Steel Design

HandBook

INSTITUTE FOR STEEL DEVELOPMENT & GROWTH

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1 HISTORICAL DEVELOPMENT AND CHARACTERISTICS OF STRUCTURAL STEELS

1.0 INTRODUCTION

According to published literature, iron was primarily used for making weapons in ancient times. The great Indian epics, Ramayana and Mahabharatha, contain evidence that our forefathers knew about the usage of iron long before many other countries knew about it! Iron is thus very native to India! This is a logical conclusion because, our war-centric epics date back to several thousand years BC. The backdrop of these epics revolves around the eastern, central and southern parts of our country, where there are still huge deposits of iron ore. Not only during Vedic times but also in medieval times, our country has been an epitome of iron wonders. A review in the subsequent sections shows that in modern times too, our country has good examples of construction in steel.

Under compelling reasons, both economic and strategic, the western countries brought about the industrial revolution during the last century. Possibly because our country was under the colonial rule at that time and also due to a mood of complacency, our country failed to catch up with the western industrial revolution. During the last 50 years our country has continued to lag behind in infrastructure development and consequently poor consumption of iron and steel. Published studies by the Steel Construction Institute (U.K) have established that countries which have a higher rate of growth in Gross Domestic Product (GDP), have proportionately higher consumption of iron and steel. Soon after independence, our country had to gear itself to meet the demands for development and industrial growth and in the first few Five Year plans made reasonable strides in the area of production and usage of iron and steel.

Due to various reasons, steel consumption in our country has been stagnating during the past 2-3 years. Further, steel industry is facing stiff global competition through imports. There is also considerable under utilisation of installed capacity for steel production. For sustenance of steel industry, extensive usage of structural steel in the construction sector is an important requirement. Our country has to live up to the global competition as we have done in Information Technology!

In this chapter, we will first discuss about the historical development of iron and steel in the world and India. Since the present days are the days of inter-disciplinary approach to engineering solutions, we will first review the metallurgical aspect of structural steels and then proceed to discuss, the mechanical properties of steel, which are very relevant to structural designers. The approach of treating the metallurgical and mechanical aspects of steel together helps the designer in structural steelwork, to use steel effectively in tune with its performance requirement. Later, we will briefly review the production process of steel, which gives an idea about the different structural steels being produced. We will also review the special variety of steels (such as stainless steels and cold rolled steels).

2.0 HISTORICAL DEVELOPMENT

Ancient Hittis were the first users of iron some 3 to 4 millenniums ago. Their language was altered to Indo – European and they were native of Asia Minor. There is archaeological evidence of usage of iron dating back to 1000 BC, when Indus valley, Egyptians and probably the Greeks used iron for structures and weapons. Thus, iron industry has a long ancestry. Wrought iron had been produced from the time of middle ages, if not before, through the firing of iron ore and charcoal in "bloomery". This method was replaced by blast furnaces from 1490 onwards. With the aid of water-powered bellows, blast furnaces were used for increased output and continuous production. A century later, rolling mill was introduced for enhanced output. The traditional use of wrought iron was principally as dowels and ties to strengthen masonry structures. As early as 6th century, iron tie-bars had been incorporated in arches of Haghia Sophia in Istanbul. Renaissance domes often relied on linked bars to reinforce their bases. A new degree of sophistication was reached in the 1770 in the design of Pantheon in Paris.

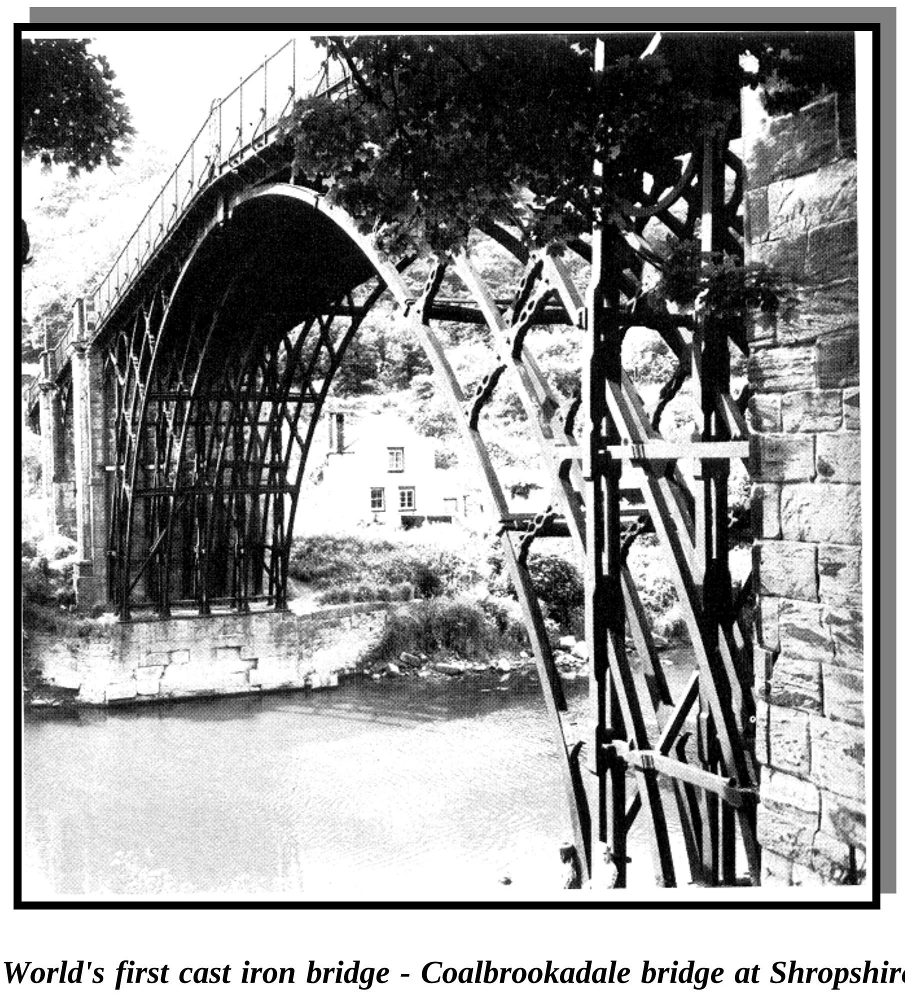


Fig. 1 World's first cast iron bridge - Coalbrookadale bridge at Shropshire, U.K (Source: John H. Stephens, The Guinness book of Structures (Bridges, towers, tunnels, dams), 1976)

Till the 18^{th} century the output of charcoal fired blast furnaces was almost fully converted to wrought iron production, with about 5% being used for casting. The most obvious cast iron items were the cannons in the early 16^{th} century. Galleries for the House of Commons in England were built of slender cast iron columns in 1706 and cast iron railings were erected around St. Paul's Cathedral in London in 1710. Abraham Darby discovered smelting of iron with coke in 1709. This led to further improvements by 1780s when workable wrought iron was developed. The iron master Henry Cort took out two patents in 1783-84, one for a coal-fired refractory furnace and the other for a method of rolling iron into standard shapes. Without the ability to roll wrought iron (into standard shapes), structural advances, which we see today, would never have taken place.

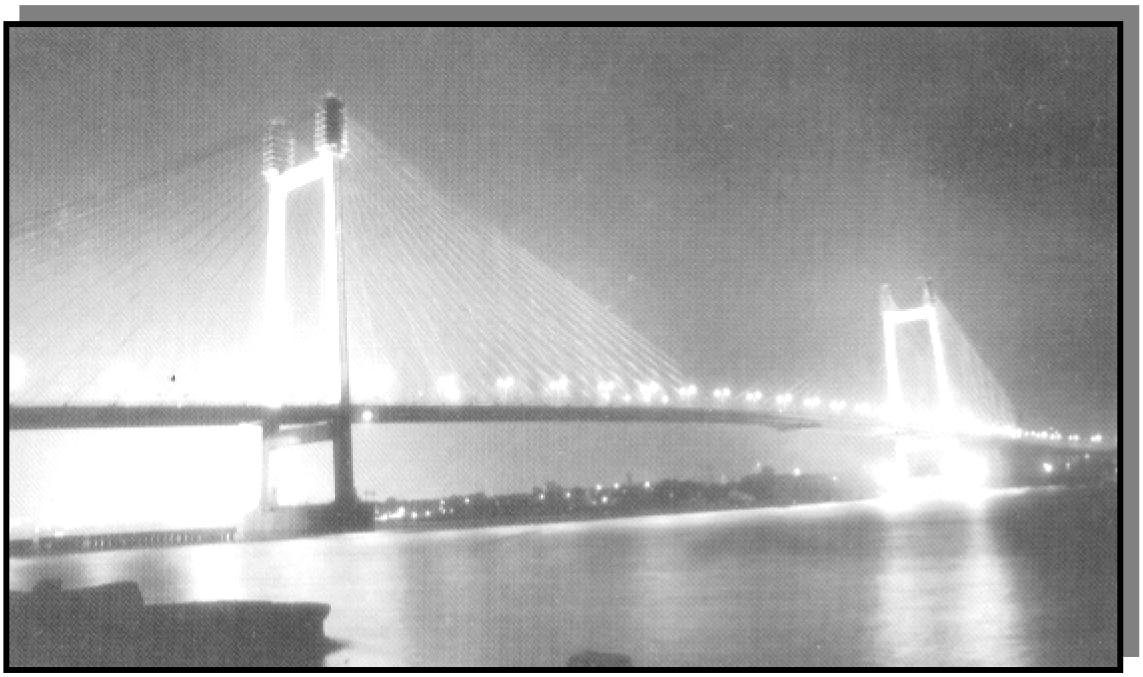


Fig.2 The second Hooghly cable stayed bridge

Technological revolution, industrial revolution and growth of mills continued in the West and this increased the use of iron in structures. Large-scale use of iron for structural purposes started in the Europe in the later part of the 18^{th} Century. The first of its kind was the 100 feet Coalbrookadale arch bridge in England (Fig.1), constructed in 1779. This was a large size cast iron bridge. The use of cast iron as a primary construction material continued up to about 1840 and then onwards, there was a preference towards wrought iron, which is more ductile and malleable. The evolution of making better steel continued with elements like manganese being added during the manufacturing process. In 1855, Sir Henry Bessemer of England invented and patented the process of making steel. It is also worth mentioning that William Kelly of USA had also developed the technique of making steel at about the same time. Until the earlier part of the 19^{th} century, the '*Bessemer process'* was very popular. Along with Bessemer process, Siemens Martin process of open-hearth technique made commercial steel popular in the 19^{th} century. In the later part of the 19^{th} century and early 20^{th} century, there had been a revolution in making better and newer grades of steel with the advent of newer technologies. This trend has continued until now and today we have very many variety of steels produced by adding appropriate quantities of alloying elements such as carbon, manganese, silicon, chromium, nickel and molybdenum etc to suit the needs of broad and diverse range of applications.

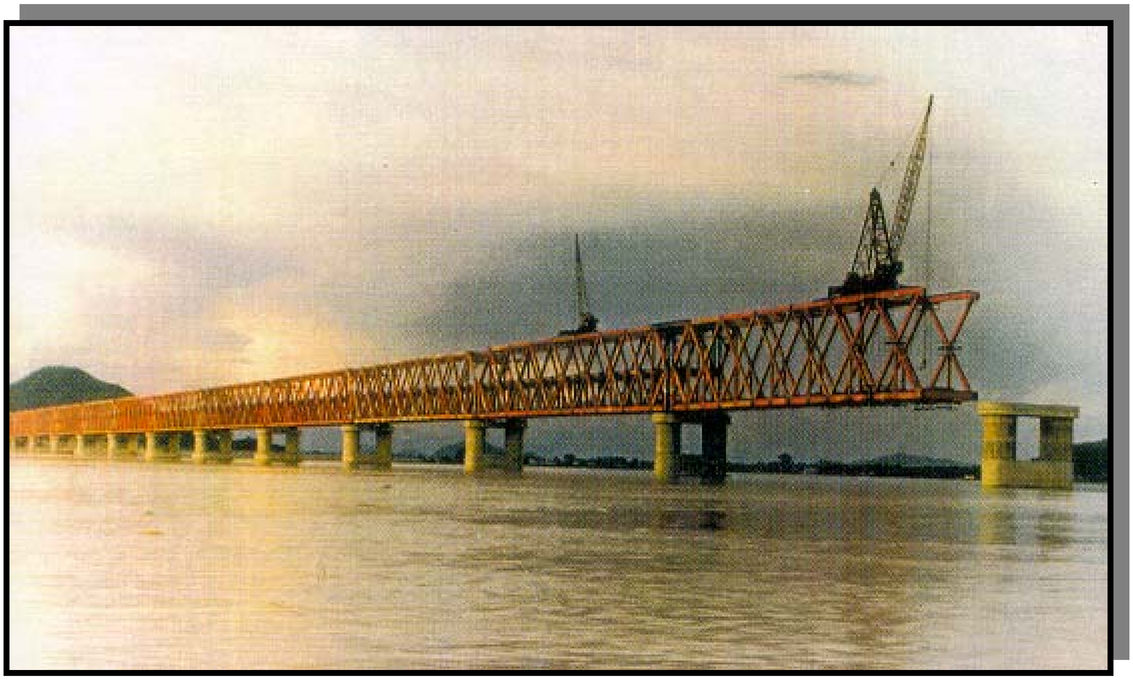


Fig. 3 Jogighopa Road-cum-rail bridge across the river Brahmaputra

2.1 Historical development of Iron and Steel in India

As mentioned earlier there are numerous examples of usage of iron in our country in the great epics Ramayana and Mahabharatha. However the archaeological evidence of usage of iron in our country, is from the Indus valley civilisation. There are evidences of iron being used as weapons and even some instruments. The iron pillar made in the 5th century (standing till today in Mehrauli Village, Delhi, within a few yards from Kutub Minar) evokes the interest and excitement of all the enlightened visitors. Scientists describe this as a "Rustless Wonder". Another example in south India is the Iron post in Kodachadri Village in Karnataka, which has 14 metres tall “Dwaja Stamba” reported to have remained without rusting for nearly 1½ millennia. The exciting aspects of these structures is not merely the obvious fact of technological advances in India at that time, but in the developments of techniques for handling, lifting, erecting and securing such obviously heavy artefacts. These two are merely examples besides several others. The usage of iron in wars during Moghul era of the history is well documented. India under the British rule experienced growth of iron and steel possibly because of the fallout of technological development of steel in the U.K. We can see several steel structures in public buildings, railway stations and bridges, which testifies the growth of steel in the colonial past. The "*Rabindra Sethu*" Howrah Bridge in Calcutta stands testimony to a marvel in steel. Even after its service life, Howrah Bridge today stands as a monument. The recent example is the Second Hooghly cable stayed bridge at Calcutta (Fig. 2), which involves 13,200 tonnes of steel. Similarly the Jogighopa rail-cum-road bridge across the river Brahmaputra (Fig. 3) is an example of steel intensive construction, which used 20,000 tonnes of steel. There are numerous bridges, especially for railways built, exclusively using steel.

As far as production of steel in India is concerned, as early as in 1907, Jamsetji Nusserwanji Tata set up the first steel manufacturing plant at Jamshedpur. Later Pandit Jawaharlal Nehru realised the potential for the usage of steel in India and authorised the setting up of major steel plants at Bhilai, Rourkela and Durgapur in the first two five year plans. In Karnataka Sir Mokshakundam Visweswarayya established the Bhadravati Steel Plant. Today we also have a number of private sector steel plants in India. The annual production of steel in 1999-2000 has touched about 25 million tonnes and this is slated to grow at a faster rate. However, when compared to countries like USA, UK, Japan, China and South Korea the per capita consumption of steel in India is extremely low at 27.5 kg/person/year. By way of comparison, rapidly growing economies like China consume about 80 kg/person/year.

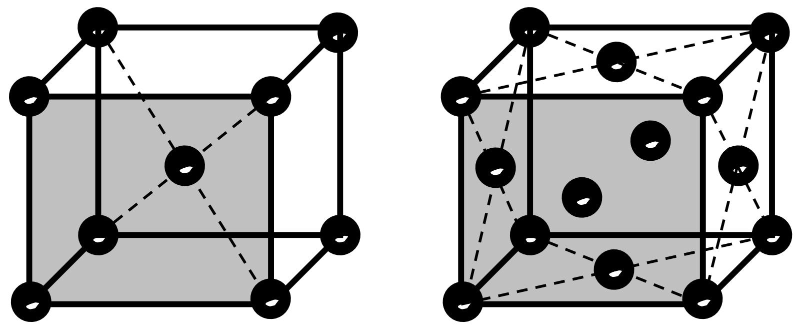
3.0 METALLURGY OF STEEL

There is a definite need for engineers involved in structural steelwork to acquaint themselves with some metallurgical aspects of steel. This will help the structural engineer to understand ductile behaviour of steel under load, welding during fabrication and erection and other important aspects of steel technology such as corrosion and fire protection. To this end, in the following sections, we shall discuss briefly the metallurgical composition of steel, its effect on heating and cooling and the effects of alloying elements such as carbon, manganese and other additive metals.

3.1 The crystal structure and the transformation of iron

Pure iron when heated from room temperature to its melting point, undergoes several crystalline transformations and exhibits two allotropic modifications such as (i) body centred cubic crystal (bcc), (ii) face centred cubic crystal (fcc) as shown in Fig.4. When iron changes from one modification to the other, it involves the 'latent heat of transformation'. If iron is heated steadily, the rise in temperature would be interrupted when the transformation starts from one phase to the other and the temperature remains constant until the transformations are completed. The flat portion of the heating/cooling curve in Fig. 5 exemplifies this. On cooling of molten iron to room temperature, the transformations are reversed and almost at the same temperature when heated as shown in Fig. 5. Iron upto a temperature of 910°C remains as '*ferrite*' or 'α-iron' with 'bcc' crystalline structure. Iron is ferromagnetic at room temperature, its magnetism decreases with increase in temperature and vanishes at about 768°C called the *Curie point*. The iron that exists between 768°C and 910°C is called the '*β-iron*' with a '*bcc*' structure. However, in the realm of metallurgy, this classification does not have much significance.

Between 910°C and 1400°C, iron transforms itself into '*austenite*' or 'γ-*iron*' with 'face centred cubic' (fcc) structure. When temperature is further increased, austenite reverts itself back to 'bcc' structure, called the 'δ-ferrite'. Iron becomes molten beyond 1539°C. The different phases of iron are summarised in Table 1.



(a) Body centred cube (bcc) *(b) Face centred cube*

Fig.4 Crystal structure of Iron

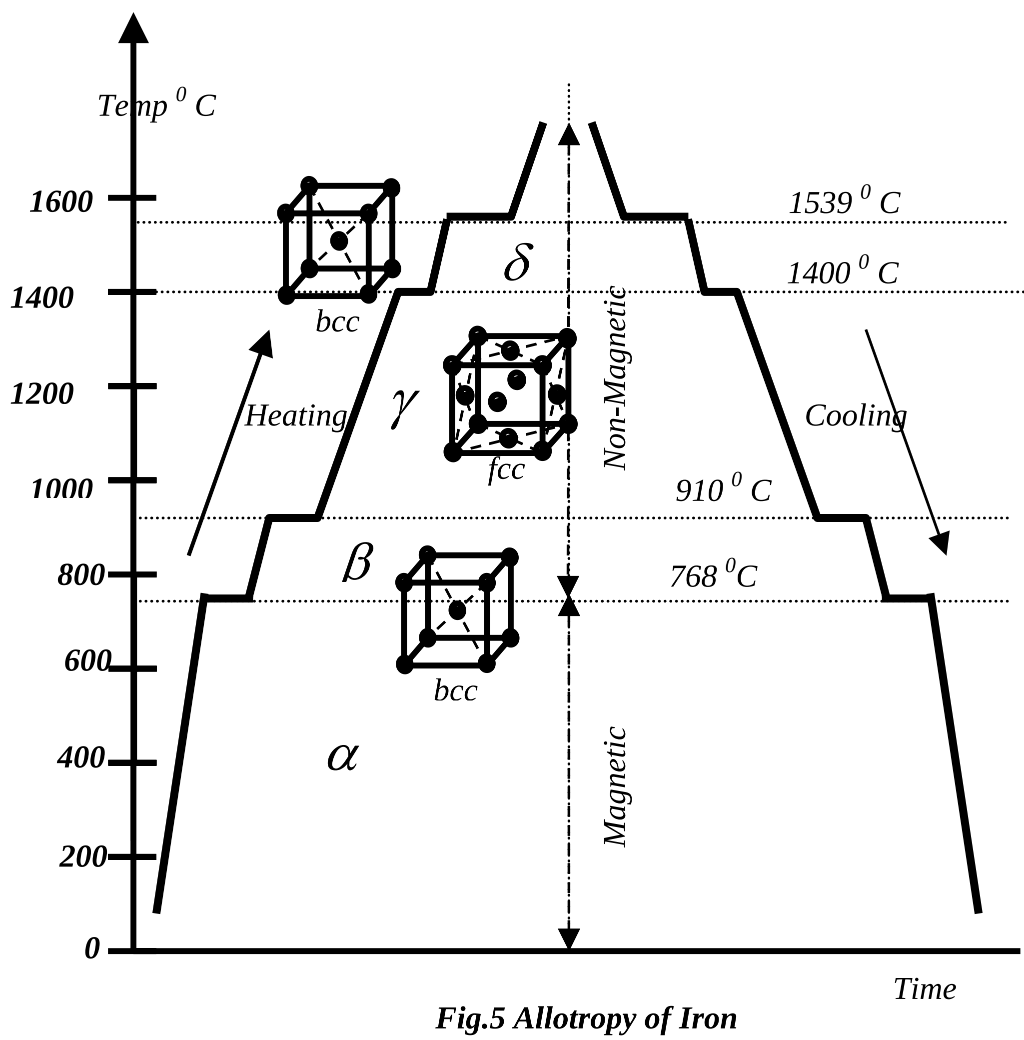


Fig.5 Allotropy of Iron

Table 1: Various forms of Iron

Stable Temp. Range ^0C	***Form of matter***	***Phase***	***Identification symbol***
>2740	Gaseous	Gas	Gas
1539-2740	Liquid	Liquid	Liquid
1400-1539	Solid	bcc	δ-ferrite
910-1400	Solid	fcc	γ-austenite
<910	Solid	bcc	α-ferrite

It is interesting to note that a given number of atoms when arranged as fcc crystals occupy slightly less volume than when arranged as bcc. Due to this reason, there would be a slight volume reduction when iron transforms itself from ferrite to austenite. As shown in Fig. 4, both bcc and fcc structures have *interstitial hole positions* (inter atomic spaces) which are at mid point of the cube for bcc and at mid point of the cube edges for fcc. In γ-iron or austenite, more volume fraction of interstitials can be accommodated than in α-iron or ferrite. Atoms of elements such as carbon, nitrogen, hydrogen and boron, whose atomic diameter is smaller, would occupy these inter atomic spaces. Such an arrangement is called an *'interstitial solid solution'* as shown in Fig. 6. In other words the solute atoms are

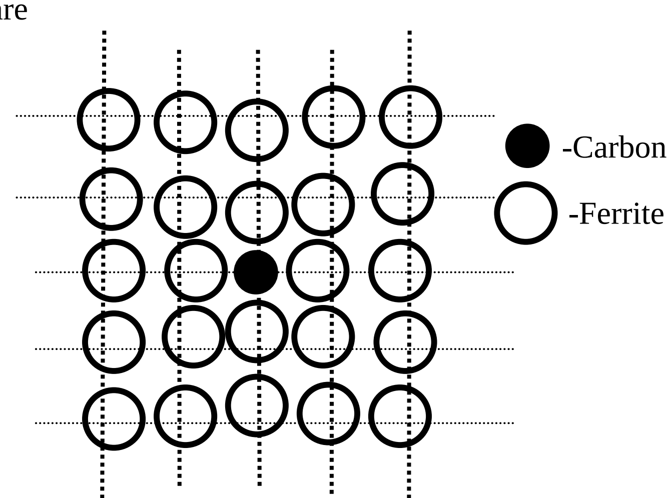


Fig.6 Interstitial solid solution of Carbon in Iron

accommodated in the interstices (inter atomic spaces) of the crystal lattice of the solvent. If we take the example of carbon, since the interstices of fcc are larger than the bcc, the solubility of carbon in austenite would be more than its solubility in ferrite.

3.2 The Iron-Carbon Constitutional Diagram

When carbon in small quantities is added to iron, 'S*teel*' is obtained. Since the influence of carbon on mechanical properties of iron is much larger than other alloying elements, we would study the fundamentals of Iron-Carbon alloy in a little elaborate way. The atomic diameter of carbon is less than the interstices between iron atoms. The carbon goes into solid solution of iron. As carbon dissolves in the interstices, it distorts the original crystal lattice of iron. The iron crystals, which were centred originally at the intersection of symmetry axes of the iron crystals, get distorted as seen from Fig. 6.

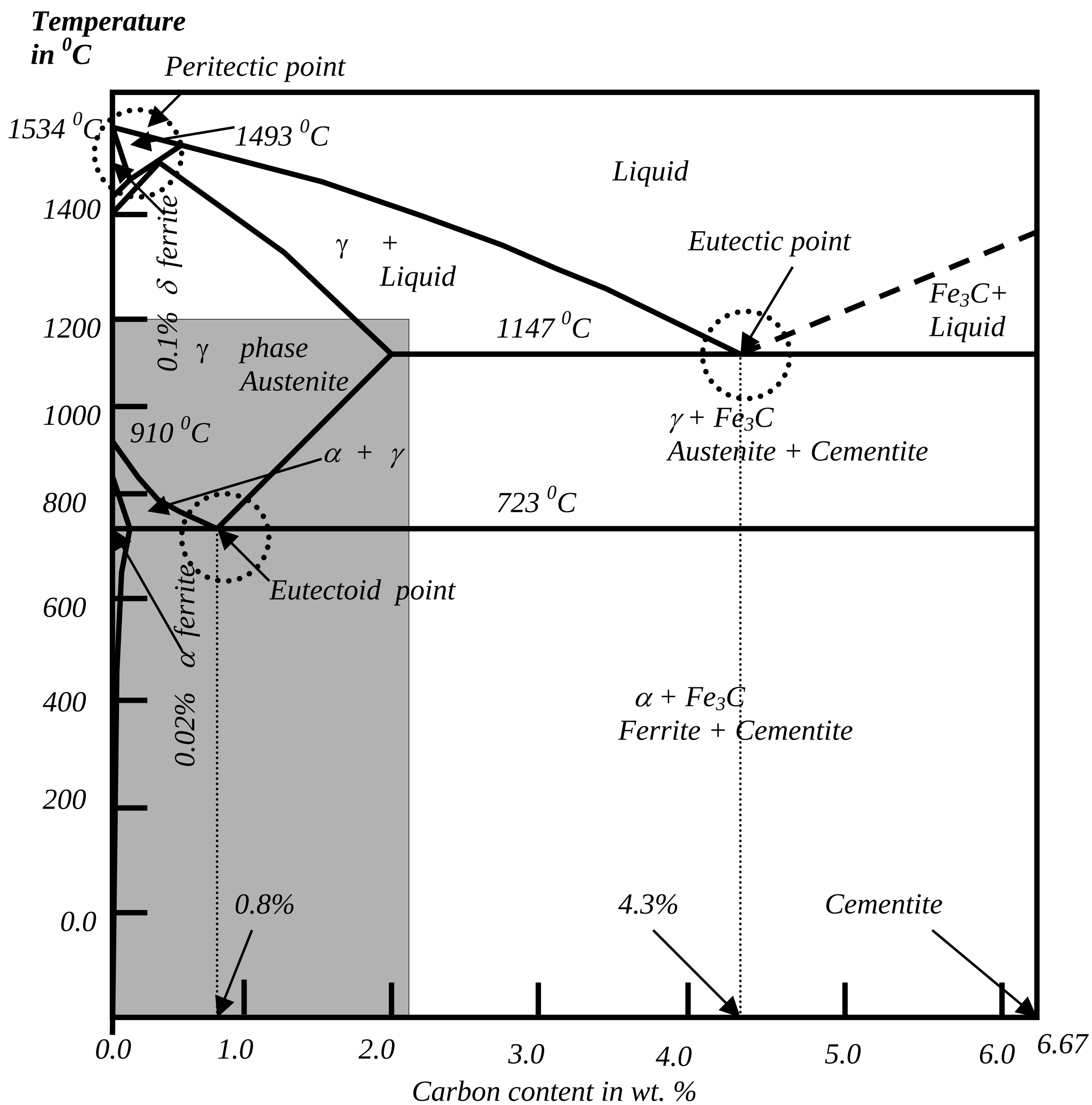


Fig.7 Iron – Iron-Carbide phase diagram

This mechanical distortion of crystal lattice interferes with the external applied strain to the crystal lattice, by mechanically blocking the dislocation of the crystal lattices. In other words, they provide mechanical strength. Obviously adding more and more carbon to iron (upto solubility of iron) results in more and more distortion of the crystal lattices and hence provides increased mechanical strength. However, solubility of more carbon influences negatively with another important property of iron called the 'ductility' (ability of iron to undergo large plastic deformation). The α-iron or ferrite is very soft and it flows plastically. Hence we see that when more carbon is added, enhanced mechanical strength is obtained, but ductility is reduced. Increase in carbon content is not the only way, and certainly not the desirable way to get increased strength of steels. More amount of carbon causes problems during the welding process. We will see later, how both mechanical strength and ductility of steel could be improved even with low carbon content. The iron-carbon equilibrium diagram, which is a plot of transformation of iron with respect to carbon content and temperature, is shown in Fig.7. This diagram is also called iron-iron carbide diagram. The important metallurgical terms, used in the diagram, are presented below.

Ferrite (α): Virtually pure iron with body centred cubic crystal structure (bcc). It is stable at all temperatures upto 910^{0}C. The carbon solubility in ferrite depends upon the temperature; the maximum being 0.02% at 723^{0}C.
Cementite: Iron carbide (Fe_3C), a compound iron and carbon containing 6.67% carbon by weight.
Pearlite: A fine mixture of ferrite and cementite arranged in lamellar form. It is stable at all temperatures below 723 ^{o}C.
Austenite (γ): Austenite is a face centred cubic structure (fcc). It is stable at temperatures above 723 ^{o}C depending upon carbon content. It can dissolve upto 2% carbon. These terms are summarised in Table 2.

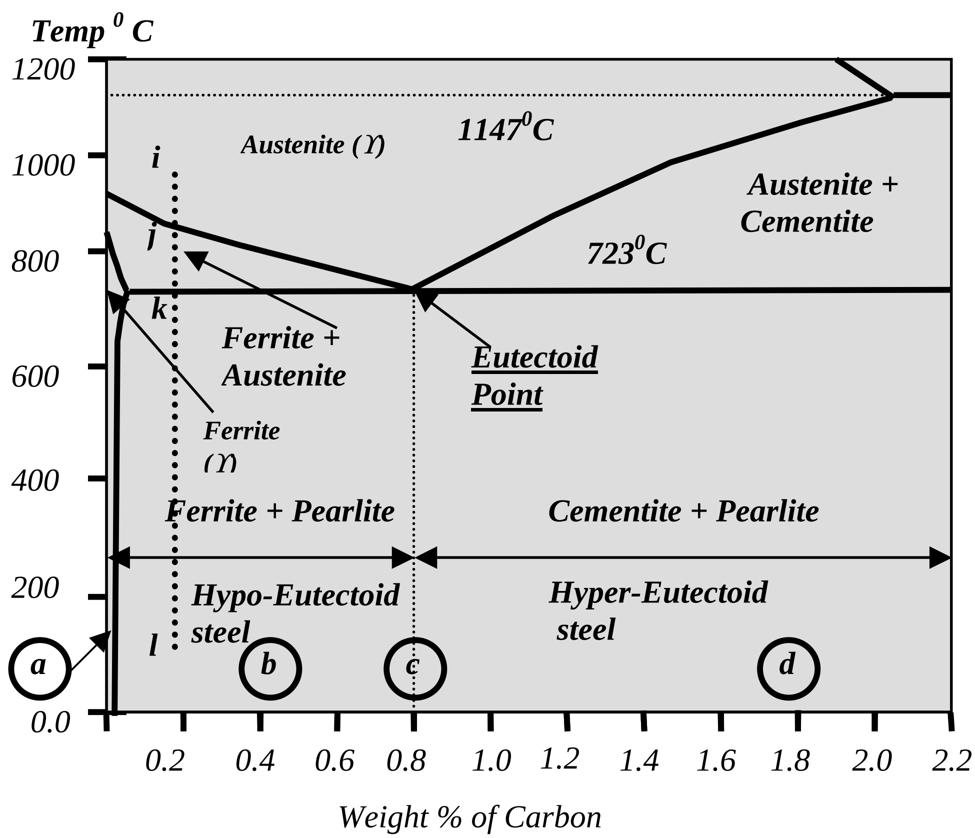


Fig.8 The Eutectoid section of the Iron – Iron Carbon phase diagram

The maximum solubility of carbon in the form of Fe_3C in iron is 6.67%. Addition of carbon to iron beyond this percentage would result in formation of free carbon or graphite in iron. At 6.67% of carbon, iron transforms completely into cementite or Fe_3C (Iron Carbide). In the iron-carbon phase diagram, there are three important points such as (1) eutectoid point (2) eutectic point and (3) peritectic point shown in dotted circles in Fig.7. Generally carbon content in structural steels is in the range of 0.12-0.25%. Upto 2% carbon, we get a structure of ferrite + pearlite or pearlite + cementite depending upon whether carbon content is less than 0.8% or beyond 0.8%. Beyond 2% carbon in iron, cast iron is formed.

Table 2: Metallurgical terms of iron

Name	***Metallurgical term***	***% Carbon(max)***	***Crystal structure***
α - Iron	Ferrite	0.02	bcc
Fe_3C	Cementite	6.67	-
Ferrite + Cementite laminar mixture	Pearlite	0.80 (overall)	-
γ - Iron	Austenite	2.0 (depends on temperature)	fcc

3.3 The Structural Steels or ferrite – Pearlite Steels

The iron-iron carbide portion of the phase diagram that is of interest to structural engineers is shown in Fig.8. The phase diagram is divided into two parts called "hypoeutectoid steels" (steels with carbon content to the left of eutectoid point [0.8% carbon]) and "hyper eutectoid steels" which have carbon content to the right of the eutectoid point. It is seen from the figure that iron containing very low percentage of carbon (0.002%) called very low carbon steels will have 100% ferrite microstructure (grains or crystals of ferrite with irregular boundaries) as shown in Fig. 9(a). Ferrite is soft and ductile with very low mechanical strength. The microstructure of 0.20% carbon steel is shown Fig. 9(b). This microstructure at ambient temperature has a mixture of what is known as 'pearlite and ferrite' as can be seen in Fig. 8. Hence we see that ordinary structural steels have a *pearlite* + *ferrite* microstructure. However, it is important to note that steel of 0.20% carbon ends up in pearlite + ferrite microstructure, *only when it is cooled very slowly from higher temperature* during manufacture. When the rate of cooling is faster, the normal pearlite + ferrite microstructure may not form, instead some other microstructure called bainite or martensite may result.

We will consider how the microstructures of structural steel are formed by the slow cooling at 0.2% carbon. At about 900^0C, this steel has austenite microstructure. This is shown as point 'i' in Fig. 8. When steel is slowly cooled, the transformation would start on reaching the point 'j'. At this point, the alloy enters a two-phase field of ferrite and austenite. On reaching the point, ferrite starts nucleating around the grain boundaries of austenite as shown in Fig. 10(*a*). By slowly cooling to point 'k', the ferrite grains grow in size and diffusion of carbon takes place from ferrite regions into the austenite regions as shown in Fig. 10(*b*), since ferrite cannot retain carbon above 0.002% at room temperature.

At this point it is seen that a network of ferrite crystals surrounds each austenite grain. On slow cooling to point 'l' the remaining austenite gets transformed into 'pearlite' as shown in Fig.10(c). Pearlite is a lamellar mixture of ferrite and cementite. The amount of 'pearlite' for a given carbon content is usually calculated using the lever rule assuming 0% carbon in ferrite as given below:

$$Volume\ \ fraction\ of\ Pearlite = \frac{\%\ of\ \ Carbon}{0.8\%\ of\ Carbon}$$

For example for microstructure of a 0.2% carbon steel would consist of a quarter of pearlite and three- quarters of ferrite. As explained earlier, ferrite is soft and ductile and pearlite is hard and it imparts mechanical strength to steel. The higher the carbon content, the higher would be the pearlite content and hence higher mechanical strength. Conversely, when the pearlite content increases, the ferrite content decreases and hence the ductility is reduced. The microstructure of ferrite-pearlite steels is given in Fig.9 (b). The white portion in Fig.9 (b) is ferrite and the black is pearlite. The constituents of the specimen are (pearlite + ferrite), but the phases are (ferrite + cementite). At the Eutectoid point where the carbon content is 0.8%, a fully lamellar pearlite structure is obtained as shown in Fig.9(c).

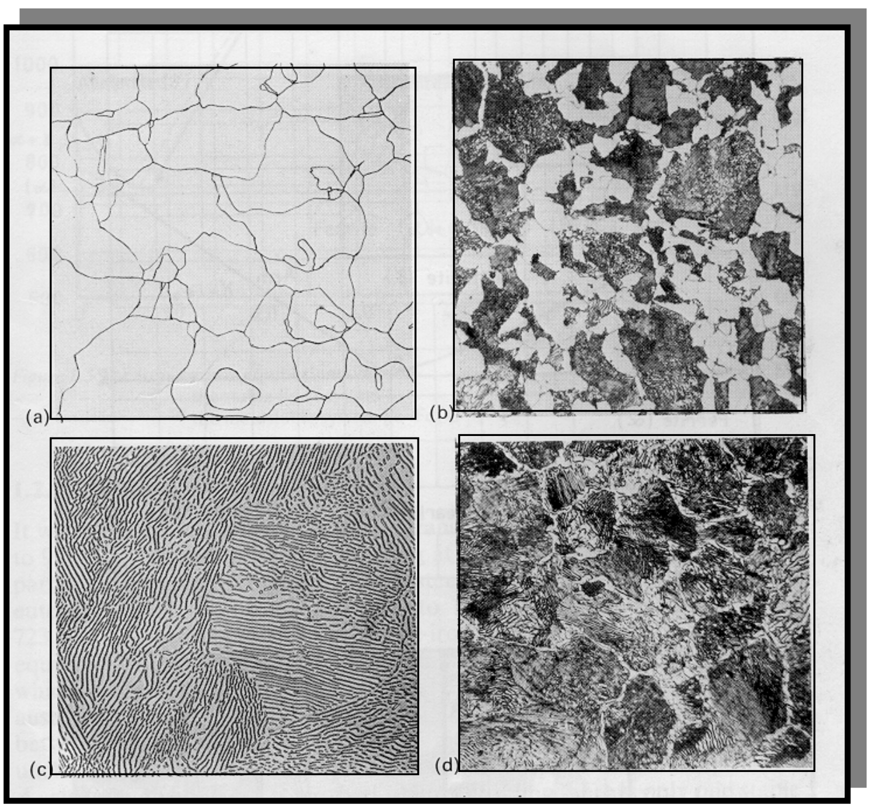


Fig.9 Microstructures of steels
(a)100% Ferrite in extra low carbon steel (b)Ferrite+Pearlite (c) 100% Pearlite in eutectoid steel (d)Pearlite+Cementite in hyper-eutectoid steel (Source: Thelning K.E,. Steel and its heat treatment, Butterworths, 1984.)

Note: Microstructures at (c) and (d) are not observed in structural steels.

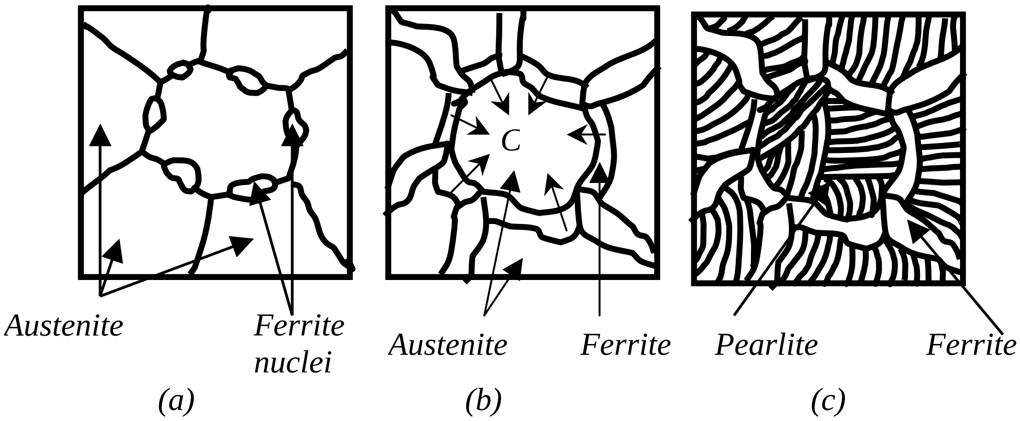


Fig.10 Different stages of formation of Pearlite

We also see from Fig.9 (d) that in the case of *hyper eutectoid steels* (steels having carbon content more than 0.8%), a microstructure of (cementite + pearlite) is obtained. Ofcourse microstructures (c) and (d) are not observed in structural steels. As mentioned earlier, increase in carbon content is not the only way to obtain increased mechanical strength. We would see in the next section, the other methods of increasing the strength of steel.

3.4 Strengthening structural steels

Cooling rate of steel from austenite region to room temperature produces different microstructures, which impart different mechanical properties. In the case of structural steels, the (pearlite + ferrite) microstructure is obtained after austenitising, by cooling it very slowly in a furnace. This process of slow cooling in a furnace is called '***annealing***'. As, mentioned in the earlier section, the formation of pearlite, which is responsible for mechanical strength, involves diffusion of carbon from ferrite to austenite. In the annealing process sufficient time is given for the carbon diffusion and other transformation processes to get completed. Hence by full annealing we get larger size pearlite crystals as shown in the cooling diagram in Fig.11. It is very important to note that the grain size of crystal is an important parameter in strengthening of steel. The yield strength of steel is related to grain size by the equation

$$f_y = f_0 + \frac{k}{\sqrt{d}} \qquad (1)$$

where f_y is the yield strength, f_0 is the yield strength of very large isolated crystals (for mild steel this is taken as 5 N/mm^2) and 'k' is a constant, which for mild steel is 38 N/mm$^{3/2}$. From Eq.1 we see that decreasing the grain size could enhance the yield strength. We will see in the following section as to how this reduction of grain size could be controlled. The grain size has an influence both in the case of mechanical strength and the temperature range of the ductile-brittle transition (temperature at which steel would become brittle from a ductile behaviour). When steel is fully annealed, there is enough time for the diffusion or shuffling of carbon atoms and larger crystallisation is possible. However, if we increase the cooling rate, then transformation that generally needs a specified time, would not keep up with the falling temperature. When we normalise (cool in air) steel, we obtain a small increase in the ferrite content and a finer lamellar pearlite as shown in the cooling curve in Fig.11. Since pearlite is responsible for mechanical strength, decrease in its grain size we get improved mechanical strength. Hence we see that another method of increasing the mechanical strength of steel is by *normalising*.

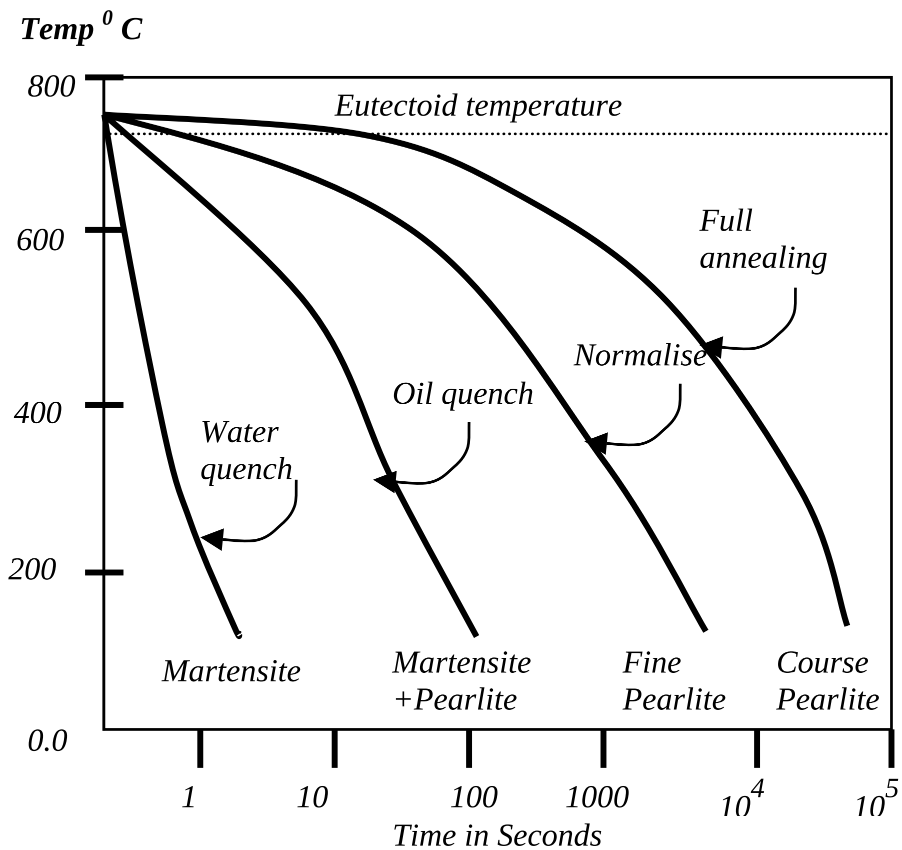


Fig.11 Variation of microstructure as a function of cooling

When structural steel sections are produced by hot rolling process, which involves the temperature range of austenite, during rolling at this high temperature, the heavy mechanical deformation results in finer size grains. In addition to that, rolling at the temperature of austenite, they are allowed to cool in air (normalising) and hence both the procedures aid the formation of smaller size crystals and hence increased mechanical strength.

3.5 Rapid cooling of steels

In the earlier section we saw that steel is made to under-cool by normalising (by giving lesser cooling time than required by the equilibrium state of the constitutional diagram), it results in finer microstructure. However, if we cool steel very rapidly, say quenching in cold water, there is insufficient time for the shuffling or diffusion of carbon atoms and hence the formation of ferrite + pearlite is prevented. However, such a fast cooling results in '*martensite*'. Slightly less rapid cooling could result in a product called '*bainite'* which is dependent on the composition of steel. The formation of martensite is shown in Fig.11 by rapid cooling. It is also seen from Fig.11 that, oil quenching where cooling rate which is slightly slower, results in a mixture of martensite + pearlite. Fig.12 shows the formation of different composition for varied cooling rates. It is seen from Fig.12 that *bainite* is formed above a temperature of about 300°C and between a cooling rate of 8.4°C/sec to 0.0062°C/sec. Martensite is formed by rapid cooling rate less than 8.4°C/sec. Very slow cooling, say full annealing does not form both Martensite and Bainite. Fig.12 clearly shows that the final microstructure of steel is dependent on cooling rate.

Martensite is very hard and less ductile. Martensitic structure is not desirable in structural steel sections used in construction, because its welding becomes very difficult. However, high strength bolts and some other important accessories have predominantly martensitic structure. The hardness of martensite is a function of carbon content. When martensite is heated to a temperature of 600°C it softens and the toughness is improved. This process of reheating martensite is called *tempering*. This process of quenching and tempering results in very many varieties of steel depending upon the requirement for hardness, wear resistance, strength and toughness.

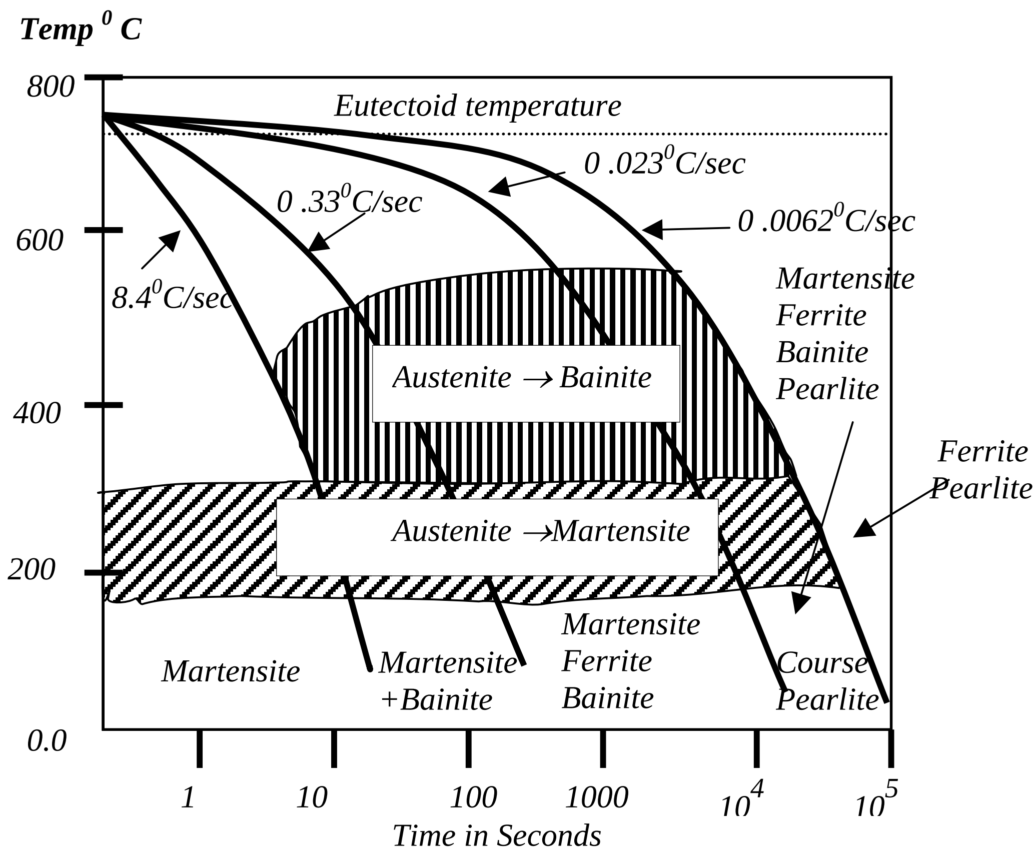


Fig.12 Rate of cooling Vs microstructure

3.6 Inclusions and alloying elements in steel

Steel contains impurities such as phosphorous and sulphur and they eventually form phosphides and sulphides which are harmful to the toughness of the steel. Hence it is desirable to keep these elements less than 0.05%. Phosphorous could be easily removed compared to sulphur. If manganese (Mn) is added to steel, it forms a less harmful manganese sulphide (MnS) rather than the harmful iron sulphide. Sometimes calcium, cerium, and other rare earth elements are added to the refined molten steel. They combine with sulphur to form less harmful elements. Steel treated this way has good toughness and such steels are used in special applications where toughness is the criteria. The addition of manganese also increases the under cooling before the start of the formation of ferrite+ pearlite. This gives fine-grained ferrite and more evenly divided pearlite. Since the atomic diameter of manganese is larger that the atomic diameter of iron, manganese exists as *'substitutional solid solution'* in ferrite crystals, by displacing the smaller iron atoms as shown in Fig.13. This improves the strength of ferrite because the distortion of crystal lattice due to the presence of manganese blocks the mechanical movement of the crystal lattices. However, manganese content cannot be increased unduely, as it might become harmful. Increased manganese content increases the formation of martensite and hence hardness and raises its ductile to brittle transition temperature (temperature at which steel which is normally ductile becomes brittle). Because of these reasons, manganese is restricted to 1.5% by weight. Based on the manganese content, steels are classified as carbon-manganese steels (Mn>1%) and carbon steels (Mn<1%). In recent years, micro alloyed steels or high strength low alloy (HSLA) steels have been developed. They are basically carbon manganese steels in which small amounts of aluminium, vanadium, mobium or other elements are used to help control the grain size.

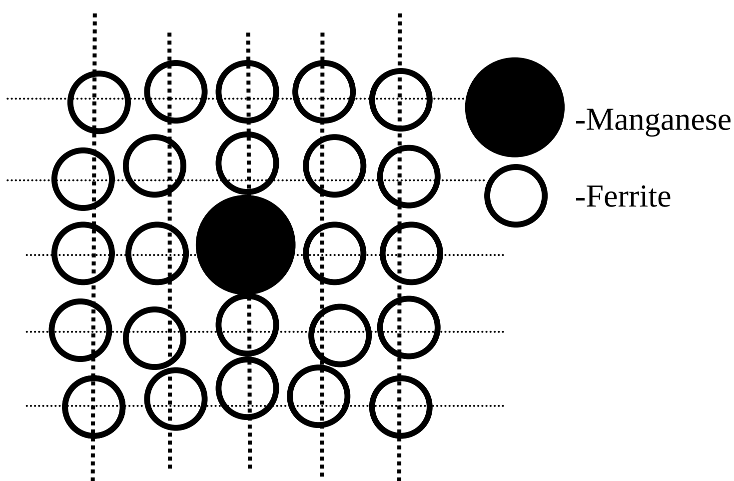


Fig.13 Substitutional solid solution of Manganese in Iron

These steels are controlled rolled and/or controlled cooled to obtain fine grain size. They exhibit a best combination of strength and toughness and also are generally weldable without precautions such as preheating or post heating. Sometimes 0.5% molybdenum is added to refine the lamellar spacing in pearlite, and to make the pearlite evenly distributed. Today steel with still higher performance are being developed all over the world to meet the following specifications such as: (a) high strength with yield strength of 480 MPa and 690 MPa, (b) excellent weldability without any need for preheating, (c) extremely high toughness with charpy V notch values of 270 N-m @ 23^{o}C compared with current bridge design requirement of 20 N.-m @ 23^{o}C, and (d) corrosion resistance comparable to that of weathering steel. (The terminology used above has been discussed later in this chapter). The micro alloyed steels are more expensive than ordinary structural steels, however, their strength and performance outweighs the extra cost.

Some typical steels with their composition range and properties and their relevant codes of practice, presently produced in India, are given in Tables 3 and 4. These steels are adequate in many structural applications but from the perspective of ductile response, the structural engineer in cautioned against using unfamiliar steel grades, without checking the producer supplied properties. Weldability of steel is closely related to the amount of carbon in steel. Weldability is also affected by the presence of other elements. The combined effect of carbon and other alloying elements on the weldability is given by *"carbon equivalent value (C_{eq})"*, which is given by

$$C_{eq} = \%C + \%\,Mn/6 + (\%\,Cr + \%\,Mo + \%\,V)/5 + (\%\,Ni + \%\,Cu)/15$$

The steel is considered to be weldable without preheating, if $C_{eq} < 0.42\%$. However, if carbon is less than 0.12% then C_{eq} can be tolerated upto 0.45%.

Table 3 Types of steel and their relevant IS standards

Type of steel	**Relevant IS standards**
Structural steel	226(withdrawn),2062,3502,1977,961,8500
Steel for bars, rivets etc.	1148,1149,1570,2073,7388,4431,4432, 5517
Steel for tubes and pipes	1239,1914,1978

Table 4 Chemical composition of some typical structural steels

Type of steel	**Designa-tion**	**IS: code**	**C**	**S**	**Mn**	**P**	**Si**	**Cr**		**Carbon equiva-lent**
Standard structural steel	Fe 410A	2062	0.23	.050	1.5	.050	-	-	SK	0.42
	Fe 410B	2062	0.22	.045	1.5	.045	0.4	-	SK	0.41
	Fe 410C	2062	0.20	.040	1.5	.040	0.4	-	K	0.39
Micro alloyed high strength steel	Fe 440	8500	0.20	.050	1.3	.050	.45			0.40
	Fe540	8500	0.20	.045	1.6	.045	.45			0.44
	Fe590	8500	0.22	.045	1.8	.045	.45			0.48
K- killed steel SK- Semi Killed steel (Explained in section 6.2)										

3.7 Thermo- Mechanically Control Process (TMCP) steels

With increase in height, size and span in buildings, higher strength, longer section and heavier thickness are required for steel products to be applied. In the conventional method, increased strength is secured by increasing addition of alloying elements. However, such an addition adversely results in deterioration of weld crack resistance due to increase in carbon equivalent (C_{eq}) and lowering of weld efficiency, due to the necessity to secure high pre-heating temperatures. To cope up with such requirements, Thermo-Mechanical Control Process (TMCP) steels with yield strength of 490 Mpa are being produced in countries like Japan. TMCP allows production of steel products having higher strength but carbon equivalent similar to those of conventional steels. Even for extra heavy sections, excellent weldabilty and stable strength can be achieved through application of TMCP. Even for thickness greater than 40 mm TMCP steels are finding wide applications.

4.0 STAINLESS STEELS

In an iron-chromium alloy, when chromium content is increased to about 11%, the resulting material is generally classified as a stainless steel. This is because at this minimum level of chromium, a thin protective passive film forms spontaneously on steel, which acts as a barrier to protect the steel from corrosion. On further increase in chromium content, the passive film is strengthened and achieves the ability to repair itself, if it gets damaged in the corrosive environment. 'Ni' addition in stainless steel improves corrosion resistance in reducing environments such as sulphuric acid. It also changes the crystal structure from bcc to fcc thereby improving its ductility, toughness and weldability. 'Mo' increases pitting and crevice corrosion in chloride environments.

Stainless steel is attractive to the architects despite its high cost, as it provides a combined effect of aesthetics, strength and durability. Now a days, stainless steel is used extensively in building construction. For example, the worlds’ tallest twin tower situated in Kuala Lampur, Malaysia used about 4000 tonnes of stainless steel made in India! Table 5 gives typical grades of stainless steel, which are used in building construction.

Table 5 Stainless Steel grades and their usage

Grade of stainless steel	Usage
316 (18% Cr)	Profiled roofing, cladding, gutters, facades and hand railings - in highly polluted environments
304 (18% Cr-(% Ni)	Decorative elements in areas near coast line. Also for kitchen and sanitary wares - coastal and less polluted areas
430 (17% Cr)	Roofing, gutters, decorative wall tiles, hallow structural sections non-polluted environments
409 (11% Cr)	Painted roofing- non-polluted environments

Stainless steels are available in variety of finishes and it enhances the aesthetics of the structure. On Life Cycle cost Analysis (LCA), stainless steel works out to be economical in many situations. Increased usage of stainless steel in the construction sector is expected, as awareness on LCA improves among architects and consulting engineers.

5.0 MECHANICAL PROPERTIES OF STEEL

5.1 Stress – strain behaviour: Tensile test

The stress-strain curve for steel is generally obtained from tensile test on standard specimens as shown in Fig.14. The details of the specimen and the method of testing is elaborated in IS: 1608 (1995). The important parameters are the gauge length 'L_c' and the initial cross section area S_o. The loads are applied through the threaded or shouldered ends. The initial gauge length is taken as 5.65 $(S_o)^{1/2}$ in the case of rectangular specimen and it is five times the diameter in the case of circular specimen. A typical stress-strain curve of the tensile test coupon is shown in Fig.15 in which a sharp change in yield point followed by plastic strain is observed. When the specimen undergoes deformation after yielding, Luder's lines or Luder's bands are observed on the surface of the specimen as shown in Fig.16.

These bands represent the region, which has deformed plastically and as the load is increased, they extend to the full gauge length. This occurs over the Luder's strain of 1 to 2% for structural mild steel. After a certain amount of the plastic deformation of the material, due to reorientation of the crystal structure an increase in load is observed with increase in strain. This range is called the *strain hardening range*. After a little increase in load, the specimen eventually fractures. After the failure it is seen that the fractured surface of the two pieces form a cup and cone arrangement. This cup and cone fracture is considered to be an indication of ductile fracture. It is seen from Fig.15 that the elastic strain is up to ε_y followed by a yield plateau between strains ε_y and ε_{sh} and a strain hardening range start at ε_{sh} and the specimen fail at ε_{ult} where ε_y, ε_{sh} and ε_{ult} are the strains at onset of yielding, strain hardening and failure respectively.

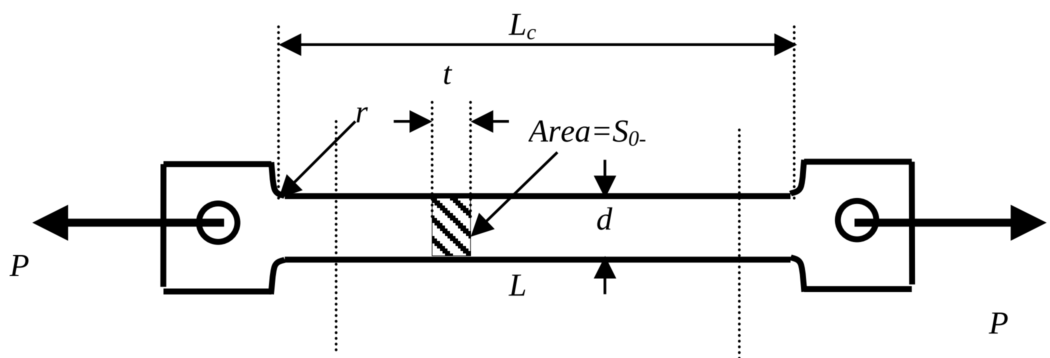


Fig.14 Standard tensile test specimen

Depending on the steel used, ε_{sh} generally varies between 5 to 15 ε_y, with an average value of 10 ε_y typically used in many applications. For all structural steels, the modulus of elasticity can be taken as 205,000 MPa and the tangent modus at the onset of strain hardening is roughly 1/30th of that value or approximately 6700 MPa.

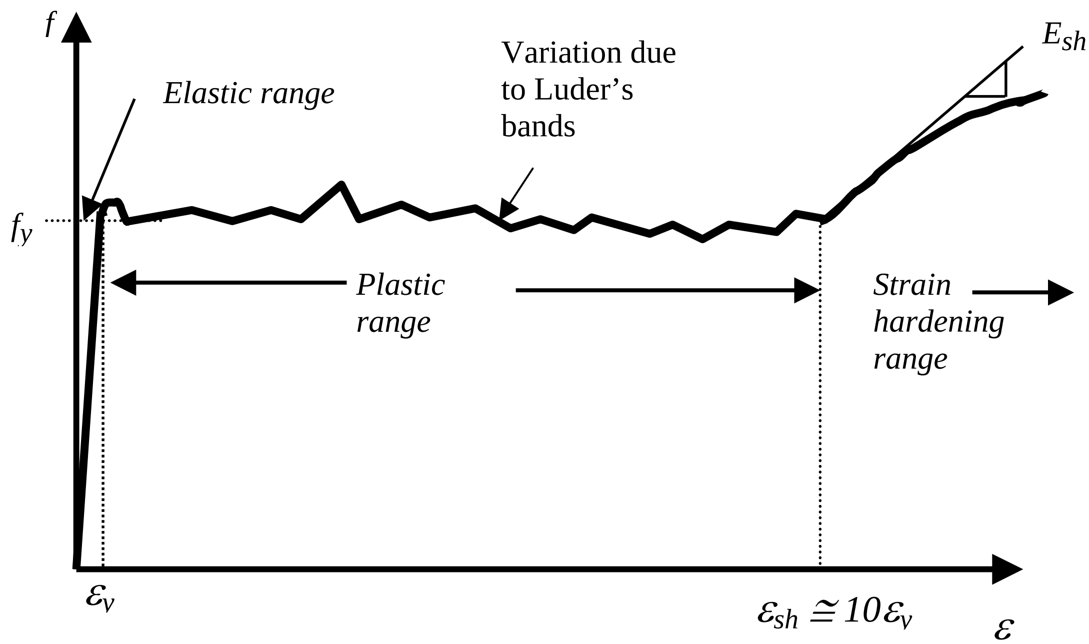


Fig.15 Stress strain curve for sharp yielding structural steels

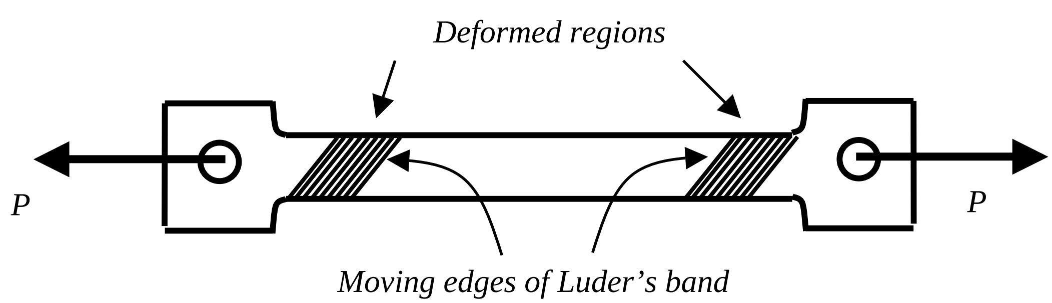


Fig.16 Luder's bands in tensile test specimen

Certain steels, due to their specific microstructure, do not show a sharp yield point but rather they yield continuously as shown in Fig. 17. For such steels the yield stress is always taken as the stress at which a line at 0.2% strain, parallel to the elastic portion, intercepts the stress strain curve. This is shown in Fig. 17.

A schematic diagram of the tensile coupon at failure is shown in Fig.18. It is seen that approximately at the mid section the area is '*S*' compared to original area S_0. Since *S* is the actual area experiencing the strain, the true stress is given by $f_t = P/S$, where *P* is the load.

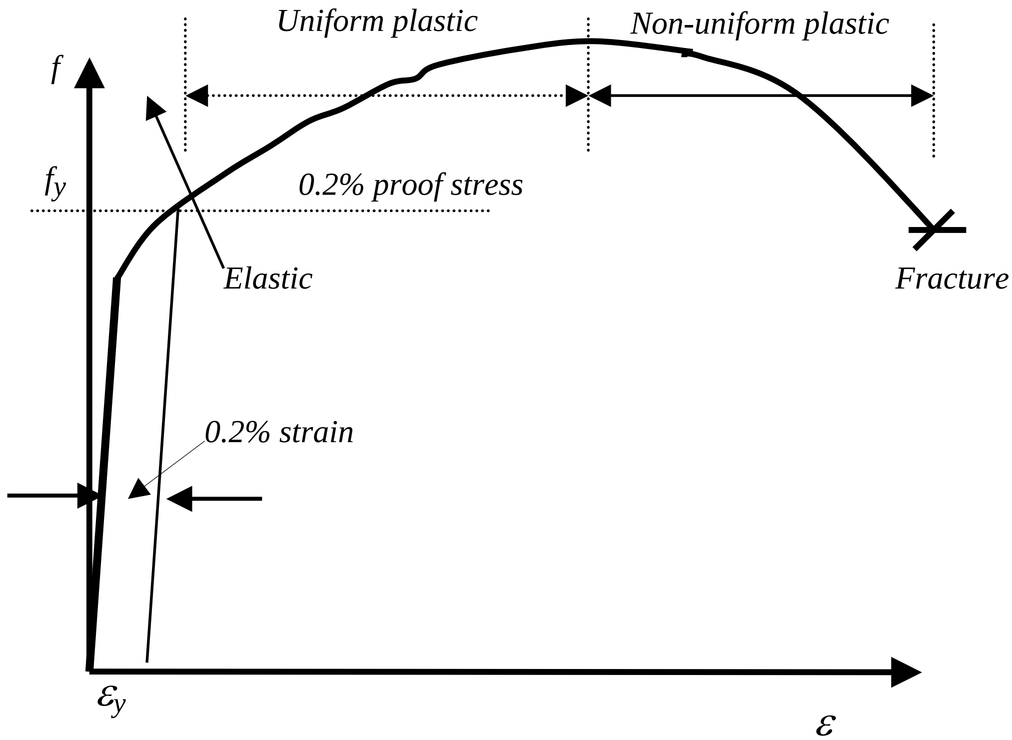


Fig. 17 Stress strain curve for continuously yielding structural steels

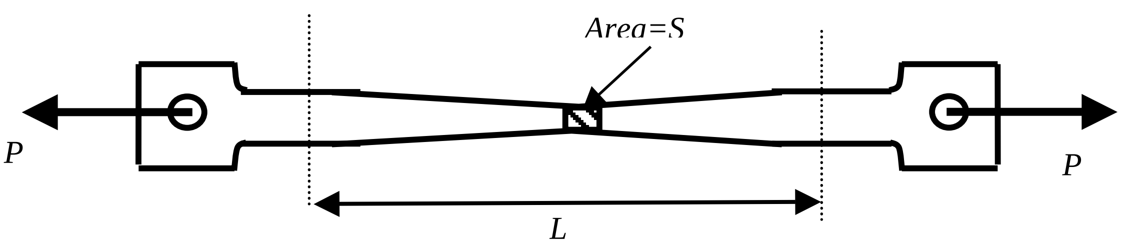


Fig.18 Tensile test specimen before rupture

However S is very difficult to evaluate compared to S_0 and the nominal stress or the engineering stress is given by $f_n = P/ S_0$. Similarly, the engineering strain is taken as the ratio of the change in length to original length. However the true strain is obtained when instantaneous strain is integrated over the whole of the elongation, given by

$$\varepsilon_t = \int_{L_0}^{L} \frac{dl}{l} = ln\left(\frac{L}{L_0}\right) \qquad (2)$$

By suitable manipulation it could be shown that

$$f_t = f_n(1+\varepsilon_n) \qquad (3)$$

and similarly

$$\varepsilon_t = ln(1 + \varepsilon_n) \tag{4}$$

where f_t and f_n are the true and nominal stresses respectively and ε_t and ε_n are the true and nominal strains respectively.

5.2 Hardness

Hardness is regarded as the resistance of a material to indentations and scratching. This is generally determined by forcing an indentor on to the surface. The resultant deformation in steel is both elastic and plastic. There are several methods using which the hardness of a metal could be found out. They basically differ in the form of the indentor, which is used on to the surface. They are presented in Table 6.

Table 6 Hardness testing methods and their indentors

	Hardness Testing Method	**Indentor**
(a)	Brinell hardness	Steel ball
(b)	Vickers hardness	Square based diamond pyramids of 135 ^{O} included angle
(c)	Rockwell hardness	Diamond core with 120 ^{O} included angle
Note: Rockwell hardness testing is not normally used for structural steels.		

In all the above cases, hardness number is related to the ratio of the applied load to the surface area of the indentation formed. The testing procedure involves forcing the indentor on to the surface at a particular road. On removal, the size of indentation is measured using a microscope. Based on the size of the indentation, hardness is worked out. For example, Brinell hardness *(BHN)* is given by the ratio of the applied load and spherical area of the indentation i.e.

$$BHN = \frac{P}{\pi(d/2)\left[D - \sqrt{D^2 - d^2}\right]} \tag{5}$$

where P is the load, D is the ball diameter, d is the indent diameter. The Vickers test gives a similar hardness value *(VHN)* as given by

$$VHN = \frac{1.854P}{L^2} \tag{6}$$

where L is the diagonal length of the indent. Some typical values of hardness of some metals are presented in Table.7.

Table 7 Hardness values of some metals

Metal	Brinell Hardness Number (BHN)	Vickers Hardness Number (VHN)
Copper (annealed)	49	53
Brass (annealed)	65	70
Steel	150-190	157-190

5.3 Effect of temperature on ductility and notch toughness

At lower temperatures below 0^0C, the yield strength of steel is only marginally affected, while there is substantial reduction in ductility and toughness. The ultimate behaviour of steel progressively changes from ductile to brittle, reaching a lowest value of toughness at a threshold temperature called "Ductile-to-Brittle-Transition-Temperature" (DBTT) range. The transition temperature for structural steel is generally well below the room temperature. However, if it is near to the ambient temperature, due to the loss of ductility, engineering components may fail under service loading. This transition temperature is affected by metallurgical aspects such as grain size and also by the presence of notches. In certain instances, due to deviations in correct processing procedure when the DBTT of steel is above room temperature or the application temperature, serious failures have been observed in ships cruising through the Arctic sea, bridges in cold climates and cryogenic gas storage facilities. The charpy "V" notch test (also known as the notch-toughness test) is used to determine the DBTT. In this test, a falling pendulum hammer fitted with a striking edge as shown in Fig.19 breaks a standard notched specimen (Fig.20). In principle, the energy absorbed by the specimen during its failure translates into a loss of potential energy of the pendulum. Thus, a rough measure of this absorbed energy can be calculated from the difference between initial height (h_1) of the pendulum when released and the maximum height (h_2) it reaches on the far side after breaking the specimen. The variation of absorbed energy with respect to temperature is shown in Fig.21. Generally, structural engineering standards and codes [IS: 1757 (1988)] will allow the use of only those steels that exhibit a minimum energy absorption capability at a pre-determined temperature say 20 N-m at 23 $\pm$5 ^{o}C.

5.4 Strain rate effect on yield strengths of steel

Strain rate is another factor that affects the strength of steel. Typically, the tensile and yield strength increases at higher strain rates as shown in Fig.22 except that at higher temperatures the reverse is true. It may also be noted that increase in strain rate causes reduction in ductility. Consideration of this phenomenon is crucial for blast resistant design of steel structures in which very high strain rates are expected but of little practical significance in earthquake engineering applications wherein the strain rate is well within the range where f_y does not change very much.

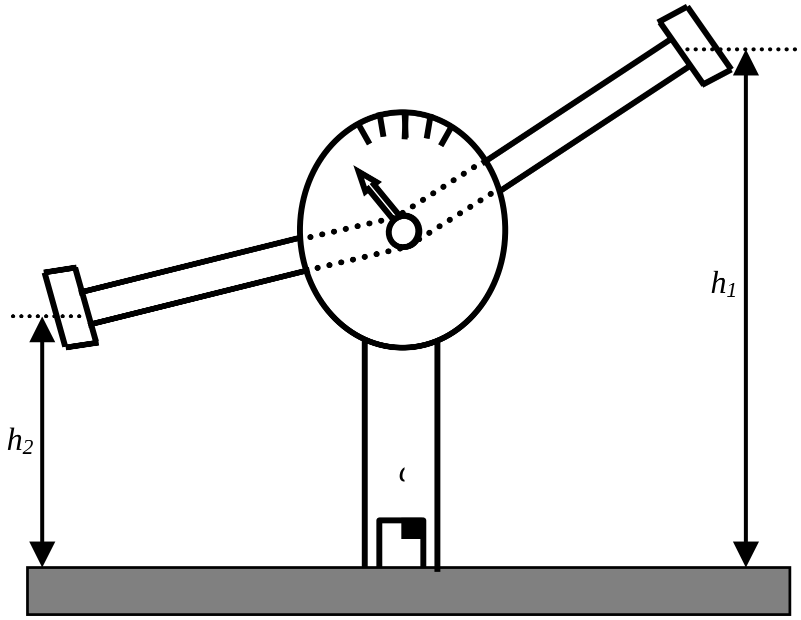


Fig. 19 Experimental set up for notch toughness test

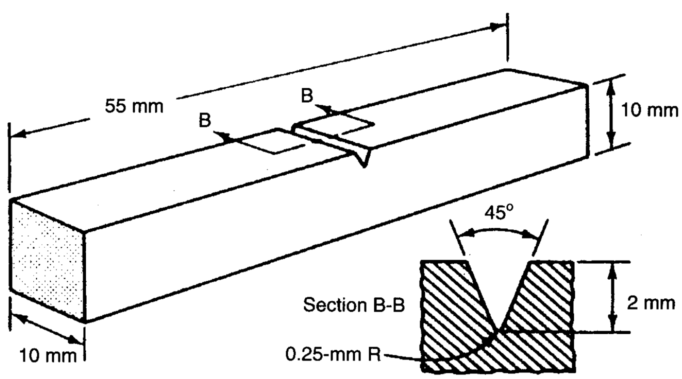


Fig. 20 Test specimen for notch toughness test

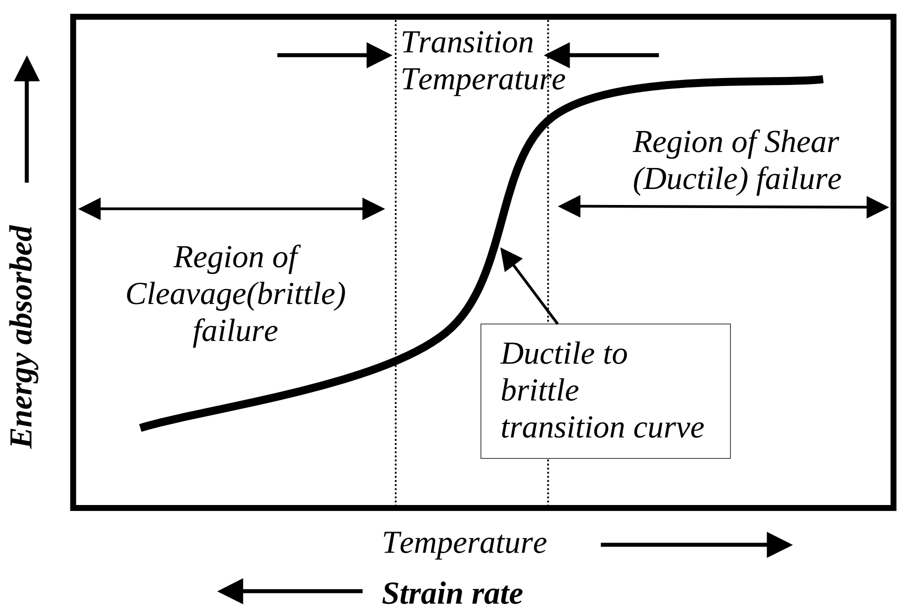


Fig.21 Effect of temperature on notch toughness of steel

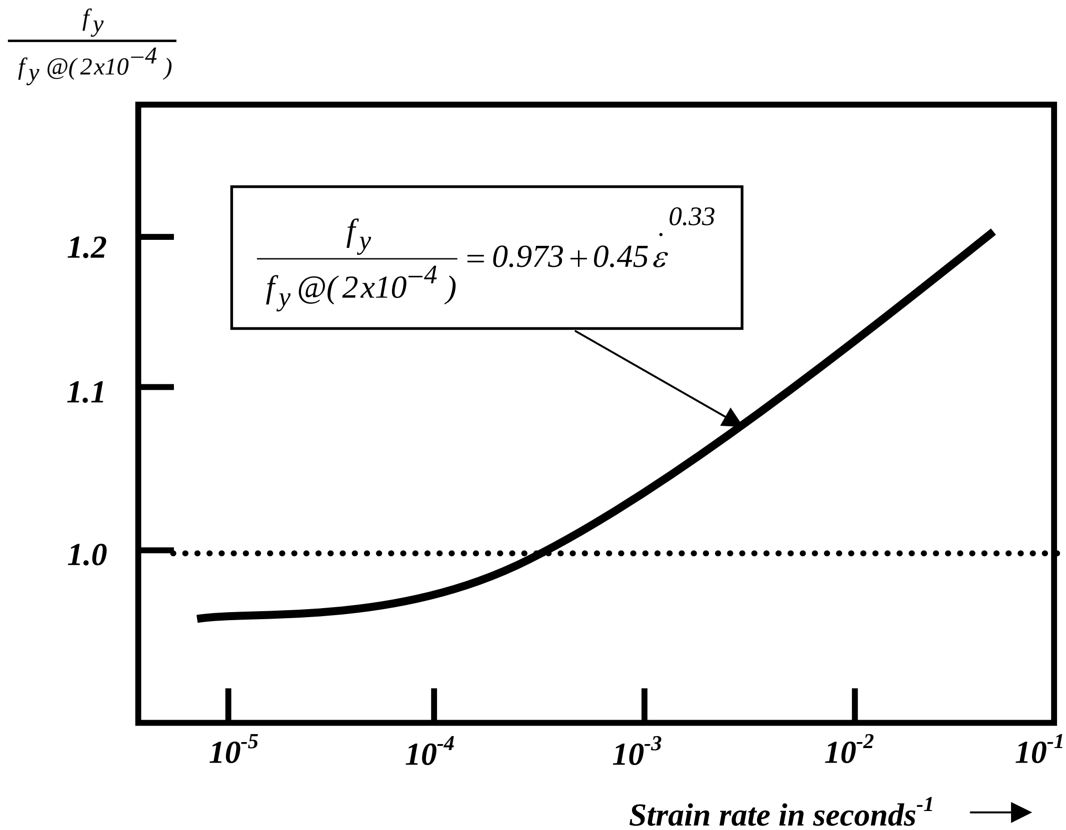


Fig.22 Strain rate effects on the yield strength of steel

Table 8 Mechanical properties of some typical structural steels

Type of steel	Design-ation	UTS (MPa)	Yield strength (MPa) Thickness (mm)			Elongation Gauge $5.65\sqrt{S_0}$	Charpy V - notch values Joules (min)
			<20	**20-40**	**>40**		
Standard structural steel	Fe 410A	410	250	240	230	23	27
	Fe 410B	410	250	240	230	23	27
	Fe 410C	410	250	240	230	23	27
			<16	**16-40**	**41-63**		
Micro alloyed high strength steel	Fe 440	440	300	290	280	22	-
	Fe 540	540	410	390	380	20	-
	Fe 590	590	450	430	420	20	-

5.5 Mechanical properties of structural steel

Table 8 summarises some of the important mechanical properties of steel produced in India. In Table 8, the UTS represents the minimum guaranteed *Ultimate Tensile Strength* at which the corresponding steel would fail.

6.0 THE MANUFACTURING PROCESS OF STRUCTURAL STEEL

For design of structures, the structural engineer uses long and flat products. The long products include: angles; channels; joists/beams; bars and rods; cold twisted deformed (CTD) bars; thermo-mechanically treated (TMT) ribbed bars, while the flat products comprise: plates; hot rolled coils (HRC) or cold rolled coils (CRC)/sheets in as annealed or galvanised condition. The starting material for the finished products is as given below:

- Blooms in case of larger diameter/cross-section long products
- Billets in case of smaller diameter/cross-section long products
- Slabs for hot rolled coils/sheets
- Hot rolled coils in case of cold rolled coils/sheets
- Hot/Cold rolled coils/sheets for cold formed sections

6.1 Electric Arc or Induction Furnace Route for Steel Making in Mini or Midi Steel Plants

The production process depends upon whether the input material to the steel plant is steel scrap or the basic raw material i.e. iron ore. In case of former, the liquid steel is produced in Electric Arc Furnace (EAF) or Induction Furnace (IF) and cast into ingots or continuously cast into blooms/billets/slabs for further rolling into desired product. The steel mills employing this process route are generally called as mini or midi steel plants. Since liquid steel after melting contains impurities like sulphur and phosphorus beyond desirable limits and no refining is generally possible in induction furnace. The structural steel produced through this process is inferior in quality. Through refining in EAF, any desired quality (i.e. low levels of sulphur and phosphorus and of inclusion content) can be produced depending upon the intended application. Quality can be further improved by secondary refining in the ladle furnace, vacuum degassing unit or vacuum arc degassing (VAD) unit.

6.2 Iron Making and Basic Oxygen Steel Making in Integrated Steel Plants

When the starting input material is iron ore, then the steel plant is generally called the integrated steel plant. In this case, firstly hot metal or liquid pig iron is produced in a vertical shaft furnace called the blast furnace (BF). Iron ore, coke (produced by carbonisation of coking coal) and limestone [Fig. 23(a)] in calculated proportion are charged at the top of the blast furnace. Coke serves two purposes in the BF(Fig.23(b)). Firstly it provides heat energy on combustion and secondly carbon for reduction of iron ore into iron. Limestone on decomposition at higher temperature provides lime, which combines with silica present in the iron ore to form slag. It also combines with sulphur in the coke and reduces its content in the liquid pig iron or hot metal collected at the bottom of the BF.

The hot metal contains very high level of carbon content around 4%; silicon in the range of 0.5-1.2%; manganese around 0.5%; phosphorus in the range 0.03-0.12%; and somewhat higher level of sulphur around 0.05%. Iron with this kind of composition is highly brittle and cannot be used for any practical purposes. Hot metal is charged in to steel making vessel called LD converter or the Basic Oxygen Furnace (BOF). Open-

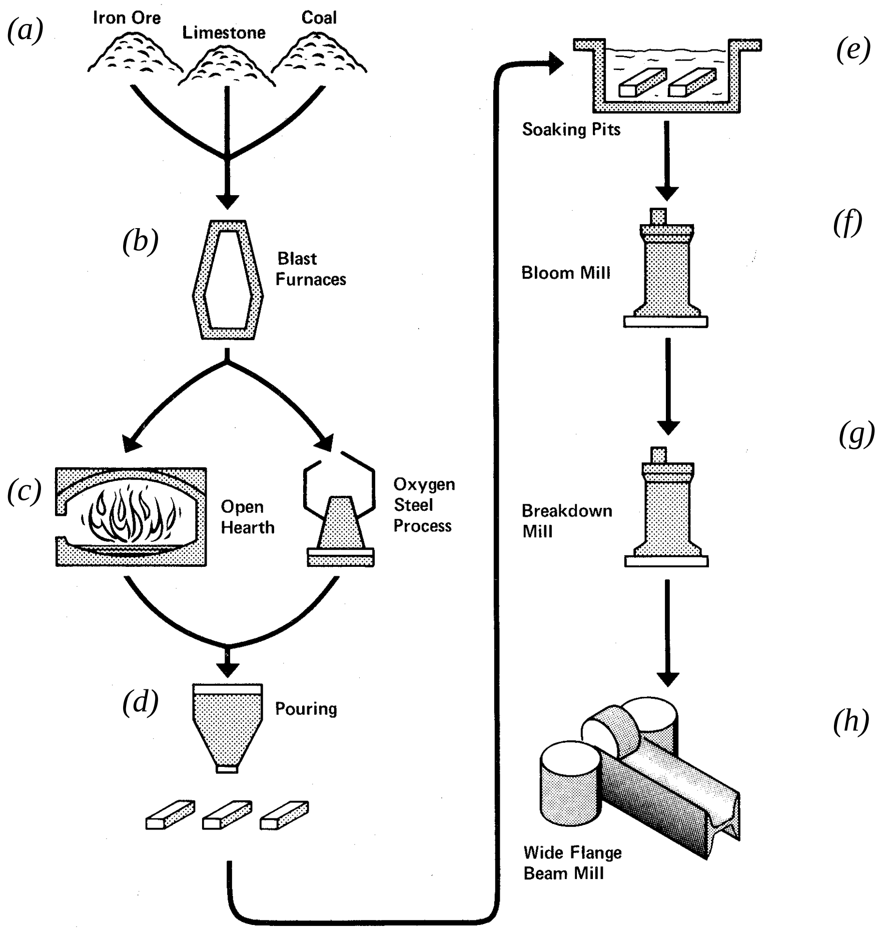


Fig.23 Schematic diagram of manufacturing of structural steel sections from Iron ore(Source: Adams P.F., Krentz H.A. and Kulak G.L., "Limit state design in structural steel – SI Units", Canadian Institute of Steel Construction (1979).)

hearth process is also used in some plants, though it is gradually being phased out [Fig.23(c)]. Oxygen is blown into the liquid metal in a controlled manner, which reduces the carbon content and oxidises the impurities like silicon, manganese, and phosphorus. Lime is charged to slag off the oxidised impurities. Ferro Manganese (FeMn), Ferro Silicon (FeSi) and/or Aluminium (Al) are added in calculated amount to deoxidise the liquid steel, since oxygen present in steel will appear as oxide inclusions in the solid state, which are very harmful. Ferro alloy addition also helps to achieve the desired composition. Generally the structural steel contains: carbon in the range 0.10-0.25%; manganese in the range 0.4-1.2%; sulphur 0.025-0.050%; phosphorus 0.025-0.050% depending upon specification and end use. Some micro alloying elements can also be added to increase the strength level without affecting its weldability and impact toughness.

If the oxygen content is brought down to less than 30 parts per million (PPM), the steel is called fully killed, whereas if the oxygen content is around 150 PPM, then the steel is called semi-killed. During continuous casting, only killed steel is used. However, both semi-killed and killed steels are cast in the form of ingots. The present trend is to go in for casting of steel through continuous casting, as it improves the quality, yield as well as the productivity.

6.3 Casting and Primary/Finish Rolling

Liquid steel is cast into ingots [Fig.(23(d)], which after soaking at 1280-1300^0 C in the soaking pits [(Fig.23(e)] are rolled in the blooming and billet mill into blooms/billets [(Fig.23(f)] or in slabbing mill into slabs. The basic shapes such as ingots, cast slabs, bloom and billets are shown in Fig.24. The blooms are further heated in the reheating furnaces at 1250-1280^0 C and rolled into billets or to large structurals[(Fig.23(h)]. The slabs after heating to similar temperature are rolled into plates in the plate mill. Even though the chemical composition of steel dictates the mechanical properties, its final mechanical properties are strongly influenced by rolling practice, finishing temperature, and cooling rate and subsequent heat treatment.

The slabs or blooms or the billets can directly be continuously cast from the liquid state and thereafter are subjected to further rolling after heating in the reheating furnaces.

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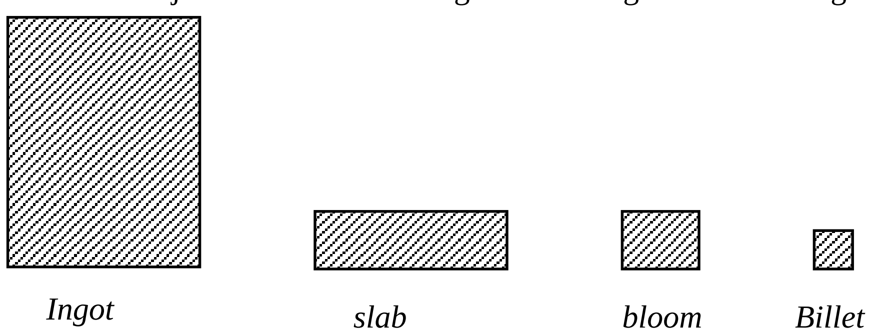


Fig. 24 Basic shapes and their relative proportions

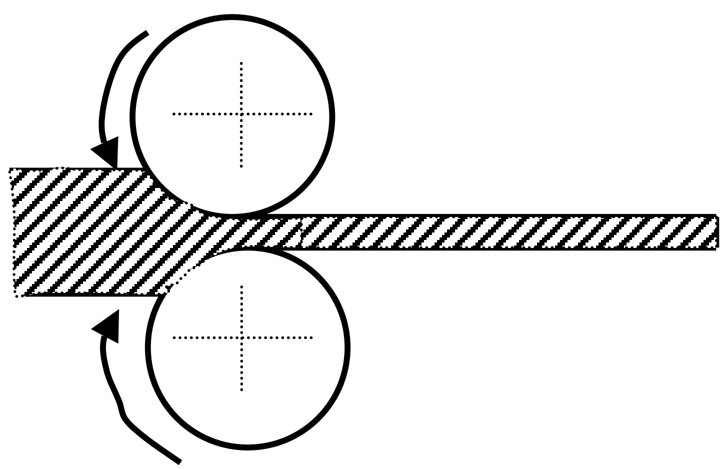


Fig.25 Primary rolls for plates

In the hot rolling operation the material passes through two rolls where the gap between rolls is lower than the thickness of the input material. The material would be repeatedly passed back and forth through the same rolls several times by reducing the gap between them during each pass. Plain rolls (Fig.25) are used for flat products such as plate, strip and sheet, while grooved rolls (Fig. 26) are used in the production of structural sections, rails, rounded and special shapes. The rolling process, in addition to shaping the steel into the required size, improves the mechanical properties by refining the grain size of the material.

Final rolling of structurals, bars/rods and HRC/CRC or sheet product is done in respective mills. In case of cold rolled sheets/coils, the material is annealed and skin passed to provide it the necessary ductility and surface finish

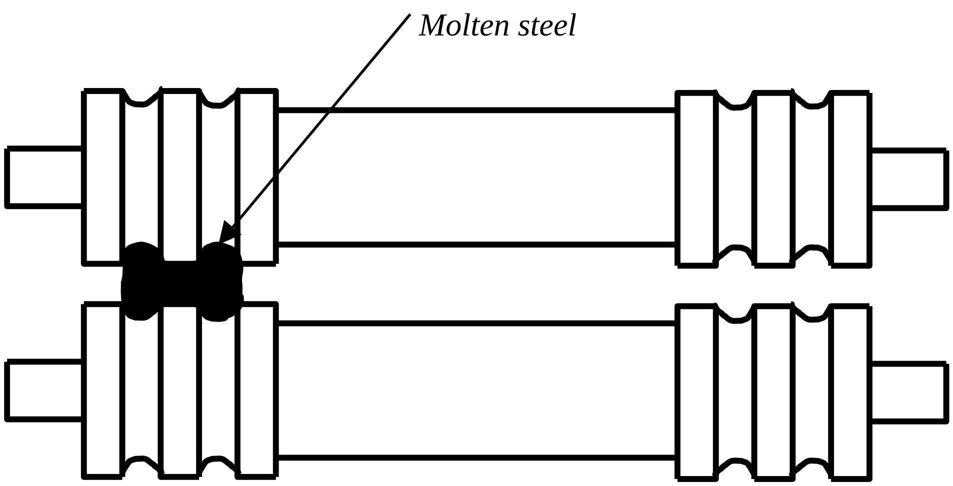


Fig.26 Primary rolls for structural shapes

6.4 Steel Products

The long products are normally used in the as-hot-rolled condition. Plates are used in hot rolled condition as well as in the normalised condition to improve their mechanical properties particularly the ductility and the impact toughness.

The structural sections produced in India include beams (classified as, light, junior, medium and heavy – defined as ISLB, ISJB, ISMB and ISHB respectively) angles (equal, unequal), channel, tees etc. Channel sections are designated as ISLC, ISMC etc. and angles are designated as ISA. Usually the member is designated along with its depth. For example ISMB 300 (300 mm depth), ISMC 250 (250 mm depth), ISA(60 x 60 x 6) (1^{st} leg breadth x 2^{nd} leg breadth x thickness) etc. Sheet products after cold rolling has high strength but very poor ductility. This product needs to be annealed at 650-680^{0}C in the hood annealing furnaces to improve its ductility.

Cold forming is done by passing the hot or cold rolled and annealed product through a series of cold forming rolls.

Now-a-days hollow sections are also becoming very popular. Hollow sections i.e. round, square or rectangular are produced either by seamless rolling process or by fusion welding or electric resistance welding after cold forming of HRC/CRC into the desired shape.

7.0 COLD ROLLING AND COLD FORMING

Cold rolling, as the term implies involves reducing the thickness of unheated material into thin sheets by applying rolling pressure at ambient temperature. The common cold rolled products are coils and sheets. Cold rolling results in smoother surface and improved mechanical properties. Cold rolled sheets could be made as thin as 0.3 mm. Cold forming is a process by which the sheets (hot rolled / cold rolled) are folded in to desired section profile by a series of forming rolls in a continuous train of roller sets. Such thin shapes are impossible to be produced by hot rolling. The main advantage of cold-formed sheets in structural application is that any desired shape can be produced. In other words it can be tailor-made into a particular section for a desired member performance. These cold formed sheet steels are basically low carbon steels (<0.1 % carbon) and after rolling these steel are reheated to about 650^0-723^0C and at this stage ferrite is recrystalised and also result in finer grain size. Because of the presence of ferrite, the ductility is enhanced.

8.0 FINISHING PROCESS

After a member is hot rolled into desired shape, a number of other services are available. These services are simply enumerated below.

- ♦ Exact cutting of length
- ♦ Line straightening of beams after rolling
- ♦ Cambering of beams
- ♦ Surface preparation such as shot blasting and application of protective paints
- ♦ Heat treatment of plates such as annealing, quenching, tempering etc.
- ♦ Pickling in acids to remove mill scale for further galvanising

Usually, information about manufacturing tolerances are supplied by the manufacturers. Even though these tolerances have little effect in normal fabrication and structural applications, they have an important effect when they are used in special applications.

9.0 SUMMARY

In this chapter, a historical review of iron and steel has been covered. The metallurgical aspects of steel, which are relevant to the designers involved in structural steelwork, are briefly presented. The mechanical aspects of structural steels have also been discussed. A mention has been made of the special steels such as stainless steels and cold-formed steels. Finally the basic elements of the manufacturing process of steel have been stated.

In brief, different aspects of steel as are important to a structural engineer, have been described.

10. REFERENCES

1. Graham W. Owens and Peter R. Knowles, "Steel designer's manual", ELBS Fifth Edition (1994).
2. Adams P.F., Krentz H.A. and Kulak G.L., "Limit state design in structural steel – SI Units", Canadian Institute of Steel Construction (1979).
3. IS:2062 Steel for general structural purposes – Specification, Fourth Revision (1992).
4. IS:961 Specification for structural steel (High Tensile) , (1962).
5. Robert E. Reed Hill ," Physical metallurgy principles", Second Ed., EWP, New Delhi, (1985).
6. IS:1608 Method of tensile testing of steel products (Second Revision) (1995).
7. IS:1757 Method for charpy impact test (V-notch) on metallic material (1988)
8. IS: 8500, Structural steel - Micro alloyed (medium and high strength qualities) - Specification, First Revision (1991).

2 CORROSION, FIRE PROTECTION AND FATIGUE CONSIDERATIONS OF STEEL STRUCTURES

1.0 INTRODUCTION

Corrosion, fire protection and fatigue failure of steel structures are some of the main concerns of an engineer involved in the design and construction of structural steel work and these aspects do warrant extra attention. A review of international literature and the state of the art in constructional steelwork would reassure the designer that many aspects of corrosion, fire and fatigue behaviour of structural steel work, are no longer the major issues. For example, the steel construction industry has developed excellent protective coatings that would retain service life even after 20 years without any serious attention! Similarly the emergence of 'fire engineering of steel structures' as a specialised discipline has addressed many of the concerns regarding the structural steel work under fire. In India 'Fire Resistant Steels (FRS)' are available which are quite effective in steelwork subjected to elevated temperatures. They are also cost effective compared to mild steel! Similarly, fatigue behaviour of steel structural systems has been researched extensively in the past few decades and has been covered excellently in the published literature. Many countries have a separate code of practice, which deals exclusively with the fatigue resistance design of steel structural systems. Today, substantial information and guidelines are available to the designers so that these three aspects could be handled in a routine manner. In this chapter we will review aspects of corrosion, fire protection and fatigue behaviour of structural steelwork briefly and outline suitable prevention methods.

2.0 CORROSION OF STEEL

There is a mindset among many Indian designers, that steel corrodes the most in India compared to other countries. This conception is very much untrue! No doubt, steel corrodes all over the world but the difference is, the problem is better tackled in the advanced countries. With the advent of new technologies of corrosion protection and better understanding of the material behaviour of steel, corrosion of steel no longer causes any undue worry for structural designers involved in structural steelwork. Nevertheless, a designer involved in structural steel work must be aware of the phenomena of corrosion and its prevention methods, both simple and detailed.

2.1 Corrosion mechanism as a miniature battery

Every metal found in nature has a characteristic electric potential, based on its atomic structure and also the ease with which the metal can produce or absorb electrons. Those metals, which provide electrons more readily, are called anodes and those that absorb electrons are called cathodes. Anodes and cathodes are called electrodes and if they get connected in the presence of an electrolyte (a conducting medium), they form a battery as shown in Fig.1.

No material individually can be called as cathode or anode, as they can serve both the functions depending on the relative potential of the material to which they are connected. For example, steel is anodic in the presence of stainless steel or brass and cathodic in the presence of zinc or aluminium. From the mechanism shown in Fig. 1, we see that two bodies of different electric potential electrically connected together in the presence of an electrolyte, the anodic body provide electrons to the cathode (To remember easily: Anodes–Away; Cathodes-Collect). In this process the anode is gradually destroyed, in other words it corrodes. On the other hand, a body will not corrode until it is immersed in or wetted by an electrolytic solution and gets electrically connected to another body having a more positive electric potential. This is the main principle called “***eliminate the electrolyte***”, using which we device many of the corrosion prevention methods, in structural steel work.

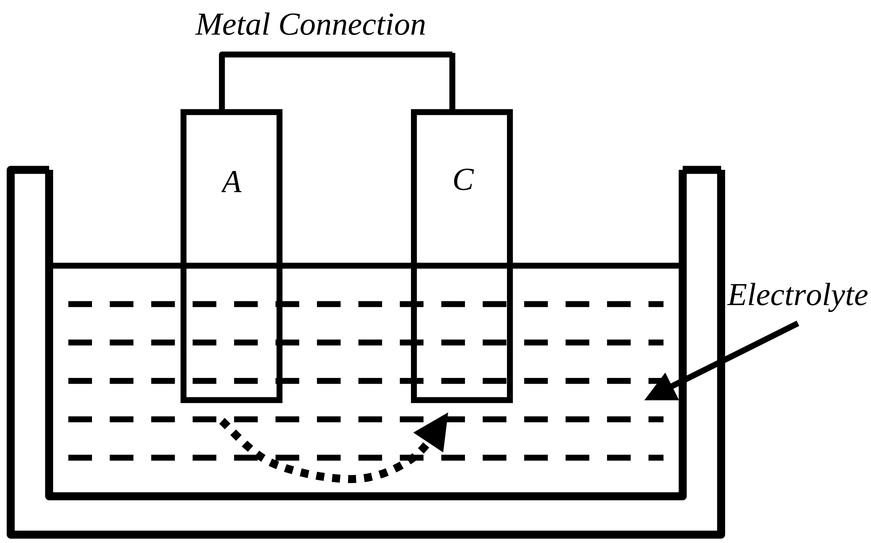


Fig.1 Mechanism of corrosion as a miniature battery

2.2 Corrosion of steel

In the case of steel, when favourable condition for corrosion occurs, the ferrous ions go into solution from anodic areas. Electrons are then released from the anode and move through the cathode where they combine with water and oxygen to form hydroxyl ions. These react with the ferrous ions from anode to produce hydrated ferrous oxide, which further gets oxidised into ferric oxide, which is known as the ‘*red rust*’. Let us consider a portion of steel member, which is slightly rusted as shown in Fig. 2.

The portion of the surface protected by the oxide film (rust) would be cathodic with respect to a portion, which is not so protected. Therefore, there will be a difference in electrical potential and hence the anode will corrode, forming rust on its surface. As rust builds up on one portion of the body, it becomes less anodic with respect to a previously rusted area. In this way they form and reform batteries and corrode the entire surface.

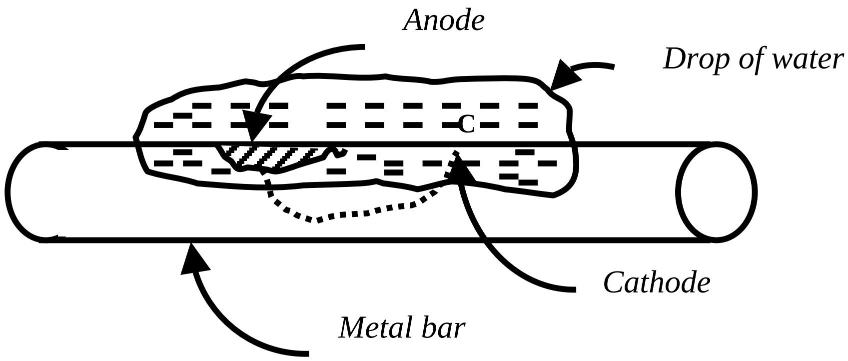


Fig. 2 Mechanism of Corrosion in steel

From the above discussion, it is clear, that the main interest of the structural designers is to prevent the formation of these "corrosion batteries". For example, if we can wipe out the 'drop of water' shown in Fig. 2, the corrosion will not takes place! Hence using the "***eliminate the electrolyte***" principle, wherever possible we need to device detailing and protection to surfaces of structural steel work to ensure that the combination of oxygen and water are avoided and hence the corrosion batteries are avoided.

2.3 Types of corrosion encountered in practice

Let us briefly review the types of corrosion encountered in structural steel elements:

Pitting corrosion: As shown in Fig. 2, The anodic areas form a corrosion pit. This can occur with mild steel immersed in water or soil. This common type of corrosion is essentially due to the presence of moisture aided by improper detailing or constant exposure to alternate wetting and drying. This form of corrosion could easily be tackled by encouraging rapid drainage by proper detailing and allowing free flow of air, which would dry out the surface.

Crevice corrosion: This again is due to improper detailing where the tops of the crevices become localised anodes and corrosion occurs at this point. The principle of crevice corrosion is exemplified in Fig. 3. The oxygen content of water trapped in a crevice is less than that of water, which is exposed to air. Because of this the crevice becomes anodic with respect to surrounding metal and hence the corrosion starts inside the crevice.

Bimetallic corrosion: When two dissimilar metals (for e.g. Iron and Aluminium) are joined together in an electrolyte, an electrical current passes between them and the corrosion occurs. This is because, metals in general could be arranged, depending on their electric potential, into a table called the 'galvanic series'. The farther the metals in the galvanic series, the greater the potential differences between them causing the anodic metal to corrode. A common example is the use of steel screws in stainless steel members and also using steel bolts in aluminium members. This type of bi-metallic corrosion is easy to spot and understand. Obviously such a contact between dissimilar metals should be avoided in detailing.

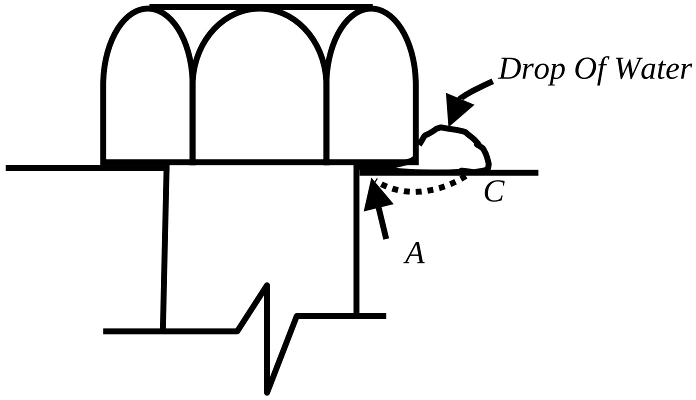


Fig.3 Mechanism of crevice corrosion

Stress corrosion: This occurs under the simultaneous influence of a static tensile stress and a specific corrosive environment. Stress makes some spots in a body more anodic (especially the stress concentrated zones) compared with the rest as shown in Fig. 4. The crack tip in Fig. 4 is the anodic part and it corrodes to make the crack wider. This corrosion is not common with ferrous metals though some stainless steels are susceptible to this.

Fretting corrosion: If two oxide coated films or rusted surfaces are rubbed together, the oxide film can be mechanically removed from high spots between the contacting surfaces as shown in Fig. 5.

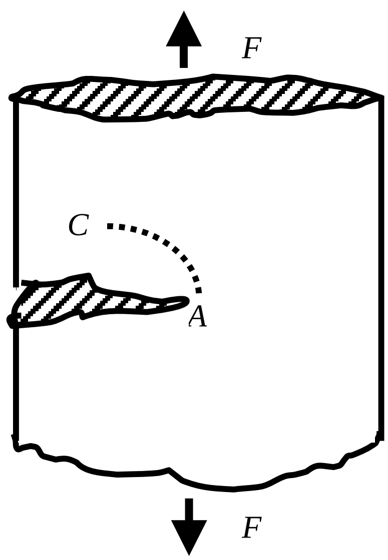


Fig. 4 Mechanism of stress corrosion

These exposed points become active anodes compared with the rest of the surfaces and initiate corrosion. This type corrosion is common in mechanical components.

Bacterial corrosion: This can occur in soils and water as a result of microbiological activity. Bacterial corrosion is most common in pipelines, buried structures and offshore structures.

Hydrogen embrittlement: This occurs mostly in fasteners and bolts. The atomic hydrogen may get absorbed into the surface of the fasteners. When tension is applied to these fasteners, hydrogen will tend to migrate to points of stress concentration. The pressure created by the hydrogen creates and/or extends a crack. The crack grows in subsequent stress cycles. Although hydrogen embrittlement is usually included in the discussion about corrosion, actually it is not really a corrosion phenomenon.

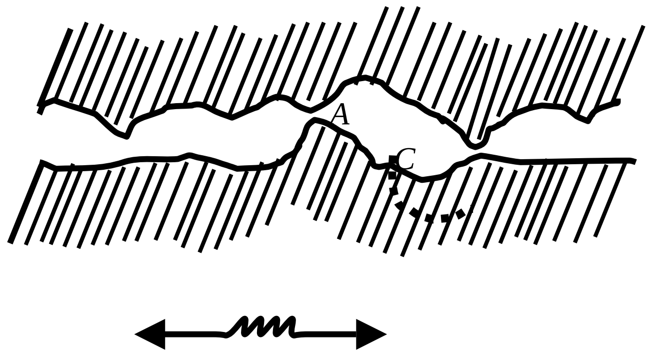


Fig. 5 The mechanism of fretting corrosion

3.0 CORROSION PROTECTION TO STRUCTURAL STEEL ELEMENTS

Taking care of the following points can provide satisfactory corrosion protection to most structural steel elements:

- Avoiding of entrapment and accumulation of moisture and dirt in components and connections by suitable detailing as shown in Fig. 6

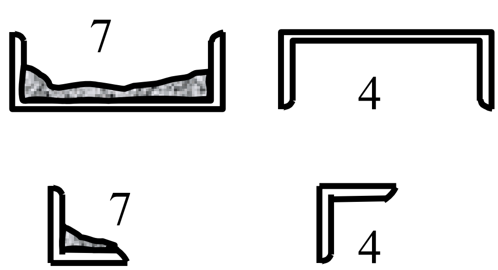


Fig.6 Simple orientation of members to avoid dirt and water entrapment

- Avoiding contact with other materials such as bimetallic connections, as explained in the earlier section.
- Detailing the structural steel work to enhance air movement and thereby keeping the surfaces dry as shown in Fig.7
- Providing suitable drain holes wherever possible to initiate easy draining of the entrapped water as shown in Fig. 8

- Providing suitable access to all the components of steel structures for periodic maintenance, cleaning and carrying out inspection and maintenance at regular intervals.
- Providing coating applications to structural steel elements. Metallic coatings such as hot-dip galvanising, metal spray coatings, etc. are very effective forms of corrosion protection. Cleaning of the surfaces and applying suitable paints is the most commonly used and reliable method of corrosion protection. This is discussed in detail in the next section.

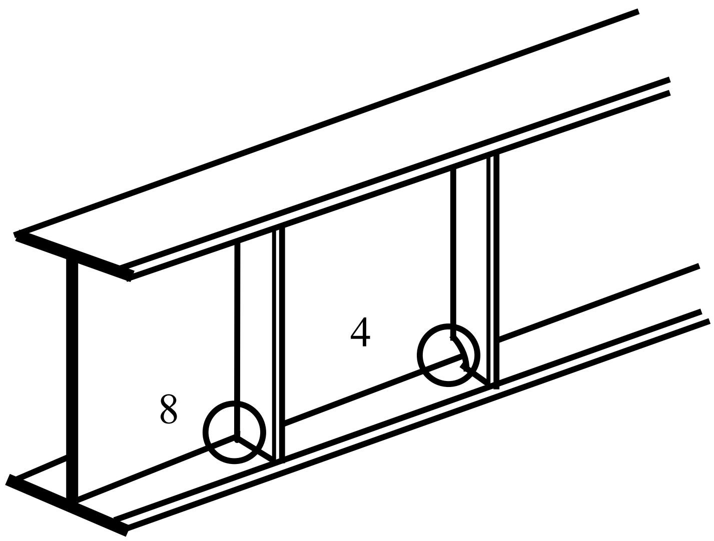


Fig.7 Detailing to enhance air movement between joints

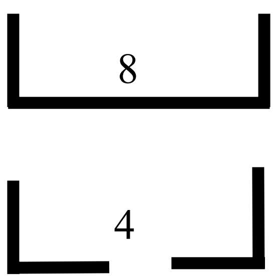


Fig.8 Provision of drain holes wherever possible.

3.1 Surface preparation

Before applying any protective coating to structural steel work, it is very essential that the surface must be free of dirt and other materials that would affect its adhesion. In this section we review the surface preparation methods which are commonly employed in structural steel work.

Structural steel comes out of the mill with a mill scale on its surface. On weathering, water penetrates into the fissures of the mill scale and rusting of the steel surface occurs. The mill scale loses its adhesion and begins to shed. Mill scale therefore needs to be removed before any protection coatings are applied. The surface of steel may also contain dirt or other impurities during storage, transportation and handling. The various surface preparation methods are briefly explained below.

***Manual preparation*:** This is a very economical surface cleaning method but only 30% of the rust and scale may be removed. This is usually carried out with a wire brush.
Mechanical preparation: This is carried out with power driven tools and up to 35% cleaning can be achieved. This method is quite fast and effective.
***Flame cleaning**:* In this process an Oxy-gas flame causes differential thermal expansion and removes mill scale more effectively.
***Acid pickling**:* This involves the immersion of steel in a bath of suitable acids to remove rust. Usually this is done before hot dip galvanising (explained in the next section).
***Blast cleaning**:* In this process, abrasive particles are projected at high speed on to the steel surface and cleaning is effected by abrasive action. The common blast cleaning method is the 'sand blasting'. However in some states of India, sand blasting is not allowed due to some environmental reasons.

3.2 Preventive coatings

The principal protective coatings applied to structural steel work are paints, metal coatings or combination of these two. Paints basically consist of a pigment, a binder and solvent. After the paint has been applied as a wet film, the solvent evaporates leaving the binder and the pigment on the surface. In codes of practices relating to corrosion protection, the thickness of the primer, the type of paints and the thickness of the paint in term of microns are specified depending upon the corrosive environment. The codes of practice also specify the frequency with which the change of paint is required. Metal coatings on structural steel work are almost either zinc or aluminium. Hot dip Zinc coatings known as "galvanising", involves dipping of the steelwork into a bath of molten Zinc at a temperature of about 450^0C. The work piece is first degreased and cleaned by pickling to enhance the wetting properties. Sometimes hot dip aluminising is also done. Alternatively, metal coating could also be applied using metal spraying.

3.3 Weathering steels

To protect steel from corrosion, some countries produce steels which by themselves can resist corrosion. These steels are called as ***"weathering steels or Corten steels"***. Weathering steels are high strength alloy weldable structural steels, which possess excellent weathering resistance in many non-polluted atmospheric conditions. They contain up to 3% of alloying elements such as chromium, copper, nickel, phosphorous, etc. On exposure to air, under suitable conditions, they form adherent protective oxide coatings. This acts as a protective film, which with time and appropriate conditions causes the corrosion rate to reduce until it is a low terminal level. Conventional coatings are, therefore, not usually necessary since the steel provides its own protection.

Weathering steels are 25% costlier than the mild steel, but in many cases the total cost of the structure can be reduced if advantage is taken of the 30% higher yield strength compared to mild steel.

3.4 Where does corrosion matter in structural steel work?

The corrosion of steel in a dry interior environment is virtually insignificant. For example, structural steel work in the interiors of offices, shops, schools, hostels, residences, airport terminals, hospitals etc. will not corrode noticeably during the expected 50-year life of the structure. Hence ***in these situations no protective coating is required and the structural steel work may be left exposed.*** Only when the structural steel work is exposed to moisture in an interior environment such as kitchens, sports halls etc. a little attention is needed in the detailing of the steel work and also thin protective coatings. Structural steel work will need protective coatings in slightly intensive corrosive environment such as some industrial buildings, dairies, laundries, breweries etc. The above mentioned situations can be termed as 'low to medium' risk categories. Structural steel work exposed to high humidity and atmosphere, chemical plants, foundries, steel bridges, offshore structures would fall into the "high risk" category. Structural steel work that is categorised into high-risk group requires better surface preparation and sufficient thickness of the anti-corrosive paints. As we review the protective coatings such as the paints available in the market to-day many of the paints can perform very satisfactorily for 5-7 years. Specially prepared epoxy paints when applied in sufficient thickness after a good surface preparation, can last as high as 20 years!! Corrosion of steel is no longer the major problem that it once was and the protective methods no longer pose any major disincentive for using steel in the building industry. For the purpose of selecting a suitable paint system, if appropriate, the risk groups of structural steel work are classified according to their location and their intended service; however the same classification can also be done depending on the exterior environment of the structural steel work as in Table.1

Table 1 Exterior environment and corrosion risk (Source: British Steel)

No.	Exterior Environment	Areas appropriate	Corrosion risk
1.	Normal Inland	Most rural and urban areas	Low
2.	Polluted Inland	High airborne sulphur dioxide	Significant
3.	Normal Coastal	As normal inland plus high airborne salt levels	High
4	Polluted Coastal	As polluted inland plus high airborne salt levels	Very high

In the aggressive environment such as the cases 2,3 and 4 in Table 1, appropriate technologies are available to counter corrosion. There is a range of corrosion protection methods, depending upon the environment and desired life of the protection method, the details of which are presented briefly in Table 2. Expert help should be obtained when the corrosion risk is "high" or "very high".

3.5 Summary of corrosion prevention methods

The mechanism of corrosion and the possible ways of its prevention has been discussed in the foregoing sections. The following are the three broad categories of corrosion prevention methods.

I. As mentioned earlier, corrosion does not occur in the absence of water. Corrosion protection can be achieved by a number of methods (e.g.)
   (a) Application of coatings to separate the metal from its environment.
   (b) Avoiding exposure to moisture and air.
   (c) Attention to detailing of the structures to encourage rapid drainage of water.

II Corrosion does not occur in the absence of Oxygen and water. This can be achieved by
   (a) Deaeration of water
   (b) De-humidification of the atmosphere
   (c) Application of certain surface coatings

III Corrosion does not occur if the basic electro-chemical reaction is suppressed
   (a) The use of corrosion inhibitors would suppress either anodic or cathodic reactions and hence the corrosion is prevented.
   (b) The other method is the application of cathodic protection, which floods the surface with free electrons and prevents formation of anodes.

4.0 STEEL STRUCTURES SUBJECTED TO FIRE

In this section a brief review of aspects of structural steel work subjected to fire is given. The strength of all engineering materials reduces as their temperature increases.

Steel is no exception. However, a major advantage of steel is that it is incombustible and it can fully recover its strength following a fire, most of the times. Fire represents a transfer of energy from a stable condition to a transient condition as combustion occurs. The common examples of fire that affects structural systems are burning of office furniture, books, and contents of filing cabinet or other materials. During the fire steel absorbs a significant amount of thermal energy. After this exposure to fire, steel returns to a stable condition after cooling to ambient temperature. During this cycle of heating and cooling, individual steel members may become slightly bent or damaged, generally without affecting the stability of the whole structure. From the point of view of economy, a significant number of steel members may be salvaged following a post-fire review of a fire affected steel structure. Using the principle *" If the member is straight after exposure to fire – the steel is O.K"*, many steel members could be left undisturbed for the rest of their service life. Steel members which have slight distortions may be made dimensionally reusable by simple straightening methods and the member may be put to continued use with full expectancy of performance with its specified mechanical properties. The members which have become unusable due to excessive deformation may simply be scrapped. In effect, it is easy to retrofit steel structures after fire.

Table 2 Corrosion protection treatment in External environment

Shop applied treatments						
	Option 1	**Option 2**	**Option 3**	**Option 4**	**Option 5**	**Option 6**
Surface preparation	Blast clean	Blast clean	Blast clean	Blast clean	Grit blast	Blast clean
Pre fabrication primer	Zinc phosphate epoxy	2 pack Zinc rich epoxy	-----	2 pack Zinc rich epoxy	----	Ethyl Zinc Silicate
Post fabrication primer	High build Zinc phosphate modified alkyd	2 pack Zinc rich epoxy	Hot dip galvanise	2 pack Zinc rich epoxy	Sprayed Zinc or Sprayed Aluminum	Ethyl Zinc Silicate
Intermediate coat	----	High build Zinc phosphate		2 pack epoxy Micaceous iron oxide	Sealer	Chlorinated rubber alkyd
Top coat	----	----	----	2 pack epoxy Micaceous Iron oxide	Sealer	----
Site applied treatments						
Surface preparation	As necessary	As necessary	No site treatment	As necessary	No site treatment	As necessary
Primer	Touch in	Touch in	----	----	----	Touch in
Intermediate coat	----	Modified Alkyd Micaceous Iron Oxide	----	Touch in	---	High build Micaceous Iron oxide chlorinated rubber
Top coat	High build Alkyd finish	Modified Alkyd Micaceous Iron Oxide	----	High build chlorinated rubber	----	High build Micaceous Iron oxide chlorinated rubber
Expected life in years						
Normal Inland	12	18	20	(+ -) 20	(+ -) 20	20+
Polluted Inland	10	15	12	(+ -) 18	(+ -) 15-20	20+
Normal coastal	10	12	20	(+ -) 20	(+ -) 20	20+
Polluted coastal	8	10	10	(+ -) 15	(+ -) 15-20	20+

In the case of concrete exposed to fire, it will start changing its colour to pink at about 285^0C and will turn into deep red at about 590^0C. Soon after that, concrete would turn into quartz aggregate and spalling would start. The degree of spalling is dependent upon the rate of temperature rise, moisture content and maximum temperature for each type of aggregate. Hence it seen that concrete exposed to fire beyond say 600^0C, may undergo an irreversible degradation in mechanical strength unlike steel where much of its original strength is regained. The above points underline the advantage of steel in terms of economy even in the case of fire.

4.1 Fire loads and fire rating of steel structures

The term 'fire load' in a compartment of a structure is the maximum heat that can be theoretically generated by the combustible items and contents of the structure. The fire load could be measured as the weight of the combustible material multiplied by the calorific value per unit weight. Fire load is conveniently expressed in terms of the floor space as MJ/m^2 or Mcal/m^2. More often it would be expressed in terms of equivalent quantity of wood and expressed as Kg wood / m^2 (1 Kg wood = 18MJ). The commonly encountered fire loads are presented in Table 3. The values are just an indication of the amount of fire load and the values may change from one environment to the other and also from country to country.

Table 3 Fire load on steel structures

Examples of fire load in various structures	
Type of steel structure	***Kg wood / m^2***
School	15
Hospital	20
Hotel	25
Office	35
Departmental store	35
Textile mill show room	>200

The fire rating of steel structures are expressed in units of time ½, 1, 2, 3 and 4 hours etc. The specified time neither represents the time duration of the real fire nor the time required for the occupants to escape. The time parameters are basically a convenient way of comparative grading of buildings with respect to fire safety.

Basically they represent the endurance of structural steel elements under standard laboratory conditions. Fig. 9 represents the performance of protected and unprotected steel in a laboratory condition of fire. The rate of heating of the unprotected steel is obviously quite high as compared to the fire-protected steel. We shall see in the following sections that these two types of fire behaviour of steel structure give rise to two different philosophies of fire design. The time equivalence of fire resistance for steel structures or the fire rating could be calculated as

$$T_{eq}(Minutes) = CWQ_f \qquad (1)$$

where Q_f is the fire load MJ/m^2 which is dependent on the amount of combustible material, 'W' is the ventilation factor relating to the area and height and width of doors and windows and '*C*' is a coefficient related to the thermal properties of the walls, floors and ceiling. As an illustration, the "*W*" value for a building with large openings could be chosen as 1.5 and for highly insulating materials "*C*" value could be chosen as 0.09.

4.2 Mechanical properties of steel at elevated temperatures

We need to know about the mechanical properties of steel at elevated temperatures in the case of fire resistant design of structural steel work. Hence in this section we review the important mechanical aspects of steel at elevated temperatures. The variations of the non-dimensional modulus of elasticity, yield strength and coefficient of thermal expansion with respect to temperature are shown in Fig. 10. The corresponding equations are given below. The variation of modulus of elasticity ratio $\overline{E}$ with respect to the corresponding value at 20^{0}C, with respect to temperature T is given by

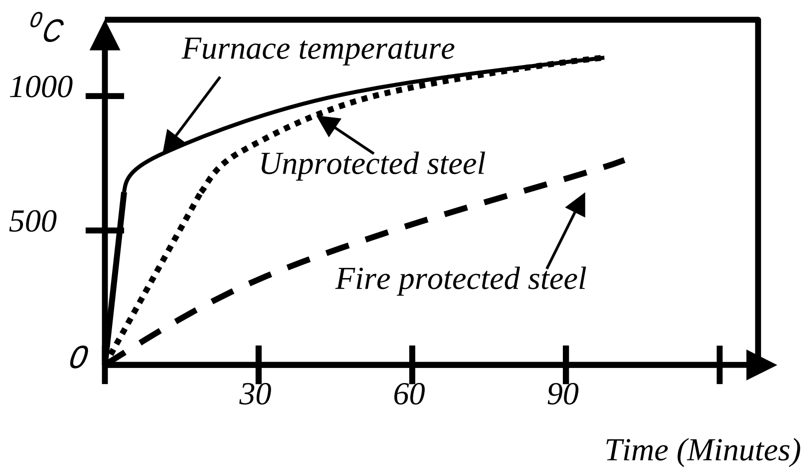


Fig. 9 Rate of heating of structural steel work

$$\bar{E} = \frac{E(T)}{E(20^0 C)} = 1.0 + \frac{T}{2000 \ln\left[\frac{T}{1100}\right]} \qquad for \quad 0^0 C < T < 600^0 C \qquad (2)$$

$$= \frac{690(1.0 - \frac{T}{1000})}{T - 53.5} \qquad for \quad 600^0 C < T < 1000^0 C$$

The yield stress of steel remains unchanged up to a temperature of about 215^{0}C and then loses its strength gradually. The yield stress ratio $\bar{f}$ (with respect to yield stress at 20^{0}C) vs. temperature T relation is given by

$$\bar{f} = \frac{f_y(T)}{f_y(20)} = 1.0 \qquad 0^0 C < T < 215^0 C$$

$$= \frac{905 - T}{690} \qquad 215^0 C < T < 905^0 C \tag{3}$$

Similarly the coefficient of thermal expansion α also varies with temperature by a simple relation

$$\alpha(T) = (12.0 + \frac{T}{100}) x10^{-6} \quad ({}^0C)^{-1} \tag{4}$$

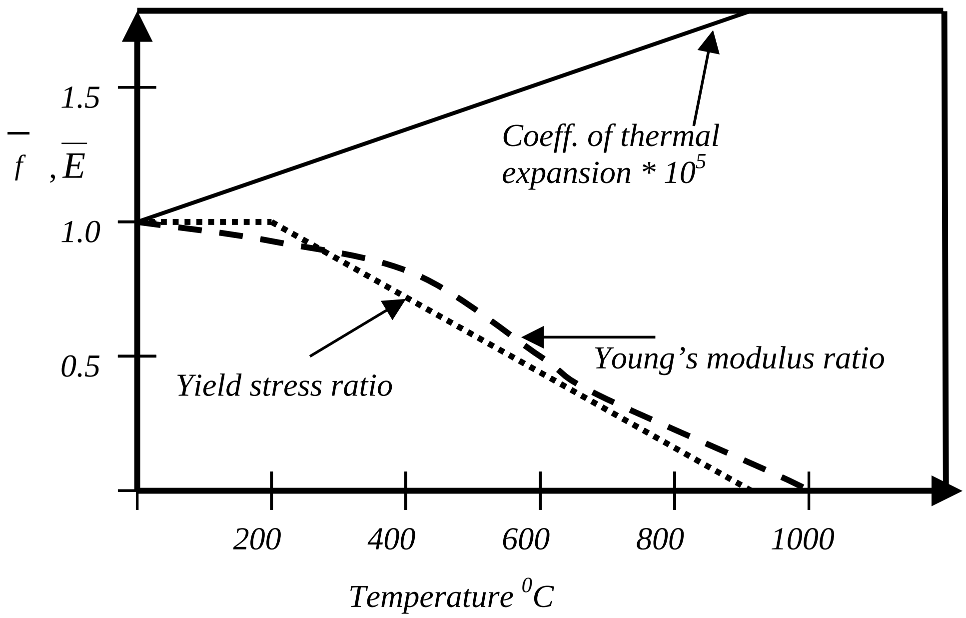


Fig.10 Mechanical properties of steel at elevated temperatures

These equations are very useful when one is interested in the analysis of steel structures subjected to fire.

In the codes of practice for steel structures subjected to fire, strength curves are generally provided for structural steel work at elevated temperatures. In these curves the strain at which the strength is assessed in an important parameter. For example the BS: 5950 part 8 has used 1.5% strain as the strain limit as against 2% for Eurocode 3 Part10. A lower strain of 0.5% may be used for columns or components with brittle fire protection materials.

4.3 Fire resistant steel

Fire safety in steel structures could also be brought about by the use of certain types of steel, which are called ‘Fire Resistant Steels (FRS)’. These steels are basically thermo-mechanically treated (TMT) steels which perform much better structurally under fire than the ordinary structural steels. These steels have the ferrite – pearlite microstructure of ordinary structural steels but the presence of Molybdenum and Chromium stabilises the microstructure even at 600^0C. The composition of fire resistant steel is presented in Table.4

Table 4 Chemical composition of fire resistant steel

	C	**Mn**	**Si**	**S**	**P**	**Mo + Cr**
FRS	≤0.20%	≤1.50%	≤0.50%	≤0.040%	≤0.040%	≤1.00%
Mild Steel	≤0.23%	≤1.50%	≤0.40%	≤0.050%	≤0.050%	-

The fire resistant steels exhibit a minimum of two thirds of its yield strength at room temperature when subjected to a heating of about 600^0C. In view of this, there is an innate protection in the steel for fire hazards. Fire resistant steels are weldable without pre-heating and are commercially available in the market as joists, channels and angles.

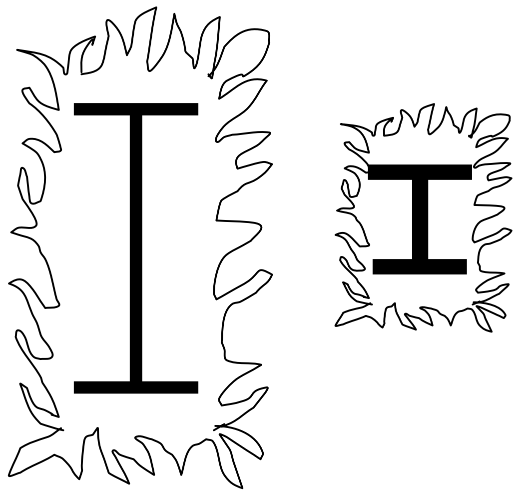
4.4 Fire engineering of steel structures

The study of steel structures under fire and its design provision are known as ‘fire engineering’. The basic idea is that the structure should not collapse prematurely without giving adequate time for the occupants to escape to safety. As briefly outlined earlier, there are two ways of providing fire resistance to steel structures. In the first method of fire engineering, the structure is designed using ordinary temperature of the material and then the important and needed members may be insulated against fire. For the purpose of fire protection the concept of ‘***section factor’*** is used. In the case of fire behaviour of structures, an important factor which affects the rate of heating of a given section, is the section factor which is defined as the ratio of the perimeter of section exposed to fire (H_p) to that of the cross-sectional area of the member (A). As seen from Fig. 11, a section, which has a low (H_p/A) value, would normally be heated at a slower rate than the one with high (H_p/A) value, and therefore achieve a higher fire resistance. Members with low H_p/A value would require less insulation. For example sections at the heavy end (deeper sections) of the structural range have low H_p/A value and hence they have slow heating rates. The section factor can be used to describe either protected or unprotected steel. The section factor is used as a measure of whether a section can be used without fire protection and also to ascertain the amount of protection that may be required. Typical values of H_p of some fire-protected sections are presented in Fig. 12.

In the second method of fire engineering, the high temperature property of steel is taken into account in design using the Equations 2,3 and 4. If these are taken into account in the design for strength, at the rated elevated temperature, then no insulation will be required for the member. The structural steel work then may be an unprotected one.

There are two methods of assessing whether or not a bare steel member requires fire protection.

The first is the load ratio method which compares the 'design temperature' i.e. maximum temperature experienced by the member in the required fire resistance time, and the 'limiting temperatures', which is the temperature at which the member fails.



High H_p / A Value *Low H_p / A Value*

Fig.11 The section factor concept

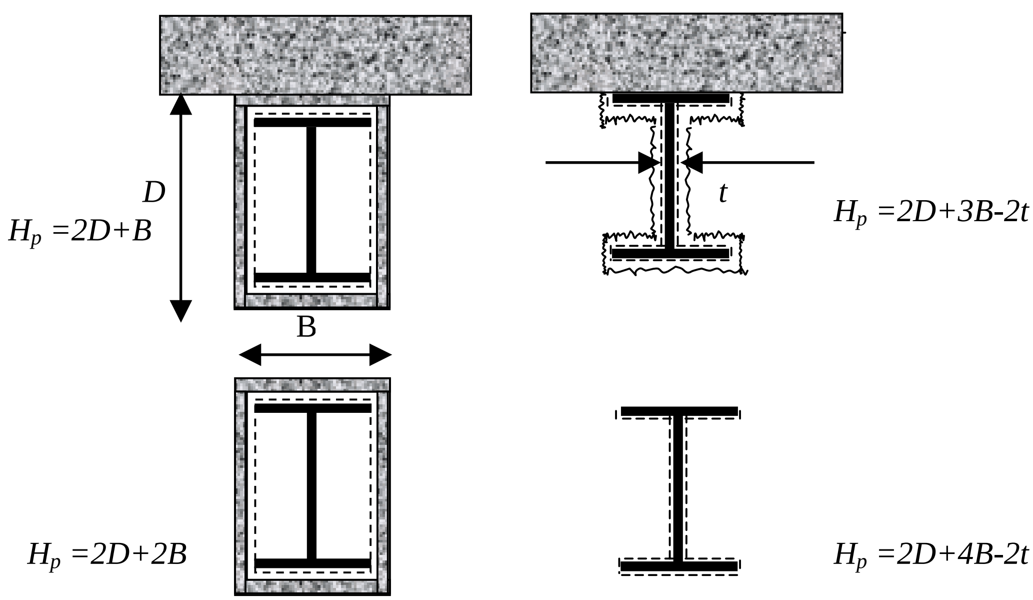


Fig. 12 Some typical values of H_P of fire protected steel sections

The limiting temperatures for various structural members are available in the relevant codes of practice. The load ratio may be defined as:

$$\text{Load ratio} = \frac{\text{Load applied at the fire limit state}}{\text{Load causing the member to fail under normal conditions}}$$

If the load ratio is less than 1, then no fire protection is required. In the second method, which is applicable to beams, the moment capacity at the required fire resistance time is compared with the applied moment. When the moment capacity under fire exceeds the applied moment, no fire protection is necessary

4.5 Methods of fire protection

Fire protection methods are basically dependent on the fire load, fire rating and the type of structural members. The commonly used fire protection methods are briefly enumerated below.

Spray protection: The thickness of spray protection depends on the fire rating required and size of the job. This is a relatively low cost system and could be applied rapidly. However due to its undulating finish, it is usually preferred in surfaces, which are hidden from the view.

Board protection: This is effective but an expensive method. Board protection is generally used on columns or exposed beams. In general no preparation of steel is necessary prior to applying the protection.

Intumescent coating: These coatings expand and form an insulating layer around the member when the fire breaks out. This type of fire protection is useful in visible steelwork with moderate fire protection requirements. This method does not increase the overall dimensions of the member. Certain thick and expensive intumescent coatings will give about 2-hour fire protection. But these type of coatings require blast cleaned surface and a priming coat.

Concrete encasement: This used to be the traditional fire proofing method but is not employed in structures built presently. The composite action of the steel and concrete can provide higher load resistance in addition to high fire resistance. However this method results in increases dead weight loading compared to a protected steel frame. Moreover, carbonation of concrete aids in encouraging corrosion of steel and the presence of concrete effectively hides the steel in distress until it is too late.

5.0 FATIGUE OF STEEL STRUCTURES

A component or structure, which is designed to carry a single monotonically increasing application of static load, may fracture and fail if the same load or even smaller load is, applied cyclically a large number of times. For example a thin rod bent back and forth beyond yielding fails after a few cycles of such repeated bending. This is termed as the 'fatigue failure'. Examples of structures, prone to fatigue failure, are bridges, cranes, offshore structures and slender towers, etc., which are subjected to cyclic loading.

The fatigue failure is due to progressive propagation of flaws in steel under cyclic loading. This is partially enhanced by the stress concentration at the tip of such flaw or crack. As we can see from Fig. 13, the presence of a hole in a plate or simply the presence of a notch in the plate has created stress concentrations at the points '*m*' and '*n*'.

The stress at these points could be three or more times the average applied stress. These stress concentrations may occur in the material due to some discontinuities in the material itself. These stress concentrations are not serious when a ductile material like steel is subjected to a static load, as the stresses redistribute themselves to other adjacent elements within the structure.

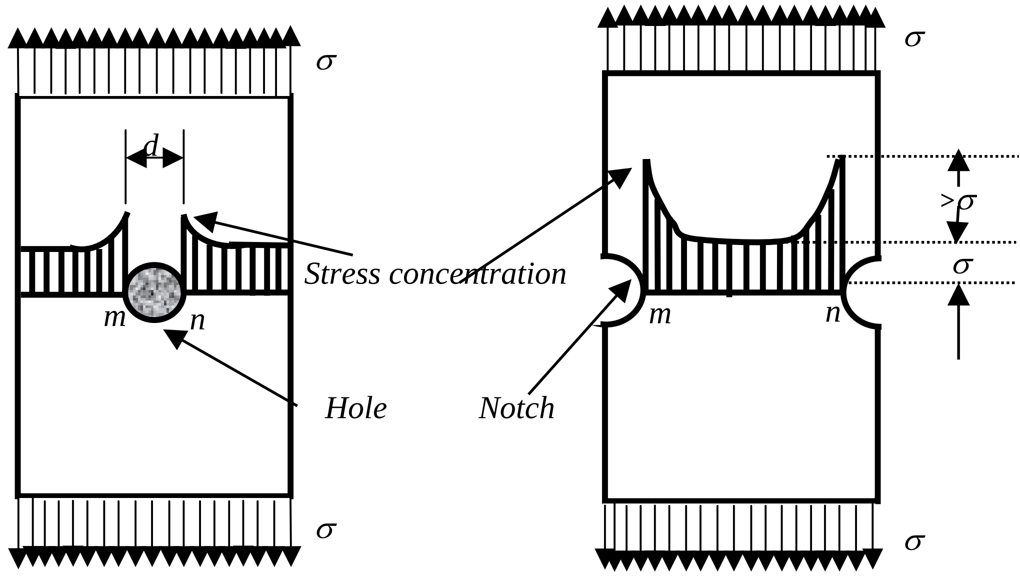


Fig. 13 Stress concentrations in the presence of notches and holes

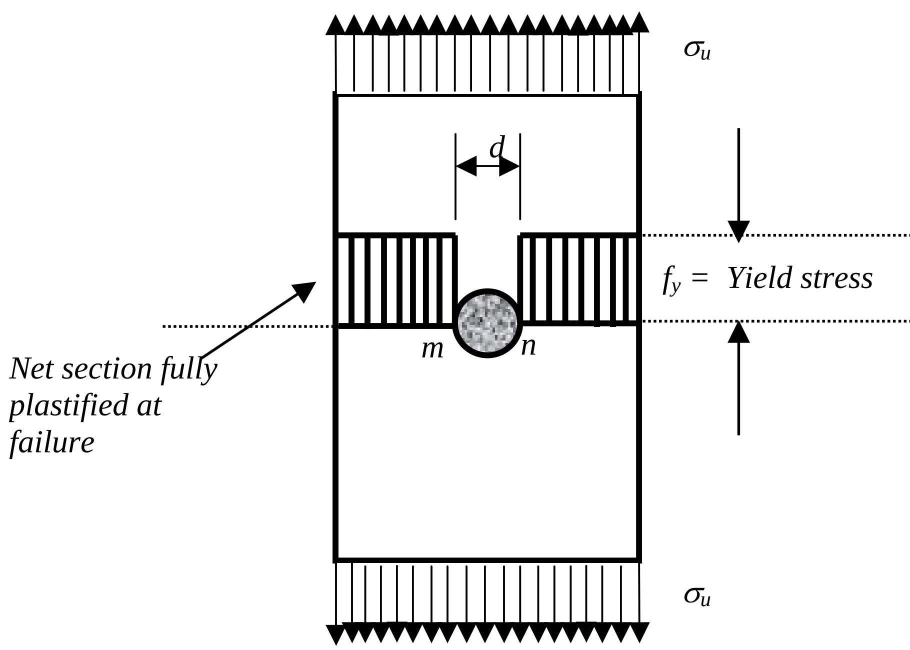


Fig. 14 Stress pattern at the point of static failure

At the time of static failure, the average stress across the entire cross section would be the yield stress as shown in Fig.14. However when the load is repeatedly applied or the load fluctuates between tension and compression, the points *m*, *n* experience a higher range of stress reversal than the applied average stress. These fluctuations involving higher stress ranges, cause minute cracks at these points, which open up progressively and spread with each application of the cyclic load and ultimately lead to rupture.

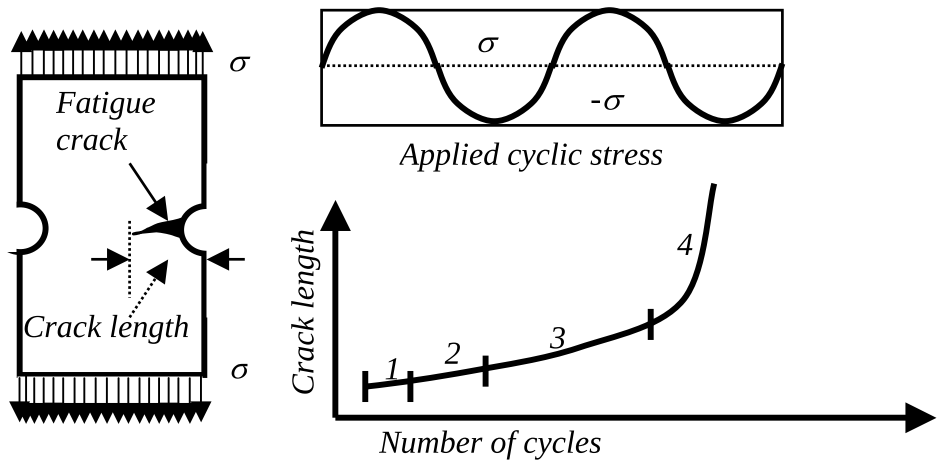


Fig.15 Crack growth and fatigue failure under cyclic load

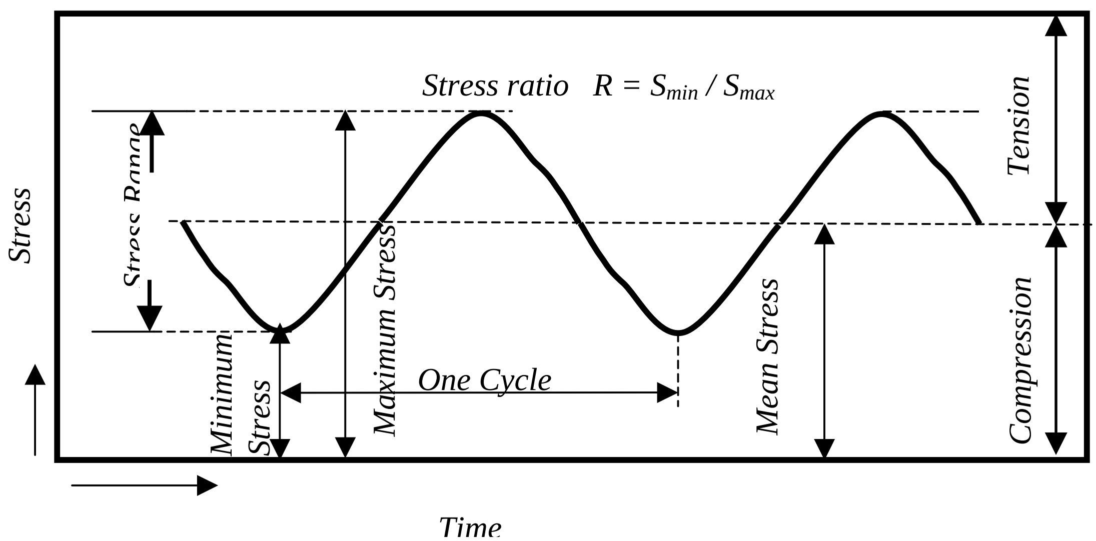


Fig.16 Terminology used in fatigue resistant design of structural steel work

The fatigue failure occurs after four different stages, namely:

1. Crack initiation at points of stress concentration
2. Crack growth
3. Crack propagation
4. Final rupture

The development of fatigue crack growth and the various stages mentioned above are symbolically represented in Fig. 15. Fatigue failure can be defined as the number of cycles and hence time taken to reach a pre-defined or a threshold failure criterion. Fatigue failures are classified into two categories namely the high cycle and low cycle fatigue failures, depending upon the number of cycles necessary to create rupture. Low cycle fatigue could be classified as the failures occurring in few cycles to a few tens of thousands of cycles, normally under high stress/ strain ranges. High cycle fatigue requires about several millions of cycles to initiate a failure. The type of cyclic stresses applied on structural systems and the terminologies used in fatigue resistant design are illustrated in Fig. 16.

5.1 S-N Curves and fatigue resistant design

The common form of presentation of fatigue data is by using the S-N curve, where the total cyclic stress (S) is plotted against the number of cycles to failure (N) in logarithmic scale. A typical S-N curve is shown in Fig. 17.

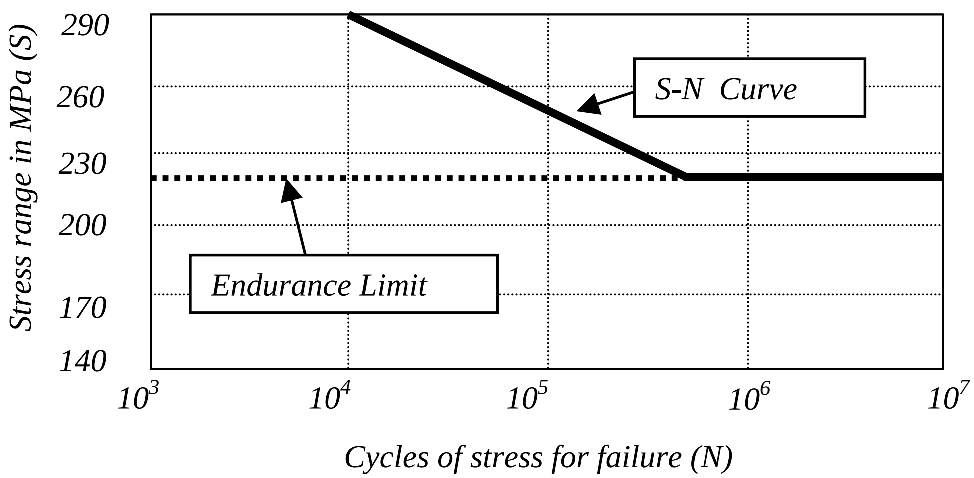


Fig. 17 S-N diagram for fatigue life assessment

It is seen from Fig. 17 that the fatigue life reduces with respect to increase in stress range and at a limiting value of stress, the curve flattens off. The point at which the S-N curve flattens off is called the 'endurance limit'. To carry out fatigue life predictions, a linear fatigue damage model is used in conjunction with the relevant S-N curve. One such fatigue damage model is that postulated by Wohler as shown in Fig. 17. The relation between stress and the number of cycles for failure could be written as

$$log\, N = log\, C - m\, log\, S \qquad (5)$$

where 'N' is the number of cycles to failure, 'C' is the constant dependant on detailing category, 'S' is the applied constant amplitude stress range and 'm' is the slope of the S-N curve. For the purpose of design it is more convenient to have the maximum and minimum stresses for a given life as the main parameters. For this reason the modified Goodman diagram, as shown in Fig. 18, is mostly used. The maximum stresses are plotted in the vertical ordinate and minimum stresses as abscissa. The line OA represents alternating cycle (R = -1), line OB represents pulsating cycle (R = 0) and OC the static load (*R* = 1). Different curves for different values of fatigue life 'N' can be drawn through point '*C*' representing the fatigue strength for various numbers of cycles. The vertical distance between any point on the '*N*' curve and the 45^0 line OC through the origin represents the stress range. As discussed earlier, the stress range is the important parameter in the fatigue resistant design. Higher the stress range a component is subjected to, lower would be its fatigue life and lower the stress range, higher would be the fatigue life.

5.2 Fatigue resistant design of structural steel work

It is seen from practical experiences that most of the fatigue failures are due to improper detailing rather than an inadequate design of the member for strength. Let us consider a lap joint using fillet weld as shown in Fig. 19. From the schematic stress diagram it is seen that the fillet weld toe becomes a point of stress concentration. As a result, if the joint is subjected to cyclic loads, the weld toe experiences a variation of larger stress range compared to the parent member. Hence, a crack may be initiated at the weld toe where there is stress concentration. This stress concentration can be eliminated by using a butt welded joint, ground flush with the plate surface.

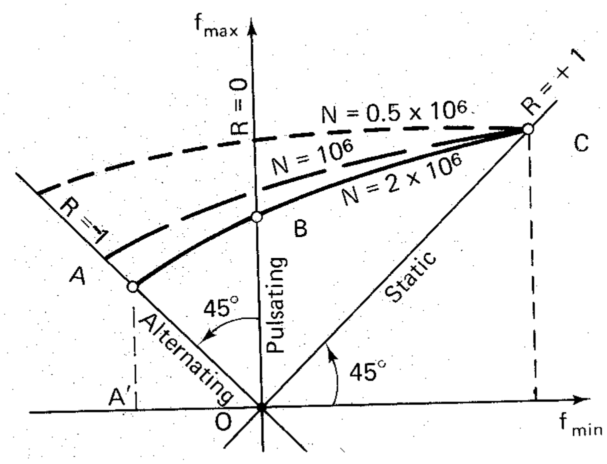


Fig.18 Modified Goodman diagram for fatigue resistant design of steel structures

It becomes very important to avoid any local structural discontinuities and notches by good design and this is the most effective means of increasing fatigue life. Where a structure is subjected to fatigue, it is important that welded joints are considered carefully. Indeed, weld defects and poor weld details are the major contributors of fatigue failures. The fatigue performance of a joint can be enhanced by the use of techniques such as proper weld geometry, improvements in welding methods and better weld quality control using non-destructive testing (NDT) methods. The following general points are important for the design of a welded structure with respect of fatigue strength: (a) use butt welds instead of fillet welds (b) use double sided welds instead of single sided fillet welds (c) pay attention to the detailing which may cause stress concentration and (d) in very important details subjected to high cyclic stresses use any non-destructive testing (NDT) method to ensure defect free details. From the point of view of fatigue design, the codes of practice classify various structural joints and details depending upon their vulnerability to fatigue cracks. For example, IS: 1024 classifies the detailing in the structural steel work in seven classes viz., A, B, C, D, E, F and G depending upon their vulnerability to stress concentrations. A typical detailing classified as 'E' is shown in Fig. 20. This class 'E' applies to members fabricated with full cruciform butt welds. Similarly, the class 'F' is applicable for members with 'T' type full penetration butt welds, members connected by transverse load – carrying fillet welds and members with stud shear connectors in composite sections. Such a typical detailing is shown in Fig. 21. The IS: 1024 (1968) provides allowable stress tables for all the classifications from A-G for different stress ratios of $R = F_{min}/F_{max}$ and different life (number of cycles N). Using these tables the allowable stress for a given life time may be linearly interpolated and the life time for a given allowable stress could be logarithmically interpolated. The accuracy of any fatigue life calculation is highly dependent on a good understanding of the expected loading sequence during the whole life of a structure. Once a global load pattern has been developed, then a more detailed inspection of particular area of a structure where the effects of loading may be more important called the 'hot spot stresses' which are basically the areas of stress concentrations.

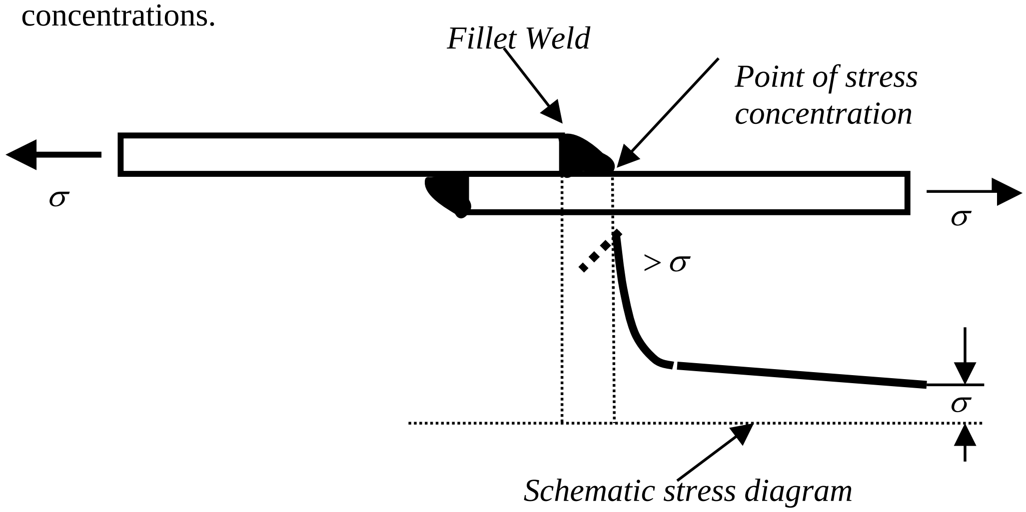


Fig. 19 Stress concentration at the weld toe

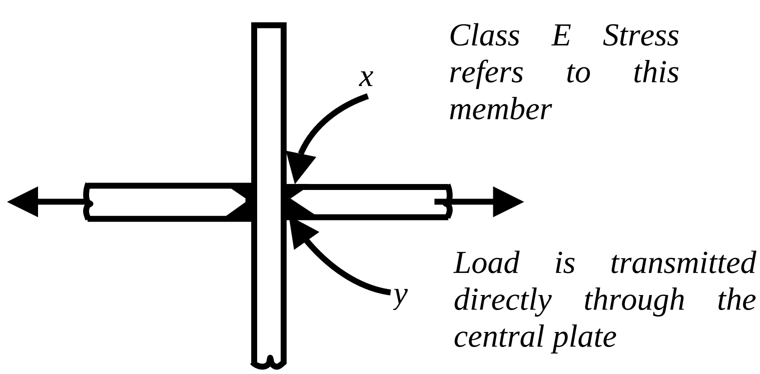


Fig. 20 Class 'E' detailing according to IS: 1024 (1968)

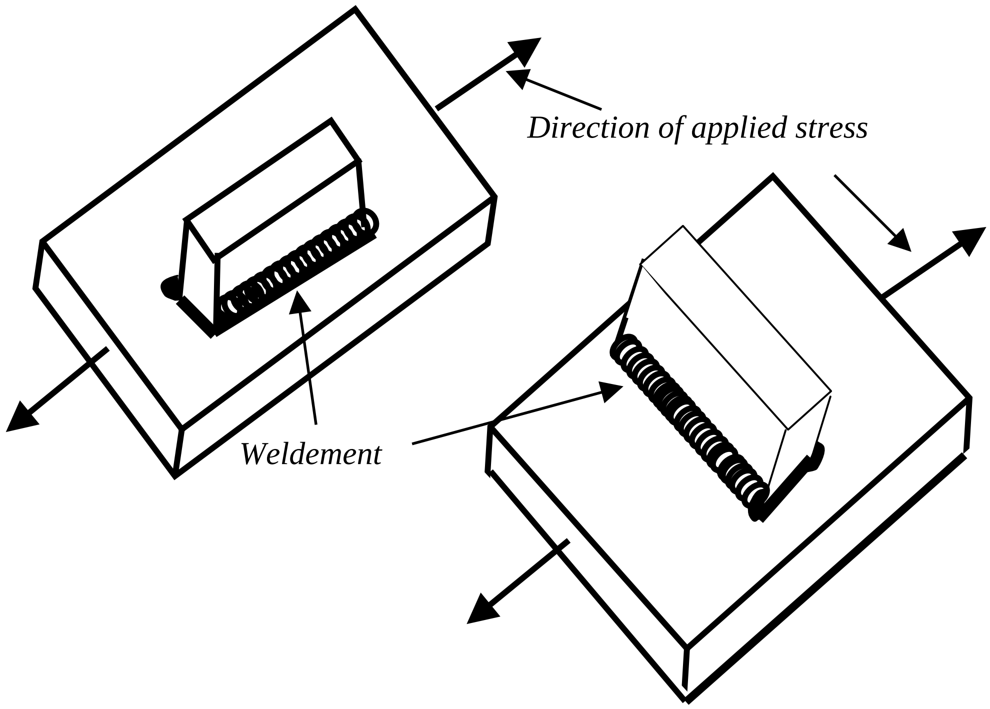


Fig. 21 Class 'F' detailing according to IS: 1024 (1968)

6.0 SUMMARY

In this chapter the three important aspects of structural steel work viz. the corrosion, fire protection, fatigue behaviour have been reviewed. Aspects of corrosion, its mechanism and means of protection of structural steel work have been discussed briefly. It was shown that the risk to structural steel work by corrosion could be effectively handled using the presently available technology. Aspects of fire resistant design of steel structures were also reviewed. Finally the fatigue failure of structural steel work and the importance of detailing in its prevention have been discussed.

7.0 FURTHER READING

1. Adams P.F., Krentz H.A. "Limit State Design in Structural Steel – SI Units", Canadian Institute of Steel Construction (1979).
2. Doran D.K., "Construction Materials Reference Book", Butterworth Heinemann (1995).
3. Graham W. Owens and Peter R. Knowles, "Steel Designer's Manual", ELBS fifth Edition (1994).
4. Jack C. McCormac, "Structural Steel Design", Harper & Row Publishers, NY (1981).
5. John H. Bickford , " An introduction to the design and behaviour of bolted joints",(Second Edition), Marcel Dekker Inc., NY,(1990)
6. Radaj D, "Design and analysis of fatigue resistant welded structures", Abington Publishing, (1990).
7. IS: 1024 – 1968, Code of Practice for use of welding in bridges and structures subjected to dynamic loading, Bureau of Indian Standards.

3 ROLE OF STRUCTURAL ENGINEER IN THE 21st CENTURY

1.0 INTRODUCTION

The term Engineer is derived from the Latin word ***ingerere***, i.e. to create. In essence, Engineers are creators of artefacts, using their ingenuity and capacity for original thinking within the constraints of affordability and practicability. Modern society expects the Engineer to understand the role of financing, project management and information technology in improving the quality of his designs. Thus, close collaboration with other specialised disciplines is vital. Full and on-going interaction between other members of the design team is essential in order to maintain effective communication across professional boundaries. Besides his principal role as an innovator**,** the designer of a constructed facility has the responsibility to ensure that ***his plan is***

- ***fit for its purpose***
- ***economical and durable***
- ***safe, both for the users and for the environment***
- ***buildable, without inconveniencing the community and***
- ***aesthetically pleasing.***

The world we left behind at the end of the 20th century was very different from what it was at the beginning of that century [1]. Dramatic changes to the scientific and engineering world have - undoubtedly - brought enhanced wealth and living standards to a small proportion of the world's population but it has also been accompanied by unpredictable upheavals in the economy of every part of the world, uncompromising social attitudes and unacceptable pollution and damage to the environment. Societal transformations and upheavals have occurred due to the insatiable consumption of the world's natural resources, uncontrolled pollution of our environment, creation of unacceptable quantities of waste, unacceptable disparity in the standards of living, unprecedented population growth and worldwide urbanisation. This complex scenario has also resulted in unbelievable damage, deterioration and destruction of our infrastructure.

India's per capita income remains very low; its per capita GNP (Gross National Product) is $436, which is even less than that of Pakistan ($492). Nevertheless the purchasing power of the country as a whole has increased. The wealth has not spread out among all the citizens, but is confined to a fifth of the population. A little more than 200 million Indians are going up the ladder. This distorted economic scenario has encouraged overcrowding in the cities, caused by people looking for work and the consequent growth of slums coexisting with affluent neighbourhoods has already caused significant environmental deterioration in major cities. Durable, eco-friendly and sustainable development alone can prevent this unredeemable environmental degradation and enable the maintenance and enhancement of good quality of life. Engineers have to develop sensitivity to these deeply felt concerns for the natural and man-made environment and face increasingly complex challenges in their everyday work for ensuring environmental sustainability. We begin by describing the challenges facing the Indian Engineer in the 21st century and proceed to discuss the various factors and impediments affecting the creation of sustainable and durable environment.

2.0 THE CHALLENGE FACING THE DESIGNER

Design problems are seldom amenable to solution by exact mathematical formulae. There is a considerable scope for exercising engineering judgement. Hence, there is no "***correct solution***" to a design problem, as there could be several so-called "correct solutions" to the same problem. This is because

- the designs are invariably subject to individual interpretation of Standards and Codes,
- the solutions are also subject to differing ideas about what is or what is NOT required from an engineering and environmental stand point, and
- the individual designers have ingrained ideas from ***their past experience***, ***which may be valid to-day only to a limited extent, or may not be valid at all.***

Thus the design problems are referred to as " open ended" problems. Nevertheless the Designer has the responsibility for ensuring that the goal of the project is achieved (i) safely, without taking any undue risks to lives and materials and without causing a liability, (ii) within time and (iii) within the (budgeted) cost. Hence, "***Engineering Design***" may be defined as a creative activity of building a new artefact which provides an optimum solution to satisfy a defined requirement or need, ***without endangering the environment.***

Herbert Hoover, a former President of the United States of America - (the massive arch dam called "Hoover Dam" in the U.S.A. is named after him), described the Engineering profession as follows (*1961*):

> ***"It is a great profession. There is the fascination of watching the figment of the imagination emerge through the aid of Science to a plan on paper. Then it moves to realisation in stone or metal or energy. Then it brings jobs and homes to men. Then it elevates the standards of living and adds to the comforts of life. That is the Engineer's high privilege. The great liability of the engineer compared to men of other professions is that his works are out on the open, where all can see them. His acts, step by step, are in hard substance. He cannot bury his mistakes in the grave like physicians. He cannot argue them into thin air or blame the judge like the lawyers. He cannot, like the architects, cover his failures with trees and vines. He cannot, like the politicians, screen his shortcomings by blaming his opponents and hope that the people will forget. The engineer simply cannot deny that he did it. If his works do not work, he is damned forever….***
>
> ***"On the other hand, unlike the doctor, his is not a life among the weak. Unlike the soldier, destruction is not his purpose. Unlike the lawyer, quarrels are not his daily bread. To the engineer falls the job of clothing the bare bones of science with life comfort and hope. No doubt, as the years go by, the people forget which engineer did it, even if they ever knew. Or some politician puts his name on it. Or they credit***

> ***it to some promoter, who used other people's money… But the engineer himself looks back at the unending stream of goodness which flows from his success with satisfaction that few other professions may know. And the verdict of his fellow professionals is all the accolade he wants."***

Thus, Hoover described the professional role and responsibilities of the Engineer succinctly. But what is meant by the "***professional***" role? The word "***profess***" has religious connotations and probably has its origin in *17th* century England [2]. Monks *professed* their vows and were generally well educated. It is from this group of religious men, erudite University educators were drawn. ("*Professors*" were those who *professed*.) Hence the term "***professional" was associated with a high degree of education and societal responsibility***.
In today's context, all professions require

- extensive specialised education of significant intellectual content
- the practitioner to provide a recognisable service to the community
- a certification (usually by government or by a chartered body).

Professions are organised into professional societies, which police themselves. People engaged in these professions are independent of external influences and cannot be coerced by their clients, employers or governments to carry out unethical instructions. Controlling governments yield power to the professionals or their Societies. (For example, in many western countries, only a physician can write prescriptions; only a registered engineer/architect can approve the plans of a structure/building). In return, the professions take on a very responsible position, vis-à-vis the public. Generally, the professional should not hurt anyone unless it is required, (e.g. a dentist!).

To eliminate conflicts developing between the roles of the professional and of the citizen, every profession has a Code of Ethics developed by the professionals themselves. For example, the Code Of Ethics, developed by the American Society of Civil Engineers, is based on three fundamental principles requiring Engineers to (i) use their skills to benefit mankind (ii) be honest and fair, and faithfully serve others; and (iii) improve the competence and prestige of the profession.

3.0 DURABILITY AND LIFE CYCLE COST ISSUES

Traditionally the professional Structural Engineer had invariably played a vital role in the design of constructed facilities, often, in close association with other professionals like Architects and others in related disciplines. As a designer, he is responsible for the complete process from the conceptual stages to the finished structure. Increasingly, the Society expects him to assume ***responsibility for the durability*** of the product. In other words, the responsibility of a professional Structural Engineer in the 21st century will not be confined merely to the immediate economic and environmental impact of his design decisions; society expects him to make rational and responsible choices by ***considering the life cycle costs and the long-term environmental effects on the community*** In the following pages, we will highlight the enhanced role of the Professional Engineer in the 21st century and explore how the two design criteria are interlinked.

3.1 The Infrastructure Crisis

The Construction Industry, with all its imperfections and limitations, is rightly perceived as the provider of the Nation's infrastructure. Clearly, it is of paramount importance to train and educate those who create and manage it, in order to ensure the economic and environmental survival of the world. While the world has witnessed some fantastic advances in Science and Technology in recent years, many of these achievements have been made at an outrageous price, plunging the world into a number of crises, which have impacted directly on the construction industry. The global effect of these dramatic changes in the world in the last 50 years can be collectively termed the "**infrastructure crisis**", which has to be encountered and managed by the construction industry.

Three-fourths of the world's population live in the (non-industrialised) developing world like India. Uncontrolled population growth, (particularly in the developing world), and evolutionary industrialisation have resulted in global urbanisation. The world population has grown from 5 bn in the late 1980's to 6 bn in 2000 and is now estimated to grow to 8 bn by 2036 and to over 9 bn. by 2050. (The population of India is now just over 1 bn). More than 95% of this increase will take place in the developing parts of the world, India included. For the first time in history, more than half the world population will live around the cities. It is estimated that there are more than 120 cities with over a million people, the majority in the developing world, thus accelerating urban decay in cities which can least afford repeated remedial action.

The magnitude of the problem in the Indian context is illustrated next by considering the "housing sector". Over the next 40 years, India is set to overtake China as the most populous country in the world. The present urban population is estimated to be 330 million, equalling the total population of the country 50 years ago. The urban population that was merely 14% of the total number of citizens 50 years ago now amounts to 33% and is set to grow to 50% by 2025. With economic liberalisation and expected enhanced growth, the rate of urbanisation in India in coming decades is likely to increase. Despite the best of efforts of well-intentioned people in Government and aid agencies, the Nation has not been unable to cope up with the ever-increasing need for shelter for every citizen. India needs some 200 million houses to accommodate all its citizens, whereas we have only167 million houses, of various types [3]. Half of these houses had mud, grass and straw walls and more than a third had grass, straw and thatch roofs. The need for upgrading the housing stock and the magnitude of the task are obvious, particularly in the context of expected urban growth.

It has been estimated that over 50% of the land in urban areas is second-hand. Much land is adversely affected by foundations from demolished buildings, which previously stored harmful chemicals, petroleum products etc. Old foundations must be viewed as contaminants and it is important to prevent land contamination by sub-structures.

Another major source of concern is water pollution. Many rivers and streams in India are not in their natural state, mainly because of industrial pollution and irresponsible drainage of sewage into them. The adverse effects of high pollution levels in our water resources are already painfully evident in India.

It is now widely recognised that much of the recent economic progress in the Western world has been at the expense of the environment and the effects of this environmental degradation are being felt globally, for instance in the form of climate change, ozone depletion, deforestation and acid rain. It is necessary therefore to assess and improve the environmental performance in all economic sectors including construction. Global warming caused by the emission of Greenhouse Gases (i.e. CO_2) into the atmosphere puts increased energy into the climate system, resulting in increases in the number and intensity of storms, rapid climatic changes, and larger, more damaging and extreme weather events. As the effects of greenhouse gases in the atmosphere take 30 years to show, the current changes in the world weather (rise in sea levels, global warming, larger deserts, severe draughts and storms) relate to emissions up to the year 1970. The effect of current pollution levels will not be evident until 2030; the present century will, therefore, be a century of disaster management. Besides the large-scale deaths and devastation of the environment that follow from these disasters, the greatest effect will be the destruction of the infrastructure and therefore its impact on the Construction Industry [1].

