

**INDIAN INSTITUTION OF BRIDGE ENGINEERS
TAMILNADU CENTRE**

SKEW BRIDGES

**A SHORT MODULAR COURSE PROGRAMME ON
SPECIAL TOPICS ON BRIDGES**

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1. SKEW BRIDGES

1.1 General

Bridge deck slabs by its nature have their supports only at two edges and the remaining 2 edges are free. They carry traffic on top and cross an obstruction. The supports for such slabs are sometimes not orthogonal for the traffic direction necessitated by many reasons. Such bridge decks are defined as skew bridge decks.

Skew bridges are sparingly chosen, mostly due to site necessity arising out of alignment constraints, land acquisition problems, etc., They are not generally preferred. From analytical point of view, knowledge on design and behaviour is limited and from practical point of view, detailing is quite involved and visibility is restricted. Several practices exist in reducing the skew effects, as there are many apprehensions about the correct prediction of the behaviour and proper designs of the skew bridges especially if the skew angle is very high. In some cases skew effects are avoided by proper choice of orientation of supports. Foundations and substructures could be oriented in the direction of flow of river or rail track in a skew crossing. But trestle cap could be provided in such a way so that deck system forms a right deck (not a skew deck). This could also be achieved in a simple way by choosing single circular column pier as shown in Fig. 1.1. In steel girder bridges of Railways, run off girders are adopted at ends to keep / place the sleepers square to steel girder. Auxiliary bearings may be provided for run off girder.

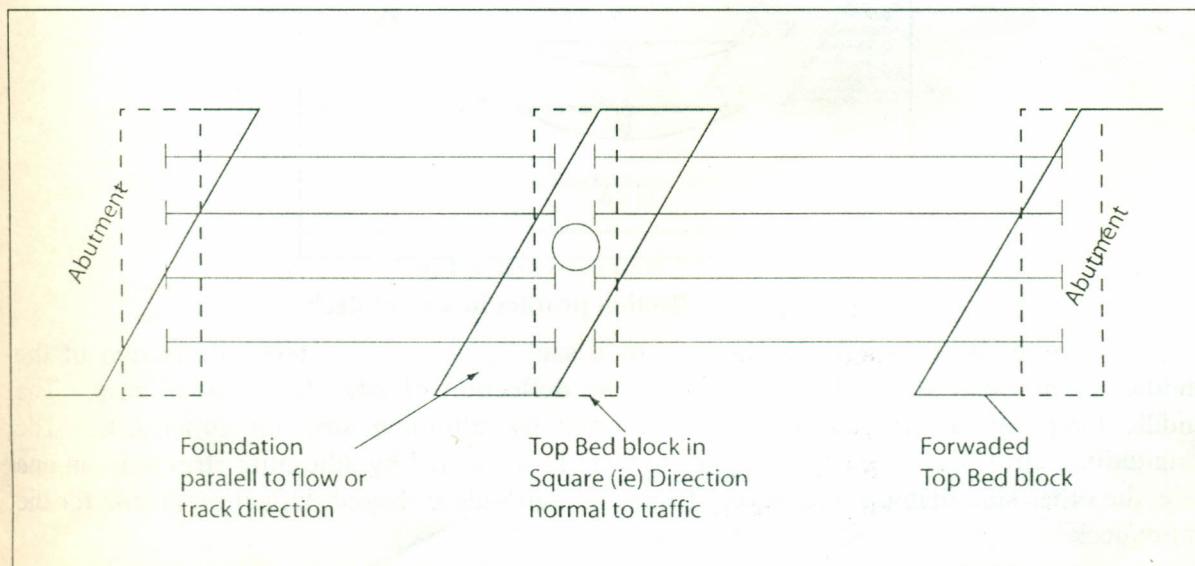


Fig. 1.1. Skew spans converted as right span girders by placement of bed block on top of pier.

In rail road skew crossing, the crossing portion of the deck is planned as a skew deck where as either side of the obligatory span (span crossing the rail track) the deck is planned, designed and executed as square deck. Ofcourse there would be one tapered deck on either side of the skew deck to bring in longitudinal plan compatibility (Fig. 1.2.).

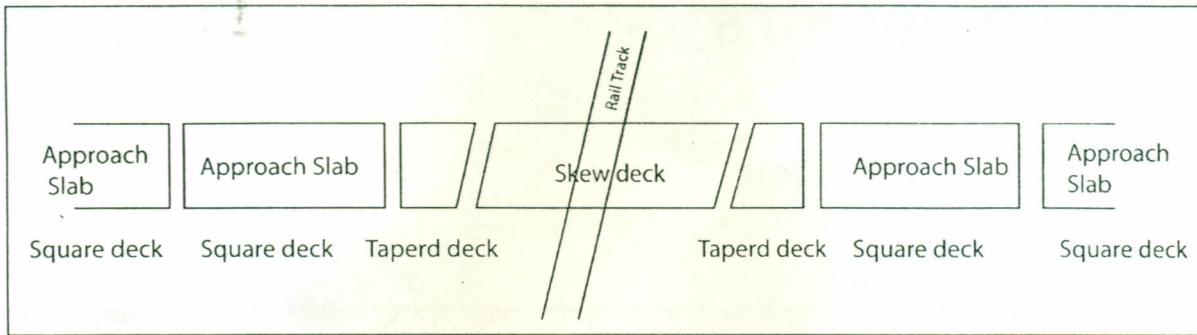


Fig. 1.2. Roadways where bridge alone is skew

1.2 Behaviour of Skew Bridge Decks

Normally a rectangular slab bridge deck behaves in flexure orthogonally in the longitudinal and transverse direction. The principal moments are also in the traffic direction and in the direction normal to the traffic. The direction and the principal moment can well be recognised by the deformation pattern as shown in Fig. 1.3, which is a reality. But stresses caused by the deformation are only conjuncture based on the relationship between deformation and stresses.

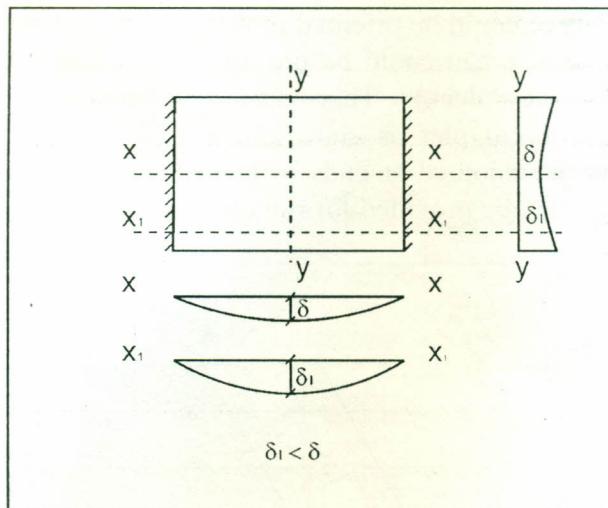


Fig. 1.3. Deflection profiles in a right deck

The slab bends longitudinally leading to a sagging moment. Hence deflection of the middle longitudinal strip will be less than the deflection of edge longitudinal strip. The middle longitudinal strip along xx is supported by adjoining strip on either side. The longitudinal strip near free edges say along x₁x₁ is supported by adjoining strip only on one side, the other side being a free edge. This results in saddle shaped deflection profile for the entire deck.

Here the load from every bit of slab is transferred to reaction line directly through flexure. There will be a small amount of twisting moment because of the bi-directional curvature and it will be negligible. Hence the rectangular slabs are designed only for transverse and longitudinal moments and reinforced accordingly. The principal moments also are in the longitudinal and the transverse directions.

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For **skew slabs** the force flow is through the **strip** of area connecting the obtuse angled corners and the slab primarily bends along the line joining the obtuse angled corners. The width of the primary bending strip is a function of skew angle and the ratio between the skew span and the width of the deck (aspect ratio). The areas on either side of the **strip** do not transfer the load to the supports directly but transfer the load only to the **strip** as cantilever. Hence the skew slab is subjected to twisting moments. This twisting moment is not small and hence cannot be neglected. Because of this, the principal moment direction also varies and it is the function of a skew angle.

The transfer of the load from the strip to support line is over a defined length along the support line from the obtuse angled corners. Then the force gets redistributed for full length. The force flow is shown in Fig. 1.4 (a & b). The thin lines in Fig. 1.4 (a) indicate deformation shape. The distribution of reaction forces along the length of the supports is also shown on both the support sides.

The deflection of the slab also is not uniform and symmetrical as it is in a right deck. There will be warping leading to higher deflection near obtuse angled corner areas and less deflection near acute angled corner areas. Fig. 1.3 & 1.5(a) show the deformation pattern of a right slab deck and also the skew slab deck.

