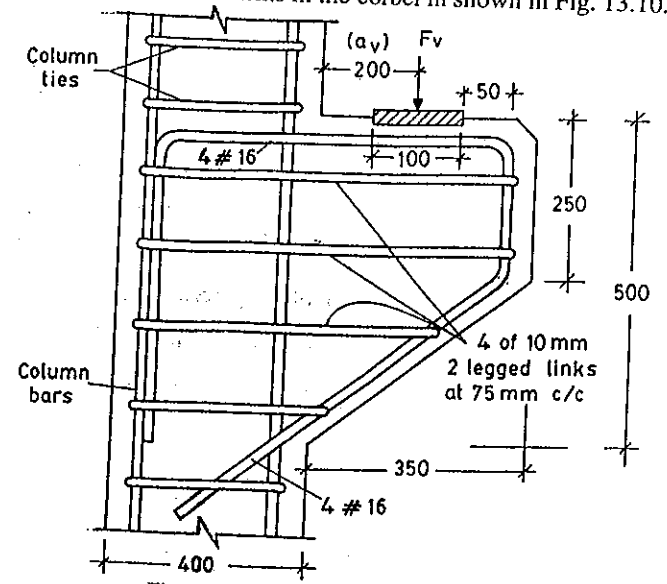


Fig. 13.10 Reinforcement Details in Corbel

2) Design a continuous nib (beam support) projecting from an R.C.C wall to support a prefabricated slab unit transmitting a service shear force of 15 kN/m, assuming the following data.

a) Data

$F_v = (1.5 \times 15) = 22.5$ kN/m
Projection of nib = 200 mm
$a_v = 100$ mm
$f_{ck} = 30$ N/mm² and $f_y = 415$ N/mm²

b) Dimensions of Nib

Since the shear force is small, adopt an overall depth of nib = D = 200 mm
And effective depth = d = 150 mm

c) Bending Moments and Shear Forces

Maximum bending moment and at the face of R.C. wall is computed as

$$M = (F_v.a_v) = (22.5 \times 10^3 \times 100) = (2.25 \times 10^6)\ \text{N.mm}$$

$$V = F_v = 22.5\ \text{kN}$$

d) Reinforcements

Assuming a lever arm depth of $z = 0.8\,d = (0.8 \times 150) = 120$ mm

$$A_{st} = \left(\frac{F_v\,a_v}{0.87\,f_y\,z}\right) = \left(\frac{22.5 \times 10^3 \times 100}{0.87 \times 415 \times 120}\right) = 52\ \text{mm}^2/\text{m}$$

Provide minimum area of reinforcement of 0.4 percent.

$$\therefore \quad A_{st} = (0.004 \times 1000 \times 150) = 600\ \text{mm}^2/\text{m}$$

Adopt 10 mm diameter bars at 130 mm centres (A_{st} = 604 mm^2) both at top and bottom of the section.

e) Check for shear stress

$$V_u = 22.5\ \text{kN}$$

$$\left(\frac{100\,A_{st}}{bd}\right) = \left(\frac{100 \times 604}{1000 \times 150}\right) = 0.40$$

From Table -19 (IS: 456) for M-30 grade concrete,

$$\tau_c = 0.45\ \text{N/mm}^2$$

But, $$\tau_v = \left(\frac{V_u}{bd}\right) = \left(\frac{22.5 \times 10^3}{1000 \times 150}\right) = 0.150\ \text{N/mm}^2$$

Since, $\tau_c > \tau_v$ shear stresses are within safe permissible limits.

f) Details of Reinforcements

The reinforcement details in the nib is shown in Fig. 13.11

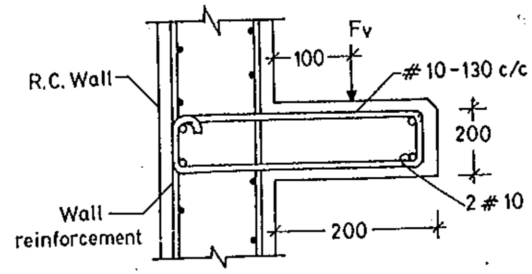


Fig. 13.11 Reinforcement Details in Nib

13.8 EXAMPLES FOR PRACTICE

1) Design a Corbel for a factory shed column 500 mm by 300 mm to support a vertical ultimate load of 500 kN, with its line of action 200 mm from the face of the column. Assume M-20 grade concrete and Fe-415 grade HYSD bars for the construction.

2) Design a Corbel to support a reaction due to a characteristic dead load of 80 kN and live load of 120 kN. This reaction acts at 200 mm from the face of the column which is 350 mm square in section. There is also a horizontal reaction of 30 kN due to shrinkage restraint of beams etc. Design the Corbel and sketch the details of reinforcement. Assume f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2.

3) A continuous concrete nib is to be provided to a reinforced concrete beam cast in situ.The nib is to support a series of precast floor units 450 mm wide and 150 mm deep. These floor units have a clear span of 3.5m and exert an ultimate total reaction of 25 kN per metre length on the nib. The dry bearing of the floor units on the beam can exert a pressure of 0.4 f_{ck}. Assuming that an allowance of 20 mm has to be provided for spilling and an allowance of 25 mm has to be made for the face of column for inaccurate dimension, design a suitable nib and sketch the details of reinforcements. Assume f_{ck} = 30 N/mm^2 and f_y = 415 N/mm^2.

CHAPTER 14

Pile and Raft Foundations

14.1 INTRODUCTION

Reinforced concrete piles are generally used in soft soils having very low bearing capacity. Buildings and bridges over tank beds are invariably built over pile foundations which are classified under deep foundations.

Concrete piles are further classified as
a) Cast in situ
b) Precast piles

In the case of cast in situ piles, a steel shell is driven first to the required depth and concreting is done after placing the reinforcement cage in the hole. If the shell is left in place, it is called a shell pile. If the shell is removed it is referred to as shelllees cast in situ pile. The world's longest shell pile is provided at Walt Disney world in Florida, U.S.A[98]. The length of the pile is 114 m. During 1950 to 1960, cast in situ piles were commonly used since the technique of precasting was not well developed. With the introduction of better quality cement and precasting techniques, now days precast piles are invariably preferred in place of cast in situ piles since multistory buildings and bridge structures generally involve foundations under water or in soils with high water table.

Precast piles can be made with a high degree of quality control regarding dimensions and strength and hence have superior structural properties in comparison with cast in situ piles. Precast piles can be cast to various shapes such as,

i) Circular,
ii) Square,
iii) Rectangular
iv) Octagonal.

Generally circular and square section piles are preferred to other shapes.

When more than one pile is used for the foundation, the group of piles are connected at the top by a pile cap to form a single unit. The piles are arranged symmetrically about the axis of the columns so that the loads are distributed uniformly to all the piles. Pile caps are invariably used to support very heavy columns of storied buildings and the piers of bridges.

14.2 DESIGN OF PILE FOUNDATIONS

Reinforced concrete piles are designed as columns to resist the loads transmitted from the structure. Structural design of reinforced concrete piles is influenced by the loads acting on the pile, the depth of the pile below the ground level, type of soil, the grade of concrete and quality and type of steel used as reinforcements. The precast piles are designed for handling and driving stresses together with loads to be sustained under service conditions.

The minimum longitudinal reinforcement in the pile should be not less than the following values:

1) 1.25 percent of the cross sectional are of the pile for piles having length upto 30 times their least later dimension.
2) 1.5 percent of the cross sectional area of the pile for piles having length between 30 to 40 times their least lateral dimension.
3) 2 percent of the cross sectional area of the pile for piles having length greater than 40 times their least lateral dimensions.

The lateral reinforcement comprises of ties or links of not less than 6 mm diameter and the spacing of the links or spirals shall be not greater than 150mm. Also the spacing of the ties should not exceed half the least lateral dimensions.

The minimum steel requirements of a typical precast concrete pile is shown in Fig. 14.1 based on the guide lines specified in SP:34[99].

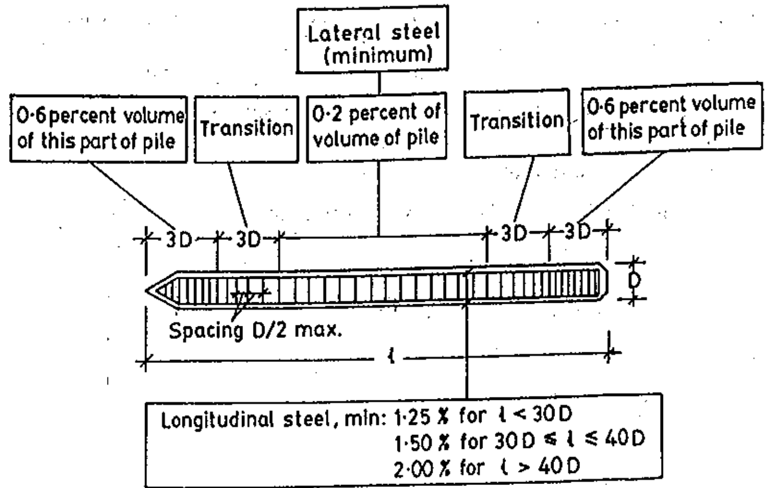


Fig. 14.1 Minimum Steel Requirements in Precast Concrete Piles

14.2.1 Design procedure of Piles

The following stepwise procedure may be followed in the design of piles.

1) The number of piles is decided based on the total load transmitted to the foundation. The service load on each pile is evaluated and the factored load is computed.
2) The size of the pile is selected depending upon the service load. For loads in the range of 400 to 600 kN, 300 mm square piles will be sufficient. Depending upon the increase in the load, the size of the pile is increased.
3) The length of the pile depends upon the depth of hard strata below ground level for bearing piles and the fiction developed in the case of cohesive soils. The length of pile above ground is generally around 0.6 m to cast the pile cap and the columns.
4) If the slenderness ratio of the pile is greater than 12, it is designed as a long column, considering the reduction co-efficient applied to the permissible stresses.
5) The longitudinal reinforcement designed should be more than the minimum percentage of steel specified in section 14.2
6) The lateral reinforcement consisting of ties or links and spirals and their percentages expressed as percent of volume of the pile should be not less than the values specified in Fig. 14.1. The detailing of the larger percentage volume of lateral reinforcement near the pile head and pile end is of particular significance due to the driving stresses developed at the pile shoe end and pile head.
7) Clear cover to all main reinforcements in pile shall be not less than 50 mm.
8) Steel forks (spacer bars) in pairs are provided at regular intervals to hold the main reinforcement in position. A steel shoe made up of mild steel plates is embedded at the pile end to facilitate easy driving of the pile into the soil strata.

Typical details of reinforcement requirements in a precast concrete pile is shown in Fig. 14.2.

14.3 DESIGN OF PILE CAPS

The load from the columns and piers in bridges are transmitted to the pile foundation from a pile cap. When a group of piles are used, they are connected together to form a single unit through pile caps above the ground level.

The salient parameters in the design of pile caps are:

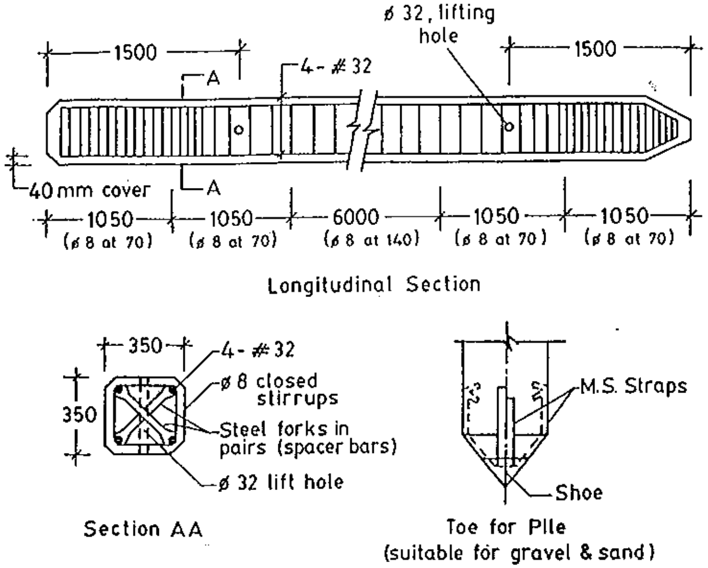


Fig. 14.2 Typical Details of Reinforcement in a Precast Concrete Pile

1) Shape of pile cap which is influenced by the number and spacing of piles.
2) Depth of pile cap, which should be sufficient to resist the bending moment and shear forces, developed due to the loads.
3) Computation of bending moment, shear force and tensile force due to strut action.
4) Amount of reinforcement and its arrangement

The following guidelines based on the British practice and Indian Standard Code (IS: 2911)[100] recommendations are useful in the design of pile and pile caps.

1) Shape of pile cap

Whenever number of piles are used symmetrically, square or rectangular shaped pile caps are commonly employed. When odd numbers of three piles spaced asymmetrically are used, triangular shaped pile caps are used.

Minimum spacing of piles = 2.5 to 3 d_p
Where, d_p = diameter of the piles

For accommodating deviations in driving of piles, the size of pile cap is made 300 mm more than the outer to outer distance of the exterior piles.

Minimum cover = 60 to 80 mm

Another criterion in arriving at the shape of the pile cap is to arrange the center of gravity of all the piles to coincide with the centroid of the pile cap. Based on these principles, the common shapes of pile caps used for two to nine piles are as shown in Fig. 14.3.

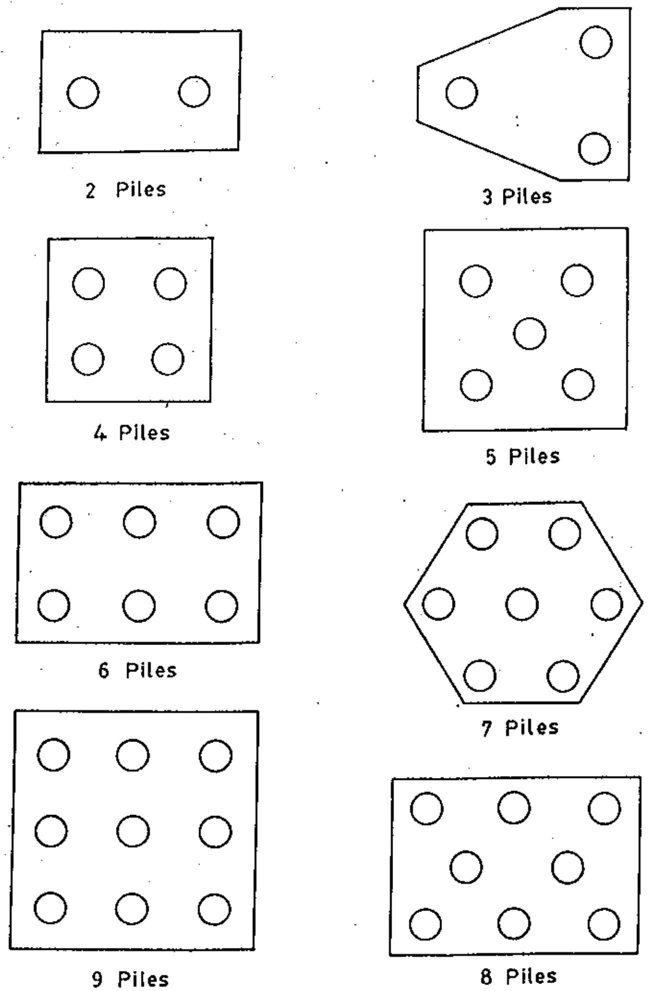


Fig. 14.3 Typical Shapes of Pile Caps

2) Depth of pile cap

Based on cost analysis an empirical relation has been recommended by Varghese[97] expressing the thickness of the pile cap as a function of diameter of the pile given by

$D = (2\,d_p + 100)$ mm for d_p not greater than 550 mm
$D = (1/3)(8\,d_p + 600)$ mm for d_p greater than or equal to 550 mm

Where, D = overall thickness of pile cap (mm)
d_p = diameter of pile (mm)

3) Design of Reinforcements in Pile Caps

The transfer of loads from the column to the pile cap and the piles depends upon the structural behaviour of the pile cap under the system of column loads and pile reactions.

The theories that are commonly used in the design of reinforcements in pile caps are grouped as,

a) Truss theory
b) Beam theory

Referring to Fig. 14.4(a), when the angle of dispersion of load 'θ' is less than 30° (tan 30° = 0.58), the value of shear span/depth ratio (a_v/d) is less than 0.6. Under these conditions, the load is transferred to the piles by strut action shown in Fig.14.4 (b) where AB is in compression and BC in tension. Experiments have shown that the truss action (similar to deep beams and corbels) is significant for ratios of (a_v/d) = 2.

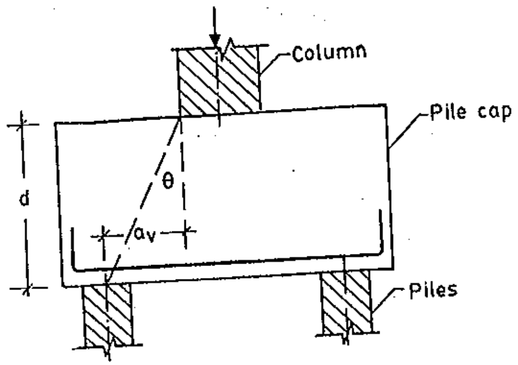
In the truss theory, the tensile force between the pile heads is assumed to be resisted by the reinforcements similar to the tie member of a truss and hence special care should be taken in detailing of the tension reinforcements and its anchorage at the ends.

When the spacing of piles is at greater intervals associated with thinner pile caps in which the shear span/depth (a_v/d) ratio is more than 2, flexural action is more predominant than truss action and hence the tensile reinforcement at the bottom of the pile cap is designed to resist the maximum bending moment as in an ordinary beam. However, the depth of pile cap should be checked for shear when designed by either of the two methods.

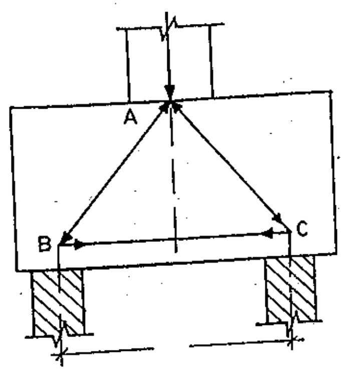
The arrangement of reinforcement in pile caps comprises of the following types of bars as shown in Fig. 14.5[96,98]

1) Main reinforcements located at the bottom of the pile cap in the direction XX bent up at the ends to provide adequate anchorage.
2) Main reinforcements placed at the bottom in the direction of YY also bent up at their ends.

3) Horizontal ties comprising of two to three layers of 16mm diameter bars as secondary reinforcement to resist bursting.
4) Vertical column starter bars which are L-shaped located at the level of the bottom main reinforcements.
5) Reinforcements of the pile are extended into the pile cap to provide the required development length in compression



(a) Load Transfer in Thick Pile Caps

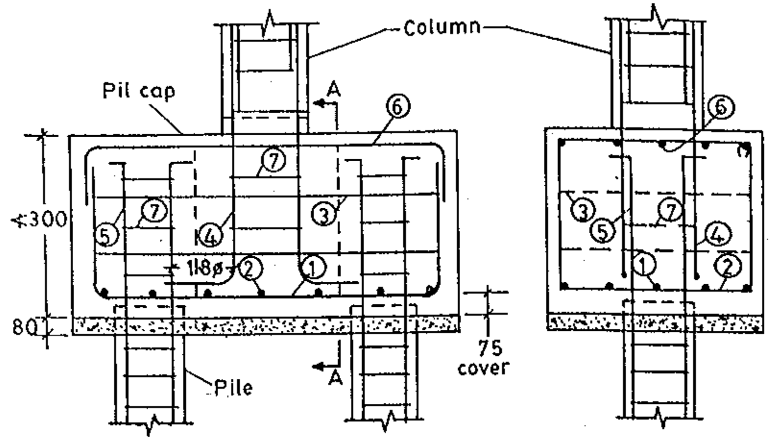


(b) Truss Action in Pile Caps

Fig. 14.4 Truss Theory of Design of Pile Caps

6) Reinforcements provided as compression steel in the pile cap at the top if required as per computations. They are tied to the bent up bottom bars to form a rigid cage before casting the pile cap.

7) Ties are provided to the pile reinforcements extended into the pile cap and column bars. The reinforcements detailing in pile caps is shown in Fig.14.5. The plan arrangement of reinforcements used in pile caps with different number of piles is shown in Fig. 14.6.



Bar mark : 1,2 - Main reinforcements
3 - Horizontal ties
4 - Column reinforcement starter bars
5 - Main bars in piles
6 - Top reinforcement in pile cap
7 - Ties for column and pile bars

Fig. 14.5 Reinforcement Details in Pile Caps

14.4 DESIGN EXAMPLES OF PILES AND PILE CAPS

1) The foundation for a structure comprising six piles of square cross section have to support a service load of 3600 kN. The piles are driven through a hard stratum and bear on hard rock. Design the reinforcements in the pile assuming the pile to be 6 m long and using M-20 grade concrete and Fe-415 HYSD bars. Sketch the details of reinforcements in the pile.

a) Data

Service load on each pile = (3600/6) = 600 kN
Ultimate load = (1.5×600) = 900 kN
Length of Pile = 6 m

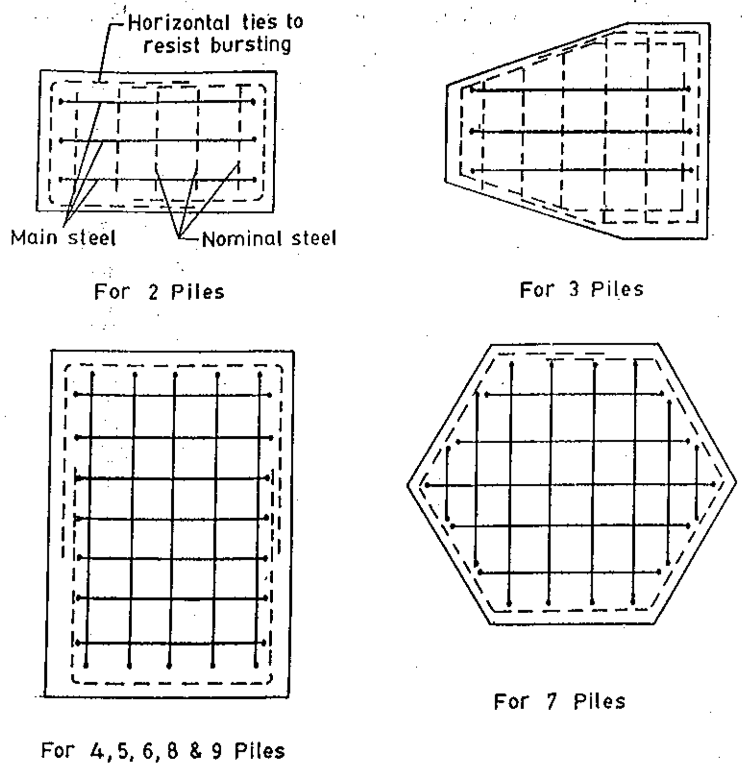


Fig. 14.6 Plan Arrangement of Reinforcement in Pile Caps

$f_{ck} = 20$ N/mm^2
$f_y = 415$ N/mm^2

b) Longitudinal Reinforcements

$$P_u = [0.4 f_{ck} A_g + (0.67 f_y - 0.4 f_{ck}) A_{sc}]$$

$$(900 \times 10^3) = (0.4 \times 20 \times 300^2) + [(0.67 \times 415) - (0.4 \times 20)] A_{sc}$$

Solving $A_{sc} = 666$ mm^2
But minimum longitudinal steel (Fig. 14.1) for piles < 30D < (30 × 300) < 9000 mm is
A_{sc} = 1.25 Percent of cross section of pile
= (1.25 × 300 × 300)/100 = 1125 mm^2
Provide 4 bars of 20mm diameter (A_{sc} = 1256 mm^2) with a clear cover of 50 mm.

c) Lateral reinforcement

Lateral reinforcement in the central portion of pile = 0.2 percent of gross volume.
Using 8 mm diameter ties,
Volume of one tie = 50 [4 (300 − 100)] = 40,000 mm^3. If p = pitch of tie,
Volume of pile per pitch length = (300 × 300 × p) = 90,000 p(mm^3)

Therefore, $$40,000 = \left(\frac{0.2}{100} \times 90000\, p\right)$$

Solving, $p = 222$ mm
Maximum permissible pitch = (D/2) = (300/2) = 150 mm
Hence, provide 8 mm diameter ties at 150 mm centres.

d) Lateral reinforcement near pile head

Spiral reinforcements is to be provided inside the main reinforcements for a length of (3 × 300) = 900 mm.
Volume of spiral = 0.6% of gross volume
Using 8mm diameter helical ties ($A_s = 50$ mm^2)

$$\text{Volume of spiral per mm length} = \left(\frac{0.6}{100} \times 300 \times 300 \times 1\right) = 540 \text{ mm}^3$$

If p = pitch of spiral with d = [300−100−40] = 160mm

$$p = \left(\frac{\text{Circumference of spiral}}{540}\right) = \left(\frac{\pi \times 160 \times 50}{540}\right) = 46.51 \text{ mm}$$

Provide 8 mm diameter spiral at a pitch of 45 mm for a length of 90mm near the pile head. The spiral is enclosed inside of the main reinforcements.

e) Lateral reinforcements near pile ends

Volume of ties = 0.6% of gross volume for a length = 3 D = (3 × 300) = 900 mm
Using 8 mm diameter ties, ($A_s = 50$ mm^2)
Volume of each tie = 50 [4 (300 − 100)] = 40,000 mm^3
If p = pitch of ties,

$$40,000 = \left(\frac{0.6}{100} \times 300 \times 300 \times p\right)$$

Solving, $p = 74.00$ mm
Provide 8mm diameter ties at 70 mm centres for a distance of 900 mm from the end of the pile both at top and bottom.

f) Spacer forks and lifting holes

Provide spacer forks in pairs of steel using 25 mm diameter bar spacers at 1500 mm centres. Provide 32 mm diameter holes at 1500 mm from ends. Reinforcement details in the pile is shown in Fig. 14.7.

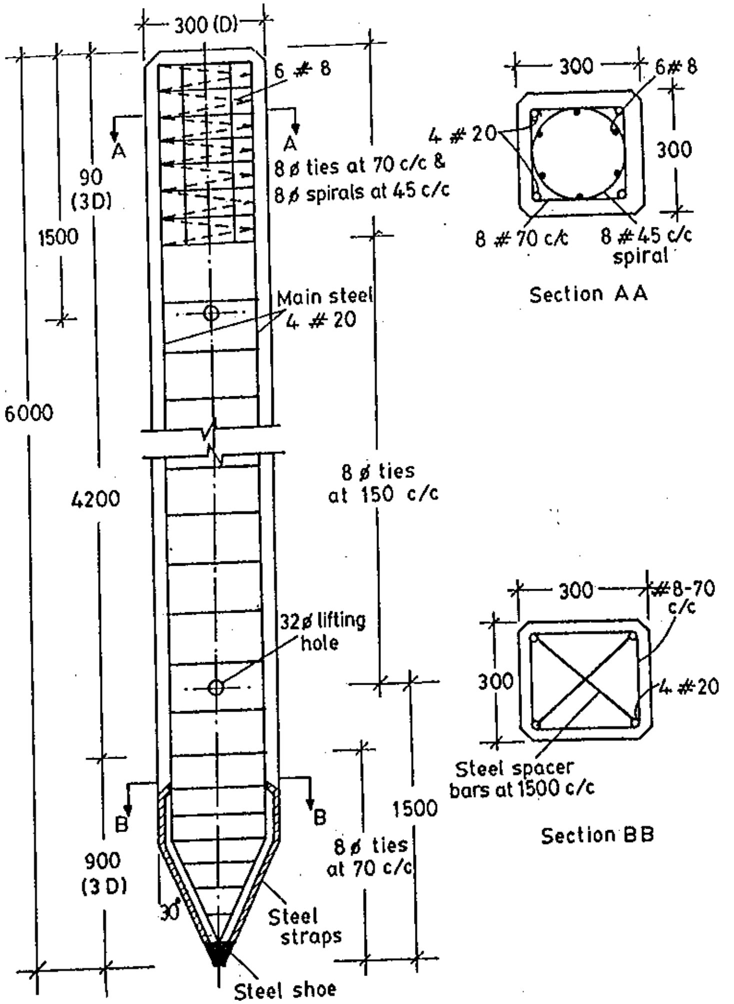


Fig. 14.7 Details of Reinforcement Details in Precast Pile

2) Design a suitable pile cap consisting of 4 piles of 300 mm by 300mm to support an R.C.C. column 500 mm by 500 mm carrying a service load of 2000 kN. The piles are arranged as shown in Fig.14.8(a). Adopt M-20 grade concrete and Fe-415 grade HYSD bars.

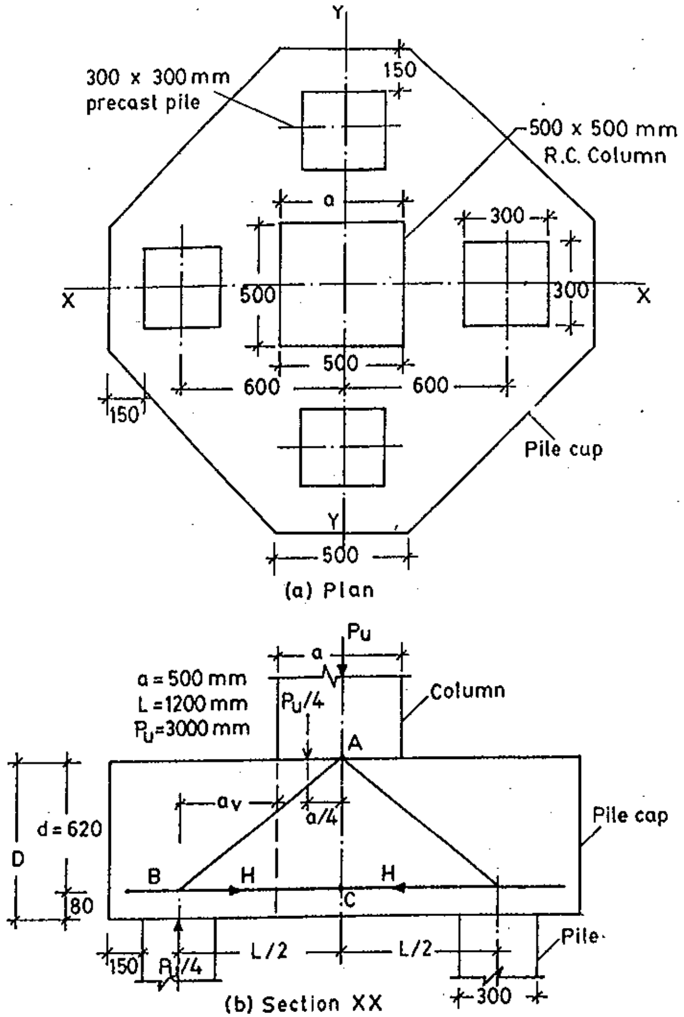


Fig. 14.8 Design of Pile Cap (Truss Action)

a) Data

Size of Column = 500 mm by 500 mm
Size of Piles = 300 mm by 300 mm
Service load on column = 2000 kN
Factored load = $P_u = (1.5 \times 2000) = 3000$ kN.
$f_{ck} = 20$ N/mm^2
$f_y = 415$ N/mm^2
Distance between c/c of piles = $L = 1200$ mm
Overall width of pile cap = (1200 + 300) = 1500 mm

b) Depth of pile cap

Total depth of pile cap = $D = (2\,d_p + 100)$
Where d_p = diameter or width of pile = 300 mm

$\therefore\ D = (2 \times 300 + 100) = 700$ mm

Assuming effective cover of 80 mm
Effective depth = $d = (700 - 80) = 620$ mm

c) Check for Truss action

Shear span = $a_v = (600 - 250) = 350$ mm
Effective depth = $d = 620$ mm

Hence, Ratio $\left(\frac{a_v}{d}\right) = \left(\frac{350}{620}\right) = 0.56 < 0.6$

Hence, Truss action is predominant

d) Design of Tension Steel

Referring to Fig. 14.8 (b)
Let H = tension in Steel
Taking moments about A

$$H.d = \frac{P_u}{4}\left[\frac{L}{2} - \frac{a}{4}\right]$$

$$H = \frac{P_u}{16d}[2L - a]$$

Substituting $P_u = 3000$ kN
$L = 1.2$ m
$a = 0.5$ m
$d = 0.62$ m

$$\therefore\quad H = \left(\frac{3000}{16 \times 0.62}\right)[(2 \times 1.2) - 0.5] = 575 \text{ kN}$$

$$\therefore\quad A_{st} = \left(\frac{575 \times 10^3}{0.87 \times 415}\right) = 1593 \text{ mm}^2$$

Adopt 4 bars of 25 mm diameter in XX and YY directions ($A_{st} = 1964$ mm^2) within a width (column) of 500 mm ($b = 500$ mm).
Percentage of steel provided is

$$p_t = \left(\frac{100 \times 1964}{500 \times 620}\right) = 0.63\% > 0.12\%$$

e) Check for shear

Nominal shear $= V_u = \left(\frac{P_u}{4}\right) = \left(\frac{3000}{4}\right) = 750$ kN

Nominal shear stress $= \tau_v = \left(\frac{V_u}{bd}\right) = \left(\frac{750 \times 10^3}{5005620}\right) = 0.24$ N/mm^2

Neglecting enhanced shear stress, refer Table-19 of (IS: 456-2000) and read out the permissible shear stress in Concrete τ_c for M-20 grade concrete as,

$\tau_c = 0.52$ N/mm^2 > τ_v. Hence Safe.

f) Check for Moment action

Maximum moment M_u at C is given by

$$M_u = \frac{P_u}{4}\left[\frac{L}{2} - \frac{a}{4}\right] = \left(\frac{3000 \times 10^3}{4}\right)\left[\frac{1200}{2} - \frac{500}{4}\right] = (356 \times 10^6) \text{ N.mm}$$

Compute parameter, $\left(\frac{M_u}{bd^2}\right) = \left(\frac{356 \times 10^6}{500 \times 620^2}\right) = 1.85$

Refer Table-2 of SP: 16 and read out the percentage of reinforcement as, $p_t = 0.584$.

$\therefore\quad A_{st} = (0.584 \times 500 \times 620)/100 = 1810$ mm^2 < A_{st} provided, hence safe.

g) The details of reinforcements as per standard practice are shown in Fig. 14.9.

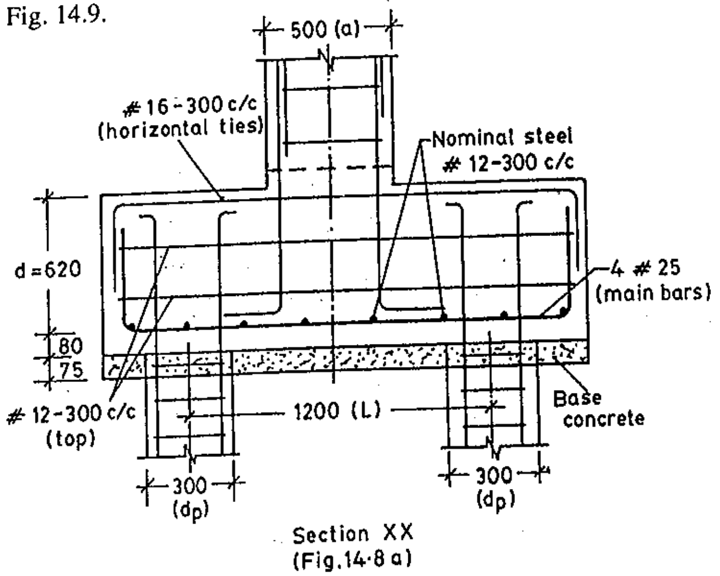


Fig. 14.9 Details of Reinforcement in Pile Cap

3) Design a pile cap for a group of two piles spaced 1.5 m apart. The piles are 400 mm diameter and the column transmits a factored load of 1000 kN and is of size 500 mm by 500mm. Adopt M-20 grade concrete and Fe-415 grade HYSD bars. Sketch the details of reinforcements.

a) Data

Size of Column = 500 × 500 mm
Size of Piles = 300 mm diameter
Factored load on Column = 1000 kN
$f_{ck} = 20$ N/mm^2
$f_y = 415$ N/mm^2
Distance between c/c of piles = L = 1500 mm

b) Pile Cap Dimensions

Allowing a cover of 150 mm from the edge of the pile and the column the overall dimension of pile cap is fixed.

Length of Pile Cap = (1500 + 300 + 300) = 2100 mm
Width of Pile Cap = (500 + 300) = 800 mm

Depth of Pile Cap = $D = (2d_p + 100) = (2 \times 300) + 100 = 700$ mm
Adopting a cover of 100 mm
Effective depth = $d = (700 - 100) = 600$ mm
The pile cap dimensions are shown in Fig. 14.10 (a)

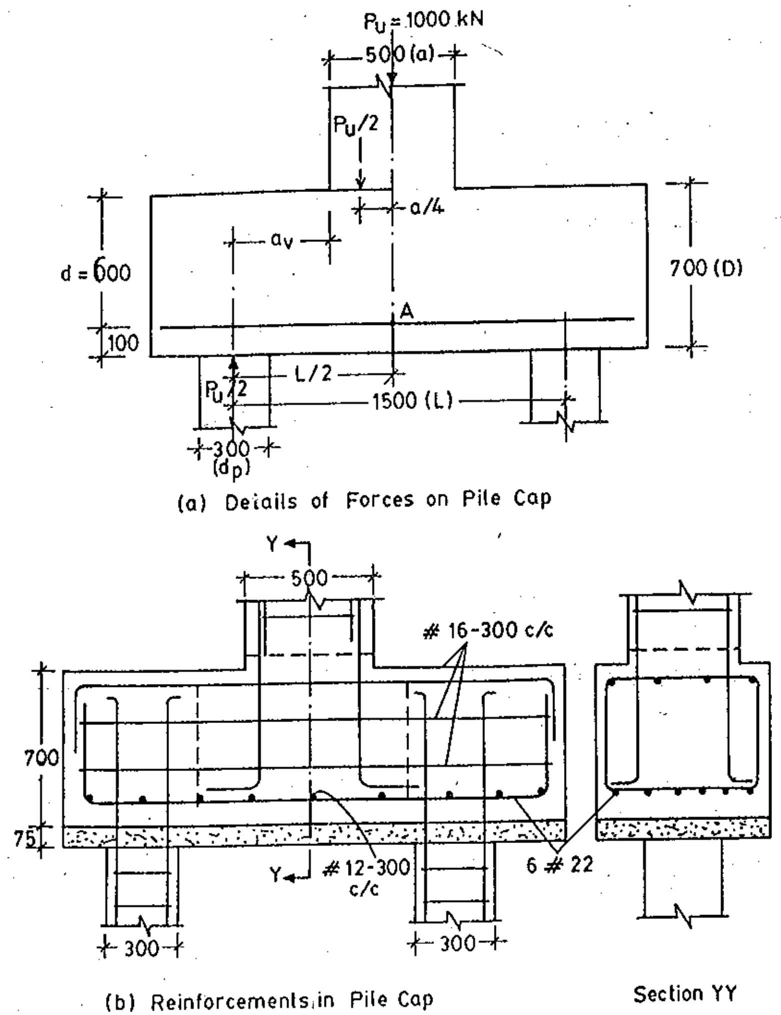


Fig. 14.10 Design of Pile Cap (Flexural action)

c) Check for Flexural and Truss action

Shear span = $a_v = (750 - 250) = 500$ mm

$$\therefore \quad \text{Ratio of } \left(\frac{a_v}{d}\right) = \left(\frac{500}{600}\right) = 0.83 > 0.6$$

Hence, flexural action will be predominant. Design the pile cap for flexure and check for shear.

d) Tension Steel

Referring to Fig.14.10(a) and taking moments about the centre of pile cap, the maximum bending moment at A is given by

$$M_u = \frac{P_u}{2}\left[\frac{L}{2} - \frac{a}{4}\right] = \frac{1000}{2}\left[\frac{1.5}{2} - \frac{0.5}{4}\right] = 312.5 \text{ kN.m}$$

$$\left(\frac{M_u}{bd^2}\right) = \left(\frac{312.5 \times 10^6}{800 \times 600^2}\right) = 1.08$$

Refer Table-2 (SP: 16) and read out the percentage of reinforcement as

$$p_t = \left(\frac{100\, A_{st}}{bd}\right) = 0.325$$

$$\therefore \quad A_{st} = \left(\frac{0.325 \times 800 \times 600}{100}\right) = 1560 \text{ mm}^2$$

Provide 6 bars 22 mm diameter with ($A_{st} = 2280$ mm^2)

e) Check for Shear stresses

$$\left(\frac{100 A_{st}}{bd}\right) = \left(\frac{100 \times 2280}{800 \times 600}\right) = 0.475$$

Refer Table-19 (IS: 456) and read out the value of permissible shear stress as

$$\tau_c = 0.47 \text{ N/mm}^2$$

$$\text{Permissible shear stress is } = \tau_c' = \tau_c\left(\frac{2d}{a_v}\right) = 0.47\left(\frac{2 \times 600}{500}\right) = 1.128 \text{ N/mm}^2$$

$$\text{Nominal shear stress } = \tau_v = \left(\frac{V_u}{bd}\right) = \left(\frac{500 \times 10^3}{800} \times 600\right) = 1.04 \text{ N/mm}^2 < \tau_c'$$

Hence, shear stresses are within safe permissible limits.

f) Reinforcement Details

The details of reinforcements in the pile cap are shown in Fig. 14.10 (b).