3.2 The "Durability Crisis"

Issues of durability have always been subjects of debates among Engineers. Is it better to spend (say) 40% more initially, in order that the life of a structure could be doubled? What is better value to the client? Spend less initially or opt for a longer life? Total neglect of durability considerations in all the infrastructure projects undertaken so far combined with primitive construction practices still prevailing in India have resulted in what can only be termed a "**durability crisis".** It is now well established that degradation of all structures has become very common in almost all the cities in India and this is particularly true of buildings and structures made of reinforced/prestressed concrete. **The great tragedy is that there have been no efforts to address this issue by the present generation of Developers, Engineers, Architects and other design professionals. As a consequence, major problems have been allowed to accumulate for future generations of owners and taxpayers to face.**

This is not to say that other parts of the world are free from this "***durability crisis*".** For example, the present total construction expenditure in the UK is 56 million British Pounds, of which 50% is spent in repairs and rehabilitation of recently completed structures. As an example, the Midlands Link Motorway around the city of Birmingham cost around 28 million British Pounds to construct; this motorway needed repairs and rehabilitation within 20 years of its completion. Between 1972 and 1989, a further 45 million British Pounds were spent in repairs. It is now estimated that another 125 million Pounds will be required in the next 15years. In Europe, the annual repair cost is estimated as 1.4 billion ECU; in the U.S. the cost of rehabilitating half a million bridges (mostly concrete) is estimated to be $100 billion. It must be noted that in many countries in the West, life cycle costing is now a mandatory requirement in the planning process. For example, International Surface Transport Efficiency Act of the US (1991) mandates that state wide and metropolitan planning processes consider Life-Cycle Costs (LCC) in the design and engineering of bridges, terminals and pavements, rather than basing decision solely **on initial costs alone** as until then.

There is no particular merit in Indian Engineers making the same mistakes and blunders as their Western counterparts did and then rectifying them. In any case, India cannot afford the luxury of these blunders. Sustainability of the Environment is NOT an Option; it is vital for the economic and environmental survival. We do not inherit the world from our ancestors - we borrow it from our children [1].

3.3 Time wasted is money wasted and opportunities lost

When a constructed facility - be it a private home or a public highway - is completed, it will be put to use immediately and this results in a return on the capital employed. Delays in the completion of a project would therefore represent a delay in the return on capital invested, besides the loss of interest, which that sum would have earned otherwise. This essential relationship between time and money is well understood in the Western world but unfortunately this is not the case in India.

The recent liberalisation and globalisation of the Indian Economy has brought with it a potential (and an opportunity) for significant growth of the construction activity particularly in the infrastructure industry. Design and construction of buildings and bridges have been major growth areas, supposedly to facilitate the expected economic upturn. In the following paragraphs, we shall discuss the factors affecting the lifetime costs and durability of a structure, in some detail. In a later section, we highlight the fallacies and errors frequently committed by professionals in India that militate against arriving at an optimum overall design.

3.4 Cost competitiveness by using Alternative Materials

Unfortunately for the Indian client, many architects and designers seldom consider the use of alternative materials of construction and the designs are invariably limited to “concrete-intensive” structures. Often the best optimal design solution is obtained by a sensible combination of reinforced and/or prestressed concrete elements with structural steel elements. Even when a “steel-intensive” solution is selected; it is very rare for limiting the selection of materials of construction to steel only.

Although India has an installed capacity to produce 35 million tonnes of steel/year, we manage to produce only 24 million tonnes/year of which the use in the construction sector accounts for around 25% - 30%. By way of comparison, China produced 120 million tonnes of steel during 1999 - 2000 and Japan, 95 million tonnes. The total per capita consumption of steel in all its forms in India is one of the lowest in the world, being 24 kg/annum, compared with 500 kg/annum in the USA and 700 kg/annum in Japan. According to the recent research by the Steel Construction Institute [4], there is a direct link between the gross national product per capita and the per capita consumption of steel.

Indeed, structural steel has inherently superior characteristics to a very significant extent, when compared with competing materials. For example, to replace one unit area of steel in tension, (with a yield stress of 450 MPa), we would need to use an equivalent plain concrete area of about 200 units. For concrete to be able to compete with Structural Steel in construction, we need to put Reinforcing Steel into it! Even then, there is no way to prevent the cracking of concrete in tension, which often encourages corrosion of reinforcement. In compression (or squash loading), one unit area of steel is the equivalent of 15-20 units of M20 concrete. A comparison of strength/weight ratio will reveal that steel is at least 3.5 times more efficient than concrete. For a given compressive loading, concrete would have 8 times the shortening of steel. Again we need reinforcing steel to prop up the plain concrete.

In structures built of Structural Steel, occasional human errors (like accidental overloading) do not usually cause any great havoc, as there is a considerable reserve strength and ductility. Steel may thus be regarded as ***a forgiving material*** whereas concrete structures under accidental overload may well suffer catastrophic collapse of the whole structure. Repair and retrofit of steel members and their strengthening at a future date (for example, to take account of enhanced loading) is a lot simpler than that of reinforced concrete members. The quality of steel-intensive construction is invariably superior, when compared with all other competing systems (including concrete structures) thus ensuring enhanced durability. This is especially true in India, where quality control in construction at site is poor.

Structural Steel is recyclable and environment-friendly. Over 400 million tonnes of steel are recycled annually worldwide, which represents 50% of all steel produced. The infrastructure and technology for the recycling of steel is very well established. Steel is the world's most versatile material to recycle. But once recycled, steel can hop from one product to another without losing its quality. Steel from cans, for instance, can as easily turn up in precision blades for turbines or super strong suspension cables. Recycling of steel saves energy and primary resources and reduces waste. A characteristic of steel buildings is that they can readily be designed to facilitate disassembly or deconstruction at the end of their useful lives. This has many environmental and economic advantages; it can mean that steel components can be re-used in future buildings without the need for recycling, and the consequent avoidance of the energy used and CO_2 emitted from the steel production processes.

Steel-intensive construction causes the least disturbance to the community in which the structure is located. Fast-track construction techniques developed in recent years using steel-intensive solutions, have been demonstrated to cause the least disruption to traffic and minimise financial losses to the community and business.

Even though “the initial cost” of a concrete intensive structure may sometimes appear to be cheaper, compared to the equivalent steel-intensive structure, it has been proved time and again that its total lifetime cost is significantly higher [5]. Thus the popular perception of the concrete-intensive structure being cheaper is NOT based on verifiable facts! There is therefore no real cost advantage either.

Except in a few special structures like tower cranes and transmission towers, it is rare to build a structure entirely in steel. Frequently the optimal solution is obtained by employing concrete elements compositely with structural steel, especially in multi-storeyed buildings and bridges. These methods ensure significant cost benefits to the developers (or owners of property) as well as to the community. Composite structural forms have been extensively developed in the western world to maximise the respective benefits of using structural steel and concrete in combination, but **this technology is largely ignored in India, despite its obvious benefits**. The sizes of composite beams and columns will be appreciably smaller and lighter than that of the corresponding reinforced or prestressed sections for resisting the same load. A direct economy in the tonnage of steel and indirect economies due to a decrease in construction depths of the floors and reduced foundation costs will, therefore, be achieved. Generally, improvements in strengths of the order of 30% can be expected by mobilising the composite action. An independent study carried out by the Central Building Research Institute (CBRI) Roorkee demonstrated that there are substantial cost savings to be achieved by the use of Composite Construction [6].

3.5 Life Cycle Costs

ASTM E917-83 (1983) describes the standard practices for evaluating LCC of buildings and building systems. The motivation for the LCC is that on any investment decision, all costs arising from the decision, both immediate and in future are potentially important. The recent development of ***fast track methods in construction*** in the western industrialised world triggered the wide spread implementation of Life Cycle Cost study, which would ensure enhanced productivity and efficient utilisation of the capital. Construction projects completed on the basis of lower initial cost alone have often proved to be far more expensive in the long run, besides causing damage to the environment and bringing poorer return on the investment. Thus the durability of structure and its life cycle cost are closely inter-linked. Enhanced durability invariably reduces or eliminates the construction-related adverse environmental impact on the community.

As pointed out already, Indian Engineers seldom give any serious consideration to vital factors like durability and lifetime costs. Environmental safety and inconvenience to the community do not seem to be given even a cursory thought. As a result, the owners (and taxpayers) do not get the most rational choice arrived at by taking into account all aspects of the design challenge. The result is that - both in the short term as well as on a long-term basis -. the construction costs in India are among the highest in the world, (despite the labour costs being very low, compared to the West) while the Construction Industry continues to pollute the environment and cause long-term damage to it.

***At this stage it is appropriate to define the Life Cycle Cost of a Structure, made up of several components listed below* [7].**

1. Initial Cost

- Actual "Cash" Cost of the project

- Cost of the Investment locked-up without Returns ("The Time Cost")
- Cost penalty to the community by traffic delays and detours; Losses suffered by local Business ("Hidden Penalty Cost")
- Cost of damage to the Environment due to Pollution ("The Environment Cost")

2. Periodic Maintenance Cost, including energy cost

3. Cost of dismantling the structure, at the end of its life

4. Less the salvage value of the construction products.

All these values are evaluated in life cycles, present value terms or Annual Value terms. A more comprehensive LCC analysis may include adjustment for taxes, adjustment of financing cost etc. The basic aspects of lifetime costs are discussed in some detail in the following paragraphs:

3.5.1 Initial Cost

(a) ***Actual "Cash" Cost***: Many government departments report the "cash cost" as the **Cost of the project**. This is both wrong and misleading. Frequently, many reputed Designers also report that the **cash costs** of Reinforced or Prestressed concrete alternatives are cheaper than Steel Options. This is because the Steel Intensive Options considered by them are based on outdated design and construction practices. For example, they do NOT employ the relatively new "Steel-concrete composite" construction, or Limit State methods of Design, possibly because Indian Codes have not kept pace with developments in technology! A recently published CBRI study has demonstrated that likely cash savings by using Steel-Intensive Designs, compared with concrete-intensive option for multi-storey buildings will be at least 3% - 16%. The experience in Europe, particularly in the United Kingdom, bears this out. (Over 90% of the new buildings in the London area are built of Steel-Concrete Composite Construction; over 60% of all bridges throughout Great Britain are built of Steel-concrete composite construction.) It is difficult to believe that the necessary expertise is unavailable in India, as the technology is not complex.

(b) ***Cost of the Investment Lock-Up without Returns ("The Time Cost")*****:** Ignoring the "time cost" has been a cultural weakness in India and needs to be overcome if we are to take our place in the community of Nations. It must be recognised that ***time does cost money***. The time taken for concrete-intensive construction would be 2-3 times it takes for steel-intensive alternative. Locking up the capital – without any return – by choosing the former results in a loss to the owner of at least 12% - 15% per year of delay. A recent study reported that even for a modest project like a flyover costing Rs. 10 crores, [See Fig. 1] the loss under this head amounts to Rs. 1.00 crore of taxpayer's money, i.e. 10% of the total cost.

(c) ***Cost penalty to the Community due to inappropriate construction planning*** **(**e.g. Traffic Delays and Detours; Losses suffered by local Businesses etc. collectively termed -

"Hidden Penalty Cost"): A prestressed concrete fly-over built in Chennai was chosen as a case study. This construction, which lasted 15 months, had resulted in all local road users and residents having to take a detour of 2 km for each trip resulting in a needless extra expenditure by the community of one crore of rupees in a project costing approximately Rs. 10 crores! This was spent in burning petrol bought by using the valuable foreign exchange. Is this a wise use of foreign exchange?

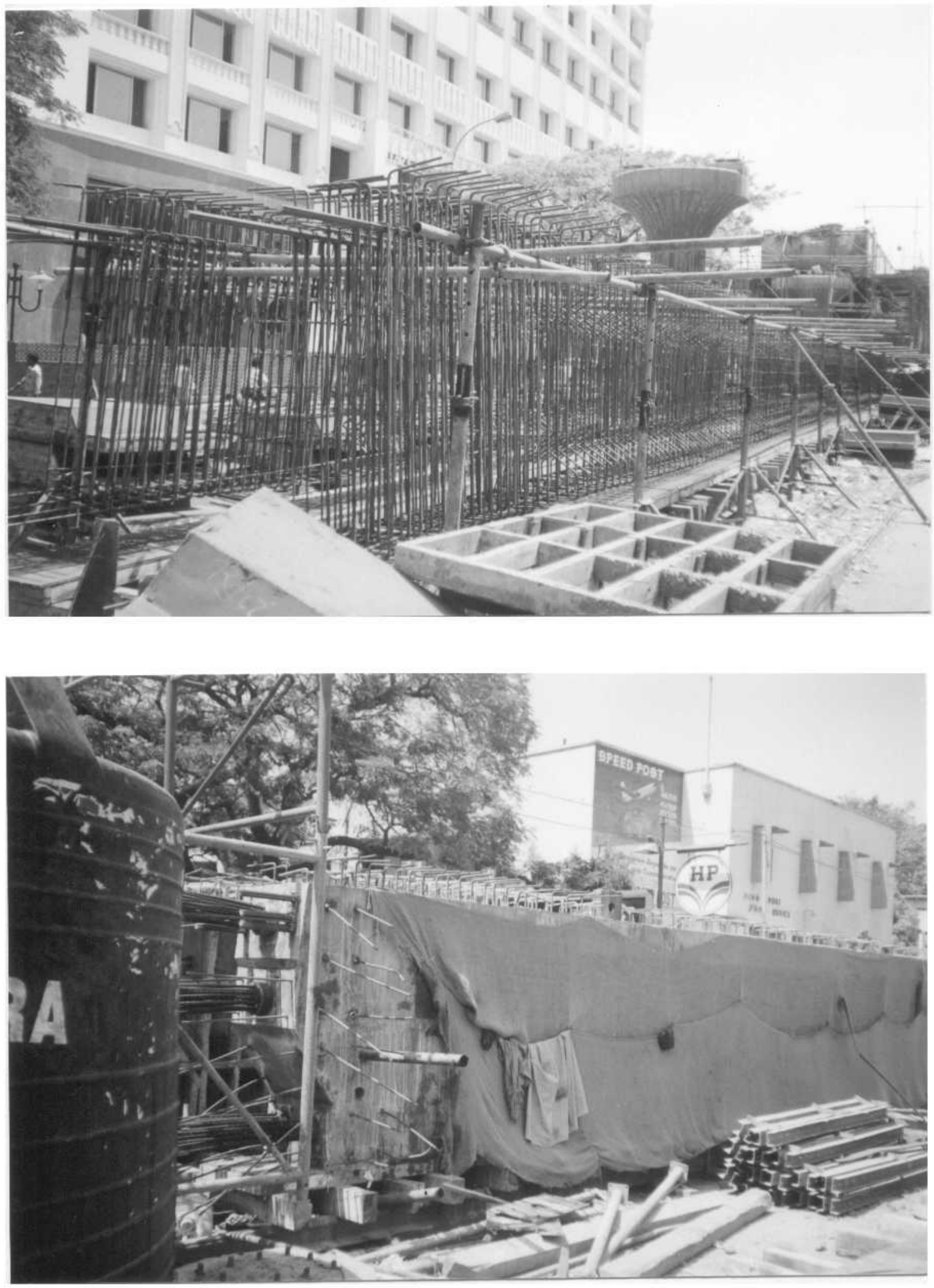


Fig. 1 Flyover construction – The Indian way

Fig. 2 Do the business need roads?

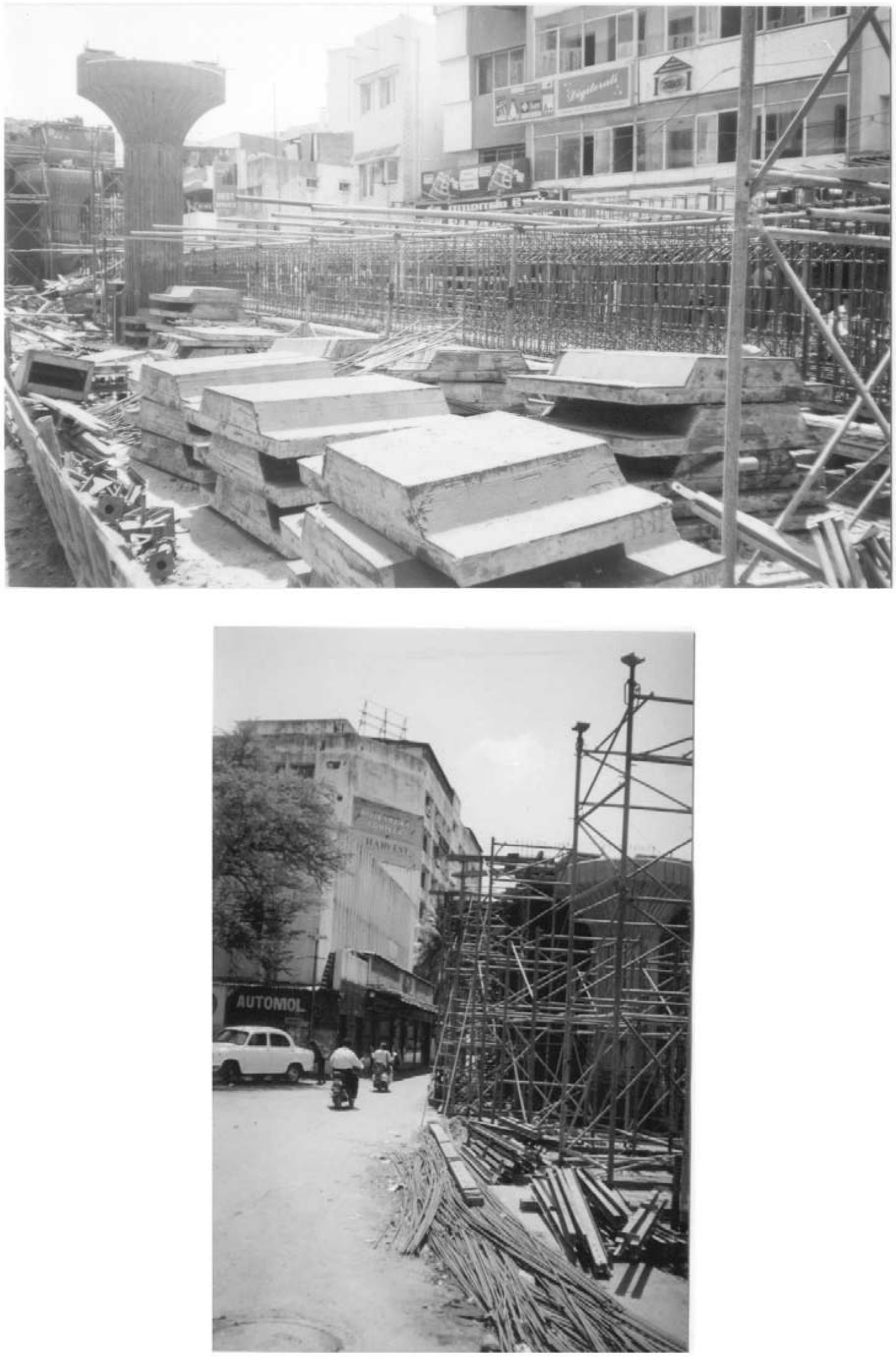


Fig. 3 Where else do we store junk?

Secondly, many businesses had lost huge sums of money because of road closures during construction and some smaller businesses had closed down, probably forever. [See Fig. 2] The study referred above estimated the total loss to the business community due to this one fly-over, to be around Rs 40,00,000, to Rs. 75,00,000. (There are 15 fly-overs currently being built in Chennai!!) A third "penalty" is the time spent by busy executives in the traffic jams and hold-ups, caused by this construction work.

There is no evidence to suggest that the Community had given its informed consent for the colossal sums being spent on their behalf. Hiding "the hidden penalty cost" (particularly in metropolitan cities) would cause long-term damage to the attractiveness of the city as a place to invest. As the public become aware that technologies do exist to create infrastructure with minimum negative impact or inconvenience to public they will increasingly demand utilisation of such technologies.

(d) *Cost Of Damage to The Environment Due To Pollution* ("The Environment Cost"): As a direct result of pollution by particulate material (construction dust, movement of heavy construction equipment etc), the penalties paid by the hundreds of residents in the locality, by way of health-care costs must be adding to staggering sums. It is, of course, impossible even to guess this figure unless a detailed study is undertaken. Many road users are horrified to observe that the highway is closed up for prolonged periods merely for storing construction materials and junk, contributing to substantial dirtying of the environment. [See Fig. 3] Traffic hold-ups due to the construction cause added air pollution in the neighbourhood. It is clear that Professional Engineers are unlikely to be admired for irresponsibly causing pollution in the neighbouring environment for prolonged periods.

(e) ***Total Initial Cost:*** From the foregoing it is clear that the total likely savings by adopting steel-concrete composite construction to the taxpayer when a building, fly-over or bridge is completed, (compared with concrete-intensive construction) will be at least 30% - and more. Substantial reduction in pollution levels, environmental damage and traffic hold-ups will be added bonuses.

3.5.2 Periodic Maintenance Cost

Periodic and preventive maintenance undoubtedly contributes to the longevity of the structure. Unfortunately this is the most neglected activity in India. Economising on periodic maintenance will invariably result in much enhanced expenditure at a later date. The problem is compounded by several myths that seem to prevail among Engineers and Architects. Some of these that affect the periodic and timely maintenance of structures are discussed below:

(a) Myth No. 1: Concrete lasts forever without maintenance: ***The reality is*** that there ***is no magical ingredient in concrete to do it****!* Concrete is subject to deterioration by the same environmental factors as steel (viz. Chloride contamination, alkali silicate reactions, sulphate attack etc.) In addition, we have problems due to poor site control, insufficient concrete cover, ineffective drainage, insufficient cement content, shrinkage, creep etc.

(b) **Myth No. 2: Concrete bridges outlast steel bridges:** ***The reality is that there is no credible statistical evidence that concrete bridges outlast steel bridges.*** Many steel bridges with over 100 years of service life are still performing well. In contrast, the first major prestressed concrete bridge in the USA (Walnut Lane Bridge in Philadelphia) had to be replaced by a steel bridge after a service life of about 40 years. The deterioration rates of 57000 bridges listed in Federal Highway Administration analysed by Lehigh University showed no correlation with the material of construction. The only factors they could identify are (1) age, (irrespective of the material of construction) and (2) the intensity of daily traffic.

Studies by the OECD (Organisation for Economic Co-operation and Development) reveal that steel bridges are expected to last much longer than prestressed concrete bridges. Indeed in Belgium and Japan they found that steel bridges outlast prestressed concrete bridges by 15 – 26 years.

(c) Myth No. 3: Concrete bridges last forever without maintenance: ***Some people believe that once in place, reinforced and prestressed concrete bridges last forever and that steel bridges are slowly corroding away.*** The perception is that concrete is an inert material, less vulnerable to the environment than structural steel. ***The fact is that Concrete deterioration is a subject, which is widely researched but not so widely discussed. Appearances can be deceiving – at least in the case of Concrete Bridges; according to US Govt. Strategic Highway Program, a bridge deck or sub-structure that appears sound, may actually be deteriorating from inside out.***

Steel is easily repairable at almost any stage of corrosion and over the years has shown a remarkable tolerance to lack of maintenance.

(d) Myth No. 4: Structural steel can not be adequately protected from corrosion: ***The reality is that there are high performance coatings available to day, which provide long term protection for EXPOSED Structural Steel at an economic price.*** Frequently, interior Steelwork does NOT require any paint or other protective coatings.

For Steelwork to corrode we need the presence of both water and air SIMULTANEOUSLY, or exposure to aggressive conditions. These conditions do NOT exist in most buildings and in many inland structures and bridges. There is certainly no evidence to suggest that many landmark structures like the London Bridge, Eiffel Tower, Empire State Building, Sears tower and many other Steel intensive structures are corroding away!

(e) Myth No. 5: A steel structure is less safe in a fire than other types of structures: ***The reality is that Steel Structures are no less safe than other structures***. The properties of **all** materials are degraded when exposed to fire. The modulus of elasticity of concrete is permanently reduced; the cross section of timber is consumed. The modulus of elasticity of steel is however, not permanently reduced and recovers once the member cools down. Steel structures can be economically fire protected to meet all building code requirements. Steel structures can be rehabilitated after a fire at a modest cost.

(f) Myth No. 6: Maintenance of Concrete intensive structures is significantly cheaper than that of Steel intensive Structures: Concrete–intensive construction is not as simple as is usually imagined**. Nor are concrete-intensive structures cheap, to maintain (contrary to popular myth).** Structural steel forgives human errors or lapses in maintenance, but not concrete! It is vital that Engineers pay attention to vital maintenance issues and not be carried away by myths.

The incidence of major concrete repairs to bridges is significantly greater than many realise and time cost of such repairs, when all costs including traffic delay costs are taken into account is very significant. It has been shown in published literature that the probability of significant repair work to be carried out within each 20 year life of concrete bridges is as high as 0.185. The direct cost of such repairs is insignificant (around 0.1 to 0.5% of the initial cost in present value) whereas the indirect cost due to the traffic delay during such longer repairs in concrete structures has been estimated to be at least 25%. Further, it is well known that repair and retrofit of steel members is a lot simpler than reinforced concrete members.

3.5.3 Cost of Dismantling the Structure at the End of its Life

Many structures in the urban environment are being demolished due to deterioration beyond repair, due to land values escalating very high, rendering the building economically unviable, or due to their inability to meet the modern functional requirements. At this stage a cost is incurred to dismantle or demolish the structure. It is well known that the cost of dismantling the steel structure is well below that of reinforced concrete structures.

3.5.4 Salvage Value of the Construction Products

Some of the products salvaged from the structure dismantled are of some economic value. This value is subtracted from the total cost after adjusting for the time of salvage. It is well known the cost of material recovered from steel intensive construction is almost equal to the original cost of the structure, although this value is to be reckoned at a later time in the life cycle, whereas in concrete intensive construction, substantial additional cost is incurred in disposing off the material. The recycling of demolished material from urban construction has become a major problem in urban areas, leading to intensive research on the subject.

3.5.5 Uncertainties in Life Cycle Costing

The effort to evaluate the life cycle cost is fraught with many difficulties listed below.

- Non-availability of reliable and consistent cost data

- Non-availability of reliable historical information on costs associated with construction, maintenance, repair, rehabilitation, and demolition (structure management system).
- Data on and experience with some of the recent technologies being only subjective and not rational and impartial (e.g. Prestressed concrete versus Steel bridge.).
- Changes in design criteria and techniques with time and location, construction materials and methods, patterns of use including magnitude and frequency of loads, maintenance methods etc.

As a direct consequence, many decision-makers experience uncertainties in arriving at reasonable estimates for LCC evaluations. Analytical techniques such as sensitivity analysis and probability analysis are sometimes used to make decisions about investments whose economic consequences are uncertain. It should, however, be noted that the LCC decisions are more influenced by life and discount; both these factors favour steel structures, since steel structures generally exhibit longer life and shorter construction and repair/rehabilitation time duration.

There is no doubt that a massive research effort is needed to assess the life cycle cost reliably and the durability aspects and environmental impact of any design decision. A sustained educational effort is also needed to persuade engineers to take a holistic approach to structural design, rather than merely confining themselves to myths and dogmas not based on rational analysis.

4.0 THE EVERYDAY LIFE OF STRUCTURAL ENGINEER

A structural engineer’s responsibility is to design the ***structural systems*** of buildings, bridges, dams, offshore platforms etc [8,9,10]. A ***system*** is an assemblage of components with specific objectives and goals and subject to certain constraints or restrictions. System components are required to co-exist and function in harmony, with each component meeting a specific performance. ***Systems design*** is the application of a scientific method to the selection and assembly of components to form the optimum system, to achieve the specified goals and objectives, while satisfying the given constraints or restrictions.

In practice, any constructed facility can be considered as a “System”. The Structural System is one of its major subsystems and is indeed its backbone. Some of the other coexisting subsystems are those connected with the mechanical, electrical, plumbing and lighting facilities.

Structural components have to meet the design requirements of adequate strength under extreme loads and required stiffness under day-today service loads, while satisfying the criteria of economy, buildability and durability.

Examples of civil engineering systems include buildings, bridges, airports, railroads, tunnels, water supply network etc. For example, a building system is an assemblage constructed to provide shelter for human activities or enclosure for stored materials. It is

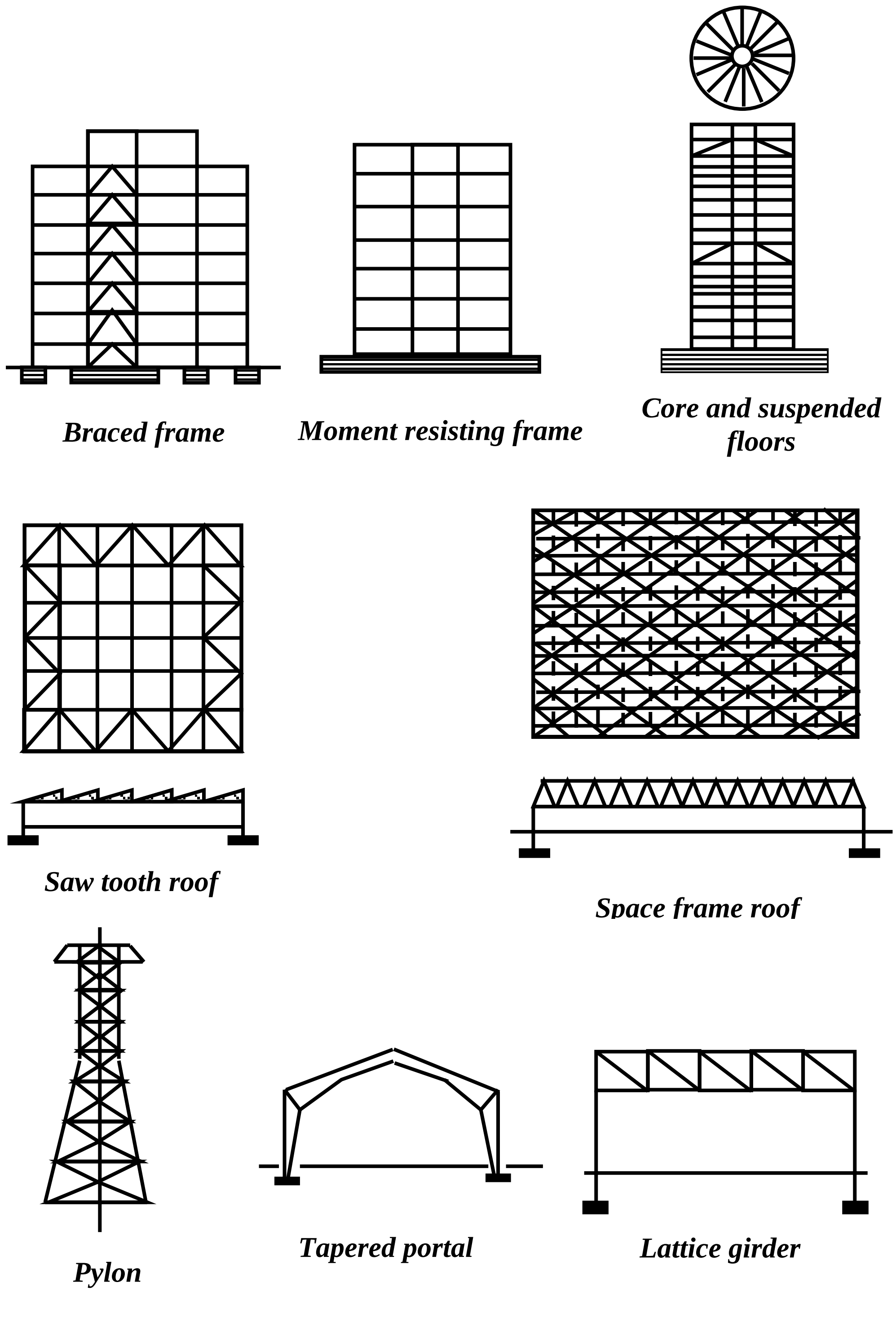


Fig. 4 Examples of steel-framed structures

subject to restrictions by building specifications on height, floor area etc. Constraints include ability to withstand loads from human activities and from natural forces like wind and earthquakes.

As pointed above, a system consists of many subsystems, i.e. components of the system. For example, in a building, major subsystems are structural framing, foundations, cladding, non-structural walls and plumbing. Each of these subsystems consists of several interrelated components. In the case of structural framing, the components include columns, beams, bracing, connections etc.

The richness and variety of structural systems can be appreciated by the available building structural types that range from massive building blocks to shell structures, from structures above or below ground or in water, to structures in outer space. Examples of a few steel-framed structures are shown in Fig. 4.

4.1 Goals

Before starting the design of a system, the designer should establish the goals for the system. These specify what the system is to accomplish and how it will affect the environment and other systems or vice versa. Goals are generally made in statements of specific design objectives such as purpose, time and cost limitation, environmental constraints etc., which would enable the generation of initial and alternative designs.

The goals for a system design applied to a subsystem serve the same purpose as for a system. They indicate the required function of the subsystem and how it affects and is affected by other subsystems.

4.2 Objectives

Having set down the goals, the designer defines the system objectives. These objectives are similar to goals but explain in detail the requirements that the system must satisfy to attain the goals. Some of the essential objectives of any project relate to health, safety and welfare requirements of the occupants, which are generally defined in local building codes or building regulations. Other special objectives include minimisation of initial costs, life-cycle costs, construction time etc.

At least one criterion (e.g. Fire resistance) must be associated with each objective. A criterion is a range of values within which the performance of the system must lie (e.g. Two hours fire rating is needed). The criterion serves as a guide in the evaluation of alternative systems to the project.

4.3 Constraints and standards

Constraints are restrictions on the values of design variables, which may or may not be under the control of the designer. For example, an I-beam section of *200 mm* depth may be desirable, but not available. There are also various legal and building code requirements. A minimum of one ***standard*** must be associated with each constraint.

4.4 Codes and Specifications

A structural engineer is guided in his design efforts by the relevant codes and specifications. Although the word '***codes***' and '***specification***' are normally used interchangeably, there is a distinction between them. A detailed set of rules and suggestions prepared by an interested party is called an ***engineering specification***.

On the other hand, ***Codes*** are frequently formulated by a group of professionals with a view to their adoption by the profession as a whole. These are revised at regular intervals based on new developments in materials, research, construction techniques etc.

Though codes offer general guidance to a certain extent, they do not provide answers to all the problems that arise in practice. Mere adherence to codes and specifications will curb all initiatives and innovative designs.

4.5 Construction and other costs

Construction cost and time are usually dominant design concerns. If the construction cost exceeds the budget, the completion of the project may be in jeopardy. Minimisation of the life cycle cost of a system would result in the most desirable solution.

5.0 DESIGN REQUIREMENTS

The principal design requirements of a structure are set out already under ***Introduction***. The primary structural safety requirement is met by ensuring that the structure has an acceptably low risk of failure during its design life. Another important requirement is that the structure must be sufficiently stiff to ensure that excessive deflection and vibrations do not affect the in-service performance of the structure.

The requirement of harmony within the structure is affected by the relationships between the different subsystems of the main system, the architectural subsystem, the mechanical and electrical subsystems, and the functional subsystems required by the use of the structure. Finally, the system should be in harmony with its environment, and should not react unfavourably with either the community or its physical surroundings.

Conceptual design refers to the task of choosing a suitable system. (As an example architect is generally concerned with the building layout, limits and parameters). In modern construction practices, a multidisciplinary team of architect, structural designer and service engineer together evolve the conceptual design. A typical organisational chart for a multidiscipline design team is seen in Fig. 5, which shows the inter-relationship between the various design professionals.

The structural engineer is charged with the task of ensuring that the structure will resist and transfer the forces and loads acting on it with adequate safety, while supporting other subsystems and making due allowance for the requirements of serviceability, economy, harmony and constructibility. The iterative process of achieving such a design is shown in Fig. 6.

Since several simplifying approximations are made in the preliminary design, it is necessary to re-check the design. The loads are recalculated more precisely and the structure is reanalysed. The performance of the structure is then re-evaluated with respect to the structural requirements, and any changes in the member and joint sizes are made [See Fig. 7].

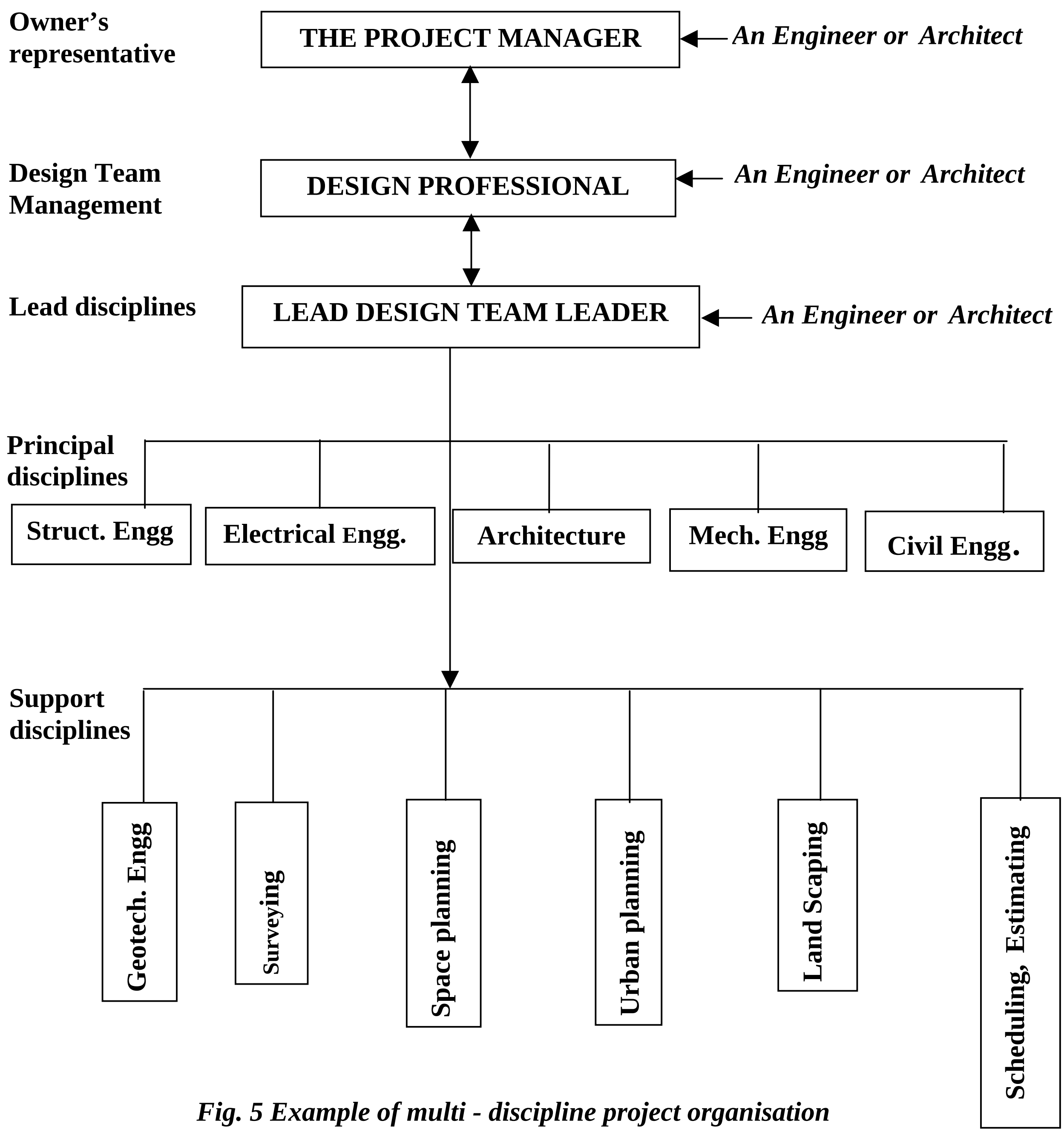


Fig. 5 Example of multi - discipline project organisation

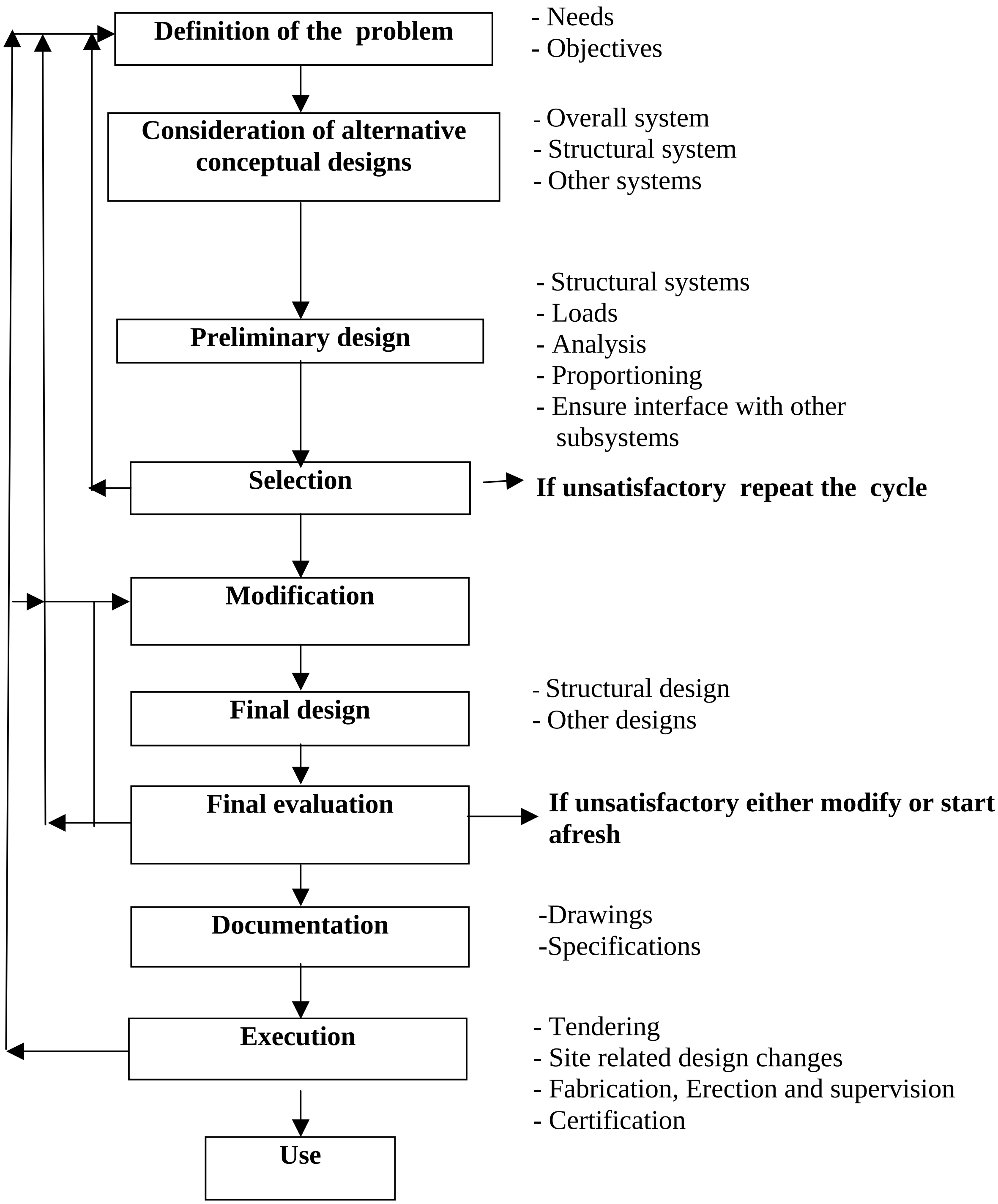


Fig: 6 The overall design process (Iterative)

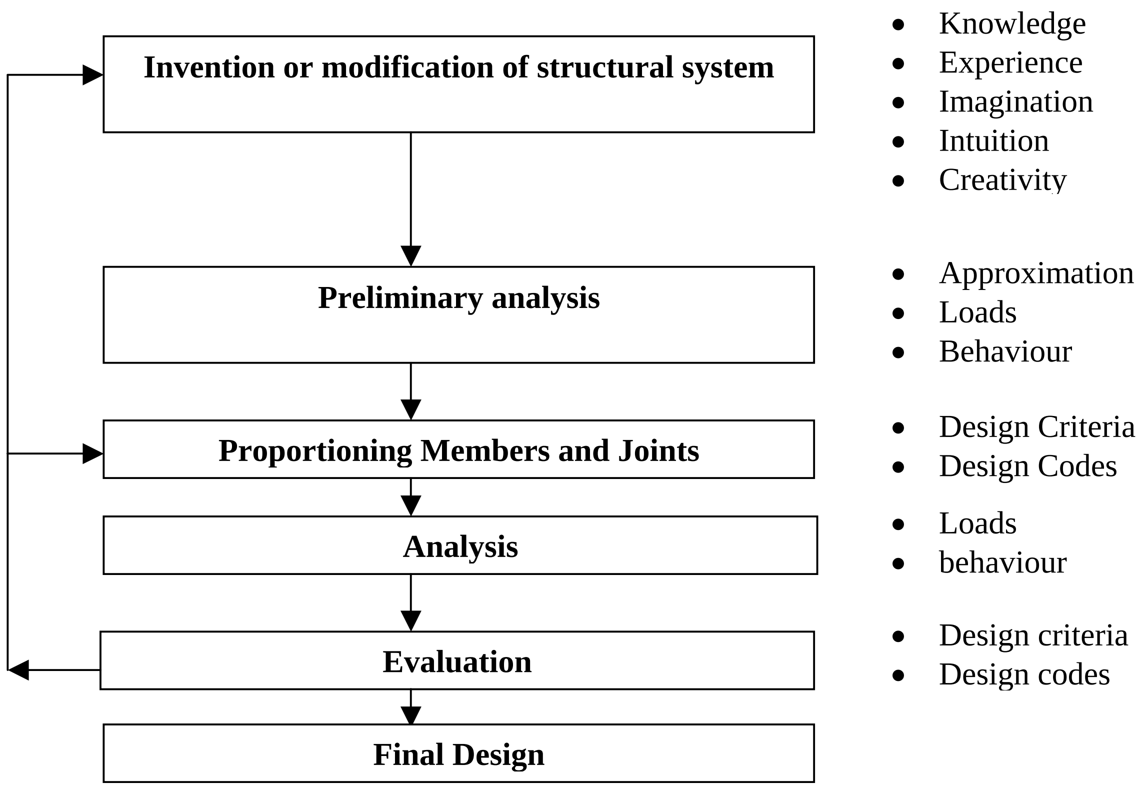


Fig. 7 The structural design process

Frequently, there is a fundamental confusion among students between a problem on Analysis and one on Design. In an analysis problem, all the parameters are known. (For example, if deflection of a loaded beam is required and the span, loading and the cross sectional properties are all known then a unique solution for the value of deflection can be arrived). The problem encountered in design (as compared to analysis) is that it involves the selection of the span, assessment of the loading, choice of the material of which the beam element is made, definition of its cross section and so on.. As a consequence, no unique solution can be offered for any design problem. It is clear that the designer has to make several decisions, each of which could affect the final result. Considerable engineering discretion of the designer is implicit in every design project.

The aim of the comparison of designs is to enable the designer to ascertain the most acceptable solution that meets the requirements for the given structure. All factors must be taken into consideration. Factors to be taken into account in a typical building project are given below by way of illustration:

1. Materials to be used
2. Arrangement and structural system (e.g. flooring system) to be adopted
3. Fabrication and type of jointing
4. Proposed method of erection of the framework
5. Type of construction for floor, walls, cladding and finishes

6. Installation of ventilating/ heating plant, lifts, water supply, power etc.
7. Corrosion protection required
8. Fire protection required
9. Operating and maintenance costs

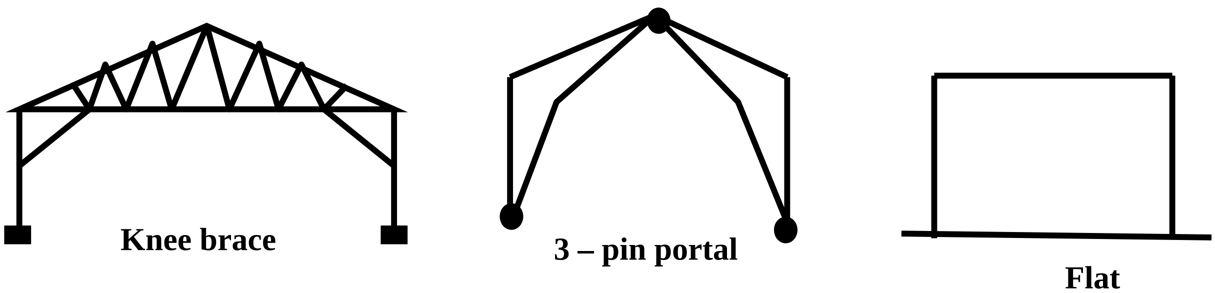


Fig: 8 Single Bay, Single-storey Structures

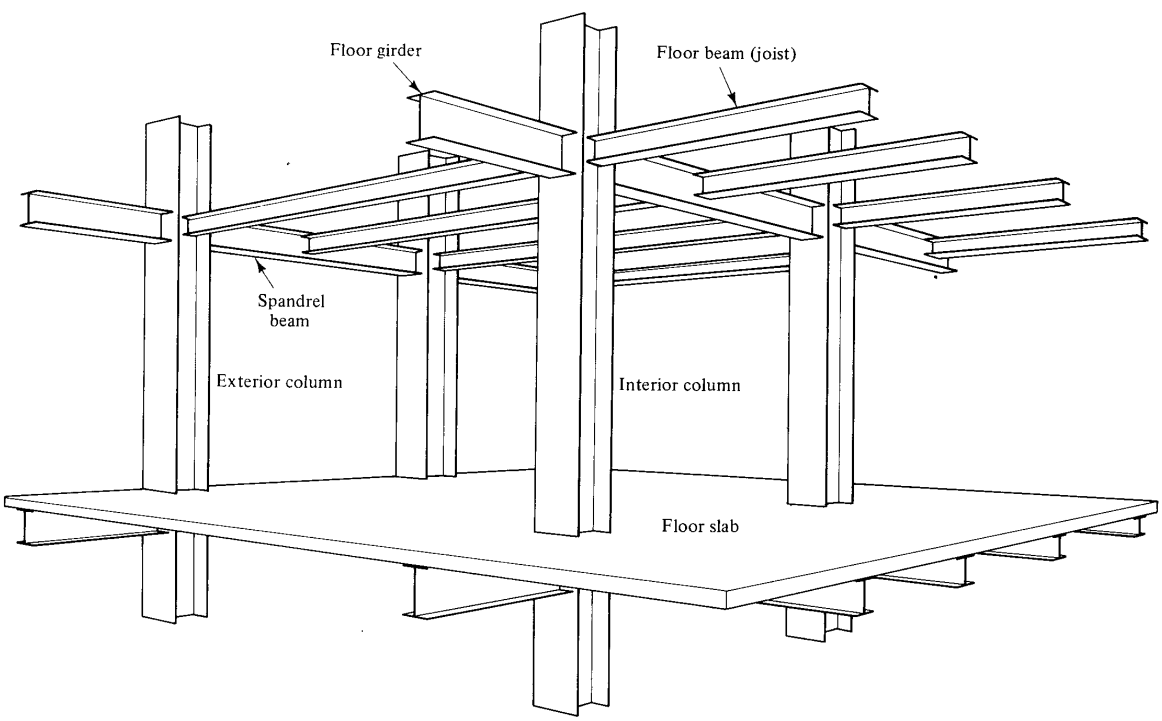


Fig.9 Beam and column construction

Aesthetic considerations are important in many cases and the choice of design may not always be based on cost alone. The weight saving may be offset by the higher cost of the stronger material or the higher cost of fabrication/construction of complicated systems. Often no one solution for a given structure is prominent or obvious to the exclusion of all other alternatives. As an example, we can illustrate several choices available to the designer for a single bay, single storey structure [See Fig. 8]. An example of beam and column system frequently used is illustrated in Fig. 9. Cable stayed structures are frequently employed in long span bridges and buildings and are shown in Fig. 10. In the following chapters, the analysis and design of steel elements are discussed in depth, followed by the analysis and design of selection of structural elements.

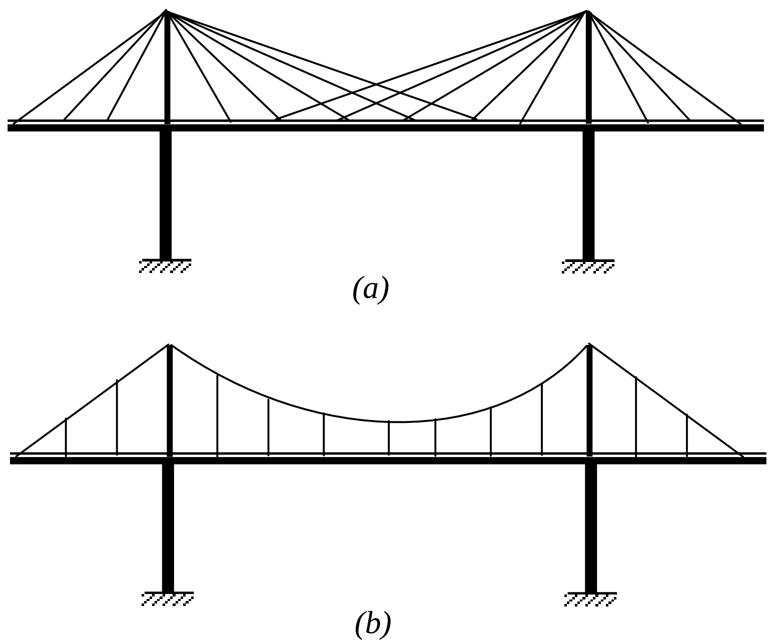


Fig. 10 Cable-stayed structures

6.0 CONCLUDING REMARKS

The paper discussed the role of a Structural Engineer in designing constructed facilities in the 21st century. The relevant environmental factors which affect his work, the durability and infrastructure crises, which face the Industry, are all discussed in detail. The importance of life cycle costing and the rational selection of appropriate materials for construction are discussed in depth.

A strong case is made for taking account of the durability and environmental considerations in the design process. They make a vital contribution to the life cycle cost of a structure. The paper concludes with a description of the structural design process in the everyday life of a structural engineer.

7.0 REFERENCES

1. Swamy, R.N. (2000) "Educating Engineers", The Structural Engineer, London, Volume 78 (17)
2. Vesiland, A.L., (Duke University, USA): Private Communication
3. Suresh. V., "Rural and Urban Housing : Opportunities for use of steel", Keynote address at the Seminar on Steel in Building Construction, (Ministry of Steel and INSDAG), September 2000
4. Owens. G. and Wood. A. (1998): "World-wide use of Steel in Construction", Journal of Constructional Steel Research, Volume 46 (1-3).
5. INSDAG and STUP CONSULTANTS (2000): " Life Cycle Cost of Viaduct Structures of Elevated Light Rail Transit System Project", INSDAG, Calcutta.
6. Naithani, K.C., Gupta, V.K., Sarvendra Kumar, Mittal, M.K., Ghosh, S.K., Karmakar. D. and Maini P.K. (1999). "Cost Economics Study of Composite Construction and its Comparison with traditional R.C. Construction, CECR.
7. Narayanan. R. and Kalyanaraman, V (2000) : Durability and Life Cycle Costs", National Seminar on Global Standards on Quality Assurance and Reliability in Structures, Association of Consulting Engineers (India), Coimbatore Centre.
8. McGuire W. (1968), "Steel Structures", Prentice Hall International Inc., London.
9. Merrit F.S. (1983), "Standard Handbook for Civil Engineers", (3rd Edition) McGraw Hill Book Company, New York.
10. Dowling P.J., Knowles P.R. and Owens G.W., (1988), "Structural Steel Design", The Steel Construction Institute, Butterworths, London.

4 INTRODUCTION TO LIMIT STATES

1.0 INTRODUCTION

A Civil Engineering Designer has to ensure that the structures and facilities he designs are (i) fit for their purpose (ii) safe and (iii) economical and durable. Thus safety is one of the paramount responsibilities of the designer. However, it is difficult to assess at the design stage how safe a proposed design will actually be – consistent with economy. There is, in fact, a great deal of uncertainty about the many factors, which influence both safety and economy. Firstly, there is a natural variability in the material strengths and secondly it is impossible to predict the loading, which a structure (e.g. a building) may be subjected to on a future occasion. Thus uncertainties affecting the safety of a structure are due to

- uncertainty about loading
- uncertainty about material strength and
- uncertainty about structural dimensions and behaviour.

These uncertainties together make it impossible for a designer to guarantee that a structure will be absolutely safe. All that the designer could ensure is that the risk of failure is extremely small, despite the uncertainties.

An illustration of the statistical meaning of safety is given in Fig. 1. Let us consider a structural component (say, a beam) designed to carry a given nominal load. Bending moments (B.M.) produced by characteristic loads are first computed. These are to be compared with the characteristic resistance or strength (R.M.) of the beam. But the characteristic resistance (R.M.) itself is <u>not</u> a fixed quantity, due to variations in material strengths that might occur between nominally same elements. The actual resistance of these elements can be expected to vary as a consequence. The statistical distribution of these member strengths (or resistances) will be as sketched in (*a*).

Similarly, the variation in the maximum loads and therefore load effects (such as bending moment) which different structural elements (all nominally the same) might encounter in their service life would have a distribution shown in (*b*). ***The uncertainty here is both due to variability of the loads applied to the structure, and also due to the variability of the load distribution through the structure.*** Thus if a particularly weak structural component is subjected to a heavy load which exceeds the strength of the structural component, clearly failure could occur.

Unfortunately it is not practicable to define the probability distributions of loads and strengths, as it will involve hundreds of tests on samples of components. Normal design calculations are made using a single value for each load and for each material property and making appropriate safety factor into the design calculations. The value used is termed as "***Characteristic Strength or Resistance***" or " ***Characteristic Load***".

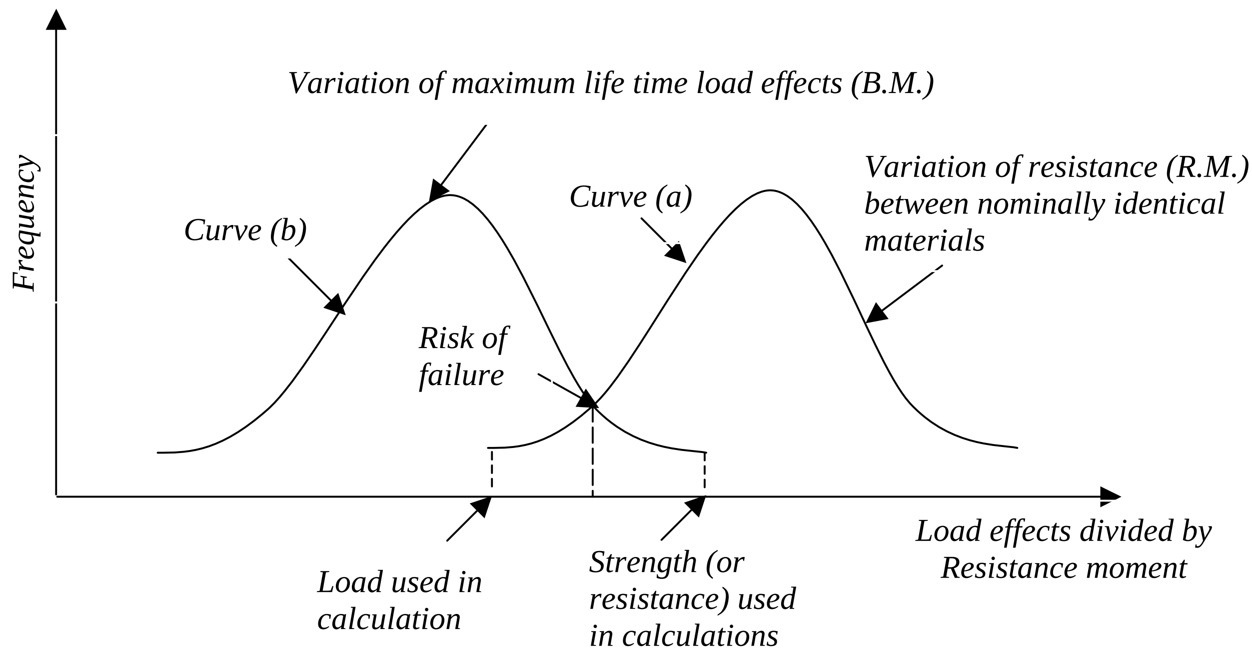


Fig. 1 Statistical Meaning of Safety

Characteristic resistance of a material (such as Concrete or Steel) is defined as that value of resistance below which not more than a prescribed percentage of test results may be expected to fall. (For example the characteristic yield stress of steel is usually defined as that value of yield stress below which not more than *5%* of the test values may be expected to fall). In other words, this strength is expected to be exceeded by *95%* of the cases.

Similarly, ***the characteristic load is that value of the load, which has an accepted probability of not being exceeded during the life span of the structure.*** Characteristic load is therefore that load which will <u>not</u> be exceeded *95%* of the time.

2.0 STANDARDISATION

Most structural designs are based on experience. Standardisation of all designs is unlikely within the foreseeable future hence design rules, based on experience, become useful. If a similar design has been built successfully elsewhere, there is no reasons why a designer may not consider it prudent to follow aspects of design that have proved successful, and adopt standardised design rules. As the consequences of bad design can be catastrophic, the society expects designers to explain their design decisions. It is therefore advantageous to use methods of design that have proved safe in the past. Standardised design methods can help in comparing alternative designs while minimising the risk of the cheapest design being less safe than the others.

Most Governments attempt to ensure structural safety through regulations and laws. Designers then attempt to achieve maximum economy within the range of designs that the regulations allow. Frequently the professions are allowed to regulate themselves; in these a cases the Regulations or ***Codes of Practices*** are evolved by consultation and consensus within the profession.

3.0 ALLOWABLE STRESS DESIGN (ASD)

With the development of linear elastic theories in the *19th* century the stress-strain behaviour of new materials like wrought iron & mild steel could be accurately represented. These theories enabled indeterminate structures to be analysed and the distribution of bending and shear stresses to be computed correctly. The first attainment of yield stress of steel was generally taken to be the onset of failure. The limitations due to non-linearity and buckling were neglected.

The basic form of calculations took the form of verifying that the stresses caused by the characteristic loads must be less than an "***allowable stress***", which was a fraction of the yield stress. Thus the allowable stress may be defined in terms of a "***factor of safety***" which represented a margin for overload and other unknown factors which could be tolerated by the structure. The allowable stress is thus directly related to yield stress by the following expression:

$$Allowable\,stress = \frac{Yield\ stress}{Factor\,of\ safety}$$

In general, each member in a structure is checked for a number of different combinations of loading. The value of factor of safety in most cases is taken to be around 1.67. Many loads vary with time and these should be allowed for. It is unnecessarily severe to consider the effects of all loads acting simultaneously with their full design value, while maintaining the same factor of safety or safety factor. Using the same factor of safety or safety factor when loads act in combination would result in uneconomic designs.

A typical example of a set of load combinations is given below, which accounts for the fact that the dead load, live load and wind load are all unlikely to act on the structure simultaneously at their maximum values:

(Stress due to dead load + live load)	< allowable stress
(Stress due to dead load + wind load)	< allowable stress
(Stress due to dead load + live load + wind)	< 1.33 times allowable stress.

In practice there are severe limitations to this approach. These are the consequences of material non-linearity, non-linear behaviour of elements in the post-buckled state and the ability of the steel components to tolerate high theoretical elastic stresses by yielding locally and redistributing the loads. Moreover the elastic theory does not readily allow for redistribution of loads from one member to another in a statically indeterminate structures.

4.0 LIMIT STATE DESIGN

An improved design philosophy to make allowances for the shortcomings in the ***"Allowable Stress Design"*** was developed in the late *1970*'s and has been extensively incorporated in design standards and codes formulated in all the developed countries. Although there are many variations between practices adopted in different countries the basic concept is broadly similar. The probability of operating conditions not reaching failure conditions forms the basis of "***Limit States Design***" adopted in all countries.

"Limit States" are the various conditions in which a structure would be considered to have failed to fulfil the purpose for which it was built. In general two limit states are considered at the design stage and these are listed in Table 1.

Table 1: Limit States

Limit State of Strength	Serviceability Limit State
Strength (yield, buckling)	Deflection
Stability against overturning and sway	Vibration
Fracture due to fatigue	Fatigue checks (including reparable
Plastic collapse	damage due to fatigue)
Brittle Fracture	Corrosion
	Fire

"Limit State of Strength" are: loss of equilibrium of the structure and loss of stability of the structure. "Serviceability Limit State" refers to the limits on acceptable performance of the structure.

Not all these limits can be covered by structural calculations. For example, corrosion is covered by specifying forms of protection (like painting) and brittle fracture is covered by material specifications, which ensure that steel is sufficiently ductile.

5.0 PARTIAL SAFETY FACTOR

The major innovation in the new codes is the introduction of the partial safety factor format. A typical format is described below:

In general calculations take the form of verifying that

$S^{*} \leq R^{*}$

where S^* is the calculated <u>factored</u> load effect on the element (like bending moment, shear force etc) and R^* is the calculated <u>factored</u> resistance of the element being checked, and is a function of the nominal value of the material yield strength.

S^* is a function of the combined effects of <u>factored</u> dead, live and wind loads.
(Other loads – if applicable, are also considered)

In accordance with the above concepts, the safety format used in Limit State Codes is based on probable maximum load and probable minimum strengths, so that a consistent level of safety is achieved. Thus, the design requirements are expressed as follows:

$$S_d \leq R_d$$

where S_d = Design value of internal forces and moments caused by the design Loads, F_d

$F_d = \gamma_f$ * Characteristic Loads.

γ_f = a load factor which is determined on probabilistic basis

$$R_d = \frac{\text{Characteristic Value of Resistance}}{\gamma_m}$$

where γ_m = a material factor, which is also determined on a '*probabilistic basis*'

It should be noted that γ_f makes allowance for possible deviation of loads and the reduced possibility of all loads acting together. On the other hand γ_m allows for uncertainties of element behaviour and possible strength reduction due to manufacturing tolerances and imperfections in the material.

Collapse is not the only possible failure mode. Excessive deflection, excessive vibration, fracture etc. also contribute to Limit States. Fatigue is an important design criterion for bridges, crane girders etc. (These are generally assessed under serviceability Limit States)

Thus the following limit states may be identified for design purposes:

- Ultimate Limit State is related to the maximum design load capacity under extreme conditions. The partial load factors are chosen to reflect the probability of extreme conditions, when loads act alone or in combination.
- Serviceability Limit State is related to the criteria governing normal use. Unfactored loads are used to check the adequacy of the structure.
- Fatigue Limit State is important where distress to the structure by repeated loading is a possibility.

The above limit states are provided in terms of partial factors reflects the severity of the risks.

An illustration of partial safety factors for applied load and materials as suggested in the revised IS: 800 for Limit States of Strength and Limit States of Serviceability are given in Table 2 and 3 respectively.

Table 2*: *Partial safety factors

Combination	**Limit State of Strength**					**Limit state of Serviceability**			
	DL	**LL’**		**WL/ EL**	**AL**	**DL**	**LL’**		**WL/ EL**
		Leading	Accompanying				Leading	Accompanying	
DL+LL+CL	1.5	1.5	1.05	—	—	1.0	1.0	1.0	—
DL+LL+CL+ WL/EL	1.2 1.2	1.2 1.2	1.05 0.53	0.6 1.2	—	1.0	0.8	0.8	0.8
DL+WL/EL	1.5 (0.9)*	—	—	1.5	—	1.0	—	—	1.0
DL+ER	1.2 (0.9)	1.2	—	—	—	—	—	—	—
DL+LL+AL	1.0	0.35	0.35	—	1.0	—	—	—	—

* This value is to be considered when the dead load contributes to stability against overturning is critical or the dead load causes reduction in stress due to other loads.

‘ When action of different live loads is simultaneously considered, the leading live load is whichever one causes the higher load effects in the member/section.

Abbreviations: DL= Dead Load, LL= Imposed Load (Live Loads), WL= Wind Load,
CL= Crane Load (Vertical/horizontal), AL=Accidental Load,
ER= Erection Load, EL= Earthquake Load.

Table 3*: *Partial safety factors

Sl. No.	**Definition**	**Partial Safety Factor**	
1	Resistance, governed by yielding γ_{m0}	1.10	
2	Resistance of member to buckling γ_{m0}	1.10	
3	Resistance, governed by ultimate stress γ_{m1}	1.25	
4	Resistance of connection γ_{m1}	Shop Fabrications	Field Fabrications
	(i) Bolts-Friction Type, γ_{mf}	1.25	1.25
	(ii) Bolts-Bearing Type, γ_{mb}	1.25	1.25
	(iii) Rivets, γ_{mr}	1.25	1.25
	(iv) Welds, γ_{mw}	1.25	1.50

Requirements for all Buildings to maintain Structural integrity are given below:

Structures should remain as complete integral units even when (due to an accident such as explosion) one of the members fail or become inoperative. This requirement provides a significant measure of safety for the occupants and is termed "Structural integrity requirement".

The buildings should be effectively tied together at each principal floor and roof level, in both directions. The recommended minimum tie strengths are *75 kN* at floor level, *40 kN* at roof level. Each section between expansion joints should be treated as a separate building. These requirements are aimed at ensuring that the collapse of one element of a structure does not trigger the failure of the structure as a whole. By tying the structure together, it is possible to ensure that there is an alternative load path that would help to enhance safety.

Suggested requirements for integrity of buildings of five storeys or more are given below:

- For sway resistance, no portion of structures should be dependent on only one bracing system.
- The minimum tie strengths to be provided are $0.5\ W_f\ S_t\ L_a$ internally and $0.25\ W_f\ S_t\ L_a$ externally.
  W_f - total factored load / unit area
  S_t - tie spacing
  L_a - distance between columns in the direction
- At the edge of the structure, columns should be restrained by horizontal ties resisting *1%* of column load.
- Columns should be continuous vertically through the floors, as far as possible.
- Collapse must not be disproportionate and the role of key elements should be identified.
- Precast floors must be anchored at both ends.

6.0 FACTORS GOVERNING THE ULTIMATE STRENGTH

Stability is generally ensured for the structure as a whole and for each of its elements. This includes overall frame stability against overturning and sway, as given below. The structure as a whole or any part of it are designed to prevent instability due to overturning, uplift or sliding under factored load as given below:

a) The actions are divided into components aiding instability and components resisting instability.

b) The permanent and variable actions and their effects causing instability are combined using appropriate load factors as per the Limit States requirements to obtain maximum destabilizing effect.

c) The permanent actions (loads) and effects contributing to resistance shall be multiplied with a partial safety factor 0.9 and added together with design resistance (after multiplying with appropriate partial safety factor). Variable actions and their effects contributing to resistance are disregarded

d) The resistance effect shall be greater than or equal to the destabilizing effect. Combination of imposed and dead loads should be such as to cause most severe effect on overall stability.

7.0 LIMIT STATE OF SERVICEABILITY

As stated in IS: 800, Serviceability Limit State is related to the criteria, governing normal use. Serviceability limit state is limit state beyond which service criteria, specified below, are no longer met:

a) Deflection Limit
b) Vibration Limit
c) Durability Consideration
d) Fire Resistance

Load factor, γ_f, of value equal to unity are used for all loads leading to Serviceability Limit States to check the adequacy of the structure under serviceability limit states, unless specified otherwise.

The deflection under serviceability loads of a building or a building component should be such that, they do not impair the strength of the structure or components or cause damage to finishing. Deflections are to be checked for the most adverse but realistic combination of service loads and their arrangement, by elastic analysis, using a load factors as per Table 3. Table 4 gives recommended limits of deflections for certain structural members and systems.

As per IS: 800, suitable provisions in the design are required to be made for the dynamic effects of live loads, impact loads and vibration due to machinery operating loads. In severe cases possibility of resonance, fatigue or unacceptable vibrations shall be investigated. Unusually flexible structures (generally the height to effective width of lateral load resistance system exceeding 5:1) need to be investigated for lateral vibration under dynamic wind loads. Structures subjected to large number of cycles of loading shall be designed against fatigue failure as discussed in Chapter 2.

Durability or Corrosion resistance of a structure is generally, under conditions relevant to their intended life as are listed below:

a) The environment
b) The degree of exposure
c) The shape of the member and the structural detail
d) The protective measure
e) Ease of maintenance

Fire resistance of a steel member is a function of its mass, its geometry, the actions to which it is subjected, its structural support condition, fire protection measures adopted and the fire to which it is exposed. Design provisions to resist fire are briefly discussed in Chapter 2.

Table 4: ***Partial safety factors [According to IS: 800 (2007)]***

Type of Building	Deflection	Design Load	Member	Supporting	Maximum Deflection
Industrial building	Vertical	Live load/Wind load	Purlins and Girts Purlins and Girts	Elastic cladding Brittle cladding	Span / 150 Span / 180
		Live load Live load	Simple span Simple span	Elastic cladding Brittle cladding	Span / 240 Span / 300
		Live load Live load	Cantilever span Cantilever span	Elastic cladding Brittle cladding	Span / 120 Span / 150
		Live load or Wind load	Rafter supporting	Profiled Metal Sheeting	Span / 180
				Plastered Sheeting	Span / 240
		Crane load (Manual operation)	Gantry	Crane	Span / 500
		Crane load (Electric operation up to 50 *t*)	Gantry	Crane	Span / 750
		Crane load (Electric operation over 50 *t*)	Gantry	Crane	Span / 1000
	Lateral	No cranes	Column	Elastic cladding	Height / 150
		No cranes	Column	Masonry/Brittle cladding	Height / 240
		Crane + wind	Gantry (lateral)	Crane(absolute)	Span / 400
				Relative displacement between rails	10 mm
		Crane+ wind	Column/frame	Gantry (Elastic cladding; pendent operated)	Height / 200
			Column/frame	Gantry (Brittle cladding; cab operated)	Height / 400
Other Buildings	Vertical	Live load	Floor & Roof	Elements not susceptible to cracking	Span / 300
		Live load	Floor & Roof	Elements susceptible to cracking	Span / 360
		Live load	Cantilever	Elements not susceptible to cracking	Span / 150
		Live load		Elements susceptible to cracking	Span / 180
	Lateral	Wind	Building	Elastic cladding Brittle cladding	Height / 300 Height / 500
		Wind	Inter storey drift	---	Storey height / 300

8.0 CONCLUDING REMARKS

This chapter reviews the provisions of safety, consequent on uncertainties in loading and material properties. The partial load factors employed in design to take into account these variations are discussed and illustrated.

9.0 REFERENCES

1. Owens G.W., Knowles P.R : "Steel Designers Manual", The Steel Construction Institute, Ascot, England, 1994
2. British Standards Institution : "BS 5950, Part-1 Structural use of steelwork in building", British Standards Institution, London, 1985
3. IS: 800 (2007), General Construction in Steel – Code of Practice, Bureau of Indian Standards, New Delhi, 2007.

Structural Steel Design Project	Job No:	Sheet *1 of 2*	Rev
	Job Title: *MAXIMUM FACTORED LOADS*		
	Worked Example - 1		
		Made by *SSSR*	Date *15-09-99*
Calculation Sheet		Checked by *RN*	Date *20-09-99*

A frame sketched in Fig. 2 is loaded by a dead load of 6 kN/m, imposed load of 20 kN/m and wind load of 10 kN/m. The example below illustrates the checks in respect of the following.

- *Imposed load + Dead load*
- *Wind load + Dead load*
- *Imposed load + Wind load + Dead load*

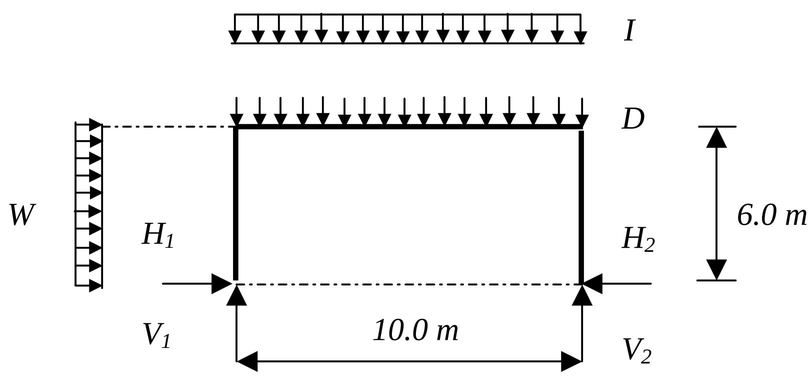


Fig. 2 Portal frame subject to loading

Dead Load (D) 6 kN/m

Imposed Load (I) 20 kN/m

Wind Load (W) 10 kN/m

Case 1- Dead plus imposed loads

$V_1 = V_2 = (1.50\, l + 1.50\, D) * span/2$
$= (1.50 * 20 + 1.50 * 6) * 5 = 195.0 kN$

$\gamma_{fDL} = 1.50$
$\gamma_{fIL} = 1.50$

Case 2 - Dead plus wind

Taking moments about right support,

$V_1 = [1.50\ D\ span^2 / 2 - 1.50\ W * height^2 / 2]\ 1/10$

$= [1.50 * 6 * 100 / 2 - 1.50 * 10 * 36 / 2]\ 1/10$

$= 18.0\ kN$

$\gamma_{fDL} = 1.50$
$\gamma_{fWL} = 1.50$

$V_2 = 1.50\ D * span - V_1$

$= 1.50 * 6 * 10 - 18.0 = 72.0\ kN$

$H_1 + H_2 = 1.50\ W * height = 1.50 * 10 * 6 = 90\ kN$

(Note: The evaluation of H_1 and H_2 will depend on the stiffness of the members as the problem is statically indeterminate)

Case 3 - Dead plus imposed plus wind

$V_1 = 1.20 * D * span/2 + 1.20 * I * span/2 - 1.20 * W * height^2 / (2 * span)$

$= 1.20 * 6 * 5 + 1.20 * 20 * 5 - 1.20 * 10 * 36/20$

$= 134.4\ kN$

$\gamma_{fDL} = 1.35$
$\gamma_{fIL} = 1.50$
$\gamma_{fWL} = 1.05$

$V_2 = 1.20 * D * span/2 + 1.20 * I * span/2 + 1.20 * W * height^2/2 * span$

$= 1.20 * 6 * 5 + 1.20 * 20 * 5 + 1.20 * 10 * 36/20$

$= 177.6\ kN$

The worst value for design purposes are;

$V_1 = 195.0\ kN\ ;\ V_2 = 177.6\ kN$

5 DESIGN OF TENSION MEMBERS

1.0 INTRODUCTION

Tension members are linear members in which axial forces act so as to elongate (stretch) the member. A rope, for example, is a tension member. Tension members carry loads most efficiently, since the entire cross section is subjected to uniform stress. Unlike compression members, they do not fail by buckling (see chapter on compression members). Ties of trusses [Fig 1(*a*)], suspenders of cable stayed and suspension bridges [Fig.1 (*b*)], suspenders of buildings systems hung from a central core [Fig.1(*c*)] (such buildings are used in earthquake prone zones as a way of minimising inertia forces on the structure), and sag rods of roof purlins [Fig 1(*d*)] are other examples of tension members.

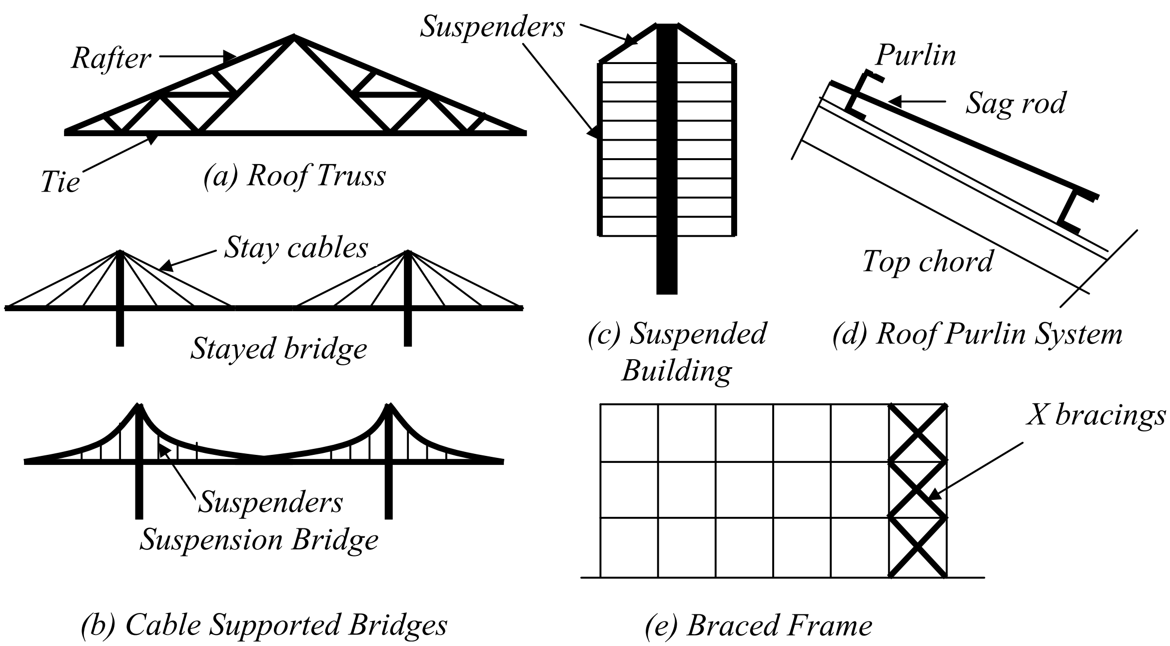


Fig. 1 Tension Members in Structures

Tension members are also encountered as bracings used for the lateral load resistance. In X type bracings [Fig.1 (*e*)] the member which is under tension, due to lateral load acting in one direction, undergoes compressive force, when the direction of the lateral load is changed and vice versa. Hence, such members may have to be designed to resist tensile and compressive forces.

The tension members can have a variety of cross sections. The single angle and double angle sections [Fig 2(*a*)] are used in light roof trusses as in industrial buildings. The tension members in bridge trusses are made of channels or I sections, acting individually or built-up [Figs. 2(*c*) and 2(*d*)]. The circular rods [Fig.2 (*d*)] are used in bracings designed to resist loads in tension only. They buckle at very low compression and are not considered effective. Steel wire ropes [Fig.2 (*e*)] are used as suspenders in the cable suspended bridges and as main stays in the cable-stayed bridges.

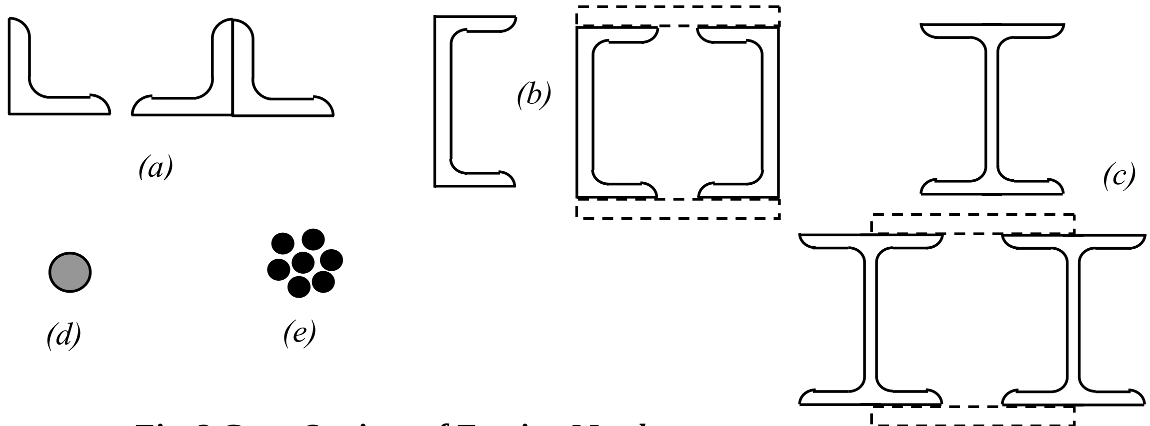


Fig. 2 Cross Sections of Tension Members

2.0 BEHAVIOUR OF TENSION MEMBERS

Since axially loaded tension members are subjected to uniform tensile stress, their load deformation behaviour (Fig.3) is similar to the corresponding basic material stress strain behaviour. Mild steel members (IS: 2062) exhibit an elastic range (a-b) ending at yielding (b). This is followed by yield plateau (b-c). In the Yield Plateau the load remains constant as the elongation increases to nearly ten times the yield strain. Under further stretching the material shows a smaller increase in tension with elongation (c-d), compared to the elastic range. This range is referred to as the strain hardening range. After reaching the ultimate load (d), the loading decreases as the elongation increases (d-e) until rupture (e). High strength steel tension members do not exhibit a well-defined yield point and a yield plateau (Fig.3). The *0.2%* offset load, *T*, as shown in Fig. 3 is usually taken as the yield point in such cases.

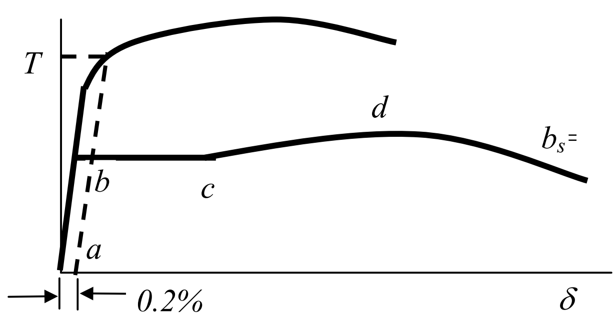


Fig. 3 Load – Elongation of Tension Members

2.1 Design strength of tension members

Although steel tension members can sustain loads up to the ultimate load without failure, the elongation of the members at this load would be nearly *10-15%* of the original length and the structure supported by the member would become unserviceable. Hence, in the design of tension members, the yield load is usually taken as the limiting load. The corresponding design strength in member under axial tension is given by

$$T_{dg} = f_y A_g / \gamma_{m0} \tag{1}$$

Where, f_y is the yield strength of the material (in *MPa*), A_g is the gross area of cross section and γ_{m0} is the partial safety factor for failure in tension by yielding. The value of γ_{m0} according to IS: 800 is *1.10*.

2.2 Plates under Tension

Frequently plates under tension have bolt holes. The tensile stress in a plate at the cross section of a hole is not uniformly distributed in the elastic range, but exhibits stress concentration adjacent to the hole [Fig 4 (*a*)]. The ratio of the maximum elastic stress adjacent to the hole to the average stress on the net cross section is referred to as the Stress Concentration Factor. This factor is in the range of 2 to 3, depending upon the ratio of the diameter of the hole to the width of the plate normal to the direction of stress.

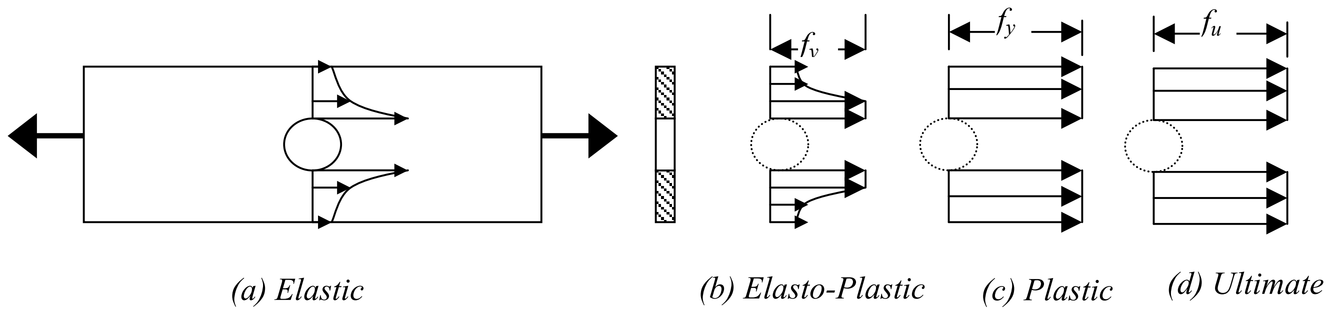


Fig. 4 Stress Distribution at a Hole in a Plate under Tension

In statically loaded tension members with a hole, the point adjacent to the hole reaches yield stress, f_y, first. On further loading, the stress at that point remains constant at the yield stress and the section plastifies progressively away from the hole [Fig.4 (*b*)], until the entire net section at the hole reaches the yield stress, f_y, [Fig. 4(*c*)]. Finally, the rupture (tension failure) of the member occurs when the entire net cross section reaches the ultimate stress, f_u, [Fig. 4(*d*)]. Since only a small length of the member adjacent to the smallest cross section at the holes would stretch a lot at the ultimate stress, and the overall member elongation need not be large, as long as the stresses in the gross section is below the yield stress. Hence, the design strength as governed by net cross-section at the hole, $T_{dn,}$ is given by

$$T_{dn} = 0.9 f_u A_n / \gamma_{m1} \tag{2}$$

where, f_u is the ultimate stress of the material, A_n is the net area of the cross section after deductions for the hole [Fig.4(*b*)] and γ_{m1} is the partial safety factor against ultimate tension failure by rupture ($\gamma_{m1} = 1.25$). Similarly threaded rods subjected to tension could fail by rupture at the root of the threaded region and hence net area, A_n, is the root area of the threaded section (Fig.5).

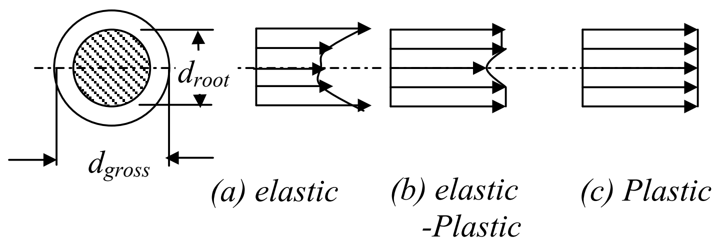


Fig. 5 Stress in a threaded Rod

The design tension of the plates with hole or the threaded rod could also be governed by yielding of the gross cross section beyond the thread (with area equal to A_g) above which the member deformation becomes large and objectionable and the corresponding design load is given by

$$T_{dg} = f_y \, A_g \, / \, \gamma_{m0} \tag{3}$$

where, $\gamma_{m0}=1.10$. The lower value of the design tension capacities, as given by Eqn. 2 and 3, governs the design strength of a plate with holes.

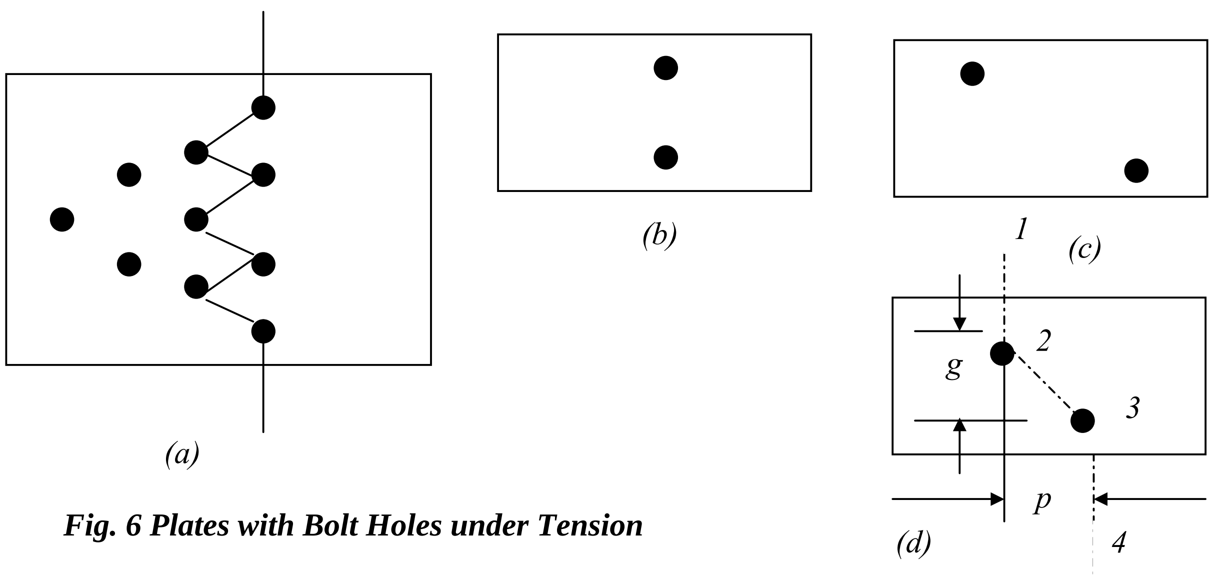


Fig. 6 Plates with Bolt Holes under Tension

Frequently, plates have more than one hole for the purpose of making connections. These holes are usually made in a staggered arrangement [Fig.6 (*a*)]. Let us consider the two extreme arrangements of two bolt holes in a plate, as shown in Fig.6 (*b*) & 6(*c*). In the case of the arrangement shown in Fig.6 (*b*), the gross area is reduced by two bolt holes to obtain the net area. Whereas, in arrangement shown in Fig.6c, deduction of only one hole is necessary, while evaluating the net area of the cross section.

Obviously the change in the net area from the case shown in Fig.6(*c*) to Fig.6 (*b*) has to be gradual. As the pitch length (the centre to centre distance between holes along the direction of the stress) p, is decreased, the critical cross section at some stage changes from straight section [Fig.6(*c*)] to the staggered section 1-2-3-4 [Fig.6 (*d*)]. At this stage, the net area is decreased by two bolt holes along the staggered section, but is increased due to the inclined leg (2-3) of the staggered section. The net effective area of the staggered section 1-2-3-4 is given by

$$A_n = (b - 2d + p^2 / 4g)\ t \tag{4}$$

where, the variables are as defined in Fig.6(*d*). In Eqn. 4 the increase of net effective area due to inclined section is empirical and is based on test results. It can be seen from Eqn.4, that as the pitch distance, *p,* increases and the gauge distance, *g*, decreases, the net effective area corresponding to the staggered section increases and becomes greater than the net area corresponding to single bolt hole. This occurs when

$$p^2 / 4g > d \tag{5}$$

When multiple holes are arranged in a staggered fashion in a plate as shown in Fig.6 (a), the net area corresponding to the staggered section in general is given by

$$A_{net} = \left(b - nd + \sum \frac{p^2}{4g} \right) t \tag{6}$$

where, *n* is the number of bolt holes in the staggered section [*n* = 7 for the zigzag section in Fig. 6(*a*)] and the summation over $p^2/4g$ is carried over all inclined legs of the section [equal to *n-1* = 6 in Fig.6(*a*)]. Normally, net area of different staggered and straight sections have to be evaluated to obtain the minimum net area to be used in calculating the design strength in tension. An example analysis of a plate with holes under tension is illustrated in Appendix I.

2.3 ANGLES UNDER TENSION

Angles are extensively used as tension members in trusses and bracings. Angles, if axially loaded through centroid, could be designed as in the case of plates. However, usually angles are connected to gusset plates by bolting or welding only one of the two legs (Fig. 7).

This leads to eccentric tension in the member, causing non-uniform distribution of stress over the cross section. Further, since the load is applied by connecting only one leg of the member there is a shear lag locally at the end connections.

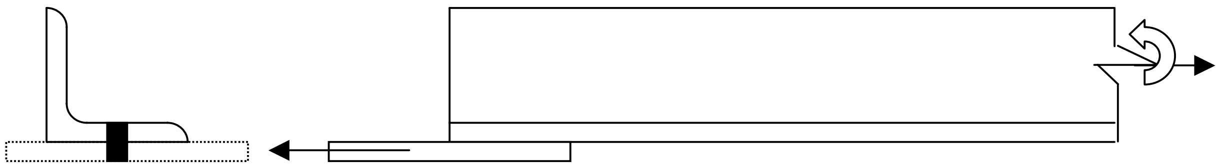


Fig. 7 Angles Eccentrically Loaded through Gussets

Kulak and Wu (1997) have reported, based on an experimental study, the results on the tensile strength of single and double angle members. Summary of their findings is:

- The effect of the gusset thickness, and hence the out of plane stiffness of the end connection, on the ultimate tensile strength is not significant.
- The thickness of the angle has no significant influence on the member strength.
- The effect of shear lag, and hence the strength reduction, is higher when the ratio of the area of the outstanding leg to the total area of cross-section increases.
- When the length of the connection (the number of bolts in end connections) increases, the tensile strength increases up to 4 bolts and the effect of further increase in the number of bolts, on the tensile strength of the member is not significant. This is due to the connection restraint to member bending caused by the end eccentric connection.
- Even double angles connected on opposite sides of a gusset plate experience the effect of shear lag.

Based on the test results, Kulak and Wu (1997) found that the shear lag due to connection through one leg only causes at the ultimate stage the stress in the outstanding leg to be closer only to yield stress even though the stress at the net section of the connected leg may have reached ultimate stress. They have suggested an equation for evaluating the tensile strength of angles connected by one leg, which accounts for various factors that significantly influence the strength. In order to simplify calculations, this formula has suggested that the stress in the outstanding leg be limited to f_y (the yield stress) and the connected sections having holes to be limited to f_u (the ultimate stress). The design tensile strength, T_d, should be the minimum of the following:

<u>Strength as governed by tearing at net section:</u>

$$T_{dn} = 0.9 A_{nc} f_u / \gamma_{m1} + \beta A_{go} f_y / \gamma_{m0} \qquad (7a)$$

Where, f_y and f_u are the yield and ultimate stress of the material, respectively. A_{nc} and A_o, are the net area of the connected leg and the gross area of the outstanding leg, respectively. The partial safety factors $\gamma_{m0} = 1.10$ and $\gamma_{m1} = 1.25$. β, accounts for the end fastener restraint effect and is given by,

$$\beta = 1.4 - 0.076\,(w/t)\,(f_y/f_u)\,(b_s/L_c) \ \leq \ (f_u \cdot \gamma_{mo} / f_y \cdot \gamma_{m1}) \text{ and } \beta \geq \ 0.7$$

where w and b_s are as shown in Fig 8

L_c = Length of the end connection, i.e., distance between the outermost bolts in the end joint measured along the length direction or length of the weld along the length direction and t = thickness of the leg

Alternatively, the rupture strength of net section may be taken as

$$T_{dn} = \alpha A_n f_u / \gamma_{m1}$$

where

α = 0.6 for one or two bolts, 0.7 for three bolts and 0.8 for four or more bolts along the length in the end connection or equivalent weld length

A_n = net area of the total cross section

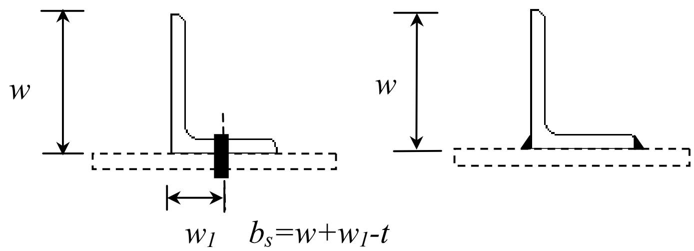


Fig 8 Angles with End Connection

<u>Strength as governed by yielding of gross section:</u>

$$T_{dg} = A_g f_y / \gamma_{m0} \quad (7\ b)$$

Where, A_g is the gross area of the angle section.

<u>Strength as governed by block shear failure:</u>

A tension member may fail along end connection due to block shear as shown in Fig. 9. The corresponding design strength can be evaluated using the following equations. If the centroid of bolt pattern is not located between the heel of the angle and the centreline of the connected leg, the connection shall be checked for block shear strength given by

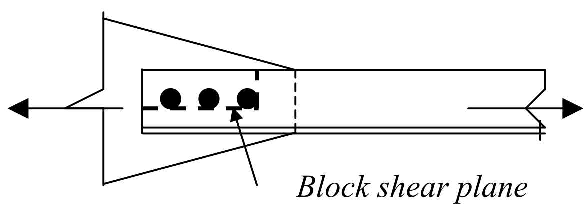


Fig. 9 Block Shear Failure

$$T_{db} = (A_{vg}\ f_y / (\sqrt{3}\ \gamma_{m0}) + \ 0.9 A_{tn} f_u / \gamma_{m1})$$

or

$$T_{db} = (0.9 A_{vn}\ f_u / (\sqrt{3}\ \gamma_{m1}) + A_{tg}\ f_y / \gamma_{m0}) \quad (7c)$$

where, A_{vg} and A_{vn} = minimum gross and net area in shear along a line of transmitted force, respectively, and A_{tg} and A_{tn} = minimum gross and net area in tension from the hole to the toe of the angle, perpendicular to the line of force, respectively.

The design strength of an angle loaded in tension through a connection in one leg is given by the smallest of the values obtained from Eqn. 7(a) to 7(c). These equations are valid for both single angle and double angles in tension, irrespective of whether they are on the same side or opposite sides of the gusset. A sample design of angle tension member is given in worked example 2.

The efficiency, η, of an angle tension member is calculated as given below:

$$\eta = T_d / (A_g\ f_y / \gamma_{m0}) \qquad (8)$$

Depending upon the type of end connection and the configuration of the built-up member, the efficiency may vary between 0.85 and 1.0. The higher value of efficiency is obtained in the case of double angles on the opposite sides of the gusset connected at the ends by welding and the lower value is usual in the bolted single angle tension members. In the case of threaded members the efficiency is around 0.85.

In order to increase the efficiency of the outstanding leg in single angles and to decrease the length of the end connections, some times a short length angle at the ends are connected to the gusset and the outstanding leg of the main angle directly, as shown in Fig. 10. Such angles are referred to as lug angles. The design of such end connections is discussed in the chapter on connections.

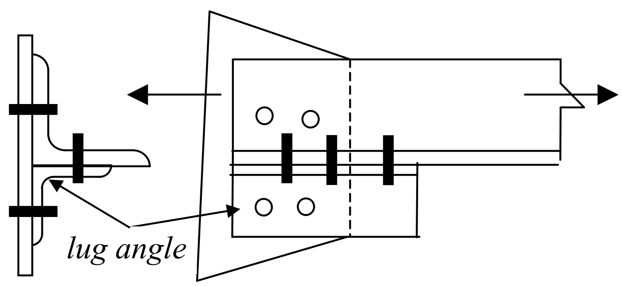


Fig. 10 Tension Member with Lug

3.0 DESIGN OF TENSION MEMBERS

In the design of a tension member, the design tensile force is given and the type of member and the size of the member have to be arrived at. The type of member is usually dictated by the location where the member is used. In the case of roof trusses, for example, angles or pipes are commonly used. Depending upon the span of the truss, the location of the member in the truss and the force in the member either single angle or double angles may be used in roof trusses. Single angle is common in the web members of a roof truss and the double angles are common in rafter and tie members of a roof truss.

Plate tension members are used to suspend pipes and building floors. Rods are also used as suspenders and as sag rods of roof purlins. Steel wires are used as suspender cables in bridges and buildings. Pipes are used in roof trusses on aesthetic considerations, in spite of fabrication difficulty and the higher cost of such tubular trusses. Built-up members made of angles, channels and plates are used as heavy tension members, encountered in bridge trusses.

3.1 Trial and Error Design Process

The design process is iterative, involving choice of a trial section and analysis of its capacity. This process is discussed in this section. Initially, the net effective area required is calculated from the design tension and the ultimate strength of the material as given below.

$$A_n = T_{dn} / (0.9 f_u / \gamma_{m1}) \qquad (9)$$

Using the net area required, the gross area required is calculated, allowing for some assumed number and size of bolt holes in plates, or assumed efficiency index in the case of angles and threaded rods. The gross area required is also checked against that required from the yield strength of the gross sections as given below.

$$A_g = T_{dg} / (f_y / \gamma_{m0}) \qquad (10)$$

A suitable trial section is chosen from the steel section handbook to meet the gross area required. The bolt holes are laid out appropriately in the member and the member is analysed to obtain the actual design strength of the trial section. The design strength of the trial section is evaluated using Eqs. 1 to 6 in the case of plates and threaded bars and using Eqs. 7 in the case of angle ties. If the actual design strength is smaller than or too large compared to the design force, a new trial section is chosen and the analysis is repeated until a satisfactory design is obtained.

3.2 Stiffness Requirement

The tension members, in addition to meeting the design strength requirement, frequently have to be checked for adequate stiffness. This is done to ensure that the member does not sag too much during service due to self-weight or the eccentricity of end plate connections. The IS: 800 imposes the following limitations on the slenderness ratio of members subjected to tension:

(a) In the case of members that are normally under tension but may experience compression due to stress reversal caused by wind / earthquake loading $\lambda/r \leq 250$.

(b) In the case of members that are designed for tension but may experience stress reversal for which it is not designed (as in X bracings) $\lambda/r \leq 350$.

(c) In the case of members subjected to tension only $\lambda/r \leq 400$

In the case of rods used as a tension member in X bracings, the slenderness ratio limitation need not be check for if they are pretensioned by using a turnbuckle or other such arrangement.

4.0 SUMMARY

The behaviour and design of various types of tension members were discussed. The important factors to be considered while evaluating the tensile strength are the reduction in strength due to bolt holes and due to eccentric application of loads through gusset plates attached to one of the elements. It was shown that the yield strength of the gross area or the ultimate strength of the net area may govern the tensile strength. The effect of connecting the end gusset plate to only one of the elements of the cross section was empirically accounted for by the reduction in the effectiveness of the out standing leg, while calculating the net effective area. The methods for accounting for these factors in the design of tension members were discussed. The iterative method of design of tension members was presented.

5.0 REFERENCES

1. AISC–LRFD. ‘*Load and resistance factor design specification for structural steel buildings’.* American Institute of Steel Construction (AISC), Chicago, III, 1993.
2. ASCE Manual No.52. ‘*Guide for design of steel transmission towers’* American Society of Civil Engineers, 1987.
3. BS-5950. ‘*Code of practice for design in simple and continuous construction: Hot rolled sections’* British Standards Institute, London, 1985.
4. CAN3-S16.1-M84. ‘*Steel structures for buildings (limit states design)’,* Canadian Standards Assoc., Rexdale, Ontario, Canada, 48, 1984.
5. Eurocode 3. *‘Design of steel structures’,* British Standards Institute 1992.
6. IS:800-2007. ‘*Code of Practice for General Construction in Steel’* Bureau of Indian Standards, New Delhi, 2007.
7. Kulak and Wu, ‘*Shear Lag in Bolted Angle Tension Members*’, ASCE, Journal of Structural Engineering, Vol.123, No.9, Sept.1997, pp.1144-1152.
8. Mueller, W.H., and Wagner, A. L. ‘*Plastic behaviour of steel angle columns*’, Res. Rept., Bonneville Power Admin., Portland, Oreg., 1985, pp 33-82.
9. Murty, Madugula and S. Mohan, ‘*Angles In Eccentric Tension’*, ASCE, Journal of Structural Engineering, Vol.114, No.10, October 1988, pp.2387-2396.
10. Nelson, H. M. ‘*Angles in Tension’,* Publication No.7, British Constructional Steelwork Assoc., United Kingdom, 1953, pp 8-18.

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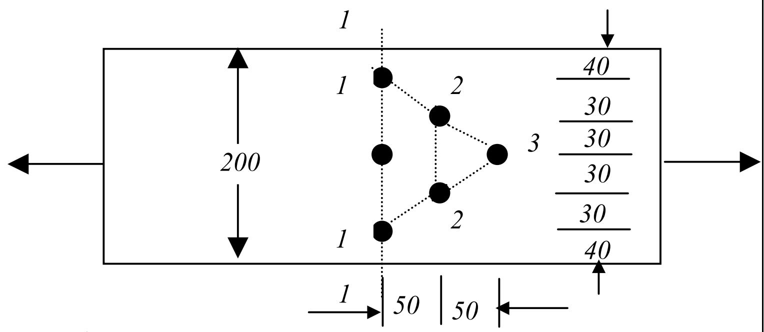
	Job Title: *Tension Member Example*		
	Worked Example*: 1*		
		Made by *SSSR*	Date: *3-1-2000*
Calculation Sheet		Checked by VK	Date

PROBLEM 1:

Determine the design tensile strength of the plate (200 X 10 mm) with the holes as shown below, if the yield strength and the ultimate strength of the steel used are 250 MPa and 420 MPa and 20 mm diameter bolts are used.

$f_y = 250$ *MPa*

$f_u = 420$ *MPa*



Calculation of net area, A_{net}*:*

$$A_n \text{ (section 11)} = (200-3*21.5)*10 = 1355\, mm^2 \text{ (governs)}$$

$$A_n \text{ (section 1221)} = \left(200-4*21.5+\frac{2*50^2}{4*30}\right)*10 = 1557\, mm^2$$

$$A_n \text{ (section 12321)} = \left(200-5*21.5+\frac{4*50^2}{4*30}\right)*10 = 1758\, mm^2$$

T_d *is lesser of*

i. $A_g.f_y / \gamma_{mo} = \dfrac{200*10*250/1.10}{1000} = 454.55$ *kN*

ii. $0.9.A_n.f_u / \gamma_{m1} = \dfrac{0.9*1355*420/1.25}{1000} = 409.75$ *kN*

Therefore $T_d = 409.75$ *kN*

Efficiency of the plate with holes $= \dfrac{T_d}{A_g\, f_y/\gamma_{m0}} = \dfrac{409.75}{454.55} = 0.90$

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Calculation Sheet</td><td>Job No:</td><td>Sheet: 1 of 4</td><td>Rev</td></tr>
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<tr><td colspan="3">Worked Example: 2</td></tr>
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<tr><td>Checked by VK</td><td>Date</td></tr>
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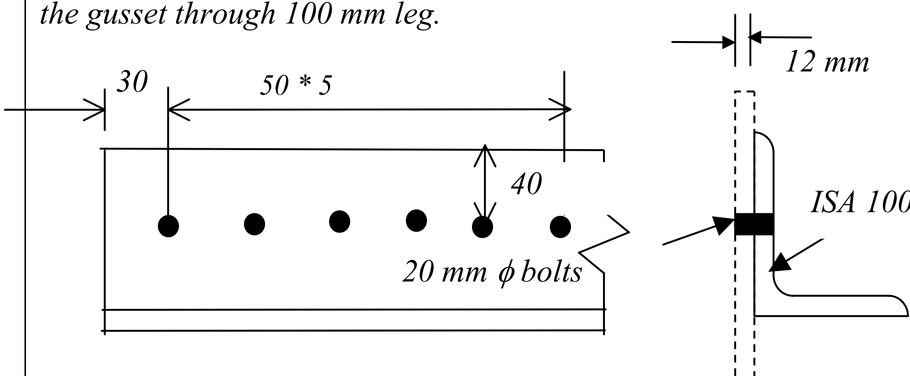
PROBLEM 2:

Analysis of single angle tension members

A single unequal angle 100x 75x 8 mm is connected to a 12 mm thick gusset plate at the ends with 6 nos. 20 mm diameter bolts to transfer tension. Determine the design tensile strength of the angle. (a) if the gusset is connected to the 100 mm leg, (b) if the gusset is connected to the 70 mm leg, (c) if two such angles are connected to the same side of the gusset through the 100 mm leg. (d) if two such angles are connected to the opposite sides of the gusset through 100 mm leg.



a) <u>*The 100mm leg bolted to the gusset*</u> :

A_{nc} = (100 - 8/2 - 21.5) *8 = 596 mm^2.

A_{go} = (75 - 8/2) * 8 = 568 mm^2

A_g = ((100-8/2) + (75 – 8/2)) * 8 =1336 mm^2

<u>*Strength as governed by tearing of net section:*</u>

$\beta = 1.4 – 0.076\ (w/t)\ (f_y/f_u)\ (b_s/L_c)$; ($b_s = w + w_1 – t = 75 + 60 – 8 = 127$)

β = 1.4 – 0.076 * (75 / 8) * (250 / 420) * (127 / 250) = 1.18

$T_{dn} = 0.9\, A_{nc}\, f_u / \gamma_{m1} + \beta\, A_{go}\, f_y / \gamma_{m0}$

= 0.9 * 596 * 420 / 1.25 + 1.18 * 568 * 250 / 1.10

= 333145 N (or) 333.1 kN

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	Job Title: *Tension Member Example*		
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Strength as governed by yielding of gross section:

$T_{dg} = A_g f_y / \gamma_{m0}$

$= 1336 * 250 / 1.10 = 303636$ N (or) 303.6 kN

Block shear strength

$T_{db} = \{ A_{vg} f_y / (\sqrt{3} \gamma_{m0}) + 0.9 A_{tn} f_u / \gamma_{m1} \}$

$= \{(5*50 + 30)*8*250 / (\sqrt{3} * 1.1) + 0.9*(40 - 21.5/2)* 8*420 / 1.25$

$= 364685$ N $= 364.7$ kN

or

$T_{db} = \{0.9 A_{vn} f_u / (\sqrt{3} \gamma_{m1}) + A_{tg} f_y / \gamma_{m0} \}$

$= \{0.9*(5*50 + 30 - 5.5*21.5)*8*420 / (\sqrt{3} *1.25) + 40*8*250 / 1.1\}$

$= 298648$ N $= 298.65$ kN

The design tensile strength of the member = 298.65 kN

The efficiency of the tension member, is given by

$$\eta = \frac{T_d}{A_g f_y} = \frac{298.5 * 1000}{(100 + 75 - 8) * 8 * 250/1.10} = 0.983$$

b) <u>*The 75 mm leg is bolted to the gusset:*</u>

$A_{nc} = (75 - 8/2 - 21.5) * 8 = 396\ mm^2$

$A_{go} = (100 - 8/2) * 8 = 768\ mm^2$

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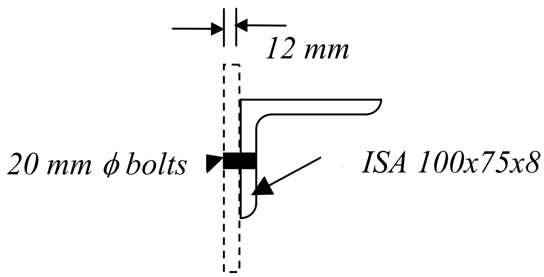

Strength as governed b tearing of net section:

$\beta = 1.4 - 0.076\ (w/t)\ (f_y/f_u)\ (b_s/L_c)$; ($b_s = w + w_1 - t = 100 + 40 - 8 = 132$)

$\beta = 1.4 - 0.076 * (100 / 8) * (250 / 420) * (132 / 250) = 1.101$

$T_{dn} = 0.9\, A_{nc} f_u / \gamma_{m1} + \beta\, A_{go}\, f_y / \gamma_{m0}$

$= 0.9 * 396 * 420 / 1.25 + 1.101 * 768 * 250 / 1.10$

$= 312000$ N (or) 312.0 kN

Strength as governed by yielding of gross section:

$T_{dg} = A_g f_y / \gamma_{m0}$

$= 1336 * 250 / 1.10 = 303636$ N (or) 303.6 kN

Block shear strength:

$T_{db} = \{ A_{vg}\, f_y / (\sqrt{3}\, \gamma_{m0}) + 0.9 A_{tn} f_u / \gamma_{m1} \}$

$= \{(5*50 + 30)*8*250 / (\sqrt{3} * 1.1) + 0.9*(35 - 21.5/2)* 8*420 / 1.25$

$= 352589$ N = 352.6 kN

or

$T_{db} = \{0.9 A_{vn}\, f_u / (\sqrt{3}\, \gamma_{m1}) + A_{tg}\, f_y / \gamma_{m0} \}$

$= \{0.9*(5*50 + 30 - 5.5*21.5)*8*420 / (\sqrt{3} *1.25) + 35*8*250 / 1.1\}$

$= 289557$ N = 289.6 kN

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The design tensile strength of the member = 289.60 kN

The efficiency of the tension member, is given by

$$\eta = \frac{T_d}{A_g\, f_y} = \frac{289.6 * 1000}{(100+75-8) * 8 * 250/1.10} = 0.954$$

Even though the tearing strength of the net section is reduced, the block shear failure still governs the design strength.

The efficiency of the tension member is 0.954

Note: *The design tension strength is more some times if the longer leg of an unequal angle is connected to the gusset (when the tearing strength of the net section governs the design strength).*

An understanding about the range of values for the section efficiency, η, is useful to arrive at the trial size of angle members in design problems.

(c & d)The double angle strength would be twice single angle strength as obtained above in case (a)

$T_d = 2 * 298.65 = 597.30$ kN

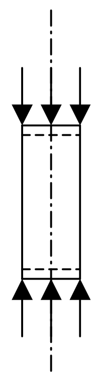
6 INTRODUCTION TO COLUMN BUCKLING

1.0 INTRODUCTION AND BASIC CONCEPTS

There are many types of compression members, the column being the best known. Top chords of trusses, bracing members and compression flanges of built up beams and rolled beams are all examples of compression elements. Columns are usually thought of as straight vertical members whose lengths are considerably greater than their cross-sectional dimensions. An initially straight strut or column, compressed by gradually increasing equal and opposite axial forces at the ends is considered first. Columns and struts are termed "*long*" or "*short*" depending on their proneness to buckling. If the strut is "short", the applied forces will cause a compressive strain, which results in the shortening of the strut in the direction of the applied forces. Under incremental loading, this shortening continues until the column "squashes". However, if the strut is "long", similar axial shortening is observed only at the initial stages of incremental loading. Thereafter, as the applied forces are increased in magnitude, the strut becomes "*unstable*" and develops a deformation in a direction normal to the loading axis. (See Fig.1). The strut is in a "*buckled*" state.

Buckling behaviour is thus characterized by deformations developed in a direction (or plane) normal to that of the loading that produces it. When the applied loading is increased, the buckling deformation also increases. Buckling occurs mainly in members subjected to compressive forces. If the member has high bending stiffness, its buckling resistance is high. Also, when the member length is increased, the buckling resistance is decreased. Thus the buckling resistance is high when the member is "*stocky*" (i.e. the member has a high bending stiffness and is short) conversely, the buckling resistance is low when the member is "*slender*".

Structural steel has high yield strength and ultimate strength compared with other construction materials. Hence compression members made of steel tend to be slender. Buckling is of particular interest while employing steel members, which tend to be slender, compared with reinforced concrete or prestressed concrete compression members. Members fabricated from steel plating or sheeting and subjected to compressive stresses also experience local buckling of the plate elements. This chapter introduces buckling in the context of axially compressed struts and identifies the factors governing the buckling behaviour. The local buckling of thin flanges/webs is not considered at this stage. These concepts are developed further in a subsequent chapter.



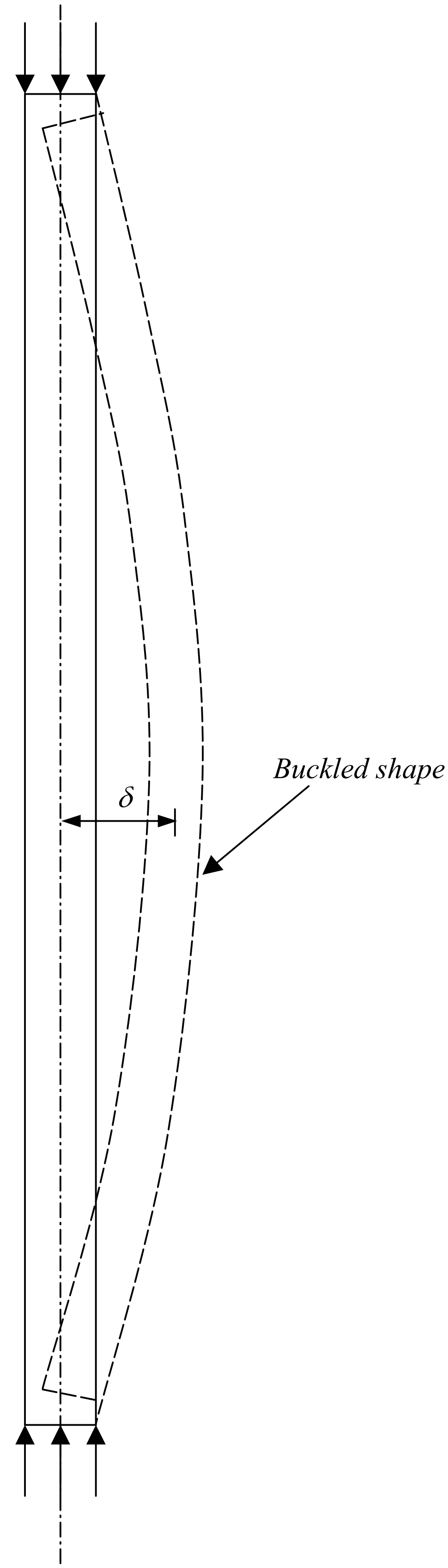


Fig 1: "short" vs "long" columns

2.0 ELASTIC BUCKLING OF AN IDEAL COLUMN OR STRUT WITH PINNED END

To begin with, we will consider the elastic behaviour of an idealized, pin-ended, uniform strut. The classical Euler analysis of this problem makes the following assumptions.

- the material of which the strut is made is homogeneous and linearly elastic (i.e. it obeys Hooke's Law).
- the strut is perfectly straight and there are no imperfections.
- the loading is applied at the centroid of the cross section at the ends.

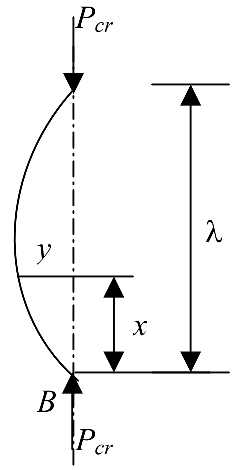


Fig. 2 Column Buckling

We will assume that the member is able to bend about one of the principal axes. (See Fig. 2). Initially, the strut will remain straight for all values of P, but at a particular value $P = P_{cr}$, it buckles. Let the buckling deformation at a section distant x from the end B be y.

The bending moment at this section = $P_{cr}.y$

The differential equation governing the small buckling deformation is given by

$$-EI\frac{d^2y}{dx^2} = P_{cr}.y$$

The general solution for this differential equation is

$$y = A_1 \cos x\sqrt{\frac{P_{cr}}{EI}} + B_1 \sin x\sqrt{\frac{P_{cr}}{EI}}$$

where A_1 and A_2 are constants.

Since $y = 0$ when $x = 0$, $A_1 = 0$.

when $x = \lambda, \quad y = 0;$

Hence $B_1 \sin \lambda \sqrt{\frac{P_{cr}}{EI}} = 0$

Either $B_1 = 0$ or $\sin \lambda\sqrt{\frac{P_{cr}}{EI}} = 0$

$B_1 = 0$ means $y = 0$ for all values of x (i.e. the column remains straight).

Alternatively $\sin\lambda\sqrt{\frac{P_{cr}}{EI}} = 0$

This equation is satisfied only when

$$\lambda\sqrt{\frac{P_{cr}}{EI}} = 0, \pi, 2\pi, \ldots\ldots..$$

$$P_{cr} = \frac{\pi^2 EI}{\lambda^2}, \frac{4\pi^2 EI}{\lambda^2} \ldots.. \frac{n^2\pi^2 EI}{\lambda^2}$$

where n is any integer.

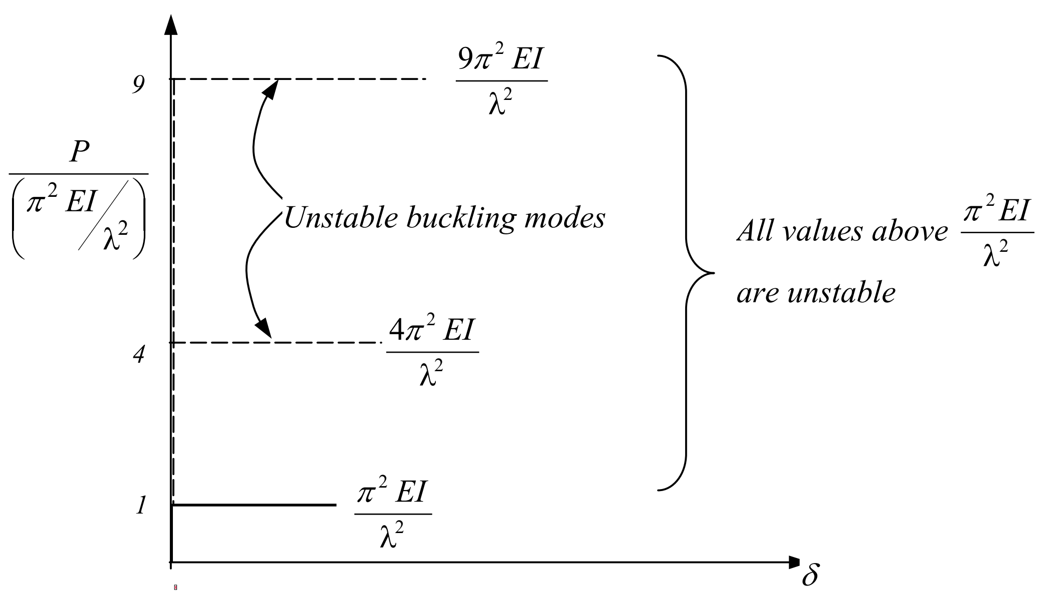


Fig. 3 Buckling load Vs Lateral deflection Relationship

While there are several buckling modes corresponding to $n = 1, 2, 3, \ldots$, the lowest *stable* buckling mode corresponds to $n = 1$. (See Fig. 3).

The lowest value of the critical load (i.e. the load causing buckling) is given by

$$P_{cr} = \frac{\pi^2 EI}{\lambda^2} \qquad (1)$$

Thus the Euler buckling analysis for a " straight" strut, will lead to the following conclusions:

1. The strut can remain straight for all values of *P*.
2. Under incremental loading, when *P* reaches a value of $P_{cr} = \dfrac{\pi^2 EI}{\lambda^2}$

   the strut can buckle in the shape of a half-sine wave; the amplitude of this buckling deflection is indeterminate.
3. At higher values of the loads given by $\dfrac{n^2 \pi^2 EI}{\lambda^2}$ other sinusoidal buckled shapes (*n* half waves) are possible. However, it is possible to show that the column will be in unstable equilibrium for all values of $P > \dfrac{\pi^2 EI}{\lambda^2}$ whether it be straight or buckled. ***This means that the slightest disturbance will cause the column to deflect away from its original position. Elastic Instability may be defined in general terms as a condition in which the structure has no tendency to return to its initial position when slightly disturbed, even when the material is assumed to have an infinitely large yield stress.*** Thus

   $$P_{cr} = \frac{\pi^2 EI}{\lambda^2} \qquad (2)$$

   represents the maximum load that the strut can usefully support.

It is often convenient to study the onset of elastic buckling in terms of the mean applied compressive stress (rather than the force). The mean compressive stress at buckling, σ_{cr} , is given by

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{A\lambda^2}$$

where *A* = area of cross section of the strut.

If *r* = radius of gyration of the cross section, then $I = Ar^2$,

Hence, $$\sigma_{cr} = \frac{\pi^2 E r^2}{\lambda^2} = \frac{\pi^2 E}{(\lambda / r)^2} = \frac{\pi^2 E}{\lambda^2} \qquad (3)$$

where λ = the slenderness ratio of the column defined by $\lambda = \lambda / r$
The equation $\sigma_{cr} = (\pi^2 E)/\lambda^2$, implies that the critical stress of a column is inversely proportional to the square of the slenderness ratio of the column (see Fig. 4).

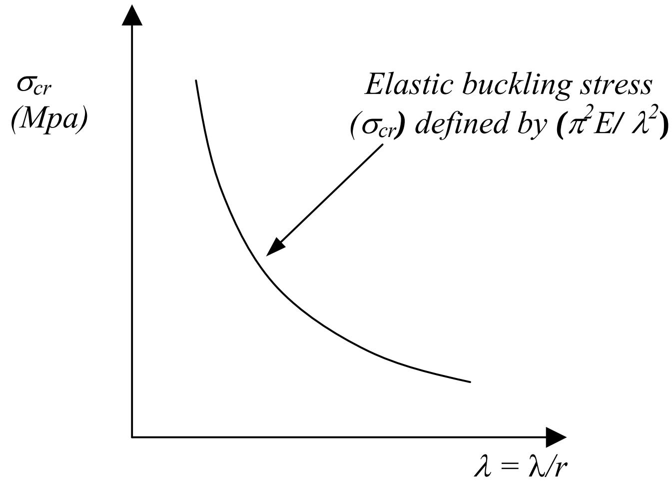


Fig. 4 Euler buckling relation between σ_{cr} and λ

3.0 STRENGTH CURVE FOR AN IDEAL STRUT

We will assume that the stress-strain relationship of the material of the column is defined by Fig. 5. A strut under compression can therefore resist only a maximum force given by $f_y.A$, when plastic squashing failure would occur by the plastic yielding of the entire cross section; this means that the stress at failure of a column can never exceed f_y , shown by A-A^1 in Fig. 6(a).

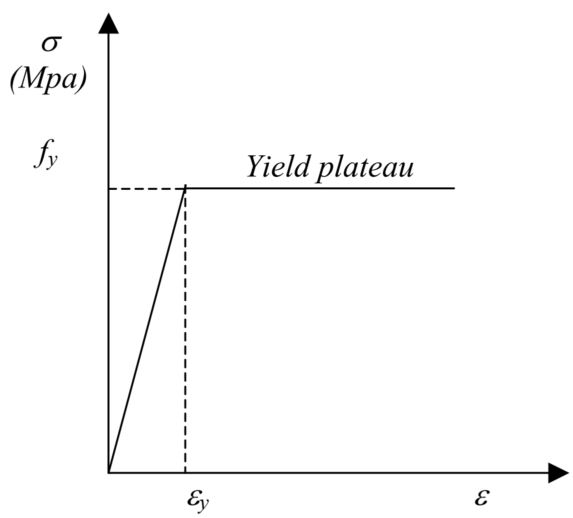


Fig. 5 Idealized elastic-plastic relationship for steel

From Fig. 4, it is obvious that the column would fail by buckling at a stress given by $\left(\dfrac{\pi^2 E}{\lambda^2}\right)$

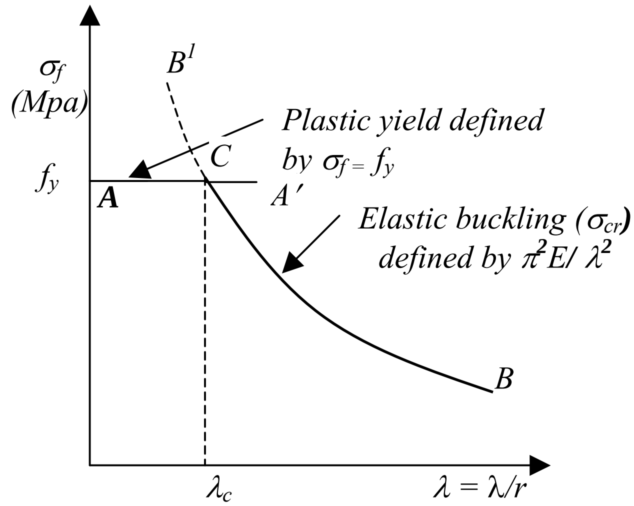


Fig. 6(a) Strength curve for an axially loaded initially straight pin-ended column

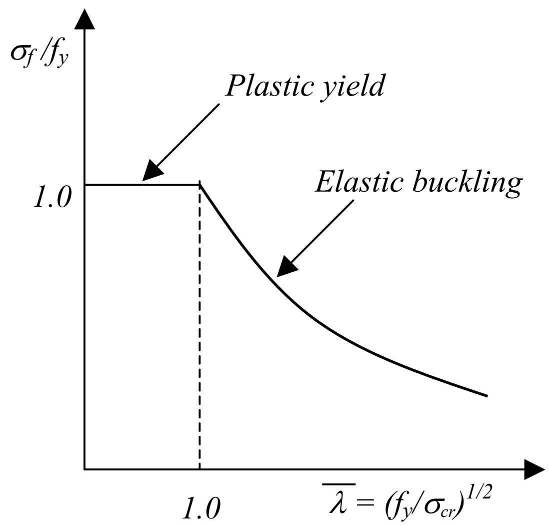


Fig. 6(b) Strength curve in a non-dimensional form

This is indicated by B-B^1 in Fig. 6(a), which combines the two types of behaviour just described. The two curves intersect at C. Obviously the column will fail when the axial compressive stress equals or exceeds the values defined by ACB. In the region AC, where the slenderness values are low, the column fails by yielding. In the region CB, the failure will be triggered by buckling. The changeover from yielding to buckling failure occurs at the point C, defined by a slenderness ratio given by λ_c and is evaluated from

$$f_y = \frac{\pi^2 E}{\lambda_c^2}$$

$$\lambda_c = \pi \sqrt{\frac{E}{f_y}} \tag{5}$$

Plots of the type Fig. 6(a) are sometimes presented in a non-dimensional form illustrated in Fig. 6*(b)*. Here (σ_f / f_y) is plotted against a generalized slenderness given by

$$\bar{\lambda} = \lambda / \lambda_c = \sqrt{f_y / \sigma_{cr}} \tag{6}$$

This single plot can be employed to define the strength of all axially loaded, initially straight columns irrespective of their E and f_y values. The change over from plastic yield to elastic critical buckling failure occurs when $\bar{\lambda} = 1$ (i.e. when $f_y = \sigma_{cr}$), the corresponding slenderness ratio $\left(\frac{\lambda}{r}\right)$ *is* $\pi \sqrt{\frac{E}{f_y}}$

4.0 STRENGTH OF COMPRESSION MEMBERS IN PRACTICE

The highly idealized straight form assumed for the struts considered so far cannot be achieved in practice. Members are never perfectly straight; they can never be loaded exactly at the centroid of the cross section. Deviations from the ideal elastic plastic behaviour defined by Fig. 5 are encountered due to strain hardening at high strains and the absence of clearly defined yield point. Moreover, residual stresses locked-in during the process of rolling also provide an added complexity.

Thus the three components, which contribute to a reduction in the actual strength of columns (compared with the predictions from the "ideal" column curve) are

(i) initial imperfection or initial bow.
(ii) Eccentricity of application of loads.
(iii) Residual stresses locked into the cross section.

4.1 The Effect of Initial Out-of-Straightness

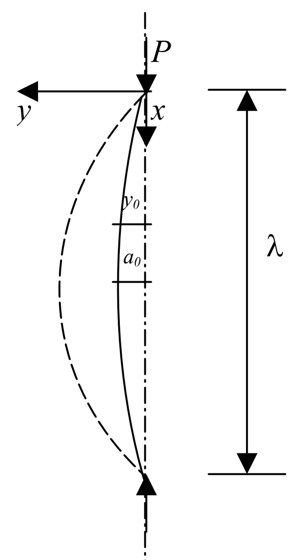


Fig. 7 Pin-ended strut with initial imperfection

Fig. 7 shows a pin-ended strut having an initial imperfection and acted upon by a gradually increasing axial load. As soon as the load is applied, the member experiences a bending moment at every cross section, which in turn causes a bending deformation. For simplicity of calculations, it is usual to assume the initial shape of the column defined by

$$y_0 = a_0 \quad \sin\frac{\pi x}{\lambda} \tag{7}$$

where a_o is the maximum imperfection at the centre, where $x = \lambda / 2$. Other initial shapes are, of course, possible, but the half sine-wave assumed above corresponding to the lowest node shape, represents the greatest influence on the actual behaviour, hence is adequate.

Provided the material remains elastic, it is possible to show that the applied force, P, enhances the initial deflection at every point along the length of the column by a multiplier factor, given

$$\frac{1}{1-(P/P_{cr})} \tag{8}$$

The deflection will tend to infinity, as P is increased to P_{cr} as shown by curve-A, see Fig. 8(a).

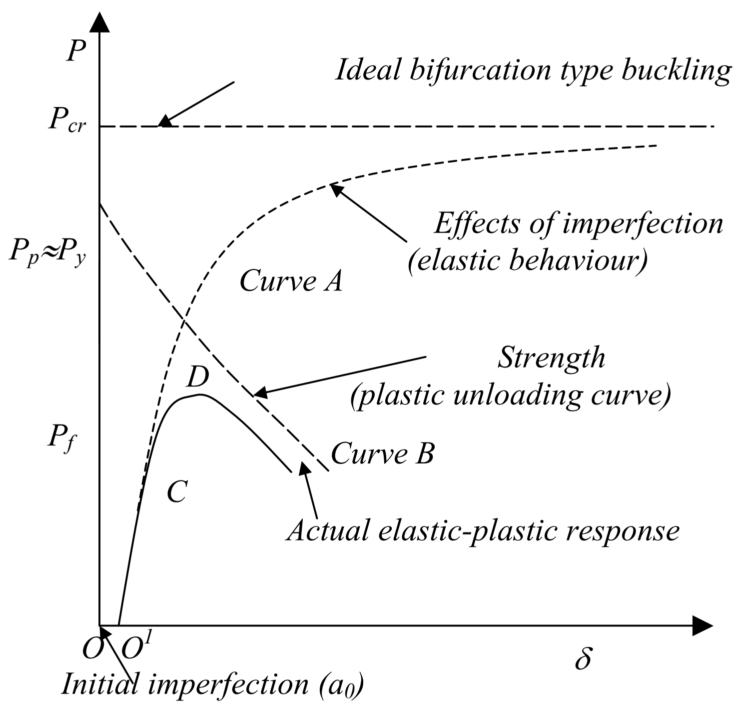


Fig. 8(a) Theoretical and actual load deflection response of a strut with initial imperfection

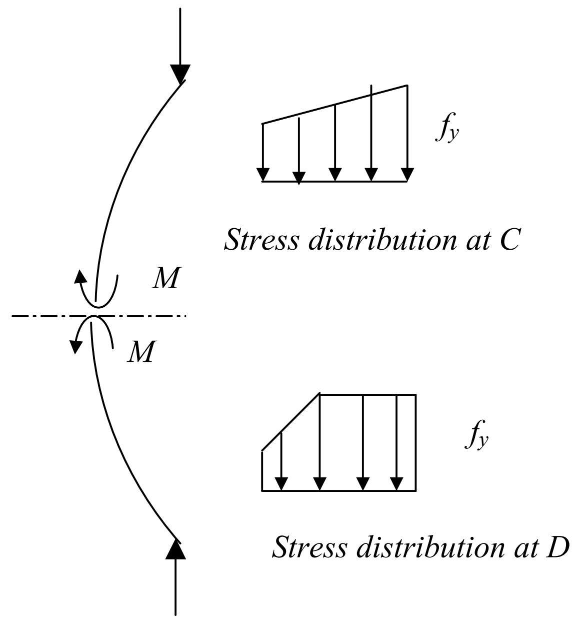


Fig. 8(b) Stress distributions at C and D

As the deflection increases, the bending moment on the cross section of the column increases. The resulting bending stress, ($M\ y/I$), on the concave face of the column is compressive and adds to the axial compressive force of P/A. As P is increased, the stress on the concave face reaches yield (f_y). The load causing first yield [point C in Fig. 8 (a)] is designated as P_y. The stress distribution across the column is shown in Fig. 8(b). The applied load (P) can be further increased thereby causing the zone of yielding to spread across the cross section, with the resulting deterioration in the bending stiffness of the column. Eventually the maximum load P_f is reached when the column collapses and the corresponding stress distribution is seen in Fig. 8 (b). The extent of the post-first-yield load increase and the section plastification depends upon the slenderness ratio of the column.

Fig. 8(a) also shows the theoretical rigid plastic response curve B, drawn assuming $P_{cr} > P_p$ (Note $P_p = A.\ f_y$). Quite obviously P_{cr} and P_p are upper bounds to the loads P_y and P_f. If the initial imperfection a_o is small, P_y can be expected to be close to P_f and P_p. If the column is stocky, P_{cr} will be very large, but P_p can be expected to be close P_y. If the column is slender, P_{cr} will be low and will often be lower than P_p or P_y. In very slender columns, collapse will be triggered by elastic buckling. Thus, for stocky columns, the upper bound is P_p and for slender columns, P_{cr} .If a large number of columns are tested to failure, and the data points representing the values of the mean stress at failure plotted against the slenderness (λ) values, the resulting lower bound curve would be similar to the curve shown in Fig. 9.

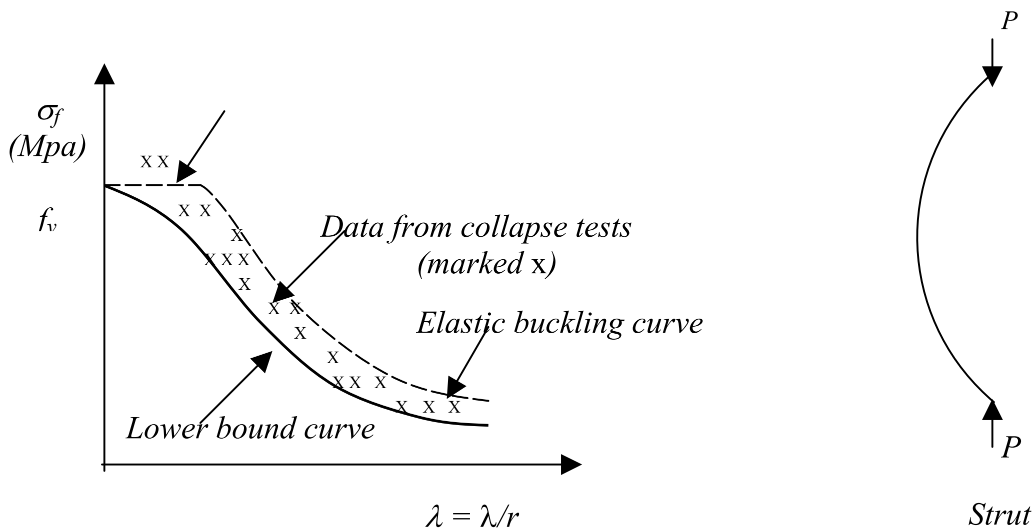


Fig 9: Strength curves for strut with initial imperfection

For very stocky members, the initial out of straightness – which is more of a function of length than of cross sectional dimensions – has a very negligible effect and the failure is by plastic squash load. For a very slender member, the lower bound curve is close to the elastic critical stress (σ_{cr}) curve. At intermediate values of slenderness the effect of initial out of straightness is very marked and the lower bound curve is significantly below the f_y line and σ_{cr} line.

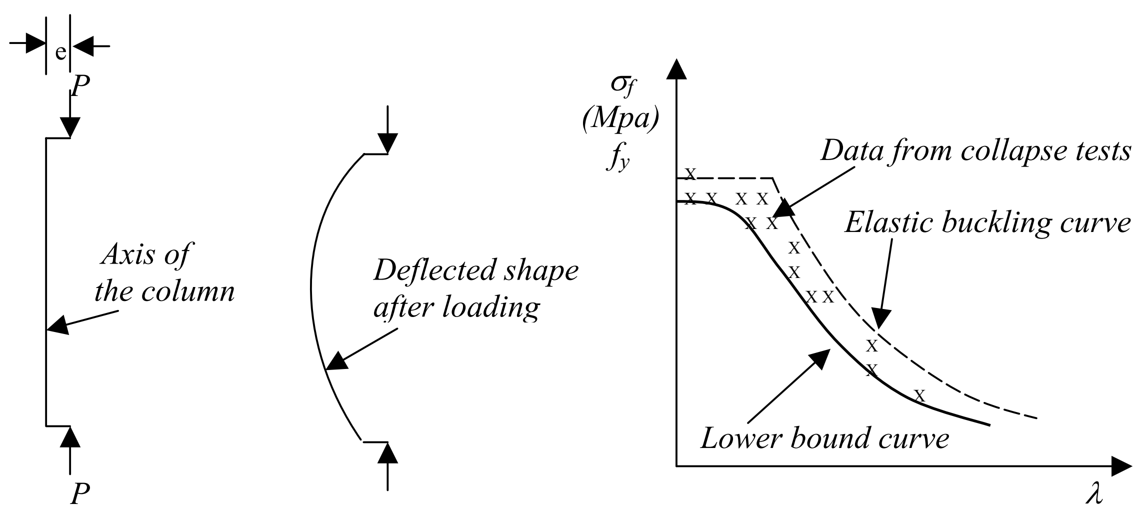


Fig. 10 Strength curve for eccentrically loaded columns

4.2 The Effect of Eccentricity of Applied Loading

As has already been pointed out, it is impossible to ensure that the load is applied at the exact centroid of the column. Fig. 10 shows a straight column with a small eccentricity (e) in the applied loading. The applied load (P) induces a bending moment ($P.e$) at every cross section. This would cause the column to deflect laterally, in a manner similar to the initially deformed member discussed previously. Once again the greatest compressive stress will occur at the concave face of the column at a section midway along its length. The load-deflection response for purely elastic and elastic-plastic behaviour is similar to those described in Fig. 8(a) except that the deflection is zero at zero load.

The form of the lower bound strength curve obtained by allowing for eccentricity is shown in Fig. 10. The only difference between this curve and that given in Fig. 9 is that the load carrying capacity is reduced (for stocky members) even for low values of λ.

4.2 The Effect of Residual Stress

As a consequence of the differential heating and cooling in the rolling and forming processes, there will always be inherent residual stresses. A simple explanation for this phenomenon follows. Consider a billet during the rolling process when it is shaped into an I section. As the hot billet shown in Fig. 11(a) is passed successively through a series of rollers, the shapes shown in 11(b), (c) and (d) are gradually obtained. The outstands (b-b) cool off earlier, before the thicker inner elements (a-a) cool down.

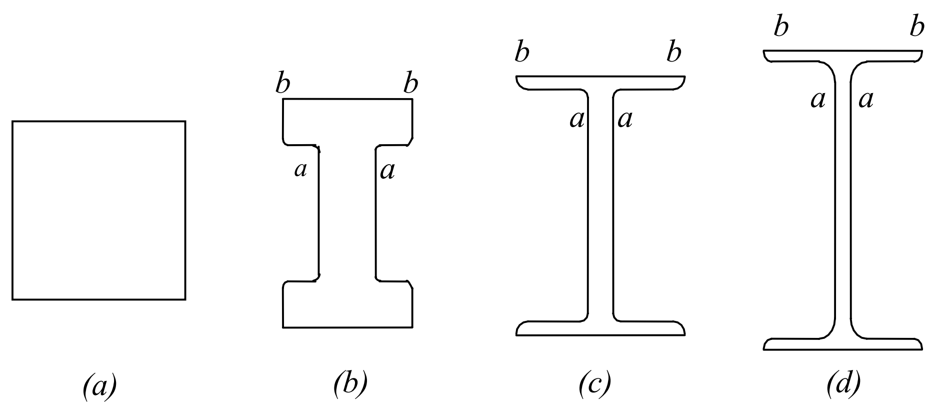
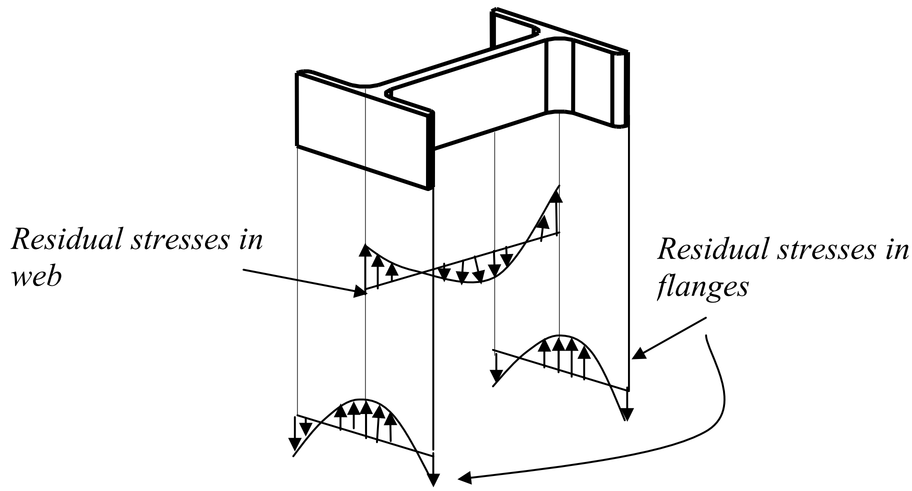


Fig. 11 Various stages of rolling a steel girder

As one part of the cross section (b-b) cools off, it tends to shrink first but continues to remain an integral part of the rest of the cross section. Eventually the thicker element (a) also cool off and shrink. As these elements remain composite with the edge elements, the differential shrinkage induces compression at the outer edges (b). But as the cross section is in equilibrium – these stresses have to be balanced by tensile stresses at inner location (a). The tensile stress can sometimes be very high and reach upto yield stress. The compressive stress induced due to this phenomenon is called "*residual compressive stress*" and the corresponding tensile stress is termed " *residual tensile stress*".



Residual stresses distribution (no applied load)

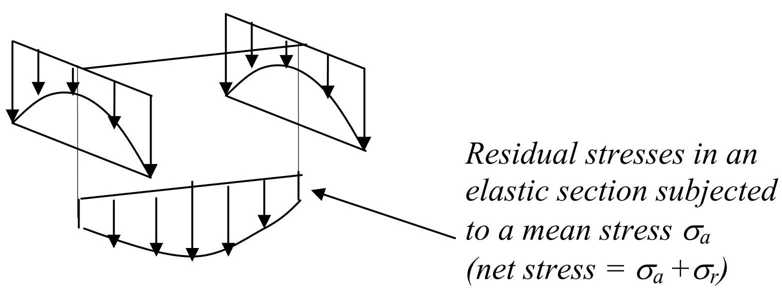


Fig. 12 The influence of residual stresses

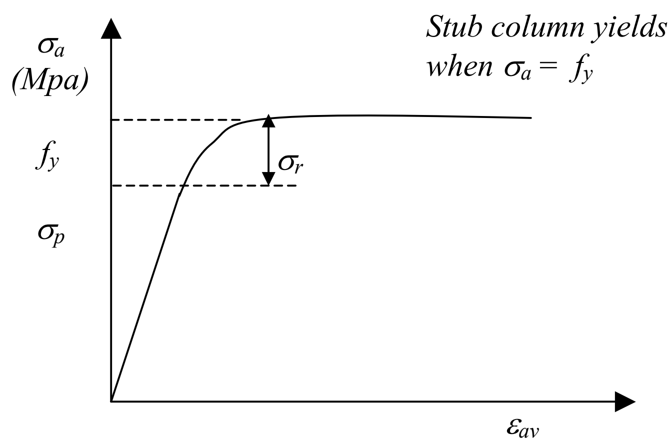


Fig. 13 Mean axial stress vs mean axial strain in a stub column test

Consider a short compression member (called a "*stub column*", Fig. 12(*a*) having a residual stress distribution as shown in Fig. 12 (*b*). When this cross section is subjected to an applied uniform compressive stress (σ_a) the stress distribution across the cross section becomes non-uniform due to the presence of the residual stresses discussed above. The largest compressive stress will be at the edges and is $(\sigma_a + \sigma_r)$

Provided the total stress nowhere reaches yield, the section continues to deform elastically. Under incremental loading, the flange tips will yield first when $[(\sigma_a + \sigma_r) = f_y]$. Under further loading, yielding will spread inwards and eventually the web will also yield. When $\sigma_a = f_y$, the entire section will have yielded. The relationship between the mean axial stress and mean axial strain obtained from the stub column test is seen in Fig. 13.

Only in a very stocky column (i.e. one with a very low slenderness) the residual stress causes premature yielding in the manner just described. The mean stress at failure will be f_y, i.e. failure load is not affected by the residual stress. A very slender strut will fail by buckling, i.e. $\sigma_{cr} << f_y$. For struts having intermediate slenderness, the premature yielding at the tips reduces the effective bending stiffness of the column; in this case, the column will buckle elastically at a load below the elastic critical load and the plastic squash load. The column strength curve will thus be as shown in Fig. 14.

Notice the difference between the buckling strength and the plastic squash load is most

pronounced when $\lambda = \frac{\lambda}{r} = \pi\left(E/f_y\right)^{1/2}$

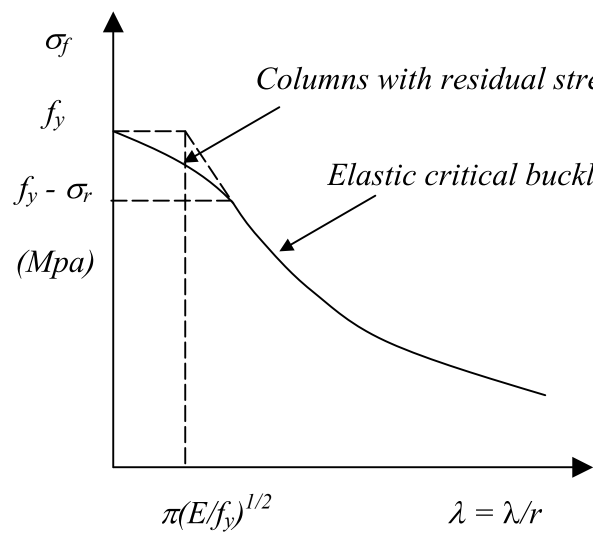


Fig. 14 Buckling of an initially straight column having residual stresses

4.4 The Effect of Strain-Hardening and the Absence of Clearly Defined Yield point

If the material of the column has a stress-strain relationship as shown in Fig. 15, the onset of first yield will not be affected, but the collapse load may be increased. Designers tend to ignore the effect of strain hardening which in fact provides a margin of safety.

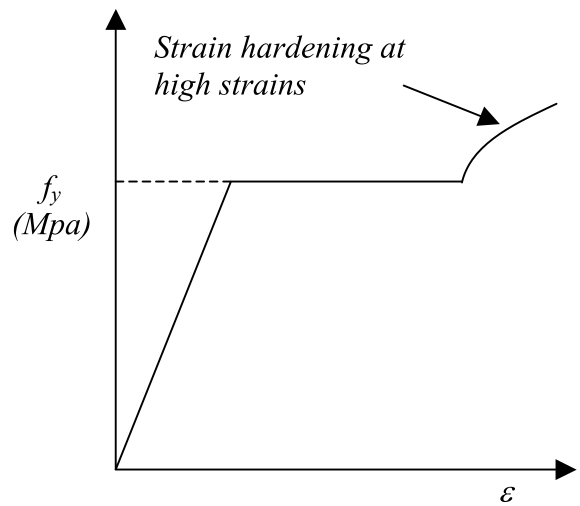


Fig. 15 Stress-strain relationship for Steels exhibiting strain hardening

High strength steels generally have stress-strain curves of the shape given in Fig. 16. At stresses above the limit of proportionality *(σ_p)*, the material behaviour is non linear and on unloading and reloading the material is linear-elastic. Most high strength structural steels Fig. 16*(a)* have an yield stress beyond which the curve becomes more or less horizontal. Some steels do not have a plastic plateau and exhibit strain-hardening throughout the inelastic range Fig. 16(*b*). In such cases, the yield stress is generally taken as the *0.2%* proof stress, for purposes of computation.

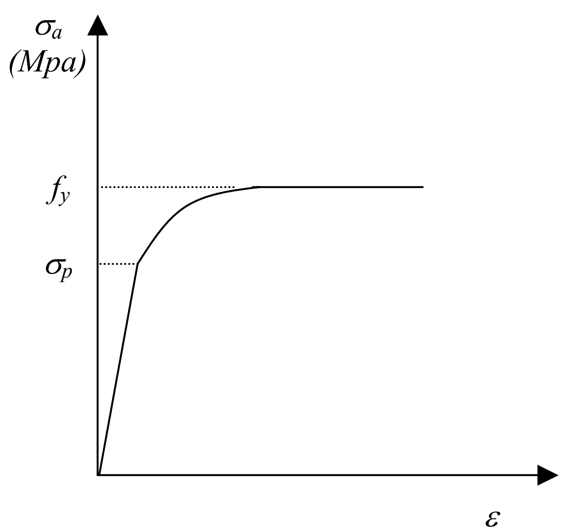


Fig.16(a)Lack of clearly defined yield

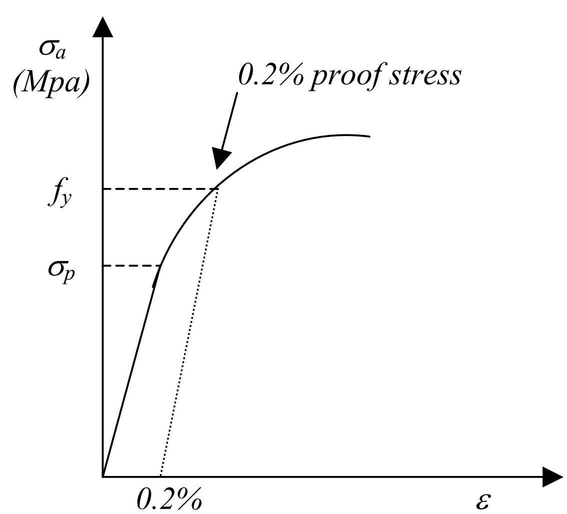


Fig.16 (b) Lack of clearly defined yield with strain hardening

4.5 The Effect of all Features Taken Together

In practice, a loaded column may experience most, if not all, of the effects listed above i.e. out of straightness, eccentricity of loading, residual stresses and lack of clearly defined yield point and strain hardening effects occurring simultaneously.

Only strain hardening tends to raise the column strengths, particularly at low slenderness values. All other effects lower the column strength values for all or part of the slenderness ratio range.

When all the effects are put together, the resulting column strength curve is generally of the form shown in Fig. 17. The beneficial effect of strain hardening at low slenderness values is generally more than adequate to provide compensation for any loss of strength due to small, accidental eccentricities in loading. Although the column strength can exceed the value obtained from the yield strength (f_y), for purposes of structural design, the column strength curve is generally considered as having a cut off at f_y, to avoid large plastic compressive deformation.

Since it is impossible to quantify the variations in geometric imperfections, accidental eccentricity, residual stresses and material properties, it is impossible to calculate with certainty, the greatest reduction in strength they might produce in practice. Thus for design purposes, it may be impossible to draw a true lower bound column strength curve. A commonly employed method is to construct a curve on the basis of specified survival probability. (For example, over *98%* of the columns to which the column curve relates, can be expected - on a statistical basis – to survive at applied loads equal to those given by the curve). All design codes provide column curves based on this philosophy. Column curves proposed for the revised Indian Code of Practice are discussed in a subsequent chapter.

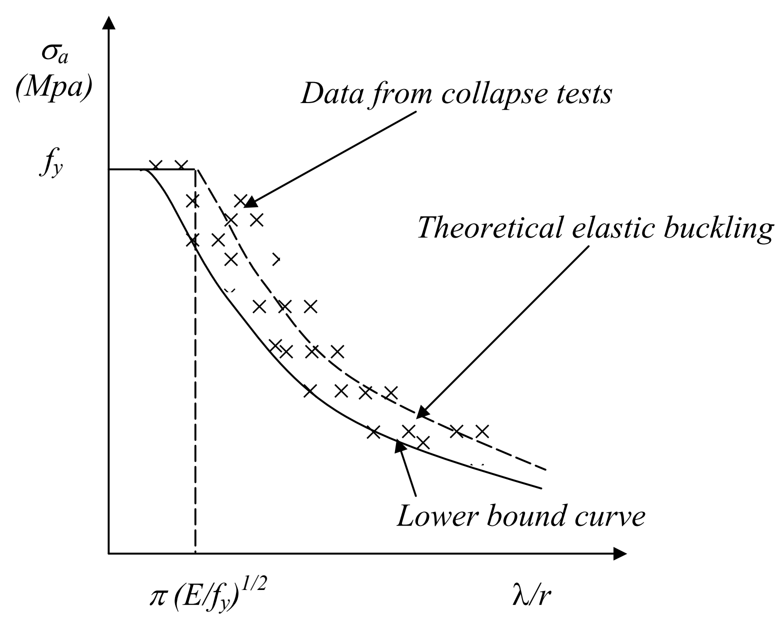


Fig. 17 General strength curves for struts with initial out of straightness,

5.0 THE CONCEPT OF EFFECTIVE LENGTHS

So far, the discussion in this chapter has been centred around pin-ended columns. The boundary conditions of a column may, however, be idealized in one the following ways

- Both the ends pin jointed (i.e. the case considered in art. 2)
- Both ends fixed.
- One end fixed and the other end pinned.
- One end fixed and the other end free.

By setting up the corresponding differential equations, expression for the critical loads as given below are obtained and the corresponding buckled shapes are given in Fig. 18.

Both ends fixed: $$P_{cr} = \frac{4\pi^2 EI}{\lambda^2} = \frac{4\pi^2 E}{\left(\frac{\lambda}{r}\right)^2}$$

One end fixed and the other end pinned: $$P_{cr} = \frac{2\pi^2 E}{\left(\frac{\lambda}{r}\right)^2}$$

One end fixed and the other end free: $$P_{cr} = \frac{\pi^2 EI}{4\lambda^2} = \frac{\pi^2 E}{4\left(\frac{\lambda}{r}\right)^2}$$

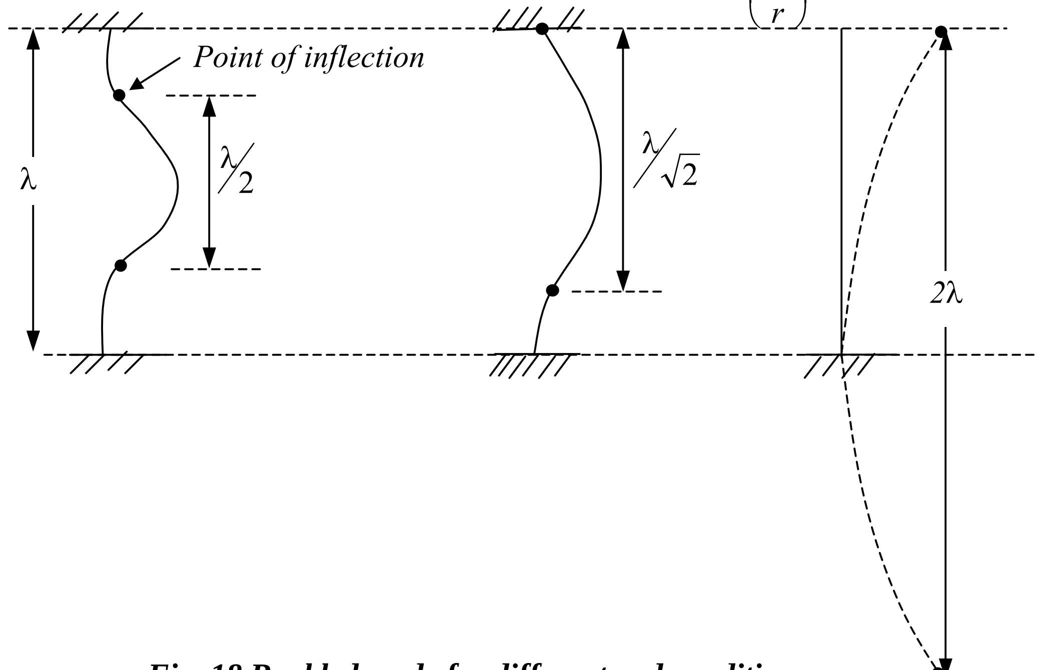


Fig. 18 Buckled mode for different end conditions

Using the column, pin ended at both ends, as the basis of comparison the critical load in all the above cases can be obtained by employing the concept of "effective length", λ_e.

It is easily verified that the calculated effective length for the various end conditions are given by

Both ends pin ended, $\lambda_e = \lambda$
Both ends fixed, $\lambda_e = \lambda / 2$

One end fixed and the other end pinned, $\lambda_e = \dfrac{\lambda}{\sqrt{2}}$

One end fixed and the other end free, $\lambda_e = 2\lambda$

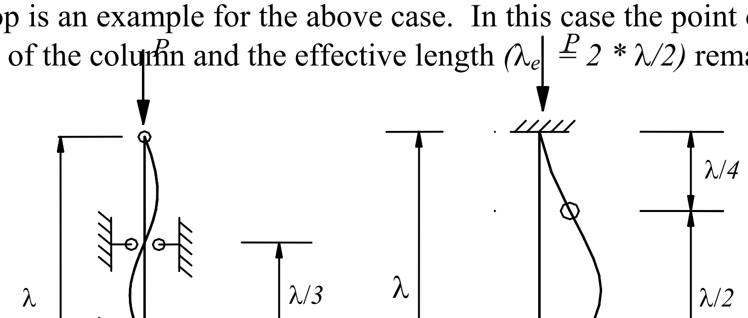
It can be seen that the effective length corresponds to the distance between the points of inflection in the buckled mode. The effective column length can be defined as the length of an equivalent pin-ended column having the same load-carrying capacity as the member under consideration. The smaller the effective length of a particular column, the smaller its danger of lateral buckling and the greater its load carrying capacity. It must be recognized that no column ends are perfectly fixed or perfectly hinged. The designer may have to interpolate between the theoretical values given above, to obtain a sensible approximation to actual restraint conditions. Effective lengths commonly employed by Designers are discussed in Chapter 10.

5.1 Effective lengths in different planes

The restraint against buckling may be different for buckling about the two column axes. Fig 19(*a*) shows a pin-ended column of *UC* section braced about the minor axis against lateral movement (but not rotationally restrained) at spacing $\lambda /3$. The minor axis buckling mode would be with an effective pin-ended column length $(\lambda_e)_y$ of $\lambda/3$. If there was no major axis bracing the effective length for buckling about the major axis $(\lambda_e)_x$ would remain as λ. Therefore, the design slenderness about the major and minor axis would be λ/r_x and $(\lambda/3)/r_y$, respectively. Generally $r_x < 3r_y$ for all UC sections, hence the major axis slenderness (λ/r_x) would be greater, giving the lower value of critical load, and failure would occur by major axis buckling. If this is not the case, checks will have to be carried out about both the axes.

Fig 19(*b*) shows a column with both ends fully restrained; the buckled shape has points of contraflexure, equivalent to pin ends, at $\lambda/4$ from either end. The central length is clearly equivalent to pin-ended column of length $\lambda/2$. This is the case, which has full rotational constraints at the ends. Fig 20 (*a*) shows the effect of partial end-restraints.

Sometimes columns are free to sway laterally, but restrained against rotation at both ends as in Fig.21 (*a*). A water tank supported on four corner columns as in Fig.21 (*b*) with rigid joints at top is an example for the above case. In this case the point of contraflexure is at mid-height of the column and the effective length $(\lambda_e = 2 * \lambda/2)$ remains λ.



5.2 Effective lengths recommended for Design

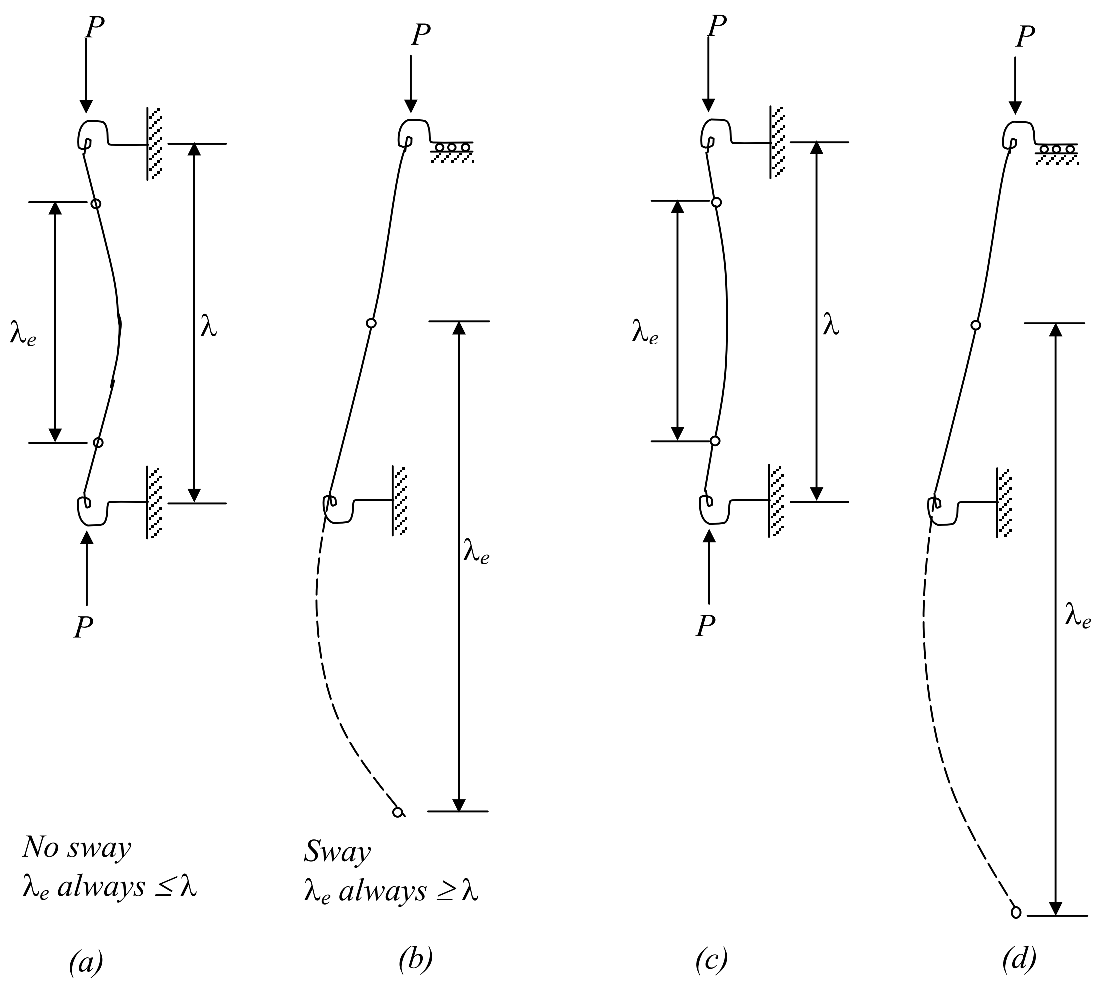


Fig. 20 Columns with partial rotational restraint

Partial end-restraints are much more common in practice than fully rigid end-constraints. The flexibility in the end-connection and (or) flexibility of the restraining members ensure partial fixity at the supports. A simple frame as shown in Fig.22 (*a*) is an example of the above case. For nodal loading, the in-plane buckling mode for this frame is shown in Fig. 22(*b*).