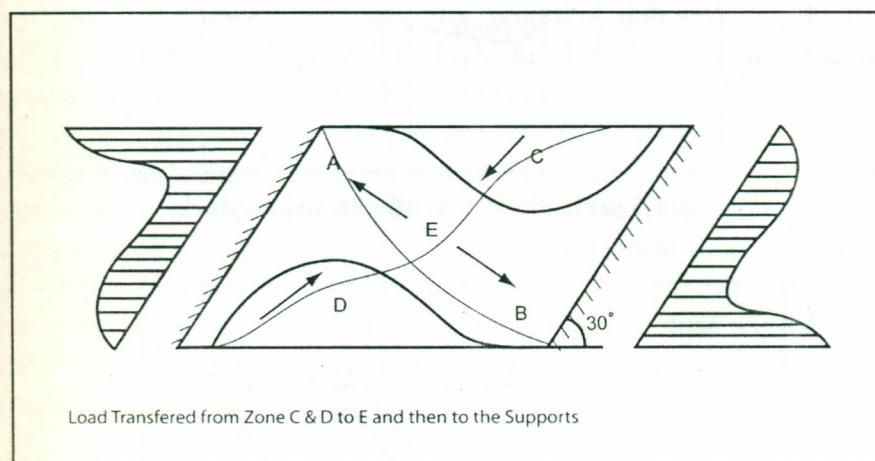
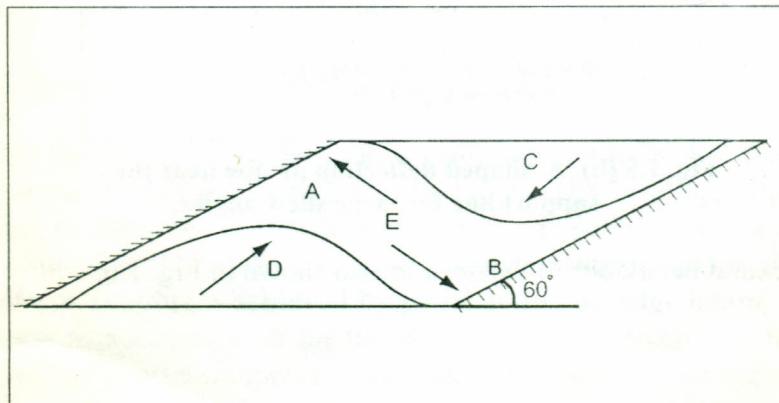


Fig. 1.4 (a) Force flow in a skew deck



Greater the skew, narrower the load transfer strip

Fig. 1.4 (b) Force flow in a skew deck

For small skew angles, both the free edges will have downward deflection but differing in magnitude. The free edge deflection profile is becoming unsymmetrical, the maximum deflection moves towards obtuse angled corners. As skew angle increases, the maximum deflection is maximum very near the obtuse angled corners and near acute angled corner there could be negative deflection resulting in S shaped deflection curve with associated twist (Fig. 1.5b).

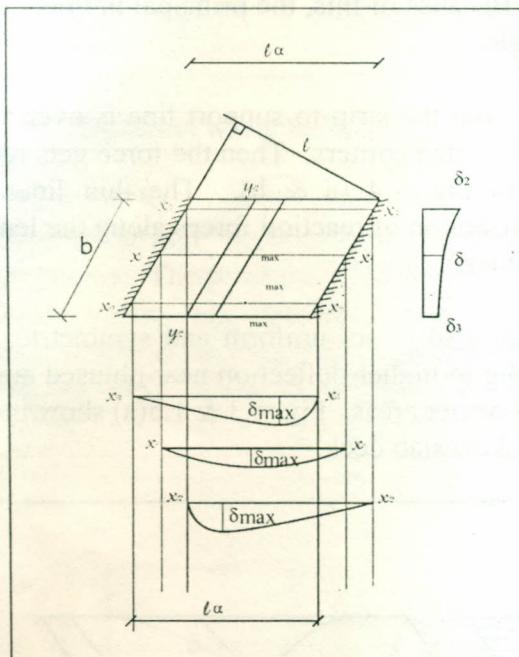


Fig. 1.5 (a) Deflection profile in a skew deck

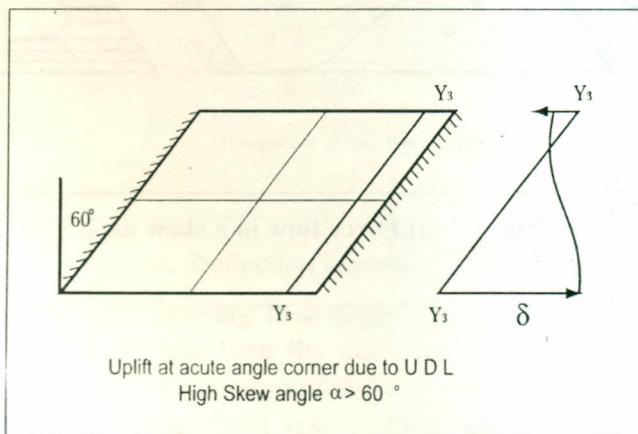


Fig. 1.5 (b) 'S' shaped deflection profile near the support line for large skew angles.

The general flexural behaviour of the deck is also shown in Fig. 1.6.

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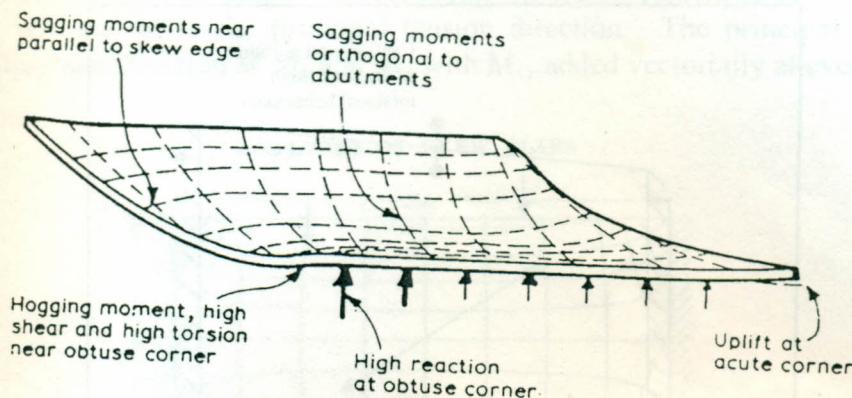


Fig. 1.6. General flexural behaviour of a skew deck [8]

Increase in skew angle results in decrease in bending moments but increase in twisting moments, the total strain energy remaining the same.

1.3 Characteristics of Skew Deck

The characteristic differences in behaviour of skew deck with respect to right deck are:

- High reaction at obtuse corners.
- Possible uplift at acute corners, specially in case of slab with very high skew angles.
- Negative moment along support line, high shear and high torsion near obtuse corners.
- Sagging moments orthogonal to abutments in central region.
- At free edges, maximum moment nearer to obtuse corners rather than at center.
- The points of maximum deflection nearer obtuse angle corners. (This shift of point of maximum deflection towards obtuse corners is more if the skew angle is more).
- Maximum longitudinal moment and also the deflection reduce with increase of skew angle for a given aspect ratio of the skew slab.
- As skew increases, more reaction is thrown towards obtuse angled corners and less on the acute angled corner. Hence the distribution of reaction forces is non-uniform over the support line.

It is generally believed that for skew angle upto 15° , effect of skew on principal moment values and its direction is very small. The analysis considering the slab as if it is a right deck with skew span as one side and right width as another side is adequate for design purposes. When skew angle increases beyond 15° , more accurate analysis is required since change in the behaviour of slab is considerable. It may be understood that behaviour is not only dependent on skew angle but also on aspect ratio, namely skew span to right width ratio.

If the width of the slab is large, the cantilevering portion from the primary bending strip connecting the obtuse angled corner will also be large. The bending strip also will be very nearly orthogonal to supports. To reduce the twisting moment on the load-bearing strip connecting the obtuse angled corners, an elastic support can be given along the free ends for the slab and this support is achieved by provision of an edge beam. If stiff edge beam is provided, it acts as a line support for the slab, which effectively extends right up to the abutment. It provides an elastic support in the transverse direction for the slab preventing the cantilever action at the triangular portion in acute angle corner zones for the full width as shown in Fig. 1.7. Behaviour of wide skew deck with and without edge beams is depicted in Fig. 1.7a & 1.7b. Provision of stiff edge beam at free edges is preferable.