14.5 DESIGN OF RAFT FOUNDATIONS

14.5.1 Introduction

Raft foundations are generally provided to support a number of heavily loaded columns situated on soils of low bearing capacity. In the case of multistorey buildings with columns based at regular intervals, the bearing area required for each column overlaps that of the adjacent column. In such cases it is advantageous and economical to provide a raft or a mat consisting of a network of beams connecting the columns with a continuous reinforced concrete slab in contact with the soil.

Fig.14.11 shows a typical raft foundation connecting all the columns with sil beams and a continuous inverted slab in contact with the soil.

The slab and the continuous beam forming the raft or mat foundation should be designed for maximum moments and shear forces developed in the members.

14.5.2 Design Principles of Raft Foundations

The structural design of raft foundations involves the computation of total area of the slab required to support the loads on the columns. The foundation area required is obtained using the safe bearing capacity of the soil and the total loads on the foundation. The slab is normally fixed or continuous at the edges and designed for the moments developed in the perpendicular directions and the design of two-way slabs has been presented in chapter-9.

The beams are normally continuous over several spans and behaves as a continuous beam with loads acting from the soffit upwards due to soil reaction. The preliminary dimensions are assumed based on span/depth ratio and the design moments and shear forces are computed using the dead and imposed loads and the bending moment and shear force coefficients given in Table-12 and 13 of IS: 456-2000. The reinforcements are computed to resist the factored moments and shear forces.

14.6 DESIGN EXAMPLE

Design a suitable raft foundation connecting the columns of a building

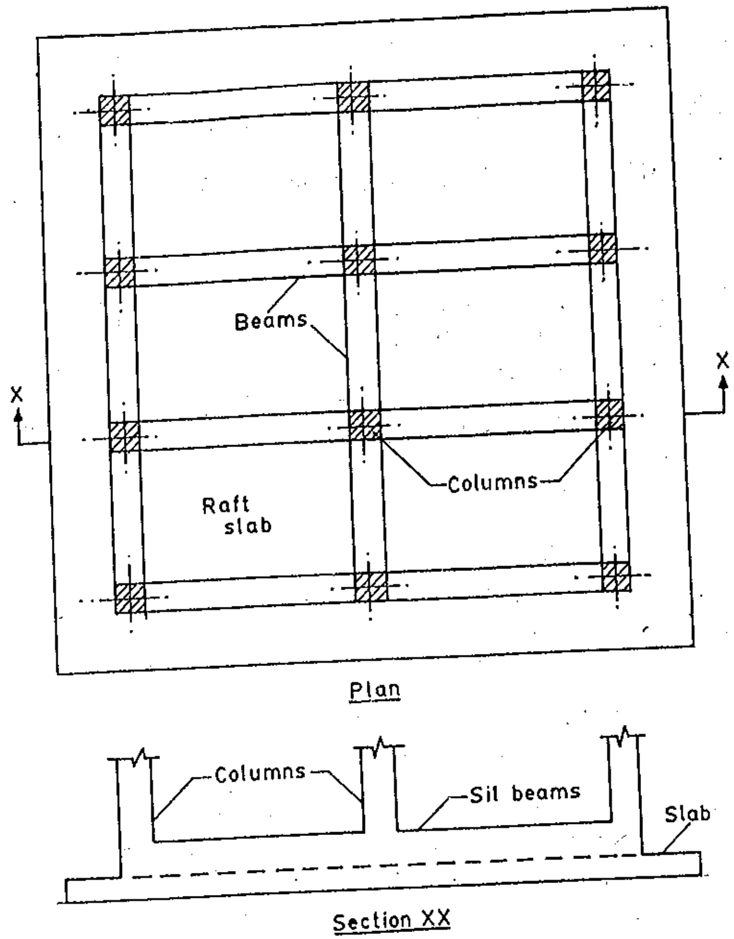


Fig. 14.11 Typical Raft Foundation

shown in Fig. 14.12 (a). The size of the building is 12 m by 12 m with the columns spaced at 4 m intervals. Adopt the following data:

a) Data

Size of building = 12 m by 12 m.
Spacing of columns all round = 4 m intervals
Service load transmitted by each column = 500 kN.
Size of Columns = 300 mm by 300 mm

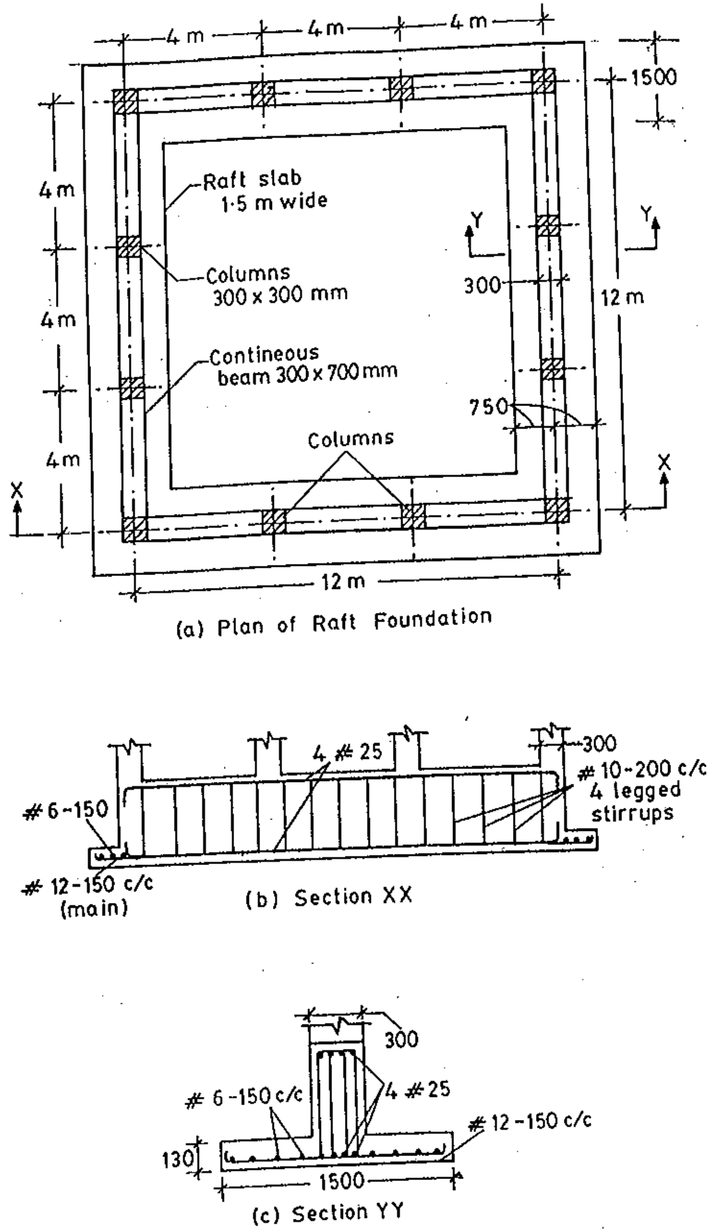


Fig. 14.12 Reinforcement Details in Raft Foundation

Safe bearing capacity of soil = 100 kN/m^2
Materials: M-20 grade concrete (f_{ck} = 20 N/mm^2)
Fe-415 HYSD bars (f_y = 415 N/mm^2)

b) Design of Raft Slab

Total service load on all columns = (12 × 500) = 6000 kN
Self weight of slab and beams at 10% = 600 kN
Total service load = 6600 kN.

$$\text{Area of raft slab} = \left(\frac{6600}{100}\right) = 66 \text{ m}^2$$

Total length of slab = (12 × 4) = 48 m
Width of slab = (66/48) = 1.375 m
Adopt a footing width of 1.5 m

$$\text{Intensity of soil pressure} = \left(\frac{6000}{1.5\times 48}\right) = 83.3 \text{ kN/m}^2$$

$$\text{Cantilever projection of slab} = \left(\frac{1500-300}{2}\right) = 600\text{mm}$$

Factored moment in Cantilever slab is computed as,

$$M_u = \left(\frac{1.5\times 83.3\times 0.6^2}{2}\right) = 22.5 \text{ kN.m}$$

Limiting depth required is given by

$$d = \sqrt{\frac{M_u}{0.138 f_{ck} b}} = \sqrt{\frac{22.5\times 10^6}{(0.138\times 20\times 10^3)}} = 90.3 \text{ mm}$$

Adopt effective depth d = 100 mm
And overall depth D = 130 mm

$$\left(\frac{M_u}{bd^2}\right) = \left(\frac{22.5\times 10^6}{10^3\times 100^2}\right) = 2.25$$

Refer Table-2 (SP: 16) and read out the percentage of reinforcement as,

$$p_t = 0.737 = \left(\frac{100A_{st}}{bd}\right)$$

$$\therefore \quad A_{st} = \left(\frac{0.737\times 10^3\times 100}{100}\right) = 737 \text{ mm}^2/\text{m}$$

Provide 12mm diameter bars at 150 mm centres (A_{st} = 754 mm^2)
Distribution bars = (0.0012 × 103 × 130) = 156 mm^2
Provide 6mm diameter bars at 150 mm centres (A_{st} = 189 mm^2)

c) Design of continuous beam over raft slab

Maximum Service load on beam

$$w = (83.3\times 1.5\times 1) = 125 \text{ kN/m}$$

$$M_u = 1.5\left(\frac{wL^2}{10}\right) = 1.5\left(\frac{125\times 4^2}{10}\right) = 300 \text{ kN.m}$$

$$V_u = 1.5\,(0.6\,wL) = 1.5(0.6\times 125\times 4) = 450 \text{ kN}$$

Assuming the width of beam = b = 300 mm,

$$\text{Effective depth} = d = \sqrt{\frac{M_u}{0.138 f_{ck}\, b}} = \sqrt{\frac{300\times 10^6}{(0.138\times 20\times 300)}} = 602 \text{ mm}$$

Adopting d = 650 mm and over all depth D = 700 mm, compute the parameter,

$$\left(\frac{M_u}{bd^2}\right) = \left(\frac{300\times 10^6}{300\times 650^2}\right) = 2.36$$

Refer Table-2 (SP: 16) and read out the percentage reinforcement as

$$p_t = 0.781 = \left(\frac{100A_{st}}{bd}\right)$$

$$\therefore \quad A_{st} = \left(\frac{0.781\times 300\times 650}{100}\right) = 1523 \text{ mm}^2$$

Provide 4 bars of 25 mm diameter both at top and bottom to resist negative moments at the supports. (A_{st} = 1963 mm^2).

d) Shear Reinforcements

V_u = 450 kN

$$\text{Nominal shear stress} = \tau_v = \left(\frac{V_u}{bd}\right) = \left(\frac{450\times 10^3}{300\times 650}\right) = 2.30 \text{N/mm}^2$$

$$\left(\frac{100A_{st}}{bd}\right) = \left(\frac{100\times 1963}{300\times 650}\right) = 1.006$$

Refer Table-19 (IS: 456) and read out the permissible shear strength of concrete as,

$$\tau_c = 0.62 \text{ N/mm}^2 < \tau_v$$

Hence, shear reinforcements are to be designed to resist the balance shear given by

$$V_s = [V_u - \tau_c bd] = [40 - (0.62 \times 300 \times 650)/10^3] = 33\text{kN}$$

Using 10 mm diameter four legged stirrups,

$$\text{Spacing } = S_v = \left[\frac{0.87 f_y A_{sv} d}{V_s}\right] = \left[\frac{0.87 \times 415 \times 4 \times 78.5 \times 650}{330 \times 10^3}\right] = 223 \text{ mm}$$

Provide 10 mm diameter four legged stirrups at 200 mm centres.

e) Reinforcement details

The reinforcement details of longitudinal and cross section of the raft is shown in Figs. 14.12 (b) and (c).

14.7 EXAMPLES FOR PRACTICE

1) A typical column of a multistoreyed building transmits a load of 3200 kN to the foundations. This load has to be supported by 4 piles having a square cross section. The piles are driven through hard stratum and rest on hard rock. 300 mm by 300 mm size Precast piles are proposed to be used for the foundations. Design the reinforcements required for a typical pile assuming the pile to be 8 m long. Adopt M-30 grade concrete and Fe-415 grade high yield strength reinforcement. Sketch the typical details of reinforcements in the pile.

2) A reinforced concrete column 400 mm by 600 mm carrying a factored load of 2400 kN is to be supported by six precast piles of length 6 m. The piles are driven through hard gravelly soil and resting on hard strata. Using M-20 grade concrete and Fe-415 HYSD bars design the reinforcements required in a typical pile and sketch the details.

3) A reinforced concrete column 400 mm by 400 mm carrying a service load of 800 kN is supported on three piles 300 by 300 mm in section. The centre to centre distance between the piles is 1500 mm. Design the reinforcements in the pile and the pile cap. The length of the piles may be assumed as 6 m bearing on hard rock. Adopt M-20 grade concrete and Fe-415 HYSD reinforcement.

4) A pile cap connecting 4 reinforced concrete piles of 300 by 300 mm is to be designed to support a reinforced concrete column 400 mm by 400 mm carrying a service load of 2000 kN. The piles are located parallel to the column faces with their centres located 800 mm from the centre of the column. Using M-30 grade concrete and Fe-500 grade reinforcement, design the pile cap and sketch the details of reinforcements.

5) Design a reinforced concrete raft foundation connecting the columns of a multistoreyed building. The columns are arranged in square grid 16 m by 16 m with their spacings 4 m apart. The safe bearing capacity of the soil at site is 100 kN/m^2. The total service load on all the columns is 4800 kN. The columns are 400 mm by 400 mm in section. Adopt M-20 grade concrete and Fe-415 HYSD bars. Sketch the details of reinforcements in the raft foundation.

6) The columns of a multistoreyed building with their centre lines forming a rectangular grid of 10.5 m by 14 m has the columns spaced at 3.5 m centres in the grid. The columns are 300 mm by 300 mm in cross section and transmit a factored load of 800 kN each to the foundations. The safe bearing capacity of soil at site is 80 kN/m^2. Adopting M-25 grade concrete and Fe-415 HYSD bars, design a suitable raft foundation for the columns and sketch the details of reinforcements in the raft beam and slab.

CHAPTER 15

Working Stress Method of Design

15.1 INTRODUCTION AND PERMISSIBLE STRESSES

The working stress or permissible stress method of design developed and widely used during the first half of the 20th century is based on the elastic theory of reinforced concrete sections outlined in detail under chapter-4. Basically, the method assumes linear elastic behavior of materials and the working stresses in the materials are obtained by applying appropriate partial safety factors to the characteristic strength. The resulting permissible stresses in concrete and steel are well within the linear elastic range of the materials.

The working stress method does not provide a realistic measure of the factor of safety against collapse of a structure, in contrast to the limit state method of design. However structures designed in accordance with the working stress method have been generally performing satisfactorily over many years. In general, the working stress method results in comparatively larger sections of the structural members with higher quantities of steel reinforcement, resulting in conservative designs. Nevertheless, the method due to its simplicity in concept as well as application was widely used by the structural engineers during the 20th century and even now, the method has been incorporated in Annexure-B of the revised Indian Standard Code IS: 456-2000 for optional use.

The permissible or working stress in concrete under compression, flexure, and bond is shown in Table-4.2 (Table-21 of IS: 456-2000) and the working stresses in steel reinforcement are compiled in Table-4.1 (Table-22 of IS: 456-2000). The design shear strength of concrete for various grades is shown in Table-15.1 (Table-23 of IS: 456-2000).

In the working stress method of design of structural concrete members, the cross sectional dimensions are generally assumed based on the basic span/depth ratios outlined in chapter-5. The service load moments and shear forces are computed at critical sections and the adequacy of the depth of the section is checked by using the relation,

$$d = \sqrt{\frac{M}{Qb}}$$

Where d = effective depth of the section
M = service load moment
b = width or breadth of section
Q = a constant depending upon the permissible stresses, neutral axis depth factor (k) and lever arm coefficient (j).

Values of design constant 'Q' for different grades of concrete and types of steel are compiled in Table-4.3. After satisfying the depth criterion, the area of reinforcement required in the section is evaluated by using the relation,

$$A_{st} = \left(\frac{M}{\sigma_{st} . j . d}\right)$$

The section is reinforced with suitable number of steel bars with due regard to spacing of bars and cover requirements.

The section is generally checked for resistance against shear by computing the nominal shear stress τ_v using the relation,

$$\tau_v = \left(\frac{V}{bd}\right)$$

Where V = service load shear force at the section

The permissible shear stress in concrete (τ_c) is influenced by the percentage reinforcement in the section and the grade of concrete as compiled in Table-15.1 (Table-23 of IS: 456-2000).

Table 15.1 Permissible Shear Stresses in Concrete (Table-23 of IS: 456-2000)

$\left(\frac{100A_s}{bd}\right)$	Permissible Shear stress in Concrete (τ_c, N/mm²)					
	M-15	M-20	M-25	M-30	M-35	M-40
≤0.15	0.18	0.18	0.19	0.20	0.20	0.20
0.25	0.22	0.22	0.23	0.23	0.23	0.23
0.50	0.29	0.30	0.31	0.31	0.31	0.32
0.75	0.34	0.35	0.36	0.37	0.37	0.38
1.00	0.37	0.39	0.40	0.41	0.42	0.42
1.25	0.40	0.42	0.44	0.45	0.45	0.46
1.50	0.42	0.45	0.46	0.48	0.49	0.49
1.75	0.44	0.47	0.49	0.50	0.52	0.52
2.00	0.44	0.49	0.51	0.53	0.54	0.55

(*Contd.*)

Table 15.1 Permissible Shear Stresses in Concrete (Table-23 of IS: 456-2000)

$\left(\frac{100A_s}{bd}\right)$	Permissible Shear stress in Concrete (τ_c, N/mm²)					
	M-15	M-20	M-25	M-30	M-35	M-40
2.25	0.44	0.51	0.53	0.55	-.56	0.57
2.50	0.44	0.51	0.55	0.57	0.58	0.60
2.75	0.44	0.51	0.56	0.58	0.60	0.62
3.00	0.44	0.51	0.57	0.60	0.62	0.63

Note: A_s is that area of longitudinal tension reinforcement which continues at least one effective depth beyond the section being considered except at supports where the full area of tension reinforcement may be used provided the detailing conforms to clauses 26.2.2 and 26.2.3 of IS: 456-2000

If the nominal shear stress, exceeds the permissible shear stress, suitable shear reinforcements are designed in members using the relation,

$$S_v = \left(\frac{\sigma_{sv} . A_{sv} . d}{V_s}\right)$$

Where S_v = spacing of stirrups
A_{sv} = cross sectional area of stirrup legs
σ_{sv} = permissible stress in steel reinforcement
d = effective depth
V_s = working load shear force at the section.

In the case of slabs, the section is revised using greater thickness since shear reinforcements are not feasible. However in slabs, the permissible shear stress in concrete is increased to a value '$k.\tau_c$' where 'k' is a multiplying factor depending upon the thickness of the slab as specified in Table-15.2 and clause 40.2.1.1 of IS: 456-2000.

Table 15.2 Values of 'k' for Solid slabs (Clause B-5.2.1.1 Of IS: 456-2000)

Overall depth of slab(mm)	300 or More	275	250	225	200	175	150 or less
k	1.00	1.05	1.10	1.15	1.20	1.25	1.30

In the case of beams, nominal shear reinforcements are provided if the nominal shear stress is less than the permissible shear stress and the spacing of such reinforcement is computed as,

$$S_v = \left(\frac{0.87 f_y A_{sv}}{0.4b}\right)$$

However, it is important to note that in slabs, the nominal shear stress (τ_v) should not exceed half the value of $\tau_{c,max}$ given in Table-15.3 (Table-24 of IS: 456-2000).

Table 15.3 Maximum Shear Stress, $\tau_{c,max}$ (N/mm²) (Table-24 of IS: 456-2000)

Concrete Grade	M-15	M-20	M-25	M-30	M-35	M-40 and above
$\tau_{c,max}$	1.6	1.8	1.9	2.2	2.3	2.5

15.2 DESIGN OF SLABS

15.2.1 Design Example of One-way Slab

Design a simply supported verandah slab to suit the following data:

a) Data

Clear span = 3 m
Slab supported on load bearing brick walls 230 mm thick.
Loading: Roof load (accessible) = 1.5 kN/m²
Materials: M-20 grade concrete and Fe-415 HYSD bars.

b) Permissible stresses

σ_{cbc} = 7 N/mm² $\quad Q$ = 0.91
σ_{st} = 230 N/mm² $\quad j$ = 0.90
m = 13

c) Depth of Slab

Assuming 0.4 percent of reinforcement in the slab, the value of K_t (Fig. 5.1) using Fe-415 HYSD bars is around 1.25.

Hence the ratio $\left(\frac{L}{d}\right) = \left(\frac{L}{d}\right)_{basic} \times K_t \times K_c$

$$= (20 \times 1.25 \times 1.00) = 25$$

$\therefore d = \left(\frac{3000}{25}\right) = 120$ mm, Using 10 mm diameter bars,

Overall depth = [120 + 5 + 15] = 140 mm. Adopt overall depth = 150 mm.

d) Effective Span

Effective span is the least of the following: -

a) Centre to centre of supports = (3 + 0.23) = 3.23 m
b) Clear span + effective depth = (3 + 0.12) = 3.12 m
∴ Effective span = L = 3.12 m

e) Loads

Dead load of slab = $(0.15 \times 25 \times 1) = 3.75$ kN/m^2
Live load on accessible roof = 1.50 kN/m^2
Roof finishes = 0.75 kN/m^2
Total load = w = 6.00 kN/m^2

Considering 1 m width of the slab, the uniformly distributed load is 6 kN/m on an effective span of 3.12 m.

f) Bending Moments and Shear Forces

$$M = (0.125\, w\, L^2) = (0.125 \times 6 \times 3.12^2) = 7.3 \text{ kN.m}$$
$$V = (0.5\, w\, L) = (0.5 \times 6 \times 3.12) = 9.36 \text{ kN}$$

g) Effective Depth

$$d = \sqrt{\frac{M}{Qb}} = \sqrt{\frac{7.3 \times 10^6}{0.91 \times 10^3}} = 90.06 \text{ mm}$$

Effective depth adopted = d = 120 mm. Hence safe.

h) Main reinforcement

$$A_{st} = \left(\frac{M}{\sigma_{st} jd}\right) = \left[\frac{7.3 \times 10^6}{230 \times 0.9 \times 120}\right] = 294 \text{ mm}^2$$

Minimum reinforcement = 0.12 percent = $(0.0012 \times 150 \times 1000)$ = 180 mm^2 < 294 mm^2

$$\text{Spacing of 10 mm diameter bars} = \left(\frac{1000 a_{st}}{A_{st}}\right) = \left(\frac{1000 \times 79}{294}\right) = 268 \text{ mm}$$

Use 10 mm diameter bars at 260 mm centres.

i) Distribution Reinforcement

$$A_{st} = (0.0012 \times 1000 \times 150) = 180 \text{ mm}^2$$

$$\text{Spacing of 6 mm diameter bars} = \left(\frac{1000 \times 28.2}{180}\right) = 156.6 \text{ mm}$$

Adopt 6 mm diameter bars at 150 mm centres

j) Check for Shear stress

$$\tau_v = \left(\frac{V}{bd}\right) = \left(\frac{9.36 \times 10^3}{10^3 \times 120}\right) = 0.078 \text{ N/mm}^2$$

Assuming 50 percent of tension reinforcement to be bent up near supports,

$$\text{Ratio}\left(\frac{100 A_{st}}{bd}\right) = \left(\frac{100 \times 147}{1000 \times 120}\right) = 0.122$$

From Table-23 of IS: 456-2000, interpolating the permissible shear stress for slab is

$$k\tau_c = (1.30 \times 0.15) = 0.195 \text{ N/mm}^2 > \tau_v$$

Hence, shear stresses are within safe permissible limits.

k) Check for Deflection Control

Percentage reinforcement = p_t = [(100 × 294)/(1000 × 120)] = 0.245
For p_t = 0.245, K_t = 1.55 [refer Fig. 4 of IS: 456-2000]

∴ $(L/d)_{max} = (20 \times 1.55) = 31$
$(L/d)_{provided} = (3120/120) = 26 < 31$, Hence safe.

15.2.2 Design Example of Two-way Slab

Design a two-way slab for a residential floor to suit the following data:

a) Data

Size of floor = 4 m by 6 m
Edge conditions: slab simply supported on all the sides without any provision for torsion at corners. Materials: M-20 grade concrete and Fe-415 HYSD bars.

b) Permissible Stresses

σ_{cbc} = 7 N/mm^2 Q = 0.91
σ_{st} = 230 N/mm^2 j = 0.90
m = 13.33

c) Type of Slab

Simply supported on all sides without any provision for torsion at corners. Hence

$L_x = 4$ m
$L_y = 6$ m Ratio $(L_y/L_x) = (6/4) = 1.50$

d) Depth of Slab

From Table 5.1,

$$\text{Overall depth} = D = \left(\frac{\text{short span}}{28}\right) = \left(\frac{4000}{28}\right) = 142.8 \text{ mm}$$

Adopt overall depth = 150 mm
Effective depth = (150 – 30) = 120 mm

e) Effective Span

Effective span is the least of the following two criteria.

a) Centre to centre of supports = (4 + 0.2) = 4.2 m
b) Clear span + effective depth = (4 + 0.12) = 4.12 m

$\therefore$ Effective span = L_{xe} = 4.12 m

f) Loads

Self weight of slab = $(0.15 \times 25) = 3.75$ kN/m²
Live load on floor = 2.00 kN/m²
Floor finishes = 0.60 kN/m²
Total service load = w = 6.35 kN/m²

g) Bending Moments

From Table-5.5, for $(L_y/L_x) = 1.5$, read out the moment coefficients,

$$\alpha_x = 0.104 \quad \text{and} \quad \alpha_y = 0.046$$

$$\therefore \quad M_x = (\alpha_x . w . L_{xe}^2) = (0.104 \times 6.35 \times 4.12^2) = 11.20 \text{ kN.m}$$

$$M_y = (\alpha_y . w . L_{xe}^2) = (0.046 \times 6.35 \times 4.12^2) = 4.96 \text{ kN.m}$$

h) Check for Depth

$$\text{Effective depth} = d = \sqrt{\frac{M}{Qb}} = \sqrt{\frac{11.2 \times 10^6}{0.91 \times 10^3}} = 111.5 \text{ mm}$$

Effective depth adopted for short span = 120 mm
Effective depth for long span(using 10 mm diameter bars) = (120 – 10) = 110 mm

i) Reinforcements

$$\text{Steel for short span} = A_{st} = \left[\frac{M}{\sigma_{st} . jd}\right] = \left[\frac{11.2 \times 10^6}{230 \times 0.9 \times 120}\right] = 451 \text{ mm}^2$$

Use 10 mm diameter bars at 170 mm centres

$$\text{Steel for long span} = \left[\frac{4.96 \times 10^6}{230 \times 0.9 \times 110}\right] = 218 \text{ mm}^2$$

Use 10 mm diameter bars at 300 mm centres

j) Shear and Bond Stresses

Shear and bond stresses in two-way slabs are very small and not generally checked since they will be within safe permissible limits.

The reinforcement details are similar to that in the design example worked out under limit state design.

15.3 DESIGN OF BEAMS

15.3.1 Design of Singly Reinforced Beam

Design a rectangular reinforced concrete beam simply supported on masonry walls 300 mm thick and 6 m apart (centre to centre) to support a distributed live load of 10 kN/m and a dead load of 5 kN/m in addition to its own weight. Assume M-20 grade concrete and Fe-415 HYSD bars.

a) Data

Span (centre to centre of supports) = 6 m
Live load = 10 kN/m
Dead load = 5 kN/m
M-20 grade concrete and Fe-415 HYSD bars

b) Permissible Stresses

σ_{cb} = 7 N/mm² $\quad Q = 0.91$
σ_{st} = 230 N/mm² $\quad j = 0.90$
m = 13.33

c) Cross Sectional Dimensions

Adopt width of beam = b = 300 mm
As the loading on beam is heavy adopt,

$$\text{Effective depth} = d = \left(\frac{\text{span}}{10}\right) = \left(\frac{6000}{10}\right) = 600 \text{ mm}$$

Overall depth = D = (600 + 50) = 650 mm

d) Effective Span

Effective span is the least of the following two criteria.

i) Centre to centre of supports = 6 m
ii) (Clear span + effective depth) = (6 − 0.3 + 0.6) = 6.3 m

Hence, adopt effective span = L = 6 m

e) Loads

Self weight of beam = (0.3 × 0.65 × 25) = 4.875 kN/m
Dead load = 5.000 kN/m
Live load = 10.000 kN/m
Finishes = 0.125 kN/m
Total load = w = 20.000 kN/m

f) Bending Moment and Shear Forces

$$M = 0.125\, w\, L^2 = (0.125 \times 20 \times 6^2) = 90 \text{ kN.m}$$
$$V = 0.5\, w\, L = (0.5 \times 20 \times 6) = 60 \text{ kN}$$

g) Check for Depth

$$d = \sqrt{\frac{M}{Qb}} = \sqrt{\frac{90 \times 10^6}{0.91 \times 300}} = 578 \text{ mm}$$

Effective depth provided = d = 600 mm, hence adequate.

h) Main Tension Reinforcement

$$A_{st} = \left(\frac{M}{\sigma_{st} j d}\right) = \left(\frac{90 \times 10^6}{230 \times 0.9 \times 600}\right) = 724 \text{ mm}^2$$

Provide 3 bars of 20 mm diameter (A_{st} = 942 mm²)

i) Shear Stresses and Reinforcements

$$\text{Nominal shear stress} = \tau_v = \left(\frac{V}{bd}\right) = \left(\frac{60 \times 10^3}{300 \times 600}\right) = 0.33 \text{N/mm}^2$$

$$\text{Ratio} \left(\frac{100 A_{st}}{bd}\right) = \left(\frac{100 \times 942}{300 \times 600}\right) = 0.52$$

Refer Table-23 (IS: 456-2000) and read out the permissible shear stress in concrete as

$$\tau_c = 0.30 \text{ N/mm}^2 < \tau_v$$

Hence, shear reinforcements in the form of stirrups are required.

Since τ_c is nearly equal τ_v, provide nominal shear reinforcements(using 6 mm diameter bars) given by the equation,

$$S_v = \left(\frac{A_{sv}.0.87 f_y}{0.4\, b}\right) = \left(\frac{2 \times 28 \times 0.87 \times 415}{0.4 \times 300}\right) = 168 \text{ mm}$$

Provide 6 mm diameter two legged stirrups at 150 mm centres up to quarter span length from supports and gradually increased to 300 mm towards the centre of span.

15.3.2 Design of Doubly Reinforced Beam

A doubly reinforced beam is to be designed having an overall cross sectional dimensions of 250 mm by 400 mm with an effective span of 4 m. The beam has to support an uniformly distributed dead load of 2.5 kN/m together with a live load of 20 kN/m in addition to its self weight. Adopting M-20 grade concrete and Fe-415 HYSD bars, design suitable reinforcements in the beam.

a) Data

Effective span = L = 4 m
Breadth of beam = b = 250 mm
Overall depth = D = 400 mm
Dead load = 2.5 kN/m
Live load = 20 kN/m
M-20 grade concrete and Fe-415 HYSD bars.

b) Permissible Stresses

$\sigma_{cb} = 7\text{ N/mm}^2$ $\quad Q = 0.91$
$\sigma_{st} = 230\text{ N/mm}^2$ $\quad j = 0.90$
$m = 13.33$

c) Loads

Self weight of beam = $(0.25 \times 0.4 \times 25) = 2.5$ kN/m
Dead load = 2.5
Live load = 20.0
Total load = $w = 25.0$ kN/m
Adopt an effective cover of 50 mm, effective depth = $(400 - 50) = 350$ mm

d) Bending Moments and Shear Forces

$$M = (0.125\, w\, L^2) = (0.125 \times 25 \times 4^2) = 50 \text{ kN.m}$$
$$V = (0.5\, w\, L) = (0.5 \times 25 \times 40) = 50 \text{ kN}$$

e) Resisting Moment

The resisting moment of singly reinforced section is computed as

$$M_1 = (Q\, b\, d^2) = (0.91 \times 250 \times 400^2)^{-6} = 27.8 \text{ kN.m} < M = 50 \text{ kN.m}$$

Balance moment = $M_2 = (M - M_1) = (50 - 27.8) = 22.2$ kN.m

f) Tension reinforcement

Tensile steel required for balanced singly reinforced section is given by

$$A_{st1} = \left[\frac{M_1}{\sigma_{st} j d}\right] = \left[\frac{27.8 \times 10^6}{230 \times 0.90 \times 350}\right] = 381 \text{ mm}^2$$

Additional tensile steel for balanced moment M_2 is computed as

$$A_{st2} = \left[\frac{M_2}{\sigma_{st}(d - d_c)}\right] = \left[\frac{27.8 \times 10^6}{230 \times 0.90 \times 350}\right] = 381 \text{ mm}^2$$

Where d_c = cover to compression steel = 50 mm

$\therefore$ Total tensile steel = $A_{st} = (A_{st1} + A_{st2}) = (381 + 322) = 703 \text{ mm}^2$

Provide 4 bars of 16 mm diameter ($A_{st} = 804 \text{ mm}^2$)

g) Compression reinforcement

$$A_{sc} = \left[\frac{m A_{st2}(d - n_c)}{(1.5m - 1)(n_c - d_c)}\right]$$

Where $n_c = 0.283\, d = (0.283 \times 350) = 99.05$ mm

$$A_{sc} = \left[\frac{13 \times 322(350 - 99.05)}{(1.5 \times 13 - 1)(99.05 - 50)}\right] = 1172 \text{ mm}^2$$

Provide 4 bars of 20 mm diameter ($A_{sc} = 1256 \text{ mm}^2$)

h) Shear Stresses and reinforcements

$$\tau_v = \left(\frac{V}{bd}\right) = \left(\frac{50 \times 10^3}{250 \times 350}\right) = 0.57 \text{ N/mm}^2$$

$$\left(\frac{100 A_{st}}{bd}\right) = \left(\frac{100 \times 804}{250 \times 350}\right) = 0.91$$

Refer Table-23 (IS: 456-2000) and read out the permissible shear stress as

$$\tau_c = 0.37 \text{ N/mm}^2 < \tau_v$$

Hence, shear reinforcements have to be designed to resist the balance shear given by

$$V_s = [V - \tau_c . b . d] = [50 - (0.37 \times 250 \times 350)/10^3] = 17.625 \text{ kN}$$

Using 6 mm diameter two legged stirrups, spacing is computed as

$$S_v = \left(\frac{A_{sv}\sigma_{sv} d}{V_s}\right) = \left(\frac{2 \times 28 \times 230 \times 350}{17.625 \times 10^3}\right) = 255 \text{ mm}$$

Provide 6 mm diameter two legged stirrups at 250 mm centres at supports gradually increased to 300 mm towards the centre of span.

15.3.3 Design of Flanged Beam

Design a Tee-beam for a commercial office floor to suit the following data:

a) Data

Clear span = 10 m
Centre to centre of wall supports = 10.5 m
Spacings of tee beams = 2.5 m
Live load (office floor) = 4 kN/m^2
Slab thickness = 150 mm
Materials: M-20 grade concrete and Fe-415 HYSD bars.

b) Permissible Stresses

$\sigma_{cb} = 7\text{ N/mm}^2$ $\quad Q = 0.91$ $\quad m = 13.33$
$\sigma_{st} = 230\text{ N/mm}^2$ $\quad j = 0.90$

c) Effective Span

$$\text{Effective depth} = d = \left(\frac{\text{span}}{15}\right) = \left(\frac{10500}{15}\right) = 700 \text{ mm}$$

Overall depth $= D = (700 + 50) = 750$ mm
Width of rib $= b_w = 300$ mm
Effective span is the least of the following two criteria.

i) (Clear span + effective depth) = (10 + 0.70) = 10.70 m
ii) Centre to centre of supports = 10.5 m

Hence, effective span $= L = 10.5$ m

d) Loads

Self weight of slab = (0.15 × 25 × 2.5) = 9.375 kN/m
Live load = (4 × 2.5) = 10.000 kN/m
Floor finish = (0.6 × 2.5) = 1.500 kN/m
Self weight of rib = (0.3 × 0.6 × 25) = 4.500 kN/m
Plaster finishes (lump sum) = 0.625 kN/m
Total load $= w = 26.000$ kN/m

e) Bending Moments and Shear Forces

$$M = (0.125\, w\, L^2) = (0.125 \times 26 \times 10.5^2) = 358 \text{ kN.m}$$
$$V = (0.5\, w\, L) = (0.5 \times 26 \times 10.5) = 137 \text{ kN}$$

f) Main Reinforcements

$$A_{st} = \left(\frac{M}{\sigma_{st} j d}\right) = \left(\frac{358 \times 10^6}{230 \times 0.90 \times 700}\right) = 2454 \text{ mm}^2$$

Provide 4 bars of 28 mm diameter ($A_{st} = 2464$ mm^2)

g) Effective Flange Width

Effective flange width is the least of the following two criteria.

i) $b_f = [(L_o/6) + b_w + 6\, D_f] = [(10500/6) + 300 + (6 \times 150)] = 2950$ mm
ii) b_f = (centre to centre of ribs) = 2500 mm

Hence, $b_f = 2500$ mm

h) Check for Stresses

Let n = depth of neutral axis

$$\left(\frac{b_f n^2}{2}\right) = m.A_{st}(d-n)$$

$$[(2950 \times n^2)/2] = [13 \times 2464(700 - n)]$$

Solving, $n = 122$ mm

Lever arm $= a = [d - (n/3)] = [700 - (122/3)] = 659.4$ mm

$$\text{Stress in steel} = \sigma_{st} = \left(\frac{358 \times 10^6}{2464 \times 659.4}\right) = 220 \text{ N/mm}^2 < 230 \text{ N/mm}^2$$

$$\text{Stress in concrete} = \sigma_{cb} = \left(\frac{220}{13} \times \frac{122}{578}\right) = 3.57 \text{ N/mm}^2 < 7 \text{ N/mm}^2$$

Hence, the stresses in steel and concrete are within safe permissible limits.

i) Shear stresses and Reinforcement

Maximum shear force $= V = 137$ kN

$$\tau_v = \left(\frac{V}{b_w d}\right) = \left(\frac{137 \times 10^3}{300 \times 700}\right) = 0.65 \text{ N/mm}^2$$

$$\left(\frac{100 A_{st}}{b_w d}\right) = \left(\frac{100 \times 2464}{300 \times 700}\right) = 1.17$$

Refer Table-23 (IS: 456-2000) and read out the permissible shear stress as,

$$\tau_c = 0.40 \text{ N/mm}^2 < \tau_v$$

Hence, shear reinforcements are to be designed to resist the balance shear given by

$$V_s = [V - \tau_c\, b_w\, d] = [137 - (0.40 \times 300 \times 700)\, 10^{-3}] = 53 \text{ kN}$$

Using 6 mm diameter two legged stirrups, the spacing is calculated as,

$$S_v = \left(\frac{A_{sv}\sigma_{sv} d}{V_s}\right) = \left(\frac{2 \times 28 \times 230 \times 700}{53 \times 10^3}\right) = 170 \text{ mm}$$

Provide 6 mm diameter two legged stirrups at 170 mm centres near supports and gradually increased to 300 mm centres towards the centre of span.