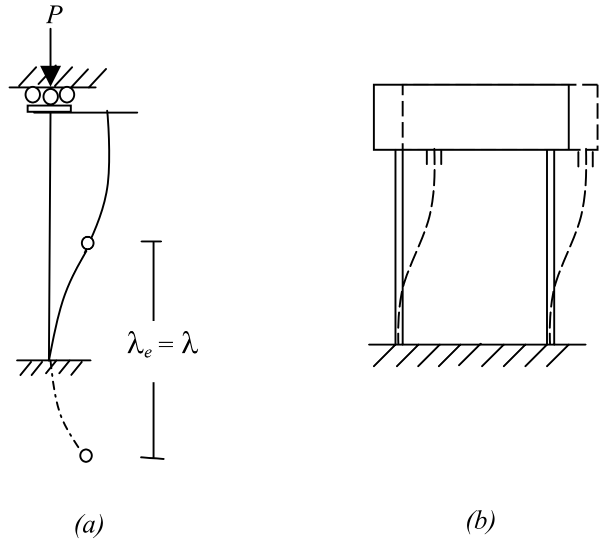


Fig.21 Columns with differing effective lengths-II

With the top beam bent in an S-shape the rotational end-restraint stiffness is given by

$$K_\theta = \frac{M}{\theta} = \frac{6EI_e}{\lambda_e}$$

For rigid beam-to-column joints this stiffness of the beam (K_θ) will control the position of the point of contraflexure in the column and thus the column effective length. These columns are represented in Fig. 22 (*c*) for which an effective length of 1.5λ is suggested.

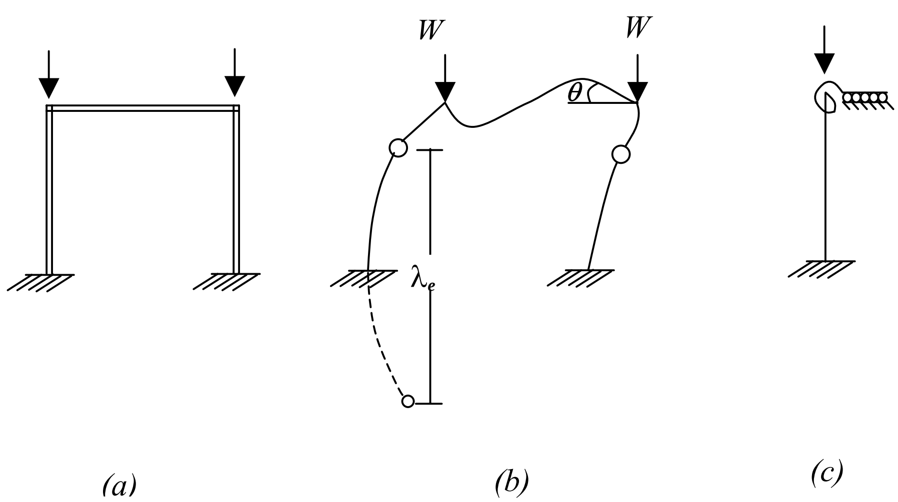


Fig. 22 Column in a simple sway frame

5.3 No-sway and sway columns

Fig. 20(*a*) and Fig. 20(*b*) represent the general cases of no-sway and sway columns with partial end-restraint. The buckled shapes will be of the form shown if the top restraint stiffness ($K_{\theta T}$) and the bottom restraint stiffness ($K_{\theta B}$) are equal. For the no sway case of Fig.20 (*a*) the position of the points of contraflexure will move within the column length as $K_{\theta T}$ and $K_{\theta B}$ vary. Fig.20*(c)* represents the situation of low $K_{\theta T}$ and high $K_{\theta B}$. However for non-sway columns λ_e is always less than or equal to λ. By contrast, for sway columns λ_e is always greater than or equal to λ. As K_θ decreases, the column end-joint rotations increase and λ_e can easily become 2λ or 3λ [Fig.20 (*d*)]. The limiting case of K_θ and $K_{\theta B} = 0$ gives $\lambda_e = \infty$.

The column design stress may be written as:

$$P_c = \frac{\pi^2 E}{\left(\lambda_e / r_y\right)^2}(area) = \left[f\left(Area\right)\right]$$

where area is dominant, the column is stocky. Otherwise the column strength is largely dependent on $(1/\lambda_e)^2$. Thus sway columns, i.e. with $\lambda_e > \lambda$, are much weaker than no-sway ones.

5.4 Accuracy in using Effective lengths

For compression members in rigid-jointed frames the effective length is directly related to the restraint provided by all the surrounding members. In a frame the interaction of all the members occurs because of the frame buckling rather than column buckling. For the design purposes, the behaviour of a limited region of the frame is considered. The limited frame comprises the column under consideration and each immediately adjacent member treated as if it were fixed at the far end. The effective length of the critical column is then obtained from a chart which is entered with two coefficients k_1, and k_2, the values of which depends upon the stiffnesses of the surrounding members k_u, k_{TL} etc. Two different cases are considered viz. columns in non-sway frames and columns in sway frames. All these cases as well as effective length charts are shown in Fig.23. For the former, the effective lengths will vary from 0.5 to 1.0 depending on the values of k_1 and k_2, while for the latter, the variation will be between 1.0 and ∞ . These end points correspond to cases of: (1) rotationally fixed ends with no sway and rotationally free ends with no sway; (2) rotationally fixed ends with free sway and rotationally free ends with free sway.

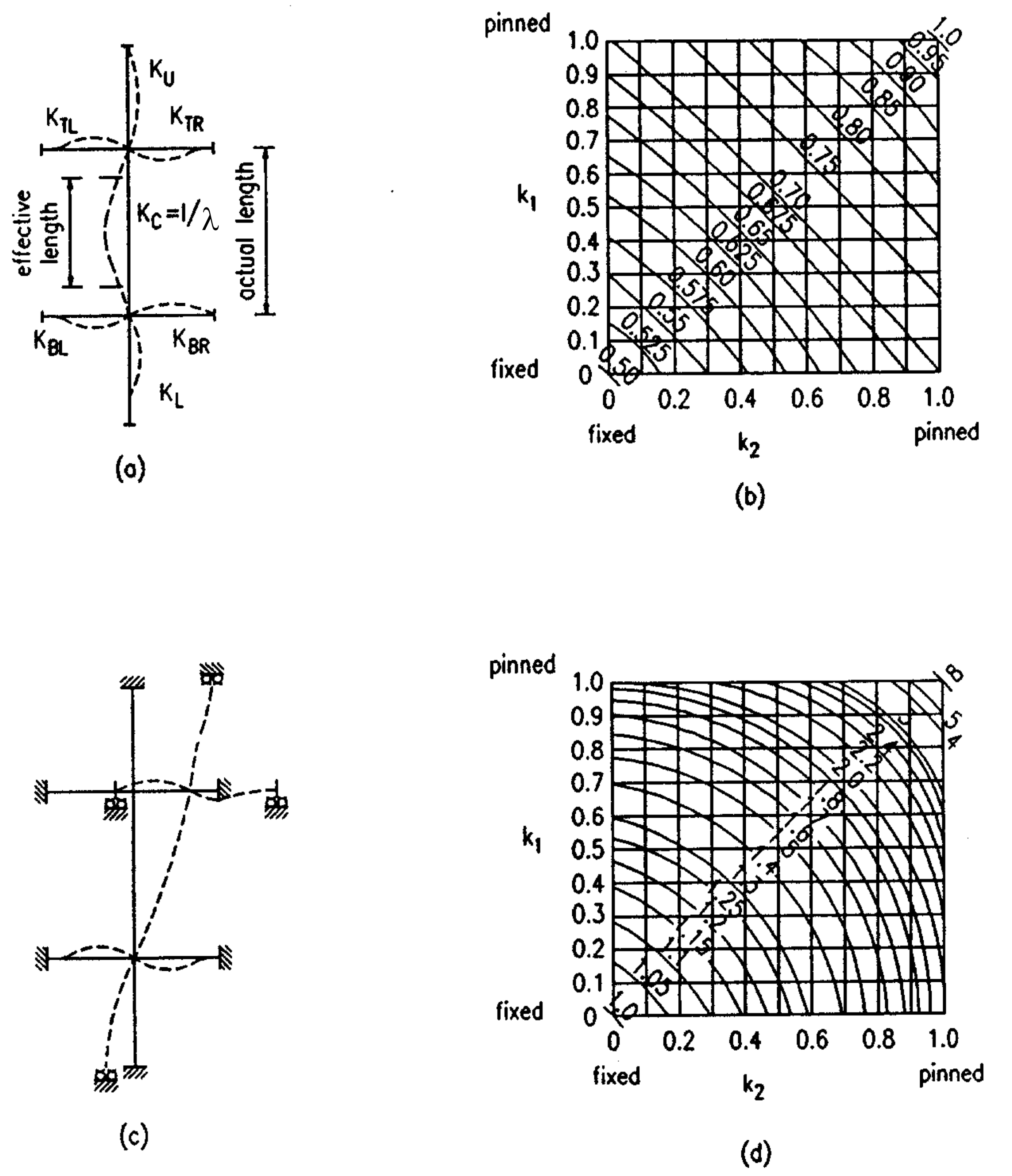


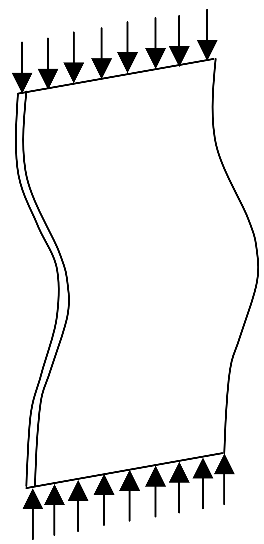
Fig. 23 Limited frames and corresponding effective length charts of BS5950: Part 1 and IS: 800.

(a) Limited frame and (b) effective length ratios ($k_3 = \infty$), for non-sway frames.

(c) Limited frame and (d) effective length ratios (without partial bracing, $k_3 = 0$), for sway frames.

6.0 TORSIONAL AND TORSIONAL-FLEXURAL BUCKLING OF COLUMNS

We have so far considered the flexural buckling of a column in which the member deforms by bending in the plane of one of the principal axes. The same form of buckling will be seen in an initially flat wide plate, loaded along its two ends, the two remaining edges being unrestrained. [See Fig. 24 (*a*)]



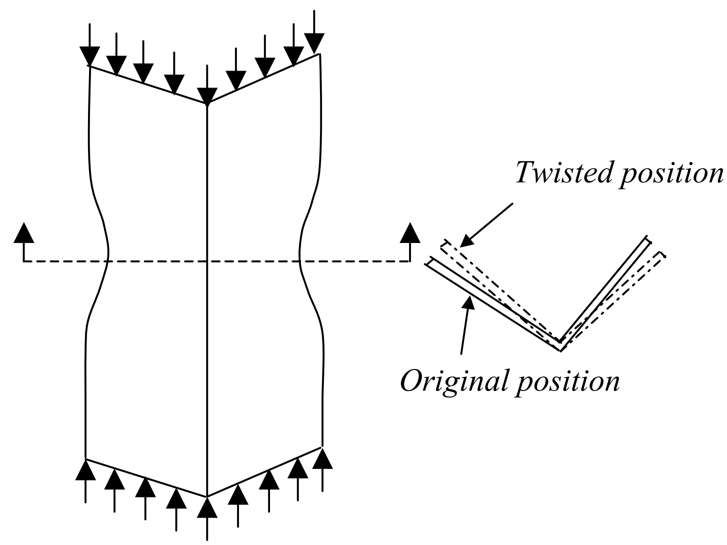


Fig.24 (a) Plate with unsupported edges ***Fig.24 (b) Folded plate twists under axial load***

On the other hand, if the plate is folded at right angles along the vertical centre-line, the resulting angular cross-section has a significantly enhanced bending stiffness. Under a uniform axial compression, the two unsupported edges tend to wave in the Euler type buckles. At the fold, the amplitude of the buckle is virtually zero. A horizontal cross-section at mid height of the strut shows that the cross-section rotates relative to the ends. This mode of buckling is essentially torsional in nature and is initiated by the lack of support at the free edges. This case illustrates buckling in torsion, due to the low resistance to twisting of the member.

Thus the column curves of the type discussed in Fig. 17 (see section 4.5) are only satisfactory for predicting the mean stress at collapse, when the strut buckles by bending in a plane of symmetry of the cross section, referred to as " flexural buckling ". Members with low torsional stiffness (eg. angles, tees etc made of thin walled members) will undergo torsional buckling before flexural buckling. Cruciform sections are generally prone to torsional buckling before flexural buckling. Singly symmetric or un-symmetric cross sections may undergo combined twisting about the shear centre and a translation of the shear centre. This is known as " torsional – flexural buckling ".

In this article we shall determine the critical load of columns that buckle by twisting or by a combination of both bending and twisting. The investigation is limited to open thin-walled sections as they are the only sections that are susceptible to torsional or torsional-flexural buckling. The study is also restricted to elastic behaviour, small deformations and concentric loading. The critical load is determined either by integrating the governing differential equations or by making use of an energy principle. The analysis presented here uses the Rayleigh-Ritz energy method to determine the critical load.

Let us consider the thin-walled open cross-section of arbitrary shape given in Fig. 25. The deformation taking place during buckling is assumed to consist of a combination of twisting and bending about two axis. To express strain energy in its simplest form the deformation is reduced to two pure translations and a pure rotation. The origin '*O*' is assumed to be the shear centre. The *x* and *y* directions are assumed to coincide with the principal axis of the section, and the *z* direction is taken along longitudinal axis through shear centre, *O*.

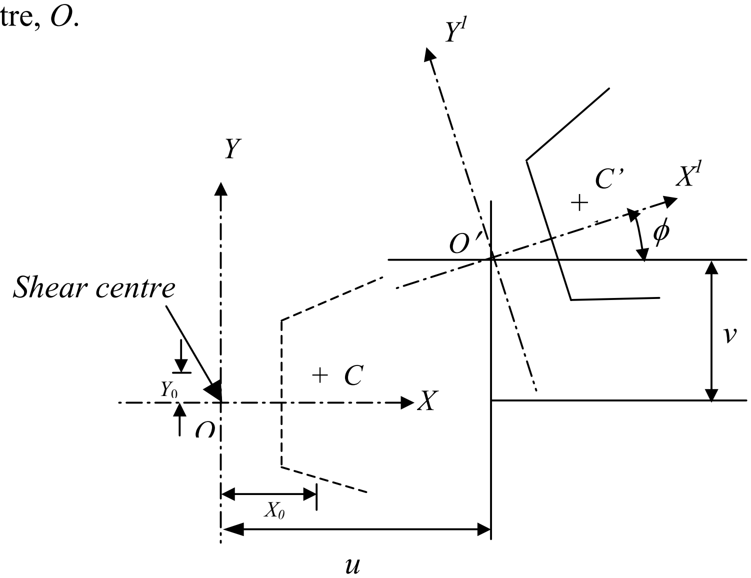


Fig. 25 Torsional -flexural buckling deformations.

(Note: In deriving Euler equations, we used x axis along the length of column; here we are using z axis along column length)

The co-ordinates of the centroid are denoted by x_o and y_o . As a result of buckling the cross section undergoes translations u and v in the x and y directions respectively, and rotation ϕ about the z-axis. The geometric shape of the cross section in the xy plane is assumed to remain undisturbed throughout.

Boundary conditions:

It is assumed that the displacements in the x and y directions and the moments about these axis vanish at the ends of the member. That is,
$u = v = 0$ at $z = 0$ and λ

$$\frac{d^2u}{dz^2} = \frac{d^2v}{dz^2} = 0 \;\; at \;\; z=0 \;\; and \; \lambda \qquad (9)$$

The torsional conditions which correspond to these flexural conditions are zero rotation and zero warping restraint at the ends of the member. Thus

$$\phi = \frac{d^2\phi}{dz^2} = 0 \quad at \quad z = 0 \;\; and \;\; \lambda \tag{10}$$

The boundary conditions will be satisfied by assuming a deflected shape of the form

$$u = C_1 \; Sin \frac{\pi z}{\lambda}$$

$$v = C_2 \; Sin \frac{\pi z}{\lambda}$$

$$\phi = C_3 \; Sin \frac{\pi z}{\lambda} \tag{11}$$

Strain energy stored in the member consists of four parts. Those are

i. energy due to bending in x-direction
ii. energy due to bending in y-direction
iii. energy of the St.Venant shear stresses.
iv. energy of the longitudinal stresses associated with warping torsion.

Thus total strain energy is given by

$$U = \frac{1}{2}\int_0^{\lambda} EI_y \left(\frac{d^2u}{dz^2}\right)^2 dz + \frac{1}{2}\int_0^{\lambda} EI_x \left(\frac{d^2v}{dz^2}\right)^2 dz$$

$$+ \frac{1}{2}\int_0^{\lambda} GJ \left(\frac{d\phi}{dz}\right)^2 dz + \frac{1}{2}\int_0^{\lambda} E\Gamma \left(\frac{d^2\phi}{dz^2}\right)^2 dz \tag{12}$$

where J and Γ are the torsional constant and warping constant of the section respectively.

Substitution of the assumed deflection function (Eqn. 11) into the strain energy expression (Eqn. 12) and then simplification gives

$$U = \frac{1}{4}\frac{\pi^2}{\lambda}\left[C_1^2 \frac{EI_y\pi^2}{\lambda^2} + C_2^2 \frac{EI_x\pi^2}{\lambda^2} + C_3^2 \left(GJ + \frac{E\Gamma\,\pi^2}{\lambda^2}\right)\right] \tag{13}$$

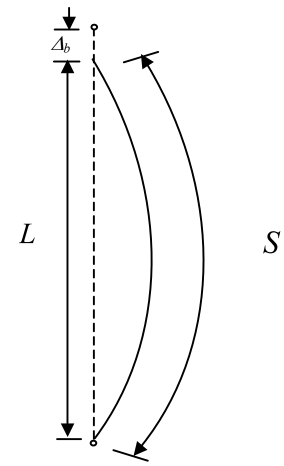
Potential Energy:

The potential energy of the external loads is equal to the negative product of the loads and the distances they move as the column deforms. Potential energy is given by

$$V = -\int_A \Delta_b \, \sigma \, dA \tag{14}$$

where dA is the cross sectional area of the fibre and the load it supports is $\sigma \, dA$
Δ_b is equal to the difference between the arc lengths and the chord length L of the fibre.
i.e. $\Delta_b = S\text{-}L$ (Fig. 26) (15)

Fig. 26 Axial shortening of longitudinal fibre due to bending



The potential energy of the external loads can be shown to be given by

$$V = -\frac{P\pi^2}{4\lambda}\left(C_1^2 + C_2^2 + C_3^2 r_0^{\,2} - 2C_1 C_3 y_0 + 2C_2 C_3 x_0\right) \tag{16}$$

where, x_o and y_o are the co-ordinates of centroid and r_o is the polar radius of gyration.

Total potential energy of the system is

$$U + V = \frac{\pi^2}{4\lambda}\left\{\begin{aligned} & C_1^2\left(\frac{\pi^2 EI_y}{\lambda^2} - P\right) + C_2^{\,2}\left(\frac{\pi^2 EI_x}{\lambda^2} - P\right) + C_3^{\,2} r_0^2\left(\frac{1}{r_0^{\,2}}\left(GJ + \frac{E\Gamma\pi^2}{\lambda^2}\right) - P\right) \\ & + 2C_1 C_3 P y_0 - 2C_2 C_3 P x_0 \end{aligned}\right\} \tag{17}$$

substituting, $$P_y = \frac{\pi^2 EI_y}{\lambda^2};\ P_x = \frac{\pi^2 EI_x}{\lambda^2} \quad and\ P_\phi = \frac{1}{r_0^2}\left(GJ + \frac{E\Gamma\,\pi^2}{\lambda^2}\right)$$

Thus, equation (17) becomes

$$U + V = \frac{\pi^2}{4\lambda}\left[C_1^2\left(P_y - P\right) + C_2^2\left(P_x - P\right) + C_3^2 r_0^2\left(P_\phi - P\right) + 2C_1 C_3 P y_0 - 2C_2 C_3 P x_0\right] \tag{18}$$

Since, *(U+V)* is a function of three variables, it will have a stationary value when its derivatives with respect to C_1, C_2 and C_3 vanish. Thus,

$$\frac{\partial(U+V)}{\partial C_1} = C_1\left(P_y - P\right) + C_3\left(Py_0\right) \qquad = 0$$

$$\frac{\partial(U+V)}{\partial C_2} = C_2\left(P_x - P\right) - C_3\left(Px_0\right) \qquad = 0$$

$$\frac{\partial(U+V)}{\partial C_3} = C_1\, Py_0 - C_2\, Px_0 + C_3\, r_0^2\left(P_\phi - P\right) \qquad = 0 \tag{19}$$

$$\begin{bmatrix} P_y - P & 0 & Py_0 \\ 0 & P_x - P & -Px_0 \\ Py_0 & -Px_0 & r_0^2(P_\phi - P) \end{bmatrix} \begin{Bmatrix} C_1 \\ C_2 \\ C_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{20.a}$$

The solution to this equation could be found by setting the determinant to be zero.

$$\begin{vmatrix} P_y - P & 0 & Py_0 \\ 0 & P_x - P & -Px_0 \\ Py_0 & -Px_0 & r_0^2(P_\phi - P) \end{vmatrix} = 0 \tag{20.b}$$

Hence, the critical load is determined by the equation,

$$\left(P_y - P\right)\left(P_x - P\right)\left(P_\phi - P\right) - \left(P_y - P\right)\frac{P^2 x_0^2}{r_0^2} - \left(P_x - P\right)\frac{P^2 y_0^2}{r_0^2} = 0 \tag{21}$$

This is a cubic equation in *P*; the three roots of the cubic equation are the critical loads of the member, corresponding to the three buckling mode shapes.

a) For cross-section with double symmetry the centroid and shear centre coincide, hence

$x_0 = 0$ and $y_0 = 0$

$$\therefore\ (P_y - P)(P_x - P)(P_\phi - P) = 0 \tag{22}$$

This equation has three roots, namely,

$$\left.\begin{aligned} P &= P_x = \frac{\pi^2 EI_x}{\lambda^2} \\ P &= P_y = \frac{\pi^2 EI_y}{\lambda^2} \end{aligned}\right\} \quad - \text{ These represent Euler loads by buckling about the } x$$

and y axes

$$P = P_\phi = \frac{1}{r_0^2}\left(GJ + \frac{E\Gamma\pi^2}{\lambda^2}\right) \quad - \text{ This represents the Torsional buckling load.} \qquad (23)$$

Depending on the cross sectional property of the member any of the critical load values would govern.

b) For singly symmetric sections (such as channel sections):-

When the cross-section has only one axis of symmetry, say the x-axis,(eg. a channel section) the shear centre will be on that axis, hence equation (22) becomes a quadratic equation,

$$y_0 = 0 \qquad P = P_y = \frac{\pi^2 EI_y}{\lambda^2} \quad \text{(This represents Euler Buckling Load)} \qquad (24)$$

$$\therefore \left(P_x - P\right)\left(P_\phi - P\right) - \frac{P^2 x_0^2}{r_0^2} = 0 \qquad (25)$$

This quadratic equation in P has two roots, which correspond to flexural-torsional buckling.

The smaller root of the above equation is

$$P_{TF} = \frac{1}{2k}\left[P_\phi + P_x - \sqrt{\left(P_\phi + P_x\right)^2 - 4kP_\phi P_x}\right] \qquad (26)$$

in which $k = \left[1 - \left(x_0/r_0\right)^2\right]$

and P_{TF} is torsional-flexural buckling load.

Thus a singly symmetric section such as an equal angle or a channel can buckle either by flexure in the plane of symmetry or by a combination of flexure and torsion. ***All centrally loaded columns have three distinct buckling loads, at least one of which corresponds to torsional or torsional - flexural mode in a doubly symmetric section. Flexural buckling load about the weak axis is almost always the lowest. Hence, we disregard the torsional buckling load in doubly symmetric sections. In non-symmetric sections, buckling will be always in torsional – flexural mode regardless of its shape and***

dimensions. However, non-symmetric sections are rarely used and their design does not pose a serious problem.

Thin-walled open sections, such as angles and channels, can buckle by bending or by a combination of bending and twisting. Which of these two modes is critical depends on the shape and dimensions of the cross-section. Hence, torsional-flexural buckling must be considered in their design.

7.0 CONCLUDING REMARKS

The elastic buckling of an ideally straight column pin ended at both ends and subjected to axial compression was considered. The elastic buckling load was shown to be dependent on the slenderness ratio (λ/r) of the column. Factors affecting the column strengths (viz. initial imperfection, eccentricity of loading, residual stresses and lack of well-defined elastic limit) were all individually considered. Finally a generalized column strength curve (taking account of all these factors) has been suggested, as the basis of column design curves employed in Design Practices. The concept of "effective length" of the column has been described, which could be used as the basis of design of columns with differing boundary conditions.

The phenomenon of Elastic Torsional and Torsional-flexural buckling of a perfect column were discussed conceptually. The instability effects due to torsional buckling of slender sections are explained and discussed. Applications to doubly and singly symmetric sections are derived.

8.0 REFERENCES

1. Timoshenko S.P. and Gere, J.M: Theory of Elastic Stability, McGraw Hill Kogakusha Ltd., New York.

2. Chajes, A.: Principles of structural Stability Theory, Prentice Hall, New Jersey,1974

3. Allen,H.G. and Bulson,P.S. : Background to Buckling, McGraw Hill Book Company, 1980

4. Owens G.W., Knowles P.R : "Steel Designers Manual", The Steel Construction Institute, Ascot, England, 1994

5. Dowling P.J., Knowles P.R., Owens G.W : Structural Steel Design, Butterworth, London, 1998

7 INTRODUCTION TO PLATE BUCKLING

1.0 INTRODUCTION

Steel plates are widely used in buildings, bridges, automobiles and ships. Unlike beams and columns, which have lengths longer than the other two dimensions and so are modeled as linear members, steel plates have widths comparable to their lengths and so are modeled as two-dimensional plane members.

Just as long slender columns undergo instability in the form of buckling, steel plates under membrane compression also tend to buckle out of their plane. The buckled shape depends on the loading and support conditions in both length and width directions.

However, unlike columns, plates continue to carry loads even after buckling in a stable manner. Their post-buckling strengths, especially in the case of slender plates, can thus be substantially greater than the corresponding buckling strengths. This property is of great interest to structural engineers as it can be utilized to their advantage.

In this chapter, the expression for the critical buckling strength, of a flat plate simply supported on all four sides, is derived. The post-buckling behaviour of plates is described in terms of both stability and strength and compared with the post-buckling behaviour of a column. The concept of effective width is introduced to tackle the non-uniform distribution of stress in practical plates before and after buckling.

Buckling of web plates in shear is described and an expression to calculate their ultimate capacity is also given for use in design. A plate buckled in shear, can also carry additional shear due to the tension field action. Interaction formulas for plates under various load combinations are also given.

2.0 CRITICAL STRESS FOR PLATE BUCKLING

2.1 Rectangular flat plate simply supported on four sides

Consider a rectangular perfectly flat plate simply supported on all four sides and subjected to uniform compressive force N_x per unit length in the x-direction (Fig.1). The equilibrium equation for such a plate is given by

$$\frac{\partial^4 w}{\partial x^4} + \frac{2\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{12(1-\nu^2)}{Et^3}\left(-N_x \frac{\partial^2 w}{\partial x^2}\right) \tag{1}$$

where, w denotes the deflection in the z-direction of any point (x,y).

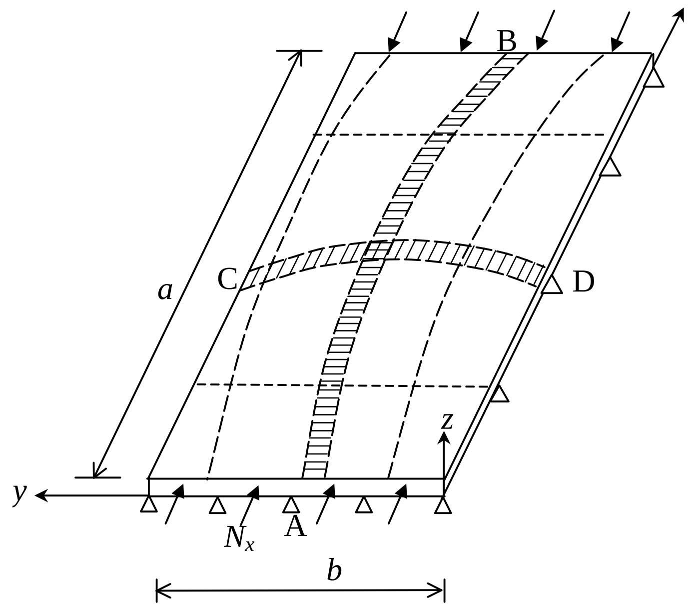


Fig.1 Buckling of Plate under Uni-axial Compression

w can be assumed as

$$w = \sum_{m=1,2,3,...} \sum_{n=1,2,3,....} w_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \tag{2}$$

The *m* and *n* in Eq. 2, indicate the number of half sine waves in the buckled mode. It may be noted that this assumed shape automatically satisfies the hinged boundary conditions for the plate, that is $w = 0$ at $x = 0$, $x = a$, $y = 0$ and $y = b$.
Substitution of Eq. (2) in Eq. (1) gives

$$\left(\frac{m^4\pi^4}{a^4} + 2\,\frac{m^2n^2\pi^4}{a^2b^2} + \frac{n^4\pi^4}{b^4} \right) = \frac{12\,(1-\nu^2)}{Et^3} (N_x)_{cr}\ \frac{m^2\pi^2}{a^2} \tag{3}$$

Therefore,

$$(N_x)_{cr} = \frac{\pi^2 Et^3}{12(1-\nu^2)} \frac{\left(m^2/a^2 + n^2/b^2\right)^2}{m^2/a^2} = \frac{\pi^2 Et^3}{12(1-\nu^2)} \left(\frac{m}{a} + \frac{n^2 a}{mb^2} \right)^2 \tag{4}$$

The lowest value of the membrane buckling stress $(N_x)_{cr}$, in Eq. (4) is obtained for $n=1$ and can also be written as follows,

$$(N_x)_{cr} = \frac{\pi^2 Et^3}{12(1-\nu^2)b^2} \left(m\frac{b}{a} + \frac{1}{m}\frac{a}{b} \right)^2 \tag{5}$$

Denoting the quantity within larger brackets by k and noting that the buckling load, N_{cr}, is the product of the buckling stress σ_{cr} and the thickness, we get the buckling stress as

$$\sigma_{cr} = \frac{k\pi^2 E}{12(1-\nu^2)(b/t)^2} \tag{6}$$

The expression for the critical buckling stress is similar to the Euler stress for columns [$\sigma_e = \pi^2 E/(\lambda/r)^2$] except for the fact that it is a function of the width-thickness ratio b/t. Why should the critical buckling stress in the x-direction be a function of the width b in the y-direction?

As the compressive load N_x on the plate is increased and reaches the critical buckling load N_{cr}, the central part of the plate such as the strip AB tends to buckle. Now, if we consider a transverse strip CD, we can realize that this strip resists the tendency of the strip AB to deflect out of the plane of the plate (z-direction). The shorter the width b, more will be the resistance offered by CD to AB. Hence the strip AB until buckling behaves like a column on elastic foundation, whose stiffness depends on b. This is the reason why the width b figures in the expression for critical buckling stress.

Next, let us consider the influence of the length 'a' of the plate on the buckling shape. Equation (5) is plotted in Fig. 3, showing the variation of buckling strength with respect to b/a ratio and for various values of m.

Consider a plate whose length a is much greater than the width b. If a longitudinal strip such as AB in Fig. 1, tends to form a single buckle, its curvature will be much less than the curvature of the transverse strip CD which tries to resist the buckling. This means that the resistance is greater than the tendency to buckle and the strength corresponding to this mode ($m=1$) is very high. Therefore, the plate prefers to buckle such that the curvatures of longitudinal and transverse strips are as equal as possible. This leads to multiple buckles in alternate directions as shown in Fig.2 such that the buckles are as square as possible. If $a = 2b$, the plate develops two buckles, if $a = 3b$, it develops three buckles and so on (Fig. 2).

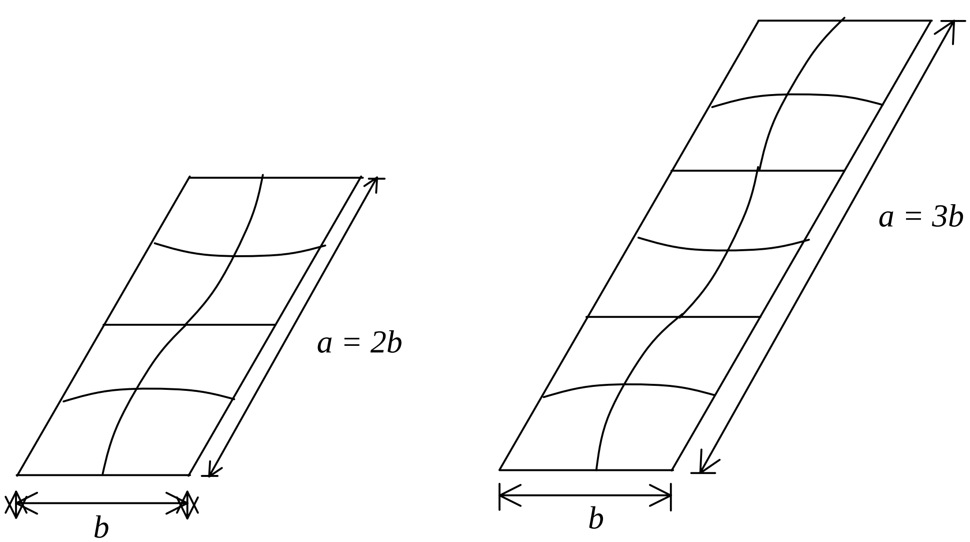


Fig. 2 Buckling Modes for Long Plates

Variation of *k*, the plate buckling coefficient, with aspect ratio (the ratio of the length, *a*, to the width, *b*) is shown in Fig. 3 for *m=1,2,3*, etc. It can be seen that the lowest value of the buckling coefficient is obtained for integral values of the aspect ratio. Correspondingly square half waves are the buckling mode shapes. Usually the plates are long in practice and for large aspect ratios the buckling coefficient is almost independent of the aspect ratio and is equal to the lowest value of 4.0. Hence the local buckling coefficient is taken to be the smallest value, independent of the aspect ratio and equals 4.0 for the case discussed.

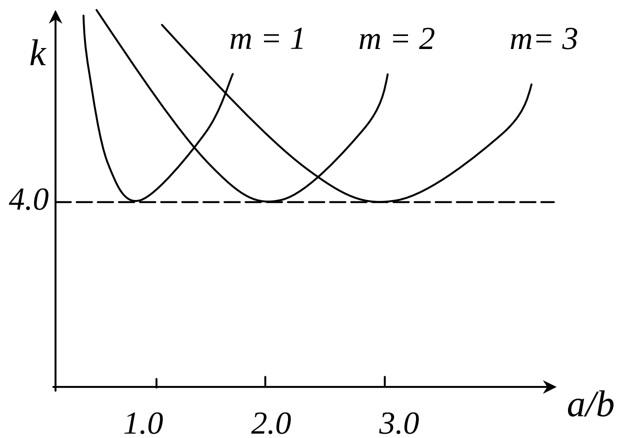


Fig. 3 k-values for a Simply Supported Plate

2.2 Plates with Other support Conditions

So far, it has been assumed that the plate is free to rotate about the longitudinal edges. Other edge conditions are of course possible. Consider for example, a box column made up of four plates as shown in Fig. 4 (*a*). If the flanges are relatively stiff, they would prevent the rotation of the corners [Fig. 4 (*b*)] and the web plate will behave as if its longitudinal edges are fixed. In this case, the bending resistance offered by the transverse strips such as *CD* will be considerably more than that of a plate with simply supported longitudinal edges and the buckling stress will be larger. If the flanges are also prone to buckling, then the corners will rotate as shown in Fig. 4c and the critical buckling stress will be the same as that for a plate with simply supported longitudinal edges. Therefore, the buckling coefficient is a function of the support condition along the longitudinal edges and the type of loading. It can be shown that the expression for the critical buckling coefficient is still valid except for the fact that the *k* values will be different. The *k* values for various common support conditions and loading cases are given in Table 1. Additional information can be found in Bulson (1970) and Timoshenko and Gere (1961).

In many rolled sections such as I-sections or channel sections, we find that flanges are similar to plates having one longitudinal edge simply supported and the other free. These are called *outstands* as against plate elements having both longitudinal edges simply supported (*internal elements*). From Table 2, we find that *k* value for outstands is *0.425* which is roughly one-tenth that for internal elements (*k=4*). The reason for such a low value is that the transverse strip (such as *CD* in Fig. 1) simply rotates and offers little bending resistance as shown in Fig. 4 (*d*).

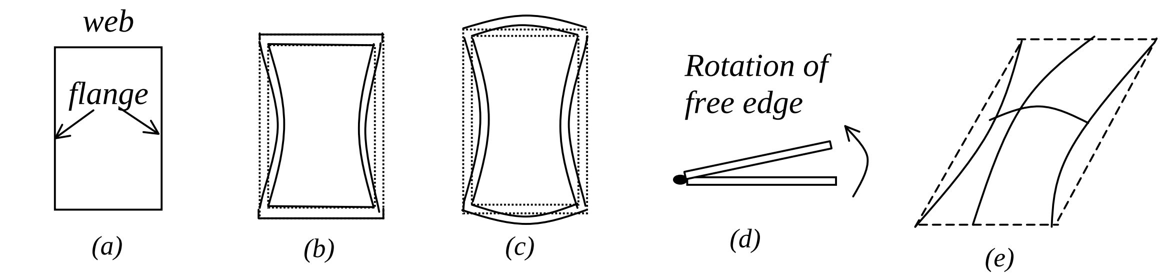


Fig. 4 Plate Elements with Different Edge Conditions

Table 1 Values of k for Different Load and Support Conditions

Load Condition	**Support Condition**	**Buckling Coefficient, *k***
Uniaxial Compressive Stress (σ_x)	Hinged-hinged	4.00
	Fixed-fixed	6.97
	Hinged-free	1.27
	Fixed free	0.43
Shear Stress (τ_{xy})	Hinged-hinged	5.35
	Fixed-fixed	8.99

The edge conditions not only affect the critical buckling stress but also influence the post-buckling behaviour. For a plate with longitudinal edges (edges parallel to the *x*-axis in Fig. 1) constrained to remain straight in the plane of the plate, the transverse stresses in the strip *CD* will be tensile and has the effect of stiffening the plate against lateral deflection. However, if the longitudinal edges are free to pull-in in the *y*-direction [Fig. 4(e)], there will be no transverse stresses in the strip *CD* and the plate will be less stiff compared to the previous case.

To ensure that a plate with a given support conditions fails by yielding rather than buckling, the corresponding critical buckling stress should be greater than the yield stress. Equating the expression given in Eq.(6) to the yield stress, the limiting value of the width-thickness ratio to ensure yielding before plate buckling can be obtained as

$$\left(\frac{b_{\lim}}{t}\right) \leq \left(\frac{k\pi^2 E}{12(1-\nu^2)\, f_y}\right)^{1/2} \tag{7}$$

The codes prescribe different limiting values for the *b/t* ratio of plate elements in structural members, in terms $C/\sqrt{f_y}$, where *C* is a constant. These are dealt with in a subsequent chapter on local buckling.

3.0 POST-BUCKLING BEHAVIOUR AND EFFECTIVE WIDTH

3.1 Post-buckling Behaviour

Consider a rectangular plate with all four edges simply supported and subjected to uniform compression along *x*-direction (Fig. 1). When the compressive stress equals the critical buckling stress σ_{cr}, the central part of the plate, such as the strip *AB*, buckles. But the edges parallel to the *x*-axis cannot deflect in the *z*-direction and so the strips closer to these edges continue to carry the load without any instability. Therefore the stress distribution across the width of the plate in the post-buckling range becomes non-uniform with the outer strips carrying more stress than the inner strips as shown in Fig. 5(*a*). However, as described before, the transverse strips such as *CD* in Fig. 1 continue to stretch and support the longitudinal strips. This ensures the stability of the plate in the post-buckling range. Increasing the axial displacement of the plate will cause an increase in the lateral displacement. When the edge stresses approach and equal the yield stress of the material, the plate deflection would be vary large and the plate, eventually, can be considered to have failed when the stresses in the edge strips reach the yield stress of the material.

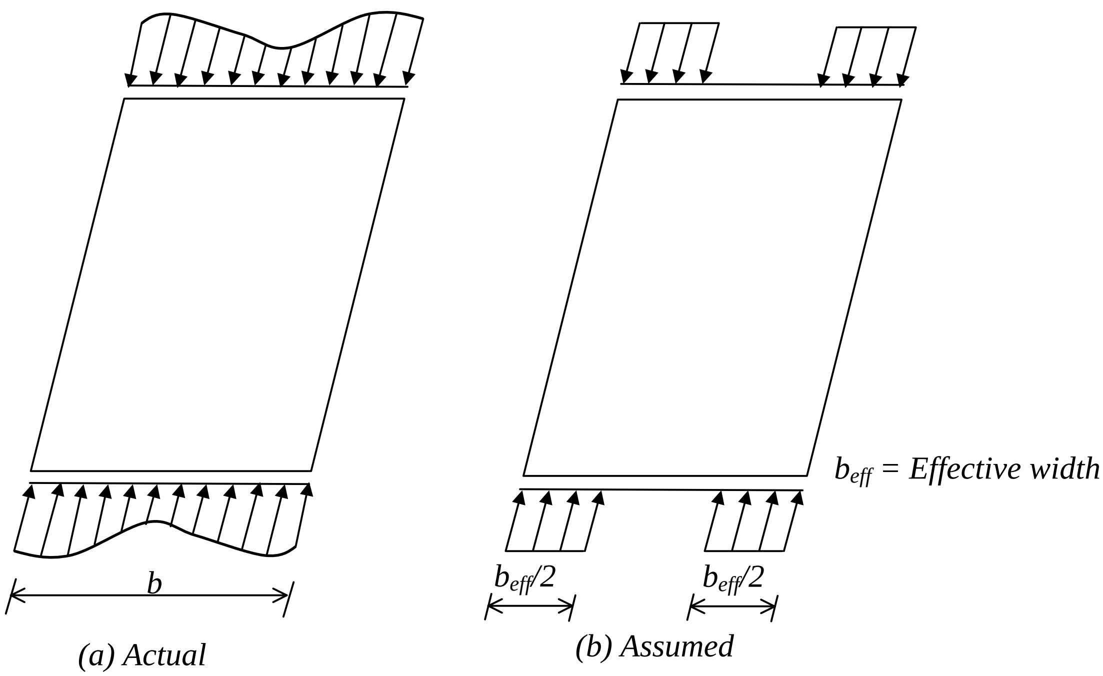


Fig. 5 Actual and Assumed Stress Distribution in the Post-buckling

3.2 Effective Width

To calculate the load carrying capacity of the plate in the post-buckling range, the concept of effective width is used. The concept was first proposed by von Karman. He realized that as the plate is loaded beyond its elastic buckling load, the central part such as strip AB deflects thereby shedding the load to the edge strips. Therefore, the non-uniform stress distribution across the width of the buckled plate, can be replaced by a uniform stress blocks of stress equal to that at the edges, over a width of $b_{eff}/2$ on either side where b_{eff} is called the *effective width* of the plate. This effective width can be calculated by equating the non-uniform stress blocks and the uniform stress blocks.

The shape of the non-uniform stress block depends on the load and support conditions. Therefore a number of formulae are available for calculating the effective width, each catering to a particular geometry of the plate. For the plate simply supported on all four sides, as the load is increased beyond the critical buckling load, the stress block becomes more and more non-uniform. When the stress at the outer strips reaches the yield stress, the corresponding effective width can be calculated using Winter's formula

$$\frac{b_{eff}}{b} = \sqrt{\left(\frac{\sigma_{cr}}{f_y}\right)\left[1 - 0.22\sqrt{\left(\frac{\sigma_{cr}}{f_y}\right)}\right]} \tag{8}$$

The yield stress f_y, multiplied by the effective width gives the ultimate strength of the plate approximately.

3.3 Plates with Initial Imperfections

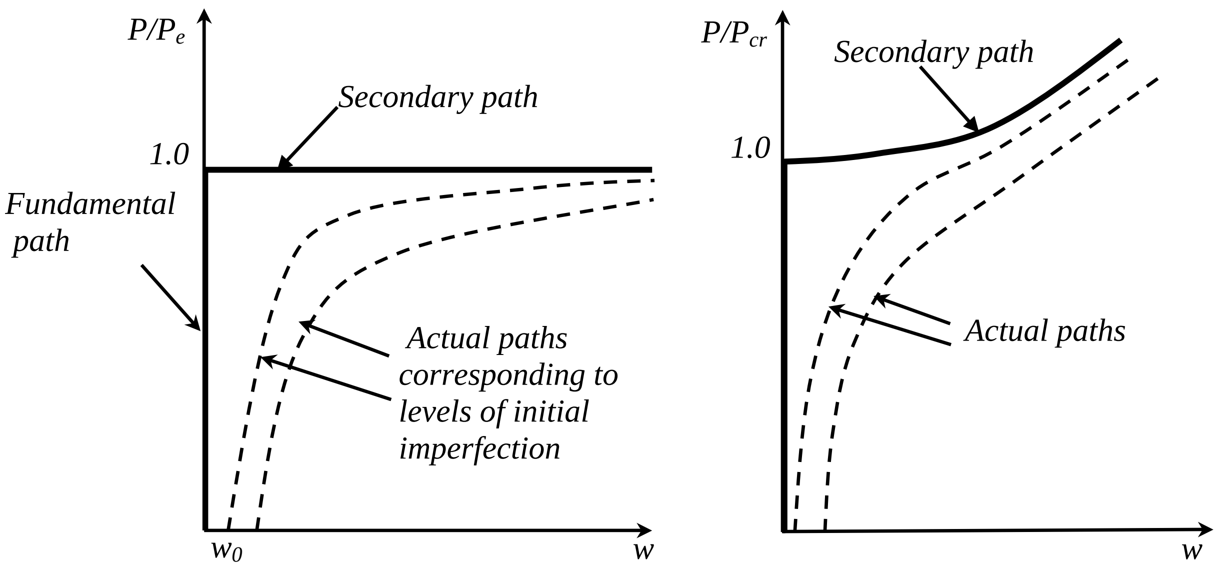
The perfectly flat plate described above represents an ideal condition. In practice, plates have initial imperfections, which (for simplicity of calculations) are normally assumed to be similar to the buckled shape. The behaviour of practical plates is broadly similar to the post-buckling behaviour of perfectly flat plates. However, the stresses across the width are non-uniform right from the beginning and so the concept of effective width can be applied to them even before the onset of elastic buckling. Other aspects of the behaviour of plates with initial imperfections such as stiffness and strength will be described in subsequent sections.

4.0 STABILITY AND ULTIMATE STRENGTH OF PLATES

4.1 Stability of Plates

It is interesting to compare the stability of a column and a plate. In the case of an ideal column, as the axial load is increased, the lateral displacement remains zero until the attainment of the critical buckling load (Euler load). If we plot the axial load versus lateral displacement, we will get a line along the load axis up to $P = P_{cr} = P_e$ (Fig. 7). This is called the fundamental path. When the axial load becomes equal to the Euler buckling load, the lateral displacement increases indefinitely at constant load. This is the secondary path, which bifurcates from the fundamental path at the buckling load. The secondary path for column represents neutral equilibrium. For practical columns, which have initial imperfections, there is a smooth transition from the stable to neutral equilibrium paths as shown by the dashed line in Fig. 6(*a*).

The fundamental path for a perfectly flat plate is similar to that of an ideal column. At the critical buckling load, this path bifurcates into a secondary path as shown in Fig. 6(*b*). The secondary path reflects the ability of the plate to carry loads higher than the elastic critical load. Unlike columns, the secondary path for a plate is stable. Therefore, elastic buckling of a plate need not be considered as collapse. However, plates having one free edge and simply supported along the other edges (outstands), have very little post-buckling strength.



(a) Column *(b) Plate*

Fig. 7 Load versus Out-of-plane Displacement Curves

Actual failure load of the columns and plates are reached when the yielding spreads from the supported edges triggering collapse and thereafter the unloading occurs.

The axial stiffness of an ideally square flat plate drops suddenly from *EA* (where *A* is the cross-sectional area and E the elastic modulus) to a smaller value (nearly *AE/2*) after buckling and remains relatively constant thereafter, as shown by the load-axial deformation curve in Fig. 8. In the case of practical plates there is a gradual loss of stiffness as shown by the dashed line in Fig. 8.

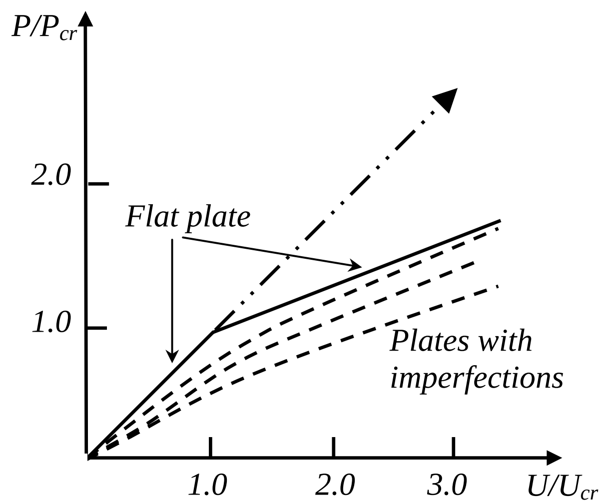


Fig. 8 Load versus Axial Deformation Diagram

4.2 Strength of Plates

Plate strength curves can also be constructed similar to the column strength curves (Fig. 9). In the case of ideal columns with low slenderness (*i.e.* stocky columns), failure is expected by squashing at the yield stress. On the other hand, if the ideal column is slender, failure will be by buckling at or near the Euler load. Tests on practical columns indicate that failure always occurs below the failure load of an ideal column of the same slenderness. If the column is stocky, then the yield stress provides an upper bound and if the column is slender then the buckling stress provides an upper bound. Also the scatter in the test results is considerable particularly in the range of intermediate slenderness ratios ($\sqrt{f_y/\sigma_e} = 1.0$).

In the case of a flat plate simply supported on all four sides, we can expect failure by squashing if the *b/t* ratio is less than the limiting value given by equation (7). Similarly, for *b/t* ratios larger than the limiting value, failure after buckling at the critical buckling stress may be expected. However, tests on practical plates indicate that for large *b/t* ratios, the failure stress is substantially greater than the critical buckling stress. This is due to the post-buckling behaviour, which is unique to plates. The load from the middle strips gets transferred to the edges and the plate continues to carry higher load in stable post-buckling range, until the edges reach the yield stress as described in the previous sections. As with columns, the scatter in the test results is considerable in the range of intermediate *b/t* ratios (at $f_y/\sigma_{cr} = 1.0$).

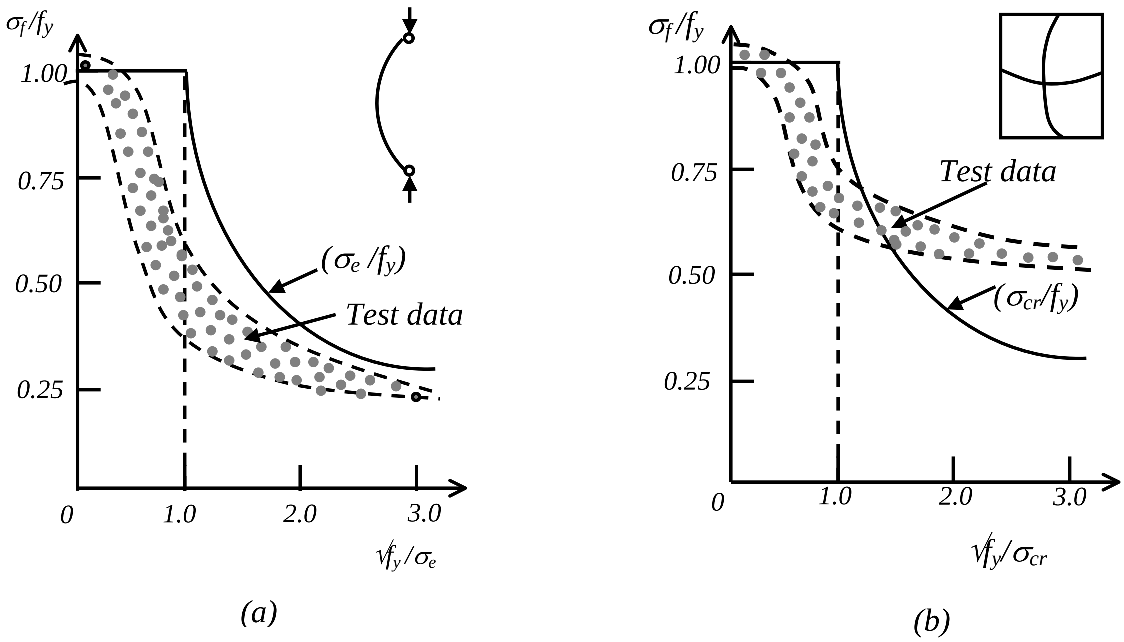


Fig. 9 Column and Plate Strength Curves

5.0 BUCKLING OF WEB PLATES IN SHEAR

Rectangular plates loaded in shear such as web plates in a plate girder, also tend to buckle. Consider a plate loaded in shear in its own plane as shown in Fig.10. A square element in the plate (Fig.10), whose edges are oriented at 45^o to the plate edges, experiences tensile stresses on two opposite edges and compressive stresses on the other two edges. The compressive stress can cause local buckling and as a result the plate develops waves perpendicular to them.

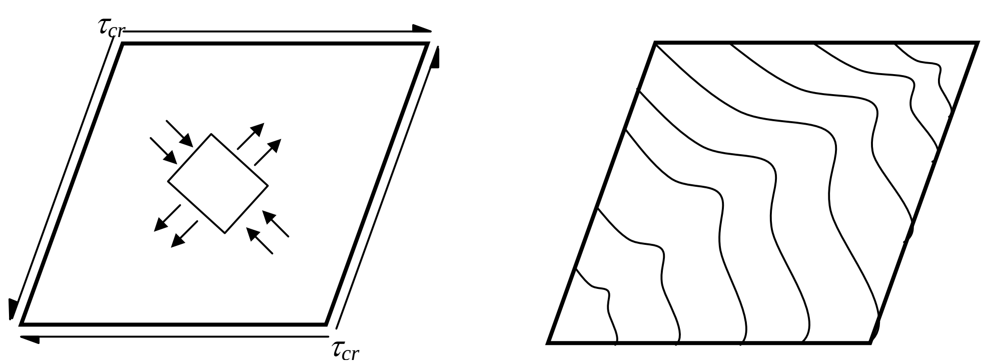


Fig. 10 Shear buckling of a plate

The critical shear stress at which this form of buckling occurs is given by the same formula as that for plate buckling under compression, except that the value for the buckling coefficient *k* is different (Table 1). The buckling coefficient varies with the aspect ratio *a/b* and increases from *5.34* for an infinitely long panel to *9.34* for a square panel. The values given in Table 1 are for the square panels.

Plates buckled in shear also can support additional loads. If we draw imaginary diagonals on the plate, the diagonal which gets loaded in compression buckles and cannot support additional load. However, the diagonal in tension continues to take more load and the plate becomes like a triangular truss with only tension diagonals. This is called ***tension field action*.**

Formulae based on theoretical and experimental works have been produced which are of the form:

$$\tau' = \tau_{cr} + \frac{\tau_y\sqrt{3}}{2}\left[\frac{1-\tau_{cr}/\tau_y}{\sqrt{1+(a/d)^2}}\right] \quad \text{when } \tau_{cr} < \tau_y \text{ and } (a/d) < 3.0$$
$$\tau' = \tau_{cr} \quad \text{when } \tau_{cr} \geq \tau_y \text{ and } (a/d) \geq 3.0 \tag{9}$$

The first term represents the critical stress of an ideally flat plate and the second term, the post-buckling reserve due to the tension field action. In practical webs, the post-buckling reserve due to tension field may be several times the critical stress τ_{cr}. Web panels are also less sensitive to usual imperfections than compression flanges.

The critical load for four-side supported plate subjected to various stress combinations is given by the following interaction formula:

$$\left(\frac{\sigma}{\sigma_{cr}}\right) + \left(\frac{\sigma_b}{\sigma_{b,cr}}\right)^2 + \left(\frac{\tau}{\tau_{cr}}\right)^2 = 1 \tag{10}$$

where, σ, σ_b and τ are the applied axial compressive, maximum bending compressive and shear stress respectively and σ_{cr}, $\sigma_{b,cr}$ and τ_{cr} are the corresponding critical stresses.

6. 0 CONCLUDING REMARKS

Like columns, plates also undergo instability and buckle under compressive and shear stresses. The critical buckling load for a plate depends upon its width-thickness ratio and support conditions. However, unlike columns, the post-buckling behaviour of plates is stable and plates will continue to carry higher loads beyond their elastic critical loads. This post-buckling range is substantial in the case of plates with high width-thickness ratios (slender plates).

In the post-buckling range, the stress distribution is non-uniform and the plate fails when the maximum stress at the supported edge in the post-buckling range reaches the yield stress. So the concept of effective width is used to calculate the strength of the plate. This concept replaces the non-uniform stress distribution by an equivalent uniform stress equal to the stress at the edges over a reduced effective width (b_{eff}) of the plate.

In practical plates having initial imperfections, the stress distribution is non-uniform right from the start and so the effective width concept can be used even before the inception of buckling. Practical plates also posses post-buckling strength and empirical formulas are available to estimate their ultimate strength. However, the critical buckling strength of three side simply supported plates such as outstands is quite small and their post-buckling strength is also negligible.

7.0 REFERENCES

1. Dowling P.J, Knowles P. and Owens G.W, 'Structural Steel Design', The Steel Construction Institute, 1988.
2. Gaylord E. H. et al, 'Design of Steel Structures', McGraw Hill, 1992.
3. Bulson P. S., 'Theory of Flat Plates', Chatto and Windus, London, 1970.
4. Bleich H., 'Buckling Strength of Metal Structures', McGraw Hill, 1952.
5. Timoshenko S.P. and Gere J.M., 'Theory of Elastic Stability', McGraw Hill Book Company, New York, 1961.

8 LOCAL BUCKLING AND SECTION CLASSIFICATION

1.0 INTRODUCTION

Sections normally used in steel structures are I-sections, Channels or angles etc. which are called open sections, or rectangular or circular tubes which are called closed sections. These sections can be regarded as a combination of individual plate elements connected together to form the required shape. The strength of compression members made of such sections depends on their slenderness ratio. Higher strengths can be obtained by reducing the slenderness ratio *i.e.* by increasing the moment of inertia of the cross-section. Similarly, the strengths of beams can be increased, by increasing the moment of inertia of the cross-section. For a given cross-sectional area, higher moment of inertia can be obtained by making the sections thin-walled. As discussed earlier, plate elements laterally supported along edges and subjected to membrane compression or shear may buckle prematurely. Therefore, the buckling of the plate elements of the cross section under compression/shear may take place before the overall column buckling or overall beam failure by lateral buckling or yielding. This phenomenon is called *local buckling*. Thus, local buckling imposes a limit to the extent to which sections can be made thin-walled.

Consider an I-section column, subjected to uniform compression [Fig. 1(*a*)]. It was pointed out in the chapter on "Introduction to Plate Buckling" that plates supported on three sides (outstands) have a buckling coefficient k roughly one-tenth that for plates supported on all four sides (internal elements). Therefore, in open sections such as I-sections, the flanges which are outstands tend to buckle before the webs which are supported along all edges. Further, the entire length of the flanges is likely to buckle in the case of the axially compressed member under consideration, in the form of waves. On the other hand, in closed sections such as the hollow rectangular section, both flanges and webs behave as internal elements and the local buckling of the flanges and webs depends on their respective width-thickness ratios. In this case also, local buckling occurs along the entire length of the member and the member develops a 'chequer board' wave pattern [Fig. 1(*b*)].

In the case of beams, the compression flange behaves as a plate element subjected to uniform compression and, depending on whether it is an outstand or an internal element, undergoes local buckling at the corresponding critical buckling stress. However, the web is partially under compression and partially under tension. Even the part in compression is not under uniform compression. Therefore the web buckles as a plate subjected to in-plane bending compression.

Normally, the bending moment varies over the length of the beam and so local buckling may occur only in the region of maximum bending moment.

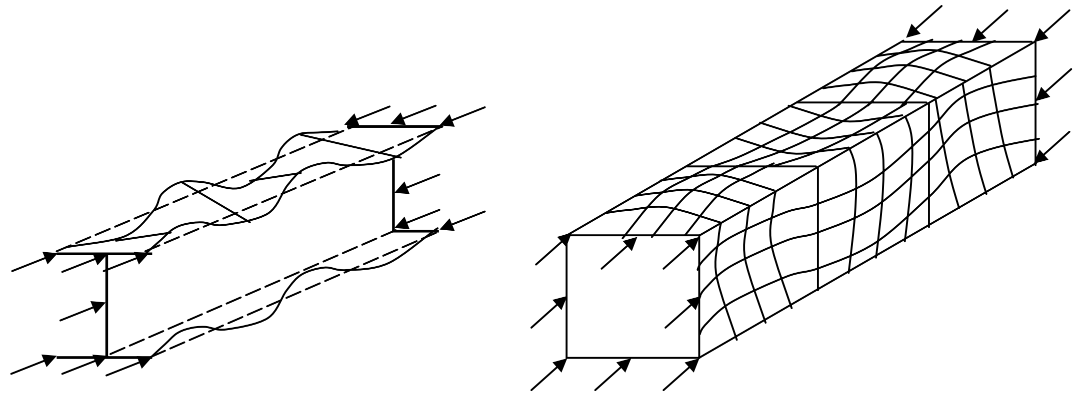


Fig. 1 Local buckling of Compression Members

Local buckling has the effect of reducing the load carrying capacity of columns and beams due to the reduction in stiffness and strength of the locally buckled plate elements. Therefore it is desirable to avoid local buckling before yielding of the member. Most of the hot rolled steel sections have enough wall thickness to eliminate local buckling before yielding. However, fabricated sections and thin-walled cold-formed steel members usually experience local buckling of plate elements before the yield stress is reached.

It is useful to classify sections based on their tendency to buckle locally before overall failure of the member takes place. For those cross-sections liable to buckle locally, special precautions need to be taken in design. However, it should be remembered that local buckling does not always spell disaster. Local buckling involves distortion of the cross-section. There is no shift in the position of the cross-section as a whole as in global or overall buckling. In some cases, local buckling of one of the elements of the cross-section may be allowed since it does not adversely affect the performance of the member as a whole. In the context of plate bucking, it was pointed out that substantial reserve strength exists in plates beyond the point of elastic buckling. Utilization of this reserve capacity may also be the objective of design. Therefore, local buckling may be allowed in some cases, provided due care is taken to estimate the reduction in the capacity of the section due to it and the consequences are clearly understood.

In what follows, first the basic concepts of plastic theory are introduced. Then the classification of cross-sections is described. The codal provisions limiting the width-thickness ratios of plate elements in a cross-section are given. Finally the implications in design are discussed.

2.0 BASIC CONCEPTS OF PLASTIC THEORY

Before attempting the classification of sections, the basic concepts of plastic theory will be introduced. More detailed descriptions can be found in subsequent chapters.

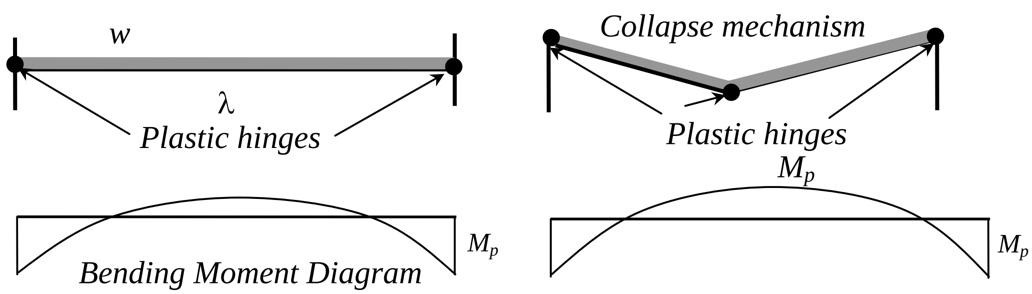


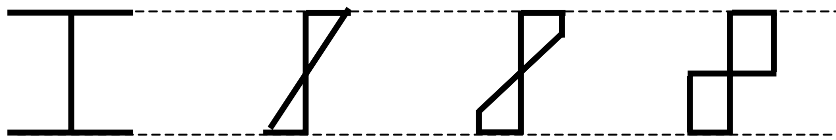
Fig. 2 Formation of a Collapse Mechanism in a Fixed Beam

Consider a beam with both ends fixed and subjected to a uniformly distributed load of *w* per meter length as shown in Fig. 2(*a*). The elastic bending moment at the ends is $w\lambda^2/12$ and at mid-span is $w\lambda^2/24$, where λ is the span. The stress distribution across any cross section is linear [Fig. 3(*a*)]. As *w* is increased gradually, the bending moment at every section increases and the stresses also increase. At a section close to the support where the bending moment is maximum, the stresses in the extreme fibers reach the yield stress. The moment corresponding to this state is called the *first yield moment* M_y, of the cross section. But this does not imply failure as the beam can continue to take additional load. As the load continues to increase, more and more fibers reach the yield stress and the stress distribution is as shown in Fig 3(*b*). Eventually the whole of the cross section reaches the yield stress and the corresponding stress distribution is as shown in Fig. 3(*c*). The moment corresponding to this state is known as the *plastic moment* of the cross section and is denoted by M_p.

The ratio of the plastic moment to the yield moment is known as the *shape factor* since it depends on the shape of the cross section. The cross section is not capable of resisting any additional moment but may maintain this moment for some amount of rotation in which case it acts like a *plastic hinge*. If this is so, then for further loading, the beam, acts as if it is simply supported with two additional moments M_p on either side, and continues to carry additional loads until a third plastic hinge forms at mid-span when the bending moment at that section reaches M_p. The beam is then said to have developed a *collapse mechanism* and will collapse as shown in Fig 2(*b*). If the section is thin-walled, due to local buckling, it may not be able to sustain the moment for additional rotations and may collapse either before or soon after attaining the plastic moment. It may be noted that formation of a single plastic hinge gives a collapse mechanism for a simply supported beam. The ratio of the ultimate rotation to the yield rotation is called the *rotation capacity* of the section. The yield and the plastic moments together with the rotation capacity of the cross-section are used to classify the sections.

3.0 SECTION CLASSIFICATION

Sections are classified depending on their moment-rotation characteristics (Fig. 4). The codes also specify the limiting width-thickness ratios $\beta = b/t$ for component plates, which enables the classification to be made.



(a) at M_y (b) $M_y < M < M_p$ (c) at M_p

Fig. 3 Plastification of Cross-section under Bending

- Plastic cross-sections: Plastic cross-sections are those which can develop their full-plastic moment M_p and allow sufficient rotation at or above this moment so that redistribution of bending moments can take place in the structure until complete failure mechanism is formed $(b/t \le \beta_1)$ (see Fig. 5).
- Compact cross-sections: Compact cross-sections are those which can develop their full-plastic moment M_p but where the local buckling prevents the required rotation at this moment to take place $(\beta_1 < b/t < \beta_2)$.
- Semi-compact cross-sections: Semi-compact cross-sections are those in which the stress in the extreme fibers should be limited to yield stress because local buckling would prevent the development of the full-plastic moment M_p. Such sections can develop only yield moment M_y $(\beta_2 < b/t \le \beta_3)$.
- Slender cross-sections: Slender cross-sections are those in which yield in the extreme fibers cannot be attained because of premature local buckling in the elastic range $(\beta_3 < b/t)$.

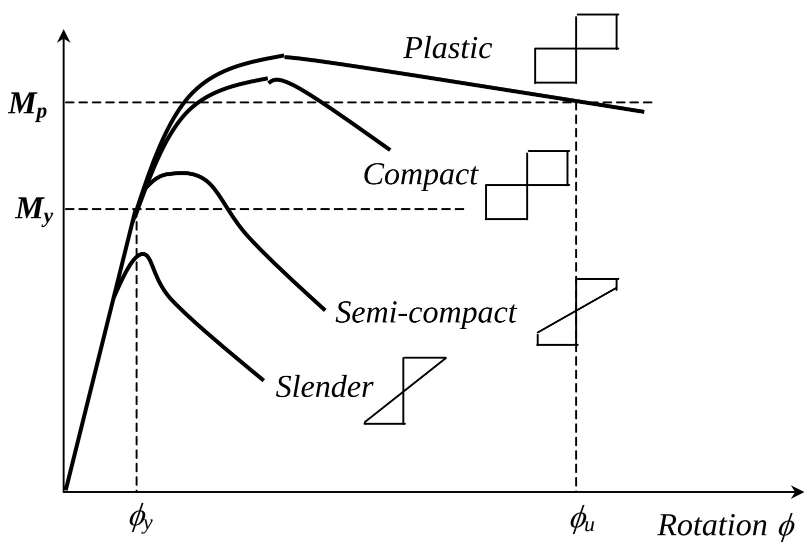


Fig. 4 Section Classification based on Moment-Rotation Characteristics

It should be remembered that even for steels with a large yield plateau, some strain hardening effects are likely to take place and the maximum moment is likely to be larger than M_p for plastic and compact sections. In such cases, the rotation capacity may be taken as the ratio of the rotation when the moment capacity drops back to M_p to the rotation at yield.

The relationship between the moment capacity M_u and the compression flange slenderness *b/t* indicating the β limits is shown in Fig. 5. In this figure, the value of M_u for semi-compact sections is conservatively taken as M_y.

In the above classification, it is assumed that the web slenderness *d/t* is such that its buckling before yielding is prevented. It should be noted that the entire web may not be in uniform compression and if the neutral axis lies in the web, a part of the web may actually be in tension. In this case, the slenderness limits are somewhat relaxed for the webs.

Since the above classification is based on bending, it cannot be used for a compression member. The only criterion required is whether the member is slender or not. However, in practice, it is considered to be prudent to use compact or plastic sections for members carrying predominantly compressive loads.

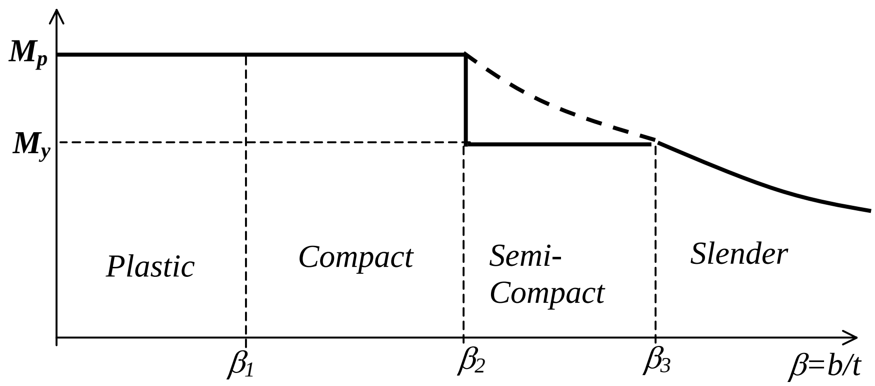


Fig. 5 Moment Capacities of Sections

4.0 LIMITS ON WIDTH-THICKNESS RATIOS

If the flanges and webs of cross-sections are considered to be plates under compression, their limiting width-thickness ratios can be obtained by equating the critical buckling stress to the yield stress. However, such an approach disregards a number of factors such as the actual support restraint provided by the adjoining plate element and the residual stresses and initial imperfections. Therefore, the limiting width-thickness ratios β_1 , β_2 and β_3 are useful for designers and are normally arrived at by validation in the testing laboratory.

The limiting width-thickness ratios for different sections as per IS: 800 are given in Table 2. The various extents of widths and thicknesses for different cross sections have been defined in Fig 6.

Local buckling can be prevented, by controlling the width-thickness ratio. One way of doing this is by adopting higher thickness of the plate. This method is adopted in rolled steel sections. However in the case of built-up sections and cold-formed sections, longitudinal stiffeners are provided which divide the total width into a number of smaller widths. The buckling of stiffened plates is beyond the scope of this chapter.

Table 1. Limits on Width to Thickness Ratio of Plate Elements

<table>
<tr><th colspan="3" rowspan="2">Compression element</th><th rowspan="2">Ratio</th><th colspan="3">Class of Section</th></tr>
<tr><th>Plastic (β_1)</th><th>Compact (β_2)</th><th>Semi-compact (β_3)</th></tr>
<tr><td rowspan="2">Outstanding element of compression flange</td><td colspan="2">Rolled section</td><td>b/t_f</td><td>9.4ε</td><td>10.5ε</td><td>15.7ε</td></tr>
<tr><td colspan="2">Welded section</td><td>b/ t_f</td><td>8.4ε</td><td>9.4ε</td><td>13.6ε</td></tr>
<tr><td rowspan="2">Internal element of compression flange</td><td colspan="2">Compression due to bending</td><td>b/ t_f</td><td>29.3ε</td><td>33.5ε</td><td rowspan="2">42ε</td></tr>
<tr><td colspan="2">Axial compression</td><td>b/ t_f</td><td colspan="2">Not applicable</td></tr>
<tr><td rowspan="4">Web of an I-H-or box section [c]</td><td colspan="2">Neutral axis at mid-depth</td><td>d/t_w</td><td>84ε</td><td>105ε</td><td>126ε</td></tr>
<tr><td rowspan="2">Generally</td><td>If r_1 is negative:</td><td>d/t_w</td><td rowspan="2">$\frac{84\varepsilon}{1+r_1}$
but $\leq 42\varepsilon$</td><td>$\frac{105.0\varepsilon}{1+r_1}$</td><td rowspan="3">$\frac{126.0\varepsilon}{1+2r_2}$
but $\leq 42\varepsilon$</td></tr>
<tr><td>If r_1 is positive :</td><td>d/t_w</td><td>$\frac{105.0\varepsilon}{1+1.5r_1}$
but $\leq 42\varepsilon$</td></tr>
<tr><td colspan="2">Axial compression</td><td>d/t_w</td><td colspan="2">Not applicable</td></tr>
<tr><td colspan="3">Web of a channel</td><td>d/t_w</td><td>42ε</td><td>42ε</td><td>42ε</td></tr>
<tr><td colspan="3">Angle, compression due to bending
(Both criteria should be satisfied)</td><td>b/t
d/t</td><td>9.4ε
$9.4\ \varepsilon$</td><td>10.5ε
10.5ε</td><td>15.7ε
15.7ε</td></tr>
<tr><td colspan="3">Single angle, or double angles with the components separated, axial compression
(All three criteria should be satisfied)</td><td>b/t
d/t
$(b+d)/t$</td><td colspan="2">Not applicable</td><td>15.7ε
15.7ε
25ε</td></tr>
<tr><td colspan="3">Outstanding leg of an angle in contact back-to-back in a double angle member</td><td rowspan="2">d/t</td><td rowspan="2">9.4ε</td><td rowspan="2">10.5ε</td><td rowspan="2">15.7ε</td></tr>
<tr><td colspan="3">Outstanding leg of an angle with its back in continuous contact with another component</td></tr>
<tr><td>Circular tube subjected to moment or axial compression</td><td colspan="2">CHS or built by welding</td><td>D/t</td><td>$44\varepsilon^2$</td><td>$55\varepsilon^2$</td><td>$88\varepsilon^2$</td></tr>
<tr><td colspan="3">Stem of a T-section, rolled or cut from a rolled I-or H-section</td><td>D/t_f</td><td>8.4ε</td><td>9.4ε</td><td>18.9ε</td></tr>
</table>

Note 1: Section having elements which exceeds semi-compact limits are to be taken as slender cross sections

Note2: $\varepsilon=(250/ f_y)^{1/2}$

Note 3: Check webs for shear buckling in accordance when d/t > 67 ε. Where, b is the width of the element may be taken as clear distance between lateral supports or between lateral support and free edge, as appropriate, t is the thickness of element, d is the depth of the web, D mean diameter of the element,

Note 4: Different elements of a cross-section can be in different classes. In such cases the section is classified based on the least favorable classification.

Note 5: The stress ratio r_1 and r_2 *are defined as*

$$r_1 = \frac{\text{actual average axial compressive stress}}{\text{design compressive stress of web alone}}, \quad r_2 = \frac{\text{actual average axial compressive stress}}{\text{design compressive stress of overall section}}$$

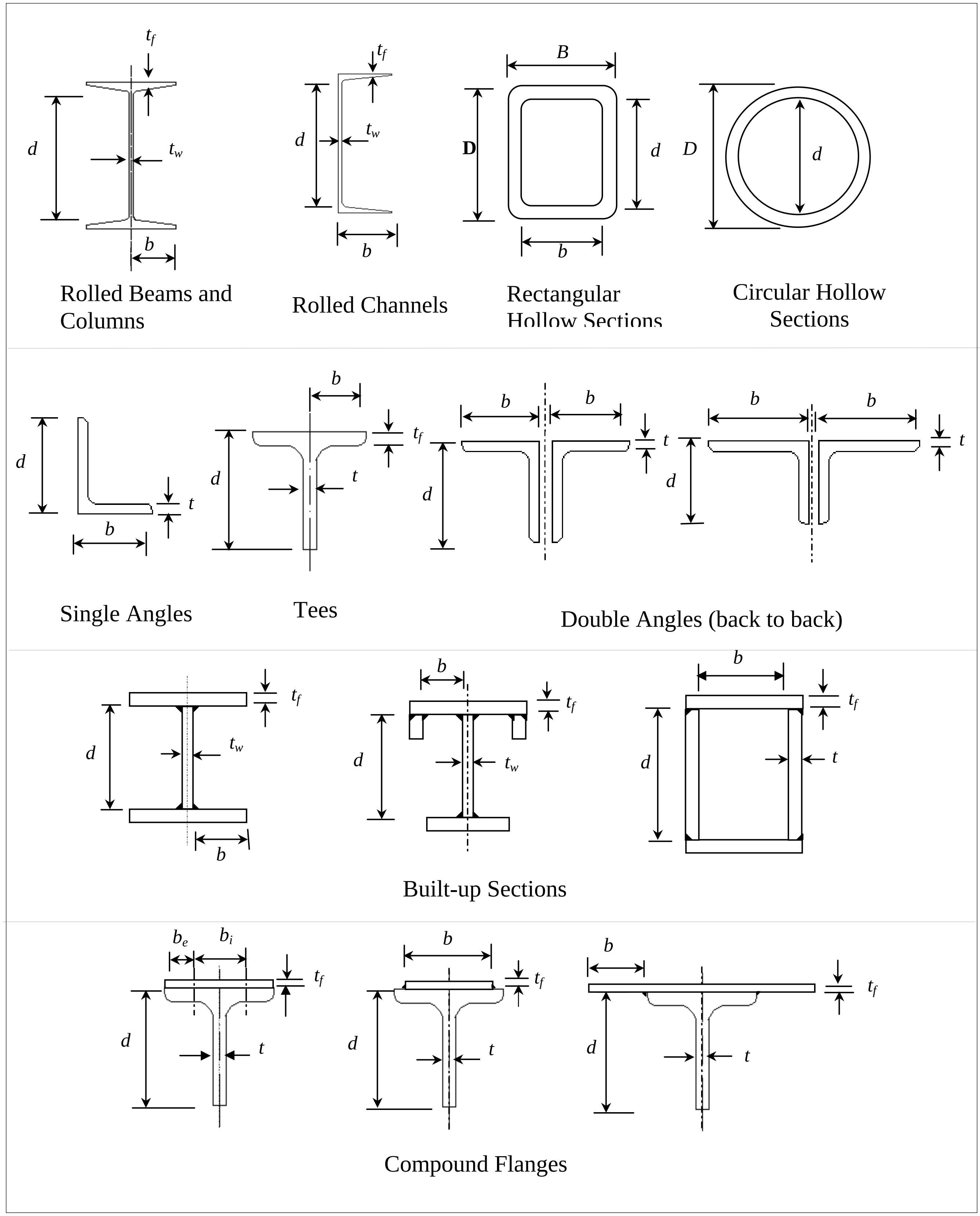


Fig 6 Dimensions of Sections

It may be noted that semi-compact and slender members cannot be used in plastic design. In fact, only plastic sections can be used in indeterminate frames forming plastic collapse mechanisms while compact sections can be used in simply supported beams failing after reaching M_p at one section. In elastic design, semi-compact sections may be used with the understanding that they will fail at M_y. Slender sections also have a stiffness problem and are normally not preferred in hot-rolled structural steel work. However, they are extensively used in cold-formed members and the manufacturer's literature may be consulted while using them. Plate girders are usually designed taking advantage of the tension field approach to achieve economy.

5.0 CONCLUDING REMARKS

Local buckling is discussed as a phenomenon controlling the strength of compression and bending members. The cross-sections are classified into plastic, compact, semi-compact and slender depending upon their moment-rotation characteristics. The limits on the width-thickness ratios of plate elements are provided to classify the section under a particular class. Only plastic and compact sections can be used if limit state design is followed and only plastic sections can be used in mechanism-forming indeterminate frames. Slender sections are to be avoided even in elastic design but are invariably used in cold-formed construction for reasons of economy. In this case, caution is required in predicting their ultimate capacities.

6.0 REFERENCES

1. Dowling P.J, Knowles P. and Owens G.W, 'Structural Steel Design', The Steel Construction Institute, 1988.
2. Bulson P. S., 'Theory of Flat Plates', Chatto and Windus, London,1970.
3. 'Steel Designers manual', 5th Edition Ed by Owens G.W and Knowles P. The Steel Construction Institute, ELBS Blackwell Scientific Publishers, London 1994.
4. Timoshenko S.P. and Gere J.M., 'Theory of Elastic Stability', McGraw Hill Book Company, New York, 1961.

9 LATERALLY RESTRAINED BEAMS

1.0 INTRODUCTION

Beams are structural members frequently used to carry loads that are transverse to their longitudinal axis. They transfer loads primarily by bending and shear. In a rectangular building frame, beams that span between adjacent columns are called '***main or primary beams/girders'***. Beams, which are used to transmit the floor loading to the main beams between columns, are called '***secondary beams/joists'***. As far as the structural steel framing in buildings is concerned, it is sufficient to consider only the bending effects for beams, as torsion is not generally predominant. For a beam (loaded predominantly by flexure) two essential requirements must be met to develop its full moment capacity:

1. The elements of the beam (i.e. flange and web) should not buckle locally and
2. The beam as a whole should not buckle laterally.

To ensure that the first condition is met, the cross sections of the flange and the web chosen must be "***plastic***" or "***compact***". (These definitions are explained in the chapter on 'Local buckling' and also in later part of this chapter). If the beam is required to have significant ductility, plastic sections must invariably be used. To avoid the lateral buckling referred to under the second condition, restraints are provided to the beam in the plane of the compression flange, and hence such beams are called ***"laterally restrained beams***". In many steel structures, especially in buildings, beams carry floor decks on top of them, and these floor decks provide restraint to the compression flange. In the absence of any such restraints, and in case the lateral buckling of beams is not accounted for in design, the designer has to provide adequate lateral supports to the compression flange. In this chapter we are concerned with laterally restrained beams, in other words beams which have adequate lateral support to the compression flange. Beams, which buckle laterally, are covered in the next chapter.

2.0 BEHAVIOUR OF STEEL BEAMS

Laterally stable steel beams can fail only by (*a*) flexure (*b*) shear or (*c*) bearing, assuming that local buckling of slender components does not occur. These three conditions are the criteria for Limit State of collapse for steel beams. Steel beams would also become unserviceable due to excessive deflection and this is classified as a limit state of serviceability. In the following sections, we review the fundamentals of these limit states.

2.1 Flexural behaviour of steel beams

It is important to recognise that only plastic sections can be used in ***"plastic design of frames"***, where ***moment redistribution is required throughout the frame***. ***"Plastic analysis"*** of the cross section is confined to the assessment of the ***behaviour of the cross section at the instant of collapse***. These two terms are not to be confused for each other.

If a flexural member is progressively loaded, it deflects and the curvature of such bending varies along its length. Initially the beam is elastic throughout its length. Let us consider a small portion of the beam at a point *A* as shown in Fig.1 (*a*) where the curvature is ρ. If we consider a small segment of the beam at *A* [Fig.1 (*b*)], then the variation of the strain across the depth of the member could be found out geometrically as

$$\varepsilon = \frac{z}{\rho} \tag{1}$$

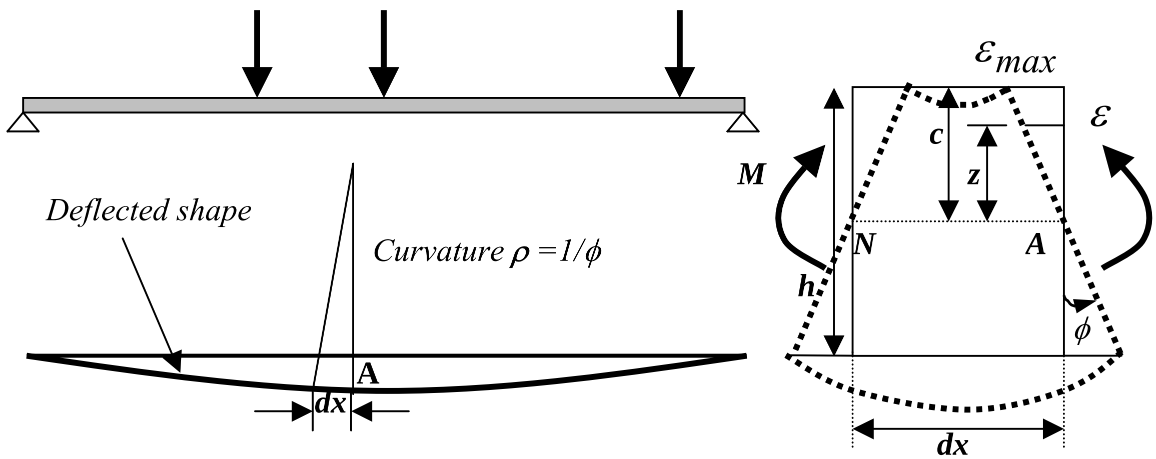


Fig.1 Curvature of bending

From Eq.1, the strain at any fibre is proportional to its distance '*z*' from the neutral axis. This is obtained from the assumption that plane sections which are normal to the longitudinal axis before bending, remains plane and normal even after bending. For each

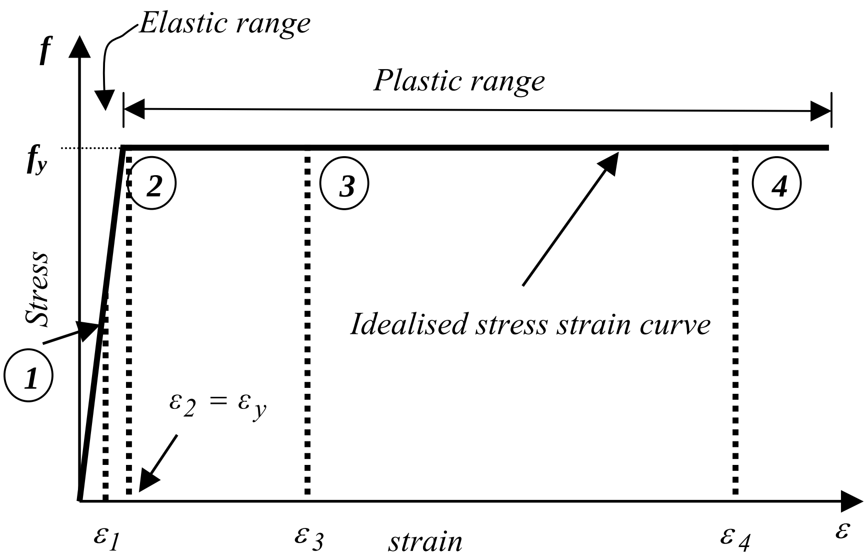


Fig.2 Idealised elasto- plastic stress- strain curve for steel

strain 'ε' one can read off the corresponding stress 'f' from the idealised stress-strain curve for steel shown in Fig. 2. (The idealised stress strain curve neglects the strain-hardening portion for all practical purposes). We choose four points 1, 2, 3, 4 on the stress-strain curve (Fig. 2) for further discussion and see how these four points are used when a simply supported beam is subjected to a mid point load.

2.2 Elastic flexural behaviour

Consider the point (1) in Fig.2 in which the strain $\varepsilon_{max} = \varepsilon_1$ which is less than the yield strain ε_y. At this stage, as seen from Figures 2 and 3, the stress is directly proportional to strain. Hence from elementary Strength of Materials, the corresponding moment of resistance (M_c) is given by

$$M_c = \frac{f_1 I}{c} \tag{2}$$

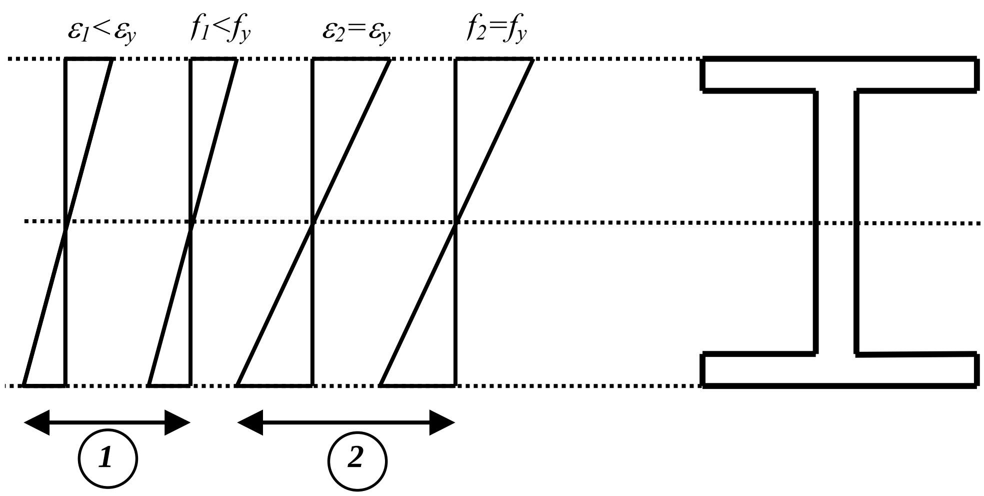


Fig.3 Strain and stress distributions in the elastic range

where 'f_1' is the extreme fibre stress, 'I' is the moment of inertia and 'c' is the extreme fibre distance from the neutral axis. The term $Z_e = I/c$ is the elastic section modulus which is a geometric property of the section. Hence Eq.2 can be rewritten in terms of elastic section modulus as

$$M_c = f_1 Z_e \tag{3}$$

2.3 Yield and plastic moment capacities

Now let us consider the point (2) in Fig. 2. The extreme fibre strain equals yield strain i.e. $\varepsilon_{max} = \varepsilon_2 = \varepsilon_y$ and also the stress $f_2 = f_y$. Where, f_y is the yield stress. Up to this stage, as shown in Fig. 3, the stress and strain are proportional to each other since the extreme fibre of the beam is stressed within the elastic range. The corresponding moment, (M_y), is just sufficient to cause yield in the extreme fibres and is given by

$$M_y = f_y Z_e \qquad (4)$$

Where M_y is called the ***"yield moment"***, i.e. the moment which just causes the extreme fibres to yield. It is evident from Fig. 5(*b*) that once the extreme fibre stresses attain yield stress they no longer take any additional stresses.

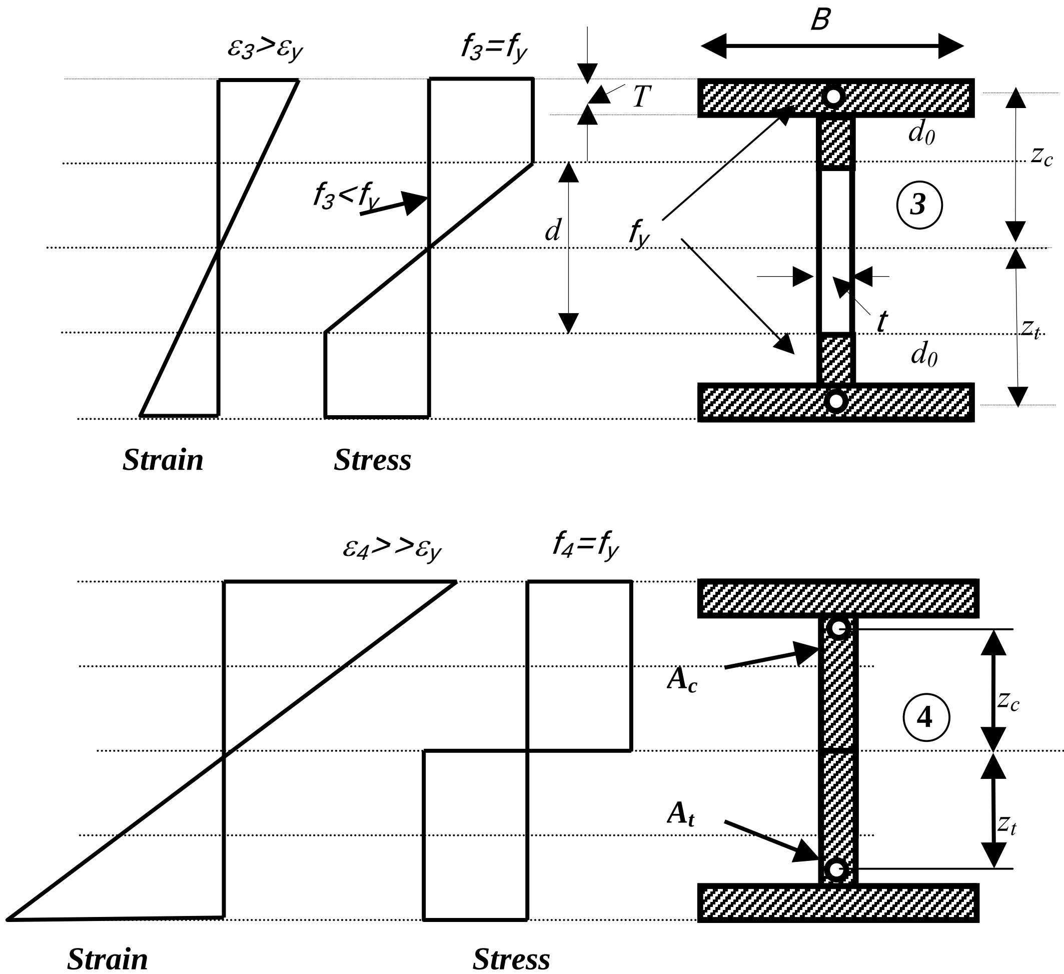


Fig. 4 Strain and stress distributions in plastic range

When the load and hence the moment is further increased, the outermost fibre strain ε_{max} near mid span of the beam (i.e. point of maximum bending moment) would attain a value say, $\varepsilon_3 > \varepsilon_y$ and this is identified as point (3) in Fig. 2. At this stage the strain is in the plastic stage, but extreme fibre stress still equals yield stress f_y. We also note that the stresses have been redistributed to the inner fibres towards the neutral axis and these fibres gradually attain a stress equal to f_y. This is shown in Fig.4. The remaining portion of the beam in the vicinity of the neutral axis is still elastic. At this stage the moment capacity is calculated by considering both the plastic portion and the elastic core as,

$$M_c = \underbrace{f_y (BT + d_0 t)(z_c + z_t)}_{Plastic} + \underbrace{\frac{f_y t d^2}{6}}_{Elastic\ core} \tag{5}$$

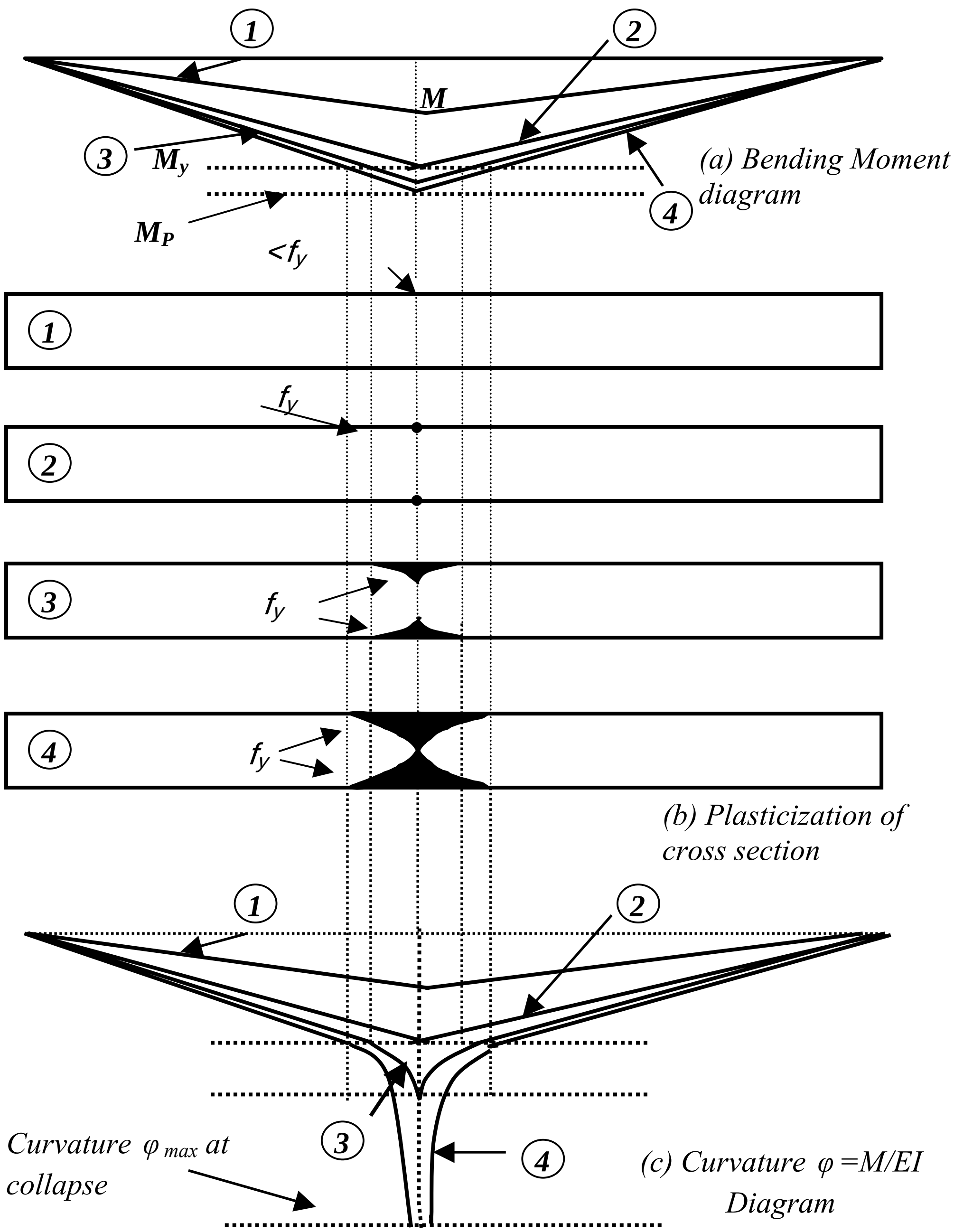


Fig.5 BM diagram and spread of plasticity across the thickness of the beam

Upon further loading, the outer fibre strain increases rapidly and attains a stage shown as point (4) in Fig.2. At this stage the elastic core in the immediate vicinity of the neutral axis becomes negligible due to the spread of plasticity into the fibres near the neutral axis. It is seen from Fig. 5(c) that the curvature φ of the beam (which was proportional to bending moment earlier) increases far more rapidly compared to the previous rate of increase, when the bending moment exceeds the yield moment value $\boldsymbol{M_y}$. When the entire cross section of the beam gets fully plastified, the curvature become infinity as shown in Fig. 5(c). Fig.4 shows such a cross-section, which is fully plastified. This also is shown in Fig.5 where the two yield zones have merged at the neutral axis. When the entire beam cross section becomes plastic, it resists any further rotation under constant moment. At this stage the beam is said to have developed a '***plastic hinge'***. In view of this rotation, deflections become very large and the beam exhibits a kink at the plastic hinge as shown in Fig 7. The magnitude of the bending moment, at which a plastic hinge is formed, is known as the '***plastic moment*** $\boldsymbol{M_p}$'. The moment- curvature relation of the cross section of the beam, at the point of maximum bending moment is shown in Fig. 6. The curvature increases enormously once the moment at the cross section reaches $\boldsymbol{M_P}$. The value of $\boldsymbol{M_p}$ could be easily determined by taking moment of the total tension and compression areas about the plastic neutral axis as

$$M_p = Cz_c + Tz_t = f_y(A_c z_c + A_t z_t) = f_y \frac{A}{2}(z_c + z_t) \tag{6}$$

as shown in Fig.4, where A_c = area under compressive yield stress and A_t = area under tensile yield stress.

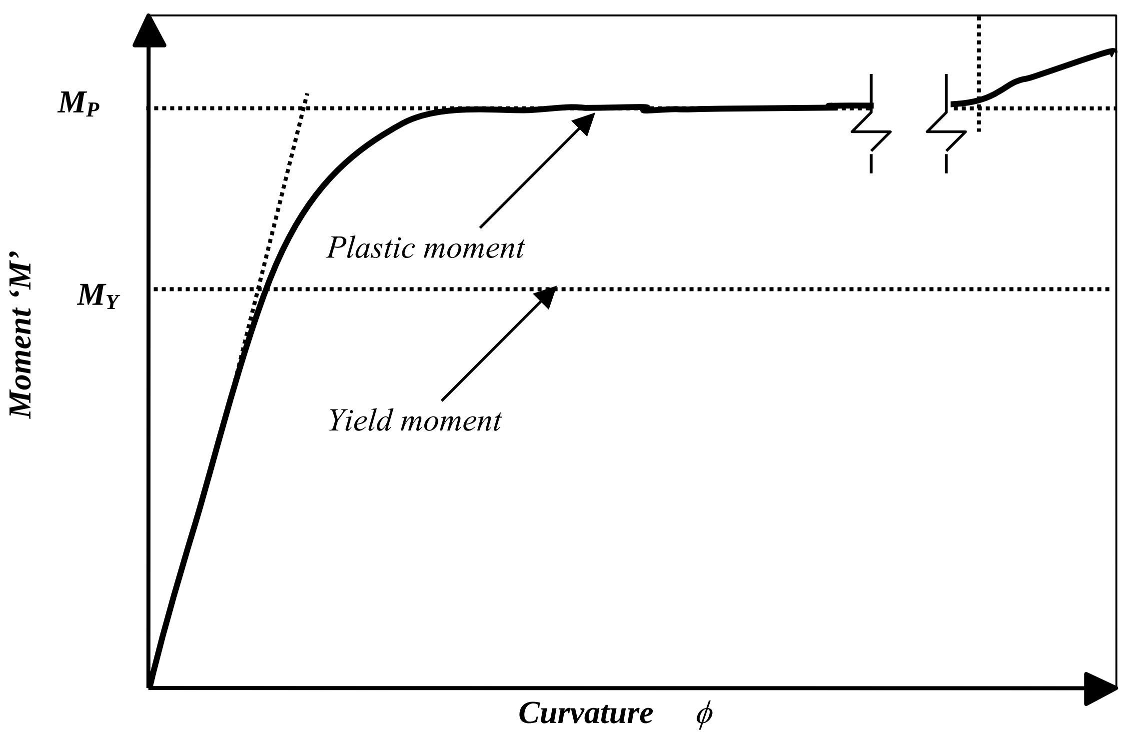


Fig. 6 Moment curvature characteristics of a simply supported beam

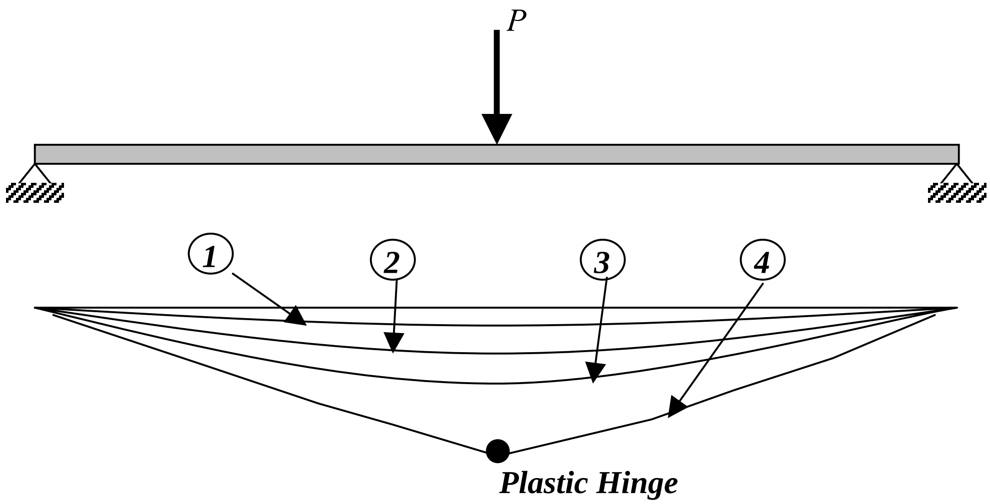


Fig.7 Simply supported beam and its deflection at various stages

In symmetrical sections the neutral axis coincides with the centroidal axis and this is not so in the case of unsymmetrical sections. However the plastic neutral axis for any cross section (also called as "***equal shear axis or equal area axis***") could be located using the condition that the tension and compression areas must be equal as

$$A_c = A_t \tag{7}$$

From Eq.6 it is seen that the plastic section modulus ***(Z_p)*** is given by

$$Z_p = (A_c z_c + A_t z_t) \tag{8}$$

The value of *S* can be obtained as the sum of the moment of the cross sectional areas above and below the plastic neutral axis. The plastic moment capacity of the beam could be written as

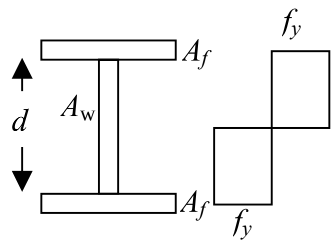
$$M_p = f_y Z_p \tag{9}$$

It is easily verified that for a rectangular section the ratio of the plastic to elastic section modulus called the '***shape factor***'is 1.5. For I-section the ratio varies between 1.07 to 1.20 and for most practical cases of I-section this is taken as 1.12. This ratio also represents the ratio between the plastic moments to the yield moment. For example for an I-beam we can write

$$\frac{Z_p}{Z_e} = 1.12 = \frac{M_p}{M_y} \tag{10}$$

where

This value 1.12 is derived as follows:



$$Z_p = A_f f_y d(1 + A_w / 4A_f)$$
$$Z_e = A_f f_y d(1 + A_w / 6A_f)$$
$$Z_p / Z_e = 1.04 \, for A_f = A_w$$
$$Z_p / Z_e = 1.12 \, for A_f = 0.5A_w$$

3.0 SHEAR BEHAVIOUR OF STEEL BEAMS

Let us take the case of an '***I***' beam subjected to the maximum shear force (at the support of a simply supported beam). The external shear '*V*' varies along the longitudinal axis '*x*' of the beam with bending moment as $V = \frac{dM}{dx}$. While the beam is in the elastic stage, the internal shear stresses τ which resist the external shear *V* can be written as,

$$\tau = \frac{VQ}{It} \qquad (11)$$

where, *V* is the shear force at the section, *I* is the moment of inertia of the entire cross section about the neutral axis, *Q* is the moment about neutral axis of the area that is beyond the fibre at which τ is calculated and '*t*' is the thickness of the portion at which τ is calculated.

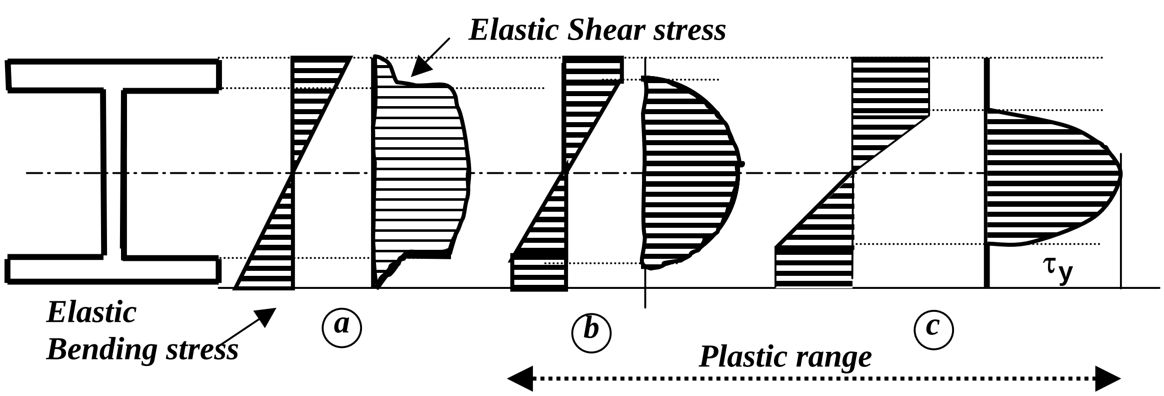


Fig. 8 Combined bending and shear in beams

Eq.11 is plotted in Fig. 8(a), which represents shear stresses in the elastic range. It is seen from Fig. 8(a) that a significant proportion of shear force is carried by the web and the shear stress distribution over the web area is nearly uniform. Hence, for the purpose of design, we can assume without much error that the average shear stress as

$$\tau_{av} = \frac{V}{t_w d_w} \tag{12}$$

where, t_w is the thickness of the web and d_w is the depth of the web. The nominal shear yielding strength of webs is based on the Von Mises yield criterion, which states that for an un-reinforced web of a beam, whose width to thickness ratio is comparatively small (so that web-buckling failure is avoided), the shear strength may be taken as

$$\tau_y = \frac{f_y}{\sqrt{3}} = 0.58 f_y \tag{13}$$

where f_y is the yield stress.

Using Eqns.12 and 13, the shear capacity of rolled beams V_c can be calculated as

$$V_c \approx 0.6 f_y t_w d_w \tag{14}$$

When the shear capacity of the beam is exceeded, the '***shear failure'*** occurs by excessive shear yielding of the gross area of the webs as shown in Fig. 9. Shear yielding is very rare in rolled steel beams.

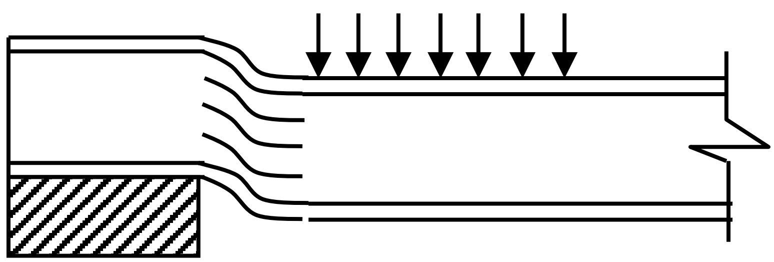
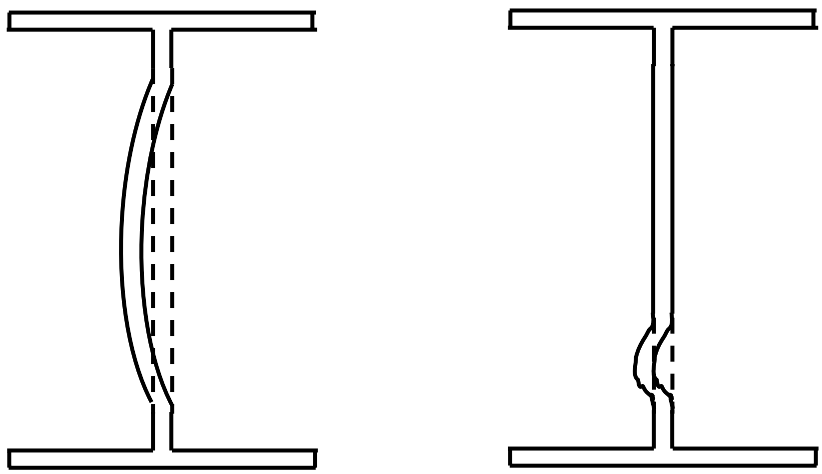


Fig.9 Shear yielding near support

4.0 WEB BUCKLING AND WEB CRIPPLING

The application of heavy concentrated loads produces a region of high compressive stresses in the web either at the support or under the load. This may cause either the web to buckle as shown in the Fig.10 (a) or the web to cripple as shown in Fig.10 (b). In the former case the web may be considered as a strut restrained by the beam flanges. Such ***'idealised struts'*** should be considered at the points of application of concentrated load or reactions at the supports as shown in Fig.11 and Fig.12.



(a) Web buckling *(b) Web crippling*

Fig.10 Local buckling of the web

In both the cases the load is spread out over a finite length of the web as shown in Fig.11. This is known as the '***dispersion length'*** and its theoretical treatment is complex. Hence empirical formulae based on experiments are used. One such assumption is that the dispersion length is taken as $(b_1 + n_1)$ where b_1 is the stiff bearing length and n_1 is the dispersion of 45° line at the mid depth of the section as shown in Fig.12. Hence the web buckling strength at the support is given by

$$P_{wb} = (b_1 + n_1)\ t\ \ f_c \tag{15}$$

where 't' is the web thickness and f_c is the allowable compressive stress corresponding to the assumed "***web strut***". The effective length of the strut is taken as $L_E = 0.7d$ where 'd' is the depth of the ***"strut"*** in between the flanges. The slenderness ratio of the idealised web strut could be written as

$$\lambda = \frac{L_E}{r_y} = \frac{0.7d}{r_y}$$

Since $$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{t^3}{12t}} = \frac{t}{2\sqrt{3}} \tag{16}$$

$$\frac{L_E}{r_y} = 0.7d\,\frac{2\sqrt{3}}{t} \approx 2.5\frac{d}{t}$$

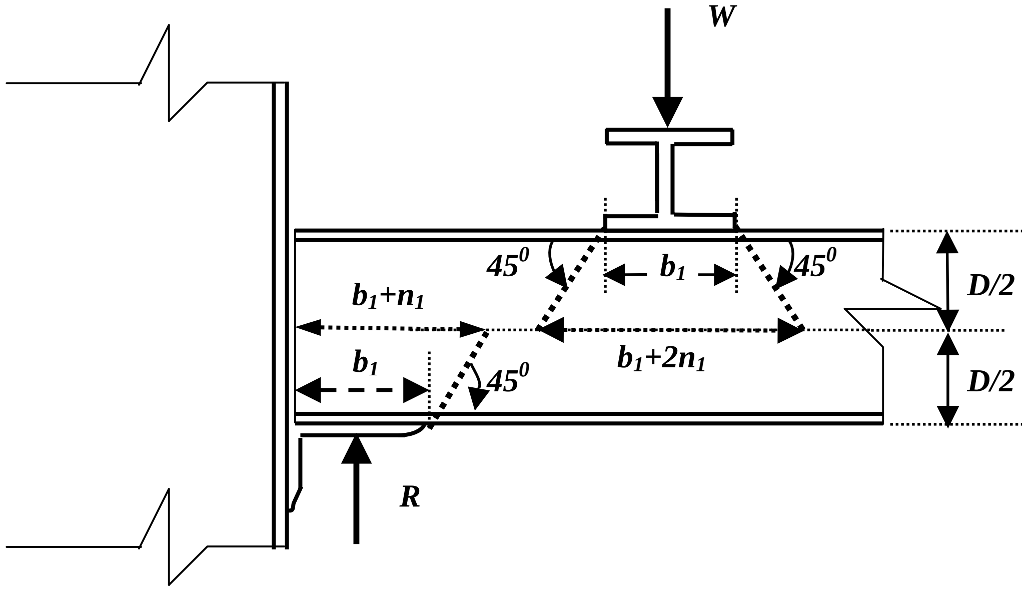


Fig.11 Dispersion of concentrated loads and reactions for evaluating web buckling

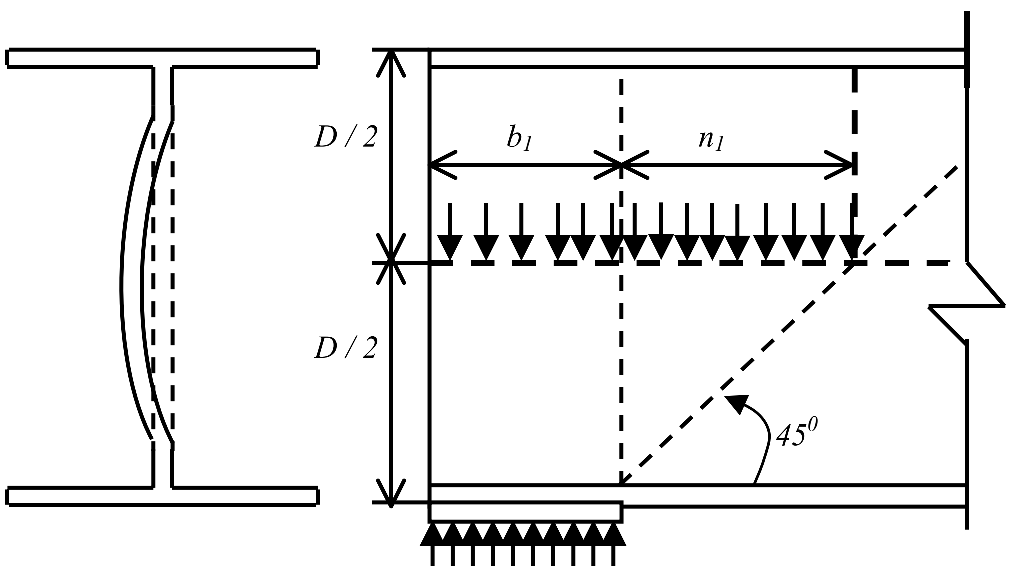


Fig. 12 Effective width for web buckling

Hence, the slenderness ratio of the idealised strut is taken as $\lambda = 2.5d / t$. Similarly the latter case of web crippling could also be calculated assuming a dispersion length of b_1+n_2, where n_2 is the length obtained by dispersion through the flange, to the flange to web connection, at a slope of 1:2.5 to the plane of the flange *(i.e.* $n_2=1.5d$*)* as shown in Fig.13. As before, the crippling strength of the web at supports is calculated as

$$P_{crip} = (b_1 + n_2)\, t\, f_{yw} \tag{17}$$

where f_{yw} is the design yield strength of the web. At an interior point where concentrated load is acting, the crippling strength is given by,

$$P_{crip} = (b_1 + 2n_1)\, t\, f_{yw} \tag{18}$$

5.0 LIMIT STATE OF SERVICEABILITY – DEFLECTION

Although excessive vibration and excessive deflection are both classified as "limit state of serviceability ", the codes usually limit only the deflection. A beam designed to have adequate strength may become unsuitable if it cannot support its loads without excessive deflection. For example, excessive deflection in a floor not only gives a feeling of insecurity, but also damages the non-structural components (such as plaster) attached to it. Excessive deflections in industrial structures often cause misalignment of the supporting machinery and cause excessive vibration. Similarly high deflections in purlins may cause damage to the roofing material. Excessive deflection in the case of flat roof results in accumulation of water during rainstorms called "***ponding***". There are instances reported in the literature where ponding had caused collapse of a flat roof. Hence the deflection in beams are restricted by codes of practice by specifying deflection limitations which are usually in terms of deflection to span ratio.

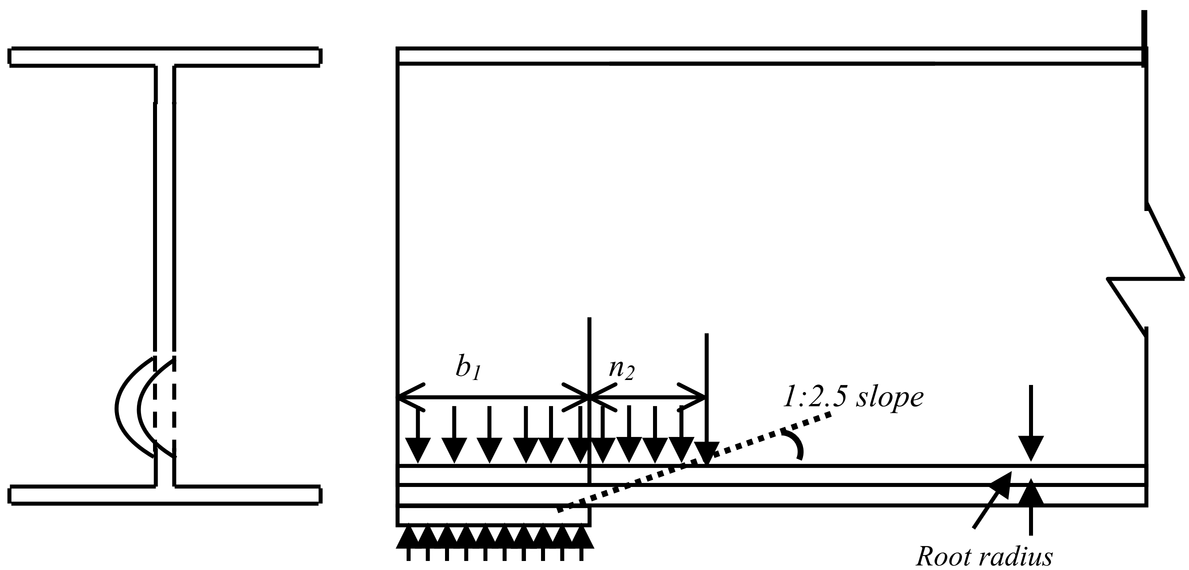


Fig. 13 Effective width of web bearing

In the case of beams (usually considered as simply supported), if the total load 'W', is assumed to be uniformly distributed, then the maximum deflection Δ is given by

$$\Delta = \frac{5}{384}\frac{WL^3}{EI} \tag{19}$$

where '*E*','*I*' are the Young's modulus and moment of inertia of a beam of length '*L*'. Since the maximum moment is $M = WL/8$ we may rewrite the Eq. (19) as

$$\Delta = \frac{5}{48}\frac{ML^2}{EI} \tag{20}$$

Substituting $\frac{M}{I} = \frac{f}{(d/2)}$ (where '*f*' is the extreme fibre flexural stress) into Eq. (20)

we get

$$\Delta = \frac{5}{24}\frac{fL^2}{Ed} \tag{21}$$

Eq. (21) can be used with sufficient accuracy for all practical deflection calculations. Eq.(21) can also be rewritten in terms of *L/d* as

$$\frac{L}{d} = \frac{24}{5}\frac{E}{f}\frac{\Delta}{L} \tag{22}$$

The above equation represents the length/depth ratio of the beam corresponding a specific ratio of deflection to span. As stipulated by the codes of practice, if we restrict the deflection to (say) $\frac{\Delta}{L} = \frac{1}{325}$, using $f = 0.6\, f_y$ (where f_y is the yield stress) we get the necessary $\frac{L}{d}$ ratio as

$$\frac{L}{d} = \frac{24}{5}\ \frac{2\times 10^5}{0.6 f_y}\frac{1}{325} = \frac{4923}{f_y} \tag{23}$$

For an yield stress of $f_y = 250\ MPa$ in the above relation, (*L/d*) ratio works out to approximately 19. In other words, if a beam is chosen for design, whose (*L/d)* value is less than 19, then the deflection criteria would automatically be satisfied. Similarly for a simply supported beam subjected to central concentrated load, the *(L/d)* ratio can be shown to be 24. This (*L/d*) value is only a guiding parameter for satisfying the Limit state of serviceability and is not mandatory in design as long as check for serviceability is separately carried out.

6.0 LIMIT STATE DESIGN OF STEEL BEAMS AS PER IS 800 (LSM VERSION)

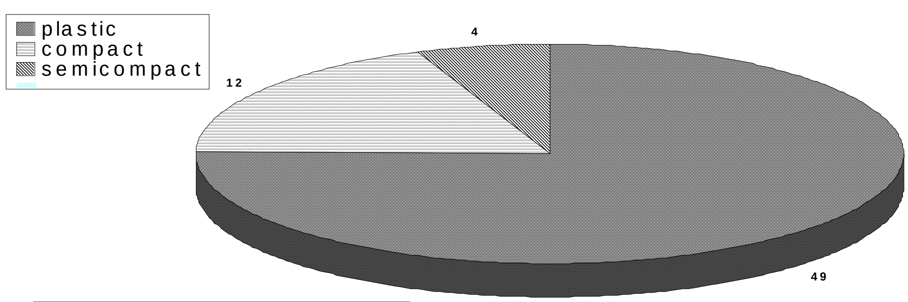
As we have outlined earlier, if the ultimate strength of the steel beams is to be mobilised, we must ensure that local buckling does not cause a premature failure. Hence in limit state design of steel beams, we pay attention to local buckling using what is known as '***section classification***'.

6.1 Concept of section classification

The critical local buckling stress of the constituent plate element of a beam, for a given material and boundary conditions is '***inversely proportional to its breadth to thickness ratio***'. Hence by suitably reducing the slenderness of the plate elements, its resistance to local buckling could be enhanced. Once the local buckling is prevented, the beam can develop its full flexural moment capacity or the limit state in flexure. Hence depending upon the slenderness of the constituent plate element of the beam, they are classified as ***slender, semi-compact, compact and plastic*** as shown in Table 1. This section classification is new to the Indian structural designers who are familiar with the code of practice for structural steelwork in India, the IS: 800 (1984). Since IS: 800(1984) is based on **'*Allowable Stress Method*'**, the extreme fibre stress in the beams is restricted to $\boldsymbol{0.66f_y}$. In addition, the 'I' sections rolled in India are found to be at least semi-compact as shown in Fig.14, in which the section classification for Indian standard *'I'* beams have been presented. In other words the flange outstands of the *'I'* beams rolled in India are so proportioned that they attain yield stress before local buckling. Because of these two reasons, there was no need for section classification in the design of steel beams using IS: 800 (1984). However in the limit state design of steel beams, section classification becomes very essential as the moment capacities of each classified section takes different values, as we will see in the later sections.

6.2 Effect of local buckling in laterally restrained "plastic" or " compact" beams

As mentioned above, laterally restrained *"plastic"* beams while carrying flexural loads sometimes fail to attain their full moment capacity, by the local buckling of the web. The local buckling of slender flanges or slender webs in *"semi-compact"* or *"slender beams",* is discussed in Chapter -8.



PLASTIC	49
COMPACT	12
SEMICOMPACT	4

Fig.14 Section classification of Indian standard rolled 'I' beams

6.3 Moment capacities of laterally restrained beams as per latest IS 800

Depending upon the flange criterion (***b / T***) and web criterion ***(d / t),*** as shown in Table 1, laterally restrained beams could be classified as (a) slender, (b) semi-compact, (c) compact, and (d) plastic sections. The flexural behaviour of such beams are presented in Fig.15. As shown in Fig.15, the section classified as 'slender' can not attain the first yield moment because of a premature local buckling of the web or flange. The next curve represents the beam classified as 'semi-compact' in which the extreme fibre stress in the beam attains yield stress but the beam fails by local buckling before further plastic re-distribution of stress could take place towards the neutral axis of the beam. The moment capacity or the design moment ***(M_d)*** of such beams can be obtained as

$$M_d = M_y = \frac{f_y}{\gamma_{mo}} Z_e \tag{24}$$

Where γ_{mo} is the partial safety factor for the material. In the Indian context γ_{mo} is taken as 1.10.

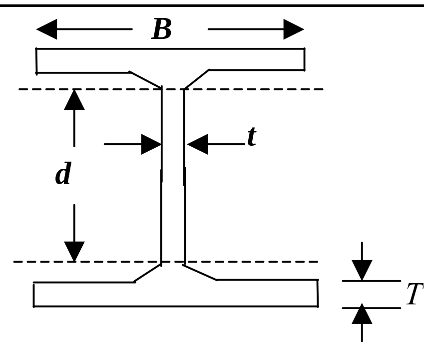
The curve shown as 'compact beam', in which the entire portion, both the compression and tension portion of the beam, attains yield stress. Because of this plastic redistribution of stress, the member has attained its plastic moment capacity ***(M_p)*** but fails by local buckling before developing plastic mechanism by sufficient plastic hinge rotation. The moment capacity of such a section can be calculated as

$$M_d = \frac{f_y}{\gamma_{mo}} Z_p \leq 1.2 \frac{f_y}{\gamma_{mo}} Z_e \tag{25}$$

where $\boldsymbol{Z_p}$ is the plastic section modulus of the cross section. An upper bound value for this moment capacity has been prescribed in codes of practice, to ensure that plasticity does not occur at working loads. (This is done by limiting Z_p value to $1.2\ Z_e$)

Table1: Sectional classification

b = B/2



$$\varepsilon = \sqrt{\frac{250}{f_y}}$$

Section type	Flange criterion (b/T)	Web criterion (d/t)
Slender	> 16 ε	> 126 ε
Semi-compact	11 ε to 16 ε	105 ε to 126 ε
Compact	9 ε to 11 ε	84 ε to 105 ε
Plastic	< 9 ε	< 84 ε

Usually for I-beams the shape factor would be less than 1.2 and only for hollow sections the value of Z_p / Z_e is greater than 1.2. The basic difference between the curves for *'plastic'* and *'compact'* sections lies in the amount of rotation they sustain at the plastic moment. Usually plastic beams sustain larger rotation at the plastic moment to develop full mechanism. The above discussion gives an idea as to how moment capacities of beams vary with different ranges of constituent plate elements as shown in Table 1.

6.4 Combined bending and shear

In *'I'* sections, the flanges predominantly resist the moment and the webs predominantly resist the shear as shown in Fig.8 (a). However, in the case of plastic redistribution of stress over the cross section, the web also is required to contribute to the flexural action as shown in Fig.8 (b)&(c). Hence the shear capacity of the web gets reduced and this becomes very important especially when the web has to carry a relatively high shear and also a high bending moment at the same cross section as in

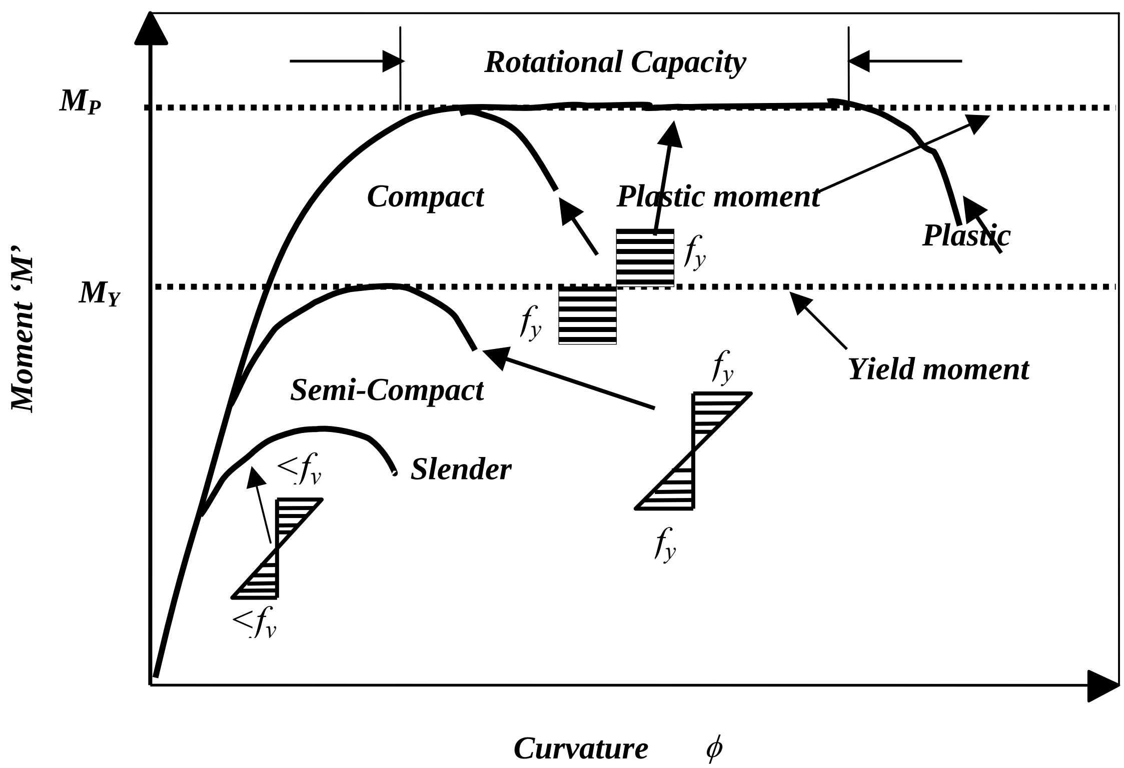


Fig. 15 Flexural member performance using section classification

the case of supports of continuous beams. As larger part of the web yields in flexure, the maximum shear stress in the remaining web reaches the yield stress in shear. To take care of this, the codes specify, that if the external shear load is greater than 0.6 times the shear capacity of the web, then the effect of shear should be considered in the calculation of plastic moment capacity of the cross section. Hence a reduction is applied to the fully plastic moment capacity (M_{dv}) as

$$M_{dv} = M_d - (2V/V_d - 1)^2 (M_d - M_{fd}) < 1.2 \frac{f_y}{\gamma_{mo}} Z \qquad (26)$$

where

M_d = plastic design moment of the whole section disregarding high shear force effect and considering web buckling effects

V = factored applied shear force.

V_d = design shear strength as governed by web yielding or web buckling

M_{fd} = plastic design strength of the area of the cross section excluding the shear area, considering partial safety factor γ_{m0}

7.0 UNSYMMETRICAL BENDING

From elementary Strength of Materials, we know that each beam cross section has a pair of mutually perpendicular axes, known as the principal axes. If bending occurs about any axis other than the principal axis, the plane of loading and plane of bending need not coincide. This is referred to as unsymmetrical bending. When the bending takes place about either of the principal axes, the plane of loading and plane of bending coincide. When loads are applied in an inclined direction (as in the case of purlins), they can be resolved into two components perpendicular to the principal axes, as shown in Fig.16, and the moment components M_x and M_y can be calculated. Thereafter it is a simple matter of calculating the two bending stresses separately and algebraically adding them.

7.1 Symmetrical sections

In the elastic design, we can write the resolved components as

$$f_x + f_y \le p_b \qquad (27)$$

where f_x and f_y are the maximum bending stresses at the cross section and p_b is the permissible bending stress. We must be careful when dealing with sections such as angles for which the principal axes are <u>not</u> the geometric axes (*i.e. x and y-axes*).

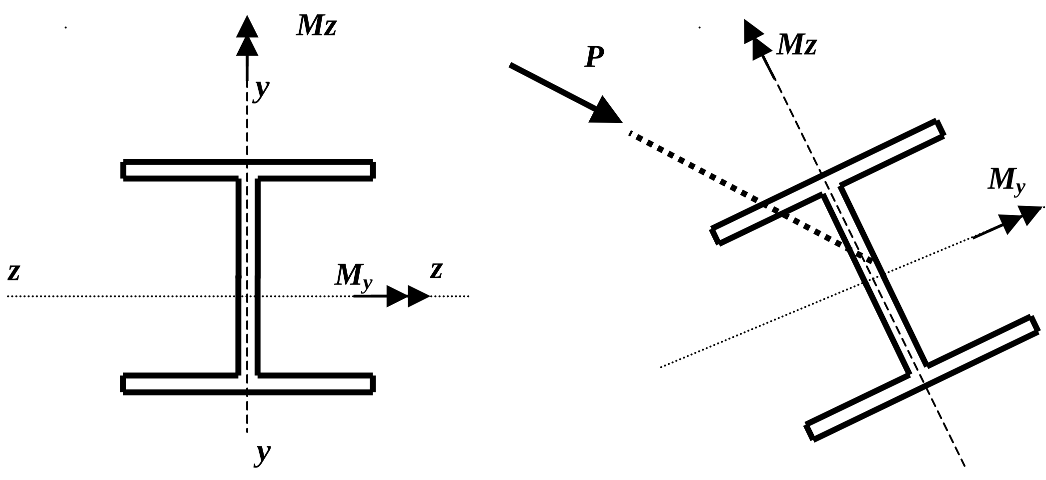


Fig 16 Unsymmetrical bending

When using the plastic strength of the cross section (in the case of ***'plastic'*** sections) the interaction between moment M_x and M_y will depend on the geometry of the cross section. As an illustration, IS:800 (2007), provides an interaction equation as

$$\left(\frac{M_z}{M_{cz}}\right)^{z_1} + \left(\frac{M_y}{M_{cy}}\right)^{z_2} \leq 1.0 \qquad (28)$$

where M_{cz} , M_{cy} are the moment resistance of the cross section about the x and y axes. z_1 and z_2 depend on the geometry of the cross section. Safe values of $z_1=z_2=1.0$ can be used as a first approximation.

Similarly IS: 800 states that for section under bi-axial bending along with axial compression the following equation needs to be satisfied.

$$\frac{N}{N_d} + \frac{M_z}{M_{dz}} + \frac{M_y}{M_{dy}} \leq 1.0 \qquad (29)$$

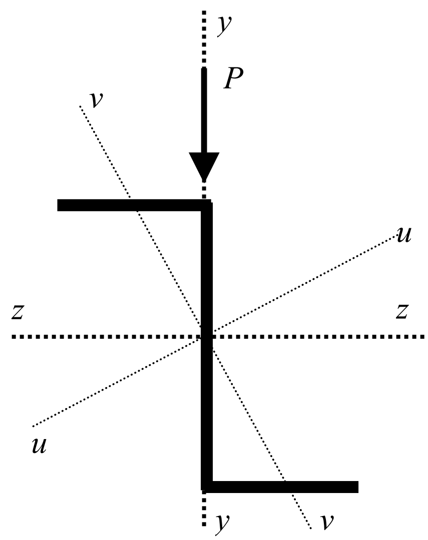
Now at zero axial compression the above equation will reduce to

$$\frac{M_y}{M_{dy}} + \frac{M_z}{M_{dz}} \leq 1.0 \qquad (30)$$

Where,
M_{dy}, M_{dz} = design strength under corresponding moment acting alone along y and z axes respectively (z-axis is equivalent of x-axis as stated above)

7.2 Unsymmetrical sections

In the previous section we described the bending of symmetrical sections, which undergo unsymmetrical bending due to inclined application of loads with respect to the principal axes. There are instances where a vertical load parallel to the x-axis could cause unsymmetrical bending, such as angles and *'Z'* sections. As shown in Fig.17, the principal axes of these sections ***u-u*** and ***v-v,*** do not coincide with the orthogonal ***x-x*** and ***y-y*** axes.



(a) Point symmetry

Fig.17 Z-section prone to unsymmetrical bending

In such cases, the same simplification as in the case of symmetrical sections can be used. However the points of maximum stresses, $f_{x,max}$ and $f_{y,max}$, may not occur at the same point. Hence the maximum stresses $f_{x,max}$ and $f_{y,max}$ must be calculated at various points. After superposition of these two stresses, the maximum value of stress, of all the points in the cross section, has to be used in the design.

7.3 Influence of plane of loading on the flexural behaviour of steel beams

When the load is applied through the centroid, (Fig.18) in the case of the I beam it deflects in the direction of the load. The channel section deflects straight down with a twist. For bending to occur without the twisting of such cross sections, the load must be applied through the '***shear centre'*** of the cross section. Shear Centre may be defined as a point through which load must pass so that twisting of the cross section does not occur during bending. This is exemplified in Fig.19, in which the section undergoes bending without twist when load is applied through the shear centre. If a cross section contains an axis of symmetry, its shear centre lies on that axis. If the cross section is symmetric about two axes or it is point symmetric, then shear centre coincides with the centroid. If the section has two elements joined together (e.g. angles) then the shear centre is at the juncture of the two elements.

Many times we encounter steel sections such as crane girders, which do not have two axes of symmetry. Such steel sections, which are prone to bending with twist, could be made to bend in a desired plane by providing physical constraints as shown in Fig.20. Such behaviour is called the '***constrained bending'***

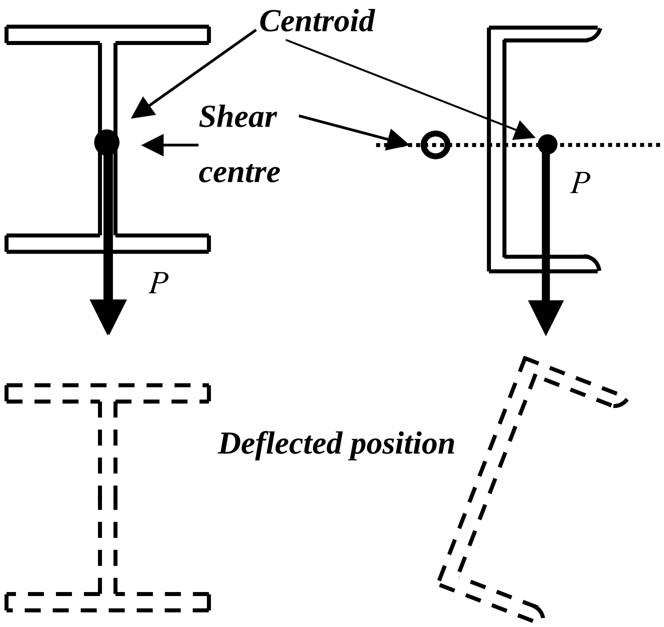


Fig:18 Deflection of beams loaded through the centroid

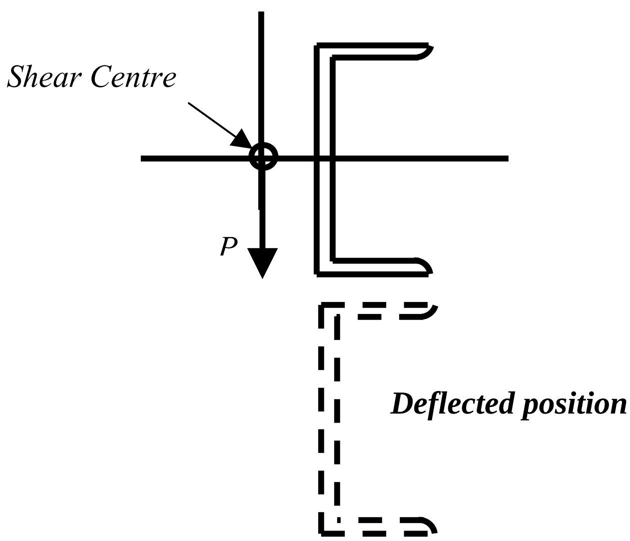


Fig.19 Deflection of channel beam loaded through the shear centre

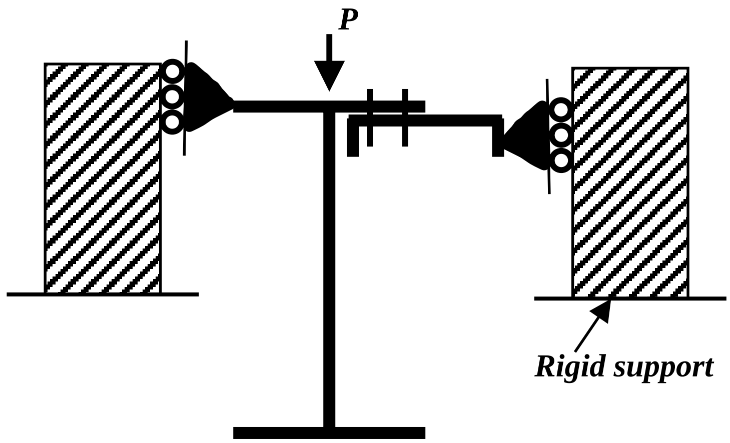


Fig.20 Constrained bending

8.0 BUILT-UP BEAMS

For many steel structures, beams may be provided from among the standard range of rolled steel sections. However, situation may arise when none of the available sections has sufficient moment capacity or there may be a restriction on the depth of the beam due to architectural considerations. Such situations may also occur when it is necessary to provide beam for longer spans or to support a heavy load. Gantry girders are the best example of such cases and strengthening of existing beams is also another example. One of the solutions to such a situation is to use a built-up section as shown in Fig.21. Consider for example, the cover-plated beam as shown in Fig.21 (a). The moment of inertia of the built-up beam is increased compared to the individual rolled section. Neglecting the moment of inertia of the added plate about its own 'x' axis, there would be an increase in moment of inertia of approximately $\boldsymbol{A(d/2)^2}$ for every plate added to the rolled beam. For the section shown in Fig.21 (a), the moment of inertia of the built-up section I_b is written as

$$I_b \approx I_s + 2A\left(\frac{d}{2}\right)^2 \qquad (29)$$

where I_S is the moment of inertia of the rolled section. It is more convenient to work in terms of section modulus than moment of inertia. The approximate value of section modulus Z_b (since $d/2$ is not the extreme fibre distance) for the built up section shown in Fig.21 (a) could be written as

$$Z_b \approx Z_s + \frac{2A(d/2)^2}{d/2} = Z_s + Ad \qquad (30)$$

where Z_S is the section modulus of the rolled section. The above expression helps in estimating the cover plate area required (although the exact value of Z_b must be verified by calculation, particularly when one plate is added to the top flange). If by design considerations, only one cover plate is to be added to rolled beam (to reduce fabrication cost), then this plate should be fastened to the compression flange. However, if the thickness of a cover plate added to only one flange exceeds about 1.5 times the thickness of the flange of the rolled section, then adding a cover plate to both the flanges is structurally more efficient. All the outstands of the cover plate is to be checked for its slenderness so as to eliminate the possibility of local buckling. Whenever one or more plates are added to form the built-up section usually the slenderness of individual plates should be considered.

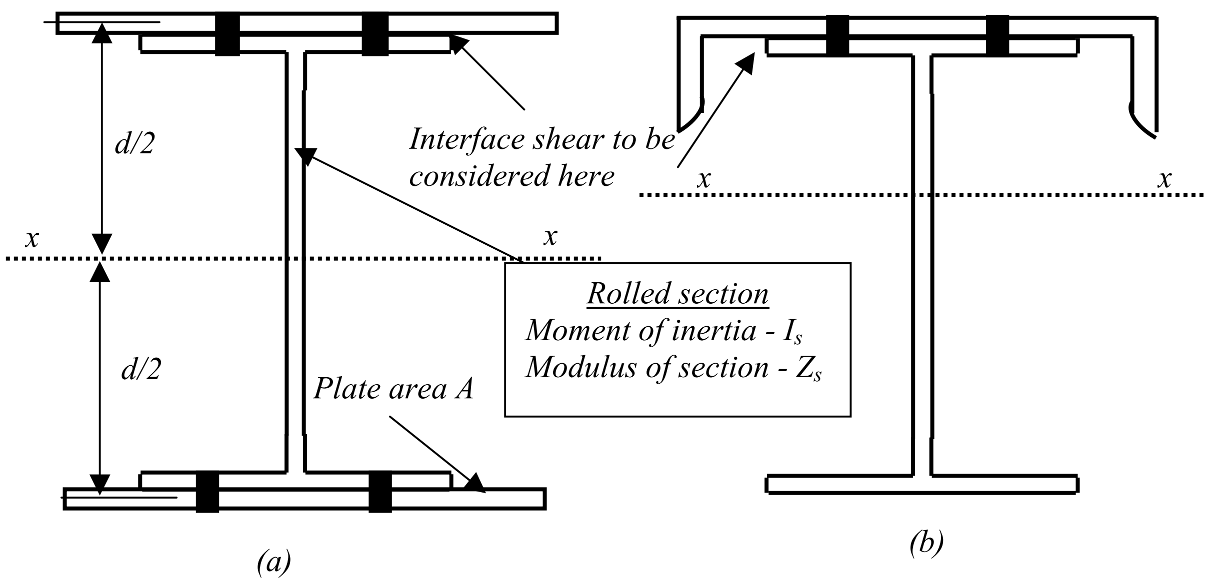


Fig.21 Example of built –up beams

The cover plate and the rolled beam should be adequately connected with welding or bolting, so as to satisfactorily transfer the interface shear between beam and plate. The longitudinal spacing of these welds or bolts must be sufficiently close so as to avoid the plate in the compression flange buckling as an individual strut between the intermittent fasteners. For connecting the cover plate and the rolled beam bolting or welding may be required. In the case of bolting, holes in the flanges become inevitable. These holes cause reduction in the flange area in the tension side. However experimental work on flexure of cover plated steel beams has shown that the failure is based primarily on the strength of the compression flange even though there are bolt holes in the tension side. The presence of these holes does not seem to be serious. Based on this reason, AISC (American Institute of Steel Construction) code suggests that no subtraction for holes need to be made for flange area, if the area of the holes is not more than 15% of the gross area of the flange. However many codes of practice have adopted the conservative procedure of accounting for reduction in flange area due to bolt holes. Likewise IS: 800 (LSM version) has laid down the a criterion which requires to be taken into account for holes in the tension zone of a beam. As per this code,

$$(A_{nf} / A_{gf}) \geq (f_y/f_u)\,(\gamma_{m1}/\gamma_{m0}) / 0.9 \qquad (31)$$

Where

A_{nf} / A_{gf} = ratio of net to gross area of the flange
f_y/f_u = ratio of yield and ultimate strength of the material
γ_{m1}/γ_{m0} = ratio of partial safety factors against ultimate to yield stress

When the A_{nf}/A_{gf} does not satisfy the above requirement, the reduced flange area, A_{nf} satisfying the above equation may be taken as the effective flange area in tension.

In practice, the stresses are worked out initially disregarding the reduction in the tension flange area due to holes. Actual tensile stress is obtained by multiplying the stresses calculated as above, by the ratio of gross to the net area (deduction made for tension only) of the respective flange sections. The flange is taken as the flange area of the rolled section and the area of the cover plate.

For the integral action of the rolled beam and the cover plate the interface shear must be adequately transferred. Using Eq.11 the longitudinal shear per unit length to be resisted by these bolts or weld could be written as

$$v = \frac{VQ}{I} \qquad (32)$$

where ***V,Q*** and ***I*** are defined in Eq.11. Using staggered bolts of bolt value '*R*' the staggered pitch of the bolts of the connection between plate and rolled beam could be calculated as

$$p = \frac{R}{(VQ / I)} \qquad (33)$$

where '***p***' is the pitch of the staggered bolts. The bolts must be spaced not less than 2.5 times diameter of the hole. The maximum spacing is 32 times the thickness of the plate or 300 mm whichever is less. In the case of welded cover plated beams, no weakening of the tension flange need be considered.

8.1 Curtailment of cover plates

The cover plate will be necessary in the middle portion of the beam where the bending moment is high. Towards the supports, the moment capacity M_c of the rolled section alone would be sufficient to resist the external bending moment. Hence in such portions, the flange plate may be cut off as shown in Fig. 22.

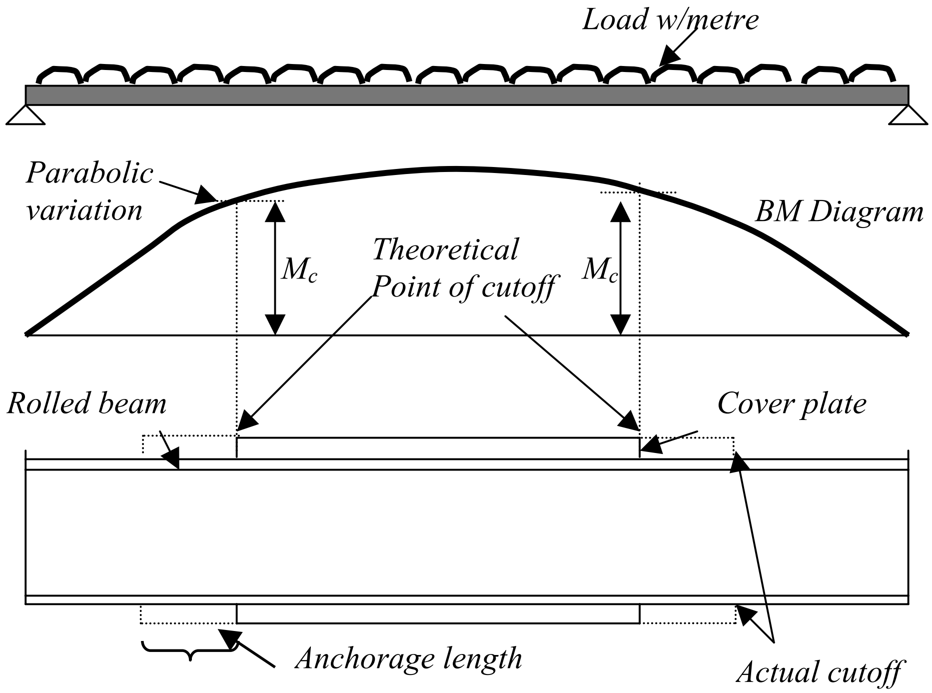


Fig.22 Curtailment of cover plate

Theoretically, the cut off point is the section at which external bending moment is equal to moment capacity of rolled section. However, in practice they are extended further, in order to accommodate bolts or welds and to develop the force in the plate for the bending moment at the point of cut off or in other words to provide anchorage length.

9.0 SUMMARY

In this chapter the fundamentals of the behaviour of laterally restrained beams have been brought out. The limit states of steel beams are discussed. The section classification of beams has been introduced with respect to flexural behaviour of steel beams. Design aspects of built-up beam have also been presented. A worked example illustrates the concept of Limit state Design as applied to beams.

10.0 FURTHER READING

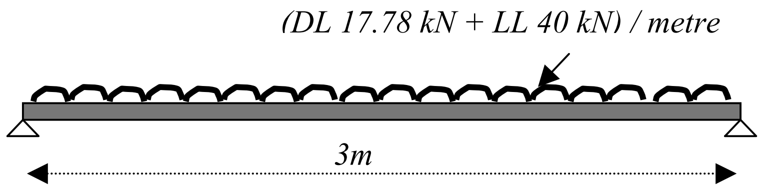
1. David Nethercot, "Limit State Design of Structural Steelwork", Van Nostrand Reinhold, (1986).
2. Introduction to Steelwork Design to BS:5950 Part I, The Steel Construction Institute, Ascot, UK (1988).
3. Samuel H. Marcus, "Basis of Structural Steel Design", Reston Publishing Co., Virginia, (1977).

Structural Steel Design Project	Job No. Ex .1	Sheet 1 of 4	Rev.
	Job. Title: *LATERALLY RESTRAINED BEAMS*		
	Worked Example -1		
	Made by *SAJ*	Date *21.03.2000*	
Calculation Sheet	Checked by *SS*	Date *26.03.2000*	

EXAMPLE: 1

Design a suitable 'I' beam for a simply supported span of 3 m and carrying a dead or permanent load of 17.78 kN/m and an imposed load of 40 kN/m. Assume full lateral restraint and stiff support bearing of 100 mm.

$\gamma_{LD} = 1.50$
$\gamma_{LL} = 1.50$



Design load calculation:

factored load = $\gamma_{LD} \times 17.78 + \gamma_{LL} \times 40$ *kN*

in this example the following load factors are chosen.

γ_{LD} *and* γ_{LL} *are taken as 1.50 and 1.50 respectively.*

γ_{LD} – *partial safety factor for dead or permanent loads*
γ_{LL} – *partial safety factor for live or imposed loads*

Total factored load = $1.50 \times 17.78 + 1.50 \times 40.0 = 86.67$ *kN / m*

Factored bending moment = $86.67 \times 3^2 / 8 = 97.504$ *kN – m*

Z—value required for $f_y = 250$ *MPa* ; $\gamma_m = 1.10$

$$Z_{reqd} = \frac{97.5 \times 1000 \times 1000 \times \gamma_m}{250}$$

$Z_{reqd} = 429.02 \ cm^3$

Structural Steel Design Project	Job No. Ex .1	Sheet 2 of 4	Rev.
	Job. Title: *LATERALLY RESTRAINED BEAMS*		
	Worked Example - 1		
Calculation sheet	Made by *SAJ*	Date *21.03.2000*	
	Checked by *SS*	Date *26.03.2000*	

Try ISMB 250

$$\varepsilon = \sqrt{\frac{250}{250}} = 1.0 \qquad D = 250\ mm$$

$B = 125\ mm$

$t = 6.9\ mm$

$T = 12.5\ mm$

$I_{zz} = 5131.6\ cm^4$

$I_{yy} = 334.5\ cm^4$

Section classification:

Flange criterion $= B/2T = 5.0$

Web criterion $= (D - 2T)/t = 32.61$

Since $B/2T < 9.4\ \varepsilon$ & $(D-2T)/t < 83.9\ \varepsilon$

*The section is classified as ' **PLASTIC** '*

Moment of resistance of the cross section:

Since the section considered is 'PLASTIC'

$$M_d = \frac{Z_p \times f_y}{\gamma_m}$$

Where Z_p *is the plastic modulus*

'Z_p' *for ISMB 250* $= 459.76\ cm^3$

$M_d = 459.76 \times 1000 \times 250 / 1.10$

$= 104.49\ kN\text{-}m > 97.504\ kN\text{-}m$

*Hence ISMB-**250** is adequate in flexure.*

Structural Steel Design Project **Calculation sheet**	Job No. Ex .1	Sheet *3 of 4*	Rev.
	Job. Title: *LATERALLY RESTRAINED BEAMS*		
	Worked Example- 1		
	Made by *SAJ*	Date *21.03.2000*	
	Checked by *SS*	Date *26.03.2000*	

Shear resistance of the cross section:

This check needs to be considered more importantly in beams where the maximum bending moment and maximum shear force may occur at the same section simultaneously, such as the supports of continuous beams. For the present example this checking is not required. However for completeness this check is presented.

Shear capacity $V_c = \dfrac{0.6 f_y A_v}{\gamma_m}$

$A_v = 250 \times 6.9 = 1725\ mm^2$

$V_c = 0.6 \times 250 \times 1725 / 1.10 = 235.3 kN$

V = factored max shear $= 86.67 \times 3 / 2 = 130.0\ kN$

$V / V_c = 130/235.3 = 0.55 < 0.6$

Hence the effect of shear need not be considered in the moment capacity calculation.

Check for Web Buckling:

The slenderness ratio of the web $= L_E/r_y = 2.5\ d/t = 2.5 \times 194.1/6.9$

$= 70.33$

The corresponding design compressive stress f_c *is found to be*

$f_c = 203\ MPa$ *(Design stress for web as fixed ended column)*

Stiff bearing length = 100 mm

45^0 *dispersion length* $n_1 = 125.0\ mm$

$P_w = (100 + 125.0) \times 6.9 \times 203.0$

$= 315.16\ kN$

$315.16 > 126$ *Hence web is safe against shear buckling*

Structural Steel Design Project	Job No. Ex .1	Sheet *4 of 4*	Rev.
	Job. Title: *LATERALLY RESTRAINED BEAMS*		
	Worked Example - 1		
	Made by *SAJ*	Date *21.03.2000*	
Calculation sheet	Checked by *SS*	Date *26.03.2000*	

Check for web crippling at support

Root radius of ISMB 250 = 13 mm

Thickness of flange + root radius = 25.5 mm

Dispersion length (1:2.5) $n_2 = 2.5 \times 25.5 = 63.75$ *mm*

$P_{crip} = (100+63.75) \times 6.9 \times 250 / 1.15$

$= 245.63\ kN > 126 kN$

Hence ISMB 250 has adequate web crippling resistance

Check for serviceability – Deflection:

Load factors for working loads γ_{LD} *and* $\gamma_{LL} = 1.0$

design load = 57.78 kN/m.

$$\delta = \frac{5 \times 57.78 \times 3000^4}{384 \times 2.1 \times 10^5 \times 5131.6 \times 10^4}$$

$$= 5.65\ mm$$

Max deflection $= \dfrac{L}{531}$

$$\frac{L}{531} < \frac{L}{200}$$

Hence serviceability is satisfied

Result: -- Use ISMB – 250.

10 DESIGN OF AXIALLY LOADED COLUMNS

1.0 INTRODUCTION

In an earlier chapter, the behaviour of practical columns subjected to axial compressive loading was discussed and the following conclusions were drawn.

- Very short columns subjected to axial compression fail by yielding. Very long columns fail by buckling in the Euler mode.
- Practical columns generally fail by inelastic buckling and do not conform to the assumptions made in Euler theory. They do not normally remain linearly elastic upto failure unless they are very slender
- Slenderness ratio (λ/r) and material yield stress (f_y) are dominant factors affecting the ultimate strengths of axially loaded columns.
- The compressive strengths of practical columns are significantly affected by (i) the initial imperfection (ii) eccentricity of loading (iii) residual stresses and (iv) lack of defined yield point and strain hardening. Ultimate load tests on practical columns reveal a scatter band of results shown in Fig. 1. A lower bound curve of the type shown therein can be employed for design purposes.

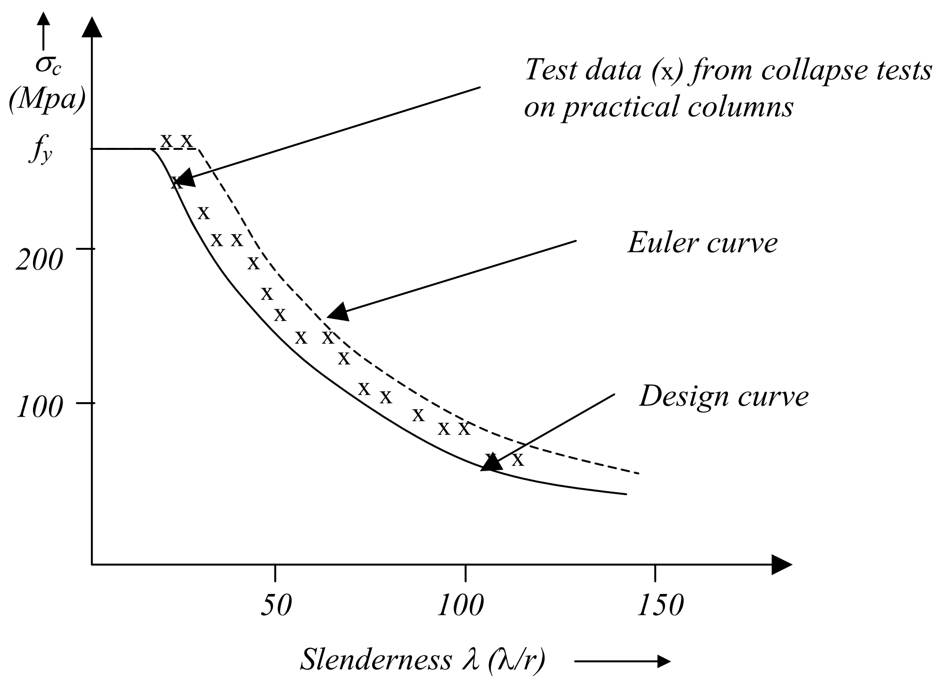


Fig. 1 Typical column design curve

2.0 HISTORICAL REVIEW

Based on the studies of Ayrton & Perry (*1886*), the British Codes had traditionally based the column strength curve on the following equation.

$$(f_y - \sigma_c)\ (\sigma_e - \sigma_c) = \eta \cdot \sigma_e \cdot \sigma_c \tag{1}$$

where

f_y = yield stress

σ_c = compressive strength of the column obtained from the positive root of the above equation

σ_e = Euler buckling stress given by $\dfrac{\pi^2 E}{\lambda^2}$ (1a)

η = a parameter allowing for the effect of lack of straightness and eccentricity of loading.

λ = Slenderness ratio given by (λ/r)

In the deviation of the above formula, the imperfection factor η was based on

$$\eta = \frac{y \cdot \Delta}{r^2} \tag{2}$$

where y = the distance of centroid of the cross section to the extreme fibre of the section.

Δ = initial bow or lack of straightness

r = radius of gyration.

Based on about *200* column tests, Robertson (*1925*) concluded that the initial bow (Δ) could be taken as *length of the column/1000* consequently η is given by

$$\eta = \left(0.001\,\frac{y}{r}\right) . \left(\frac{\lambda}{r}\right)$$

$$= \alpha\left(\frac{\lambda}{r}\right) = \alpha \cdot \lambda \tag{3}$$

where α is a parameter dependent on the shape of the cross section.

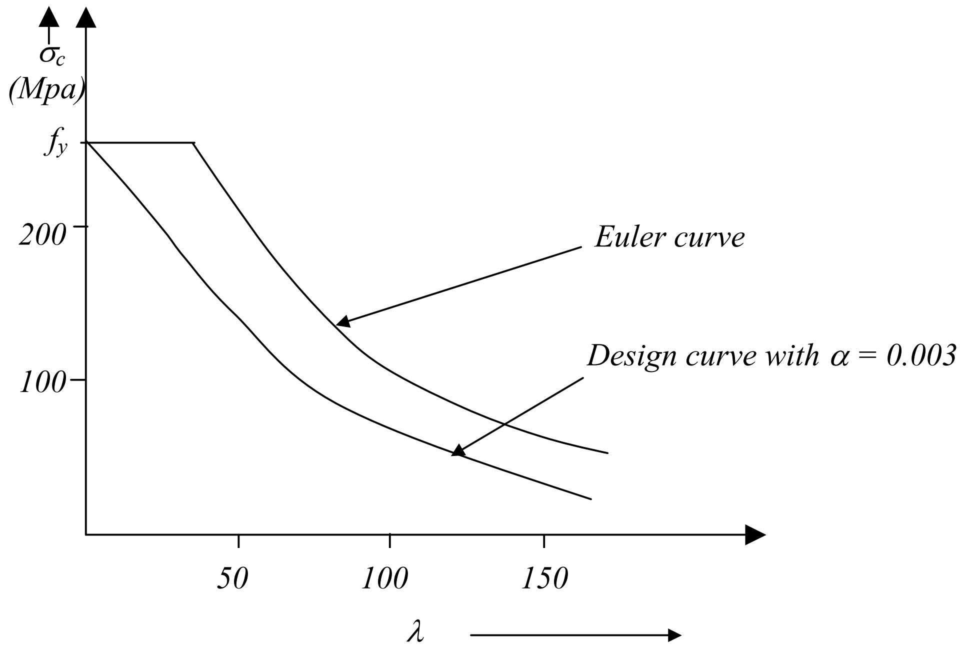


Fig.2 Robertson's Design Curve

Robertson evaluated the mean values of α for many sections as given in Table 1:

Table1: α values Calculated by Robertson

Column type	***α Values***
Beams & Columns about the major axis	0.0012
Rectangular Hollow sections	0.0013
Beams & Universal columns about the minor axis	0.0020
Tees in the plane of the stem	0.0028

He concluded that the lower bound value of $\alpha = 0.003$ was appropriate for column designs. This served as the basis for column designs in Great Britain until recently. The design curve using this approach is shown in Fig. 2. The Design method is termed "Perry-Robertson approach".