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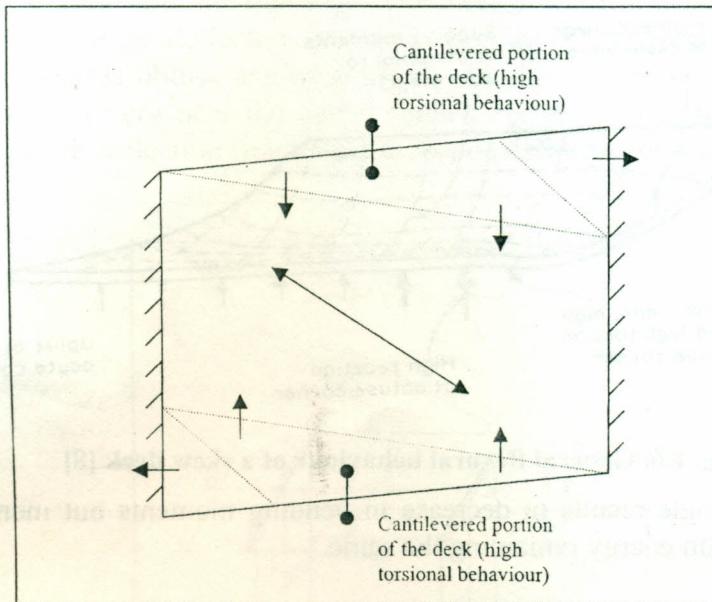


Fig. 1.7 (a) Wide skew deck without edge beam

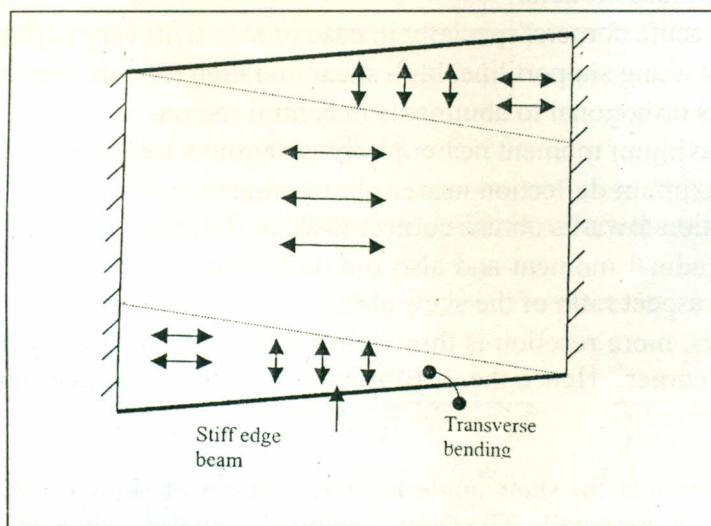


Fig. 1.7 (b) Wide skew deck with edge beam

The principal stress trajectories are shown in Fig. 1.8.

Zone A, E, B in Fig. 1.4 is subjected to high degree of flexure and relatively low degree of torsion. Zones C and D are subjected to high torsion and less of flexure.

It is already explained that central portion of slab predominately has sagging BM with **bending direction nearly along the line joining obtuse angle corners**, whereas end triangular regions do have different bending directions with associated twist.

If triangular portions are small compared to central rectangular region, skew effects are limited to ends only as shown in Fig. 1.9 (b).

Normally the designers and field engineers prefer the reinforcement to be only in a regular pattern and it will not be in the principal tension direction. The principal moments are resolved in orthogonal direction as M_x and M_y with M_{xy} added vectorially at every section.

ANALYSES OF SKEW SLABS

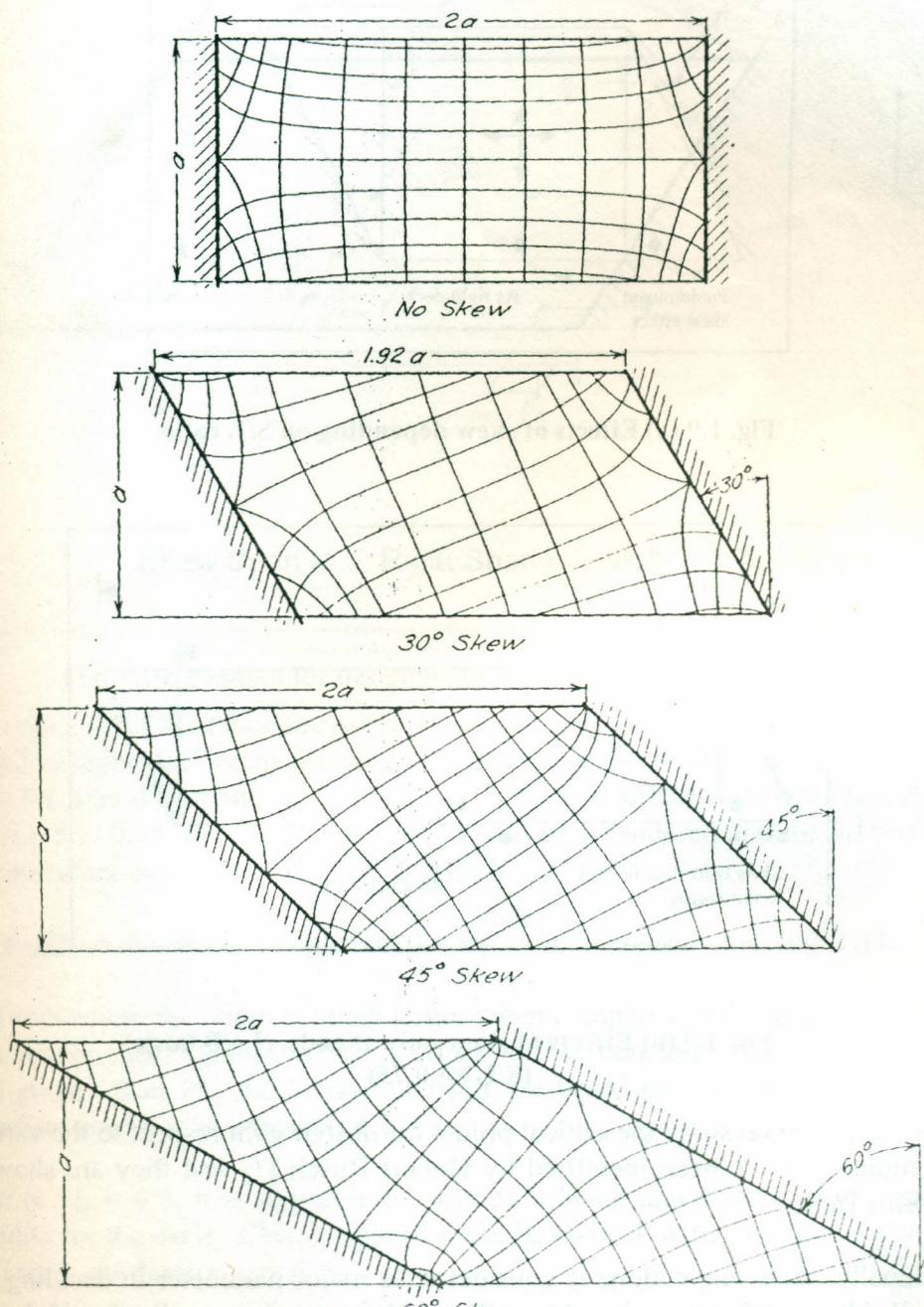


Fig. 1.8. Principal stress trajectories for various skew angles

Skew bridges can be defined as nonconcurrent deck girder bridges that have an angle other than 90° between the longitudinal axis of the bridge and the transverse axis of the bridge. In this chapter, we will discuss the effects of skew on the behaviour of the bridge.

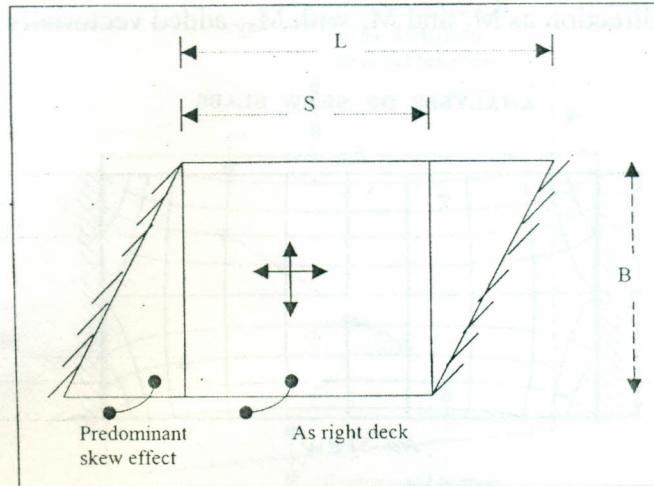


Fig. 1.9 (a) Effects of skew depending on S/L ratio.

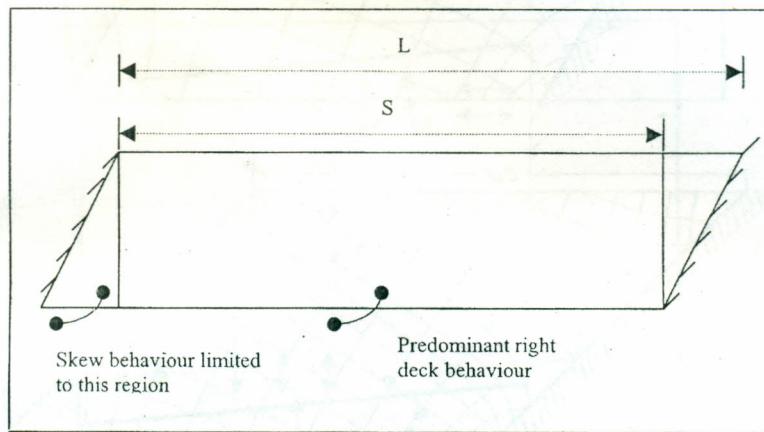


Fig. 1.9 (b) Effects of skew only at ends. (L/B large)
($S/L \gg 0.75$)

Based on experimental study the critical points for design with respect to the various types of bending moments have been identified by Hubert Rusch [1] and they are shown in Fig. 1.10 along with Table 1.1.