15.4 Design of Column and footings

Design a suitable R.C.C. column of rectangular section and a suitable footing to support an axial service load of 1000 kN. Size of the column is

300 mm by 500 mm. Safe bearing capacity of the soil is 200 kN/m^2. Adopt M-20 grade concrete and Fe-415 HYSD bars.

a) Data

Axial load = P = 1000 kN
Size of column = 300 mm by 500 mm
Safe bearing capacity of soil = 200 kN/m^2
Materials: M-20 grade concrete and Fe-415 HYSD bars.

b) Permissible Stresses

$\sigma_{cc} = 5$ N/mm^2 $\quad Q = 0.91$
$\sigma_{cb} = 7$ N/mm^2 $\quad j = 0.90$
$\sigma_{st} = 230$ N/mm^2 $\quad m = 13$
$\sigma_{sc} = 190$ N/mm^2

c) Main Column Reinforcement

$$P = \sigma_{sc} A_{sc} + \sigma_{cc}(A_c - A_{sc})$$

Area of concrete = $A_c = (300 \times 500) = (15 \times 10^4)$ mm^2

$$(1000 \times 10^3) = 190\,A_{sc} + 5\,[(15 \times 10^4) - A_{sc}]$$

Solving, the area of steel reinforcement = A_{sc} = 1350 mm^2
Minimum steel area = 0.8% = (0.008 × 300 × 500) = 1200 mm^2
Provide 4 bars of 22 mm diameter at corners ($A_{sc} \doteq 1520$ mm^2)

d) Ties

Greater of the diameters of

i) (22/4) = 5.5 mm
ii) 6 mm

Adopt 6 mm diameter ties
Pitch of the ties is the least of the following:

i) Least lateral dimension = 300 mm
ii) (16 × diameter of smallest longitudinal bar) = (16 × 22) = 352 mm
iii) 300 mm

Adopt 6 mm diameter ties at 300 mm centres.

e) Size of footing

Working load on Column = 1000 kN
Self weight of footing (10%) = 100 kN
Total load = 1100 kN
Area of footing = (1100/2000) = 5.5 m^2

Proportioning the footing in the same proportion as the sides of the column,

$$(3\,x \times 5\,x) = 5.5 \qquad \therefore\ x = 0.604$$

Short side of footing = (3 × 0.604) = 1.814 m
Long side of footing = (5 × 0.604) = 3.02 m
Adopt a rectangular footing of size 3 m by 2 m
Upward soil pressure = [1000/(2 × 3)] = 167 kN/m^2

f) Bending Moments

Cantilever projection from the face of short side of column = 0.5(3−0.5)
= 1.25 m

Cantilever projection from the face of long side of column = 0.5(2−0.3)
= 0.85 m

Bending moment at short side face of column is computed as,

$$M = \left(\frac{wL^2}{2}\right) = \left(\frac{167 \times 1.25^2}{2}\right) = 130 \text{ kN.m}$$

Bending moment at long side face of column is calculated as,

$$M = \left(\frac{wL^2}{2}\right) = \left(\frac{167 \times 0.85^2}{2}\right) = 60 \text{ kN.m}$$

g) Depth of Footing

i) From bending moment considerations, effective depth is computed as,

$$d = \sqrt{\frac{M}{Qb}} = \sqrt{\frac{130 \times 10^6}{0.91 \times 10^3}} = 380 \text{ mm}$$

ii) Depth required from shear stress considerations is very much larger and nearly double the value obtained from moment considerations.

Hence assume an overall depth = D = 600 mm and effective depth = d = 550 mm.

h) Reinforcements in footing

i) Longer direction

$$A_{st} = \left(\frac{M}{\sigma_{st} j d}\right) = \left(\frac{130 \times 10^6}{230 \times 0.90 \times 550}\right) = 1134 \text{ mm}^2$$

Adopt 20 mm diameter bars at 200 mm centres (A_{st} = 1571 mm2)

ii) Shorter direction

$$A_{st} = \left(\frac{M}{\sigma_{st} j d}\right) = \left(\frac{60 \times 10^6}{230 \times 0.90 \times 534}\right) = 539 \text{ mm}^2$$

But minimum reinforcement = $A_{st,min} = (0.0012 \times 10^3 \times 600) = 720 \text{ mm}^2/\text{m}$
Adopt 12 mm diameter bars at 150 mm centres in the shorter direction

i) Check for Shear stresses

Shear force at a distance 'd' from the face of the column is computed as

$$V = (167 \times 0.7 \times 1) = 116.9 \text{ kN}$$

$$\tau_v = \left(\frac{V}{bd}\right) = \left(\frac{116.9 \times 10^3}{1000 \times 550}\right) = 0.21 \text{ N/mm}^2$$

$$\left(\frac{100 A_{st}}{bd}\right) = \left(\frac{100 \times 1571}{1000 \times 550}\right) = 0.28$$

Refer Table-23 (IS: 456-2000) and read out the permissible shear stress as,

$$\tau_c = 0.22 \text{ N/mm}^2 > \tau_v$$

Hence, shear stresses are within safe permissible limits.
The reinforcement detailing in the column and footing is similar to that shown in the example under limit state design of columns and footings.

15.5 DESIGN OF RETAINING WALLS

Design a cantilever type retaining wall to retain an earth embankment 4 m high above ground level. The density of earth is 18 kN/m^3 and its angle of repose is 30°. The embankment is horizontal at its top. The safe bearing capacity of the soil is 200 kN/m^2 and the coefficient of friction between soil and concrete is 0.5. Adopt M-20 grade concrete and Fe-415 HYSD bars.

a) Data

Embankment height above ground level = 4 m
Safe bearing capacity of soil = 200 kN/m^2
Angle of repose = 30°
Density of soil = 18 kN/m^3
Coefficient of friction between soil and concrete = 0.5
Materials: M-20 grade concrete and Fe-415 HYSD bars.

b) Permissible stresses

σ_{cb} = 7 N/mm^2 Q = 0.91
σ_{st} = 230 N/mm^2 j = 0.90
m = 13

c) Dimensions of Retaining Wall

$$\text{Minimum depth of foundation} = \frac{p}{\gamma_e}\left[\frac{1-\sin\phi}{1+\sin\phi}\right] = \frac{200}{18}\left(\frac{1}{3}\right)^2 = 1.2$$

Provide depth of foundation = 1.2 m
Overall depth of wall = H = (4 + 1.2) = 5.2 m

Thickness of base slab = (H/12) = (5200/12) = 433 mm
Adopt thickness of base slab = 450 mm
(Height of stem = h = (5.2–0.45) = 4.75 m
Coefficient of active earth pressure is expressed by the relation,

$$C_a = \left[\frac{1-\sin\phi}{1+\sin\phi}\right] = \left[\frac{1-\sin 30°}{1+\sin 30°}\right] = 0.333$$

∴ Width of heel slab = $x_h = H\sqrt{C_a/3} = 5.2\sqrt{0.333/3} = 1.73$ m
Width of base slab = $B = 1.5\, x_h = (1.5 \times 1.73) = 2.6$ m
Adopt width of base slab = B = 3 m
Width of toe slab = (1/3) B = (1/3)3 = 1 m
Width of heel slab = 2 m

Fig. 15.1 shows the overall dimensions of the retaining wall.

d) Design of Stem

Height of stem = h = 4.75 m
Maximum bending moment in stem at the base is given by the expression,

$$M = \left(C_a \cdot \frac{wh^3}{6}\right) \quad \text{where} \quad C_a = \left[\frac{1-\sin\phi}{1+\sin\phi}\right] = \frac{1}{3}$$

$$= \left(\frac{1}{3} \times \frac{18 \times 4.75^3}{6}\right) = 107.17 \text{ kN.m}$$

$$\text{Effective depth } = d = \sqrt{\frac{M}{Qb}} = \sqrt{\frac{107.17 \times 10^6}{0.91 \times 10^3}} = 346\text{mm}$$

Adopt $d = 400$ mm and overall depth $= D = 450$ mm
Top width of stem = 200 mm

$$A_{st} = \left(\frac{M}{\sigma_{st} j d}\right) = \left(\frac{107.17 \times 106}{230 \times 0.90 \times 400}\right) = 1295 \text{ mm}^2$$

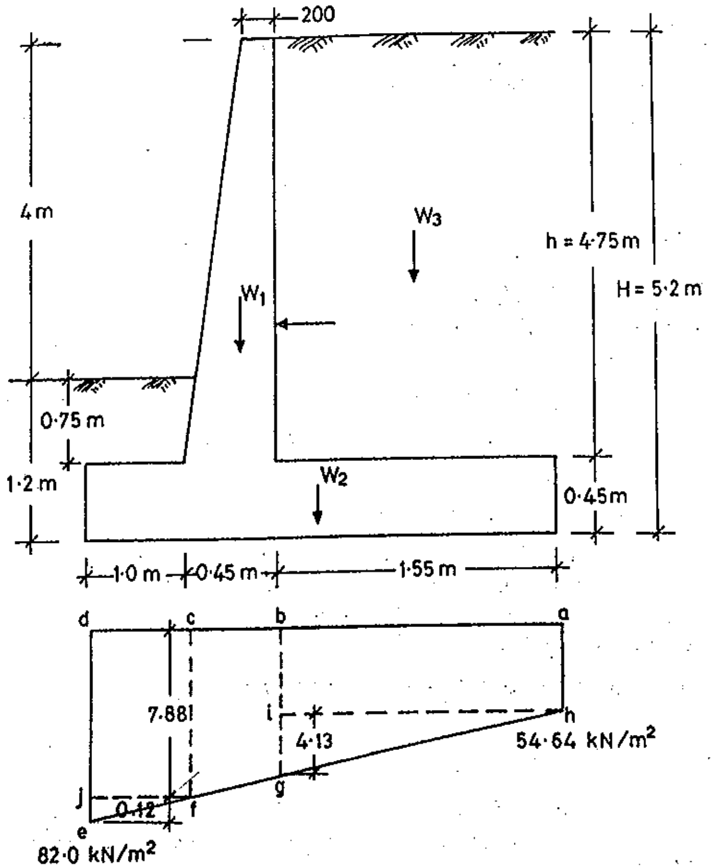


Fig. 15.1 Forces Acting on Cantilever Retaining Wall

Provide 20 mm diameter bars at 200 mm centres ($A_{st} = 1571$ mm^2)
Distribution reinforcement = $(0.0012 \times 1000 \times 450) = 540$ mm^2/m
Provide 10 mm diameter bars at 300 mm centres on both faces.

e) Stability Computations

The soil pressure distribution at base is computed by stability calculations. The overall dimensions of the wall are shown in Fig. 15.1.

The stability computations are compiled in Table-15.4.

Distance of point of application of resultant force from end 'a' is given by

$$z = \left(\frac{\Sigma M}{\Sigma W}\right) = \left(\frac{327.45}{204.90}\right) = 1.6 \text{ m}$$

Table 15.4 Stability calculations for one metre length of wall

Loads	Magnitude of load (kN)	Distance from 'a' (m)	Moment (kN.m)
$W_1 = (0.2 \times 4.75 \times 25)$ $(0.5 \times 0.25 \times 4.75 \times 25)$	23.80 14.84	1.65 1.83	39.27 27.15
$W_2 = (3 \times 0.45 \times 25)$	33.75	1.50	50.62
$W_3 = (1.55 \times 4.75 \times 18)$	132.51	0.78	103.35
Moment of earth Pressure = $(C_a wh^3/6)$ $\left(\frac{1}{3}\right)\left(\frac{18 \times 4.75^3}{6}\right)$	–	–	107.06
Total $\Sigma W =$	204.90	$\Sigma M =$	327.45

Eccentricity = $e = (z - 0.5B) = [1.6-(0.5 \times 3)] = 0.1$ m
$(B/6) = (3/6) = 0.5$. Hence $e < (B/6)$
Since eccentricity is less than $(B/6)$, no tension develops at base.
Maximum and minimum pressures developed at base are computed as,

$$p_{(\max,\min)} = \frac{\Sigma W}{B}\left[1 \pm \frac{6e}{B}\right] = \frac{204.90}{3}\left[1 \pm \frac{(6 \times 0.1)}{3}\right] \text{kN/m}^2$$

$$\therefore \quad p_{\max} = 82.00 \text{ kN/m}^2$$

$$p_{\min} = 54.64 \text{ kN/m}^2$$

The maximum pressure developed is less than the S.B.C of soil = 200 kN/m^2.

Fig. 15.1 shows the pressure distribution at the base of the retaining wall.

f) Design of Heel Slab

Moment computations for one metre length of heel slab are shown in Table-15.5.

Maximum bending moment in heel slab at 'b' = (116.18–71.28) = 44.90 kN.m

$$\therefore \quad A_{st} = \left(\frac{M}{\sigma_{st} j\, d}\right) = \left(\frac{44.90 \times 10^6}{230 \times 0.90 \times 400}\right) = 538 \text{ mm}^2$$

Minimum reinforcement = $(0.0012 \times 450 \times 1000) = 540$ mm²/m

Adopt 12 mm diameter bars at 200 mm centres ($A_{st} = 565$ mm²)

Table 15.5 Moment Computations in Heel Slab

Loads	Magnitude of load (kN)	Distance from 'b' (m)	Bending moment (kN.m)
$W_3 = (1.55 \times 4.75 \times 18)$	132.50	0.775	102.68
Self weight of heel slab $(1.55 \times 0.45 \times 25)$	17.43	0.775	13.50
Total			116.18
Deduct for upward pressure (abih) = (54.64×1.55)	84.69	0.775	65.63
Upward pressure (ghi) $(0.5 \times 1.55 \times 14.13)$	10.95	0.516	5.65
Total deduction			71.28

g) Design of Toe Slab

Moment computations in toe slab for 1 m length is compiled in Table-15.6

Table 15.6 Moment computations in Toe Slab

Loads	Magnitude of load (kN)	Distance from 'c' (m)	Moment about 'c' (kN.m)
Upward pressure (cdif) (72.88×1)	72.88	0.50	36.44
Upward Pressure (jfe) $(0.5 \times 1 \times 9.12)$	4.56	0.67	3.05
Total			39.49
Deduct self weight of Toe slab $(1 \times 0.45 \times 25)$	11.25	0.50	5.62
Dead weight of soil over Toe slab $(0.75 \times 1 \times 18)$	13.50	0.50	6.75
Total deduction			12.37
Maximum B.M in Toe slab			27.12

$$\therefore \quad A_{st} = \left(\frac{M}{\sigma_{st} j\, d}\right) = \left(\frac{27.12 \times 10^6}{230 \times 0.90 \times 400}\right) = 325 \text{ mm}^2$$

Minimum reinforcement = $(0.0012 \times 1000 \times 450) = 540$ mm²/m

Hence, adopt 12 mm diameter bars at 200 mm centres as main reinforcement and also distribution steel.

h) Check for Sliding

$$\text{Total horizontal earth pressure} = \left(C_a \cdot \frac{\gamma H^2}{2}\right) = \left(\frac{1}{3} \times 18 \times \frac{5.2^2}{2}\right) = 81.12 \text{ kN}$$

Maximum possible frictional force = $\mu W = (0.5 \times 204.90) = 102.45$ kN

$$\therefore \quad \text{Factor of safety against sliding} = \left(\frac{102.45}{81.12}\right) = 1.26 < 1.5$$

The wall is unsafe against sliding.

A shear key is to be designed below the stem to increase the factor of safety.

i) Design of Shear Key

Let p_p = Intensity of passive pressure developed just in front of the shear key

This passive pressure intensity depends upon the soil pressure 'p' just in front of shear key.

$$p_p = C_p . p \quad \text{where} \quad C_p = \left(\frac{1+\sin\phi}{1-\sin\phi}\right) = \left(\frac{1}{C_a}\right) = 3$$

And $p = 72.88$ kN/m² (Refer Fig. 15.1)

$$\therefore \quad p_p = (3 \times 72.88) = 218.64 \text{ kN/m}^2$$

If a = depth of shear key

Total passive force = $P_p = (p_p \,.\, a) = (218.64 \times 0.45) = 98.38$ kN

$\therefore$ Factor of safety against sliding is computed as,

$$\text{F.S. aginst sliding} = \left[\frac{\mu W + P_p}{\Sigma P}\right] = \left[\frac{102.45 + 98.38}{81.12}\right] = 2.47 > 1.5$$

Hence, the retaining wall is safe against failure due to sliding.

The reinforcement in the stem is extended up to the shear key.

j) Check for Shear stress at junction of Stem and Base slab

Net shear force = $[1.5 \Sigma P - W] = [(1.5 \times 81.12) - 102.45] = 19.23$ kN

$$\tau_v = \left(\frac{19.23 \times 10^3}{1000 \times 400}\right) = 0.048 \text{ N/mm}^2$$

$$\left(\frac{100 A_{st}}{bd}\right) = \left(\frac{100 \times 1571}{1000 \times 400}\right) = 0.39$$

From Table-23 (IS: 456-2000), read out the permissible shear stress as

$$\tau_c = 0.25 \text{ N/mm}^2 > \tau_v$$

Hence, shear stresses are within safe permissible limits.
The details of reinforcements in the cantilever retaining wall is similar to that shown in limit state method of design in chapter-11.

15.6 DESIGN OF STAIRCASE

Design one of the flights of a doglegged staircase of a multi-storied building using the following data:

a) Data

Number of steps (Risers) in the Going = 10
Tread (T) = 300 mm, Rise (R) = 150 mm
Width of landing slab on either side = 1 m
Materials: M-20 grade concrete and Fe-415 HYSD bars.

b) Permissible Stresses

$\sigma_{cb} = 7 \text{ N/mm}^2$ $\quad Q = 0.91$
$\sigma_{st} = 230 \text{ N/mm}^2$ $\quad j = 0.90$
$m = 13$

c) Effective span

According to IS: 456-2000 code Fig. 17,
Effective span = $L = (G + X + Y)$
G = Going = Number of treads = $(9 \times 300) = 2700$ mm and $X = Y = 0.5$ m
$\therefore$ Effective span = $L = (2.7 + 0.5 + 0.5) = 3.7$ m
Fig. 15.2 shows the flight of staircase.
Assume thickness of waist slab based on span/depth ratio of 20.

$$\text{Depth} = \left(\frac{\text{span}}{20}\right) = \left(\frac{3700}{20}\right) = 185 \text{ mm}$$

Adopt overall depth = D = 185 mm and effective depth = d = 160 mm

d) Loads

Self weight of waist slab (on slope) = $w_1 = (0.185 \times 1 \times 25) = 4.62$ kN/m
Self weight of one step = $(0.5 \times 0.15 \times 0.3 \times 25) = 0.56$ kN/m

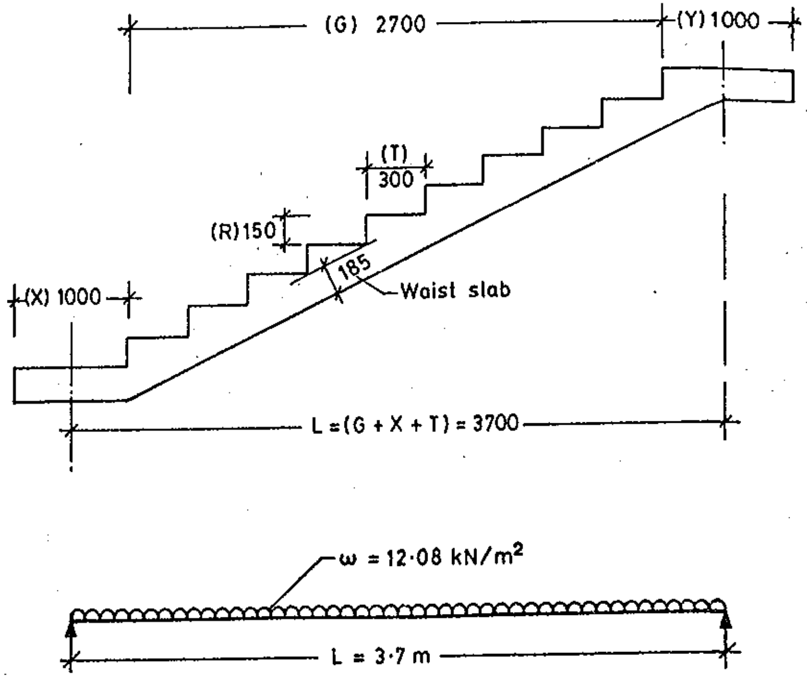


Fig. 15.2 Dimensions and Loading on Stair Case Flight

Load of steps/m length = $[0.56 \times (1000/300)] = 1.86$ kN/m
Weights of finishes = 0.60 kN/m
$\therefore$ Total dead load = $(4.62 + 1.86 + 0.60) = 7.08$ kN/m
Live load (Liable for over crowding) = 5.00 kN/m
Total design working load = w = 12.08 kN/m

e) Bending Moments and Shear Forces

$M = (0.125\, w\, L^2) = (0.125 \times 12.08 \times 3.7^2) = 20.67$ kN.m
$V = (0.5\, w\, L) = (0.5 \times 12.08 \times 3.7) = 22.35$ kN

f) Check for effective depth

$$d = \sqrt{\frac{M}{Qb}} = \sqrt{\frac{20.67 \times 10^6}{0.91 \times 10^3}} = 151.8 \text{ mm}$$

Effective depth provided = d = 160 mm (Hence, safe)

g) Reinforcements

$$A_{st} = \left(\frac{M}{\sigma_{st} j\, d}\right) = \left(\frac{20.67 \times 10^6}{230 \times 0.90 \times 160}\right) = 620 \text{ mm}^2/\text{m}$$

Provide 12 mm diameter bars at 180 mm centres ($A_{st} = 628$ mm^2)
Distribution reinforcement = $(0.0012 \times 1000 \times 185) = 222$ mm^2/m
Use 8 mm diameter bars at 200 mm centres ($A_{st} = 251$ mm^2)

h) Check fr- Shear stress

$$\tau_v = \left(\frac{V}{bd}\right) = \left(\frac{22.35 \times 10^3}{1000 \times 160}\right) = 0.139 \text{ N/mm}^2$$

$$\left(\frac{100A_{st}}{bd}\right) = \left(\frac{100 \times 628}{1000 \times 160}\right) = 0.39$$

Refer Table-23 (IS: 456-2000) and read out the permissible shear stress as

$$k_s\tau_c = (1.23 \times 0.25) = 0.30 \text{ N/mm}^2 > \tau_v$$

Hence, shear stresses are within safe permissible limits.
The reinforcement details are similar to that shown in the example 12.4 presented under limit state design of stairs in chapter-12.

15.7 DESIGN OF WATER TANKS

15.7.1 Introduction

Reinforced concrete water tanks are widely used to store large quantities of water in urban and rural water supply schemes. Water tightness is an important criterion in water tanks and to achieve this, richer concrete mixes of grades M-20 to M-30 are commonly used in the construction of water tanks. High quality concrete, in addition to providing water tightness, also has higher resistance to tensile stresses developed in the tank walls.There are three main types of reinforced concrete water tanks which are grouped as

a) Tanks resting on ground
b) Underground tanks
c) Elevated water tanks.

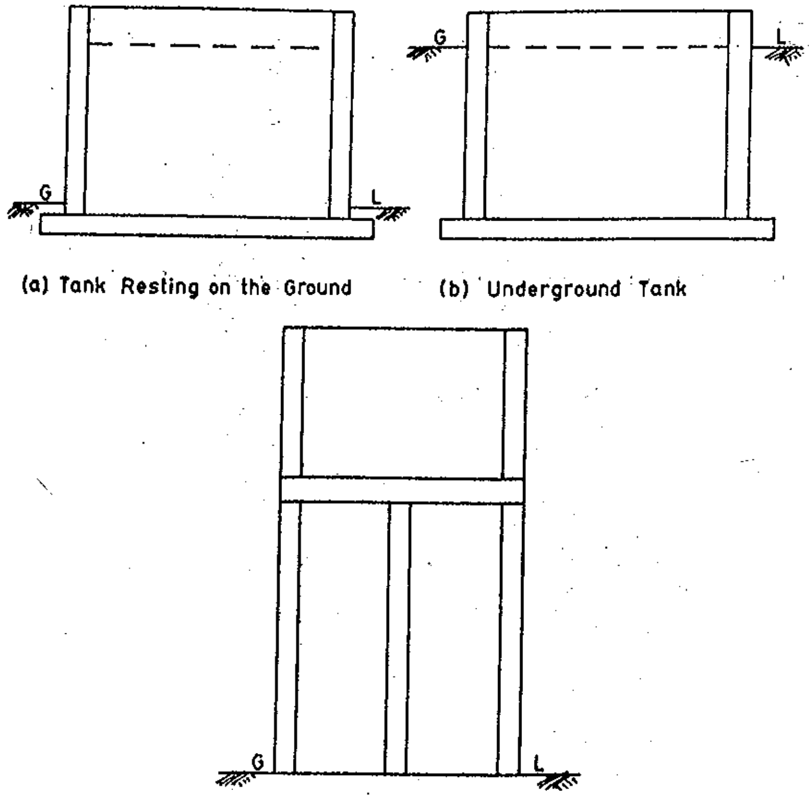
Fig. 15.3 shows the typical details of various types of water tanks.

The most common shapes of tanks are

(i) Square
(ii) Rectangular
(iii) Circular

Conical shaped tanks are also adopted due to aesthetic considerations but the storage capacity of these tanks are comparatively smaller than other types.

Elevated water tanks of large capacity are circular in shape or of Intz type. Circular tanks are generally preferred to other shapes since the walls of such tanks are subjected to hoop tension and the whole cross section is effective in resisting the tensile forces, leading to efficient utilization of material and reduction of costs.



(a) Tank Resting on the Ground (b) Underground Tank

(c) Elevated Water Tank

Fig. 15.3 Types of Water Tanks

15.7.2 Permissible Stresses and Reinforcement Details

a) Permissible Stresses

The permissible stresses in water retaining structures like tanks are specified in the Indian Standard Code IS: 3370-1965[101]. In water tanks, it is essential to ensure proper crack control which is achieved by reducing the permissible tensile stresses both in concrete and steel reinforcement.

The permissible stresses in concrete and steel in calculations relating to resistance to cracking in water retaining structures are compiled in Tables-15.7 and 15.8 respectively.

Table 15.7 Permissible Concrete Stresses in calculations relating to Resistance to Cracking in Water retaining Structures (IS: 3370-Part-II-1965)

Stress (N/mm^2)	Grade of Concrete					
	M-15	M-20	M-25	M-30	M-35	M-40
Direct Tension	1.1	1.2	1.3	1.5	1.6	1.7
Bending Tension	1.5	1.7	1.8	2.0	2.2	2.4

Table 15.8 Permissible Stresses in Steel reinforcement for Strength Calculations in Water retaining Structures (IS: 3370-Part-II-1965)

Stress (N/mm^2)	Plain Mild Steel bars	HYSD bars
Tensile stresses in members Under direct tension	115	150
Tensile stresses in members in Bending		
a) On liquid retaining face of members	115	150
b) On face away from liquid for members less than 225 mm thick	115	150
c) On face away from liquid for members 225 mm or more in thickness	125	190
d) Compressive stresses in columns subjected to direct load	125	175

b) Reinforcement Details

Minimum area of steel is 0.3 percent of gross area of section up to 100 mm thick, reduced to 0.2 percent in sections up to 450 mm thick. For sections above 225 mm thick, provide two layers of reinforcement. The percentage of reinforcement in base or floor slab resting directly on ground must be

not less than 0.15 percent of the concrete section.

The minimum cover to all reinforcement should be not less than 25 mm or the diameter of the bar whichever is greater.

15.7.3 Junctions of Tank wall and Base Slab

The joint between the walls of the tank and floor slab may be any one of the following three types as shown in Fig. 15.4.

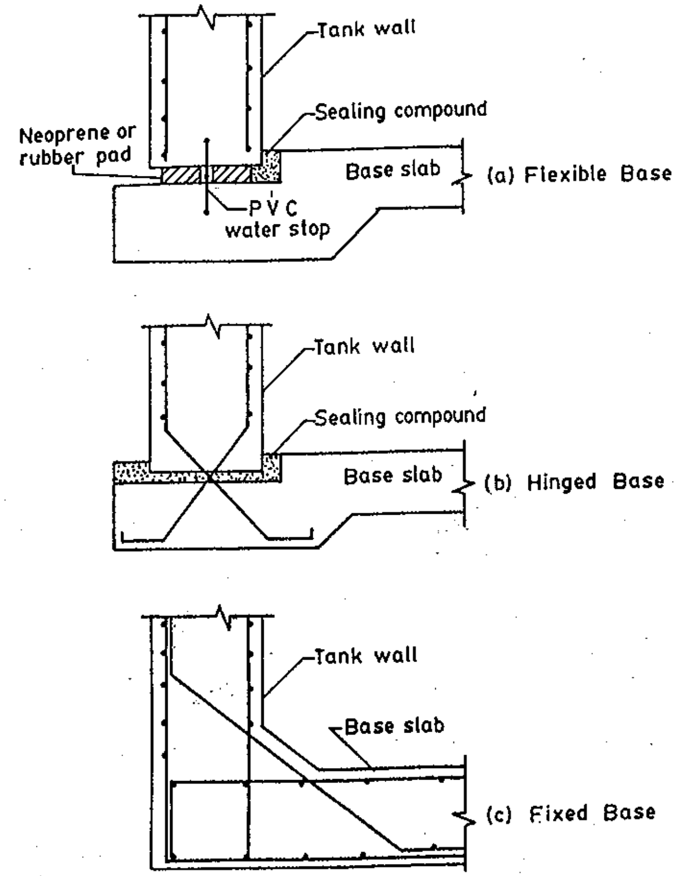


Fig. 15.4 Junctions of Tank Wall and Base Slab

a) Flexible or Free base b) Hinged base c) Fixed base

The type of junction between the tank walls and base slab influences the hoop tension and bending moments developed in the tank walls.

In the case of free or flexible base between tank wall and base slab, the walls are free to slide and expand and the hoop tension developed in the circular walls can be calculated easily due to the hydrostatic pressure. However for hinged and fixed bases, the coefficients for moments and ring tension are compiled in Tables-15.9 to 15.12 as recommended in IS: 3370(Part-4). These coefficients are expressed as a function of the non dimensional parameter (H^2/Dt),

Where H = height of water tank
D = diameter of tank
t = thickness of the tank wall

15.7.4 Ring Tension and Bending Moments in Cylindrical Tank Walls

In cylindrical tanks subjected to hydrostatic pressure, the walls of the tank develop hoop tension and bending moments depending upon the following factors.

1) Type of fixity between tank wall and base slab
2) Diameter of the tank
3) Thickness of the wall
4) Elastic constants of the wall material

The analysis is generally based on Timoshenko's general theory of cylindrical shells[102] with the valid assumption that the thickness of the tank wall is small in relation to the diameter. The bending moments M_w and the hoop tension N_d, developed in the tank walls. At a distance 'x' from the base of the tank (Refer Fig. 15.5) are expressed as,

$$M_w = \left(\frac{1}{\beta}\right).e^{-\beta x}\{M_o\beta(\cos\beta x + \sin\beta x) + N_o \sin\beta_x\}$$

$$N_d = \left(\frac{Et}{D\beta^3 K}\right)e^{-\beta x}\{M_o\beta(\cos\beta x - \sin\beta x) + N_o\cos\beta x\}$$

Where D = diameter of the tank
t = thickness of the tank wall
E = modulus of elasticity of material of the tank wall

The parametric constants β and K are expressed by the relation,

$$\beta = \sqrt[4]{\frac{12(1-\nu_c^2)}{D^2t^2}}$$

Table 15.9 Moments in Cylindrical Walls-Fixed Base Free at Top IS: 3370–Part IV) (Table-10 of IS:3370-Part-IV)
Moment M_w = (Coefficient) × (wH^3) kN m/m Positive sign indicates tension at the outside face

$\frac{H^2}{Dt}$	Coefficients at point									
	0.1 H	0.2 H	0.3 H	0.4 H	0.5 H	0.6 H	0.7 H	0.8 H	0.9 H	1.0 H
0.4	+.0005	+.0014	+.0021	+.0007	−.0042	−.0150	−.0302	−.0529	−.0816	−.1205
0.8	+.0011	+.0037	+.0063	+.0080	+.0070	+.0023	−.0068	−.0224	−.0465	−.0795
1.2	+.0012	+.0042	+.0077	+.0103	+.0112	+.0090	+.0022	−.0108	−.0311	−.0602
1.6	+.0011	+.0041	+.0075	+.0107	+.0121	+.0111	+.0058	−.0051	−.0232	−.0505
2.0	+.0010	+.0035	+.0068	+.0099	+.0120	+.0115	+.0075	−.0021	−.0185	−.0436
3.0	+.0006	+.0024	+.0047	+.0071	+.0090	+.0097	+.0077	+.0021	−.0119	−.0333
4.0	+.0003	+.0015	+.0028	+.0047	+.0066	+.0077	+.0069	+.0023	−.0080	−.0268
5.0	+.0002	+.0008	+.0016	+.0029	+.0046	+.0059	+.0059	+.0028	−.0058	−.0222
6.0	+.0001	+.0003	+.0008	+.0019	+.0032	+.0046	+.0051	+.0029	−.0041	−.0187
8.0	.0000	+.0001	+.0002	+.0008	+.0016	+.0028	+.0038	+.0029	−.0022	−.0146
10.0	.0000	.0000	+.0001	+.0004	+.0007	+.0019	+.0029	+.0028	−.0012	−.0122
12.0	.0000	−.0001	+.0001	+.0002	+.0003	+.0013	+.0023	+.0026	−.0005	−.0104
14.0	.0000	.0000	.0000	.0000	.0001	+.0008	+.0019	+.0023	+.0001	−.0090
16.0	.0000	.0000	−.0001	−.0002	−.0001	+.0004	+.0013	+.0019	+.0001	−.0079

	Coefficients at point				
	.80 H	.85 H	.90 H	.95 H	1.00 H
20.0	+.0015	+.0014	+.0005	−.0018	−.0063
24.0	+.0012	+.0012	+.0007	−.0013	−.0053
32.0	+.0007	+.0003	+.0007	−.0008	−.0040
40.0	+.0002	+.0005	+.0005	−.0005	−.0032
48.0	.0000	+.0001	+.0005	−.0003	−.0026
56.0	.0000	.0000	+.0004	−.0001	−.0023

Table 15.10 Ring Tension Cylindrical Walls-Fixed Base Free at Top (IS:3370–Part IV) ((Table-9 of IS:3370-Part-IV)
Ring tension N_d = (Coefficient) (wHR) kN/m

$\frac{H^2}{Dt}$	Coefficients at point									
	0.0 H	0.1 H	0.2 H	0.3 H	0.4 H	0.5 H	0.6 H	0.7 H	0.8 H	0.9 H
0.4	+0.149	+0.134	+0.120	+0.101	+0.082	+0.066	+0.049	+0.029	+0.014	+0.004
0.8	+0.263	+0.239	+0.215	+0.190	+0.160	+0.130	+0.096	+0.063	+0.034	+0.010
1.2	+0.283	+0.271	+0.254	+0.234	+0.209	+0.180	+0.142	+0.099	0.054	+0.016
1.6	+0.265	+0.268	+0.268	+0.266	+0.250	+0.226	+0.185	+0.134	+0.075	+0.023
2.0	+0.234	+0.251	+0.273	+0.285	+0.285	+0.274	+0.232	+0.172	+0.104	+0.031
3.0	+0.134	+0.203	+0.267	+0.322	+0.357	+0.362	+0.330	+0.262	+0.157	+0.052
4.0	+0.067	0.164	+0.256	+0.339	0.403	+0.429	+0.409	+0.334	+0.210	+0.073
5.0	+0.025	+0.137	+0.245	+0.346	+0.428	+0.477	+0.469	+0.398	+0.259	+0.092
6.0	+0.018	+0.119	+0.234	+0.344	+0.441	+0.504	+0.514	+0.447	+0.301	+0.112
8.0	−0.011	+0.104	+0.218	+0.335	0.443	+0.534	+0.575	+0.530	+0.381	+0.151
10.0	−0.011	+0.098	+0.208	+0.323	+0.437	+0.542	+0.608	+0.589	+0.440	+0.179
12.0	−0.005	+0.097	+0.202	0.312	+0.429	0.543	+0.628	+0.633	+0.494	+0.211
14.0	−0.002	+0.098	+0.200	+0.306	+0.420	+0.539	+0.639	+0.666	+0.541	+0.241
16.0	−0.000	+0.099	+0.199	+0.304	+0.412	+0.531	+0.641	+0.687	+0.582	+0.265

	Coefficients at point				
	.75 H	.80 H	.85 H	.90 H	.95 H
20.0	+0.716	+0.654	+0.520	+0.325	+0.115
24.0	+0.746	+0.702	+0.577	+0.372	+0.137
32.0	+0.782	+0.768	+0.663	+0.459	+0.182
40.0	+0.800	+0.805	+0.731	+0.530	+0.217
48.0	+0.791	+0.828	+0.785	+0.593	+0.254
56.0	+0.763	+0.838	+0.824	+0.536	+0.285

Table 15.11 Moments in Cylindrical Walls-Hinged Base Free at Top (IS: 3370–Part IV) (Table-13 of IS:3370-Part-IV) Moment M_w = (Coefficient) × (wH^3) kN m/m Positive sign indicates tension at the outside

$\frac{H^2}{Dt}$	Coefficients at point									
	0.1 H	0.2 H	0.3 H	0.4 H	0.5 H	0.6 H	0.7 H	0.8 H	0.9 H	1.0 H
0.4	+.0020	+.0072	+.0151	+.0230	+.0301	+.0348	+.0357	+.0312	+.0197	0
0.8	+.0019	+.0064	+.0133	+.0207	+.0271	+.0319	+0.329	.0292	+.0187	0
1.2	+.0016	+.0058	+.0111	+.0177	+.0237	+.0280	+.0296	+.0263	+.0187	0
1.6	+.0012	+.0044	+.0091	+.0145	+.0195	+.0236	+.0255	+.0232	+.0171	0
2.0	+.0006	+.0033	+.0073	+.0114	+.0158	+.0199	+.0219	+.0205	+.0155	0
3.0	+.0004	+.0018	+.0040	+.0063	+.0092	+.0127	+.0152	+.0153	+.0145	0
4.0	+.0001	+.0007	+.0016	+.0033	+.0057	+.0083	+.0109	+.0118	+.0111	0
5.0	.0000	+.0001	+.0006	+.0016	+.0034	+.0057	+.0080	+.0094	+.0092	0
6.0	.0000	.0000	+.0002	.0000	+.0019	+.0039	+.0062	+.0078	+.0078	0
8.0	.0000	.0001	−.0002	.0000	+.0007	+.0020	+.0038	+.0057	+.0068	0
10.0	.0000	.0000	−.0002	−.0001	+.0002	+.0011	+.0025	+.0043	+0054	0
12.0	.0000	.0000	−.0001	.0002	.0000	+.0005	+.0017	+.0032	+.0045	0
14.0	.0000	.0000	.0000	−.0001	.0001	.0000	+.0012	+.0026	+.0039	0
16.0	.0000	.0000	.0000	−.0001	−.0002	−.0004	+.0008	+.0022	+.0033	0
									+.0029	0

	Coefficients at point				
	.75 H	.80 H	.85 H	.90 H	.95 H
20.0	+.0008	+.0014	+.0020	.0024	+.0020
24.0	+.0005	+.0010	+.0015	+.0020	+.0017
32.0	.0000	+.0005	+.0009	+.0014	+.0013
40.0	.0000	+.0003	+.0006	+.0011	+.0011
48.0	.0000	+.0001	+.0004	+.0008	+.0010
56.0	.0000	.0000	+.0003	+.0007	+.0008

Table 15.12 Ring Tension Cylindrical Walls-Hinged Base Free at Top (IS:3370–Part IV) ((Table-12 of IS:3370-Part-IV) Ring tension N_d = (Coefficient) × (wHR) kN/m Positive sign indicates tension

$\frac{H^2}{Dt}$	Coefficients at point									
	0.0 H	0.1 H	0.2 H	0.3 H	0.4 H	0.5 H	0.6 H	0.7 H	0.8 H	0.9 H
0.4	+0.474	+0.440	+0.395	+0.352	+0.305	+0.264	+0.215	+0.165	+0.111	+0.057
0.8	+0.423	+0.402	+0.381	+0.358	+0.330	+0.297	+0.249	+0.202	+0.145	+0.076
1.2	+0.350	+0.355	+0.361	+0.362	+0.358	+0.343	+0.309	+0.256	+0.186	+0.098
1.6	+0.271	+0.303	+0.341	+0.369	+0.385	+0.385	+0.362	+0.314	+0.233	+0.124
2.0	+0.205	+0.260	+0.321	+0.373	+0.411	+0.434	+0.419	+0.369	+0.280	+0.151
3.0	+0.074	+0.179	+0.281	+0.375	+0.449	+0.506	+0.519	+0.479	+0.375	+0.210
4.0	+0.017	+0.137	+0.253	+0.367	+0.469	+0.545	+0.579	+0.553	+0.447	+0.256
5.0	-0.008	+0.114	+0.235	+0.356	+0.469	+0.562	+0.617	+0.606	+0.503	+0.294
6.0	-0.011	+0.103	+0.223	+0.343	+0.463	+0.566	+0.639	+0.643	+0.547	+0.327
8.0	-0.015	+0.096	+0.208	+0.324	+0.443	+0.564	+0.661	+0.697	+0.621	+0.386
10.0	-0.008	+0.095	+0.200	+0.311	+0.428	+0.552	+0.666	+0.730	+0.678	+0.433
12.0	-0.002	+0.097	+0.197	+0.302	+0.417	+0.541	+0.664	+0.750	+0.720	+0.477
14.0	0.000	+0.098	+0.197	+0.299	+0.408	+0.531	+0.659	+0.761	+0.752	+0.513
16.0	+0.002	+0.100	+0.198	+0.299	+0.403	+0.521	+0.650	+0.764	+0.776	+0.543

	Coefficients at point				
	.75 H	.80 H	.85 H	.90 H	.95 H
20.0	+0.812	+0.817	+0.755	+0.603	+0.344
24.0	+0.816	+0.839	+0.793	+0.647	+0.377
32.0	+0.814	+0.861	+0.847	+0.721	+0.436
40.0	+0.802	+0.866	+0.880	+0.778	+0.483
48.0	+0.791	+0.854	+0.900	+0.820	+0.527
56.0	+0.781	+0.859	+0.911	+0.852	+0.563

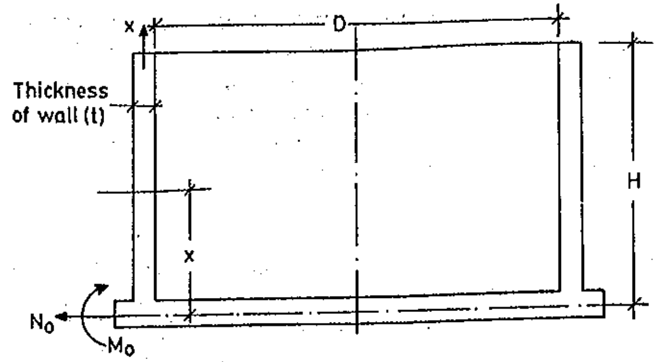


Fig. 15.5 Analysis of Circular Cylindrical Tank

$$K = \left[\frac{Et^3}{12(1-\nu_c^2)}\right]$$

Where ν_c = Poisson's ratio of wall material and M_o and N_o are the moment and shear acting at the base of the tank with their values depending upon the pressure distribution and the conditions of fixity at the base. A diagrammatic representation of the variation of bending moments and hoop tension in the walls of the tank for different types of bases is shown in Fig. 15.6.