3.0 MODIFICATION TO THE PERRY ROBERTSON APPROACH

3.1 Stocky Columns

It has been shown previously that very stocky columns (e.g. stub columns) resisted loads in excess of their squash load of $f_y.A$ (i.e. theoretical yield stress multiplied by the area of the column). This is because the effect of strain hardening is predominant in low values of slenderness *(λ)*. Equation (1) will result in column strength values lower than f_y even in very low slenderness cases. To allow empirically for this discrepancy, recent British and European Codes have made the following modification to equation 3 given by

$\eta = \alpha\ \lambda$

In the unmodified form this will cause a drop in the calculated value of column strength even for very low values of slenderness. Such columns actually fail by squashing and there is no drop in observed strengths in such very short columns. By modifying the slenderness, λ to $(\lambda - \lambda_0)$ we can introduce a plateau to the design curve at low slenderness values. In generating the British Design (BS: 5950 Part-1) curves

$\lambda_0 = 0.2\left(\pi\ \sqrt{E/f_y}\right)$ was used as an appropriate fit to the observed test data, so that we

obtain the failure load (equal to squash load) for very low slenderness values. Thus in calculating the elastic critical stress, we modify the formula used previously as follows:

$$\sigma_e = \frac{\pi^2 E}{(\lambda - \lambda_0)^2}\ \textit{for all values of}\ \lambda > \lambda_0 \tag{4}$$

Note that no calculations for σ_e is needed when $\lambda \le \lambda_e$ as the column would fail by squashing at f_y.

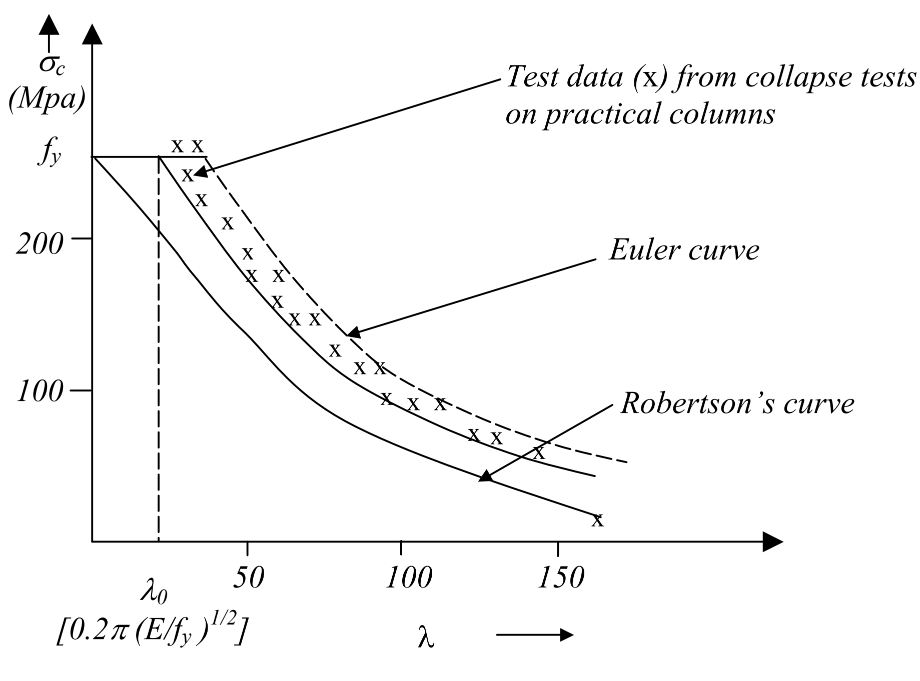


Fig.3 Strut curve with a plateau for low slenderness values

3.2 Influence of Residual Stresses

Reference was made earlier to the adverse effect of locked-in residual stresses on column strengths (see Fig. 4). Studies on columns of various types carried out by the European Community have resulted in the recommendation for adopting a family of design curves rather than a single "*Typical Design Curve*" shown in Fig. 3. Typically four column curves are suggested in British and European codes for the different types of sections commonly used as compression members [See Fig. 5(*a*)]. In these curves, $\eta = \alpha(\lambda - \lambda_0)$

where $\lambda_0 = 0.2\pi\sqrt{E/f_y}$ and the α values are varied corresponding to various sections.

Thus all column designs are to be carried out using the strut curves given in [Fig. 5(*a*)].

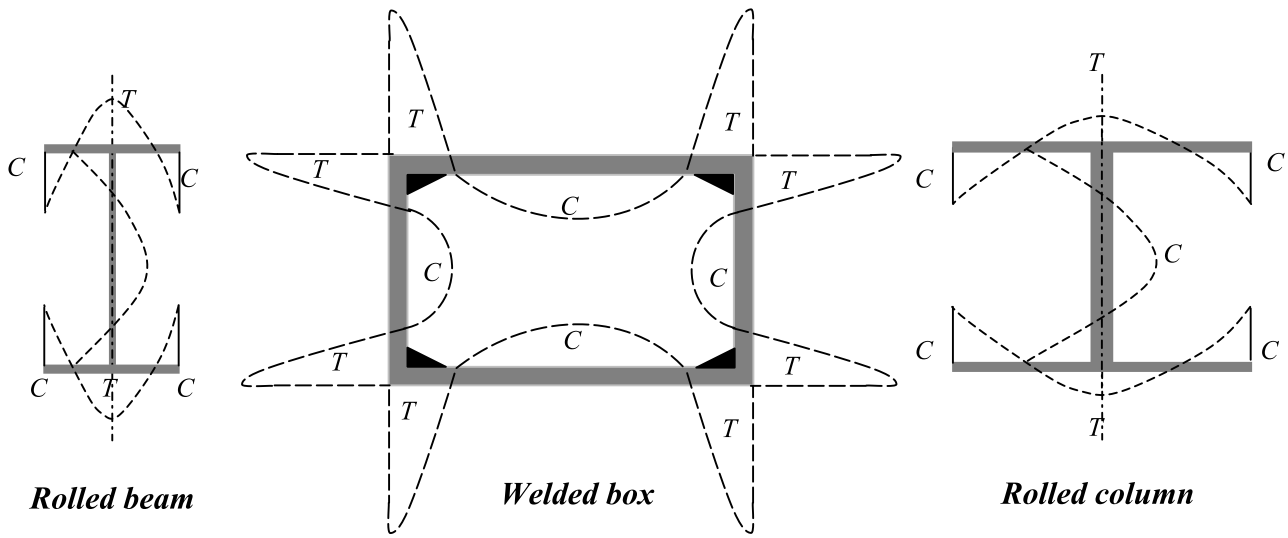


Fig. 4 Distribution of residual stresses

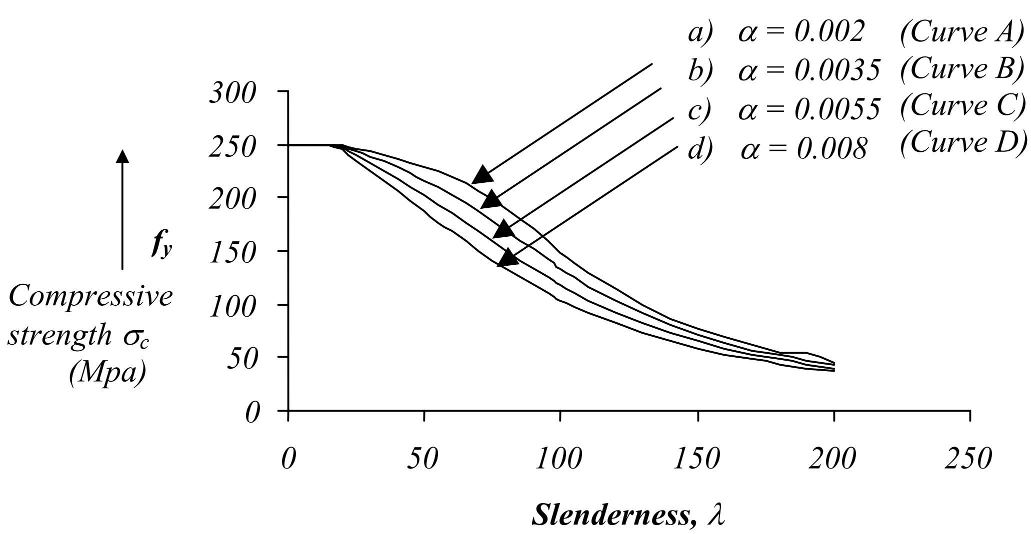


Fig. 5(a) Compressive strength curves for struts for different values of α

The selection of an appropriate curve is based on cross section and suggested curves are listed in Table 2.

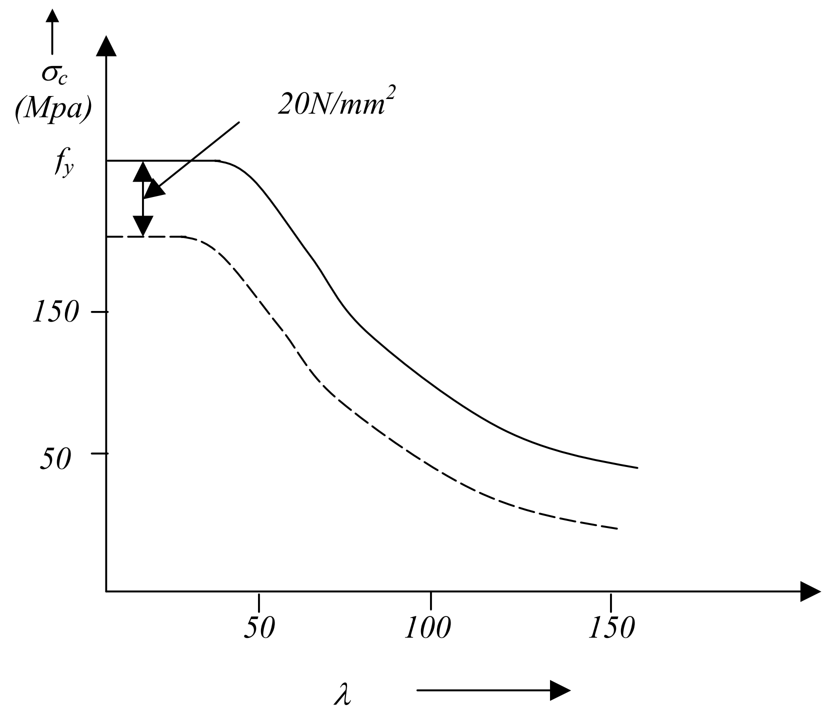


Fig. 5(b) Compressive strength of welded sections

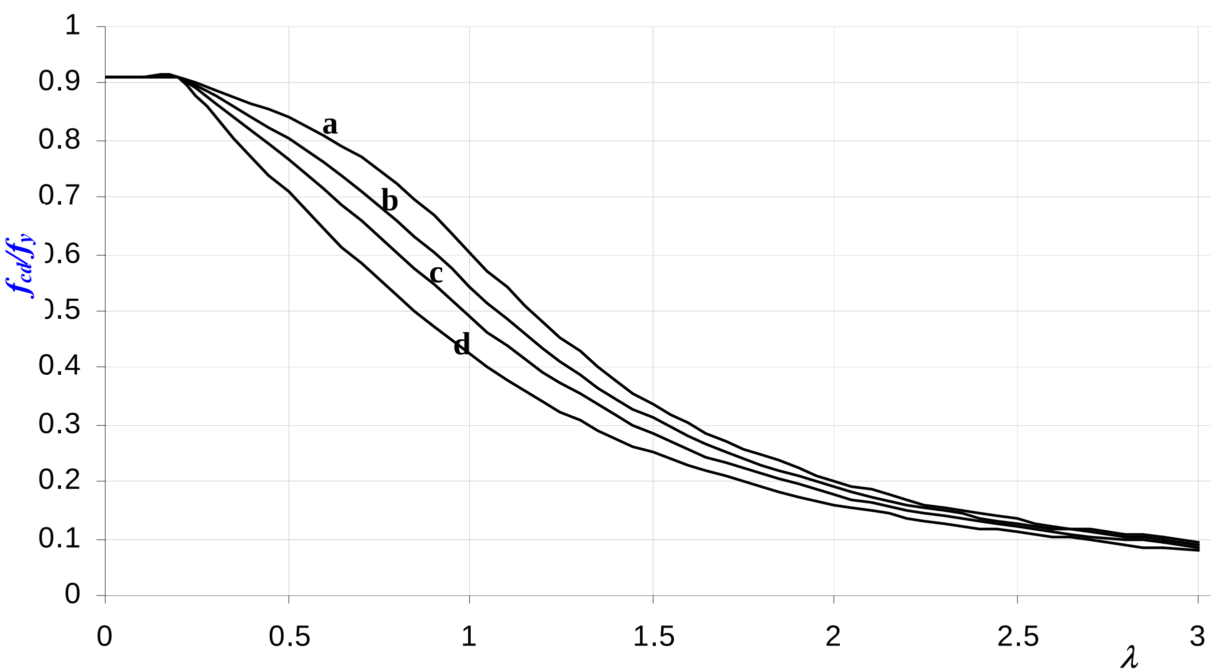


Fig. 5(c) Column Buckling Curves as per IS: 800

Table 2: Choice of appropriate values of α

Sections	**Axis of buckling**	
	X -X	***Y - Y***
Hot rolled structural hollow sections	α = 0.002 (Curve A)	α = 0.002 (Curve A)
Hot rolled I section	α = 0.002 (Curve A)	α = 0.0035 (Curve B)
Welded plate I section (up to 40 mm thick) I section (above 40 mm thick)	 α = 0.0035 (Curve B) α = 0.0035 (Curve B)	 α = 0.0055 (Curve C) α = 0.008 (Curve D)
Welded Box Section (Up to 40 mm thick) (Over 40 mm thick)	 α = 0.0035 (Curve B) α = 0.0055 (Curve C)	 α = 0.0035 (Curve B) α = 0.0055 (Curve C)
Rolled I section with Welded cover plates (Up to 40mm thick) (Over 40mm thick)	α = 0.0035 (Curve B) α = 0.0055 (Curve C)	α = 0.002 (Curve A) α = 0.0035(Curve B)
Rolled angle, Channel, T section Compound sections - Two rolled sections back to back, Battened or laced sections	Check buckling about ANY axis With α = 0.0055 (Curve C)	

Note: For sections fabricated by plates by welding the Value of f_y should be reduced by 20 N/mm^2 [See Fig. 5(*b*)].

For computational convenience, formulae linking σ_c and λ are required. The lower root of equation (1) [based on Perry - Robertson approach] represents the strut curves given Fig. 3 and BS: 5950 Part - 1.

$$\sigma_c = \phi - \sqrt{\phi^2 - f_y \sigma_e} \leq f_y \tag{5}$$

$$where,\ \phi = \frac{f_y + (\eta + 1)\sigma_e}{2} \tag{6a}$$

$$and\ \ \eta = (\lambda - \lambda_0)\alpha \tag{6b}$$

3.3 Types of Column Sections

Steel suppliers manufacture several types of sections, each type being most suitable for specific uses. Some of these are described below. It is important to note that columns may buckle about *Z*, *Y*, *V* or *U* axis. It is necessary to check the safety of the column about several axes, so that the lowest load that triggers the onset of collapse may be identified.

Universal Column *(UC)* sections have been designed to be most suitable for compression members. They have broad and relatively thick flanges, which avoid the problems of local buckling. The open shape is ideal for economic rolling and facilitates easy beam-to-column connections. The most optimum theoretical shape is in fact a ***circular hollow section (CHS),*** which has no weak bending axis. Although these have been employed in large offshore structures like oil platforms, their use is somewhat limited because of high connection costs and comparatively weaker in combined bending and axially compressive loads. ***Rectangular Hollow Sections (RHS)*** have been widely used in multi-storey buildings satisfactorily. For relatively light loads, (e.g. Roof trusses) ***angle sections*** are convenient as they can be connected through one leg. Columns, which are subjected to bending in addition to axial loads, are designed using ***Universal beams (UB)***.

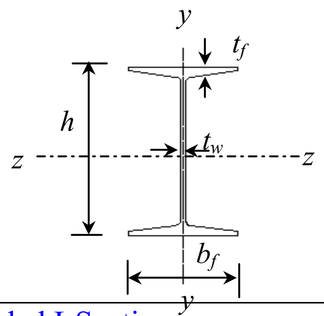
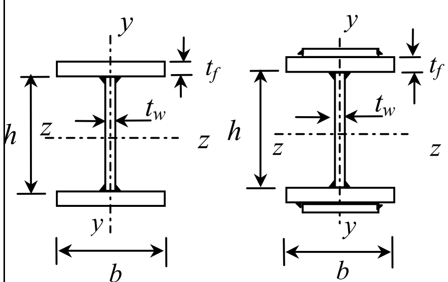
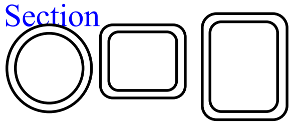
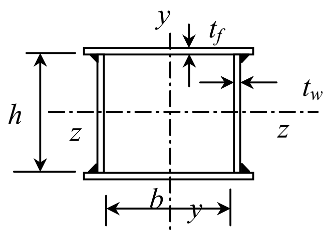
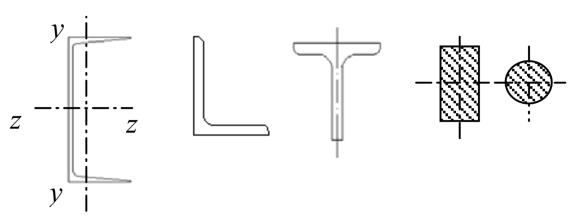
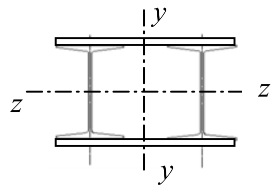
3.4 Heavily Welded Sections

Although both hot rolled sections and welded sections have lock-in residual stresses, the distribution and magnitude differ significantly. Residual stresses due to welding are very high and can be of greater consequence in reducing the ultimate capacity of compression members.

3.5 Stipulations of IS: 800

The stipulations of IS: 800 follow the same methodology as detailed above. For various types of column cross sections including Indian Standard rolled steel sections (as against Universal Column sections), CHS, SHS, RHS and Heavily Welded sections, IS: 800 recommends the classifications following the column buckling curves a, b, c and d as detailed in fig. 5(c) above. Apart from types of column cross-sections, the respective geometric dimensions of individual structural elements and their corresponding limits also guide Buckling Class. For example, if the ratio of overall height is to the overall width of the flange of rolled I section i.e. *h/b* is greater than 1.2 and the thickness of flange is less than 40 mm, the buckling class corresponding to axis z-z will be guided by Curve a of Fig. 5 (c). This shows for same slenderness ratio, more reserve strength is available for the case described above in comparison to built-up welded sections. For various types of column cross-sections, Table 3 (Table 7.2 of Revised IS: 800) defines the buckling classes with respect to their respective limits of height to width ratio, thickness of flange and the axis about which the buckling takes place (to be determined based on the column buckling curves as indicated in fig. 5 (c) corresponding to both major and minor axis):

TABLE 3: BUCKLING CLASS OF CROSS SECTIONS

Cross Section	Limits	Buckling about axis	Buckling Class
Rolled I-Sections 	$h/b_f > 1.2$: $t_f \leq 40$ mm	z-z y-y	a b
	40 mm< $t_f \leq 100$ mm	z-z y-y	b c
	$h/b_f \leq 1.2$: $t_f \leq 100$ mm	z-z y-y	b c
	t_f>100 mm	z-z y-y	d d
Welded I-Section 	$t_f \leq 40$ mm	z-z y-y	b c
	t_f >40 mm	z-z y-y	c d
Hollow Section 	Hot rolled	Any	a
	Cold formed	Any	b
Welded Box Section 	Generally (Except as below)	Any	b
	Thick welds and b/t_f < 30	z-z	c
	h/t_w < 30	y-y	c
Channel, Angle, T and Solid Sections 		Any	c
Built-up Member 		Any	c

4.0 EFFECTIVE LENGTH OF COLUMNS

The effective length, KL, is calculated from the actual length, L, of the member, considering the rotational and relative translational boundary conditions at the ends. The actual length shall be taken as the length from center to center of its intersections with the supporting members in the plane of the buckling deformation, or in the case of a member with a free end, the free standing length from the center of the intersecting member at the supported end.

Effective Length – Where the boundary conditions in the plane of buckling can be assessed, the effective length, KL, can be calculated on the basis of Table 7.5. Where frame analysis does not consider the equilibrium of a framed structure in the deformed shape (Second-order analysis or Advanced analysis), the effective length of compression members in such cases can be calculated using the procedure given in Appendix E.1. The effective length of stepped column in individual buildings can be calculated using the procedure given in Appendix E.2.

Eccentric Beam Connection – In cases where the beam connections are eccentric in plan with respect to the axes of the column, the same conditions of restraint as in concentric connection, shall be deemed to apply, provided the connections are carried across the flange or web of the columns as the case may be, and the web of the beam lies within, or in direct contact with the column section. Where practical difficulties prevent this, the effective length shall be taken as equal to the distance between points of restraint, in non-sway frames.

Stipulations of IS: 800 – Method for determining Effective Length for Stepped Columns

Single Stepped Columns – Effective length in the plane of stepping (bending about axis z-z) for bottom and top parts for single stepped column shall be taken as given in Table E.2

Note: The provisions of E.2.1 are applicable to intermediate columns as well with stepping on either side, provided appropriate values of I1and I2 are taken

Double Stepped Columns – Effective lengths in the plane of stepping (bending about axis z-z) for bottom, middle and top parts for a double stepped column shall be taken as per the stipulations of Appendix E3

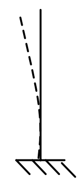
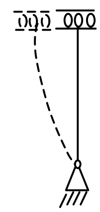
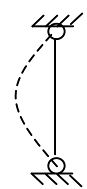
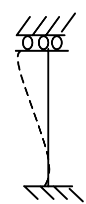
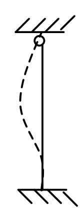
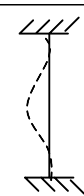
Coefficient K_1 for effective length of bottom part of double stepped column shall be taken from the formula:

$$K_1 = \sqrt{\frac{t_1 K_1{}^2 + \left(t_2 K_2{}^2 + K_3{}^2\right) \times \left(1 + n_2\right)^2 \times \dfrac{I_1}{I'_{av}}}{1 + t_1 + t_2}}$$

where

K_1, K_2, and K_3 are taken from Table E.6,

TABLE 4: EFFECTIVE LENGTH OF PRISMATIC COMPRESSION MEMBERS

Boundary Conditions				Schematic representation	Effective Length
At one end		At the other end			
Translation	Rotation	Translation	Rotation		
Restrained	Restrained	Free	Free		2.0L
Free	Restrained	Restrained	Free		
Restrained	Free	Restrained	Free		1.0L
Restrained	Restrained	Free	Restrained		1.2L
Restrained	Restrained	Restrained	Free		0.8L
Restrained	Restrained	Restrained	Restrained		0.65 L

Note – L is the unsupported length of the compression member (7.2.1).

Designs of columns have to be checked using the appropriate effective length for buckling in both strong and weak axes. A worked example illustrating this concept is appended to this chapter.

5.0 STEPS IN DESIGN OF AXIALLY LOADED COLUMNS AS PER IS: 800

The procedure for the design of an axially compressed column as stipulated in IS: 800 is as follows:

(i) Assume a suitable trial section and classify the section in accordance with the classification as detailed in Table 3.1 (Limiting Width to Thickness Ratios) of the Chapter 3 of IS: 800. (If, the section is slender then apply appropriate correction factor).

(ii) Calculate effective sectional area, A_e as defined in Clause 7.3.2 of IS: 800

(iii) Calculate effective slenderness ration, KL/r, ratio of effective length KL, to appropriate radius of gyration, r

(iv) Calculate λ from the equation, λ = non-dimensional effective slenderness ratio = $\sqrt{f_y / f_{cc}} = \sqrt{f_y \left(KL/r\right)^2 / \pi^2 E}$

(v) Calculate ϕ from the equation, $\phi = 0.5[1+\alpha(\lambda - 0.2)+ \lambda^2]$
Where,
α = Imperfection factors for various Column Buckling Curves a, b, c and d are given in the following Table: (Table 7.1 of IS: 800)

TABLE 5: IMPERFECTION FACTOR, α

Buckling Class	a	b	c	d
α	0.21	0.34	0.49	0.76

(vi) Calculate χ from equation, $\chi = \left[1 \Big/ \left[\phi + \left(\phi^2 - \lambda^2\right)^{0.5}\right] \right]$

(vii) Choose appropriate value of Partial safety factor for material strength, γ_{m0} from Table 5.2 of Chapter 5 of IS: 800

(viii) Calculate design stress in compression, f_{cd} , as per the following equation (Clause **7.1.2.1** of IS: 800):

$$f_{cd} = \frac{f_y / \gamma_{m0}}{\phi + \left[\phi^2 - \lambda^2\right]^{0.5}} \quad = \quad \chi f_y / \gamma_{m0} \quad \le f_y / \gamma_{m0}$$

(ix) Compute the load P_d, that the compression member can resist $P_d = A_e f_{cd}$

(x) Calculate the factored applied load and check whether the column is safe against the given loading. The most economical section can be arrived at by trial and error, i.e. repeating the above process.

6.0 CROSS SECTIONAL SHAPES FOR COMPRESSION MEMBERS AND BUILT- UP COLUMNS

Although theoretically we can employ any cross sectional shape to resist a compressive load we encounter practical limitations in our choice of sections as only a limited number of sections are rolled by steel makers and there are sometimes problems in connecting them to the other components of the structure. Another limitation is due to the adverse impact of increasing slenderness ratio on compressive strengths; this virtually excludes the use of wide plates, rods and bars, as they are far too slender. It must be specially noted that all values of slenderness ratio referred to herein are based on the least favourable value of radius of gyration, so that (λ/r) is the highest value about any axis.

6.1 Rolled Steel Sections

Some of the sections employed as compression members are shown in Fig. 6. Single angles [Fig 6(*a*)] are satisfactory for bracings and for light trusses. Top chord members of roof trusses are usually made up of twin angles back to back [Fig 6(*b*)]

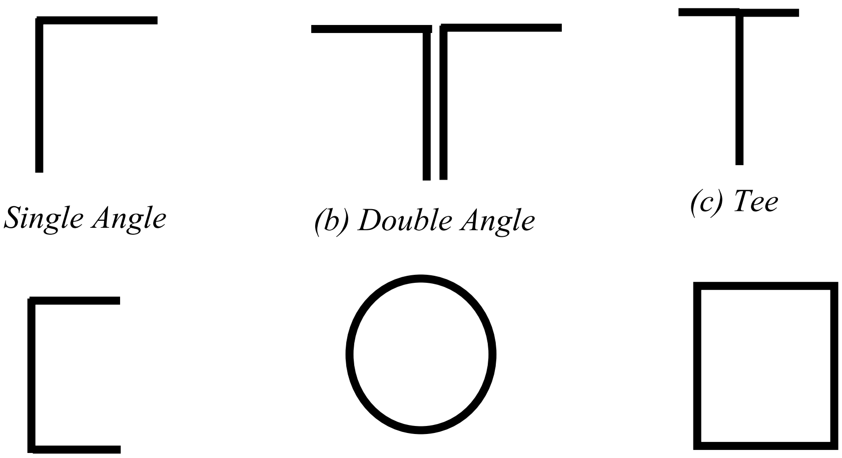
Double angle sections shown in Fig. 6(*b*) are probably the most commonly used members in light trusses. The pair of angles used has to be connected together, so they will act as one unit. Welds may be used at intervals – with a spacer bar between the connecting legs. Alternately " stitch bolts", washers and "ring fills" are placed between the angles to keep them at the proper distance apart (e.g. to enable a gusset to be connected).

When welded roof trusses are required, there is no need for gusset plates and T sections [Fig 6(*c*)] can be employed as compression members.

Single channels or C-sections [Fig. 6(*d*)] are generally not satisfactory for use in compression, because of the low value of radius of gyration. They can be used if they could be supported in a suitable way in the weak direction.

Circular hollow sections [Fig. 6(*e*)] are perhaps the most efficient as they have equal values of radius of gyration about every axis. But connecting them is difficult but satisfactory methods have been evolved in recent years for their use in tall buildings.

The next best in terms of structural efficiency will be the square hollow sections (SHS) and rectangular hollow sections, [Fig. 6(*f*)] both of which are increasingly becoming popular in tall buildings, as they are easily fabricated and erected. Welded tubes of circular, rectangular or square sections are very satisfactory for use as columns in a long series of windows and as short columns in walkways and covered warehouses. For many structural applications the weight of hollow sections required would be only *50%* of that required for open profiles like I or C sections.



(a) Single Angle *(b) Double Angle* *(c) Tee*

(d) Channel *(e) Circular Hollow Section (CHS)* *(f) Rectangular Hollow Section (RHS)*

Fig 6: Cross Section Shapes for Rolled Steel Compression Members

The following general guidance is given regarding connection requirements:

When compression members consist of different components, which are in contact with each other and are bearing on base plates or milled surfaces, they should be connected at their ends with welds or bolts. When welds are used, the weld length must be not less than the maximum width of the member. If bolts are used they should be spaced longitudinally at less than *4* times the bolt diameter and the connection should extend to at least *1 ½* times the width of the member.

Single angle discontinuous struts connected by a single bolt are rarely employed. When such a strut is required, it may be designed for *1.25* times the factored axial load and the effective length taken as centre to centre of the intersection at each end. Single angle discontinuous struts connected by two or more bolts in line along the member at each end may be designed for the factored axial load, assuming the effective length to be *0.85* times the centre to centre distance of the intersection at each end.

For double angle discontinuous struts connected back to back to both sides of a gusset or section by not less than two bolts or by welding, the factored axial load is used in design, with an effective length conservatively chosen. (A value between *0.7* and *0.85* depending upon the degree of restraint provided at the ends).

All double angle struts must be tack bolted or welded. The spacing of connectors must be such that the largest slenderness ratio of each component member is neither greater than *60* nor less than *40*. A minimum of two bolts at each end and a minimum of two additional connectors spaced equidistant in between will be required. Solid washers or packing plates should be used in-between if the leg width of angles exceed *125 mm*.

For member thickness upto *10 mm*, *M16* bolts are used; otherwise *M20* bolts are used. Spacing of tack bolts or welds should be less than *600 mm.*

The following guide values are suggested for initial choice of members:

(i) Single angle size : *1/30* of the length of the strut $\left(\lambda/r \approx 150\right)$

(ii) Double angle size : *1/35* of the length of strut $\left(\lambda/r \approx 100-120\right)$

(iii) Circular hollow sections diameter = *1/40* length $\left(\lambda/r \approx 100\right)$

6.2 Built-up or fabricated Compression Members

When compression members are required for large structures like bridges, it will be necessary to use built-up sections. They are particularly useful when loads are heavy and members are long (e.g. top chords of Bridge Trusses). Built up sections [illustrated in Fig. 7(*a*) and 7(*b*)] are popular in India when heavy loads are encountered. The cross section consists of two channel sections connected on their open sides with some type of lacing or latticing (dotted lines) to hold the parts together and ensure that they act together as one unit. The ends of these members are connected with "batten plates" which tie the ends together. Box sections of the type shown in Fig. 7(*a*) or 7(*b*) are sometimes connected by solid plates also (represented by straight lines).

A pair of channels connected by cover plates on one side and latticing on the other [Fig.7(c)] is sometimes used as top chords of bridge trusses. The gussets at joints can be conveniently connected to the inside of the channels. Plated I sections or built-up I sections are used when the available rolled I sections do not have sufficient strengths to resist column loads [Fig 7(*d*) and 7(*e*)]. For very heavy column loads, a welded built up section [See Fig. 7(*f*)] is quite satisfactory.

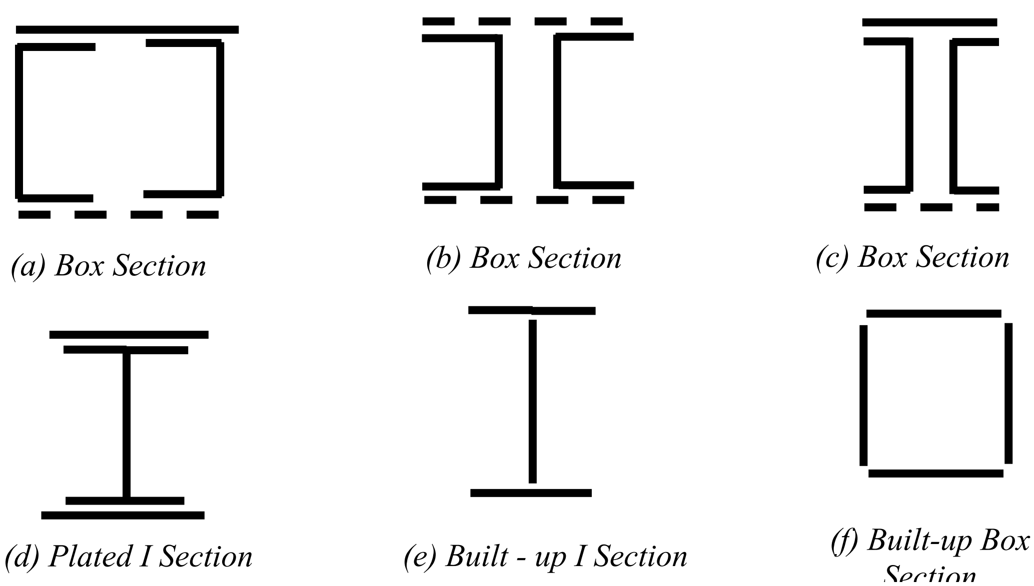


Fig 7: Cross Section Shapes for Built - up or fabricated Compression Members

Built up columns made up of solid webs:

In these columns the webs are solid and continuous [See Fig 7(*d*)*,* 7(*e*)*,* and 7(*f*)]. Flange plates or channels may be used in combination with rolled sections to enhance the load resistance of the commonly available sections, which are directly welded or bolted to each other. For preliminary calculations, approximate values of radii of gyration given in Fig. 8 for various built-up sections may be employed.

The lateral dimension of the column is generally chosen at around *1/10* to *1/15* of the height of the column. For purposes of detailing the connection between the flange cover plates or the outer rolled sections to the flanges of the main rolled section, it is customary to design the fasteners for a transverse shear force equal to *2.5%* of the compressive load of the column. (Connection Design is dealt later in this resource).

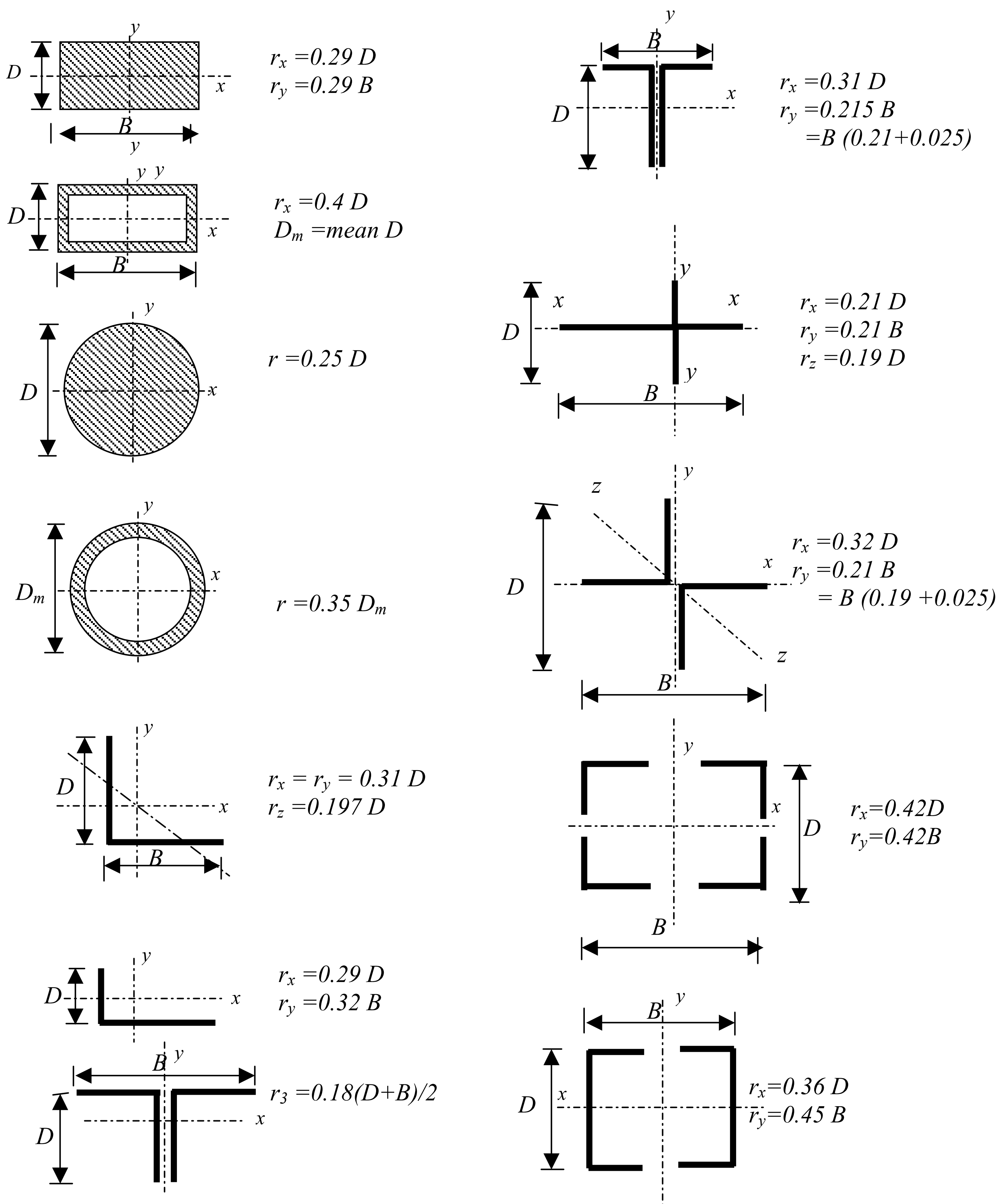


Fig 8: Approximate radii of gyration
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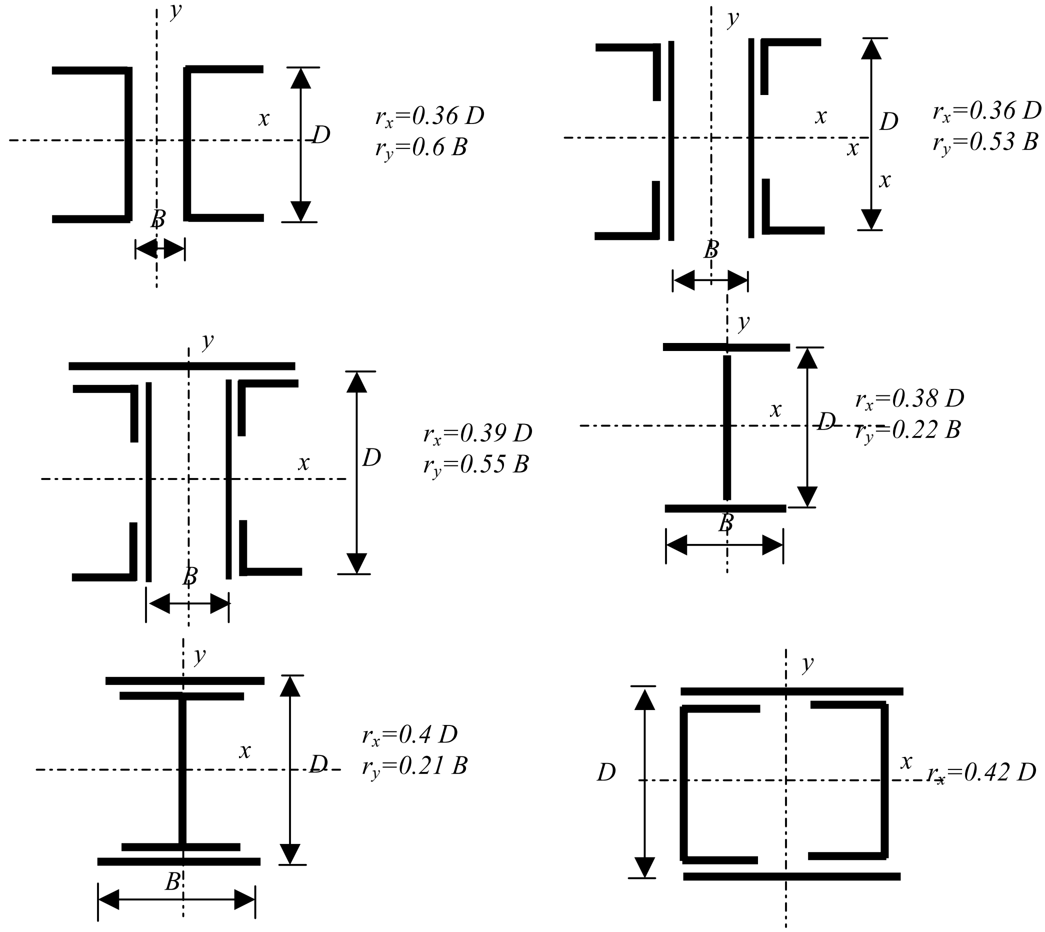


Fig 8: Approximate radii of gyration

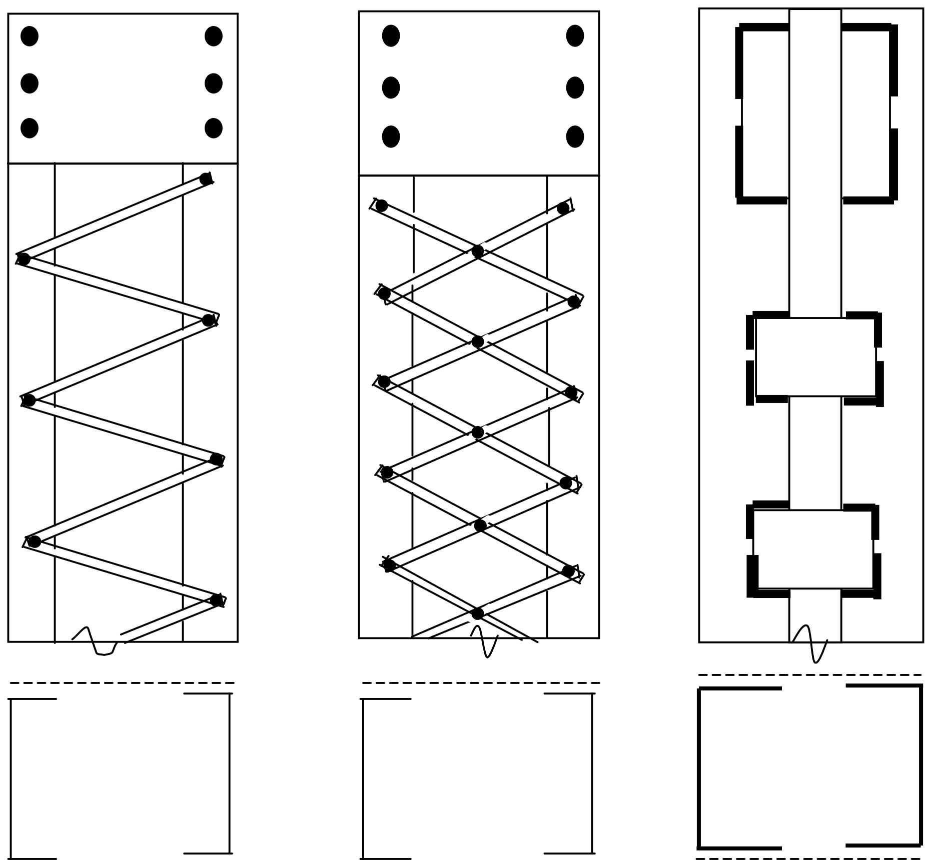
Open Web Columns:

In Fig. 9 the two channel sections of the column are connected together by batten plates or laces which are shown by dotted lines. A typical lacing or batten plate is shown in Fig. 9. Laced columns (also called latticed columns) generally carry *10%* more load than battened columns for the same area of cross section. This necessitates a *10%* increase in the slenderness ratio for battened columns. All columns should be tied at the ends by tie plates or end battens to ensure a satisfactory performance.

6.3 Design Considerations for Laced and Battened Columns

The two channel constituents of a laced column, shown in Fig. 9*(a)* and 9 *(b)* have a tendency to buckle independently. Lacing provides a tying force to ensure that the channels do not do so. The load that these tying forces cause is generally assumed to cause a shearing force equal to *2.5%* of axial load on the column. (Additionally if the columns are subjected to moments or lateral loading the lacing should be designed for the additional bending moment and shear). To prevent local buckling of unsupported lengths between the two constituent lattice points (or between two battens), the slenderness ratio of individual components should be less than *50* % or *70%* of the slenderness ratio of the built up column (whichever is less).

In laced columns, the lacing should be symmetrical in any two opposing faces to avoid torsion. Lacings and battens are not combined in the same column. The inclination of lacing bars from the axis of the column should not be less than 40^0 nor more than 70^0. The slenderness ratio of the lacing bars should not exceed *145*. The effective length of lacing bars is the length between bolts for single lacing and *0.7* of this length for double lacing. The width of the lacing bar should be at least *3* times the diameter of the bolt. Thickness of lacing bars should be at least $1/40^{th}$ of the length between bolts for single lacing and *1/60* of this length for double lacing (both for welded and bolted connections).



(a) Single Lacing *(b) Double Lacing* *(c) Battens*

Fig. 9 Built-up column members

In the Western world, it was common practice to "build-up" the required cross sectional area of steel compression members from a number of smaller sections. Since then, the increasing availability of larger rolled steel sections and the high fabrication costs have resulted in a very large drop in the use of built-up compression members. The disrepute of built-up compression members arises from the unrealistic expectation of many designers that a built-up member should have the same capacity of a solid member and also behaves in every respect in an identical manner to the latter. Early designers were disappointed to discover that this was just not possible. A contributory cause to this decline was the restrictive clauses introduced in many western design codes, following the Quebec Bridge Failure in *1907*. However there is a continuing use of built-up members, where stiffness and lightness are required, as in Transmission line Towers. There is, however, a wide spread use of built up compression members in the developing world, India included, largely because of the non-availability of heavier rolled sections and the perception that fabrication of members is cheaper.

For Laced and Battened Columns, Section 7.6 and 7.7 of IS: 800 shall be followed. In practical columns, the battens are very stiff and as they are normally welded to the vertical members, they can be considered as rigid connectors.

To allow approximately for this behaviour, a modified formula for calculating the effective slenderness (λ_b) of battened columns has been widely employed. This ensures that the Perry-Robertson approach outlined earlier could be used with a modified value for slenderness given by λ_b defined below.

$$\lambda_b = \sqrt{\lambda_f^2 + \lambda^2} \tag{7}$$

where,

λ_f = lower value of slenderness of the individual vertical members between batten intervals and

λ = slenderness of the overall column, using the radius of gyration of the whole built up section.

This equation, though an approximation, has been shown by Porter and Williams of Cardiff University actually to give accurate and safe values over the entire range of practical parameters for uniform columns with normal depth battens. In calculating the values of σ_e in the Perry Robertson equation, [equation 1 and 1(a)] λ_b is to be employed in place of λ, using slenderness ratio in defined in equation (3). The imperfection (η_b) is calculated from

$$\eta_b = 0.0055(\lambda_b) \tag{8}$$

The strength of the battened column is evaluated from

$$\sigma_c = \frac{f_y + (\eta_b + 1)\sigma_e}{2} - \sqrt{\left[\frac{f_y + (\eta_b + 1)\sigma_e}{2}\right]^2 - f_y \cdot \sigma_e} \tag{9}$$

λ_b = effective slenderness with η_b computed as given in equation (8)

σ_e = calculated using η_b values given in equation (7)

7.0 BASE PLATES FOR CONCENTRICALLY LOADED COLUMNS

The design compressive stress in a concrete footing is much smaller than it is in a steel column. So it becomes necessary that a suitable base plate should be provided below the column to distribute the load from it evenly to the footing below. The main function of the base plate is to spread the column load over a sufficiently wide area and keep the footing from being over stressed.

For a purely axial load, a plain square steel plate or a slab attached to the column is adequate. If uplift or overturning forces are present, a more positive attachment is necessary. These base plates can be welded directly to the columns or they can be fastened by means of bolted or welded lug angles. These connection methods are illustrated in Fig. 10.

A base plate welded directly to the columns is shown in Fig. 10(*a*). For small columns these plates will be shop-welded to the columns, but for larger columns, it may be necessary to ship the plates separately and set them to the correct elevations. For this second case the columns are connected to the footing with anchor bolts that pass through the lug angles which have been shop-welded to the columns. This type of arrangement is shown in Fig. 10(*b*).

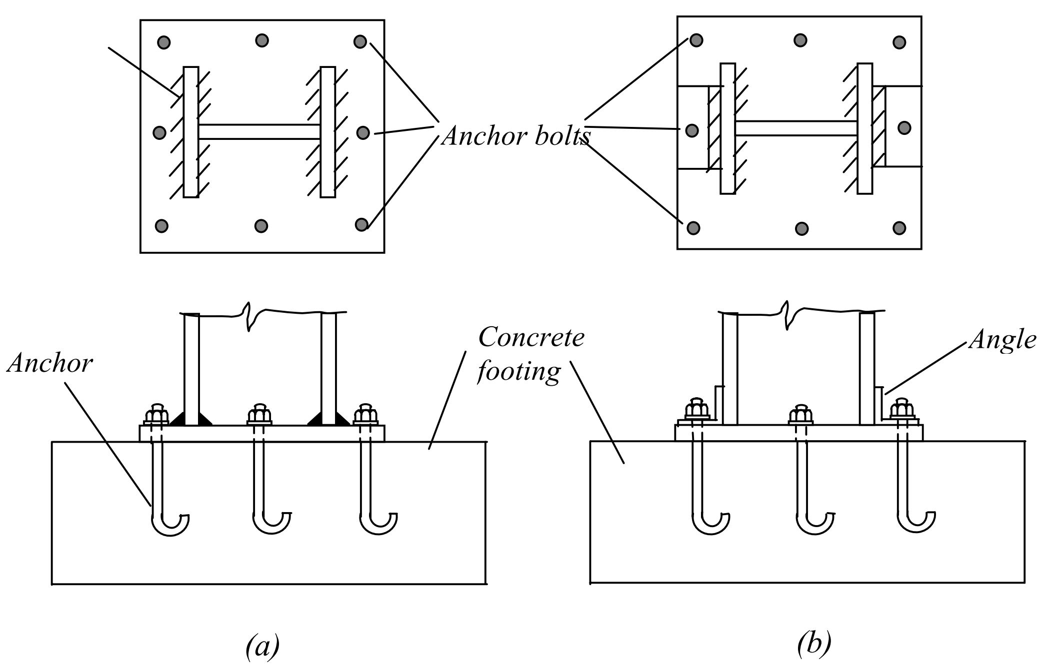


Fig. 10 Column base

Sometimes, when there is a large moment in relation to the vertically applied load a gusseted base may be required. This is intended to allow the lever arm from the holding down bolts to be increased to give maximum efficiency while keeping the base plate thickness to an acceptable minimum.

A critical phase in the erection of a steel building is the proper positioning of column base plates. If they are not located at their correct elevations, serious stress changes may occur in the beams and columns of the steel frame. In many cases, levelling plates of the same dimensions as the base plate are carefully grouted in place to the proper elevations first and then the columns with attached base plates are set on the levelling plates.
The lengths and widths of column base plates are usually selected in multiples of *10 mm* and the thickness chosen to conform to rolled steel plates. Usually the thickness of base plates is in the range of *40-50 mm*. If plates of this range are insufficient to develop the applied bending moment or if thinner plates are used, some form of stiffening must be provided.

Concrete support area should be significantly larger than the base plate area so that the applied load can disperse satisfactorily on to the foundation. To spread the column loads uniformly over the base plates, and to ensure there is good contact between the two, it is customary not to polish the underside of the base plate, but grout it in place.

Columns supporting predominantly axial loads are designed as being pin-ended at the base. The design steps for a base plate attached to an axially loaded column with pinned base is explained below.

Procedure for empirical design of a slab base plate for axial load only (pinned connection)

1. Determine the factored axial load and shear at the column base.
2. Decide on the number and type of holding down bolts to resist shear and tension. The chosen number of bolts are to be arranged symmetrically near corners of base plate or next to column web, similar to the arrangement sketched in Fig. 10.
3. Maximum allowable bearing strength $= 0.4 f_{cu}$ (where f_{cu} = cube strength of concrete) Actual bearing pressure to be less than or equal to $0.4 f_{cu}$.
4. Determine base plate thickness t;

For *I, H*, *channel, box* or *RHS columns*

$t = \sqrt{\dfrac{2.5w}{f_{yp}}\left(a^2 - 0.3b^2\right)}$ but not less than the thickness of the flange of the supported column.

w = pressure in N/mm^2 on underside of plate, assuming a uniform distribution.
a = larger plate projection from column [See Fig. 11]
b = smaller plate projection from column
f_{yp} = design strength of plate, but not greater than $250\ N/mm^2$ divided by γ_m

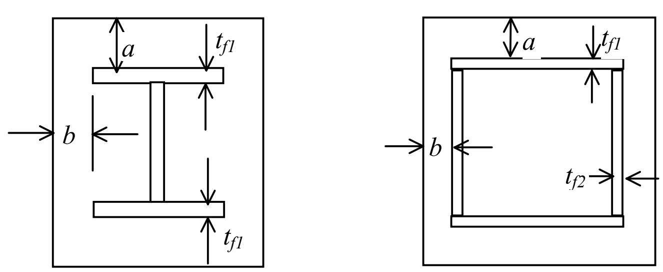


Fig. 11 Base plates subjected to concentric forces

5. Check for adequacy of weld. Calculate the total length of weld to resist axial load.
6. Select weld size.
7. Check shear stress on weld.
8. Vector sum of all the stresses carried by the weld must not exceed $p_{w,}$ the design strength, of the weld.
9. Check for bolt. Check maximum co-existent factored shear and tension, if any, on the holding down bolts.
10. Check the bolts for adequacy (see a later chapter for bolt design).

Stipulations of IS: 800 for Column Bases

Column bases should have sufficient, stiffness and strength to transmit axial force, bending moments and shear forces at the base of the columns to their foundation without exceeding the load carrying capacity of the supports. Anchor bolts and shear keys should be provided wherever necessary. Shear resistance at the proper contact surface between steel base and concrete/grout may be calculated using a friction coefficient of 0.45.

The nominal bearing pressure between the base plate and the support below may be determined on the basis of linearly varying distribution of pressure. The maximum bearing pressure should not exceed the bearing strength equal to $0.6f_{ck}$, where f_{ck} is the smaller of characteristic cube strength of concrete or bedding material.

If the size of the base plate is larger than that required to limit the bearing pressure on the base support, an equal projection, *c*, of the base plate beyond the face of the column and gusset may be taken as effective in transferring the column load (Fig 7.2), such that beam pressure as the effective area does not exceed bearing capacity of concrete base.

Gusseted Bases – For stanchion with gusseted bases, the gusset plates, angle cleats, stiffeners, fastenings, etc., in combination with the bearing area of the shaft, shall be sufficient to take the loads, bending moments and reactions to the base plate without exceeding specified strength. All the bearing surfaces shall be machined to ensure perfect contact.

Where the ends of the column shaft and the gusset plates are not faced for complete bearing, the welding, fastenings connecting them to the base plate shall be sufficient to transmit all the forces to which the base is subjected.

Column and Base Plate Connections – Where the end of the column is connected directly to the base plate by means of full penetration butt welds, the connection shall be deemed to transmit to the base all the forces and moments to which the column is subjected.

Slab Bases – Columns with slab bases need not be provided with gussets, but sufficient fastenings shall be provided to retain the parts securely in place and to resist all moments and forces, other than direct compression, including those arising during transit, unloading and erection.

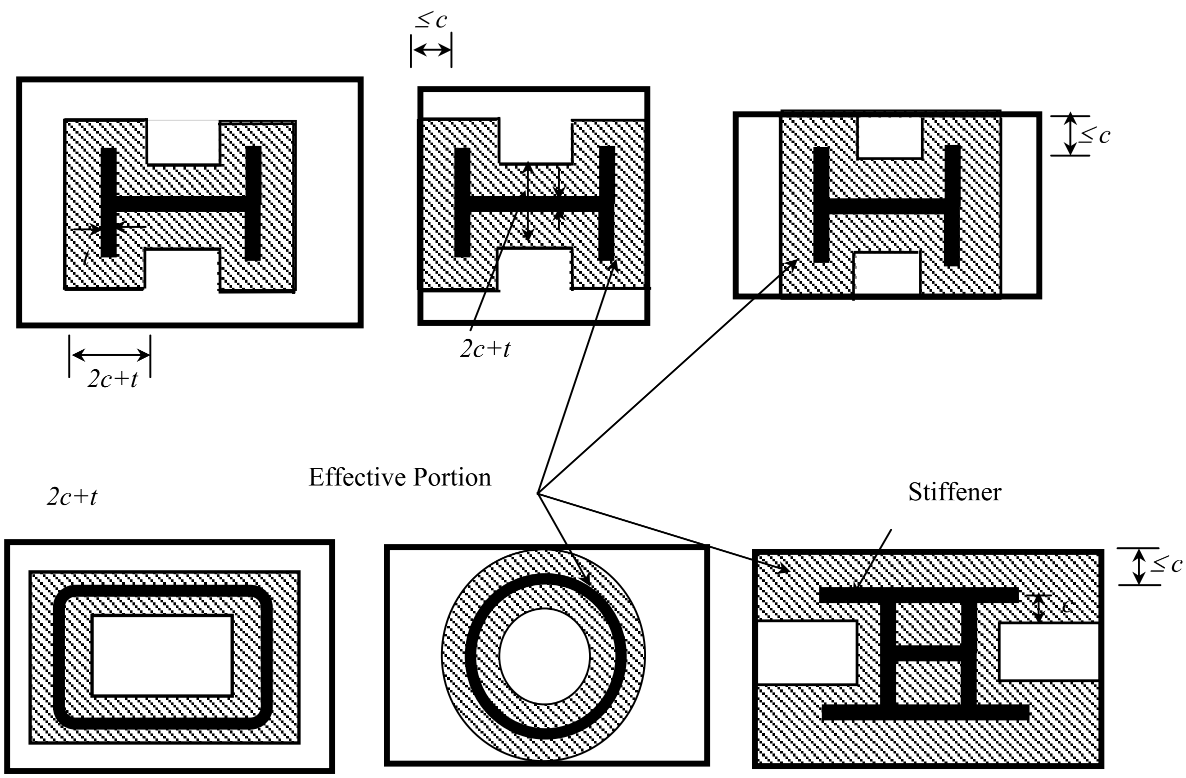


FIG 7.2 EFFECTIVE AREA OF A BASE PLATE

The minimum thickness, t_s, of rectangular slab bases, supporting columns under axial compression shall be

$$t_s = \sqrt{2.5\, w (a^2 - 0.3 b^2) \gamma_{m0} / f_y} \quad > t_f$$

where,

w = uniform pressure from below on the slab base under the factored load axial compression

a, b = larger and smaller projection of the slab base beyond the rectangle circumscribing the column, respectively

t_f = flange thickness of compression member

When only the effective area of the base plate is used as in section 7.4.1.1instead of $(a^2 - 0.3b^2)$, c^2 may be used in the above equation (see Fig 7.2)

When the slab does not distribute the column load uniformly, due to eccentricity of the load etc, special calculation shall be made to show that the base is adequate to resist the moment due to the non-uniform pressure from below.

Bases for bearing upon concrete or masonry need not be machined on the underside.

In cases where the cap or base is fillet welded directly to the end of the column without boring and shouldering, the contact surfaces shall be machined to give a perfect bearing and the welding shall be sufficient to transmit the forces as required in **7.4.3**. Where full strength butt welds are provided, machining of contact surfaces is not required.

7.0 CONCLUDING REMARKS

Design of columns using multiple column curves is discussed in this chapter. Additional provision required for accounting for heavily welded sections are detailed. Built-up fabricated members frequently employed (when rolled sections are found inadequate) are discussed in detail. Design guidance is provided for laced/battened columns. Effective lengths for various end conditions are listed and illustrative worked examples are appended. A simple method of designing a base plate for an axially loaded column is proposed and illustrated by a worked example. Wherever required, provisions of IS: 800 (latest Version) have been highlighted and worked examples have been prepared based on the stipulations of IS: 800 (Latest Version).

8.0 REFERENCES

1. Owens G.W., Knowles P.R (1994): "Steel Designers Manual", The Steel Construction Institute, Ascot, England.
2. Dowling P.J., Knowles P.R., Owens G.W (1998): "Structural Steel Design", Butterworths, London.
3. British Standards Institution (1985): "BS 5950, Part-1 Structural use of steelwork in building", British Standards Institution, London.
4. Bureau of Indian Standard, IS: 800,

Table3: Ultimate Compressive stress (σ_c) values in compression members (f_y = 250 N/mm^2)

λ	$\alpha = 0.002$	$\alpha = 0.0035$	$\alpha = 0.0055$	$\alpha = 0.008$
15	250	250	250	250
20	249	248	247	245
25	246	243	240	235
30	243	239	233	225
35	240	234	225	216
40	237	228	218	206
45	233	223	210	196
50	229	216	202	187
55	225	210	194	177
60	219	203	185	168
65	213	195	176	160
70	206	187	168	150
75	198	178	159	141
80	189	169	150	133
85	180	160	141	125
90	170	151	133	118
95	159	142	125	111
100	149	133	118	104
110	130	117	104	92
120	114	103	92	82
130	99	91	82	73
140	87	80	73	66
150	77	71	65	59
160	69	64	59	53
170	62	57	53	48
180	55	52	48	44
190	55	47	44	40
200	45	43	40	37

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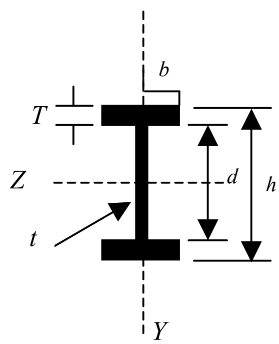
	Job Title: *AXIALLY COMPRESSED COLUMN*		
	Worked Example - 1		
		Made by *GC*	Date *23-02-07*
Calculation Sheet		Checked by *TKB*	Date *28-02-07*

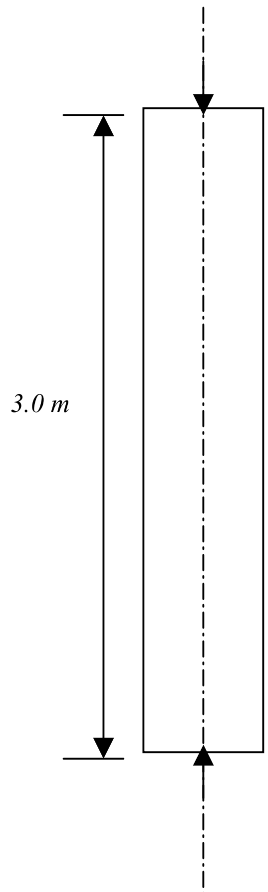
Obtain factored axial load on the column section ISHB400. The height of the column is 3.0m and it is pin-ended.

[$f_y = 250$ N/mm^2 ; $E = 2 \times 10^5$ N/mm^2 ; $\gamma_m = 1.10$]

Table 5.2 of IS: 800

CROSS-SECTION PROPERTIES:





<table>
<tr><td rowspan="5">Structural Steel Design Project

Calculation Sheet</td><td>Job No:</td><td>Sheet 2 of 4</td><td>Rev</td></tr>
<tr><td colspan="3">Job Title: AXIALLY COMPRESSED COLUMN</td></tr>
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<tr><td rowspan="2"></td><td>Made by GC</td><td>Date 23-02-07</td></tr>
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</table>

Flange thickness = T = *12.7 mm*

Overall height of ISHB400 = h = *400 mm*

Clear depth between flanges = d = *400 – (12.7 * 2) = 374.6 mm*

Thickness of web = t = *10.6 mm*

Flange width = $2b$ = b_f = *250 mm*

Hence, half Flange Width = b = *125 mm*

Self –weight = w = *0.822 kN/m*

Area of cross-section = A = *10466 mm²*

Radius of gyration about x = r_x = *166.1 mm*

Radius of gyration about y = r_y = *51.6 mm*

(i) Type of section:

$$\frac{b}{T} = \frac{125}{12.7} = 9.8 < 10.5\varepsilon$$

$$\frac{d}{t} = \frac{374.6}{10.6} = 35.3 < 42\varepsilon$$

Table 3.1 of IS: 800

$$where, \varepsilon = \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1.0$$

Hence, cross- section can be classified as "COMPACT"

(ii) Effective Sectional Area, A_e = *10466 mm²*

(Since there is no hole, no reduction has been considered)

Clause 7.3.2 of IS: 800

Structural Steel Design Project	Job No:	Sheet *3 of 4*	Rev
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(iii) Effective Length:

As, both ends are pin-jointed effective length $KL_x = KL_y = 1.0 \times L_x = 1.0 \times L_y = 1.0 \times 3.0\ m = 3.0\ m$

Clause 7.2 and Table 7.5 of IS: 800

(iv) Slenderness ratios:

$$KL_x / r_x = \frac{3000}{166.1} = 18.1$$

$$KL_y / r_y = \frac{3000}{51.6} = 58.1$$

(v) Non-dimensional Effective Slenderness ratio, λ :

$$\lambda = \sqrt{f_y / f_{cc}} = \sqrt{f_y \left(KL/r\right)^2 / \pi^2 E} = \sqrt{250x(58.1)^2 / \pi^2 x2x10^5}$$

= 0.654

Clause 7.1.2.1 of IS: 800

(vi) Value of ϕ from equation $\phi = 0.5[1+\alpha(\lambda - 0.2)+\lambda^2]$:

Where, α = Imperfection Factor which depends on Buckling Class

Now, from Table 7.2 of Chapter 7, for *h/b_f = 400 / 250 = 1.6 > 1.2 and also thickness of flange, T = 12.7 mm, hence for z-z axis buckling class 'a' and for y-y axis buckling class 'b' will be followed.*

Table 7.1 of IS: 800

Hence, α = 0.34 for buckling class 'b' will be considered.

Hence, ***$\phi = 0.5 \times [1+0.34 \times (0.654-0.2)+0.654^2] = 0.791$***

(vii) Calculation of χ from equation $\chi = \left[1 \Big/ \left[\phi + \left(\phi^2 - \lambda^2\right)^{0.5}\right] \right]$:

$$\chi = \left[1 \Big/ \left[\phi + \left(\phi^2 - \lambda^2\right)^{0.5}\right] \right] = \left[1 \Big/ \left[0.791 + \left(0.791^2 - 0.654^2\right)^{0.5}\right] \right]$$

= 0.809

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(viii) Calculation of f_{cd} from the following equation:

$$f_{cd} = \frac{f_y / \gamma_{m0}}{\phi + \left[\phi^2 - \lambda^2\right]^{0.5}} = \chi\, f_y / \gamma_{m0} = 0.809x250/1.10 = 183.86 \text{ N/mm}^2$$

(ix) Factored Axial Load in kN, P_d:

$P_d = A_e f_{cd} = 10466 \text{ x } 183.86/1000 = 1924.28$ ***kN***

<table>
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Calculation Sheet</td><td>Job No:</td><td>Sheet 1 of 2</td><td>Rev</td></tr>
<tr><td colspan="3">Job Title: AXIALLY COMPRESSED COLUMN</td></tr>
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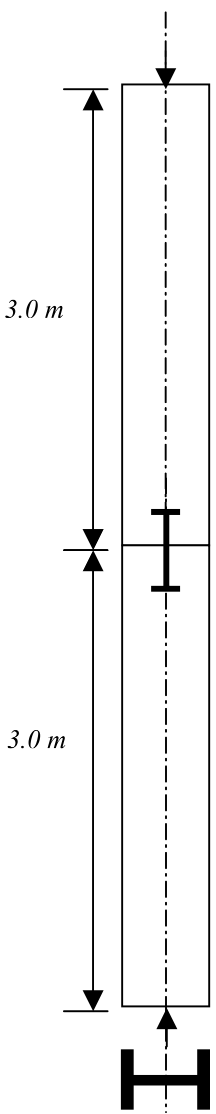
</table>

Obtain maximum axial load carried by the column shown when ISHB 400 is employed. The column is effectively restrained at mid-height in the y-direction, but is free in x-axis. The data is the same as in problem1.

[$f_y = 250\ N/mm^2$; $E = 2.0 \times 10^5\ N/mm^2$; $\gamma_m = 1.10$]



(i) Type of section:

Section is "COMPACT" from previous example.

(ii) Effective lengths:

As, both ends are pin-jointed effective length $KL_x = 1.0 \times 6000 = 6000$ mm
$KL_y = 1.0 \times 3000 = 3000$ mm

(iii) Slenderness ratios:

$$KL_x / r_x = \frac{6000}{166.1} = 36.12$$

$$KL_y / r_y = \frac{3000}{51.6} = 58.1$$

(iv) Non-dimensional Effective Slenderness ratio, λ :

As calculated in previous example,
*λ = **0.654** (for worst slenderness ratio)*

(v) Value of ϕ:

As calculated in previous example,
*ϕ = **0.791** (for λ = **0.654**)*

(vi) Value of χ:

As calculated in previous example,
*χ = **0.809** (for ϕ = **0.791**)*

Structural Steel Design Project	Job No:	Sheet *2 of 2*	Rev
	Job Title: *AXIALLY COMPRESSED COLUMN*		
	Worked Example - 2		
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Calculation Sheet		Checked by *TKB*	Date *28-02-07*

(vii) Calculation of f_{cd}:

As calculated in previous example,

f_{cd} **= 183.86** N/mm^2

(viii) Calculation of Factored Load:

Factored Load = $f_{cd} \, x A_e / \gamma_m$ = 183.86/1.10 x10466/1000= 1924.28 kN

<table>
<tr><td rowspan="5">Structural Steel Design Project

Calculation Sheet</td><td>Job No:</td><td>Sheet 1 of 1</td><td>Rev</td></tr>
<tr><td colspan="3">Job Title: BASE PLATE</td></tr>
<tr><td colspan="3">Worked Example - 3</td></tr>
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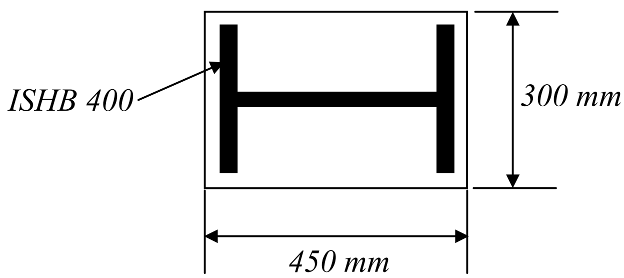
</table>

Design a simple base plate for a ISHB400 @ 0.822 kN/m column to carry a factored load of 1800 kN.

[f_{cu} = 40 N/mm^2 ; f_y = 250 N/mm^2 ; γ_m = 1.10] *Table 5.2 of IS: 800*



Thickness of Flange for ISHB400 = T = 12.7 mm

Bearing strength of concrete = $0.4f_{cu} = 0.4 * 40 = 16$ N/mm^2

Area required = $1800*10^3/16 = 112500$ mm^2

Use plate of 450 X 300 mm (135000 mm^2)

Assuming projection of 25 mm on each side, a = b = 25 mm

$$w = \left(1800 \times 10^3 / 450 \times 300\right) = 13.33 \text{ N/mm}^2$$

Now thickness of Slab Base, t_s

$$t_s = \sqrt{2.5w\,(a^2 - 0.3b^2)\gamma_{m0} / f_y} \quad > \quad T$$

Clause 7.4.3.1 of IS: 800

$$= \sqrt{\frac{2.5w\left(a^2 - 0.3b^2\right)\times 1.10}{f_y}} = \sqrt{\frac{2.5 \times 13.33 \times \left(25^2 - 0.3 \times 25^2\right) \times 1.10}{250}} = 8.01 \text{ mm}$$

< T = 12.7 mm, Hence provide a base plate of thickness not less than 12.7 mm and since the available next higher thickness of plate is 16 mm

Use 450 X 300 X 16 mm plate.

11 UNRESTRAINED BEAM DESIGN – I

1.0 INTRODUCTION

Generally, a beam resists transverse loads by bending action. In a typical building frame, main beams are employed to span between adjacent columns; secondary beams when used – transmit the floor loading on to the main beams. In general, it is necessary to consider only the bending effects in such cases, any torsional loading effects being relatively insignificant. The main forms of response to uni-axial bending of beams are listed in Table 1.

Under increasing transverse loads, beams of category 1 [Table1] would attain their full plastic moment capacity. This type of behaviour has been covered in an earlier chapter. Two important assumptions have been made therein to achieve this ideal beam behaviour. They are:

- ♦ The compression flange of the beam is restrained from moving laterally, and
- ♦ Any form of local buckling is prevented.

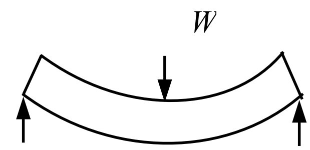
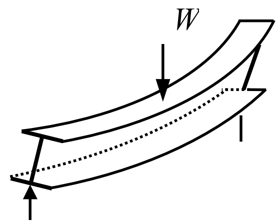
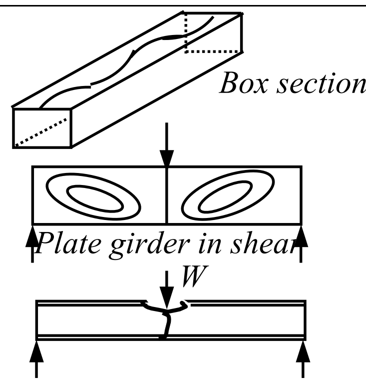
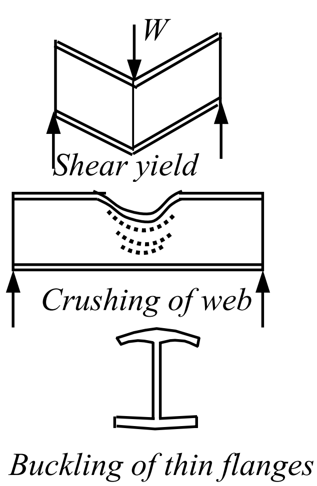
If the laterally unrestrained length of the compression flange of the beam is relatively long as in category 2 of Table 1, then a phenomenon, known as *lateral buckling* or *lateral torsional buckling* of the beam may take place. The beam would fail well before it could attain its full moment capacity. This phenomenon has a close similarity to the Euler buckling of columns, triggering collapse before attaining its squash load (full compressive yield load).

Lateral buckling of beams has to be accounted for at all stages of construction, to eliminate the possibility of premature collapse of the structure or component. For example, in the construction of steel-concrete composite buildings, steel beams are designed to attain their full moment capacity based on the assumption that the flooring would provide the necessary lateral restraint to the beams. However, during the erection stage of the structure, beams may not receive as much lateral support from the floors as they get after the concrete hardens. Hence, at this stage, they are prone to lateral buckling, which has to be consciously prevented.

Beams of category 3 and 4 given in Table 1 fail by local buckling, which should be prevented by adequate design measures, in order to achieve their capacities. The method of accounting for the effects of local buckling on bending strength was discussed in an earlier chapter.

In this chapter, the conceptual behaviour of laterally unrestrained beams is described in detail. Various factors that influence the lateral buckling behaviour of a beam are explained. The design procedure for laterally unrestrained beams is also included.

Table 1 Main failure modes of hot-rolled beams

Category	**Mode**		**Comments**
1	Excessive bending triggering collapse		This is the basic failure mode provided (1) the beam is prevented from buckling laterally,(2) the component elements are at least compact, so that they do not buckle locally. Such "stocky" beams will collapse by plastic hinge formation.
2	Lateral torsional buckling of long beams which are not suitably braced in the lateral direction.(i.e. "un restrained" beams)		Failure occurs by a combination of lateral deflection and twist. The proportions of the beam, support conditions and the way the load is applied are all factors, which affect failure by lateral torsional buckling.
3	Failure by local buckling of a flange in compression or web due to shear or web under compression due to concentrated loads		Unlikely for hot rolled sections, which are generally stocky. Fabricated box sections may require flange stiffening to prevent premature collapse. Web stiffening may be required for plate girders to prevent shear buckling. Load bearing stiffeners are sometimes needed under point loads to resist web buckling.
4	Local failure by (1) shear yield of web (2) local crushing of web (3) buckling of thin flanges.		Shear yield can only occur in very short spans and suitable web stiffeners will have to be designed. Local crushing is possible when concentrated loads act on unstiffened thin webs. Suitable stiffeners can be designed. This is a problem only when very wide flanges are employed. Welding of additional flange plates will reduce the plate b / t ratio and thus flange buckling failure can be avoided.

2.0 SIMILARITY OF COLUMN BUCKLING AND LATERAL BUCKLING OF BEAMS

It is well known that slender members under compression are prone to instability. When slender structural elements are loaded in their strong planes, they have a tendency to fail by buckling in their weaker planes. Both axially loaded columns and transversely loaded beams exhibit closely similar failure characteristics due to buckling.

Column buckling has been dealt with in detail in an earlier chapter. In this section, lateral buckling of beams is described and its close similarity to column buckling is brought out.

Consider a simply supported and laterally unsupported (except at ends) beam of "short-span" subjected to incremental transverse load at its mid section as shown in Fig.1 (*a*). The beam will deflect downwards i.e. in the direction of the load [Fig. 1(*b*)].

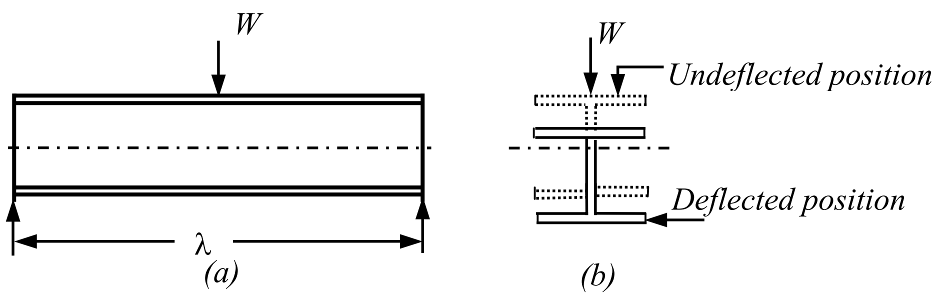


Fig. 1(a) Short span beam, (b) Vertical deflection of the beam.

The direction of the load and the direction of movement of the beam are the same. This is similar to a short column under axial compression. On the other hand, a "long-span" beam [Fig.2 (*a*)], when incrementally loaded will first deflect downwards, and when the load exceeds a particular value, it will tilt sideways due to instability of the compression flange and rotate about the longitudinal axis [Fig. 2(*b*)].

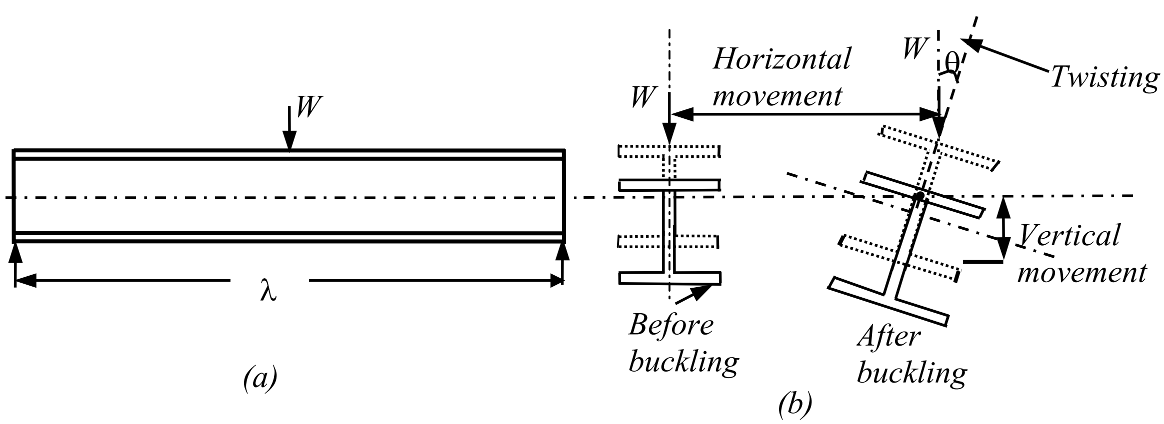


Fig. 2(a) Long span beam, (b) Laterally deflected shape of the beam

The three positions of the beam cross-section shown in Fig. 2(*b*) illustrate the displacement and rotation that take place as the midsection of the beam undergoes lateral torsional buckling. The characteristic feature of lateral buckling is that the entire cross section rotates as a rigid disc without any cross sectional distortion. This behaviour is very similar to an axially compressed long column, which after initial shortening in the axial direction, deflects laterally when it buckles. The similarity between column buckling and beam buckling is shown in Fig. 3.

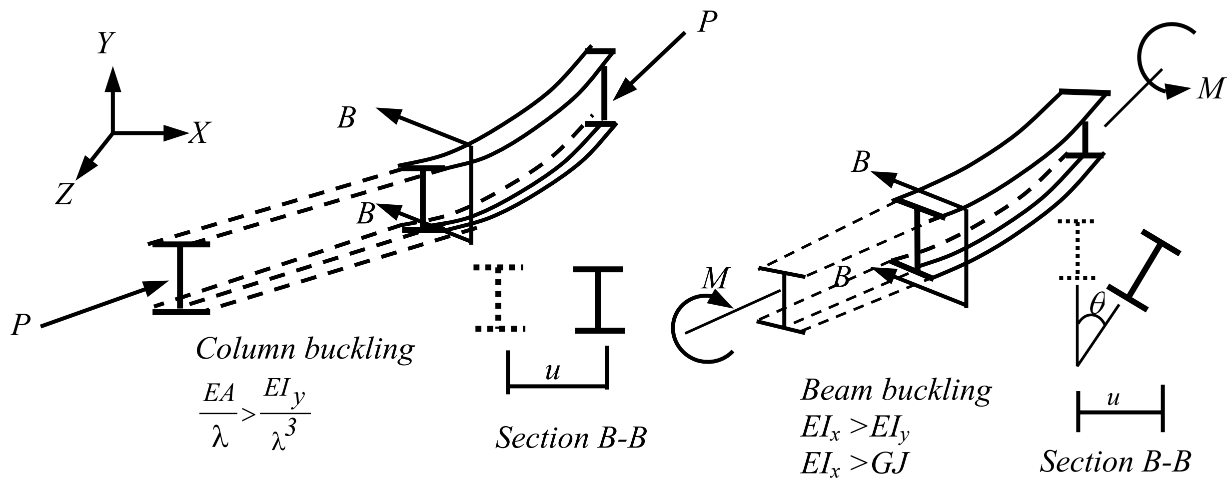


Fig. 3 Similarity of column buckling and beam buckling

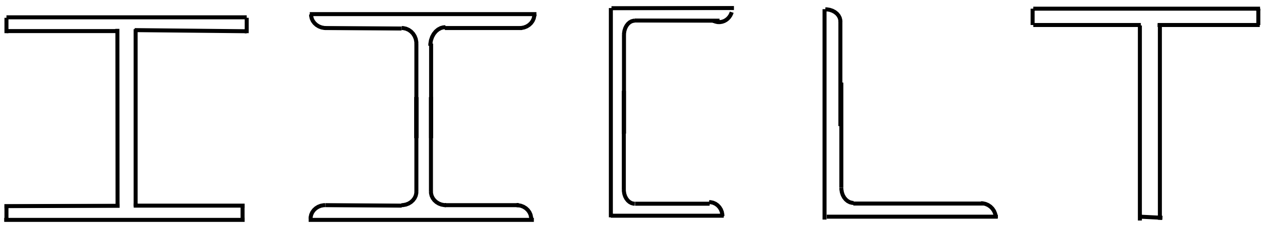
In the case of axially loaded columns, the deflection takes place sideways and the column buckles in a pure flexural mode. A beam, under transverse loads, has a part of its cross section in compression and the other in tension. The part under compression becomes unstable while the tensile stresses elsewhere tend to stabilize the beam and keep it straight. Thus, beams when loaded exactly in the plane of the web, at a particular load, will fail suddenly by deflecting sideways and then twisting about its longitudinal axis [Fig.3]. This form of instability is more complex (compared to column instability) since the lateral buckling problem is 3-dimensional in nature. It involves coupled lateral deflection and twist i.e., when the beam deflects laterally, the applied moment exerts a torque about the deflected longitudinal axis, which causes the beam to twist. The bending moment at which a beam fails by lateral buckling when subjected to a uniform end moment is called its *elastic critical moment* (M_{cr}). In the case of lateral buckling of beams, the elastic buckling load provides a close upper limit to the load carrying capacity of the beam. It is clear that lateral instability is possible only if the following two conditions are satisfied.

- The section possesses different stiffness in the two principal planes, and
- The applied loading induces bending in the stiffer plane (about the major axis).

Similar to the columns, the lateral buckling of unrestrained beams, is also a function of its slenderness.

3.0 INFLUENCE OF CROSS SECTIONAL SHAPE ON LATERAL TORSIONAL BUCKLING

Structural sections are generally made up of either open or closed sections. Examples of open and closed sections are shown in Fig. 4.



Wide Flange Beam *Standard beam* *Channel* *Angle* *Tee*

Open sections

Tubular *Box*

Closed sections

Fig. 4 Open and closed sections

Cross sections, employed for columns and beams (I and channel), are usually open sections in which material is distributed in the flanges, i.e. away from their centroids, to improve their resistance to in-plane bending stresses. Open sections are also convenient to connect beams to adjacent members. In the ideal case, where the beams are restrained laterally, their bending strength about the major axis forms the principal design consideration. Though they possess high major axis bending strength, they are relatively weak in their minor axis bending and twisting.

The use of open sections implies the acceptance of low torsional resistance inherent in them. No doubt, the high bending stiffness (EI_x) available in the vertical plane would result in low deflection under vertical loads. However, if the beam is loaded laterally, the deflections (which are governed by the lower EI_y rather than the higher EI_x) will be very much higher. From a conceptual point of view, the beam has to be regarded as an element having an enhanced tendency to fall over on its weak axis.

In contrast, closed sections such as tubes, boxes and solid shafts have high torsional stiffness, often as high as 100 times that of an open section. The hollow circular tube is the most efficient shape for torsional resistance, but is rarely employed as a beam element on account of the difficulties encountered in connecting it to the other members and lesser efficiency as a flexural member. The influence of sectional shapes on the lateral strength of a beam is further illustrated in a later Section.

4.0 LATERAL TORSIONAL BUCKLING OF SYMMETRIC SECTIONS

As explained earlier, when a beam fails by lateral torsional buckling, it buckles about its weak axis, even though it is loaded in the strong plane. The beam bends about its strong axis up to the critical load at which it buckles laterally [Fig. 5(*a*) and 5(*b*)].

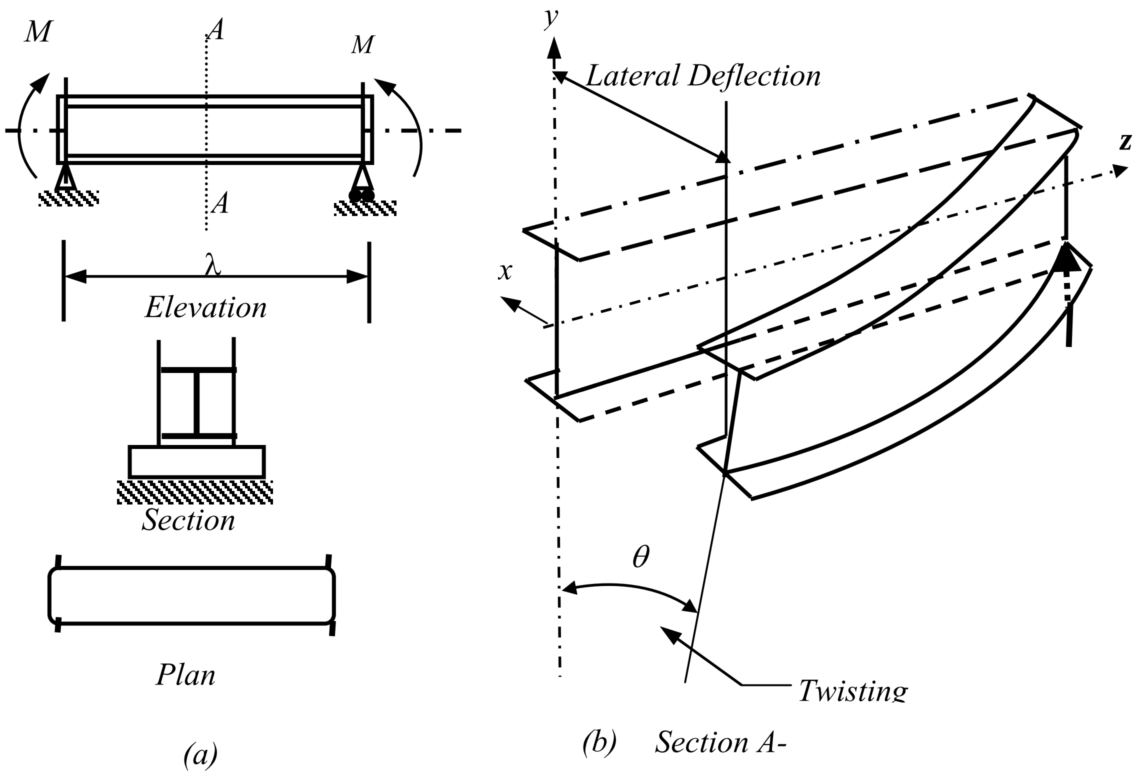


Fig. 5(a) Original beam (b) laterally buckled beam

For the purpose of this discussion, the lateral torsional buckling of an I-section is considered with the following assumptions.

1. The beam is initially undistorted
2. Its behaviour is elastic (no yielding)
3. It is loaded by equal and opposite end moments in the plane of the web.
4. The loads act in the plane of the web only (there are no externally applied lateral or torsional loads)
5. The beam does not have residual stresses
6. Its ends are simply supported vertically and laterally.

Obviously, in practice, the above ideal conditions are seldom met. For example, rolled sections invariably contain residual stresses. The effects of the deviations from the ideal case are discussed in a later Section.

The critical bending moment capacity attained by a symmetric I beam subjected to equal end moments undergoing lateral torsional buckling between points of lateral or torsional support is a function of two torsional characteristics of the specific cross-section: the pure torsional resistance under uniform torsion and the warping torsional resistance

M_{cr} = [(torsional resistance)^2 + (warping resistance)^2]$^{1/2}$

$$M_{cr} = \frac{\pi}{\lambda}\left[EI_y\,GJ + \frac{\pi^2 EI_y\,\Gamma}{\lambda^2}\right]^{\frac{1}{2}} \qquad 1(a)$$

This may be rewritten as

$$M_{cr} = \frac{\pi}{\lambda}\left(EI_y\,GJ\right)^{\frac{1}{2}}\left[1 + \frac{\pi^2 E\Gamma}{\lambda^2 GJ}\right]^{\frac{1}{2}} \qquad 1(b)$$

where, EI_y is the minor axis flexural rigidity
GJ is the torsional rigidity
$E\Gamma$ is the warping rigidity

The torsion that accompanies lateral buckling is always non-uniform. The critical bending moment, M_{cr} is given by Eqn.1 (a).

It is evident from Eqn.1 (*a*) that the flexural and torsional stiffness of the member relate to the lateral and torsional components of the buckling deformations. The magnitude of the second square root term in Eqn.1 (*b*) is a measure of the contribution of warping to the resistance of the beam. In practice, this value is large for short deep girders. For long shallow girders with low warping stiffness, $\Gamma \approx 0$ and Eqn. 1(*b*) reduces to

$$M_{cr} = \frac{\pi}{\lambda}\left(E\,I_y\,G\,J\right)^{\frac{1}{2}} \qquad (2)$$

An I-section composed of very thin plates will posses very low torsional rigidity (since J depends on third power of thickness) and both terms under the root will be of comparable magnitude. The second term is negligible compared to the first for the majority of hot rolled sections. But light gauge sections derive most of the resistance to torsional deformation from the warping action. The beam length also has considerable influence upon the relative magnitudes of the two terms as shown in the term $\pi^2 E\Gamma / \lambda^2 GJ$. Shorter and deep beams ($\pi^2 E\Gamma / \lambda^2 GJ$ term will be large) demonstrate more warping resistance, whereas, the term will be small for long and shallow beams. Eqn. (1) may be rewritten in a simpler form as given below.

$$M_{cr} = \alpha\left(E\,I_y\,G\,J\right)^{\frac{1}{2}}\left[\frac{\pi}{\lambda}\left(1 + \frac{\pi^2}{B^2}\right)^{\frac{1}{2}}\right] \qquad (3)$$

where $B^2 = \lambda^2\,G\,J / E\,\Gamma$ 3(a)

$$M_{cr} = \alpha(E\, I_y\, G\, J)^{1/2}\, \gamma \tag{4}$$

where $\gamma = \pi/\lambda\ (1+\pi^2 / B^2)^{1/2}$ 4(a)

Eqn. (4) is a product of three terms: the first term, α, varies with the loading and support conditions; the second term varies with the material properties and the shape of the beam; and the third term, γ, varies with the length of the beam. Eqn. (4) is regarded as the basic equation for lateral torsional buckling of beams. The influence of the three terms mentioned above is discussed in the following Section.

4.1 LATERAL TORSIONAL BUCKLING AS STIPULATED IN NEW IS: 800:

New IS: 800 take into account the effect of elastic critical moment for consideration of lateral torsional buckling and the stipulations are as follows:

a) Simplified equation for prismatic members made of standard rolled I-sections and welded doubly symmetric I-sections:

$$M_{cr} = \frac{\pi^2 E I_y h_f}{2L_{LT}^2}\left[1+\frac{1}{20}\left(\frac{L_{LT}/r_y}{h_f/t_f}\right)^2\right]^{0.5} = \beta_b Z_p f_{cr,b}$$

I_t = torsional constant = $\sum b_i t_i^3 / 3$ for open section

I_w = warping constant

I_y, r_y = moment of inertia, radius of gyration about the weak axis, respectively

L_{LT} = effective length for lateral torsional buckling (**8.3**)

h_f = Center to center distance between flanges

t_f = thickness of the flange

b) For doubly symmetric prismatic beams

The elastic critical moment corresponding to lateral torsional buckling of a doubly symmetric prismatic beam subjected to uniform moment in the unsupported length and torsionally restraining lateral supports is given by

$$M_{cr} = \frac{\pi^2 E I_y}{(KL)^2}\left[\frac{I_w}{I_y} + \frac{G\, I_t\, (KL)^2}{\pi^2 E\, I_y}\right]^{0.5}$$

Where

I_y, I_w, I_t = Moment of inertia about the minor axis, warping constant and St. Venants torsion constant of the cross section, respectively

G = Modulus of rigidity

KL = Effective length against lateral torsional buckling

c) For sections symmetric about minor axis:

In case of a beam which is symmetrical only about the minor axis, and bending about major axis, the elastic critical moment for lateral torsional buckling is given by the general equation,

$$M_{cr} = c_1 \frac{\pi^2 E I_y}{(KL)^2} \left\{ \left[\left(\frac{K}{K_w} \right)^2 \frac{I_w}{I_y} + \frac{G I_t (KL)^2}{\pi^2 E I_y} + \left(c_2\, y_g - c_3\, y_j \right)^2 \right]^{0.5} - \left(c_2\, y_g - c_3\, y_j \right) \right\}$$

[c_1, c_2, c_3 = factors depending upon the loading and end restraint conditions (Table F.1)

K, K_w = effective length factors of the unsupported length accounting for boundary conditions at the end lateral supports].

The effective length factors K varies from 0.5 for complete restraint against rotation about weak axis to 1.0 for free rotate about weak axis, with 0.7 for the case of one end fixed and other end free. It is analogous to the effective length factors for compression members with end rotational restraint.

The K_w factor refers to the warping restraint. Unless special provisions to restrain warping of the section at the end lateral supports are made, K_w should be taken as 1.0.

y_g is the y-distance between the point of application of the load and the shear centre of the cross section and is positive when the load is acting towards the shear centre from the point of application

$y_j = y_s - 0.5 \int_A (z^2 - y^2)\, y\, dA / I_z$

y_s is the coordinate of the shear centre with respect to centroid, positive when the shear centre is on the compression side of the centroid

y, z are coordinates of the elemental area with respect to centroid of the section

The z_j can be calculated by using the following approximation

a) Plain flanges

$y_j = 0.8\,(2\beta_f - 1)\, h_y / 2.0$ (when $\beta_f > 0.5$)

$y_j = 1.0\,(2\beta_f - 1)\, h_y / 2.0$ (when $\beta_f \leq 0.5$)

b) Lipped flanges

$y_j = 0.8\,(2\beta_f - 1)\,(1 + h_L/h)\, h_y/2$ (when $\beta_f > 0.5$)

$y_j = (2\beta_f - 1)\,(1 + h_L/h)\, h_y/2$ (when $\beta_f \leq 0.5$)

Where,

h_L = height of the lip

h = overall height of the section

h_y = distance between shear centre of the two flanges of the cross section

The torsion constant I_t is given by

$I_t = \sum b_i t_i^3 / 3$ for open section

$= 4 A_e^{\,2} / \sum (b/t)$ for hollow section

Where,

A_e = area enclosed by the section

b, t = breadth and thickness of the elements of the section respectively

The warping constant, I_w, is given by

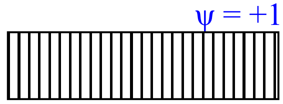
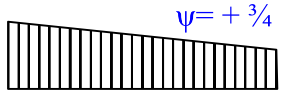
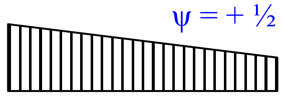
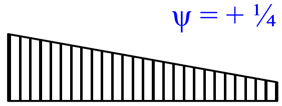
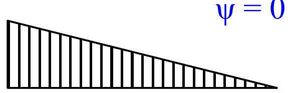
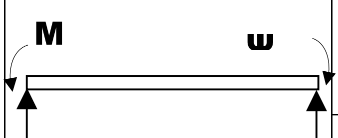
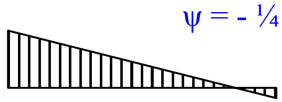
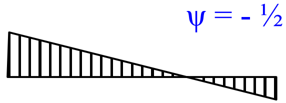
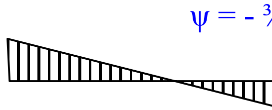
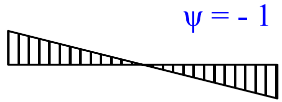
$I_w = (1-\beta_f)\, \beta_f\, I_y\, h_y^2$ for I sections mono-symmetric about weak axis

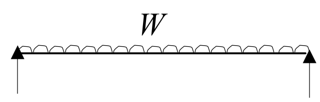
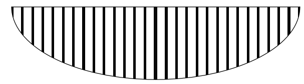
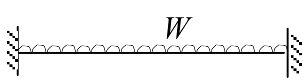
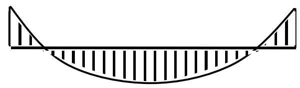
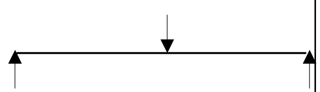
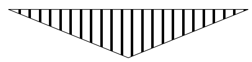
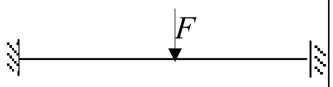
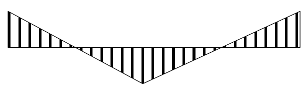
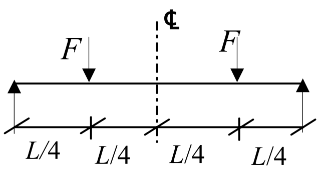
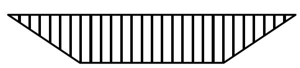
$= 0$ for angle, Tee, narrow rectangle section and approximately for hollow sections

$\beta_f = I_{fc} / (I_{fc} + I_{ft})$ where I_{fc}, I_{ft} are the moment of inertia of the compression and tension flanges, respectively, about the minor axis of the entire section

TABLE F.1 CONSTANTS c_1, c_2, AND c_3

(*Section* F.1.2)

Loading and Support Conditions	**Bending Moment Diagram**	**Value of K**	**Constants**		
			c_1	c_2	c_3
	$\psi = +1$ 	1.0 0.7 0.5	1.000 1.000 1.000	---	1.000 1.113 1.144
	$\psi = +\frac{3}{4}$ 	1.0 0.7 0.5	1.141 1.270 1.305	---	0.998 1.565 2.283
	$\psi = +\frac{1}{2}$ 	1.0 0.7 0.5	1.323 1.473 1.514	---	0.992 1.556 2.271
	$\psi = +\frac{1}{4}$ 	1.0 0.7 0.5	1.563 1.739 1.788	---	0.977 1.531 2.235
	$\psi = 0$ 	1.0 0.7 0.5	1.879 2.092 2.150	---	0.939 1.473 2.150
	$\psi = -\frac{1}{4}$ 	1.0 0.7 0.5	2.281 2.538 2.609	---	0.855 1.340 1.957
	$\psi = -\frac{1}{2}$ 	1.0 0.7 0.5	2.704 3.009 3.093	---	0.676 1.059 1.546
	$\psi = -\frac{3}{4}$ 	1.0 0.7 0.5	2.927 3.009 3.093	---	0.366 0.575 0.837
	$\psi = -1$ 	1.0 0.7 0.5	2.752 3.063 3.149	---	0.000 0.000 0.000

Loading and Support Conditions	Bending Moment Diagram	Value of K	Constants		
			c_1	c_2	c_3
		1.0 0.5	1.132 0.972	0.459 0.304	0.525 0.980
		1.0 0.5	1.285 0.712	1.562 0.652	0.753 1.070
		1.0 0.5	1.365 1.070	0.553 0.432	1.780 3.050
		1.0 0.5	1.565 0.938	1.257 0.715	2.640 4.800
		1.0 0.5	1.046 1.010	0.430 0.410	1.120 1.390

5.0 FACTORS AFFECTING LATERAL STABILITY

The elastic critical moment, M_{cr}, as obtained in the previous Section, is applicable only to a beam of I section which is simply supported and subjected to end moments. This case is considered as the basic case for future discussion. In practical situations, support conditions, beam cross section, loading etc. vary from the basic case. The following sections elaborate on these variations and make the necessary modifications to the basic case for design purposes.

5.1 Support conditions

The lateral restraint provided by the simply supported conditions assumed in the basic case is the lowest and therefore M_{cr} is also the lowest. It is possible, by other restraint conditions, to obtain higher values of M_{cr}, for the same structural section, which would result in better utilization of the section and thus saving in weight of material. As lateral buckling involves three kinds of deformations, namely *lateral bending, twisting* and *warping*, it is feasible to think of various types of end conditions. But, the supports should either completely prevent or offer no resistance to each type of deformation. Solutions for partial restraint conditions are complicated. The effect of various support conditions is taken into account by way of a parameter called *effective length,* which is explained, in the next Section.

5.2 Effective length

The concept of effective length incorporates the various types of support conditions. For the beam with simply supported end conditions and no intermediate lateral restraint, the effective length is equal to the actual length between the supports. When a greater amount of lateral and torsional restraints is provided at supports, the effective length is less than the actual length and alternatively, the length becomes more when there is less restraint. The effective length factor would indirectly account for the increased lateral and torsional rigidities provided by the restraints.

For simply supported beams and girders of span length, *L,* where no lateral restraint to the compression flanges is provided, but where each end of the beam is restrained against torsion, the effective length L_{LT} of the lateral buckling shall be taken as in Table 2 (Table 8.3 of New IS: 800).

Table 2 Effective length for Simply Supported Beams
(Table 8.3 of New IS: 800, Effective Length of Simply Supported Beams, L_{LT})

Conditions of restraint at supports		Loading condition	
Torsional restraint[1]	Warping Restraint[2]	Normal	Destabilising
Fully restrained	Both flanges fully restrained	0.70 L	0.85 L
Fully restrained	Compression flange fully restrained	0.75 L	0.90 L
Fully restrained	Both flanges fully restrained	0.80 L	0.95 L
Fully restrained	Compression flange partially restrained	0.85 L	1.00 L
Fully restrained	Warping not restrained in both flanges	1.00 L	1.20 L
Partially restrained by bottom flange support connection	Warping not restrained in both flanges	1.0 L+ 2D	1.2 L+2D
Partially restrained by bottom flange bearing support	Warping not restrained in both flanges	1.2 L+2D	1.4 L+2D

[1] Torsional restraint prevents rotation about the longitudinal axis
[2] Warping restraint prevents rotation of the flange in its plane
[3] D is the overall depth of the beam

In simply supported beams with intermediate lateral restraints against lateral torsional buckling, the effective length for lateral torsional buckling L_{LT}, shall be taken as the length of the relevant segment in between the lateral restraints. The effective length shall be equal to 1.2 times the length of the relevant segment in between the lateral restraints.

Restraint against torsional rotation at supports in these beams may be provided by:

i) web or flange cleats, or
ii) bearing stiffeners acting in conjunction with the bearing of the beam, or
iii) lateral end frames or external supports provide lateral restraint to the compression flanges at the ends, or
iv) their being built into walls

5.3 Level of application of transverse loads

The lateral stability of a transversely loaded beam is dependent on the arrangement of the loads as well as the level of application of the loads with respect to the centroid of the cross section. Fig. 6 shows a centrally loaded beam experiencing either destabilising or restoring effect when the cross section is twisted.

A load applied above the centroid of the cross section causes an additional overturning moment and becomes more critical than the case when the load is applied at the centroid. On the other hand, if the load is applied below the centroid, it produces a stabilising effect. Thus, a load applied below or above the centroid can change the buckling load by *± 40%.* The location of the load application has no effect if a restraint is provided at the load point. For example, New IS: 800 takes into account the destabilising effect of top flange loading by using a notional effective length of 1.2 times the actual span to be used in the calculation of effective length (see Table 2).

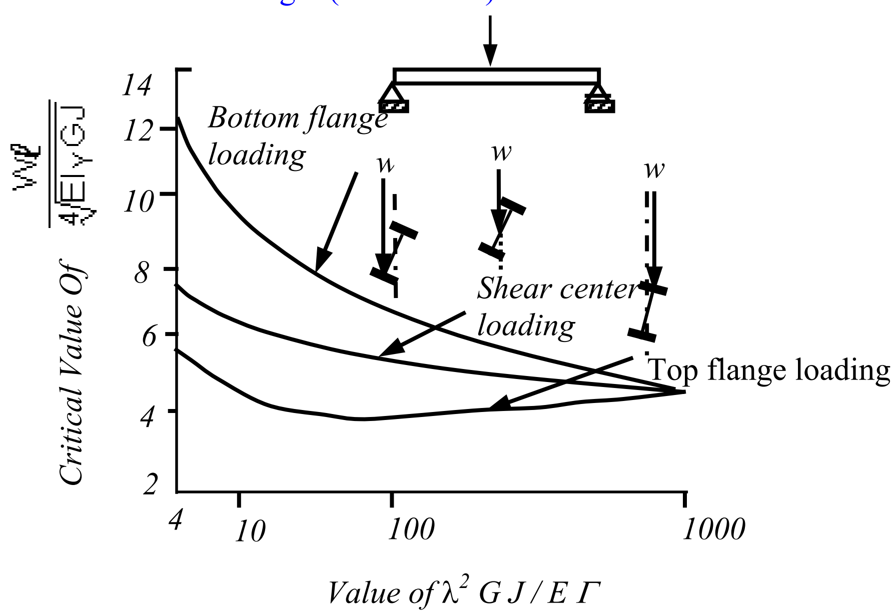


Fig.6 Effect of level of loading on beam stability

Provision of intermediate lateral supports can conveniently increase the lateral stability of a beam. With a central support, which is capable of preventing lateral deflection and twisting, the beam span is halved and each span behaves independently. As a result, the rigidity of the beam is considerably increased. This aspect is dealt in more detail in a later chapter.

5.4 Influence of type of loading

So far, only the basic case of beams loaded with equal and opposite end moments has been considered. But, in reality, loading patterns would vary widely from the basic case. The two reasons for studying the basic case in detail are: (1) it is analytically amenable, and (2) the loading condition is regarded as the most severe. Cases of moment gradient, where the end moments are unequal, are less prone to instability and this beneficial effect is taken into account by the use of ***"equivalent uniform moments".*** In this case, the basic design procedure is modified by comparing the elastic critical moment for the actual case with the elastic critical moment for the basic case. This process is similar to the effective length concept in strut problems for taking into account end fixity.

5.4.1 Loading applied at points of lateral restraint

While considering other loading cases, the variation of the bending moment within a segment (i.e. the length between two restraints) is assumed to be linear from M_{max} at one end to M_{min} at the other end as shown in Fig. 7.

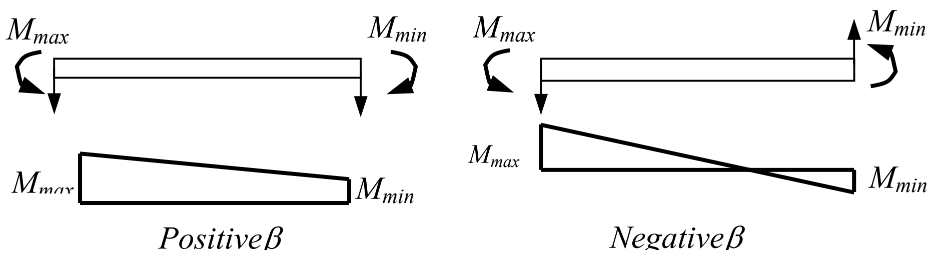


Fig. 7 Non uniform distribution of bending moment

The value of β is defined as $\beta = M_{min} / M_{max}$ $\quad (1.0 \geq \beta \geq -1.0)$ (5)

The value of β is positive for opposing moments at the ends (single curvature bending) and negative for moments of the same kind (double curvature bending). For a particular case of β, the value of M at which elastic instability occurs can be expressed as a ratio *'m'* involving the value of M_{cr} for the segment i.e. the elastic critical moment for $\beta = 1.0$. The ratio may be expressed as a single curve in the form:

$$m = 0.57 + 0.33\beta + 0.1\beta^2 \nless 0.43 \qquad (6)$$

The quantity '*m'* is usually referred to as the *equivalent uniform moment factor.*

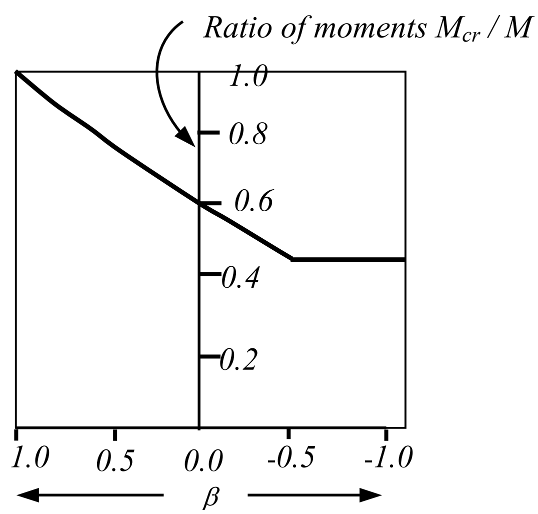


Fig. 8 'm' factor for equivalent uniform moment

The relationship is also expressed in Fig. 8. As seen from the figure, $m = 1.0$ for uniform moment and $m < 1.0$ for non uniform moment; therefore, beam with variation of moment over the unsupported length is less vulnerable to lateral stability as compared to that subjected to uniform moment. Its value is a measure of the intensity of the actual pattern of moments as compared with the basic case. In many cases, its value is dependent only on the shape of the moment diagram and a few examples are presented in Fig.9.

A good estimate of the critical moment due to the actual loading may be found using the proper value of m in the equation

$$M = (1 / m)\, M_{cr} \tag{7}$$

This approximation helps in predicting the buckling of the segments of a beam, which is loaded through transverse members preventing local lateral deflection and twist. Each segment is treated as a beam with unequal end moments and its elastic critical moments may be determined from the relationship given in Eqn.7. The critical moment of each segment can be determined and the lowest of them would give a conservative approximation to the actual critical moment.

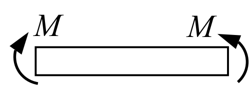
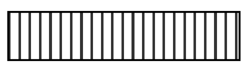
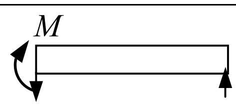
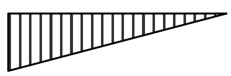
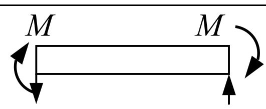
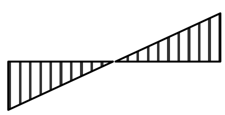
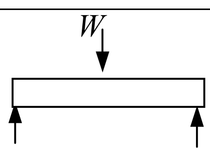
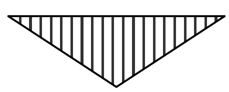
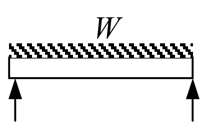
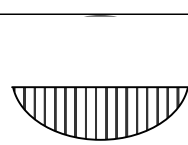
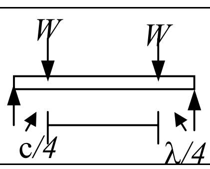
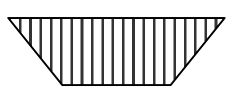
Beam and loads	Actual bending moment	M_{max}	m	Equivalent uniform moment
		M	1.0	
		M	0.57	
		M	0.43	
		$W\lambda/4$	0.74	
		$W\lambda^2/8$	0.88	
		$W\lambda/4$	0.96	

Fig. 9 Equivalent uniform moment

It may be noted here that the values of *'m'* apply only when the point of maximum moment occurs at one end of the segments of the beams with uniform cross section and equal flanges. In all other cases $m=1.0$. For intermediate values of β, m can be determined by Eqn. 6 or can be interpolated from Fig 8. The local strength at the more heavily stressed end also may be checked against plastic moment capacity, M_p as in Eqn. 8.

$M_{max} \le M_p$. (8)

5.4.2 Use of *m* factors in design

As discussed earlier, the shape of the moment diagram influences the lateral stability of a beam. A beam design using uniform moment loading will be unnecessarily conservative. In order to account for the non-uniformity of moments, a modification of the moment may be made based on a comparison of the elastic critical moment for the basic case. This can be done in two ways. They are:

(i) Use equivalent uniform moment value $\overline{M} = m\,M_{max}$ (M_{max} is the larger of the two end moments) for checking against the buckling resistance moment M_b.

(ii) M_b value is determined using an effective slenderness ratio $\lambda'_{LT} = \lambda_{LT}\sqrt{m}$.
(where λ_{LT} is the lateral torsional slenderness ratio and λ'_{LT} is the effective lateral torsional slenderness ratio).

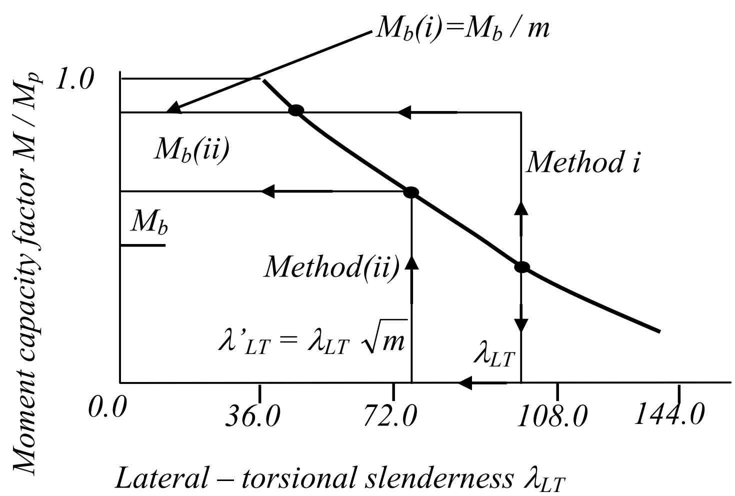


Fig. 10 Moment capacity of beams

The idea of lateral torsional slenderness λ_{LT} is introduced here to write the design capacity M_b as

$$\frac{M_b}{M_p} = f\left(\frac{1}{\lambda_{LT}^2}\right) \qquad (9)$$

where M_p is the fully plastic moment

The quantity λ_{LT} is defined by

$$\lambda_{LT} = \sqrt{\frac{\pi^2 E}{p_y}}\sqrt{\frac{M_p}{M_{cr}}} \tag{10}$$

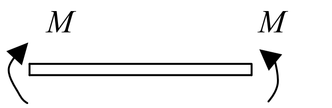
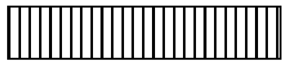
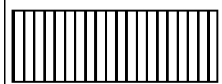
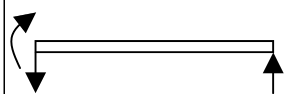
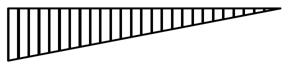
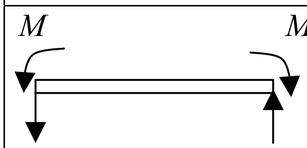
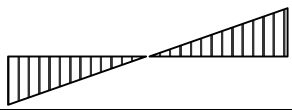
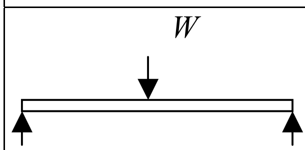
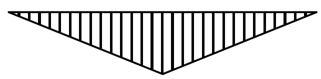
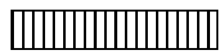
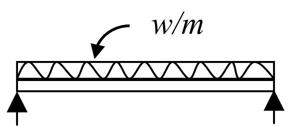
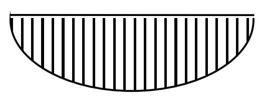
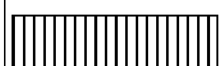
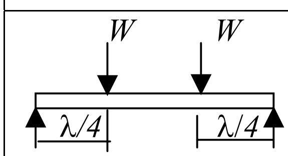
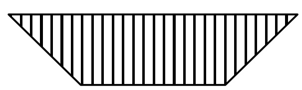
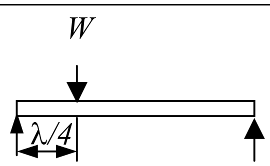
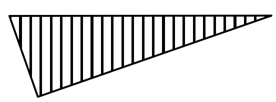
For a particular material (i.e particular E and p_y) the above equation can be considered as a product of c constant and $\sqrt{\frac{M_p}{M_{cr}}}\left(\overline{\lambda}_{LT}\right)$. The quantity $\overline{\lambda}_{LT}$ is called as the new defined slenderness ratio.

Buckling resistance moment, M_b is always less than the elastic critical moment, M_{cr}. Therefore, the second method is more conservative especially for low values of λ_{LT}. The two methods are compared in Fig. 10, where for the first case M_{max} is to be checked against $M_b\ / m$ and for the second case against M_b only. Method (i) is more suitable for cases where loads are applied only at points of effective lateral restraint. Here, the yielding is restricted to the supports; consequently, results in a small reduction in the lateral buckling strength. In order to avoid overstressing at one end, an additional check, $M_{max} < M_p$ should also be satisfied. In certain situations, maximum moment occurs within the span of the beam. The reduction in stiffness due to yielding would result in a smaller lateral buckling strength. In this case, the prediction according to method (i) based on the pattern of moments would not be conservative; here the method (ii) is more appropriate. In the second method, a correction factor n is applied to the slenderness ratio λ_{LT} and design strength is obtained for $n\lambda_{LT}$. It is clear from the above that $n = \sqrt{m}$. The slenderness correction factor is explained in the next section.

5.4.3 Slenderness correction factor

For situations, where the maximum moment occurs away from a braced point, e.g. when the beam is uniformly loaded in the span, a modification to the slenderness, λ_{LT}, may be used. The allowable critical stress is determined for an effective slenderness, $n\lambda_{LT}$, where n is the slenderness correction factor, as illustrated in Fig. 11 for a few cases of loading.

For design purposes, one of the above methods – either the moment correction factor method (m method) or slenderness correction factor method (n method) may be used. If suitable values are chosen for m and n, both methods yield identical results. The difference arises only in the way in which the correction is made; in the n factor method the slenderness is reduced to take advantage of the effect of the non- uniform moment, whereas, in the m factor method, the moment to be checked against lateral moment capacity, M_b, is reduced from M_{max} to $\overline{M}$ by the factor m. It is always safe to use $m = n$ =1 basing the design on uniform moment case. In any situation, either m = 1 or n= 1, i.e. any one method should be used.

Slenderness correction factor, *n*			
Load pattern	Actual bending moment	*n*	Equivalent uniform moment
		1.0	
		0.77	
		0.65	
		0.86	
		0.94	
		0.94	
		0.94	

5.5 Effect of cross-sectional shape

The shape of the cross-section of a beam is a very important parameter while evaluating its lateral buckling capacity. In other words, lateral instability can be reduced or even avoided by choosing appropriate sections. The effect of cross-sectional shape on lateral instability is illustrated in Fig. 12 for different type of section with same cross sectional area.

The figure shows that the I-section with the larger in-plane bending stiffness does not have matching stability. Box sections with high torsional stiffness are most suitable for beams. However, I-sections are commonly used due to their easy availability and ease of connections. Box sections are used as crane girders where the beam must be used in a laterally unsupported state.

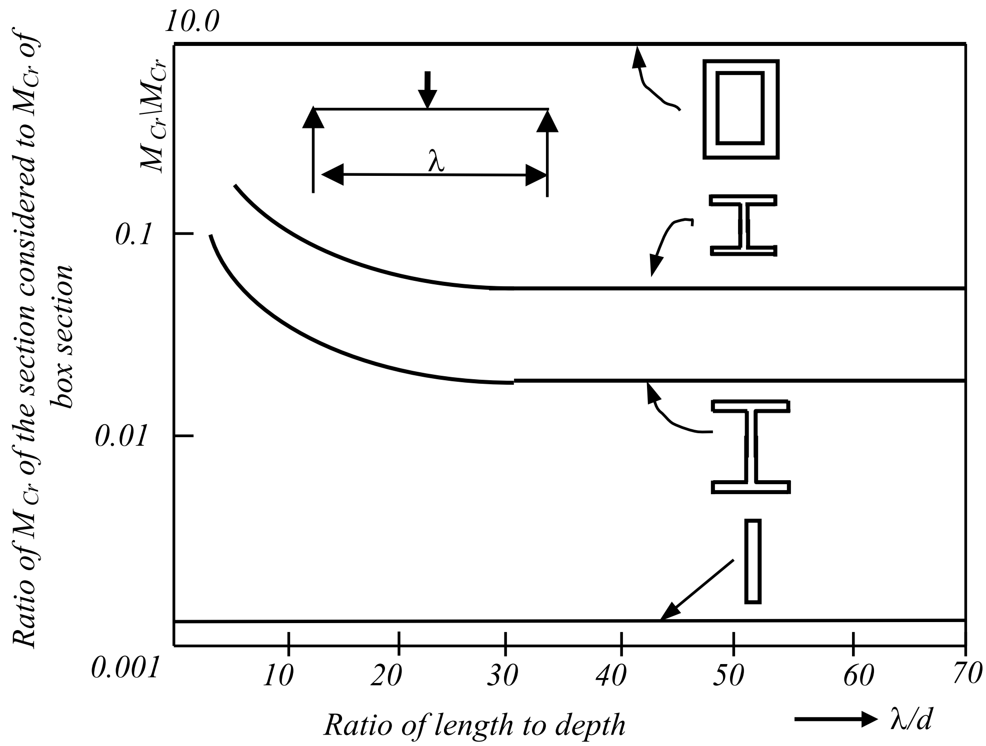


Fig. 12 Effect of type of cross section

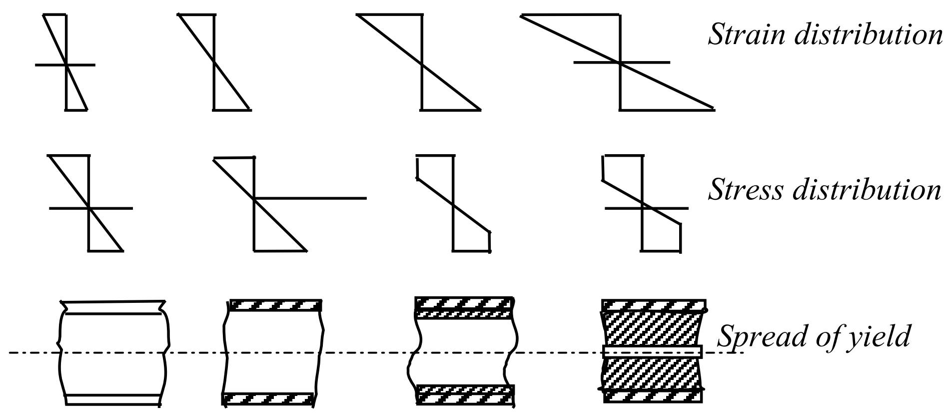
6.0 BUCKLING OF REAL BEAMS

The theoretical assumptions made in section 4.0 are generally not realised in practice. In this section, the behaviour of real beams (which do not meet all the assumptions of the buckling theory) is explained. Effects of plasticity, residual stresses and imperfections are described in the following sections.

6.1 Plasticity effects

Initially, the case, where buckling is not elastic is considered. All other assumptions hold good. As the beam undergoes bending under applied loads, the axial strain distribution at a point in the beam varies along the depth as shown in Fig.13.

With the increase in loading, yielding of the section is initiated at the outer surfaces of the top and bottom flanges. If the M_{cr} of the section as calculated by Eqn.1 is less than M_y, then the beam buckles elastically. In the case where M_{cr} is greater than M_y, some amount of plasticity is experienced at the outer edges before buckling is initiated. If the beam is sufficiently stocky, the beam section attains its full plastic moment capacity, M_p. The interaction between instability and plasticity is shown in Fig. 14.



(Elastic –perfectly plastic material behaviour is assumed)

Fig 13 Strain / Stress Distribution and yielding of section

There are three distinct regions in the curve as given below.

1. Beams with high slenderness ($\sqrt{\frac{M_p}{M_{cr}}} > 1.2$). The failure of the beam is by elastic lateral buckling at M_{cr}
2. Beams of intermediate slenderness 0.4 < $\sqrt{\frac{M_p}{M_{cr}}} < 1.2$), where failure occurs by inelastic lateral buckling at loads below M_p and above M_{cr}
3. Stocky beams ($\sqrt{\frac{M_p}{M_{cr}}} < 0.4$)), which attain M_p without buckling.

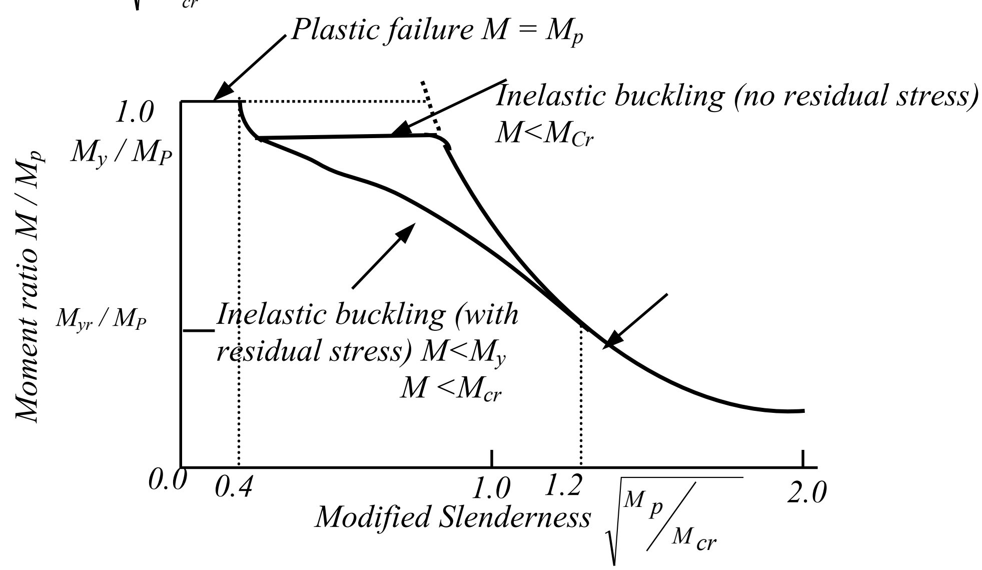


Fig. 14 Interaction between instability and plasticity

6.2 Residual stresses

It is normally assumed that a structural section in the unloaded condition is free from stress and strain. In reality, this is not true. During the process of manufacture of steel sections, they are subjected to large thermal expansions resulting in yield level strains in the sections. As the subsequent cooling is not uniform throughout the section, self-equilibrating patterns of stresses are formed. These stresses are known as *residual stresses*. Similar effects can also occur at the fabrication stage during welding and flame cutting of sections. A typical residual stress distribution in a hot rolled steel beam section is shown in Fig.15.

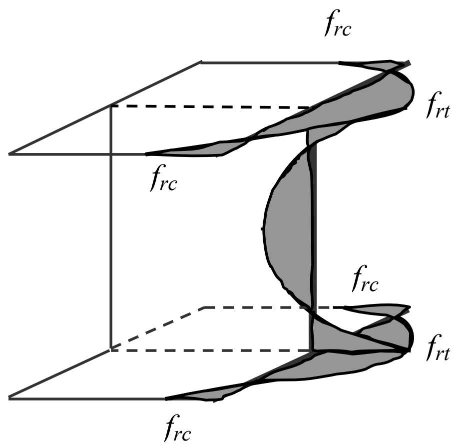


Fig. 15 Residual stresses in I beams

Due to the presence of residual stresses, yielding of the section starts at lower moments. Then, with the increase in moment, yielding spreads through the cross-section. The in-elastic range, which starts at M_{yr} increases instead of the elastic range. The plastic moment value M_p is not influenced by the presence of residual stresses.

6.3 Imperfections

The initial distortion or lack of straightness in beams may be in the form of a lateral bow or twist. In addition, the applied loading may be eccentric inducing more twist to the beam. It is clear that these initial imperfections correspond to the two types of deformations that the beam undergoes during lateral buckling. Assuming $M_{cr} < M_y$, the lateral deflection and twist increase continuously from the initial stage of loading assuming large proportion as M_{cr} is reached. The additional stresses, thus produced, would cause failure of the beam as the maximum stress in the flange tips reaches the yield stress. This form of failure by limiting the stress to yield magnitude is shown in Fig. 16. In the case of beams of intermediate slenderness, a small amount of stress redistribution takes place after yielding and the prediction by the limiting stress approach will be conservative. If residual stresses were also included, the failure load prediction would be conservative even for slender beams.

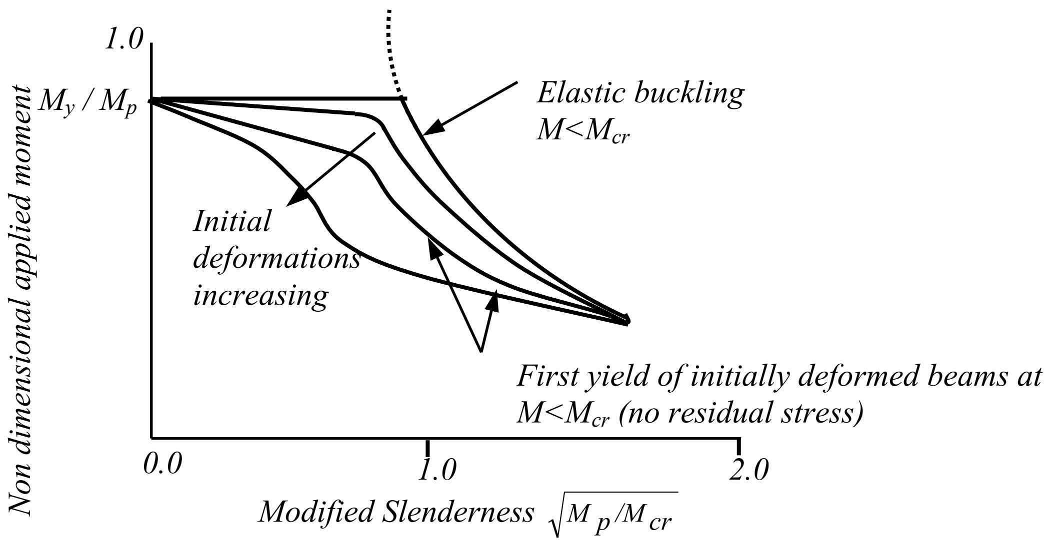


Fig. 16 Beam failure curve

While studying the behaviour of beams, it is necessary to account for the combined effects of the various factors such as instability, plasticity, residual stresses and geometrical imperfections.

7.0 DESIGN APPROACH

Lateral instability is a prime design consideration for all laterally unsupported beams except for the very stocky ones. The value M_{cr} is important in assessing their load carrying capacity. The non-dimensional modified slenderness $\bar{\lambda}_{LT} = \sqrt{M_p / M_{cr}}$ indicates the importance of instability and as a result the governing mode of failure.

For design purposes, the application of the theoretical formula is too complex. Further, there is much difference between the assumptions made in the theory and the real characteristics of the beams. However, as the theoretical prediction is elastic, it provides an upper bound to the true strength of the member. A non-dimensional plot with abscissa as $\sqrt{M_p / M_{cr}}$ and the ordinate as M/M_p, where M_p is the plastic moment capacity of section and M is the failure moment shows clearly the lateral torsional behaviour of the beam. Such a non-dimensional plot of lateral torsional buckling moment and the elastic critical moment is shown in Fig 17. Experiments on beams validate the use of such a curve as being representative of the actual test data.

Three distinct regions of behaviour may be noticed in the figure. They are:

- Stocky, where beams attain M_p, with values of $\bar{\lambda}_{LT} < 0.4$
- Intermediate, the region where beams fail to reach either M_P or M_{cr} ; $0.4 < \bar{\lambda}_{LT} < 1.2$
- Slender, where beams fail at moment M_{cr}; $\bar{\lambda}_{LT} > 1.2$

As pointed out earlier, lateral stability is not a criterion for stocky beams. For beams of the second category, which comprise of the majority of available sections, design is based on inelastic buckling accounting for geometrical imperfections and residual stresses.

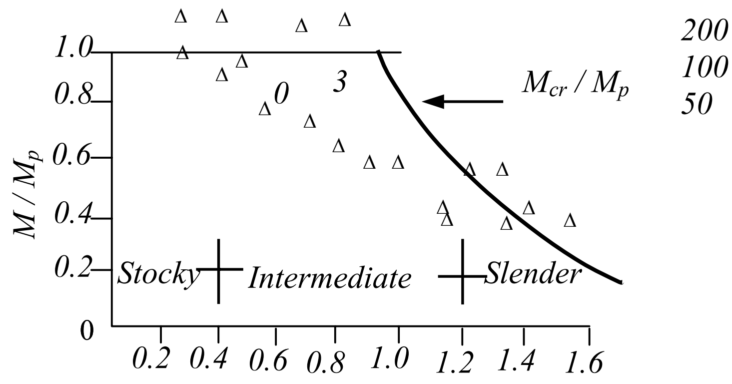


Fig 17. Theoretical elastic critical moment

7.1 Conservative design procedure

The lateral buckling moment capacity of a section can be expressed as

$$M_b = p_b S_x \tag{11}$$

where, p_b is the bending strength accounting for lateral instability
S_x is the appropriate plastic section modulus

The slenderness of the beam λ_{LT} is defined as:

$$\lambda_{LT} = \sqrt{\frac{\pi^2 E}{p_y}}\ \bar{\lambda}_{LT} \tag{12}$$

This has close similarity to the slenderness associated with compressive buckling of a column. The relation between p_b and λ_{LT} is shown in Fig.18.

In the case of slender beams, p_b is related to λ_{LT} . λ_{LT} can be determined for a given section by the following relationship

$$\lambda_{LT} = n\, u\, v\, \lambda_e / r_y \tag{13}$$

where, n is the slenderness correction factor

u is buckling parameter from steel tables (= 0.9 for rolled beams and channels and1.0 for other sections)

v is slenderness factor and $f(\lambda/r_y, x)$, given in Table 14 of BS 5950 part 1; but pproximated to 1.0 for preliminary calculations

x is the torsional index which is provided in BS 5950 part 1

$x = 0.566\, h \left(A / J\right)^{1/2}$ for bi-symmetric sections and sections symmetric about minor axis, and

$x = 1.132 \left(A\, H / I_y J\right)^{1/2}$ for sections symmetric about major axis.

where

A is the cross sectional area of the member.
I_y is the second moment of the area about the minor axis
H is the warping constant
J is the torsion constant
h is the distance between the shear center of the flanges.

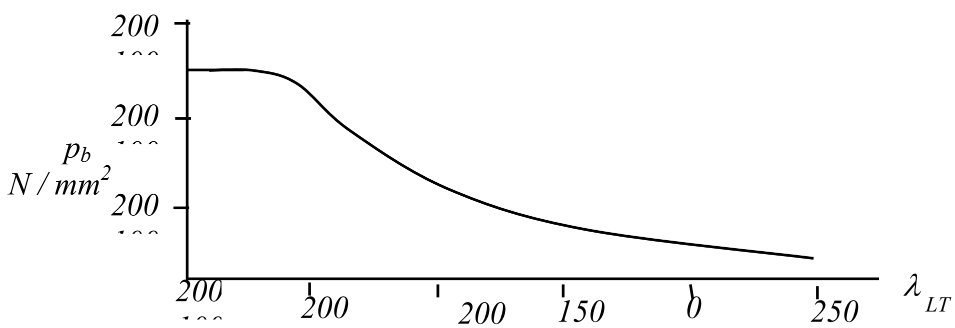


Fig. 18 Bending strength for rolled sections of design strength 240 N / mm²

For compact sections, full plasticity is developed at the most heavily stressed section. Unlike plastic design, moment redistribution is not considered here. For example, for a particular grade of steel and for $\bar{\lambda}_{LT} < 0.4$, when p_b attains the value of p_y, $\lambda_{LT} = 37$. Hence, this is the value of maximum slenderness for which instability does not influence strength.

A good design can be achieved by determining the value of λ_{LT} and thereby p_b more accurately. M_b can be determined using Eqn.11. Effective lengths of the beam may be adopted as per the guidelines given in Table 2. For beams, and segments of beams between lateral supports, equivalent uniform moments may be calculated to determine their relative severity of instability. The lateral stability is checked for an equivalent moment $\bar{M}$ given by

$$\bar{M} = m\, M_{max} \qquad (14)$$

where m is the equivalent uniform moment factor.

If $M_b > \bar{M}$, the section chosen is satisfactory. At the heavily stressed locations, local strength should be checked against development of M_p.

$$M_{max} \ngtr M_p \qquad (15)$$

7.2 Design approach as per New IS: 800:

The New IS: 800 follows the same design philosophy with certain alterations in the parameters for calculating design bending strength governed by lateral torsional buckling. The step by step design procedure has been detailed below:

The design bending strength of laterally unsupported beam as governed by lateral torsional buckling is given by:

$M_d = \beta_b Z_p f_{bd}$

[β_b = 1.0 for plastic and compact sections

= Z_e/Z_p for semi-compact sections

Z_p, Z_e = plastic section modulus and elastic section modulus with respect to extreme compression fibre.]

f_{bd} = design bending compressive stress, obtained as given below:

f_{bd} = $\chi_{LT} f_y / \gamma_{m0}$

χ_{LT} = bending stress reduction factor to account for lateral torisonal buckling

$$\chi_{LT} = \frac{1}{\left\{\phi_{LT} + \left[\phi_{LT}^2 - \lambda_{LT}^2\right]^{0.5}\right\}} \le 1.0$$

$$\phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\lambda_{LT} - 0.2\right) + \lambda_{LT}^{\ 2}\right]$$

α_{LT}, the imperfection parameter is given by:

α_{LT} = 0.21 for rolled steel section

α_{LT} = 0.49 for welded steel section

The non-dimensional slenderness ratio, λ_{LT}, is given by

$$\lambda_{LT} = \sqrt{\beta_b Z_p f_y / M_{cr}}$$

$$= \sqrt{f_y / f_{cr,b}}$$

M_{cr} = elastic critical moment to be calculated as per **8.2.2.1**

$f_{cr,b}$ = extreme fibre bending compressive stress corresponding to elastic lateral buckling moment (**8.2.2.1**, Table 8.1)

8.0 SUMMARY

Unrestrained beams that are loaded in their stiffer planes may undergo lateral torsional buckling. The prime factors that influence the buckling strength of beams are: the un braced span, cross sectional shape, type of end restraint and the distribution of moment. For the purpose of design, the simplified approach as given in BS: 5950 Part-1 has been presented. The effects of various parameters that affect buckling strength have been accounted for in the design by appropriate correction factors. The behaviour of real beams (which do not comply with the theoretical assumptions) has also been described. In order to increase the lateral strength of a beam, bracing of suitable stiffness and strength has to be provided.

9.0 REFERENCES

1. Timoshenko S., ‘Theory of elastic stability’ McGraw Hill Book Co., 1st Edition 1936.

2. Clarke A.B. and Coverman, ‘Structural steel work-Limit state design’, Chapman and Hall, London, 1987

3. Martin L.H. and Purkiss J.A., ‘Structural design of steel work to BS 5950, Edward Arnold, 1992.

4. Trahair N.S., ‘The behaviour and design of steel structures’, Chapman and Hall London, 1977

5. Kirby P.A and Nethercot D.A.,’Design for structural stability’, Granada Publishing, London, 1979

<table>
<tr><td rowspan="4">Structural Steel Design Project

Calculation sheet</td><td>Job No.</td><td>Sheet 1 of 4</td><td>Rev.</td></tr>
<tr><td colspan="3">Job title: UNRESTRAINED BEAM DESIGN</td></tr>
<tr><td colspan="3">Worked example: 1</td></tr>
<tr><td></td><td>Made by. GC
Checked by. TKB</td><td>Date.23/02/07
Date. 26/02/07</td></tr>
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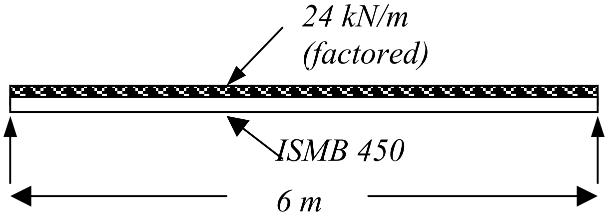
Problem - 1

Check the adequacy of ISMB 450 to carry a uniformly distributed load of 24 kN / m over a span of 6 m. Both ends of the beam are attached to the flanges of columns by double web cleat.

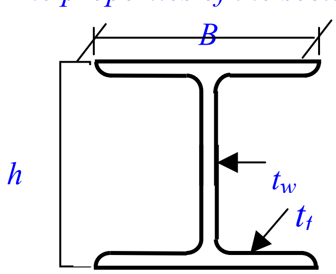


Design check:

For the end conditions given, it is assumed that the beam is simply supported in a vertical plane, and at the ends the beam is fully restrained against lateral deflection and twist with, no rotational restraint in plan at its ends.

Section classification of ISMB 450:

The properties of the section are:



Depth, $h = 450$ *mm*

Width, $B = 150$ *mm*

Web thickness, $t_w = 9.4$ *mm*

Flange thickness, $t_f = 17.4$ *mm*

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	Job title: *UNRESTRAINED BEAM DESIGN*		
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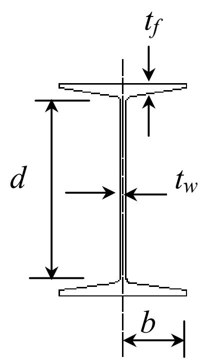

Rolled Steel Beams

Depth between fillets, d = 379.2 mm

Radius of gyration about minor axis, r_y = 30.1 mm

Plastic modulus about major axis, Z_p = 1533.36 x 10^3 mm^3 — *Appendix I of IS: 800*

Assume f_y = 250 N/mm^2, E=200000 N/mm^2, γ_m = 1.10,

(I) Type of section

Flange criterion:

$$b = \frac{B}{2} = \frac{150}{2} = 75\ mm$$

$$\frac{b}{t_f} = \frac{75.0}{17.4} = 4.31$$

$$\frac{b}{t_f} < 9.4\varepsilon \quad where\ \varepsilon = \sqrt{\frac{250}{f_y}}$$

Hence O.K.

Web criterion:

$$\frac{d}{t_w} = \frac{379.2}{9.4} = 40.3$$

$$\frac{d}{t_w} < 84\varepsilon$$

Hence O.K.

Since $\frac{b}{t_f} < 9.4\varepsilon$ *and* $\frac{d}{t_w} < 84\varepsilon$, *the section is classified as 'plastic'* — *Table 3.1 (Section 3.7.2) of IS: 800*

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(II) Check for lateral torsional buckling:

Check for Slenderness Ratio:

Effective length criteria:

With ends of compression flanges fully restrained for torsion at support but both the flanges are not restrained against Warping, effective length of simply supported beam $L_{LT} = 1.0\ L$, where L is the span of the beam.

Table 8.3 of IS: 800

Hence, $L_{LT} = 1.0 \times 6.0\ M = 6000\ mm$, $L_{LT}/r = 6000/30.1 = 199.33$

And $h/t_f = 450/17.4 = 25.86$,

Corresponding value of Critical Stress, $f_{cr,b} = 99.47\ N/mm^2$

Table 8.2 of IS: 800

For $f_{cr,b} = 99.47\ N/mm^2$, $f_{bd} = 76.94\ N/mm^2$ *for* $\alpha_{LT} = 0.21$ *for rolled steel section*

Table 8.1a of IS: 800

Now, $M_d = \beta_b Z_p f_{bd}$

where

β_b = 1.0 for plastic and compact sections
= Z_e/Z_p for semi-compact sections

Z_p, Z_e = plastic section modulus and elastic section modulus with respect to extreme compression fibre.

f_{bd} = design bending compressive stress, obtained as given in Table 8.1a of New IS: 800

Hence, $M_d = \beta_b Z_p f_{bd} = 1.0 \times 1533.36 \times 76.94/1000 = 117976.72/1000$

$= 117.98\ kN\text{-}m$

Hence, Bending strength, $M_d = 117.98\ kN\text{-}m$

Structural Steel Design Project	Job No.	Sheet *4* of *4*	Rev.
	Job title: *UNRESTRAINED BEAM DESIGN*		
	Worked example*: 1*		
Calculation sheet	Made by. *GC*	*Date.23/02/07*	
	Checked by. *TKB*	*Date. 26/02/07*	

For the simply supported beam of 6.0 m span with a factored load of 24.0 KN/m

$$M_{max} = \frac{w\lambda^2}{8} = \frac{24*6^2}{8}$$

$$= 108.0 \text{ KN m} < 117.98 \text{ kN m}$$

$$\text{Hence } M_d > M_{max}$$

$\therefore$ *ISMB 450 is adequate against lateral torsional buckling.*

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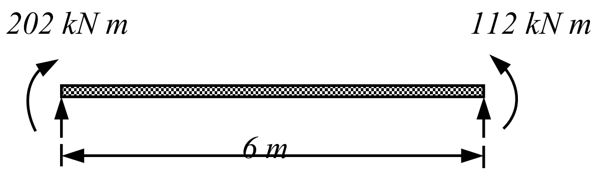
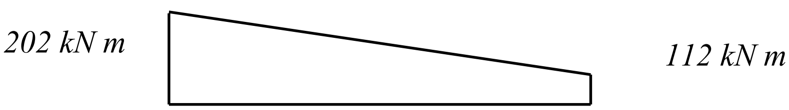
	Worked example: *2*		
		Made by. *GC*	Date. *23/02/07*
Calculation sheet		Checked by. *TKB*	Date. *28/02/07*

Problem-2

(i) A simply supported beam of span 6 m is subjected to end moments of 202 kN m (clockwise) and 112 k N m (anticlockwise) under factored -applied loading. Check whether ISMB 450 is safe with regard to lateral buckling.





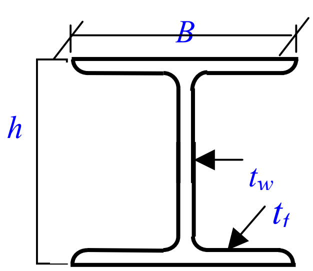
B.M Diagram

Design check:

For the end conditions given, it is assumed that the beam is simply supported in a vertical plane, and at the ends the beam is fully restrained against lateral deflection and twist with, no rotational restraint in plan at its ends.

Section classification of ISMB 450

The properties of the section are:



Depth, h = 450 mm

Width, B = 150 mm

Web thickness, t_w = 9.4 mm

Flange thickness, t_f = 17.4 mm

$I_y = 834 \times 10^4\ mm^4$

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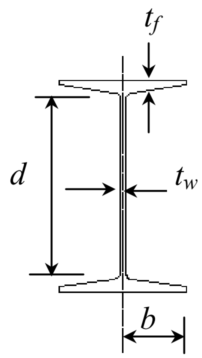
Calculation sheet		Made by. *GC*	Date. *23/02/07*
		Checked by. *TKB*	Date. *28/02/07*



Rolled Steel Beams

Depth between fillets, d = 379.2 mm

Radius of gyration about minor axis, r_y = 30.1 mm

Plastic modulus about major axis, Z_p = 1533.36 x 10^3 mm^3 — *Appendix I of IS: 800*

Assume f_y = 250 N/mm^2, E=200000 N/mm^2, γ_m = 1.10,

(II) Type of section

Flange criterion:

$$b = \frac{B}{2} = \frac{150}{2} = 75\ mm$$

$$\frac{b}{t_f} = \frac{75.0}{17.4} = 4.31$$

$$\frac{b}{t_f} < 9.4\varepsilon \quad where\ \varepsilon = \sqrt{\frac{250}{f_y}}$$

Hence O.K.

Web criterion:

$$\frac{d}{t_w} = \frac{379.2}{9.4} = 40.3$$

$$\frac{d}{t_w} < 84\,\varepsilon$$

Hence O.K.

Since $\frac{b}{t_f} < 9.4\,\varepsilon$ *and* $\frac{d}{t_w} < 84\,\varepsilon$, *the section is classified as 'plastic'* — *Table 3.1 (Section 3.7.2) of IS: 800*

<table>
<tr><td rowspan="5">Structural Steel Design Project

Calculation sheet</td><td>Job No.</td><td>Sheet 3 of 6</td><td>Rev.</td></tr>
<tr><td colspan="3">Job title: UNRESTRAINED BEAM DESIGN</td></tr>
<tr><td colspan="3">Worked example: 2</td></tr>
<tr><td rowspan="2"></td><td>Made by. GC</td><td>Date.23/02/07</td></tr>
<tr><td>Checked by. TKB</td><td>Date.28/02/07</td></tr>
</table>

(II)Check for lateral torsional buckling:

Check for Slenderness Ratio:

Effective length criteria:

With ends of compression flanges fully restrained for torsion at support but both the flanges are not restrained against Warping, effective length of simply supported beam $L_{LT} = 1.0\ L$, where L is the span of the beam.

Table 8.3 of IS: 800

Hence, $L_{LT} = 1.0 \times 6.0\ M = 6000\ mm$, $L_{LT}/r = 6000/30.1 = 199.33$

Since the moment is varying from 155 k-Nm to 86 k-Nm, there will be moment gradient. So for calculation of f_{bd}*, Critical Moment,* M_{cr} *is to be calculated.*

Now, Critical Moment,

Clause F.1.2 of Appendix F of IS: 800

$$M_{cr} = c_1 \frac{\pi^2 EI_y}{(KL)^2} \left\{ \left[\left(\frac{K}{K_w} \right)^2 \frac{I_w}{I_y} + \frac{GI_t (KL)^2}{\pi^2 EI_y} + \left(c_2\, y_g - c_3\, y_j \right)^2 \right]^{0.5} - \left(c_2\, y_g - c_3\, y_j \right) \right\}$$

Where,

c_1, c_2, c_3 = factors depending upon the loading and end restraint conditions (Table F.1)

K, K_w = effective length factors of the unsupported length accounting for boundary conditions at the end lateral supports,

Here, both K *and* K_w can be taken as 1.0.

and

y_g = *y distance between the point of application of the load and the shear centre of the cross section and is positive when the load is acting towards the shear centre from the point of application*

Structural Steel Design Project Calculation sheet	Job No.	Sheet *4* of 6	Rev.
	Job title: *UNRESTRAINED BEAM DESIGN*		
	Worked example: *2*		
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$y_j = y_s - 0.5 \int_A (z^2 - y^2)\, y\, dA / I_z$

y_s = *coordinate of the shear centre with respect to centroid, positive when the shear centre is on the compression side of the centroid*

Here, for plane and equal flange I section,

$y_g = 0.5 \times h = 0.5 \times 0.45 = 0.225\ M = 225\ mm$

$y_j = 1.0\ (2\beta_f - 1)\ h_y / 2.0$ (when $\beta_f \leq 0.5$)

h_y = distance between shear centre of the two flanges of the cross section = $h - t_f$

here, $\beta_f = 0.5$, *and* $h_y = h - t_f = 450 - 17.4 = 432.6\ mm$

hence, $y_j = 1.0 \times (2 \times 0.5 - 1) \times 432.6/2.0 = 0$

and $y_s = 0$

$I_t = \sum b_i t_i^3 / 3$, for open section

$= 2 \times 150 \times 17.4^3 + (450 - 2 \times 17.4) \times 9.4^3 = 192.527 \times 10^4\ mm^4$

The warping constant, I_w, is given by

$I_w = (1-\beta_f)\ \beta_f\ I_y\ h_y^2$ for I sections mono-symmetric about weak axis

$= (1-0.5) \times 0.5 \times 834 \times 10^4 \times 432.6^2 = 39019265.46 \times 10^4\ mm^6$

Modulus of Rigidity, $G = 0.769 \times 10^5 N/mm^2$

Here, $\psi = 86/155 = 0.555$ *and* $K = 1.0$, *for which* $c_1 = 1.283$, $c_2 = 0$ *and* $c_3 = 0.993$

Table F.1 of Appendix F of IS: 800

Hence, Critical Moment,

$$M_{cr} = c_1 \frac{\pi^2 E I_y}{(KL)^2} \left\{ \left[\left(\frac{K}{K_w} \right)^2 \frac{I_w}{I_y} + \frac{G I_t (KL)^2}{\pi^2 E I_y} + \left(c_2\, y_g - c_3\, y_j \right)^2 \right]^{0.5} - \left(c_2\, y_g - c_3\, y_j \right) \right\}$$

Structural Steel Design Project	Job No.	Sheet *5* of 6	Rev.
	Job title: *UNRESTRAINED BEAM DESIGN*		
	Worked example*: 2*		
Calculation sheet		Made by. *GC*	Date.*23/02/07*
		Checked by. *TKB*	Date.*28/02/07*

$$= 1.283\frac{\pi^2 x200000x834x10^4}{(1.0x6000)^2}\left\{\left[\left(\frac{1}{1}\right)^2\frac{39019265x10^4}{834x10^4}+\frac{0.769x10^5 x192.527x10^4 x6000^2}{\pi^2 x200000x834x10^4}\right]^{0.5}\right\}$$

= 357142.72 x 10³ N-mm

Calculation of f_{bd}:

Now, $\lambda_{LT} = \sqrt{\beta_b Z_p f_y / M_{cr}} = \sqrt{1.0x1533.36x10^3 x250/357142.72x10^3}$ *= 1.036* — *Clause 8.2.2 of IS: 800*

for which, $\phi_{LT} = 0.5x\left[1+\alpha_{LT}(\lambda_{LT}-0.2)+\lambda_{LT}^2\right]$

$= 0.5x\left[1+0.21(1.036-0.2)+1.036^2\right] = 1.124$

for which, $\chi_{LT} = \frac{1}{\left\{\phi_{LT}+\left[\phi^2_{LT}-\lambda^2_{LT}\right]^{0.5}\right\}} = \frac{1}{\left\{1.124+\left[1.124^2-1.036^2\right]^{0.5}\right\}}$

$= 0.641$

$f_{bd} = \chi_{LT} f_y / \gamma_{m0}$ *= 0.641 x 250 / 1.10 = 145.68 N/mm²*

Hence, $M_d = \beta_b Z_p f_{bd}$ *=1.0 x 1533.36 x 145.68/1000 = 223379.88/1000*

= 223.38 kN-m

Max. Bending Moment, M_{max} *= 202 kN-m*

Hence, $M_d > M_{max}$ *(223.38 > 202)*

∴ *ISMB 450 is adequate against lateral torsional buckling for the applied bending moments.*

(ii) If the beam of problem (i) is subjected to a central load producing a maximum factored moment of 202 kN m, check whether the beam is still safe.

For this problem with zero bending moments at the supports and central max bending moment being 202 kN-m, — *Table F.1 of Appendix F of IS: 800*

For the value of K = 1.0, $c_1 = 1.365$; $c_2 = 0.553$ *and* $c_3 = 1.780$

Structural Steel Design Project	Job No.	Sheet 6 of 6	Rev.
	Job title: *UNRESTRAINED BEAM DESIGN*		
	Worked example*: 2*		
Calculation sheet	Made by. *GC*		Date.*23/02/07*
	Checked by. *TKB*		Date.*28/02/07*

Hence, Critical Moment,

$$M_{cr} = c_1 \frac{\pi^2 EI_y}{(KL)^2}\left\{\left[\left(\frac{K}{K_w}\right)^2 \frac{I_w}{I_y} + \frac{GI_t (KL)^2}{\pi^2 EI_y} + (c_2\, y_g - c_3\, y_j)^2\right]^{0.5} - (c_2\, y_g - c_3\, y_j)\right\}$$

$$= 1.365 \frac{\pi^2 x 2x10^5 x 834x10^4}{(1.0x6000)^2}\left\{\left[\frac{390.19x10^9}{834x10^4} + \frac{0.769x10^5 x 192527x10^4 x 36x10^6}{\pi^2 x 2x10^5 x 834x10^4} + (0.553x225)^2\right]^{0.5} - 0.553x225\right\}$$

=310158.31 x 10^{3}N-mm

Calculation of f_{bd}:

Now, $\lambda_{LT} = \sqrt{\beta_b Z_p f_y / M_{cr}} = \sqrt{1.0x1533.36x10^3 x250/310158.31x10^3}$ *= 1.112* — *Clause 8.2.2 of IS: 800*

for which, $\phi_{LT} = 0.5x\left[1 + \alpha_{LT}(\lambda_{LT} - 0.2) + \lambda_{LT}^2\right]$

$= 0.5x\left[1 + 0.21(1.112 - 0.2) + 1.112^2\right]$ = 1.214

for which, $\chi_{LT} = \dfrac{1}{\left\{\phi_{LT} + \left[\phi^2_{LT} - \lambda^2_{LT}\right]^{0.5}\right\}} = \dfrac{1}{\left\{1.214 + \left[1.214^2 - 1.112^2\right]^{0.5}\right\}}$

= 0.588

$f_{bd} = \chi_{LT} f_y / \gamma_{m0}$ = 0.588 x 250 / 1.10 = 133.64 N/mm^2

Hence, $M_d = \beta_b Z_p f_{bd}$ =1.0 x 1533.36 x 133.64/1000 = 204918.23/1000

= 204.92 kN-m

Therefore the M_d > M_{max} (204.92 > 202)

Therefore the section ISMB 450 is adequate against lateral torsional buckling for this case also.

12 UNRESTRAINED BEAM DESIGN – II

1.0 INTRODUCTION

The basic theory of beam buckling was explained in the previous chapter. Doubly symmetric I- section has been used throughout for the development of the theory and later discussion. It was established that practical beams fail by:

(i) Yielding, if they are short
(ii) Elastic buckling, if they are long, or
(iii) Inelastic lateral buckling, if they are of intermediate length.

A conservative method of designing beams was also explained and its limitations were outlined.

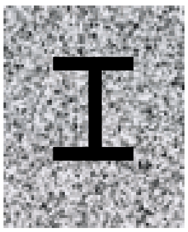
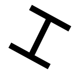
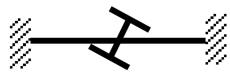
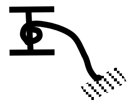
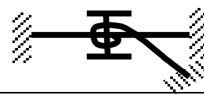
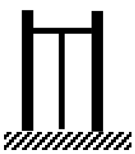
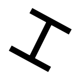
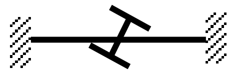
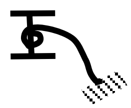
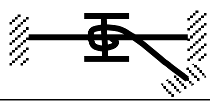
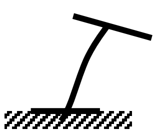
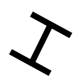
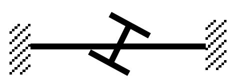
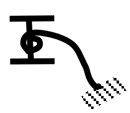
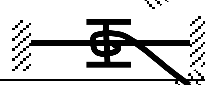
In this chapter a few cases of lateral buckling strength evaluation of beams encountered in practice would be explained. Cantilever beams, continuous beams, beams with continuous and discrete lateral restraints are considered. Cases of monosymmetric beams and non-uniform beams are covered. The buckling strength evaluation of non-symmetric sections is also described.

2.0 CANTILEVER BEAMS

A cantilever beam is completely fixed at one end and free at the other. In the case of cantilevers, the support conditions in the transverse plane affect the moment pattern. For design purposes, it is convenient to use the concept of *notional effective length, k,* which would include both loading and support effects. The notional effective length is defined as the length of the notionally simply supported (in the lateral plane) beam of similar section, which would have an elastic critical moment under uniform moment equal to the elastic critical moment of the actual beam under the actual loading conditions. Recommended values of *'k'* for a number of cases are given in Table 1. It can be seen from the values of '*k*' that it is more effective to prevent twist at the cantilever edge rather than the lateral deflection.

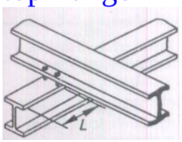
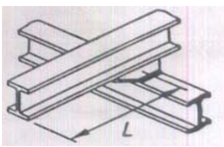
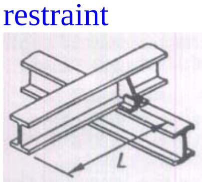
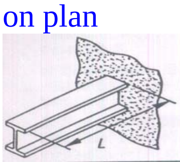
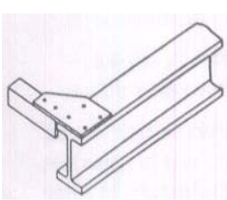
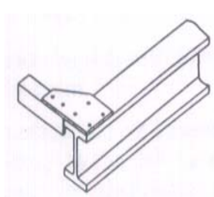
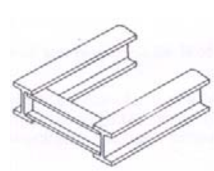
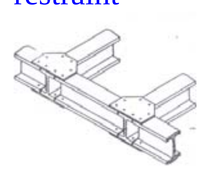
Generally, in framed structures, continuous beams are provided with overhang at their ends. These overhangs have the characteristics of cantilever beams. In such cases, the type of restraint provided at the outermost vertical support is most significant. Effective prevention of twist at this location is of particular importance. Failure to achieve this would result in large reduction of lateral stability as reflected in large values of '*k'*, in Table 1.

Table 1 Recommended values of 'k'

Restraint conditions		Loading condition	
At support	At tip	Normal	Destabilizing
Built in laterally and torsionally 	Free 	0.8λ	1.4λ
	Lateral restraint only 	0.7λ	1.4λ
	Torsional Restraint only 	0.6λ	0.6λ
	Lateral and Torsional Restraint 	0.5λ	0.5λ
Continuous with lateral and torsional restraint 	Free 	1.0λ	2.5λ
	Laterally restraint only 	0.9λ	2.5λ
	Torsional Restraint only 	0.8λ	1.5λ
	Laterally and Torsional Restraint 	0.7λ	1.2λ
Continuous, with lateral restraint only 	Free 	3.0λ	7.5λ
	Lateral restraint only 	2.7λ	7.5λ
	Torsional Restraint only 	2.4λ	4.5λ
	Laterally and Torsional Restraint 	2.1λ	3.6λ
For continuous cantilevers λ not less than λ_1 where λ_1 is the length of the adjacent span			

For cantilever beams of projecting length, L, the effective length L_{LT} to be used shall be taken as in Table 3 (Table 8.4 of New IS: 800) for different support conditions.

Table 3 Effective length for Cantilever Beams
(Table 8.4 of New IS: 800, Effective Length, L_{lt} , for Cantilever of Length L)
(Section 8.3.3)

Restraint condition		**Loading conditions**	
At Support	**At tip**	**Normal**	**destabilizing**
a) Continuous, with lateral restraint to top flange 	1) Free 2) Lateral restraint to top flange 3) Torsional restraint 4) Lateral and torsional restraint	3.0L 2.7L 2.4L 2.1L	7.5L 7.5L 4.5L 3.6L
b) Continuous, with partial torsional restraint 	1) Free 2) Lateral restraint to top flange 3) Torsional restraint 4) Lateral and torsional restraint	2.0L 1.8L 1.6L 1.4L	5.0L 5.0L 3.0L 2.4L
c) Continuous, with lateral and torsional restraint 	1) Free 2) Lateral restraint to top flange 3) Torsional restraint 4) Lateral and torsional restraint	1.0L 0.9L 0.8L 0.7L	2.5L 2.5L 1.5L 1.2L
d) Restrained laterally, torsionally and against rotation on plan 	1) Free 2) Lateral restraint to top flange 3) Torsional restraint 4) Lateral and torsional restraint	0.8L 0.7L 0.6L 0.5L	1.4L 1.4L 0.6L 0.5L
Top restraint conditions			
1) Free 	2) Lateral restraint to top flange 	3) Torsional restraint 	4) Lateral and torsional restraint 

3.0 CONTINUOUS BEAMS

Beams, extending over a number of spans, are normally continuous in vertical, lateral or in both planes. In the cases, where such continuity is not provided lateral deflection and twisting may occur. Such a situation is typically experienced in roof purlins before sheeting is provided on top of them and in beams of temporary nature. For these cases, it is always safe to make no assumption about possible restraints and to design them for maximum effective length.

Another case of interest with regard to lateral buckling is a beam that is continuous in the lateral plane i.e. the beam is divided into several segments in the lateral plane by means of fully effective braces. The buckled shapes for such continuous beams include deformation of all the segments irrespective of their loading. Effective length of the segments will be equal to the spacing of the braces if the spacing and moment patterns are similar. Otherwise, the effective length of each segment will have to be determined separately.

To illustrate the behaviour of continuous beams, a single-span beam provided with equally loaded cross beams is considered (see Fig. 1).

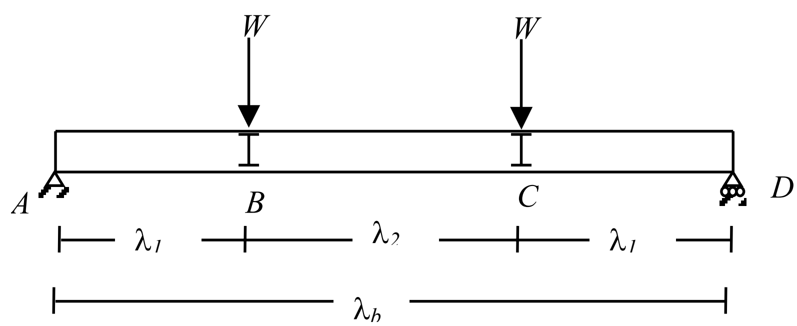


Fig. 1 Single - span beam

Two equally spaced, equally loaded cross beams divide the beam into three segments laterally. In this case, true M_{cr} of the beam and its buckling mode would depend upon the spacing of the cross beams. The critical moment M_{crB} for any ratio of λ_1 / λ_b would lie in between the critical moment values of the individual segments. The critical moments for the two segments are obtained using the basic equation given in the earlier chapter.

$$M_{cr1} = 1.75 \frac{\pi}{\lambda_1} \sqrt{(E\,I_y\,GJ)} \sqrt{1 + \frac{\pi^2 E\,\Gamma}{\lambda_1^{\,2}\;GJ}} \tag{1}$$

(In the outer segment, m = 0.57. Using 1 / m and the basic moment, the critical moment is determined)

$$M_{cr2} = \frac{\pi}{\lambda_2} \sqrt{(EI_y\,GJ)} \sqrt{1 + \frac{\pi^2 E\,\Gamma}{\lambda_2^{\,2}\;GJ}} \tag{2}$$

(This segment is loaded by uniform moments at its ends – basic case)

M_{cr1} and M_{cr2} values are plotted against λ_1 / λ_b and shown in Fig. 2 for the particular case considered with equal loading and a constant cross section throughout.