Conventionally, skew angle alone is considered as major parameter in deciding about the behaviour of bridge and for assessing skew effects. It is stated generally that if skew angle $< 15^\circ$, skew effect is negligible. This is acceptable only if the width of the bridge is relatively small (far less than skew span), which was the case in the yester years. But as a recent development, it is understood that skew angles as well as the aspect ratio namely skew span to right width ratio are governing parameters for predicting the skew effects. Combining the two parameters namely skew angle and aspect ratio, skew behaviour can be assessed with a single parameter namely S/L as described below.

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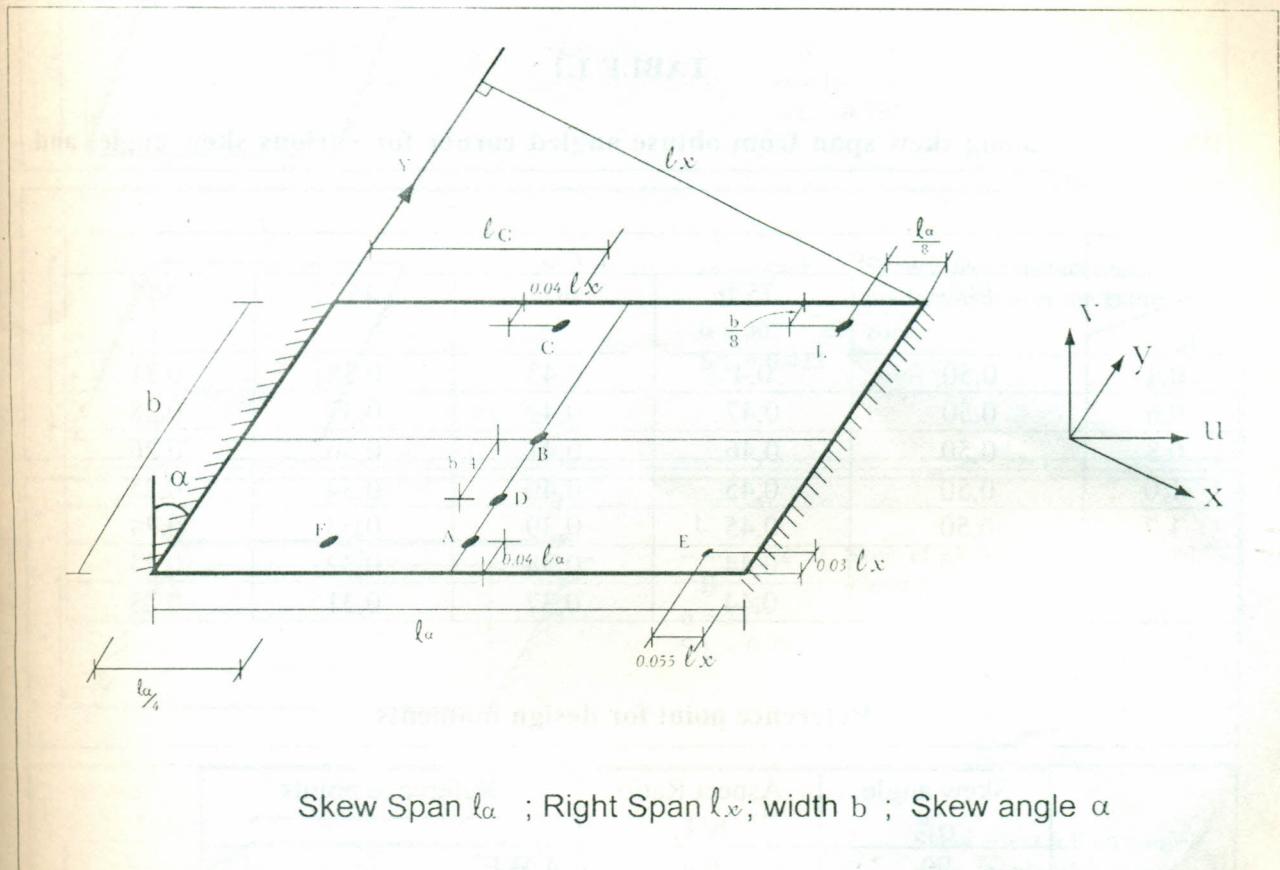


Fig. 1.10. Position for design bending moments in a skew deck

If L is the skew span (distance measured along the free edge) and B is the right width, S can be defined as right deck span [projected length of the slab between the obtuse angled corners along the span direction – ref. Fig. 1.9 (a & b)] skew effect is insignificant if $S/L \geq 0.75$. If S/L is 0.5 to 0.75, skew effect is necessary to be considered and for $S/L < 0.5$, skew behaviour is dominant over entire region of the deck.

Different combination of skew angle and the aspect ratio are shown in Fig. 1.11.

In case of slab where the width is much larger when compared to the span, the effect of skew becomes less predominant. Hence the skew effect need be considered upto a value of $L_e/B = 0.5$. If B is greater than $2L$, the behaviour itself are based on unidirectional bending and hence skew effect need not be considered.

In case of $S / L < 0.5$, it is desirable to provide a beam and slab deck to provide high torsional rigidity for the deck. Closely spaced multi beam deck with cast-in-situ slab can also be adopted if there are depth restrictions.

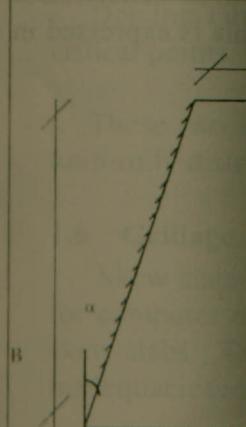
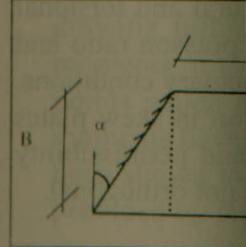
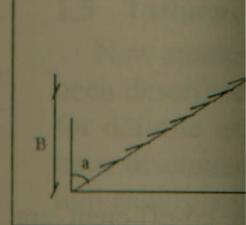
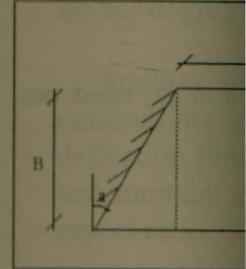
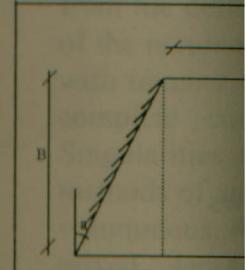
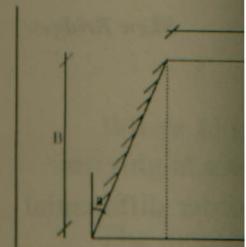
TABLE 1.1

Distance of l_c along skew span from obtuse angled corner for various skew angles and aspect ratios.

$\frac{b}{l_\alpha}$	$\frac{l_c}{l_\alpha}$				
	90°	75°	60°	45°	30°
0,4	0,50	0,4	0,43	0,38	0,31
0,6	0,50	0,47	0,42	0,37	0,28
0,8	0,50	0,46	0,41	0,36	0,26
1,0	0,50	0,45	0,40	0,34	0,25
1,2	0,50	0,45	0,39	0,33	0,24
1,4	0,50	0,44	0,38	0,32	0,23
1,6	0,50	0,44	0,37	0,31	0,23

Reference point for design moments

Skew angle α°	Aspect Ratio b/l_α	Reference points
90	0,4	A,B,E
	0,6	A,B,C,E
	1,0	A,B,C,D,E
	1,6	A,B,C,D,E
60	0,4	A,B,E
	0,6	A,B,C,E
	1,0	A,B,C,D,E
	1,6	A,B,C,D,E
45	0,4	A,B,C,E
	0,6	A,B,C,E
	0,67	A,B,C,D,E,F,G,H,I,K,L
	1,0	A,B,C,D,E
30	0,4	A,B,C,E
	0,6	A,B,C,E
	1,0	A,B,C,D,E
	1,6	A,B,C,D,E