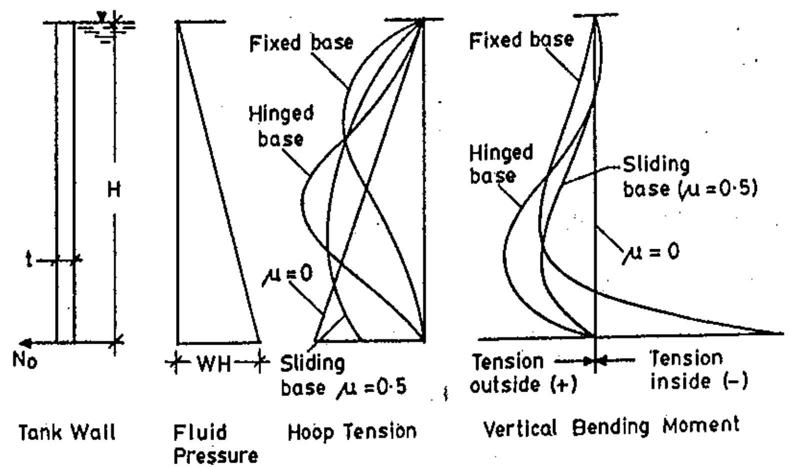


Fig. 15.6 Hoop Tension and Bending Moments in Cylindrical Tank Walls

The hoop tension is maximum at base in the case of tanks with sliding or free base while the bending moment is maximum for tanks with a fixed base. In the case of tank walls resting on neoprene or rubber pads, a comparatively smaller magnitude of bending moments are generated due to the radial frictional force developed at the base junction. Investigations by Crom[103], indicates that the base shear can be estimated for a maximum coefficient of friction 'μ' of 0.5, which is not likely to be exceeded.

The maximum bending moment developed in the tank wall due to a base shear force N_o has a value of $0.247\, N_o \sqrt{Rt}$, where 'R' is the internal radius of the tank and 't' is the thickness of the tank wall. The maximum moment generally develops at approximately one-fifth the height of the wall under pressure, measured from the base. The design moments and hoop tension in cylindrical walls can be computed by using the coefficients compiled in Tables-15.9 to 15.12 recommended in IS: 3370[104]. The design coefficients are expressed as a function of the non-dimensional parameter (H^2/Dt), where 'H' is the depth of water stored. The coefficients are tabulated for various points from $0.1H$ to H.

15.7.5 Design Example of Circular Tank with Freebase

Design an R.C.C. circular water tank resting on the ground with a flexible base and a spherical dome for storing 500 000 liters of water. The depth of storage is to be 4 m. Freeboard = 200 mm. Adopt M-20 grade concrete and Fe-415 HYSD bars. Permissible stresses should comply with the values recommended in IS: 3370 and IS: 456-2000 codes. Sketch the details of reinforcements in the dome, tank walls, and the floor slab.

a) Data

Capacity of circular tank = 500,000 litres
Depth of water storage = 4 m
Free board = 200 mm
Materials: M-20 grade concrete and Fe-415 HYSD bars.

b) Permissible Stresses

σ_{ct} = 1.2 N/mm^2 (for tank walls)
= 2.8 N/mm^2 (for dome ring beam)
σ_{cc} = 5 N/mm^2
σ_{st} = 150 N/mm^2
m = 13

c) Dimensions of Tank

Let D = diameter of the circular tank (Refer Fig. 15.7)

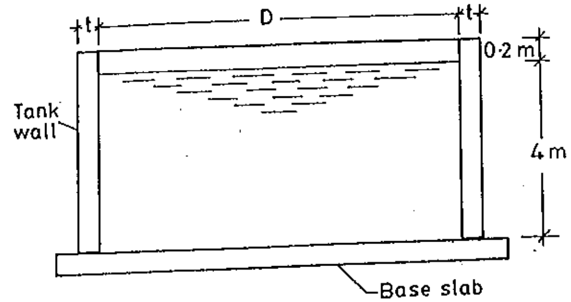


Fig. 15.7 Circular Water Tank

$$\left(\frac{\pi D^2}{4} \times 4\right) = \left(\frac{500000 \times 10^3}{10^6}\right)$$

Solving, the diameter of the tank = D = 12.6 m

d) Design of Spherical Dome

Diameter of dome at base = 12.6 m
Central rise = (1/5) Diameter = (1/5)(12.6) = 2.5 m
Let R = radius of the dome
Referring to Fig. 15.8, we have the relation,

$$(R - 2.5)^2 = (R^2 - 6.3^2)$$

Solving, R = 9.2 m
Semi-central angle = θ = 43.2 degrees

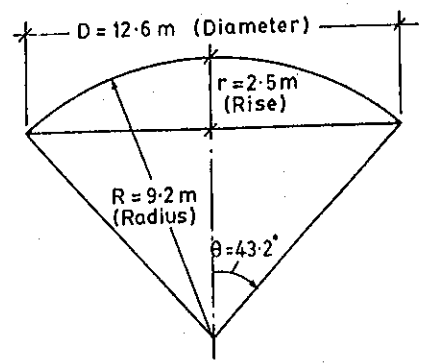


Fig. 15.8 Details of Dome of Circular Water Tank

Hence, $\sin\theta = 0.6847$

$\cos\theta = 0.7289$

Assume the thickness of dome $= t = 100$ mm

i) Loads

Self weight of dome $= (0.1 \times 25) = 2.5$ kN/m^2

Live load and Finishes $= 2.0$

Total load $= w = 4.5$ kN/m^2

ii) Stresses in Dome

$$\text{Meridional thrust} = T_1 = \left(\frac{wR}{1+\cos\theta}\right) = \left(\frac{4.5\times 9.2}{1+0.7289}\right) = 23.94 \text{ kN/m}^2$$

$$\text{Meridional compressive stress} = \left(\frac{23.94\times 10^3}{1000\times 100}\right) = 0.2394 \text{ N/mm}^2 < 5\text{N/mm}^2$$

$$\text{Hoop stress} = \frac{wR}{t}\left[\cos\theta - \frac{1}{(1+\cos\theta)}\right]$$

$$= \frac{(4.5\times 9.2)}{0.1})\left[0.7289 - \frac{1}{1.7289}\right] = 62.1 \text{ kN/m}^2 = 0.0621 \text{ N/mm}^2 < 5 \text{ N/mm}^2$$

Hence, the stresses are within safe permissible limits.

iii) Reinforcements in Dome

Since the stresses are very low, nominal reinforcement of 0.3 percent of the gross cross sectional area is provided.

$$A_{st} = (0.003 \times 1000 \times 100) = 300 \text{ mm}^2$$

Spacing of 8 mm diameter bars $= [(1000 \times 50)/300] = 166$ mm

Provide 8 mm diameter bars at 150 mm centres both meridionally and circumferentially.

iv) Ring Beam

Horizontal component of thrust $= T_1 \cos\theta = (23.94 \times 0.7289) = 17.45$ kN/m

Hoop tension in ring beam $= F_t = [(17.45 \times 12.6)/2] = 110$ kN

$$\therefore \quad A_{st} = [(110\times 1000)/150 = 734 \text{ mm}^2$$

Provide 4 bars of 16 mm diameter ($A_{st} = 804$ mm^2)

Let A_c = cross sectional area of ring beam.

Allowing a tensile stress of 2.8 N/mm^2 in concrete, we have the relation,

$$\left[\frac{F_t}{A_c + (m-1)A_{st}}\right] = \left[\frac{110\times 10^3}{A_c + (13-1)804}\right] = 2.8$$

Solving $A_c = 29638$ mm^2

Adopt a ring beam of size 200 mm by 200 mm with 4 bars of 16 mm diameter as hoop reinforcement and stirrups of 6 mm diameter at 150 mm centres.

e) Reinforcements in Tank Walls

Maximum hoop tension $= (0.5.w.H.D) = (0.5 \times 10 \times 4.2 \times 12.6)$
$= 264.6$ kN

Tension reinforcement per metre height of tank wall is computed as,

$$A_{st} = \left(\frac{264.6\times 10^3}{150}\right) = 1764 \text{ mm}^2$$

Using 12 mm diameter bars on both faces,

$$\text{Spacing} = \left(\frac{1000\times 113\times 2}{1764}\right) = 128 \text{ mm}$$

Provide 12 mm diameter bars at 120 mm centres at the base section on either face of the wall.

f) Thickness of Tank Wall

If t = thickness of tank wall, from cracking considerations we have the relation,

$$\left[\frac{0.5wHD}{1000t + (m-1)A_{st}}\right] = \left[\frac{264.6\times 10^3}{1000t + (13-1)1764}\right] = \sigma_{ct} = 1.2$$

Solving we have, $t = 199.3$ mm

Provide 200 mm thick wall

g) Curtailment of Reinforcement in Tank Walls

Minimum reinforcement at the top of tank wall = 0.3 percent of cross sectional area.

$$A_{st} = \left(\frac{0.3 \times 1000 \times 200}{100}\right) = 600 \text{ mm}^2$$

Spacing of 10 mm diameter hoops on both faces is computed as,

$$\text{Spacing of bars} = \left(\frac{1000 \times 79 \times 2}{600}\right) = 263\text{mm}^2$$

Provide 10 mm diameter bars at 250 mm centres on both faces for a height of 1 m from the top of tank.

Area of reinforcement required at mid height (2 m below top) is given by

$$A_{st} = \left(\frac{0.5\, wHD}{150}\right) = \left(\frac{0.5 \times 10 \times 2 \times 12.6 \times 10^3}{150}\right) = 882 \text{ mm}^2$$

Spacings of 12 mm diameter bars are given by the relation,

$$\text{Spacing of bars} = \left(\frac{1000 \times 113 \times 2}{882}\right) = 256\text{mm}$$

Provide 12 mm diameter bars at 250 mm centres on both faces.
Distribution and temperature reinforcement is provided in the vertical direction.
Area of vertical reinforcement = 0.3 percent = (0.003 × 1000 × 200) = 600 mm^2
Spacings of 10 mm diameter bars on both faces is computed as,

$$\text{Spacing of bars} = \left(\frac{1000 \times 78.5 \times 2}{600}\right) = 261\text{mm}$$

Provide 10 mm diameter bars at 250 mm centres on both faces in the vertical direction.

h) Design of Tank Floor Slab

Provide nominal thickness of 150 mm for the base slab over a layer of M-10 lean concrete of 75 mm thick. Reinforcement is provided in the form of a mat with bars in the mutually perpendicular directions.

Area of reinforcement = 0.3 percent = (0.003 × 150 × 1000) = 450 mm^2 in each direction.
Provide half the reinforcement near each face.

Hence, $A_{st} = (0.5 \times 450) = 225 \text{ mm}^2$

$$\text{Spacing of 8 mm diameter bars} = \left(\frac{1000 \times 50}{225}\right) = 220 \text{ mm}$$

Provide 8 mm diameter bars at 200 mm centres in both directions at the top and bottom faces of the tank floor slab. The details of reinforcements in the dome, tank walls and floor slab are shown in Fig. 15.9.

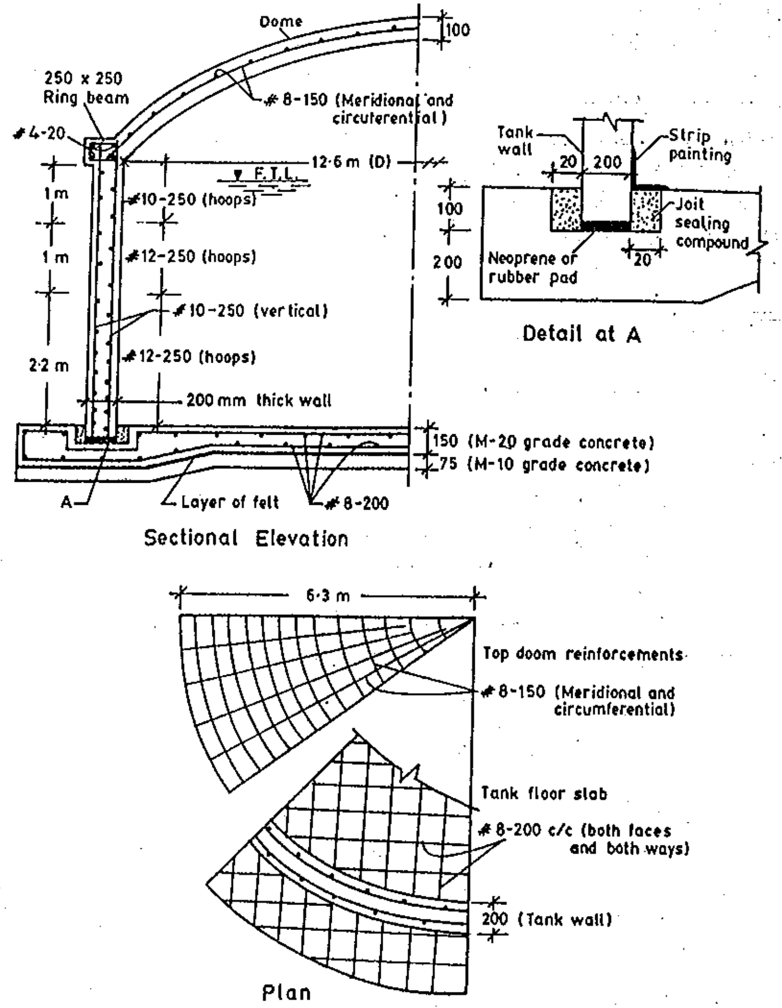


Fig. 15.9 Reinforcement Details in Circular Water Tank

15.7.6 Design of Rectangular Tanks

Rectangular tanks are frequently used for storage of water of small capacities due to ease of construction where form work costs of circular tanks is prohibitive. The tank walls are subjected to moments and direct tension due to hydrostatic pressure. The exact analysis is rather complex being three dimensional in nature and hence rectangular tanks are designed by approximate methods.

The following parameters are used in the design of tanks.
Longer side of tank wall (length) = L
Shorter side of tank wall (Breadth) = B
Height of tank wall = H
The moments developed in the tank walls depends upon the ratio (L/B).

a) Tanks of Ratio (L/B) < 2

For rectangular tanks in which the ratio of length to breadth is less than 2 as shown in Fig. 15.10, the tank walls are designed as continuous frame subjected to water pressure varying from zero at top to maximum at ($H/4$) or 1 m whichever is more.

For the bottom portion ($H/4$) or 1 m, the bending is in the vertical plane and this portion is designed as a cantilever. The corners are designed for the maximum moment obtained after moment distribution with the intensity of pressure $p = w\,(H-h)$. In the absence of moment distribution, the bending moments may be computed by the following approximate expressions.

Bending moment at centre of span $= \left(\dfrac{pB^2}{16}\right)$ and $\left(\dfrac{pL^2}{16}\right)$ respectively producing tension on the outer face.

Bending moment at centre of span $= \left(\dfrac{pB^2}{12}\right)$ and $\left(\dfrac{pL^2}{12}\right)$ respectively producing tension on water face.

In addition to the bending moments, the tank walls are subjected to direct tension computed using the following expressions.
Direct tension in long walls = $T_L = [w(H-h)B/2]$
Direct tension in short walls = $T_B = [w(H-h)L/2]$
Design moment = $(M - T.x)$
Where x = distance of steel reinforcement from the centre of section.
The total steel area $A_{st} = (A_{st1} + A_{st2})$ is computed as that required for bending moment and direct tension using the following expressions.

$$A_{st1} = \left[\frac{M - T.x}{\sigma_s j d}\right] \quad \text{and} \quad A_{st2} = \left[\frac{T}{\sigma_{st}}\right]$$

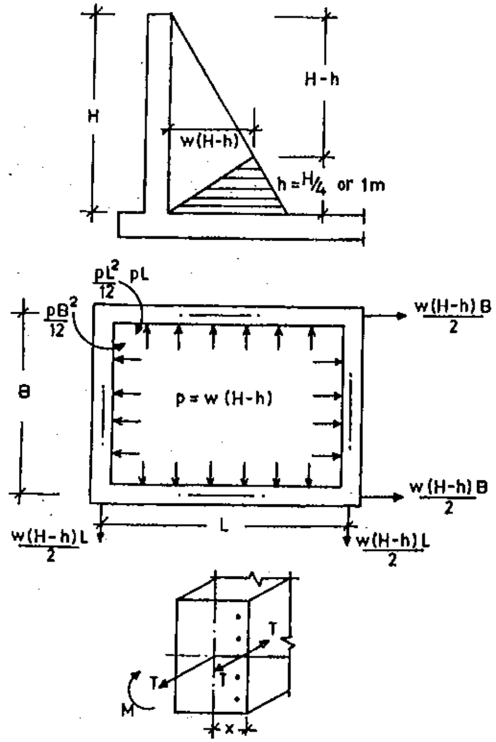


Fig. 15.10 Forces in Walls of Rectangular Tanks

b) Tanks of Ratio (L/B) > 2

In this case, the long walls are assumed to bend vertically and hence designed as cantilevers. Short walls are assumed to bend horizontally supported by long walls above ($H/4$) or 1 m from the bottom of the tank.

Bending moment for long walls $= \left(\dfrac{wH^3}{6}\right)$

Bending moment for short walls (above 1 m from base) $= \left[\dfrac{w(H-h)B^2}{16}\right]$

Maximum cantilever moment for short wall $= \left(\dfrac{w.H.h^2}{6}\right)$ or $\left(\dfrac{wH \times 1}{6}\right)$

whichever is greater.

In addition to bending moment, short walls and long walls are subjected to direct tension.

15.7.2 Design Example

A rectangular R.C.C water tank with an open top is required to store 80,000 liters of water. The inside dimensions of the tank may be taken as 6 m by 4 m. The tank rests on walls on all the four sides. Design the side walls of the tank using M-20 grade concrete and Fe-415 HYSD bars.

a) Data

Volume of tank = 80,000 litres

Size of tank = 6 m by 4 m

Free board = 150 mm

Materials: M-20 grade concrete and Fe-415 HYSD bars

b) Permissible Stresses

$\sigma_{cb} = 7\ \text{N/mm}^2$ $\qquad Q = 1.20$

$\sigma_{st} = 150\ \text{N/mm}^2$ $\qquad j = 0.86$

$m = 13$

c) Dimensions of Tank

Referring to Fig. 15.11 (a)

$$\text{Height of water} = \left(\frac{80000 \times 10^3}{600 \times 400}\right) = 335\ \text{cm}$$

Height of tank walls = $H = (335 + 15) = 350$ cm = 3.5 m

$$L = 6\ \text{m} \ \text{ and } \ B = 4\ \text{m} \quad \therefore \quad \text{Ratio } (L/B) = (6/4) = 1.2 < 2$$

Hence walls are designed as continuous slab subjected to water pressure above (H/4) or 1 m from bottom, whichever is greater.

$$\therefore \qquad (H - h) = (3.5 - 1) = 2.5\ \text{m}$$

At section XX, the intensity of pressure = p = w (H − h) = 10(2.5) = 25 kN/m^2

d) Moments in Sidewalls

The moments in sidewalls is determined by moment distribution.

Fixed end moments:

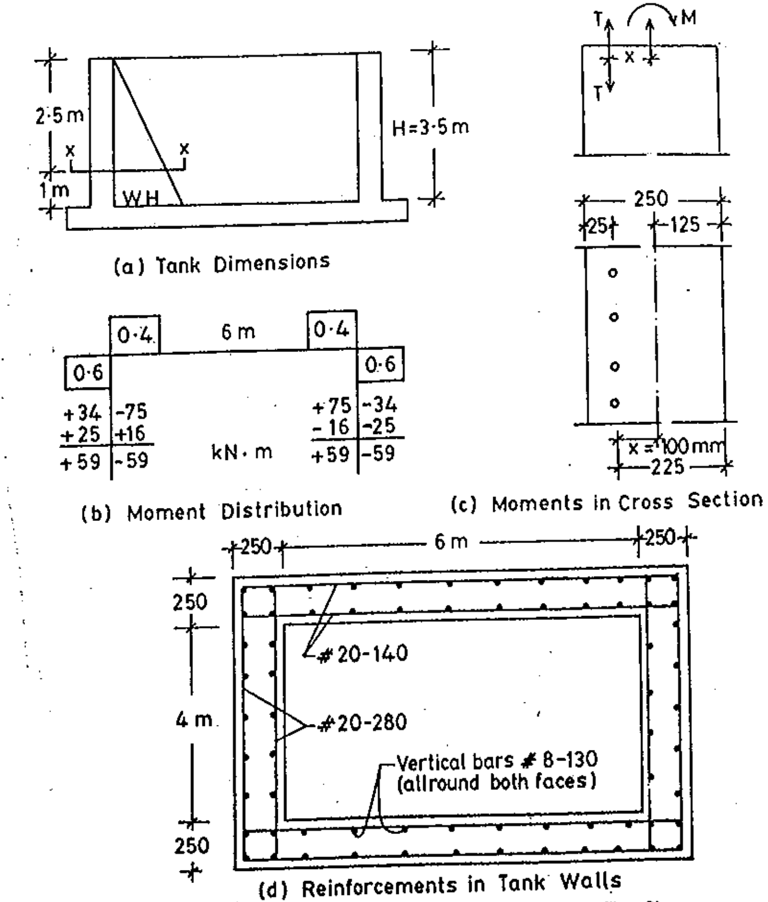


Fig. 15.11 Design of Rectangular Tank (L/B < 2)

i) Long walls:

$$\left(\frac{pL^2}{12}\right) = \left(\frac{25 \times 6^2}{12}\right) = 75\ \text{kN.m}$$

$$\left(\frac{pL^2}{8}\right) = \left(\frac{25 \times 6^2}{8}\right) = 112.5\ \text{kN.m}$$

ii) Short walls

$$\left(\frac{pB^2}{12}\right) = \left(\frac{25 \times 4^2}{12}\right) = 34\ \text{kN.m}$$

$$\left(\frac{pB^2}{8}\right) = \left(\frac{25 \times 4^2}{8}\right) = 50 \text{ kN.m}$$

Moment distribution is shown in Fig. 15.11(b)
Moment at support = 59 kN.m
Moment at centre (Long walls) = (112 – 59) = 53 kN.m
Moment at centre (Short walls) = (50 – 59) = –9 kN.m

e) Design of Long and Short walls

Maximum moment = 59 kN.m

$$d = \sqrt{\frac{59 \times 10^6}{1.2 \times 10^3}} = 222 \text{ mm}$$

Adopt effective depth = d = 225 mm and overall depth = D = 250 mm
Direct tension in long wall = T_L = [(25 × 4)/2] = 50 kN
Direct tension in short wall = T_B = [(25 × 6)/2] = 75 kN

$$A_{st} \text{ (Long wall corners)} = \left[\frac{M - T.x}{\sigma_{st} j\, d}\right] + \left[\frac{T}{\sigma_{st}}\right]$$

Referring to Fig. 15.11(c),

$$A_{st} = \left[\frac{(59 \times 10^6) - (50 \times 10^3 \times 100)}{150 \times 0.86 \times 225}\right] + \left[\frac{50 \times 10^3}{150}\right] = 2194 \text{ mm}^2$$

Provide 20 mm diameter bars at 140 mm centres (A_{st} = 2244 mm²/m) at corners.
Reinforcement at centre of span (Long walls) is computed as,

$$A_{st} = \left[\frac{(53 \times 10^6) - (50 \times 10^3)}{150 \times 0.86 \times 225}\right] + \left[\frac{50 \times 10^3}{150}\right] = 1987 \text{ mm}^2$$

Provide 20 mm diameter bars at 140 mm centres (A_{st} = 2244 mm²)

For short walls, the moment being small, provide 50 percent of the bars at corners i.e. 20 mm diameter bars at 280 mm centres at centre of span.

f) Reinforcement for Cantilever moment
(For 1 m height from bottom)

Cantilever moment = (3.5 × 10 × 0.5 × 0.333) = 5.833 kN.m

$$\therefore \quad A_{st} = \left(\frac{5.833 \times 10^6}{150 \times 0.86 \times 225}\right) = 201 \text{ mm}^2$$

Minimum reinforcement in the vertical direction is computed as,

$$A_{st,min} = 0.3 \text{ percent} = (0.003 \times 1000 \times 250) = 750 \text{ mm}^2$$

Steel on each face = (0.5 × 750) = 375 mm²

$$\text{Spacing of 8 mm diameter bars} = \left(\frac{1000 \times 50}{375}\right) = 130 \text{ mm}$$

Adopt 8 mm diameter bars at 130 mm centres on both faces as shown in Fig. 15.11 (d).

15.7.3 Design Example

Design the sidewalls of a rectangular reinforced concrete water tank of dimensions 6 m by 2 m and having a maximum depth of 2.5 m, using M-20 grade concrete and Fe-415 HYSD bars.

a) Data

Size of tank = 6 m by 2 m
Length = L = 6 m and breadth = B = 2 m
Depth of tank = H = 2.5 m
Materials: M-20 grade concrete and Fe-415 HYSD bars

b) Permissible Stresses

σ_{cb} = 7 N/mm² $\qquad Q$ = 1.20
σ_{st} = 150 N/mm² $\qquad j$ = 0.86

c) Design of Long walls

L = 6 m and B = 2 m
(Ratio (L/B) = (6/2) = 3 > 2

Long walls are designed as vertical cantilevers and short wall as a slab spanning horizontally between long walls.
Maximum bending moment at base of long wall is computed as,

$$M_L = \left(\frac{wH^3}{6}\right) = \left(\frac{10 \times 2.5^3}{6}\right) = 26.04 \text{ kN.m}$$

$$d = \sqrt{\frac{M}{Qb}} = \sqrt{\frac{26.04 \times 10^6}{1.20 \times 10^3}} = 147.3 \text{ mm}$$

Adopt effective depth = d = 150 mm and overall depth = D = 180 mm

$$\therefore \quad A_{st} = \left(\frac{26.04 \times 10^6}{1.20 \times 10^3}\right) = 1340 \text{ mm}^2$$

Provide 16 mm diameter bars at 150 mm centres (A_{st} = 1341 mm^2) at the bottom of the tank. Spacing increased to 170 mm for the top 1 m portion of the tank.

Intensity of pressure 1 m above the base is computed as

$$P = w(H - h) = 10(2.5 - 1) = 15 \text{ kN/m}^2$$

Direct tension in long walls = $T_L = [(15 \times 2)/2] = 15$ kN

$$\therefore \quad A_{st} = \left(\frac{15 \times 10^3}{150}\right) = 100 \text{ mm}^2$$

Minimum area of steel = 0.3% = (0.003 × 180 × 1000) = 540 mm^2

$$\text{Spacing of bars} = \left(\frac{1000 \times 79}{540}\right) = 146 \text{ mm}$$

Since steel is distributed on both faces, provide 10 mm diameter bars at 280 mm centres on both faces in the horizontal direction.

d) Design of Short walls

Intensity of pressure = p = 15 kN/m^2
Effective span of horizontally spanning slab = (2 + 0.18) = 2.18 m

$$\text{Bending moment (corner section)} = \left(\frac{pL^2}{12}\right) = \left(\frac{15 \times 2.18^2}{12}\right) = 5.94 \text{ kN.m}$$

Tension transferred per metre height of short wall = (15 × 1) = 15 kN

$$\therefore \quad A_{st} = \left[\frac{M - T.x}{\sigma_{st}\, j\, d}\right] + \left[\frac{T}{\sigma_{st}}\right]$$

$$A_{st} = \left[\frac{(5.94 \times 10^6) - (15 \times 10^3)(150 - 90)}{(150 \times 0.86 \times 150)}\right] + \left[\frac{15 \times 10^3}{150}\right] = 360 \text{ mm}^2$$

Minimum reinforcement = 0.3 percent = (0.003 × 180 × 1000) = 540 mm^2
Hence, provide 10 mm diameter bars at 280 mm centres on both faces in the horizontal direction with an effective cover of 30 mm

e) Design for Cantilever effect of short wall

Maximum bending moment at bottom of wall is computed as,

$$M = (0.5 \times 10 \times 2.5 \times 1 \times 0.333) = 4.2 \text{ kN.m}$$

Effective depth using 10 mm diameter bars = (180 − 40) = 140 mm

$$A_{st} = \left(\frac{4.2 \times 10^6}{150 \times 0.86 \times 140}\right) = 232 \text{ mm}^2$$

Minimum reinforcement = 0.3% = (0.003 × 180 × 1000) = 540 mm^2
Provide 10 mm diameter bars at 280 mm centres in the vertical direct on both faces.
The details of reinforcements in the tank walls are shown in Fig. 15.12.

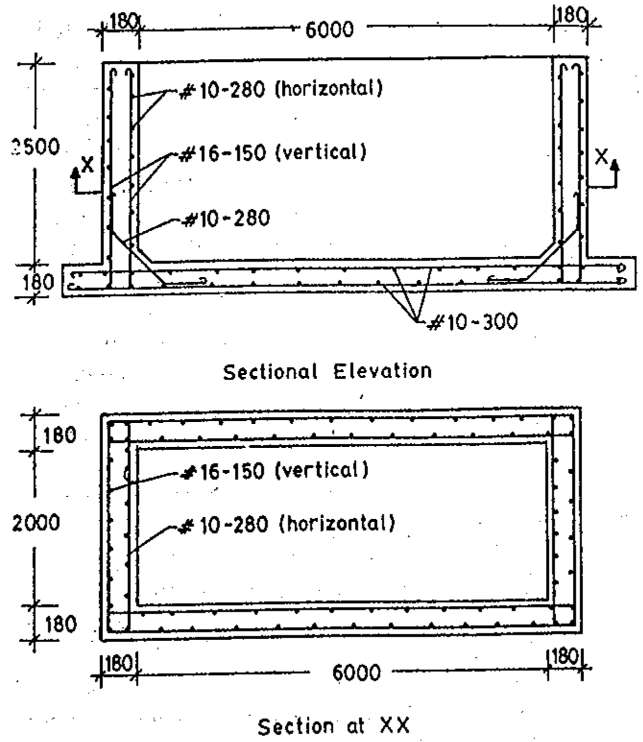


Fig. 15.12 Reinforcement Details in Rectangular Tank

15.8 EXAMPLES FOR PRACTICE

1) A simply supported reinforced concrete slab with a clear span of 3.5 m is supported on concrete masonry walls 300 mm thick along the edges. If the service live load on the slab is 2 kN/m^2 and the floor finish is 0.6 kN/m^2, design the slab using M-20 grade concrete and Fe-415 HYSD bars.

2) Design a simply supported slab supported on masonry walls 200 mm thick and having a clear span of 2.5 m. Live load = 4 kN/m^2, Floor finish = 0.6 kN/m^2. Assume the permissible stresses in steel and concrete as σ_{cb} = 7 N/mm^2, σ_{st} = 230 N/mm^2 and the modular ratio '*m*' = 13.
3) A simply supported verandah slab of clear span 3.3 m is supported on brick walls 400 mm thick on one side and 200 mm thick on the other side. Adopting M-20 grade concrete and Fe-415 grade HYSD bars, design the slab and sketch the details of reinforcements in the slab.
4) Design a two-way reinforced concrete slab for a room having clear dimensions of 3.5 m by 4.5 m. The slab is supported on masonry walls 300 mm thick on all the four sides and the corners are held down. Assume the live load on the slab inclusive of finishes as 3 kN/m^2. Adopt M-25 grade concrete and Fe-500 grade reinforcements.
5) Design a two way R.C.C. slab for a an office floor building having clear dimensions of 4.5 m by 5.5 m. The slab is continuous on all the four edges being supported on reinforced concrete beams 300 mm wide. The live load on the slab inclusive of finishes may be taken as 5 kN/m^2. Use M-20 grade concrete and Fe-415 HYSD bars.
6) Design a singly reinforced concrete beam to support a class room floor over a clear span of 6 m. The beam is supported on 300 mm thick stone masonry walls. The beams are spaced at 3 m intervals. The thickness of the slab is 150 mm. Adopt M-20 grade concrete and Fe-415 HYSD bars.
7). A singly reinforced concrete beam of effective span 6 m has a rectangular section 300 mm wide by 650 mm deep. The beam is reinforced with 4 bars of 25 mm diameter at an effective depth of 600 mm. The super imposed dead load on the beam is 6 kN/m. Calculate the maximum permissible live load on the beam. Adopt M-25 grade concrete and Fe-415 HYSD bars.
8) Design a balanced singly reinforced concrete beam section having an effective depth twice that of the width to support a uniformly distributed total (dead + live) load of 10 kN/m over an effective span of 5 m. Assume cover to tensile steel as 50 mm. Adopt M-20 grade concrete and Fe-415 HYSD bars.
9) A doubly reinforced concrete beam of overall dimensions 250 mm by 600 mm is simply supported over an effective span of 6 m and has to support a uniformly distributed live load of 25 kN/m. Assuming the effective cover to tensile and compression reinforcement as 50 mm, design the steel reinforcements in the beam using M-25 grade concrete and Fe-500 grade reinforcement.

10) A reinforced concrete Tee beam with an effective flange width of 2 m and slab thickness of 120 mm and rib width 300 mm has an overall depth of 600 mm. The beam is reinforced with 6 steel bars of 25 mm diameter with an effective cover of 60 mm. If M-20 grade concrete and Fe-415 HYSD bars are used, estimate the moment of resistance of the section.
11) The floor of an educational institution is made up of a Tee beam and slab having the following details:
Effective span = 6 m, Effective width of flange = 2.5 m
Thickness of flange = 150 mm, width of rib = 300 mm
Depth of rib = 550 mm
Tension reinforcement = 8 bars of 25 mm diameter
Effective depth = 600 mm
Materials: M-20 grade concrete and Fe-415 HYSD bars
Estimate the moment of resistance of the tee beam section and calculate the maximum permissible live load on the beam.
12) Design a Tee beam for an office floor to suit the following data:
Clear span = 11.5 m
Centre to centre of supports = 12 m
Loading (Office floor) = 4 kN/m^2
Thickness of slab = 150 mm
Spacings of tee beams = 3 m
Materials: M-20 grade concrete and Fe-415 HYSD bars.
Sketch the details of reinforcements in the tee beam.
13) The cross section of an R.C.C. column is 400 mm by 400 mm. The service axial load on the column is 1600 kN. The safe bearing capacity of the soil at site is 150 kN/m^2. Using M-25 grade concrete and Fe-415 HYSD bars, design the reinforcements in the column and footing and sketch their details.
14) A rectangular column 400 mm by 600 mm in section is required to support an axial service load of 2000 kN. Design suitable reinforcements in the column. Also design a suitable footing for the column assuming the safe bearing capacity of soil at site as 200 kN/m^2. Adopt M-20 grade concrete and Fe-415 HYSD bars.
15) Design a cantilever type R.C.C. retaining wall to retain earth level with the top of the wall to a height of 5 m. The safe bearing capacity of soil at site is 200 kN/m^2. The density of earth-fill is 18 kN/m^3. Angle of shearing resistance = 30° and the coefficient of resistance between soil and concrete is 0.5. Sketch the details of reinforcements in the stem, heel and toe slabs.

16) A stair case room measures 4 m by 2.5 m and the height between the floors is 3 m. design a suitable doglegged stair case with mid landing slab. Assume the tread as 270 mm and rise as 150 mm. Adopt M-20 grade concrete and Fe-415 HYSD bars. Sketch the details of reinforcements in one of the flights.