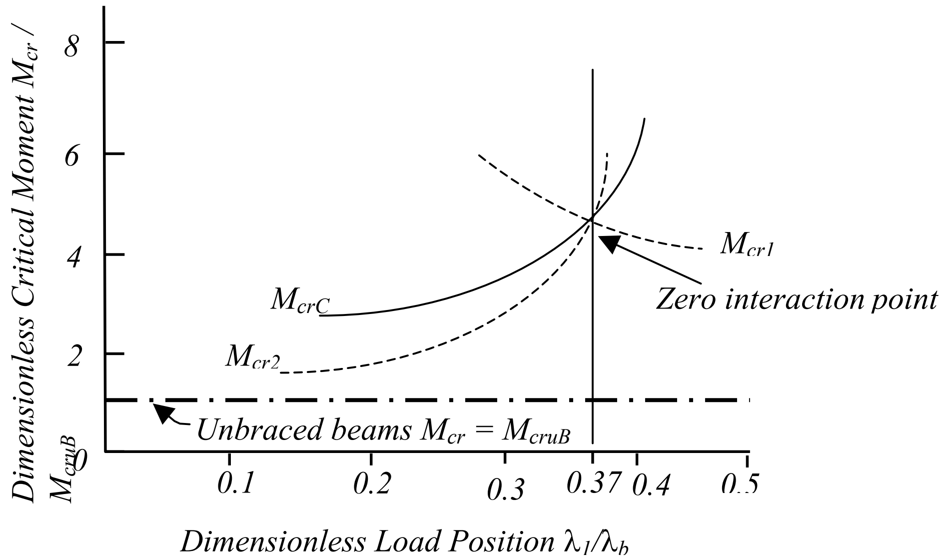


Fig.2 Interaction between M_{cr1} and M_{cr2}

It is seen that for $\lambda_1 / \lambda_b = 0.37$, M_{cr1} *and* M_{cr2} are equal and the two segments are simultaneously critical. The beam will buckle with no interaction between the two segments. For any other value of λ_1 / λ_b there will be interaction between the segments and the critical load would be greater than the individual values, as shown in the figure. For values of $\lambda_1 / \lambda_b < 0.37$, outer segments will restrain the central segment and vice-versa when $\lambda_1 / \lambda_b > 0.37$.

The safe load for a laterally co....nuous beam may be obtained by calculating all segmental critical loads individually and choosing the lowest value assuming each segment as simply supported at its ends.

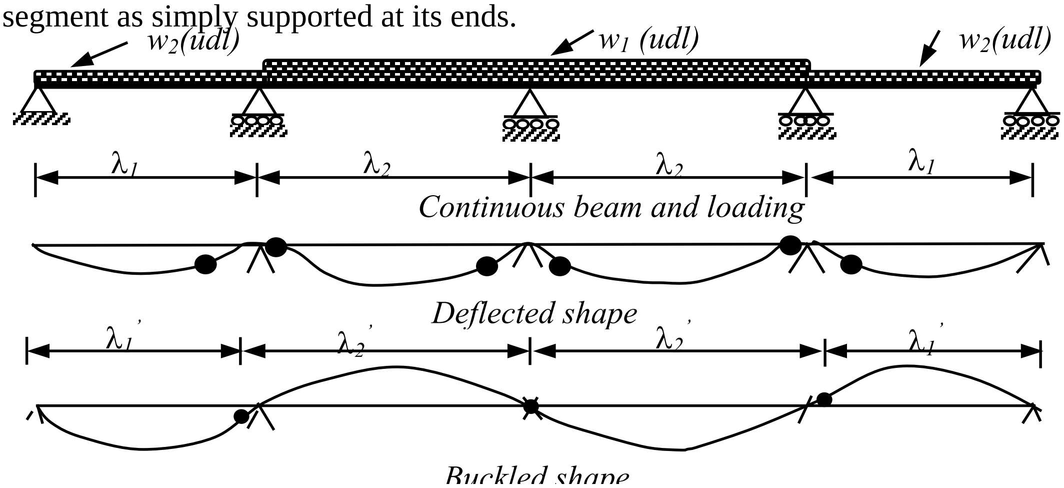


Fig. 3 Continuous beam – deflected shape and buckled shape

It is of interest to know the behaviour of beams, which are continuous in both transverse and lateral planes. Though the behaviour is similar to the laterally unrestrained beams, their moment patterns would be more complicated. The beam would buckle in the lateral plane and deflect in the vertical plane. There is a distinct difference between the points of contraflexure in the *buckled shape* and points of contraflexure in the *deflected shape*. These points will not normally occur at the same location within a span, as shown in Fig. 3. Therefore, it is wrong to use the distance between the points of contraflexure of the deflected shape as the effective length for checking buckling strength.

3.1 PROVISIONS OF NEW IS: 800 (LSM VERSION)

For beams, which are provided with members giving effective lateral restraint to the compression flange at intervals along the span, in addition to the end torsional restraint required, the effective length for lateral torsional buckling shall be taken as the distance, centre-to-centre, of the restraint members in the relevant segment under normal loading condition and 1.2 times this distance, where the load is not acting on the beam at the shear and is acting towards the shear centre so as to have destabilizing effect during lateral torsional buckling deformation.

Where a member is provided as intermediate lateral supports to improve the lateral buckling strength, these restraints should have sufficient strength and stiffness to prevent lateral movement of the compression flange at that point, relative to the end supports. The intermediate lateral restraints should be either connected to an appropriate bracing system capable of transferring the restraint force to the effective lateral support at the ends of the member, or should be connected to an independent robust part of the structure capable of transferring the restraint force. Two or more parallel member requiring such lateral restraint shall not be simply connected together assuming mutual dependence for the lateral restraint.

The intermediate lateral restraints should be connected to the member as close to the compression flange as practicable. Such restraints should be closer to the shear centre of the compression flange than to the shear centre of the section. However, if torsional restraint preventing relative rotation between the two flanges is provided, they intermediate lateral restraint may be connected at any appropriate level.

For beams which are provided with members giving effective lateral restraint at intervals along the span, the effective lateral restraint shall be capable of resisting a force of 2.5 percent of the maximum force in the compression flange taken as divided equally between the points at which the restraint members are provided. Further, each restraint point should be capable of resisting 1 percent of the maximum force in the compression flange.

For Purlins which are adequately restrained by sheeting need not be normally checked for the restraining forces required by rafters, roof trusses or portal frames that carry predominately roof loads provided there is bracing of adequate stiffness in the plane of rafters or roof sheeting which is capable of acting as a stressed skin diaphragm.

4.0 EFFECTIVE LATERAL RESTRAINT

Providing proper lateral bracing may increase the lateral stability of a beam. Lateral bracing may be either discrete (e.g. cross beams) or continuous (e.g. beam encased in concrete floors). The lateral buckling capacity of the beams with discrete bracing may be determined by using the methods described in a later Section. For the continuously restrained beams, assuming lateral deflection is completely prevented, design can be based on in-plane behaviour. It is important to note that in the hogging moment region of a continuous beam, if the compression flange (bottom flange) is not properly restrained, a form of lateral deflection with cross sectional distortion would occur.

4.1 Discrete bracing

In order to determine the behaviour of discrete braces, consider a simply supported beam provided with a single lateral support of stiffness K_b at the centroid, as shown in Fig. 4.

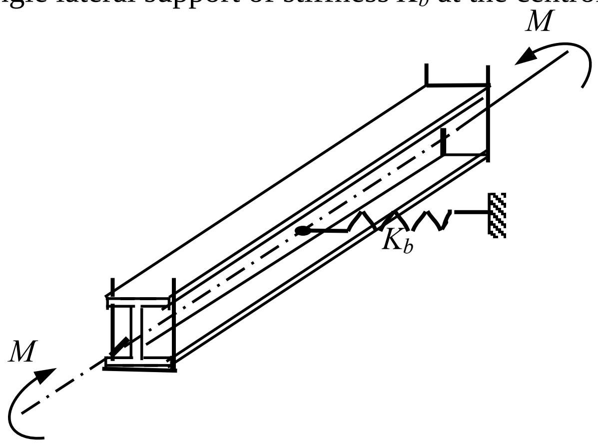


Fig. 4 Beam with single lateral support

The relationship between K_b and M_{cr} is shown in Fig. 5.

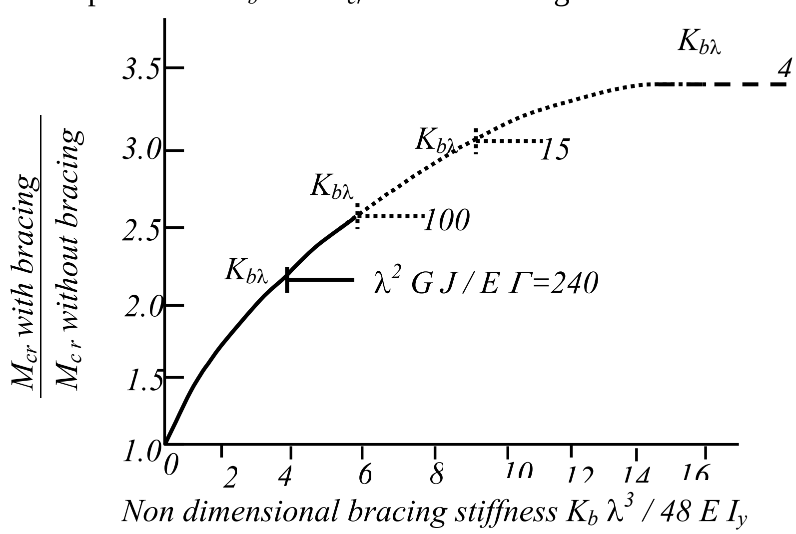


Fig. 5 Relationship between K_b and M_{cr}

It is seen that M_{cr} value increases with K_b, until K_b is equal to a limiting value of $K_{b\lambda}$. The corresponding M_{cr} value is equal to the value of buckling for the two segments of the beam. M_{cr} value does not increase further as the buckling is now governed by the individual M_{cr} values of the two segments.

Generally, even a light bracing has the ability to provide substantial increase in stability. There are several ways of arranging lateral bracing to improve stability. The limiting value of the lateral bracing stiffness, $K_{b\lambda}$, is influenced by the following parameters.

- Level of attachment of the brace to the beam i.e. top or bottom flange.
- The type of loading on the beam, notably the level of application of the transverse load
- Type of connection, whether capable of resisting lateral and torsional deformation
- The proportion of the beam.

Provision of bracing to tension flanges is not so effective as compression flange bracing. Bracing provided below the point of application of the transverse load would not be able to resist twisting and hence full capacity of the beam is not achieved. For the design of effective lateral bracing systems, the following two requirements are essential.

- Bracing should be of sufficient stiffness so that buckling occurs between the braces
- Lateral bracing should have sufficient strength to withstand the force transferred by the beam.

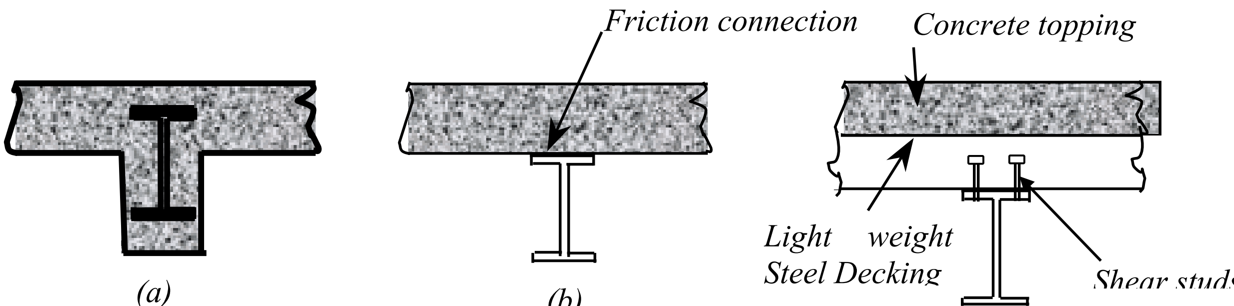
A general rule is that lateral bracing can be considered as fully effective if the stiffness of the bracing system is at least 25 times the lateral stiffness of the member to be braced. Provisions in BS 5950 stipulate that adequate lateral and torsional restraints are provided if they are capable of resisting 1) a lateral force of not less than 1% of the maximum factored force in the compression flange for lateral restraints, 2) and a couple with lever arm equal to the depth between centroid of flanges and a force not less than 1% of the maximum factored compression flange force.

4.2 Continuous restraint

In a framed building construction, the concrete floor provides an effective continuous lateral restraint to the beam. As a result, the beam may be designed using in-plane strength. A few examples of fully restrained beams are shown in Fig. 6.

The lateral restraint to the beam is effective only after the construction of the floor is completed. The beam will have to be temporarily braced after its erection till concreting is done and it has hardened.

For the case shown in Fig.6 (*a*), the beam is fully encased in concrete, and hence there will be no lateral buckling. In the arrangement shown in Fig.6 (*b*), the slab rests directly upon the beam, which is left unpainted. Full restraint is generally developed if the load transmitted and the area of contact between the slab and the beam are adequate to develop the needed restraint by friction and bond.



(a) *Beam fully encased in concrete*   *(b)* *Beam in friction connection with concrete*   *(c)* *Beam with metal decking*

Fig. 6 Beams with continuous lateral restraint

For the case shown in Fig.6(*c*), the metal decking along with the concrete provides adequate bracing to the beam. However, the beam is susceptible to buckling before the placement of concrete due to the low shear stiffness of the sheeting. Shear studs are provided at the steel-concrete interface to enhance the shear resistance. The codal provisions require that for obtaining fully effective continuous lateral bracing, it must withstand not less than 1% of the maximum force in the compression flange.

5.0 BUCKLING OF MONOSYMMETRIC BEAMS

For beams symmetrical about the major axis only e.g. unequal flanged I- sections, the non-coincidence of the shear centre and the centroid complicates the torsional behaviour of the beam. The monosymmetric I-sections are generally more efficient in resisting loads provided the compressive flange stresses are taken by the larger flange.

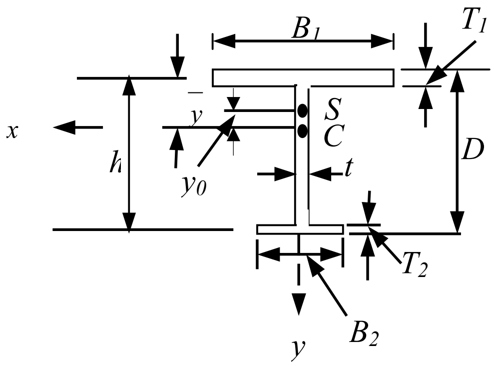
When a monosymmetric beam is bent in its plane of symmetry and twisted, the longitudinal bending stresses exert a torque, which is similar to torsional buckling of short concentrically loaded compression members. The longitudinal stresses exert a torque, T_M given by

$$T_M = M_x \beta_x \, d\phi / dz \tag{3}$$

where $$\beta_x = 1/I_x \int_A (x^2 y + y^3)\, dA - 2y_0 \tag{4}$$

is the monosymmetry property of the cross section. Explicit expression for β_x for a monosymmetric I-section is given in Fig. 7.

The torque developed, T_m, changes the effective torsional rigidity of the section from GJ to $(GJ+M_x \beta_x)$. In doubly symmetric beams the torque exerted by the compressive bending stresses is completely balanced by the restoring torque due to the tensile stresses and therefore β_x is zero. In monosymmetric beams, there is an imbalance of torque due to larger stresses in the smaller flange, which is farther from the shear centre.



$$h = D - (T_1 + T_2)\,/\,2$$

$$\bar{y} = \frac{B_2 T_2 h + (D - T_1 - T_2)(D - T_2)t/2}{B_1 T_1 + B_2 T_2 + (D - T_1 - T_2)t}$$

$$y_0 = \alpha h - \bar{y}$$

$$\alpha = \frac{1}{1 + (B_1/B_2)^3 (T_1/T_2)}$$

$$\Gamma = \alpha B_1^3\, T_1\, h^2 / 12$$

$$\beta_x = \frac{1}{I_x}\left\{ \begin{array}{l} (h-\bar{y})\left[B_2^{\,3}\, T_2 \big/ 12 + B_2 T_2 (h-\bar{y})^2 \right] \\ -\bar{y}\left[B_1^{\,3}\, T_1 \big/ 12 + B_1 T_1 \bar{y}^2 \right] + \left[(h-\bar{y}-T_2/2)^4 - (\bar{y}-T_1/2)^4 \right] t/4 \end{array} \right\}$$

Fig. 7 Properties of monosymmetric I-sections

Hence, when the smaller flange is in compression there is a reduction in the effective torsional rigidity; $M_x \beta_x$ is negative and when the smaller flange is in tension $M_x\ \beta_x$ is positive. Thus, the principal effect of monosymmetry is that the buckling resistance is increased when the larger flange is in compression and decreased when the smaller flange is in compression. This effect is similar to the Wagner effect in columns. The value of critical moment for unequal flange I beam is given by.

$$M_c = \frac{\pi}{\lambda}\sqrt{E\, I_y\, GJ}\left\{ \sqrt{\left[1 + \frac{\pi^2 E \Gamma}{G J \lambda^2} + \left(\frac{\pi \rho_m}{2}\right)^2 \right]} + \frac{\pi \rho_m}{2} \right\} \qquad (5)$$

Where $\rho_m = \dfrac{I_{yc}}{I_y}$ (6)

I_{yc} is the section minor axis second moment of area of the compression flange.

The monosymmetry property is approximated to

$$\beta_x = 0.9h\,(2\,\rho_m - 1)\,(1 - I_y^2 / I_x^2) \qquad (7)$$

and the warping constant Γ by

$$\Gamma = \rho_m\,(1 - \rho_m)\, I_y\, h^2 \qquad (8)$$

Very little is known of the effects of variations in the loading and the support conditions on the lateral stability of monosymmtric beams. However, from the available results, it is established that for top flange loading higher critical loads are always obtained when the larger flange is used as the compression flange. Similarly for bottom flange loading higher critical loads can be obtained when this is the larger flange. For Tee- sections β_x can be obtained by substituting the flange thickness T_1 or T_2 equal to zero; also for Tee sections the warping constant, Γ is zero.

6.0 BUCKLING OF NON-UNIFORM BEAMS

Non-uniform beams are often used in situations, where the strong axis bending moment varies along the length of the beam. They are found to be more efficient than beams of uniform sections in such situations. The non-uniformity in beams may be obtained in several ways. Rectangular sections generally have taper in their depths. I-beams may be tapered in their depths or flange widths; flange thickness is generally kept constant. However, steps in flange width or thickness are also common.

Tapering of narrow rectangular beams will produce considerable reduction in minor axis flexural rigidity, EI_y, and torsional rigidity, GJ; consequently, they have low resistance to lateral torsional buckling. Reduction of depth in I-beams does not affect EI_y, and has only marginal effect on GJ. But warping rigidity, $E\Gamma$, is considerably reduced. Since the contribution of warping rigidity to buckling resistance is marginal, depth reduction does not influence significantly the lateral buckling resistance of I beams. However, reduction in flange width causes large reduction in GJ, EI_y and $E\Gamma$. Similarly, reduction in flange thickness will also produce large reduction in EI_y, $E\Gamma$, and GJ in that order. For small degrees of taper there is little difference between width-tapered beams and thickness tapered beams. But for highly tapered beams, the critical loads of thickness tapered ones are higher. Thus, the buckling resistance varies considerably with change in the flange geometry.

Based on the analysis of a number of beams of different cross sections with a variety of loading and support conditions, the elastic critical load for a tapered beam may be determined approximately by applying a reduction factor r to the elastic critical load for an equivalent uniform beam possessing the properties of the cross section at the point of maximum moment

$$r = \frac{7+\gamma}{5+3\gamma} \tag{9}$$

$$\gamma = \frac{S_{x0}}{S_{x1}}\left[\left(\frac{D_1}{D_0}\right)^3\left(\frac{B_0}{B_1}\right)^3\left(\frac{T_0}{T_1}\right)^2\right] \tag{10}$$

S_x = section modulus.

T = flange thickness.

D = depth of the section.

B =flange width.

Subscripts 0 and 1 relate to the points of maximum and minimum moment respectively.

For the design of non-uniform sections, BS 5950 provides a simple method, in which the properties where the moment is maximum may be used and the value of n is suitably adjusted. The value of n is given by

$$n = 1.5 - 0.5\, A_{sm} / A_{lm} \geq 1.0 \tag{11}$$

Where A_{sm} and A_{lm} are flange areas at the points of the smallest and largest moment, and m = 1.0

7.0 BEAMS OF UNSYMMETRICAL SECTIONS

The theory of lateral buckling of beams developed so far is applicable only to doubly symmetrical cross sections having uniform properties throughout its length. Many lateral buckling problems encountered in design practice belong to this category. However, cases may arise where the symmetry property of the section may not be available. Such cases are described briefly in this Section.

The basic theory can also be applied to sections symmetrical about minor axis only e.g. Channels and Z-sections. In this section, the shear centre is situated in the axis of symmetry although not at the same point as the centroid. In the case of channel and Z sections, instability occurs only if the loading produces pure major axis bending. The criterion is satisfied for the two sections if: (1) for the channel section, the load must act through the shear centre Fig.8 (a) and, (2) for Z-section in a direction normal to the horizontal principal plane [Fig.8 (b)].

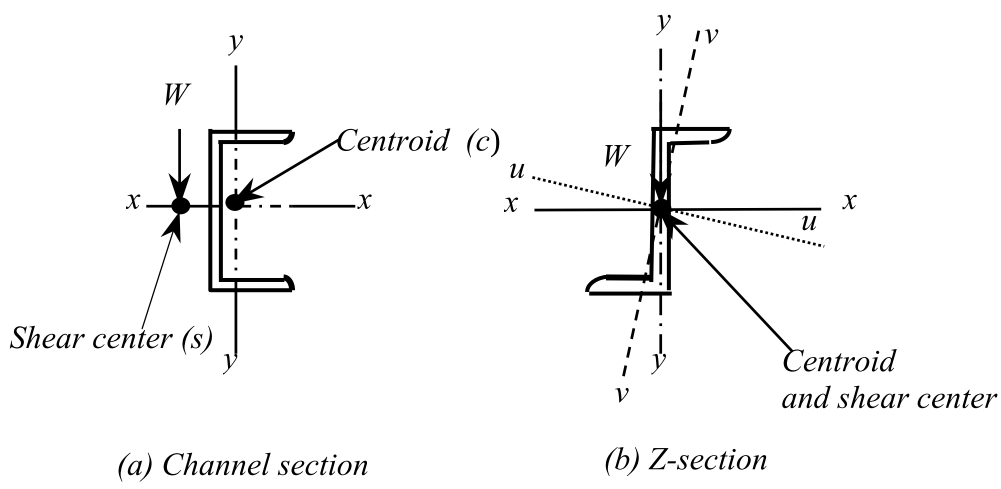


Fig. 8 Loading through shear centre

If these conditions are satisfied, M_{cr} of these sections can be obtained using their properties and the theoretical equation. The warping constant Γ for the sections are:

$$\Gamma = \frac{T B^3 h}{12}\left[\frac{3BT + 2ht}{6BT + ht}\right] \text{ for a channel} \tag{12}$$

$$\Gamma = \frac{B^3 h^2}{12(2Bth)^2}\left[2t(B^2 + Bh + h^2) + 3tBh\right] \text{for a Z-section} \tag{13}$$

where,

h = distance between flange centroids
t = thickness of web
B = total width of flange
T = flange thickness.

It is very difficult to obtain such loading arrangements so as to satisfy the restrictions mentioned above. In such cases, the behaviour may not be one of lateral stability; instead a combination of bending and twisting or bi-axial bending.

For sections, which have symmetry about minor axis only, their shear centre does not coincide with the centroid. This results in complicated torsional behaviour and theoretical predictions are not applicable. Fig. 9 shows the instability behaviour of sections with flanges of varying sizes and positions (top or bottom). It can be seen from the figure that sections with flange in the compression region are more advantageous.

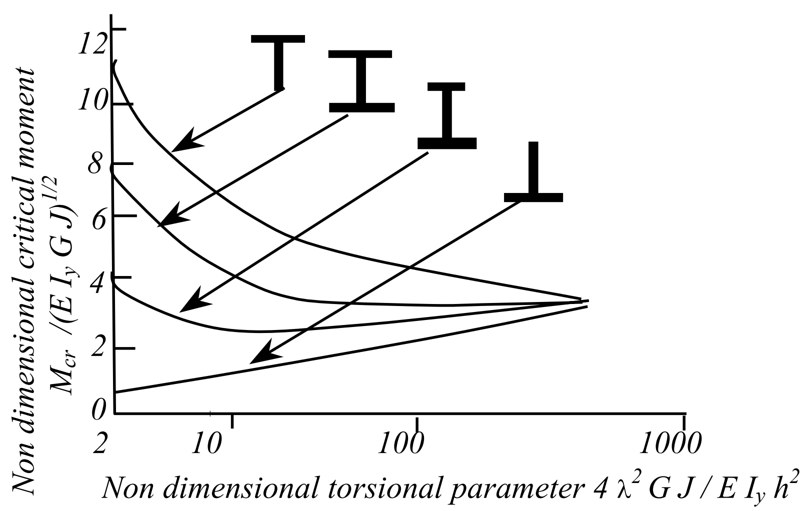


Fig. 9 Effect of flange position and proportion on lateral stability

While considering the case of tapered beams, it has been established, based on lateral stability studies, that variation of the flange properties can cause large changes in the lateral buckling capacity of the beam, whereas tapering of depth has insignificant influence on the buckling capacity.

8.0 SUMMARY

In this chapter, lateral torsional buckling of some practical cases of beams has been explained. It is pointed out that for cantilever beams the type of restraint provided at fixed -end plays a significant role in their buckling capacities. Torsional restraint of the cantilever beam has been found to be more beneficial than lateral restraint. In the case of beams with equally spaced and loaded cross beams the critical moment of the main beam and the associated buckling mode will depend on the spacing of the cross beams. Requirements for effective lateral restraint have been presented. Continuous restraint provided by concrete floors to beams in composite constructions of buildings is discussed. As discussed in an earlier chapter, the local buckling effects should be taken into account by satisfying the minimum requirements of the member cross-section. Cases of monosymmetric beams and non- uniform beams are also briefly explained. Finally cases of beams with un-symmetric sections are discussed and concluded that beams with flanges in the compression zone are more advantageous from the point of view of lateral torsional buckling.

9.0 REFERENCES

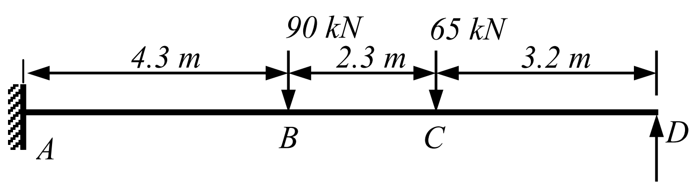
1. Trahair N.S., 'The behaviour and design of steel structures', Chapman and Hall London, 1977

2. Kirby P.A. and Nethercot D.A.,'Design for structural stability', Granada Publishing, London, 1979

Structural Steel Design Project	Job No.	Sheet *1* of *10*	Rev.
	Job title: *UNRESTRAINED BEAM DESIGN*		
	Worked example: *1*		
Calculation sheet		Made by. *GC*	Date.26*/02/07*
		Checked by. *TKB*	Date.28*/02/07*

Problem I :

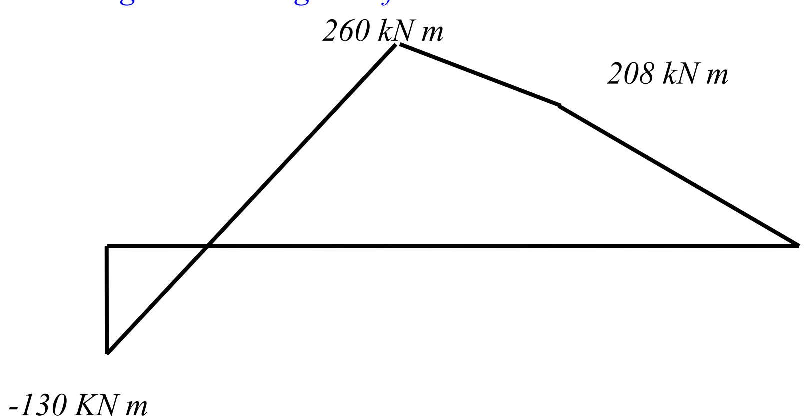
A propped cantilever has a span of 9.8 m. it is loaded by cross beams at 4.3 m and 6.6 m from its left hand end. The ends of the beam and the loaded points are assumed to be fully braced laterally and torsionally.



The loads as given are factored.

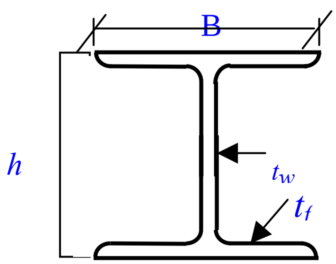
Design a suitable section for the beam:

The bending moment diagram of the beam is as shown below.



Section classification of ISMB 450:

The properties of the section are:



Depth, h = 450 mm

Width, B = 150 mm

Web thickness, t_w = 9.4 mm

Flange thickness, t_f = 17.4 mm

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Depth between fillets, d = 379.2 mm.

Radius of gyration about minor axis, $r_y = 30.1$ *mm.*

Plastic modulus about major axis, $Z_p = 1533.36 \times 10^3\ mm^3$ — *Appendix I of IS: 800*

Assume $f_y = 250\ N/mm$, $E = 200000\ N/mm^2$, $\gamma_m = 1.10$

(1) ***Type of section***

(i) flange criterion:

$$b = \frac{B}{2} = \frac{150}{2} = 75\ mm$$

$$\frac{b}{T} = \frac{75}{17.4} = 4.31$$

$$\frac{b}{T} < 9.4\varepsilon, \ where\ \ \varepsilon = \sqrt{\frac{250}{f_y}}$$

Hence o.k.

(ii) Web criterion:

$$\frac{d}{t} = \frac{379.2}{9.4} = 40.3$$

$$\frac{d}{t} < 84\,\varepsilon$$

Hence o.k.

Since $\frac{b}{t_f} < 9.4\varepsilon$ *and* $\frac{d}{t_w} < 84\varepsilon$, *the section is classified as 'plastic'* — *Table 3.1 (Section 3.7.2) of IS: 800*

Now the moment gradients are different for different segments of the entire span viz. for AB, BC and CD.

So based on the moment gradients, each segment will be checked against Lateral Torsional Buckling and the corresponding safe bending stress will also be checked for each segment.

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(iii) Lateral torsional buckling for segment AB:

The beam length AB = 4.3 m

Check for Slenderness Ratio:

Effective length criteria:

With ends of compression flanges fully restrained for torsion at support but both the flanges are not restrained against Warping, effective length of simply supported beam $L_{LT} = 1.0\, L$, where L is the span of the beam.

Table 8.3 of IS: 800

Hence, $L_{LT} = 1.0 \times 4.3\ M = 4300\ mm$, $L_{LT}/r = 4300/30.1 = 142.86$

Since the moment is varying from -130 k-Nm to 260 k-Nm, there will be moment gradient. So for calculation of f_{bd}, Critical Moment, M_{cr} is to be calculated.

Now, Critical Moment,

$$M_{cr} = c_1 \frac{\pi^2 E I_y}{(KL)^2}\left\{\left[\left(\frac{K}{K_w}\right)^2 \frac{I_w}{I_y} + \frac{G I_t (KL)^2}{\pi^2 E I_y} + \left(c_2\, y_g - c_3\, y_j\right)^2\right]^{0.5} - \left(c_2\, y_g - c_3\, y_j\right)\right\}$$

Clause F.1.2 of Appendix F of IS: 800

Where,

c_1, c_2, c_3 = factors depending upon the loading and end restraint conditions (Table F.1)

K, K_w = effective length factors of the unsupported length accounting for boundary conditions at the end lateral supports,

Here, both K *and* K_w can be taken as 1.0.

and

y_g = *y distance between the point of application of the load and the shear centre of the cross section and is positive when the load is acting towards the shear centre from the point of application*

$y_j = y_s - 0.5 \int_A (z^2 - y^2)\, y\, dA / I_z$

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y_s = *coordinate of the shear centre with respect to centroid, positive when the shear centre is on the compression side of the centroid*

Here, for plane and equal flange I section,

$y_g = 0.5 \times h = 0.5 \times 0.45 = 0.225\ M = 225\ mm$

$y_j = 1.0\ (2\beta_f - 1)\ h_y/2.0$ (when $\beta_f \leq 0.5$)

h_y = distance between shear centre of the two flanges of the cross section = $h - t_f$

here, $\beta_f = 0.5$, and $h_y = h - t_f = 450 - 17.4 = 432.6\ mm$

hence, $y_j = 1.0 \times (2 \times 0.5 - 1) \times 432.6/2.0 = 0$

and $y_s = 0$

$I_t = \sum b_i t_i^3 / 3$, for open section

$= 2 \times 150 \times 17.4^3 + (450 - 2 \times 17.4) \times 9.4^3 = 192.527 \times 10^4\ mm^4$

The warping constant, I_w, *is given by*

$I_w = (1-\beta_f)\ \beta_f\ I_y\ h_y^2$ *for I sections mono-symmetric about weak axis*

$= (1-0.5) \times 0.5 \times 834 \times 10^4 \times 432.6^2 = 39019265.46 \times 10^4\ mm^6$

Modulus of Rigidity, $G = 0.769 \times 10^5 N/mm^2$

Here, $\psi = -130/260 = -0.5$ *and* $K = 1.0$, *for which*

$c_1 = 2.704$, $c_2 = 0$ and $c_3 = 0.676$

Table F.1 of Appendix F of IS: 800

Hence, Critical Moment,

$$M_{cr} = c_1 \frac{\pi^2 E I_y}{(KL)^2}\left\{\left[\left(\frac{K}{K_w}\right)^2 \frac{I_w}{I_y} + \frac{G I_t (KL)^2}{\pi^2 E I_y} + \left(c_2\, y_g - c_3\, y_j\right)^2\right]^{0.5} - \left(c_2\, y_g - c_3\, y_j\right)\right\}$$

$$= 2.704 \frac{\pi^2 x 200000 x 834 x 10^4}{(1.0 x 4300)^2}\left\{\left[\left(\frac{1}{1}\right)^2 \frac{39019265 x 10^4}{834 x 10^4} + \frac{0.769 x 10^5 x 192.527 x 10^4 x 4300^2}{\pi^2 x 200000 x 834 x 10^4}\right]^{0.5}\right\}$$

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= 1111296062.60 N-mm

Calculation of f_{bd}:

Now, $\lambda_{LT} = \sqrt{\beta_b Z_p f_y / M_{cr}} = \sqrt{1.0x1533.36x10^3 x250/1111296.06x10^3}$

$= 0.587$

Clause 8.2.2 of IS: 800

for which, $\phi_{LT} = 0.5x\left[1+\alpha_{LT}(\lambda_{LT}-0.2)+\lambda_{LT}^{\ 2}\right]$

$= 0.5x\left[1+0.21(0.587-0.2)+0.587^2\right] = 0.713$

for which, $\chi_{LT} = \dfrac{1}{\left\{\phi_{LT}+\left[\phi^2{}_{LT}-\lambda^2{}_{LT}\right]^{0.5}\right\}} = \dfrac{1}{\left\{0.713+\left[0.713^2-0.587^2\right]^{0.5}\right\}}$

$= 0.895$

$f_{bd} = \chi_{LT}\, f_y / \gamma_{m0} = 0.895 \text{ x } 250 / 1.10 = 203.41 \text{ N/mm}^2$

Hence, $M_d = \beta_b Z_p f_{bd} = 1.0 \text{ x } 1533.36 \text{ x } 203.41/1000 = 311900.76/1000$

$= 311.90$ *kN-m*

Max. Bending Moment, $M_{max} = 260$ *kN-m*

Hence, $M_d > M_{max}$ *(311.90 > 260)*

∴ *ISMB 450 is adequate against lateral torsional buckling for the applied bending moments for the segment AB of the beam.*

(iv) Lateral torsional buckling for segment BC:

The beam length BC = 2.3 m

Check for Slenderness Ratio:

Effective length criteria:

With ends of compression flanges fully restrained for torsion at support but both the flanges are not restrained against Warping, effective length of simply supported beam $L_{LT} = 1.0\, L$, where L is the span of the beam.

Table 8.3 of IS: 800

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Hence, $L_{LT} = 1.0 \times 2.3\ M = 2300\ mm$, $L_{LT}/r = 2300/30.1 = 76.41$

Since the moment is varying from 260 k-Nm to 208 k-Nm, there will be moment gradient. So for calculation of f_{bd}, Critical Moment, M_{cr} is to be calculated.

Now, Critical Moment,

$$M_{cr} = c_1 \frac{\pi^2 EI_y}{(KL)^2}\left\{\left[\left(\frac{K}{K_w}\right)^2 \frac{I_w}{I_y} + \frac{GI_t (KL)^2}{\pi^2 EI_y} + (c_2\ y_g - c_3\ y_j)^2\right]^{0.5} - (c_2\ y_g - c_3\ y_j)\right\}$$

Clause F.1.2 of Appendix F of IS: 800

Where,

c_1, c_2, c_3 = factors depending upon the loading and end restraint conditions (Table F.1)

K, K_w = effective length factors of the unsupported length accounting for boundary conditions at the end lateral supports,

Here, both K and K_w can be taken as 1.0.

and

y_g = *y distance between the point of application of the load and the shear centre of the cross section and is positive when the load is acting towards the shear centre from the point of application*

$y_j = y_s - 0.5 \int_A (z^2 - y^2)\, y\, dA / I_z$

y_s = *coordinate of the shear centre with respect to centroid, positive when the shear centre is on the compression side of the centroid*

Here, for plane and equal flange I section,

$y_g = 0.5 \times h = 0.5 \times 0.45 = 0.225\ M = 225\ mm$

$y_j = 1.0\ (2\beta_f - 1)\ h_y/2.0$ (when $\beta_f \leq 0.5$)

h_y = distance between shear centre of the two flanges of the cross section = $h - t_f$

here, $\beta_f = 0.5$, *and* $h_y = h - t_f = 450 - 17.4 = 432.6\ mm$

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hence, y_j = 1.0 x (2 x 0.5 – 1) x *432.6*/2.0 = 0

and $y_s = 0$

$I_t = \sum b_i t_i^3 / 3$, for open section

= 2 x 150 x 17.4^3 + (450 – 2 x 17.4) x 9.4^3 = 192.527 x 10^4 mm^4

The warping constant, I_w, is given by

$I_w = (1-\beta_f)\,\beta_f\, I_y\, h_y^2$ *for I sections mono-symmetric about weak axis*

= (1-0.5) x 0.5 x 834 x 10^4 x 432.6^2 = 39019265.46 x 10^4 mm^6

Modulus of Rigidity, G = 0.769 x 10^{5}N/mm^2

Here, ψ = 208/260 =- 0.5 and K = 1.0, for which
c_1 = 1.113, c_2 = 0 and c_3 = 0.998 — *Table F.1 of Appendix F of IS: 800*

Hence, Critical Moment,

$$M_{cr} = c_1 \frac{\pi^2 EI_y}{(KL)^2}\left\{\left[\left(\frac{K}{K_w}\right)^2 \frac{I_w}{I_y} + \frac{GI_t (KL)^2}{\pi^2 EI_y} + \left(c_2\, y_g - c_3\, y_j\right)^2\right]^{0.5} - \left(c_2\, y_g - c_3\, y_j\right)\right\}$$

$$= 1.113 \frac{\pi^2 x 200000 x 834 x 10^4}{(1.0 x 2300)^2}\left\{\left[\left(\frac{1}{1}\right)^2 \frac{39019265 x 10^4}{834 x 10^4} + \frac{0.769 x 10^5 x 192.527 x 10^4 x 2300^2}{\pi^2 x 200000 x 834 x 10^4}\right]^{0.5}\right\}$$

= 1063972985.92 N-mm

Calculation of f_{bd}:

Now, λ_{LT} = $\sqrt{\beta_b Z_p f_y / M_{cr}} = \sqrt{1.0 x 1533.36 x 10^3 x 250 / 1063972.98 x 10^3}$ — *Clause 8.2.2 of IS: 800*

= 0.600

for which, ϕ_{LT} $= 0.5 x\left[1 + \alpha_{LT}(\lambda_{LT} - 0.2) + \lambda_{LT}^{\,2}\right]$

$= 0.5 x\left[1 + 0.21(0.600 - 0.2) + 0.600^2\right]$ *= 0.722*

for which, χ_{LT} = $\dfrac{1}{\left\{\phi_{LT} + \left[\phi^2{}_{LT} - \lambda^2{}_{LT}\right]^{0.5}\right\}} = \dfrac{1}{\left\{0.722 + \left[0.722^2 - 0.600^2\right]^{0.5}\right\}}$

= 0.89

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$f_{bd} = \chi_{LT} f_y / \gamma_{m0} = 0.89 \times 250 / 1.10 = 202.27\ N/mm^2$

Hence, $M_d = \beta_b Z_p f_{bd} = 1.0 \times 1533.36 \times 202.27/1000 = 310152.73/1000$

$= 310.15\ kN\text{-}m$

Max. Bending Moment, $M_{max} = 260\ kN\text{-}m$

Hence, $M_d > M_{max}\ (310.15 > 260)$

$\therefore$ *ISMB 450 is adequate against lateral torsional buckling for the applied bending moments for the segment BC of the beam.*

(v) Lateral torsional buckling for segment CD:

The beam length CD = 3.2 m

Check for Slenderness Ratio:

Effective length criteria:

With ends of compression flanges fully restrained for torsion at support but both the flanges are not restrained against Warping, effective length of simply supported beam $L_{LT} = 1.0\ L$, where L is the span of the beam.

Table 8.3 of IS: 800

Hence, $L_{LT} = 1.0 \times 3.2\ M = 3200\ mm$, $L_{LT}/r = 3200/30.1 = 106.31$

Since the moment is varying from 208 k-Nm to 0 k-Nm, there will be moment gradient. So for calculation of f_{bd}, Critical Moment, M_{cr} is to be calculated.

Now, Critical Moment,

$$M_{cr} = c_1 \frac{\pi^2 EI_y}{(KL)^2} \left\{ \left[\left(\frac{K}{K_w} \right)^2 \frac{I_w}{I_y} + \frac{GI_t (KL)^2}{\pi^2 EI_y} + \left(c_2\, y_g - c_3\, y_j \right)^2 \right]^{0.5} - \left(c_2\, y_g - c_3\, y_j \right) \right\}$$

Clause F.1.2 of Appendix F of IS: 800

Where,

c_1, c_2, c_3 = factors depending upon the loading and end restraint conditions (Table F.1)

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K, K_w = effective length factors of the unsupported length accounting for boundary conditions at the end lateral supports,

Here, both K *and* K_w can be taken as 1.0.

and

y_g = *y distance between the point of application of the load and the shear centre of the cross section and is positive when the load is acting towards the shear centre from the point of application*

$y_j = y_s - 0.5 \int_A (z^2 - y^2)\, y\, dA / I_z$

y_s = *coordinate of the shear centre with respect to centroid, positive when the shear centre is on the compression side of the centroid*

Here, for plane and equal flange I section,

$y_g = 0.5 \times h = 0.5 \times 0.45 = 0.225\ M = 225\ mm$

$y_j = 1.0\ (2\beta_f - 1)\ h_y / 2.0$ (when $\beta_f \le 0.5$)

h_y = distance between shear centre of the two flanges of the cross section = $h - t_f$

here, $\beta_f = 0.5$, *and* $h_y = h - t_f = 450 - 17.4 = 432.6\ mm$

hence, $y_j = 1.0 \times (2 \times 0.5 - 1) \times 432.6/2.0 = 0$

and $y_s = 0$

$I_t = \sum b_i t_i^3 / 3$, for open section

$= 2 \times 150 \times 17.4^3 + (450 - 2 \times 17.4) \times 9.4^3 = 192.527 \times 10^4\ mm^4$

The warping constant, I_w, *is given by*

$I_w = (1-\beta_f)\ \beta_f\ I_y\ h_y^2$ *for I sections mono-symmetric about weak axis*

$= (1-0.5) \times 0.5 \times 834 \times 10^4 \times 432.6^2 = 39019265.46 \times 10^4\ mm^6$

Modulus of Rigidity, $G = 0.769 \times 10^5 N/mm^2$

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	Worked example:*1*		
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	Checked by. *TKB*		Date.28*/02/07*

Here, $\psi = 0/208 = 0$ *and* $K = 1.0$*, for which* $c_1 = 1.879$*,* $c_2 = 0$ *and* $c_3 = 0.939$ — *Table F.1 of Appendix F of IS: 800*

Hence, Critical Moment,

$$M_{cr} = c_1 \frac{\pi^2 EI_y}{(KL)^2}\left\{\left[\left(\frac{K}{K_w}\right)^2 \frac{I_w}{I_y} + \frac{GI_t (KL)^2}{\pi^2 EI_y} + (c_2\, y_g - c_3\, y_j)^2\right]^{0.5} - (c_2\, y_g - c_3\, y_j)\right\}$$

$$= 1.879 \frac{\pi^2 x200000x834x10^4}{(1.0x3200)^2}\left\{\left[\left(\frac{1}{1}\right)^2 \frac{39019265x10^4}{834x10^4} + \frac{0.769x10^5 x192.527x10^4 x3200^2}{\pi^2 x200000x834x10^4}\right]^{0.5}\right\}$$

= 1125742175.56 N-mm

Calculation of f_{bd}**:**

Now, $\lambda_{LT} = \sqrt{\beta_b Z_p f_y / M_{cr}} = \sqrt{1.0x1533.36x10^3 x250 / 1125742.18x10^3}$ — *Clause 8.2.2 of IS: 800*

= 0.584

for which, $\phi_{LT} = 0.5x\left[1 + \alpha_{LT}(\lambda_{LT} - 0.2) + \lambda_{LT}{}^2\right]$

$= 0.5x\left[1 + 0.21(0.584 - 0.2) + 0.584^2\right]$ *= 0.71*

for which, $\chi_{LT} = \dfrac{1}{\left\{\phi_{LT} + \left[\phi^2{}_{LT} - \lambda^2{}_{LT}\right]^{0.5}\right\}} = \dfrac{1}{\left\{0.71 + \left[0.71^2 - 0.584^2\right]^{0.5}\right\}}$

= 0.898

$f_{bd} = \chi_{LT}\, f_y / \gamma_{m0}$ *= 0.898 x 250 / 1.10 = 204.09 N/mm*^{2}

Hence, $M_d = \beta_b Z_p f_{bd}$ *=1.0 x 1533.36 x 204.09/1000 = 312943.44/1000*

= 312.94 kN-m

Max. Bending Moment, M_{max} *= 208 kN-m*

Hence, $M_d > M_{max}$ *(312.94 > 208)*

ISMB 450 is adequate against lateral torsional buckling for the applied bending moments for the segment CD of the beam.

13 DESIGN OF BEAM-COLUMNS - I

1.0 INTRODUCTION

Columns in practice rarely experience concentric axial compression alone. Since columns are usually parts of a frame, they experience both bending moment and axial force. The frames, in which columns are members, may be classified as *braced* or *unbraced.* In braced frames the resistance to lateral loads at floor levels is provided either by bracings [Fig. 1(*b*)] or shear walls. In case of unbraced frames [Fig. 1(*d*)] the resistance to lateral loads is obtained from the members of the frames with moment resisting connections between them. Thus the relative translation between the ends of a column in a braced frame is prevented, whereas in unbraced frames the columns are free to sway causing relative translation between their ends. More details on classification of frames as braced and unbraced are given in the chapter on frames. Thus columns in practice experience bending about one or both axis in addition to axial compression, due to one or more of the following reasons.

- The compressive force may be eccentrically transferred to the column [Fig. 1(*a*)]. When this eccentric force is transferred to the centre line of the column, an equivalent axial compression and bending moment act on the column.

- When the beams in braced rigid portal frames are subjected to gravity loads, the rotation of the beams at their junction with the column causes rotation of the column also at the junction due to rigid connection [Fig. 1(*b*)]. Hence beam transfers bending moments to the column in addition to axial load [Fig. 1(*c*)].

- When a multi-storey multi-bay un-braced frame is subjected to gravity loads and lateral loads due to wind or earthquake, the columns are subjected to sway deflection and bending [Fig 1(*d*)]. In such cases, the columns experience axial compression as well as bending moments [Fig.1 (*e*)].

- Beams may frame from two orthogonal directions in corner columns in buildings [Fig. 1(*f*)]. In such cases the columns may be subjected to bending about both principal axes in addition to axial compression [Fig. 1(*f*)].

Columns subjected to combined axial force and bending moment are referred to as beam-columns. A beam-column may be subjected to single curvature bending over its length [Fig. 1(*c*)]. In this case the nature of the bending stress (compressive or tensile) at a point in the cross section and sign of the bending moment diagram over its entire length of the beam-column remains the same. Consequently, the curvature has the same sign over the entire length of the column. On the other hand, the columns in a sway frame [Fig. 1(*d*)] experience reverse curvature bending as shown in Fig. 1(*e*), causing variation of the nature (positive or negative) of the bending moment and curvature over the length of the column.

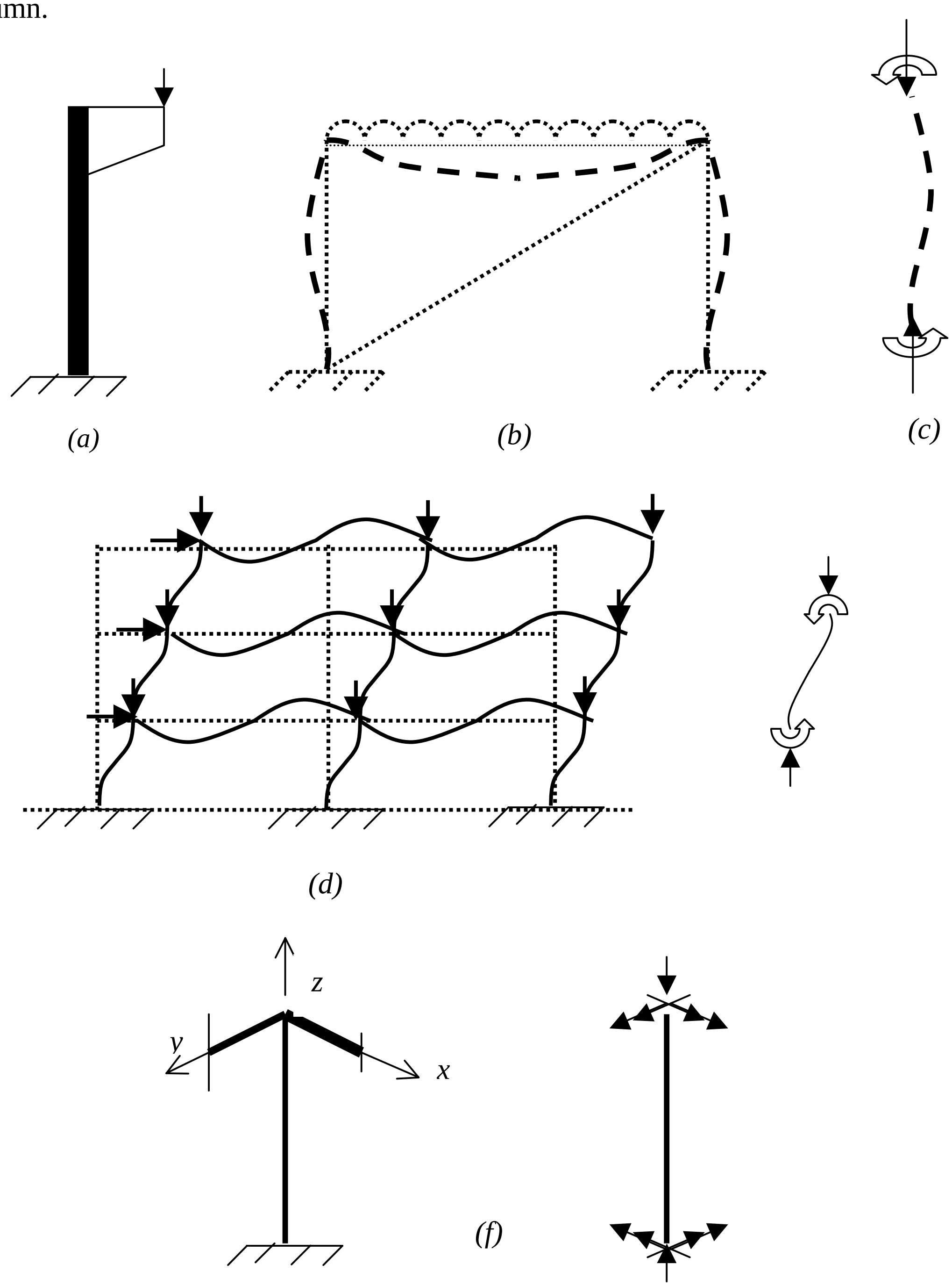


Fig. 1 Beam-Columns in Frames

Presence of bending moments in the beam-columns reduces the axial force at which they fail. This topic presented in two parts, deals with the behaviour, and design of beam-columns. In Part I initially, the behaviour and strength of short beam-column members under combined compression and bending moment are discussed. In such short beam column the failure is due to the strength of the material being reached (material failure). Subsequently, the behaviour and strength of practical, long beam-columns, as affected by stability and deformation, are discussed. In long columns the failure may be either due to material strength being reached at the ends of the column or instability of the overall column. In Part II equations for the design of beam-columns subjected to combination of axial compression and biaxial bending are presented. A design example of a beam-column also is presented in Part II.

2.0 SHORT BEAM-COLUMNS

A short member (stub column), made of non-slender (plastic, compact or semi-compact) section under axial compression, fails by yielding (due to large deformation) at the squash load, P_d, given by [Fig. 2(*a*)]

$$P_d = A_g f_y \tag{1}$$

where, f_y is the yield strength of the material, and A_g is the gross area of the cross section.

If the stub column is made of *slender cross section*, the plate elements of the cross section undergo local buckling before reaching the yield stress. This causes reduction in the effective area of the cross section to a value below the gross area, A_g, and the member fails at a load below P_d, given by Eqn.1.

Similarly a short member made of plastic or compact section and subjected to only bending moment fails at the plastic moment capacity, M_p, given by [Fig. 2(*b*)]

$$M_p = Z_p . f_y \tag{2}$$

where, Z_p = plastic section modulus of the cross section, in the case of plastic and compact sections.

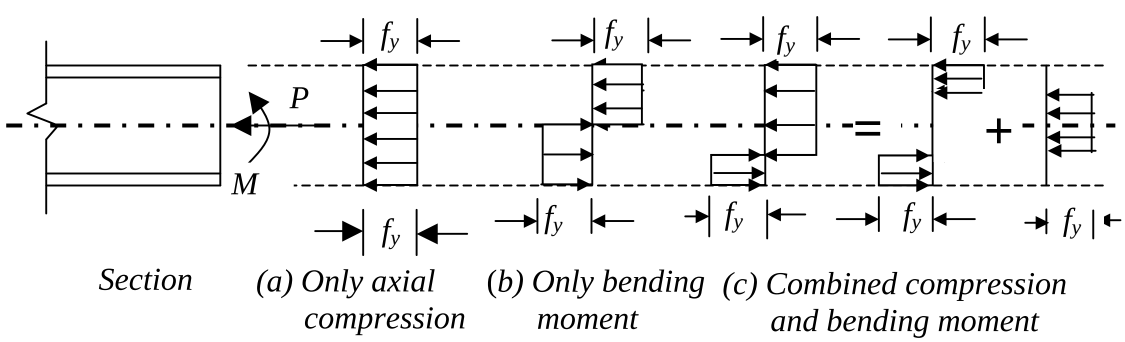


Fig. 2 Stresses in Short Beam-Columns

A semi-compact section subjected to bending moment only fails by buckling of a plate element of the cross section before the plastification of the entire section as shown in Fig. 2 (*b*) but after the stress at the extreme fibre in compression reaches the yield stress. In a slender section, the plate elements buckle even before the extreme fibre stress in bending reaches the yield stress. Consequently, the semi-compact and slender sections fail under bending even before reaching the plastic moment, M_p, given by Eqn. 2.

The discussions that follow generally assume that the cross section is either plastic or compact. In the case of slender and semi-compact sections the effect of earlier failure before complete section yielding has to be considered. The strength of such members may be analysed by following the procedure discussed in the chapters on *cold-formed steel members.*

The stress distribution at failure over the (plastic or compact) cross-section of a beam-column under combined compression and bending moment is shown in Fig. 1(*c*). It can be modeled as superposition of only compressive stress over an area of the cross section close to the neutral axis of the cross section and the balance of the section subjected to compressive and tensile stress due to bending. Hence such beam-columns fail before reaching the squash load, P_d, given by equation 1 or the plastic moment, M_p, given by Eqn. 2. The typical failure envelope diagram of a stub beam-column made of I section and subjected to axial compression P and bending moment M is shown in Fig. 3, in a non-dimensional form. At smaller values of axial compression, only a small area of the cross section closer to the neutral axis is necessary to equilibrate the external compression, P. Since the area closer to the neutral axis contributes very little to the plastic moment capacity, M_p, of the cross section, the reduction in the moment capacity, M, is negligible when the axial compression is small. It is seen in the failure envelope that for smaller axial compression ($P / P_d < 0.15$) the reduction in the moment capacity is negligible ($M/M_p \approx 1.0$)

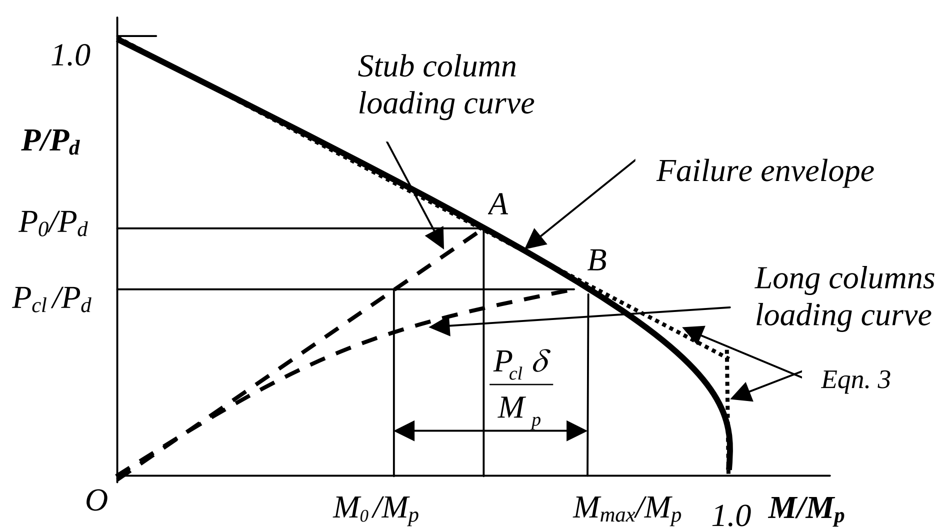


Fig. 3 Beam-Column Failure Envelope

Eqn. 3 in a non-dimensional form gives the failure envelope under the major axis bending and the axial compression, as shown in Fig. 3.

$$\frac{P}{P_d} + \frac{0.85M}{M_p} \leq 1.0$$
$$\frac{M}{M_p} \qquad \leq 1.0 \tag{3}$$

Although there is a small reduction in the bending moment at lower values of axial compression as seen in Fig. 3, in Eqn. 3 this has been disregarded. The interaction equation (Eqn. 3) is linear up to the plastic moment capacity, M_p, of the member.

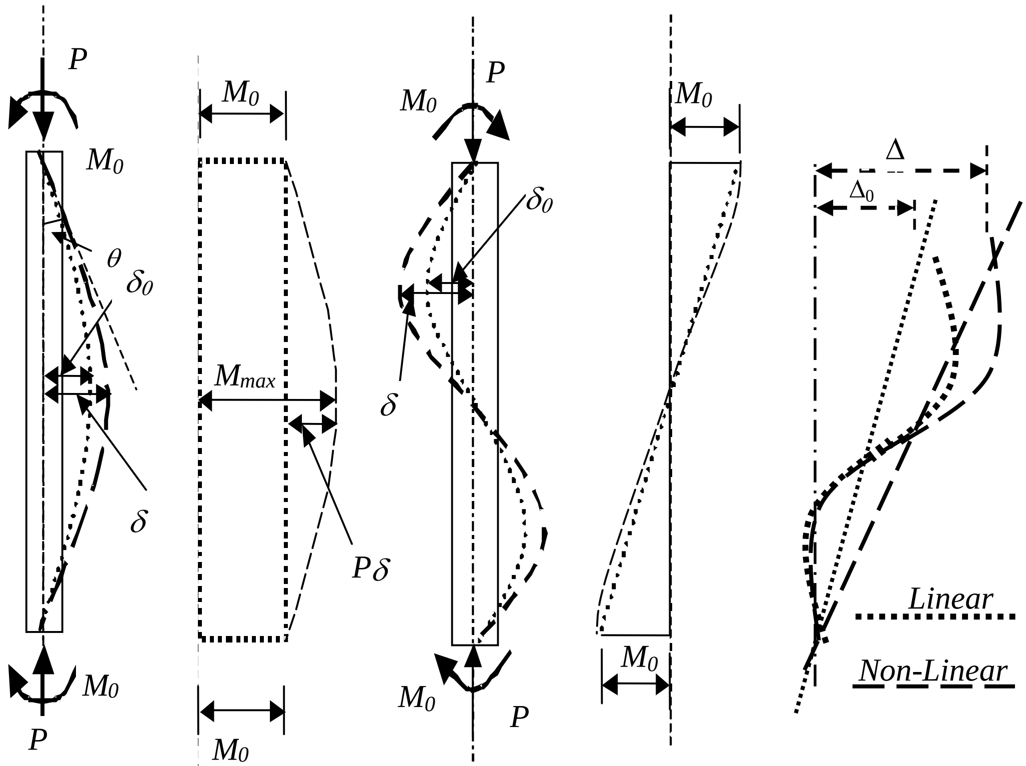
The loading curve of a short beam-column by a compressive force at a constant eccentricity, is indicated by a straight line, such as OA (Fig. 3) having a constant slope. The slope is dictated by the eccentricity. When this loading path OA intersects the failure envelope curve at A, the beam-column strength is reached. The values of P and M corresponding to point A are the compression and moment capacity of short the beam-column under the given eccentricity. This may be calculated from Eqn. 3 by substituting $P.e$ for M, where e is the eccentricity of the compressive force, P.

3.0 LONG BEAM-COLUMNS

Typically steel columns in practice are long and slender. Such slender columns when axially compressed tend to fail by buckling rather than yielding, as discussed in the chapter on *Introduction to Column Buckling*. Similarly, slender I sections subjected to bending moment about the major axis (z-axis) when not laterally supported, may fail by lateral-torsional buckling, as discussed earlier in the chapter on *Unrestrained Beam Design*. However, under minor axis (y-axis) bending, the plastic and compact sections will reach the plastic moment capacity, M_{py}, without undergoing premature lateral buckling or local buckling. Such long slender members subjected to combined axial compression and bending may experience different modes of instability or material failure. These are discussed in this section.

Consider a slender beam-column subjected only to equal and opposite end moment, M_o, as shown in Fig. 4(a). The beam-column is bent into a single curvature with a maximum deflection δ_0, as shown by the dotted line in Fig. 4(a). If the axial compression is applied at the ends of the column now, additional bending moment is caused due to the axial load acting on the deformed shape. This additional bending moment causes additional deflection and so on, until the final maximum deflection δ is reached at the stage of equilibrium under combined axial force and bending moments. This is referred to as P-δ effects. The final deflected shape and the final bending moment diagram, considering the P-δ effect, are shown by dashed curves in Fig. 4(a). It is seen that due to P-δ effect, the maximum moment in the beam-column, M_{max}, is larger than the externally applied end moments, M_o.

The same beam-column, when subjected to equal end moments acting in the same direction, experiences double curvature bending in addition to axial compression as shown in Fig. 4(b). The deflected shape as well as the bending moment diagram of the beam column, not considering $P\text{-}\delta$ effects, are shown in Fig. 4(*b*) by dotted curves and after considering the $P\text{-}\delta$ effects is indicated by the dashed curve. It is seen that although δ is greater than δ_o, in this case, the magnified moment considering the $P\text{-}\delta$ effects need not be greater than the end moments M_o. Thus, it is seen that the $P\text{-}\delta$ effects and the magnified moments depend upon the moment gradient over the length of the member. The discussion so far was about beam-columns in frames braced against sway.



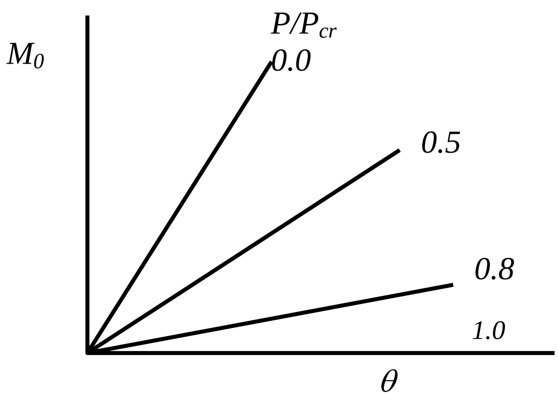
(a) Single curvature *(b) Double curvature* *(c) Sway Deformation*

Fig. 4 Deflection and Moment Magnification

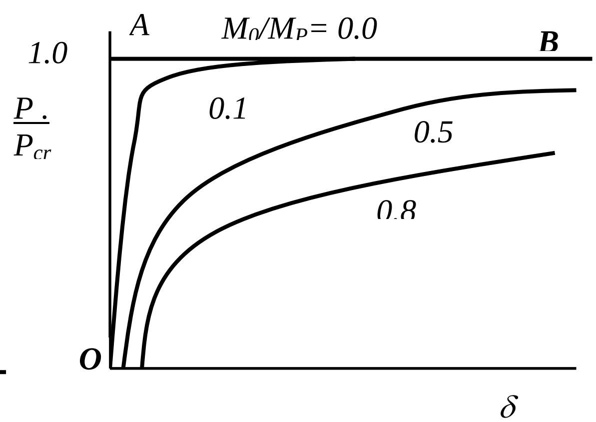
If a frame is not braced to prevent lateral sway, linear elastic analysis for lateral loads may indicate that column ends translate relative to one another by a distance Δ_0, in addition to end rotations and reverse curvature deformation as indicated by dotted curve in Fig. 4(*c*). The axial forces, *P*, acting on the frame with sway displacement, Δ_0, increase the sway to Δ, as shown by dashed curve and the column and beam moments also increase. This additional displacement can be obtained only by a non-linear analysis of the frame considering the equilibrium of the frame in the deformed configuration. This increase in sway deformation and bending moments, due to the load acting on the deformed structure, are referred to as P-Δ effects.

Let us see the load-deformation behaviour of a beam-column, when the axial compression and end moments are applied one after another. If an axial compression, *P*, is initially applied so that P/P_{cr} is maintained constant while the moments at the two ends of the beam-column are proportionately increased in the elastic range of the material, the moment-end rotation diagram is linear as shown by Fig. 5(*a*). However, it is seen that the stiffness of the beam-column (the slope of the moment-rotation line) decreases with increase in the initially applied axial compression. When the $P/P_{cr} = 1.0$, the end rotation, θ [Fig. 4(*a*)] increases to infinity even for $M=0$, indicating instability under axial compression itself.

On the other hand, if the ratios of the initially applied end moments, M_0/M_p, is maintained constant and the axial compression alone is increased, the compressive load versus lateral deformation behaviour of the beam-column in the elastic range is as per Fig. 5(*b*). In this case the load deformation behaviour is non-linear. It is seen that a perfect column subjected to axial compression without any end moments undergoes bifurcation type of buckling [OAB in Fig.5 (*b*)]. If some end moments are applied initially, the member undergoes initial deflection, δ_o, the magnitude of which depends upon the magnitude of the end moments. Subsequently, as the axial compression is increased gradually, the lateral deflection increases even from the very beginning. Initially such an increase is seen to be at a slower rate, but nearer to the critical load it increases rapidly before failure occurs [Fig.5 (*b*)]. Usually the failure is triggered by yielding under the combined effect of axial compression and maximum moment. It is seen that the end moments modify the behaviour of an axially loaded column in a way, similar to the initial bow type of imperfections.



(a) At constant axial load, P



(b) At constant end moments, M_0

Fig. 5 Elastic Behaviour of Beam-Columns

Conventional first order linear elastic analyses of frames do not reflect these additional bending moments in beam-columns due to P-δ and P-Δ effects, since the equilibrium equations in these analyses methods are derived for the un-deformed structure. In the linear elastic analysis of a frame, the axial force and bending moments in a beam-column increase linearly, with the increase in the load on frame, as shown by straight line OA in Fig.3 drawn for short column. However, if we look at the equilibrium of the beam-column in the deformed configuration [Fig.4], (through a non-linear analysis), bending moments are magnified by P-Δ and P-δ effects in a nonlinear fashion, as the load increases.

The non-linear variation of the maximum bending moment, M_{max}, due to increase in the axial load, P, acting at a constant end eccentricity (M_0/P = constant end eccentricity) of a long column is also shown in Fig.3 (curve OB). When this non-linear P versus M loading curve intersects the failure envelope at B, the long beam-column would fail. It is seen that if the moment magnification of the long column due to P-δ and P-Δ effects were not considered, the compressive strength of the member would be obtained as P_0, whereas the actual compressive strength of the long beam-column is reduced to P_{cl}, due to the moment magnification effect. The corresponding linear analysis moment is M_o, whereas the actual magnified moment is M_{max}. The additional deflection and bending moment are due to the axial load acting on the deformed column as given below.

- in a column within a floor (P-δ effect) [Figs. 4(a) and 4 (b)]
- between the ends of the columns (sway) at adjacent floors (P-Δ effect) [Fig. 4(c)].

The magnified deflection and bending moment can be approximately obtained from the following equations.

$$\delta = \frac{\delta_0}{\left(1 - P/P_E\right)} \tag{4a}$$

$$\Delta = \frac{\Delta_0}{\left(1 - P/P_E\right)} \tag{4b}$$

$$M_{\max} = \frac{C_m M_0}{\left(1 - P/P_E\right)} \tag{4c}$$

where C_m is a coefficient that accounts for the moment gradient effects explained in greater detail later in this chapter. P_E is the Euler buckling strength of the column in the plane of bending. It is seen that as the applied axial load approaches the Euler buckling load, both deflection, δ, and magnified moment, M_{max}, increase rapidly and tend to approach infinity, indicating that even if M_0 is very small, as P approaches P_E, failure is imminent.

3.1 Beam-columns at Ultimate Load

An axially loaded I section long column fails by buckling about the slender axis. The member (beam) bent about the major axis fail by either formation of plastic hinge at plastic moment M_{pz} or by lateral buckling at a value of bending moment less than M_{pz} depending upon the laterally unsupported length. The member bent about the minor axis fails by formation of plastic hinge at plastic moment, M_{py}.

A beam-column becomes axially loaded compression member when the eccentricity of the applied compression is equal to zero. When the eccentricity of the applied compression is very large (tending to infinity) the beam column tends to behave like a beam, since the axial compression effect is negligible. Thus these two cases define the two limits of a beam-column. In between, a beam-column covers a range of combination of axial load and bending moment. Due to this, various combinations of buckling and plastic failures are exhibited by beam-columns, depending upon the relative values of the axial force, bending moment, buckling strength and bending strength of the member. Further, the bending may be about the minor axis only, causing flexural yielding type of failure or about the major axis only, causing torsional flexural buckling, or a combination of bending moments about both the axes.

Figs. 6(*a*) and 6(*b*) show the strength of typical beam-columns made of I sections, subjected to axial compression and uniaxial bending about the minor and major axis, respectively. The solid curves represent the strength envelope of beam-columns in a frame, considering the different long column and sway effects. The curves PBQ in Figs. 6(*a*) and 6(*b*) represent the strength envelopes of a stub column without considering the P-δ and P-Δ effects. Therefore, if these short column strength envelopes are used, the actual bending moments and axial force used in evaluating the strength should be based on a non-linear analysis accounting for the P-δ and P-Δ effects. Thus the loading path from such an analysis would be represented by the dashed curve OB. Similarly, the strength envelopes PCR account for the P-δ effects only. Hence, if these strength envelope curves are used, actual moment and axial force evaluation should be based on an analysis method that accounts for P-Δ effects. The loading path (P versus M) from such an analysis is represented by dashed line OC. The strength envelopes PAS account for both P-δ and P-Δ effects. Hence a linear analysis is adequate to obtain the axial forces and moments, while using these strength curves.

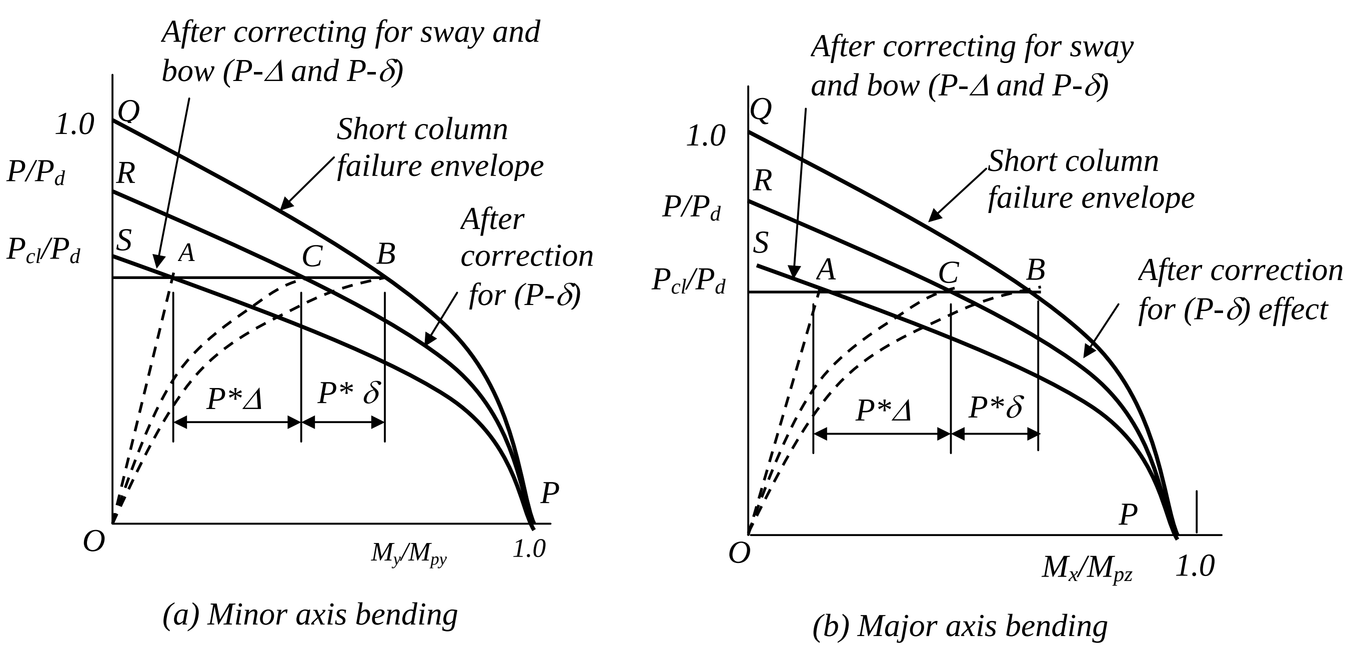


Fig. 6 Uniaxial Bending of Slender Beam-Columns

It is seen in Figs. 6(*a*) and 6(*b*) that the strength under axial compression decreases when the P-δ and P-Δ effects are considered. It is also seen that under pure bending, the major axis bending strength is affected due to the lateral buckling and that pure bending strength can be less than the plastic moment capacity M_{pz}. The bending strength, when the axial compression is zero, is given by M_z, which is less than M_{pz}. The dashed curves represent the loading path corresponding to different levels of analysis. The dashed curves OA represent the linear analysis path and hence their intersections with the strength envelopes corrected for P-δ and P-Δ effects (PAS), give the member strengths, P_{cl}. The dashed curves OB represent the nonlinear analysis paths, considering P-δ and P-Δ effects and hence intersections of these curves with the short column strength curve (PBQ) give the member strengths, P_{cl}. The dashed curves OC represent the loading paths from nonlinear analysis considering P-Δ effects only and hence their intersections with the strength curves (PCR) corrected for P-δ effect give the member strengths, P_{cl}.

3.2 Effects of Slenderness Ratio and Axial Force on Modes of Failure

Beam-columns may fail by flexural yielding or torsional flexural buckling. The actual mode of failure would depend upon the magnitude of the axial load and eccentricity as well as the slenderness ratio. The sub-ultimate and failure behaviour of beam-columns, as affected by different parameters, are briefly reviewed in the following sections (Dowling et al., 1988).

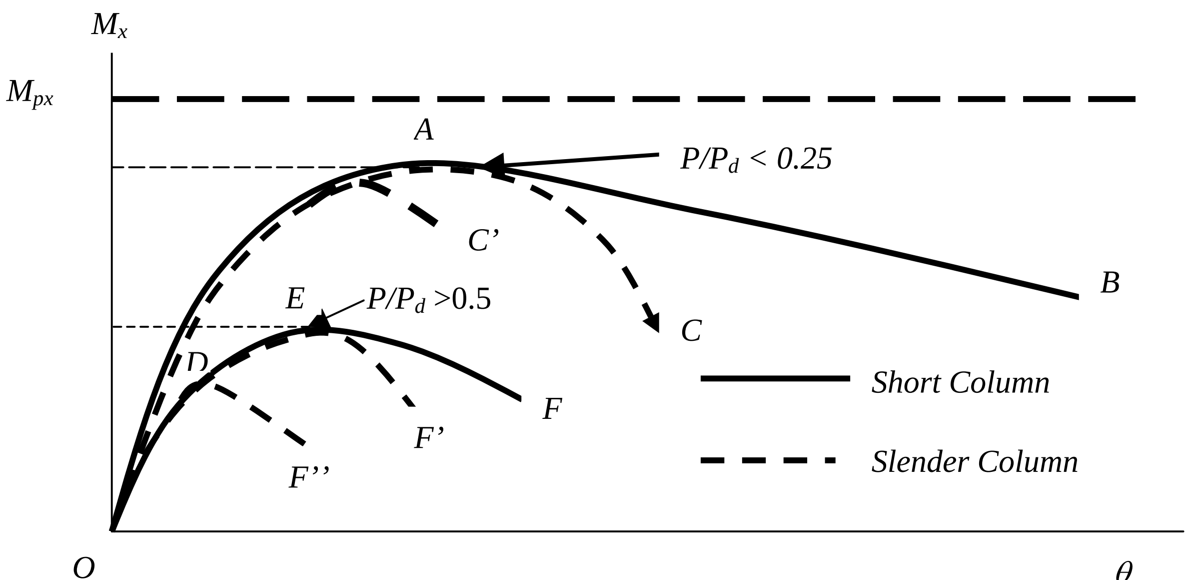


Fig. 7 Beam-column Moment Rotation Behaviour

3.2.1 Low axial load ratio (P/P_d< 0.33)

Beam-columns having lower slenderness ratios (λ/r <50): When subjected to moments about the major axis at both ends of the beam, the moment-curvature relationship at the sub-ultimate stage may be linear or non-linear depending upon whether the axial force is applied first followed by bending moment or both of them are increased proportionately (Figs. 5). The deformation is only in the plane of bending moment. At the penultimate stage, yielding of the compression flange occurs first, which spreads through the section on further loading. The ultimate strength is reached, when the plastic hinge is formed at one or both the ends. Thus the failure is due to the section strength being reached at one or both the ends. Under proportional loading, the loading path is indicated by the line OAB in Fig. 7. It is seen that there would be a small reduction in the moment capacity below M_{pz} due to the presence of the axial compression in the member. Further, there would be a reduction in the moment (unloading) beyond point A, with increasing end rotation, due to the spread of plasticity from the end sections to other sections along the length.

Beam-columns having a higher slenderness ratio (λ/r.>80): A more slender column, under smaller axial compression combined with end moments as before, would fail by buckling out-of-the-plane of the bending moment. If bending is predominant, lateral buckling of compression flange as in unrestrained beams occurs (OAC in Fig. 7). The axial force could cause minor axis deformation and hence the failure can be by minor axis bending and twisting (OC'), at moments below the full in plane strength obtained in the case of short/stocky columns. The moment rotation behaviour at the ultimate stage is indicated by dashed line in Fig.7. The failure would be after plastic hinge formation unless the slenderness ratio is very large.

3.2.2 High axial load ratio (P/P_d > 0.5)

Beam-columns having a lower slenderness ratio (λ/r <50): Under high axial load combined with bending moment, yielding can occur over a larger segment of the member, due to combined axial stress, bending stress and residual stress. Moment magnification at the mid-length of the column occurs due to single curvature bending deflection caused by equal end moments [Fig.4 (*a*)]. In the case of short/stocky member, the failure is due to the yield strength being reached at a section over the length of the member under combined axial force and magnified bending moment. The corresponding curve is shown by line ODEF in Fig.7. The main differences in the behaviour of stocky beam-column under larger axial compression, compared to smaller axial compression are the moment magnification, larger reduction in moment capacity due to larger axial compression and the drastic unloading in the penultimate stage (EF in Fig. 7).

Beam-columns having a higher slenderness ratio (λ/r.>80): In the case of slender beam-columns with a larger axial compression, the P-δ effect is larger both in the plane of and out of plane of the moment. In longer beam-columns, the moment may drop drastically when yielding starts under combined axial compression and magnified moment (OEF'). The weak axis buckling/flexural torsional buckling, causing out-of-plane deformation, could occur earlier than that corresponding to the section strength under combined axial force and magnified moment (ODF" in Fig. 7).

Thus design of beam-columns having higher slenderness ratio requires investigation of in- plane bending failure by flexural yielding and out-of-plane buckling failure. The behavior of beam-columns subject to bending about the minor axis is similar to that subjected to major axis bending as discussed, but for the following differences:

- In the case of slender members under smaller axial load, there is very little reduction of moment capacity below M_p, since the lateral torsional buckling is not a problem in weak axis bending.
- The moment magnification is larger in the case of beam-columns bending about the weak axis.
- The failure of short/stocky members is either due to section strength being reached at the ends (under smaller axial load) or at the section of larger magnified moment (under larger axial load).
- The failure of even slender members is due to buckling about the weak axis only and no torsional deformation is experienced.
- The *M-P* failure envelope varies as shown in Fig.6 (*a*), due to the variations of slenderness ratio and axial force.

3.3 Beam-Column under Biaxial Bending

The ultimate behaviour of beam-columns under biaxial bending is complicated by the effect of plastification, moment magnification and lateral torsional buckling. Typical failure envelope diagram is as shown in Fig.8. It is seen that the increase in the slenderness ratio of the member tends to reduce the strength of the member, except in the case of nearly pure bending about the weak axis (y-axis). Further, but for very small axial force ranges, increases in the axial compression tend to decrease the bending strength about both axes.

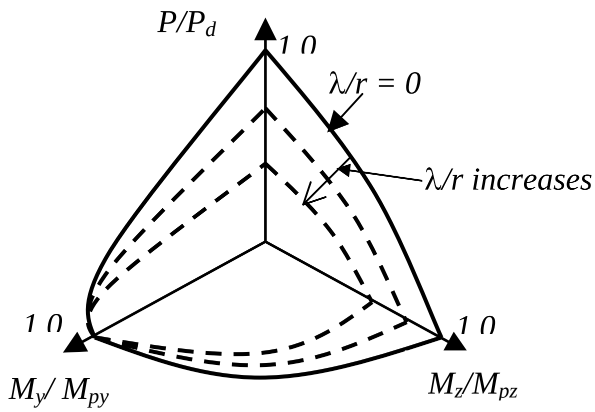


Fig. 8 beam-columns under Biaxial Bending

4.0 SUMMARY

Behaviour of beam-columns, which are members subjected to axial compression and bending simultaneously, was discussed in this chapter. The following were the main issues discussed in the chapter.

- The moment magnification due to P-δ and P-Δ effects are important considerations affecting their strength, particularly in case of slender members subjected to larger axial compression.
- Under major axis bending lateral buckling is also an important consideration.

- Flexural yielding, flexural buckling, torsional flexural buckling are the different modes of failure, depending upon the slenderness ratio, axis of bending and extent of axial compression.

In the next chapter, II of this topic, evaluation of strength of beam-columns will be dealt with and steps in designing beam-columns will be presented supported by an example.

5.0 REFERENCES

1. Dowling P.J, Knowles, P.R. and Owens, G.W., “Structural Steel Design”, Butterworths, London, 1998.

14 DESIGN OF BEAM-COLUMNS - II

1.0 INTRODUCTION

Beam-columns are members subjected to combined bending and axial compression. Their behaviour under uniaxial bending, biaxial bending and torsional flexural buckling were discussed in Part I on this topic in the previous chapter. It was shown that a range of behaviour varying from flexural yielding to torsional flexural or flexural buckling is possible.

In this chapter evaluation of strength of beam-columns is discussed. The steps in the analysis of strength of beam-column are presented along with an example.

2.0 STRENGTH OF BEAM-COLUMNS

The discussions in the part I of this topic, clearly indicated that the behaviour of beam-columns is fairly complex, particularly at the ultimate stage and hence exact evaluation of the strength would require fairly complex analysis. However, for design purposes, simplified equations are available, using which it is possible to obtain the strength of members, conservatively. These are discussed below.

2.1 Modes of Failure

The following are the possible modes of failure of beam-columns

2.1.1 *Local section failure*

This is usually encountered in the case of short, stocky beam columns ($\lambda/r << 50$) with relatively smaller axial compression ratio ($P/P_d < 0.33$) and beam-columns bent in reverse curvature.

- The strength of the end section reached under combined axial force and bending moment, governs the failure.
- The strength of the section may be governed by plastic buckling of plate elements in the case of plastic, compact and semi-compact sections or the elastic buckling of plate elements in the case of slender sections (see the chapter on plate buckling).

2.1.2 Overall instability failure under flexural yielding

This type of failure is encountered in the case of all members subjected to larger compression ($P/P_d > 0.5$) and single curvature bending about the minor axis as well as not very slender members subjected to axial compression and single curvature bending about the major axis.

- The member fails by reaching the strength of the member at a section over the length of the member, under the combined axial compression and magnified bending moment.
- In the case of weak axis bending of slender members ($\lambda/r > 80$), the failure may be by weak axis buckling, or failure of the maximum moment section under the combined effect of axial force and magnified moment.
- The section failure may be due to elastic or plastic plate buckling depending on the slenderness ratio (b/t) of the plate (See the chapter on plate buckling).

2.1.3 Overall instability by torsional flexural buckling

This is common in slender members ($\lambda/r>80$) subjected to large compression ($P/P_d>0.5$) and uniaxial bending about the major axis or biaxial bending.

- At the ultimate stage the member undergoes biaxial bending and torsional instability mode of failure.

2.1.4 Design Equations as per IS: 800

The design code specifies, as given below, the linear interaction equations to check the section strength to prevent local section failure as well as member failure by flexural yielding and torsional flexural buckling. These are conservative simplifications of the non-linear failure envelopes discussed in the previous chapter.

2.2.1 *Local section failure*

The Indian Standard Specification IS: 800 clearly deals with all types of members and mainly classifies the members in two groups, namely Plastic and *Compact Sections* and *Semi-Compact* sections. The strength of *Slender Members* may be analysed by following the procedure discussed in the chapters on *cold-formed steel members.*

IS: 800 states that under combined axial force and bending moment section strength as governed by material failure and member strength as governed by buckling failure have to be checked as given below;

1. *Section Strength*

Plastic and Compact Sections ─ In the design of members subjected to combined axial force (tension or compression) and bending moment, the following should be satisfied

$$\left(\frac{M_y}{M_{ndy}}\right)^{\alpha_1} + \left(\frac{M_z}{M_{ndz}}\right)^{\alpha_2} \le 1.0 \qquad (1)$$

Conservatively, the following equation may be used under combined axial force and bending moment

$$\frac{P}{P_d} + \frac{M_y}{M_{dy}} + \frac{M_z}{M_{dz}} \le 1.0 \qquad (2)$$

where

M_y, M_z = factored applied moments about the minor and major axis of the cross section, respectively

M_{ndy}, M_{ndz} = design reduced flexural strength under combined axial force and the respective uniaxial moment acting alone

P= factored applied axial force

P_d = design strength in compression due to yielding given by

$$P_d = A_g f_y / \gamma_{m0}$$

M_{dy}, M_{dz} = design strength under corresponding moment acting alone

A_g = gross area of the cross section

α_1, α_2 = constants as given in Table 1

$n = N / N_d$

Table: 1 Constants α_1 and α_2

Section	α_1	α_2
I and Channel	$5n \geq 1$	2
Circular tubes	2	2
Rectangular tubes	$1.66/(1-1.13n^2) \leq 6$	$1.66/(1-1.13n^2) \leq 6$
Solid rectangles	$1.73+1.8n^3$	$1.73+1.8\, n^3$

For plastic and compact sections without bolts holes, the following approximations may be used for evaluating M_{ndy} and M_{ndz}:

a) *Plates*

$M_{nd} = M_d\,(1-n^2)$

b) *Welded I or H sections*

$$M_{ndy} = M_{dy}\left[1-\left(\frac{n-a}{1-a}\right)^2\right] \leq M_{dy}$$

$M_{ndz} = M_{dz}\,(1-n) / (1-0.5a) \leq M_{dz}$

where $n = P / P_d$ and $a = (A - 2\, b\, t_f) / A \leq 0.5$

c) *For standard I or H sections*

for $n \leq 0.2$ $\quad M_{ndy} = M_{dy\backslash}$

for $n > 0.2$ $\quad M_{ndy} = 1.56\, M_{dy}\,(1-n)\,(n+0.6)$

$\quad M_{ndz} = 1.11\, M_{dz}\,(1-n) \leq M_{dz}$

d) *For Rectangular Hollow sections and Welded Box sections* –

When the section is symmetric about both axes and without bolt holes

$M_{ndy} = M_{dy}\,(1-n) / (1-0.5a_f) \leq M_{dy}$

$M_{ndz} = M_{dz}\,(1\text{-}n)\,/\,(1\text{-}0.5a_w) \le M_{dz}$

where

$a_w = (A - 2\,b\,t_f)\,/\,A \quad \le 0.5$

$a_f = (A\text{-}2\,h\,t_w)\,/\,A \;\le 0.5$

e) *Circular Hollow Tubes without Bolt Holes*

$M_{nd} = 1.04\,M_d\,(1\text{-}n^{1.7}) \le M_d$

<u>*Semi-compact section*</u> – In the absence of high shear force semi-compact section design is satisfactory under combined axial force and bending, if the maximum longitudinal stress under combined axial force and bending, *fBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB*$_{x,}$ satisfies the following criteria.

$f_x \le f_y/\gamma_{m0}$

For cross section without holes, the above criteria reduces to

$$\frac{P}{P_d} + \frac{M_y}{M_{dy}} + \frac{M_z}{M_{dz}} \le 1.0 \qquad (3)$$

where

P_d, M_{dy}, M_{dz} are as defined earlier

2. *Overall Member Strength*

Members subjected to combined axial compression and moment shall be checked for overall buckling failure as given below:

$$\frac{P}{P_{dy}} + k_y \frac{C_{my} M_y}{M_{dy}} + k_{LT} \frac{M_z}{M_{dz}} \le 1.0$$

$$\frac{P}{P_{dz}} + 0.6 k_y \frac{C_{my} M_y}{M_{dy}} + k_z \frac{C_{mz} M_z}{M_{dz}} \le 1.0 \qquad (4)$$

where

C_{my}, C_{mz} =	Equivalent uniform moment factor as per Table 2
P =	applied axial tension or compression under factored load
M_y, M_z =	maximum factored applied bending moments about y and z-axis of the member, respectively.
P_{dy}, P_{dz} =	design strength under axial tension or compression as governed by buckling about minor (y) and major (z) axis respectively.
M_{dy}, M_{dz} =	design bending strength about y (minor) or z (major) axis of the cross section

$K_y = \quad 1+(\lambda_y\text{-}0.2)n_y \le 1+0.8\,n_y$

$K_z = \quad 1+(\lambda_z\text{-}0.2)n_z \le 1+0.8\,n_z$

$$K_{LT} = 1 - \frac{0.1\lambda_{LT} n_y}{(C_{mLT} - 0.25)} \geq 1 - \frac{0.1 n_y}{(C_{mLT} - 0.25)}$$

where,

n_y, n_z = ratio of actual applied axial force to the design axial strength for buckling about the y and z axis, respectively and

C_{mLT} = Equivalent uniform moment factor for lateral torsional buckling as per Table 2 corresponding to the actual moment gradient between lateral supports against torsional deformation in the critical region under consideration.

More accurate evaluation of beam-column strength is possible by resorting to non-linear P-Δ analysis. In this case, the actual axial compression and bending moments as obtained from such an analysis may be used in the interaction equation and the sway effects may be disregarded in evaluation of P_u, P_{ex} and P_{ey}. These methods of analysis and design are beyond the scope of this chapter and are not discussed herein.

3.0 STEPS IN ANALYSING A BEAM-COLUMN

(i) Calculate the cross section properties.

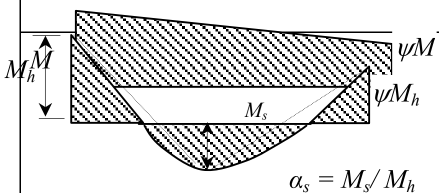
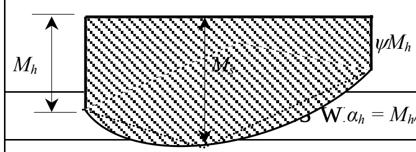
Area, principal axes moments of inertia, section moduli, radii of gyration, effective lengths and slenderness ratios.

(ii) Evaluate the type of section based on the (*b/t*) ratio of the plate elements, as plastic, compact, semi-compact, or slender.

(iii) Check for resistance of the cross-section under the combined effects as governed by yielding (Eq. 1,2 or 3).

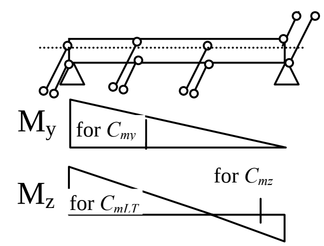
(iv) Check for resistance of member under the combined effects as governed by buckling (Eq. 4).

Table: 2 Equivalent Uniform Moment Factor

Bending moment diagram	**Range**		C_{my}, C_{mz}, C_{mLT}	
			Uniform loading	***Concentrated load***
	$-1 \le \psi \le 1$		$0.6 + 0.4\ \psi \ge 0.4$	
	$0 \le \alpha_s \le 1$	$-1 \le \psi \le 1$	$0.2 + 0.8\ \alpha_s \ge 0.4$	$0.2 + 0.8\ \alpha_s \ge 0.4$
	$-1 \le \alpha_s \le 0$	$0 \le \psi \le 1$	$0.1 - 0.8\ \alpha_s \ge 0.4$	$- 0.8\ \alpha_s \ge 0.4$
		$-1 \le \psi \le 0$	$0.1(1-\psi) -0.8\ \alpha_s \ge 0.4$	$0.2(1-\psi) -0.8\ \alpha_s \ge 0.4$
	$0 \le \alpha_s \le 1$	$-1 \le \psi \le 1$	$0.095 - 0.05\ \alpha_h$	$0.90 + 0.10\ \alpha_h$
	$-1 \le \alpha_s \le 0$	$0 \le \psi \le 1$	$0.095 + 0.05\ \alpha_h$	$0.90 + 0.10\ \alpha_h$
		$-1 \le \psi \le 0$	$0.95 + 0.05\ \alpha_h\ (1+2\ \psi)$	$0.90 + 0.05\ \alpha_h\ (1+2\ \psi)$
s w buckling mode the equivalent uniform moment factor $C_{my} = C_{mz} = 0.9$.				

C_{my}, C_{mz}, C_{mLT} shall be obtained according to the bending moment diagram between the relevant braced points

Moment factor	*Bending axis*	*Points braced in direction*
C_{my}	*z-z*	*y-y*
C_{mz}	*y-y*	*z-z*
C_{mLT}	*z-z*	*z-z*



4.0 SUMMARY

This chapter presented equations for the design of beam-columns and an example design. The behaviour and design of beam-columns are contained in the two parts on this topic. The following are the important points discussed in these chapters.

- The beam-column may fail by reaching either the ultimate strength of the section (in the case of smaller axial load and shorter members) or by the buckling strength as governed by weak axis buckling or lateral torsional buckling.
- At lower loads, the failure is likely to be after the formation of the plastic hinges, especially in the case of shorter members.
- In slender beam-columns with larger axial compression, either weak axis or lateral torsional buckling would control failure.
- The interaction formulae given for the design are conservative and simple, considering the complicated nature of beam-column failure.
- In the design of beam-columns in frames, the magnification of moment due to P-δ and P-Δ effects are to be considered.

5.0 REFERENCES

1. IS: 800(2007), "General Construction in Steel – Code of Practice", Bureau of Indian Standards, New Delhi, 2007.

2. Dowling P.J, Knowles and Owens, G.W., "Structural Steel Design", Butterworth, London, 1998.

3. Eurocode 3: 1992, "Design of Steel structures", British Standards Institution.

15 PLATE GIRDERS - I

1.0 INTRODUCTION

A fabricated plate girder shown diagrammatically in Fig. 1 is employed for supporting heavy loads over long spans. The bending moments and shear forces produced in such girders are well beyond the bending and shear resistance of rolled steel girders available. In such situations the designer has the choice of one of the following solutions:

- Use two or more regularly available sections, side-by-side. This is an expensive solution and may still not satisfy deflection limitation.
- Use a cover-plated beam; i.e. weld a plate of adequate thickness to beef up each flange. This would enhance the bending resistance, in circumstances where the web has adequate shear resistance and the rolled steel section is only marginally inadequate.
- Use a fabricated plate girder, wherein the designer has the freedom (within limits) to choose the size of web and flanges, or
- Use a steel truss or a steel-concrete-composite truss.

This chapter is concerned with plate girders only. Plate girders are large I – shaped sections built up from plates, as shown in Fig. 2.

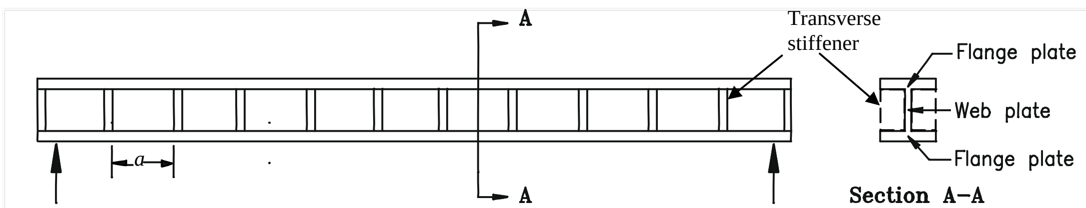


Fig. 1 Typical plate girder with intermediate and end stiffeners

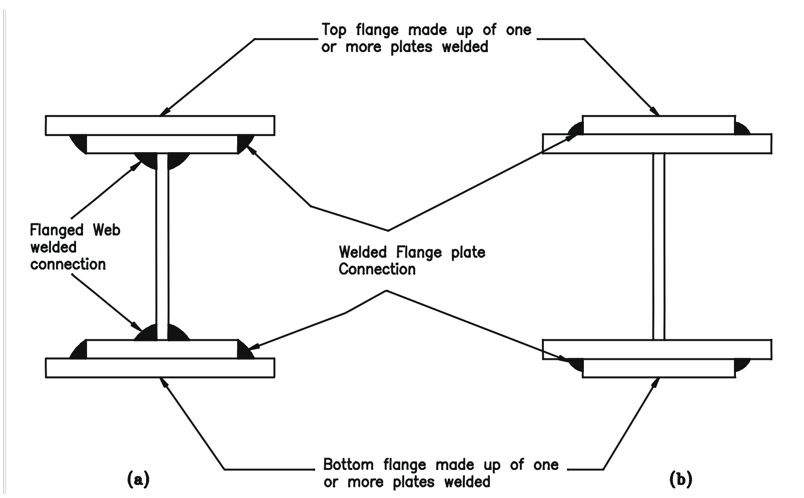


Fig. 2 Cross-section of fabricated plate girders

Nearly all plate girders built today are welded, although they may use bolted field splices. In the West, plate girders are invariably fabricated in fabrication shops, using numerically controlled welding machines [See Fig. 2(*a*)]. If the plate girders are to be fabricated in the field, the type sketched in Fig. 2(*b*) is used to minimise overhead welding.

The primary function of the flange plates is to resist bending moments by developing axial compressive and tensile stresses. The web plate resists the shear. For a given applied bending moment, the axial force decreases, as the depth of the girder (d) increases. From this point of view it is economical to keep the flanges as far apart as possible. This would ensure that the flanges would have to resist smaller axial forces. Thus a smaller area of cross section would suffice than would be the case if a smaller depth were chosen. However, this would also mean that the web would be deep. To reduce the self-weight (and the corresponding self-weight bending moment), the web thickness (t) would have to be limited to slender proportions, (the web proportions are normally expressed in terms of the web slenderness ratio, d/t). Slender webs (with large d/t values) would buckle at relatively low values of applied shear loading. (It is also important to note that webs having a span/depth ratio smaller than *12* would result in a *"deep"* beam, wherein the structural behaviour can no longer be described by conventional simple beam theories).

Efficient and economical design usually results in slender members. Hence advantage must be taken of the post buckling capacity of the web i.e. the ability of the girder to withstand transverse loads considerably in excess of the load at which the web buckles under shear. A girder of high strength to weight ratio can be designed by incorporating the post buckling strength of the web in the design method employed. This would be particularly advantageous where the reduction of self-weight is of prime importance. Examples of such situations arise in long span bridges, ship girders, transfer girders in building etc.

2.0 SHEAR RESISTANCE OF TRANSVERSELY STIFFENED PLATE GIRDERS

Webs of plate girders are usually stiffened transversely as shown in Figure 1. This helps to increase the ultimate shear resistance of the webs, as will be seen later. The stiffener spacing *(a)* influences both buckling and post buckled behaviour of the web under shear. In order to allow for this, the parameter (a/d) which accounts for the geometry of the web panel is important. Obviously, a long span girder will have various web panels and each panel will have different combinations of bending moments and shear forces. In a long plate girder, panels close to the support will be subjected to predominant shear and those close to the centre, to predominant bending moments.

In what follows, the effect of shear will be considered first, followed by the effect of co-existing bending moment and shear forces.

2.1 Shear resistance of a web

2.1.1 Pre-buckling behaviour (Stage 1)

When a web plate is subjected to shear, we can visualise the structural behaviour by considering the effect of complementary shear stresses generating diagonal tension and diagonal compression.

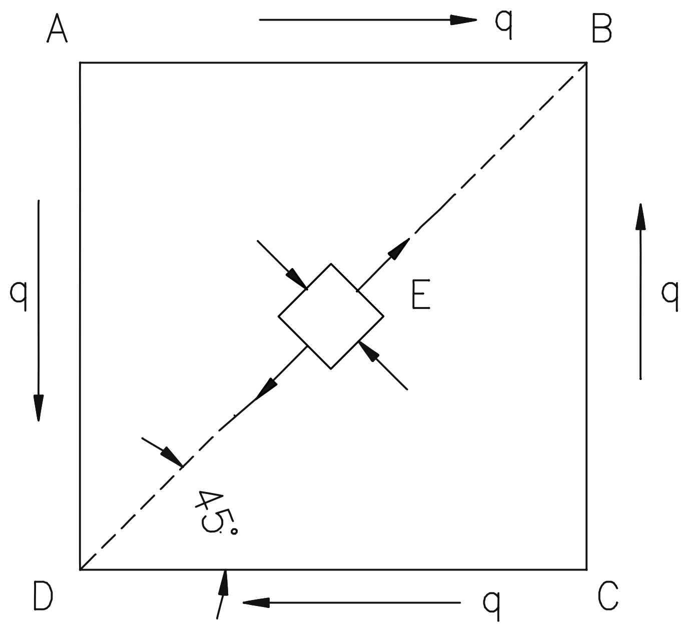


Fig. 3 Unbuckled shear panel

Consider an element *E* in equilibrium inside a square web plate subject to a shear stress *q*. The requirements of equilibrium result in the generation of complementary shear stresses as shown in Fig. 3. This results in the element being subjected to principal compression along the direction *AC* and tension along the direction of *BD*. As the applied loading is incrementally enhanced, with corresponding increases in *q*, very soon, the plate will buckle along the direction of compressive diagonal *AC*. The plate will lose its capacity to any further increase in compressive stress; the corresponding shear stress in the plate is the "*critical shear stress*" q_{cr}. The value of q_{cr} can be determined from classical stability theory if the boundary conditions of the plate are known. As the true boundary conditions of the plate girder web are difficult to establish due to restraint offered by the flanges and stiffeners we may conservatively assume them to be simply supported. The critical shear stress in such a case is given by

$$q_{cr} = k_s \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{d}\right)^2 \qquad (1)$$

where, k_s is the shear buckling coefficient given by

$$k_s = 5.35 + 4\left(\frac{d}{a}\right)^2 \quad where\ \frac{a}{d} \geq 1,\ \ i.e.\ for\ wide\ panels$$

$$k_s = 5.35\left(\frac{d}{a}\right)^2 + 4\ where\ \frac{a}{d} \leq 1,\ \ i.e.\ for\ webs\ with\ closely\ spaced\ transverse\ stiffeners$$

Values of q_{cr} for various values of web aspect ratios are tabulated in Table 1.

Table 1: q_{cr} (MPa) Values

d/t \ a/d	**1.0**	**1.5**	**2.0**	**0.5**
100	169	129	115	Buckling does not govern
125	108	83	73	Buckling does not govern
150	75	57	51	204
175	55	42	37	150
200	42	32	29	115
225	33	25	23	91
250	27	21	18	72

$E = 200{,}000\ MPa \qquad \nu = 0.3 \qquad \pi = 3.1412$

When the value of (d/t) is sufficiently low ($d/t < 85$) q_{cr} increases above the value of yield shear stress and the web will yield under shear before buckling.

2.1.2 Post buckled behaviour (Stage 2)

The compression diagonal (*AC*) is unable to resist any more loading beyond the one corresponding to the elastic critical stress. Once the web has lost its capacity to sustain increase in compressive stresses, a new load–carrying mechanism is developed. Applications of any further increases in the shear load are supported by a *tensile membrane field*, anchored to the boundaries, viz. the top and bottom flanges and the adjacent stiffener members on either side of the web. The angle of inclination of the membrane stress (θ) is unknown at this stage (See Fig. 4). Thus the total state of stress in this web plate may be obtained by superimposing the post-buckled membrane tensile stresses (p_t) upon those set up when the applied shear stress reached the critical value q_{cr}.

The state of stress in the web in the post-buckled stage is shown in Fig. 5.

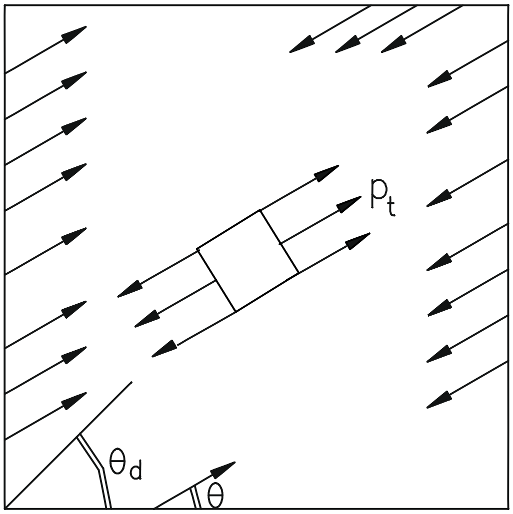


Fig. 4 Post buckled behaviour

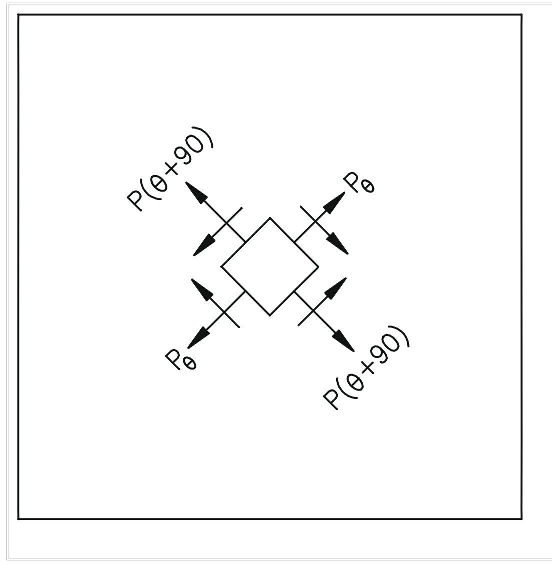


Fig. 5 State of stress in the web in the post buckled stage

Resolving these stresses in the direction along and perpendicular to the inclination θ we get,

$$\begin{aligned} p_\theta &= q_{cr}.\sin 2\theta + p_t \\ p_{(\theta+90)} &= -q_{cr}\sin 2\theta \\ q_\theta &= -q_{cr}.\cos 2\theta \end{aligned} \qquad (2)$$

Since the flanges are of finite rigidity, the pull exerted by the tensile membrane stresses in the web will cause the flanges to bend inwards.

2.1.3 Collapse behaviour (Stage 3)

When the load is further increased, the tensile membrane stress *(p_t)* developed in the web continues to exert an increasing pull on the flanges. Eventually the resultant stress *(p_θ)* (obtained by combining the buckling stress in Equation (1) and the membrane stress p_t) reaches the yield value for the web.

This value (of the membrane stress at yield) may be denoted by p_{yw}, and may be determined by Von-Mises yield criterion.

$$p_\theta^2 + p_{(\theta+90)}^2 - p_\theta . p_{(\theta+90)} + 3q_\theta^2 = p_{yw}^2 \qquad (3)$$

where, p_{yw} = tensile yield stress of the web

By substituting the values of p_θ, $p_{(\theta+90)}$ *and* q_θ from Equation (2), the above equation may be presented in a non-dimensional form as follows:

$$\frac{p_{yt}}{p_{yw}} = \sqrt{\left(1-\frac{q_{cr}}{q_{yw}}\right)^2\left(1-\frac{3}{4}\sin^2 2\theta\right)} - \frac{\sqrt{3}}{2}.\frac{q_{cr}}{q_{yw}}\sin 2\theta \qquad (4)$$

where, q_{cr} is obtained from Equation (1)

$$q_{yw} = \frac{p_{yw}}{\sqrt{3}} \qquad i.e.\, shear\ yield = \frac{tensile\ yield}{\sqrt{3}} \qquad (5)$$

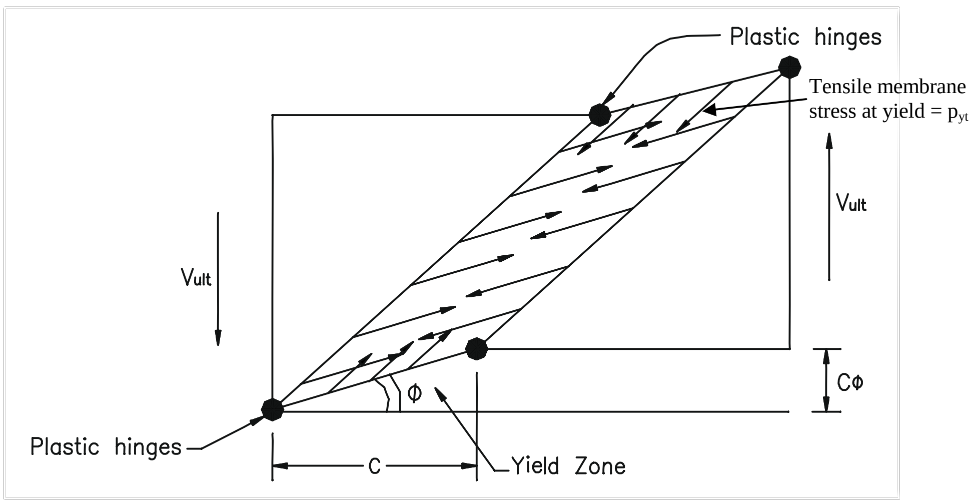


Fig. 6 Collapse of the panel

Once the web has yielded, final collapse of the girder will occur when four plastic hinges are formed in the flanges as shown in Fig. 6. The plastic moment capacity of the flange plate is M_{pf}.

By using the virtual work method, Rockey and his team at Cardiff have shown that the failure load can be computed from

$$V_S = q_{cr}.d.t + p_{yt}.t\sin^2\theta\left(d\cot\theta - a + c\right) + 4\frac{M_{pf}}{c} \qquad (6)$$

where c = distance between the hinges given by

$$c = \frac{2}{\sin\theta}\sqrt{\frac{M_{pf}}{p_{yt}.t}} \qquad (7)$$

This equation can be non-dimensionalised by using the shear load required to produce yielding in the entire web *($V_{yw} = q_{yw}.\ d.t.$)*

$$\frac{V_S}{V_{yw}} = \frac{q_{cr}}{q_{yw}} + \sqrt{3}\sin^2\theta\left(\cot\theta - \frac{a}{d}\right)\frac{p_{yt}}{p_{yw}} + 4\sqrt{3}\sin\theta\sqrt{\frac{p_{yt}}{p_{yw}}\cdot\frac{M_{pf}}{d^2 t.p_{yw}}} \qquad (8)$$

Equation (6) or (8) can be solved if θ is known. As these equations are based on Energy Method, the correct solution will be obtained by maximising V_s with respect to θ. By a systematic set of parametric studies, Evans has established that θ is approximately equal to *2/3* of the inclination of diagonal of the web.

$$\theta \cong \frac{2}{3}\tan^{-1}\left(\frac{d}{a}\right) \qquad (9)$$

Notice that Equations (6) and (8) are obtained by adding 3 quantities: web-buckling strength, post-buckling membrane strength of the web plate and the plastic moment capacity of the flange.

In this context, it must be noted that in order for the flanges to develop hinges, the flanges must be classifiable as "***plastic***" sections. If flanges can not develop plastic hinges because they are compact, semi-compact or slender, this method of analysis is NOT applicable.

2.1.4 "Weak" flanges

When a plate girder has weak flanges, M_{pf} is a small quantity in comparison with the other terms. Hence

$$\frac{V_S}{V_{yw}} = \frac{q_{cr}}{q_{yw}} + \sqrt{3}\sin^2\theta\left(\cot\theta - \frac{a}{d}\right)\frac{p_{yt}}{p_{yw}} \qquad (10)$$

In this case the tension field is NOT supported by flanges.

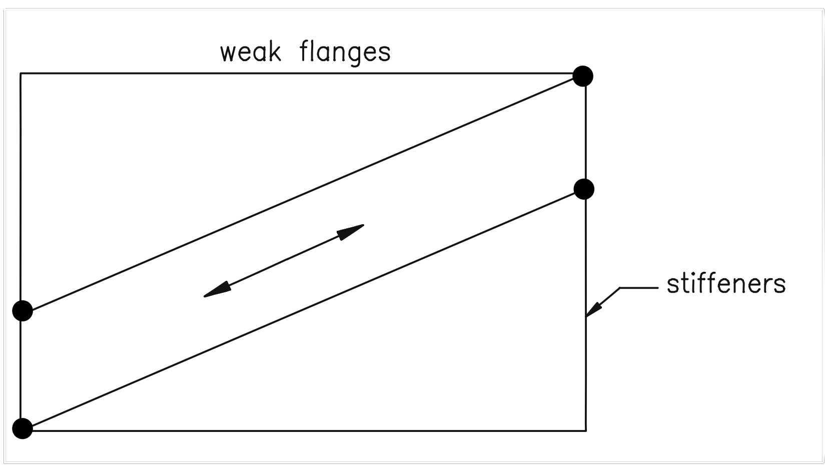


Fig. 7 Weak flange

The field anchors entirely on transverse stiffeners as shown in Fig. 7.

2.1.5 Very "Strong" flanges

When *"Very Strong"* flanges are employed, the distance *(c)* of the plastic hinge away from the end panel increases. When $c = a$, the hinges will form at the four corners, constituting a *"picture frame"* type mechanism (See Fig. 8) and the tension field angle (θ) is $45°$. Ultimate shear in this case is given by:

$$\frac{V_S}{V_{yw}} = \frac{1}{4}\frac{q_{cr}}{q_{yw}} + \frac{\sqrt{3}}{2}\sqrt{\left(1 - \frac{1}{4}\left(\frac{q_{cr}}{q_{yw}}\right)^2\right)} + 4\sqrt{3}\,\frac{d}{a}.\frac{M_{pf}}{d^2 t\, p_{yw}} \qquad (11)$$

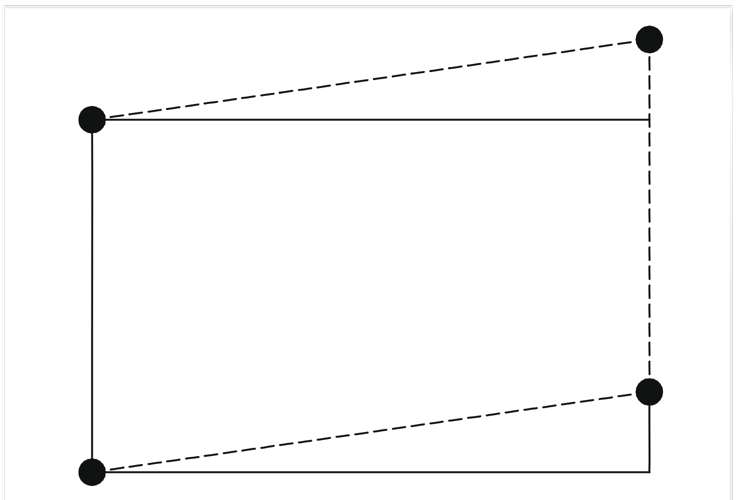


Fig. 8 Picture frame mechanism of strong flange

2.1.6 Very "Thick" webs

In case the web is thick, it will yield before buckling failure will form by a picture frame mechanism, with $q_{cr} = q_{yw}$.

$$\frac{V_S}{V_{yw}} = 1 + 4\sqrt{3}\left(\frac{d}{a}\right).\frac{M_{pf}}{d^2 t\, p_{yw}} \qquad (12)$$

2.1.7 Very "Slender" webs

Very slender webs are rarely used as they cause anxieties for the users due to high levels of buckling.

In very slender webs q_{cr}/q_{yw} is extremely small and there will be significant post-buckled tension field, the value of membrane stress p_{yt} will be very large. The general expression given in Equation (8) is valid and θ can be evaluated using equation (9).

3.0 WEBS SUBJECTED TO CO-EXISTENT BENDING AND SHEAR

When a girder is subjected to predominant bending moments and low shear, its ultimate capacity is conditioned by the interaction between the effects of the bending moment and shear force.

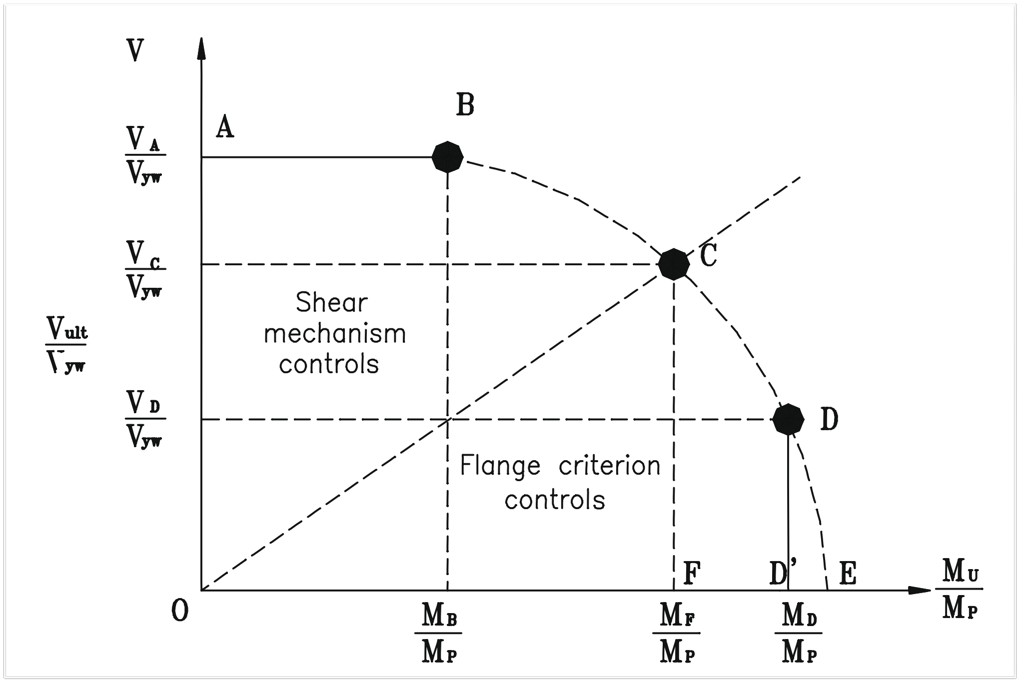


Fig . 9(a)

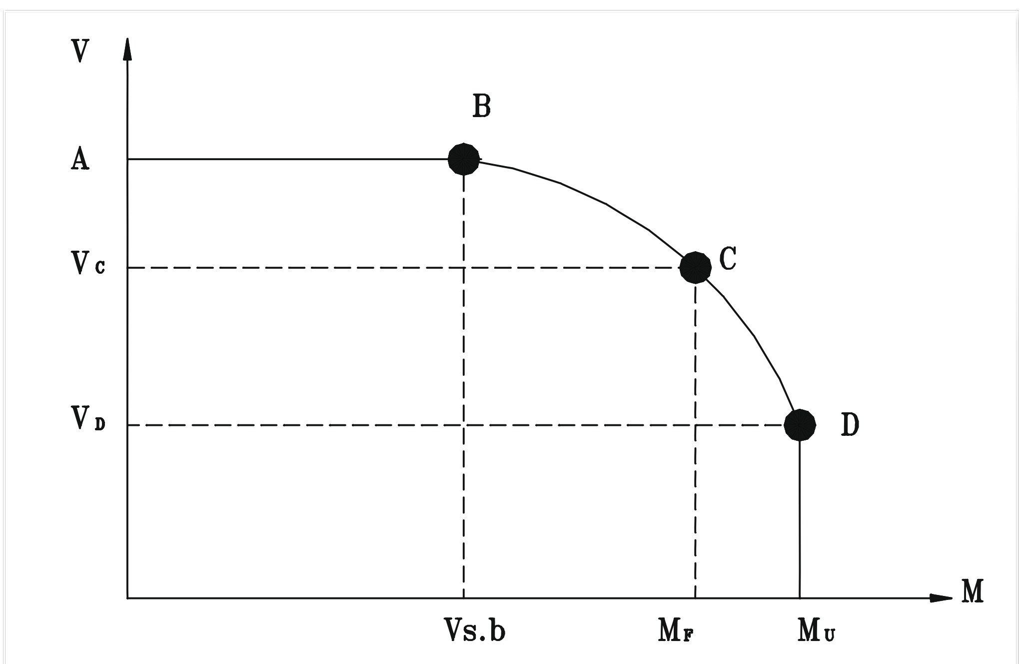


Fig. 9(b)

Fig. 9 Interaction between bending and shear effects

The interaction diagram is generally expressed in the form seen in Fig. 9, where the shear capacity is plotted in the *Y*-axis and the bending capacity in the *X*-axis. Any point in the interaction diagram shows the co-existent values of shear and bending moment that the girder can sustain. The vertical ordinates are non-dimensionalised using V_{yw} (Yield shear of the web) and the horizontal ordinates by M_p (the fully plastic moment resistance of the cross section). The portion of the curve between points *A* and *C* is the region in which the girder will fail by predominant shear, i.e. shear mechanism of the type represented in Fig. 6 will develop at collapse.

The vertical ordinate at A presents the shear capacity (V_s) given by Equation 8. This shear capacity will reduce gradually due to the presence of co-existent bending moment. Beyond point *C*, when the applied moment is high, the failure will be triggered by the collapse of flanges by one of the following: (i) by yielding of flange material or (ii) by inward buckling of the compression flange or (iii) by lateral buckling of the flange. Thus there is a distinct change in failure criterion represented by line *OC* in Fig. 9(*a*); the left of *OC* represents shear failure and the right of *OC*, flexural failure. Generally the flange failure mode will be triggered, when the applied bending moment is approximately equal to the plastic moment resistance M_F, provided by the flange plates only, neglecting the contribution from the web.

$$M_F = b_f \,.T\,.\, p_{yf} \,(d + T) \qquad (13)$$

where, b_f - Breadth of flange
T - Thickness of flange
p_{yf} - Design stress of flange
d - depth of web plate

This value represents the horizontal co-ordinate of the point *C*, i.e. the point *F*. In zone *ABC*, the presence of additional bending moment requires the following three factors to be considered.

- The reduction in the web buckling stress due to the presence of bending stresses.
- The influence of bending stresses on the value of membrane stress required causing yield in the web.
- The reduction of plastic moment capacity of flanges due to the presence of axial flange stresses caused by bending moment.

3.1 Modified web buckling stress

The modified web buckling stress due to coincident bending stress may be computed from the following interaction Equation:

$$\left(\frac{q_{crm}}{q_{cr}}\right)^2 + \left(\frac{f_{mb}}{f_{crb}}\right) = 1 \qquad (14)$$

where, q_{crm} = modified shear buckling stress in web

q_{cr} = elastic critical shear stress in web (pure shear case as defined previously)

f_{mb} = compressive bending stress in the extreme fibre at the mid panel due to the bending moment.

f_{crb} = buckling stress for the plate due to a pure bending moment given by

$$f_{crb} = 23.9\ \frac{\pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t}{d}\right)^2 \qquad (15)$$

3.2 Modified membrane stress for web yielding

The bending membrane stress *(p_{yt})* to be added to the critical shear stress *(q_{cr})* was calculated in the pure shear case, using Equations (4) and (1) respectively. The modified expression for the membrane stress p_{ytm}, in the presence of applied bending moment is given by

$$p_{ytm} = -\frac{1}{2}A + \frac{1}{2}\sqrt{\left(A^2 - 4\left(p_b^2 + 3q_{crm}^2 - p_{yw}^2\right)\right)} \qquad (16)$$

where, $A = 3\ q_{crm} \sin 2\theta + p_b \sin^2\theta - 2p_b \cos^2\theta$

p_b = The value of bending stress, which varies over both the depth and width of the web panel.

As p_{ytm} varies for various values of p_b, it may be necessary to compute p_{ytm} at a number of locations in order to compute the resultant of the membrane stresses. This could be time-consuming. To simplify the design calculations an adequately accurate approximate procedure is suggested a little later.

3.3 Reduction of plastic moment capacity of flanges

When high axial forces are developed in the flanges due to bending moments, their effects in reducing plastic moment capacity of flange plates must be taken into account. From plasticity theory, the reduced capacity *(M'_{pf})* is given by

$$M'_{pf} = M_{pf}\left[1 - \left(\frac{p_f}{p_{yt}}\right)^2\right] \qquad (17)$$

where, p_f is the average axial stress for the portion of the flange between hinges.

3.4 Design procedure

The simplified design procedure suggested by Rockey et. al (*1978*) is validated by them by experiments and parametric studies. This procedure is summarised below:

The shear load capacity at point C of the interaction diagram may be obtained approximately from an empirical relationship given below.

$$\left(\frac{V_c}{V_{yw}}\right) = \frac{q_{cr}}{q_{yw}} + \frac{p_{yt}}{p_{yw}}\sin\left(\frac{4}{3}\theta_d\right)\left[0.554 + 36.8\,\frac{M_{pf}}{M_F}\right]\left[2 - \left(\frac{b}{d}\right)^{\frac{1}{8}}\right] \qquad (18)$$

This equation gives the vertical ordinate of the point C in the interaction diagram [Fig. 9(a)]. The horizontal ordinate as stated previously is given by the value of M_F (See Equation 13).

The interaction diagram is constructed in stages as follows [See Fig. 9(b)]:

(i) Between A and B, the curve is horizontal. The horizontal ordinate B is given by maximum bending moment in the end panel given by $V_s.b$, but limited to a value of $0.5M_P$.

(ii) Between B and C, the curve may be straight (for simplicity). The moment corresponding to C is given by

$$M_F = b_f \,.\, T \,.\, p_{yf} \; (d + T)$$

(iii) The point D represents nearly the ultimate capacity of the flanges (M_u) and the shear values when high bending is present. This is discussed in the next section.

3.5 Webs subjected to pure bending

The region beyond C of the interaction diagram represents a high bending moment, so the failure is by bending moment, rather than by shear mode. In a thin walled girder, the web subjected to compressive bending stress will buckle, thereby losing its capacity to carry further compressive stresses. The compression flange will therefore carry practically all the compressive stresses, as the web is unable to be fully effective. Consequently the girder is unable to develop full plastic moment of resistance (M_p) of the cross section.

If no lateral buckling occurs (e.g. by provision of adequate lateral supports), the girder will fail by inward collapse of compression flange at an applied moment (M_u) which is approximately equal to the moment required to produce first yield in the extreme fibres of compression flange. This moment is – of course – reduced because of the effects of web buckling. Though the concept is simple, the resulting calculations are complex. The research of Cooper in *1971* enables the ultimate moment capacity to be determined by a simple formula:

$$\frac{M_u}{M_y} = 1 - 0.0005\,\frac{A_w}{A_f}\left[\frac{d}{t} - 5.7\sqrt{\frac{E}{p_{yf}}}\right] \geq \frac{M_F}{M_y} \qquad (19)$$

M_y = Bending moment required to produce yield in the extreme fibre of flange assuming fully effective web (i.e. neglecting web buckling)

This value of M_u is the moment required to produce yield in the extreme fibres of the flange. The corresponding stresses in the web will be below yield. (Point *D* in the interaction diagrams). The ordinate of *D* can be calculated approximately from

$$\frac{V_D}{V_C} = \sqrt{\frac{M_p - M_u}{M_{pw}}} \qquad (20)$$

where, M_p is the fully plastic moment capacity of the complete cross section
M_{pw} = plastic moment resistance of the web plate alone.
$= 0.25\, td^2 .\, p_{yw}$
The complete interaction diagram can now be drawn.

4.0 ULTIMATE BEHAVIOUR OF TRANSVERSE WEB STIFFENERS

The shear failure mechanism described so far has been extensively verified by experiments. Before post-buckling action in webs can develop the members in the boundary (viz. the flanges and the stiffeners) must be able to support the forces imposed on them by the web tension field.

The transverse stiffeners play an important role in allowing the full ultimate capacity of the girder to be achieved (a) by increasing the web buckling stress (b) by supporting the tension field after web buckling and (c) by preventing the tendency of flanges to get pulled towards each other. The stiffeners must therefore possess sufficient rigidity to ensure that they remain straight, while restricting buckling to the individual web panels.

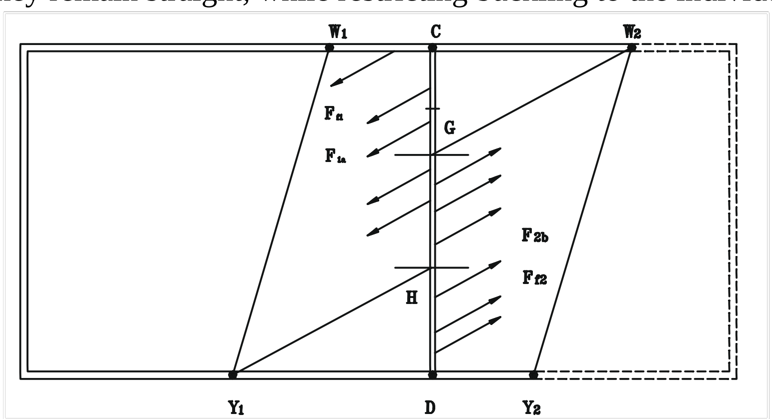


Fig. 10 Force imposed on transverse stiffeners by tension field

4.1 Analysis of loads imposed on the transverse stiffener

Fig. 10 represents the loads acting on typical stiffener *CD* positioned between two adjacent panels, each of which have developed shear failure mechanism. This is perhaps the most critical form of loading of an intermediate stiffener.

Let us consider the loads on the intermediate stiffener

The resultant of the loads acting on portion W_1C of top flanges is F_{fw}, inclined at an angle of θ_1 and DY_2 of bottom flange is $F_{f.2}$, inclined at an angle of θ_2. The vertical component of these forces will tend to pull the flanges together and the stiffener will resist this by developing end loads (V_C and V_D).

$$V_C = -F_{f.1}\sin\theta_1$$
$$V_D = +F_{f.2}\sin\theta_2 \qquad (21)$$

Moreover, the loading imposed directly upon the stiffener by web tension field can be split into *3* zones: the top part *CG* is subject to a pull to the left by the left hand panel, the bottom part *HD* is similarly subject to a pull to the right by the right hand panel. The part *GH* is pulled to the left and to the right (these two forces more or less balancing each other). Thus the central region remains virtually unloaded by the tension field action.

The vertical component of forces on zones CG and *DH* are respectively obtained as

$$V_1 = -p_{yt1}.t.CG.\sin\theta_1.\cos\theta_1$$
$$V_2 = -p_{yt2}.t.HD.\sin\theta_2.\cos\theta_2 \qquad (22)$$

Thus once the ultimate shear loading and the geometry of failure mechanism has been determined, calculations of forces on the stiffeners by employing Equations (21) and (22) may be made.

Unfortunately, the actual behaviour of stiffeners (as evidenced by experimental studies by Rockey in 1981 and Puthali in 1979) is somewhat different from the simple model described above.
Firstly, a portion of the web plate acts with the stiffeners in resisting the axial load, despite the fact the web has theoretically yielded due to tension field. The effective cross section is in the form of a *T* section or a cruciform section, if the stiffeners are on both sides of the web plate.

Secondly the theory explained previously assumed that the axial loading is applied to the stiffener cross section at its mid thickness. The true position of load application is unknown and some degree of eccentricity of application of loads is inevitable.

Thus the stiffener is subjected to both axial load *(P)* and bending moment arising due to the eccentricity of the applied load from the centroidal axis ($\bar{x}$) and is given by $P\bar{x}$. There will–inevitably be–some imperfections (δ_o) in the stiffener giving rise to consequent moments of $P\delta_o$. The disturbing action on the stiffener due to the web buckling is difficult to quantify but nevertheless is present.

Horne (*1979*) has proposed a suitable expression to define the combination of axial load and bending moment (making allowance for the above complexities) which can be sustained by a stiffener:

$$\frac{M}{M_{ps}} = 1.0 - \frac{p_{ys}\, t_s\, \left(b_s - \bar{x} + 0.5t\right)^2}{M_{ps}}\, \frac{P}{P_s} \qquad (23)$$

where, M_{ps} = Full plastic moment capacity of the section where there is no axial loading
P_s = Squash load (i.e. full axial yield load)
b_s = Width of stiffener
t_s = Thickness of stiffener
p_{ys} = Design stress of stiffener
t = Thickness of web

For any girder, the axial load to which the stiffener is subjected can be computed from Equations (21) and (22). Then, provided the co-existent moment is less than the allowable moment defined by Equation (23), the stiffener will be able to support the loads to which it is subjected.

The theory governing the design of stiffeners is given above; but the design codes make simplifications to ease the task of the designers and enable quick sizing of the stiffeners.

5.0 GENERAL BEHAVIOUR OF LONGITUDINALLY STIFFENED GIRDERS

In order to obtain greater economy and efficiency in the design of plate girders, slender webs are often reinforced both longitudinally and transversely. The longitudinal stiffeners are generally located in the compression zones of the girder. The main function of the longitudinal stiffeners is to increase the buckling resistance of web. The longitudinal stiffener remains straight thereby subdividing the web and limiting the web buckling to smaller web panels. In the past it was usually thought that the resulting increase in ultimate strengths could be significant. Recent studies have shown that this is not always the case, as the additional cost of welding the longitudinal stiffeners invariably offsets any economy resulting in their use.

The main effect of longitudinal stiffeners is to increase the elastic critical buckling strength. Studies by Rockey et al have shown that a longitudinally reinforced plate girder subject predominantly to shear would develop a collapse mechanism, similar to the one described previously, provided the stiffeners remained rigid up to failure. Once a tension field develops it extends over the complete depth of the girder. In other words, once one of the sub panels has buckled, the post buckling tension field develops over the whole depth of the web panel and the influence of the stiffeners may be neglected. Thus the equations established previously are valid, keeping in mind q_{cr} values are enhanced due to the smaller panel dimensions.

In the design of longitudinal stiffeners, linear buckling theories are often used to establish the minimum value of stiffener rigidity required to ensure that the longitudinal stiffeners remain straight at first buckling. This would ensure that the buckling is limited to individual sub-panels of the web. This invariably involves the provision of stiffeners of substantial sizes, just so that they would remain straight without themselves buckling.

Normally the rigidity of such stiffeners has to be increased by *4-6* times, to satisfy this requirement. The heavy stiffeners thus designed, though adequate, will naturally increase the weight of steel used and therefore the cost. It seems that it is more sensible to increase the web thickness in these cases.

In general, the extra cost of providing longitudinal stiffeners is rarely justified. In western countries numerically controlled machines are used extensively for fabricating plate girders. In these countries the provision of longitudinal (and transverse) stiffeners involves manual welding and contributes to the rise in the cost of fabrication. Indeed, in Scandinavian countries, the current trend is to eliminate or minimise the use of transverse stiffeners as far as possible. The use of longitudinal stiffeners has also been discontinued for this reason in these countries. In other words, if they can help it, they do not use any stiffeners at all!

Longitudinal stiffeners are rarely – if ever – used in buildings. They are sometimes used in bridges, particularly when the Elastic Design is employed. Fig. 11 shows a typical plate girder with longitudinal and transverse stiffeners.

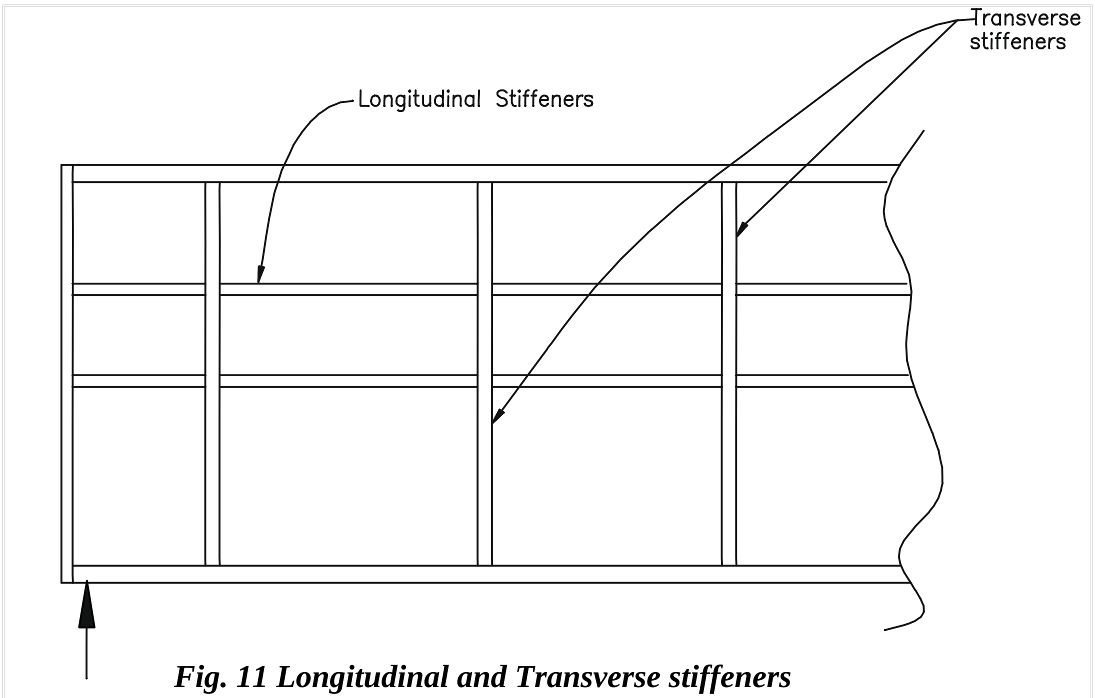


Fig. 11 Longitudinal and Transverse stiffeners

6.0 CONCLUSIONS

This chapter has considered the ultimate behaviour of plate girders in some detail. Fundamental theoretical relationship based on buckling and post-buckling theories have been established. In some case, semi-empirical procedures have been suggested to ease the tedium of lengthy calculations. Transverse stiffeners have been considered in some depth. The use of longitudinal stiffeners has also been described and the reasons for not using these extensively have been discussed.

7.0 REFERENCES

1. Narayanan R. *Plated Structures, Stability & Strength*. *Chapter 1 Longitudinally and transversely reinforced Plate Girders,* (by EVANS H.R.) Elsevier Applied Science Publishers, London, 1983.
2. Owens D.R.G. Skaloud, M. and Rockey K.C. *Ultimate Load behaviour of longitudinally reinforced web plates*, IABSE Memories 1970 Zurich 113 – 148.
3. Cooper P.B *The ultimate bending moment for Plate Girders,* Proceedings of IABSE Colloqum, London, 1971.
4. Mele M and Puthali R : *University of Rome Reports* CECAR / 136, 1979.
5. Rockey K.C., Evans H.R. and Porter D.M. *A design method for predicting collapse behaviour of Plate Girders,* Proceedings of the Institution of Civil Engineers 1978, pp 85-112.
6. Rockey K.C. and Skaloud M : *Influence of flange stiffeners on load carrying capacity of webs in shear*. Proceedings of the Eighth Congress IABSE, New York, 429-439.

16 PLATE GIRDERS – II

1.0 INTRODUCTION

This chapter describes the current practice for the design of plate girders adopting meaningful simplifications of the equations derived in the chapter on Plate Girders – I, as per provisions of BS 5950: Part – 1 for buildings.

It is important to choose appropriate sections for various components of the plate girder. In these girders, the bending moments are assumed to be carried by the flanges by developing compressive and tensile forces. To effect economy, the web depth '*d*' (See Fig. 1) is chosen to be large enough to result in low flange forces for the design bending moment.

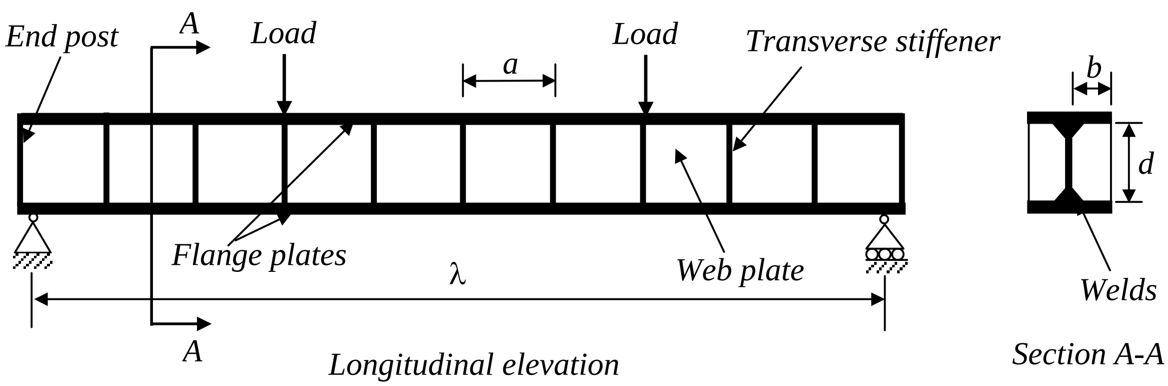


Fig. 1 A typical Plate Girder

1.1 Span to depth ratios

The recommended span / depth (λ/d) ratios for initial choice of cross-section in a plate girder used in a building is given below:

i.	Constant depth beams used in simply-supported composite and non-composite girders with concrete decking	*12 < λ/d < 20*
ii.	Constant depth beams in continuous composite and non-composite girders	*15 < λ/d < 25*
iii.	Simply-supported crane girders	*10 < λ/d < 15*

1.2 Recommended proportions for web

When $d/t \leq 66.2\varepsilon$, where $\varepsilon = (250/f_y)^{1/2}$ the web plate will not buckle because the shear stress 'q' is less than critical buckling stress 'q_{cr}'. The design, in such cases, is similar to rolled steel beams. Here we consider plate girders having thin webs with $d/t > 66.2\varepsilon$. In the design of these webs, shear buckling should be considered. In general we may have an un-stiffened web, a web stiffened by transverse stiffeners (Fig. 1) or a web stiffened by both transverse and longitudinal stiffeners (Fig. 2).

By choosing a minimum web thickness 't', the self-weight is reduced. However, the webs are vulnerable to buckling and hence are stiffened if necessary. The web thicknesses recommended are:

i. For un-stiffened web $\quad t \geq d/250$

ii. For stiffened web with $a/d > 1$; $\quad t \geq d/250$

and with $a/d \leq 1$; $\quad t \geq (d/250)(a/d)^{1/2}$

where a is the horizontal spacing between the transverse stiffeners in a web of depth d and thickness t.

In practice, however, $a/d < 1$ is rarely used - if at all - in plate girders used in buildings and bridges.

To avoid flange buckling into web, BS 5950: Part - 1, specifies

i. For un-stiffened web $\quad t \geq (d/294)(p_{yf}/250)^{1/2}$

where p_{yf} is the design stress of flange material.

ii. For stiffened web with $a/d > 1.5$; $\quad t \geq (d/294)(p_{yf}/250)^{1/2}$

and with $a/d \leq 1.5$; $\quad t \geq (d/337)(p_{yf}/250)^{1/2}$

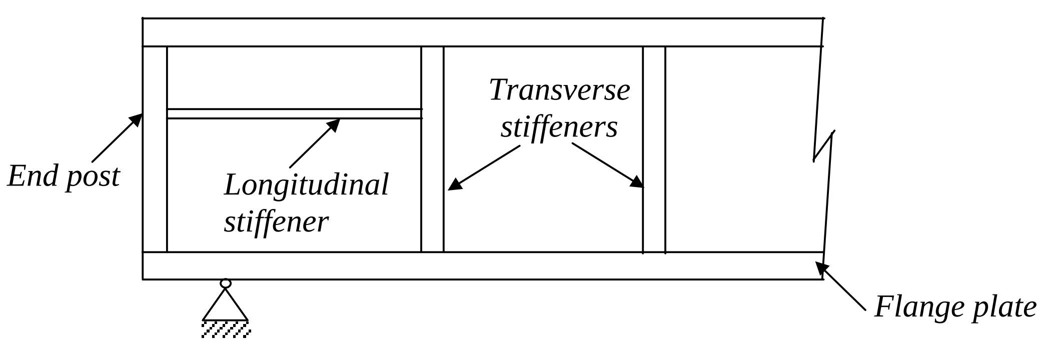


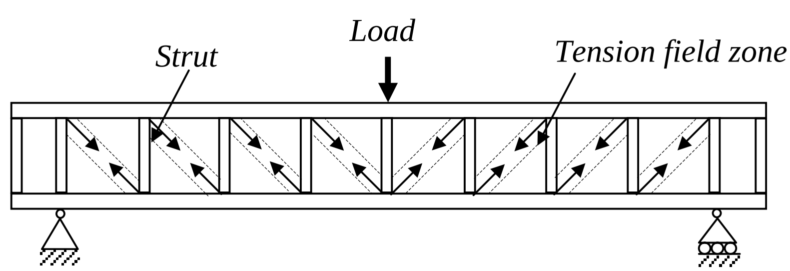
Fig. 2 End panel strengthened by longitudinal stiffener

1.3 Recommended proportion of flanges

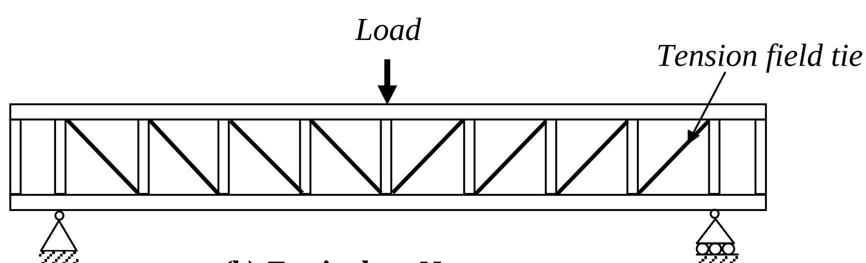
Generally the thickness of flange plates is not varied along the spans for plate girders used in buildings. For non-composite plate girder the width of flange plate is chosen to be about *0.3* times the depth of the section as a thumb rule. It is also necessary to choose the breadth to thickness ratio of the flange such that the section classification is generally limited to plastic or compact sections only ($b/T \leq 8.9\varepsilon$). This is to avoid local buckling before reaching the yield stress. For preliminary sizing, the overall flange width-to-thickness ratio may be limited to *24*. For the tension flanges (i.e. bottom flange of a simply supported girder) the width can be increased by *30%*.

1.4 Stiffener spacing

Vertical stiffeners are provided close to supports to increase the bearing resistance and to improve shear capacity. Horizontal stiffeners are generally not provided in plate girders used in buildings. Intermediate stiffeners also may not be required in the mid-span region. When vertical stiffeners are provided, the panel aspect ratio a/d (see Fig.1) is chosen in the range of *1.2* to *1.6*. The web is able to sustain shear in excess of shear force corresponding to q_{cr} because of vertical stiffeners. Vertical stiffeners help to support the tension field action of the web panel between them. Where the end panel near support is designed without using the tension field action a smaller spacing of $a/d = 0.6$ *-1.0* is adopted. Sometimes double stiffeners are adopted near the bearing (see Fig. 3) and in such cases the overhangs beyond the supports are limited to *1/8* of the depth of the girder.



(a) Tension field in individual sub-panel



(b) Equivalent N - truss

Fig. (3) Tension field action and the equivalent N - truss

2.0 PROVISIONS FOR MOMENT AND SHEAR CAPACITY AS PER BS 5950-PART 1

Any cross section of the plate girder will have to resist shear force and bending moment. The design may be based on any one of the following assumptions:

1) The moment is resisted by the flanges and the shear is resisted by the web only.
2) The moment is resisted by the entire section while the web is designed to resist shear and longitudinal stresses due to bending.
3) A combination of (1) and (2) above by approximating a percentage of the shear to the web and remaining to the entire section. This method is rarely used.

The assumptions made in method (1) leads to mathematical simplification and a good and simple visualisation of load transfer mechanism. This method is used for computing the moment and shear capacity of a section as indicated below.

2.1 Moment Capacity

Moment capacity M_c is computed from the plastic capacity of the flanges. Thus,

$$M_c = p_{yf}\ Z_{pf} \qquad (1)$$

where, p_{yf} = The design stress of the flange steel ($= f_{yf}/\gamma_m$)
Z_{pf} = Plastic section modulus of flanges about the transverse axis of the section.
γ_m = Material safety factor for steel (= 1.15)

2.2 Shear Capacity

Thin webs are designed either with or without stiffeners. These two cases are described individually below.

2.2.1 Webs without intermediate stiffeners

The shear capacity of un-stiffened webs is limited to its shear buckling resistance. Hence,

$$V_{cr} = q_{cr}\, d\, t \qquad (2)$$

The elastic pre-buckling behaviour was described in section 2.1.1 of the previous chapter. Based on this theory the code gives the following values for q_{cr}, for webs, which are not too slender. These values depend on the slenderness parameter λ_w defined as

$$\lambda_w = (0.6\ p_{yw} / q_e)^{1/2} \qquad (3)$$

where, q_e = Elastic critical shear strength values to be used in design for different values of a/d and d/t are tabulated in Table - 1.

p_{yw} = Design strength of web ($= f_{yw}/\gamma_m$)

γ_m = Material safety factor for steel (= *1.15*)

The elastic critical stress (Refer Table 1 of the previous chapter) has been simplified and given based on *a/d* and *t/d* as given in Table - 1.

Table 1: Elastic critical stress related to aspect ratio

Aspect ratio	***Elastic critical stress***
$a/d \leq 1$	$q_e = [0.75 + 1/(a/d)^2]\ [1000/(d/t)]^2$
$a/d > 1$	$q_e = [1 + 0.75/(a/d)^2]\ [1000/(d/t)]^2$

where, *a* is the stiffener spacing

d is the depth of web

Table - 2 gives the values of q_{cr} recommended by the code for design purposes.

Table 2: Elastic critical stress for design purposes

$\lambda_w \leq 0.8$	$0.8 < \lambda_w < 1.25$	$\lambda_w \geq 1.25$
$q_{cr} = 0.6\ p_{yw}$	$q_{cr} = 0.6\ p_{yw}[1 - 0.8(\lambda_w - 0.8)]$	$q_{cr} = q_e$

Note that for very slender webs q_{cr} is limited to elastic critical shear stress. In other cases the value of q_{cr} is a function of design stress of web steel, p_{yw}.

2.2.2 Webs with intermediate stiffeners

Design of the plate girders with intermediate stiffeners, as indicated in Fig. 2, can be done by limiting their shear capacity to shear buckling strength. However this approach is uneconomical and does not account for the mobilisation of the additional shear capacity, as indicated earlier. The shear resistance is improved in the following two ways.

i) Increase in buckling resistance due to reduced *a/d* ratio.

ii) The web develops tension field action and thus resists considerably larger stress than the elastic critical strength of web in shear.

Fig.3 shows the diagonal tension fields anchored between top and bottom flanges and against transverse stiffeners on either side of the panel. With the stiffeners acting as struts and the tension field acting as ties the plate girder behaves similar to an N-truss [Fig. 3(*b*)]. As indicated in the previous chapter, the failure occurs when the web yields and plastic hinges form in flanges, *2* at top and *2* at bottom, developing a sway mechanism.

The full shear buckling resistance is calculated as,

$$V_b = [\, q_b + q_f\,(k_f)^{1/2}\,]\; d\, t \quad but \le 0.6\, p_y\, dt \qquad (4)$$

The first term q_b comprises of critical elastic stress q_{cr} and the tension field strength of the panel i.e.,

$$q_b = q_{cr} + \frac{y_b}{2\left[\frac{a}{d} + \sqrt{1 + \left(\frac{a}{d}\right)^2}\right]} \qquad (5)$$

where, $y_b = (p_{yw}^{\,2} - 3q_{cr}^{\,2} + \phi_t^{\,2})^{1/2} - \phi_t$

$$\phi_t = \frac{1.5 q_{cr}}{\sqrt{1 + \left(\frac{a}{d}\right)^2}}$$

a – spacing of transverse stiffeners

d – depth of girder

t – web thickness

p_{yw} – design web strength ($= f_{yw}/\gamma_m$)

Note that this is a simplified version of equation (4) derived in the previous chapter based on yield criteria.

The term $q_f(k_f)^{1/2}$ represents the contribution of the flanges to the post buckling strength and depends on plastic moment capacity of the flanges M_{pf} [Equation (8) of the previous chapter]

The flanges support the pull exerted by the tension field. When the flanges reach their ultimate capacity they form hinges. k_f is a parameter that relates to the plastic moment capacity of the flange M_{pf}, and the web M_{pw}, described later.

The flange-dependent shear strength is simplified and given as

$$q_f = 0.6\, p_{yw}\left[4\sqrt{3}\,\left(\frac{y_b}{p_{yw}}\right)^{1/2} \sin\frac{\theta}{2}\right] \qquad (6)$$

where, $\theta = tan^{-1}(d/a)$

When the girder is to resist pure shear, then

$$M_{pf} = \frac{2b}{4} T^2 \, p_{yf} \qquad (7)$$

However in presence of overall bending moment, the contribution of flange to shear resistance will be reduced by the longitudinal stress *f* induced because of overall bending moment, by the factor $(1 - f/p_{yf})$. When *f* approaches p_{yf} at maximum moment region, the factor nearly becomes zero and hence the contribution of flanges to shear resistance will become negligible.

The plastic moment capacity of the web, M_{pw}, is given by

$$M_{pw} = 0.25\, d^2\, t\, p_{yw} \qquad (8)$$

and

$$k_f = M_{pf} / 4\, M_{pw} \qquad (9)$$

Fig. 4 shows typical variations of shear strength with web stiffness as contributed by

a) critical shear strength
b) post buckling strength due to web tension field adequately resisted by transverse vertical stiffeners
c) the plastic moment capacity of the flanges.

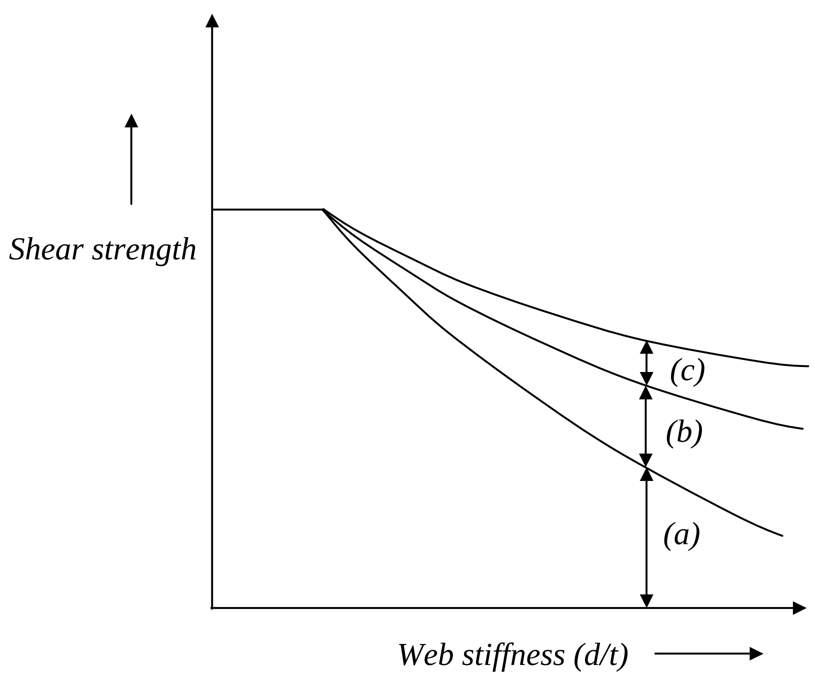


Fig. 4 Typical variation of shear strength with web stiffness as contributed by shear strength of panel, post buckling strength due to tension field and flange mechanism

3.0 END PANELS

For tension field action to develop in the end panels, adequate anchorage should be provided all around the end panel. The anchor force H_q required to anchor the tension field force is

$$H_q = 0.75\, d\, t\, p_y \left(1 - \frac{q_{cr}}{0.6\, p_y}\right)^{1/2} \tag{10}$$

The end panel, when designed for tension field will impose additional loads on end post and hence it will become stout [Fig. 5(*a*)]. For a simple design it may be assumed that the capacity of the end panel is restricted to V_{cr} so that no tension field develops in it [Fig. 5 (*c*)]. In this case, end panel acts as a beam spanning between the flanges to resist shear and moment caused by H_q and produced by tension field of penultimate panel.

This approach is conservative, as it does not utilise the post-buckling strength of end panel especially where the shear is maximum. This will result in the *a/d* value of end panel spacing to be less than that of other panels. The end stiffener should be designed for compressive forces due to bearing and the moment, M_{tf}, due to tension field in the penultimate panel.

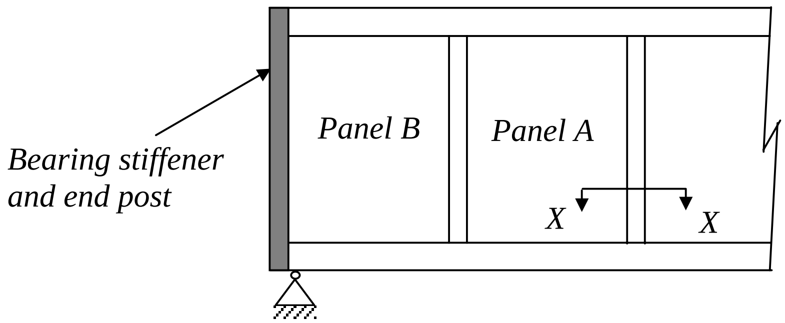
In order to be economical the end panel also may be designed using tension field action. In this case the bearing stiffener and end post are designed for a combination of stresses resulting from compression due to bearing and a moment equal to *2/3* caused due to tension in the flanges, M_{tf}. The stiffener will be stout. Instead of one stout stiffener we can use a double stiffener as shown in Fig. 5(*d*). Here the end post is designed for horizontal shear and the moment M_{tf}.

4.0 STIFFENERS

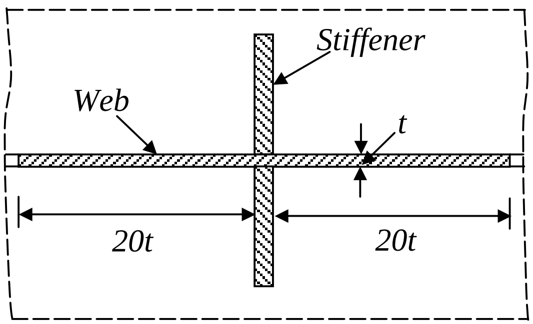
Stiffeners are provided to transfer transverse concentrated compressive force on the flange into the web and are essential for desired performance of web panels. These are referred to as bearing stiffeners. Intermediate web stiffeners are provided to improve its shear capacity. Design of these stiffeners is discussed below.

4.1 Load bearing stiffeners

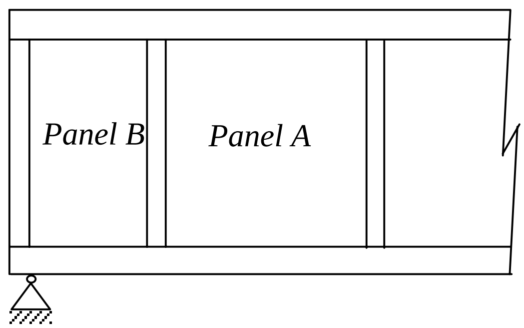
Whenever there is a risk of the buckling resistance of the web being exceeded, especially owing to concentrated loads, load-bearing stiffeners are provided. Normally a web width of *20 t* on both sides as shown in Fig. 5 (*b*) is assumed to act along with the stiffener provided to resist the compression as an equivalent cruciform shaped strut of effective length 0.7 times its actual length between the top and bottom flanges. The bearing stress in the stiffener is checked using the area of that portion of the stiffener in contact with the flange through which compressive force is transmitted.



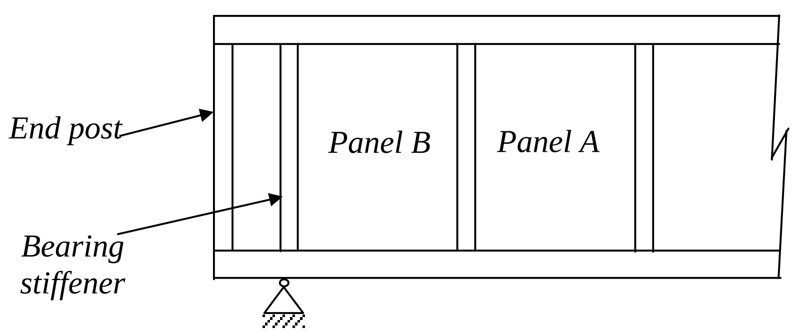
(a) End panel designed using tension field action and end post designed for both bearing and to resist tension field



(b) Section at X-X



(c) End panel designed without using tension field action



(d) End panel designed using tension field strengthened by additional stiffener (Double stiffener)

Fig. 5 Various treatments for end panel

4.2 Intermediate stiffeners

The intermediate stiffeners are provided to prevent out of plane buckling of web at the location of stiffeners. The buckling resistance P_q of the stiffener acting as a strut (with a cruciform section as described earlier) should be not less than $(V_t - V_s)$ where V_t is the maximum shear force in the panel and V_s is the buckling resistance of web without considering tension field action. In its limit V_s will be equal to V_{cv} of the web without stiffeners.

Sometimes the stiffeners are provided for more than one of the above purposes. In such cases stiffeners are considered for their satisfactory resistance under combined load effects. Such combined loads are common.

4.3 Longitudinal stiffeners

Longitudinal stiffeners are hardly used in building plate girders, but sometimes they are used in highway bridge girders for aesthetic reasons. They are not as effective as transverse stiffeners. Nowadays, the use of longitudinal stiffeners is rare due to welding problems.

For design of longitudinal stiffeners there are two requirements:

- A moment of inertia to ensure adequate stiffness to create a nodal line along the stiffener
- An area adequate to carry axial compression stress while acting integrally with the web.

5.0 CURTAILMENT OF FLANGE PLATES

For a plate girder subjected to external loading, the maximum bending moment occurs at one section usually, e.g. when the plate girder is simply supported at the ends, and subjected to the uniformly distributed load, then, maximum bending moment occurs at the centre. Since the values of bending moment decreases towards the end, the flange area designed to resist the maximum bending moment is not required at other sections. Therefore the flange plates may be curtailed at a distance from the centre of span greater than the distance where the plate is no longer required as the bending moment decreases towards the ends. It gives economy as regards to the material and cost. At least one flange plate should be run for the entire length of the girder.

6.0 SPLICES

6.1 Web splices

A joint in the web plate provided to increase its length is known as web splice. The plates are manufactured up to a limited length. When the maximum manufactured length of the plate is insufficient for full length of the plate girder, web splice becomes essential. It also becomes essential when the length of plate girder is too long to handle conveniently during transportation and erection. Generally, web splices are not used in buildings. They are mainly used in bridges.

Splices in the web of the plate girder are designed to resist the shear and moment at the spliced section. The splice plates are provided on each side of the web.

6.2 Flange splices

A joint in the flange element provided to increase the length of flange plate is known as flange splice. The flange splices should be avoided as far as possible. Generally, the flange plates can be obtained for full length of the plate girder. In spite of the availability of full length of flange plates, sometimes it becomes necessary to make flange splices. Flange joints should not be located at the points of maximum bending moment.

7.0 CONCLUSIONS

This chapter has outlined the procedure for design of plate girders as specified in BS 5950: Part - I for buildings. It shows how the reserve strength due to post buckling behaviour explained in Plate Girder I chapter can be advantageously used by the designer without performing mathematically involved calculations.

8.0 REFERENCES

1. Narayanan. R: Plate Girders, Steel Designer's Manual (Fifth Edition). The Steel Construction Institute UK 1992.

2. Evans H.R: Introduction to Steelwork Design to BS 5950 Part 1 SCI Publication 069. The Steel Construction Institute UK 1988.

17 BEAMS SUBJECTED TO TORSION AND BENDING -I

1.0 INTRODUCTION

When a beam is transversely loaded in such a manner that the resultant force passes through the longitudinal shear centre axis, the beam only bends and no torsion will occur. When the resultant acts away from the shear centre axis, then the beam will not only bend but also twist.

When a beam is subjected to a pure bending moment, originally plane transverse sections before the load was applied, remain plane after the member is loaded. Even in the presence of shear, the modification of stress distribution in most practical cases is very small so that the Engineer's Theory of Bending is sufficiently accurate.

If a beam is subjected to a twisting moment, the assumption of planarity is simply incorrect except for solid circular sections and for hollow circular sections with constant thickness. Any other section will warp when twisted. Computation of stress distribution based on the assumption of planarity will give misleading results. Torsional stiffness is also seriously affected by this warping. If originally plane sections remained plane after twist, the torsional rigidity could be calculated simply as the product of the polar moment of inertia ($I_p = I_{xx} + I_{yy}$) multiplied by (G), the shear modulus, viz. $G. (I_{xx} + I_{yy})$. Here I_{xx} and I_{yy} are the moments of inertia about the principal axes. This result is accurate for the circular sections referred above. For all other cases, this is an overestimate; in many structural sections of quite normal proportions, the true value of torsional stiffness as determined by experiments is only *1% - 2%* of the value calculated from polar moment of inertia.