$\frac{l}{B} \leq 0,66$
 $\alpha \approx 1$

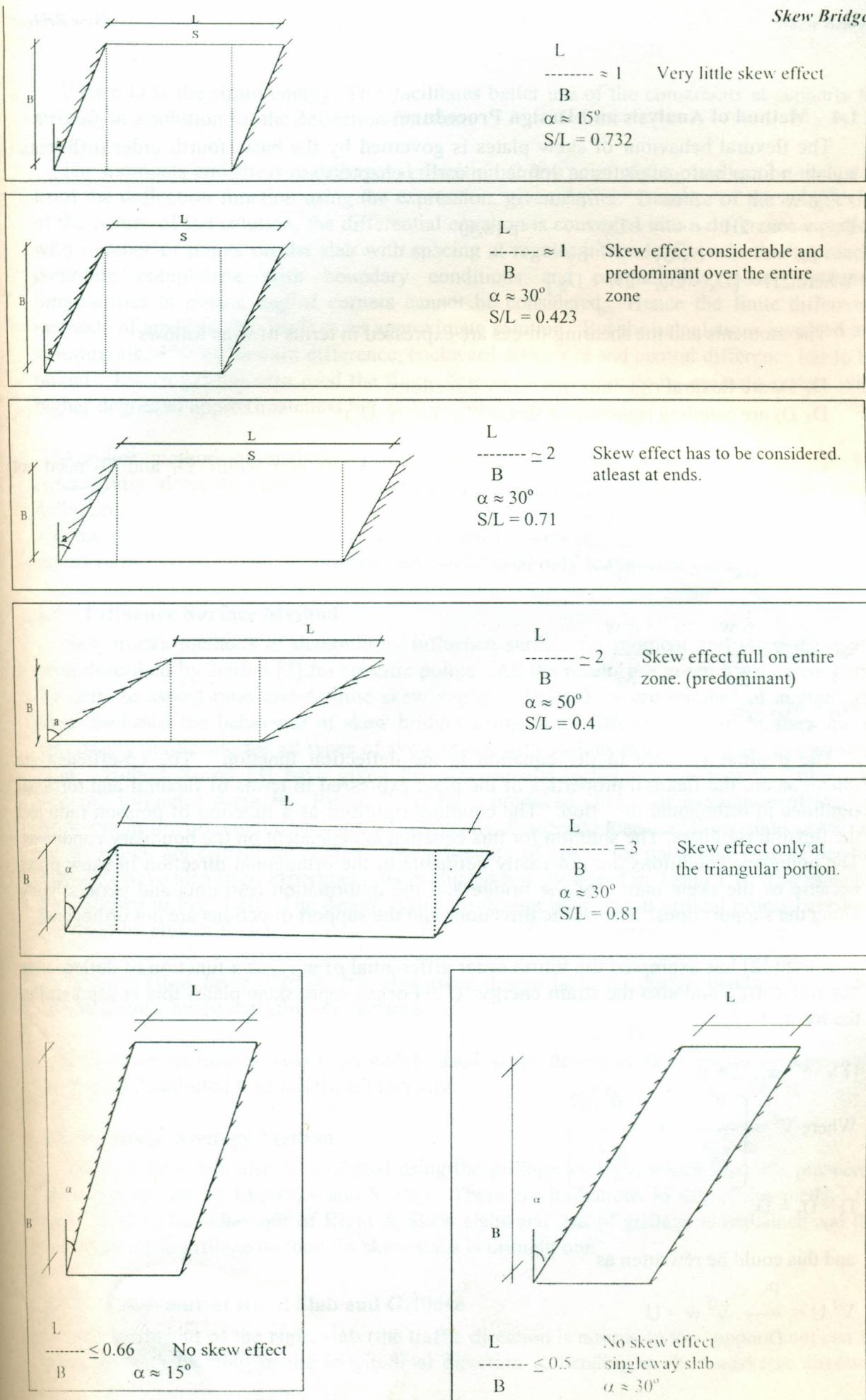


Fig. 1.11. Effect of skew angle and aspect ratio on skew slabs.

1.4 Method of Analysis and Design Procedure

The flexural behaviour of skew plates is governed by the basic fourth order differential equation for orthotropic plates as defined in earlier chapters as:

$$D_x \frac{\delta^4 w}{\delta x^4} - 2H \frac{\delta^4 w}{\delta x^2 \delta y^2} + D_y \frac{\delta^4 w}{\delta y^4} = p(x, y)$$

$$\text{Where } 2H = [D_x + D_{yx} - D_1 + D_2]$$

The moments and the shearing forces are expressed in terms of w as follows

D_x, D_y are flexural rigidities in x and y directions.

D_1, D_2 are coupling rigidities in flexure ($\eta_y D_x, \eta_x D_y$)

D_{xy}, D_{yx} are called torsional rigidities of the deck per unit width. D_1 and D_2 need not necessarily be equal, but normally considered being so.

$$M_x = -[D_x \frac{\delta^2 w}{\delta x^2} + D_1 \frac{\delta^2 w}{\delta y^2}]$$

$$M_y = -[D_y \frac{\delta^2 w}{\delta y^2} + D_2 \frac{\delta^2 w}{\delta x^2}]$$

$$M_{xy} = M_{yx}$$

The explicit variable in the equation is the deflection function. The co-efficient and functions are the flexural properties of the plate expressed in terms of flexural and torsional rigidities in orthogonal direction. The coupling rigidities as a function of poisson ratio and the flexural rigidities. The solution for this equation is dependent on the boundary conditions. The boundary conditions are not easily definable in the orthogonal direction in skew plates because of the skew nature of the bridge and the deformation restraints and permissibility along the support lines. The traffic directions and the support directions are not orthogonal.

Jensen [2] has expressed the fourth order differential of $w_{(x,y)}$ as a function of deflection at any point $w_{(x,y)}$ and also the strain energy 'U'. For isotropic skew plates this is expressed in the form of

$$D \nabla^2 \nabla^2 w_{(x,y)} = p_{(x,y)}$$

Where $\nabla^2 = \left[\frac{\delta^2}{\delta x^2} + \frac{\delta^2}{\delta y^2} \right]$

$$D = D_x = D_y$$

and this could be rewritten as

$$\nabla^2 U = \frac{p}{D}, \nabla^2 w = U$$

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Where U is the strain energy. This facilitates better use of the constraints at supports for arriving at a solution for the deflection function.

The moments and shear in orthogonal direction at any point of the plate can be evaluated from the deflection function using the expression, given earlier. Because of the complexity of the nature of the solution, the differential equation is converted into a difference equation with number of points on the slab with spacing at regular interval. Even in this approach, complete compliance with boundary conditions and compatibility pose problems. Singularities at obtuse angled corners cannot be considered. Hence the finite difference methods of analysis also leads to an approximate solution. But the calculations involved are voluminous. Use of forward difference, backward difference and central difference has to be mixed. Jensen [2] has also used the finite difference with oblique co-ordinate system with higher degree of approximateness but with less of complications.

Another method of analysis is finite strip method using individual strips with the connectivity along the edges of the strips and using trigonometric series as the loading and deflection function. Jensen's approach of finite difference method has yielded the deflection and moment at the specific points as a function of skew angle and skew span for a given aspect ratio. These values are discrete and can be used only for specific cases.

1.5 Influence Surface Method

New marks methods of determining influence surface for moment and stress have also been described by Jensen [2] for specific points. All the results are given for discrete points for definite aspect ratio and definite skew angles. While there are number of authors who have discussed the behaviour of skew bridges using finite difference methods, they are not universally applicable for all types of skew plates with various skew angles of various skew ratio. Hubert Rusch [3] have given for experimental techniques on isotropic plates for drawing influence surfaces. Rusch H [1] have produced charts for evaluation of bending moments in the principal directions for uniformly distributed loads on skew slabs at various points which are critical for design. The criticality of points for particular type of bending moments have also been experimentally evaluated and presented in Table 1.1. The same has been shown in Fig. 1.10. The design tables for design moments at critical points have been provided by Hubert Rusch.

Doc.Ing.Tibor Javor, Csc [4] has also given design table for design bending moment at critical points, based on influence surfaces.

These two literatures have been widely used in the design of skew plates for equivalent uniformly distributed load (UDL) till recently.

1.6 Grillage Analogy Method

Skew slabs could also be analyzed using the grillage analogy, which also was pioneered for computer use by Lightfoot and Sawko. There are limitations in use of this method for skew slabs. The behaviour of Right & Skew slabs and that of grillage is explained and the inadequacies in grillage method for skew slabs is brought out.

1.6.1 Behaviour of Right Slab and Grillage

The behaviour of the **right slab** (the traffic direction is normal to the support line) can be idealized with bending in the longitudinal direction & bending in the transverse direction

with interaction between the two. Bending in one direction is also being resisted as torsion in the other direction. The resistance for these structural actions is provided by the flexural stiffness in the longitudinal direction, flexural stiffness in the transverse direction, torsional stiffness for unit width strip in the longitudinal direction and in the transverse direction.

In case of a slab, the depth is relatively small (when compared to the width), whereas in beam, the depth is large when compared to the width of the beam. Hence the torsional resistance of a slab is much less than that of a beam represented by $k \frac{bt^3}{G}$.

$$k \frac{bt^3}{G}$$

Even for a pure torsion there could be a distortion in case of slab and there could be warping.

But if the strip of the slab is analyzed as a beam, the torsion is resisted by a shear flow as shown in Fig. 1.12 and the same would be understood as the sum of the torque due to

- 1) the opposed horizontal shear flows near the top and bottom faces and
- 2) the opposed vertical shear flows near the two edges.

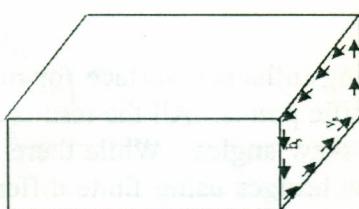


Fig.1.12 : Torsion of slab-like beam

In contrast, if the strip is wide enough and the same is analyzed as a slab, then the torque is defined as only due to the opposed horizontal shear flows near the top and bottom faces (Fig. 1.13). The vertical shear flows at the edges constitute local high values of the vertical shear force v_s since it is not over the entire width of the relatively wider slab. The opposed vertical shear flows provide half the total torque if the slab is considered as a Beam. The two definitions of torque, though different, are equivalent. The slab has half the torsional capacity (and hence half strain energy) when compared to that of a beam, attributed to longitudinal torsion only whereas the beam has full torsional capacity attributed both by longitudinal torsion caused by shear flow on top and bottom and transverse torsion caused by vertical shear flow.