17) Design a single flight straight stair-case, with 10 risers each of 150 mm and with the tread of 300 mm. The upper and lower landings are 1.25 m wide in the direction of the stair case flight. The edges of the two landings are simply supported on masonry walls 300 mm thick. Design the waist slab type stair case flight assuming M-20 grade concrete and Fe-415 HYSD bars. Adopt the live loads as specified in IS: 875 for an office building.

18) A circular water tank resting on ground with a sliding base is required to store four hundred thousand liters of water. The depth of storage is to be 5 m. Free board = 300 mm. Adopting M-20 grade concrete and Fe-415 HYSD bars design the tank walls and a suitable spherical dome and ring beam and base slab. Sketch the details of reinforcements in the various structural elements.

19) A rectangular reinforced concrete water tank is to be designed to store 100,000 liters of water. The inside dimensions of the tank may be assumed as 6 m by 5 m. The tank rests on reinforced concrete beams on all the four sides. Design the side walls of the tank adopting M-25 grade concrete and Fe-500 grade reinforcements.

20) Design the sidewalls of a rectangular reinforced concrete water tank of interior dimensions 5 m by 2 m with the depth of storage water being 2 m. Adopt M-20 grade concrete and Fe-415 HYSD bars. Sketch the details of reinforcements in the tank walls.

CHAPTER 16

Reinforcement Detailing in Structural Concrete Members

16.1 INTRODUCTION

The primary aim of structural design is to produce economical, safe, serviceable and durable structures conforming to the national codes. The design should not only produce aesthetic structures but also should serve the intended function for which they are designed. While analysis and design form the first phase, the more important job is the process of structural construction which translates the design into a sound and solid high quality structure. To facilitate the construction process, good detailing of reinforcements with proper drawings are essential at the site of construction.

The prominence given to the limit state design in the revised Indian standard code IS: 456-2000 implies that the structural concrete members should satisfy the limit states of failure as well as serviceability.

Very rarely concrete members fail due to the limit state of collapse since the failure loads are significantly higher than the service loads. However, in most of the cases, the structural concrete members do not perform satisfactorily under service loads due to local damage in the form of cracks and excessive deflections leading to distress of floors and partition walls.

The working stress method with lower permissible stresses and approximate methods of analysis usually resulted in larger sections of the members with increased quantities of steel reinforcement. The conservative designs of the working stress method generally assured safety against both collapse and serviceability limit states even with minor deviations in detailing of reinforcements.

The introduction of limit state method of design using higher strength materials associated with higher stresses and with partial safety factors, lower than the factor of safety adopted in the working stress method has resulted in slender members. Consequently, any minor deviation in detailing of reinforcements and lack of quality control of concrete and supervision of construction process may seriously affect the serviceability criteria of the structures. Hence, it is very important to note that while using the limit state method of design, the structural engineers should pay special attention not only to plan and design but also detailing, fabrication and construction of structural concrete members.

It is important to note that the cost of concrete and reinforcement constitutes a fraction of the total cost of any project. Hence, proper care has to be taken about the durability and serviceability criteria of the structure with judicious use of materials and quality construction. The reader may refer to the specialist literature[105-115], for exhaustive information on quality control, detailing and construction practices, types of failure, methods of prevention of cracks in structural concrete members.

16.2 STRUCTURAL DISTRESS and COLLAPSE

Improper design and detailing results in failure of buildings due to serviceability exhibited in the form of local cracks and large deflections or ultimate collapse of the structural elements. The various reasons for serviceability distress are attributed to the following factors:

a) Large deflections of floor slabs and beams affecting the partition walls.
b) In sufficient cover leading to corrosion of reinforcement and spilling of concrete.
c) Improper slopes on roofs resulting in ponding of water and dampness due to poor drainage.
d) Local cracking of beams and slabs.
e) Growth of algae and moss on wet surfaces of roof slabs and chajjas leading to discoloration and dampness.

The ultimate failure or collapse of the structural concrete elements are due to the following reasons:-

a) Improper design and detailing leading to primary failure of load bearing members.
b) Lack of quality control during construction may significantly reduce the design strength of concrete leading to sudden collapse of the members.
c) Use of poor quality materials may lead to the collapse of the members.
d) Failure may also occur due to over loading or due to natural calamities like bomb blasts and earthquakes.
e) Improper maintenance may lead to progressive collapse of the structure.

16.3 COMMON SHAPES OF REINFORCEMENTS

Reinforcements used in the structural concrete members may be in different shapes such as straight or cranked bars, single or double legged stirrups, or bundled bars. The most common shapes used in structural concrete members like slabs, beams and columns are compiled in Fig. 16.1 as reported by Kalgal and Jayasimha[116]

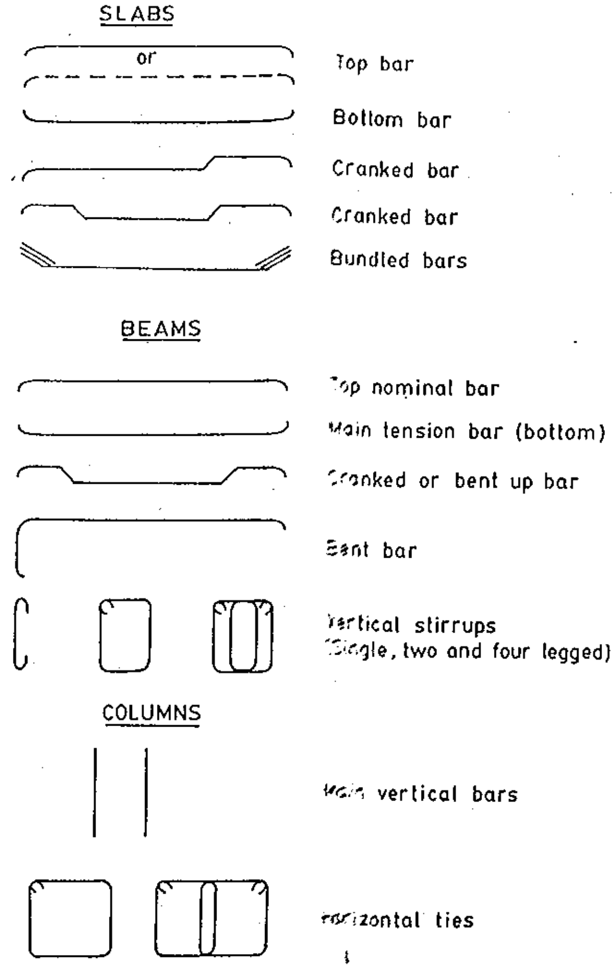


Fig. 16.1 Common Shapes of Bars

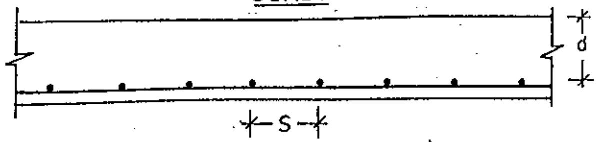
Straight bars are commonly used in slabs and beams. Sometimes the bars are cranked up near the supports to resist the negative moments that may develop due to fixity at the supports. Two legged stirrups are commonly used in most of the beams. In bridge beams, where the shear forces near the supports are large, four legged stirrups are required. In the case of very thin beams such as I-sections, single legged stirrups are used nominally throughout the length of the member.

Main column reinforcements are normally straight with single or double ties depending upon the spacing of longitudinal bars.

16.4 DETAILING OF REINFORCEMENTS IN SLABS

Details of Reinforcement requirements in reinforced concrete slabs are compiled in Fig. 16.2. Based on durability and ease of construction the minimum and maximum ~~diameters~~ of bars, the percentage or reinforcement and their spacing in the section is shown in Fig. 16.2

SLABS



REINFORCEMENTS

	Minimum	Maximum
Diameter	8 mm (main) 6 mm (distribution)	⅛ x t (thickness of slab)
Quantity	0·12 % (HYSD bars) Fe-415 0·15 (plain bars) Fe-250	For M-20 concrete 0·96% (Fe-415 steel) 1·76 % Fe-250 steel)
Spacing	1·5 (aggregate size)	3 d (main) 5 d (distribution) or 450 mm

Fig. 16.2 Reinforcement Details in Slab (IS: 456-2000 and SP-34)

Slabs are generally singly reinforced members and are generally reinforced with main steel on the tension face to resist the tensile stresses developed due to loads in the direction of span. In the case of one-way slabs, distribution reinforcement should be provided in the transverse direction to resist the effects of shrinkage and temperature.

Indian standard code of practice IS: 456-2000, has specified the rules regarding the detailing of reinforcements in the slabs under clauses 26.3.2 and 26.5.2. The essence of these specifications are compiled in Fig. 16.2.

Typical reinforcement detailing in cantilever, simply supported and continuous slabs is shown in Fig. 16.3

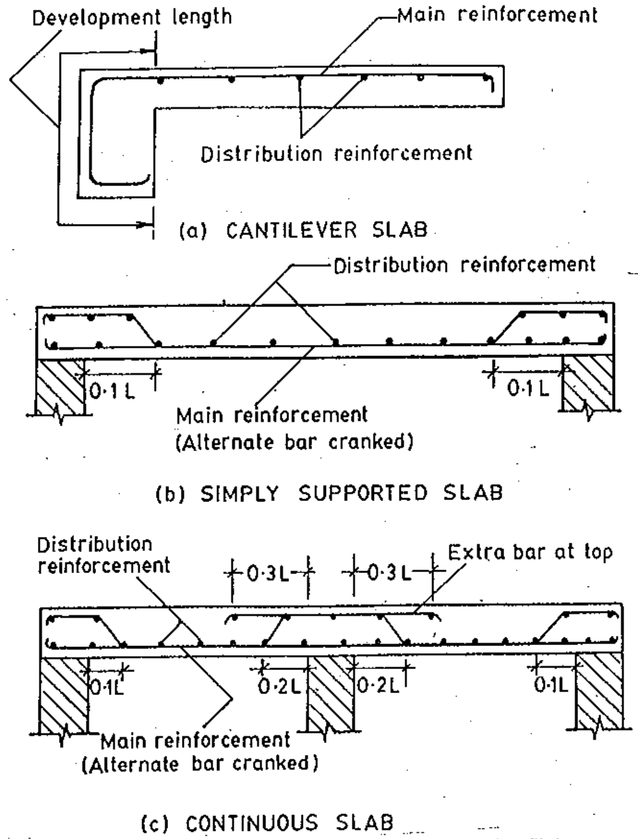


Fig. 16.3 Reinforcement Detailing in One Way Slabs

In the case of one-way simply supported slabs, the main bars are cranked up at a distance of $0.1L$ from the inside edge of the supports or $0.15L$ from the center line of the supports to resist any negative moments which may develop due to partial fixity at the supports. Typical detailing in one-way slab as per the specification of SP: 34 is shown in Fig. 16.4. The distribution bars are generally straight as shown in Section BB. In two way slabs spanning in mutually perpendicular directions as shown in the plan of Fig. 16.5, the main bars are cranked near the supports in both directions as shown in sections AA and BB along the short and long span directions respectively.

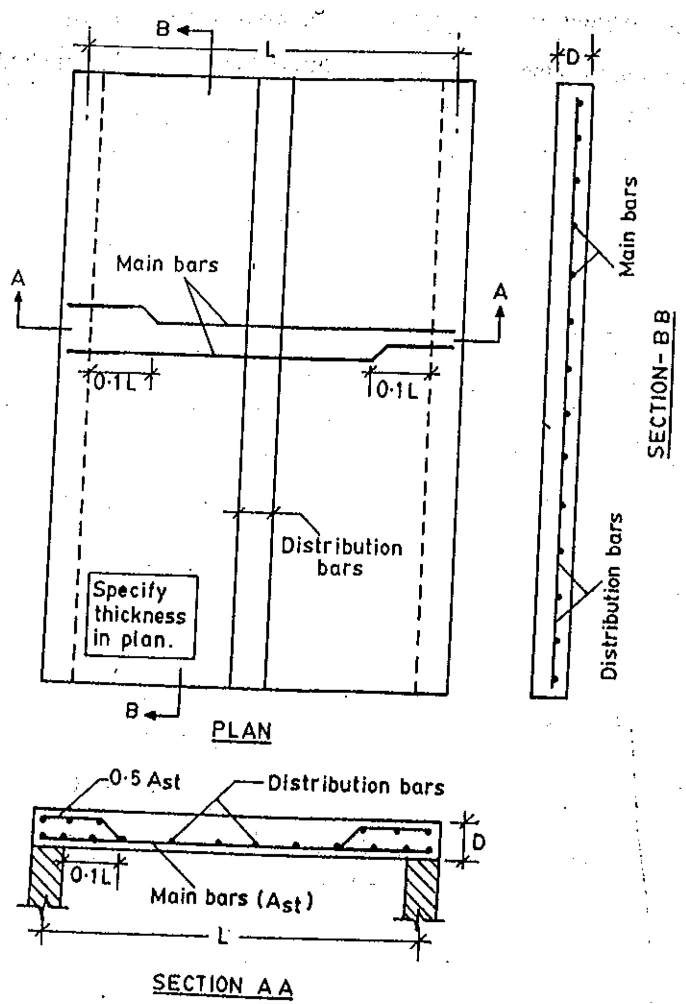


Fig. 16.4 Typical Detailing in One Way Slab (SP-34)

In the case of two way slabs, simply supported on both edges meeting at the corner, torsion reinforcement in the form of a mesh should be provided with bars placed parallel to the sides of the slab and extending from the edges to a minimum distance of one fifth of the shorter span. The area of reinforcement in each of these four layers shall be three quarters of the area required for the maximum mid span moment of the slab according to the clause D-1.8, 1.9 and 1.10 of I.S: 456-2000.

In the case of continuous slabs shown in Fig. 16.6, the main bars provided at the mid span tension face are cranked up at a distance of 0.25L

and extended up to 0.3L as shown in section AA to resist the hogging moments developed at the interior supports.

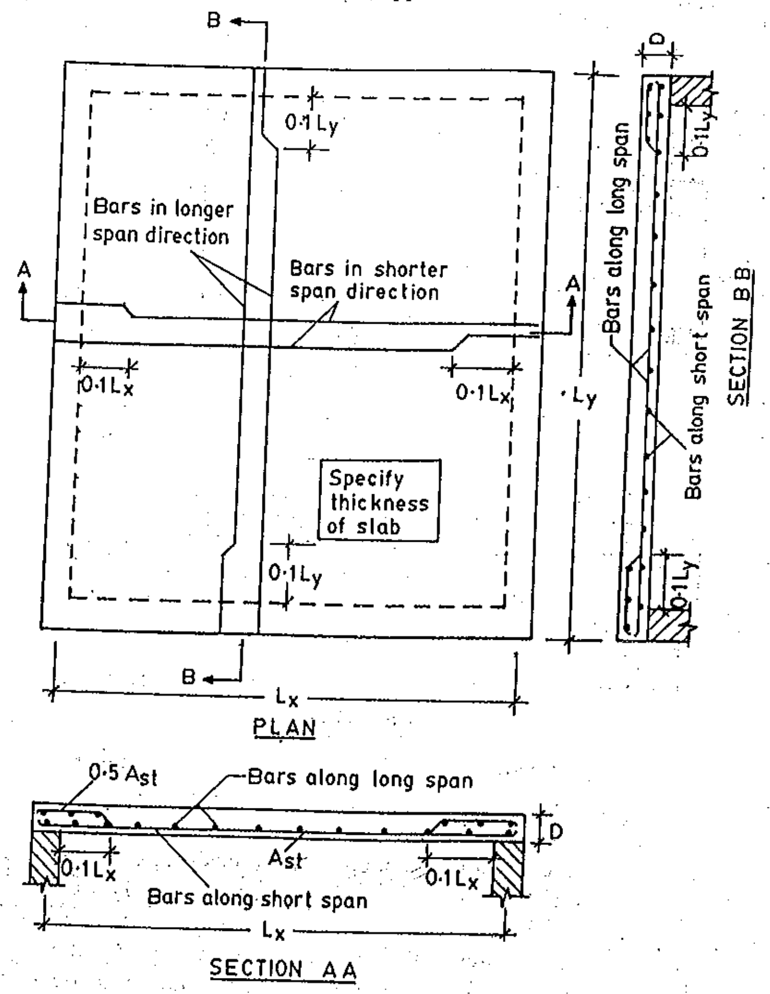


Fig. 16.5 Typical Detailing in Two Way Slabs (SP-34)

In multistorey buildings comprising reinforced concrete columns and beams, the floor slabs are generally continuous over beams and built with edge beams. Hence, torsion reinforcement at corners is not required. In the case of simply supported continuous slabs, torsion reinforcement shall be provided at corners as per the specifications laid down in IS: 456-2000.

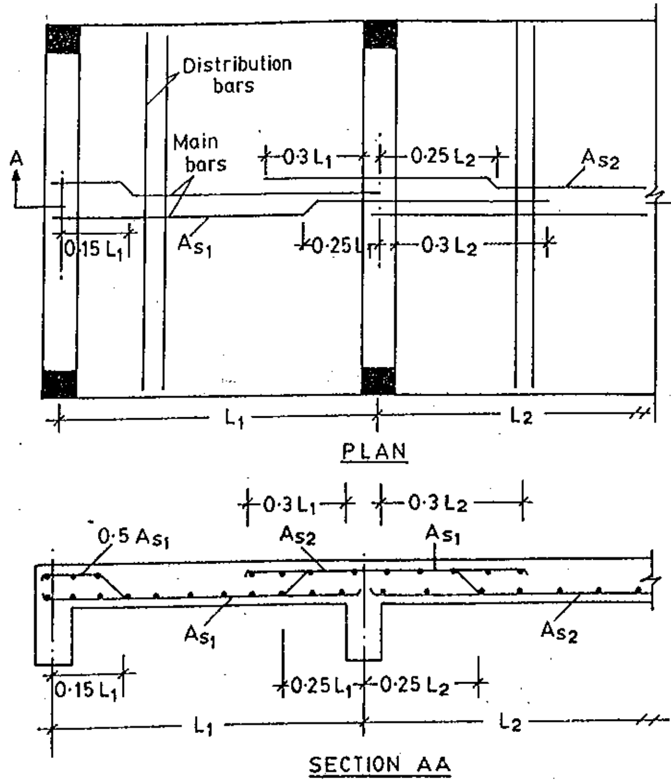


Fig. 16.6 Typical Detailing in One Way Continuous Slab (SP-34)

16.5 DETAILING OF REINFORCEMENTS IN BEAMS

Reinforcements are provided in beams to resist tensile stresses due to flexure and shear in the case of singly reinforced members. However, if the depth of the beam is restricted as in the case of basement floors, reinforcements are required at the compression face rendering the beam as doubly reinforced. Beams subjected to torsion in addition to flexure and shear require additional longitudinal and shear reinforcements to resist the equivalent shear and flexural stresses developed due to torsion.

Reinforcements comprising straight bars are generally provided in most of the beams resisting tensile and or compressive stresses. In rectangular sections, the tension and compression reinforcements may be provided in different layers if the width of the beam is insufficient to accommodate all the bars in a single layer.

A typical arrangement of two layers of reinforcements in a Tee beam at mid span and support sections is shown in Fig. 16.7. Beams are subjected to heavy shear forces in the vicinity of the supports and hence shear reinforcements in the form of stirrups and or bent up bars are provided to resist the diagonal tension developed due to the shear forces. Stirrups can be either vertical or inclined. However, vertical stirrups are preferred due to ease of fabrication and detailing. Stirrups can also be of open or closed type, single or multilegged as shown in Fig. 16.8.

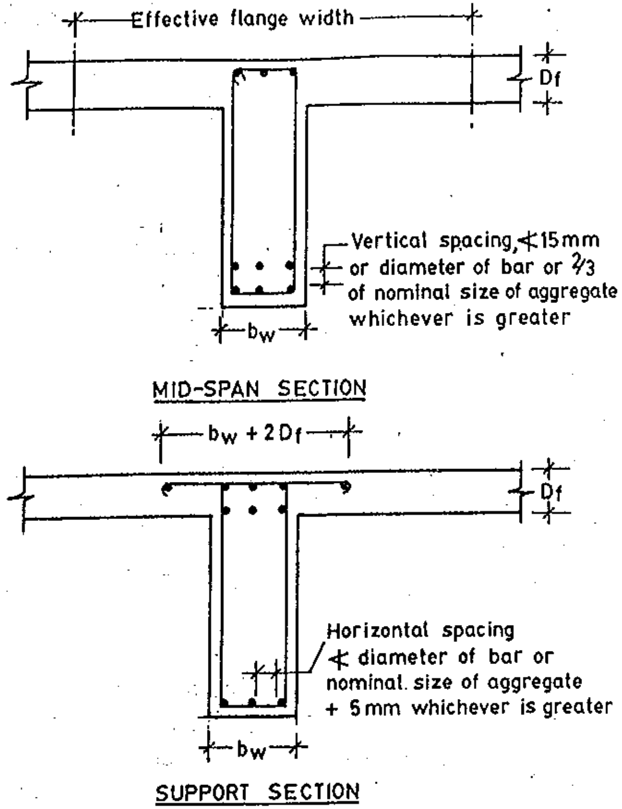


Fig. 16.7 Detailing of Bars in Tee Section

Standard bends and hooks required for reinforcing bars is shown in Fig. 16.9. The specifications regarding detailing of reinforcements in beams are given in clause 26.5.1 of IS: 456-2000. The important provisions regarding the minimum and maximum percentages of main reinforcements and stirrups in beams are compiled in Table. 16.1.

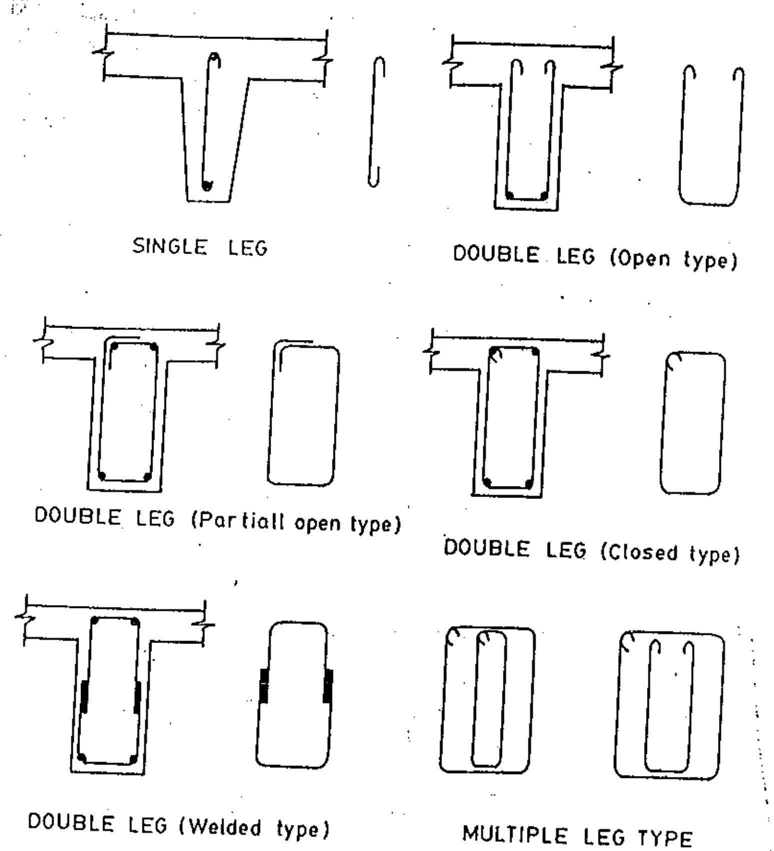
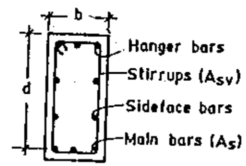


Fig. 16.8 Types of Stirrups

In beams, the reinforcement can be curtailed along its length depending upon the magnitude of bending moment at the section. Typical details of curtailment of bars in cantilever and continuous beams are shown in Fig. 16.10. Another salient aspect in detailing of reinforcement in beams is the anchorage or development length required at supports. The anchorage length required for main reinforcement in tension and compression outlined in clause 26.2 of IS: 456-2000 is shown in Fig. 16.11 for structural concrete members subjected to tension and compression.

Table 16.1 Detailing of Reinforcements in Beams



Diameter of Bars

Diameter	Min.	Max. (Limit state)	Max. (Heavy struct.)
Main bars	10 mm	32 mm	50 mm
Hanger bars	8 mm	16 mm	20 mm
Stirrups	6 mm	16 mm	16 mm

Percentage Reinforcement $A_s \nless \left(\frac{0.85\,bd}{f_y}\right)$

Fe-250 Plain M S bars	Min: 0·34 %	Max: 4 %
Fe-415 HYSD bars	Min: 0·20 %	Max: 4 %
Stirrups	$A_{sv} > \left(\frac{0.45\,b\,s_v}{f_y}\right)$	
Side Face	Min: 0·10 %	(D > 750 mm

Spacing of Bars

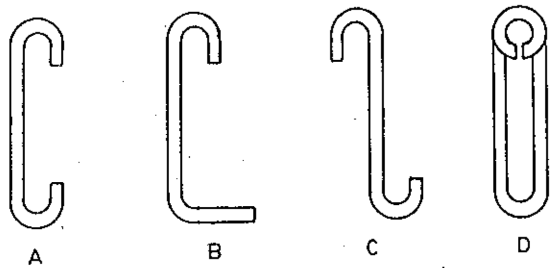
Main bars	Min: ≮ 0; ≯ 5mm + Size of C A	Max: Refer table 15 (IS: 456-2000)
Stirrups	Min: 50 mm (S P: 16)	Max: 0·75 d; 300 mm
Side Face	-	Max: 300 mm

16.6 DETAILING OF REINFORCEMENTS IN COLUMNS

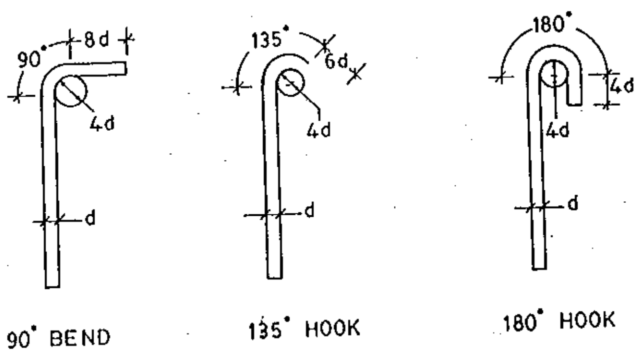
Column reinforcements are generally subjected to compression and in cases of brackets attached to columns, eccentric compression may develop tensile stresses in some bars of the column. Ties or transverse reinforcements are invariably provided to prevent buckling of the main longitudinal bars at regular intervals. Typical cross sections of columns with main and transverse reinforcements are shown in Figs. 16.12 and 16.13.

Table 16.2 Detailing of Reinforcements in Columns

Diameter	Min.	Max.	
		Building	Bridge
Diameter	12mm	32mm	50 mm
Quantum	0.8%	4.0%	4.0%
Number of Bars	4–in Rectangular Columns 3–In triangular Columns 6–In Circular Columns		
Spacing of bars	$\frac{2}{3}$ Aggregate Size	300 mm	
Column Ties	Min.	Max.	
Diameter	$\frac{1}{4}$ bar dia or 5 mm	12 mm	165 mm
	150 mm	Least Lateral Dimension or 16 (Column bar diameter) or 300 mm	

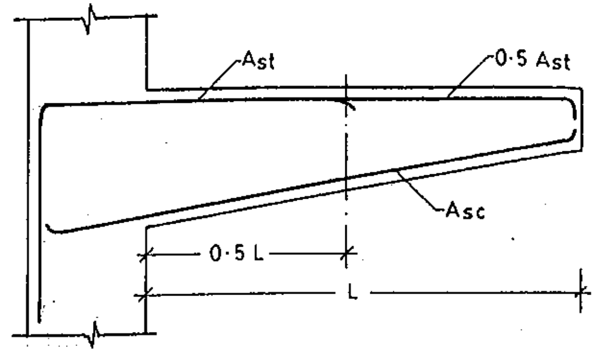


VARIOUS FORMS OF LINKS

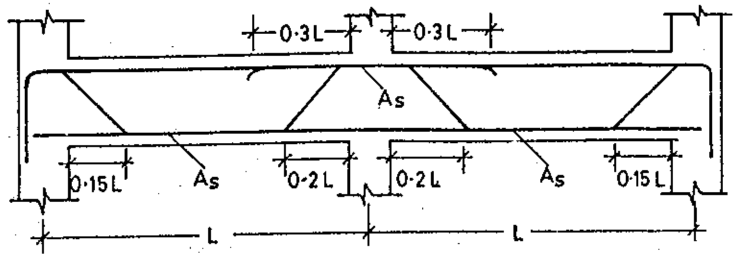


STANDARD BENDS AND HOOKS

Fig. 16.9 Standard Bends, Hooks and Links



CURTAILMENT IN CANTILEVER BEAM



CURTAILMENT IN CONTINUOUS BEAMS

Fig. 16.10 Curtailment in Beams

Notes:-

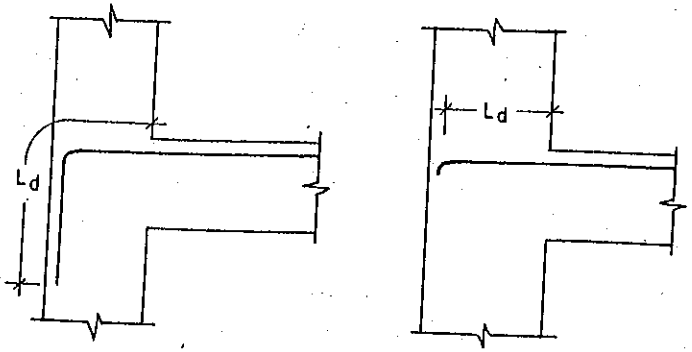
a) If distance between bars ≤ 75 mm, only alternate bars to be tied in both ways.

b) If distance between bar > 75 mm, all bars to be tied both ways.

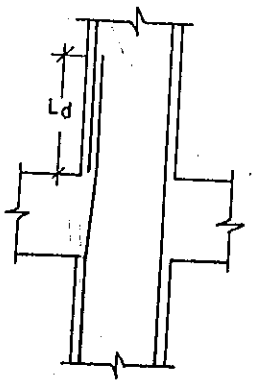
Detailing provisions specified in clause 26.5.3 of IS: 456-2000 is compiled in Table-16.2 and Fig. 16.14. In the case of circular columns with helical ties, the pitch of the ties should not exceed 75 mm and it should not be less than 25 mm nor less than three times the diameter of the steel bar forming the helix.

16.7 DETAILING OF REINFORCEMENTS IN FOUNDATIONS AND WALLS

Isolated column footings are widely used as foundations for columns due to economy and ease of construction. Combined footings are used when columns are closely spaced and the bearing capacity of soil is less so that the bearing area of individual footings overlap. Combined footings are designed as slab with beam tied between the columns. Hence, the guide lines and specifications for detailing of reinforcements in foundations are similar to that for slabs and beams. A typical detail of an isolated column footing is shown in Fig. 16.15.



ANCHORAGE LENGTH (Tension)

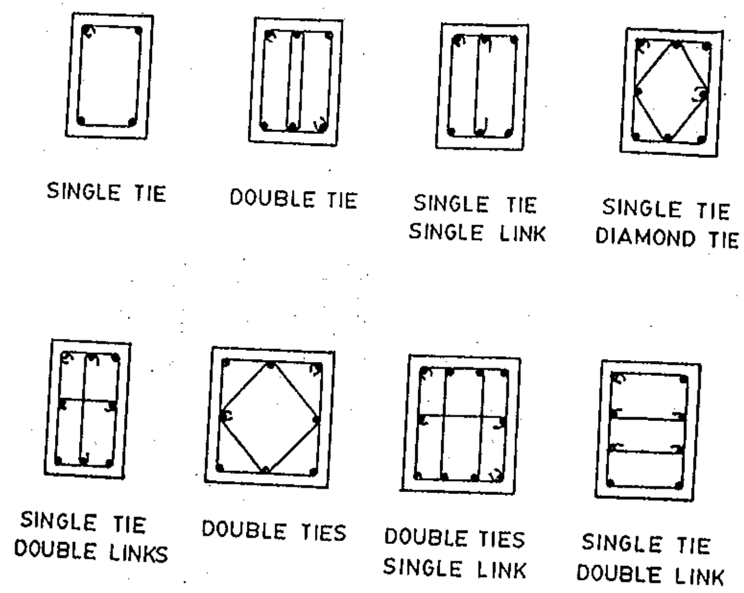


ANCHORAGE LENGTH (Compression)

Anchorage length (L_d)			
Grade of Concrete	M-20	M-25	M-30
Tension	47 d	40 d	38 d
Compression	38 d	32 d	30 d
d = Diameter of bar			
Fe-415 HYSD bars			

Fig. 16.11 Anchorage Length of Bars (IS 456-2000)

Reinforced concrete wall is a structural member whose length exceeds 4 times its thickness. R.C.Walls may be used as partitions in which case they are non-load bearing members. They may also be designed as load bearing walls. The thickness of walls should generally be not less than



1	Tie diameter ≮ 1/4 bar diameter nor less than 5 mm
2	Tie spacing ≯ 16 x bar diameter ≯ 48 x tie diameter ≯ Least lateral dimension of member

Fig. 16.12 Typical Lay Out of Ties in Columns

100mm. Walls are intended to carry vertical loads and are designed in accordance with the recommendations specified for columns The reinforcements in walls comprise of vertical and horizontal bars provided in single or two layers near the faces. The specifications regarding the detailing of reinforcements in walls given in clause 32.5 of IS: 456-2000 code is compiled in Fig. 16.16

16.8 DETAILING AT JUNCTIONS

In Structural concrete members particular care should be taken to arrange the reinforcements at junctions of beam to beam, column to beam and at corners where beams in perpendicular directions meet the columns.

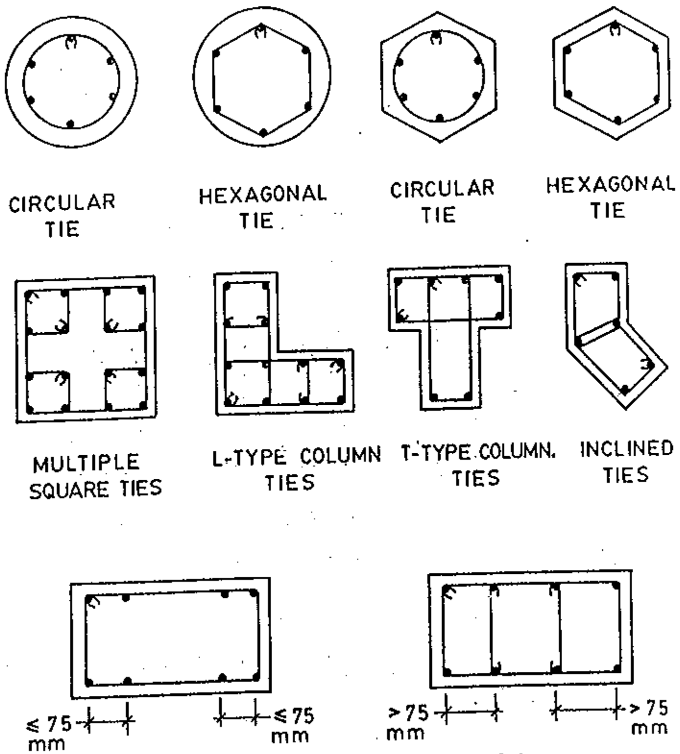


Fig. 16.13 Typical Lay Out of Ties in Columns

16.8.1 Beam-to-Beam Junctions

In multistorey reinforced concrete buildings, the following main types of beam-to beam junctions are encountered.

a) Secondary beam shallower than main beam.
b) Main beam and secondary beam of same depth.

The detailing of reinforcements for case(a) is shown in Fig. 16.17(a) as recommended in SP: 34. In this case, the bottom layer reinforcements of secondary beam should be placed above the bottom layer reinforcements of main beam. The secondary beams are generally shallower than the main beam and placing of main steel of secondary beam over that of main beam will not pose any problems.

In the case of main and secondary beams having the same depth, the detailing of reinforcements at the junction of the beams can be as shown in Figs. 16.17 (b) or (c).To provide continuity and integrity at the junction, it

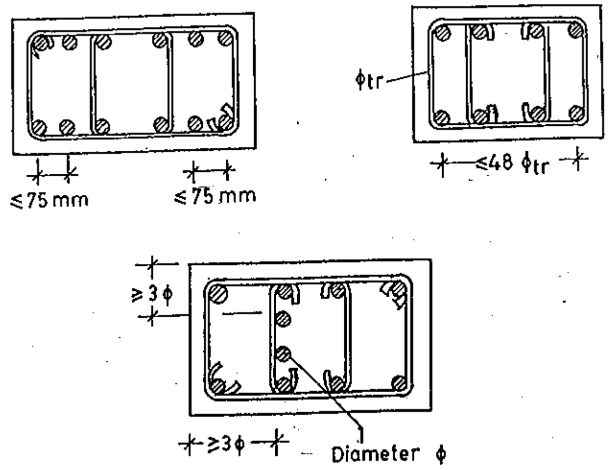
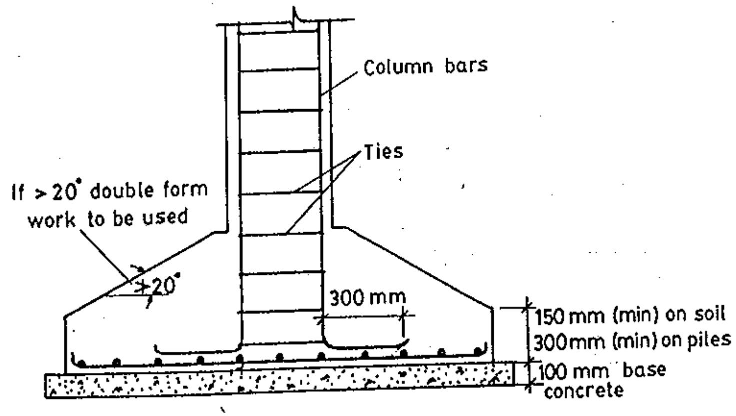


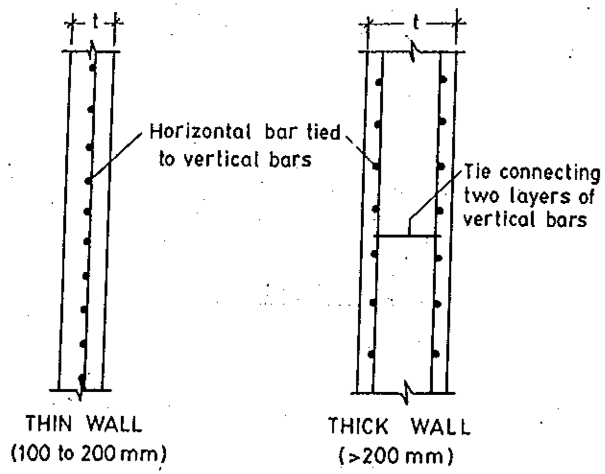
Fig. 16.14 Arrangement of Ties in Columns (IS: 456-2000)



SPECIFICATIONS FOR FOOTINGS (IS: 456-2000)		
Diameter	10 mm (min)	—
Quantum	0·12 % (min) 0·15 % (min)	—
Spacing	1·5 (Aggregate size) (min)	3 d 300 mm (max)

Fig. 16.15 Detailing of Footings

is necessary to provide extra diagonal open stirrups and horizontal loops at all beam to beam junctions as shown in Fig. 16.18.