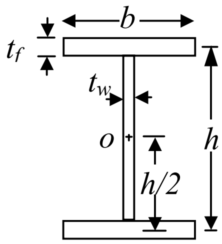
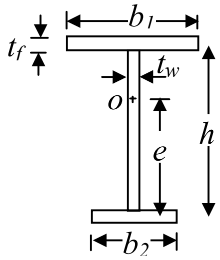
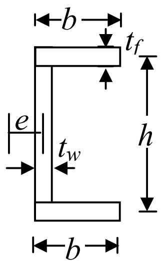
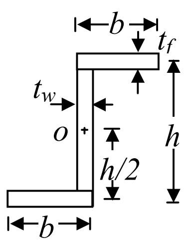
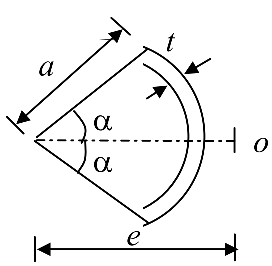
It should be emphasised that the end sections of a member subjected to warping may be modified by constraints. If the central section remains plane, for example, due to symmetry of design and loading, the stresses at this section will differ from those based on free warping. Extreme caution is warranted in analysing sections subjected to torsion.

2.0 UNIFORM AND NON-UNIFORM TORSION

2.1 Shear Centre and Warping

Shear Centre is defined as the point in the cross-section through which the lateral (or transverse) loads must pass to produce bending without twisting. It is also the centre of rotation, when only pure torque is applied. ***The shear centre and the centroid of the cross section will coincide, when section has two axes of symmetry. The shear centre will be on the axis of symmetry, when the cross section has one axis of symmetry.***

Table 1: Properties of Sections

	$J=\dfrac{2bt_f^3+ht_w^3}{3}$ $C_w=\dfrac{t_f h^2 b^3}{24}$	If $t_f=t_w=t$: $J=\dfrac{t^3}{3}(2b+h)$
	$e=h\,\dfrac{b_1^3}{b_1^3+b_2^3}$ $J=\dfrac{(b_1+b_2)t_f^3+ht_w^3}{3}$ $C_w=\dfrac{t_f h^2}{12}\,\dfrac{b_1^3 b_2^3}{b_1^3+b_2^3}$	If $t_f=t_w=t$: $J=\dfrac{t^3}{3}(b_1+b_2+h)$
	$e=\dfrac{3b^2t_f}{6bt_f+ht_w}$ $J=\dfrac{2bt_w^3+ht_w^3}{3}$ $C_w=\dfrac{t_f b^3 h^2}{12}\,\dfrac{3bt_f+2ht_w}{6bt_f+ht_w}$	If $t_f=t_w=t$: $e=\dfrac{3b^2t_f}{6b+h}$ $J=\dfrac{t^3}{3}(2b+h)$ $C_w=\dfrac{tb^3h^2}{12}\,\dfrac{3b+2h}{6b+h}$
	$J=\dfrac{2bt_f^3+ht_w^3}{3}$ $C_w=\dfrac{b^3h^2}{12(2b+h)^2}$ $\times[2t_f(b^2+bh+h^2)+3t_w bh]$	If $t_f=t_w=t$: $J=\dfrac{t^3}{3}(2b+h)$ $C_w=\dfrac{tb^3h^2}{12}\,\dfrac{b+2h}{2b+h}$
	$e=2a\,\dfrac{\sin\alpha-\alpha\cos\alpha}{\alpha-\sin\alpha\cos\alpha}$ $J=\dfrac{2a\alpha t^3}{3}$ $C_w=\dfrac{2ta^5}{3}$ $\times\left[\alpha^3-\dfrac{6(\sin\alpha-\alpha\cos\alpha)^2}{\alpha-\sin\alpha\cos\alpha}\right]$	If $2\alpha=\pi$: $e=\dfrac{4a}{\pi}$ $J=\dfrac{\pi a t^3}{3}$ $C_w=\dfrac{2ta^5}{3}\left(\dfrac{\pi^3}{8}-\dfrac{12}{\pi}\right)=0.0374ta^5$

where O = shear centre; J = torsion constant; C_w = warping constant

If the loads are applied away from the shear centre axis, torsion besides flexure will be the evident result. The beam will be subjected to stresses due to torsion, as well as due to bending.

The effect of torsional loading can be further split into two parts, the first part causing twist and the second, *warping.* These are discussed in detail in the next section.

Warping of the section does not allow a plane section to remain as plane after twisting. This phenomenon is predominant in Thin Walled Sections, although consideration will have to be given to warping occasionally in hot rolled sections. An added characteristic associated with torsion of non-circular sections is the in-plane distortion of the cross-section, which can usually be prevented by the provision of a stiff diaphragm. Distortion as a phenomenon is not covered herein, as it is beyond the scope of this chapter.

Methods of calculating the position of the shear centre of a cross section are found in standard textbooks on Strength of Materials.

2.2 Classification of Torsion as Uniform and Non-uniform

As explained above when torsion is applied to a structural member, its cross section may warp in addition to twisting. If the member is allowed to warp freely, then the applied torque is resisted entirely by torsional shear stresses (called ***St. Venant's torsional shear stress***). If the member is not allowed to warp freely, the applied torque is resisted by St. Venant's torsional shear stress and warping torsion. This behaviour is called ***non-uniform torsion.***

Hence (as stated above), the effect of torsion can be further split into two parts:

- Uniform or Pure Torsion (called St. Venant's torsion) - T_{sv}
- Non-Uniform Torsion, consisting of St.Venant's torsion (T_{sv}) and warping torsion (T_w).

2.3 Uniform Torsion in a Circular Cross Section

Let us consider a bar of constant circular cross section subjected to torsion as shown in Fig. 1. In this case, *plane cross sections normal to the axis of the member remain plane after twisting*, i.e. there is no warping. The torque is solely resisted by circumferential shear stresses caused by St. Venant's torsion. Its magnitude varies as its distance from the centroid.

For a circular section, the St. Venant's torsion is given by

$$T_{sv} = I_p G \frac{d\phi}{dz} \tag{1}$$

where, ϕ - angle of twist
G - modulus of rigidity
T_{sv} - St. Venant's torsion.
I_p - the polar moment of inertia
z - direction along axis of the member.

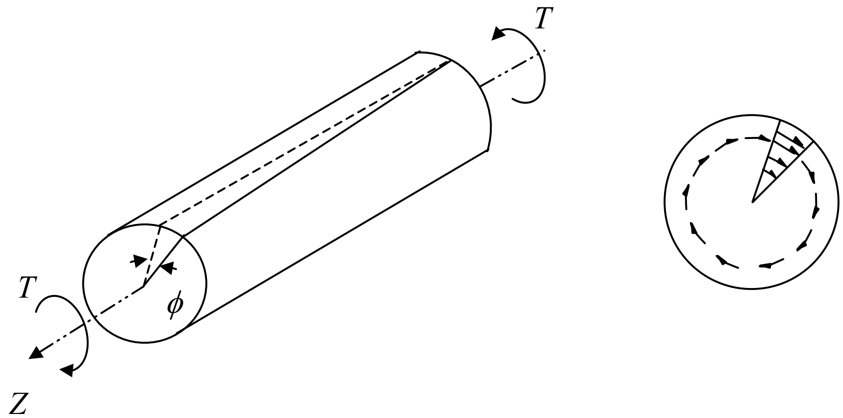


Fig. 1Twisting of circular section.

2.4 Uniform Torsion in Non-Circular Sections

When a torque is applied to a non-circular cross section (e.g. a rectangular cross section), the transverse sections which are plane prior to twisting, warp in the axial direction, as described previously, so that a plane cross section no longer remains plane after twisting. However, so long as the warping is allowed to take place freely, the applied load is still resisted by shearing stresses similar to those in the circular bar. The St.Venant's torsion (T_{sv}) can be computed by an equation similar to equation (1) but by replacing I_p by J, the torsional constant. The torsional constant *(J)* for the rectangular section can be approximated as given below:

$$J = C.\, bt^3 \tag{1.a}$$

where b and t are the breadth and thickness of the rectangle. C is a constant depending upon (b/t) ratio and tends to *1/3* as b/t increases.

Then ,
$$T_{sv} = JG\,\frac{d\phi}{dz} \tag{1.b}$$

2.4.1 Torsional Constant (J) for thin walled open sections made up of rectangular elements

Torsional Constant *(J)* for members made up of rectangular plates (see Fig. 2) may be computed approximately from

$$J = \frac{1}{3} \sum_i b_i (t_i)^3 \quad \text{(1.c)}$$

in which b_i and t_i are length and thickness respectively of any element of the section.

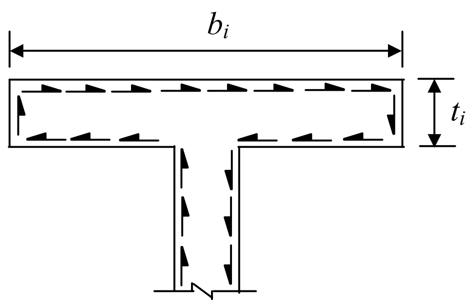


Fig. 2. Thin walled open section made of rectangular elements

In many cases, only uniform (or St. Venant's) torsion is applied to the section and the rate of change of angle of twist is constant along the member and the ends are free to warp (See Fig. 3)

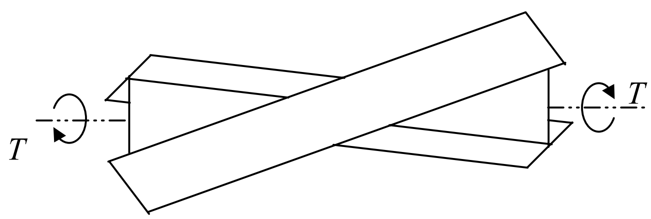


Fig.3 Uniform Torsion (Constant Torque : Ends are free to warp)

In this case the applied torque is resisted entirely by shear stresses and ***no warping stresses result.***

The total angle of twist ϕ is given by

$$\phi = \frac{T\ z}{GJ} \quad \text{(2)}$$

where T = Applied Torsion = T_{sv}

(Note: in this case only St.Venant's Torsion is applied)

The maximum shear stress in the element of thickness t is given by

$$\tau_t = G t \phi' \quad \text{(3)}$$

Fig. 4 gives the corresponding stress pattern for an I section.

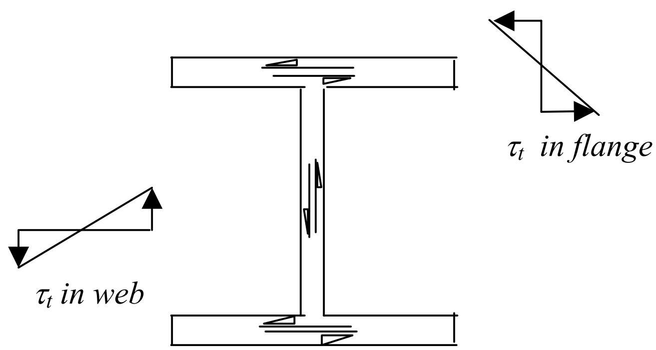


Fig.4 Stress pattern due to pure torsion
(Shear stresses are enlarged for clarity)

3.0 NON-UNIFORM TORSION

When warping deformation is constrained, the member undergoes non-uniform torsion. Non-uniform torsion is illustrated in Fig. 5 where an *I*-section fixed at one end is subjected to torsion at the other end. Here the member is restrained from warping freely as one end is fixed. The warping restraint causes bending deformation of the flanges in their plane in addition to twisting. The bending deformation is accompanied by a shear force in each flange.

The total non-uniform torsion (T_n) is given by

$$T_n = T_{sv} + T_w \tag{4}$$

where T_w is the warping torsion.

Shear force V_f in each flange is given by

$$V_f = -\frac{dM_f}{dz} \tag{5}$$

where M_f is the bending moment in each flange. Since, the flanges bend in opposite directions, the shear forces in the two flanges are oppositely directed and form a couple. This couple, which acts to resist the applied torque, is called ***warping torsion.***

For the *I*-section shown in Fig. 5, warping torsion is given by

$$T_w = V_f.h \tag{6}$$

The bending moment in the upper flange is given by

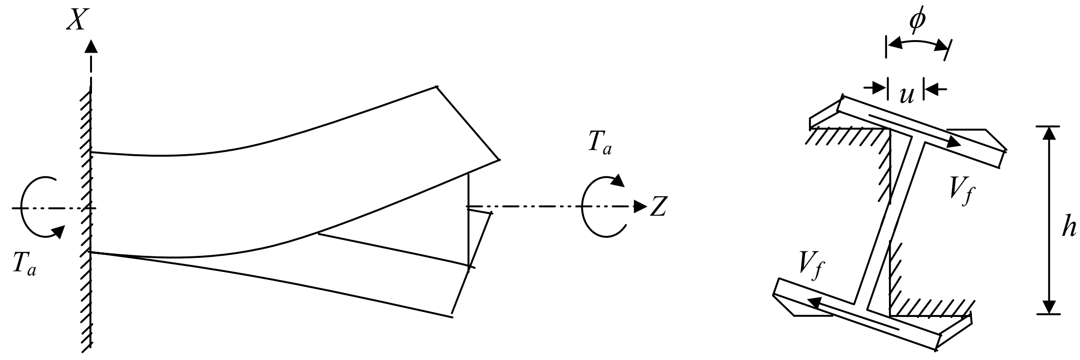


Fig. 5 Non uniform Torsion:Twisting of Non-Circular Section restrained against free warping (Constant Torque : End warping is prevented)

$$M_f = EI_f \frac{d^2u}{dz^2} \tag{7}$$

in which I_f is the moment of inertia of flange about its strong axis (i.e. the vertical axis) and u, the lateral displacement of the flange centreline which is given by

$$u = \phi h/2 \tag{8}$$

On substituting eq. 8 in eq. 7 we get

$$M_f = \frac{EI_f h}{2}\frac{d^2\phi}{dz^2} = \frac{EI_f\ h}{2}\ \phi'' \tag{9}$$

On simplification by substituting eqn.(9) into eqn. (6), we obtain the value of warping torsion as,

$$T_w = -\frac{EI_f h^2}{2}\frac{d^3\phi}{dz^3} = -\frac{EI_f h^2}{2}\phi''' \tag{10}$$

The term $I_f h^2/2$ is called the warping constant (Γ) for the cross-section.

then, $$T_w = -E\Gamma\frac{d^3\phi}{dz^3} = -E\Gamma\,\phi''' \tag{11}$$

in which $$\Gamma = \frac{I_f \cdot h^2}{2} \quad \text{(for an I-section)} \tag{12}$$

$E\Gamma$ is termed as the warping rigidity of the section, analogous to GJ, the St. Venant's torsional stiffness. The torque will be resisted by a combination of St.Venant's shearing stresses and warping torsion. Non-uniform torsional resistance (T_n) at any cross-section is therefore given by the sum of St.Venant's torsion (T_{sv}) and warping torsion (T_w).

Thus, the differential equation for non-uniform torsional resistance $T_n(z)$ can be written as the algebraic sum of the two effects, due to St.Venant's Torsion and Warping Torsion.

$$T_n(z) = GJ\frac{d\phi}{dz} - E\Gamma\frac{d^3\phi}{dz^3} = GJ\phi' - E\Gamma\,\phi''' \tag{13a}$$

$$or,\ T_n(z) = GJ\phi' - EI_f.\frac{h^2}{2}.\phi''' \quad \text{(for an } I\text{-section)} \tag{13b}$$

In the above, the first term on the right hand side (depending on GJ) represents the resistance of the section to twist and the second term represents the resistance to warping and is dependent on $E\Gamma$.

In the example considered (Fig. 5), the applied torque T_a is constant along the length, λ, of the beam . For equilibrium, the applied torque, T_a, should be equal to torsional resistance T_n.

The boundary conditions are: (i) the slope of the beam is zero when $z = 0$ and (ii) the BM is zero when $z = \lambda$ i.e. at the free end.

$$\frac{d\phi}{dz} = 0 \qquad when \qquad z = 0$$

$$\frac{d^2\phi}{dz^2} = 0 \qquad when \qquad z = \lambda$$

The solution of equation (13.a) is

$$\frac{d\phi}{dz} = \frac{T_n}{GJ}\left(1 - \frac{Cos\,h\frac{\lambda - z}{a}}{Cos\,h\frac{\lambda}{a}}\right) \tag{14}$$

in which $$a^2 = \frac{E\Gamma}{GJ} \tag{15}$$

Since the flexural rigidity EI_f and torsional rigidity GJ are both measured in the same units ($N.mm^2$), equation (15) shows that a has the dimensions of length and *depends on the proportions of the beam*. Because of the presence of the second term in equation (14) the angle of twist per unit length varies along the length of the beam even though the applied torsion, T_a ,remains constant. When $\frac{d\phi}{dz}$ is known, the St. Venant's torsion (T_{sv}) and the warping torsion (T_w) may be calculated or any cross section. At the built-in section ($z = 0$) and $\frac{d\phi}{dz} = 0$, hence we obtain from eq.(1) that $T_{sv} = 0$. At this point, the entire torque is balanced by the moment of the shearing forces in each of the flanges.

$$\therefore V_f = -T_n/h \tag{16}$$

At the end $z = \lambda$, using equation (14) , we obtain

$$\frac{d\phi}{dz} = \frac{T_n}{GJ}\left(1 - \frac{1}{Cos\,h\left(\lambda/a\right)}\right) \tag{17}$$

If the length of the beam is large in comparison with the cross sectional dimensions, $\left(1 - \frac{1}{Cosh\left(\frac{\lambda}{a}\right)}\right)$ tends to approach 1, as the second term is negligible. Hence $\frac{d\phi}{dz}$ approaches T_n/GJ .

The bending moment in the flange is found from

$$V_f = \frac{dM_f}{dz} = EI_f \cdot \frac{d^3\phi}{dz^3} \cdot \frac{h}{2} \tag{18}$$

where M_f is the bending moment in each flange.

$$M_f = EI_f \cdot \frac{h}{2} \cdot \frac{d^2\phi}{dz^2} \tag{19}$$

Substituting for $\frac{d\phi}{dz}$ from eq. (14) we obtain

$$M_f = \frac{a}{h} . T_n . \frac{Sin\,h\left(\frac{\lambda - z}{a}\right)}{Cos\,h\,\lambda/a} \tag{20}$$

The maximum bending moment at the fixed end is given by

$$M_{f\,\max} = \frac{a}{h} \cdot T_n \cdot tanh\left(\frac{\lambda}{a}\right) \tag{21}$$

When λ is several times larger than a, $tan\,h\,(\lambda/a)$ approaches 1, so that

$$M_{f\,\max} \cong \frac{a.T_n}{h} \tag{22}$$

In other words, the maximum bending moment in each of the flanges will be the same as that of cantilever of length *a*, and loaded at the free end by a force of $\left(\frac{T_n}{h}\right)$. For a short beam λ is small in comparison with *a*, so $tanh\left(\frac{\lambda}{a}\right) \cong \left(\frac{\lambda}{a}\right)$

Hence $$M_{f\,max} = \frac{T_n \cdot \lambda}{h} \tag{23}$$

The range of values for $M_{f\,max}$ therefore varies from $\frac{T_n}{h}(\lambda)$ *to* $\frac{T_n}{h}(a)$ as the length of the beam varies from a "short" to a "long" one.

To calculate the angle of twist, ϕ, we integrate the right hand side of equation (14)

$$\phi = \frac{T_n}{GJ}\left[z + \frac{a \sin h\left(\frac{\lambda - z}{a}\right)}{Cos\, h\left(\frac{\lambda}{a}\right)} - a \tan h\left(\frac{\lambda}{a}\right)\right] \tag{24}$$

From equation (24), we obtain the value of ϕ at the end (i.e.) when $z = \lambda$

$$(\phi)_{z=\lambda} = \frac{T_n}{GJ}\left(\lambda - a \tan h \frac{\lambda}{a}\right) \tag{25}$$

For long beams $\tan h\left(\frac{\lambda}{a}\right) \cong 1$, so equation (25) becomes

$$(\phi)_{z=\lambda} = \frac{T_n}{GJ}(\lambda - a) \tag{26}$$

The effect of the warping restraint on the angle of twist is equivalent to diminishing the length λ of the beam to *(λ - a).*

Certain simple cases of the effect of Torsion in simply supported beams and cantilever are illustrated in Figures 6 and 7.

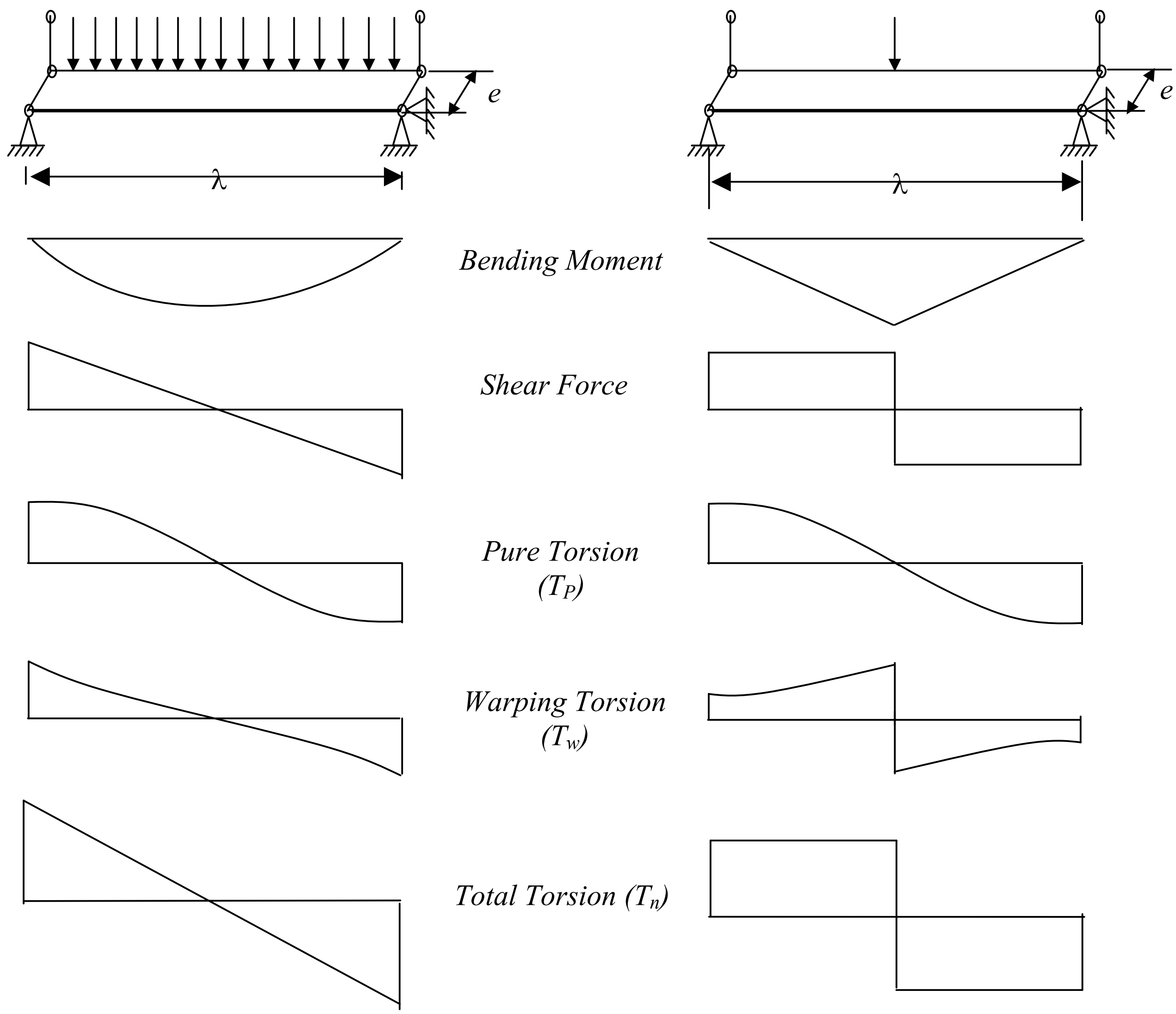


Fig.6 Torsion in simply supported beam with free end warping

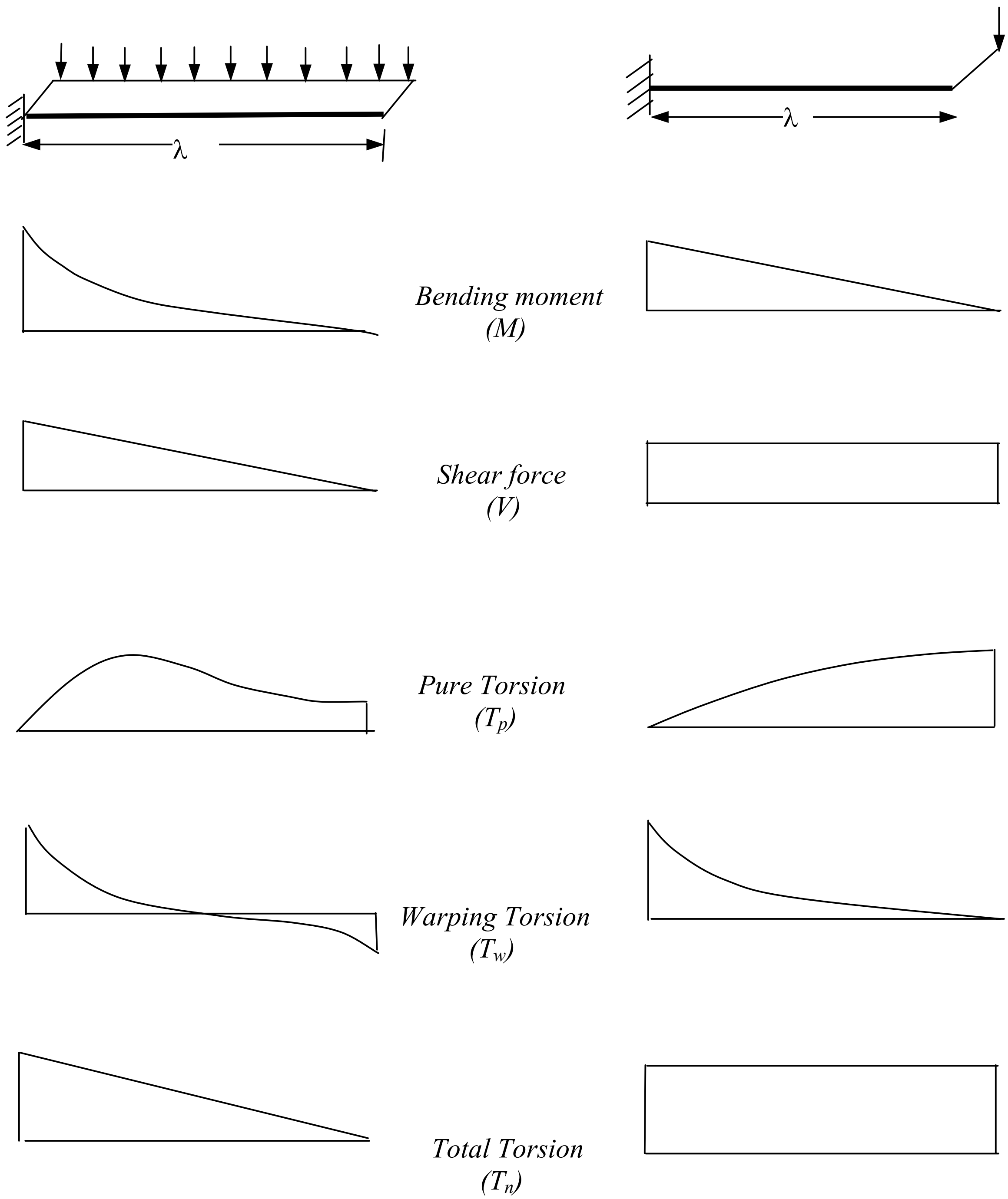


Fig.7 Torsion in Cantilevers

4.0 AN APPROXIMATE METHOD OF TORSION ANALYSIS

A simple approach is often adopted by structural designers for rapid design of steel structures subjected to torsion. This method (called ***the bi-moment method***) is sufficiently accurate for practical purposes. The applied torque is replaced by a couple of horizontal forces acting in the plane of the top and bottom flanges as shown in Fig. 8 and Fig. 9.

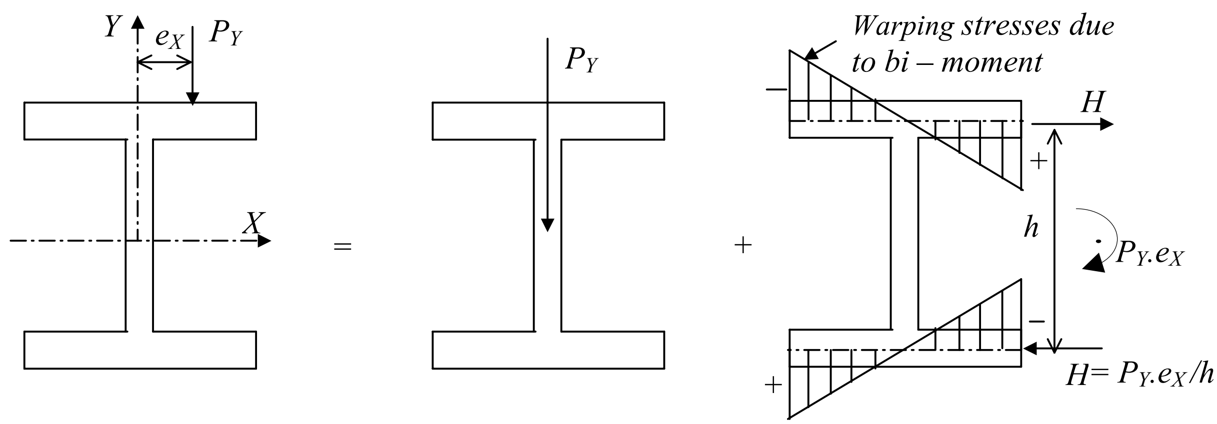


Fig 8: Load P_Y acting eccentrically w.r.t. y – axis and causing torsion.

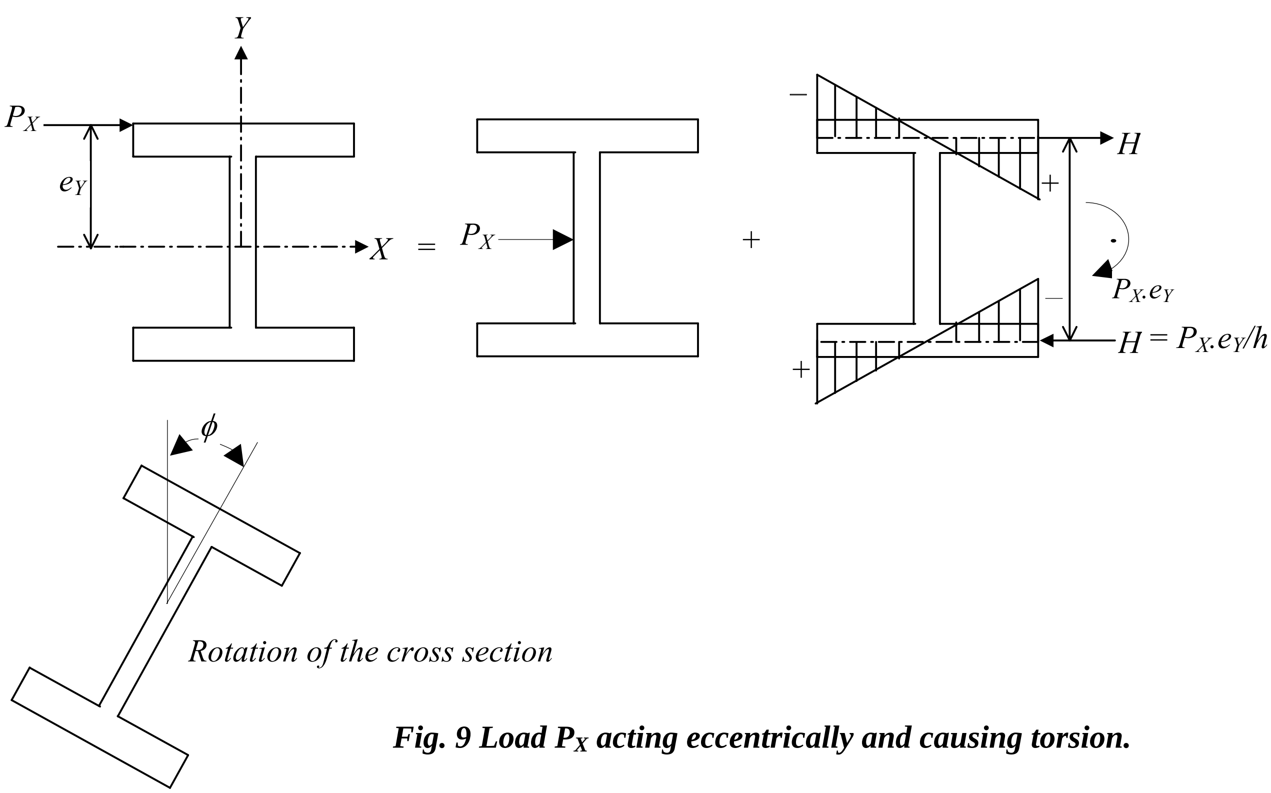


Fig. 9 Load P_X acting eccentrically and causing torsion.

When a uniform torque is applied to an open section restrained against warping, the member itself will be in non-uniform torsion. The angle of twist, therefore, varies along the member length. The rotation of the section will be accompanied by bending of flanges in their own plane. The direct and shear stresses caused are shown in Fig.10.

For an *I* section, the warping resistance can be interpreted in a simple way. The applied torque T_a is resisted by a couple comprising the two forces *H*, equal to the shear forces in each flange. These forces act at a distance equal to the depth between the centroids of each flange.

Each of these flanges can be visualized as a beam subjected to bending moments produced by the forces *H*. This leads to bending stresses σ_w in the flanges. These are termed ***Warping Normal Stresses.***

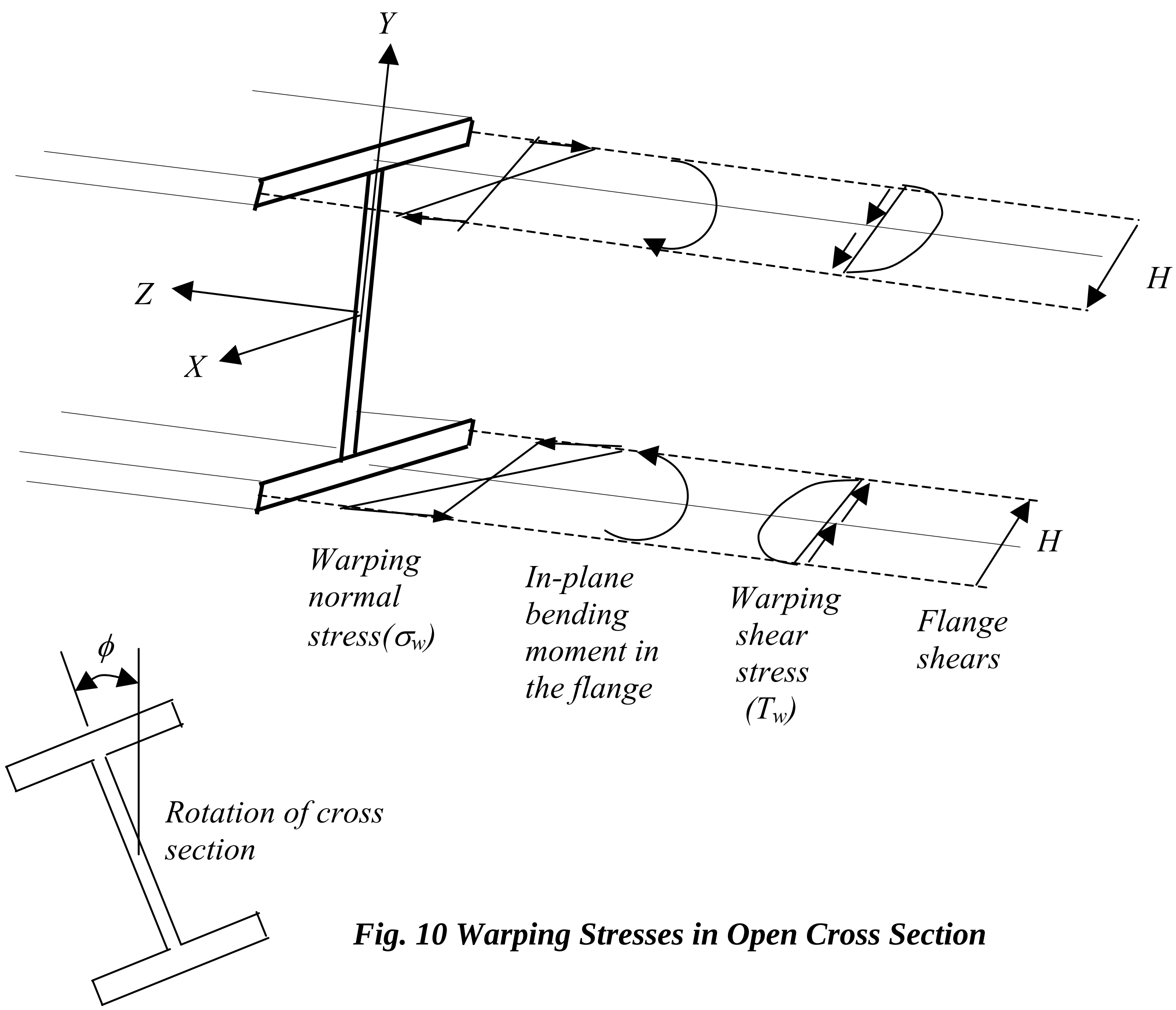


Fig. 10 Warping Stresses in Open Cross Section

The magnitude of the warping normal stress at any particular point *(σ_w)* in the cross section is given by

$$\sigma_w = - EW_{nwfs}\, \phi'' \qquad (27)$$

where W_{nwfs} = normalised warping function at a particular point S in the cross section.

An approximate method of calculating the normalised warping function for any section is described in Reference 3. The value of W_{nwfs} for an I -section is given in section 5.3. The in-plane shear stresses are called ***Warping shear stresses***. They are constant across the thickness of the element. Their magnitude varies along the length of the element. The magnitude of the warping shear stress at any given point is given by

$$\tau_w = -\frac{ES_{wms}\,\phi'''}{t} \tag{28}$$

where S_{wms} = Warping statical moment of area at a particular point S. Values of warping normal stress and in-plane shear stress are tabulated in standard steel tables produced by steel makers. Section 5.3 gives these values for I and H sections.

5.0 THE EFFECT OF TORSIONAL RIGIDITY *(GJ)* AND WARPING RIGIDITY *(EΓ)*

The warping deflections due to the displacement of the flanges vary along the length of the member. Both direct and shear stresses are generated in addition to those due to bending and pure torsion. As discussed previously, the stiffness of the member associated with the former stresses is directly proportional to the warping rigidity, $E\Gamma$.

When the torsional rigidity (GJ) is very large compared to the warping rigidity, $E\Gamma$, then the section will effectively be in "uniform torsion". Closed sections (eg. rectangular or square hollow sections) angles and Tees behave this way, as do most flat plates and all circular sections. Conversely if GJ is very small compared with $E\Gamma$, the member will effectively be subjected to warping torsion. Most thin walled open sections fall under this category. Hot rolled I and H sections as well as channel sections exhibit a torsional behaviour in between these two extremes. In other words, the members will be in a state of non-uniform torsion and the loading will be resisted by a combination of uniform (St.Venant's) and warping torsion.

5.1 End Conditions

The end support conditions of the member influence the torsional behaviour significantly; three ideal situations are described below. (It must be noted that torsional fixity is essential at least in one location to prevent the structural element twisting bodily). Warping fixity cannot be provided without also ensuring torsional fixity.

The following end conditions are relevant for torsion calculations

- ***Torsion fixed, Warping fixed***: This means that the twisting along the longitudinal (Z) axis and also the warping of cross section at the end of the member are prevented. ($\phi = \phi' = 0$ at the end). This is also called "fixed" end condition.

- ***Torsion fixed, Warping free:*** This means that the cross section at the end of the member cannot twist, but is allowed to warp. ($\phi = \phi'' = 0$). This is also called "pinned" end condition.

- ***Torsion free, Warping free***: This means that the end is free to twist and warp. The unsupported end of cantilever illustrates this condition. (This is also called "free" end condition).

Effective warping fixity is difficult to provide. It is not enough to provide a connection which provides fixity for bending about both axes. It is also necessary to restrain the flanges by additional suitable reinforcements. It may be more practical to assume "warping free" condition even when the structural element is treated as "fixed" for bending. On the other hand, torsional fixity can be provided relatively simply by standard end connections.

5.2 Procedures for checking adequacy in Flexure

These procedures have been described in an earlier chapter dealing with "unrestrained bending". Particular attention should be paid to lateral torsional buckling by evaluating the equivalent uniform moment $\overline{M}$, such that

$$\overline{M} < M_b$$

where $\overline{M}$ = equivalent uniform moment
M_b = lateral-torsional buckling resistance moment.

If the beam is stocky (eg. due to closely spaced lateral restraints), the design will be covered by moment capacity M_c.

In addition to bending stresses the shear stresses, τ_b, due to plane bending have to be evaluated.

Shear stress at any section is given by, $\tau = \dfrac{V A \bar{y}}{I t} = \dfrac{V Q}{I t}$

where Q = Statical moment of area of the shaded part (Fig. 11).

For the web, $\tau_{bw} = \dfrac{VQ_w}{I.t}$

For the flange, $\tau_{bf} = \dfrac{VQ_f}{I.T}$

where V = applied shear force
I = moment of inertia of the whole section

T	=	flange thickness
Q_w	=	statical moment of area for the web
Q_f	=	statical moment of area for the flange.
t	=	web thickness

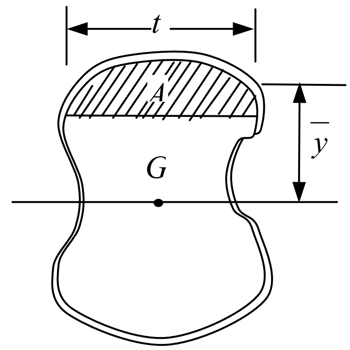


Fig.11

5.3 Cross Sectional Properties for Symmetrical *I* and *H* Sections

For an I or H section subjected to torsion, the following properties will be useful (see Fig. 12).

$$J = \frac{1}{3}\left[2BT^3 + (D-2T)t^3\right]$$

$$W_{nwfs} = \frac{hB}{4}$$

$$S_{wms} = \frac{hB^2T}{16}$$

$$\Gamma = \frac{I_y h^2}{4}$$

$$Q_f = A_f \cdot y_f$$

$$Q_w = \frac{A}{2} y_w$$

where	A_f	=	area of half the flange
	y_f	=	distance of neutral axis to the centroid of the area A_f
	A	=	total cross sectional area
	y_w	=	the distance from the neutral axis to the centroid of the area above neutral axis.

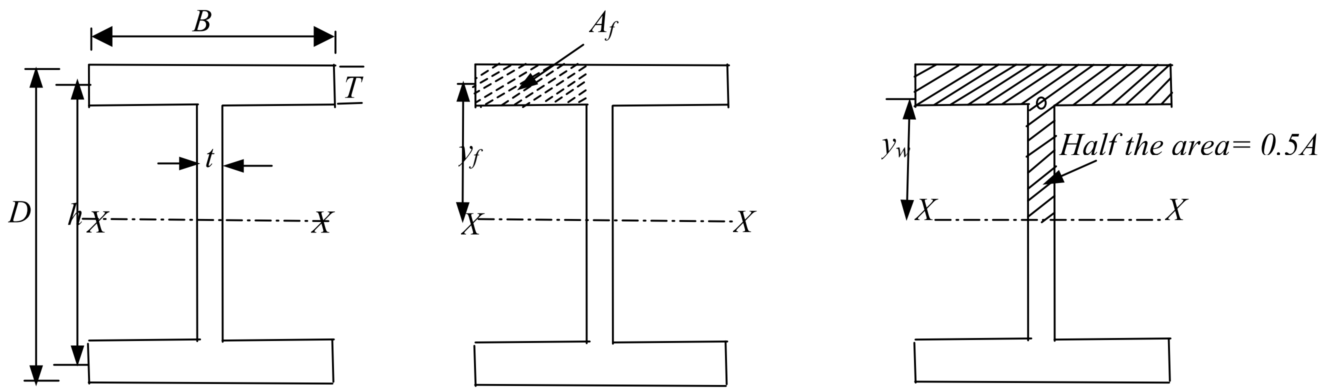


Fig.12

6.0 CONCLUSIONS

Analysis of a beam subjected to torsional moment is considered in this chapter. Uniform torsion (also called St.Venant's torsion) applied to the beam would cause a twist. Non-uniform torsion will cause both twisting and warping of the cross section. Simple methods of evaluating the torsional effects are outlined and discussed.

7.0 REFERENCES

1. Trahair, N. S, The Behaviour and Design of Steel Structures, Chapman & Hall, London, 1977
2. Mc Guire, W. , Steel Structures, Prentice Hall, 1968
3. Nethercot, D. A. , Salter, P. R. and Malik, A. S., Design of members subjected to Combined Bending and Torsion, The Steel Construction Institute, 1989.

18 BEAMS SUBJECTED TO TORSION AND BENDING - II

1.0 INTRODUCTION

In the previous chapter, the basic theory governing the behaviour of beams subjected to torsion was discussed. A member subjected to torsional moments would twist about a longitudinal axis through the shear centre of the cross section. It was also pointed out that when the resultant of applied forces passed through the longitudinal shear centre axis no torsion would occur. In general, torsional moments would cause twisting and warping of the cross sections.

When the torsional rigidity (GJ) is very large compared with its warping rigidity $(E\Gamma)$, the section would effectively be in uniform torsion and warping moment would unlikely to be significant from the designer's perspective. Examples of this behaviour are closed hot-rolled sections (e.g. rectangular or square hollow sections) and rolled angles and Tees. Note that warping moment is developed only if warping deformation is restrained. Warping deformation in angle and *T*-sections are not small, only warping moment would be small. On the other hand, most thin walled open sections have much smaller torsional rigidity (GJ) compared with warping rigidity $(E\Gamma)$ values and these sections will be exhibiting significant warping moment. Hot rolled *I* sections and *H* sections would exhibit torsional behaviour in-between these two extremes and the applied loading is resisted by a combination of uniform torsion and warping torsion.

2.0 DESIGNING FOR TORSION IN PRACTICE

Any structural arrangement in which the loads are transferred to an *I* beam by torsion is <u>not</u> an efficient one for resisting loads. The message for the designers is "Avoid Torsion - if you can ". In a very large number of practical designs, the loads are usually applied in a such a manner that their resultant passes through the centroid. If the section is doubly symmetric (such as *I* or *H* sections) this automatically eliminates torsion, as the shear centre and centroid of the symmetric cross section coincide. Even otherwise load transfer through connections may - in many cases - be regarded as ensuring that the loads are effectively applied through the shear centre, thus eliminating the need for designing for torsion. Furthermore, in situations where the floor slabs are supported on top flanges of channel sections, the loads may effectively be regarded as being applied through the shear centre since the flexural stiffness of the attached slab prevents torsion of the channel.

Where significant eccentricity of loading (which would cause torsion) is unavoidable, alternative methods of resisting torsion efficiently should be investigated. These include design using box sections, tubular (hollow) sections or lattice box girders which are fully triangulated on all faces. All these are more efficient means of resisting torsional moments compared with *I* or *H* sections. Unless it is essential to utilise the torsional resistance of an *I* section, it is not necessary to take account of it. The likely torsional effects due to a particular structural arrangement chosen should be considered in the early stages of design, rather than left to the final stages, when perhaps an inappropriate member has already been chosen.

3.0 PURE TORSION AND WARPING

In the previous chapter, the concepts of uniform torsion and warping torsion were explained and the relevant equations derived.

When a torque is applied only at the ends of a member such that the ends are free to warp, then the member would develop only pure torsion.
The total angle of twist (ϕ) over a length of z is given by

$$\phi = \frac{T_q \cdot z}{G J} \qquad (1)$$

where T_q = applied torque
GJ = Torsional Rigidity

When a member is in non-uniform torsion, the rate of change of angle of twist will vary along the length of the member. The warping shear stress (τ_w) at a point is given by

$$\tau_w = -\frac{E S_{wms} \phi'''}{t} \qquad (2)$$

where E = Modulus of elasticity
S_{wms} = Warping statical moment at a particular point S chosen.

The warping normal stress (σ_w) due to bending moment in-plane of flanges (bi-moment) is given by

$$\sigma_w = -E \, . W_{nwfs} \, . \, \phi''$$

where W_{nwfs} = Normalised warping function at the chosen point S.

4.0 COMBINED BENDING AND TORSION

There will be some interaction between the torsional and flexural effects, when a load produces both bending and torsion. The angle of twist ϕ caused by torsion would be amplified by bending moment, inducing additional warping moments and torsional shears. The following analysis was proposed by Nethercot, Salter and Malik in reference (2).

4.1 Maximum Stress Check or "Capacity check"

The maximum stress at the most highly stressed cross section is limited to the design strength (f_y / γ_m). Assuming elastic behaviour and assuming that the loads produce bending about the major axis in addition to torsion, the longitudinal direct stresses will be due to three causes.

$$\left.\begin{aligned} \sigma_{bx} &= \frac{M_x}{Z_x} \\ \sigma_{byt} &= \frac{M_{yt}}{Z_y} \\ \sigma_w &= E.W_{nwfs}.\phi'' \end{aligned}\right\} \tag{3}$$

σ_{byt} is dependent on M_{yt}, which itself is dependent on the major axis moment M_x and the twist ϕ.

$$M_{yt} = \phi \ M_x \tag{4}$$

Thus the "capacity check" for major axis bending becomes:

$$\sigma_{bx} + \sigma_{byt} + \sigma_w \le f_y / \gamma_m. \tag{5}$$

Methods of evaluating ϕ, ϕ', ϕ'' and ϕ''' for various conditions of loading and boundary conditions are given in reference (2).

4.2 Buckling Check

Whenever lateral torsional buckling governs the design (i.e. when p_b is less than f_y) the values of σ_w and σ_{byt} will be amplified. Nethercot, Salter and Malik have suggested a simple "buckling check" along lines similar to *BS 5950, part 1*

$$\frac{\overline{M}_x}{M_b} + \frac{(\sigma_{byt} + \sigma_w)}{(f_y / \gamma_m)}\left[1 + 0.5\frac{\overline{M}_x}{M_b}\right] \le 1 \tag{6}$$

where $\overline{M}_x$, equivalent uniform moment = $m_x M_x$

and M_b, the buckling resistance moment = $\dfrac{M_E M_p}{\phi_B + \left(\phi_B{}^2 - M_E \ M_p\right)^{1/2}}$

in which $\phi_B = \dfrac{M_p + (\eta_{LT}+1)M_E}{2}$

M_P, the plastic moment capacity $= f_y \,.\, Z_p / \gamma_m$

Z_p = the plastic section modulus

M_E, the elastic critical moment $= \dfrac{M_p\ \pi^2\ E}{\lambda_{LT}^2 \,.\, f_y/\gamma_m}$

where λ_{LT} is the equivalent slenderness.

4.3 Applied loading having both Major axis and Minor axis moments

When the applied loading produces both major axis and minor axis moments, the "capacity checks" and the "buckling checks" are modified as follows:

Capacity check:

$$\sigma_{bx} + \sigma_{byt} + \sigma_w + \sigma_{by} \le f_y/\gamma_m \tag{7}$$

Buckling check:

$$\frac{\overline{M}_x}{M_b} + \frac{\overline{M}_y}{f_y Z_y/\gamma_m} + \frac{(\sigma_{byt}+\sigma_w)}{(f_y/\gamma_m)}\left[1 + 0.5\frac{\overline{M}_x}{M_b}\right] \le 1 \tag{8}$$

$$\text{where } \overline{M}_y = m_y M_y$$
$$\sigma_{byt} = M_y / Z_y$$

4.4 Torsional Shear Stress

Torsional shear stresses and warping shear stresses should also be amplified in a similar manner:

$$\tau_{vt} = (\tau_t + \tau_w)\left(1 + 0.5\,\frac{\overline{M}_x}{M_b}\right) \tag{9}$$

This shear stress should be added to the shear stresses due to bending in checking the adequacy of the section.

5.0 DESIGN METHOD FOR LATERAL TORSIONAL BUCKLING

The analysis for the lateral torsional buckling is very complex because of the different types of structural actions involved. Also the basic theory of elastic lateral stability cannot be directly used for the design purpose because

- the formulae for elastic critical moment M_E are too complex for routine use and
- there are limitations to their extension in the ultimate range

A simple method of computing the buckling resistance of beams is given below. In a manner analogous to the Perry-Robertson Method for columns, the buckling resistance moment, M_b, is obtained as the smaller root of the equation

$$(M_E - M_b)(M_p - M_b) = \eta_{LT} . M_E M_b \tag{10}$$

As explained in page 3, M_b is given by,

$$M_b = \frac{M_E M_p}{\phi_B + \left(\phi_B{}^2 - M_E M_p\right)^{1/2}}$$

where $$\phi_B = \frac{M_p + (\eta_{LT} + 1) M_E}{2}$$

[As defined above, M_E = Elastic critcal moment

M_p = $f_y . Z_p / \gamma_m$

η_{LT} = Perry coefficient, similar to column buckling coefficient

Z_p = Plastic section modulus]

In order to simplify the analysis, *BS5950: Part 1* uses a curve based on the above concept (Fig. 1) (similar to column curves) in which the bending strength of the beam is expressed as a function of its slenderness (λ_{LT}). The design method is explained below.

The buckling resistance moment M_b is given by

$$M_b = p_b . Z_p \tag{11}$$

where p_b = bending strength allowing for susceptibility to lateral -torsional buckling.

Z_p = plastic section modulus.

It should be noted that $p_b = f_y$ for low values of slenderness of beams and the value of p_b drops, as the beam becomes longer and the beam slenderness, calculated as given below, increases. This behaviour is analogous to columns.

The beam slenderness (λ_{LT}) is given by,

$$\lambda_{LT} = \sqrt{\pi^2 \frac{E}{f_y}} \cdot \bar{\lambda}_{LT} \tag{12}$$

where $$\bar{\lambda}_{LT} = \sqrt{\frac{M_p}{M_E}}$$

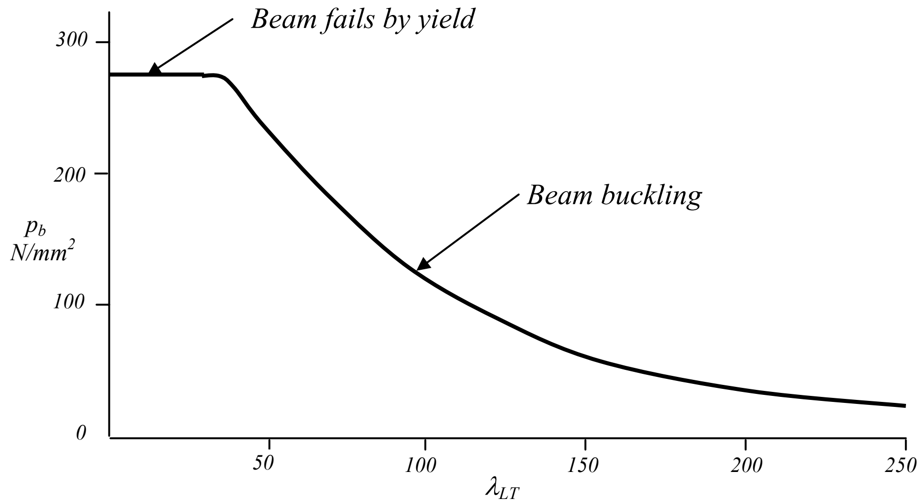


Fig.1 Bending strength for rolled sections of design strength 275 N/mm² according to BS 5950

Fig. 2 is plotted in a non-dimensional form comparing the observed test data with the two theoretical values of upper bounds, viz. M_p and M_E. The test data were obtained from a typical set of lateral torsional buckling data, using hot-rolled sections. In Fig. 2 three distinct regions of behaviour can be observed:-

- stocky beams which are able to attain the plastic moment M_p, for values of $\bar{\lambda}_{LT}$ below about 0.4.

- Slender beams which fail at moments close to M_E, for values of $\bar{\lambda}_{LT}$ above about 1.2

- beams of intermediate slenderness which fail to reach either M_p or M_E . In this case $0.4 < \bar{\lambda}_{LT} < 1.2$

Beams having short spans usually fail by yielding. So lateral stability does not influence their design. Beams having long spans would fail by lateral buckling and these are termed "slender". For the practical beams which are in the intermediate range without lateral restraint, design must be based on considerations of inelastic buckling.

In the absence of instability, eqn. 11 permits that the value of f_y can be adopted for the full plastic moment capacity p_b for $\lambda_{LT} < 0.4$. This corresponds to λ_{LT} values of around 37 (for steels having $f_y = 275\ N/mm^2$) below which the lateral instability is <u>NOT</u> of concern.

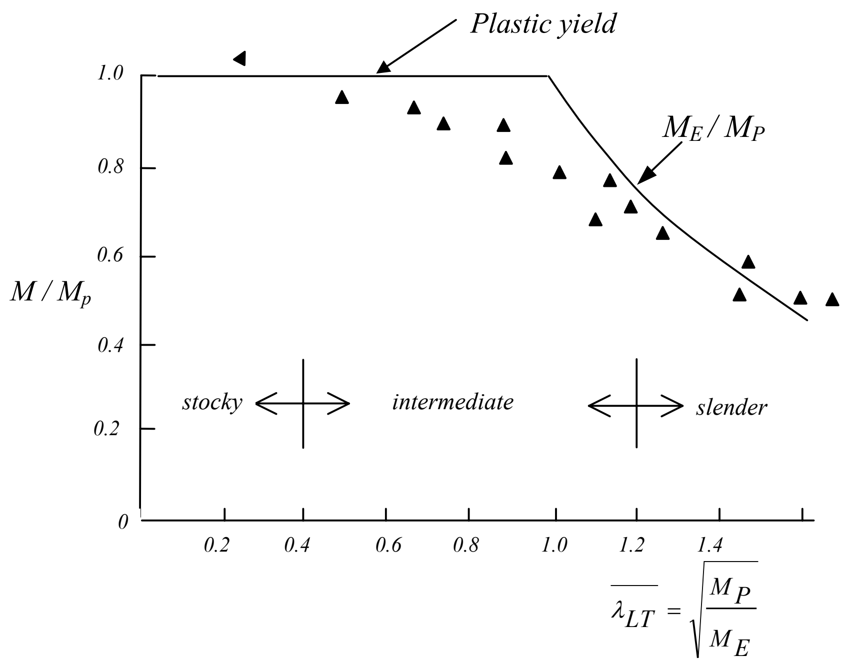


Fig.2 Comparison of test data (mostly I sections) with theoretical elastic critical moments

For more slender beams, p_b is a function of λ_{LT} which is given by ,

$$\lambda_{LT} = uv\frac{\lambda}{r_y} \tag{13}$$

u is called the buckling parameter and x, the torsional index.

For flanged sections symmetrical about the minor axis,

$$u = \left(\frac{4\,Z_p^{\,2}\,\gamma}{A^2\,h_s^{\,2}}\right)^{1/4} \quad \text{and} \quad x = 0.566\ h_s\left(A/J\right)^{1/2}$$

For flanged sections symmetrical about the major axis

$$u = \left(\frac{I_y \cdot Z_p^{\,2}\gamma}{A^2\Gamma}\right)^{1/4} \quad \text{and} \quad x = 1.132\left(\frac{A\Gamma}{I_y\,J}\right)^{1/2}$$

In the above Z_p = plastic modulus about the major axis

$$\gamma = \left(1 - \frac{I_y}{I_x}\right)$$

A = cross sectional area of the member

Γ = torsional warping constant $\cong \dfrac{h_s^{2}\, t_1 t_2 b_1^{3} b_2^{3}}{12\left(t_1 b_1^{3} + t_2 b_2^{3}\right)}$

J = the torsion constant

h_s = the distance between the shear centres of the flanges

t_1, t_2 = flange thicknesses

b_1, b_2 = flange widths

We can assume

u = *0.9* for rolled *UBs, UCs, RSJs* and channels
= *1.0* for all other sections.

$v = a\ function\ of \left(\dfrac{\lambda}{r_y}, x\right)$ is given in Table 14 of *BS5950: Part I*

(for a preliminary assessment $v = 1$)

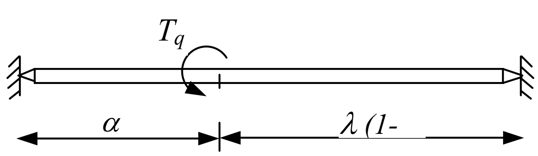
$x = D/T$ providing the above values of u are used.

5.1 Unequal flanged sections

For unequal flanged sections, eqn. 11 is used for finding the buckling moment of resistance. The value of λ_{LT} is determined by eqn.13 using the appropriate section properties. In that equation u may be taken as *1.0* and v includes an allowance for the degree of monosymmetry through the parameter $N = I_c / (I_c + I_t)$. Table 14 of *BS5950: Part* 1 must now be entered with $(\lambda_E / r_y)/x$ and N .

5.2 Evaluation of differential equations

For a member subjected to concentrated torque with torsion fixed and warping free condition at the ends (torque applied at varying values of αL), the values of ϕ and its differentials are given by



For $0 \le z \le \alpha\lambda$, $\quad \phi = \dfrac{T_q .a}{GJ}\left\{(1-\alpha)\dfrac{z}{a} + \left[\dfrac{\sinh\dfrac{\alpha\lambda}{a}}{\tanh\dfrac{\lambda}{a}} - \cosh\dfrac{\alpha\lambda}{a}\right]\sinh\dfrac{z}{a}\right\}$

$$\phi' = \frac{T_q}{GJ}\left\{(1-\alpha) + \left[\frac{\sinh\frac{\alpha\lambda}{a}}{\tanh\frac{\lambda}{a}} - \cosh\frac{\alpha\lambda}{a}\right]\cosh\frac{z}{a}\right\}$$

$$\phi'' = \frac{T_q}{G\,J\,a}\left[\frac{\sinh\frac{\alpha\lambda}{a}}{\tanh\frac{\lambda}{a}} - \cosh\frac{\alpha\lambda}{a}\right]\sinh\frac{z}{a}$$

$$\phi''' = \frac{T_q}{G\,J\,a^2}\left[\frac{\sinh\frac{\alpha\lambda}{a}}{\tanh\frac{\lambda}{a}} - \cosh\frac{\alpha\lambda}{a}\right]\cosh\frac{z}{a}$$

Similar equations are available for different loading cases and for different values of $\alpha\lambda$. Readers may wish to refer Ref. (2) for more details. We are unable to reproduce these on account of copyright restrictions.

6.0 SUMMARY

This chapter is aimed at explaining a simple method of evaluating torsional effects and to verify the adequacy of a chosen cross section when subjected to torsional moments. The method recommended is consistent with BS 5950: Part 1.

7.0 REFERENCES

(1) British Standards Institution, BS 5950: Part 1: 1985. Structural use of steelwork in Building part 1: Code of Practice for design in simple and continuous construction: hot rolled sections. BSI, 1985.

(2) Nethercot, D. A., Salter, P. R., and Malik, A. S. Design of Members Subject to Combined Bending and Torsion , The Steel construction Institute , 1989.

(3) Steelwork design guide to BS 5950: Part 1 1985, Volume 1 Section properties and member capacities. The Steel Construction Institute, 1985.

(4) Introduction to Steelwork Design to BS 5950: Part 1,
The Steel Construction Institute, 1988.

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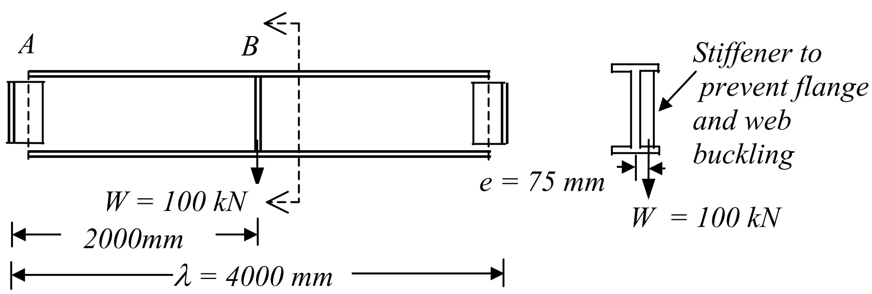
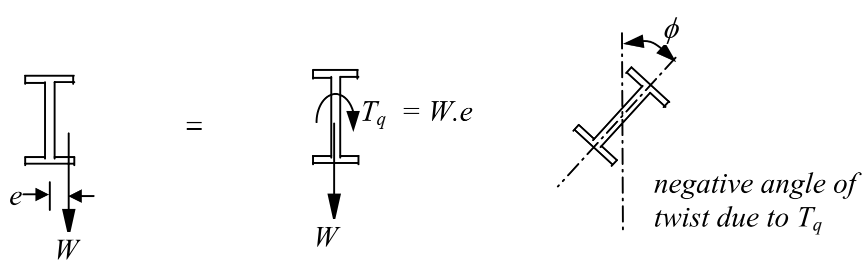
<u>Example 1</u>

The beam shown below is unrestrained along its length. An eccentric load is applied to the bottom flange at the centre of the span in such a way that it does not provide any lateral restraint to the member.
The end conditions are assumed to be simply supported for bending and fixed against torsion but free for warping. For the factored loads shown, check the adequacy of the trial section.



Replace the actual loading by an equivalent arrangement, comprising a vertical load applied through the shear centre and a torsional moment as shown below.



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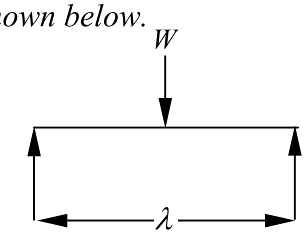
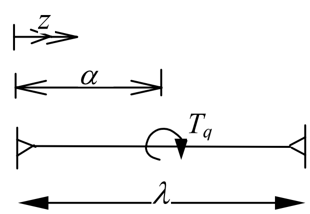
Loadings due to plane bending and torsion are shown below.



(i) Plane

+



(ii) Torsional

Loading (Note: These are factored loads and are not to be multiplied by γ_f)

Point load, $W = 100$ kN
Distributed load (self weight), $w = 1$ kN/m *(say)*
Eccentricity, $e = 75$ mm

Bending effects (at U.L.S)

Moment at B, $M_{xB} = 102$ kNm
Shear at A, $F_{vA} = 52$ kN
Shear at B, $F_{vB} = 50$ kN

Torsional effects (at U.L.S)

Torsional moment, $T_q = W.e$

$$T_q = 100 \times 75 \times 10^{-3} = 7.5 \text{ kNm}$$

This acts in a negative sense, $\therefore T_q = -7.5$ kNm

Generally wide flange sections are preferable to deal with significant torsion. In this example, however, an ISWB section will be tried.

Try ISWB 500 × 250 @ 95.2 kg/m
Section properties from steel tables.

Depth of section $D = 500$ mm
Width of section $B = 250$ mm

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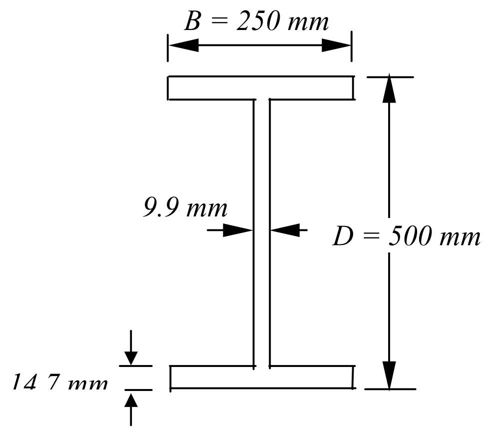

Web thickness $t = 9.9\ mm$
Flange thickness $T = 14.7\ mm$
Moment of inertia $I_{xx} = 52291\ cm^4$
Moment of inertia $I_{yy} = 2988\ cm^4$
Radius of gyration $r_y = 49.6\ mm$
Elastic modulus $Z_x = 2092\ cm^3$
Elastic modulus $Z_y = 239\ cm^3$
Cross sectional area $A = 121.2\ cm^2$

Additional properties

Torsional constant, $$J = \frac{1}{3}\left[2BT^3 + (D - 2T)\,t^3\right]$$

$$= \frac{1}{3}\left[2\times 250\times 14.7^3 + (500 - 2\times 14.7)\,9.9^3\right] = 682\times 10^3\ mm^4$$

Warping constant, $$\Gamma = \frac{I_y \cdot h^2}{4}$$

$$= \frac{2988\times 10^4\times(500 - 14.7)^2}{4} = 1.76\times 10^{12}\ mm^6$$

Shear modulus, $$G = \frac{E}{2(1+\upsilon)}$$

$$= \frac{2\times 10^5}{2(1+0.3)} = 76.9\ kN/mm^2$$

Torsional bending constant, $$a = \left(\frac{E\Gamma}{GJ}\right)^{1/2}$$

$$= \left(\frac{2\times 10^5\times 1.76\times 10^{12}}{76.9\times 10^3\times 682\times 10^3}\right)^{1/2} = 2591\ mm$$

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Normalized warping function,

$$W_{nwfs} = \frac{h\,B}{4} = \frac{(500 - 14.7) \times 250}{4} = 30331 \ \ mm^2$$

Warping statical moment,

$$S_{wms} = \frac{h\,B^2\,T}{16} = \frac{485.3 \times 250^2 \times 14.7}{16} = 2787 \times 10^4 \ mm^4$$

Statical moment for flange,

$$Q_f = A_f \, . \, y_f = (120.05 \times 14.7) \times 242.7 = 428.2 \times 10^3 \ mm^3$$

Statical moment for web,

$$Q_w = (A/2) \times y_w$$

$$y_w = \frac{\left[14.7 \times 250 \times 242.7 \ + \ 9.9 \times 235.3 \times \frac{235.3}{2}\right]}{14.7 \times 250 \ + \ 9.9 \times 235.3} = 194.2 \ mm$$

$$\therefore \quad Q_w = 6061 \times 194.2 = 1166 \times 10^3 \ mm^3$$

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Material Properties $$\frac{250}{1.15} \times \left[\frac{250 \times 500^2}{4} - \frac{240.1 \times 470.6^2}{4} \right] = 507 kNm$$

Shear modulus, $G = 76.9\ kN/mm^2$

Design strength, $p_y = 250/\gamma_m = 250/1.15 = 217\ N/mm^2$

<u>Check for Combined bending and torsion</u>

(i) *<u>Buckling check (at Ultimate Limit State)</u>*

$$\frac{\overline{M}_x}{M_b} + \frac{(\sigma_{byt} + \sigma_w)}{f_y / \gamma_m} \left[1 + 0.5\, \frac{\overline{M}_x}{M_b} \right] \leq 1$$

$$\overline{M}_x = m \times M_{xB}$$

$$m = 1.0$$

$$\therefore \quad \overline{M}_x = 1.0 \times M_{xB} = 102\ kNm$$

Effective length $\lambda_E = 1.0\, L$ $\qquad \lambda_E = 4000\ mm$

The buckling resistance moment, $$M_b = \frac{M_E M_p}{\phi_B + \left(\phi_B^2 - M_E M_p\right)^{1/2}}$$

$$\phi_B = \frac{M_p + (\eta_{LT} + 1) M_E}{2}$$

BS 5950: Part I App.B.2

where

M_E = *elastic critical moment*

M_p = *plastic moment capacity*

$= f_y . Z_p / \gamma_m =$

<table>
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Elastic critical moment, $M_E = \dfrac{M_p\ \pi^2\ E}{\lambda_{LT}{}^2 \cdot p_y}$

BS 5950: Part I App.B.2.2

λ_{LT} = *the equivalent slenderness* $= nuv\ \lambda$

λ = *the minor axis slenderness* $= \lambda_E / r_y = 4000 / 49.6 = 80.7$

$n = 0.86, \quad u = 0.9$

BS 5950: Part I Table 14

v = *slenderness factor (according to N and* λ/x*)*

$$N = \frac{I_{cf}}{I_{cf} + I_{tf}} = 0.5 \text{ (for equal flanged sections)}$$

BS 5950: Part 1 App.B.2.5

$$x = 1.132\left(\frac{A\Gamma}{I_y . J}\right)^{1/2}$$

$$= 1.132\left(\frac{12122 \times 1.76 \times 10^{12}}{2988 \times 10^4 \times 681.6 \times 10^3}\right)^{1/2} = 36.63$$

$$\lambda / x = 80.7 / 36.6 = 2.2$$

$$v = 0.948$$

$$\lambda_{LT} = nuv\lambda$$

$$= 0.86 \times 0.9 \times 0.948 \times 80.7 = 59.2$$

$$M_E = \frac{583 \times 10^6 \times \pi^2 \times 2 \times 10^5}{59.2^2 \times 217}$$

$$= 1143\ kNm$$

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BS 5950: Part 1 App.B.2.3

$$\phi_B = \frac{M_p + (\eta_{LT} + 1)M_E}{2}$$

The Perry coefficient, $\eta_{LT} = \alpha_b(\lambda_{LT} - \lambda_{LO})$

Limiting equivalent slenderness, $\lambda_{LO} = 0.4\left(\frac{\pi^2 E}{p_y}\right)^{1/2}$

$$= 0.4\left(\frac{\pi^2 \times 2 \times 10^5}{217}\right)^{1/2} = 38.2$$

$$\eta_{LT} = 0.007\,(59.2 - 38.2) = 0.15$$

$$\therefore \quad \phi_B = \frac{507 + (0.15 + 1) \times 1143}{2} = 911 \text{ kNm}$$

$$\therefore \quad M_b = \frac{M_E M_p}{\phi_B + \left(\phi_B^2 - M_E M_p\right)^{1/2}}$$

$$= \frac{1143 \times 507}{911 + \left(911^2 - 1143 \times 507\right)^{1/2}} = 411 \text{ kNm}$$

$$M_{yt} = M_x \,.\, \phi$$

<u>*To calculate* ϕ</u>

$$\lambda / a = 4000/2591 = 1.54$$
$$z = \alpha\lambda, \quad \alpha = 0.5$$
$$= 0.5 \times 4000 = 2000$$
$$\alpha\lambda / a = 0.77$$

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$$\phi = \frac{T_q\,.a}{GJ}\left\{(1-\alpha)\frac{z}{a} + \left[\frac{\sinh\frac{\alpha\lambda}{a}}{\tanh\frac{\lambda}{a}} - \cosh\frac{\alpha\lambda}{a}\right]\sinh\frac{z}{a}\right\}$$

Ref. 2.0 App. B

$$= \frac{-7.5\times10^6\times 2591}{76.9\times10^3\times 681.6\times10^3}\left\{(1-0.5)0.77 + \left[\frac{\sinh0.77}{\tanh1.54} - \cosh0.77\right]\sinh0.77\right\}$$

$$= -0.023\ \text{rads}$$

$$M_{yt} = 102\times 0.023 = 2.36\ \text{kNm}$$

$$\sigma_{byt} = \frac{M_{yt}}{Z_y} = \frac{2.36\times10^6}{239\times10^3} = 9.89\ \ N/mm^2$$

$$\sigma_w = E\,.\,W_{nwfs}\,.\phi''$$

<u>*To calculate ϕ''*</u>

$$\phi'' = \frac{T_q}{G\,J\,a}\left[\frac{\sinh\frac{\alpha\lambda}{a}}{\tanh\frac{\lambda}{a}} - \cosh\frac{\alpha\lambda}{a}\right]\sinh\frac{z}{a}$$

Ref. 2.0 App. B

$$= \frac{-7.5\times10^6}{76.9\times10^3\times 681.6\times10^3\times 2591}\left[\frac{\sinh0.77}{\tanh1.54} - \cosh\ 0.77\right]\ \sinh0.77$$

$$= 1.8\times10^{-8}$$

$$\sigma_w = 2\times10^5\times30331\times1.8\times10^{-8} = 109\ \ N/mm^2$$

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$$\frac{\overline{M_x}}{M_b} + \frac{(\sigma_{byt} + \sigma_w)}{f_y / \gamma_m}\left[1 + 0.5\frac{\overline{M_x}}{M_b}\right] \leq 1$$

$$\frac{102 \times 10^6}{411 \times 10^6} + \frac{(9.9 + 109.2)}{(250/1.15)}\left[1 + 0.5 \times \frac{102 \times 10^6}{411 \times 10^6}\right] = 0.86 < 1$$

$\therefore$ *Buckling is O. K*

(i) *Local "capacity" check*

$$\sigma_{bx} + \sigma_{byt} + \sigma_w \leq f_y / \gamma_m$$

$$\sigma_{bx} = M_x / Z_x = 102 \times 10^6 / 2092 \times 10^3 = 48.8\ N/mm^2$$

$$\therefore\ 48.8 + 9.9 + 109.2 = 168\ N/mm^2 < 217\ N/mm^2$$

$\therefore$ *O. K*

Strictly the shear stresses due to combined bending and torsion should be checked, although these will seldom be critical.

Shear stresses due to bending (at Ultimate Limit state)

At support:-

In web,
$$\tau_{bw} = \frac{F_{VA} \cdot Q_w}{I_x . t} = \frac{52 \times 10^3 \times 1166 \times 10^3}{52291 \times 10^4 \times 9.9}$$

$$= 11.7\ N/mm^2$$

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In flange, $\tau_{bf} = \dfrac{F_{VA} \cdot Q_f}{I_x . T} = \dfrac{52 \times 10^3 \times 428.2 \times 10^3}{52291 \times 10^4 \times 14.7}$

$= 2.9 \ N/mm^2$

At midspan :-

In web, $\tau_{bw} = 11.3 \ N/mm^2$

In flange, $\tau_{bf} = 2.8 \ N/mm^2$

<u>Shear stresses due to torsion (at Ultimate Limit state)</u>

Stress due to pure torsion, $\tau_t = G.t.\phi'$

Stress due to warping, $\tau_w = \dfrac{-E . S_{wms} . \phi'''}{t}$

<u>To calculate ϕ' and ϕ'''</u>

$$\phi' = \frac{T_q}{G J}\left\{(1-\alpha) + \left[\frac{\sinh\frac{\alpha\lambda}{a}}{\tanh\frac{\lambda}{a}} - \cosh\frac{\alpha\lambda}{a}\right]\cosh\frac{z}{a}\right\}$$

Ref. 2.0 App.B

$$\phi''' = \frac{T_q}{G J a^2}\left[\frac{\sinh\frac{\alpha\lambda}{a}}{\tanh\frac{\lambda}{a}} - \cosh\frac{\alpha\lambda}{a}\right]\cosh\frac{z}{a}$$

At $\alpha = 0.5$,

$$\frac{\alpha\lambda}{a} = \frac{0.5 \times 4000}{2591} = 0.77$$

$$\sinh\frac{\alpha\lambda}{a} = 0.851, \quad \cosh\frac{\alpha\lambda}{a} = 1.313, \quad \tanh\frac{\lambda}{a} = 0.913$$

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At support, $z = 0$

$$\cosh\frac{z}{a} = \cosh(0) = 1.0$$

At midspan, $z = 2000$

$$\cosh\frac{z}{a} = \cosh(0.77) = 1.313$$

At support

$$\phi''' = \frac{-7.5\times 10^6}{76.9\times 10^3 \times 681.6\times 10^3 \times 2591^2}\left[\frac{0.851}{0.913} - 1.313\right]\times 1$$

$$\therefore\ \phi''' = 0.812\times 10^{-11}$$

$$\phi' = \frac{-7.5\times 10^6}{76.9\times 10^3\times 681.6\times 10^3}\left[(1-0.5)+\left[\frac{0.851}{0.913} - 1.313\right]\times 1\right]$$

$$= -1.7\times 10^{-5}$$

Stresses due to pure torsion.

In web, $\tau_{tw} = G.t.\phi'$

$$\tau_{tw} = 76.9\times 10^3\times 9.9\times(-1.7\times 10^{-5})$$

$$= -12.95\ N/mm^2$$

In flange, $\tau_{tf} = G.\,T.\,\phi'$

$$\tau_{tf} = 76.9\times 10^3\times 14.7\times(-1.7\times 10^{-5})$$

$$= -19.22\ N/mm^2$$

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Stresses due to warping in flange,

$$\tau_{wf} = \frac{-E.S_{wms}.\phi'''}{T}$$

$$\tau_{wf} = \frac{-2\times10^{5}\times2787\times10^{4}\times0.812\times10^{-11}}{14.7} = -3.1\,N/mm^{2}$$

<u>At midspan</u>

$$\phi' = 0$$

$$\phi''' = \frac{-7.5\times10^{6}}{76.9\times10^{3}\times681.6\times10^{3}\times2591^{2}}\left[\frac{0.851}{0.913}-1.313\right]\times1.313$$

$$= 1.06\times10^{-11}$$

Stresses due to pure torsion,

In web, $\tau_{tw} = G.t.\phi' = 0$

In flange, $\tau_{tf} = G.T.\phi' = 0$

Stresses due to warping in flange,

$$\tau_{wf} = \frac{-E.S_{wms}.\phi'''}{T}$$

$$\tau_{wf} = \frac{-2\times10^{5}\times2787\times10^{4}\times1.06\times10^{-11}}{14.7} = -4.02\,N/mm^{2}$$

By inspection the maximum combined shear stresses occur at the support.

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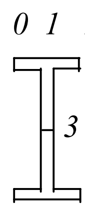

At support

$$\tau_{vt} = (\tau_t + \tau_w)\left(1 + 0.5\ \frac{\overline{M}_x}{M_b}\right)$$

In web at 3, $\tau_{tw} = -12.95\ N/mm^2$

$$\therefore \quad \tau_{vt} = -12.95\left(1 + 0.5 \times \frac{102}{411}\right) = -14.6\ N/mm^2$$

This must be added to the shear stresses due to plane bending.

$$\tau = \tau_{bw} + \tau_{vt}$$

$$\tau = \pm\ 11.7 - 14.6 = -26.3\ N/mm^2 \text{(acting downwards)}$$

In the top flange at 1, $\tau_{tf} = -19.2\ N/mm^2$

$$\tau_{wf} = -3.1\ N/mm^2$$

$$\therefore \quad \tau_{vt} = (-19.2 - 3.1)\left(1 + 0.5\ \frac{102}{411}\right) = -25.1\ N/mm^2$$

$$\tau = \tau_{bf} + \tau_{vt} = -27.9\ N/mm^2 \text{ (acting left to right)}$$

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</table>

Shear strength, $f_v = 0.6 f_y / \gamma_m = 0.6 \times 250 / 1.15 = 130 \; N/mm^2$

Since $\tau < f_v \qquad 27.9 < 130 \; N/mm^2$

Section is adequate for shear

Referring back to the determination of the maximum angle of twist ϕ*, in order to obtain the value at working load it is sufficient to replace the value of torque* T_q *with the working load value as* ϕ *is linearly dependent on* T_q*. Since* T_q *is due to solely the imposed point load W, dividing by the appropriate value of* γ_f *will give :-*

$\therefore$ *Working load value of* T_q *is* $\dfrac{7.5}{1.6} = 4.7 \; kNm$

the corresponding value of $\phi = \dfrac{0.026}{1.6} = 0.016 \; rads = 0.93°$

On the assumption that a maximum twist of 2° is acceptable at working load, in this instance the beam is satisfactory.

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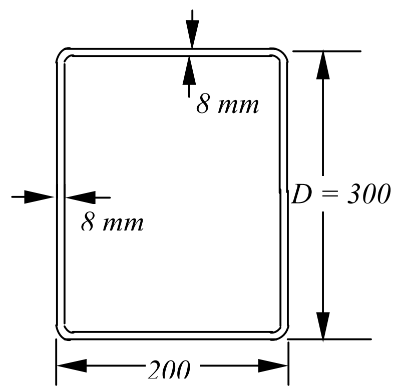
Example 2

Redesign the member shown in example 1, using a rectangular hollow section.

Try $300 \times 200 \times 8$ @ 60.5 kg / m R. H. S

Section properties.

Depth of section	D	$= 300$ mm
Width of section	B	$= 200$ mm
Web thickness	t	$= 8$ mm
Flange thickness	T	$= 8$ mm
Area of section	A	$= 77.1$ cm^2
Moment of inertia	I_x	$= 9798$ cm^4
Radius of gyration	r_y	$= 8.23$ cm
Elastic modulus	Z_x	$= 653$ cm^3
Elastic modulus	Z_y	$= 522$ cm^3
Plastic modulus	Z_p	$= 785$ cm^3



Additional properties

Torsional constant $J = \frac{t^3 h}{3} + 2K \cdot A_h$

Area enclosed by the mean perimeter of the section, $A_h = (B - t)(D - T)$
(neglecting the corner radii)

$= (200 - 8)(300 - 8)$

$= 56064 \text{ mm}^2$

The mean perimeter, $h = 2[(B - t) + (D - T)]$

$= 2[(200 - 8) + (300 - 8)] = 968$ mm

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$$K = \frac{2\,A_h \cdot t}{h} = \frac{2\times 56064\times 8}{968} = 927\ mm^2$$

$$\therefore \quad \text{Torsional constant,} \quad J = \frac{8^3\times 968}{3} + 2\times 927\times 56064$$

$$= 104\times 10^6\ mm^4$$

$$\text{Torsional modulus constant,} \quad C = \frac{J}{t + K/t}$$

$$= \frac{104\times 10^6}{8 + 927/8} = 840\times 10^3\ mm^3$$

Material properties

$$\text{Shear modulus,} \quad G = \frac{E}{2(1+\upsilon)} = \frac{2\times 10^5}{2(1+0.3)} = 76.9\ kN/mm^2$$

$$\text{Design strength,} \ p_y = 250/\gamma_m = 250/1.15 = 217\ N/mm^2$$

<u>Check for combined bending and torsion</u>

(i) *<u>Buckling check</u>*

$$\frac{\overline{M_x}}{M_b} + \frac{(\sigma_{byt} + \sigma_w)}{f_y/\gamma_m}\left[1 + 0.5\,\frac{\overline{M_x}}{M_b}\right] \le 1$$

Since slenderness ratio ($\lambda_E / r_y = 4000/82.3 = 48.6$) is less than the limiting value $\left(350\times\frac{275}{250}\times\frac{250}{f_y} = 385\right)$ *given in BS 5950 Part 1, table 38, lateral torsional buckling need not be considered..*

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Hence $M_b = M_{cx}$

Shear capacity $P_v = 0.6\, f_y / \gamma_m \,.\, A_v$

Shear area $A_v = \left(\dfrac{D}{D+B}\right) A = \left(\dfrac{300}{300+200}\right) 77.1 = 46.3\ cm^2$

$\therefore\ P_v = 0.6 \times (250/1.15) \times 46.3 \times 10^2 \times 10^{-3} = 604.3\ kN$

Since $F_{VB} < 0.6\, P_v \qquad 50 < 363$

$M_{cx} = f_y . Z_p / \gamma_m \le 1.2\, f_y / \gamma_m . Z_x$ *(for plastic sections)*

$\therefore\ M_{cx} = 1.2 \times (250/1.15) \times 653 \times 10^{-3} = 170\ kNm$ — *BS 5950: Part 1 4.2.5*

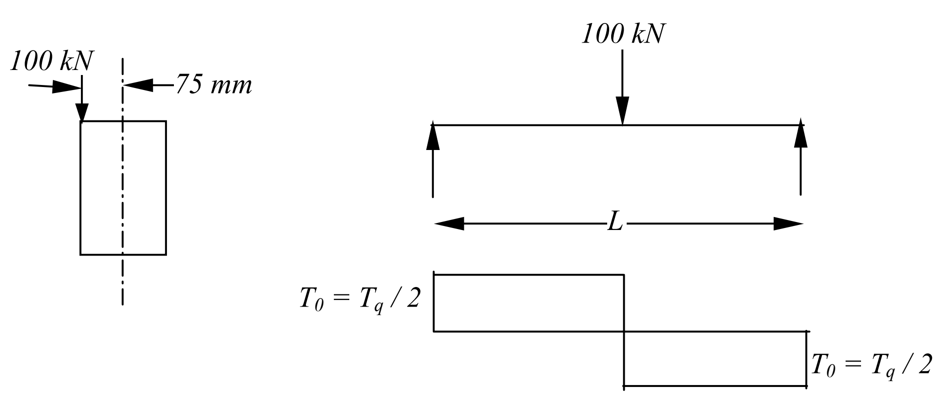
$\overline{M} = m \cdot M_{xB}$

$m = 1.0$

$\overline{M} = 1.0 \times 102 = 102\ kNm$ — *BS 5950: Part 1 4.3.7.2 table 13*

To calculate ϕ

The 100 kN eccentric load gives a value of $T_q = 100 \times 0.75 = 7.5\ kNm$



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$$\phi = \frac{T_0}{GJ} \cdot z$$

$$T_0 = \frac{T_q}{2} = \frac{7.5}{2} = 3.75\ kNm$$

At centre of span, $z = \lambda / 2 = 2000\ mm$

$$\phi = \frac{3.75 \times 10^6 \times 2000}{76.9 \times 10^3 \times 104 \times 10^6} = 0.001\ radians$$

$$M_{yt} = \phi \, . \, M_{xB} = 0.001 \times 102 = 0.102\ kNm$$

$$\sigma_{byt} = \frac{M_{yt}}{Z_y} = \frac{0.102 \times 10^6}{522 \times 10^3} = 0.195\ N/mm^2$$

Warping stresses (σ_w) are insignificant due to the type of section employed.

Check becomes

$$\frac{\overline{M}_x}{M_b} + \frac{\sigma_{byt}}{f_y/\gamma_m}\left[1 + 0.5\,\frac{\overline{M}_x}{M_b}\right] \le 1$$

$$\frac{102}{170} + \frac{0.195}{250/1.15}\left[1 + 0.5 \times \frac{102}{170}\right] = 0.6 < 1$$

$\therefore$ **<u>O. K</u>**

(ii) <u>*Local capacity check*</u>

$$\sigma_{bx} + \sigma_{byt} + \sigma_w \le f_y/\gamma_m$$

$$\sigma_{bx} = M_{xB} / Z_y$$

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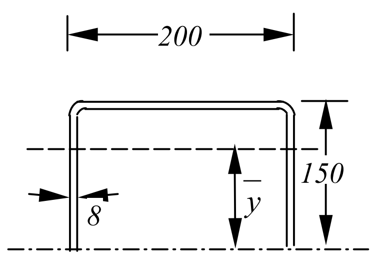
$$\sigma_{bx} = \frac{102 \times 10^6}{522 \times 10^3} = 196 \ N/mm^2$$

$$196 + 0.195 + 0 = 196.2 < 217 \ N/mm^2$$

$\therefore$ ***O. K***

Shear stresses due to bending (at Ultimate Limit state)

Maximum value occurs in the web at the support.



$$\tau_{bw} = \frac{F_{VA} \cdot Q_w}{I_x \cdot t_1}$$

$$Q_w = A_1 \cdot \bar{y}$$

$$A_1 = \frac{A}{2}$$

$$\bar{y} = \frac{150 \times 8 \times \frac{150}{2} \times 2 + 184 \times 8 \times 146}{A_1} \times 10^{-3} = \frac{395}{A_1} cm$$

$$\therefore Q_w = A_1 \times \frac{395}{A_1} = 395 \ cm^3$$

$$\tau_{bw} = \frac{52 \times 10^3 \times 395 \times 10^3}{9798 \times 10^4 \times 2 \times 8} = 13.1 \ N/mm^2$$

Shear stresses due to torsion (at Ultimate limit State)

$$\tau_t = \frac{T_0}{C} = \frac{T_q}{2C} = \frac{7.5 \times 10^6}{2 \times 837 \times 10^3} = 4.5 \ N/mm^2$$

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Total shear stress (at Ultimate Limit State)

$$\tau = \tau_{bw} + \tau_{vt}$$

$$\tau_{vt} = (\tau_t + \tau_w)\left(1 + 0.5\frac{\overline{M_x}}{M_b}\right)$$

$$= (4.5 + 0)\left(1 + 0.5 \times \frac{102}{170}\right) = 5.9\ N/mm^2$$

$$\tau = 13.1 + 5.9 = 19\ N/mm^2$$

Shear strength $p_v = 0.6 f_y / \gamma_m = 0.6 \times 250 / 1.15 = 130\ N/mm^2$ — *BS 5950: Part 1 4. 2. 3*

Since $\tau < p_v$ $\quad 19 < 130\ N/mm^2$

$\therefore$ ***the section is adequate for shear.***

19 COLD FORMED STEEL SECTIONS – I

1.0 INTRODUCTION

Thin sheet steel products are extensively used in building industry, and range from purlins to roof sheeting and floor decking. Generally these are available for use as basic building elements for assembly at site or as prefabricated frames or panels. These thin steel sections are ***cold-formed,*** i.e. their manufacturing process involves forming steel sections in a cold state (i.e. without application of heat) from steel sheets of ***uniform*** thickness. These are given the generic title ***Cold Formed Steel Sections***. Sometimes they are also called ***Light Gauge Steel Sections*** or ***Cold Rolled Steel Sections***. The thickness of steel sheet used in cold formed construction is usually *1* to *3* mm. Much thicker material up to *8 mm* can be formed if pre-galvanised material is not required for the particular application. The method of manufacturing is important as it differentiates these products from ***hot rolled steel*** sections. Normally, the yield strength of steel sheets used in cold-formed sections is at least $280\ N/mm^2$, although there is a trend to use steels of higher strengths, and sometimes as low as $230\ N/mm^2$.

Manufacturers of cold formed steel sections purchase steel coils of *1.0* to *1.25 m* width, slit them longitudinally to the correct width appropriate to the section required and then feed them into a series of roll forms. These rolls, containing male and female dies, are arranged in pairs, moving in opposite direction so that as the sheet is fed through them its shape is gradually altered to the required profile. The number of pairs of rolls (called ***stages***) depends on the complexity of the cross sectional shape and varies from *5* to *15*. At the end of the rolling stage a flying shearing machine cuts the member into the desired lengths.

An alternative method of forming is by press - braking which is limited to short lengths of around *6 m* and for relatively simple shapes. In this process short lengths of strip are pressed between a male and a female die to fabricate, one fold at a time and obtain the final required shape of the section. Cold rolling is used when large volume of long products are required and press breaking is used when small volume of short length products are produced.

Galvanizing (or zinc coating) of the preformed coil provides very satisfactory protection against corrosion in internal environments. A coating of $275\ g/m^2$ (total for both faces) is the usual standard for internal environments. This corresponds to zinc coating of 0.04 *mm*. Thicker coatings are essential when moisture is present for long periods of time. Other than galvanising, different methods of pre-rolling and post-rolling corrosion protection measures are also used.

Although the cold rolled products were developed during the First World War, their extensive use worldwide has grown only during the last 20 years because of their versatility and suitability for a range of lighter load bearing applications. Thus the wide range of available products has extended their use to primary beams, floor units, roof trusses and building frames. Indeed it is difficult to think of any industry in which Cold Rolled Steel products do not exist in one form or the other. Besides building industry, they are employed in motor vehicles, railways, aircrafts, ships, agricultural machinery, electrical equipment, storage racks, house hold appliances and so on. In recent years, with the evolution of attractive coatings and the distinctive profiles that can be manufactured, cold formed steel construction has been used for highly pleasing designs in practically every sector of building construction.

In this chapter, the background theory governing the design of cold formed steel elements is presented in a summary form. Design of cold formed steel sections are dealt with in IS: 801-1975 which is currently due under revision. In the absence of a suitable Limit State Code in India, the Code of Practice for Cold Formed Sections in use in the U.K.(BS 5950, Part 5) is employed for illustrating the concepts with suitable modifications appropriate to Indian conditions.

2.0 ADVANTAGES OF COLD FORMED SECTIONS

Cold forming has the effect of increasing the yield strength of steel, the increase being the consequence of cold working well into the strain-hardening range. These increases are predominant in zones where the metal is bent by folding. The effect of cold working is thus to enhance the mean yield stress by *15% - 30%.* For purposes of design, the yield stress may be regarded as having been enhanced by a minimum of *15%.*

Some of the main advantages of cold rolled sections, as compared with their hot-rolled counterparts are as follows:

- Cross sectional shapes are formed to close tolerances and these can be consistently repeated for as long as required.
- Cold rolling can be employed to produce almost any desired shape to any desired length.
- Pre-galvanised or pre-coated metals can be formed, so that high resistance to corrosion, besides an attractive surface finish, can be achieved.
- All conventional jointing methods, (i.e. riveting, bolting, welding and adhesives) can be employed.
- High strength to weight ratio is achieved in cold-rolled products.
- They are usually light making it easy to transport and erect.

It is possible to displace the material far away from the neutral axis in order to enhance the load carrying capacity (particularly in beams).

There is almost no limit to the type of cross section that can be formed. Some typical cold formed section profiles are sketched in Fig.1.

In Table 1 hot rolled and cold formed channel section properties having the same area of cross section are shown. From Table 1, it is obvious that thinner the section walls, the larger will be the corresponding moment of inertia values (I_{xx} and I_{yy}) and hence capable of resisting greater bending moments. The consequent reduction in the weight of steel in general applications produces economies both in steel costs as well as in the costs of handling transportation and erection. This, indeed, is one of the main reasons for the popularity and the consequent growth in the use of cold rolled steel. Also cold form steel is protected against corrosion by proper galvanising or powder coating in the factory itself. Usually a thickness limitation is also imposed, for components like lipped channels.

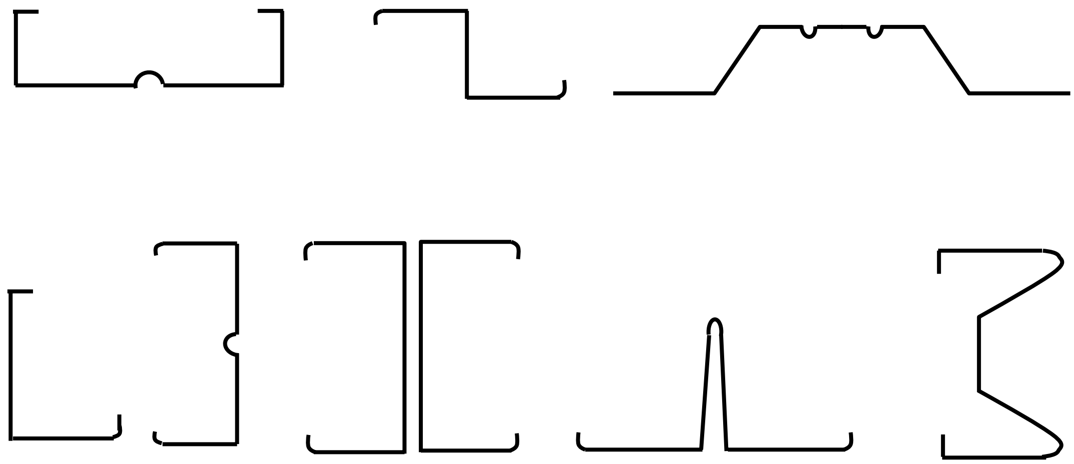
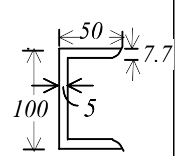
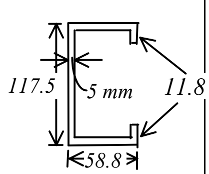
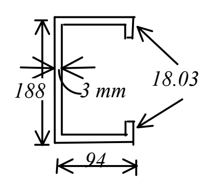
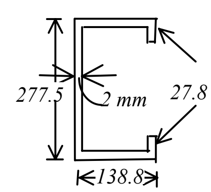


Fig. 1 Typical Cold Formed Steel Profiles

Table - 1 Comparison of Hot Rolled and Cold Rolled sections

	 *Hot rolled channel*	 *Cold rolled channel*	 *Cold rolled channel*	 *Cold rolled channel*
A	$1193\ mm^2$	$1193\ mm^2$	$1193\ mm^2$	$1193\ mm^2$
I_{xx}	$1.9 \times 10^6\ mm^4$	$2.55 \times 10^6\ mm^4$	$6.99 \times 10^6\ mm^4$	$15.53 \times 10^6\ mm^4$
Z_{xx}	$38 \times 10^3\ mm^3$	$43.4 \times 10^3\ mm^3$	$74.3 \times 10^3\ mm^3$	$112 \times 10^3\ mm^3$
I_{yy}	$0.299 \times 10^6\ mm^4$	$0.47 \times 10^6\ mm^4$	$1.39 \times 10^6\ mm^4$	$3.16 \times 10^6\ mm^4$
Z_{yy}	$9.1 \times 10^3\ mm^3$	$11.9 \times 10^3\ mm^3$	$22 \times 10^3\ mm^3$	$33.4 \times 10^3\ mm^3$

While the strength to weight ratios obtained by using thinner material are significantly higher, particular care must be taken to make appropriate design provisions to account for the inevitable buckling problems.

2.1 Types of Stiffened and Unstiffened Elements

As pointed out before, cold formed steel elements are either ***stiffened*** or ***unstiffened***. An element which is supported by webs along both its longitudinal edges is called a ***stiffened*** element. An ***unstiffened*** element is one, which is supported along one longitudinal edge only with the other parallel edge being free to displace. Stiffened and unstiffened elements are shown in Fig. 2.

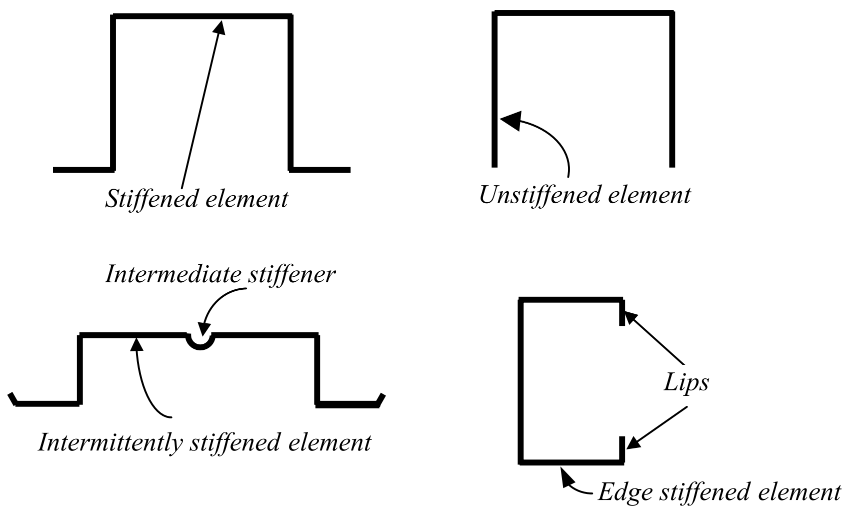


Fig. 2 Stiffened and Unstiffened elements

An ***intermittently stiffened element*** is made of a very wide thin element which has been divided into two or more narrow sub elements by the introduction of intermediate stiffeners, formed during rolling.

In order that a flat compression element be considered as a ***stiffened element***, it should be supported along one longitudinal edge by the web and along the other by a web or lip or other edge stiffener, (eg. a bend) which has sufficient flexural rigidity to maintain straightness of the edge, when the element buckles on loading. A rule of thumb is that the depth of simple "lips" or right angled bends should be at least one-fifth of the adjacent plate width. More exact formulae to assess the adequacy of the stiffeners are provided in Codes of Practice. If the stiffener is adequate, then the edge stiffened element may be treated as having a local buckling coefficient (*K*) value of *4.0*. If the edge stiffener is inadequate (or only partially adequate) its effectiveness is disregarded and the element will be regarded as ***unstiffened***, for purposes of design calculations.

3.0 LOCAL BUCKLING

Local buckling is an extremely important facet of cold formed steel sections on account of the fact that the very thin elements used will invariably buckle before yielding. Thinner the plate, the lower will be the load at which the buckles will form.

3.1 Elastic Buckling of Thin Plates

It has been shown in the chapter on "Introduction to Plate Buckling" that a flat plate simply supported on all edges and loaded in compression (as shown in Fig. 3(a)) will buckle at an elastic critical stress given by

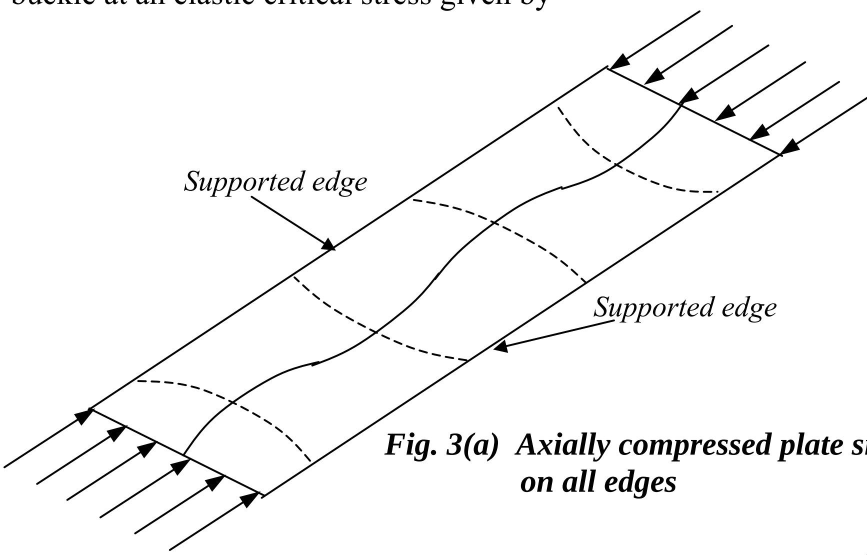


Fig. 3(a) Axially compressed plate simply supported on all edges

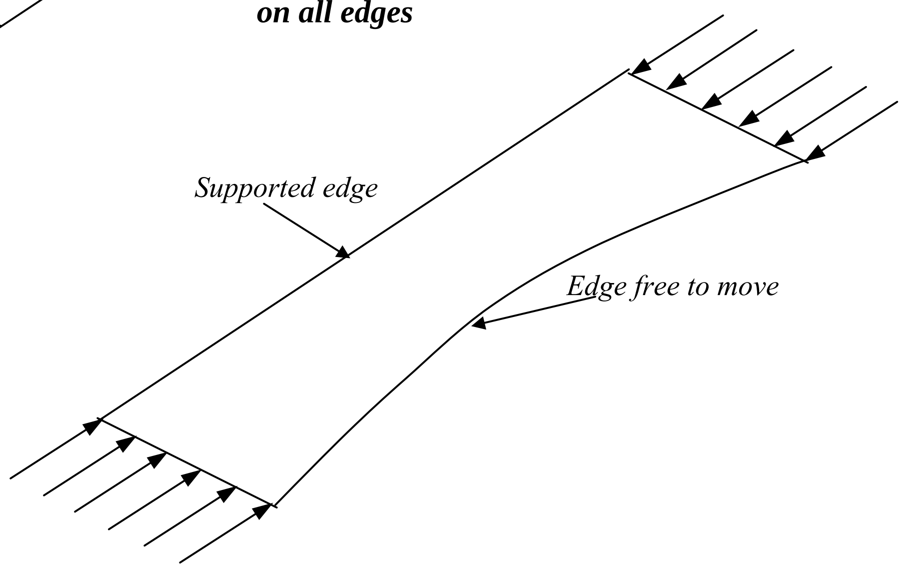


Fig. 3(b) Axially compressed plate with one edge supported and the other edge free to move

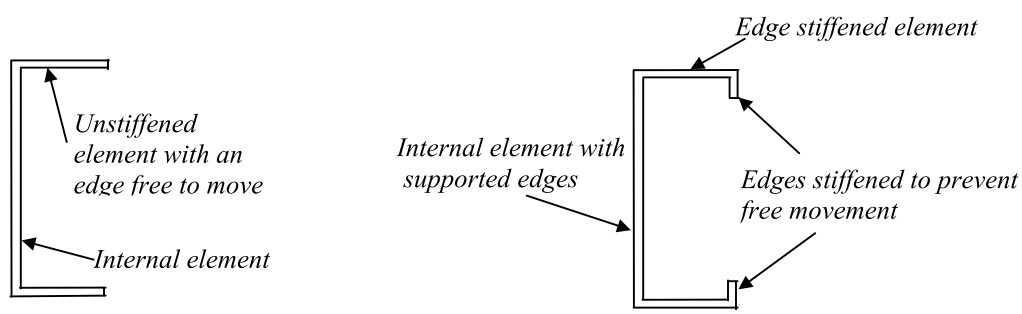
$$p_{cr} = \frac{K\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2 \tag{1}$$

Substituting the values for π, $\nu = 0.3$ and $E = 205\ kN/mm^2$, we obtain the value of p_{cr} as

$$p_{cr} \approx 185\times 10^3 \times K\left(\frac{t}{b}\right)^2 \text{ with units of } N/mm^2 \tag{1a}$$

The value of K is dependent on support conditions. When all the edges are simply supported K has a value of 4.0.

When one of the edges is free to move and the opposite edge is supported, (as shown in Fig. 3b), the plate buckles at a significantly lower load, as K reduces dramatically to 0.425. This shows that plates with free edges do not perform well under local buckling. To counter this difficulty when using cold formed sections, the free edges are provided with a lip so that they will be constrained to remain straight and will not be free to move. This concept of stiffening the elements is illustrated in Fig. 4.



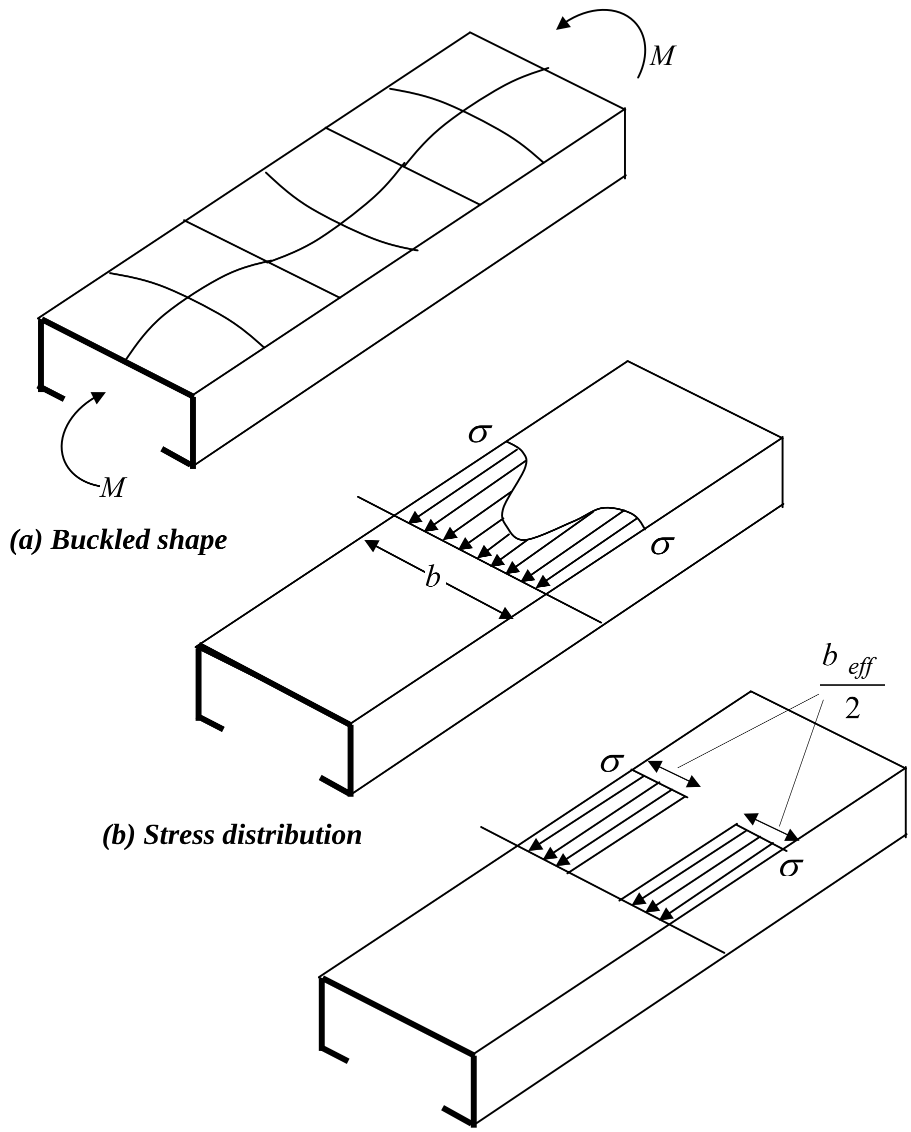
Section with unstiffened element

The same section with stiffened outstands

Fig. 4 The technique of stiffening the element

3.2 Post - critical behaviour

Let us consider the channel subjected to a uniform bending by the application of moments at the ends. The thin plate at the top is under flexural compression and will buckle as shown in Fig. 5 (a). This type of buckling is characterised by ripples along the length of the element. The top plate is supported along the edges and its central portion, which is far from the supports, will deflect and shed the load to the stiffer edges. The regions near the edges are prevented from deflecting to the same extent. The stresses are non uniform across the section as shown in Fig. 5 (b). It is obvious that the applied moment is largely resisted by regions near the edges (i.e. elements which carry increased stresses) while the regions near the centre are only lightly stressed and so are less effective in resisting the applied moment.



(a) Buckled shape

(b) Stress distribution

(c) Effective width

Fig. 5 Local Buckling Effects

From a theoretical stand point, flat plates would buckle instantaneously at the elastic critical load. Under incremental loading, plate elements which are not perfectly flat will begin to deform out of plane from the beginning rather than instantaneously at the onset of buckling and fail at a lower load. This means that a non-uniform state of stress exists throughout the loading regime. The variation of mean stress with lateral deflection for flat plates and plates with initial imperfection, under loading are shown in Fig. 6.

This tendency is predominant in plates having *b/t* (breadth/thickness) ratios of *30-60*. For plates having a *b/t* value in excess of *60*, the in-plane tensile stresses or the "membrane stresses" (generated by the stretching of the plates) resist further buckling and cause an increase in the load-carrying capacity of wide plates.

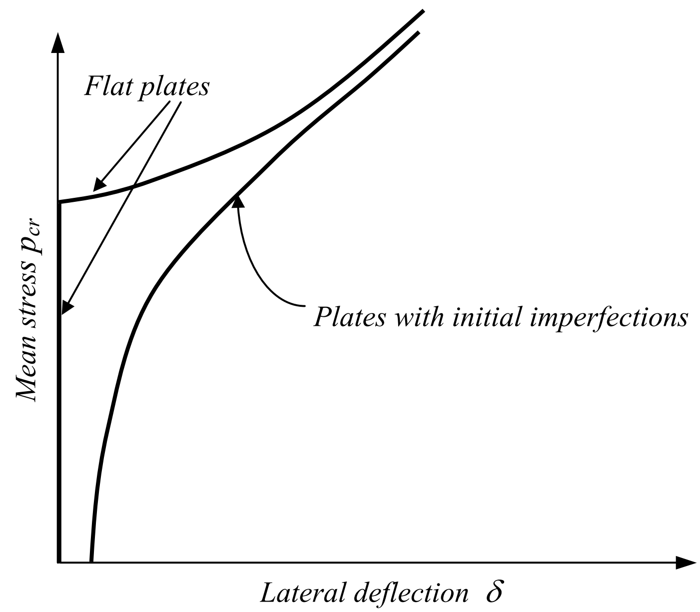


Fig. 6 Mean stress Vs Lateral deflection relation

3.3 Effective Width Concept

The effects of local buckling can be evaluated by using the concept of ***effective width***. Lightly stressed regions at centre are ignored, as these are least effective in resisting the applied stresses. Regions near the supports are far more effective and are taken to be fully effective. The section behaviour is modelled on the basis of the effective width (b_{eff}) sketched in Fig. 5(c).

The effective width, (b_{eff}) multiplied by the edge stress (σ) is the same as the mean stress across the section multiplied by the total width (b) of the compression member.

The ***effective width*** of an element under compression is dependent on the magnitude of the applied stress f_c, the width/thickness ratio of the element and the edge support conditions.

3.4 Code Provisions on "Local Buckling of Compressed Plates"

The effective width concept is usually modified to take into account the effects of yielding and imperfection. For example, BS5950: Part 5 provides a semi-empirical formula for basic effective width, b_{eff}, to conform to extensive experimental data.

When $f_c > 0.123\ p_{cr}$, then

$$\frac{b_{eff}}{b} = \left[1 + 14 \left\{ \left[\frac{f_c}{p_{cr}} \right]^{0.5} - 0.35 \right\}^4 \right]^{-0.2} \qquad (2a)$$

When $f_c < 0.123\, p_{cr}$, then $b_{eff} = b$ (2b)

where

f_c = compressive stress on the effective element, N/mm^2
p_{cr} = local buckling stress given by
$p_{cr} = 185{,}000\, K\, (t/b)^2\ N/mm^2$
K = load buckling coefficient which depends on the element type, section geometry etc.
t = thickness of the element, in *mm*
b = width of the element, in *mm*

The relationship given by eqn. 2 (a) is plotted in Fig.7

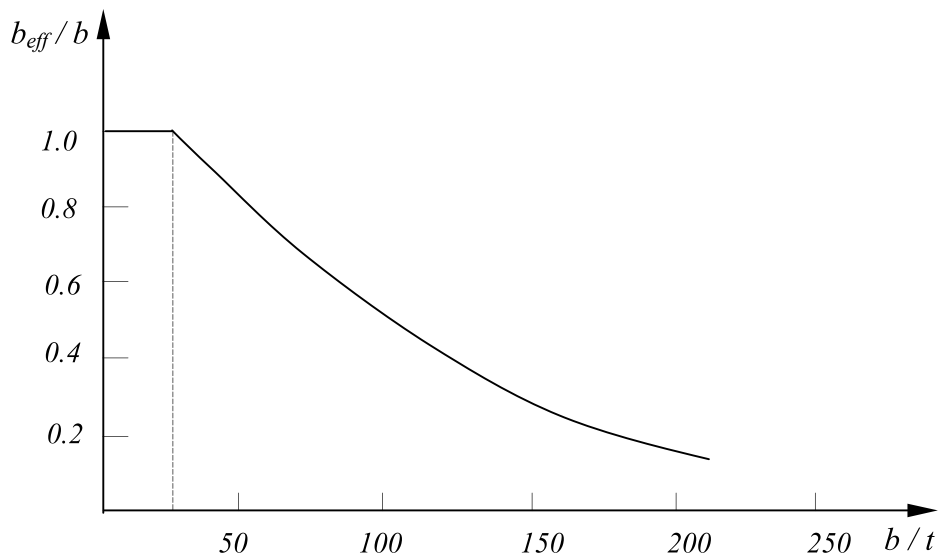


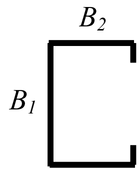
Fig. 7 Ratio of effective width to flat width (f_y = 280 N/mm^2) of compression plate with simple edge supports

It is emphasised that in employing eqn. (2a), the value of K (to compute p_{cr}) could be *4.0* for a stiffened element or *0.425* for an unstiffened element.

BS5950, part 5 provides for a modification for an unstiffened element under uniform compression (Refer clause 4.5.1). The code also provides modifications for elements under combined bending and axial load (ref. Clause 4.5.2). Typical formula given in

BS 5950, Part 5 for computing K values for a channel element is given below for illustration. (see BS 5950, Part 5 for a complete list of buckling coefficients).

1. Lipped channel.



The buckling coefficient K_1 for the member having a width of B_1 in a lipped channel of the type shown above is given by

$$K_1 = 7 - \frac{1.8h}{0.15+h} - 1.43h^3 \qquad (3a)$$

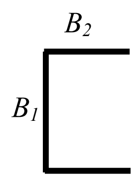
where $h = B_2 / B_1$

For the member having the width of B_2 in the above sketch.

$$K_2 = K_1 h^2 \left(\frac{t_1}{t_2}\right)^2 \qquad (3b)$$

where t_1 and t_2 are the thicknesses of element width B_1 and B_2 respectively. (Note: normally t_1 and t_2 will be equal). The computed values of K_2 should not be less than *4.0* or *0.425* as the case may be.

2) Plain channel (without lips)



The buckling coefficient K_1 for the element of width B_1 is given by

$$K_1 = \frac{2}{\left(1+15h^3\right)^{0.5}} + \frac{2 + 4.8h}{\left(1+15h^3\right)} \qquad (4)$$

K_2 is computed from eqn.. 3(b) given above.

3.4.1 Maximum width to thickness ratios

IS: 801 and *BS 5950, Part 5* limit the maximum ratios of *(b/t)* for compression elements as follows:

- Stiffened elements with one longitudinal edge connected to a flange or web element and the other stiffened by a simple lip — 60
- Stiffened elements with both longitudinal edges connected to other stiffened elements — 500
- Unstiffened compression elements — 60

However the code also warns against the elements developing very large deformations, when *b/t* values exceed half the values tabulated above.

3.5 Treatment of Elements with Stiffeners

3.5.1 Edge Stiffeners

As stated previously, elements having $b/t \le 60$ and provided with simple lip having one fifth of the element width may be regarded as a stiffened element. If $b/t > 60$, then the width required for the lip may become too large and the lip itself may have stability problems. Special types of lips (called "compound" lips) are designed in such cases and these are outside the scope of this chapter.

3.5.2 Intermediate stiffeners

A wide and ineffective element may be transformed into a highly effective element by providing suitable intermediate stiffeners (having a minimum moment of inertia (I_{min}) about an axis through the element mid surface). The required minimum moment of inertia of the stiffener about the axis *0-0* in Fig. 8 is given by:

$$I_{min} = 0.2\, t^4 \cdot \left(\frac{w}{t}\right)^2 \cdot \left(\frac{f_y}{280}\right) \qquad (5)$$

where w = larger flat width of the sub element (see Fig. 8) between stiffeners (in *mm*)
t = thickness of the element (*mm*)
f_y = yield stress (N/mm^2)

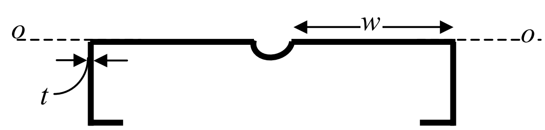


Fig. 8 Intermediate stiffener

If the sub-element width/thickness ratio (*w/t)* does not exceed *60,* the total effective area of the element may be obtained by adding effective areas of the sub-elements to the full areas of stiffeners.

When (*w/t*) is larger than *60,* the effectiveness of the intermediately stiffened elements is somewhat reduced due to shear lag effects. (Refer to BS5950, Part 5, clauses 4.7.2 and 4.7.3) If an element has a number of stiffeners spaced closely ($b/t \le 30$), *and* then generally all the stiffeners and sub elements can be considered to be effective. To avoid introducing complexities at this stage, shear lag effects are not discussed here.

3.6 Effective Section Properties

In the analysis of member behaviour, the effective section properties are determined by using the effective widths of individual elements. As an example, let us consider the compression member *ABCDEF* shown in Fig.9. The effective portions of the member are shown darkened (i.e. *1-B, B-2, 3-C, C-4, 5-D, D-6, 7-E*, and *E-8*). The parts *A-1, 2-3, 4-5, 6-7* and *8-F* are regarded as being ineffective in resisting compression. As a general rule, the portions located close to the supported edges are effective (see Fig. 5c). Note that in the case of compression members, all elements are subject to reductions in width.

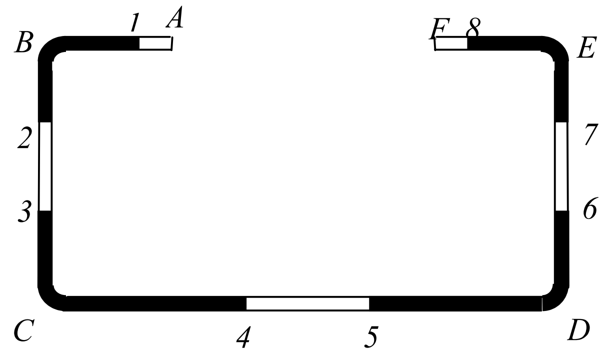


Fig. 9 Effective widths of compression elements

In the case of flexural members, in most cases, only the compression elements are considered to have effective widths. Some typical effective sections of beams are illustrated in Fig.10.

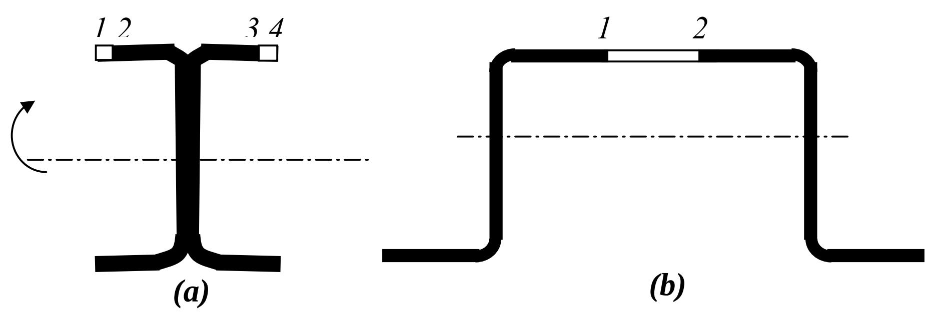


Fig. 10 Effective flexural sections

As in the previous example, fully effective sections in compression elements are darkened in Fig.10. The portions *1-2* and *3-4* in Fig. 10(a) and the portion *1-2* in Fig. 10 (b) are regarded as ineffective in resisting compression. Elements in tension are, of course, not subject to any reduction of width, as the full width will resist tension.

3.7 Proportioning of Stiffeners

The performance of unstiffened elements could be substantially improved by introducing stiffeners (such as a lip). Similarly very wide elements can be divided into two or more narrower sub elements by introducing intermediate stiffeners formed during the rolling process; the sum of the "effective widths" of individual sub elements will enhance the efficiency of the section.
According to BS 5950, Part 5 an unstiffened element (when provided with a lip) can be regarded as a stiffened element, when the lip or the edge stiffener has a moment of inertia about an axis through the plate middle surface equal to or greater than

$$I_{\min} = \frac{b^3 t}{375} \tag{6}$$

where t and b are the thickness and breadth of the full width of the element to be stiffened.

For elements having a full width b less than or equal to $60\,t$, a simple lip of one fifth of the element width (i.e. $b/5$) can be used safely. For lips with $b > 60\,t$, it would be appropriate to design a lip to ensure that the lip itself does not develop instability.

A maximum b/t ratio of 90 is regarded as the upper limit for load bearing edge stiffeners.

The Indian standard IS: 801-1975 prescribes a minimum moment of inertia for the lip given by

$$I_{\min} = 1.83\ t^4 \sqrt{\left(w/t\right)^2 - 281200/F_y} \quad \text{but not less than } 9.2\ t^4 .$$

where I_{min} = minimum allowable moment of inertia of stiffener about its own centroidal axis parallel to the stiffened element in cm^4

w/t = flat width - thickness ratio of the stiffened element.

F_y = Yield stress in kgf/cm^2

For a simple lip bent at right angles to the stiffened element, the required overall depth d_{min} is given by

$$d_{\min} = 2.8t\ \sqrt[6]{\left(w/t\right)^2 - 281200/F_y} \quad \text{but not less than } 4.8\ t$$

Note that both the above equations given by the Indian standard are dependent on the units employed.

3.7.1 Intermediate Stiffeners.

Intermediate stiffeners are used to split a wide element into a series of narrower and therefore more effective elements. The minimum moment of inertia about an axis through the element middle surface required for this purpose (according to BS 5950, Part 5) is given in equation (5) above.

The effective widths of each sub element may be determined according to equation 2 (a) and eqn..2 (b) by replacing the sub element width in place of the element width *b*.

When $w / t < 60$, then the total effective area of the element is obtained as the sum of the effective areas of each sub element to the full areas of stiffeners.

When the sub elements having a larger w / t values are employed ($w/ t > 60$), the performance of intermittently stiffened elements will be less efficient. To model this reduced performance , the sub element effective width must be reduced to b_{er} given by,

$$\frac{b_{er}}{t} = \frac{b_{eff}}{t} - 0.1\left(\frac{w}{t} - 60\right) \tag{7}$$

The effective stiffener areas are also reduced when $w / t > 90$ by employing the equation:

$$A_{eff} = A_{st} \cdot \frac{b_{er}}{w} \tag{8}$$

where A_{st} = the full stiffener area and
A_{eff} = effective stiffener area.

For w / t values between 60 and 90, the effective stiffener area varies between A_{st} and A_{eff} as given below:

$$A_{eff} = A_{st}\left[3 - 2\frac{b_{er}}{w} - \frac{1}{30}\left(1 - \frac{b_{er}}{w}\right)\frac{w}{t}\right] \tag{9}$$

It must be noted that when small increases in the areas of intermediate stiffeners are provided , it is possible to obtain large increases in effectiveness and therefore it is advantageous to use a few intermediate stiffeners , so long as the complete element width does not exceed $500\, t$.

When stiffeners are closely spaced , i.e. $w < 30\, t$, the stiffeners and sub elements may be considered to be fully effective. However there is a tendency for the complete element (along with the stiffeners) to buckle locally. In these circumstances, the complete element is replaced for purposes of analysis by an element of width *b* and having a fictitious thickness t_s given by

$$t_s = \left(\frac{12\ I_s}{b}\right)^{1/3} \tag{10}$$

where I_s = Moment of inertia of the complete element including stiffeneres, about its own neutral axis.

IS: 801- 1975 also suggests some simple rules for the design of intermediate stiffeners.

When the flanges of a flexural member is unusually wide, the width of flange projecting beyond the web is limited to

$$w_f = \sqrt{\frac{126500\,t\,d}{f_{av}}} \times \sqrt[4]{\frac{100\,c_f}{d}} \qquad 10(a)$$

where t = flange thickness
d = depth of beam
c_f = the amount of curling
f_{av} = average stress in kgf/cm^2 as specified in IS: 801 – 1975.

The amount of curling should be decided by the designer but will not generally exceed 5 % of the depth of the section.

Equivalent thickness of intermediate stiffener is given by

$$t_s = \sqrt[3]{\frac{12\,I_s}{w_s}} \qquad 10(b)$$

4.0 BEAMS

As stated previously, the effect of local buckling should invariably be taken into account in thin walled members, using methods described already. Laterally stable beams are beams, which do not buckle laterally. Designs may be carried out using simple beam theory, making suitable modifications to take account of local buckling of the webs. This is done by imposing a maximum compressive stress, which may be considered to act on the bending element. The maximum value of the stress is given by

$$p_0 = \left[1.13 - 0.0019\frac{D}{t}\sqrt{\frac{f_y}{280}}\right] p_y \ngeq f_y \qquad (11)$$

where p_o = the limiting value of compressive stress in N/mm^2
D/t = web depth/thickness ratio
f_y = material yield stress in N/mm^2.
p_y = design strength in N/mm^2
For steel with $f_y = 280$ N/mm^2, $p_0 = f_y$ when $(D/t) \leq 68$.

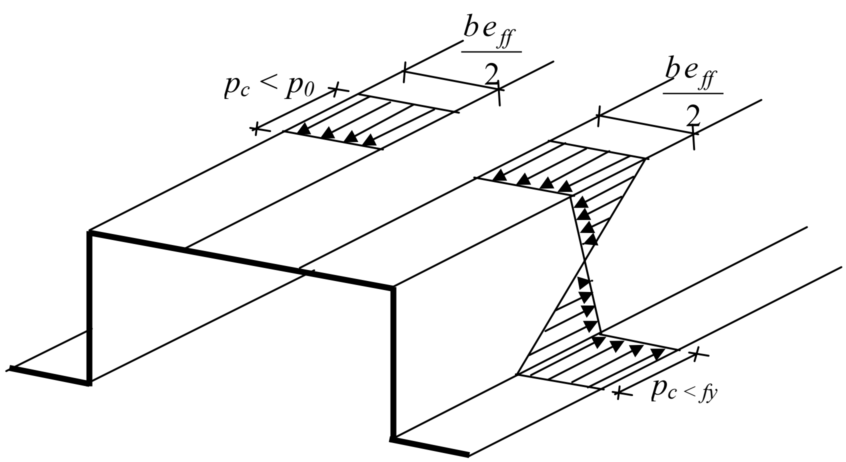
For greater web slenderness values, local web buckling has a detrimental effect. The moment capacity of the cross section is determined by limiting the maximum stress on the web to p_0. The effective width of the compression element is evaluated using this stress and the effective section properties are evaluated. The ultimate moment capacity (M_{ult}) is given by

$$M_{ult} = Z_c . p_0 \qquad (11.a)$$

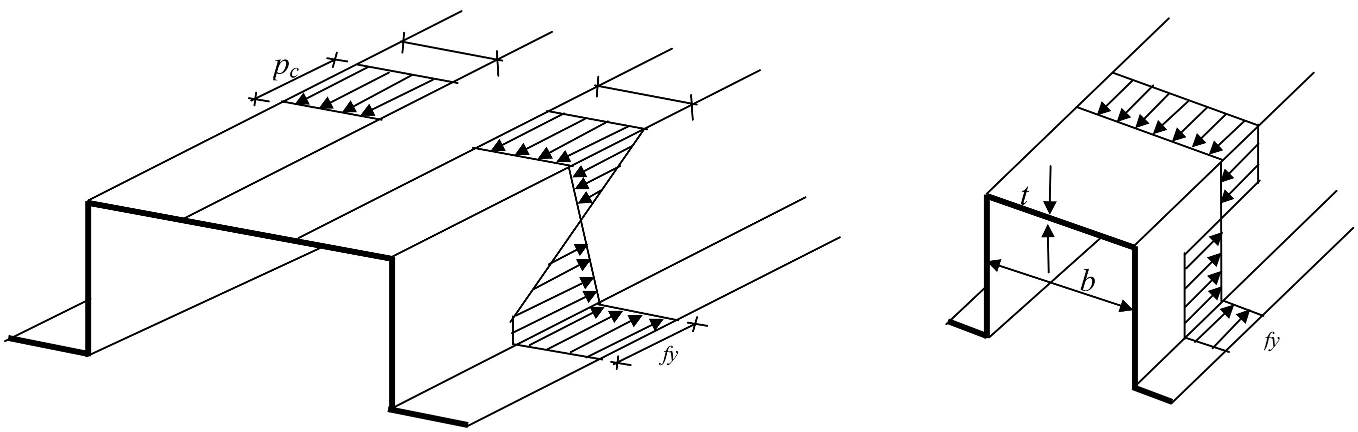
where Z_c = effective compression section modulus

This is subject to the condition that the maximum tensile stress in the section does not exceed f_y (see Fig. 11a).

If the neutral axis is such that the tensile stresses reach yield first, then the moment capacity is to be evaluated on the basis of elasto-plastic stress distribution (see Fig.11b). In elements having low (width/thickness) ratios, compressive stress at collapse can equal yield stress (see Fig. 11c). In order to ensure yielding before local buckling, the maximum (width/thickness) ratio of stiffened elements is $\leq 25\sqrt{\frac{280}{f_y}}$ and for unstiffened elements, it is $\leq 8\sqrt{\frac{280}{f_y}}$



(a) Failure by compression
(Tensile stresses elastic)



(b) Tensile stresses reach yield before failure-
(Elasto plastic stress distribution)

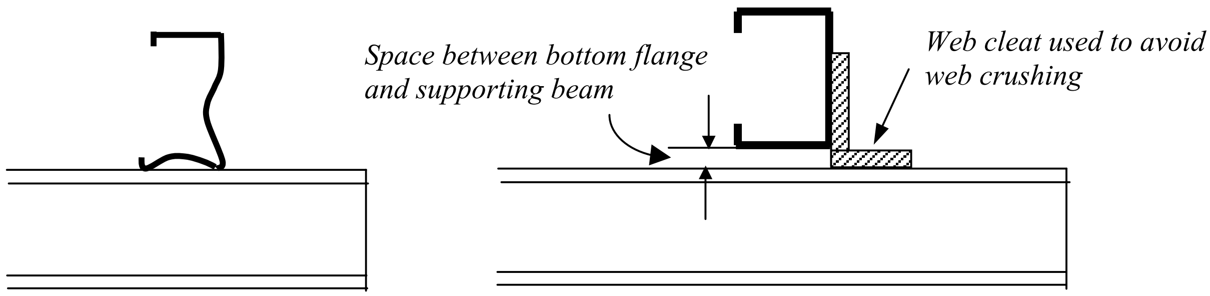
(c) Fully plastic stress distribution
(Thick elements)

Fig. 11 Laterally Stable Beams: Possible stress patterns

4.1 Other Beam Failure Criteria

4.1.1 Web Crushing

This may occur under concentrated loads or at support point when deep slender webs are employed. A widely used method of overcoming web crushing problems is to use web cleats at support points (See Fig.12).



(a) Web crushing *(b) Cleats to avoid web crushing*

Fig.12 Web crushing and how to avoid it

4.1.2 Shear Buckling

The phenomenon of shear buckling of thin webs has been discussed in detail in the chapter on "Plate Girders". Thin webs subjected to predominant shear will buckle as shown in Fig. 13. The maximum shear in a beam web is invariably limited to *0.7* times yield stress in shear. In addition in deep webs, where shear buckling can occur, the average shear stress (p_v) must be less than the value calculated as follows:

$$p_v \le \left(\frac{1000\ t}{D}\right)^2 \tag{12}$$

where p_v = average shear stress in N/mm^2.
t and D are the web thickness and depth respectively (in *mm*)

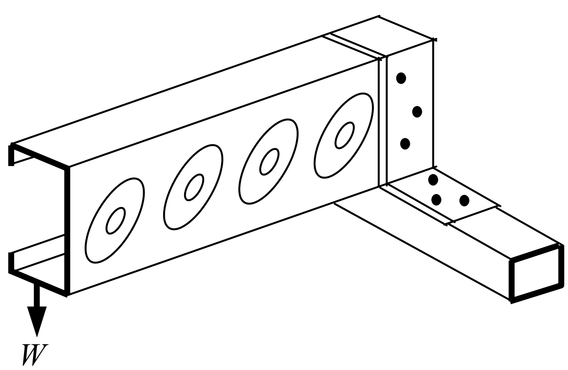


Fig. 13 Web buckling

4.2 Lateral Buckling

The great majority of cold formed beams are (by design) restrained against lateral deflections. This is achieved by connecting them to adjacent elements, roof sheeting or to bracing members. However, there are circumstances where this is not the case and the possibility of lateral buckling has to be considered.

Lateral buckling will not occur if the beam under loading bends only about the minor axis. If the beam is provided with lateral restraints, capable of resisting a lateral force of *3%* of the maximum force in the compression flange, the beam may be regarded as restrained and no lateral buckling will occur.

As described in the chapter on "Unrestrained Beams'", lateral buckling occurs only in "long" beams and is characterised by the beam moving laterally and twisting when a transverse load is applied. This type of buckling is of importance for long beams with low lateral stiffness and low torsional stiffness (See Fig. 14); such beams under loading will bend about the major axis.

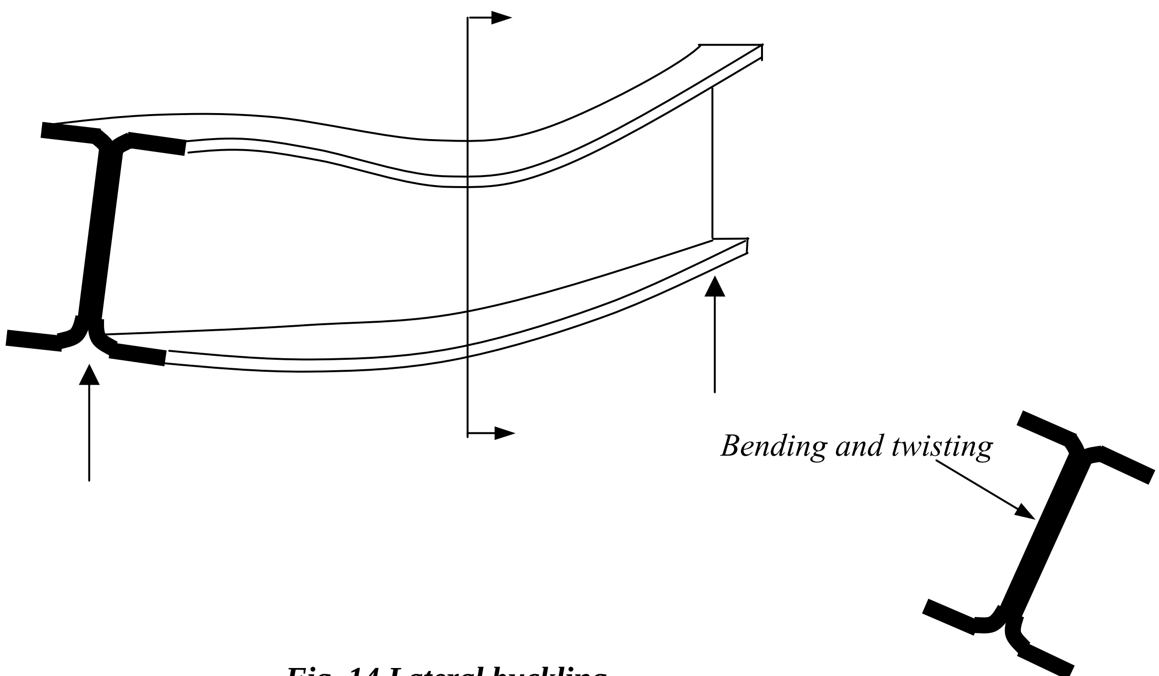


Fig. 14 Lateral buckling

The design approach is based on the "effective length" of the beam for lateral buckling, which is dependent on support and loading conditions. The effective length of beams with both ends supported and having restraints against twisting is taken as *0.9* times the length, provided the load is applied at bottom flange level. If a load is applied to the top flange which is unrestrained laterally, the effective length is increased by *20%.* This is considered to be a "destabilising load", i.e. a load that encourages lateral instability.

The elastic lateral buckling moment capacity is determined next. For an *I* section or symmetrical channel section bent in the plane of the web and loaded through shear centre, this is

$$M_E = \frac{\pi^2 A \cdot E \cdot D}{2\left(\lambda_e / r_y\right)^2} \cdot C_b \sqrt{1 + \frac{1}{20}\left(\frac{\lambda_e}{r_y} \cdot \frac{t}{D}\right)^2} \qquad (13)$$

where,

A = cross sectional area, in mm^2
D = web depth, in mm
t = web thickness, in mm
r_y = radius of gyration for the lateral bending of section
$C_b = 1.75 - 1.05\,\beta + 0.3\,\beta^2 \leq 2.3.$

where β = ratio of the smaller end moment to the larger end moment M in an unbraced length of beam. β is taken positive for single curvature bending and negative for double curvature (see Fig. 15)

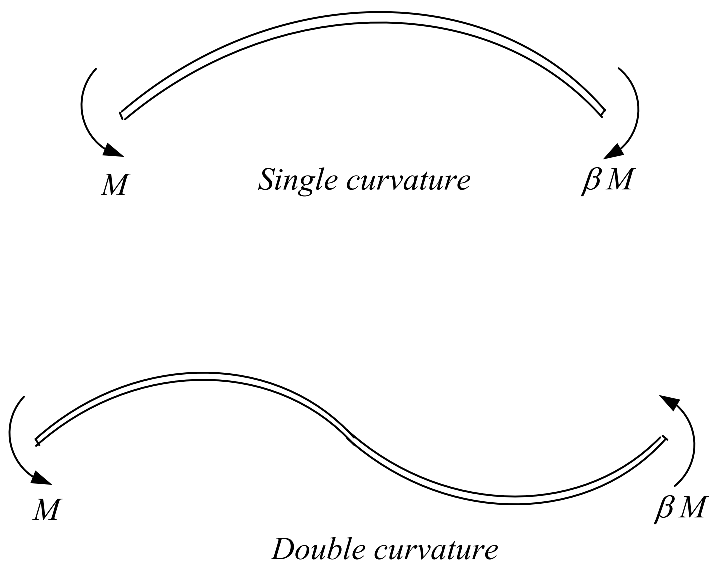


Fig. 15 Single and double curvature bending

To provide for the effects of imperfections, the bending capacity in the plane of loading and other effects, the value of M_E obtained from eq. (13) will need to be modified. The basic concept used is explained in the chapter on Column Buckling where the failure load of a column is obtained by employing the Perry-Robertson equation for evaluating the collapse load of a column from a knowledge of the yield load and Euler buckling load.

A similar Perry-Robertson type equation is employed for evaluating the Moment Resistance of the beam

$$M_b = \frac{1}{2}\left[\left\{M_y + (1+\eta)M_E\right\} - \sqrt{\left[M_y + (1+\eta)M_E\right]^2 - 4M_y \cdot M_E}\right] \tag{14}$$

M_y = First yield moment given by the product of yield stress (f_y) and the Elastic Modulus (Z_c) of the gross section.

M_E = Elastic lateral buckling resistance moment given by equation (13)

η = Perry coefficient, given by

$$When\ \frac{\lambda_e}{r_y} < 40\,C_b,\ \eta = 0.$$

$$When\ \frac{\lambda_e}{r_y} > 40\,C_b,\ \eta = 0.002\left(\frac{\lambda_e}{r_y} - 40\,C_b\right)$$

λ_e = effective length
r_y = radius of gyration of the section about the y - axis.

When the calculated value of M_b exceed M_{ult} calculated by using equation (11.a), then M_b is limited to M_{ult}. This will happen when the beams are "short".

5.0 CONCLUDING REMARKS

In this chapter the difference between cold rolled steel and hot rolled steel has been discussed and the merits of the former are outlined. The concepts of "effective width" and "effective section" employed in the analysis and design of cold rolled section have been explained. The difference between "stiffened" and "unstiffened" elements has been explained. Considerations in the design of cold rolled beams have been explained and formulae employed for the limit state design of beams made of cold rolled sections have been provided.

6.0 REFERENCES

1. BS5950, Part 5: Structural Use of Steelwork in Building, British Standards Institution, London 1987.
2. J. Rhodes and R.M. Lawson "Design of Structures using Cold Formed Steel Sections, SCI Publication 089, The Steel Construction Institute, U.K. 1992.

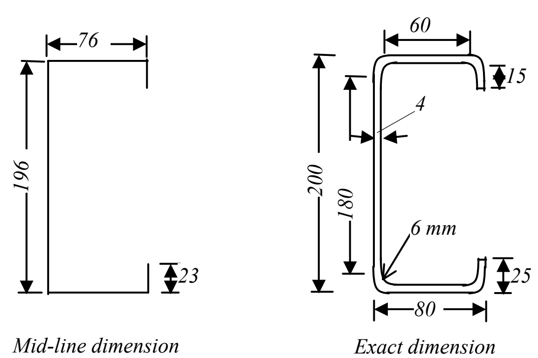
Structural Steel Design Project	Job No.	Sheet *1* of *5*	Rev.
	Job title: ***Section Properties Calculation***		
	Worked Example. ***1***		
CALCULATION SHEET		Made by ***RSP***	Date ***April 2000***
		Checked by ***RN***	Date ***April 2000***

ANALYSIS OF EFFECTIVE SECTION UNDER COMPRESSION

To illustrate the evaluation of reduced section properties of a section under axial compression.

Section : $200 \times 80 \times 25 \times 4.0$ *mm*

Using mid-line dimensions for simplicity. Internal radius of the corners is 1.5t.



Effective breadth of web (flat element)

$$h = B_2 / B_1 = 60/180 = 0.33$$

$$K_1 = 7 - \frac{1.8\ h}{0.15 + h} - 1.43\ h^3$$

$$= 7 - \frac{1.8 \times 0.33}{0.15 + 0.33} - 1.43 \times 0.33^3 = 5.71 \text{ or } 4 \text{ (minimum)}$$

$$= 5.71$$

App. B Fig . 13 BS 5950: Part 5

Structural Steel Design Project	Job No.	Sheet **2** of **5**	Rev.
	Job title: ***Section Properties Calculation***		
	Worked Example. ***1***		
		Made by ***RSP***	Date ***April 2000***
CALCULATION SHEET		Checked by ***RN***	Date ***April 2000***

$$p_{cr} = 185000\, K_1\, (t/b)^2$$

$$= 185000 \times 5.71 \times (4/180)^2 = 521.7\ N/mm^2$$

$$\frac{f_{cr}}{p_{cr} \times \gamma_m} = \frac{240}{521.7 \times 1.15} = 0.4 > 0.123$$

$$\frac{b_{eff}}{b} = \left[1 + 14 \left\{ \sqrt{f_{cr} / (p_{cr} \times \gamma_m)} - 0.35 \right\}^4 \right]^{-0.2}$$

$$= \left[1 + 14 \left\{ \sqrt{0.4} - 0.35 \right\}^4 \right]^{-0.2} = 0.983$$

or $b_{eff} = 0.983 \times 180 = 176.94\ mm$

<u>Effective width of flanges (flat element)</u>

$$K_2 = K_1 h^2 (t_1/t_2)^2$$

$$= K_1 h^2 \qquad (\Theta\, t_1 = t_2)$$

$$= 5.71 \times 0.33^2 = 0.633 \text{ or } 4 \text{ (minimum)}$$

$$= 4$$

$$p_{cr} = 185000 \times 4 \times (4/60)^2 = 3289\ N/mm^2$$

$$\frac{f_c}{p_{cr} \times \gamma_m} = \frac{240}{3289 \times 1.15} = 0.063 > 0.123$$

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$$\therefore \quad \frac{b_{eff}}{b} = 1$$

$b_{eff} = 60\ mm$

<u>Effective width of lips (flat element)</u>

$K = 0.425$ *(conservative for unstiffened elements)*

$$p_{cr} = 185000 \times 0.425 \times (4/15)^2 = 5591\ N/mm^2$$

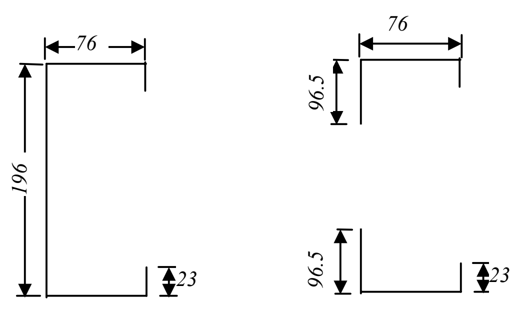
$$\frac{f_c}{p_{cr} \times \gamma_m} = \frac{240}{5591 \times 1.15} = 0.04 < 0.123$$

$$\therefore \quad \frac{b_{eff}}{b} = 1$$

$b_{eff} = 15\ mm$

<u>Effective section in mid-line dimension</u>

As the corners are fully effective, they may be included into the effective width of the flat elements to establish the effective section.



Gross section *Reduced section*

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The calculation for the area of gross section is tabulated below:

	A_i (mm^2)
Lips	$2 \times 23 \times 4 = 184$
Flanges	$2 \times 76 \times 4 = 608$
Web	$196 \times 4 = 784$
Total	1576

The area of the gross section, A $= 1576\ mm^2$

The calculation of the area of the reduced section is tabulated below:

	A_i (mm^2)
Lips	$2 \times 15 \times 4 = 120$
Corners	$4 \times 45.6 = 182.4$
Flanges	$2 \times 60 \times 4 = 480$
Web	$176.94 \times 4 = 707.8$
Total	1490.2

The area of the effective section , $A_{eff} = 1490.2\ mm^2$

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Therefore, the factor defining the effectiveness of the section under compression,

$$Q = \frac{A_{eff}}{A} = \frac{1490}{1576} = 0.95$$

The compressive strength of the member $= Q\,A\,f_y / \gamma_m$

$$= 0.95 \times 1576 \times 240 / 1.15$$

$$= \underline{\ 313\ kN}$$

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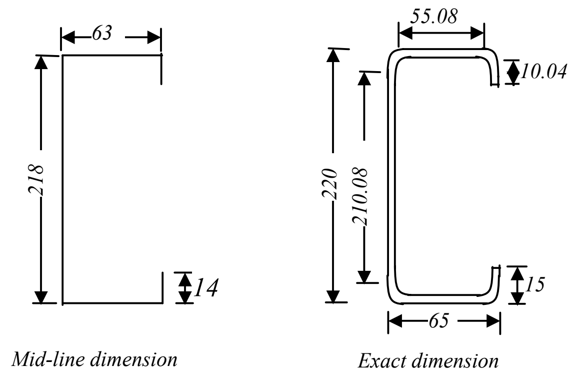
ANALYSIS OF EFFECTIVE SECTION UNDER BENDING

To illustrate the evaluation of the effective section modulus of a section in bending.

We use section : 220 × 65 × 2.0 mm Z28 Generic lipped Channel (from "Building Design using Cold Formed Steel Sections", Worked Examples to BS 5950: Part 5, SCI PUBLICATION P125)

Only the compression flange is subject to local buckling.

Using mid-line dimensions for simplicity. Internal radius of the corners is 1.5t. *Cl.5.2.2.3 BS 5950: Part 5*



Thickness of steel(ignoring galvanizing), t = 2 - 0.04 = 1.96 mm
Intenal radius of the corners = 1.5 × 2 = 3 mm

App. B Fig. 14 BS 5950: Part 5

Limiting stress for stiffened web in bending

$$p_0 = \left\{ 1.13 - 0.0019\,\frac{D}{t}\sqrt{\frac{f_y}{280}} \right\} p_y$$

$$\text{and } p_y = 280 / 1.15 = 243.5 \ N/mm^2$$

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$$p_0 = \left\{ 1.13 - 0.0019 \times \frac{220}{1.96}\sqrt{\frac{280}{280}} \right\} \frac{280}{1.15}$$

$$= 223.2\ N/mm^2$$

which is equal to the maximum stress in the compression flange, i.e.,

$$f_c = 223.2\ N/mm^2$$

<u>Effective width of compression flange</u>

$$h = B_2/B_1 = 210.08/55.08 = 3.8$$

$$K_1 = 5.4 - \frac{1.4\,h}{0.6 + h} - 0.02\,h^3$$

$$= 5.4 - \frac{1.4 \times 3.8}{0.6 + 3.8} - 0.02 \times 3.8^3$$

$$= 3.08 \text{ or } 4\ (\text{minimum}) = 4$$

$$p_{cr} = 185000 \times 4 \times \left(\frac{1.96}{55.08}\right)^2 = 937\ N/mm^2$$

$$\frac{f_c}{p_{cr}} = \frac{223.2}{937} = 0.24 > 0.123$$

$$\frac{b_{eff}}{b} = \left[1 + 14\left\{ \sqrt{f_c/p_{cr}} - 0.35 \right\}^4 \right]^{-0.2}$$

$$= \left[1 + 14\left\{ \sqrt{0.24} - 0.35 \right\}^4 \right]^{-0.2} = 0.998$$

$$b_{eff} = 0.99 \times 55 = 54.5$$

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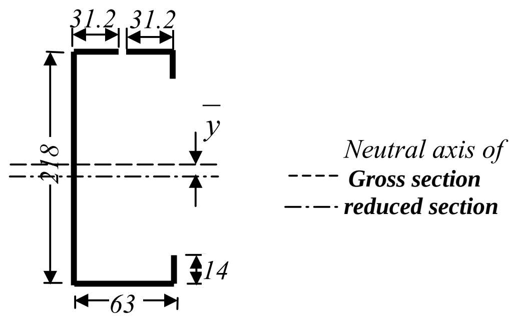
Effective section in mid-line dimension:

The equivalent length of the corners is $2.0 \times 2.0 \quad = \ 4\ mm$

The effective width of the compression flange $= \ 54.5 \ + 2 \times 4 \ = \ 62.5$



The calculation of the effective section modulus is tabulated as below:

Elements	A_i (mm^2)	y_i (mm)	$A_i\, y_i$ (mm^3)	$I_g + A_i\, y_i^2$ (mm^4)
Top lip	27.44	102	2799	448 + 285498
Compression flange	122.5	109	13352.5	39.2 + 1455422.5
Web	427.3	0	0	1692171.2 + 0
Tension flange	123.5	-109	-13459.3	39.5 + 1467064
Bottom lip	27.4	-102	-2799	448 + 285498
Total	728.2		-106.8	5186628.4

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The vertical shift of the neutral axis is

$$\bar{y} = \frac{-106.8}{728.2} = -0.15\ mm$$

The second moment of area of the effective section is

$$I_{xr} = (5186628.4 + 728.2 \times 0.15^2) \times 10^{-4}$$

$$= 518.7\ cm^4 \quad at\ p_0 = 223.2\ N/mm^2$$

$$or \quad = 518.7 \times \frac{223.2 \times 1.15}{280} = 475.5\ cm^4 \quad at\ p_y = 280/1.15\ N/mm^2$$

The effective section modulus is,

$$Z_{xr} = \frac{475.5}{(109 + 0.15)/10} = 43.56\ cm^3$$

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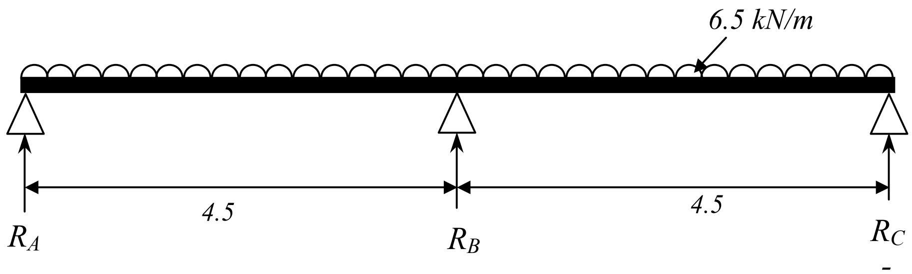
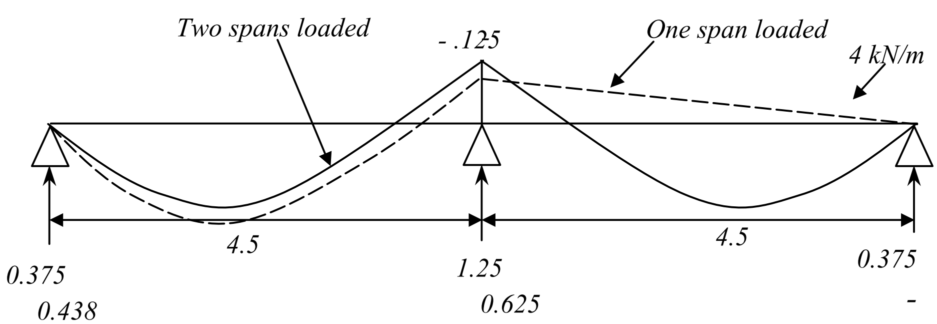
Design a two span continuous beam of span 4.5 m subject to a UDL of 4kN/m as shown in Fig.1.



Factored load on each span $= 6.5 \times 4.5 = 29.3\ kN$

Bending Moment



Coefficients for reactions

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Maximum hogging moment $= 0.125 \times 29.3 \times 4.5 = 16.5$ *kNm*

Maximum sagging moment $= 0.096 \times 29.3 \times 4.5 = 12.7$ *kNm*

Shear Force

Two spans loaded : $R_A = 0.375 \times 29.3 = 11$ *kN*

$R_B = 1.25 \times 29.3 = 36.6$ *kN*

One span loaded : $R_A = 0.438 \times 29.3 = 12.8$ *kN*

$\therefore$ *Maximum reaction at end support,* $F_{w,max} = 12.8$ *kN*

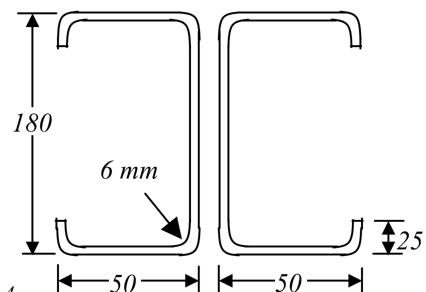
Maximum shear force , $F_{v,max} = 29.3 - 11 = 18.3$ *kN*

Try $180 \times 50 \times 25 \times 4$ *mm Double section (placed back to back)*

Material Properties : $E = 205$ kN/mm^2

$p_y = 240 / 1.15$

$= 208.7$ N/mm^2



Section Properties : $t = 4.0$ *mm*

$D = 180$ *mm*

$r_{yy} = 17.8$ *mm*

$I_{xx} = 2 \times 518 \times 10^4$ mm^4

$Z_{xx} = 115.1 \times 10^3$ mm^3

Only the compression flange is subject to local buckling

Limiting stress for stiffened web in bending

$$p_0 = \left\{ 1.13 - 0.0019 \frac{D}{t} \sqrt{\frac{f_y}{280}} \right\} p_y$$

and $p_y = 240 / 1.15 = 208.7$ N/mm^2

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$$p_0 = \left\{ 1.13 - 0.0019 \times \frac{180}{4} \sqrt{\frac{240}{280}} \right\} \times 208.7$$

$$= 219.3\ N/mm^2$$

which is equal to the maximum stress in the compression flange, i.e.,

$$f_c = 219.3\ N/mm^2$$

Effective width of compression flange

$$h = B_2 / B_1 = 160/30 = 5.3$$

$$K_1 = 5.4 - \frac{1.4\,h}{0.6 + h} - 0.02\,h^3$$

$$= 5.4 - \frac{1.4 \times 5.3}{0.6 + 5.3} - 0.02 \times 5.3^3$$

$$= 1.1 \text{ or } 4 \text{ (minimum)} = 4$$

$$p_{cr} = 185000 \times 4 \times \left(\frac{4}{30}\right)^2 = 13155\ N/mm^2$$

$$\frac{f_c}{p_{cr}} = \frac{219.3}{13155} = 0.017 < 0.123$$

$$\frac{b_{eff}}{b} = 1$$

$$b_{eff} = 30\ mm$$

i.e. the full section is effective in bending.

$$I_{xr} = 2 \times 518 \times 10^4\ mm^4$$
$$Z_{xr} = 115.1 \times 10^3\ mm^3$$

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Moment Resistance

The compression flange is fully restrained over the sagging moment region but it is unrestrained over the hogging moment region, that is, over the internal support. However unrestrained length is very short and lateral torsional buckling is not critical.

The moment resistance of the restrained beam is:

$$M_{cx} = Z_{xr}\, p_y$$

$$= 115.1 \times 10^3 \times (240 / 1.15)\, 10^{-6} = 24 \text{ kNm} > 16.5 \text{ kNm}$$

$\therefore$ *O.K*

Shear Resistance

Shear yield strength, BS 5950: Part 5 Cl. 5.4.3

$$p_v = 0.6\, p_y = 0.6 \times 240 / 1.15 = 125.2 \text{ N/mm}^2$$

Shear buckling strength, $q_{cr} = \left(\dfrac{1000\, t}{D}\right)^2 = \left(\dfrac{1000 \times 4}{180}\right)^2$

$$= 493.8 \text{ N/mm}^2$$

Maximum shear force, $F_{v,max} = 18.3$ *kN*

Shear area $= 180 \times 4 = 720 \text{ mm}^2$

Average shear stress $f_v = \dfrac{18.3 \times 10^3}{720} = 25.4 \text{ N/mm}^2 < q_{cr}$

$\therefore$ *O.K*

Web crushing at end supports

Check the limits of the formulae.

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$$\frac{D}{t} = \frac{180}{4} = 45 \le 200 \qquad \therefore \; O.K$$

$$\frac{r}{t} = \frac{6}{4} = 1.5 \le 6 \qquad \therefore \; O.K$$

At the end supports, the bearing length, N is 50 mm (taking conservatively as the flange width of a single section)

For c=0, $N/t = 50/4 = 12.5$ *and restrained section .*
C is the distance from the end of the beam to the load or reaction.

Use — *Table 8 BS 5950: Part 5*

$$P_w = 2 \times t^2 C_7 \frac{f_y}{\gamma_m} \left\{ 8.8 + 1.11 \sqrt{N/t} \right\}$$

$$C_7 = 1 + \frac{D/t}{750}$$

$$= 1 + \frac{45}{750} = 1.06$$

$$P_w = 2 \times 4^2 \times 1.06 \times \frac{240}{1.15} \left\{ 8.8 + 1.11\sqrt{12.5} \right\} 10^{-3}$$

$$= 89.8\ kN > R_A\ (= 12.8\ kN) \qquad \therefore \; O.K$$

Web Crushing at internal support

At the internal support, the bearing length, N, is 100mm (taken as the flange width of a double section)

Table 8 BS 5950: Part 5

For $c > 1.5D$, $N/t = 100/4 = 25$ *and restrained section.*

$$P_w = t^2 C_5 C_6 \frac{f_y}{\gamma_m} \left\{ 13.2 + 1.63 \sqrt{N/t} \right\}$$

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$$k = \frac{f_y}{228 \times \gamma_m} = \frac{240}{1.15 \times 228} = 0.92$$

$$C_5 = (1.49 - 0.53\,k) = 1.49 - 0.53 \times 0.92 = 1.0 > 0.6$$

$$C_6 = (0.88 - 0.12\,m)$$

$$m = t / 1.9 = 4 / 1.9 = 2.1$$

$$C_6 = 0.88 - 0.12 \times 2.1 = 0.63$$

$$\therefore \quad P_w = 2 \times 4^2 \times 1 \times 0.63 \times \frac{240}{1.15} \left\{13.2 + 1.63\sqrt{25}\right\} 10^{-3}$$

$$= 89.8\ kN > R_B\ (= 36\ kN)$$

<u>Deflection Check</u>

A coefficient of $\frac{3}{384}$ *is used to take in account of unequal loading on a double span. Total unfactored imposed load is used for deflection calculation.*

$$\delta_{\max} = \frac{3}{384} \frac{W L^3}{E I_{av}}$$

$$I_{av} = \frac{I_{xx} + I_{xr}}{2} = \frac{1036 + 1036}{2} = 1036 \times 10^4\ mm^4$$

$$W = 29.3 / 1.5 = 19.5\ kN$$

$$\delta_{\max} = \frac{3}{384} \frac{19.5 \times 10^3 \times 4500^3}{205 \times 10^3 \times 1036 \times 10^4} = 6.53mm$$

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Deflection limit $= L \ / \ 360$ *for imposed load*

$= 4500 / 360 = 12.5 \text{ mm} > 6.53 \text{ mm} \quad \therefore \text{O.K}$

$\therefore$ *In the double span construction :*

Use double section $180 \times 50 \times 25 \times 4.0$ *mm lipped channel placed back to back.*

20 COLD FORMED STEEL SECTIONS- II

1.0 INTRODUCTION

In the last chapter, the special features and attractions of cold formed steel sections for many industrial applications were presented and discussed. In view of the use of very thin steel sheet sections, (generally in the 1 mm - 3 mm range), particular attention has to be paid to buckling of these elements. Stiffened and unstiffened elements were compared and the concept of effective width to deal with the rapid design of compression elements together with suitable design simplifications, outlined. Finally, the methods adopted for the design of laterally restrained beams and unrestrained beams were discussed. The techniques of eliminating lateral buckling in practice, by providing lateral braces or by attachment to floors etc were described so that the compression flanges would not buckle laterally.

In this chapter the design of columns for axial compression, compression combined with bending as well as for torsional-flexural buckling will be discussed. The diversity of cold formed steel shapes and the multiplicity of purposes to which they are put to, makes it a difficult task to provide general solution procedures covering all potential uses. Some design aspects are nevertheless included to provide a general appreciation of this versatile product. It is not unusual to design some cold formed steel sections on the basis of prototype tests or by employing empirical rules. These are also discussed in a summary form herein.

2.0 AXIALLY COMPRESSED COLUMNS

As pointed out in the last chapter, local buckling under compressive loading is an extremely important feature of thin walled sections. It has been shown that a compressed plate element with an edge free to deflect does not perform as satisfactorily when compared with a similar element supported along the two opposite edges. Methods of evaluating the effective widths for both edge support conditions were presented and discussed.

In analysing column behaviour, the first step is to determine the effective area (A_{eff}) of the cross section by summing up the total values of effective areas for all the individual elements.