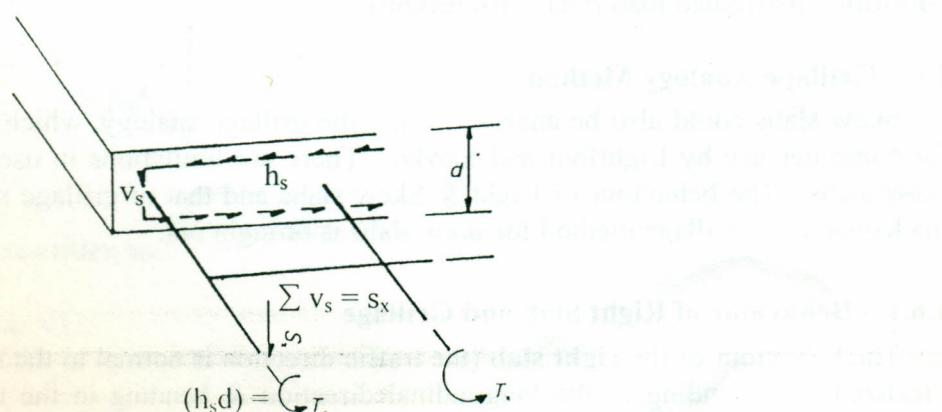


Fig.1.13 : Torsion forces at edge of grillage [8]

All the properties of the slab are defined per unit width in orthogonal direction (like D_x , D_y etc.,) as if it is a beam of unit width. Hence the slab could be idealized as grillage with beam in the longitudinal and transverse direction representing a definite width of the slab in both the directions. Ofcourse D_1 , D_2 components of the plate equation are omitted in a grillage system. The grillage analysis will provide very nearly correct design values for a design of a **right slab**, if torsional resistance is considered as equivalent to $\frac{1}{2}$ that of a beam based on earlier reasoning.

1.6.2 Behaviour of Skew Slab and Grillage

In case of **skew slabs**, the predominant bending is not in the longitudinal and transverse direction, orthogonally, but it is in the direction connecting the obtuse angled corners and a direction normal to it.

Bending of the strip between the obtuse angled corners cannot be represented by bending in direction of traffic and a direction orthogonal to it using trigonometric functions since the boundary condition will not be the same for the two representative bending (the skew bridge slab is supported only on two sides).

If the grillage is chosen in the direction of principal trajectories then grillage will be representing truly the behaviour of a skew slab. But in the grillage analysis the grillage beams are chosen

- a) Parallel to the free edges and
 - b) Parallel to supports
- or
- Orthogonal to line, joining the supports.

In both the cases they are not representing the prime bending behaviour of the skew slab. If any other line is chosen for grillage other than the prime bending direction, the torsional behaviour is not properly reflected.

In skew slabs of a skew angle less than 15° or $S/L > 0.75$, grillage beams can be provided in the traffic direction and parallel to supports (Fig. 1.14 (a)). Since the skew angle is small these grillage beams will deviate from orthogonality only marginally and the results got from grillage analysis will be applicable for the slab for design purposes. If the skew angle is large or S/L is 0.5 to 0.75 it is preferable to have the main grillage beams running from support to support and the transverse running orthogonal to it (Fig. 1.14 (b)). In this case there will be triangular portion in the grid as shown in Fig. 1.14 (b) and in this portion of the deck the properties and the cross section is represented more than what it should be and hence the results which are obtained of this in the triangular portion and the grid are not in order.

If the skew angle is large or $S/L < 0.5$ and if the grillage is formed by the beams running from support to support and parallel to the supports, the intersecting angle is far away from right angle and the properties of the beam are not truly representing the properties for the behaviour of the deck.

If the skew angle is large or $S/L < 0.5$ the grillage analysis may not yield acceptable results since the properties of the grillage beams, in whatever way they are placed, will not fully depict the representative properties of the deck. It would be specifically so when torsion effect is predominant (cases explained earlier)

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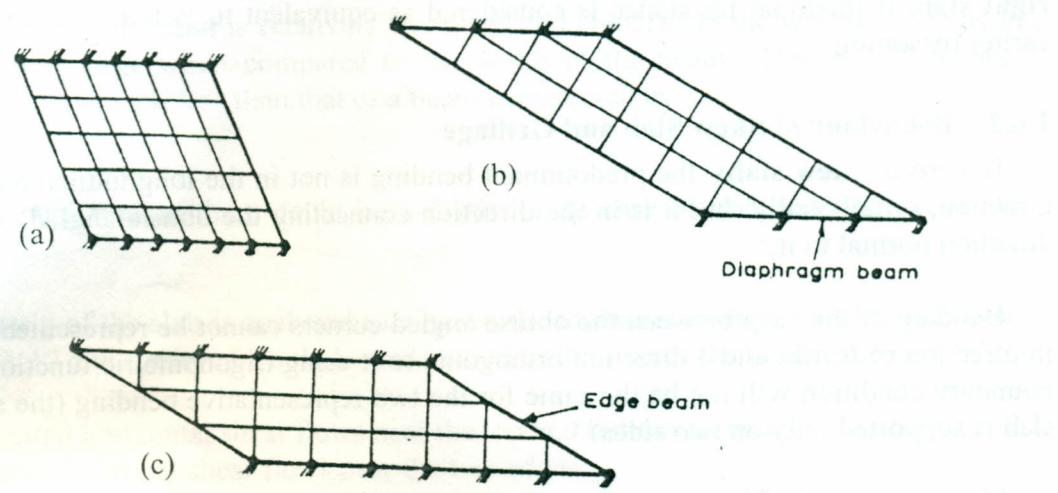


Fig.1.14 : Grillages for skew decks: (a) skew mesh (b) mesh orthogonal to span and (c) mesh orthogonal to support

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1.7 Finite Element Method

With the advent of computers and the finite element method of analysis, the facility of analyzing the various types of skew plates of various boundary conditions is enhanced. Hence in the modern days, finite element analysis has been widely used with commercially developed software like STAD PRO, SAP 2000, STRUDL, ANSYS, NASTRAN, ADINA, etc., These software have the capacity to consider non-linear constitutive relationship. Hence are more easily adaptable for concrete bridges. The facility of including reinforcement as individual elements or smeared elements is also possible.

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For skew plates it is desirable to use parallelogram elements at the centre and triangular elements at the ends near the supports. Depending on the thickness of the plate, the plate can be divided into number of layers and solutions can also be obtained using layered elements. Care has to be taken while interpreting the results coming out of the finite element analysis. The outcome from the finite element method of analysis is fully dependent on the input in the programme and hence the input has to be carefully processed before feeding into the computer. It is necessary to make an equilibrium check before the results from the computer analysis is accepted. Bearings could be considered as elastic supports at specific points.

Choice of element is equally important. A multi layered plate shell element is reasonable for analysis of flexurally controlled skew bridges. In case of thick plates where shear deformation may also control the deflection pattern in addition to the flexural deformation, a 3D element or plate element with shear deformation may be appropriate choice. But 3D elements are comparatively costly and analysis using 3D elements leads to more number of degrees of freedom, problem size becomes large and solution is time consuming. They are also stiff elements and may under estimate the deflection. Unless warranted 3D elements are not to be chosen.

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1.8 Reinforcement for Flexure

It is already explained that principal direction of bending is along the line joining the obtuse corners and hence it is more logical to provide main reinforcement in that direction and secondary reinforcement perpendicular to it as shown in Fig. 1.15.

Such a detailing introduces problems in giving adequate development length for the bars ending the free edges. The bar bending is complicated using bits of reinforcement of different lengths. Under practical consideration it is easy to provide main reinforcement parallel to free edges as well as parallel to supports as components of principal moment reinforcement in these two directions. [Fig. 1.15]

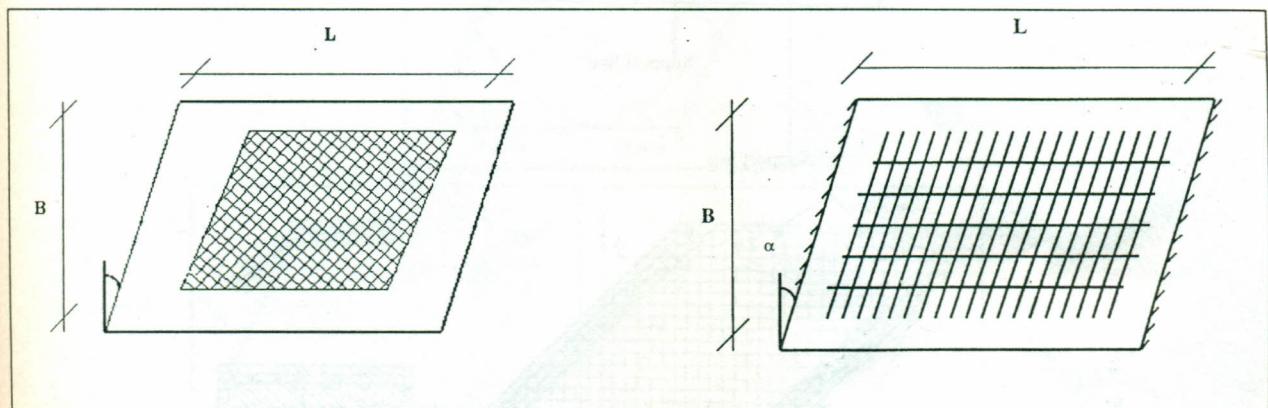


Fig. 1.15. Placement of reinforcement

Such a reinforcement detailing will result in more quantity of reinforcement compared to earlier pattern. This has the advantages of easy detailing, easy tying of reinforcement, satisfactory development length in all corner regions, reduced quantity of cut bits, easy placement of more quantity reinforcement near free edges etc., This detailing is a convenient one even though not orienting in force flow direction. This kind of detailing will be acceptable for skew angles less than 30° . For skew angles greater than 30° , if orthogonal pattern of reinforcement is to be chosen for construction eases, the reinforcement should be parallel and normal to support line (Fig. 1.16) (Ref. 5).