MINIMUM REINFORCEMENT IN WALLS

	Load bearing	Non load bearing
Vertical bars	0·12 % 0·40 % (preferred	0·12 %
Horizontal bars	0·20 %	0·20 %

SPACING OF BARS

	Min.	Max.
Vertical bars	75 mm	3t or 450 mm whichever is less
Horizontal bars	75 mm	3t or 450 mm whichever is less

Fig. 16.16 Detailing of Walls

16.8.2 Beam to Column Junctions

In multistorey framed structures, column-beam junctions require special attention regarding detailing since the reinforcements from beams and columns crowd together at the junction. Detailing should be done with care to ensure proper integrity of the connections.

The following three types are identified in beam-column junctions.

a) The breadth of beam and column are the same at the junction.
b) Breadth of beam is smaller than column side at the junction.
c) Rare cases in which the breadth of the beam is larger than the size of the column at the junction.

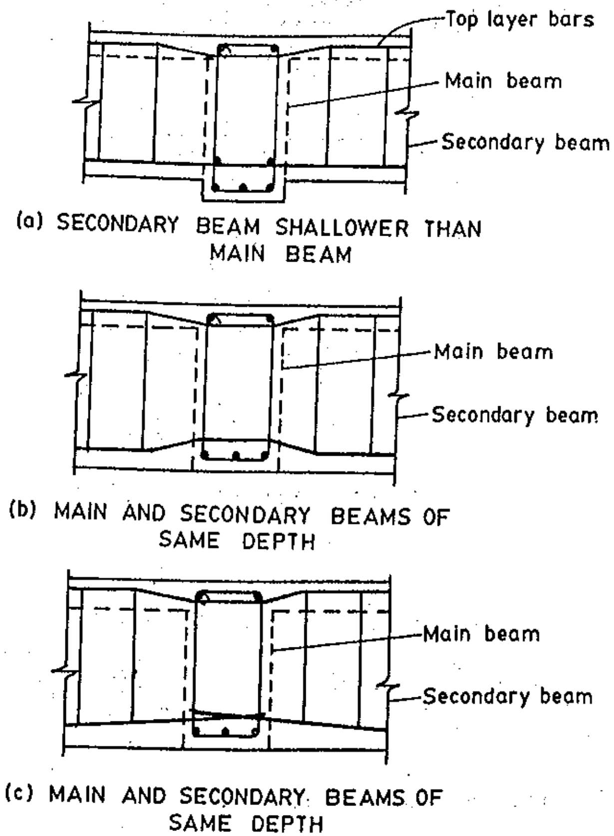


Fig. 16.17 Typical Detailing at Beam to Beam Junctions

The reinforcement detailing for case(a) is shown in Fig. 16.19. The beam bars should be housed inside the corner reinforcements of the column and the ties of columns should be continued within the junction.

In case(b) where the breadth of the beam is smaller than the side of the columns, the beam reinforcement should be housed within the column reinforcement cage as shown in Fig. 16.20, where the beam is located at the edge of the column. The outer face bars of the beam should be bent so that they are enclosed within the cage of the column bars. The column ties should be continued within the junction.

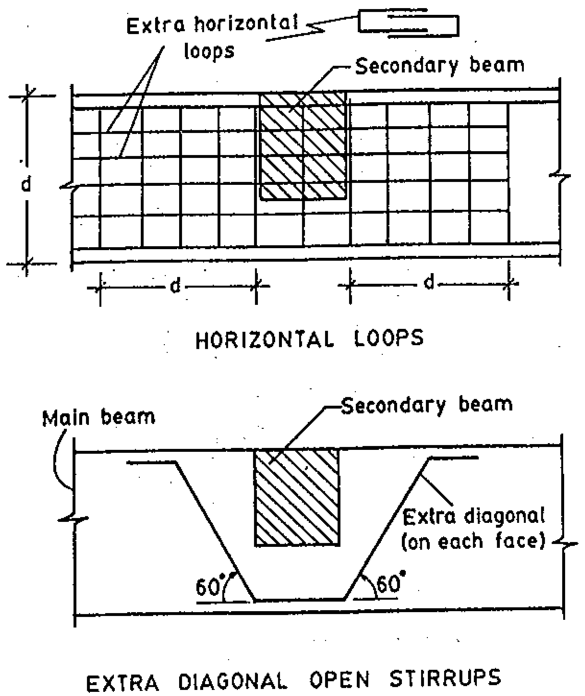


Fig. 16.18 Reinforcement Detailing at Junction of Beams

Fig. 16.21 illustrates the detailing of reinforcements when the beam width exceeds that of column side. In this case, the column bars are taken within the outer face bars of the beam with the beam stirrups continued within the junction. In all these cases it is important to note that column bars should no be bent or kinked within the junction..

16.8.3 Corner Junctions

Corner junctions arise when structural members meet at a joint at corners. Typical examples being the vertical wall and base slab or the side walls of a tank or corners of a building frame or the landing slab and sloping slab of a stair case flight or the sloping members of a gable bent.

Junctions may be of opening type or closing type depending upon the nature of moments at the junction of the members. Typical examples of opening types where the members meeting at a joint tend to move apart developing cracks at the inner corner are:-

a) Junctions of beams and columns in a framed building.
b) Junction of sloping members of a gable bent.
c) Junction of horizontal and vertical members of a box culvert.

Typical examples of closing types where the members meeting at a joint tend to move towards each other developing a crack at the outer face are:-

a) Junctions of vertical walls of a rectangular tank.
b) Junction of vertical wall and base slab of an aqueduct carrying water.

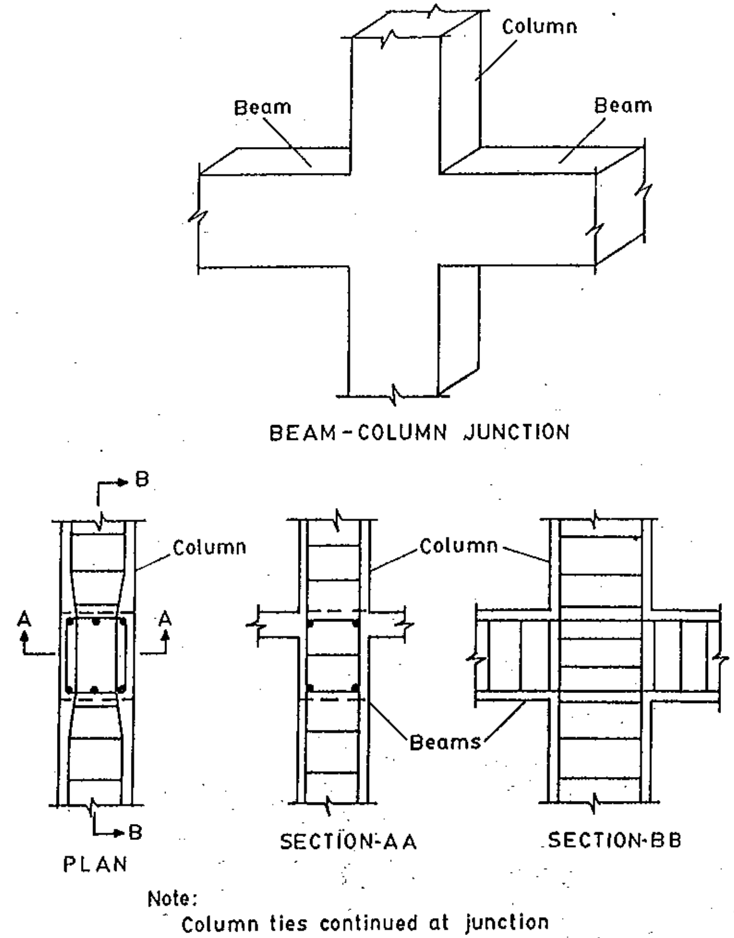


Fig. 16.19 Detailing at Column-Beam Junction

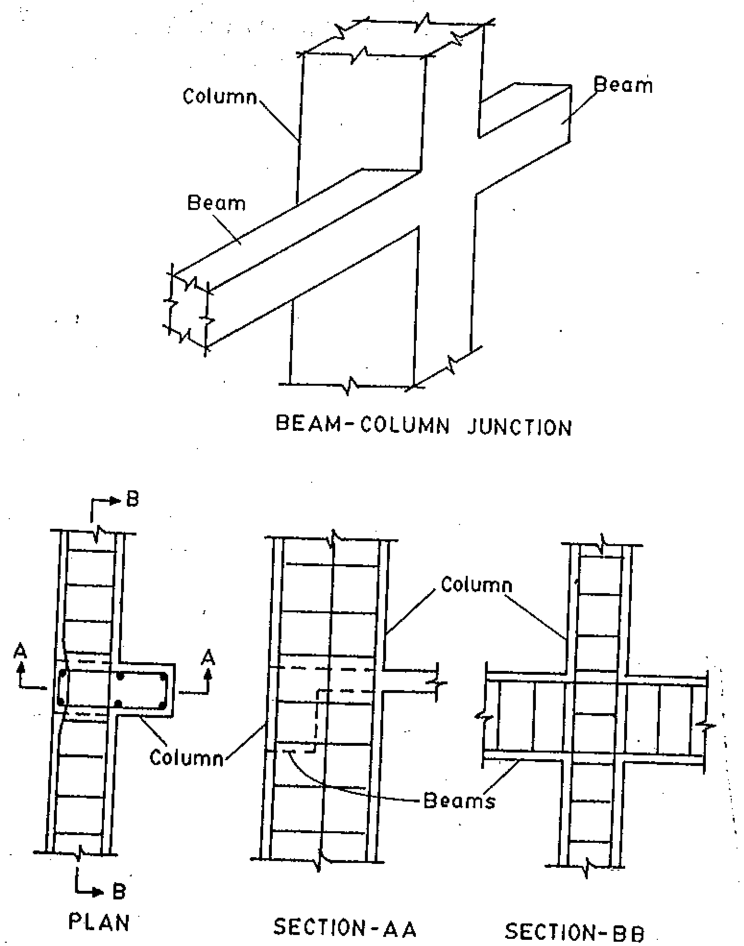


Fig. 16.20 Detailing at Column-Beam Junction

The reinforcement detailing at the junction of opening and closing corners should be able to arrest the initiation and propagation of cracks at these joints. Fig. 16.22(a) illustrates the layouts of reinforcements at corner junctions in a box culvert where slabs form the structural members. In the case of knee bent or portal frame where the horizontal beam and vertical columns are relatively thick, it is important to provide inclined stirrups in addition to the main reinforcements to resist the diagonal tension cracks as shown in Fig. 16.22(b).

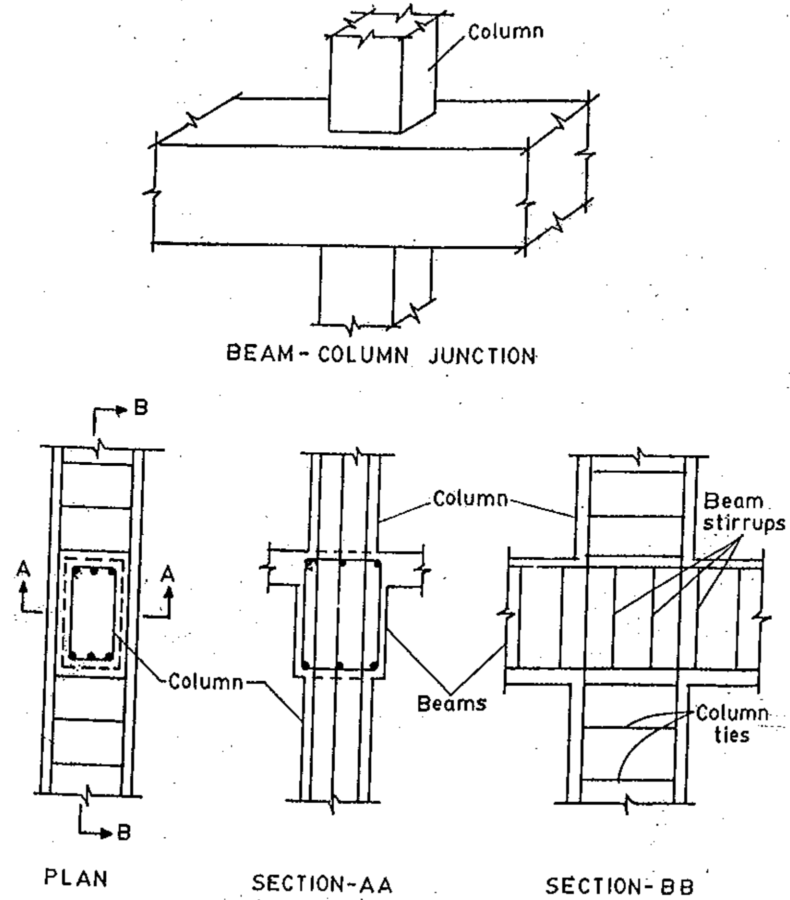


Fig. 16.21 Detailing at Column-Beam Junction

Fig. 16.23 shows the detailing of reinforcements at the crown of a mill bent and at the junction of landing and sloping waist slab of a staircase flight involving opening corners. Typical examples of detailing at junctions of walls of water tanks and aqueducts involving closing corners is shown in Fig. 16.24.

16.9 DETAILING IN CORBELS

Corbels or brackets are provided to columns in industrial structures to support the tracks of overhead gantry cranes. The design aspects of such corbels has been dealt in detail in Chapter-13.

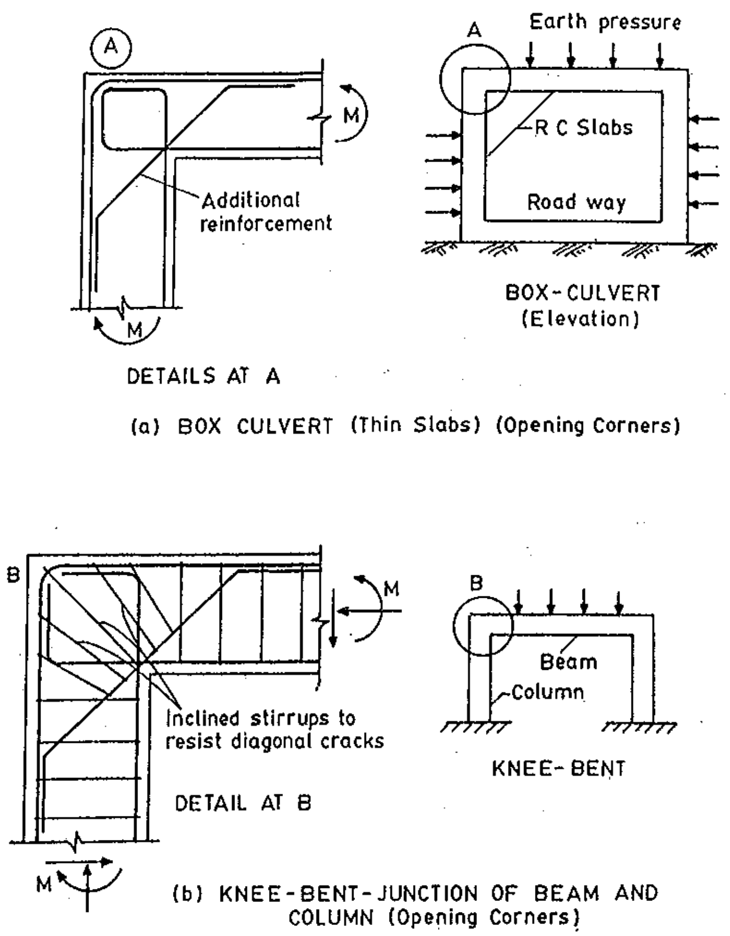


Fig. 16.22 Detailing at Opening Corners

The shear stresses developed in brackets being very high, adequate reinforcement is required in the form of

a) Horizontal loops
b) Inclined stirrups
c) Vertical stirrups and
d) Combinations of the above types.

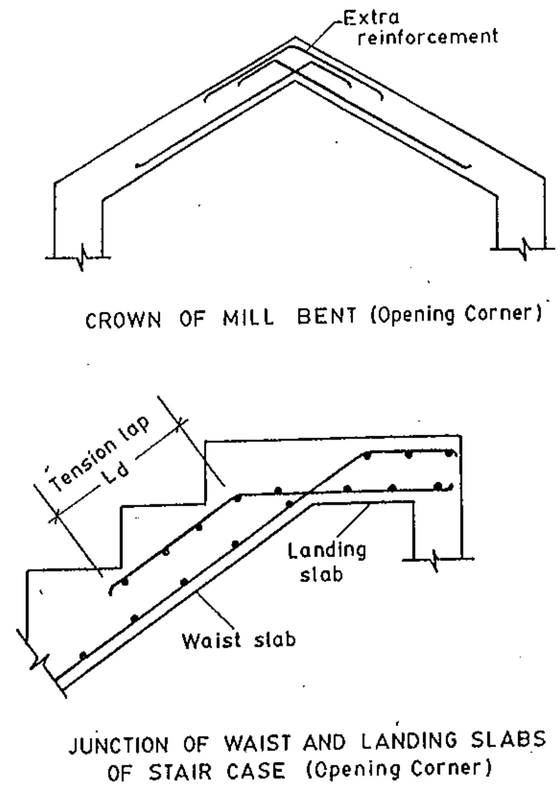


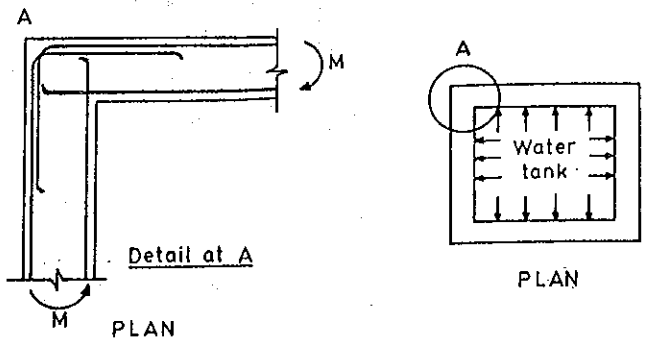
Fig. 16.23 Detailing at Opening Corners

Typical detailing of different types of reinforcements in single and double type brackets are shown in Fig. 16.25. Experimental investigations have shown that horizontal loops are superior to the other types in resisting the forces developed in corbels.

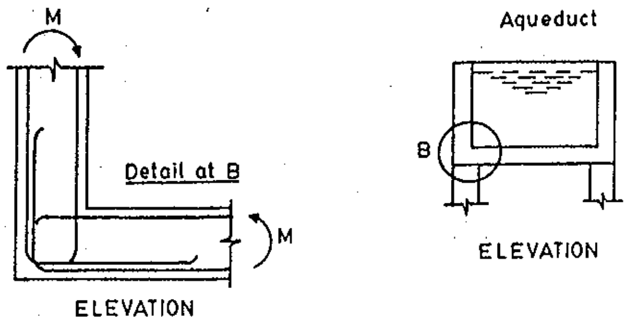
16.10 LAPPING, SPLICING AND WELDING OF REINFORCEMENTS

In the case of continuous beams of long spans, it becomes necessary to join reinforcement bars by tieing the ends of one bar with the other so that the lap length is equal to the anchorage length. In the case of larger diameter bars, to avoid wastage, the bars may be welded together using a lap weld or butt weld as shown in Figs. 16.26 and 16.27 respectively.

The following guide lines have to be followed while lapping the bars:-



(a) DETAILING AT JUNCTION OF WATER TANK WALLS
(Closing Corner)



(b) DETAILING AT JUNCTION OF VERTICAL AND BASE SLAB OF AQUEDUCT (Closing Corner)

Fig. 16.24 Detailing at Closing Corners

a) Lapping should be avoided at points of maximum tensile stress such as the center of beams and slabs.
b) In structural concrete members, no more than 50 percent of the bars should be lapped at one place.
c) The lap length provided should be sufficient to transfer the entire force from one bar to the other.
d) The lap length should be based on the basis of smaller diameter bar when two bars of different diameters are lapped.

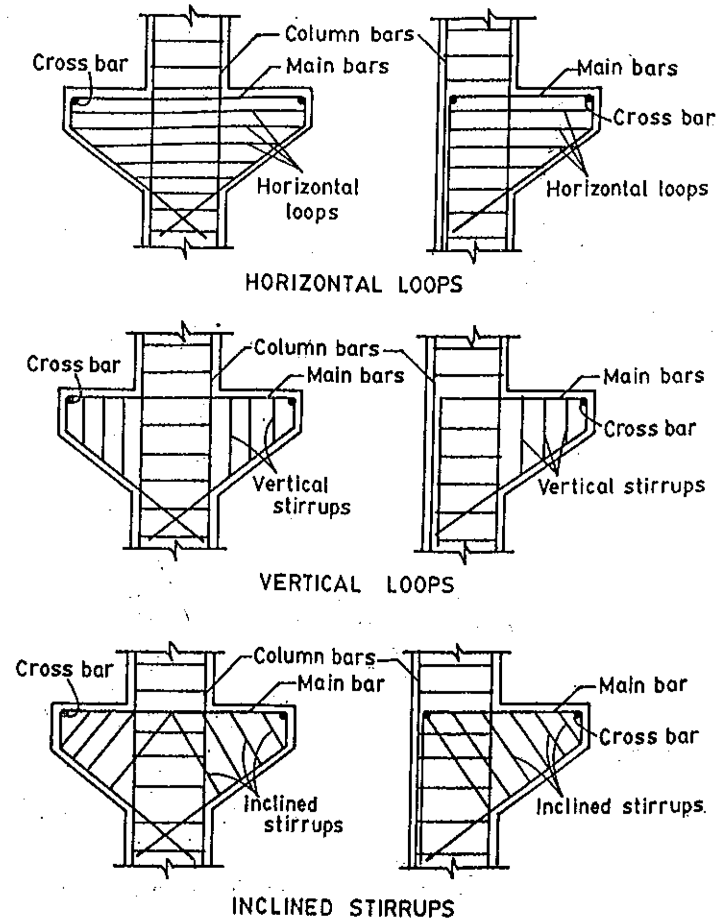


Fig. 16.25 Detailing in Corbels

In multistorey buildings, the columns towards the top floors are of smaller cross section when compared to the lower storey columns. Proper detailing of reinforcements is necessary in the case of transitions in columns. Detailing at column transitions is shown in Fig. 16.28.

In the case of junctions of heavily reinforced columns, lapping may lead to congestion of reinforcements. Alternatively splicing may be used to join the bars. The splices may be bonded type or strap type or coupler type as shown in Fig. 16.29.

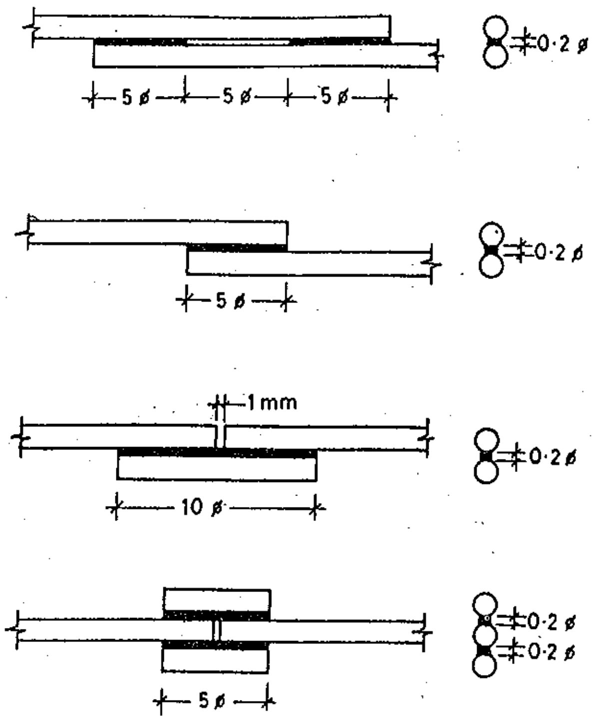


Fig. 16.26 Typical Lap Welds

16.11 BUNDLING OF REINFORCING BARS

In the case of heavily reinforced sections such as the columns in ground floors of multistorey buildings, it is advantageous to bundle the bars in groups of 2, 3 and 4 to prevent congestion of reinforcements and effective concreting of the members. Bundling is generally preferred for bars in the range of 20 to 40mm. The commonly accepted method of bundling bars in columns and beams is shown in Figs. 16.30 and 16.31 respectively. The bars in each bundle should be tied together firmly so that they act as a unit and the specifications regarding ties in columns and stirrups in beams is the same as that when individual bars are used. Bundling helps for proper compaction of concrete by using poker vibrators resulting in dense concrete.

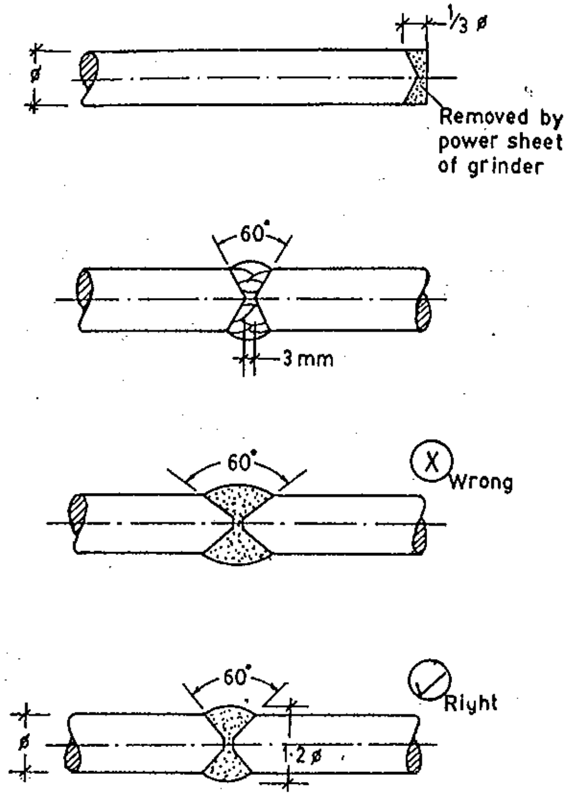


Fig. 16.27 Typical Butt Welds

16.12 COVER TO REINFORCEMENT AND REINFORCEMENT SUPPORTS

In structural concrete members, it is very important to ensure proper cover to reinforcements to prevent corrosion and deterioration due to aggressive atmospheric and chemical actions. Proper cover all-round the bars acts as a protective barrier and safe gaurds the reinforcement.

The Indian Standard code IS: 456-2000, clause 26.4 specifies the cover requirements for different types of environmental conditions. Table-16 and 16A of IS: 456-2000 code gives the nominal cover requirements for different types of exposure and for specified periods of fire resistance and the same has been detailed in chapter-2 of the text.

The cover specified is achieved only when the reinforcing bars are properly supported at regular intervals by supporting devices before the concrete is poured. There are different types of supporting devices made

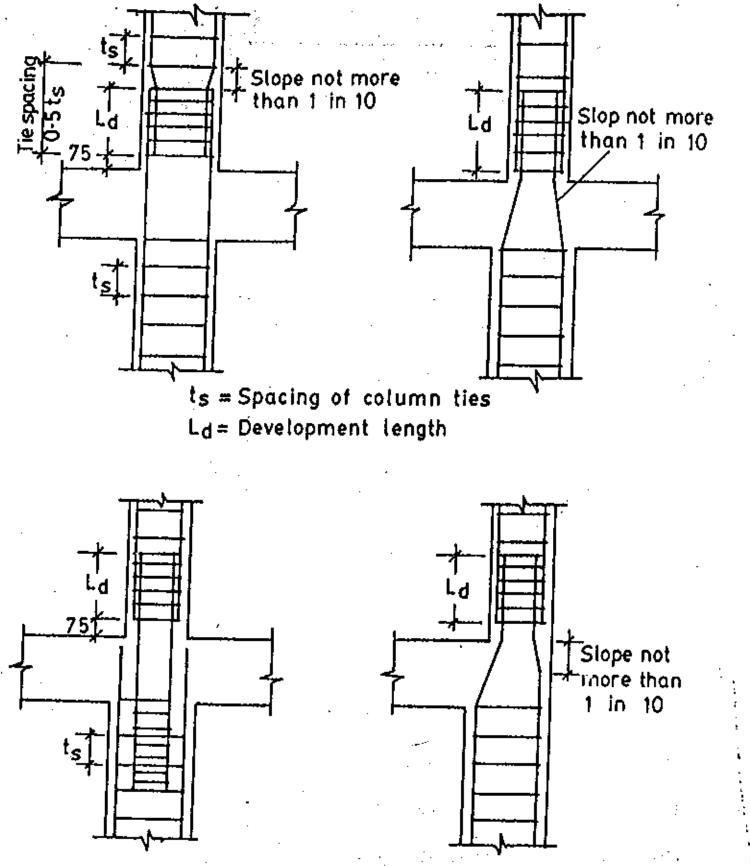


Fig. 16.28 Column Transitions

up of concrete blocks, steel chairs and moulded plastic supporting devices. The different types of reinforcement supports commonly used are shown in Fig. 16.32.

16.13 EXPANSION JOINTS IN CONCRETE STRUCTURES

In the construction of large structural concrete structures, joints are inevitable due to suspension of concreting work at the end of the day. These joints are termed as construction or cold joints. When concreting work is resumed, special care in the form of scrapping and removing loose material at the joint and coating with rich cement slurry on the old surface is essential before regular concreting is started at the construction joint. In addition, proper care has to be taken to select the location of the construction joint.

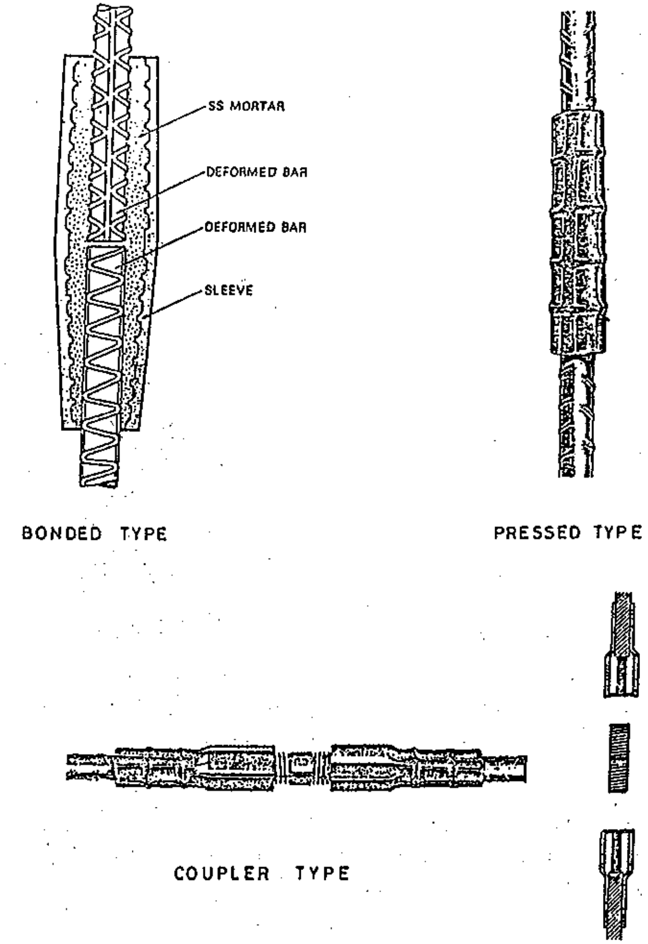


Fig. 16.29 Various Types of Splices

In general construction joints should be avoided at location of maximum tensile stress (Ex: Centre of slabs and beams) the construction joints should preferably be located in the vicinity of sections of contra flexure(where moments change from positive to negative) in the case of continuous members and far away from sections of maximum moment in simply supported members. Typical location of construction joint in a one way slab is shown in Fig. 16.33.

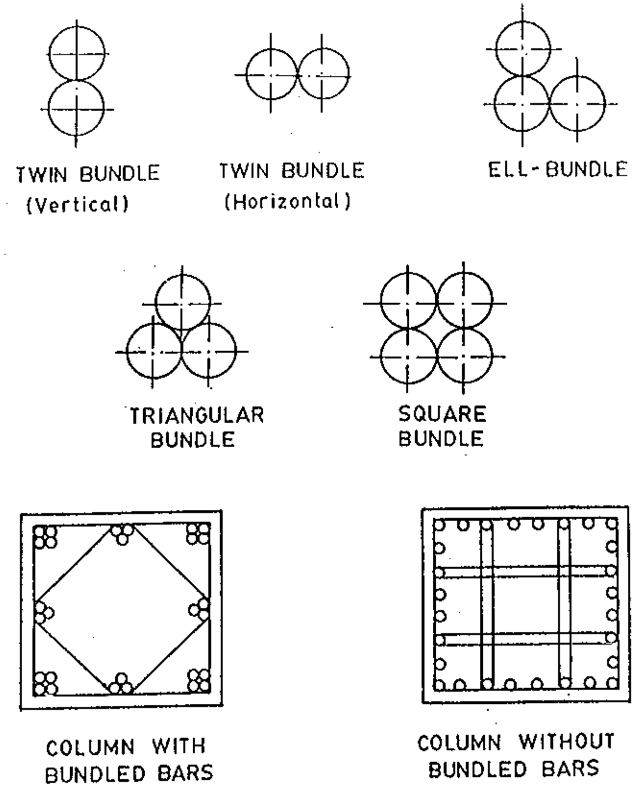


Fig. 16.30 Bundling of Bars in Columns

In the case of large buildings of considerable length and abrupt changes in plan dimensions, it is essential to provide expansion joints which accommodate the movement of the structure due to changes in temperature. According to the Indian Standard Code clause 27 of IS: 456-2000, expansion joints should be provided at junctions of structures with abrupt changes in plan dimensions.

Expansion joints should facilitate the necessary movements of the structures on either side of the joint due tc variations in temperature. The structure adjacent should preferably be supported on separate columns or walls but not necessarily on separate foundations. Reinforcements should not extend across the expansion joint and there should be a complete break between the adjacent structure.

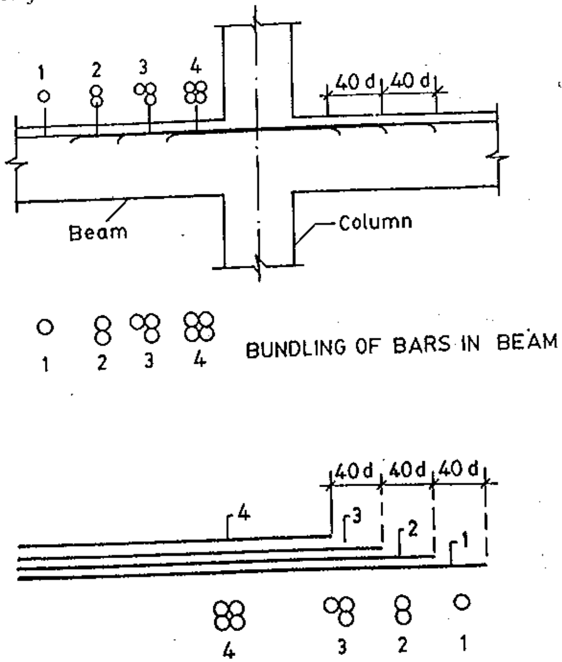


Fig. 16.31 Bundling and Curtailment of Bars in Beams

Normally structures exceeding 45m in length should be provided with one or more expansion joints. Fig. 16.34 and 16.35 shows the location and salient details of expansion joints. The thickness of the expansion joint depends upon the length of the structure, variation of temperature at the site, type of building and several other factors. The Indian Standard Code, gives the guidelines for the design of expansion joints in reinforced concrete structures.

16.14 DO's AND DONT's IN DETAILING OF REINFORCEMENTS

Detailing of reinforcements in structural concrete members involves many operations such as bending of bars, formation of end hooks, lapping, splicing, welding and positioning, the reinforcements at appropriate locations as per detailed structural drawings.

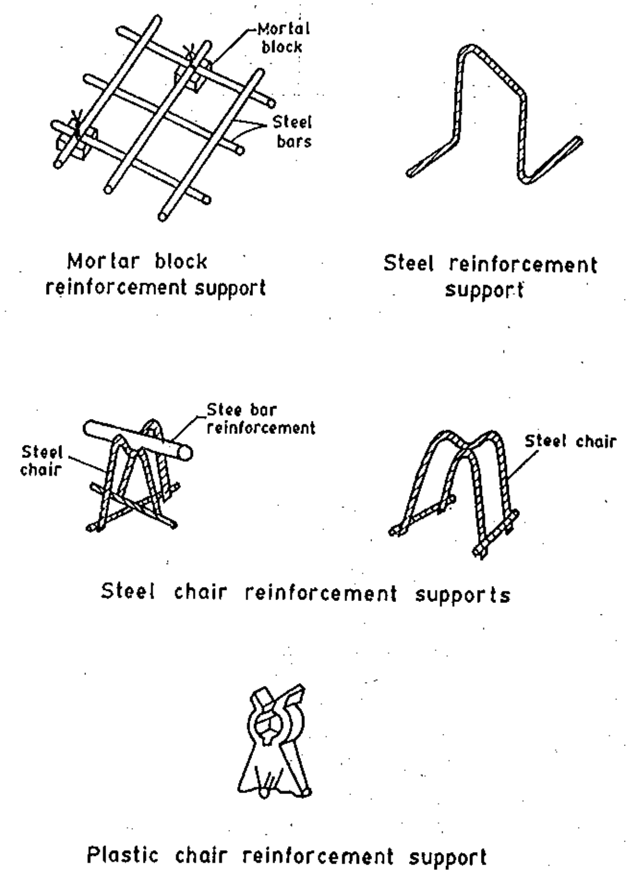


Fig. 16.32 Typical Reinforcement Supporting Devices

It requires special skill on the part of the bar bender to translate the information given in structural drawings to the actual structure at the worksite. Inspite of all the precautions, wrong detailing of reinforcing bars is still a common feature in many construction sites leading to serious problems affecting the strength and serviceability of the structure.

Sudarshan[118] has identified some common, mistakes based on his field supervision over a number of years and has suggested a few do's

and dont's in fabrication and detailing of reinforcements in structural concrete members and these are compiled in various Fig. 16.36 to 16.48 for the benefit of the site engineer.