The ultimate load (or squash load) of a short strut is obtained from

$$P_{cs} = A_{eff} \, . \, f_{yd} \quad = \quad Q . \, A . \, f_{yd} \tag{1}$$

where P_{cs} = ultimate load of a short strut

A_{eff} = sum of the effective areas of all the individual plate elements

Q = the ratio of the effective area to the total area of cross section at yield stress

In a long column with doubly - symmetric cross section, the failure load (P_c) is dependent on Euler buckling resistance (P_{EY}) and the imperfections present. The method of analysis presented here follows the Perry-Robertson approach presented in the chapter on "Introduction to Column Buckling". Following that approach, the failure load is evaluated from

$$P_c = \frac{1}{2}\left\{ \left[P_{cs} + (1+\eta)P_{EY}\right] - \sqrt{\left[P_{cs} + (1+\eta)P_{EY}\right]^2 - 4P_{cs}\cdot P_{EY}} \right\} \tag{2}$$

$$where\ \eta = 0.002\left(\frac{\lambda_e}{r_y} - 20\right), \qquad for\ \frac{\lambda_e}{r_y} > 20 \tag{2a}$$

$$\eta = 0, \qquad for\ \frac{\lambda_e}{r_y} \le 20 \tag{2b}$$

P_{EY} = the minimum buckling load of column = $\dfrac{\pi^2 EI_{min}}{\lambda_e^{\,2}}$

and r_y = radius of gyration corresponding to P_{EY}.

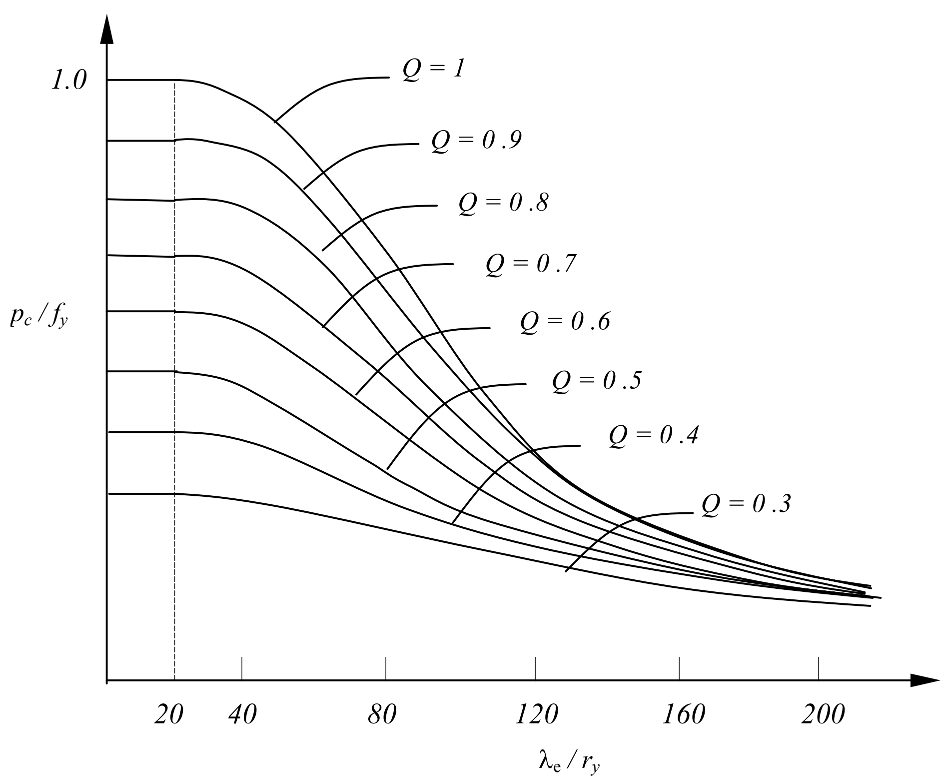
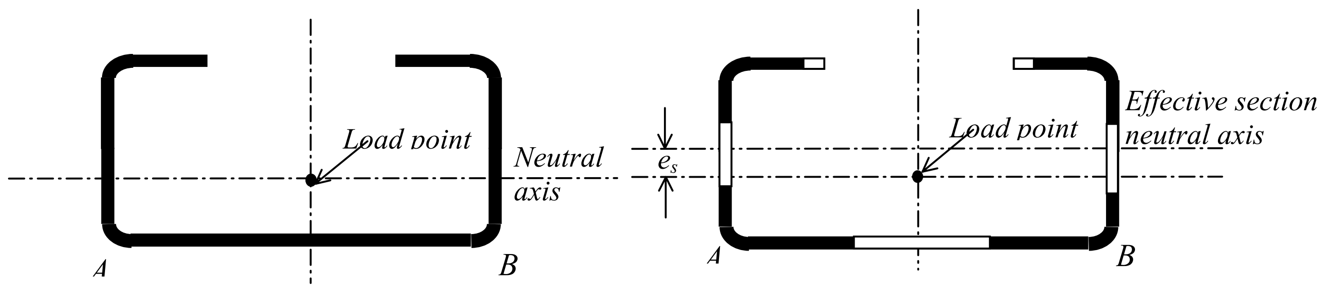


Fig. 1 Column Strength (non- dimensional) for different Q factors

Fig. 1 shows the mean stress at failure ($p_c = P_c$ / cross sectional area) obtained for columns with variation of λ_e / r_y for a number of "Q" factors. (The y-axis is non dimensionalised using the yield stress, f_y and "Q" factor is the ratio of effective cross sectional area to full cross sectional area). Plots such as Fig. 1 can be employed directly for doubly symmetric sections.



(a) Channel section loaded through its centroid

(b) The move of the neutral axis (due to plate buckling) causes an eccentricity e_s and a consequent moment $P. e_s$. This would cause an additional compression on flange AB

Fig. 2 Effective shift in the loading axis in an axially compressed column

2.1 Effective shift of loading axis

If a section is not doubly symmetric (see Fig. 2) and has a large reduction of effective widths of elements, then the effective section may be changed position of centroid. This would induce bending on an ***initially concentrically loaded section***, as shown in Fig. 2. To allow for this behaviour, the movement of effective neutral axis (e_s) from the geometric neutral axis of the cross section must be first determined by comparing the gross and effective section properties. The ultimate load is evaluated by allowing for the interaction of bending and compression using the following equation:

$$P_{ult} = \frac{P_c \cdot M_c}{M_c + P_c . e_s} \tag{3}$$

where P_c is obtained from equation (2) and M_c is the bending resistance of the section for moments acting in the direction corresponding to the movement of neutral axis; e_s is the distance between the effective centroid and actual centroid of the cross section.

2.2 Torsional - flexural buckling

Singly symmetric columns may fail either (a) by Euler buckling about an axis perpendicular to the line of symmetry (as detailed in 2.1 above) or (b) by a combination of bending about the axis of symmetry and a twist as shown in Fig. 3. This latter type of behaviour is known as Torsional-flexural behaviour. Purely torsional and purely flexural failure does not occur in a general case.

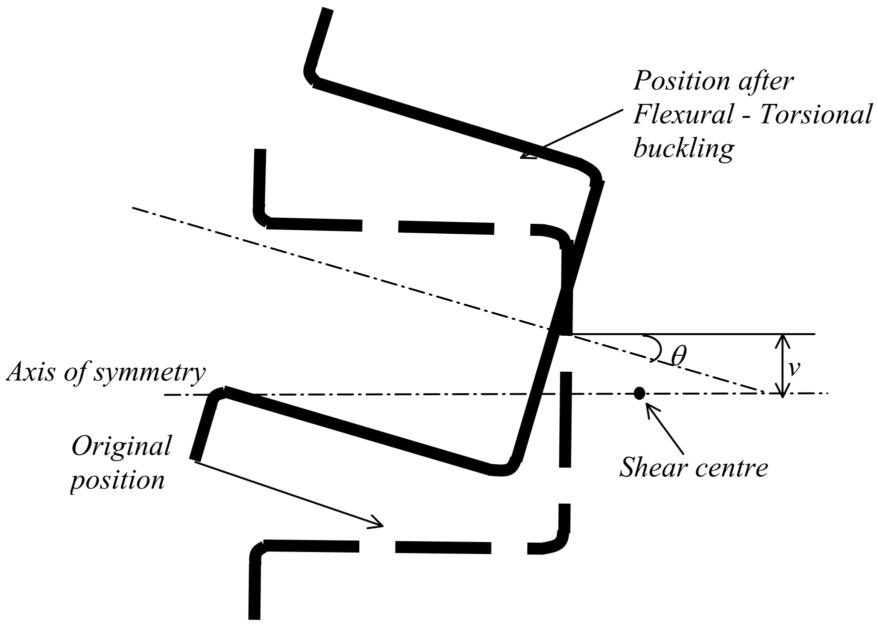


Fig. 3 Column displacements during Flexural - Torsional buckling

Theoretical methods for the analysis of this problem was described in the chapters on Beam Columns. Analysis of torsional-flexural behaviour of cold formed sections is tedious and time consuming for practical design. Codes deal with this problem by simplified design methods or by empirical methods based on experimental data.

As an illustration, the following design procedure, suggested in BS5950, Part 5 is detailed below as being suitable for sections with at least one axis of symmetry (say x - axis) and subjected to flexural torsional buckling.

Effective length multiplication factors (known as α factors) are tabulated for a number of section geometries. These α factors are employed to obtain increased effective lengths, which together with the design analysis prescribed in 2.1 above can be used to obtain torsional buckling resistance of a column.

$$For\ P_{EY} \leq P_{TF}, \quad \alpha = 1$$

$$For\ P_{EY} > P_{TF}, \ \alpha = \sqrt{\frac{P_{EY}}{P_{TF}}} \qquad (4)$$

α values can be computed as follows:

where P_{EY} is the elastic flexural buckling load (in Newtons) for a column about the y-axis, i.e. $\dfrac{\pi^2 EI_y}{\lambda_e^{\ 2}}$

λ_e = effective length (in mm) corresponding to the minimum radius of gyration
P_{TF} = torsional flexural buckling load (in Newtons) of a column given by

$$P_{TF} = \frac{1}{2\beta}\left[\left(P_{EX} + P_T\right) - \left\{\left(P_{EX} + P_T\right)^2 - 4\beta P_{EX} P_T\right\}^{1/2}\right] \tag{5}$$

where P_{EX} = Elastic flexural buckling load of the column (in Newtons) about the x- axis given by $\dfrac{\pi^2 EI_x}{\lambda_e^{\ 2}}$

P_T = Torsional buckling load of a column (In Newtons) given by

$$P_T = \frac{1}{r_0^2}\left(GJ + \frac{2\pi^2 \cdot E\,\Gamma}{\lambda_e^{\ 2}}\right) \tag{6}$$

β is a constant given by $\beta = 1 - \left(\dfrac{x_0}{r_0}\right)^2$ (7)

In these equations,

r_o = polar radius of gyration about the shear centre (in mm) given by

$$r_0 = \left(r_x^2 + r_y^2 + x_0^2\right)^{1/2} \tag{8}$$

where

r_x, r_y are the radii of gyration (in mm) about the x and y- axis
G is the shear modulus (N/mm^2)
x_0 is the distance from shear centre to the centroid measured along the x axis (mm)

J St Venants' Torsion constant (mm^4) which may be taken as $\sum \dfrac{bt^3}{3}$, summed up for all elements, where b = flat width of the element and t = thickness (both of them measure in mm)

I_x the moment of inertia about the x axis (mm^4)
Γ Warping constant for all section.

2.3 Torsion Behaviour

Cold formed sections are mainly formed with "open" sections and do not have high resistance to torsion. Hence the application of load which would cause torsion should be avoided where possible. Generally speaking, by adjusting the method of load application, it is possible to restrain twisting so that torsion does not occur to any significant extent.

In general, when examining torsional behaviour of thin walled sections, the total torsion may be regarded as being made up of two effects:

- St. Venant's Torsion or Pure Torsion
- Warping torsion.

St.Venant's torsion produces shear stresses, which vary linearly through the material thickness. Warping torsion produces in-plane bending of the elements of a cross section, thus inducing direct (i.e. normal) stresses and the angle of twist increases linearly.

Since cold formed sections are thin walled, they have very little resistance to St. Venant's Torsion and will twist substantially. The extent of warping torsion in a thin walled beam is very much dependent on the warping restraint afforded by the supports as well as the loading conditions and the type of section.

If the beam ends are restrained from warping, then short beams exhibit high resistance to warping torsion and the total torque acting on such a beam will be almost completely devoted to overcoming warping resistance, the St Venant's Torsion being negligible. Conversely, the resistance to warping torsion becomes low for long beams and warping stresses and degrees of twist become very large.

A detailed theoretical treatment of beams subject to bending and torsion is given in another chapter. As stated previously, particular care and attention should be paid to the detailing of the connections and the method of load application so that the design for torsion does not pose a serious problem.

3.0 COMBINED BENDING AND COMPRESSION

Compression members which are also subject to bending will have to be designed to take into account the effects of interaction. The following checks are suggested for members which have at least one axis of symmetry: (i) the local capacity at points of greatest bending moment and axial load and (ii) an overall buckling check.

3.1 Local Capacity Check

The local capacity check is ascertained by satisfying the following at the points of greatest bending moment and axial load:

$$\frac{F_c}{P_{cs}} + \frac{M_x}{M_{cx}} + \frac{M_y}{M_{cy}} \leq 1 \qquad (9)$$

F_c = applied axial load
P_{cs} = short strut capacity defined by $A_{eff}.P_{yd}$ (eqn. 1)
M_x, M_y = applied bending moments about x and y axis
M_{cx} = Moment resistance of the beam about x axis in the absence of F_c and M_y
M_{cy} = Moment resistance of the beam about y axis in the absence of F_c and M_x.

3.2 Overall buckling check

For members not subject to lateral buckling, the following relationship should be satisfied:

$$\frac{F_c}{P_c} + \frac{M_x}{C_{bx} \cdot M_{cx}\left(1-\frac{F_c}{P_{EX}}\right)} + \frac{M_y}{C_{by} \cdot M_{cy}\left(1-\frac{F_c}{P_{EY}}\right)} \leq 1 \tag{10}$$

For beams subject to lateral buckling, the following relationship should be satisfied:

$$\frac{F_c}{P_c} + \frac{M_x}{M_b} + \frac{M_y}{C_{by} \cdot M_{cy}\left(1-\frac{F_c}{P_{EY}}\right)} \leq 1 \tag{11}$$

where

P_c = axial buckling resistance in the absence of moments (see eq. 2)
P_{EX}, P_{EY} = flexural buckling load in compression for bending about the x- axis and for bending about the y-axis respectively.
C_{bx}, C_{by} = C_b factors (defined in the previous chapter) with regard to moment variation about x and y axis respectively.
M_b = lateral buckling resistance moment about the x axis defined in the previous chapter.

4.0 TENSION MEMBERS

If a member is connected in such a way as to eliminate any moments due to connection eccentricity, the member may be designed as a simple tension member. Where a member is connected eccentrically to its axis, then the resulting moment has to be allowed for.

The tensile capacity of a member (P_t) may be evaluated from

$$P_t = A_e \; . \; P_y \tag{12}$$

where

A_e is the effective area of the section making due allowance for the type of member (angle, plain channel, Tee section etc) and the type of connection (eg. connected through one leg only or through the flange or web of a T- section).

p_y is design strength (N/mm^2)

Guidance on calculation of A_e is provided in Codes of Practice (eg. BS 5950, Part 5). The area of the tension member should invariably be calculated as its gross area less deductions for holes or openings. (The area to be deducted from the gross sectional area of a member should be the maximum sum of the sectional areas of the holes in any cross section at right angles to the direction of applied stress).

Reference is also made to the chapter on "Tension Members" where provision for enhancement of strength due to strain hardening has been incorporated for hot rolled steel sections. The Indian code IS: 801-1975 is in the process of revision and it is probable that a similar enhancement will be allowed for cold rolled steel sections also.

When a member is subjected to both combined bending and axial tension, the capacity of the member should be ascertained from the following:

$$\frac{F_t}{P_t} + \frac{M_x}{M_{cx}} + \frac{M_y}{M_{cy}} \leq 1 \tag{13}$$

and

$$\frac{M_x}{M_{cx}} \leq 1 \tag{14}$$

and

$$\frac{M_y}{M_{cy}} \leq 1 \tag{15}$$

where F_t = applied load

P_t = tensile capacity (see eqn. 12)

M_x, M_y, M_{cx} and M_{cy} are as defined previously.

5.0 DESIGN ON THE BASIS OF TESTING

While it is possible to design many cold formed steel members on the basis of analysis, the very large variety of shapes that can be formed and the complex interactions that occur make it frequently uneconomical to design members and systems completely on theoretical basis. The behaviour of a component or system can often be ascertained economically by a test and suitable modifications incorporated, where necessary.

Particular care should be taken while testing components, that the tests model the actual loading conditions as closely as possible. For example, while these tests may be used successfully to assess the material work hardening much caution will be needed when examining the effects of local buckling. There is a possibility of these tests giving misleading information or even no information regarding neutral axis movement. The specimen lengths may be too short to pick up certain types of buckling behaviour.

Testing is probably the only realistic method of assessing the strength and characteristics of connections. Evaluating connection behaviour is important as connections play a crucial role in the strength and stiffness of a structure.

In testing complete structures or assemblies, it is vital to ensure that the test set up reflects the in-service conditions as accurately as possible. The method of load application, the type of supports, the restraints from adjacent structures and the flexibility of connections are all factors to be considered carefully and modeled accurately.

Testing by an independent agency (such as Universities) is widely used by manufacturers of mass produced components to ensure consistency of quality. The manufacturers also provide load/span tables for their products, which can be employed by structural designers and architects who do not have detailed knowledge of design procedures. An advantage to the manufacturers in designing on the basis of proof testing is that the load/span tables obtained are generally more advantageous than those obtained by analytical methods; they also reassure the customers about the validity of their load/span tables.

6.0 EMPIRICAL METHODS

Some commonly used members such as Z purlins are sometimes designed by time-tested empirical rules; such rules are employed when theoretical analysis may be impractical or not justified and when prototype test data are not available. (Members designed by proven theoretical methods or by prototype testing need not comply with the empirical rules). As an illustration the empirical rules permitted by BS 5950, Part 5 is explained below.

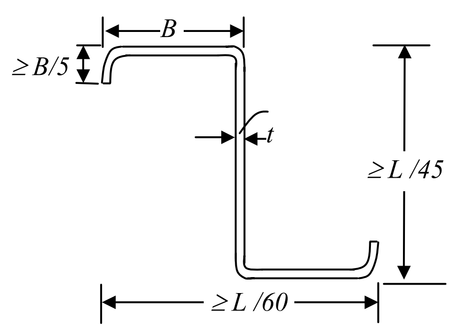


Fig. 4 Z Purlins

6.1 Z Purlins

A Z purlin used for supporting the roofing sheet is sketched in Fig. 4. In designing Z purlins with lips using the simplified empirical rules the following recommendations are to be complied with:

- Unfactored loads should be used for designing purlins
- Imposed loads should be taken to be at least $0.6\ kN/mm^2$
- Claddings and fixings should be checked for adequacy to provide lateral restraint to the purlin and should be capable of carrying the component of load in the plane of the roof slope.
- The purlin should be considered to carry the load normal to roof slope (and a nominal axial load due to wind or restraint forces)
- These rules apply to purlins up to 8 m span in roof slopes up to $22\frac{1}{2}^o$
- Antisag bars should be provided to ensure that laterally unsupported length of the purlin does not exceed 3.8 m. These should be anchored to rigid apex support or their forces should be transferred diagonally to main frames.
- Purlin cleats should provide adequate torsional restraint.

6.2 Design rules

The following design rules apply with reference to Fig. 4

- The overall depth should be greater than $100\ t$ and not less than $L/45$.
- Overall width of compression flange / thickness ratio should be less than 35.
- Lip width should be greater than $B/5$
- Section Modulus $\geq \dfrac{WL}{1400}\ cm^3$ for simply supported purlins

  and $\geq \dfrac{WL}{1800}\ cm^3$ for continuous or semi rigidly jointed purlins.

In the above,

L = span of the purlin (in mm)
W = Normal component of unfactored (distributed dead load+imposed load) in kN
B = Width of the compression flange in mm
T = thickness of the purlin in mm.

- The net allowable wind uplift in a direction normal to roof when purlins are restrained is taken as *50%* of the (dead + imposed) load.

7.0 CONCLUDING REMARKS

In the two preceeding chapters on cold rolled steel, a detailed discussion of design of elements made from it has been provided, the major differences between the hot rolled steel products and cold rolled steel products outlined and the principal advantages of using the latter in construction summarized. Design methods, including methods based on prototype testing and empirical design procedures have been discussed in detail.

Thin steel products are extensively used in building industry in the western world and this range from purlins and lintels to roof sheeting and decking. Light steel frame construction is often employed in house building and is based on industrialized manufacture of standardized components, which ensure a high quality of materials of construction. The most striking benefit of all forms of light steel framing is their speed of construction, ease of handling and savings in site supervision and elimination of wastage in site, all of which contribute to overall economy.

In the Indian context, industrialized methods of production and delivery of cold rolled steel products to site have the potential to build substantially more houses than is otherwise possible, with the same cash flow, thus freeing capital and financial resources for other projects. Other advantages include elimination of shrinkage and movement cracks, greater environmental acceptability and less weather dependency. Properly constructed light steel frames are adaptable to future requirements and will provide high acoustic performance and a high degree of thermal insulation. Provided the sheets are pre-galvanised, the members provide adequate corrosion protection when used close to the boundaries of the building envelope. The design life of these products exceeds 60 years.

8.0 REFERENCES

1. Rhodes, J. and Lawson, R. M., Design of Structures using Cold Formed Steel Sections, The Steel Construction Institute, Ascot, 1992.
2. Chung, K. F Worked examples to BS 5950, Part 5, 1987, The Steel Construction Institute, Ascot, 1993.
3. Wei-Wen Yu , Cold Formed Steel Structures by, Mc Graw Hill Book Company, 1973.

Structural Steel Design Project	Job No.	Sheet ***1*** of ***2***	Rev.
	Job title: ***Column Design***		
	Worked Example. ***1***		
		Made by ***RSP***	Date ***April 2000***
CALCULATION SHEET		Checked by ***RN***	Date ***April 2000***

COLUMN DESIGN

Design a column of length 2.7 m for an axial load of 550 kN.

Axial load $P = 550\ kN$

Length of the column, $L = 2.7\ m$

Effective length, $\lambda_e = 0.85L = 0.85 \times 2.7 = 2.3\ m$

Try $200 \times 80 \times 25 \times 4.0$ *mm Lipped Channel section*

Material Properties:

$$E = 205\ kN/mm^2$$
$$f_y = 240\ N/mm^2$$
$$p_y = 240 / 1.15 = 208.7\ N/mm^2$$

Section Properties:

$$A = 2 \times 1576 = 3152\ mm^2$$
$$I_{xx} = 2 \times 903 \times 10^4\ mm^4$$
$$I_{yy} = 2\,[124 \times 10^4 + 1576 \times 24.8^2\,] = 442 \times 10^4\ mm^4$$
$$r_{min} = \sqrt{\frac{442 \times 10^4}{2 \times 1576}} = 37.4\ mm$$

Cl. 6.2.2 BS 5950: Part 5

Load factor Q $= 0.95$ *(from worked example 1)*

The short strut resistance, $P_{cs} = 0.95 \times 2 \times 1576 \times 240 / 1.15 = 625\ kN$

$\therefore\ P = 550\ kN < 625\ kN$ $\therefore$ *O. K*

Axial buckling resistance

Check for maximum allowable slenderness

$$\frac{\lambda_e}{r_y} = \frac{2.3 \times 10^3}{37.4} = 61.5 < 180$$

$\therefore$ *O. K*

Structural Steel Design Project **CALCULATION SHEET**	Job No.	Sheet ***2*** of ***2***	Rev.
	Job title: ***Column design***		
	Worked Example. ***1***		
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		Checked by ***RN***	Date ***April 2000***

In a double section, torsional flexural buckling is not critical and thus $\alpha = 1$

Modified slenderness ratio,

$$\bar{\lambda} = \frac{\alpha \dfrac{\lambda_e}{r_y}}{\lambda_y}$$

Ref.2 Table 6

$$\lambda_y = \pi\sqrt{\frac{E}{p_y}} = \pi\sqrt{\frac{2.05 \times 10^5}{208.7}} = 98.5$$

$$\therefore \bar{\lambda} = \frac{1 \times 61.5}{98.5} = 0.62$$

$$\frac{P_c}{P_{cs}} = 0.91$$

$$P_c = 0.91 \times 625 = 569\ kN > P \qquad \therefore O.K$$

21 COMPOSITE BEAMS – I

1.0 INTRODUCTION

In conventional composite construction, concrete slabs rest over steel beams and are supported by them. Under load these two components act independently and a relative slip occurs at the interface if there is no connection between them. With the help of a deliberate and appropriate connection provided between the beam and the concrete slab, the slip between them can be eliminated. In this case the steel beam and the slab act as a "***composite beam***" and their action is similar to that of a monolithic Tee beam. Though steel and concrete are the most commonly used materials for composite beams, other materials such as pre-stressed concrete and timber can also be used. Concrete is stronger in compression than in tension, and steel is susceptible to buckling in compression. By the composite action between the two, we can utilise their respective advantages to the fullest extent. Generally in steel-concrete composite beams, steel beams are integrally connected to prefabricated or cast in situ reinforced concrete slabs. There are many advantages associated with steel concrete composite construction. Some of these are listed below:

- The most effective utilisation of steel and concrete is achieved.
- Keeping the span and loading unaltered; a more economical steel section (in terms of depth and weight) is adequate in composite construction compared with conventional non-composite construction.
- As the depth of beam reduces, the construction depth reduces, resulting in enhanced headroom.
- Because of its larger stiffness, composite beams have less deflection than steel beams.
- Composite construction provides efficient arrangement to cover large column free space.
- Composite construction is amenable to "*fast-track*" construction because of using rolled steel and pre-fabricated components, rather than cast-in-situ concrete.
- Encased steel beam sections have improved fire resistance and corrosion.

2.0 ELASTIC BEHAVIOUR OF COMPOSITE BEAMS

The behaviour of composite beams under transverse loading is best illustrated by using two identical beams, each having a cross section of $b \times h$ and spanning a distance of λ, one placed at the top of the other. The beams support a uniformly distributed load of w/unit length as shown in Fig 1. For theoretical explanation, two extreme cases of no interaction and 100% (full) interaction are analysed below:

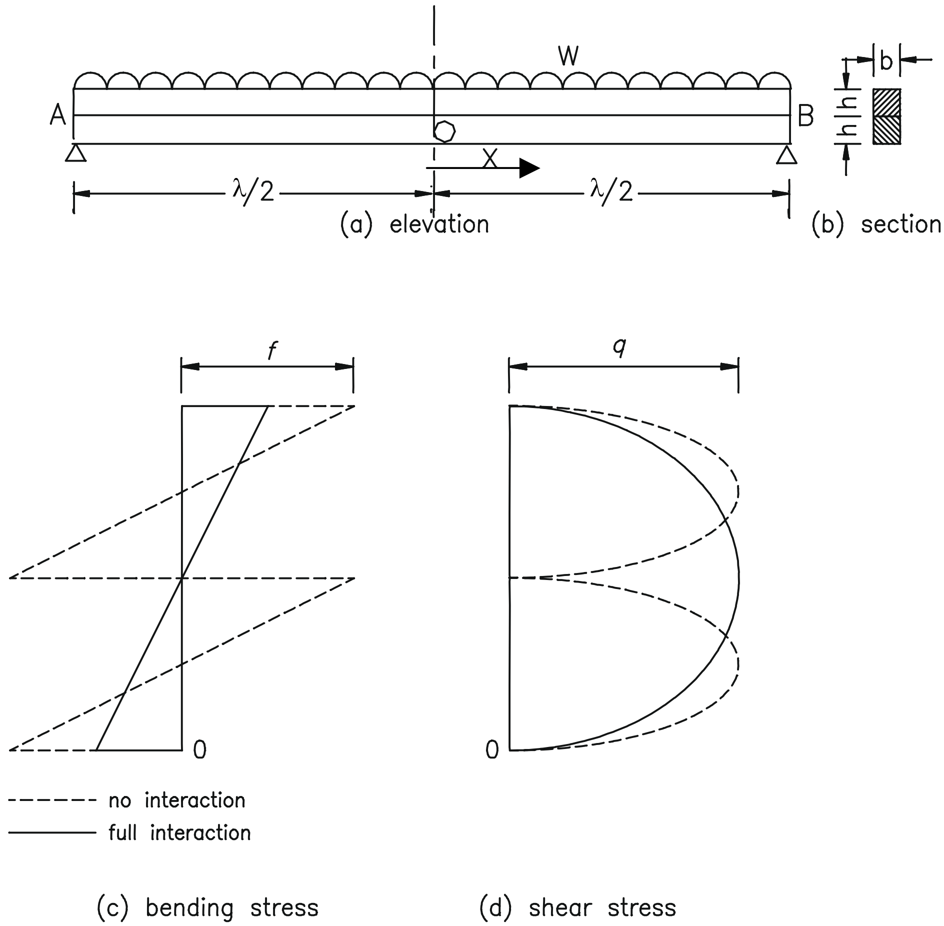


Fig. 1. Effect of shear connection on bending and shear stresses

2.1 No Interaction Case

It is first assumed that there is no shear connection between the beams, so that they are just seated on one another but act independently. The moment of inertia *(I)* of each beam is given by $bh^3/12$. The load carried by each beam is $w/2$ per unit length, with mid span moment of $w\lambda^2/16$ and vertical compressive stress of $w/2b$ at the interface. From elementary beam theory, the maximum bending stress in each beam is given by,

$$f = \frac{My_{\max}}{I} = \frac{3w\lambda^2}{8bh^2} \tag{1}$$

where, M is the maximum bending moment and y_{max} is the distance to the extreme fibre equal to $h/2$.

The maximum shear stress (q_{max}) *that* occurs at the neutral axis of each member near support is given by

$$q_{\max} = \frac{3}{2}\frac{w\lambda}{4}\frac{1}{bh} = \frac{3w\lambda}{8bh} \tag{2}$$

and the maximum deflection is given by

$$\delta = \frac{5(w/2)\lambda^4}{384EI} = \frac{5w\lambda^4}{64Ebh^3} \tag{3}$$

The bending moment in each beam at a distance *x* from mid span is,

$$M_x = w(\lambda^2 - 4x^2)/16 \tag{4}$$

So, the tensile strain at the bottom fibre of the upper beam and the compression stress at the top fibre of the lower beam is,

$$\varepsilon_x = \frac{My_{\max}}{EI} = \frac{3w\,(\lambda^2 - 4x^2)}{8Ebh^2} \tag{5}$$

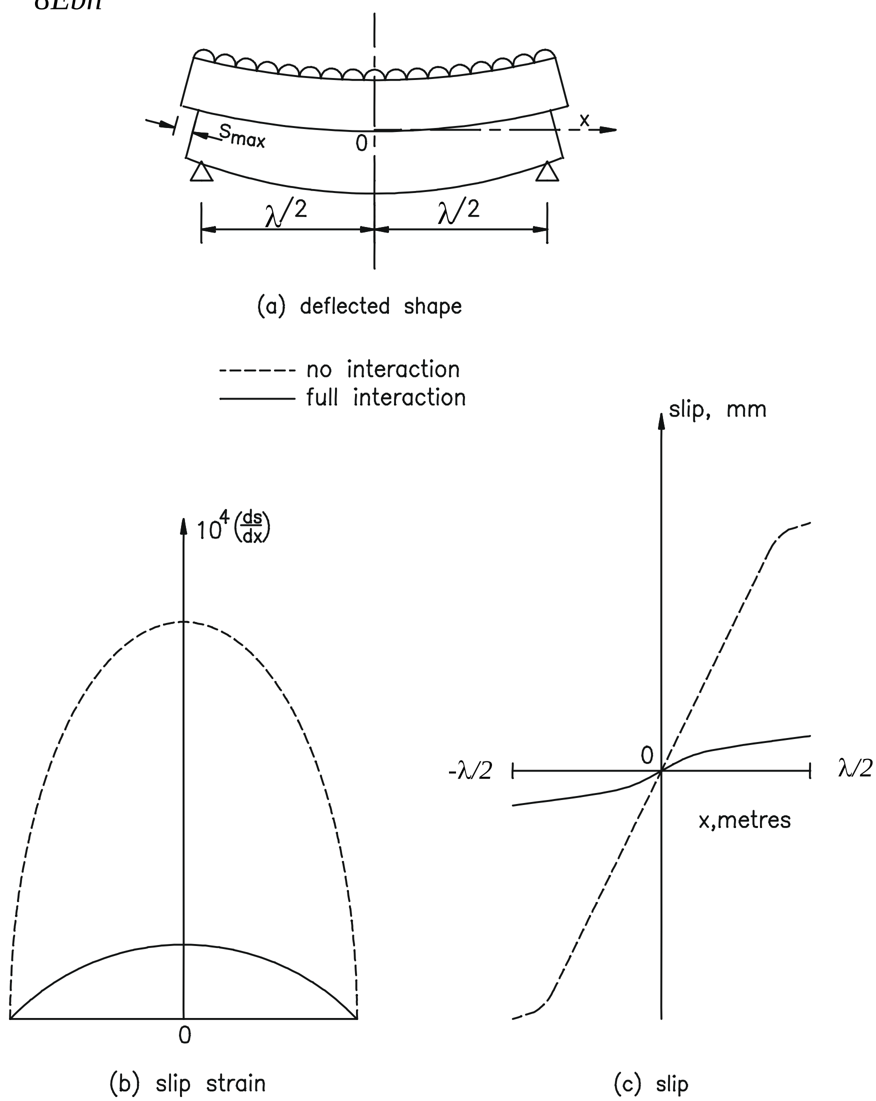


Fig. 2. Typical Deflections, slip strain and slip.

Hence the top fibre of the bottom beam undergoes slip relative to the bottom fibre of the top beam. The slip strain i.e. the relative displacement between adjacent fibres is therefore $2\varepsilon_x$. Denoting slip by *S*, we get,

$$\frac{dS}{dx} = 2\varepsilon_x = \frac{3w\,(\lambda^2 - 4x^2)}{4Ebh^2} \tag{6}$$

Integrating and applying the symmetry boundary condition *S = 0* at *x* = 0 we get the equation

$$S = \frac{w\,(3\lambda^2 x - 4x^3)}{4Ebh^2} \tag{7}$$

The Eqn. (6) and Eqn. (7) show that at $x = 0$, slip strain is maximum whereas the slip is zero, and at $x=\lambda/2$, slip is maximum whereas slip strain is zero. This is illustrated in Fig 2. The maximum slip (i.e. $S_{max} = w\lambda^3/4Ebh^2$) works out to be $3.2h/\lambda$ times the maximum deflection of each beam derived earlier. If $\lambda/(2h)$ of beams is *20*, the slip value obtained is *0.08* times the maximum deflection. This shows that slip is a very small in comparison to deflection of beam. In order to prevent slip between the two beams at the interface and ensure bending strain compatibility shear connectors are frequently used. Since the slip at the interface is small these shear connections, for full composite action, have to be very stiff.

2.2 Full (100%) interaction case

Let us now assume that the beams are joined together by infinitely stiff shear connection along the face *AB* in Fig. 1. As slip and slip strain are now zero everywhere, this case is called *"full interaction"*. In this case the depth of the composite beam is *2h* with a breadth *b*, so that $I = 2bh^3/3$. The mid-span moment is $w\lambda^2/8$. The maximum bending stress is given by

$$f_{\max} = \frac{My_{\max}}{I} = \frac{w\lambda^2}{8}\,\frac{3}{2bh^3}\,h = \frac{3w\lambda^2}{16bh^2} \tag{8}$$

This value is half of the bending stress given by Eqn. (1) for *"no interaction case"*. The maximum shear stress q_{max} remains unaltered but occurs at mid depth. The mid span deflection is

$$\delta = \frac{5w\lambda^4}{256Ebh^3} \tag{9}$$

This value of deflection is one fourth of that of the value obtained from Eqn. (3).

Thus by providing full shear connection between slab and beam, the strength and stiffness of the system can be significantly increased, even though the material consumption is essentially the same.
The shear stress at the interface is

$$V_x = q_x b = \frac{3wx}{4h}$$

where x is measured from the centre of the span. Fig.3 shows the variation of the shear stress. The design of the connectors has to be adequate to sustain the shear stress. In elastic design, connections are provided at varying spacing normally known as *"triangular spacing"*. In this case the spacing works out to be

$$S = \frac{4Ph}{3wx}$$

where, P is the design shear resistance of a connector.

The total shear force in a half of the span is

$$V = \int_0^{\lambda/2} \frac{3wx}{4h}\, dx = \frac{3w\lambda^2}{32h}$$

With a value of $\lambda/(2h) \approx 20$, the total shear in the whole span works out to be

$$2V = 2 \times \frac{3\lambda}{32\,h}\, w\lambda \approx 8w\lambda$$ i.e. eight times the total load carried by the beam.

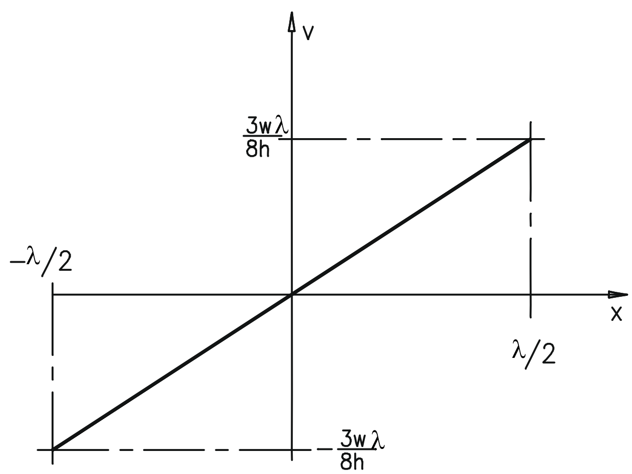


Fig.3. Shear stress variation over span length

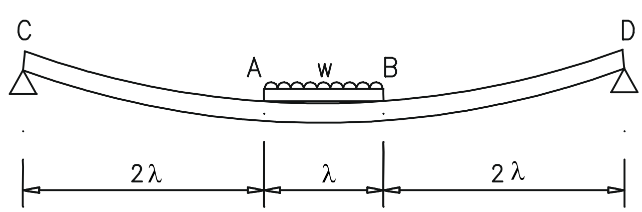


Fig. 4. Uplift forces

2.3 Uplift

Vertical separation between the members occurs, if the loading is applied at the lower edge of the beam. Besides, the torsional stiffness of reinforced concrete slab forming flanges of the composite beam and tri-axial state of stress in the vicinity of shear connector also tend to cause uplift at the interface. Consider a composite beam with partially completed flange or a non-uniform section as in Fig 4. *AB* is supported on *CD*, without any connection between them and carries a uniformly distributed load of magnitude *w*. If the flexural rigidity of *AB* is larger even by *10%* than that of *CD*, the whole load on *AB* is transferred to *CD* at *A* and *B* with a separation of the beams between these two points. If *AB* was connected to *CD*, there will be uplift forces at mid span. This shows that shear connectors are to be designed to give resistance to slip as well as uplift.

3.0 SHEAR CONNECTORS

From the previous example it is also found that the total shear force at the interface between a concrete slab and steel beam is approximately eight times the total load carried by the beam. Therefore, mechanical shear connectors are required at the steel-concrete interface. These connectors are designed to (a) transmit longitudinal shear along the interface, and (b) Prevent separation of steel beam and concrete slab at the interface.

3.1 Types of shear connectors

3.1.1 Rigid type

As the name implies, these connectors are very stiff and they sustain only a small deformation while resisting the shear force. They derive their resistance from bearing pressure on the concrete, and fail due to crushing of concrete. Short bars, angles, T-sections are common examples of this type of connectors. Also anchorage devices like hooped bars are attached with these connectors to prevent vertical separation. This type of connectors is shown in Fig 5(*a*).

3.1.2 Flexible type

Headed studs, channels come under this category. These connectors are welded to the flange of the steel beam. They derive their stress resistance through bending and undergo large deformation before failure. Typical flexible connectors are shown in Fig 5(*b*). The stud connectors are the types used extensively. The shank and the weld collar adjacent to steel beam resist the shear loads whereas the head resists the uplift.

3.1.3 Bond or anchorage type

These connectors derive their resistance through bond and anchorage action. These are shown in Fig 5(*c*). The dimensions of typical shear connectors as per IS: 11384 –1985 are given in Fig 6.

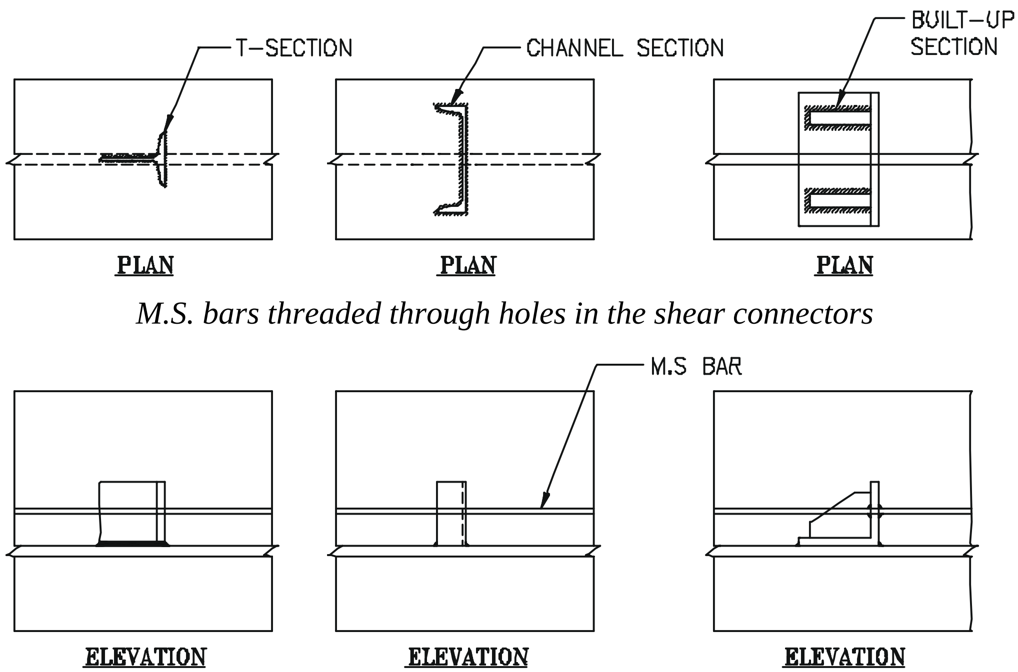


Fig. 5(a). Typical rigid connectors with anchorage device to hold down the concrete slab against uplift

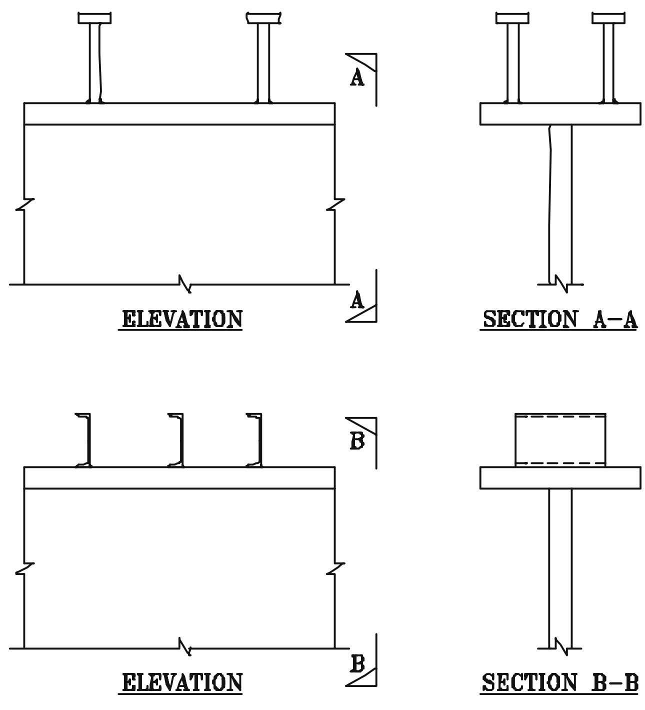
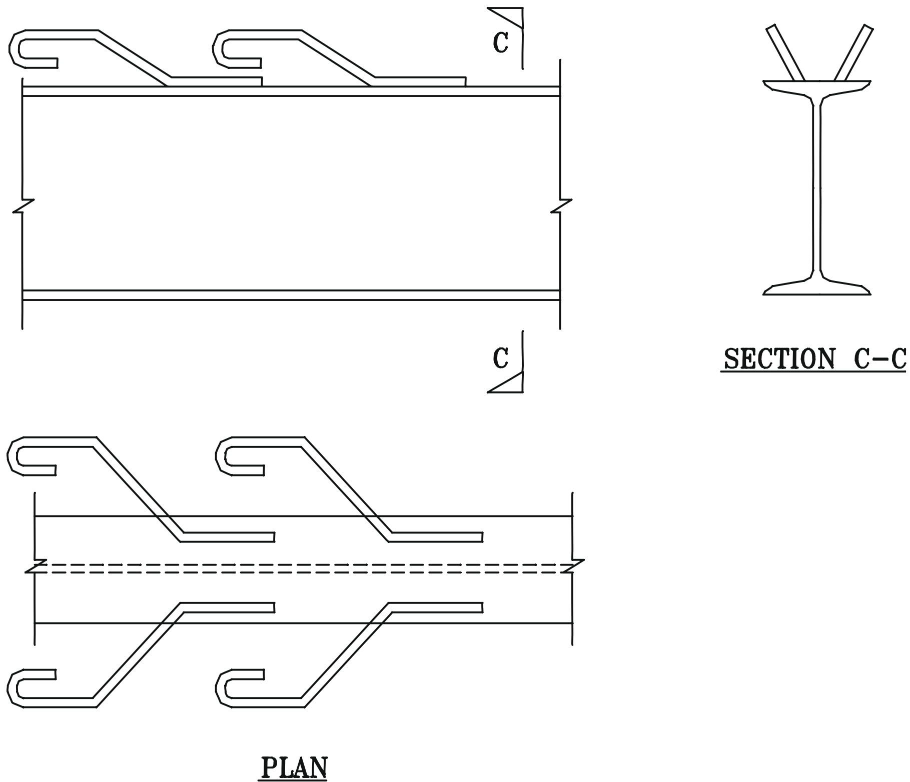
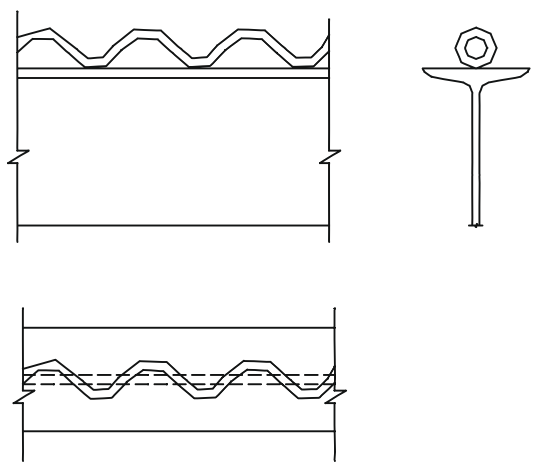


Fig. 5(b). Typical flexible connectors

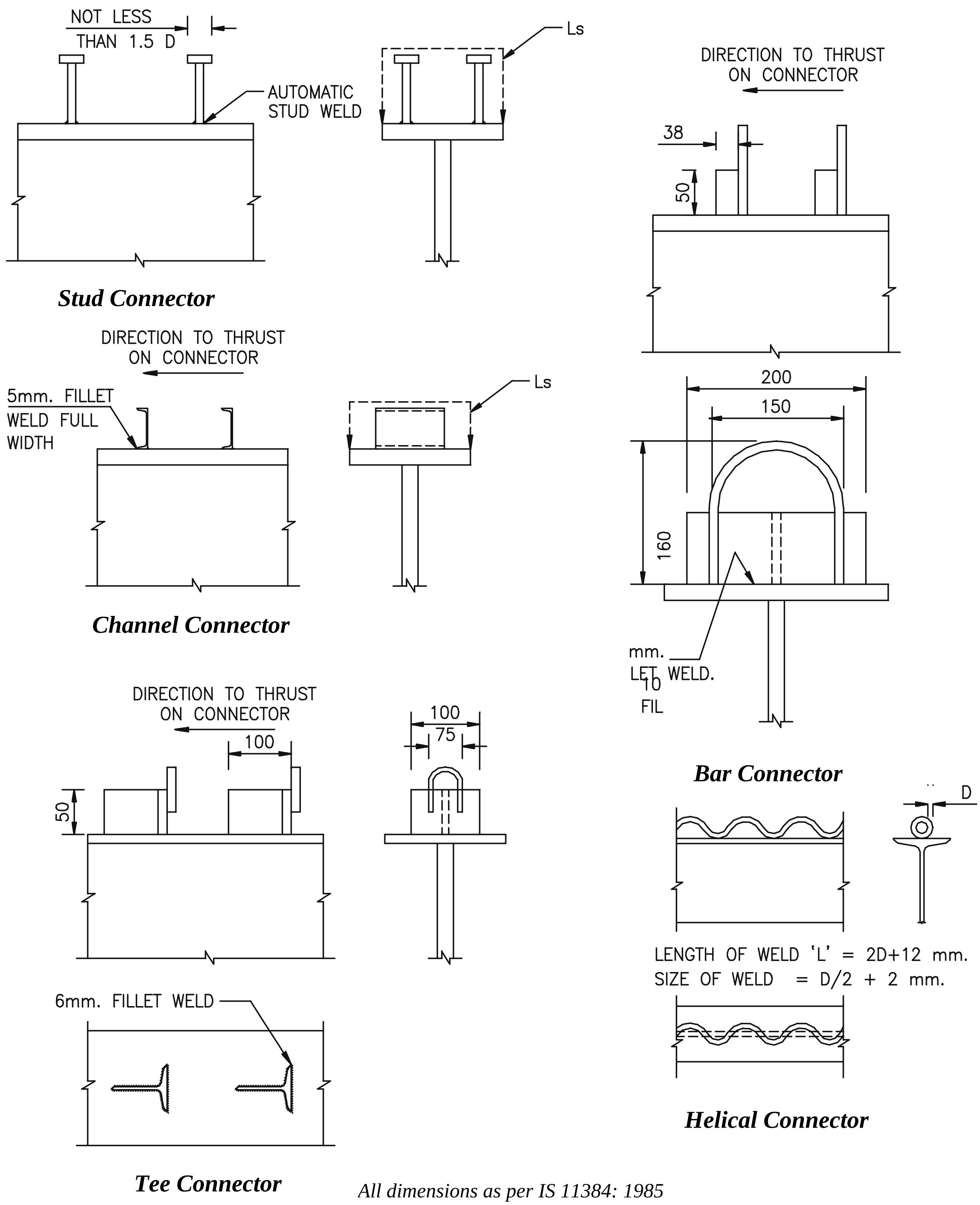


(i). Inclined mild steel bars welded to the top flange of steel unit



(ii). Helical connector

Fig. 5(c). Typical bond or anchorage connectors



All dimensions as per IS 11384: 1985

Fig. 5(d) Typical shear connectors

Fig. 5. Typical rigid connectors with anchorage device to hold down the concrete slab against uplift

3.2 Characteristics of shear connectors

Though in the discussion of full interaction it was assumed that slip was zero everywhere, results of tests have proved that even at the smallest load, slip occurs. This load-slip characteristic of shear connectors affects the design considerably. To obtain the load-slip curve "***push- out***" tests are performed. Arrangements for these tests as per Eurocode 4 and IS: 11384-1985 are shown in Fig. 6(*a*) and 6(*b*) respectively.

In "push-out" test two small slabs are connected to the flanges of an *I* section. The slabs are bedded onto the lower platen of a compression-testing machine and load is applied on the upper end of the steel section. A load-slip curve is obtained by plotting the average slip against the load per connector.

To perform the test, IS:11384-1985 suggests that,

- at the time of testing, the characteristic strength of concrete used n_p / n_f should not exceed the characteristic strength of concrete in the beams for which the test is designed.

- a minimum of three tests should be made and the design values should be taken as *67%* of the lowest ultimate capacity.

Fig 7(*a*) shows trend of some of the results of "***push -out***" tests on different shear connectors. The brittle connectors reach their peak resistance with relatively small slip and then fail suddenly, but the ductile connectors maintain their shear carrying capacity over large displacements. Based on the load slip curve two important parameters can be obtained- the plastic plateau and the connector stiffness *k*. While ultimate strength analysis is based on plastic behaviour of shear connectors, the *'k'* value is required for serviceability analysis and to find slip strain and stresses at partial interaction. In the ultimate analysis it is assumed that concrete slab, steel beam and the dowel are fully stressed, which is known as "*rigid plastic*" condition. In this condition the flexural strength of the section is determined from equilibrium equation. This can be seen from the idealised load-slip characteristics of connectors as in Fig 7(*c*).

Fig 7(*c*) shows an idealised load-slip characteristic of three different types of interaction that can arise depending on the type of connectors used. Note that full interaction occurs when $k=\infty$ represented by an arrow along Y-axis. This occurs when very stiff connectors are used. When there is partial interaction the load slip relationship is assumed to be bilinear. The ultimate capacity is reached at a shear load of D_{max} and only, thereafter slip occurs even without increase in shear load. The stiffness *k* for this partial interaction is assumed to be constant from zero shear loads up to D_{max}.

The dotted lines in the Fig 7(*c*) show the rigid plastic interaction with finite slip (S_{ult}) behaviour. This has a definite plastic plateau and assume $k=\infty$ at occurring at the maximum shear load indicated by D_{max}.

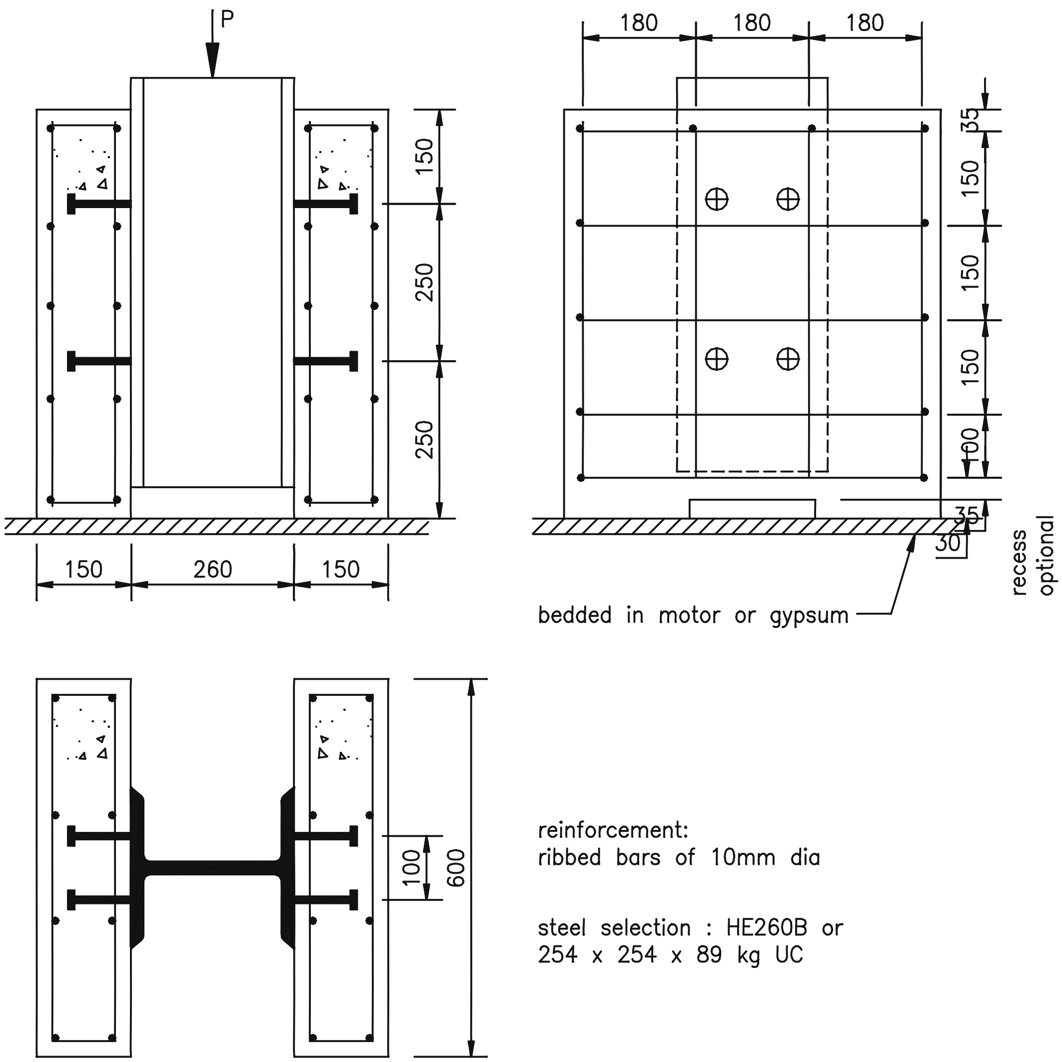


Fig. 6(a). Standard push test *[Eurocode – 4]*

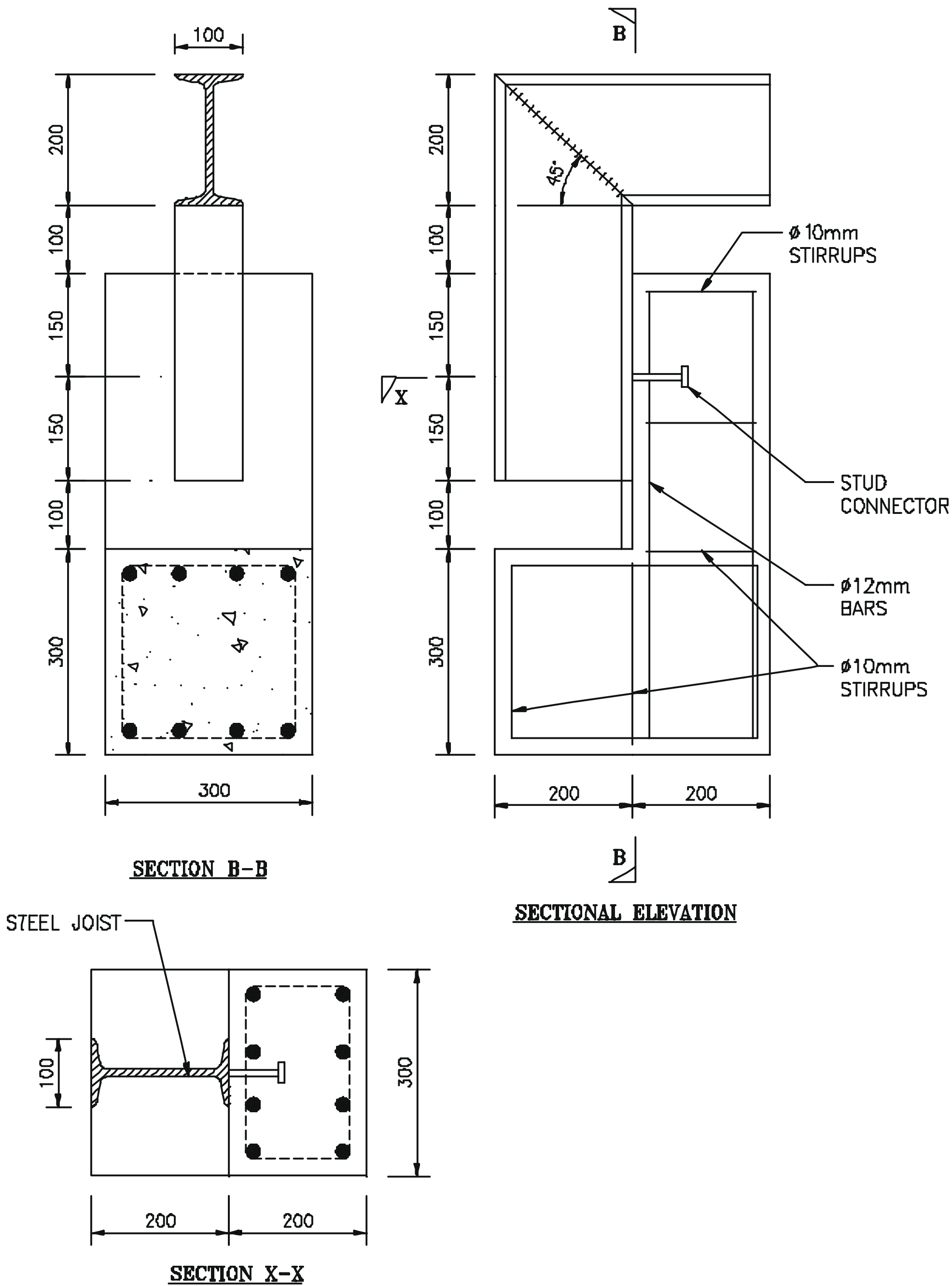


Fig. 6(b) Standard test for shear connectors *(As per IS: 11384-1985)*

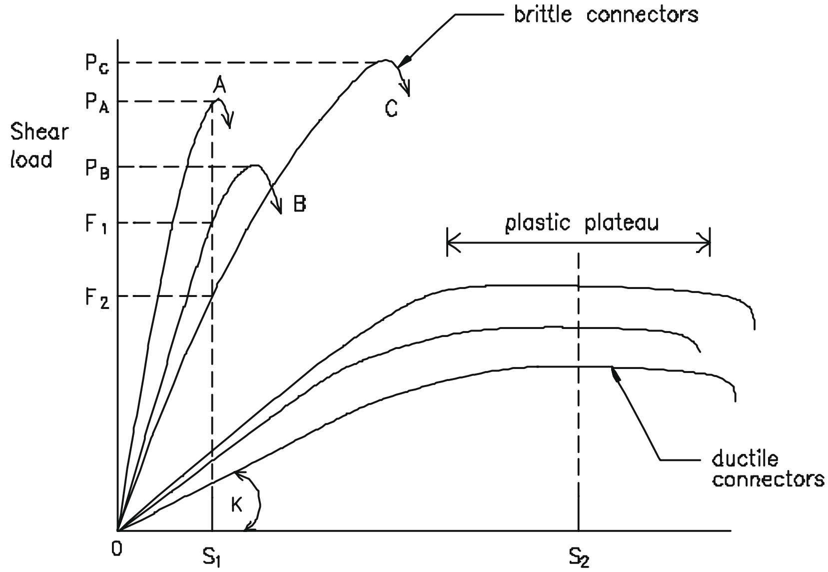


Fig. 7(a). Load/Slip characteristics

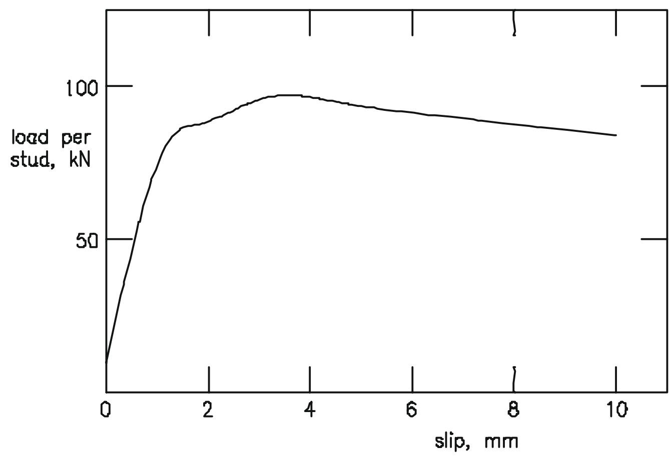


Fig. 7(b). Typical load-slip curve for 19mm stud connectors

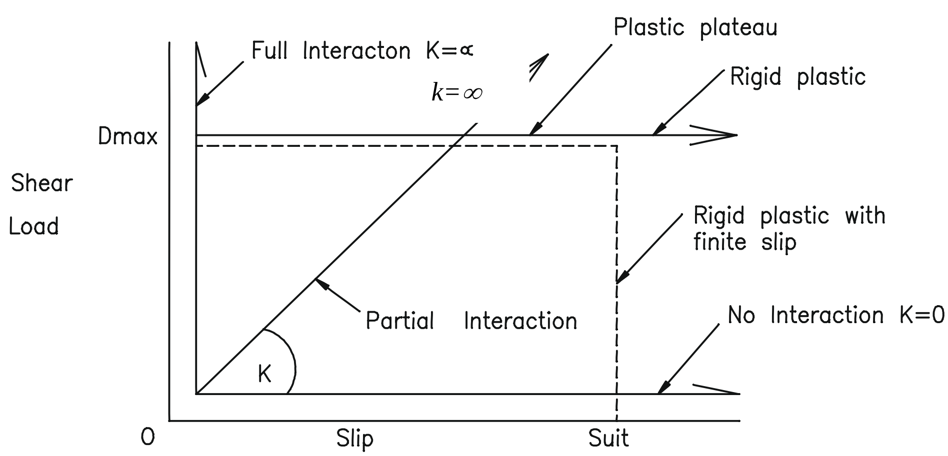


Fig. 7(c). Idealized load-slip characteristics

3.3 Load bearing mechanism of shear connectors

In the course of resisting the shear load, the connectors deform and transfer the load to concrete through bearing. This is illustrated in Fig 8. The dispersion of load can cause tensile cracks in concrete by ripping, shear and splitting action, shown in Fig 9. However the steel dowel may also fail before concrete fails.

Though the transfer of longitudinal shear through mechanical shear connectors is a very complex mechanism, it is shown in an idealised manner in Fig 10. Here the resultant force *F* acts at an eccentricity *'e'* from the interface. It has been found by research that the bearing stress on a shank is concentrated near the base as in Fig 11. Assuming that the force is distributed over a length of *2d* where *d* is the shank diameter, it can be shown that concrete has to withstand a bearing stress of about five times its cube strength. This high strength is possible, because concrete bearing on the connector is confined laterally by the steel element, reinforcement and surrounding concrete. Referring to Fig. 10, we find that, for equilibrium, horizontal shear force as well as a moment is induced at the base of the connector. So, the steel dowel must resist shear as well as flexural forces that cause high tensile stress in the steel failure zone.

However, in a better approach, considering the frictional force between the dowel and the concrete, it has been found that eccentricity *'e'* depends upon E_s/E_c also. As E_s increases, the bearing pressure on dowel becomes more uniform, increasing the eccentricity *'e'*. As a result, flexural force *F.e* increases reducing the dowel strength. On the other hand with decrease in E_s, dowel strength increases.

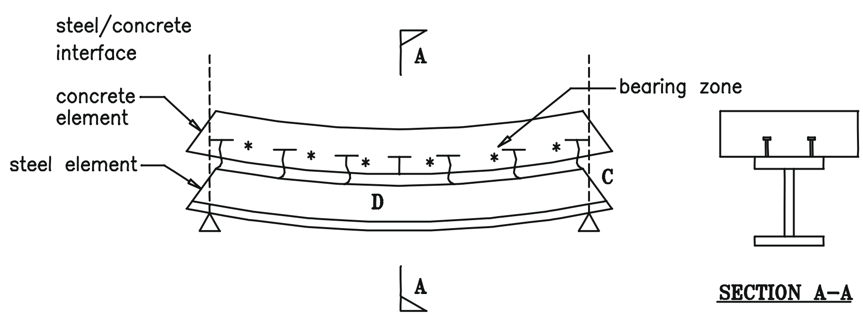


Fig.8 Load Bearing Mechanism

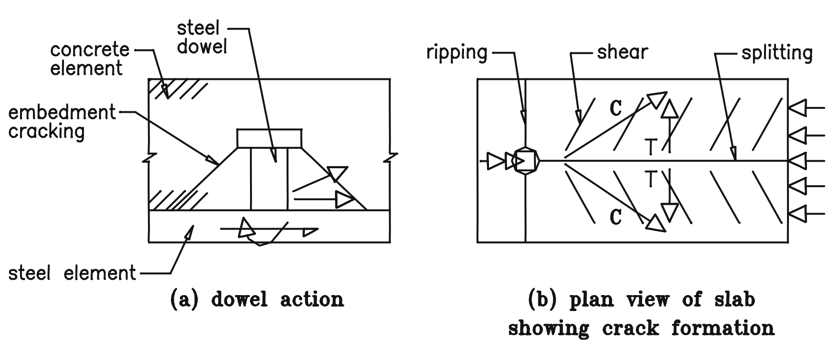


Fig. 11 Bearing stress on the shank of a stud connector

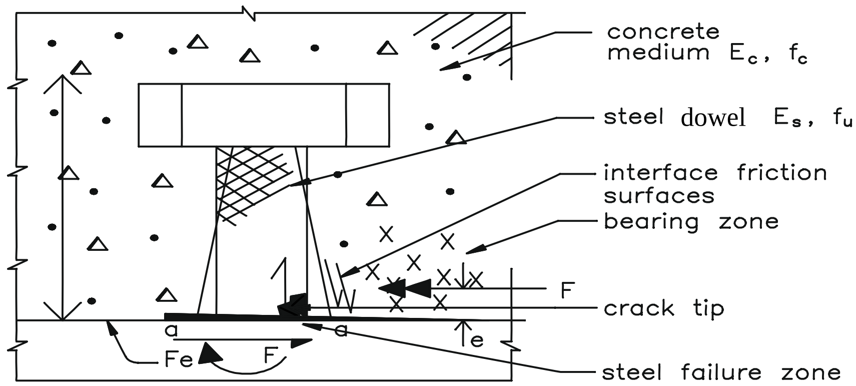


Fig.10. Dowel mechanism of shear studs

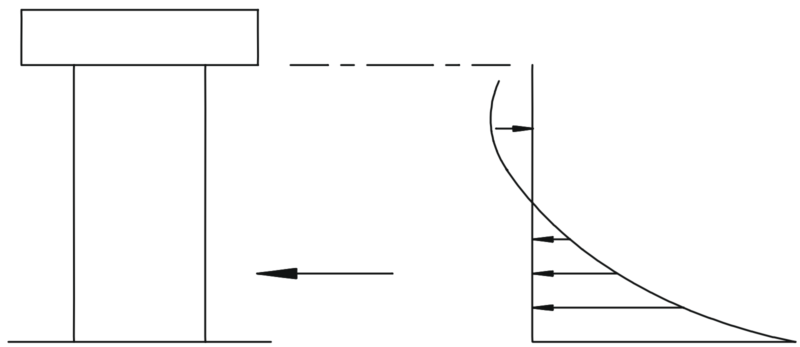


Fig. 11 Bearing stress on the shank of a stud connec

3.4 Strength of connectors

From the above discussion, it can be inferred that dowel strength *(D*) is a function of the following parameters: -

$D = f\,[A_d, f_u, (f_{ck})_{cy}, E_c/E_s]$

where

A_d = cross section area of dowel

f_u = tensile strength of steel

$(f_{ck})_{cy}$ =characteristic compressive (cylinder) strength of concrete

E_c/E_s = ratio of modulus of elasticity of concrete to that of steel.

Table 1: Design Strength of Shear Connectors for Different Concrete Strengths

Type of Connectors		**Connector Material**	**Design Strength of Connectors for Concrete of Grade**		
			M-20	**M-30**	**M-40**
I. Headed stud		IS:961-1975* Fe 540-HT	Load per stud (P_c), kN		
Diameter mm	Height mm				
25	100		86	101	113
22	100		70	85	94
20	100		57	68	75
20	75		49	58	64
16	75		47	49	54
12	62		23	28	31
II. Bar Connector		IS:2261975	Load per bar KN		
50mm x 38 mm x 200mm			318	477	645
III. Channel connector		IS226-1975	Load per channel	(P_c)kN	
125mm x 65mm x 12.7kg x 150mm			184	219	243
100mm x 50mm x 9.2kg x 150 mm			169	204	228
75mm x 40mm x 6.8 kg x 150mm			159	193	218
IV. Tee connector		IS:226-1975	Load per connector (P_c)kN		
100 mm x 100 mm x 10 mm Tee x 50mm			163	193	211
V. Helical connector		IS:226-1975	Load per pitch	(P_c)kN	
Bar diameter mm	Pitch circle diameter mm	IS:226-1975	Load per pitch	(P_c)kN	
20	125		131	154	167
16	125		100	118	96
12	100		70	83	90
10	75		40	48	52

A typical load slip curve for 19 mm-stud connector is shown in Fig. 7(*b*). Eurocode 4 has given the following two empirical formulae to find design resistance of shear studs with $h/d \geq 4$.

$$P_{Rd} = \frac{0.8 f_u (\pi d^2 / 4)}{\gamma_v} \qquad (10)$$

$$P_{Rd} = \frac{0.29\, d^2\, ((f_{ck})_{cy}\, E_{cm})^{1/2}}{\gamma_v} \qquad (11)$$

f_u = ultimate tensile strength of steel *(≤500 N/mm²)*
$(f_{ck})_{cy}$ = cylinder strength of concrete
E_{cm} = mean secant (elastic) modulus of concrete.
γ_v = partial safety factor for stud connector = *1.25*

Equation. (*10*) is based on failure of the shank whereas Equation. (*11*) is based on failure in concrete. The lower of the above two values governs the design.

The design strength of some commonly used shear connectors as per IS:11384-1985 is given in Table (1).

It is to be noted that as per this code the design value of a shear connector is taken as *67%* of the ultimate capacity arrived at by testing.

4.0 ULTIMATE LOAD BEHAVIOUR OF COMPOSITE BEAM

The design procedure of composite beams depends upon the class of the compression flange and web. Table (2) shows the classification of the sections suggested in Eurocode 4 based upon the buckling tendency of steel flange or web. The resistance to buckling is a function of width to thickness ratio of compression members. Table (2) shows that for sections falling in Class 1 & 2 (in EC 4, See Table 2), plastic analysis is recommended. For simply supported composite beams the steel compression flange is restrained from local as well as lateral buckling due to its connection to concrete slab. Moreover, the plastic neutral axis is usually within the slab or the steel flange for full interaction. So, the web is not in compression. This allows the composite section to be analysed using plastic method. Results obtained from plastic analysis have been found to be in close agreement with those obtained from test.

Table 2: Classification of sections, and methods of analysis (according to Eurocode4)

Slenderness class and name	1 plastic	2 compact	3 semi-compact	4 slender
Method of global analysis Analysis of cross-sections	plastic$^{(4)}$ plastic$^{(4)}$	elastic plastic$^{(4)}$	elastic elastic$^{(1)}$	elastic elastic$^{(2)}$
Maximum ratio of *c/t* for flanges of rolled *I*-section: (3)				
Uncased web	8.14	8.95	12.2	no limit
Encased web	8.14	12.2	17.1	no limit

Notes: (1) hole-in-the-web method enables plastic analysis to be used:
(2) with reduced effective width or yield strength

(3) for Grade *50* steel ($f_y = 355\ N/mm^2$): *c* is half the width of a flange of thickness *t;*

(4) Elastic analysis may be used, but is more conservative.

The *assumptions* made for the analyses of the Ultimate Moment Capacity of the section (according to Eurocode 4) are as follows: -

- The tensile strength of concrete is ignored.
- Plane sections of both structural steel and reinforced concrete remain plane after bending.
- The effective area of concrete resists a constant stress of $0.85\ (f_{ck})_{cy}/\gamma_c$ (where $(f_{ck})_{cy}$)=cylinder strength of concrete; and γ_c =partial safety factor for concrete) over the depth between plastic neutral axis and the most compressed fibre of concrete.
- The effective area of steel member is stressed to its design yield strength f_y/γ_a where f_y is the yield strength of steel and γ_a is the material safety factor for steel.

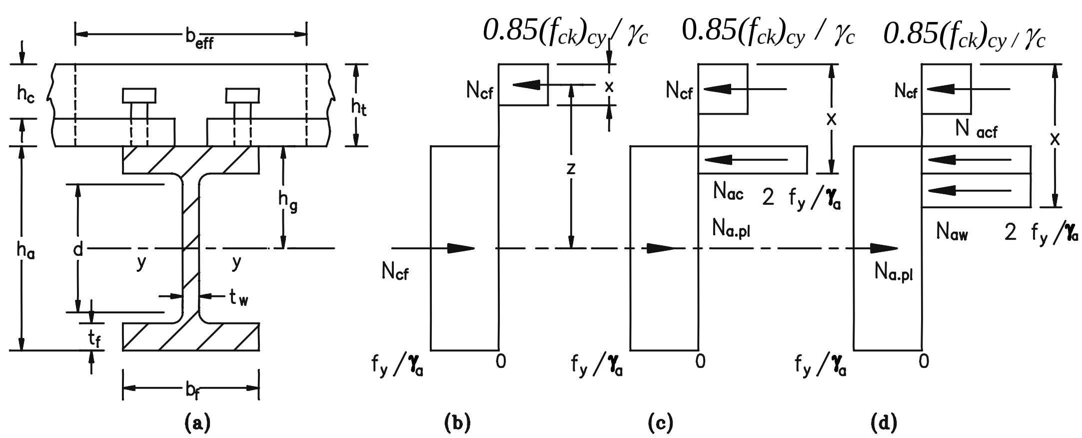


Fig.12. Resistance to sagging bending of composite section in class 1 or 2 for full interaction.

The notations used here are as follows: -

A_a =area of steel section
γ_a = partial safety factor for structural steel
γ_c = partial safety factor for concrete
b_{eff} =effective width of flange of slab
f_y =yield strength of steel
$(f_{ck})_{cy}$ =characteristic (cylinder) compressive strength of concrete
h_c =distance of rib from top of concrete
h_t =total depth of concrete slab
h_g =depth of centre of steel section from top of steel flange

Note: Cylinder strength of concrete $(f_{ck})_{cy}$ is usually taken as *0.8* times the cube strength.

4.1 Full shear connection

Assuming full interaction following three cases may arise.

1) Neutral axis within the concrete slab [see Fig. 12(*b*)].

This occurs when

$$0.85\,\frac{(f_{ck})_{cy}}{\gamma_c}\,b_{eff}\,h_c \geq \frac{A_a f_y}{\gamma_a} \qquad (12)$$

The depth of plastic neutral axis can be found by using force equilibrium.

$$N_{cf} = \frac{A_a f_y}{\gamma_a} = b_{eff}\;x\,\frac{0.85(f_{ck})_{cy}}{\gamma_c} \qquad (13)$$

$$\therefore \quad x = \frac{\dfrac{A_a f_y}{\gamma_a}}{\dfrac{0.85(f_{ck})_{cy}}{\gamma_c}\,b_{eff}} \qquad (14)$$

This expression is valid for $x \leq h_c$.

The plastic moment of resistance of the section,

$$M_p = \frac{A_a f_y}{\gamma_a}\,(h_g + h_t - x/2) \qquad (15)$$

2) Neutral axis within the steel top flange [see Fig.12(*c*)]

This case arises when

$N_{cf} < N_{a \cdot pl}$

$$i.e. \quad b_{eff}\,h_c\,\frac{0.85(f_{ck})_{cy}}{\gamma_c} < \frac{A_a\,f_y}{\gamma_a} \qquad (16)$$

To simplify the calculation it is assumed that strength of steel in compression is $2\,f_y/\gamma_a$, so that, the force $N_{a \cdot pl}$ and its line of action remain unchanged. Note that the compression flange is assumed to have a tensile stress of f_y/γ_a and a compressive stress of $2f_y/\gamma_a$, giving a net compressive stress of f_y/γ_a.

So, the plastic neutral axis will be within steel flange if

$N_{a \cdot pl} - N_{cf} \leq \; 2\, b_f\, t_f\, f_y/\gamma_a$

Equating tensile force with compressive,

$N_{a \cdot pl} = N_{cf} \; + N_{ac}$

i.e. $$\frac{A_a f_y}{\gamma_a} = \frac{0.85 (f_{ck})_{cy}}{\gamma_c} b_{eff} h_c + 2 b_f \,(x - h_t\,) \frac{f_y}{\gamma_a} \qquad (17)$$

The value of x is found from the above expression.
The plastic moment of resistance is found from

$$M_p = N_{a.pl}\,(h_g + h_t - h_c/2) - N_{ac}\,(x - h_c + h_t)/2 \qquad (18)$$

3) The neutral axis lies within web (see Figure 12d).

If the value of x exceeds $(h_c + t_f)$, then the neutral axis lies in the web. In design this case should be avoided, otherwise the web has to be checked for slenderness.

In similar procedure as the previous one, here x can be found from

$$\begin{aligned} N_{a.pl} &= N_{cf} + N_{acf} + N_{aw} \\ &= N_{cf} + 2 b_f t_f\, f_y/\gamma_a + 2 t_w\,(x - h_t - t_f)\, f_y/\gamma_a \end{aligned} \qquad (19)$$

Plastic moment of resistance

$$\begin{aligned} M_p = N_{a.pl}\,(h_g + h_t - h_c/2) &- N_{acf}\,(h_t + t_f/2 - h_c/2) \\ &- N_{a.w}\,(x + h_t + t_f - h_c)/2 \end{aligned} \qquad (20)$$

4.2 Partial shear connection

Sometimes due to the problem of accommodating shear connectors uniformly or to achieve economy, partial shear connections are provided between steel beam and concrete slab. Then the force resisted by the connectors are taken as their total capacity $(F_c < F_{cf})$ between points of zero and maximum moment. Here F_{cf} refers to the term of N_{cf} as used earlier. If n_f and n_p are the number of shear connectors required for full interaction and partial interaction respectively, then the degree of shear connection is defined as, n_p / n_f. Therefore,

$$\textit{Degree of shear connection} = \frac{n_p}{n_f} = \frac{F_c}{F_{cf}}$$

Assuming all connectors have same resistance to shear, The depth of compressive stress block in slab is

$$x_c = \frac{F_c}{0.85b(f_{ck})_{cy}/\gamma_c} \tag{21}$$

which is less than h_c.

The neutral axis of steel section may be in the flange or in the web. In case the neutral axis is within top flange, the moment of resistance can be found out using stress block shown in Figure *12c*. Here the block of N_{cf} is replaced by F_c therefore,

$$M_{Rd} = N_{a.pl}(h_g + h_t - \frac{x_c}{2}) - F_c\frac{x_a + h_t - x_c}{2} \tag{22}$$

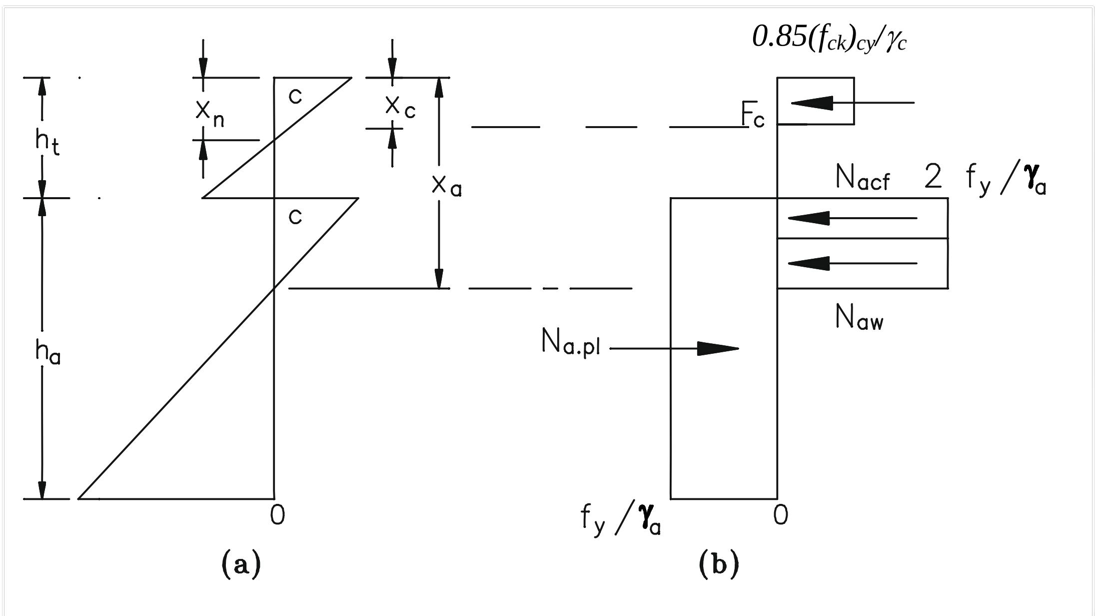


Fig.13. Resistance to sagging bending of composite section in class 1 or 2 for partial interaction

If the neutral axis lies in web (refer Fig. 13) the moment of resistance is determined by taking moment about top surface

$$M_{Rd} = N_{a.pl}(h_g + h_t) - \frac{F_c x_c}{2} - N_{cf}(h_t + \frac{t_f}{2}) - N_{aw}\frac{x_a + h_t - t_f}{2} \tag{23}$$

where $N_{acf} = 2b_f\, t_f\, \dfrac{f_y}{\gamma_a}$

$N_{aw} = N_{a.pl} - F_c$ - N_{acf}

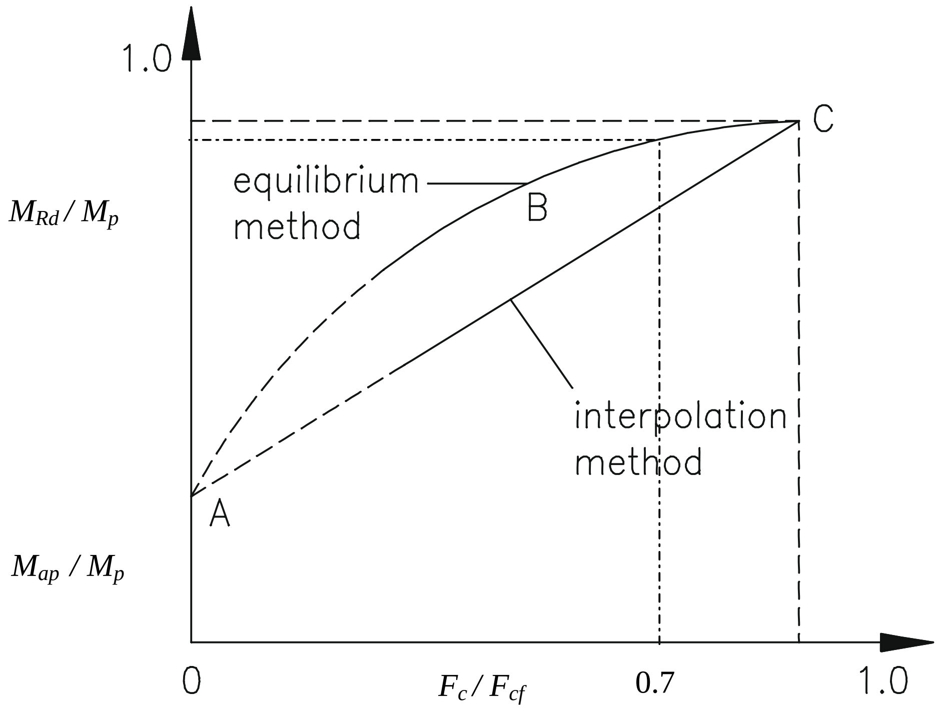


Fig.14. Design methods of partial shear connection

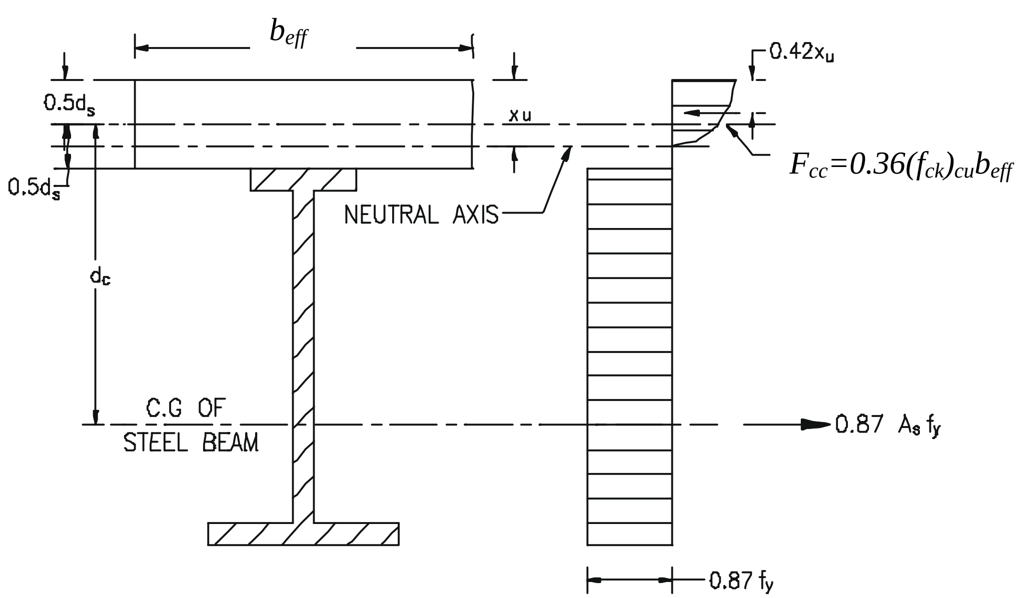


Fig 15 Stress distribution in a composite beam with neutral axis within concrete slab

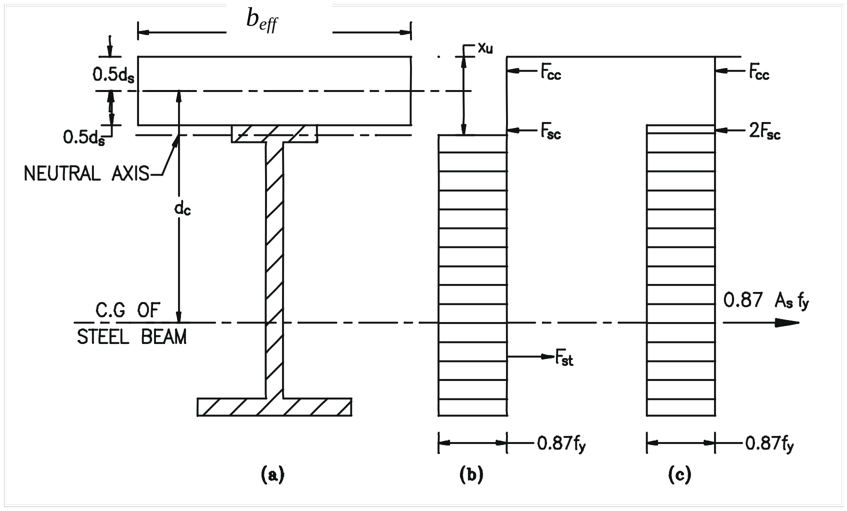


Fig.16. Stress distribution in a composite beam with neutral axis with in flange of steel beam.

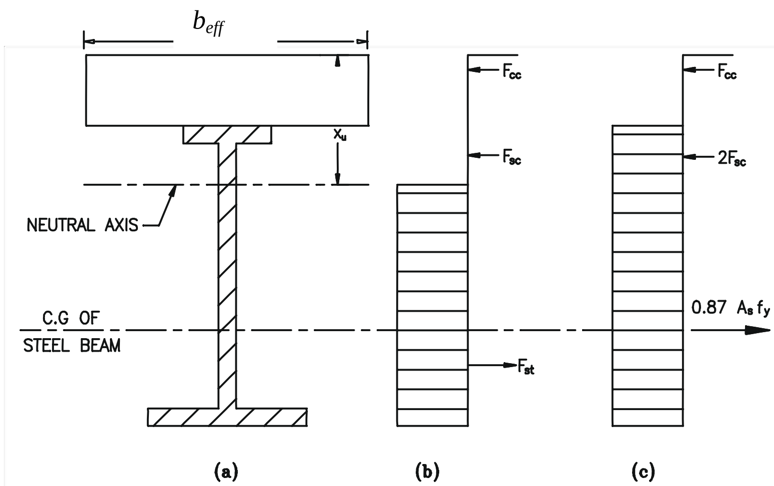


Fig.17 Stress distribution in a composite beam with neutral axis within the web of the steel beam.

Moment of resistance reduces due to partial shear connection. The relation between M_{Rd}/M_p with degree of shear connection F_c/F_{cf} is shown in Fig.14. The curve *ABC* is not valid for very low value of shear connection. It can be seen from the curve that at $F_c/F_{cf} = 0.7$, the required bending resistance is slightly below M_p. Using this a considerable saving in the cost of shear connectors can be achieved without unduly sacrificing the moment capacity. However for design purpose the curve *ABC* is replaced by a straight-line *AC* given by

$$S_c = \frac{M - M_{ap}}{M_p - M_{ap}} F_{cf} \qquad (24)$$

where, M is the required bending resistance of the section; and

M_{ap} is the plastic moment of resistance of steel section only.

Using the values of M_{sd}, M_{ap}, M_p and F_c, the values of F_{cf} can be found. With the value of F_{cf}, number of shear connectors per span is determined.

However, the Indian Code of Practice for Composite Construction in Structural Steel and Concrete (IS: 11384 – 1985), has adopted a parabolic stress block in concrete slab for plastic analysis of the section. Here a stress factor $a = 0.87\, f_y/0.36(f_{ck})_{cu}$ is applied to convert the concrete section into steel. The additional assumptions made by IS: 11384 – 1985 (compared to those of Eurocode) are given below:

- The maximum strain in concrete at outermost compression member is taken as *0.0035* in bending.
- The total compressive force in concrete is given by $F_{cc} = 0.36\, (f_{ck})_{cu}\, bx_u$ and this acts at a depth of $0.42\, x_u$, not exceeding d_s.
- The stress strain curve for steel section and concrete are as per IS: 456-1978.

The notations used here are

A_f = area of top flange of steel beam of a composite section.
A_s = cross sectional area of steel beam of a composite section.
b_{eff} = effective width of concrete slab.
b_f = width of top flange of steel section.
d_c = vertical distance between centroids of concrete slabs and steel beam in a composite section.
t_f = average thickness of the top flange of the steel section.
x_u = depth of neutral axis at ultimate limit state of flexure
M_u = ultimate bending moment.

The three cases that may arise are given below with corresponding M_u.

Case I: Plastic neutral axis within the slab (Refer Fig.15)

This occurs when $b_{eff}\, d_s \geq a\, A_s$.

Taking moment about centre of concrete compression

$$M_u = 0.87\, A_s\, f_y\, (d_c + 0.5 d_s - 0.42\, x_u) \qquad (25)$$

where,

$$x_u = aA_s / b_{eff}$$

$$a = \frac{0.87 f_y}{0.36\,(f_{ck})_{cu}}$$

Case II: Plastic neutral axis within the top flange of steel section (Refer Fig. 16).

This happens when

$b_{eff} d_s < aA_s < (b_{eff} d_s + 2a\, A_f)$

Equating forces as in Eqn. (17) we get

$$x_u = d_s + \frac{aA_s - b_{eff}\; d_s}{2b_f\, a} \tag{26}$$

Taking moment about centre of concrete compression.

$$M_u = 0.87\, f_y \left\lfloor A_s(d_c + 0.08 d_s) - b_f(x_u - d_s)(x_u + 0.16 d_s) \right\rfloor \tag{27}$$

Case III: Plastic neutral axis lies within web (Refer Fig. 17.) This occurs when

$a\,(A_s - 2A_f) > b_{eff} d_s$

Equating area under tension and compression as in Equation. (19) we get

$$x_u = d_s + t_f + \frac{a(A_s - 2A_f) - b_{eff}\; d_s}{2at_w} \tag{28}$$

Taking moment about the centre of concrete compression

$$\begin{aligned} M_u =\ & 0.87 f_y A_s (d_c + 0.08 d_s) - 2A_f\,(0.5 t_f + 0.58 d_s) \\ & - 2t_w (x_u - d_s - t_f)(0.5\, x_u + 0.08\, d_s + 0.5 t_f) \end{aligned} \tag{29}$$

Note: ***In IS: 11384 – 1985 no reference has been made to profiled deck slab and partial shear connection. Therefore the above equations can be used only for composite beams without profiled deck sheeting (i.e., steel beam supporting concrete slabs).***

5.0 SERVICEABILITY LIMIT STATES

For simply supported composite beams the most critical serviceability Limit State is usually deflection. This would be a governing factor in design for un-propped construction. Besides, the effect of vibration, cracking of concrete, etc. should also be checked under serviceability criteria. Often in exposed condition, it is preferable to design to obtain full slab in compression to avoid cracking in the shear connector region.

5.1 Stresses and deflection in service

As structural steel is supposed not to yield at service load, elastic analysis is employed in establishing the serviceability performance of composite beam. In this method the concrete area is converted into equivalent steel area by applying modular ratio $m = (E_s/E_c)$. The analysis is done in terms of equivalent steel section. It is assumed that full interaction exists between steel beam and concrete slab. The effect of reinforcement in compression, the concrete in tension and the concrete between rib of profiled sheeting are ignored.

Refer to Fig.18, where a transformed section is shown.

When neutral axis lies within the slab

$$A_a(Z_g - h_c) < \tfrac{1}{2} b_{eff} \frac{h_c^2}{m} \tag{30}$$

The actual neutral axis depth can be found from

$$A_a(Z_g - x) = \tfrac{1}{2} b_{eff} \frac{x^2}{m} \tag{31}$$

and the moment of inertia of the transformed section is

$$I = I_a + A_a(Z_g - x)^2 + \left(\frac{b_{eff}}{m}\right)\frac{x^3}{3} \tag{32}$$

When neutral axis depth exceeds h_c, its depth x is found from the following equation.

$$A_a(Z_g - x) = \frac{b_{eff}}{m} h_c \left(x - \frac{h_c}{2}\right) \tag{33}$$

and moment of inertia of the transformed section

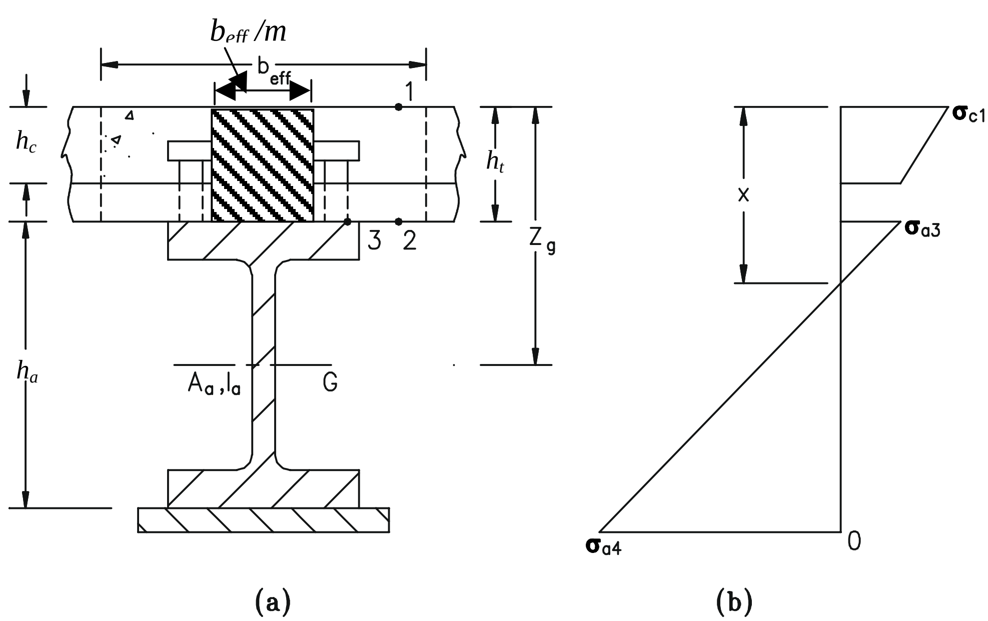


Fig.18. Elastic analysis of composite beam section in sagging bending

$$I = I_a + A_a \left(Z_g - x\right)^2 + \frac{b_{eff}\ h_c}{m}\left(\frac{h_c^{\ 2}}{12} + \left(x - h_c/2\right)^2\right) \qquad (34)$$

For distributed load *w* over a simply supported composite beam, the deflection at mid-span is

$$\delta_c = \frac{5\,wL^4}{384\ E_a I} \qquad (35)$$

where E_a = Young's Modulus for structural steel.
I = moment of inertia found from Equation. (32) and Equation. (34) as applicable.

The beam can be checked for stresses under service load using the value of *'I'* as determined above.

When the shear connections is only partial the increase in deflection occurs due to longitudinal slip. This depends on method of construction. Total deflection,

$$\delta = \delta_c\left(1 + k\left(1 - \frac{N}{N_f}\right)\left(\frac{\delta_a}{\delta_c} - 1\right)\right) \qquad (36)$$

with $k = 0.5$ for propped construction

and $k = 0.3$ for un-propped construction

δ_a = deflection of steel beam acting alone

The expression gives acceptable results when $n_p/n_f \geq 0.4$

The increase in deflection can be disregarded where:

- either $n_p/n_f \geq 0.5$ or when force on connector does not exceed $0.7\ P_{RK}$ where P_{RK} is the characteristic resistance of the shear connector; and
- when the transverse rib depth is less than *80* mm.

The empirical nature of the above rules stipulated by BS.5950 is because of the difficulty in predicting the deflection accurately.

5.2 Effects of shrinkage of concrete and of temperature

In a dry condition, an unrestrained concrete slab is expected to shrink by *0.03%* of its length or more. But in case of composite beam the slab is restrained by steel beam. The shear connectors resist the force arising out of shrinkage, by inducing a tensile force on concrete. This reduces the apparent shrinkage of composite beam than the free shrinkage. Moreover no account of this force is taken in design as it acts in the direction opposite to that caused by load. However, the increase in deflection due to shrinkage may be significant. In an approximate approach the increase in deflection in a simply supported beam is taken as the long-term deflection due to weight of the concrete slab acting on the composite member.

Generally the span/depth ratios specified by codes take care of the shrinkage deflection. However, a check on shrinkage deflection should be done in case of thick slabs resting on small steel beams, electrically heated floors and concrete mixes with high "*free shrinkage*". Eurocode 4 recommends that the effect of shrinkage should be considered when the span/depth ratio exceeds *20* and the free shrinkage strain exceeds *0.04%*. For dry environments, the limit on free shrinkage for normal- weight concrete is *0.0325%* and for lightweight concrete *0.05%.*

5.3 Vibration

5.3.1 General

Generally, human response to vibration is taken as the yardstick to limit the amplitude and frequency of a vibrating floor. The present discussion is mainly aimed at design of an office floor against vibration. To design a floor structure, only the source of vibration near or on the floor need be considered. Other sources such as machines, lift or cranes should be isolated from the building. In most buildings following two cases are considered-

i) People walking across a floor with a pace frequency between *1.4 Hz* and *2.5 Hz*.
ii) An impulse such as the effect of the fall of a heavy object.

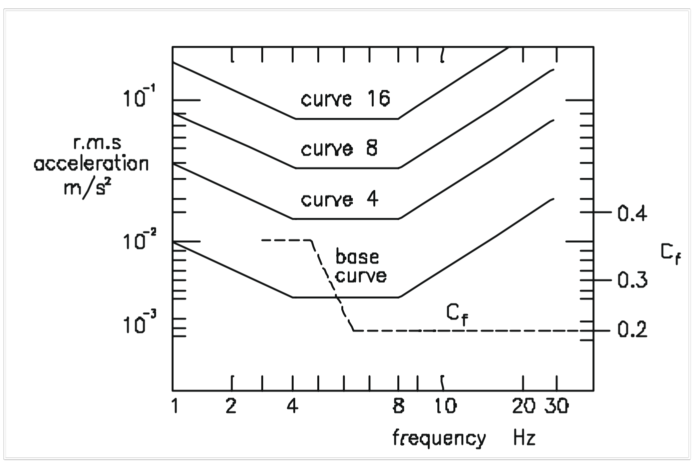


Fig.19. Curves of constant human response to vibration, and Fourier component factor

BS 6472 present models of human response to vibration in the form of a base curve as in Fig. (19). Here root mean square acceleration of the floor is plotted against its natural frequency f_0 for acceptable level R based on human response for different situations such as, hospitals, offices etc. The human response $R=1$ corresponds to a *"minimal level of adverse comments from occupants"* of sensitive locations such as hospital, operating theatre and precision laboratories. Curves of higher response *(R)* values are also shown in the Fig. (19). The recommended values of R for other situations are

R = 4 for offices
R = 8 for workshops

These values correspond to continuous vibration and some relaxation is allowed in case the vibration is intermittent.

5.3.2 Natural frequency of beam and slab

The most important parameter associated with vibration is the natural frequency of floor. For free elastic vibration of a beam or one way slab of uniform section the fundamental natural frequency is,

$$f_0 = K\left(\frac{EI}{mL^4}\right)^{1/2} \qquad (37)$$

where, $K = \pi/2$ for simple support; and
$K = 3.56$ for both ends fixed.
EI = Flexural rigidity (per unit width for slabs)
L = span
m = vibrating mass per unit length (beam) or unit area (slab).

The effect of damping, being negligible has been ignored.

Un-cracked concrete section and dynamic modulus of elasticity should be used for concrete. Generally these effects are taken into account by increasing the value of I by *10%* for variable loading. In absence of an accurate estimate of mass (m), it is taken as the mass of the characteristic permanent load plus *10%* of characteristic variable load. The value of f_0 for a single beam and slab can be evaluated in the following manner.
The mid-span deflection for simply supported member is,

$$\delta_m = \frac{5mgL^4}{384\,EI} \qquad (38)$$

Substituting the value of 'm' from Eqn. (38) in Eqn. (37) we get,

$$f_0 = \frac{17.8}{\sqrt{\delta_m}} \qquad (39)$$

where, δ_m is in millimeters.

However, to take into account the continuity of slab over the beams, total deflection δ in considered to evaluate f_0, so that,

$$f_0 = \frac{17.8}{\sqrt{\delta}} \tag{40}$$

where, $\delta = \delta_b + \delta_s$
δ_s – deflection of slab relative to beam
δ_b- deflection of beam.

From Equation. (39) and (40)

$$\frac{1}{f_0^2} = \frac{1}{f_{0s}^2} + \frac{1}{f_{0b}^2} \tag{41}$$

where f_{os} and f_{ob} are the frequencies for slab and beam each considered alone.

From Eqn. (41) we get,

$$f_{ob} = \frac{\pi}{2}\left(\frac{EI_b}{msL^4}\right)^{1/2} \tag{42}$$

$$f_{os} = 3.56\left(\frac{EI_S}{ms^4}\right)^{1/2} \tag{43}$$

where, s is the spacing of the beams.

A typical vibrating profile of a floor structure is shown in Fig. (20).

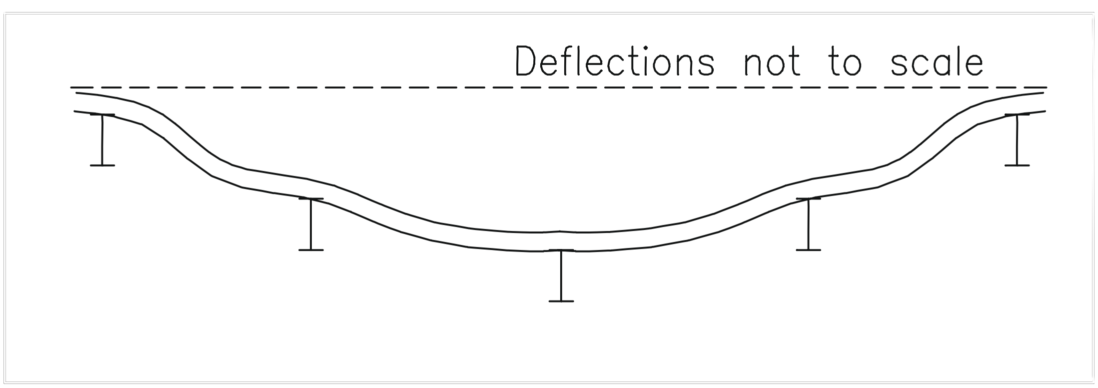


Fig.20. Cross-section of vibrating floor structure showing typical fundamental mode

5.3.3 Response factor

Reactions on floors from people walking have been analyzed by Fourier Series. It shows that the basic fundamental component has amplitude of about *240N*. To avoid resonance with the first harmonics it is assumed that the floor has natural frequency $f_0 > 3$, whereas the excitation force due to a person walking has a frequency *1.4* Hz to *2* Hz. The effective force amplitude is,

$$\overline{F} = 240 C_f \qquad (44)$$

where C_f is the Fourier component factor. It takes into account the differences between the frequency of the pedestrians' paces and the natural frequency of the floor. This is given in the form of a function of f_0 in Fig. (19).

The vertical displacement y for steady state vibration of the floor is given approximately by,

$$y = \frac{\overline{F}}{2 k_e \zeta} \sin 2\pi \, f_0 t \qquad (45)$$

$$\begin{aligned} \text{where } \frac{\overline{F}}{k_e} &= \textit{Static deflection of floor} \\ \frac{1}{2\zeta} &= \textit{magnification factor at resonance} \\ &= 0.03 \ \textit{for open plan offices with composite floor} \\ f_0 &= \textit{steady state vibration frequency of the floor} \end{aligned}$$

R.m.s value of acceleration

$$a_{r.m.s} = 4\pi^2 \, f_0^2 \, \frac{\overline{F}}{2\sqrt{2}\, k_e \, \zeta} \qquad (46)$$

The effective stiffness k_e depends on the vibrating area of floor, $L \times S$. The width S is computed in terms of the relevant flexural rigidities per unit width of floor which are I_s for slab and I_b/s for beam.

$$S = 4.5 \left(\frac{EI_s}{m f_0^2} \right)^{1/4} \qquad (47)$$

As f_{0b} is much greater than f_{0s}, the value of f_{0b} can be approximated as f_0. So, replacing mf_0^2 from Eqn. (42) in Eqn. (45), we get,

$$\frac{S}{L} = 3.6\left(\frac{I_s s}{I_b}\right)^{1/4} \qquad (48)$$

Eqn. (48) shows that the ratio of equivalent width to span increases with increase in ratio of the stiffness of the slab and the beam.

The fundamental frequency of a spring-mass system,

$$f_0 = \frac{1}{2\pi}\left(\frac{k_e}{M_e}\right)^{1/2} \qquad (49)$$

where, M_e is the effective mass = $mSL/4$ (approximately)

From Eqn. (51),

$$k_e = \pi^2 f_0^{\,2}\, mSL \qquad (50)$$

Substituting the value of k_e from Eqn. (50) and F from Eqn. (44) into Eqn. (46)

From definition, Response factor,

$$a_{rms} = 340\frac{C_f}{msL\zeta} \qquad (51)$$

$$a_{rms} = 5\times 10^{-3} R \quad m/s^2 \qquad (52)$$

Therefore, from Equation (52),

To check the susceptibility of the floor to vibration the value of R should be compared with the target response curve as in Fig. (19).

$$R = 68000\frac{C_f}{msL\zeta} \qquad in\ \ MKS\ \ units \qquad (53)$$

6.0 CONCLUSION

This chapter mainly deals with the theory of composite beam and the underlying philosophy behind its evolution. This comparatively new method of construction quickly gained popularity in the Western World because of its applicability in bridges, multistoried buildings, car parks etc with reduced construction time. There were valuable research studies, to support the design basis. It has been reported that saving in high yield strength steel can be up to 40% in composite construction. However this method is cost effective for larger span and taller buildings. The design procedures of simply supported as well as continuous beams have been elaborately discussed with examples in the next chapter.

7.0 REFERENCES

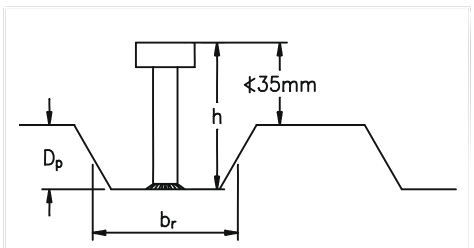
1) R.P.Johnson: *Composite Structure of Steel and Concrete (Volume 1)*, Blackwell Scientific Publication (Second Edition), U.K., 1994.
2) G. W. Owens and P. Knowles: *Steel Designer's Manual (Fifth edition)*, The steel construction Institute (U.K), Oxford Blackwell Scientific Publication,1992
3) IS: 11384-1985, *Code of Practice for Composite Construction in Structural Steel and Concrete*.

22 COMPOSITE BEAMS – II

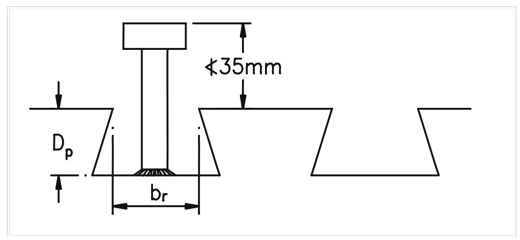
1.0 INTRODUCTION

A steel concrete composite beam consists of a steel beam, over which a reinforced concrete slab is cast with shear connectors, as explained in the previous chapter. Since composite action reduces the beam depth, rolled steel sections themselves are found adequate frequently (for buildings) and built-up girders are generally unnecessary. The composite beam can also be constructed with profiled sheeting with concrete topping, instead of cast-in place or precast reinforced concrete slab. The profiled sheets are of two types

- Trapezoidal profile
- Re-entrant profile



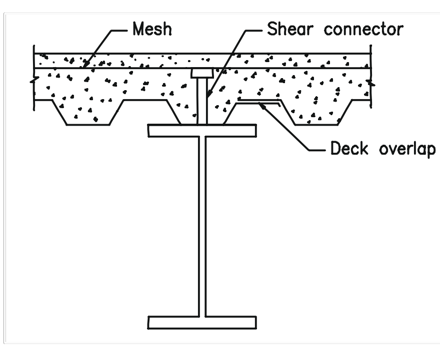
(a) Trapezoidal profile



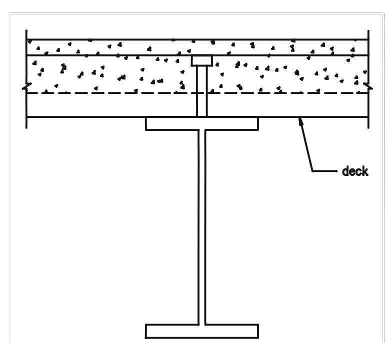
(b) Re-entrant profile

Fig. 1 Types of profile deck

These two types are shown in Fig *1*. The profiled steel sheets are provided with indentations or embossments to prevent slip at the interface. The shape of the re-entrant form, itself enhances interlock between concrete and the steel sheet. The main advantage of using profiled deck slab is that, it acts as a platform and centering at construction stage and also serves the purpose of bottom reinforcement for the slab.



(a) Ribs parallel to the beam



(b) Ribs perpendicular to the beam

Fig. 2. Orientation of Profiled deck slab in a composite beam

The deck slab with profiled sheeting is of two types (see Fig *2*).

- The ribs of profiled decks running parallel to the beam
- The ribs of profiled decks running perpendicular to the beam.

2.0 PROVISION FOR SERVICE OPENING IN COMPOSITE BEAMS

There is now a growing demand for longer spans, either for open plan offices, or to permit greater flexibility of office layout, or for open exhibition and trading floors. For these longer spans, the choice of structural form is less clear cut largely on account of the need for providing for services satisfactorily. Service openings can be easily designed in conventional rolled steel beams. Conventional construction may still be appropriate, but other, more novel, structural forms may offer economy or other overriding advantages, besides easy accommodation of services. Open web joist floor system may be one such solution for longer span (see the chapter on trusses). In fact, many of these were developed in Great Britain and a number of Design Guides have been produced by the Steel Construction Institute.

2.1 Simple Construction with Rolled Sections(1)

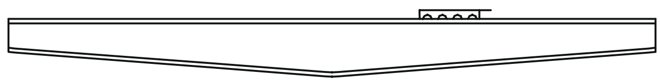
For spans in the range of *6* to *10 m*, perhaps the most appropriate form of construction is rolled sections and simple, shear only connections. Secondary beams at *2.4 m* or *3.0 m* centres support lightweight composite floor slabs and span onto primary beams, which in turn frame directly into the columns. The same form of construction may also be used for longer span floors but beam weights and costs increase to the point where other forms of construction may be more attractive. Of increasing concern to developers is the provision of web openings as these are inflexible and they can create difficulties in meeting the specific needs of tenants or in subsequent reservicing during the life of the structure.

2.2 Fabricated Sections(2)

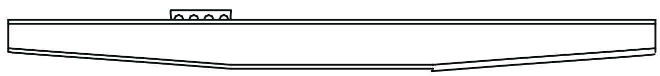
The use of fabricated sections for multi-storey buildings has been explored by some U.K.designers. This usage became economic with advances in the semi-automatic manufacture of plate girder sections. Different approaches to manufacture have been developed by different fabricators. Significant savings in weight can be achieved due to the freedom, within practical limits, to tailor the section to suit its bending moment and shear force envelopes. Depth, taper and shape flange size and web thickness may all be selected independently by the designer.

Fabricated sections are most likely to be economic for spans above *12 m*. Above this span length, rolled sections are increasingly heavy and a fine-tuned fabricated section is likely to be able to save on both flange size and web thickness. With some manufacturing processes, asymmetric sections with narrow top flanges can be adopted, achieving further weight savings.

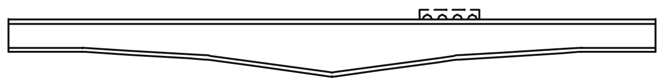
The freedom to tailor the fabrication to the requirements of the designer allows the depth of the girder to be varied along its length and to allow major services to run underneath the shallower regions. A range of shapes is feasible (see Fig.*3*) of which the semi-tapered beam is the most efficient structurally but can only accommodate relatively small ducts. The straight-tapered beams shown in Fig *3(a)* offers significantly more room for ducts, at the expense of some structural efficiency, and has proved to be the most popular shape to date. Cranked taper beams can also be used, providing a rectangular space under the beams at their ends. Fabricated beams are often employed to span the greater distance, and supporting shorter span primary beams of rolled sections.



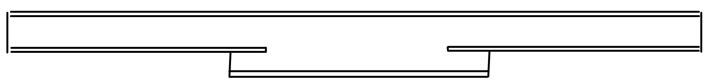
(a) Straight Taper



(b) Semi-Taper



(c)Cranked Taper



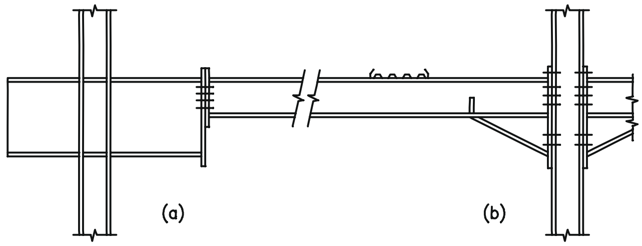
(d) Stepped Beam
(where automatic welding is not crucial)

Fig. 3. Fabricated sections for commercial buildings

2.3 Haunched Beams[3]

In traditional multi-storey steel frames, the conventional way to achieve economy is to use ‘simple’ design. In a long span structure, there is perhaps twice the length of primary beams compared to the columns and for a low rise building their mass/metre will be comparable. In these circumstances the economic balance may shift in favour of sacrificing column economy in order to achieve greater beam efficiency by having moment resisting connections. The benefits of continuity are particularly significant when stiffness rather than the strength governs design, and this is increasingly likely as spans increase. Where fully rigid design is adopted, the beam to column connection is likely to have to develop the hogging bending capacity of the composite section. Until our design concepts on composite connections are more fully developed, designers have to rely on an all-steel connection and this will usually require substantial stiffening and could prove to be expensive.

The most straightforward way to reduce connection costs is to use some form of haunched connection (Fig. *4*); they occupy the region below the beam, which is anyway necessary for the main service ducts. (With haunched beams, the basic section is usually too shallow for holes to be formed in its web that are sufficiently large to accommodate main air-conditioning ducts). Thus the haunches simplify beams of column connection significantly and improve beam capacity and stiffness without increasing the overall floor depth.



(a) sections of different size
(b) haunches cut from main beam

Fig. 4 Haunched beams: Two types of haunches

2.4 Parallel Beam Approach(4)

In the parallel beam approach, it is the secondary beams that span the greater distance. A very simple form of construction results as they run over the primary beams and achieve continuity without complex connections (see Fig. *5*).

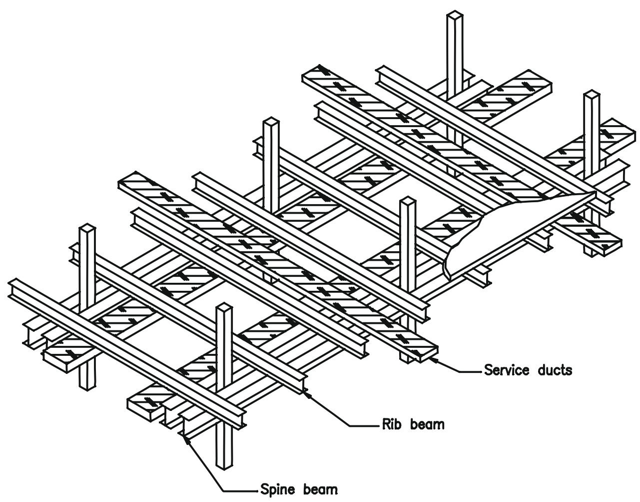


Fig. 5. Parallel beam grillage

The primary or spine beams also achieve continuity by being used in pairs with one beam passing on either side of the columns. Shear is transferred into the columns by means of brackets. This 'offset' construction, where members are laid out in the three orthogonal directions deliberately to miss each other enable continuity of the beams to be achieved without the high cost of moment resisting connections; this improves the structural efficiency and (of particular importance for long span construction) stiffness. There is also a considerable saving of both erection time and erection cost. Because continuity is such an integral part of the approach, it is primarily applicable for multi-bay layouts.

Superficially, the approach appears to lead to deeper construction. However, because of continuity, the primary and secondary beams can both be very shallow for the spans and overall depths are comparable with conventional construction. Most importantly, the separation of the two beam directions into different planes creates an ideal arrangement for the accommodation of services.

2.5 Castellated Sections [5]

Castellated beams are made from Rolled Steel beams by fabricating openings in webs, spaced at regular intervals. Castellated sections have been used for many years (see Fig.*6*) as long span roof beams where their attractive shape is often expressed architecturally. The combination of high bending stiffness and strength per unit weight with relatively low shear capacity is ideal for carrying light loads over long spans. As composite floor beams, their usage is limited by shear capacity. These are generally unsuitable for use as primary beams in a grillage, because the associated shears would require either stiffening to or infilling of the end openings, thereby increasing the cost to the point that other types of beams become economical. However, if the castellated sections are used to span the longer direction directly, then the shear per beam drops to the level at which the unstrengthened castellated sections can be used.

The openings in the castellated beams allow the accommodation of circular ducts used for many air-conditioning systems. There are, in addition, plenty of openings for all the other services, which can be distributed throughout the span effectively without any consideration of their interaction with the structure. It is also possible, near mid-span, to cut out one post and thereby create a much larger opening encompassing two conventional castellations. The shear capacity of this opening will need careful checking, taking due account of eccentric part span loading and associated midspan shears. If this opening needs strengthening then longitudinal stiffeners at top and bottom are likely to be adequate.

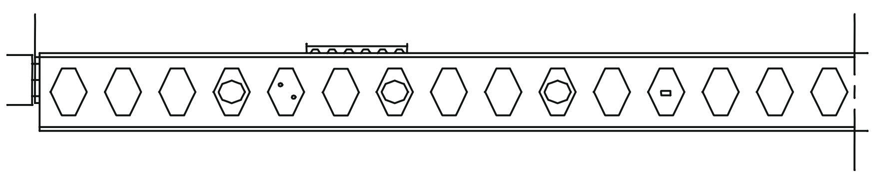


Fig. 6. Castellated beams

2.6 Stub Girders [6]

Stub Girders comprise a steel bottom chord with short stubs connecting it to the concrete or profiled sheet slab (Fig. *7*). Openings for services are created adjacent to the stubs. Bottom chords will need to be propped during construction, if this method is used.

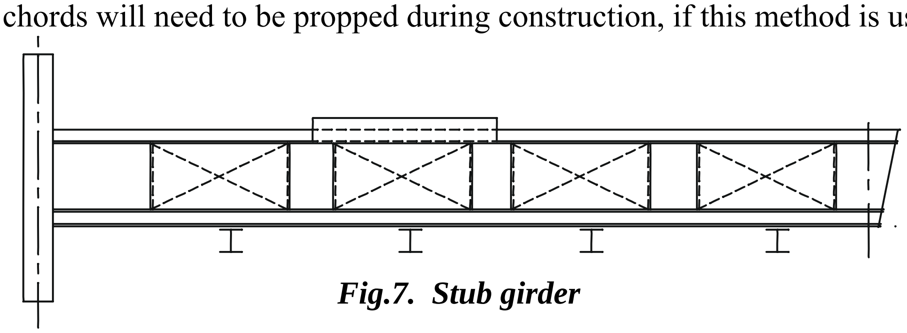


Fig.7. Stub girder

2.7 Composite Trusses [7]

Consider a steel truss acting compositely with the floor slab (Fig. *8*). Bracing members can be generally eliminated in the central part of the span, so that – if needed – large rectangular ducts can pass between bracing members. The chords are fabricated from T sections or cold formed shapes and bracing members from angles. As is obvious from the above discussion, several innovative forms of composite beam using profiled steel deck have been developed in recent years. The designer has, therefore, a wide choice in selecting an appropriate form of flooring using these concepts.

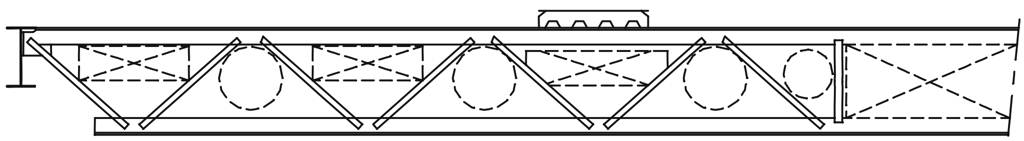


Fig. 8. Composite trusses

3.0 BASIC DESIGN CONSIDERATIONS

3.1 Design Method suggested by Eurocode 4[8]

For design purpose, the analysis of composite section is made using Limit State of collapse method. *IS:11384-1985* Code deals with the design and construction of only simply supported composite beams. Therefore, the method of design suggested in this chapter largely follows *EC4.* Along with this, *IS:11384-1985* Code provisions and its limitations are also discussed.

The ultimate strength of composite section is determined from its plastic capacity, provided the elements of the steel cross section do not fall in the semi-compact or slender category as defined in the section on plate buckling. The serviceability is checked using elastic analysis, as the structure will remain elastic under service loading. Full shear connection ensures that full moment capacity of the section develops. In partial shear connection, although full moment capacity of the beam cannot be achieved, the design will have to be adequate to resist the applied loading. This design is sometimes preferred due to economy achieved through the reduced number of shear connector to be welded at site.

3.2 Span to depth ratio

EC4 specifies the following span to depth (total beam and slab depth) ratios for which the serviceability criteria will be deemed to be satisfied.

Table 1 Span to Depth ratio as according to EC4

	EC4
Simply supported	15-18 (Primary Beams) 18-20 (Secondary Beams)
Continuous	18-22 (Primary Beams) 22-25 (end bays)

3.3 Effective breadth of flange

A composite beam acts as a T-beam with the concrete slab as its flange. The bending stress in the concrete flange is found to vary along the breadth of the flange as in Fig *9*, due to the shear lag effect. This phenomenon is taken into account by replacing the actual breadth of flange *(B)* with an effective breadth *(b_{eff}),* such that the area *FGHIJ* nearly equals the area *ACDE*. Research based on elastic theory has shown that the ratio of the effective breadth of slab to actual breadth (b_{eff}/B) is a function of the type of loading, support condition, and the section under consideration. For design purpose a portion of the beam span *(20% - 33%)* is taken as the effective breadth of the slab.

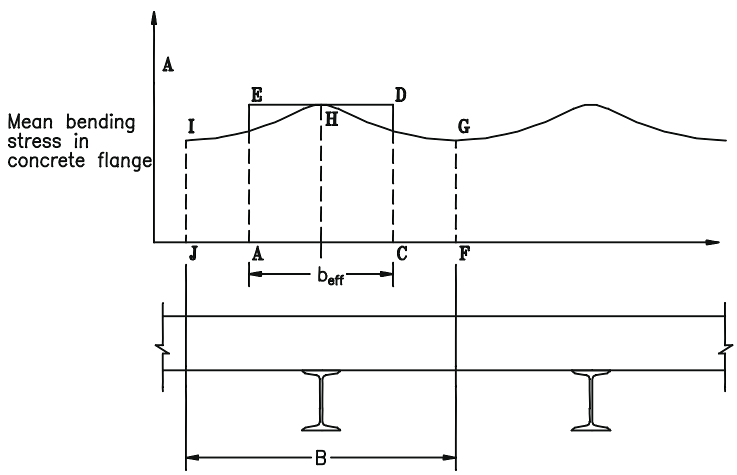


Fig. 9. Use of effective width to allow for shear lag

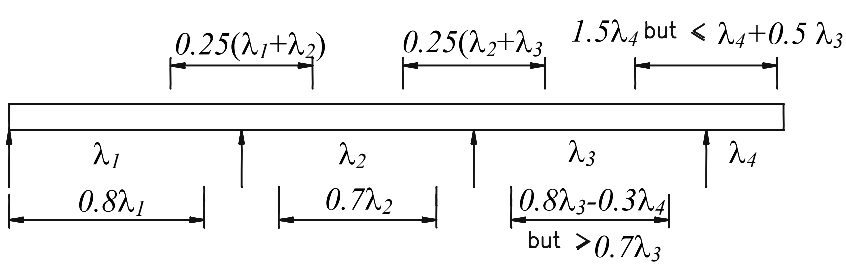


Fig. 10 Value of λ_0 for continuous beam as per EC4

In *EC4,* the effective breadth of simply supported beam is taken as $\lambda_o/8$ on each side of the steel web, but not greater than half the distance to the next adjacent web. For simply supported beam $\lambda_o = \lambda$ Therefore,

$$b_{eff} = \frac{\lambda}{4} \quad but \leq B$$

where,

λ_o = The effective span taken as the distance between points of zero moments.
λ = Actual span
B = Centre to centre distance of transverse spans for slab.

For continuous beams λ_o is obtained from Fig *10*.

3.4 Modular ratio

Modular ratio is the ratio of elastic modulus of steel (E_s) to the time dependent secant modulus of concrete (E_{cm}). While evaluating stress due to long term loading (dead load etc.) the time dependent secant modulus of concrete should be used. This takes into account the long-term effects of creep under sustained loading. The values of elastic modulus of concrete under short term loading for different grades of concrete are given in Table *2*.

IS:11384 -1985 has suggested a modular ratio of *15* for live load and *30* for dead load, for elastic analysis of section. It is to be noted that a higher value of modular ratio for dead load takes into account the larger creep strain of concrete for sustained loading. In *EC 4* the elastic modulus of concrete for long-term loads is taken as one-third of the short-term value and for normal weight concrete, the modular ratio is taken as *6.5* for short term loading and *20* for long term loading.

Table 2 Properties of concrete

Grade Designation	*M25*	*M30*	*M35*	*M40*
$(f_{ck})_{cu}$ (N/mm^2)	25	30	35	40
$E_{cm}=5700\sqrt{(fck)_{cu}}(N/mm^2)$	28500	31220	33720	36050

3.5 Shear Connection

The elastic shear flow at the interface of concrete and steel in a composite beam under uniform load increases linearly from zero at the centre to its maximum value at the end. Once the elastic limit of connectors is reached, redistribution of forces occurs towards the less stressed connectors as shown in Fig *11* in the case of flexible shear connectors (such as studs). Therefore at collapse load level it is assumed that all the connectors carry equal force, provided they have adequate shear capacity and ductility. In *EC4,* the design capacity of shear connectors is taken as *80%* of their nominal static strength. Though, it may be considered as a material factor of safety, it also ensures limit condition to be reached by the flexural failure of the composite beam, before shear failure of the interface.

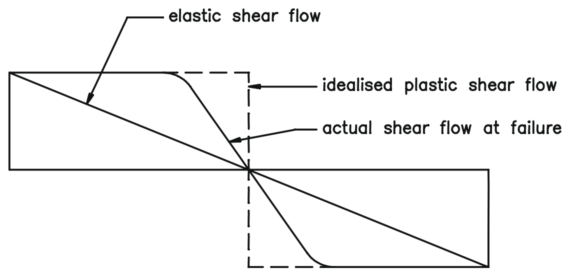


Fig 11. Shear flow at interface

The design strength of some commonly used shear connectors as per *IS:11384-1985* is given in Table *1* of the previous chapter (Composite Beam-I).

3.6 Partial Safety Factor

3.6.1 Partial safety factor for loads and materials **–** The suggested partial safety factors for load, γ_f and for materials, γ_m are shown in Table *3*.

Table 3 Partial safety factors as per the proposed revisions to IS: 800

Load	**Partial safety factor, γ_f**
Dead load Live load	1.35 1.5
Materials	**Partial safety factor, γ_m**
Concrete Structural Steel Reinforcement	1.5 1.15 1.15

3.7 Section Classifications

Local buckling of the elements of a steel section reduces its capacity. Because of local buckling, the ability of a steel flange or web to resist compression depends on its slenderness, represented by its breadth/thickness ratio. The effect of local buckling is therefore taken care of in design, by limiting the slenderness ratio of the elements i.e. web and compression flange. The classification of web and compression flange is presented in the Table *4*.

Table 4 Classification of Composite Section

Type of Element	Type of Section	Class of Section		
		Plastic	**Compact**	**Semi-compact**
Outstand element of compression flange	Built up by welding	$b/T \leq 7.9\in$	$b/T \leq 8.9\in$	$b/T \leq 13.6\in$
	Rolled section	$b/T \leq 8.9\in$	$b/T \leq 9.9\in$	$b/T \leq 15.7\in$
Web, with neutral axis at mid-depth	All sections	$d/t \leq 83\in$	$d/t \leq 103\in$	$d/t \leq 126\in$
Web, generally	All section	$\frac{d}{t} \leq \frac{83\in}{0.4+0.6\alpha}$	$\frac{d}{t} \leq \frac{103\in}{\alpha}$	when $R > 0.5$ for welded section $\frac{d}{t} \leq (109-80R)\in$ for rolled section $\frac{d}{t} \leq (98-57R)\in$ when $R \leq 0.5$ but > -0.45 $\frac{d}{t} \leq \frac{126\in}{1+1.6R}$

where,

b = half width of flange of rolled section
T = Thickness of top flange
d = clear depth of web

$$\alpha = \frac{2\,Y_c}{d} \geq 0$$

where, Y_c is the distance from the plastic neutral axis to the edge of the web connected to the compression flange. But if $\alpha > 2$, the section should be taken as having compression through out.

$$\in = \text{constant} = \sqrt{\frac{250}{f_y}}$$

t = thickness of web

R = is the ratio of the mean longitudinal stress in the web to the design strength.

f_y with compressive stress taken as positive and tensile stress negative.

If the compression flange falls in the plastic or compact category as per the above classification, plastic moment capacity of the composite section is used provided the web is not slender. For compression flange, falling in semi-compact or slender category elastic moment capacity of the section is used.

4.0 DESIGN OF COMPOSITE BEAMS

4.1 Moment Resistance

4.1.1 Reinforced Concrete Slabs, supported on Steel beams

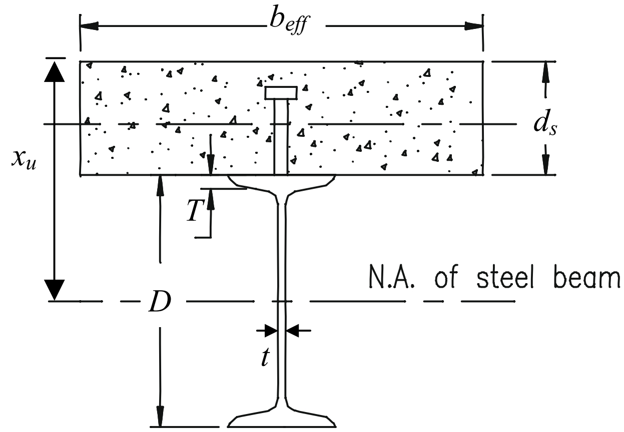


Fig. 12. Notations as per IS: 11384-1985

Reinforced concrete slab connected to rolled steel section through shear connectors is perhaps the simplest form of composite beam. The ultimate strength of the composite beam is determined from its collapse load capacity. The moment capacity of such beams can be found by the method given in *IS:11384-1985*. In this code a parabolic stress distribution is assumed in the concrete slab. The equations used are explained in detail in the previous chapter (Composite Beam-I) and are presented in Table *5*. Reference can be made to Fig. *12* for the notations used in *IS:11384-1985.*

IS: 11384 – 1985, gives no reference to profiled deck slab and partial shear connection. Therefore the equations given in Table *5* can be used only for composite beams without profiled deck sheeting (i.e., steel beam supporting concrete slabs).

Note: 1) Total compressive force in concrete is taken to be $F_{cc}=0.36\,(f_{ck})_{cu}\,b_{eff}\,x_u$ and acting at a depth of $0.42x_u$ from top of slab, where x_u is the depth of plastic neutral axis.

2) $a=\dfrac{0.87\,f_y}{0.36\,(f_{ck})_{cu}}$

Table 5 Moment capacity of composite Section with full shear interaction (according to IS:11384 - 1985)

Position of Plastic Neutral Axis	**Value of x_u**	**Moment Capacity M_p**
Within slab	$x_u = a\,A_a / b_{eff}$	$M_p = 0.87 A_a f_y\,(d_c + 0.5 d_s - 0.42\,x_u)$
Plastic neutral axis in steel flange	$x_u = d_s + \dfrac{(aA_a - b_{eff} d_s)}{2Ba}$	$M_p = 0.87 f_y\,[A_a\,(d_c + 0.08\,d_s) - B(x_u - d_s)(x_u + 0.16\,d_s)]$
Plastic neutral axis in web	$x_u = d_s + T + \dfrac{a(A_a - 2A_f) - b_{eff} d_s}{2at}$	$M_p = 0.87 f_y\,A_s\,(d_c + 0.08\,d_s) - 2A_f\,(0.5T + 0.58\,d_s) - 2t(x_u - d_s - T)(0.5\,\mathbf{x_u} + 0.08\,d_s + 0.5\,T)$

4.1.2 Reinforced concrete slabs, with profiled sheeting supported on steel beams

A more advanced method of composite beam construction is one, where profiled deck slabs are connected to steel beams through stud connectors. In this case the steel sheeting itself acts as the bottom reinforcement and influences the capacity of the section. Table *6* presents the equations for moment capacity according to *EC4*. These equations are largely restricted to sections, which are capable of developing their plastic moment of resistance without local buckling problems. These equations are already discussed in the previous chapter. Fig *13* shows the stress distribution diagram for plastic and compact sections for full interaction according to *EC4*. Fig *14* shows the stress distribution for hogging bending moment.

The notations used here are as follows: -

A_a =area of steel section
γ_a = partial safety factor for structural steel
γ_c = partial safety factor for concrete
b_{eff} =effective width of flange of slab
f_y =yield strength of steel
$(f_{ck})_{cy}$ =characteristic (cylinder) compressive strength of concrete
(f_{sk}) = yield strength of reinforcement.
h_c =distance of rib from top of concrete
h_t =total depth of concrete slab
h_g =depth of centre of steel section from top of steel flange

Note: Cylinder strength of concrete $(f_{ck})_{cy}$ is usually taken as *0.8* times the cube strength $(f_{ck})_{cu}$.

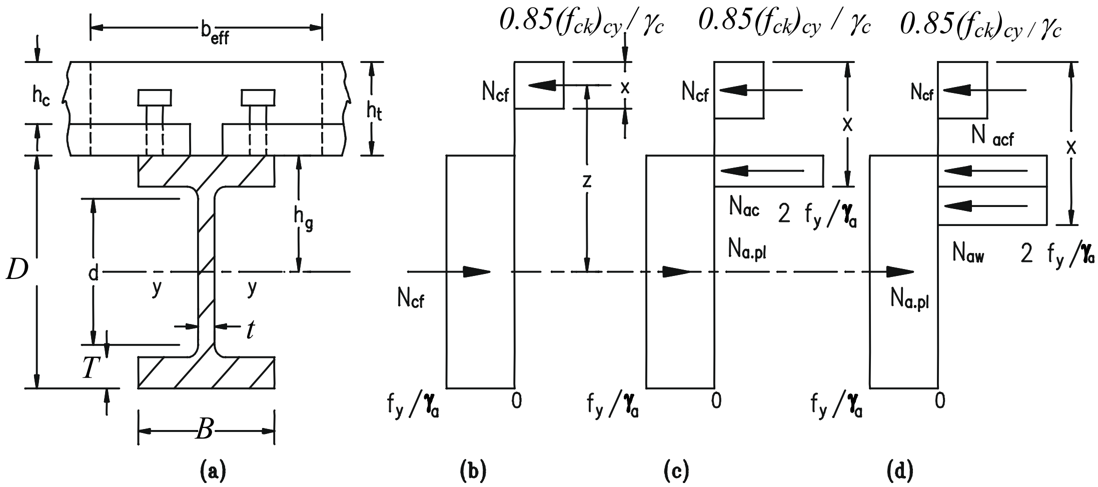


Fig.13. Resistance to sagging bending moment in plastic or compact sections for full interaction.

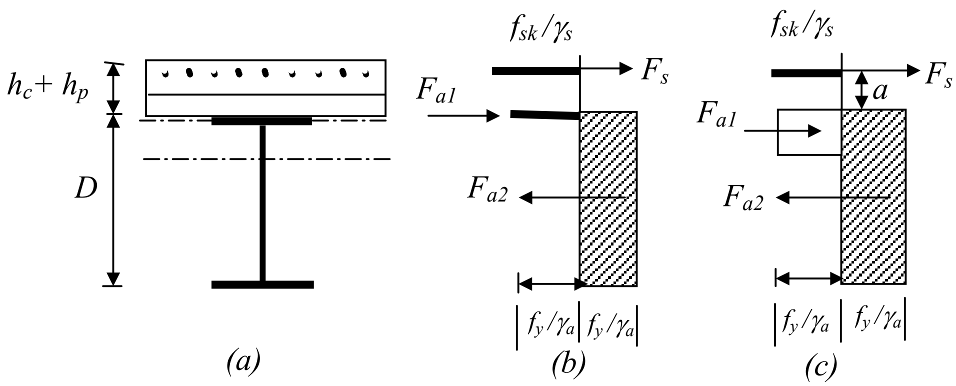


Fig. 14 Resistance to hogging Bending Moment

Table 6 Positive moment capacity of section with full shear connection(According to EC4)

Position of Plastic Neutral Axis	Condition	Moment Capacity M_p
Plastic neutral axis in concrete slab (Fig.*13b*)	$0.85\,\frac{(f_{ck})_{cy}}{\gamma_c}\,b_{eff}\,h_c \geq \frac{A_a f_y}{\gamma_a}$	$M_p = \frac{A_a f_y}{\gamma_a}\,(h_g + h_t - x/2)$
Plastic neutral axis in steel flange (Fig. *13c)*	$b_{eff}\,h_c\,\frac{0.85(f_{ck})_{cy}}{\gamma_c} < \frac{A_a f_y}{\gamma_a}$	$M_p = N_{a.pl}\,(h_g + h_t - h_c/2) - N_{ac}\,(x - h_c + h_t)/2$
Plastic neutral axis in web (Fig.*13d)*	$b_{eff}\,h_c\,\frac{0.85(f_{ck})_{cy}}{\gamma_c} + B*T*f_y/\gamma_a < \frac{A_a f_y}{\gamma_a}$	$M_p = N_{a.pl}\,(h_g + h_t - h_c/2) - N_{acf}\,(h_t + T/2 - h_c/2) - N_{a.w}\,(x + h_t + T - h_c)/2$

Table 7 Negative moment capacity of section with full shear connection (according EC4)

Position of Plastic Neutral Axis	Condition	Moment Capacity M_p
Plastic neutral axis in steel flange (Fig.*14b*)	$\frac{A_{aw} f_y}{\gamma_a} < \frac{A_s f_{sk}}{\gamma_s} < \frac{A_a f_y}{\gamma_a}$	$M_p \approx \frac{A_a f_y}{\gamma_a}\frac{D}{2} + \frac{A_s f_{sk}}{\gamma_s}a$
Plastic neutral axis in web (Fig. *14c)*	$\frac{A_s f_{sk}}{\gamma_s} < \frac{A_{aw} f_y}{\gamma_a}$	$M_p = M_{ap} + \frac{A_s f_{sk}}{\gamma_s}\left(\frac{D}{2} + a\right) + \left(\frac{A_s f_{sk}}{\gamma_s}\right)^2 \Big/ 4*t*f_y/\gamma_a$

4.2 Vertical Shear

In a composite beam, the concrete slab resists some of the vertical shear. But there is no simple design model for this, as the contribution from the slab is influenced by whether it is continuous across the end support, by how much it is cracked, and by the local details of the shear connection. It is therefore assumed that the vertical shear is resisted by steel beam alone, exactly as if it were not composite.

The shear force resisted by the structural steel section should satisfy:

$$V \le V_p \qquad (1)$$

where, V_p is the plastic shear resistance given by,

$$V_p = 0.6\,D\,t\,\frac{f_y}{\gamma_a} \qquad \text{(for rolled I, H, C sections)} \qquad (2)$$

$$= d\,t\,\frac{f_y}{\gamma_a\sqrt{3}} \qquad \text{(for built up I sections)} \qquad (3)$$

In addition to this the shear buckling of steel web should be checked.

The shear buckling of steel web can be neglected if following condition is satisfied

$$\frac{d}{t} \le 67\,\in \qquad \text{for web not encased in concrete} \qquad (4)$$

$$\frac{d}{t} \le 120\,\in \qquad \text{for web encased in concrete} \qquad (5)$$

where, $\in = \sqrt{\dfrac{250}{f_y}}$

d is the depth of the web considered in the shear area.

4.3 Resistance of shear connectors

The design shear resistance of shear connectors for slab without profiled steel decking according to *EC4* and *IS:11384-1985* was already explained in the previous chapter.

4.3.1 Effect of shape of deck slab on shear connection.

The profile of the deck slab has a marked influence on strength of shear connector. There should be a *45°* projection from the base of the connector to the core of the solid slab for smooth transfer of shear. But the profiled deck slab limits the concrete around the connector. This in turn makes the centre of resistance on connector to move up, initiating a local concrete failure as cracking. This is shown in Fig *15*. *EC 4* suggests the following reduction factor *k* (relative to solid slab).

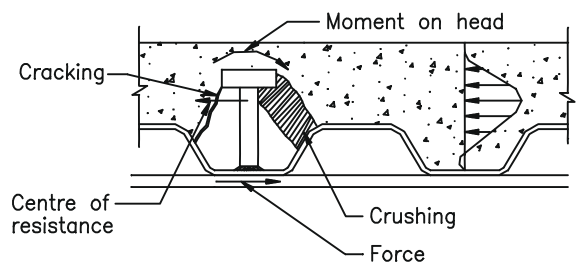


Fig. 15. Behaviour of a shear connection fixed through profile sheeting

(1) Profiled steel decking with the ribs <u>parallel</u> to the supporting beam.

$$k_p = 0.6\frac{b_0}{h_p}\left(\frac{h-h_p}{h_p}\right) \le 1.0 \ \ where \ \ h \le h_p + 75 \qquad (6)$$

(2) Profiled steel decking with the ribs <u>transverse</u> to the supporting beam.

For studs of diameter not exceeding *20 mm,*

$$k_t = \frac{0.7}{\sqrt{N_r}}\frac{b_0}{h_p}\left(\frac{h-h_p}{h_p}\right) \le 1.0 \ \ where \ \ h_p \le 85 \ and \ b_0 \ge h_p \qquad (7)$$

where,

- b_0 is the average width of trough
- h is the stud height
- h_p is the height of the profiled decking slab
- N_r is the number of stud connectors in one rib at a beam intersection (should not greater than *2*).

For studs welded through the steel decking, k_t should not be greater than *1.0* when $N_r=1$, and not greater than *0.8* when $N_r \ge 2$

4.4 Longitudinal Shear Force

4.4.1 Full Shear Connection

(1) Single span beams

For single span beams the total design longitudinal shear, V_λ to be resisted by shear connectors between the point of maximum bending moment and the end support is given by:

$$V_\lambda = F_{cf} = A_a f_y/\gamma_a \quad \text{or} \quad V_\lambda = 0.85\,(f_{ck})_{cy}\, b_{eff}\, h_c/\gamma_c \qquad (8)$$

whichever is smaller.

(2) Continuous Span Beams

For continuous span beams the total design longitudinal shear, V_λ to be resisted by shear connectors between the point of maximum positive bending moment and an intermediate support is given by:

$$V_\lambda = F_{cf} + A_s f_{sk}/\gamma_s \qquad (9)$$

where, A_s - the effective area of longitudinal slab reinforcement

The number of required shear connectors in the zone under consideration for full composite action is given by:

$$n_f = V_\lambda/P$$

where

V_λ is the design longitudinal shear force as defined in equation (8)
P design resistance of the connector.

The shear connectors are usually equally spaced.

4.4.2 Minimum degree of shear connection

Ideal plastic behaviour of the shear connectors may be assumed if a minimum degree of shear connection is provided, as the opportunity for developing local plasticity are greater in these cases

The minimum degree of shear connection is defined by the following equations:

(1) $n/n_f \geq 0.4 + 0.03\lambda \quad where\ 3A_t \geq A_b$
(2) $n/n_f \geq 0.25 + 0.03\lambda \quad where\ A_t = A_b$
(3) $n/n_f \geq 0.04\lambda \quad where\ A_t = A_b$

where

A_t is the top flange area and
A_b is the bottom flange area.
λ beam span in metres

4.5 Interaction between shear and moment

Interaction between bending and shear can influence the design of continuous beam. Fig. *16* shows the resistance of the composite section in combined bending (hogging or sagging) and shear. When the design shear force, V exceeds $0.5V_p$(point *A* in the Fig.), moment capacity of the section reduces non-linearly as shown by the parabolic curve *AB*, in the presence of high shear force. At point *B* the remaining bending resistance M_f is that contributed by the flanges of the composite section, including reinforcement in the slab. Along curve *AB*, the reduced bending resistance is given by

$$M \le M_f + (M_p - M_f)\left[1-\left(2\frac{V}{V_p}-1\right)^2\right] \qquad (10)$$

where

M design bending moment
M_f plastic resistance of the flange alone
M_p plastic resistance of the entire section
V design shear force
V_p plastic shear resistance as defined in equation *(2)* and equation *(3)*.

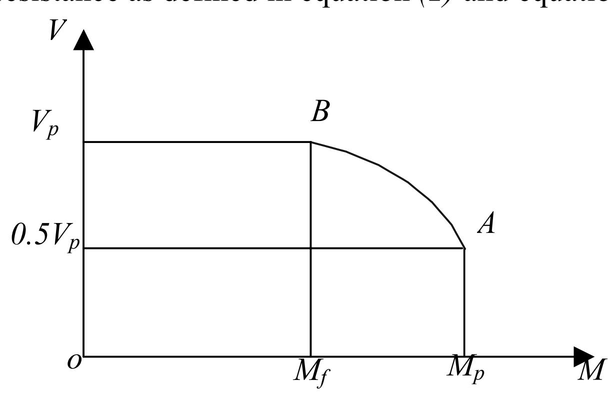


Fig16 Resistance to combined bending and vertical shear

4.6 Transverse reinforcement

Shear connectors transfer the interfacial shear to concrete slab by thrust. This may cause splitting in concrete in potential failure planes as shown in Fig *17*. Therefore reinforcement is provided in the direction transverse to the axis of the beam. Like stirrups in the web of a reinforced T beam, the reinforcement supplements the shear strength of the concrete. A truss model analysis [See Fig. *18*] shows, how the design shear force per unit length V_λ is transferred through concrete struts *AC* and *AB*, causing tension in reinforcement *BC*. Here v_r is the shear resistance of a failure plane as *B-B*. The model gives a design equation of the form

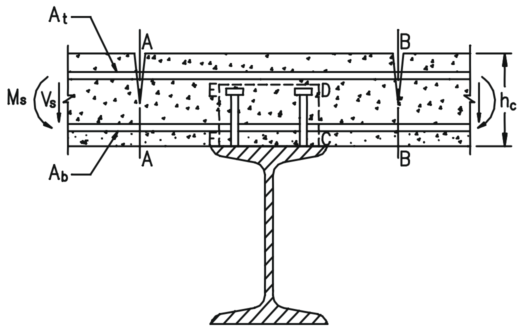


Fig.17. Surfaces of potential shear failure

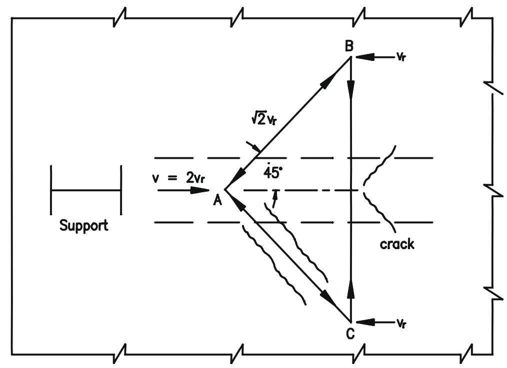


Fig.18. Truss model analysis

$$\frac{V_\lambda}{2} = v_r = A_{cv}\, f\left(\frac{(f_{ck})_{cy}}{\gamma_c}\right) + \frac{A_{sv} f_{sk}}{\gamma_s} \qquad (11)$$

where,

A_{cv} = cross sectional area of concrete shear surface per unit length of beam

A_{sv} = Area of transverse reinforcement.

The formulae suggested by *EC4* and *IS:11384 – 1985* are given in Table 8.

5.0 EFFECT OF CONTINUITY

The above design formulae are applicable to simply supported beams as well as to continuous beams. Besides these, a continuous beam necessitates the check for the stability of the bottom flange, which is in compression due to hogging moments at supports.

5.1 Moment and Shear Coefficients for continuous beam

In order to determine the distribution of bending moments under the design loads, Structural analysis has to be performed. For convenience, the *IS: 456-1978* lists moment coefficients as well as shear coefficients that are close to exact values of the maximum load effects obtainable from rigorous analysis on an infinite number of equal spans on point supports. Table *9* gives the bending moment coefficients and Table *10* gives the shear coefficients according to *IS: 456-1978*. These coefficients are applicable to continuous beams with at least three spans, which do not differ by more than *15* percent of the longest. These values are also applicable for composite continuous beams.

Table8 Comparison of EC4 and IS:11384 – 1985 provisions for transverse reinforcement

EC4	***IS 11384 – 1985***
$v_r = 2.5\, A_{cv} \eta \tau + A_e f_{sk} / \gamma_s + v_{pd}$ or $v_r = 0.2\, A_{cv} \eta (f_{ck})_{cy} / \gamma_c + V_{pd} / \sqrt{3}$ A_e is the sum of the cross sectional areas of transverse reinforcement (assumed to be perpendicular to the beam) per unit length of beam crossing the shear surface under consideration including any reinforcement provided for bending of the slab. A_{cv} mean cross sectional area per unit length of the beam of the concrete shear surface under consideration. $\eta = 1$ for normal weight concrete $\eta = 0.3 + 0.7(\rho/24)$ for light weight concrete τ basic shear strength to be taken as 0.25 f_{ctk}/γ_c, where f_{ctk} is the characteristic tensile strength of concrete. V_{pd} contribution of profiled steel sheeting, if any $= A_p f_{yp} / \gamma_{ap}$ (for ribs running perpendicular to the beam) $= P_{pb} / s$ but $\leq A_p f_{yp} / \gamma_{ap}$ (for ribs running parallel to the beam) P_{pd} design resistance of the headed stud against headed stud against tearing through the steel sheet. A_p cross-sectional area of the profile steel sheeting per unit length of the beam f_{yp} yield strength of steel sheeting.	$v_r = N_c . F_c / s < 0.232\, L_s \sqrt{(f_{ck})_{cu}} +$ $0.1\, A_{sv} f_y n < 0.623\, L_s \sqrt{(f_{ck})_{cu}}$ where, N_c is the number of a shear connector at a section F_c – Load in kN on one connector at ultimate load s – Spacing of connectors in m L_s - Length of shear surface (mm as shown in Fig.(5d) of previous chapter but $2d_s$ for T - beam d_s for L – beam A_{sv} = Area of transverse reinforcement in cm per metre of beam. $n = 2$ for T beam $n = 1$ for L – beams [n is the number of times each lower transverse reinforcement intersects shear surface.]

s is the spacing centre to centre of the studs along the beam	

Table9 Bending moment coefficients according to IS: 456-1978

TYPE OF LOAD	SPAN MOMENTS		SUPPORT MOMENTS	
	Near middle span	At middle of interior span	At support next to the end support	At other interior supports
Dead load + Imposed load (fixed)	+ 1/12	+1/24	- 1/10	- 1/12
Imposed load (not fixed)	+1/10	+1/12	- 1/9	- 1/9
For obtaining the bending moment, the coefficient shall be multiplied by the total design load and effective span.				

Table 10 Shear force coefficients

TYPE OF LOAD	At end support	At support next to the end support		At all other interior supports
		Outer side	Inner side	
Dead load + Imposed load(fixed)	0.40	0.60	0.55	0.50
Imposed load(not fixed)	0.45	0.60	0.60	0.60
For obtaining the shear force, the coefficient shall be multiplied by the total design load				

5.2 Lateral Torsional Buckling of Continuous Beams

The concrete slab is usually assumed to prevent the upper flange of the steel section from moving laterally. In negative moment regions of continuous composite beams the lower flange is subjected to compression. Hence, the stability of bottom flange should be checked at that region. The tendency of the lower flange to buckle laterally is restrained by the distortional stiffness of the cross section. The tendency for the bottom flange to displace laterally causes bending of the steel web, and twisting at top flange level, which is resisted by bending of the slab as shown in Fig. *19*.

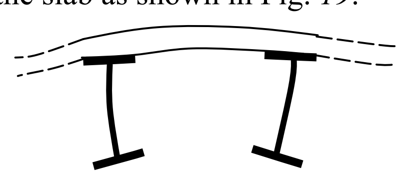


Fig 19 Inverted – U frame Action

Local-Torsional Buckling of Continuous Beams can be neglected if following conditions are satisfied.

1. adjacent spans do not differ in length by more than *20%* of the shorter span or where there is a cantilever, its length does not exceed *15%* of the adjacent span.
2. the loading on each span is uniformly distributed and the design permanent load exceeds *40%* of the total load.

3. the shear connection in the steel-concrete interface satisfies the requirements of section *4.4*

4. $h_a \leq 550\ mm$

6.0 SERVICEABILITY

Composite beams must also be checked for adequacy in the Serviceability Limit State. It is not desirable that steel yields under service load. To check the composite beams serviceability criteria, elastic section properties are used.

IS:11384-1985 limits the maximum deflection of the composite beam to $\lambda/325$. The total elastic stress in concrete is limited to $(f_{ck})_{cu} / 3$ while for steel, considering different stages of construction, the elastic stress is limited to $0.87\, f_y$. Unfortunately this is an error made in the Code as the same limits are applied for steel in determining the ultimate resistance of the cross section. Since *EC4* gives explicit guidance for checking serviceability Limit State, therefore the method described below follows *EC 4*.

6.1 Deflection

The elastic properties relevant to deflection are section modulus and moment of inertia of the section. Applying appropriate modular ratio *m* the composite section is transformed into an equivalent steel section. The moment of inertia of uncracked section is used for calculating deflection. Normally unfactored loads are used for for serviceability checks. No stress limitations are made in *EC 4*.

Under positive moment the concrete is assumed uncracked, and the moment of inertia is calculated as:

$$I = \frac{A_a\left(h_c + 2h_p + h_a\right)^2}{4\left(1 + mr\right)} + \frac{b_{eff}\, h_c^{\ 3}}{12m} + I_a \qquad (12)$$

where

m is the ratio of the elastic moduli of steel to concrete taking into account creep.

$$r = \frac{A_a}{b_{eff}\, h_c}$$

I_a is the moment of inertia of steel section.

6.1.1 Simply supported Beams

The mid-span deflection of simply supported composite beam under distributed load w is given by

$$\delta_c = \frac{5w\lambda^4}{384\, E_a I} \tag{13}$$

where, E_a is the modulus of elasticity of steel.
I is the gross uncracked moment of inertia of composite section.

6.1.2 Influence of partial shear connection

Deflections increase due to the effects of slip in the shear connectors. These effects are ignored in composite beams designed for full shear connection. To take care of the increase in deflection due to partial shear connection, the following expression is used.

$$\frac{\delta}{\delta_c} = 1 + 0.5\left(1 - \frac{n_p}{n_f}\right)\left(\frac{\delta_a}{\delta_c} - 1\right) \quad \text{for propped construction} \tag{14}$$

$$\frac{\delta}{\delta_c} = 1 + 0.3\left(1 - \frac{n_p}{n_f}\right)\left(\frac{\delta_a}{\delta_c} - 1\right) \quad \text{for unpropped construction} \tag{15}$$

where

δ_a and δ_c are deflection of steel beam and composite beam respectively with proper serviceability load.

Note: For $\frac{n_p}{n_f} \geq 0.5$, this additional simplification can usually be ignored

6.1.3 Shrinkage induced deflections

For simply supported beams, when the span to depth ratio of beam exceeds *20*, or when the free shrinkage strain of the concrete exceeds 400×10^{-6} shrinkage, deflections should be checked. In practice, these deflections will only be significant for spans greater than *12 m* in exceptionally warm dry atmospheres. The shrinkage induced deflection is calculated using the following formula:

$$\delta_s = 0.125 K_s \lambda^2 \tag{16}$$

where

λ is the effective span of the beam.

K_s is the curvature due to the free shrinkage strain, $\in_s$ given by

$$K_s = \frac{\in_s \left(h + h_c + 2h_p\right)A_a}{2\left(1 + mr\right)I_c} \qquad (17)$$

m modular ratio appropriate for shrinkage calculations (*m=20*)
Note: This formula ignores continuity effects at the supports.

6.1.4 Continuous Beams

In the case of continuous beam, the deflection is modified by the influence of cracking in the hogging moment regions (at or near the supports). This may be taken into account by calculating the second moment of area of the cracked section under negative moment (ignoring concrete). In addition to this there is a possibility of yielding in the negative moment region. To take account of this the negative moments may be further reduced. As an approximation, a deflection coefficient of *3/384* is usually appropriate for determining the deflection of a continuous composite beam subject to uniform loading on equal adjacent spans. This may be increased to *4/384* for end spans. The second moment of area of the section is based on the uncracked value.

6.1.5 Crack Control

Cracking of concrete should be controlled in cases where the functioning of the structure or its appearance would be affected. In order to avoid the presence of large cracks in the hogging moment regions, the amount of reinforcement should not exceed a minimum value given by,

$$p = \frac{A_s}{A_c} = k_c * k * \frac{f_{ct}}{\sigma_s} \qquad (18)$$

where

p is the percentage of steel
k_c is a coefficient due to the bending stress distribution in the section($k_c \approx 0.9$)
k is a coefficient accounting for the decrease in the tensile strength of concrete ($k \approx 8$)
f_{ct} is the effective tensile strength of concrete. A value of $3\ N/mm^2$ is the minimum adopted.
σ_s is the maximum permissible stress in concrete.

7.0 CONCLUSION

This chapter summarises the method of design of composite beams, connected to solid slab, as well as profiled deck slab. Two design examples follow this chapter, where designs of simply supported and continuous composite beams have been presented in detail. The design of simply supported beam follows *IS:11384-1985* whereas, the design of continuous beam follows *EC4*.

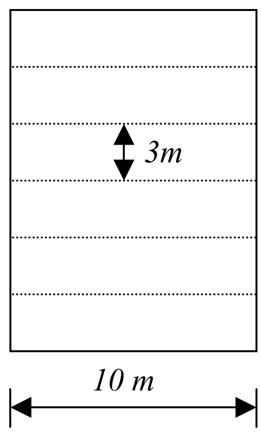
8.0 REFERENCES

1. Lawson, R.M., " Design of composite slabs and beams with steel decking", SCI Publications, P050, 1993.
2. Owens, G. W., " Design of fabricated composite beams in buildings", SCI Publications P059, 1989.
3. Lawson, R.M. and Rackham, J. W., " Design of haunched composite beams in buildings", SCI Publications, P060, 1989.
4. Brett, P., and Rushton, J., " Parallel beam approach- a design guide", SCI Publications, P074, 1990.
5. Knowles, P.R., "Design of castellated beams", SCI Publication P005, 1985.
6. Lawson, R. M. and McConnel, R., " Design of stub girders", SCI Publications, P118, 1993.
7. Merrill, S. K.," Design of composite trusses", SCI Publications, P083, 1992.
8. ENV 1994-1-1: EC4: Design of composite steel and concrete structures, Part 1.1: General rules and rules for buildings.
9. Johnson, R. P., "Composite Structures of Steel and Concrete", Oxford Blackwell Scientific Publications, London, 1994
10. *IS:11384-1985*, Code of Practice for Composite Construction in Structural Steel and Concrete.

Structural Steel Design Project Calculation Sheet	Job No:	Sheet ***1 of 11***	Rev
	Job Title: ***Design of simply supported Beam***		
	Worked Example: ***1***		
		Made By ***IB***	Date
		Checked By *PU*	Date

PROBLEM 1

Design a simply supported composite beam with 10m span shown (dotted line) in the figure below. The thickness of slab is 125 mm. The floor is to carry an imposed load of 3.0 kN/m^2, partition load of 1.5 kN/m^2 and a floor finish load of 0.5 kN/m^2



Given Data

Imposed load	*3.0 kN/m^2*
Partition load	*1.5 kN/m^2*
Floor finish load	*0.5 kN/m^2*
Construction load	*0.75 kN/m^2*

Data assumed

$(f_{ck})_{cu}$	*30 N/mm^2*
f_y	*250 N/mm^2*
Density of concrete	*24 kN/m^3*

Partial safety factors

Load Factor, γ_f

for LL	*1.5*
for DL	*1.35*

Material Factor, γ_m

Steel	*1.15*
Concrete	*1.5*
Reinforcement	*1.15*

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Step 1: Load Calculation

Construction stage

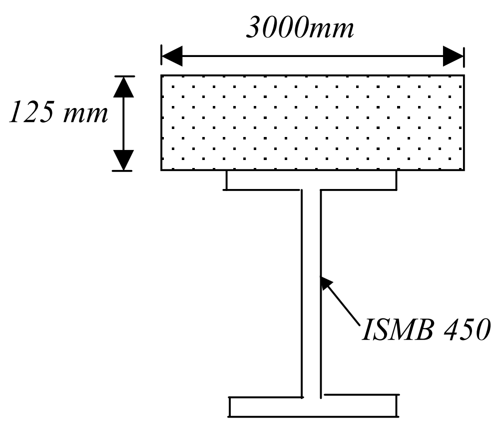
i) Self weight of slab = 3 * 0.125 * 24 = 9 kN/m

ii) Self weight of beam = 0.71 kN/m (assuming ISMB 450)

iii) Construction load = 0.75 * 3 = 2.25 kN/m

Total design load at Construction Stage

= {1.5 * 2.25 + 1.35 * (9 + 0.71) =16.5 kN/m



Composite stage

Dead Load

i) Self weight of slab = 9 kN/m

ii) Self weight of beam = 0.71 kN/m

iii) Load from floor finish = 0.5 * 3 =1.5 kN/m

Total Dead Load = 11.2 kN/m

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Live Load

*i) Imposed load = 3 * 3 = 9.0 kN/m*
*ii) Load from partition wall = 1.5 * 3 = 4.5 kN/m*

Total Live Load = 13.5 kN/m

*Design load carried by composite beam= (1.35 * 11.2 + 1.5 * 13.5) = 35.4 kN/m*

Step 2: Calculation of Bending Moment

Construction Stage

$M = 16.5 * 10^2/8 = 206$ kNm

Composite Stage

$M = 35.4 * 10^2/8 = 442$ kNm

Step 3: Classification of Composite Section

Sectional Properties

$T = 17.4$mm;
$D = 450$ mm;
$t = 9.4$ mm
$I_x = 303.9 * 10^6$ mm^4
$I_y = 8.34 * 10^6$ mm^4
$Z_x = 1350*10^3$ mm;
$r_y = 30.1$ mm

Classification of composite section — *Refer Table 4*

$0.5\ B/T=0.5*150/17.4=4.3< 8.9\varepsilon$
$d/t=(450-2*17.4)/9.4=44.2 < 83\varepsilon$

Therefore the section is a plastic section.