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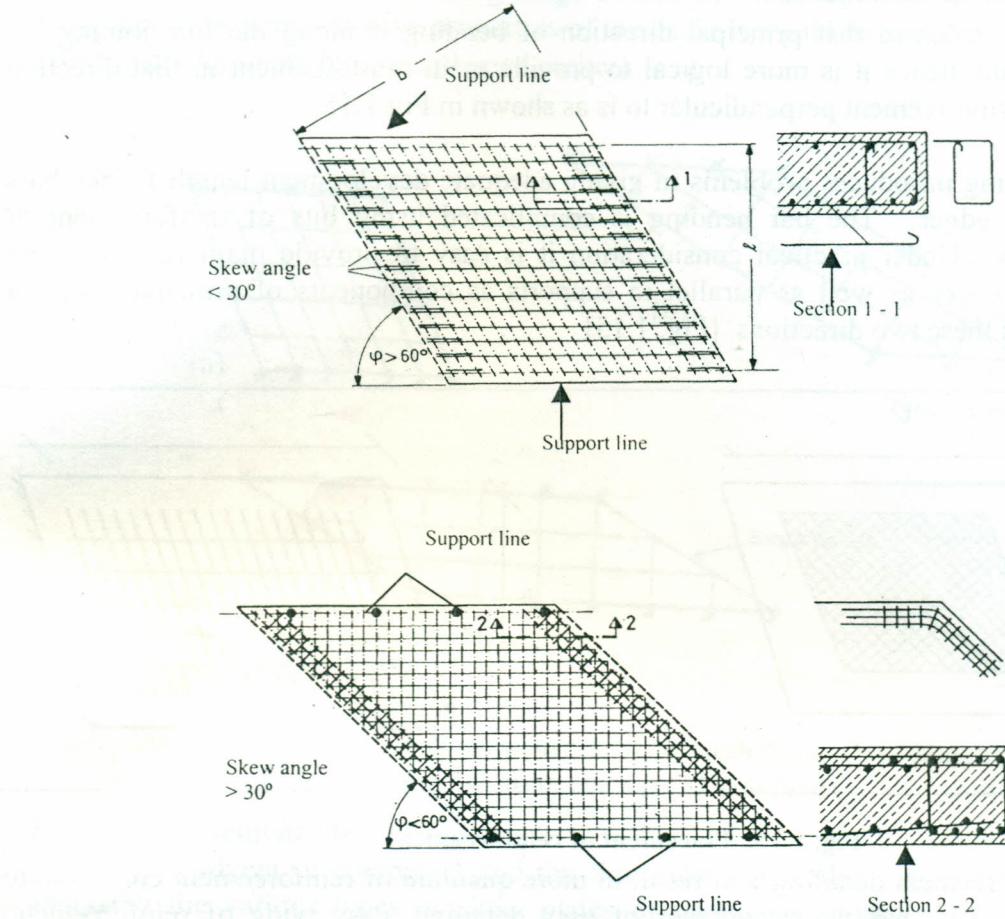


Fig. 1.16. Placement of Reinforcement with edge stiffening

It is essential that tension reinforcement zone shall be given adequate development length and hooked into compression zone.

1.9 Stiffening Edges

Torsional moment along free edges increases as skew angle increases and hence it is necessary to stiffen the edges to increase torsional moment of criteria. There are two ways of achieving this.

- i) By increasing of depth of section at free ends.
- ii) By providing additional longitudinal reinforcement at free edges (it will act as steel beam or so-called concealed beam (Fig. 1.16)). Closed stirrups shall be provided to take care of torsion.

First option is generally preferable, as it not only gives higher torsional stiffness but also changes principal moment directions from parallel to edges to perpendicular to supports. Such a choice is automatic for high skew angles.

The second choice is for generally $L/B \sim 1$ and skew angle 15° to 45° and wherever depth restrictions are there. Bandwidth in which additional longitudinal reinforcement is to be provided has to be decided. Generally $L/5$ or $L/7$ is adopted but $L/7$ is adequate.

1.10 Torsional Reinforcement

In the zones G and E near acute corners (Fig. 1.17), lifting up corners, especially when the self-weight of the slab is less, has to be catered for. To counteract this, corner zone is to be stiffened by providing top main reinforcement parallel to AC and bottom main reinforcement parallel to BD. Secondary reinforcement is to be provided in perpendicular directions.

Similarly to stiffen the obtuse corner, bottom main reinforcement parallel to BD and top main reinforcement parallel to AC are to be provided. Secondary reinforcement is to be provided in perpendicular direction.

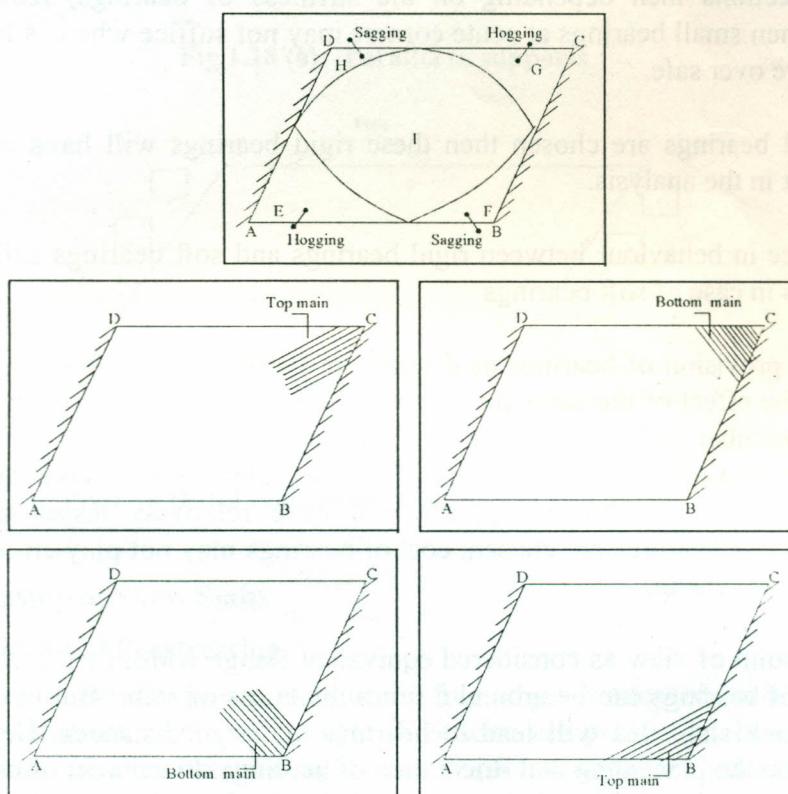


Fig.1.17: Torsional Reinforcement

1.11 Bearings

Bearings are provided to accommodate deflection and rotations. If the rotations are small, then the slab can be placed directly on to supports with chamfered corners at supports as is applicable for small spans. For larger spans, bearings are required whereas for smaller spans, edge chamfering of bed blocks will suffice.

In skew bridges, reactions over abutment length vary and more reaction is thrown towards obtuse corner and possible uplift at acute corner.

Resilient of bearings has a considerable effect on the distribution of bearing reactions. Use of highly resilient rubber bearings reduce the obtuse corner reactions at the expense of increased principal moments in the mid span region.

The detrimental effects of skew can be reduced by supporting the deck on soft bearings. A portion of the reaction on the bearing at the obtuse corner is shed to neighbouring bearings. In addition to reducing the magnitude of the maximum reaction, this also reduces

the shear stresses in the slab and it reduces the hogging support line moment at the obtuse corner. Uplift at the acute corner can also be eliminated. However, this redistribution of forces along the abutment is accompanied by an increase in sagging moment in the span.

In finite element analysis, if support deflections are idealized as zeros, (fully constrained) then obtuse reactions are far more than the acute reactions and bearing sizes are accordingly chosen.

But while providing supports at reaction points, if elastomeric bearings are provided which accommodate deflections then depending on the stiffness of bearings, redistribution of reactions occur. Then small bearings at acute corners may not suffice whereas large bearings at obtuse corners are over safe.

If stainless steel bearings are chosen then these rigid bearings will have same reaction pattern as arrived at in the analysis.