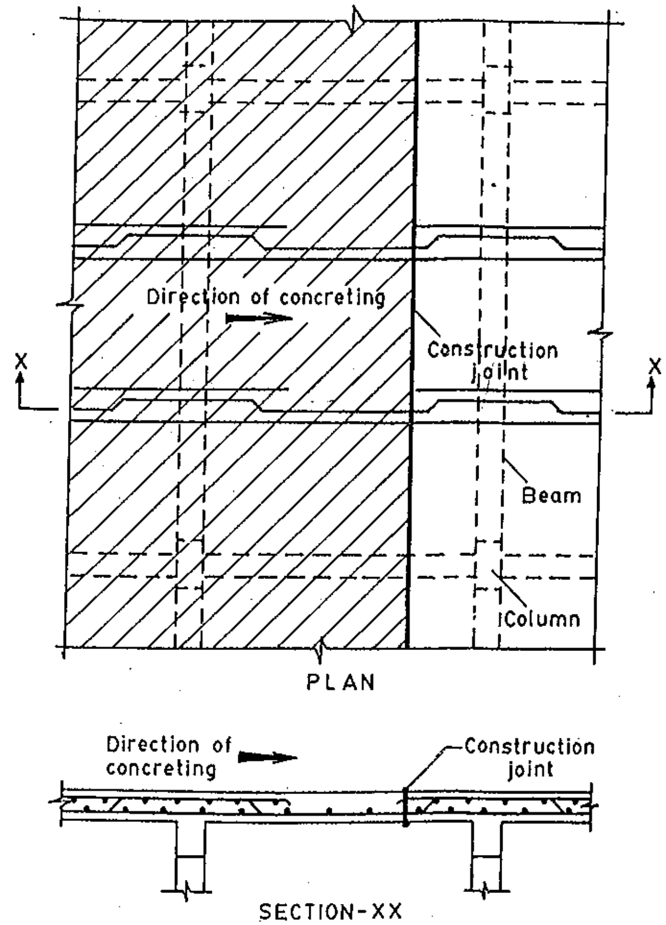
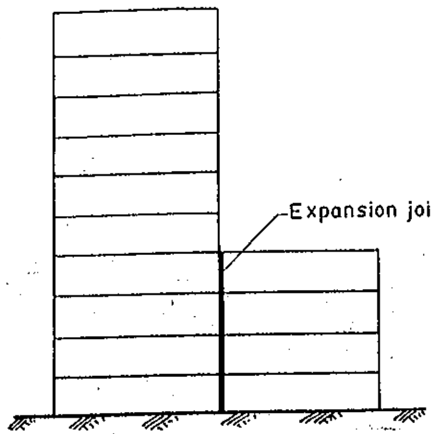
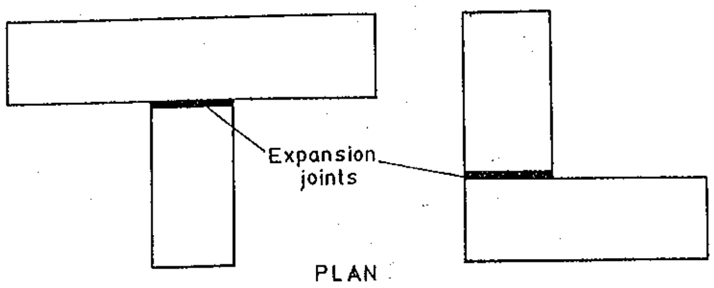


Fig. 16.33 Typical Construction Joint in One Way Slab



(a) EXPANSION JOINT AT JUNCTION OF TALL AND SHORT BUILDINGS



(b) EXPANSION JOINTS AT MAJOR CHANGES OF BUILDINGS

Fig. 16.34 Location of Expansion Joints in Buildings

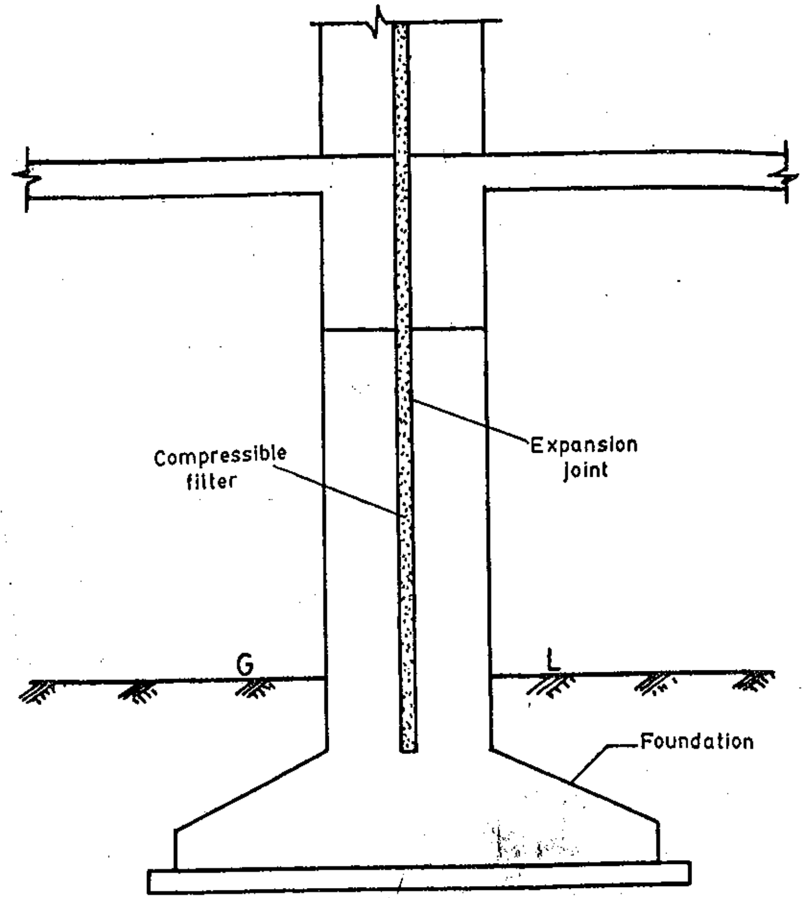


Fig. 16.35 Expansion Joint Between Building Blocks

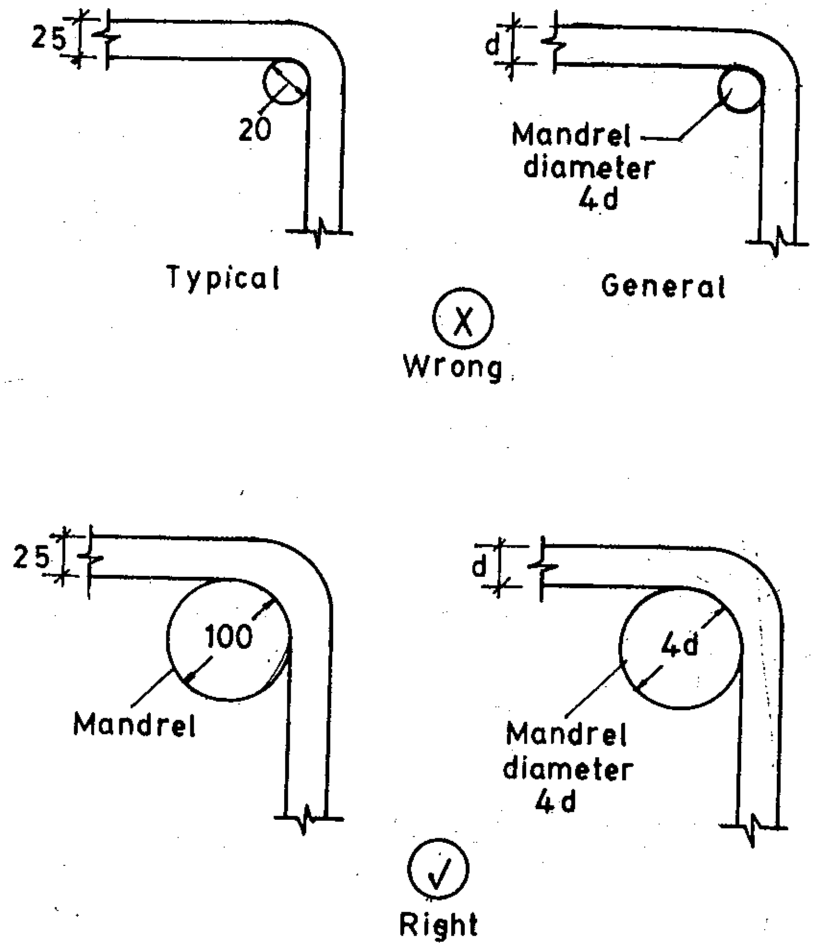


Fig. 16.36 Choice of Mandrel For Bar Bending

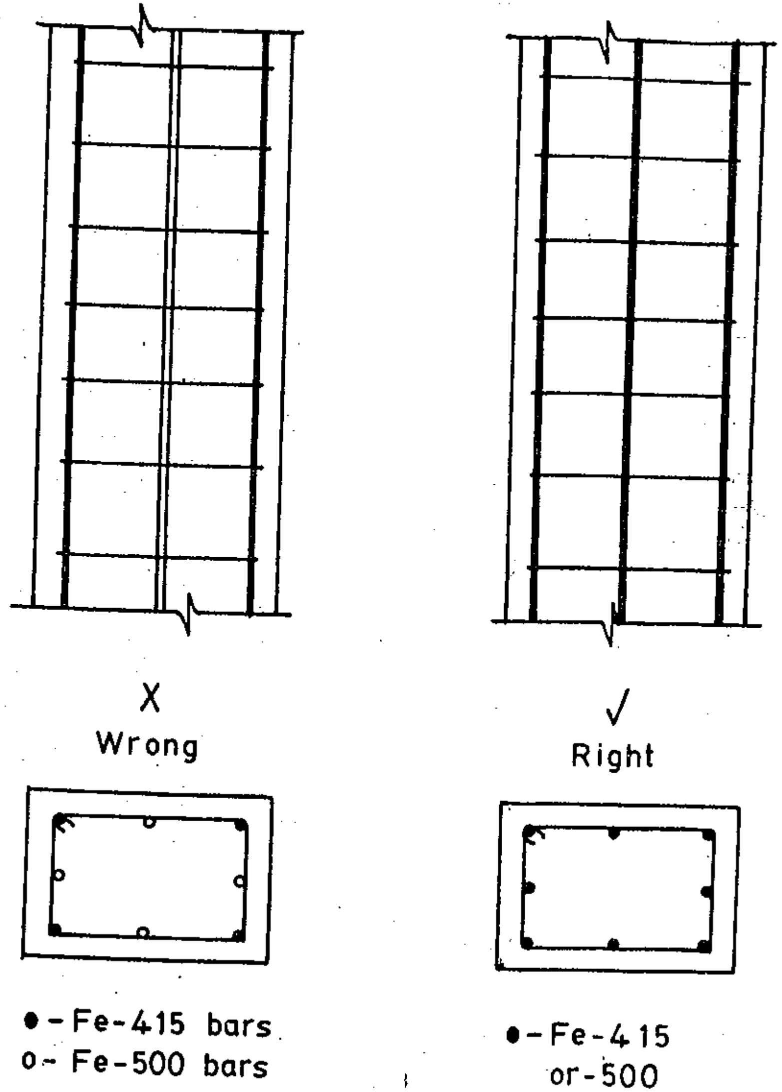


Fig. 16.37 Mix-up of Different Grade Reinforcements

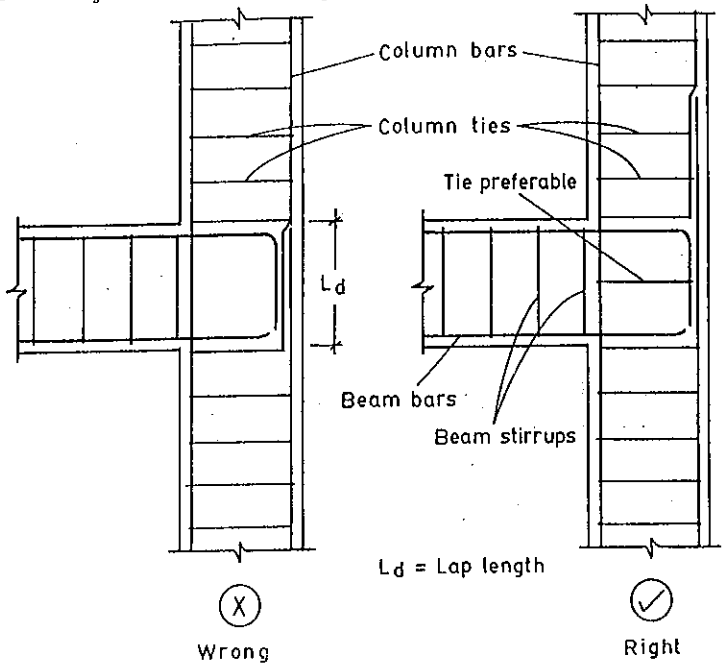


Fig. 16.38 Lapping of Bars at Column-Beam Junction

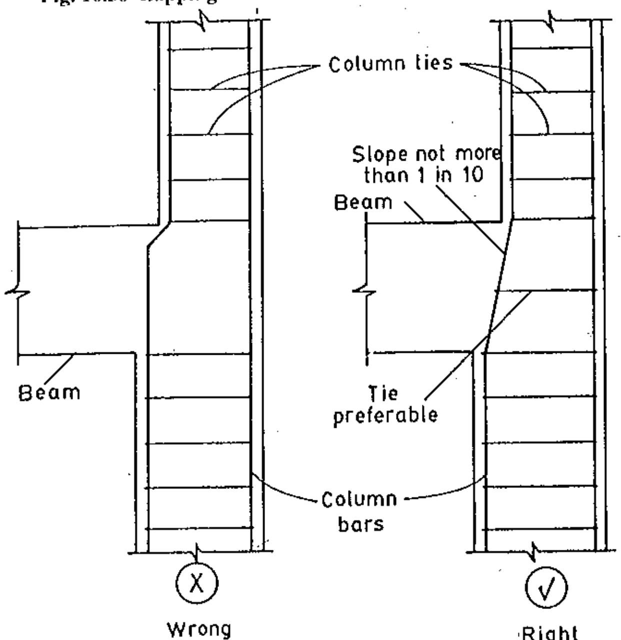


Fig. 16.39 Abrupt Kinking of Bars in Columns

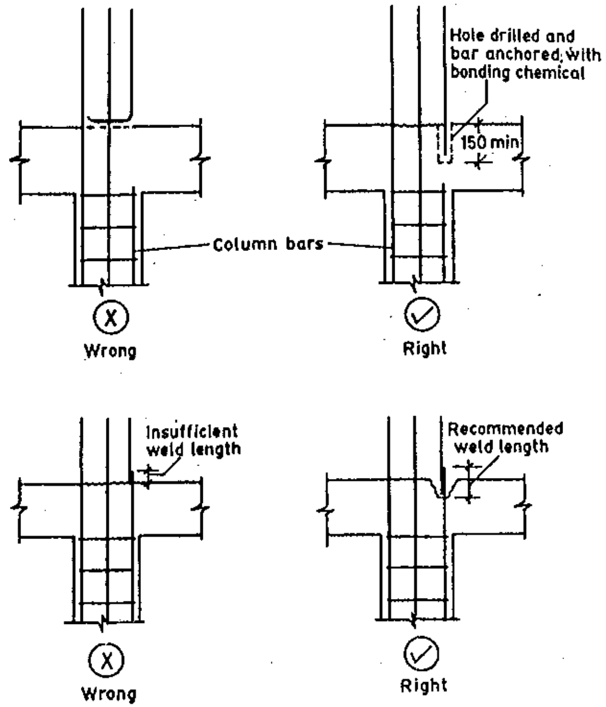


Fig. 16.40 Discontinuity of Bars in Columns

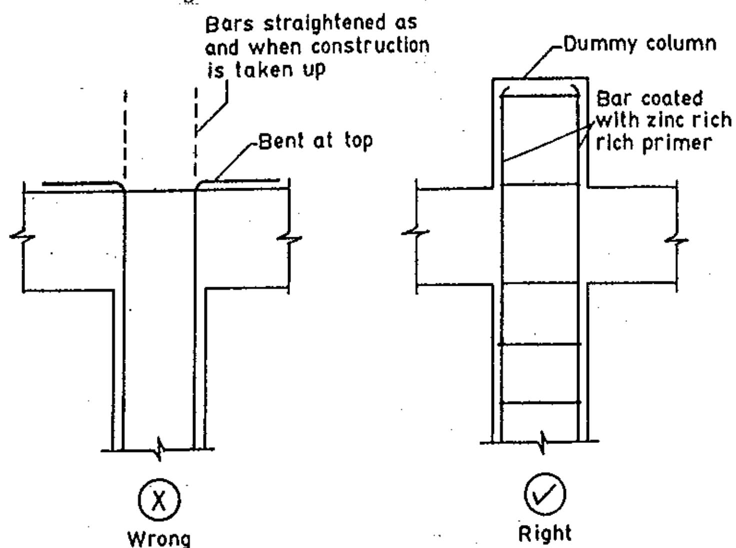


Fig. 16.41 Profile of Bars For Future Expansion

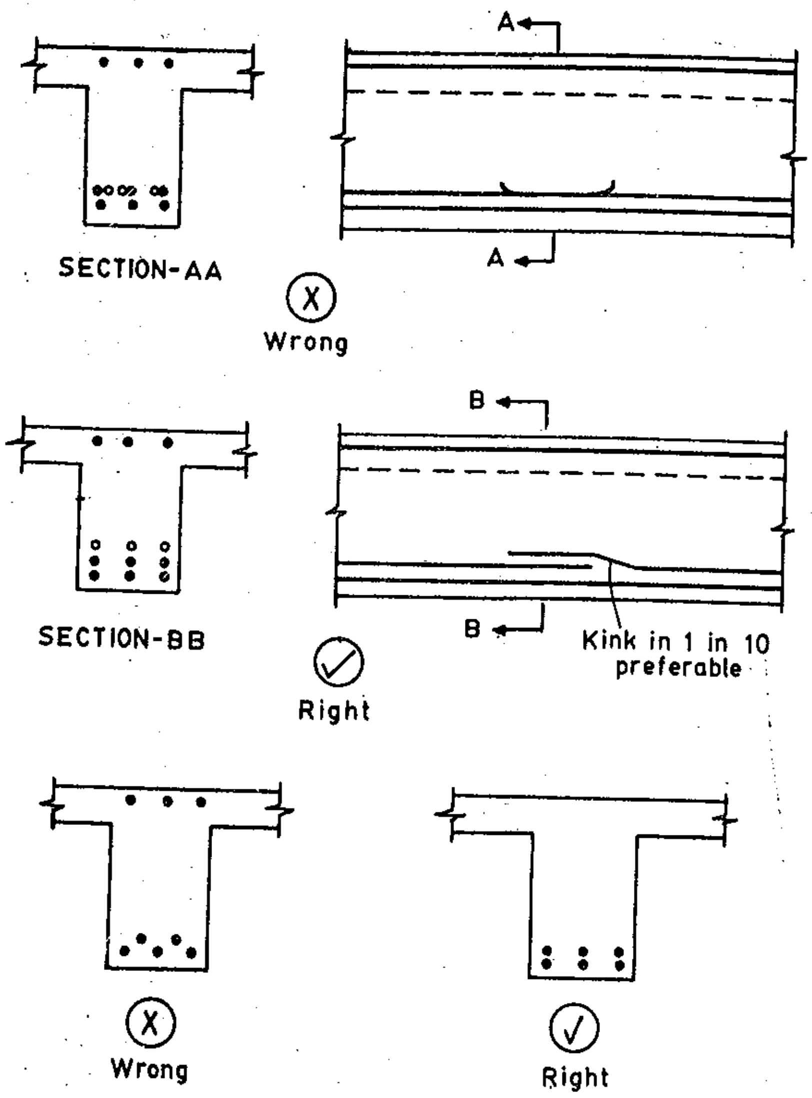


Fig. 16.42 Lapping Arrangement of Bars in Beams

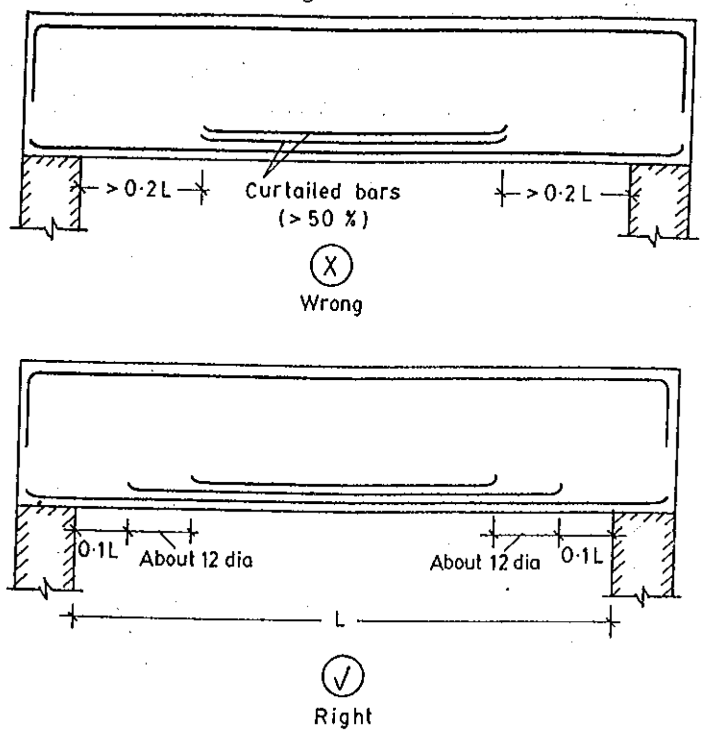


Fig. 16.43 Curtailment of Bars in Beams

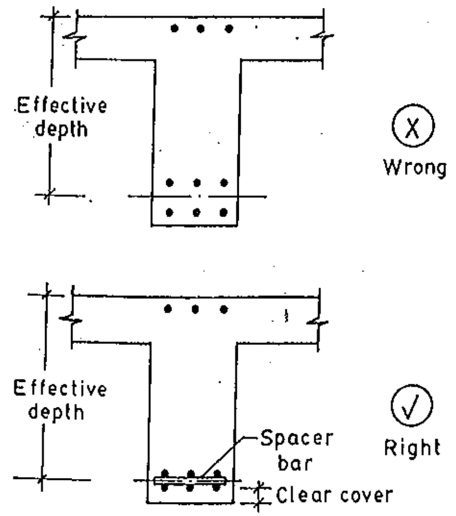


Fig. 16.44 Defective Positioning of Bars in Beams

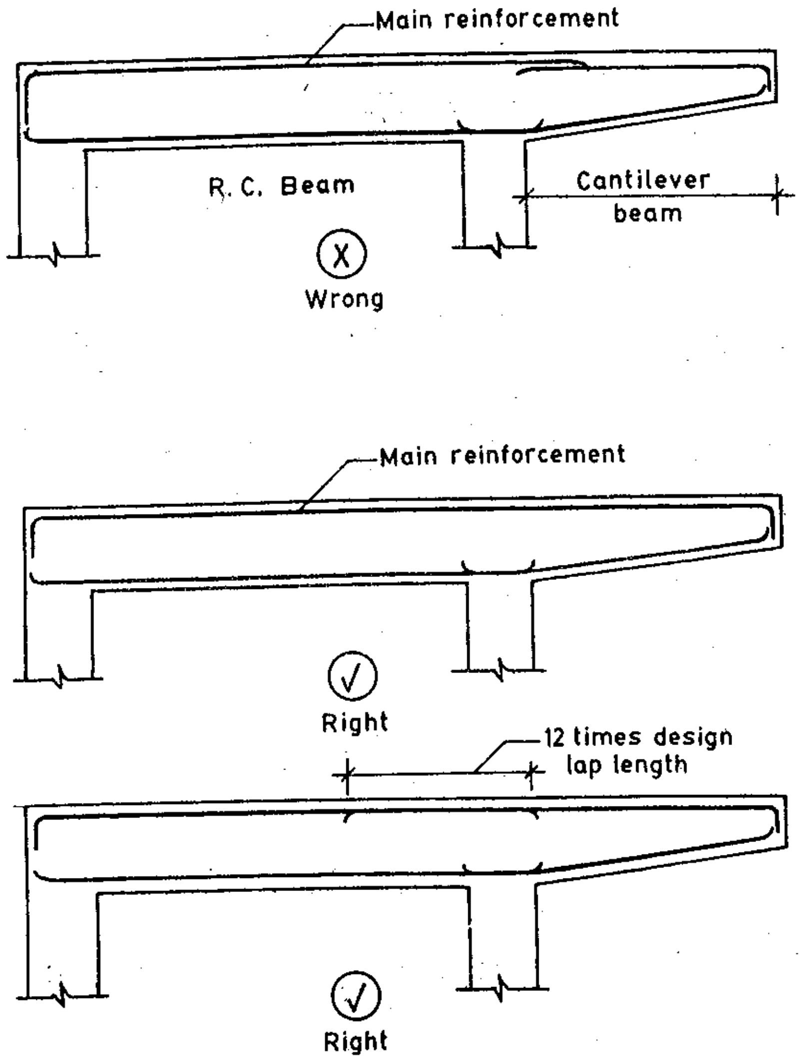


Fig. 16.45 Lapping of Bars in Cantilever Beams

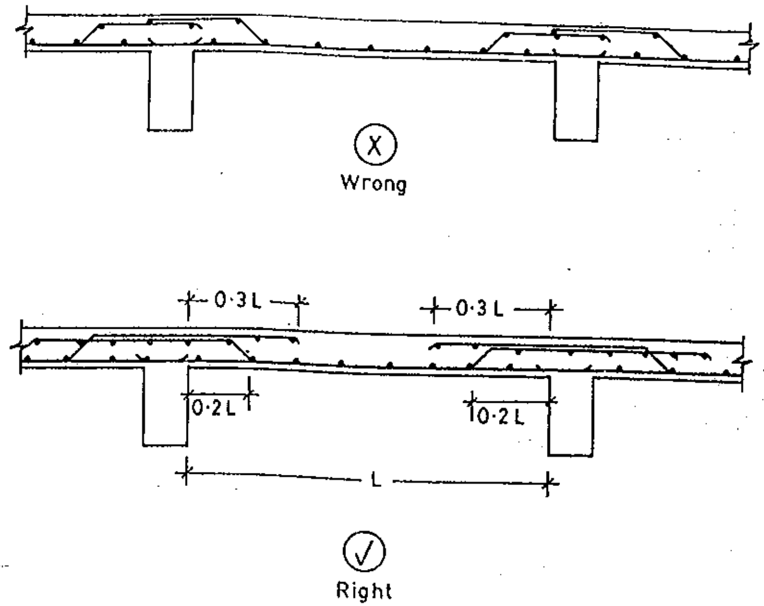


Fig. 16.46 Placement of Bars at Slab Support

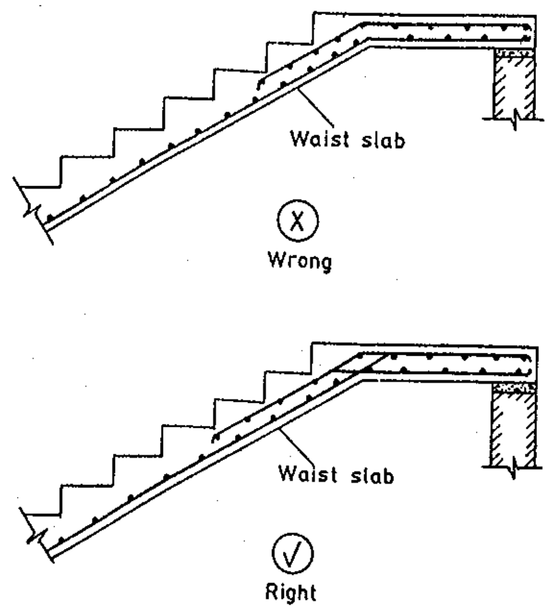


Fig. 16.47 Placement of Bars in Waist Slab

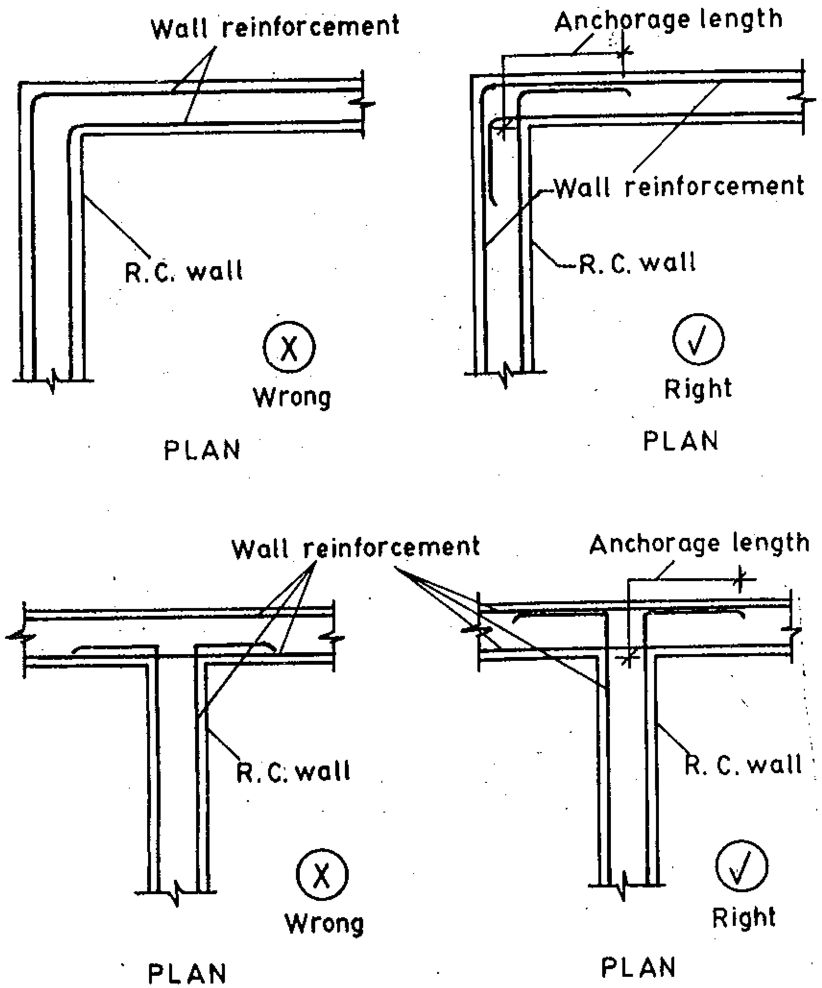


Fig. 16.48 Reinforcements at Wall Intersections

CHAPTER 17

Earthquake Resistant Design and Detailing

17.1 INTRODUCTION

Earthquakes of destructive intensity can be classified as major natural disasters confined to a relatively few areas of the world but resulting in considerable damage to buildings. During the last decade of 20^{th} century, four major earth quakes of magnitude 6 or more on the Richter scale have already occurred in India. The earthquake which occurred in Uttarakashi in 1991 with a magnitude of 6.6 on the Richter scale, damaged hundreds of buildings.

The Latur earthquake of 1993 with a magnitude of 6.4 on Richter scale resulted in the destruction of thousands of houses. The Jabalpur earthquake of 1997 measuring 6 on the Richter scale destroyed thousands of houses. The 1999 earthquake at Chamoli district of UttarPradesh with a magnitude of 6.8 destroyed more than 4000 houses and partially damaged 25,000 dwellings. Recently in January 2001, a major earthquake of intensity 6.9 destroyed several towns in Gujarat with considerable loss of life and property.

During an earthquake, ground motions develop in a random manner both horizontally and vertically in all directions radiating from the epicentre. The ground motions develop vibrations in the structure inducing inertial forces on them. Hence structures located in seismic zones should be suitably designed and detailed to ensure strength, serviceability and stability with acceptable levels of safety under seismic forces.

The satisfactory performance of a large number of reinforced concrete structures subjected to severe earthquakes in various parts of the world has demonstrated that it is possible to design such structures to successfully withstand the destructive effects of major earthquakes.

The Indian standard codes IS: 1893-1984[119], IS: 4236-1976[120] and IS: 13920-1993[121] have specified the minimum design requirements of earthquake resistant design based on the probability of occurrence of earthquakes, the characteristics of the structure and the foundation and the acceptable magnitude of damage. The A.C.I. Committee-318[122] and other national codes[123,124] give details of codal requirements in several earthquake zones of the world.

The main criteria enshrined in the codes for fixing the level of the design seismic loading based on the SEAOC[125] report are as follows: -

1) The structures should be able to resist **minor** earthquakes without damage which implies that the structural behaviour under minor earthquake shocks should be within the elastic range of stress.
2) The structures should be able to resist **moderate** earthquakes with minor structural and some non-structural damage. With proper design and construction, it should be possible to restrict the damage so that it is repairable.
3) The structure should be able to resist **major** earthquakes without collapse but with some structural and non-structural damage.

Depending upon the intensity of earthquake, forces are induced in the structural system. These forces are influenced by the damping, ductility and energy-dissipation capacity of the structure[126].

The basic principle of design of earthquake resistant structures is by

a) Enhancing the ductility (rotation capacity) of the structural members.
b) Increasing the energy dissipation capacity of the structure.

If the structure is designed to have the above mentioned properties, the induced seismic forces are considerably reduced resulting in an economical structure with the added advantage of reduced probability of collapse of the structural system.

In general buildings designed to resist the lateral loads are subjected to very low seismic forces. Typical structural systems, which are generally used to resist earthquakes, are

a) Ductile moment resisting space frame.
b) Dual system, comprising ductile moment resisting frame and ductile flexural (shear) wall.

According to Fintel[127], a ductile structural system is one in which the members undergo significant inelastic deformations beyond the initial yield and the loads are resisted by the redistribution of moments so that there is no decrease in the ultimate load resistance of the structural system.

The present day codes recommend proper ductility requirements in reinforced concrete members to reduce the seismic forces and their destructive effect.

17.2 EARTHQUAKE FORCES

Determination of design earthquake forces is computed by the following methods:-

a) Equivalent static lateral loading.
b) Dynamic Analysis.

In the former method, different partial safety factors are applied to dead, live, wind and earthquake forces to arrive at the design ultimate load. In the IS: 456-2000 code, while considering earthquake effects, wind loads are replaced by earthquake loads assuming that both severe wind and earthquake do not act simultaneously. The American and Australian code recommendations are similar but with different partial safety factors.

The dynamic analysis involves the rigorous analysis of the entire structural system by studying the dynamic response of the structure by considering the total response in terms of component modal responses, which is outside the scope of this text, and the reader may refer to specialist texts by Clough and Penzien[128], New Mark[129] and various other authors[130-135]

This chapter mainly deals with the major codal specifications of IS: 13920-1993[121] regarding the designing and detailing for ductility in moment resisting frames and shear walls. These provisions are essential for the design of structural systems located in high intensity earthquake zones categorized as Zones III, IV and V in the Indian standard code IS: 1893-94[119]

17.3 DUCTILITY OF REINFORCED CONCRETE MEMBERS

17.3.1 Concept of Ductility

Ductility of reinforced concrete members is defined as its ability to accommodate large magnitude of inelastic deformations beyond the first yield deformation without any decrease in its collapse strength or resistance to loads. Ductility can also be defined with respect to strains, rotations, curvatures or deflections.

Ductility based on deflections comprises the entire configuration of the structural member which includes the material and section properties, loading, support configuration and span of the member. Fig. 17.1 (a) shows the Force-deformation behaviour of a flexural member under increasing loads[136]. The force may be load, moment or stress while the deformation comprises of elongation, rotation, curvatures or strain as shown in Fig. 17.1 (b)

Let Δ_u, ϕ_u and θ_u are the ultimate deformation, curvature and rotation respectively and Δ_y, ϕ_y and θ_y are the corresponding values at first yield, then a quantitative measure of ductility 'μ' can be expressed as the ratio,

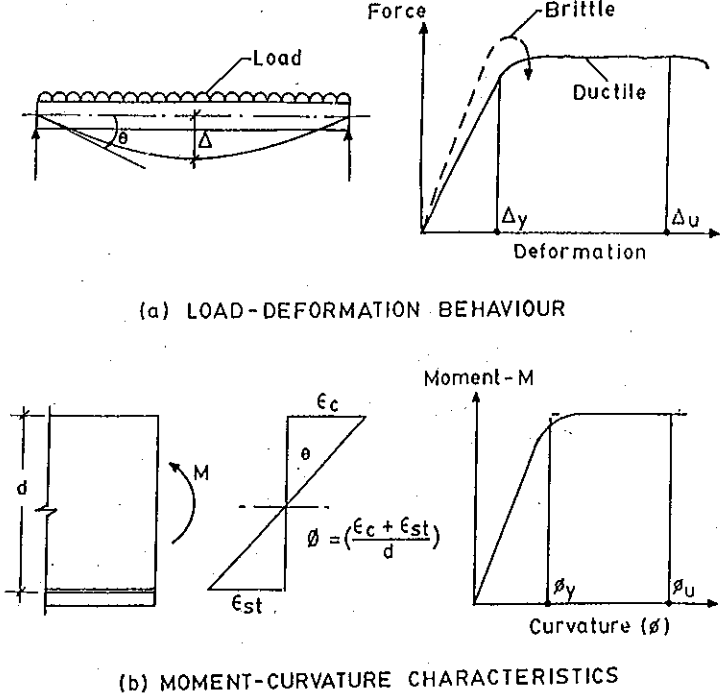


Fig. 17.1 Ductility of Reinforced Concrete Members

$$\mu = \left(\frac{\Delta_u}{\Delta_y}\right) \text{.................Displacements}$$

$$= \left(\frac{\phi_u}{\phi_y}\right) \text{.................Curvatures}$$

$$= \left(\frac{\theta_u}{\theta_y}\right) \text{.................Rotations}$$

Also the curvature at any section can be expressed as

$$\phi = \left[\frac{\varepsilon_c + \varepsilon_{st}}{d}\right]$$

Where ε_c = strain at compression face
ε_{st} = strain in the tension zone.
d = effective depth.

Structural concrete members are subjected to several cycles of reversed cyclic loading under seismic forces. Fig. 17.2 shows typical load-deflection curves for a cantilever reinforced concrete beam subjected to reversed cyclic loading. The beam is reinforced on both faces to resist tension under reversed cyclic loading. The structure responds in an elasto-plastic (ductile) manner with increased deflection compared to the elastic stage. The stiffness of the beam decreases with number of cycles. The load-deflection curves tend to pinch-in near Zero Load. The stiffness degradation and pinch-in effects are characteristic features of reinforced concrete beams and columns[137]

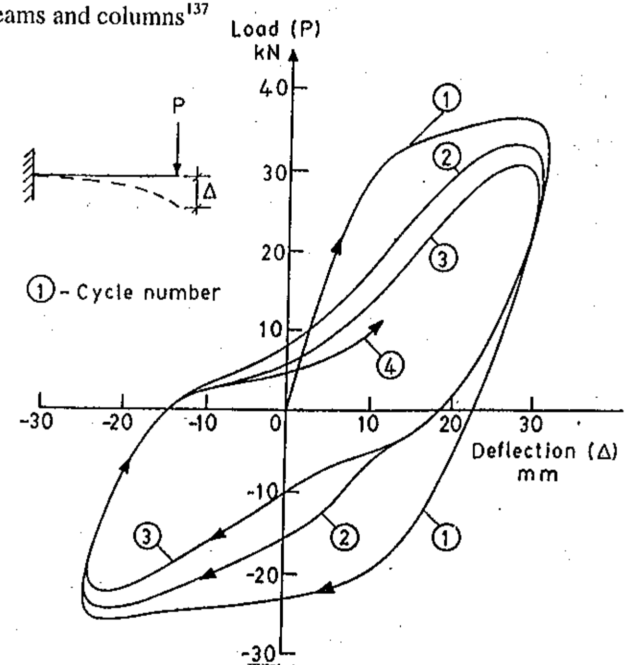


Fig. 17.2 Hysteresis Behaviour of a Reinforced Concrete Cantilever Beam

The elasto-plastic behaviour of reinforced concrete is influenced mainly by the degree of cracking in concrete, strain hardening and Bauschinger effect in steel reinforcements, bond and anchorage between concrete and steel reinforcements and the presence of shear.

The stiffness of the structural member gradually decreases with increasing cycles of loading. The primary aim in designing structural concrete members should be to increase the ductility so that the structure can sustain large inelastic deformations in order to avoid sudden collapse.

A ductile structure under overloading deforms inelastically and redistributes the excess load to elastic parts of the structure. In the case of ductile structures, there will be sufficient warnings in the form of cracks and deflections of members before impending failure so that the probability of loss of life is reduced in the event of collapse of the structure.

In the limit state design procedure, it is assumed that all the critical sections in the structure will reach their maximum capacities at the design collapse load for the structure. To fulfill this requirement, it is essential that the structure should be ductile to withstand forces and deformations corresponding to the yielding of the reinforcements.

17.3.2 Ductility Computations

The ductility of reinforced concrete beams is influenced by the behaviour of the cross sectional properties involving the following parameters:-

a) Shape of cross section(Rectangular or Flanged)
b) Tension reinforcement ratio.
c) Compression reinforcement ratio.
d) Grade of concrete.
e) Grade of steel reinforcement.
f) Ultimate compressive strain in concrete.
g) Yield and Ultimate strain of reinforcement.