Such a difference in behaviour between rigid bearings and soft bearings calls for greater maintenance efforts in case of soft bearings.

Decision on the provision of bearings as discrete or distributed over the length has to be taken considering the effect of the same on cost and the behaviour of slab. Discrete support means only fewer number of bearings whereas distributed supports means more number of bearings simulating continuous elastic supports. Discrete supports will have larger size and thicker bearings whereas distributed supports will have relatively lesser size and thin bearings. If elastomeric bearings are chosen, cost of bearings may not play an important role since they are relatively cheap.

From shear lag point of view as considered equivalent flange width in T beam design, the maximum spacing of bearings can be around 6 times thickness of slab. But in case of larger skew angles and thick slab, this will lead to bearings at far distances. Next aspect for consideration shall be the placement and orientation of bearings. Placement of bearings needs special considerations.

Two alternatives are possible.

- i) Parallel to supports
- ii) Perpendicular to span or free edges.

Plan dimensions of bearings will be different for the above two cases as θ_x and θ_y rotations are different which will make the designs different.

It is advisable to orient the bearings in the traffic direction perpendicular to the free edge or skew span. It is a superior solution even though it is convenient to provide the bearings parallel to supports Fig. 1.18(a). But if the bearings are placed oriented in the traffic direction, they may pose problems of tearing of edges Fig. 1.18(b). It is desirable that bearings are provided on pedestal. Lifting points for replacement of bearings shall be reinforced in the form of layered mesh for taking jack reactions as concentrated forces.

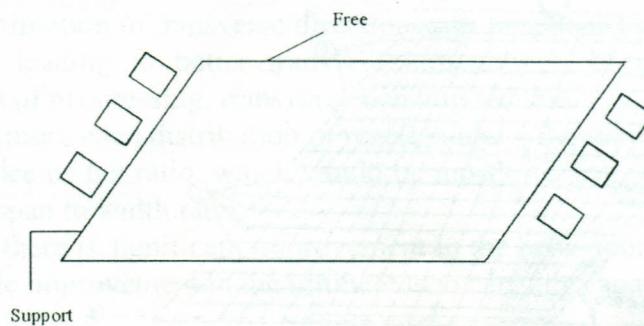


Fig.1.18 (a): Parallel to supports

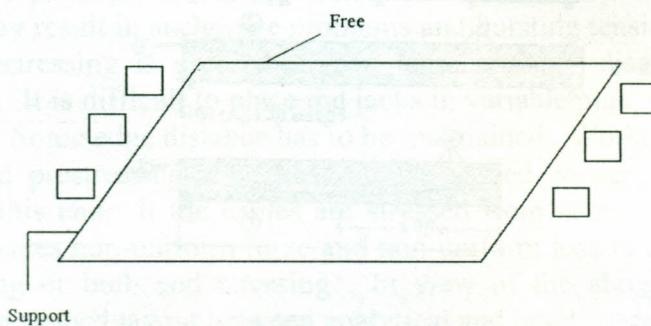


Fig.1.18 (b): Perpendicular to span or free edges

1.12 Prestressing of Skew Slabs

1.12.1 Longitudinal Prestressing

Conventionally parallel prestressing to the free edges is adopted for skew slabs as shown in Fig. 1.19. But the principal moment directions are parallel to line joining the obtuse corners. Hence parallel prestressing is not conforming to analytical requirements even though it is a convenient way from practical point of view. Since the maximum moments are concentrated around obtuse corner zone and less at acute corner zone, there is a need for differential prestressing.

Cables should be closely spaced in obtuse corner zone and widely spaced in acute corner zone which result in fan shaped spacing of cables (Fig. 1.19). This would produce counter moments similar to the moments caused by UDL on skew slabs and hence the final deformed profile of the plate will be very similar to right angled slab.

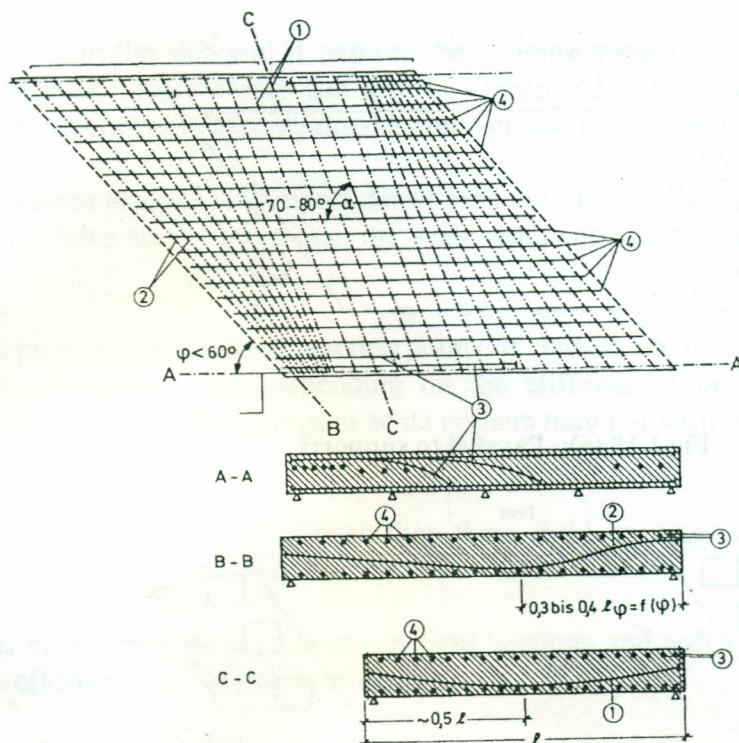


Fig.1.19: Fan type of prestressing

For convenience, support lengths can be divided into four parts, one part in the obtuse zone shall have 50% of total prestressing force and the remaining three parts balance 50% of total prestressing force. This spacing of cables conforms to analytical requirements, i.e. conforming to force flow as observed in the experimental investigations carried out at IIT-Madras (6&7). By this kind of concentration of prestressing the upward deflection caused at that place shall move the maximum deflection towards the centre of slab which is the case in a right-angled slab. Such an orientation will cause in built twist during stage prestressing – sequence of prestressing in practice and finally may vanish after completing prestressing. Sequence of prestressing to be decided to reduce in plane twist. In such cases it is preferable to have stressing from both ends or stressing alternate cable from each end.

In conclusion it could be seen that:

- i) By fanning the cables, skew slab behaviour is given as right slab behaviour.
- ii) The fact that more of moment is thrown towards the obtuse angled corner shows the greater necessity of higher amount of prestress there and hence calls for fan shaped arrangement of cables covering towards the corners.
- iii) By prestressing the skew slabs, the stiffness of the slabs is increased and hence deformations during service load are reduced.
- iv) With a fan shaped layout for prestressing tendons the points of maximum deflection due to prestressing alone along the free edge shift away from obtuse angled corners towards the centre.
- v) Providing prestressing and providing it in fan shaped layout counteracts the effects due to live load very effectively.
- vi) At service load stage, in case of prestressed concrete slabs with fan shaped tendons, the deflections are small, more uniform and a symmetrical deflection profile was observed even along the free edge.

- vii) The deformation in transverse direction with fan shaped prestressing is more or less uniform leading to better transverse distribution of moments. By increasing quantum of prestressing, transverse bending reduces.
- viii) There is more even distribution of reaction along the support line.
- ix) The choice of fan ratio, which would be most effective, depends on the skew angle and the span to width ratio.
- x) Though there is significant improvement in the behaviour at service stage, there is a very little improvement in the ultimate load carrying capacity of slab.

There are certain practical difficulties in providing fan shaped layout. Concentrating 50% of prestress in one-fourth length near obtuse creates problem of working space for providing anchorages and jacking. Stressing in different directions with steep plan inclinations in obtuse zone may result in anchorage problems and bursting tension problems. Small quantity distributed prestressing is preferable than large quantity discrete prestressing (as in fan shaped layout). It is difficult to place the jacks in variable plan and elevation angles as in fan shaped layout. Some edge distance has to be maintained. Working space will be very limited in concentrated prestressing zone near obtuse angled corner and hence cables cannot be stressed from this end. If the cables are stressed from acute zone side, then only one end prestressing creates non-uniform force and non-uniform loss in cables, compared to alternate end prestressing or both end stressing. In view of the above reasons, a compromise is required for fan shaped layout between analytical and practical requirements.

1.12.2 Transverse Prestressing

In case of parallel prestressing, cables are in traffic direction, which do not take care of transverse moments. Transverse reinforcement should be provided to take care of transverse moment. In case of wide skew slabs with aspect ratio ≤ 1 , transverse prestressing may be required. Since prestressing has to be done from the (unsupported free edges), there are certain difficulties in providing stable staging at elevated level.

If precast multi 'I' beams and box beams are adopted for deck then also transverse prestressing is required. Erection of beams is easy but transverse prestressing of beam to take care of transverse bending has to be done carefully. To avoid this, cast in situ slab over precast beams is designed for taking transverse moments.

Wherever transverse prestressing is quite involved, it is better to go in for fan shaped longitudinal prestressing which analytically proved to reduce transverse bending effects. In such a case firm supports are available for stressing and controlled grouting.

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