An expression for the curvature ductility of a reinforced concrete beam can be derived using the above parameters. Referring to Fig. 17.3 and using the following notations:-

d = effective depth.
A_{st} = area of tension reinforcement.
A_{sc} = area of compression reinforcement.
nd = depth of neutral axis based on elastic theory.
m = modular ratio = $[280/3\,\sigma_{cb}]$
p_t = percentage tension steel = $[100\,A_{st}/bd]$
p_c = Percentage compression steel $[100\,A_{sc}/bd]$
ε_y = Yield strain of Tension steel.
μ = Curvature ductility = $\left(\dfrac{\phi_u}{\phi_y}\right)$
ε_{cu} = ultimate compressive strain in concrete.
x_u = neutral axis depth at ultimate stage.
ϕ_y = yield curvature.
ϕ_u = ultimate curvature.
E_s = modulus of elasticity of concrete.

f_{ck} = characteristics compressive strength of concrete.
f_y = yield stress of tension reinforcement.
f_y' = stress in compression reinforcement = $0.87\,\sigma_y$
n = $-mp_t + \sqrt{m^2 p_t^2 + 2mp_t}$

From Fig. 17.3 (a)

$$\phi_y = \left(\frac{\varepsilon_y}{n - nd}\right) \quad \text{and} \quad \phi_u = \left(\frac{\varepsilon_{cu}}{x_u}\right)$$

Where $$x_u = \left(\frac{0.87 f_y A_{st}}{0.36 f_{ck} b}\right) = \left(\frac{0.87 f_y p_t d}{0.36 f_{ck}}\right) \le x_m$$

Where x_m = maximum permissible neutral axis depth depending upon the grade of steel.
= 0.53 d for Fe-250 grade steel.
= 0.48 d for Fe-415 grade steel.

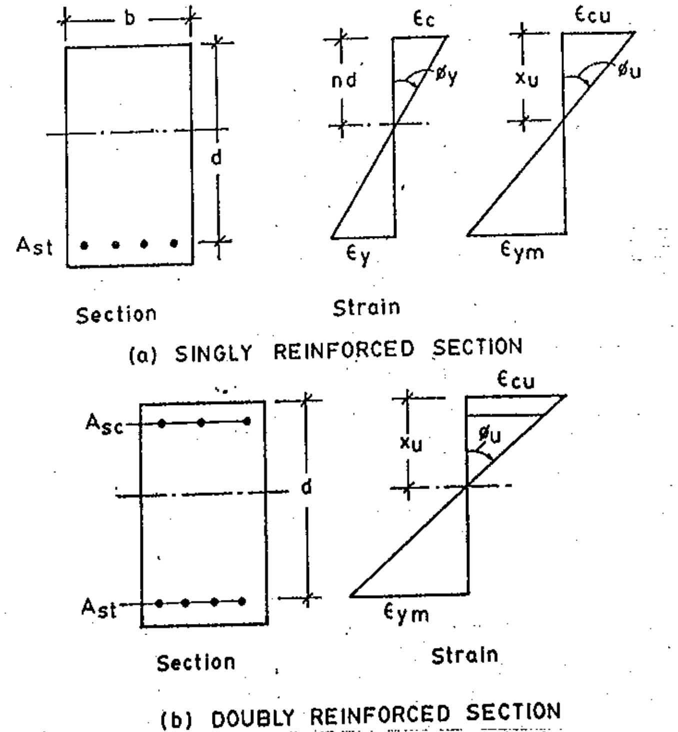


Fig. 17.3 Ductility of Reinforced Concrete Sections

$\therefore$ Curvature ductility $= \mu = \left(\dfrac{\phi_u}{\phi_y}\right) = \dfrac{\varepsilon_{cu}}{\phi_y}\left(\dfrac{d-nd}{x_u}\right)$

or $$\mu = \frac{\varepsilon_{cu}}{(f_y/E_s)}\left[\frac{1+mp_t-\sqrt{m_2p_t^2+2mp_t}}{(x_u/d)}\right] \quad ...(17.1)$$

Using this equation, the variation of curvature ductility μ with the tension steel ratio p_t can be studied.

Similarly for doubly reinforced concrete beams, a similar expression for ductility factor can be derived.

Stress in compression steel at ultimate stage $= f'_y$

Then from force equilibrium, we have

$$[0.36 f_{ck}\, b.x_u + f'_y A_{sc}] = 0.87 f_y A_{st}$$

Or $$\left(\frac{x_u}{d}\right) = \left[0.87\, p_t - \left(\frac{f'_y}{f_y}\right) p_c\right]\frac{f_y}{0.36 f_{ck}}$$

Since $f'_y = 0.87 f_y$

$$\left(\frac{x_u}{d}\right) = (p_t - p_c)\left(\frac{0.87 f_y}{0.36 f_{ck}}\right)$$

and $$x_u \le x_m$$

$\therefore$ $$(x_u) = (p_t - p_c)\left(\frac{0.87 f_y d}{0.36 f_{ck}}\right)$$

and $$\phi_u = \left(\frac{\varepsilon_{cu}}{x_u}\right)$$

$$\mu = \left(\frac{\phi_u}{\phi_y}\right) \quad \text{and} \quad \varepsilon_y = (f_y/E_s)$$

$$= \left(\frac{\varepsilon_{cu}}{x_u}\right)\left(\frac{d-nd}{\varepsilon_y}\right)$$

$$\mu = \left[\frac{(0.00035)(0.36 f_{ck})(1-n)E_s}{(p_t - p_c)(0.87 f_y)}\right] \quad ...(17.2)$$

Equations 17.1 and 17.2 can be used to evaluate the curvature ductility of singly reinforced and doubly reinforced sections respectively.

The variations of curvature ductility with steel ratio p_t and $(p_t - p_c)$ is shown for different grades of steel Fe-250 and Fe-415 and for different grades of concrete from M-15 to M-30 in Figs. 17.4 (a) and (b)

17.3.3 Factors Influencing Ductility

a) Tension steel ratio (p_t)

The ductility of a reinforced concrete section increases with the decrease in steel ratio p_t or $(p_t - p_c)$ as shown in Figs. 17.4 (a) and (b). Excessive steel reinforcement will result in the crushing of concrete before the steel yields resulting in brittle failure corresponding to $\mu = 1.0$. Hence it is advantageous to design the sections as under reinforced and the code IS: 456-2000 prohibits the use of over reinforced sections, which are prone to brittle failure.

b) Grade of steel and concrete

The ductility is influenced by the grade of concrete and steel. The Ultimate strain in concrete (ε_{cu}) is a function of the characteristic strength of concrete, rate of loading and strengthening effect of stirrups. The IS: 456-2000 code recommends a value of $\varepsilon_{cu} = 0.0035$. Figs 17.4(a) and (b) clearly indicate that the ductility increases with the characteristic strength of concrete (f_{ck}) and decreases with the increase in characteristic strength of steel (f_y). According to Jain[137], ductility is inversely proportional to the square of σ_y. From the ductility point of view, it is more desirable to use Fe-250 grade steel having a larger percentage of elongation than the Fe-415 grade deformed bars, which have comparatively lower value of percentage elongation.

Lower grade steel has defined and longer yield plateau and hence the plastic hinges developed at critical sections will have larger rotation capacity leading to greater energy dissipation. It is important to note that lower grade steels have a higher ratio of ultimate to yield strength and the higher ratio of (f_u/f_y), is desirable since it results in an increased length of plastic hinge and also increased plastic rotation capacity of the critical sections.

Based on these factors, Fe-250 grade mild steel is better suited for use as reinforcements in earth-quake resistant design. However the use of Fe-250 grade mild steel results in larger cross sections of flexural members. Hence the Indian Standard Code IS: 13920-1993[121] permits the use of commonly used Fe-415 grade steel but prohibits the use of grades higher than Fe-415. The code also limits the minimum grade of concrete to M-20. High strength concrete is also undesirable due to its lower ultimate compressive strain (ε_{cu}) which reduces the ductility. Low density concrete is also undesirable due to its relatively poor performance under reversed cyclic loading. Based on this factor, the American[122] and Canadian[124] codes limit the maximum cylinder strength of low density concrete to a value of M-30 for use in earth quake resistant design.

c) Compression steel ratio (p_c)

The curvature ductility (μ) is inversely proportional to the parameters ($p_t - p_c$) as shown in Eq. 17.2 the ductility increases with the decrease in the value of ($p_t - p_c$). Hence the ductility increases with the increase in compression steel. Most of the national codes prescribe limits for the minimum and maximum steel ratios to be used in earthquake resistant design of structural members.

d) Shape of cross section

In the case of flanged (Tee) beams, due to the enlarged compression face, the neutral axis at collapse stage falls within the flange resulting in the increase of ductility. In such cases the ductility can be estimated using Fig. 17.4.

e) Lateral Reinforcements

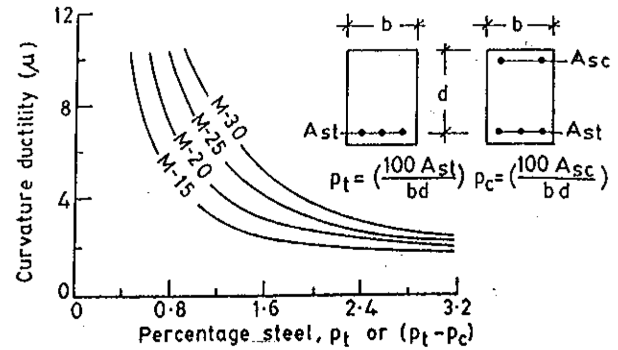
The provision of Lateral reinforcements in the form of stirrups and the use of circular hoops to confine the concrete in the compression zone tends to improve the ductility by preventing premature shear and compression failure, thus increasing the deformation capacity of reinforced concrete beams.

17.4 DESIGN PRINCIPLES and CODE PROVISIONS

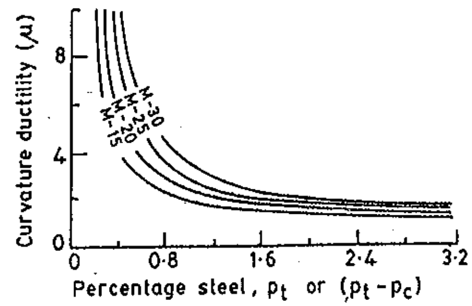
The basic principles of modern code provisions dealing with earthquake resistant design have evolved from rather simplified concepts of the dynamic behaviour of structures and have been influenced to a large extent by field observations of the performance of structures subjected to actual earthquakes.

It is pertinent to note that many structures built in the 1930's and designed on the basis of more or less arbitrarily chosen lateral forces have successfully with stood severe earthquakes. The satisfactory performance of such structures is attributed to one or more of the following factors.[138,139]

1) Yielding at critical sections of members, which increased the period of vibration of such structures but allowed them to absorb greater amounts of input energy resulting from an earthquake.



(a) CURVATURE DUCTILITY IN BEAMS WITH Fe-250 STEEL



(b) CURVATURE DUCTILITY IN BEAMS WITH Fe-415 STEEL

Fig. 17.4 Ductility Characteristics of Reinforced Concrete Beams

2) Greater actual strength of such structures resulting from non-structural elements, which were generally ignored, in the analysis.
3) The significant energy dissipation capacity of non-structural elements.
4) The reduced response of the structure due to yielding of the foundations.

The buildings designed under the present code provisions would be expected to undergo fairly large inelastic deformations absorbing the energy resulting from the earthquake. The codes have accepted the fact that it is uneconomical to design buildings to resist major earthquakes elastically and the recognition of the capacity of structures possessing adequate strength and ductility to withstand major earthquakes by inelastic behavior.

The capacity of a structure to deform in a ductile manner beyond the yield limit without significant loss of strength results in absorption of major portion of energy from an earthquake without serious damage.

Extensive Laboratory investigations[140-144] have demonstrated that reinforced concrete members and their connections designed and detailed according to the provisions of the present day codes do possess the necessary ductility required to respond inelastically to earthquakes of major intensity without significant loss of ultimate strength.

The major objective of the special design and detailing provisions of the Indian standard code IS:13920 is to ensure adequate ductility without loss of strength for structural members such as beams, columns and walls and to prevent sudden or non ductile types of failure.

The principal design considerations to be followed to ensure sufficient ductility of the members are,

a) Use of low percentage of tensile reinforcement of relatively low grade and use of compression reinforcement.
b) Prevent shear failures by using adequate stirrups and ensuring flexural failure associated with under reinforced sections.
c) Use of confined concrete in the compression zone by using closely spaced hoops or spirals to avoid compression failure of concrete associated with over reinforced sections.
d) Reinforcement detailing with regard to anchorage, splicing and quantity of minimum reinforcement in the section.
e) Continuity in construction to improve the inelastic behaviour of the structure with moment redistribution and energy dissipation at several plastic hinges.

Structural stability and stiffness are also important factors to be considered in the design of earthquake resistant structures. The structural system should be so designed such that the plastic hinges are formed at suitable locations resulting in the failure of individual structural elements only without leading to the instability or progressive collapse of the structure. The use of redundant structures such as continuous rigid frames will ensure the development of alternative load paths thus helping the redistribution of forces and dissipation of energy, preventing progressive collapse of the structure.

In order to ensure ductile behaviour with minimal damage it is important that the foundations should not yield prior to the failure of the super structure. Hence the moments, shear and axial forces transferred from the super structure to the foundation system under conditions of actual yielding, should be resisted by the foundations with the margin of safety of 1.5 applied to the loads and 1.15 applied to the materials. Correspondingly the ultimate moment corresponding to actual yielding at a section in super structure is obtained as its characteristic or nominal moment capacity without applying the partial safety factors. This will ensure that the foundation is relatively stronger when compared to the super structure. This

recommendation is yet to be incorporated in the Indian Codes[120] while the American[122] and Canadian[124] codes contain this concept which will result in ductile behaviour of the superstructure without any serious distress in the foundation system.

17.5 INDIAN STANDARD CODE PROVISIONS FOR EARTHQUAKE RESISTANT DESIGN

The salient features of the specifications of the Indian Standard code IS: 13920-1993 for design and detailing of flexural members, columns and shear walls in earthquake resistant design are summarized in the following sections[126].

17.5.1 Flexural Members (Beams)

1) Flexural members should have an overall depth 'h' not exceeding one fourth of the clear span mainly to limit shear deformations. The width of the member (b) should be not less than 200mm and (b/h) ratio of more than 0.3 to avoid lateral instability and greater resistance to torsion.
2) The percentage tensile reinforcement in the section (p_t) should not exceed 2.5 to ensure ductile behaviour of the member under reversals of displacements in the inelastic range and to avoid congestion of reinforcements and to limit shear stresses.
3) Minimum reinforcement ratio of $p_{min} = 0.24\sqrt{f_{ck}/f_y}$ is specified at both top and bottom faces of the member for the entire length (with minimum of 2 bars placed at each face) to prevent sudden brittle failure due to cracking of concrete.
4) Reinforcement detailing in flexural members of a ductile-framed structure in which the beams are assumed to yield under design earthquake forces with the development of reversible plastic hinges in the vicinity of beam-column junctions should comply with the following specifications.
   a) The reinforcements resisting positive moments at a joint face should be not less than half the negative moment reinforcement at that joint face.
   b) The top and bottom face reinforcement at any section along the length of the member should be not less than one-fourth of the negative moment reinforcement at the joint face on either side.

c) The top and bottom reinforcements should be taken through column and extended to the adjacent span so that continuity is maintained in the case of interior junctions. In the case of end junctions of a beam and column, the beam reinforcements must be extended to the far face of the column core and provided with an anchorage length of ($L_d + 10\phi$) where L_d is the development length of the bars of diameter ϕ in tension as shown in Fig. 17.5.

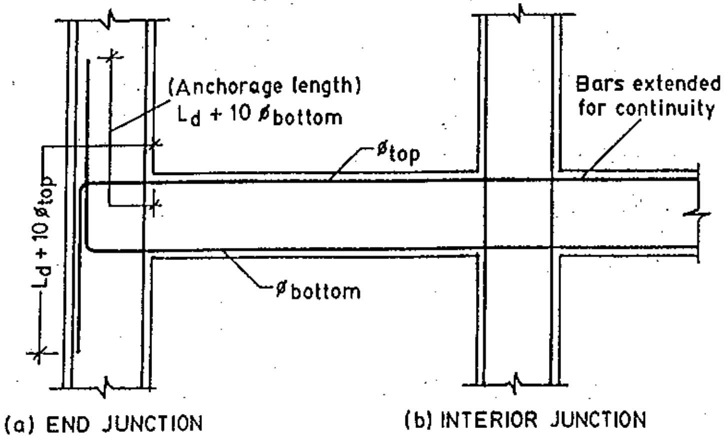


Fig. 17.5 Arrangement of Reinforcements at End and Interior Junctions

d) Lap splices of flexural reinforcement is not permitted in the vicinity of plastic hinges to safeguard against spalling of concrete under large reversed strains. However welded splices or mechanical connections are permitted in locations far away from plastic hinge zone.

e) Lap splices if required are provided at regions other than plastic hinge zones. Concrete is confined in these zones by transverse reinforcement in the form of closed hoops (135° hook and 10ϕ extension) spread over the entire splice length at a spacing not exceeding 150 mm as shown in Fig. 17.6

f) The reinforcements provided should account for possible shifts in the contraflexure points, which occur under the combined effects of dead and earthquake loads.

g) In earthquake resistant design of flexural members, shear failures should not occur before the development of plastic hinges due to flexure. Hence the shear forces are suitably over estimated by considering the plastic moment capacities of 1.4 M_{ul} and 1.4 M_{ur} at the ends of the beam as shown in Fig. 17.7 (a) and (b). The maximum design shear forces V_{ul} and V_{ur} at the left and right support

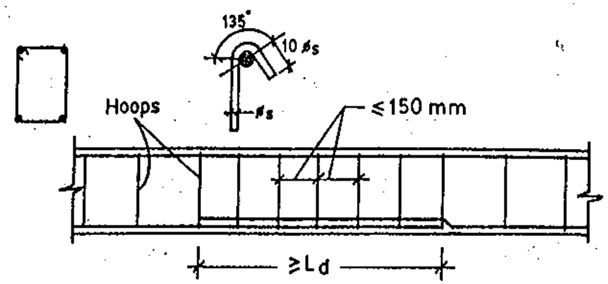


Fig. 17.6 Lap Splice in a Flexural Member

faces is computed by considering the equilibrium of forces shown in Figs. 17.7(b) and (c) for the right and left sway conditions respectively. Fig. 17.7 (a) shows the uniformly distributed dead (g) and live loads (q) on the beam of clear span L_n. Accordingly, the maximum design shear forces are computed as follows:-

$$w_u = 1.2(g + q)$$

$$V_{ul} = 0.5 w_u L_n + 1.4(M_{ul} + M_{ur})/L_n$$

$$V_{ur} = 0.5 w_u L_n + 1.4(M_{ul} + M_{ur})/L_n$$

The sign conventions and shear force diagrams are shown in Figs. 17.7(b) and (c)

h) Due to the alternate direction of shear forces developed due to earthquake forces, the directions of diagonal tension also changes and hence inclined bars are generally not allowed as shear reinforcement. Only vertical stirrups can effectively resist the shear whether it is positive or negative depending upon the sway in the member as shown in Fig. 17.8(a) and (b)

i) Web reinforcement in earthquake resistant structures must be in the form of closed stirrups or hoops place perpendicular to the longitudinal reinforcement and provided throughout the length of the member. The hoops should have a minimum diameter of $\phi_s = 8$ mm in beams with a clear span exceeding 5m. The free ends of the hoops should be bent at 135° with a minimum bar extension of 10ϕ as shown in Fig. 17.8 (a) to ensure proper anchoring of the bar ends in the core of concrete.

j) The purpose of the hoops is to confine the concrete and prevent buckling of the longitudinal bars particularly in the vicinity

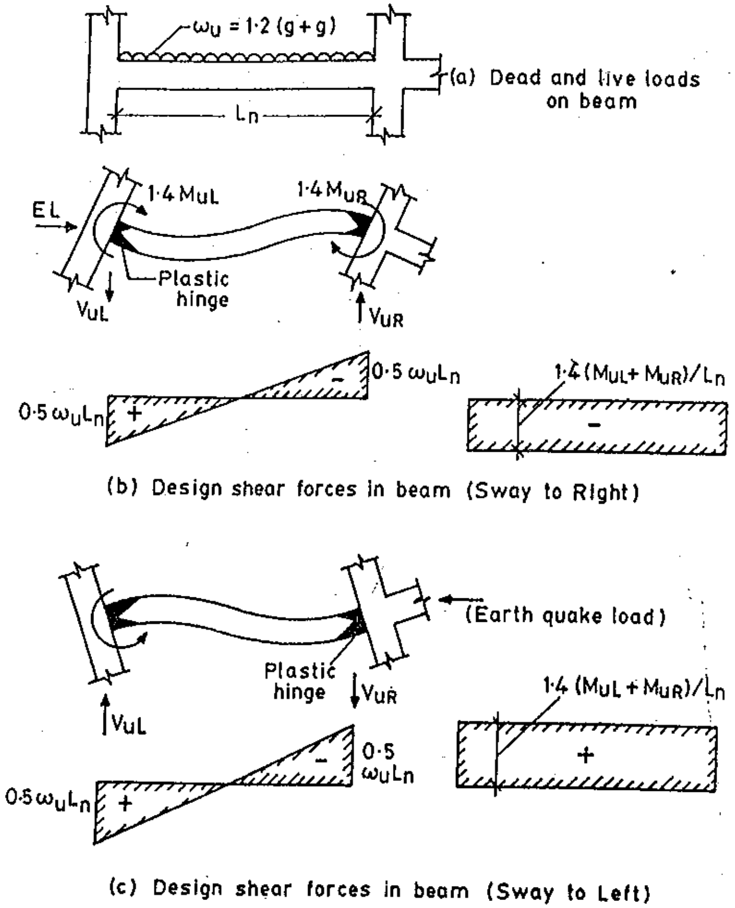


Fig. 17.7 Combination of Design Shear Forces in Beams

of the beam-column junctions where reversible plastic hinges develop with the likely hood of the concrete cover spalling after a few cycles of inelastic rotations. The I.S Code-13920[121] specifies closer spacing of hoops over a length equal to twice the effective depth from the face of the column. The spacing of the hoops should not exceed (d/4) or 8 times the diameter of the smallest longitudinal bar, with the first hoop located at a distance not exceeding 50 mm from the column face. The spacing of the hoops beyond 2d should not exceed half the effective depth as shown in Fig. 17.8(b).

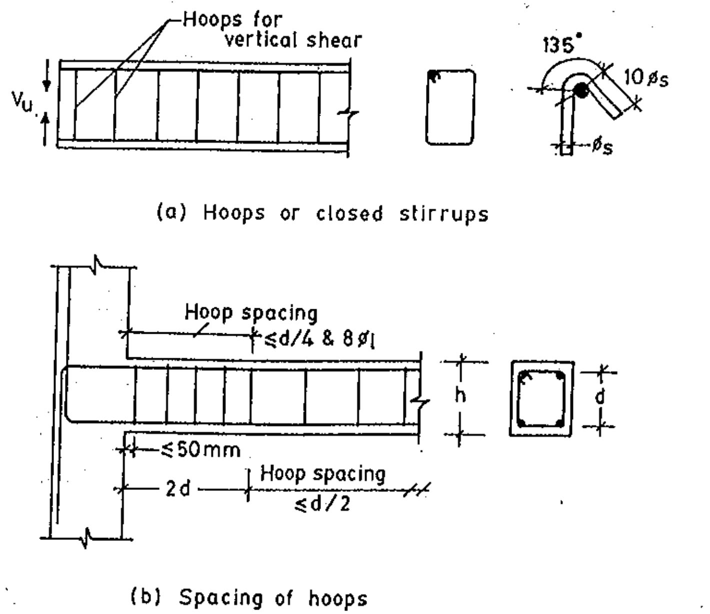


Fig. 17.8 Type of Web Reinforcement for Reversed Shear Condition

17.5.2 Framed Members subjected to flexure and axial loads (Columns)

a) Columns subjected to a factored axial stress greater that 0.1 f_{ck} under the effect of earthquake forces are grouped under this category. The minimum cross sectional dimension of the member should be not less than 200mm and the ratio of short cross sectional dimension to the perpendicular (large) dimension should preferably be not less than 0.4. Also for columns with unsupported length exceeding 4m, the shortest dimension should be not less than 300mm

b) In earthquake resistant design of framed members, the combined flexural resistance of the columns should be greater than the beams at the column-beam junctions, to ensure that plastic hinges develop first at the beam ends rather than the column ends facilitating ductile behaviour of the frame. In this regard, the I.S.Code 13920 does not make any specific recommendations. However the American code ACI 318-89 and the Canadian code CAN-A23.3-M-84 specify that the sum of the factored moment resistances of the columns framing into the joint should be at least 10 percent higher than the sum of the characteristic moment resistances of the beams framing into the joint as shown in Fig. 17.9(a).

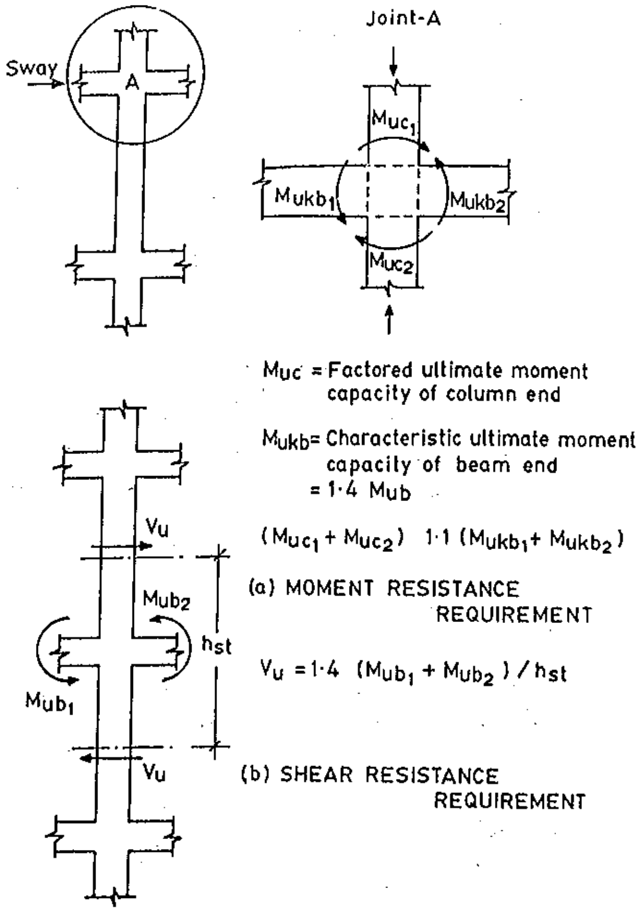


Fig. 17.9 Column Moment and Shear Resistance Requirements

c) Lap splices are permitted in the central half of the column length and not at the ends where spalling of the concrete due to plastic hinge is likely to develop in the member. Hoops are recommended over the splice length with spacing of 150 mm and at any section, only 50 percent of the bars should be spliced.

d) The design shear force in a column should be taken as the larger of the following computations: -

1) The shear force due to the factored loads.
2) The shear force in the column due to the development of plastic moments in the beams framing into the column computed as

$$V_u = 1.4\,(M_{ub1} + M_{ub2})/h_{st}$$

Where M_{ub1} and M_{ub2} are the factored moments of resistance of beam ends 1 and 2 framing into the column from opposite faces and 'h_{st}' is the height of storey as shown in Fig. 17.9 (b).

e) In the case of column joints, special confining reinforcements in the form of hoops on both sides extending over a length L_o from the joint face as shown in Fig. 17.10(a) should be provided.

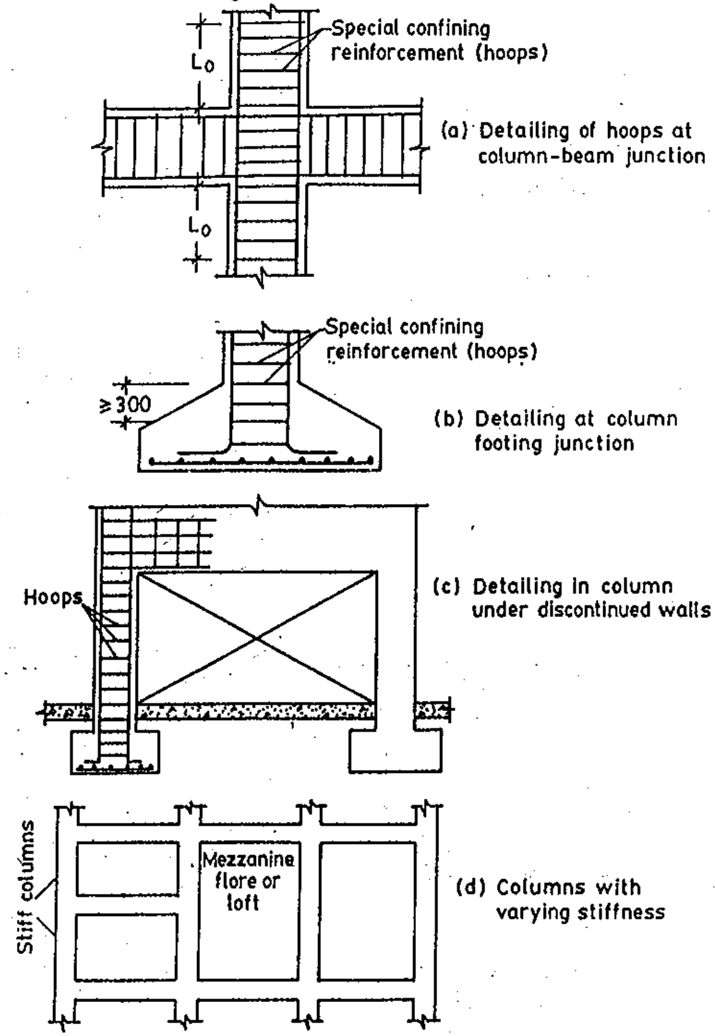


Fig. 17.10 Detailing of Columns in Earthquake Resistant Design

The length L_o should be not less than the following:-

1) The larger lateral dimension of the member at the section where yielding may occur.
2) 1/6 of the clear length of the column.
3) 450mm

The spacing of the hoops used as confining reinforcement should not exceed one-fourth of the minimum dimension of the member but need not be less than 75mm or more than 100 mm. The cross sectional area (A_{st}) of the bar to be used as confining reinforcement is given by the relation,

$$A_{st} \geq \begin{cases} 0.09\, s D_k \left(\frac{f_{ck}}{f_y}\right)\left[\frac{A_g}{A_k} - 1\right] & \text{for circular hoops / spiral} \\ 0.185\, s D_h \left(\frac{f_{ck}}{f_y}\right)\left[\frac{A_g}{A_k} - 1\right] & \text{for rectangular hoops} \end{cases}$$

Where

s = pitch of spiral or spacing of hoops.
D_k = Diameter of core, measured to the outside of the spiral or hoop.
D_h = Longer dimension of the rectangular hoop measured to its outer face, but not to exceed 300mm.
A_g = gross area of the column section, and
A_k = area of the concrete core (contained within the outer dimension of the hoop/spiral)

f) At the junction of the column and footing or mat, special confining reinforcement should be provided to a distance of at least 300mm, to account for possible development of plastic hinges at the column base as shown in Fig. 17.10 (b)

The following design and detailing specifications of IS:13920-1993 code should be followed in the case of ductile shear walls.

a) The thickness of any part of the wall should preferably be not less than 150 mm. Thinner walls are susceptible to local buckling at regions of high compressive strain. The walls can be thickened at highly compressed zones or by providing flanges or cross walls to improve bending resistance and ductility. The I.S. Code restricts the effective flange width of flanged walls to

1) Half the distance to an adjacent shear wall web and
2) One-tenth of the total wall height.

b) Shear walls should be reinforced with a minimum percentage reinforcement of 0.25 percent of the gross cross section in both vertical and horizontal directions. The diameter of the bar used should not exceed one-tenth of the wall thickness and the spacing of the bars should not exceed,

1) 1/5 of the horizontal length of wall.
2) Thrice the wall(web) thickness.
3) 450mm

The vertical reinforcement comprising both the distributed reinforcement and concentrated reinforcement near wall ends should be designed to resist the flexural and axial forces.

c) In the case of walls, which do not have flanges(boundary elements), concentrated vertical reinforcement should be provided towards each end face of the wall in addition to the uniformly distributed reinforcements. A minimum of 4 bars of 12 mm diameter arranged in at least two layers should be provided near each end face of wall. The concentrated vertical flexural reinforcement near the ends of the wall should be tied together by transverse ties as in a column. This will confine the concrete in the core and ensure yielding without buckling of the compression bars when a plastic hinge is formed.

d) The shear wall must be designed to prevent premature brittle shear failure before the development of its full plastic resistance in bending. Similar to column design, it is desirable to design the shear resistance of the wall for an over estimated shear force. Due to severe shear cracking under reversed cyclic loading, the shear resisted by concrete in the plastic hinge zone is neglected.

In the case of columns supporting discontinued stiff members such as walls or trusses, special confining reinforcement should be provided over the entire height of the column as shown in Fig. 17.10(c).

Special confining reinforcement over the full height of column is necessary where there is significant variation of stiffness along the height of columns, which are provided in mezzanine floors or lofts as shown in Fig. 17.10(d)

17.5.3 Detailing of Joints in Ductile Frames

a) The joints of beam-columns in ductile frames should be designed to have adequate shear strength and ductility to accommodate large inelastic reversible rotations under severe earthquakes. The shear strength of joints depends primarily on the grade of concrete and is not sensitive to the magnitude of shear reinforcement[122]. Hence the ACI-ASCE Committee140 recommends the use of high strength concrete at joints of ductile frames resulting in good compaction and higher strength.

b) When the beams and columns meet at a joint, the special confining reinforcement (hoops) should be provided near the column ends as shown in Fig. 17.10 (a). In the case of externally confined joint where the beams frame into all the vertical faces of the joint, having a beam width at least three-fourths of the column width, the spacing of hoops to be provided in the joint region may be taken as twice that required at the end of the column but the spacing is limited to 150 mm according to the codal specifications.

c) In the case of beams joining the columns from one side only, development length requirement of the flexural reinforcement within the joint as shown in Fig. 17.8(b) should be provided. The junction zone of beams and columns is an area of high concentration of beam, column and hoop bars. Hence extreme care should be taken in detailing of reinforcements at joints to provide for proper stress transfer and to avoid congestion and facilitate placing of concrete with proper compaction.

d) In the case of precast construction subject to seismic loading, the most critical location is the beam-column connection. Investigations by Pillai and Kirk[141] has shown that by careful detailing it is possible to produce ductile beam-column connections having adequate strength, stiffness and ductility and energy-dissipating capacity.

17.5.4 Detailing of shear wall structures

Shear wall or more appropriately termed flexural wall structures are vertical cantilevers from the foundations and form the lateral load resisting system. Shear walls are subjected to axial load, bending moment and shear forces. A typical shear wall deforming under lateral loads is shown in Fig. 17.11. Unlike beams, shear wall is relatively thin and deep and is subjected to large axial forces. The wall is generally designed as an axially loaded beam, capable of forming reversible plastic hinges with sufficient rotation capacity at the base.

The I.S. Code also provides guide lines regarding the design of boundary elements, coupled shear walls with openings and the reader may refer to the specified various references[142-146] for further information on the design and detailing of structures subjected to seismic forces.

17.6 Isolation Concepts in Earthquake Resistant Design

Recent advances in Earthquake resistant design and the results of observation of structures subjected to recent earthquakes have brought about the examination of earthquake-adaptive systems and the development of

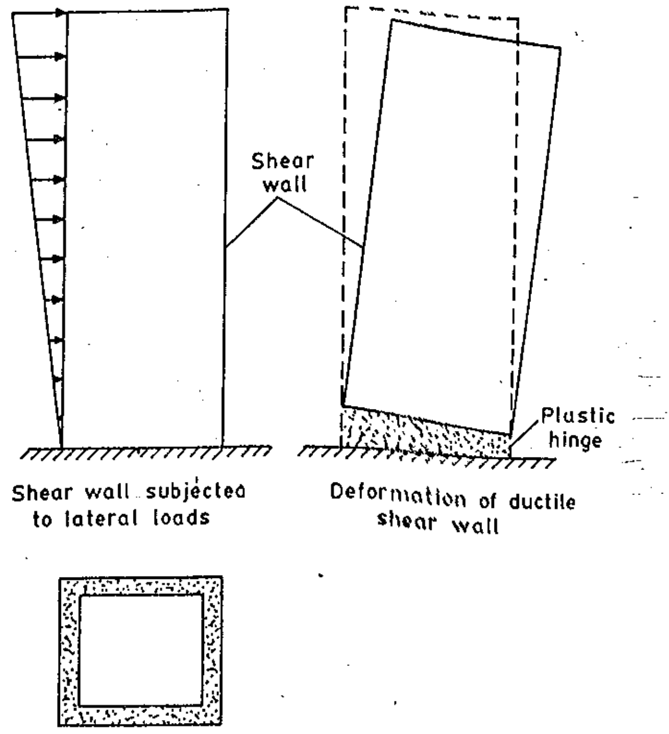


Fig. 17.11 Shear Wall Structure

shock-isolation concepts as applied to earthquake resistant structures. This concept is a radical departure from current seismic design practice. Its successful implementation on a large scale promises significant simplifications in the design of tall reinforced concrete structures in earthquake zones.

In a significant deviation from the present philosophy of designing an entire structure to withstand the distortions resulting from earthquake motions, an adaptive system is designed to isolate the upper portions of a structure from destructive vibrations by confining the severe distortions to a specially designed portion at its base.

A number of isolation devices or mechanisms have been proposed including a soft storey with hinging columns[147], a combination ball bearing and rod system[148], steel balls on ellipsoidal cavities[149]. An exhaustive analysis of aseismic base isolation is presented by Kelly[150] and the reader may refer to this reference for upto date information on the subject.

Basically an isolation system for a multistorey structure comprises a resisting element which exhibits linear elastic behavior under the maximum wind loading, but yields under earthquake forces slightly greater than that corresponding to the maximum wind loading. By allowing the isolation mechanism at the base of the structure as shown in Fig. 17.12 to yield at a predetermined lateral load, the structure above it is effectively isolated or shielded from the forces that would otherwise cause inelastic deformations. In this way, the isolating mechanism sets an upper limit to the forces that can be transmitted to the structure from the foundation. The structure supported on an isolating mechanism need only to be designed for vertical and wind loads, with special attention for earthquake resistance focused only on the isolating mechanism at its base. This concept clearly offers economic and technical advantages when compared to the traditional method of analyzing a complex structure under earth quake motion and providing ductile members and connections throughout the entire structure.

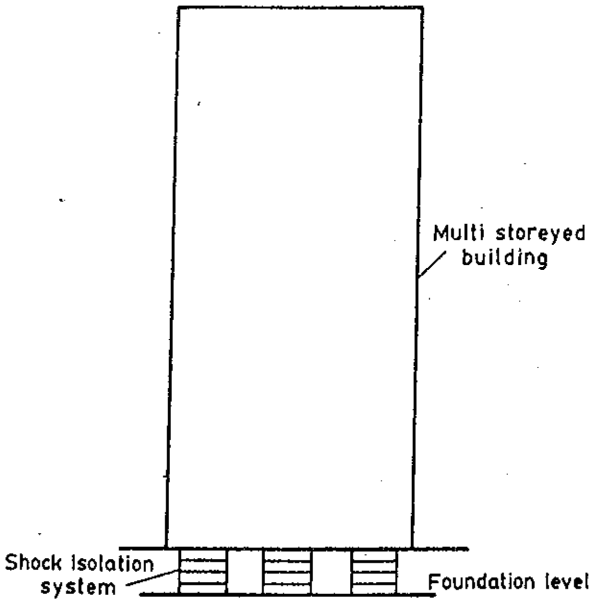


Fig. 17.12 Shock Isolation Mechanism

Field observations of buildings in earth-quake damaged areas has conclusively shown that an effective isolation system not only allows the structure above to remain elastic during a strong earthquake but spares the non structural elements from extensive distress. The non structural elements comprising partitions, glazing, mechanical equipment etc in a typical multistorey structure account for about 80% of the building's cost and hence a positive means of preventing distress in such elements during

strong earthquakes would result in significant savings in the repair and replacement costs. Developed countries like U.S.A and Japan have developed and used isolation systems in multistoreyed buildings located in earth quake zones.

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