Step 4: Check for the adequacy of the section at construction stage

Design moment in construction stage = 206 kNm

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Moment of resistance of steel section

$= f_{yd} * Z_p$

$= [(250/1.15) * 1.14 * 1350.7 * 10^3]/10^6 \text{ kNm}$

$= 334.7 \text{ kNm} > 206 \text{ kNm}$

<u>As the top flange of the steel beam is unrestrained and under compression, stability of the top flange should be checked.</u>

Step 5: Check for Lateral Buckling of the top flange

From clause 6.2.4, IS:800-1984

Elastic critical stress, f_{cb} is given by

$$f_{cb} = k_1 \frac{c_2}{c_1} \frac{26.5*10^5}{\left(\frac{\lambda}{r_y}\right)^2} \left[\sqrt{1 + \frac{1}{20}\left(\frac{\lambda}{r_y}\frac{T}{D}\right)^2} + k_2 \right]$$

$k_1 = 1$ (as $\Psi = 1.0$)

$k_2 = 0$ (as $\phi = 0.5$)

$c_2 = c_1 = 225$ mm;

$T = 17.4$ mm;

$D = 450$ mm;

$\lambda = 10{,}000$ mm;

$r_y = 30.1$ mm

$$f_{cb} = \frac{26.5*10^5}{\left(\frac{10000}{30.1}\right)^2} \left[\sqrt{1 + \frac{1}{20}\left(\frac{10000*17.4}{30.1*450}\right)^2} \right] = 73\, N/mm^2$$

Therefore the bending compressive stress in beams

$$F_{cb} = \frac{f_{cb}*f_y}{\left[(f_{cb})^{1.4} + (f_y)^{1.4}\right]^{\frac{1}{1.4}}} = 64.9\, N/mm^2$$

Moment at construction stage = 206 kNm

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Maximum stress at top flange of steel section

$$F_{cb} = \frac{206*10^6*225}{303.9*10^6} = 152.5\ N/mm^2 > 64.9\ N/mm^2$$

So, we have to reduce the effective length of the beam.

Provide 2 lateral restraints with a distance of approximately 3330 mm between them

From clause 6.2.4, IS:800-1984

$$f_{cb} = \frac{26.5*10^5}{\left(\frac{3330}{30.1}\right)^2}\left[\sqrt{1+\frac{1}{20}\left(\frac{3330*17.4}{30.1*450}\right)^2}\right] = 299.6\ N/mm^2$$

Therefore the bending compressive stress in beams

$$F_{cb} = \frac{299.6*250}{\left[(299.6)^{1.4}+(250)^{1.4}\right]^{\frac{1}{1.4}}} = 165.9\ N/mm^2$$

F_{cb}=165.9 >152.5 N/mm^2

Note: These restraints are to be kept till concrete hardens.

Step6: Check for adequacy of the section at Composite stage

Bending Moment at the composite Stage, M = 442 kNm

Effective breadth of slab is smaller of

I. span /4 = 10000/4 = 2500 mm

II. C/C distance between beams = 3000 mm

Hence, b_{eff} = 2500 mm

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Position of neutral axis

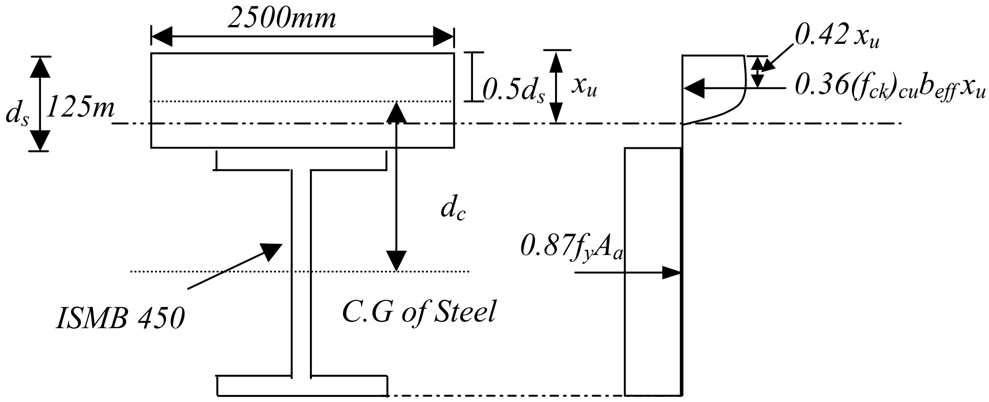
$$a = \frac{0.87 f_y}{0.36\left(f_{ck}\right)_{cu}} = \frac{0.87*250}{0.36*30} = 20.1$$

$A_a = 9227\ mm^2$

$a\,A_a = 20.1*\,9227 = 1.85 * 10^5\ mm^2$

$b_{eff}\,d_s = 2500 * 125 = 3.13 * 10^5\ mm^2 > aA_a$

Hence PNA lies in concrete



Position of neutral axis

$$x_u = \frac{0.87*9227*250}{0.36*30*2500} = 74.3\ mm \quad \text{from the top of the slab}$$

Moment Resistance of the section, M_p

$M_p = 0.87A_a f_y(d_c+0.5d_s-0.42x_u)$

$= 0.87*9227*250(287.5+0.5*125-0.42*74.3)$

$= 640\ \ kNm > 442\ kNmm$

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Step7 : Design of shear connectors

The position of neutral axis is within slab.

$\therefore$ *Total load carried by connectors*

$$F_{cc} = 0.36(f_{ck})_{cu}\, b_{eff}\, x_u = (0.36 * 30 * 2500 * 74.3)/1000 \text{ kN}$$
$$= 2006 \text{ kN}$$

As per Table 1(Composite Beam-II), the design strength of 20 mm (dia) headed stud for M30 concrete is 58 kN

$\therefore$ *Number of shear connectors required for 10/2 m = 5 m length*

$$= 2006 / 58 \approx 34$$

These are spaced uniformly

Spacing = 5000/34 = 147 mm $\approx$ *145 mm*

*If two connectors are provided in a row the spacing will be = 145 * 2 = 290 mm*

Step8: Serviceability check

Modular ratio for live load = 15

Modular ratio for deal load = 30

(1) <u>Deflection</u>

For <u>dead load</u> deflection is calculated using moment of inertia of steel beam only

$$\delta_d = \frac{5*9.71*(10000)^4}{384*2*10^5*303.91*10^6} = 20.8 \text{ mm}$$

For <u>live load</u> deflection is calculated using moment of inertia of composite section

To find the moment of inertia of the composite section we have to first locate the position of neutral axis.

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<u>Position of neutral axis</u>

$A\,(d_g - d_s) < \frac{1}{2}(b_{eff}/\alpha_e)\,d_s^2$

$9227\,(350 - 125) < \frac{1}{2} * 2500/15 * 125^2$

$2.08 * 10^6 < 1.3 * 10^6$ *which is not true*

$\therefore$ *N.A. depth exceeds* d_s

$$A_a\left(d_g - x_u\right) = \frac{b_{eff}}{m} d_s \left(x_u - \frac{d_s}{2}\right)$$

$$9227\left(\frac{450}{2} + 125 - x_u\right) = \frac{2500}{15} * 125 * \left(x_u - \frac{125}{2}\right)$$

$$x_u = 150.75\,mm$$

Moment of inertia of the gross section, I_g

$$I_g = I_x + A_a\left(d_g - x_u\right)^2 + \frac{b_{eff}}{\alpha_e} d_s\left[\frac{d_s^{\ 2}}{12} + \left(x_u - d_s\right)^2\right]$$

$$= 303.91*10^6 + 9227\left(350 - 150.75\right)^2 + \frac{2500*125}{15}\left[\frac{125^2}{12} + \left(150.75 - \frac{125}{2}\right)^2\right]$$

$$= 859.6*10^6\ mm^4$$

$$\delta_l = \frac{5*15*\left(10000\right)^4}{384*2*10^5*859.6*10^6} = 11.4\ mm$$

$$\therefore Total\ Deflection = \delta_d + \delta_l = 20.8 + 11.4\ mm$$

$$= 32.2\ mm > \frac{\lambda}{325}$$

The section fails to satisfy the deflection check.

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(2) Stresses

Composite Stage

Dead Load

In composite stage, dead load W_d

$W_d = 11.2$ kN/m

$M = 11.2 * 10^2/8 = 140$ kNm

Position of neutral axis

Assuming neutral axis lies within the slab

$A\,(d_g - d_s) < ½\, b_{eff}.\, d_s^2/\alpha_e$

$9227\,(350 - 125) < ½ * 2500/30 * 125^2$

$2.07 * 10^6 > 6.5 * 10^5$

$\therefore$ N.A. depth exceeds d_s

Modular ratio for dead load, $\alpha_e = 30$

Location of neutral axis

$$A_a\left(d_g - x_u\right) = \frac{b_{eff}}{m} d_s\left(x_u - \frac{d_s}{2}\right)$$

$$9227\left(\frac{450}{2} + 125 - x_u\right) = \frac{2500}{30} * 125 * \left(x_u - \frac{125}{2}\right)$$

$x_u = 197.5$mm

Moment of Area of the section

$$I_g = I_x + A_a\left(d_g - x_u\right)^2 + \frac{b_{eff}\,d}{m}{}_s\left[\frac{d_s^{\,2}}{12} + \left(x_u - d_s\right)^2\right]$$

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$$I_g = 303.91*10^6 + 9227(350-197.5)^2 + \frac{2500*125}{30}\left[\frac{125^2}{12} + \left(197.5 - \frac{125}{2}\right)^2\right]$$

$$= 721.9*10^6\ mm^4$$

$$\text{Stress in steel flange} = \frac{140*10^6(450+125-197.5)}{721.9*10^6} = 73.2\ N/mm^2$$

Live load

In composite stage stress in steel for live load

W_l = 13.5 kN/m

$M = 13.5*10^2/8 = 168.75\ kNm$

$$\text{Stress in steel flange} = \frac{168.75*10^6(450+125-150.75)}{859.6*10^6} = 83.29\ N/mm^2$$

$\therefore$ *Total stress in steel* $= 73.2 + 83.29 = 156.5\ N/mm^2 <$ *allowable stress in steel*

In a similar procedure the stress in concrete is found.

$$\frac{1}{30}\left\{\frac{140*10^6*197.54}{721.9*10^6}\right\} + \frac{1}{15}\left\{\frac{168.75*10^6*150.75}{859.6*10^6}\right\} = 3.25 < \frac{(f_{ck})_{cu}}{3} = 10\ N/mm^2$$

The section is safe.

Since the section does not satisfy the deflection check, therefore trial can be made with higher steel section

Step 9: Transverse reinforcement

Shear force transferred per metre length

$$v_r = \frac{2*58}{0.29}\ kN/m \quad (n = 2, \text{Since there are two shear studs})$$

$$= 400\ kN/m$$

Refer Table 6

$$v_r \le 0.232 L_s \sqrt{(f_{ck})_{cu}} + 0.1 A_{sv} f_y n$$

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or

$0.632 L_s \sqrt{(f_{ck})_{cu}}$

$L_s = 2*125 = 250\,mm$

$f_y = 250\,mm$

$n = 2$

$\therefore 0.232 L_s \sqrt{(f_{ck})_{cu}} + 0.1 A_{sv} f_y n = 0.232*250\sqrt{30} + 0.1*A_{sv}*250*2$

$= 317.7 + 50 A_{sv}$

or

$0.632*250\sqrt{30} = 865\,kN/m$

$\therefore 400 = 317.7 + 50 A_{sv}$

$= 165\ mm^2/m$

Minimum reinforcement

$= 250 v_r / f_y\ mm^2/m$

$= 400\ mm^2/m$

Provide 12 mm φ @280 mm c/c.

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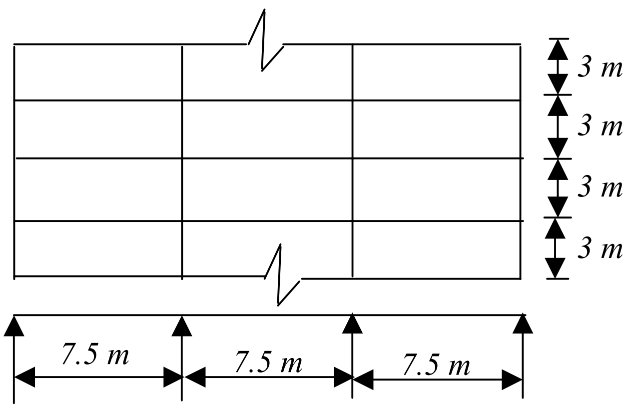
PROBLEM 2

A composite floor slab is supported on three span continuous composite beams spaced at 3 m centres. The effective length of each span being 7.5 m. The thickness of composite slab is 130 mm. The floor has to carry an imposed load of 3.5 kN/m^2, partition load of 1.0 kN/mm^2 and a floor finish load of 0.5 kN/m^2. Design the continuous beam.



Step 1: List of Datas

Given:

Imposed Load $=3.5\ kN/m^2$
Partition Load $=1.0\ kN/m^2$
Floor finish Load $=0.5\ kN/m^2$
Construction Load $=0.5\ kN/m^2$

Assumed:

$(f_{ck})_{cu}=30\ N/mm^2$; $f_y = 250\ N/mm^2$; $f_{sk}=415\ N/mm^2$

Density of concrete$=24\ kN/m^2$

Partial Safety factors:

Load Factor γ_f
for LL 1.5; *for DL* 1.35

Material Factor, γ_m

Steel, $\gamma_a=1.15$; *Concrete,* $\gamma_c=1.5$; *Reinforcement,* $\gamma_s=1.15$

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Step2: Load Calculation

Construction stage

Dead Load

*Self weight of slab=3*0.13*24=9.36 kN/m*

Self weight of beam =0.44 kN/m (assuming ISMB 300)

Total dead load = 9.8 kN/m

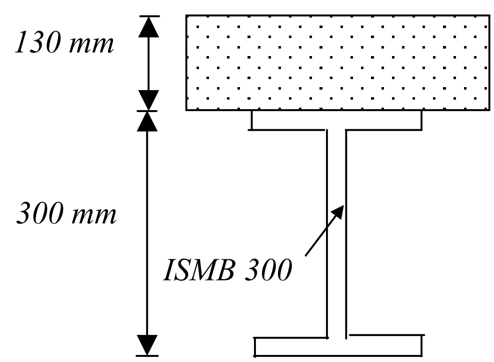
Total design dead load =1.35(9.8)=13.2 kN/m*

Live Load

Construction Load =0.5 3=1.5 kN/m*

*Total design live load =1.5*1.5=2.25 kN/m*

Composite Stage



Dead load

*Self weight of slab=3*0.13* 24=9.36 kN/m*

Self weight of beam=0.44 kN/m

Load from floor finish =0.5 3=1.5 kN/m*

Total dead load = 11.3 kN/m

*Total design dead load =1.35*11.3=15.3 kN/m*

Live Load

*Imposed Load =3.5*3=10.5 kN/m*

*Partition Load =1.0*3=3.0 kN/m*

Total design live load =1.5(13.5)=20.3 kN/m*

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Step3: Bending Moment and Shear Force Calculation

Construction Stage

$$Maximum\ Positive\ Moment\ = \frac{w_d\lambda^2}{12} + \frac{w_l\lambda^2}{10}$$

$$= \frac{13.2*7.5^2}{12} + \frac{2.25*7.5^2}{10} = 74.5\,kNm$$

$$Maximum\ Negative\ Moment\ = -\left(\frac{w_d\lambda^2}{10} + \frac{w_l\lambda^2}{9}\right)$$

$$= -\left(\frac{13.2*7.5^2}{10} + \frac{2.25*7.5^2}{9}\right) = -88.3\,kNm$$

$$Maximum\ Shear\ force\ = 0.6(w_d\lambda + w_l\lambda)$$

$$= 0.6*7.5*(13.2 + 2.25) = 69.5\ kN$$

Composite Stage

$$Maximum\ Positive\ Moment\ = \left(\frac{w_d\lambda^2}{12} + \frac{w_l\lambda^2}{10}\right)$$

$$= \frac{15.3*7.5^2}{12} + \frac{20.3*7.5^2}{10} = 185.9\,kNm$$

$$Maximum\ Negative\ Moment\ = -\left(\frac{w_d\lambda^2}{10} + \frac{w_l\lambda^2}{9}\right)$$

$$= -\left(\frac{15.3*7.5^2}{10} + \frac{20.3*7.5^2}{9}\right) = -212.9\ kNm$$

$$Maximum\ Shear\ force\ = 0.6(w_d\lambda + w_l\lambda)$$

$$= 0.6*7.5*(15.3 + 20.3) = 160.2kN$$

Step4: Selection of steel section

Assuming span/depth =22

Depth of Composite Section=7500 /22=341

Let us take ISMB 300 @ 0.44 kN/m

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Section Properties: $T = 12.4$ mm; $B = 140$ mm $D = 300$ mm; $t = 7.5$ mm $I_x = 86*10^6$ mm^4; $I_y = 4.53*10^6$ mm^4 $r_x = 123.7$ mm ; $r_y = 28.4$ mm $Z_x = 573.6 * 10^3$ mm^3; $Z_y = 64.8 * 10^3$ mm^3	
Classification of composite section $0.5\ B/T = 0.5*140/12.4 = 5.65 < 8.9\in$ $d/t = (300-2*12.4) / 7.5 = 36.7 < 83\in$ Here, $\in = \sqrt{\frac{250}{f_y}}$ *Therefore the section is a plastic section.*	*Refer Table 4*
Step 5: Ultimate Limit State ***[A] Construction Stage*** ***(1) Plastic Moment Resistance of the Steel Section*** $M_{ap} = \frac{f_y}{\gamma_a} Z_{px}$ $= \left(\frac{250}{1.15} * 1.14 * 573.6 * 10^3\right) \Big/ 10^{-6} = 142.2\ kNm > 88.3\ kNm$	$Z_{px} = 1.14 * Z_x$
(2) Plastic Shear Resistance $V_p = 0.6 * D * t * \frac{f_y}{\gamma_a}$ $= \left[0.6*300*7.5*\frac{250}{1.15}\right] / 1000 = 293.5\ kN > 69.5\ kN$	*Refer Section 4.2*

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Bending Moment and Vertical Shear Interaction

Refer Section 4.5

Bending Moment and Vertical Shear Interaction can be neglected if

$V < 0.5\ V_p$

$69.5 < 0.5 * 293.5$

< 146.7 kN

Therefore, vertical shear has no effect on the plastic moment resistance.

(3) Check for Lateral torsional buckling of the steel Beam

The design buckling resistance moment of a laterally unrestrained beam is given by

$$M_b = \chi_{LT} \beta_w Z_{px} \frac{f_y}{\gamma_m}$$

where

χ_{LT} *is the reduction factor for lateral torsional buckling.*

$$= \frac{1}{\left(\phi_{LT} + \sqrt{\phi_{LT}^2 - \bar{\lambda}_{LT}^2}\right)} \le 1.0$$

where

$$\phi_{LT} = 0.5\left(1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0.2\right) + \bar{\lambda}_{LT}^2\right)$$

Here

α_{LT} = *imperfection factor (= 0.21for rolled section)*

$$\bar{\lambda}_{LT} = \sqrt{\frac{\beta_w * Z_{px} * f_y}{M_{cr}}} \text{ (non dimensional slenderness ratio)}$$

where

β_w *is a constant which is equal to 1.0 for plastic section*

M_{cr} *is the elastic critical moment for lateral torsional buckling given by*

$$M_{cr} = \frac{\pi^2 EI_y}{\lambda_e^2}\left[\frac{I_w}{I_y} + \frac{\lambda_e^2 * G * I_t}{\pi^2 EI_y}\right]^{0.5}$$

where

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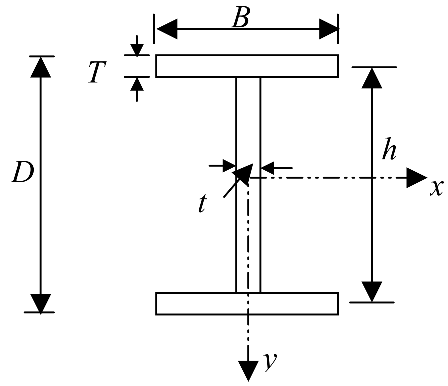
$$G = \frac{E}{2(1+m)} = \frac{2*10^5}{2(1+0.3)} = 76.9*10^3\ N/mm^2$$

$$I_t = \text{torsion constant} = \frac{1}{3}\left(2BT^3 + (D-2T)t^3\right)$$

$$= \frac{1}{3}\left(2*140*12.4^3 + (300 - 2*12.4)*7.5^3\right)$$

$$= 216.6*10^3\ mm^4$$



$$I_w = \text{warping constant} = \frac{I_y h^2}{4}$$

$$= \frac{4.53*10^6*(287.6)^2}{4}$$

$$= 93.9*10^9\ mm^6$$

Assuming two lateral supports @ 2500 mm

$$M_{cr} = \frac{\pi^2*2*10^5*4.53*10^6}{(2500)^2}\left[\frac{93.9*10^9}{4.53*10^6} + \frac{(2500)^2*76.9*10^3*216.6*10^3}{\pi^2*2*10^5*4.53*10^6}\right]^{0.5}$$

$$= 257.7\ kNm$$

$$\bar{\lambda}_{LT} = 0.796;\ \phi_{LT} = 0.879;\ \chi_{LT} = 0.79$$

$$M_b = [\,0.79*1.0*1.14*573.6*10^3*250/1.15]*10^{-6}$$

$$= 112.3 kNm > 88.3\ kNm$$

[B] Composite Stage

(1) Moment Resistance of the cross section

Negative Bending Moment

At internal support negative bending moment of resistance is obtained by considering the tensile resistance of the reinforcement. Concrete area is neglected.

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a) effective width of the concrete flange (Refer section 3.3)

$$b_{eff} = \frac{\lambda_o}{4}$$

$$= \frac{1}{4}\left(0.25(\lambda_1 + \lambda_2)\right) = \frac{1}{4}\left(0.25(7.5 + 7.5)\right) * 1000$$

$$\approx 935 \text{ mm}$$

Let us provide 12mm ϕ bar @ 100 mm c/c

$A_s = 1050 \ mm^2$

(b) Location of neutral axis

$$F_a = A_a \frac{f_y}{\gamma_a} = \left(5626 * \frac{250}{1.15}\right) \Big/ 1000 = 1223 \, kN$$

$$F_s = A_s \frac{f_{sk}}{\gamma_s} = \left(1050 * \frac{415}{1.15}\right) \Big/ 1000 = 379 \, kN < F_a$$

$$\text{Depth of web in tension} = \frac{D}{2} - \frac{F_s}{2t_w * f_y/\gamma_a} = \frac{300}{2} - \frac{379*1000}{2*7.5*250/1.15}$$

$$= 33.8 \text{ mm}$$

Therefore NA lies in the web.

Negative Moment of resistance of the section (Refer Table 6)

$$M_p = p_y * Z_{px} + \frac{A_s f_{sk}}{\gamma_s}\left(\frac{D}{2} + a\right) - \left(\frac{A_s f_{sk}}{\gamma_s}\right)^2 \Big/ 4t_w f_y/\gamma_a$$

$$= \frac{250}{1.15} * 1.14 * 573.6 * 10^3 + \frac{1050 * 415}{1.15}\left(\frac{300}{2} + 109\right) - \left(\frac{1050 * 415}{1.15}\right)^2 \Big/ 4 * 7.5 * \frac{250}{1.15}$$

$$= 218.3 \, kN > 212.9 \text{ kNm}$$

(Assuming clear cover to reinforcement 15 mm)

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Positive Bending Moment

(a) Effective width of the concrete flange

Refer Section 3.3

$$b_{eff} = \frac{\lambda_o}{4}$$

$$= \frac{1}{4}(0.8(\lambda)) = \frac{1}{4}(0.8*7500)$$

$$= 1500\,mm$$

b) Location of neutral axis

$$F_a = A_a \frac{f_y}{\gamma_a} = \left(5626*\frac{250}{1.15}\right) \Big/ 1000 = 1223\ kN$$

$$F_c = 0.85\frac{(0.8*(f_{ck})_{cu})}{\gamma_c}*b_{eff}*h_c = \left(0.85\frac{25}{1.5}*1500*130\right) \Big/ 1000 = 2763\,kN$$

$$F_c > F_a\text{, Hence neutral axis lies in the slab}$$

Depth of neutral axis

$$x_u = A_a \frac{f_y}{\gamma_a} \Big/ 0.85\frac{(0.8*(f_{ck})_{cu})}{\gamma_c}*b_{eff} = 5626*\frac{250}{1.15} \Big/ 0.85\frac{25}{1.5}*1500 = 57.6\ mm$$

Positive Moment of resistance of the section

Refer Table 6

$$M_p = \frac{A_a f_y}{\gamma_a}\left(\frac{D}{2} + h_c - \frac{x_u}{2}\right)$$

$$= \frac{5626*250}{1.15}\left(\frac{300}{2} + 130 - \frac{57.6}{2}\right)*10^{-6}$$

$$= 307.3\,kNm > 185.9\,kNm$$

(2) Check for vertical shear and bending moment and shear force interaction

Refer section 4.5

Vertical shear force, V=160.2 kN < 293.5 kN

Hence safe.

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Bending Moment and Vertical Shear Interaction can be neglected if
$V < 0.5\, V_p$

$V = 162 > 0.5 * 293.5$ *kN*

$$M \leq M_f + \left(M_p - M_f\right)\left[1 - \left(2\frac{V}{V_p} - 1\right)^2\right]$$

refer section 4.4

$$= 2*B*T\left(\frac{D-T}{2}\right)\frac{f_y}{\gamma_a} + \left(M_p - 2*B*T\left(\frac{D-T}{2}\right)\frac{f_y}{\gamma_a}\right)*\left[1 - \left(2\frac{V}{V_p} - 1\right)^2\right]$$

$$= \left(2*140*12.4*\left[\frac{300-12.4}{2}\right]*\frac{250}{1.15}\right)*10^{-6} + \left(236.9 - 108.5\right)*\left[1 - \left(2*\frac{162}{293.5} - 1\right)^2\right]$$

$212.9 < 235.5$ *kNm*

(2) Check for shear buckling

refer section 4.2

$d/t_w = (300-2*12.4)/7.5 = 36.7 < 67\epsilon$, *Hence safe*

[C] Design of shear connectors

<u>Longitudinal shear force</u>

(a) Between simple end support and point of maximum positive moment

Length$=0.4\lambda = 0.4*7500 = 3000$ *mm*

$V_\lambda = F_a = 1223$ *kN*

(b) Between point of maximum positive moment and internal support

Length$=7500-3000 = 4500$ *mm*

$V_\lambda = F_a + A_s f_{sk}/\gamma_s = 1223 + 379 = 1602$ *kN*

Design resistance of shear connectors

Let us provide 22 mm dia. studs 100 mm high, P = 85 kN

Table 1 (composite Beam-I)

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No. of shear connectors

(a) Between simple end support and point of maximum positive moment

Assuming full shear connection,

No. of shear connectors, $n_f = 1223/85 = 15$

$\therefore$ *Spacing* $= 3000 / 15 = 200$ *mm*

b) Between point of maximum positive moment and internal support.

Assuming full shear connection,

No. of shear connectors, $n_f = 1602 /85 = 19$

$\therefore$ *Spacing* $= 4500 / 19 = 230$ *mm*

Let us provide 22 mm dia. Shear Studs @ 200 mm c/c throughout the span.

[D] Transverse reinforcement

Refer Table 6

Assuming a 0.2% reinforcement (perpendicular to the beam) for solid slab

$A_e = 0.002 A_c = 0.002*130*1000 = 265\,mm^2/m$

Provide 8 mm dia. bar @ 190 mm c/c in 2 layers

$A_e = 2*265\ mm^2/m$

Longitudinal shear force in the slab

$v_r = 2.5\ A_{cv}\eta\tau + A_e f_{sk}/\gamma_s + v_p$

or

$v_r = 0.2\ A_{cv}\eta(f_{ck})_{cy}/\gamma_c + v_p/\sqrt{3}$ *, whichever is smaller.*

$A_{cv} = 130*1000 = 130*10^3\ mm^2$

$\eta = 1.0$

$\tau = 0.3\ N/mm^2$

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$f_{sk}=415\ N/mm^2$
$\gamma_s=1.15$
$A_e=2*265\ mm^2/m$
$v_p=0$

$v_r=2.5*130*10^3*1*0.3+2*265*415/1.15=288.76\ kN/m$
or
$v_r=0.2*130*10^3*1*25/1.5=433.3\ kN/m$

Therefore, $v_r=288.76\ kN/m$

The longitudinal design shear force

$V_\lambda = 85*1000 / 200=425\ kN/m$

For each shear plane

$v_r=425 / 2= 212.5 < 288.76\ kN/m$

Hence safe.

23 COMPOSITE FLOORS - I

1.0 INTRODUCTION

Traditional steel - concrete composite floors consist of rolled or built-up structural steel beams and cast in-situ concrete floors connected together using shear connectors in such a manner that they would act monolithically (Fig.1). The principal merit of steel-concrete composite construction lies in the utilisation of the compressive strength of concrete slabs in conjunction with steel beams, in order to enhance the strength and stiffness of the steel girder.

More recently, composite floors using profiled sheet decking have become very popular in the West for high rise office buildings. Composite deck slabs are particularly competitive where the concrete floor has to be completed quickly and where medium level of fire protection to steel work is sufficient. However, composite slabs with profiled decking are unsuitable when there is heavy concentrated loading or dynamic loading in structures such as bridges. The alternative composite floor in such cases consists of reinforced or pre-stressed slab over steel beams connected together to act monolithically.
A typical composite floor system using profiled sheets is shown in Fig.2. There is presently no Indian standard covering the design of composite floor systems using profiled sheeting.

Designing a reinforced concrete slab or pre-stressed concrete slab in composite construction is not different from any conventional R.C. or pre-stressed structures; hence, this is not discussed any further here. In this chapter, concrete floors using profiled decks are treated in depth. The structural behaviour of these floors is similar to a reinforced concrete slab, with the steel sheeting acting as the tension reinforcement. The main structural and other benefits of using composite floors with profiled steel decking are:

- Savings in steel weight are typically *30%* to *50%* over non-composite construction
- Greater stiffness of composite beams results in shallower depths for the same span. Hence lower storey heights are adequate resulting in savings in cladding costs, reduction in wind loading and savings in foundation costs.
- Faster rate of construction.

The steel decking performs a number of roles, such as:
- It supports loads during construction and acts as a working platform
- It develops adequate composite action with concrete to resist the imposed loading
- It transfers in-plane loading by diaphragm action to vertical bracing or shear walls
- It stabilises the compression flanges of the beams against lateral buckling, until concrete hardens.
- It reduces the volume of concrete in tension zone
- It distributes shrinkage strains, thus preventing serious cracking of concrete.

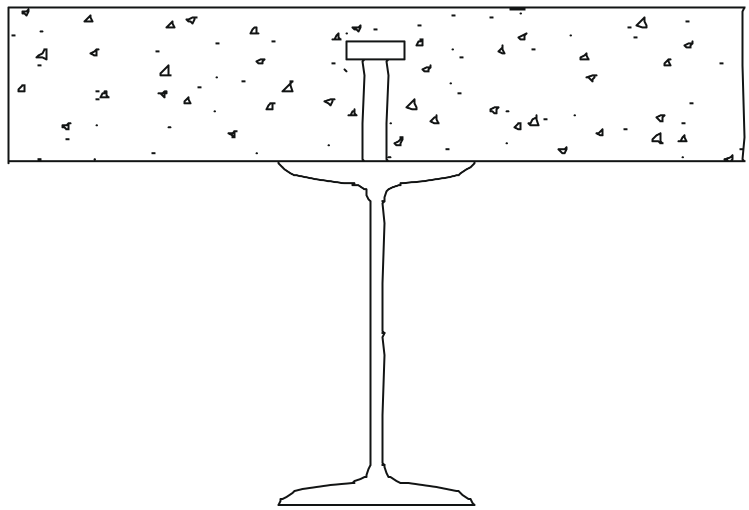


Fig. 1 Steel beam bonded to concrete slab with shear connectors

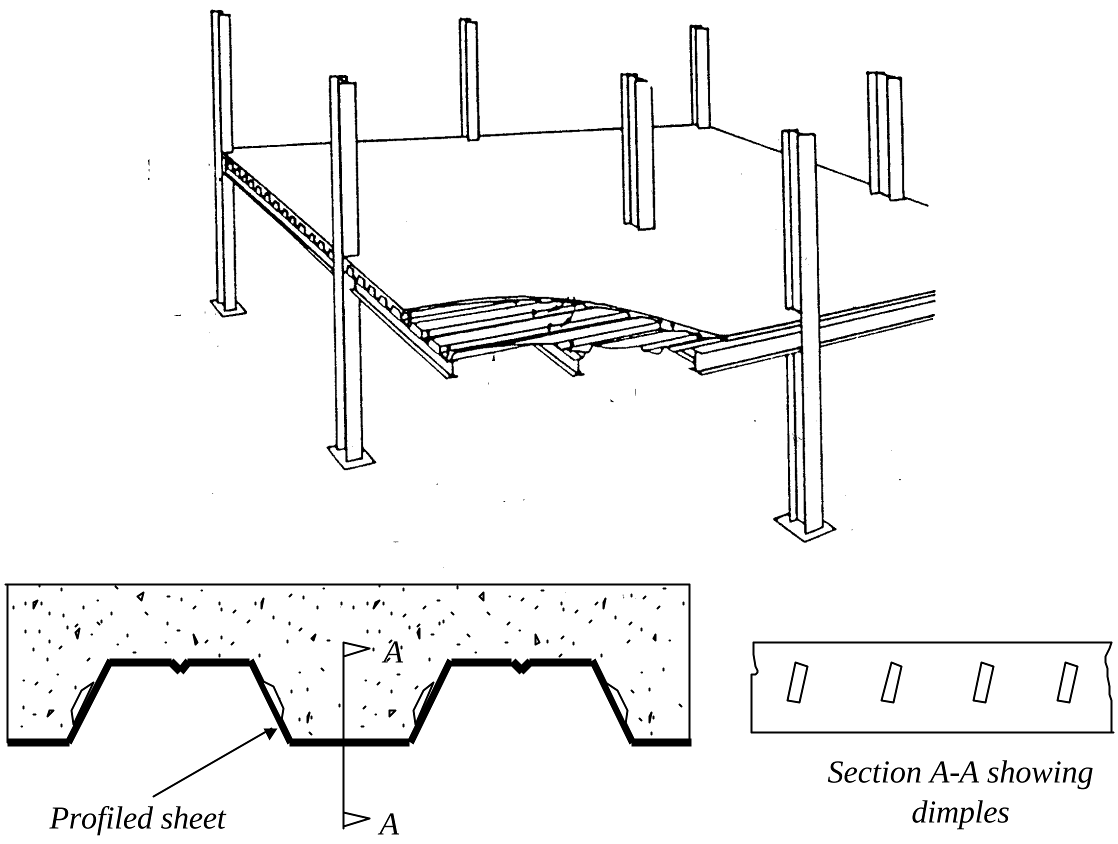


Fig. 2 Composite floor system using profiled sheets

Care has to be taken in the construction of composite floors with profiled decking to prevent excessive *'ponding'*, especially in the case of long spans. The profiled sheet deflects considerably requiring additional concrete at the centre that may add to the concreting cost. Thus, longer spans will require propping to eliminate substantial deflection or need significant quantities of concrete. Fig. 3 shows ponding of the profiled deck.

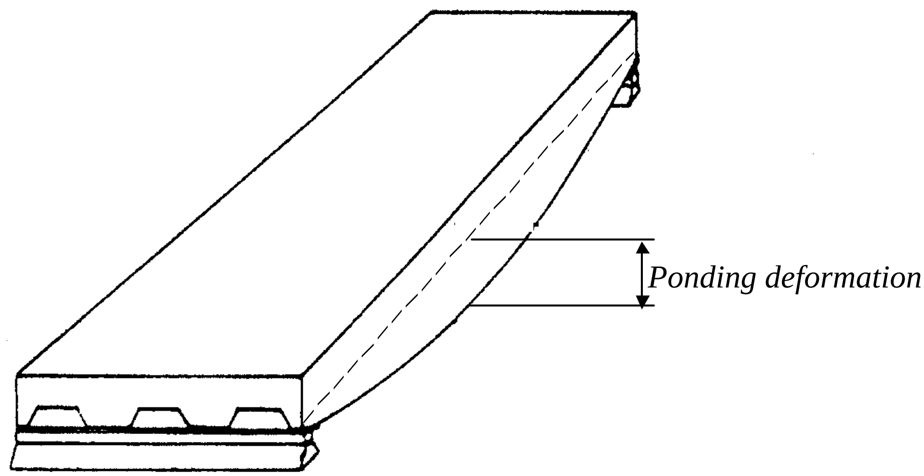


Fig. 3 Ponding in profiled decking, due to the weight of concrete

2.0 THE STRUCTURAL ELEMENTS

Composite floors with profiled decking consists of the following structural elements along with in-situ concrete and steel beams:

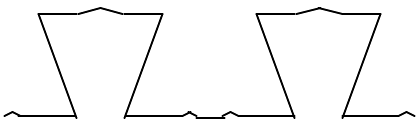
- Profiled decking
- Shear connectors
- Reinforcement for shrinkage and temperature stresses

Connections between the structural steel elements are generally designed as “simple” i.e. not moment resisting. Stud shear connectors are welded through the sheeting on to the top flange of the beam. Insulation requirements for fire usually control the slab thickness above the profile. Thickness values between *65* and *120 mm* are sufficient to give a fire rating of up to *2* hours. Lightweight concrete is popular, despite its slightly higher initial cost, because of the consequent reduction in weight and enhanced fire-insulation properties.

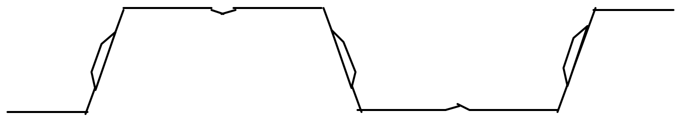
2.1 Profiled sheet decking

The steel deck is normally rolled into the desired profile from *0.9 mm* to *1.5 mm* galvanised coil. It is profiled such that the profile heights are usually in the range of *38-75 mm* and the pitch of corrugations is between *150 mm* and *350 mm*. Generally, spans of the order of *2.5 m* to *3.5 m* between the beams are chosen and the beams are designed to span between *6 m* to *12 m*. There are two well-known generic types of profiles.

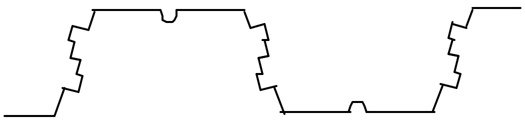
- Dovetail profile
- Trapezoidal profile with web indentations



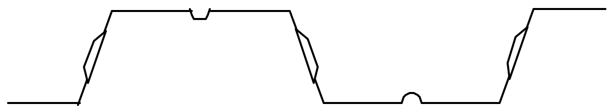
Dovetail profile



Chevron indents

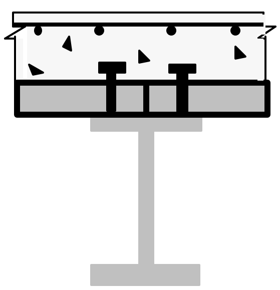


Horizontal indents

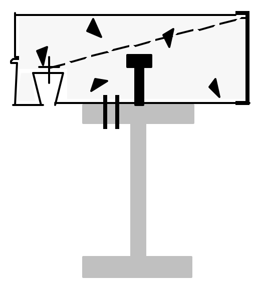


Circular indents

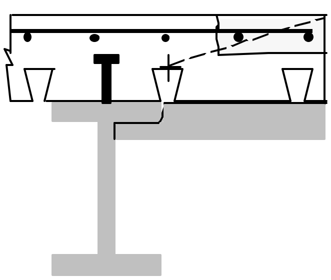
(a) Different profiles used



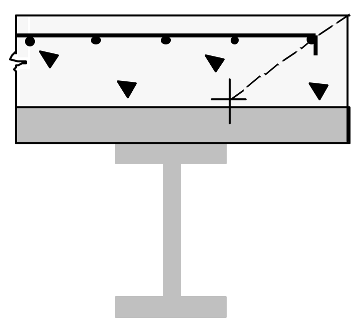
(b) Double stud butt joint



(c) Typical edge detail



(d) Side cantilever with stud bracket



(e) Typical end cantilever

Fig. 4 Deck profiles and typical details

Their shapes are generally chosen as a compromise between enhancing the bond at the steel-concrete interface and providing stability while supporting wet concrete and other construction loads. Indentations and protrusions into the rib mobilise the bearing resistance in addition to adhesion and also provide the shear transfer in composite slabs. The shear resistance capacity will depend on the parameters like re-entrant angle, lug spacing, lug width etc. Fig. 4 shows typical examples of deck profiles and the details of their attachments to the steel beams.

2.1.1 Profiled sheeting as permanent form work

Construction stage: During construction, the profiled steel deck acts alone to carry the weight of wet concrete, self weight, workmen and equipments. It must be strong enough to carry this load and stiff enough to be serviceable under the weight of wet concrete only. In addition to structural adequacy, the finished slab must be capable of satisfying the requirements of fire resistance.

Composite Beam Stage: The composite beam formed by employing the profiled steel sheeting is different from the one with a normal solid slab, as the profiling would influence its strength and stiffness. This is termed *'composite beam stage'*. In this case, the profiled deck, which is fixed transverse to the beam, results in voids within the depth of the associated slab. Thus, the area of concrete used in calculating the section properties can only be that depth of slab above the top flange of the profile. In addition, any stud connector welded through the sheeting must lie within the area of concrete in the trough of the profiling. Consequently, if the trough is narrow, a reduction in strength must be made because of the reduction in area of constraining concrete. In current design methods, the steel sheeting is ignored when calculating shear resistance; this is probably too conservative.

Composite Slab Stage: The structural behaviour of the composite slab is similar to that of a reinforced concrete beam with no shear reinforcement. The steel sheeting provides adequate tensile capacity in order to act with the concrete in bending. However, the shear between the steel and concrete must be carried by friction and bond between the two materials. The mechanical keying action of the embossments is of great importance. This is especially so in open trapezoidal profiles, where the embossments must also provide resistance to vertical separation. The predominant failure mode is one of shear bond rupture that results in slip between the concrete and steel.

2.1.2 Design method

As there is no Indian standard covering profiled decking, we refer to Eurocode 4 (EC4) for guidance. The design method defined in EC4 requires that the slab be checked firstly for bending capacity, assuming full bond between concrete and steel, secondly for shear bond capacity and, finally, for vertical shear. The analysis of the bending capacity of the slab may be carried out as though the slab was of reinforced concrete with the steel deck setting as reinforcement. However, no satisfactory analytical method has been developed so far for estimating the value of shear bond capacity. Depending upon the test data available, the loads at the construction stage often govern the allowable span rather than at the composite slab stage.

2.2 Shear connectors

Shear connectors are steel elements such as studs, bars, spiral or any other similar devices welded to the top flange of the steel section and intended to transmit the horizontal shear between the steel section and the cast in-situ concrete and also to prevent vertical separation at the interface. This topic is discussed in detail in the chapter titled composite beams - I

2.3 Reinforcement for shrinkage and temperature stresses

In buildings, temperature difference in the slabs is negligible; thus there is no need to provide reinforcement to account for temperature stresses. The effect of shrinkage is considered and the total shrinkage strain for design may be taken as *0.003* in the absence of test data.

3.0 BENDING RESISTANCE OF COMPOSITE SLAB

The structural properties of profiled sheet along with reinforcement provided and concrete with a positive type of interlock between concrete and steel deck is the basis of a composite floor. Some loss of interaction and hence slip may occur between concrete - steel interface. Such a case is known as *'Partial interaction'*. Failure in such cases occurs due to a combination of flexure and shear.

The width of the slab 'b' shown in Fig. 5(a) is one typical wavelength of profiled sheeting. But, for calculation purpose 'b' is taken as *1.0 m*. The overall thickness is h_t and the depth of concrete above main flat surface h_c. Normally, h_t is not less than *80 mm* and h_c is not less than *40 mm* from sound and fire insulation considerations.

The neutral axis normally lies in the concrete in case of full shear connection; but in regions of partial shear connection, the neutral axis may be within the steel section. The local buckling of steel sections should then be considered. For sheeting in tension, the width of embossments should be neglected. Therefore, the effective area 'A_p' per meter and height of centre of area above bottom 'e' are usually based on tests. The plastic neutral axis e_p is generally larger than e.

The simple plastic theory of flexure is used for analysis of these floors for checking the design at Limit State of collapse load. Eurocode assumes the equivalent ultimate stress of concrete in compression as $0.85(f_{ck})_{cy}/\gamma_c$ where $(f_{ck})_{cy}$ is the characteristic cylinder compression strength of concrete. However, IS 456: 1978 uses an average stress of $0.36 (f_{ck})_{cu}$ accommodating the value of γ_c and considering $(f_{ck})_{cu}$ as characteristic ***cube strength of concrete***. IS code is on the conservative side. Comparison of two values is shown in the appendix. Note that in this chapter f_{ck} refers to cube strength of concrete

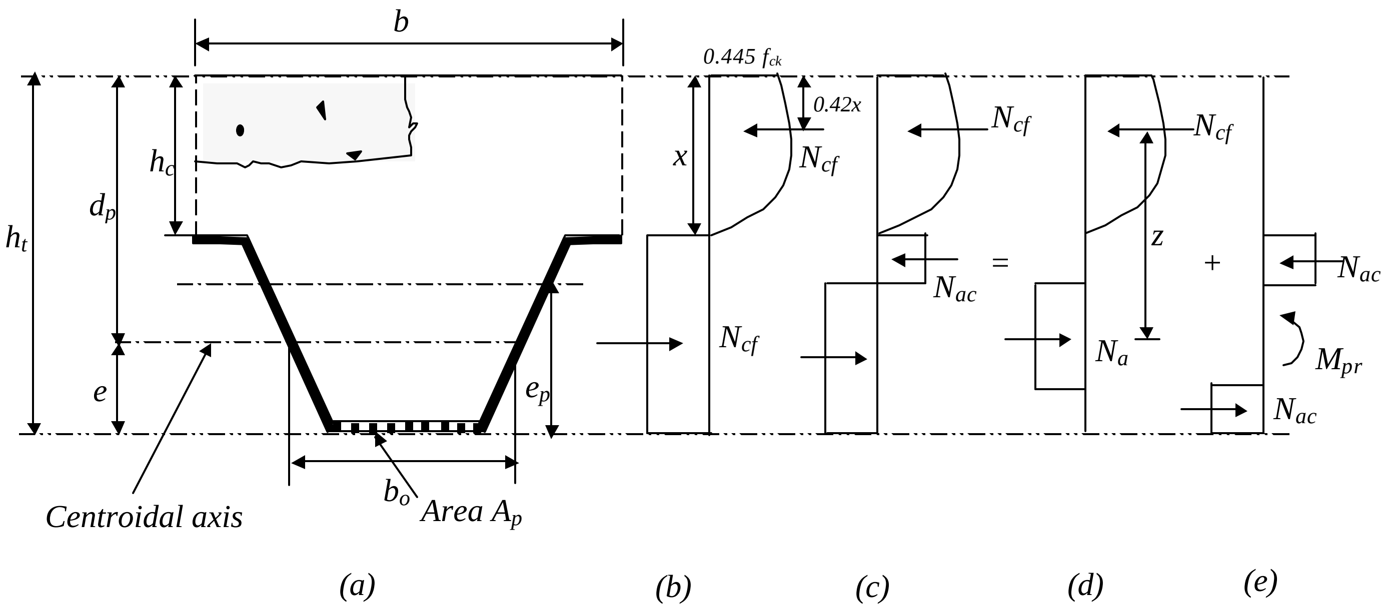


Fig. 5 Resistance of composite slab to sagging bending

3.1 Neutral axis above the sheeting [Fig. 5(b)]

Full shear connection is assumed. Hence, compressive force N_{cf} in concrete is equal to steel yield force N_{pa}.

$$N_{cf} = N_{pa} = \frac{A_p \, f_{yp}}{g_{ap}} \tag{1}$$

$N_{cf} = 0.36\, f_{ck}.b.x$

where A_p = Effective area per meter width
f_{yp} = Yield strength of steel
g_{ap} = Partial safety factor (*1.15*)

The neutral axis depth x is given by

$$x = \frac{N_{cf}}{b\,(0.36\ f_{ck})} \tag{2}$$

This is valid when $x \le h_c$, i.e. when the neutral axis lies above steel decking.

$$M_{p.Rd} = N_{cf}\,(d_p - 0.42\,x) \tag{3}$$

Note that centroid of concrete force lies at $0.42\,x$ from free concrete surface.

$M_{p.Rd}$ is the design resistance to sagging bending moment.

3.2 Neutral axis within sheeting and full shear connection [Fig. 5(c)]

$$N_{cf} = \left(bh_c \times 0.36\ f_{ck}\right) \qquad (4)$$

The compression of concrete within rib is neglected. The force N_{cf} is less than N_{pa}. The tensile force in sheeting is split into N_a (equal to compressive force N_{cf}) plus N_{ac}.

$N_a = N_{cf}$

and the remaining force N_{ac} such that the total tensile force is $N_{ac} + N_a$. The equal and opposite force N_{ac} provide resisting moment M_{pr}. Note this M_{pr} will be less than M_{pa}, the flexural capacity of steel sheeting. The relationship between $\frac{M_{pr}}{M_{pa}}$ and $\frac{N_{cf}}{N_{pa}}$ is shown in Fig. 6(*a*) in the dotted line. For design this can be approximated by line ADC that can be expressed as

$$M_{pr} = 1.25 M_{pa}\left[1 - \frac{N_{cf}}{N_{pa}}\right] \le M_{pa} \qquad (5)$$

The moment of resistance is given by

$$M_{p.Rd} = (N_{cf})z + M_{pr} \qquad (6)$$

Sum of resistance is shown in Fig. 5(*d*) and Fig. 5(*e*), which is equal to the resistance shown in Fig. 5(*c*).

The lever arm z can be found by examining the two extreme cases. For case (i) where $N_{cf} = N_{pa}$ or $N_{cf} / N_{pa} = 1.0$, $N_{ac} = 0$ and hence $M_{pr} = 0$.

$$M_{p.Rd} = N_{pa}\,(d_p - 0.42\,h_c) \qquad (7)$$

Hence, $z = d_p - 0.42h_c = h_t - e - 0.42\,h_c$ (8)

This is indicated by point F in Fig. 6(*b*).

For case (ii), on the other hand

N_{cf} @ $0;\ N_a = 0.$

$M_{pr} = M_{pa}$. The neutral axis is at a height e_p above the bottom. Then

$$z = h_t - 0.42h_c - e_p \qquad (9)$$

This is represented by point E. Thus the equation to the line EF is

$$z = h_t - 0.42\,h_c - e_p\;\frac{(e_p - e)N_{cf}}{N_{pa}} \tag{10}$$

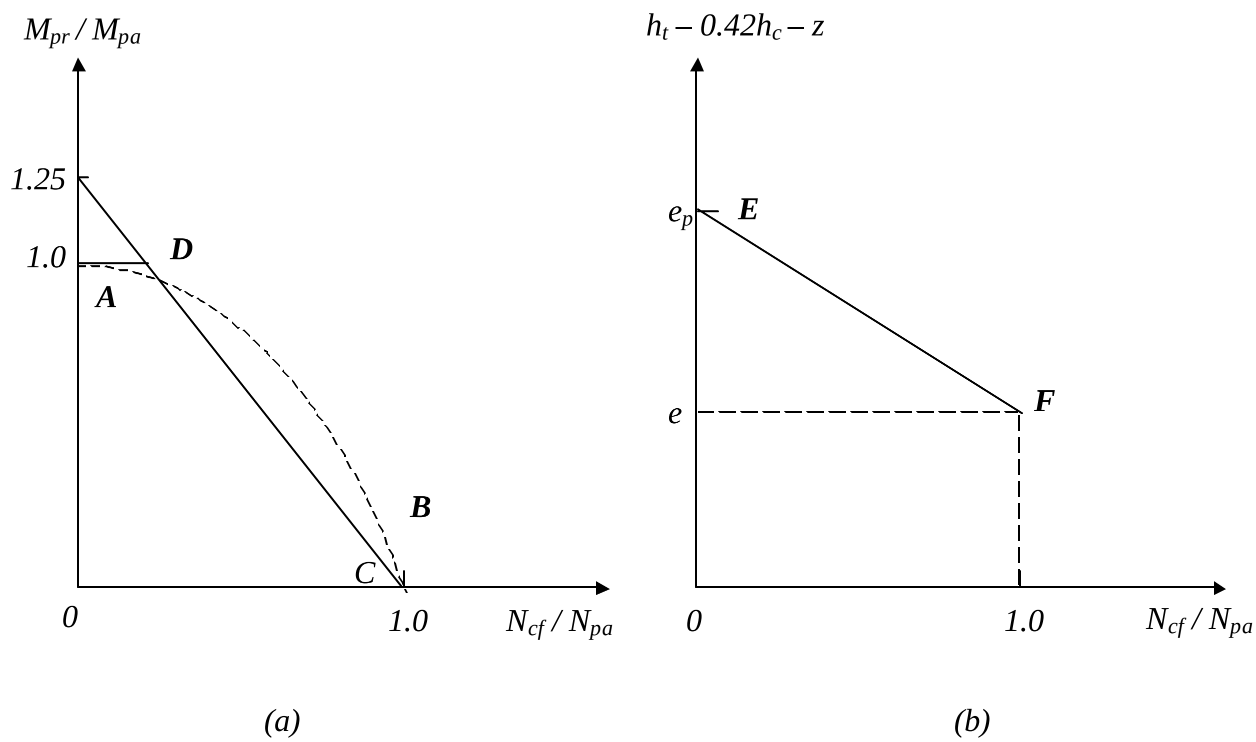


Fig. 6 Resistant moment of profiles

3.3 Partial shear connection ($N_c < N_{cf}$)

In this case, the compressive force in the concrete N_c is less than N_{cf} and depends on the strength of shear connection and the stress blocks are as shown in Fig. 5(*b*) for the slab (with N_c in place of N_{cf}) and Fig. 5(c) for sheeting.

The depth of stress block is,

$$x = \frac{N_c}{b\,(0.36\,f_{ck})} \le h_c \tag{11}$$

In this case, equations 5, 6 and 10 get modified by substituting

N_c	=	N_{cf}
N_{cf}	=	N_{pa}
x	=	h_c

Thus,

$$z = h_t - 0.42x - e_p \frac{(e_p - e)N_c}{N_{cf}} \tag{12}$$

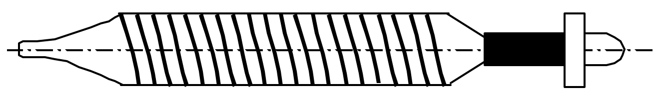
$$M_{pr} = 1.25 M_{pa} \left[1 - \frac{N_c}{N_{cf}} \right] \leq M_{pa} \tag{13}$$

$$M_{p.Rd} = N_c z + M_{pr} \tag{14}$$

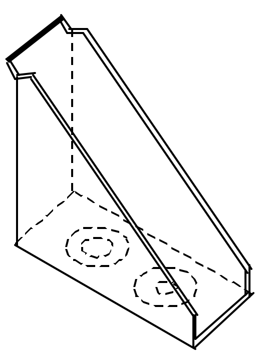
4.0 SHEAR RESISTANCE OF COMPOSITE SLAB

The shear resistance of composite slab largely depends on connection between profiled deck and concrete. The following three types of mechanisms are mobilised:

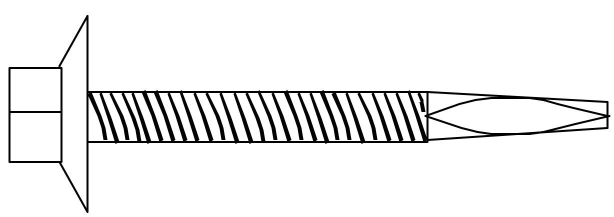
(i) Natural bond between concrete and steel due to adhesion
(ii) Mechanical interlock provided by dimples on sheet and shear connectors
(iii) Provision of end anchorage by shot fired pins or by welding studs (Fig. 7) when sheeting is made to rest on steel beams.



(a) Shot fired stud



(b) Angle bracket for two pin fixing



(c) Self drilling and tapping screws

Fig. 7 Connector details

Natural bond is difficult to quantify and unreliable, unless separation at the interface between the sheeting and concrete is prevented. Dimples or ribs are incorporated in the sheets to ensure satisfactory mechanical interlock. These are effective only if the embossments are sufficiently deep. Very strict control during manufacture is needed to ensure that the depths of embossments are consistently maintained at an acceptable level. End anchorage is provided by means of shot-fired pins, when the ends of a sheet rests on a steel beam, or by welding studs through the sheeting to the steel flange.

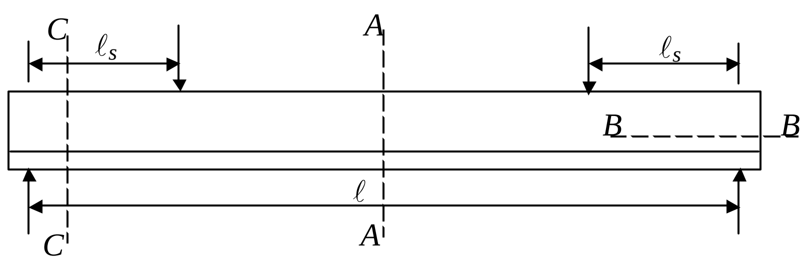
Quite obviously the longitudinal shear resistance is provided by the combined effect of frictional interlock, mechanical interlock and end anchorage. No mathematical model could be employed to evaluate these and the effectiveness of the shear connection is studied by means of load tests on simply supported composite slabs as described in the next section.

4.1 Resistance to longitudinal shear

If the shear connection is partial, slip occurs between decking and concrete. The effectiveness of the shear connection is tested using an *m-k* shear bond test. The test is described below. The failure of the beam is initiated by one of the following three modes [See Fig. 8].

(i) Flexure
(ii) Shear at support
(iii) Shear bond mode

Note that ℓ_s is the shear span and ℓ is effective span.



A-A Flexure mode
B-B Shear bond mode
C-C Vertical Shear

Fig. 8 Failure modes and critical sections

4.1.1 Evaluation of shear capacity of profiled sheets using m – k test

Specifications for tests to evaluate shear capacity of profiled sheets are given in Eurocode 4 and BS 5950: Part 4. The influence of bond is minimised in the standard test, by the application of several thousand cycles of repeated loading up to *1.5* times the service load, before loading to failure. The length of each shear span (ℓ_s) is usually (*span /4*) for uniformly distributed loading. The span is typically 3 metres. The evaluation of shear span is illustrated below.

Illustration of evaluation of shear spans:

For uniformly distributed load on a span ℓ, the length ℓ_s is taken as $\ell/4$. The principle that is used when calculating ℓ_s for other loading is now illustrated by an example.

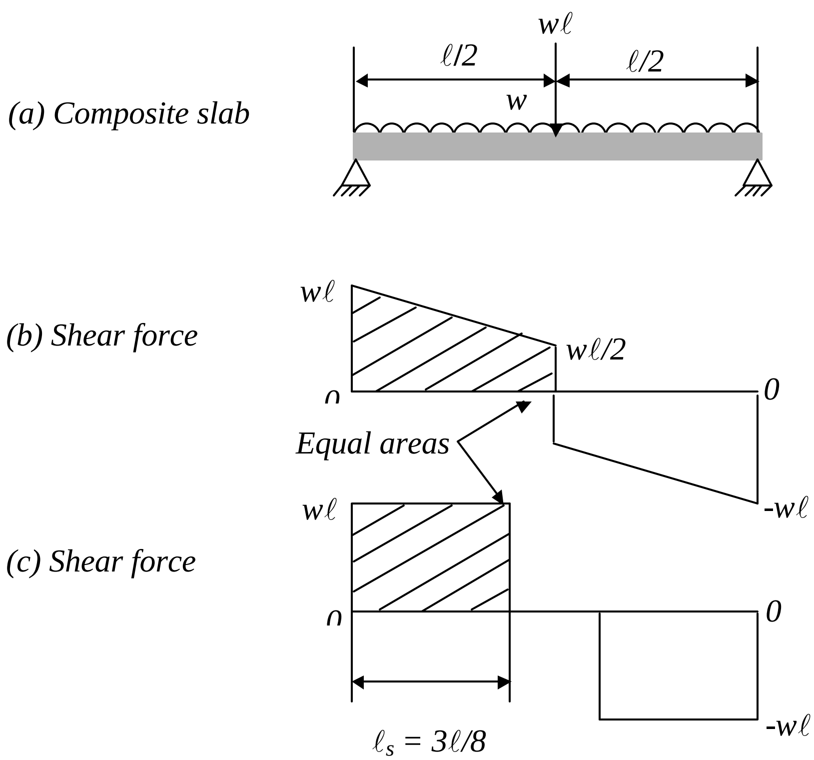


Fig. 9 Calculation of ℓ_s for composite slab

The composite slab shown in Fig. 9(a) has a distributed load w per unit length and a centre point load $w\ell$, so the shear force diagram is as shown in Fig. 9(b). A new shear force diagram is constructed for a span with two point loads only, and the same two end reactions, such that the areas of the positive and negative parts of the diagram equal to those of the original diagram. This is shown in Fig. 9(c), in which each shaded area is $3w\ell^2/8$. The positions of the point loads define the lengths of the shear spans. Here, each one is $3\ell/8$.

The expected mode of failure in a test depends on the ratio of (ℓ_s) to the effective depth (d_p) of the slab. The test specimen of breadth *"b"* should include four or five complete wavelengths of sheeting. The total cross sectional area of the sheeting is A_p. Fig. 10 shows arrangement for m - k test.

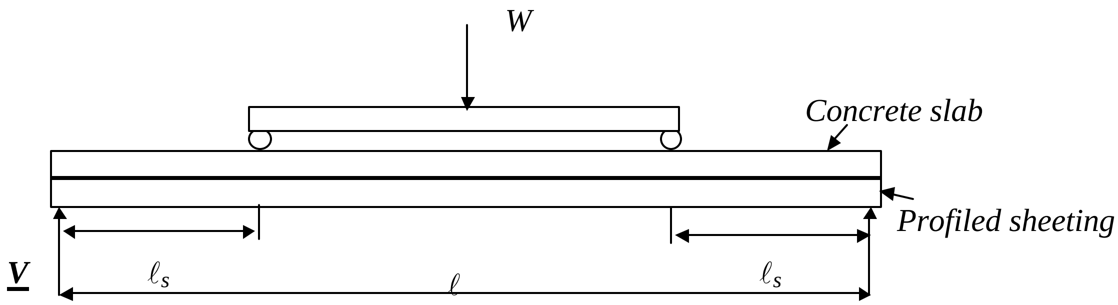


Fig. 10 A typical test arrangement

In typical Eurocode 4 tests, the results are plotted on a diagram with axis V/bd_p and $A_p/b\ell_s$ (See Fig. 11)

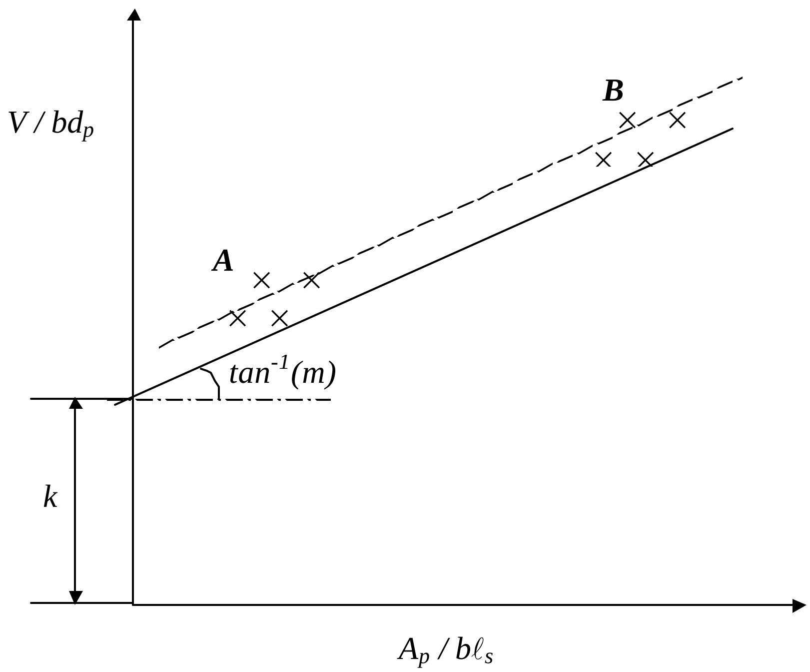


Fig. 11 m-k test details

The empirical constants m and k are determined from prototype slab tests to failure and are calculated from the slope and intercept of a regression line of Fig. 11. Tests are carried out under two or four point loads to stimulate a uniform load. The regression line is to be lowered by *15%* if less than eight slab tests are performed over a range of spans. Physically *"m"* is a broad measure of the mechanical interlock and k represents the friction load. The conceptual background to these tests is described below. At high values of ℓ_s/d_p, flexural failure occurs. The maximum bending moment (M_u) is given by

$$M_u = V.\ell_s \tag{15}$$

where, V is the maximum vertical shear.

Flexural failure is modelled by simple plastic theory with all the steel at its yield stress (f_{yp}). Concrete is stressed to average compressive stress of $0.36\ (f_{ck})_{cu}$, where $(f_{ck})_{cu}$ is the cube strength of concrete. The lever arm is approximately equal to d_p.

$$M_u \text{ is proportional to } A_p f_{yp} d_p \tag{16}$$

From equation (*15*),

$$\frac{V}{bd_p} = \frac{M_u}{bd_p \cdot \ell_s} \text{ is proportional to } \frac{A_p \cdot f_{yp}}{b\ell_s} \tag{17}$$

In tests f_{yp} is not varied. Hence flexural failure should show as a line through the origin.

At low values of (ℓ_s/d_p), vertical shear failure occurs. The mean vertical shear stress on the concrete is roughly (V/bd_p). Longitudinal shear failure occurs at intermediate values (ℓ_s/d_p) and be on the line

$$\frac{V}{bd_p} = m\left(\frac{A_p}{b\ell_s}\right) + k \tag{18}$$

A typical set of tests consists of two groups of three or four each; one of these has ℓ_s/d_p values chosen in such a manner that the results be near the point *A* in Fig. 11. Second group is chosen with a lower ℓ_s/d_p values such that the results lie near the point *B*. Values of *m* and *k* are found for a line drawn below the lowest result in each group, at a distance that allows for the scatter of test data. The behaviour is controlled by the two parameters of the straight line, namely

- *m* - the slope of the line
- *k* - the intercept of *y*-axis.

The specifications require that all tests have to be in longitudinal shear. Typically the failure is initiated when a crack occurs in concrete under one of the load points, associated with loss of bond along the shear span. If this leads to failure of the slab - the shear connection is classified as "brittle", as they occur suddenly. These are penalised EC4 by *20%* reduction in design resistance. When the eventual failure load exceeds the load causing the first end slip by more than *10%*, the failure is classed as "ductile".

Because of the rather complex nature of the prescribed tests, manufacturers of profiled sheets generally provide *m* and *k* values based on tests carried out by independent laboratories.

4.2 Resistance to vertical shear

The resistance to vertical shear is mainly provided by the concrete ribs. For open profiles b_o [Fig. 5(*a*)] should be taken as effective width. The resistance of a concrete slab with ribs of effective width b_0 at a spacing of *b* is

$$V_{v.Rd} = (b_0/b)d_p\, \tau_{Rd}\, k_v\ (1.2 + 40\rho)\ \text{per unit width} \qquad (19)$$

where d_p is the depth to the centroidal axis

τ_{Rd} is the basic shear strength of concrete

k_v allows higher shear strength for shallow members

$k_v = (1.6 - d_p) \geq 1$ with d_p in m

ρ allows a small contribution due to shearing

$$\rho = A_p / b_0\, d_p < 0.02 \qquad (20)$$

A_p = effective area of shearing within width b_0

5.0 SERVICEABILITY CRITERIA

The composite slab is checked for the following serviceability criteria:

(i) Cracking
(ii) Deflection
(iii) Fire endurance

5.1 Cracking

The crack width is calculated for the top surface in the negative moment region using standard methods prescribed for reinforced concrete. The method is detailed in the next chapter. Normally crack width should not exceed *3 mm*. IS 456: 2000 gives a formula to calculate the width of crack. Provision of *0.4 %* steel will normally avoid cracking problems in propped construction and provision *0.2 %* of steel is normally sufficient in un-propped construction. If environment is corrosive it is advisable to design the slab as continuous and take advantage of steel provided for negative bending moment for resisting cracking during service loads.

5.2 Deflection

The IS 456: 2000 gives a stringent deflection limitation of $\ell/350$ which may be un-realistic for un-propped construction. The Euro code gives limitations of $\ell/180$ or *20 mm* which ever is less. It may be worth while to limit span to depth ratio in the range of *25* to *35* for the composite condition, the former being adopted for simply supported slabs and the later for continuous slabs. The deflection of the composite slabs is influenced by the slip-taking place between sheeting and concrete. Tests seem to be the best method to estimate the actual deflection for the conditions adopted.

5.3 Fire endurance

The fire endurance is assumed based on the following two criteria:

- Thermal insulation criterion concerned with limiting the transmission of heat by conduction
- Integrity criterion concerned with preventing the flames and hot gases to nearby compartments.

It is met by specifying adequate thickness of insulation to protect combustible materials. *R* (time in minutes) denotes the fire resistance class of a member or component. For instance, *R60* means that failure time is more than *60* minutes. It is generally assumed that fire rating is *R60* for normal buildings. Reader can refer to reference 1 for further details.

6.0 CONCLUSION

This chapter described various behavioural aspects to be considered for composite floors using profiled sheeting. The design equations and application to simply supported and continuous slabs are included in the next chapter.

7.0 REFERENCES

1. R.P. Johnson " Composite Structures of Steel and Concrete" Volume 1, Blackwell Scientific Publications, UK, 1994.

2. R. Narayanan "Composite Steel Structures" Advances, Design and Construction, Elsevier, Applied science, UK, 1987.

3. R.M. Lawson, D.L Mullett and FPD Ward "Good practice in Composite floor Construction". The Steel Construction Institute, 1990.

APPENDIX

ULTIMATE STRESS OF CONCRETE

Eurocode assumes ultimate stress of concrete as $0.85\ (f_{ck})_{cy}/g_c$ where $(f_{ck})_{cy}$ is the characteristic cylinder compression strength of concrete and g_c is partial safety factor that is equal to 1.50. They adopt rectangular stress block with dimensions is shown in Fig. 11. For the sake of comparative study cylinder strength is changed to cube strength, $(f_{ck})_{cu}$. The ratio of cylinder strength to cube strength is adopted as 0.80.

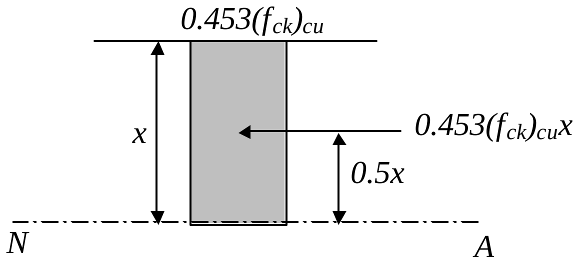


Fig. 11 Stress block for concrete strength according to Eurocode

$0.85(f_{ck})_{cy}/g_m = 0.8 * 0.85(f_{ck})_{cu}/1.5 = 0.453(f_{ck})_{cu}$

Moment capacity $= 0.453\ (f_{ck})_{cu}\ x * 0.5x = 0.227\ (f_{ck})_{cu}\ x^2$

In contrast, IS: 456 - 2000 assumes parabolic stress block as shown in Fig. 12. The dimensions of the stress block are also shown in Fig. 12.

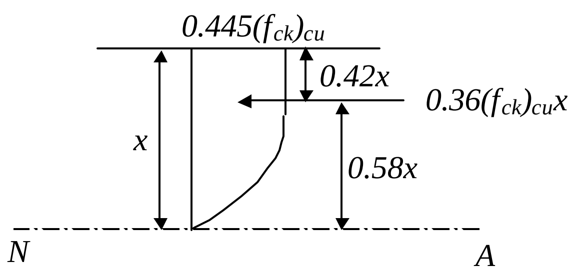


Fig. 12 Stress block for concrete strength according to IS: 456 - 2000

Moment capacity $= 0.36\ (f_{ck})_{cu}\ x * 0.58x = 0.209\ (f_{ck})_{cu}\ x^2$

IS code is $7 - 8\ \%$ conservative compared to Eurocode, it is accounted for difference in quality control of the concrete at site. If, designer is confident about quality control of the concrete he can make proper choice between the two and arrive at an economic design.

24 COMPOSITE FLOORS - II

1.0 INTRODUCTION

This chapter describes the basis for design of composite floors using profiled deck sheets adopting the equations described in the chapter on composite floors - I based on limit state design philosophy. To make it applicable to practice in India IS 456: 2000 has been followed wherever it is applicable.

The main economy in using profiled deck is achieved due to speed in construction. Normally *3* to *4 m* spans can be handled without propping and spans in excess of *4 m* will require propping. The yield strength of decking steel is in the range of *220* to $460\ N/mm^2$. Though light - weight concrete is preferable both from reducing the effect of ponding deflection as well as increasing the fire resistance, the normal practice in India is to use concrete of grade *M 20* to *M 40*. With the availability of ready mixed concrete, pumped pours become possible to the extent of $500\ m^3$ a day.

The profiled deck depth normally available ranges from *40* to *85 mm* and the metal thickness *0.6 mm* to *2.5 mm*. The normal span/depth values for continuous composite slab should be chosen to be less than *35*. The overall depth of the composite slab should not be less than *90 mm* and thickness of concrete, h_c, shall not be less than *50 mm*.

2.0 DESIGN SITUATIONS

The most important aspect of designer is to ensure an adequate degree of safety and serviceability of structure. The structure should therefore be checked for ultimate and serviceability limit states. In the design of composite floors with profiled decking the following situations are considered.

2.1 Profiled steel sheeting as shuttering

Verification is required for the profiled steel sheeting at the construction stage when it is acting as formwork for the wet concrete, construction loads and storage loads if any. While calculating the loads on the profiled sheet, increased depth of concrete due to deflection of the sheeting i.e., ponding effect has to be considered. Account should be taken of the effect of props, if any.

If the central deflection (d) of the profiled deck in non-composite stage is less than $\ell/325$ or *20 mm*, whichever is smaller, then the ponding effect may be ignored in the design of profiled deck.

2.1.1 Loads on profiled sheeting

Design should make appropriate allowances for construction loads, which include the weight of operatives, concreting plant and any impact or vibration that may occur during construction. These loads should be arranged in such a way that they cause maximum bending moment and shear. In any area of *3 m* by *3 m* (or the span length, if less), in addition to weight of wet concrete, construction loads and weight of surplus concrete should be provided for by assuming a load of *1.5* kN/m^2. Over the remaining area a load of *0.75* kN/m^2 should be added to the weight of wet concrete.

2.1.2 Effective span

The continuous slab is designed as a series of simply - supported spans, for simplicity. The effective span can be taken as the lesser of the two following:

- Distance between centres of supports
- The clear span plus the effective depth of the slab

If, profiled deck sheet is propped during construction then, effective span is calculated using the formula. (This rule is taken from BS 5950:Part 4 as there is no provision in Eurocode). The width of the prop is neglected here.

$$\ell_e = \frac{\ell - B + d_{ap}}{2} \tag{1}$$

where, B – Width of top flanges of the supporting steel beams
d_{ap} - The depth of the sheeting
ℓ - Actual span of the composite floor

2.2 Composite slab

Verification is required for the floor slab after composite behaviour has commenced and any props have been removed. Total loading that is acting on the composite slab is considered in the design checks for the ultimate Limit State. The loads are applied in such a way that the load combination is most unfavourable. Load factors of *1.35* for dead load and *1.5* for imposed load are employed in design calculations.

Generally it is sufficient to consider the following load combinations in buildings mostly subjected to uniformly distributed loads:

- Alternate spans carrying total factored loading due to imposed and dead loads. Other spans carrying only factored loading due to dead load.
- Any two adjacent spans carrying total factored load due to imposed and dead load and all other spans carrying only factored dead load.

3.0 ANALYSIS FOR INTERNAL FORCES AND MOMENTS

3.1 Profiled steel sheeting as shuttering

Elastic analysis shall be used where sheeting is considered. The design based on elastic distribution of bending moment is conservative, as it does not take into account redistribution of moments that can occur between support and mid span sections. The following treatment is based on moment redistribution at the ultimate stages.

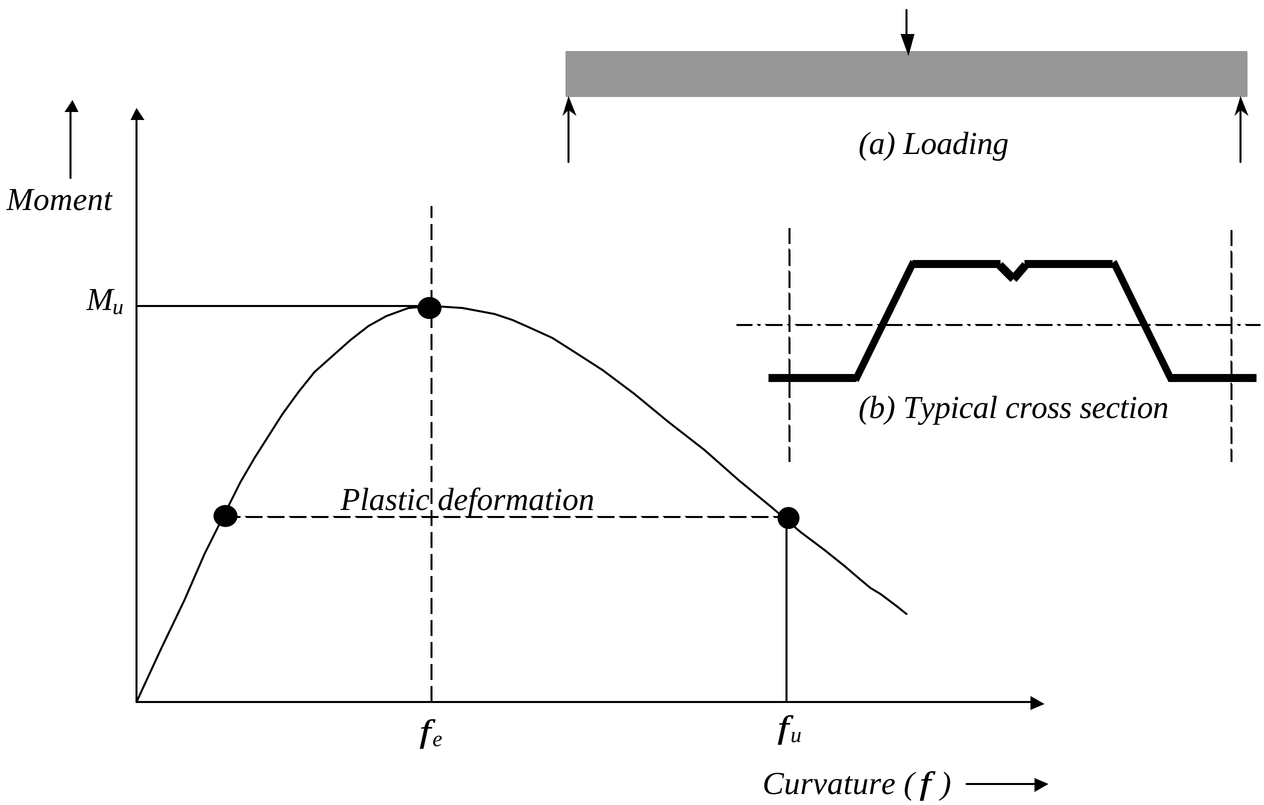
A moment - curvature relationship for typical section of metal deck is shown in Fig. 1.

Fig. 2 shows the elastic moment at the onset of yield and the moment at failure utilising the plastic deformation of the hinges formed earlier for a typical two span continuous beam.

If M_p and M_n are the moment capacities of deck at mid-span and support sections respectively, then at failure only a portion of M_n i.e. kM_n at support can be realised because of the available ductility. Thus,

$$w = \frac{8}{\ell^2}\left(M_p + 0.46kM_n\right) \tag{2}$$

The value of k can be determined experimentally for the type of profile used.



(c) Moment – curvature variation of critical cross section

Fig. 1 Moment – curvature relationship for a metal deck floor

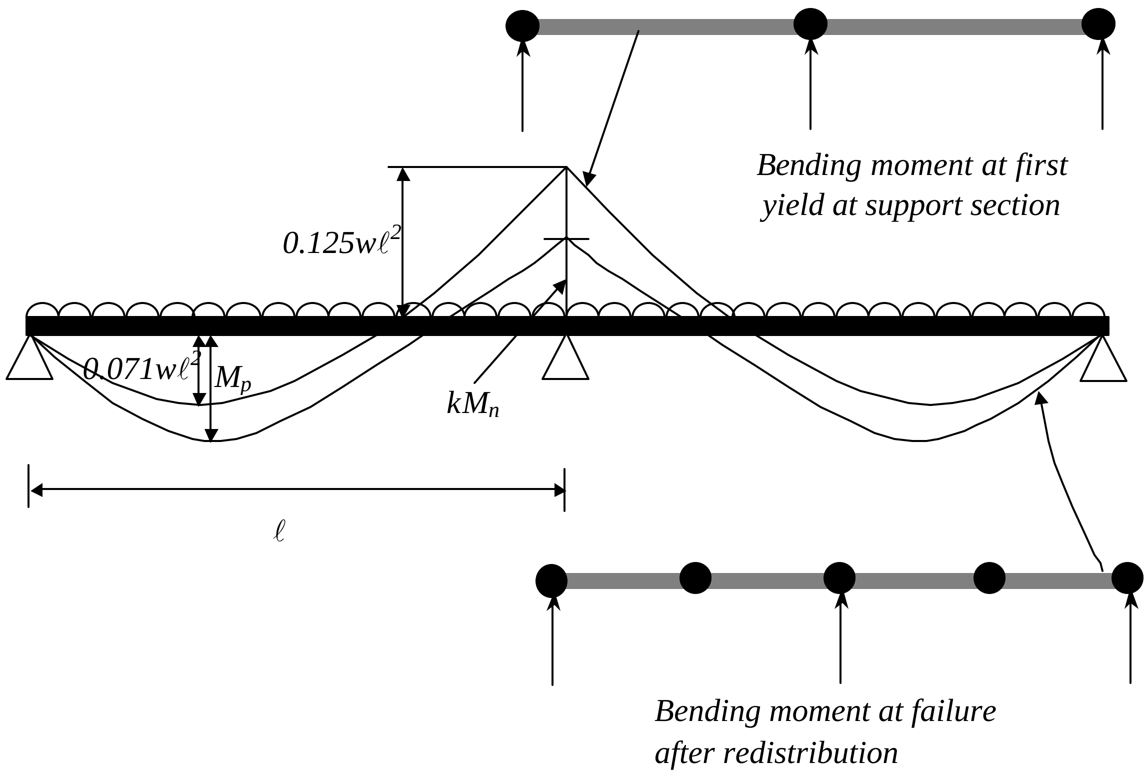


Fig. 2 Bending moment variation on two span decking

The load span tables given by manufactures of the profile deck are based on tests and hence take advantage of post elastic strength. They are satisfactory for spans *10* to *15%* in excess of the corresponding values based on elastic design. However, IS: 456 – 1978 gives coefficients for bending moments and shear forces for continuous beams which can be used for calculating moments and shear forces for composite floors also. Table –7 and table – 8 in the Chapter on Composite Beams – II gives moment and shear coefficients respectively. These coefficients do not make any allowance for redistribution and may be too conservative.

3.2 Composite slab

The following methods of analysis may be used:

- Linear analysis with or without redistribution
- Rigid-plastic global analysis based either on the kinematic method or on the static method provided that it is shown that sections where plastic rotations are required have sufficient rotation capacity
- Elastic-plastic analysis taking into accounts the non-linear material properties.

The application of linear methods of analysis is suitable for the serviceability limit states as well as for the ultimate limit states. Plastic methods, with their high degree of simplification, shall only be used in the ultimate Limit State.

Equations for evaluating moment, longitudinal shear and vertical shear are given in the previous chapter.

4.0 DESIGN TABLES

The manufacturers of the steel profiled sheets normally provide design tables for different decks made by them. These tables give information regarding recommended slab depth and profile thickness to be adopted for different type of support conditions such as single span without prop, with prop or for multiple spans for different imposed load rating. The spans that could be achieved with propped condition are generally greater because the governing criteria will be composite condition. However it is necessary to check deflections in such cases for serviceability.

5.0 SERVICEABILITY LIMIT STATES FOR COMPOSITE SLABS WITH PROFILED DECKS

5.1 Cracking of concrete

The profiled deck sheeting protects the lower surface of the slab. Cracking will occur in the top surface where the slab is continuous over a supporting beam in the hogging moment regions. Crack width will be wider over the supports if each span of the slab is designed as simply supported, rather than continuous, and if the spans are propped during construction

To counter cracking, longitudinal reinforcement should be provided above internal supports. The minimum recommended amounts are as *0.2%* of the area of concrete above the sheeting, for unpropped construction, and *0.4%* if propping is used. (According to Eurocode 4). If the environment is corrosive, the slabs should be designed as continuous, with cracking controlled by providing additional reinforcement and ensuring that the concrete cover for reinforcement is suitably enhanced.

5.2 Deflection

The limitations on deflection for composite slabs are not explicitly provided for in IS: 11384 – 1985. Eurocode 4 gives explicit guidance, which is explained below. The deflection of profiled sheeting due to its own weight and the wet concrete slab should not exceed $\ell_e/180$ or *20 mm*, where ℓ_e is the effective span.

For the composite slab stage, the rare loading combinations described in section 2.2 are normally used. The maximum deflection below the level of the supports should not exceed *span/250*, and the increase of deflection after construction (due to creep and to variable load) should not exceed *span/300*, or *span/350* if the floor supports brittle finishes or partitions.

The deflections may not be excessive when span-to-depth ratios are kept within certain limits. These values are given in Eurocode 4 as *25* for simply supported slabs, *32* for spans with one end continuous and *35* for internal spans. These limitations are regarded as "deemed to satisfy" the serviceability deflection limits. 'Depth' limits relate to effective depths, so for composite slabs the depth should be taken as depth of composite slab over centroidal axis of the profiled deck sheet rather than total depth of the slab.

Broadly speaking, slip occurs after reaching working load in a well-designed slab. If the slip occurs earlier it will cause an increase in deflection even in the Serviceability State, which is not desirable. So, slip factor to be taken care of while designing the slab.

6.0 FIRE RESISTANCE

In general no fire tests are required before the design of profiled steel sheeting, as their manufacturers carry out these tests for general use in buildings, before releasing the products in the market.

Standard fire-resistance tests are carried out by them, using independent testing centres, to ensure

- Strength or stability under load
- Ability to transmit smoke and flame
- Insulation so that upper part of the slab is not excessively heated.

Adequacy is checked by ensuring that deflection does not exceed *span/20* under fire tests. The load carrying capacity is checked by considering only embedded steel. In buildings the in-plane resistance and negative reinforcements at points of continuity add to the strength under fire condition. While simply supported slab under fire test exhibits an endurance period of *30* minutes, the continuous slab withstands *30* minutes or even more with an imposed load of $6.7\ kN/m^2$. Under fire condition with this imposed load and with reduced strength of elements and elevated temperature the safety is checked so that plastic capacity gives a load factor of *1.0* or more. For larger fire protection using fire engineering method, the area of embedded mesh reinforcement is suitably increased by additional emergency reinforcement as per the increase in the fire resistance period desired. Simple design tables for common cases of fire resistance of composite deck slabs are given in reference 3. These tables can be judiciously converted and used in the absence of test results for specific cases required.

7.0 DIAPHRAGM ACTION OF DECK SLAB

Deck slab transmits in-plane loads for ensuring lateral stability of the building system. For this the deck slab is attached on all the four sides at spacing exceeding *600 mm* on either the beams or supporting walls. The diaphragm action is excellent if through deck welding is resorted to. The steel decking also provides lateral support to the steel beams it supports. However, beams running parallel to the decking are laterally supported only at transverse beam connections.

8.0 STEPS IN THE DESIGN OF PROFILED DECKING

The following are the steps for design of profiled decking sheets:

(i) List the decking sheet data (Preferably from manufacturer's data)

(ii) List the loading

(iii) Design the profiled sheeting as shuttering
- Calculate the effective length of the span
- Compute factored moments and vertical shear
- Check adequacy for moment
- Check adequacy for vertical shear
- Check deflections

(iv) Design the composite slab – Generally the cross sectional area of the profiled decking that is needed for the construction stage provides more than sufficient reinforcement for the composite slab. So, the design of short span continuous slabs can be done as series of simply supported slabs and top longitudinal reinforcement is provided for cracking as given in section 5.1. However, long-span slabs are designed as continuous over supports.
- Calculate the effective length of the span
- Compute factored moments and vertical shear
- Check adequacy for moment
- Check adequacy for vertical shear
- Check adequacy for longitudinal shear
- Check for serviceability, i.e. cracking above supports and deflections

9.0 CONCLUSIONS

'Floors using profiled deck sheets' is a very new design concept in the Indian context and hence no appropriate codes are available. These chapters on composite floors are intended to provide the most upto-date information and hence largely Eurocodes are followed. A design example is included to illustrate the method employed in the choice of profiled decking and for verifying its adequacy.

10.0 REFERENCES

1. Mark Lawson and Peter Wickens "Composite Deck Slab", Steel Designers Manual (Fifth edition), The Steel Construction Institute, UK, 1992.

2. Bryan E.R. and Leach. P "Design of Profiled sheeting as Permanent Formwok", Construction Industry Research and Information Association (CIRIA), Technical Note 116, 1984.

3. Data Sheet: Fire resistance of Composite Slabs with Steel Decking, Construction Industry Research and Information Association (CIRIA), Special Publication 42

25 STEEL-CONCRETE COMPOSITE COLUMNS-I

1.0 INTRODUCTION

A steel-concrete composite column is a compression member, comprising either a concrete encased hot-rolled steel section or a concrete filled tubular section of hot-rolled steel and is generally used as a load-bearing member in a composite framed structure. Typical cross-sections of composite columns with fully and partially concrete encased steel sections are illustrated in Fig. *1*. Fig. *2* shows three typical cross-sections of concrete filled tubular sections. Note that there is no requirement to provide additional reinforcing steel for composite concrete filled tubular sections, except for requirements of fire resistance where appropriate.

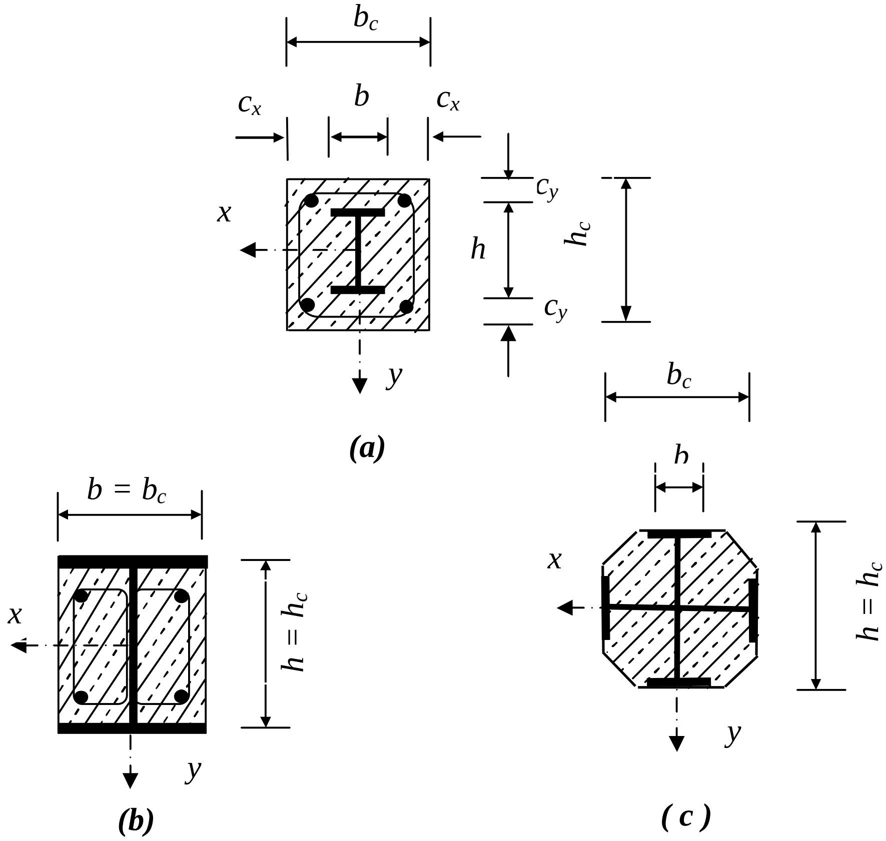


***Fig. 1*: Typical cross - sections of fully and partially concrete encased columns**

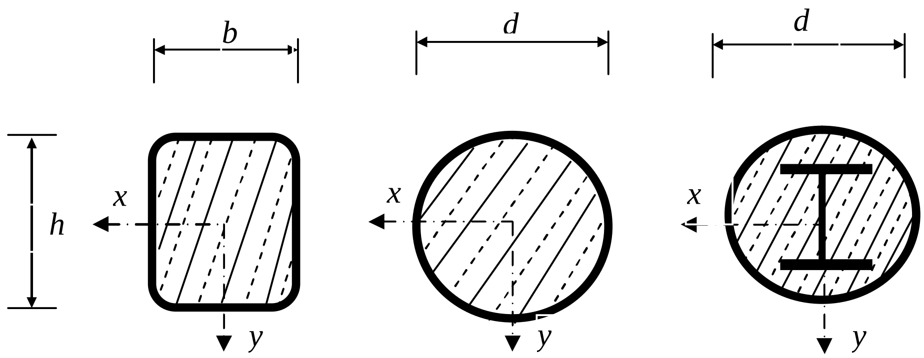


Fig. 2*: *Typical cross-sections of concrete filled tubular sections

In a composite column both the steel and concrete would resist the external loading by interacting together by bond and friction. Supplementary reinforcement in the concrete encasement prevents excessive spalling of concrete both under normal load and fire conditions.

In composite construction, the bare steel sections support the initial construction loads, including the weight of structure during construction. Concrete is later cast around the steel section, or filled inside the tubular sections. The concrete and steel are combined in such a fashion that the advantages of both the materials are utilised effectively in composite column. The lighter weight and higher strength of steel permit the use of smaller and lighter foundations. The subsequent concrete addition enables the building frame to easily limit the sway and lateral deflections.

With the use of composite columns along with composite decking and composite beams it is possible to erect high rise structures in an extremely efficient manner. There is quite a vertical spread of construction activity carried out simultaneously at any one time, with numerous trades working simultaneously. For example

- One group of workers will be erecting the steel beams and columns for one or two storeys at the top of frame.
- Two or three storeys below, another group of workers will be fixing the metal decking for the floors.
- A few storeys below, another group will be concreting the floors.
- As we go down the building, another group will be tying the column reinforcing bars in cages.
- Yet another group below them will be fixing the formwork, placing the concrete into the column moulds etc.

The ***advantages*** of composite columns are:

- increased strength for a given cross sectional dimension.
- increased stiffness, leading to reduced slenderness and increased buckling resistance.
- good fire resistance in the case of concrete encased columns.
- corrosion protection in encased columns.

- significant economic advantages over either pure structural steel or reinforced concrete alternatives.
- identical cross sections with different load and moment resistances can be produced by varying steel thickness, the concrete strength and reinforcement. This allows the outer dimensions of a column to be held constant over a number of floors in a building, thus simplifying the construction and architectural detailing.
- erection of high rise building in an extremely efficient manner.
- formwork is not required for concrete filled tubular sections.

2.0 MATERIALS

2.1 Structural Steel

All structural steels used shall, before fabrication conform to *IS: 1977-1975*, *IS: 2062-1992*, and *IS: 8500-1977* as appropriate. Some of the structural steel grade commonly used in construction as per *IS: 961-1975* and *IS: 1977-1975* are given in Table *1*.

***Table1(a)*: Yield strength f_y of steel sections**

Nominal steel grade	Nominal thickness/diameter (*mm*)	Yield stress, f_y (*MPa*)
Fe 570-HT	$t<6$	350
	$6\leq t\leq 28$	350
	$28< t\leq 45$	340
Fe 540W-HT	$t<6$	350
	$6\leq t\leq 16$	350
	$16< t\leq 32$	340
Fe 410-O (not subjected to dynamic loading other than wind)	$t<6$	250
	$6\leq t\leq 20$	250
	$20< t\leq 40$	240

***Table1(b)*: Yield strength f_y of steel sections as per IS 2062:1992**

Nominal steel grade	Nominal thickness/diameter (*mm*)	Yield stress, f_y (*MPa*)
Fe 410W A	< 20	250
	20 - 40	240
	> 40	230
Fe 410W B	< 20	250
	20 - 40	240
	> 40	230
Fe 410W C	< 20	250
	20 - 40	240
	> 40	230

2.2 Concrete

Concrete strengths are specified in terms of the characteristic cube strengths, $(f_{ck})_{cu}$, measured at *28* days. Table *2* gives the properties of different grades of concrete according to *IS: 456-2000* and the corresponding *EC4* values.

Table 2: Properties of concrete

Grade Designation	*M*25	*M*30	*M*35	*M*40
$(f_{ck})_{cu}$ (N/mm^2)	25	30	35	40
$(f_{ck})_{cy}$ (N/mm^2)	20	25	28	32
$f_{ctm}(N/mm^2)$	2.2	2.6	2.8	3.3
$E_{cm}=5700\sqrt{(fck)_{cu}}(N/mm^2)$	28500	31220	33720	36050

where, $(f_{ck})_{cu}$ characteristic compressive (cube) strength of concrete

$(f_{ck})_{cy}$ characteristic compressive (cylinder) strength of concrete, given by 0.8 times 28 days cube strength of concrete according to EC4

f_{ctm} mean tensile strength of concrete

For lightweight concrete, the E_{cm} values are obtained by multiplying the values from Table *2* by ρ */2400*,where ρ is the unit mass (kg/m^3)

2.3 Reinforcing Steel

Steel grades commonly used in construction are given in Table *3*. It should be noted that although the ductility of reinforcing bars has a significant effect on the behaviour of continuous composite beams, this property has little effect on the design of composite columns. Concrete filled tubular sections may be used without any reinforcement except for reasons of fire resistance, where appropriate.

Table 3: Characteristic strengths of reinforcing steel

Type of steel	Indian Standard	Nominal size (*mm*)	Yield Stress, f_{sk} (N/mm^2)
Mild steel Grade I (plain bars)	*IS:432(Part1)-1982*	$d \leq 20$	250
		$20 < d \leq 50$	240
Mild steel Grade II (plain bars)	*IS:432(Part1)-1982*	$d \leq 20$	225
		$20 < d \leq 50$	215
Medium tensile steel (plain bars)	*IS:432(Part1)-1982*	$d \leq 16$	540
		$16 < d \leq 32$	540
		$32 < d \leq 50$	510
Medium tensile steel (Hot-rolled deformed bars and Cold-twisted deformed bars)	*IS:1786-1985*	for bars of all sizes	415
			500
			550

Note: This chapter is confined to steel concrete composite columns made up of hot rolled steel sections having yield strengths within the range $250\ N/mm^2$ to $350\ N/mm^2$ and reinforcement with steel rods of *415* or $500\ N/mm^2$. This limitation is considered necessary at the present time on account of the lower ductility of steels having higher yield strengths.

2.4 Partial safety factors

2.4.1 Partial safety factor γ_f for loads - The suggested partial safety factor γ_f for different load combinations is given below in Table *4*.

Table 4* : *Partial safety factors (According to proposed revisions to IS 800)

Loading	γ_f		
	DL	LL	WL
Dead Load (unfavourable effects)	1.35	-	-
Dead load restraining uplift or overturning	1.0	-	-
Imposed Load + Dead Load	1.35	1.5	-
Dead Load + Wind Load	1.35	-	1.5
Dead Load + Imposed Load + wind Load (Major Load)	1.35	1.05	1.5
Dead Load + Imposed Load (Major Load) + wind Load	1.35	1.5	1.05

2.4.2 Partial safety factor for materials

The partial safety factor γ_m for structural steel, concrete and reinforcing steel is given in Table*5*.

Table 5: Partial safety factor for materials

Material	γ_m*
Steel Section	1.15
Concrete	1.5
Reinforcement	1.15

**IS: 11384-1985* Code for composite construction has prescribed $\gamma_m = 1.15$ for structural steel. (By contrast, *EC4* has prescribed $\gamma_m = 1.10$ for structural steel).

3.0 COMPOSITE COLUMN DESIGN

3.1 General

As in other structural components, a composite column must also be designed for the Ultimate Limit State. For structural adequacy, the internal forces and moments resulting from the most unfavourable load combination should not exceed the *design resistance* of the composite cross-sections. While local buckling of the steel sections may be eliminated, the reduction in the compression resistance of the composite column due to overall buckling should definitely be allowed for, together with the effects of residual stresses and initial imperfections. Moreover, the second order effects in slender columns as well as the effect of creep and shrinkage of concrete under long term loading must be considered, if they are significant. The reduction in flexural stiffness due to cracking of the concrete in the tension area should also be considered.

3.2 Method of Design

At present, there is no Indian Standard covering Composite Columns. The method of design suggested in this chapter largely follows *EC4*, which incorporates the latest research on composite construction. Isolated symmetric columns having uniform cross sections in braced or non-sway frames may be designed by the *Simplified design method* described in the next section. This method also adopts the European buckling curves for steel columns as the basis of column design. It is formulated in such a way that only hand calculation is required in practical design. This method cannot be applied to sway columns.

When a sufficiently stiff frame is subjected to in–plane horizontal forces, the additional internal forces and moments due to the consequent horizontal displacement of its nodes can be neglected, and the frame is classed as "non-sway".

3.3 Fire resistance

Due to the thermal mass of concrete, composite columns always possess a higher fire resistance than corresponding steel columns. (It may be recalled that composite columns were actually developed for their inherent high fire resistance). Composite columns are usually designed in the normal or 'cool' state and then checked under fire conditions. Additional reinforcement is sometimes required to achieve the target fire resistance. Some general rules on the structural performance of composite columns in fire are summarised as follows:

- The fire resistance of composite columns with fully concrete encased steel sections may be treated in the same way as reinforced concrete columns. The steel is insulated by an appropriate concrete cover and light reinforcement is also required in order to maintain the integrity of the concrete cover. In such cases, two-hour fire resistance can usually be achieved with the minimum concrete cover of *40 mm.*

- For composite columns with partially concrete encased steel sections, the structural performance of the columns is very different in fire, as the flanges of the steel sections are exposed and less concrete acts as a 'heat shield'. In general, a fire resistance of up to one hour can be achieved if the strength of concrete is neglected in normal design. Additional reinforcement is often required to achieve more than one-hour fire resistance.

- For concrete filled tubular sections subjected to fire, the steel sections are exposed to direct heating while the concrete core behaves as ‘heat sink’. In general, sufficient redistribution of stress occurs between the hot steel sections and the relatively cool concrete core, so that a fire resistance of one hour can usually be achieved.

For longer periods of fire resistance, additional reinforcement is required, which is not provided in normal design. Steel fibre reinforcement is also effective in improving the fire resistance of a concrete filled column. It is also a practice in India to wrap the column with ferrocement to increase the fire rating

4.0 PROPOSED DESIGN METHOD

The simplified method is formulated for prismatic composite columns with doubly symmetrical cross-sections. The calculations of various design parameters are covered and the checks for structural adequacy of a composite column under applied loads are presented below.

4.1 Resistance of cross-section to compression

The plastic compression resistance of a composite cross-section represents the maximum load that can be applied to a short composite column. Concrete filled circular tubular sections exhibit enhanced resistance due to the tri-axial confinement effects. Fully or partially concrete encased steel sections and concrete filled rectangular tubular sections do not achieve such enhancement.

4.1.1 Encased steel sections and concrete filled rectangular/square tubular sections:-

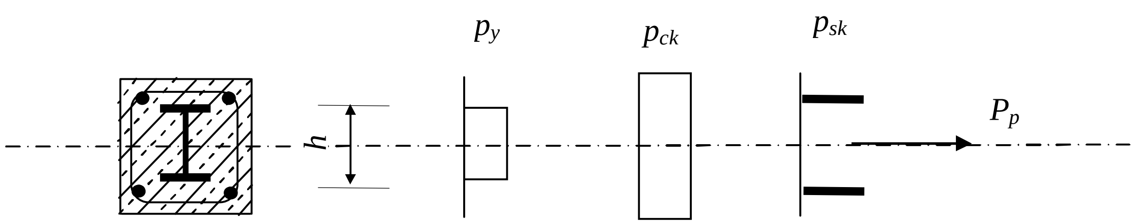


Fig. 3 Stress distribution of the plastic resistance to compression of an encased I section

The plastic resistance of an encased steel section or concrete filled rectangular or square section (i.e. the so-called “squash load”) is given by the sum of the resistances of the components as follows:

$$P_p = A_a.f_y / \gamma_a + \alpha_c.A_{c.} (f_{ck})_{cy} / \gamma_c + A_s .f_{sk} / \gamma_s$$

$$P_p = A_a.f_y / \gamma_a + \alpha_c.A_{c.}[0.80^* (f_{ck})_{cu}] / \gamma_c + A_s .f_{sk} / \gamma_s \qquad (1)$$

where

A_a, A_c and A_s are the areas of the steel section, the concrete and the reinforcing steel respectively

f_y, $(f_{ck})_{cy}$ and f_{sk} are the yield strength of the steel section, the characteristic compressive strength (cylinder) of the concrete, and the yield strength of the reinforcing steel respectively.

$(f_{ck})_{cu}$ the characteristic compressive strength (cube) of the concrete

α_c strength coefficient for concrete, which is *1.0* for concrete filled tubular sections, and *0.85* for fully or partially concrete encased steel sections.

For ease of expression, $\dfrac{f_y}{\gamma_a}, \dfrac{\alpha_c (f_{ck})_{cy}}{\gamma_c}$ *and* $\dfrac{f_{sk}}{\gamma_s}$ are presented as the design strengths of the respective materials such as p_y , p_{ck} and p_{sk}. Eqn. (*1*) can therefore be rewritten as follows:

$$P_p = A_a p_y + A_c p_{ck} + A_s p_{sk} \tag{2}$$

At this stage it should be pointed out that the Indian Standards for composite construction (*IS:11384-1985*) does not make any specific reference to composite columns. The provisions contained in *IS: 456 - 2000* are often invoked for design of composite structures. Extension of *IS: 456 - 2000* to composite columns will result in the following equation:

$$P_p = A_a p_y + A_c p_{ck} + A_s p_{sk} \tag{2a}$$

where

$$p_y = 0.87 f_y \text{ ; } p_{ck} = 0.4 (f_{ck})_{cu} \text{ and } p_{sk} = 0.67 f_y \tag{2b}$$

An important design parameter is the steel contribution ratio, β_a which is defined in *EC4* as follows:

$$\beta_a = \frac{A_a \times p_y}{P_p} \tag{3}$$

IS: 456 - 2000 is also to be employed for the spacing and design of ties.

4.1.2 Concrete filled circular tubular sections: Special Provisions

The method described above is valid for rectangular and square tubular sections. For composite columns using circular tubular sections, there is an increased resistance of concrete due to the confining effect of the circular tubular section. However, this effect on the resistance enhancement of concrete is significant only in stocky columns. For composite columns with a non-dimensional slenderness of $\bar{\lambda} \leq 0.5$ (where $\bar{\lambda}$ is defined in Eqn.*8*,in section *4.3*), or where the eccentricity, *e* [defined in Eqn. 4 below], of the applied load does not exceed the value *d/10*, (where *d* is the outer dimension of the circular tubular section) this effect has to be considered.

The eccentricity, *e*, is defined as follows:

$$e = \frac{M}{P} \leq \frac{d}{10} \tag{4}$$

where

e is the eccentricity
M is the maximum applied design moment (second order effects are ignored)
P is the applied design load

The plastic compression resistance of concrete filled circular tubular sections is calculated by using two coefficients η_1 and η_2 as given below.

$$P_p = A_a \eta_2 \, p_y + A_c \, p_{ck} \left[1 + \eta_1 \frac{t}{d} \frac{f_y}{f_{ck}}\right] + A_s \, p_{sk} \tag{5}$$

where

t is the thickness of the circular tubular section.
η_1 and η_2 two coefficients given by

$$\eta_1 = \eta_{10} \left[1 - \frac{10e}{d}\right] \tag{6}$$

and

$$\eta_2 = \eta_{20} + (1 - \eta_{20}) \frac{10\,e}{d} \tag{7}$$

In general, the resistance of a concrete filled circular tubular section to compression may increase by *15%* under axial load only when the effect of tri-axial confinement is considered. Linear interpolation is permitted for various load eccentricities of $e \leq d/10$. The basic values η_{10} and η_{20} depend on the non-dimensional slenderness $\bar{\lambda}$, which can be read off from Table *5*. Non-dimensional slenderness is described in section 4.1.3.

If the eccentricity *e* exceeds the value *d/10*, or if the non-dimensional slenderness exceeds the value *0.5* then $\eta_1 = 0$ and $\eta_2 = 1.0$.

Table 5: Basic value η_{10} and η_{20} to allow for the effect of tri-axial confinement in concrete filled circular tubular sections, as provided in EC 4 applicable for concrete grades $(f_{ck})_{cy}$ = 25 to 55 N/mm^2

	$\bar{\lambda} = 0.0$	$\bar{\lambda} = 0.1$	$\bar{\lambda} = 0.2$	$\bar{\lambda} = 0.3$	$\bar{\lambda} = 0.4$	$\bar{\lambda} \geq 0.5$
η_{10}	4.90	3.22	1.88	0.88	0.22	0.00
η_{20}	0.75	0.80	0.85	0.90	0.95	1.00

4.1.3 Non-dimensional slenderness

The plastic resistance to compression of a composite cross-section P_p, represents the maximum load that can be applied to a short column. For slender columns with low elastic critical load, overall buckling may be critical. In a typical buckling curve for an ideal column as shown in Fig. 4(*a*), the horizontal line represents P_p, while the curve represents P_{cr}, which is a function of the column slenderness. These two curves limit the compressive resistance of ideal column.

For convenience, column strength curves are plotted in non dimensionalised form as shown in Fig. 4(*b*) the buckling resistance of a column may be expressed as a proportion χ of the plastic resistance to compression, P_p thereby non-dimensioning the vertical axis of Fig. 4(*a*), where χ is called the reduction factor. The horizontal axis may be non-dimensionalised similarly by P_{cr} as shown in Fig. 4(*b*).

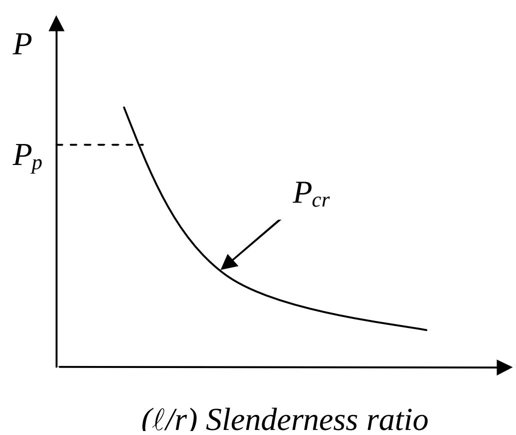


Fig. 4(a): Idealised column buckling curve

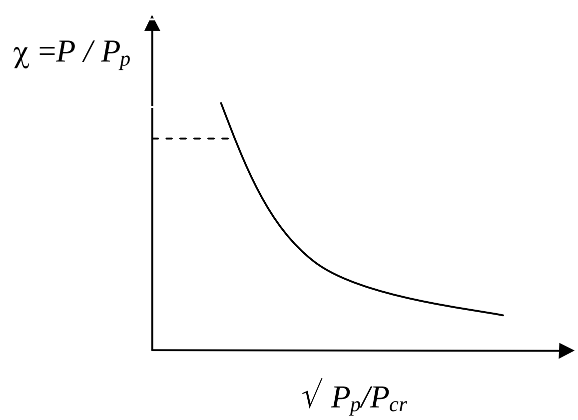


Fig. 4(b) Non-dimensionalised column buckling curve

Practical columns have strength curves different from ideal columns due to residual stresses and geometric imperfections. The European buckling curves have been drawn after incorporating the effects of both residual stresses and geometric imperfections. They form the basis of column buckling design for both steel and composite columns in *EC 3* and *EC4*. For using the European buckling curves, the non-dimensional slenderness of the column should be first evaluated as follows:

$$\bar{\lambda} = \sqrt{\frac{P_{pu}}{P_{cr}}} = \sqrt{\frac{f_y}{\pi^2 E}}\ \frac{\ell}{r} = f\left(\frac{\ell}{r}\right) \tag{8}$$

where

P_{pu} plastic resistance of the cross-section to compression, according to Eqn (2) or Eqn. *(5)* with $\gamma_a = \gamma_c = \gamma_s = 1.0$

P_{cr} is the elastic buckling load of the column as defined in Eqn. (11).

Once the $\bar{\lambda}$ value of a composite column is established, the buckling resistance to compression of the column may be evaluated as given below.

4.1.4 Local buckling of steel sections

Both Eqns. (2) and (5) are valid provided that local buckling in the steel sections does not occur. To prevent premature local buckling, the width to thickness ratio of the steel sections in compression must satisfy the following limits:

- $\frac{d}{t} \leq 85 \in^2$ for concrete filled circular tubular sections
- $\frac{h}{t} \leq 50 \in$ for concrete filled rectangular tubular sections
- $\frac{b}{t_f} \leq 43 \in$ for partially encased I sections (9)

where

$$\in = \sqrt{\frac{250}{f_y}} \tag{10}$$

f_y is the yield strength of the steel section in $N/mm^2 (MPa)$.

For fully encased steel sections, no verification for local buckling is necessary as the concrete surrounding effectively prevents local buckling. However, the concrete cover to the flange of a fully encased steel section should not be less than *40 mm*, nor less than one-sixth of the breadth, *b*, of the flange for it to be effective in preventing local buckling.

Local buckling may be critical in some concrete filled rectangular tubular sections with large h/t ratios. Designs using sections, which exceed the local buckling limits for semi-compact sections, should be verified by tests.

4.2 Effective elastic flexural stiffness

Composite columns may fail in buckling and one important parameter for the buckling design of composite columns is its elastic critical buckling load (Euler Load), P_{cr}, which is defined as follows:

$$P_{cr} = \frac{\pi^2 (EI)_e}{\ell^2} \tag{11}$$

where

$(EI)_e$ is the effective elastic flexural stiffness of the composite column (defined in the next section).

ℓ is the effective length of the column, which may be conservatively taken as system length L for an isolated non-sway composite column.

However, the value of the flexural stiffness may decrease with time due to creep and shrinkage of concrete. Two design rules for the evaluation of the effective elastic flexural stiffness of composite columns are given below.

4.2.1 Short term loading

The effective elastic flexural stiffness, $(EI)_e$, is obtained by adding up the flexural stiffness of the individual components of the cross-section:

$$(EI)_e = E_a I_a + 0.8\, E_{cd} I_c + E_s I_s \tag{12}$$

where

I_a , I_c and I_s are the second moments of area of the steel section, the concrete (assumed uncracked) and the reinforcement about the axis of bending considered respectively.

E_a and E_s are the moduli of elasticity of the steel section and the reinforcement

$0.8\, E_{cd} I_c$ is the effective stiffness of the concrete; the factor *0.8* is an empirical multiplier (determined by a calibration exercise to give good agreement with test results). Note I_c is the moment of inertia about the centroid of the uncracked column section.

$$E_{cd} = E_{cm} / \gamma^*_c \tag{13}$$

E_{cm} is the secant modulus of the concrete, see Table *2* of the text.

γ^*_c is reduced to *1.35* for the determination of the effective stiffness of concrete according to Eurocode *2.*

Note: Dividing the Modulus of Elasticity by γ_m is unusual and is included here to obtain the effective stiffness, which conforms to test data.

4.2.2 Long term loading

For slender columns under long-term loading, the creep and shrinkage of concrete will cause a reduction in the effective elastic flexural stiffness of the composite column, thereby reducing the buckling resistance. However, this effect is significant only for slender columns. As a simple rule, *the effect of long term loading should be considered if the buckling length to depth ratio of a composite column exceeds 15.*

If the eccentricity of loading as defined in Eqn. *4* is more than twice the cross-section dimension, the effect on the bending moment distribution caused by increased deflections due to creep and shrinkage of concrete will be very small. Consequently, it may be neglected and no provision for long-term loading is necessary. Moreover, no provision is also necessary if the non-dimensional slenderness, $\bar{\lambda}$ of the composite column is less than the limiting values given in Table *6*

.

Table 6*: *Limiting values of* $\bar{\lambda}$ *for long term loading

	Braced Non-sway systems	Unbraced and/or sway systems
Concrete encased cross-sections	0.8	0.5
Concrete filled cross sections	$\frac{0.8}{1-\delta}$	$\frac{0.5}{1-\delta}$

Note: δ is the steel contribution ratio as defined in Eqn. *3*.

However, when $\bar{\lambda}$ exceeds the limits given by Table-*6* and *e/d* is less than *2*, the effect of creep and shrinkage of concrete should be allowed for by employing the modulus of elasticity of the concrete $E_{c\infty}$ instead of E_{cd} in Eqn. *13*, which is defined as follows:

$$E_{c\infty} = E_{cd}\left[1 - \frac{0.5\,P_d}{P}\right] \tag{14}$$

where

P is the applied <u>design</u> load.

P_d is the part of the applied <u>design</u> load permanently acting on the column.

The effect of long-term loading may be ignored for concrete filled tubular sections with $\bar{\lambda} \leq 2.0$ provided that δ is greater than *0.6* for braced (or non-sway) columns, and *0.75* for unbraced (and/or sway) columns.

4.3 Resistance of members to axial compression

For each of the principal axes of the column, the designer should check that

$$P \leq \chi P_p \tag{15}$$

where

P_p is the plastic resistance to compression of the cross-section, from Eqn. (2) or Eqn. (5)

χ *i*s the reduction factor due to column buckling and is a function of the non-dimensional slenderness of the composite column.

The European buckling curves illustrated in Fig. 4 (*c*) are proposed to be used for composite columns. They are selected according to the types of the steel sections and the axis of bending:

curve *a* for concrete filled tubular sections

curve *b* for fully or partially concrete encased I-sections buckling about the strong axis of the steel sections (*x*-*x* axis).

curve *c* for fully and partially concrete encased I-sections buckling about the weak axis of the steel sections (*y*-*y* axis).

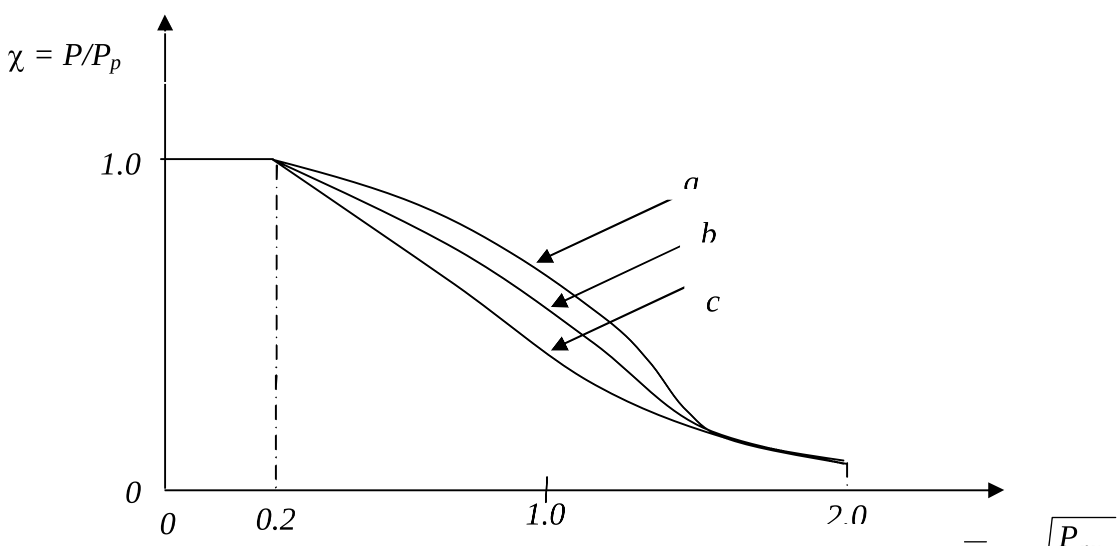


Fig. 4(c) European buckling curves

These curves can also be described mathematically as follows:

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} \quad but\ \chi \leq 1.0 \tag{16}$$

$$\phi = 0.5\left[1 + \alpha(\bar{\lambda} - 0.2) + \bar{\lambda}^2\right] \tag{17}$$

where

The factor α allows for different levels of imperfections and residual stresses in the columns corresponding to curves *a*, *b*, and *c*. Table *7* gives the value of α for each buckling curve. Note that the second order moment due to imperfection, has been incorporated in the method by using multiple buckling curves; no additional considerations are necessary.

(It should be noted by way of contrast that *IS: 456-2000* for reinforced concrete columns specifies a *2 cm* eccentricity irrespective of column geometry. The method suggested here allows for an eccentricity of load application by the term α**.** No further provision is necessary for steel and composite columns)

Using the values of $\bar{\lambda}$ determined from Eqn. (*8*) and the reduction factor χ calculated from Eqn. (*16*), the design buckling resistance of the composite column to compression, P_b or χP_p may thus be evaluated.

Table 7*: *Imperfection factor α for the buckling curves

European buckling curve	*a*	*b*	*c*
Imperfection factor α	0.21	0.34	0.49

The isolated non-sway composite columns need not be checked for buckling, if anyone of the following conditions is satisfied:

(*a*) The axial force in the column is less than $0.1\ P_{cr}$ where P_{cr} is the elastic buckling load of the column given by Eqn (*11*)

(*b*) The non-dimensional slenderness, $\bar{\lambda}$ given by Eqn. (*8*) is less than *0.2*.

5.0 STEPS IN DESIGN

5.1 Design Steps for columns with axial load

5.1.1 List the composite column specifications and the design value of forces and moments.

5.1.2 List material properties such as f_y, f_{sk}, $(f_{ck})_{cy}$, E_a, E_s, E_c

5.1.3 List sectional properties A_a, A_s, A_c, I_a, I_s, I_c of the selected section.

5.1.4 Design checks

(1) Evaluate plastic resistance, P_p of the cross-section from equation,

$$P_p = A_a f_y/\gamma_a + \alpha_c A_c (f_{ck})_{cy}/\gamma_c + A_s f_{sk}/\gamma_s$$

(2) Evaluate effective flexural stiffness, $(EI)_{ex}$ and $(EI)_{ey}$, of the cross- section for short term loading from equations,

$$(EI)_{ex} = E_a I_{ax} + 0.8\, E_{cd} I_{cx} + E_s I_{sx}$$

$$(EI)_{ey} = E_a I_{ay} + 0.8\, E_{cd} I_{cy} + E_s I_{sy}$$

(3) Evaluate non-dimensional slenderness, $\bar{\lambda}_x$ and $\bar{\lambda}_y$ from equation,

$$\bar{\lambda}_x = \left(\frac{P_{pu}}{(P_{cr})_x}\right)^{\frac{1}{2}}$$

$$\bar{\lambda}_y = \left(\frac{P_{pu}}{(P_{cr})_y}\right)^{\frac{1}{2}}$$

where

$$P_{pu} = A_a f_y + \alpha_c A_c (f_{ck})_{cu} + A_s f_{sk} \qquad (\gamma_a = \gamma_c = \gamma_s = 1.0\ ;\ \text{Refer Eqn (8)})$$

$$P_{crx} = \frac{\pi^2 (EI)_{ex}}{\ell^2}$$

and $P_{cry} = \dfrac{\pi^2 (EI)_{ey}}{\ell^2}$

(4) Check the resistance of the section under axial compression about both the axes.

Design against axial compression is satisfied if following conditions are satisfied:

$$P < \chi_x P_p$$

$$P < \chi_y P_p$$

where

$$\chi_x = \frac{1}{\left(\phi_x + \left\{\phi_x^2 - \bar{\lambda}_x^2\right\}^{\frac{1}{2}}\right)}$$

$$\text{and } \phi_x = 0.5\left[1 + \alpha_x\left(\bar{\lambda}_x - 0.2\right) + \bar{\lambda}_x^2\right]$$

$$\chi_y = \frac{1}{\left(\phi_y + \left\{\phi_y^2 - \bar{\lambda}_y^2\right\}^{\frac{1}{2}}\right)}$$

$$\text{and } \phi_y = 0.5\left[1 + \alpha_y\left(\bar{\lambda}_y - 0.2\right) + \overline{\lambda_y}^2\right]$$

6.0 CONCLUSION

In this chapter the design of steel-concrete composite column subjected to axial load using simplified design method suggested in EC 4 is discussed. The use of European buckling curve in the design of steel-concrete composite column is described. The advantages of steel–concrete composite column and the properties of materials used are also discussed.

A worked example illustrating the use of the above design procedure is appended to this chapter.

NOTATION

A	cross-sectional area
b	breadth of element
d	diameter, depth of element.
e	eccentricity of loading
e_o	initial imperfections
E	modulus of elasticity
$(EI)_e$	effective elastic flexural stiffness of a composite cross-section.
$(f_{ck})_{cu}$	characteristic compressive (cube) strength of concrete
f_{sk}	characteristic strength of reinforcement
f_y	yield strength of steel
$(f_{ck})_{cy}$	characteristic compressive (cylinder) strength of concrete, given by 0.80 times 28 days cube strength of concrete.
f_{ctm}	mean tensile strength of concrete
p_{ck}, p_y, p_{sk}	design strength of concrete, steel section and reinforcement respectively
I	second moment of area (with subscripts)
ℓ	effective length
L	length or span
P	axial force

P_p	plastic resistance to compression of the cross section.
P_{pu}	plastic resistance to compression of the cross section with $\gamma_a = \gamma_c = \gamma_s = 1.0$
P_{cr}	elastic critical load of a column
P_c	axial resistance of concrete, $A_c\, p_{ck}$
t	thickness of element
Z_e	elastic section modulus
Z_p	plastic section modulus

Greek letters

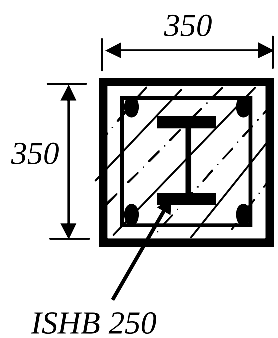
γ_f	partial safety factor for loads
γ_m	partial safety factor for materials (with subscripts)
γ^*_c	reduction factor(1.35) used for reducing E_{cm} value
$\bar{\lambda}$	slenderness ($\bar{\lambda}$ = non-dimensional slenderness)
ε	coefficient $=\sqrt{250/f_y}$
α	imperfection factor
α_c	strength coefficient for concrete
χ	reduction factor buckling
η_c	reduction factor on axial buckling resistance P_b for long term loading
β_a	steel contribution ratio, $A_a\, p_y / P_p$
s	reinforcement
w	web of steel section
d	dead load
l	live load

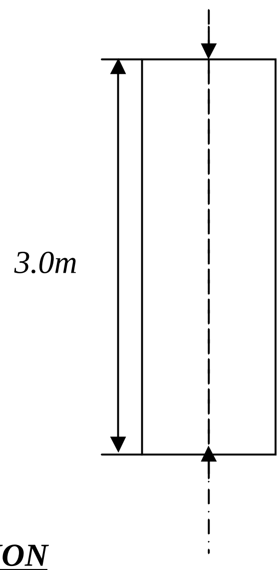
Note-The subscript x, y denote the x-x and y-y axes of the section respectively. x-x denotes the major axes whilst y-y denotes the minor principal axes.

Structural Steel Design Project	Job No:	Sheet *1 of 6*	Rev
	Job Title: ***Composite Column Design***		
	Worked Example: ***1***		
		Made By ***PU***	Date
Calculation Sheet		Checked By ***RN***	Date

PROBLEM1

Obtain plastic resistance of a steel section made of ISHB 250 encased in concrete. The height of the column is 3.0m and is pin ended.





5.1.1 DETAILS OF THE SECTION

Column dimension	*350 X 350 X 3000*
Concrete Grade	*M30*
Steel Section	*ISHB 250*
Reinforcement steel area	*Fe 415* *0.5% of gross concrete area.*
Cover from the flanges	*50 mm*
Height of the column	*3000 mm*

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5.1.2 LIST MATERIAL PROPERTIES

(1) Structural steel

Steel section ISHB 250
Nominal yield strength $f_y = 250\ N/mm^2$
Modulus of elasticity $E_a = 200\ kN/mm^2$

(2) Concrete

Concrete grade M30
Characteristic strength (cube), $(f_{ck})_{cu} = 30\ N/mm^2$
Characteristic strength (cylinder), $(f_{ck})_{cy} = 25\ N/mm^2$
Secant modulus of elasticity for short term loading, $E_{cm} = 31220\ N/mm^2$

(3) Reinforcing steel

Steel grade Fe 415
Characteristic strength $f_{sk} = 415\ N/mm^2$
Modulus of elasticity $E_s = 200\ kN/mm^2$

(4) Partial safety factors

$\gamma_a = 1.15$

$\gamma_c = 1.5$

$\gamma_s = 1.15$

5.1.3 LIST SECTION PROPERTIES OF THE GIVEN SECTION

(1) Steel section

$A_a = 6971\ mm^2$

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</table>

$h = 250$ mm
$t_w = 8.8$ mm
$I_{ax} = 79.8 * 10^6$ mm^4
$I_{ay} = 20.1 * 10^6$ mm^4

(2) Reinforcing steel

Area reinforcement = 0.5% of gross concrete area $= 0.5/100 * (115529)$
$= 578$ mm^2

Provide 4 bars of 14 mm dia., $A_s = 616$ mm^2

(3) Concrete

$A_c = A_{gross} - A_a - A_s$

$= 350 * 350 - 6971 - 616$

$= 114913$ mm^2

5.1.4 DESIGN CHECKS

(1)Plastic resistance of the section

$P_p = A_a f_y/\gamma_a + \alpha_c A_c (f_{ck})_{cy}/\gamma_c + A_s f_{sk}/\gamma_s$
$P_p = A_a f_y/\gamma_a + \alpha_c A_c (0.80 * (f_{ck})_{cu})/\gamma_c + A_s f_{sk}/\gamma_s$

$= [6971 * 250/1.15 + 0.85 * 114913 * 25/1.5 + 616 * 415/1.15]/1000$
$= 3366$ kN

$P_p = 3366$ kN

(2) Calculation of Effective elastic flexural stiffness of the section

<u>About the major axis</u>

$(EI)_{ex} = E_a I_{ax} + 0.8\, E_{cd} I_{cx} + E_s I_{sx}$

$I_{ax} = 79.8 * 10^6$ mm^4

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$I_{sx} = Ah^2$
$= 616 * [\ 350/2-25-7]^2$
$= 12.6 * 10^6\ mm^4$

$I_{cx} = (350)^4/12 - [\ 79.8 + 12.6] * 10^6$
$= 1158 * 10^6\ mm^4$

$E_{cd} = E_{cm}/\gamma^*_c = 31220/1.35 = 23125\ N/mm^2$

$(EI)_{ex} = 2.0 * 10^5 * 79.8 * 10^6 + 0.8 * 23125 * 1158.09 * 10^6 + 2.0 * 10^5 * 12.6 * 10^6$
$= 39.4 * 10^{12}\ N\ mm^2$

<u>*About minor axis*</u>

$(EI)_{ey} = 28.5 * 10^{12}\ N\ mm^2$

(3) Non dimensonal slenderness

$\bar{\lambda} = (P_{pu}/P_{cr})^{1/2}$

Value of P_{pu} $(\gamma_a = \gamma_c = \gamma_s = 1.0)$

$P_{pu} = A_a f_y + \alpha_c A_c (f_{ck})_{cy} + A_s f_{sk}$

$P_{pu} = A_a f_y + \alpha_c A_c * 0.80 * (f_{ck})_{cu} + A_s f_{sk}$

$= 6971 * 250 + 0.85 * 114913 * 25 + 415 * 616$

$= 44.40 * 10^5 N$

$= 4440\ kN$

$P_{pu} = 4440\ kN$

$$(P_{cr})_x = \frac{\pi^2 (EI)_{ex}}{\ell^2}$$

$$= \frac{\pi^2 * 39.4 * 10^{12}}{(3000)^2}$$

$= 43207\ kN$

$(P_{cr})_x = 43207\ kN$

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$$(P_{cr})_y = \frac{\pi^2 * 28.5 * 10^{12}}{(3000)^2} = 31254kN$$

$(P_{cr})_y$ = $31254\ kN$

$$\bar{\lambda}_x = (44.40/432.07)^{\frac{1}{2}} = 0.320$$

$$\bar{\lambda}_y = (44.40/312.54)^{\frac{1}{2}} = 0.377$$

(4) Resistance of the composite column under axial compression

Buckling resistance of the section should satisfy the following condition

$\boldsymbol{P_b < \chi P_p}$

where

P_b= *buckling load*

χ = *reduction factor for column buckling*

P_p = *plastic resistance of the section*

= *3366 kN*

χ ***values :***

<u>*About major axis*</u>

$\alpha_x = 0.34$

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$\chi_x = 1 / \{\phi_x + (\phi_x^2 - \bar{\lambda}_x^2)^{1/2}\}$

$\phi_x = 0.5 [1 + \alpha_x (\bar{\lambda}_x - 0.2) + \bar{\lambda}_x^2]$

$= 0.5 [1 + 0.34(0.320-0.2) + (0.320)^2] = 0.572$

$\chi_x = 1 / \{0.572 + [(0.572)^2 - (0.320)^2]^{1/2}\}$

$= 0.956$

<u>About minor axis</u>

$\alpha_y = 0.49$

$\phi_y = 0.61$

$\chi_y = 0.918$

$(P_b)_x = \chi_x P_P$

$= 0.956 * 3366 = 3218$ kN

$(P_b)_y = \chi_y P_P$

$= 0.918 * 3366 = 3090$ kN

Hence, the lower value of plastic resistance against buckling, $P_b = 3090$ kN

Lower value of plastic resistance = 3090 kN

26 STEEL-CONCRETE COMPOSITE COLUMNS-II

1.0 INTRODUCTION

In a previous chapter, the design of a steel-concrete composite column under axial loading was discussed. This chapter deals with the design of steel-concrete composite columns subjected to both axial load and bending. To design a composite column under combined compression and bending, it is first isolated from the framework, and the end moments which result from the analysis of the system as a whole are taken to act on the column under consideration. Internal moments and forces within the column length are determined from the structural consideration of end moments, axial and transverse loads. For each axis of symmetry, the buckling resistance to compression is first checked with the relevant non-dimensional slenderness of the composite column. Thereafter the moment resistance of the composite cross-section is checked in the presence of applied moment about each axis, e.g. x-x and y-y axis, with the relevant non-dimensional slenderness values of the composite column. For slender columns, both the effects of long term loading and the second order effects are included.

2.0 COMBINED COMPRESSION AND UNI-AXIAL BENDING

The design method described here is an extension of the simplified design method discussed in the previous chapter for the design of steel-concrete composite columns under axial load.

2.1 Interaction Curve for Compression and Uni-axial Bending

The resistance of the composite column to combined compression and bending is determined using an interaction curve. Fig. *1* represents the non-dimensional interaction curve for compression and uni-axial bending for a composite cross-section.

In a typical interaction curve of a column with steel section only, it is observed that the moment of resistance undergoes a continuous reduction with an increase in the axial load. However, a short composite column will often exhibit increases in the moment resistance beyond plastic moment under relatively low values of axial load. This is because under some favourable conditions, the compressive axial load would prevent concrete cracking and make the composite cross-section of a short column more effective in resisting moments. The interaction curve for a short composite column can be obtained by considering several positions of the neutral axis of the cross-section, h_n, and determining the internal forces and moments from the resulting stress blocks.

(It should be noted by way of contrast that *IS: 456-1978* for reinforced concrete columns specifies a *2 cm* eccentricity irrespective of column geometry. The method suggested here, using EC4, allows for an eccentricity of load application by the term α and therefore no further provision is necessary for steel columns. Another noteworthy feature is the prescription of strain limitation in *IS: 456-1978,* whereas EC4 does not impose such a limitation. The relevant provision in the Indian Code limits the concrete strain to *0.0035* minus *0.75* times the strain at the least compressed extreme fibre)

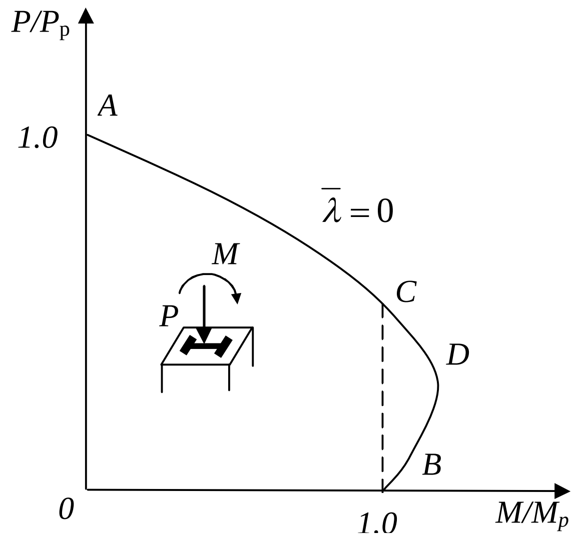

Fig. 1 Interaction curve for compression and uni-axial bending

Fig. *2* shows an interaction curve drawn using simplified design method suggested in the *UK* National Application Document for EC 4 *(NAD)*. This neglects the increase in moment capacity beyond M_P discussed above, (under relatively low axial compressive loads).

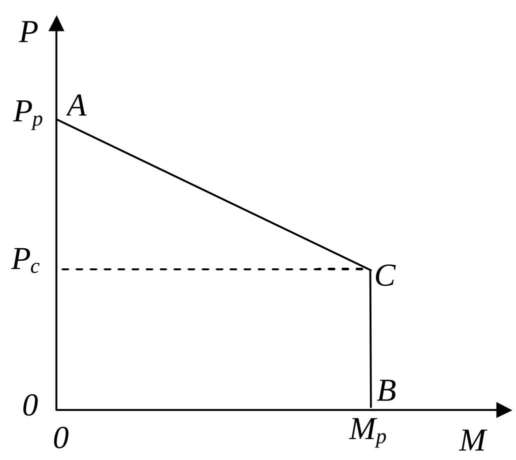

Fig. 2 Interaction curve for compression and uni-axial bending using the simplified method

Fig. *3* shows the stress distributions in the cross-section of a concrete filled rectangular tubular section at each point, *A*, *B* and *C* of the interaction curve given in Fig. *2*. It is important to note that:

- Point *A* marks the plastic resistance of the cross-section to compression (at this point the bending moment is zero).

$$P_A = P_p = A_a.f_y / \gamma_a + \alpha_c.A_c.(f_{ck})_{cy} / \gamma_c + A_s.f_{sk} / \gamma_s \qquad (1)$$

$$M_A = 0 \qquad (2)$$

- Point *B* corresponds to the plastic moment resistance of the cross-section (the axial compression is zero).

$$P_B = 0 \qquad (3)$$

$$M_B = M_p = p_y (Z_{pa} - Z_{pan}) + p_{sk}(Z_{ps} - Z_{psn}) + p_{ck}(Z_{pc} - Z_{pcn}) \qquad (4)$$

where

Z_{ps}, Z_{pa}, and Z_{pc} are plastic section moduli of the reinforcement, steel section, and concrete about their own centroids respectively.

Z_{psn}, Z_{pan} and Z_{pcn} are plastic section moduli of the reinforcement, steel section, and concrete about neutral axis respectively.

- At point *C*, the compressive and the moment resistances of the column are given as follows;

$$P_C = P_c = A_c\, p_{ck}. \qquad (5)$$

$$M_C = M_p \qquad (6)$$

- The expressions may be obtained by combining the stress distributions of the cross-section at points *B* and *C*; the compression area of the concrete at point *B* is equal to the tension area of the concrete at point *C*. The moment resistance at point *C* is equal to that at point *B*, since the stress resultants from the additionally compressed parts nullify each other in the central region of the cross-section. However, these additionally compressed regions create an internal axial force, which is equal to the plastic resistance to compression of the concrete, P_c alone.

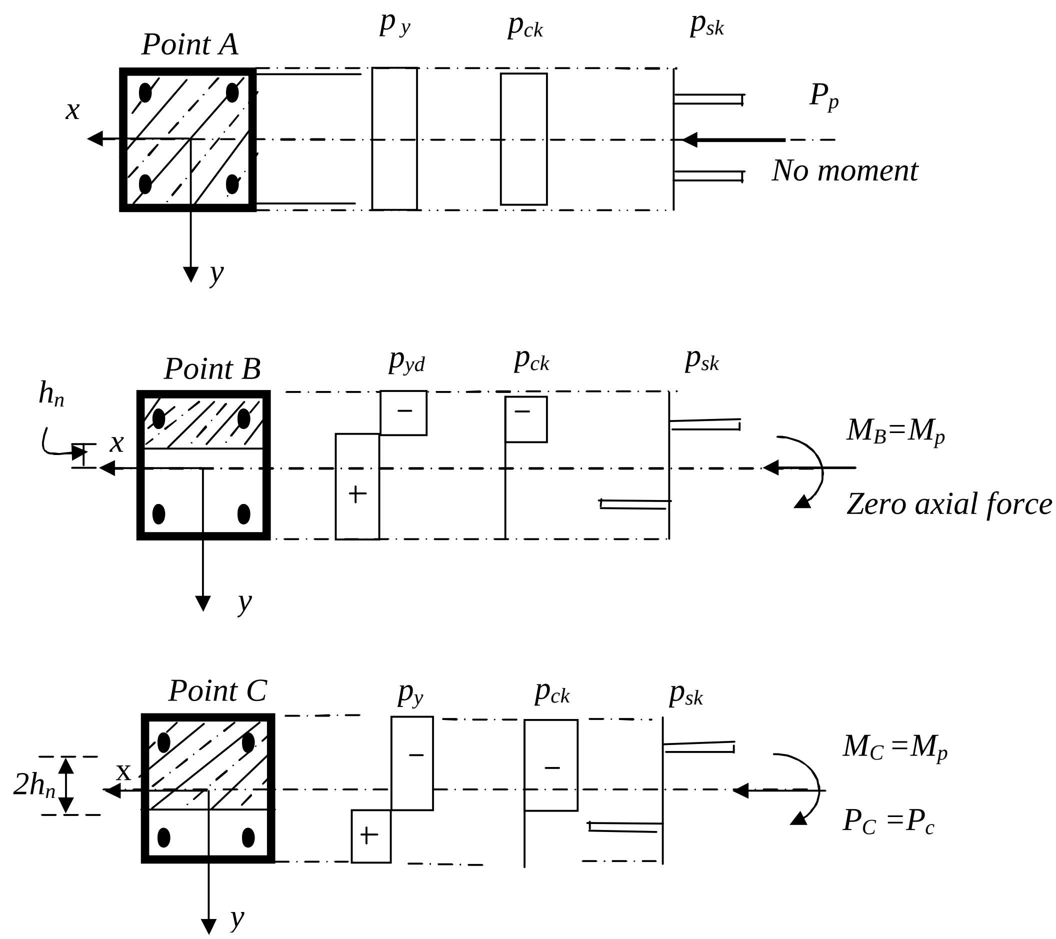

Fig. 3 Stress distributions for the points of the interaction curve for concrete filled rectangular tubular sections

It is important to note that the positions of the neutral axis for points *B* and *C*, h_n, can be determined from the difference in stresses at points *B* and *C*. The resulting axial forces, which are dependent on the position of the neutral axis of the cross-section, h_n, can easily be determined as shown in Fig. *4*. The sum of these forces is equal to P_c. This calculation enables the equation defining h_n to be determined, which is different for various types of sections.

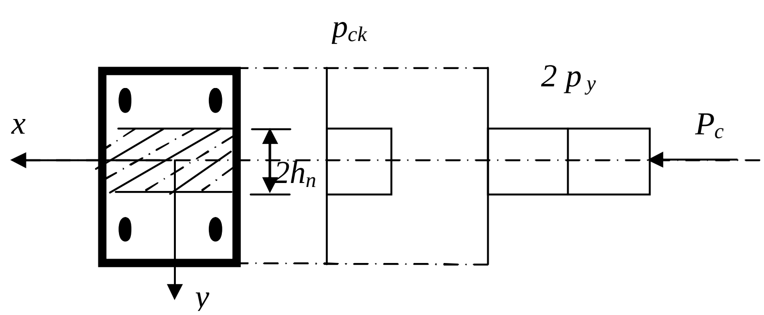

Fig. 4(a) Variation in the neutral axis positions

***(1)* For concrete encased steel sections:**

Major axis bending

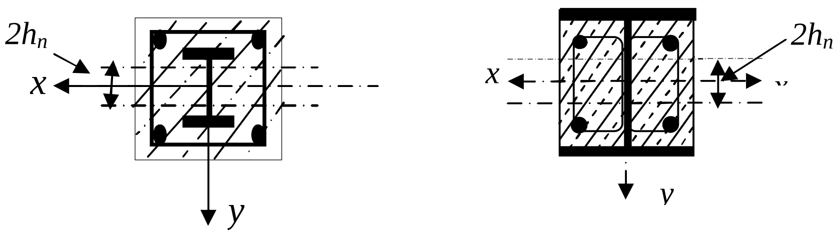

Fig. 4(b)

(1) Neutral axis in the web*:* $h_n \leq [\, h/2 - t_f \,]$

$$h_n = \frac{A_c p_{ck} - A_s'(2p_{sk} - p_{ck})}{2b_c p_{ck} + 2t_w(2p_y - p_{ck})}$$

(2) Neutral axis in the flange: $[h/2 - t_f\,] \leq h_n \leq h/2$

$$h_n = \frac{A_c p_{ck} - A_s'(2p_{sk} - p_{ck}) + (b - t_w)(h - 2t_f)(2p_y - p_{ck})}{2b_c p_{ck} + 2b(2p_y - p_{ck})}$$

(3) Neutral axis outside the steel section: $h/2 \leq h_n \leq h_c/2$

$$h_n = \frac{A_c p_{ck} - A_s'(2p_{sk} - p_{ck}) + A_a(2p_y - p_{ck})}{2b_c p_{ck}}$$

Minor axis bending

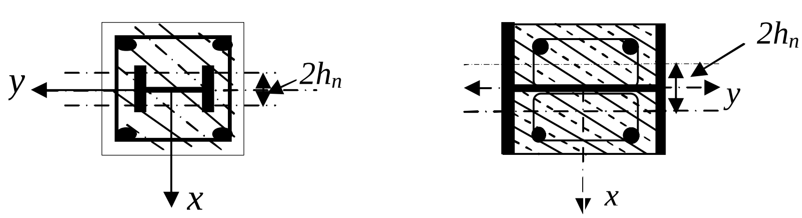

Fig. 4(c)

(1) Neutral axis in the web: $h_n \leq t_w/2$

$$h_n = \frac{A_c p_{ck} - A_s'(2p_{sk} - p_{ck})}{2h_c p_{ck} + 2h(2p_y - p_{ck})}$$

(2) Neutral axis in the flange: $t_w/2 < h_n < b/2$

$$h_n = \frac{A_c p_{ck} - A'_s(2p_{sk} - p_{ck}) + t_w(2t_f - h)(2p_y - p_{ck})}{2h_c p_{ck} + 4t_f(2p_y - p_{ck})}$$

(3) Neutral axis outside the steel section: $b/2 \leq h_n \leq b_c/2$

$$h_n = \frac{A_c p_{ck} - A'_s(2p_{sk} - p_{ck}) - A_a(2p_y - p_{ck})}{2h_c p_{ck}}$$

Note: A'_s is the sum of the reinforcement area within the region of $2h_n$

(2) For concrete filled tubular sections

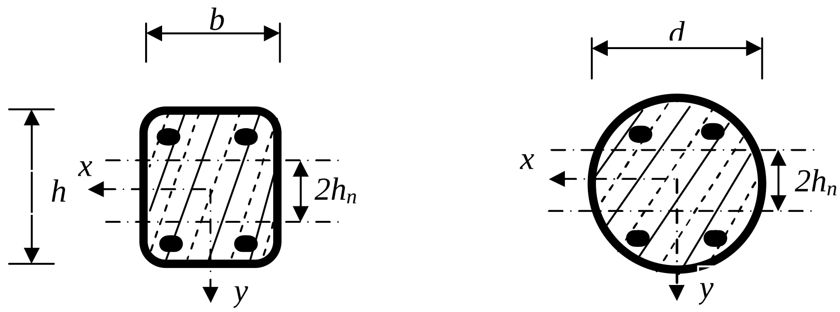

Fig. 4(d)

Major axis bending

$$h_n = \frac{A_c p_{ck} - A'_s(2p_{sk} - p_{ck})}{2b_c p_{ck} + 4t(2p_y - p_{ck})}$$

Note:

- For circular tubular section substitute $b_c = d$
- For minor axis bending the same equations can be used by interchanging h and b as well as the subscripts x and y.

2.2 Analysis of Bending Moments due to Second Order Effects

Under the action of a design axial load, P, on a column with an initial imperfection, e_o, as shown in Fig. 5, there will be a maximum internal moment of $P.e_o$. It is important to note that this second order moment, or 'imperfection moment', does not need to be considered separately, as its effect on the buckling resistance of the composite column is already accounted for in the European buckling curves.

However, in addition to axial forces, a composite column may be also subject to end moments as a consequence of transverse loads acting on it, or because the composite column is a part of a frame. The moments and the displacements obtained initially are referred to as 'first order' values. For slender columns, the 'first order' displacements may be significant and additional or 'second order' bending moments may be induced under the actions of applied loads. As a simple rule, the second order effects should be considered if the buckling length to depth ratio of a composite column exceeds *15*.

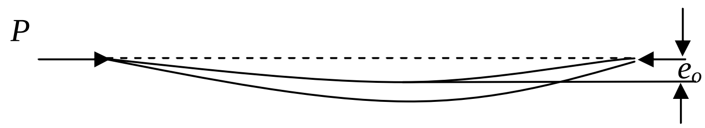

Fig. 5 Initially imperfect column under axial compression

The second order effects on bending moments for isolated non-sway columns should be considered if both of the following conditions are satisfied:

$$(1)\quad \frac{P}{P_{cr}} > 0.1 \tag{7}$$

where

P is the design applied load, and

P_{cr} is the elastic critical load of the composite column.

(2) Elastic slenderness conforms to:

$$\bar{\lambda} > 0.2 \tag{8}$$

where

$\bar{\lambda}$ is the non-dimensional slenderness of the composite column

In case the above two conditions are met, the second order effects may be allowed for by modifying the maximum first order bending moment (moment obtained initially), M_{max}, with a correction factor k, which is defined as follows:

$$k = \frac{1}{1-\frac{P}{P_{cr}}} \geq 1.0 \tag{9}$$

where

P is the applied design load.
P_{cr} is the elastic critical load of the composite column.

2.3 **Resistance of Members under Combined Compression and Uni-axial Bending**

The graphical representation of the principle for checking the composite cross-section under combined compression and uni-axial bending is illustrated in Fig. *6*.

The design checks are carried out in the following stages:

(1) The resistance of the composite column under axial load is determined in the absence of bending, which is given by $\chi\ P_p$. The procedure is explained in detail in the previous chapter.

(2) The moment resistance of the composite column is then checked with the relevant non-dimensional slenderness, in the plane of the applied moment. As mentioned before, the initial imperfections of columns have been incorporated and no additional consideration of geometrical imperfections is necessary.

The design is adequate when the following condition is satisfied:

$$M \le 0\cdot 9\mu\, M_p \tag{10}$$

where

M	is the design bending moment, which may be factored to allow for second order effects, if necessary
μ	is the moment resistance ratio obtained from the interaction curve.
M_p	is the plastic moment resistance of the composite cross-section.

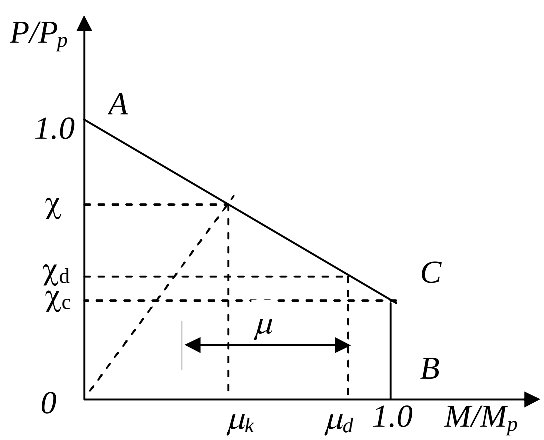

Fig. 6 Interaction curve for compression and uni-axial bending using the simplified method

The interaction curve shown in Fig. *6* has been determined without considering the strain limitations in the concrete. Hence the moments, including second order effects if necessary, are calculated using the effective elastic flexural stiffness, $(EI)_e$, and taking into account the entire concrete area of the cross-section, (i.e. concrete is uncracked).

Consequently, a reduction factor of *0.9* is applied to the moment resistance as shown in Equation *(10)* to allow for the simplifications in this approach. If the bending moment and the applied load are independent of each other, the value of μ must be limited to *1.0.*

Moment resistance ratio μ can be obtained from the interaction curve or may be evaluated. The method is described below.

Consider the interaction curve for combined compression and bending shown in Fig. *6*. Under an applied force P equal to χP_p, the horizontal coordinate $\mu_k M_p$ represents the second order moment due to imperfections of the column, or the 'imperfection moment'. It is important to recognise that the moment resistance of the column has been fully utilised in the presence of the 'imperfection moment'; the column cannot resist any additional applied moment.

χ_d represents the axial load ratio defined as follows:

$$\chi_d = P/P_p \qquad (11)$$

By reading off the horizontal distance from the interaction curve, the moment resistance ratio, μ, may be obtained and the moment resistance of the composite column under combined compression and bending may then be evaluated.

In accordance with the *UK NAD*, the moment resistance ratio μ for a composite column under combined compression and uni-axial bending is evaluated as follows:

$$\mu = \frac{(\chi - \chi_d)}{(1-\chi_c)\chi} \qquad \text{when } \chi_d \geq \chi_c \qquad (12)$$

$$= 1 - \frac{(1-\chi)\chi_d}{(1-\chi_c)\chi} \qquad \text{when } \chi_d < \chi_c \qquad (13)$$

where

χ_c = axial resistance ratio due to the concrete, $\dfrac{P_c}{P_p}$

χ_d = design axial resistance ratio, $\dfrac{P}{P_p}$

χ = reduction factor due to column buckling

The expression is obtained from geometry consideration of the simplified interaction curve illustrated in Fig. *6*. A worked example illustrating the use of the above design procedure is appended to this chapter.

3.0 COMBINED COMPRESSION AND BI-AXIAL BENDING

For the design of a composite column under combined compression and bi-axial bending, the axial resistance of the column in the presence of bending moment for each axis has to be evaluated separately. Thereafter the moment resistance of the composite column is checked in the presence of applied moment about each axis, with the relevant non-dimensional slenderness of the composite column. Imperfections have to be considered only for that axis along which the failure is more likely. If it is not evident which plane is more critical, checks should be made for both the axes.

The moment resistance ratios μ_x and μ_y for both the axes are evaluated as given below:

$$\mu_x = \frac{(\chi_x - \chi_d)}{(1-\chi_c)\chi_x} \qquad \text{when } \chi_d \geq \chi_c \qquad (14)$$

$$= 1 - \frac{(1-\chi_x)\chi_d}{(1-\chi_c)\chi_x} \qquad \text{when } \chi_d < \chi_c \qquad (15)$$

$$\mu_y = \frac{(\chi_y - \chi_d)}{(1-\chi_c)\chi_y} \qquad \text{when } \chi_d \geq \chi_c \qquad (16)$$

$$= 1 - \frac{(1-\chi_y)\chi_d}{(1-\chi_c)\chi_y} \qquad \text{when } \chi_d < \chi_c \qquad (17)$$

where

χ_x and χ_y are the reduction factors for buckling in the *x* and *y* directions respectively.

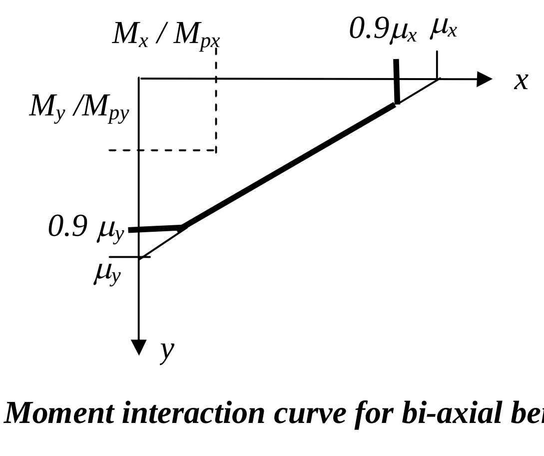

Fig. 7 Moment interaction curve for bi-axial bending

In addition to the two conditions given by Equations *(18)* and *(19)*, the interaction of the moments must also be checked using moment interaction curve as shown in Fig. *7*. The linear interaction curve is cut off at $0.9\mu_x$ and $0.9\mu_y$. The design moments, M_x and M_y related to the respective plastic moment resistances must lie within the moment interaction curve.

Hence the three conditions to be satisfied are:

$$\frac{M_x}{\mu_x M_{px}} \leq 0.9 \tag{18}$$

$$\frac{M_y}{\mu_y M_{py}} \leq 0.9 \tag{19}$$

$$\frac{M_x}{\mu_x M_{px}} + \frac{M_y}{\mu_y M_{py}} \leq 1.0 \tag{20}$$

When the effect of geometric imperfections is not considered the moment resistance ratio is evaluated as given below:

$$\mu = \frac{(1-\chi_d)}{(1-\chi_c)} \qquad \text{when } \chi_d > \chi_c \tag{21}$$

$$= 1.0 \qquad \text{when } \chi_d \leq \chi_c \tag{22}$$

A worked example on combined compression and bi-axial bending is appended to this chapter.

4.0 STEPS IN DESIGN

4.1 Design Steps for columns with axial load and uni-axial bending

4.1.1 List the composite column specifications and the design values of forces and moments.

4.1.2 List material properties such as f_y, f_{sk}, $(f_{ck})_{cy}$, E_a, E_s, E_c

4.1.3 List section properties A_a, A_s, A_c, I_a, I_s, I_c of the selected section

4.1.4 Design checks

(1) Evaluate plastic resistance, P_p of the cross-section from equation,

$P_p = A_a f_y/\gamma_a + \alpha_c A_c (f_{ck})_{cy} / \gamma_c + A_s f_{sk}/ \gamma_s$

(2) Evaluate effective flexural stiffness, $(EI)_e$ of the cross-section for short term loading in *x* and *y* direction using equation,

$(EI)_e = E_a I_a + 0.8\, E_{cd} I_c + E_s I_s$

(3) Evaluate non-dimensional slenderness, $\overline{\lambda}_x$ and $\overline{\lambda}_y$ in x and y directions from equation,

$$\overline{\lambda} = \left(\frac{P_{pu}}{(P_{cr})}\right)^{\frac{1}{2}}$$

where

$P_{pu} = A_a f_y + \alpha_c A_c (f_{ck})_{cy} + A_s f_{sk}$

Note: P_{pu} is the plastic resistance of the section with $\gamma_a = \gamma_c = \gamma_s = 1.0$

and $P_{cr} = \dfrac{\pi^2 (EI)_e}{\ell^2}$

(4) Check for long-term loading

The effect of long term loading can be neglected if following conditions are satisfied:

- Eccentricity, *e* given by

$e = M/P \geq 2$ *times the cross section dimension in the plane of bending considered*

- the non-dimensional slenderness $\overline{\lambda}$ in the plane of bending being considered exceeds the limits given in Table 6 of the previous chapter (Steel Concrete Composite Column-I)

(5) Check the resistance of the section under axial compression for both *x* and *y* axes.

Design against axial compression is satisfied if following condition is satisfied for both the axes:

$P < \chi P_p$

where

χ= reduction factor due to column buckling.

$$= \frac{1}{\left(\phi + \left\{\phi^2 - \bar{\lambda}^2\right\}^{\frac{1}{2}}\right)}$$

where

$$and\ \phi = 0.5\left[1 + \alpha\left(\bar{\lambda} - 0.2\right) + \bar{\lambda}^2\right]$$

(6) Check for second order effects

Isolated non – sway columns need not be checked for second order effects if following conditions are satisfied for the plane of bending under consideration:

- $P / P_{cr} \leq 0.1$
- $\bar{\lambda} \leq 0.2$

(7) Evaluate plastic moment resistance of the composite column about the plane of bending under consideration.

$$M_p = p_y (Z_{pa}-Z_{pan}) + 0.5\ p_{ck} (Z_{pc}-Z_{pcn}) + p_{sk} (Z_{ps}- Z_{psn})$$

where

Z_{ps}, Z_{pa}, and Z_{pc} are plastic section modulus of the reinforcement, steel section, and concrete about their own axes respectively.

Z_{psn},, Z_{pan}, and Z_{pcn} are plastic section modulus of the reinforcement, steel section, and concrete about neutral axis respectively.

(8) Check the resistance of the composite column under combined axial compression and uni-axial bending

The design against combined compression and uni-axial bending is adequate if following condition is satisfied:

$$M \leq 0.9\ \mu\ M_P$$

where
M design bending moment
M_p plastic moment resistanc
μ moment resistance ratio

4.2 Design Steps for columns with axial load and bi-axial bending

4.2.1 List the composite column specifications and the design values of forces and moments.

4.2.2 List material properties such as f_y, f_{sk}, $(f_{ck})_{cy}$, E_a, E_s, E_c

4.2.3 List section properties A_a, A_s, A_c, I_a, I_s, I_c of the selected section.

4.2.4 Design checks

(1) Evaluate plastic resistance, P_p of the cross-section from equation,

$$P_p = A_a f_y / \gamma_a + \alpha_c A_c (f_{ck})_{cy} / \gamma_c + A_s f_{sk} / \gamma_s$$

(2) Evaluate effective flexural stiffness, $(EI)_{ex}$ and $(EI)_{ey}$, of the cross- section for short term loading from equation,

$$(EI)_{ex} = E_a I_{ax} + 0.8\, E_{cd} I_{cx} + E_s I_{sx}$$

$$(EI)_{ey} = E_a I_{ay} + 0.8\, E_{cd} I_{cy} + E_s I_{sy}$$

(3) Evaluate non-dimensional slenderness, $\overline{\lambda}_x$ and $\overline{\lambda_y}$ from equation,

$$\overline{\lambda}_x = \left(\frac{P_{pu}}{(P_{cr})_x} \right)^{\frac{1}{2}}$$

$$\overline{\lambda}_y = \left(\frac{P_{pu}}{(P_{cr})_y} \right)^{\frac{1}{2}}$$

where

$$P_{pu} = A_a f_y + \alpha_c A_c (f_{ck})_{cy} + A_s f_{sk}$$

Note: P_{pu} is the plastic resistance of the section with $\gamma_a = \gamma_c = \gamma_s = 1.0$

$$(P_{cr})_x = \frac{\pi^2 (EI)_{ex}}{\ell^2}$$

and $$(P_{cr})_y = \frac{\pi^2 (EI)_{ey}}{\ell^2}$$

(4) Check for long term loading.

The effect of long-term loading can be neglected if following conditions are satisfied:

- Eccentricity, e given by

$e = M / P \geq 2$ times cross section dimension in the plane of bending considered.
$e_x \geq 2b_c$
and $e_y \geq 2h_c$

- the non-dimensional slenderness $\bar{\lambda}$ in the plane of bending being considered exceeds the limits given in Table 6 of the previous chapter (Steel Concrete Composite Column –I).

(5) Check the resistance of the section under axial compression about both the axes. Design against axial compression is satisfied if following conditions are satisfied:

$P < \chi_x P_p$

$P < \chi_y P_p$

where

$$\chi_x = \frac{1}{\left(\phi_x + \left\{\phi_x^2 - \bar{\lambda}_x^2\right\}^{\frac{1}{2}}\right)}$$

$$and\ \phi_x = 0.5\left[1 + \alpha_x\left(\bar{\lambda}_x - 0.2\right) + \bar{\lambda}_x^2\right]$$

$$\chi_y = \frac{1}{\left(\phi_y + \left\{\phi_y^2 - \bar{\lambda}_y^2\right\}^{\frac{1}{2}}\right)}$$

$$and\ \phi_y = 0.5\left[1 + \alpha_y\left(\bar{\lambda}_y - 0.2\right) + \bar{\lambda}_y^2\right]$$

(6) Check for second order effects

Isolated non – sway columns need not be checked for second order effects if:

$P / (P_{cr})_x \leq 0.1$ for bending about x-x axis

$P / (P_{cr})_y \leq 0.1$ for bending about y-y axis

(7) Evaluate plastic moment resistance of the composite column under axial compression and bi-axial bending about both the axes.

About x-x axis

$M_{px} = [p_y\ (Z_{pa}\text{-}Z_{pan}) + 0.5\ p_{ck}\ (Z_{pc}\text{-}Z_{pcn}\ ) + p_{sk}\ (Z_{ps}\text{-}\ Z_{psn})\ ]_x$

where

M_{px} plastic moment resistance about x-x axis

Z_{psx}, Z_{pax}, and Z_{pcx} are plastic section modulus of the reinforcement, steel section, and concrete about their own axes in *x* direction respectively.
Z_{psn}, Z_{pan},and Z_{pcn} are plastic section modulus of the reinforcement, steel section, and concrete about neutral axis in x direction respectively.

About y-y axis

$M_{py} =[\ p_y\ (Z_{pay}\text{-}Z_{pan}) + 0.5\ p_{ck}\ (Z_{pcy}\text{-}Z_{pcn}\ ) + p_{sk}\ (Z_{psy}\text{-}\ Z_{psn})\ ]_y$

where

M_{py} plastic moment resistance about y-y axis

Z_{psy}, Z_{pay}, and Z_{pcy} are plastic section moduli of the reinforcement, steel section, and concrete about their own axes in *y* direction respectively.

Z_{psn}, Z_{pan},and Z_{pcn} are plastic section modulus of the reinforcement, steel section, and concrete about neutral axis in *y* direction respectively.

(8) Evaluate resistance of the composite column under combined axial compression and bi-axial bending

The design against combined compression and bi-axial bending is adequate if following conditions are satisfied:

(1) $M_x \le 0.9\ \mu_x\ M_{Px}$

(2) $M_y \le 0.9\ \mu_y\ M_{Py}$

$$(3)\frac{M_x}{\mu_x M_{px}} + \frac{M_y}{\mu_y M_{py}} \le 1.0$$

where

μ_x and μ_y are the moment resistance ratios in the *x* and *y* directions respectively.

5.0 CONCLUSION

In this chapter the design of steel-concrete composite column subjected to axial load and bending is discussed. The use of interaction curve in the design of composite column subjected to both uni-axial bending and bi-axial bending is also described. Worked out example in each case is also appended to this chapter.

NOTATION

A	cross-sectional area
b	breadth of element
d	diameter, depth of element.
e	eccentricity of loading
e_o	initial imperfections
E	modulus of elasticity
$(EI)_e$	effective elastic flexural stiffness of a composite cross-section.
$(f_{ck})_{cu}$	characteristic compressive (cube) strength of concrete
$(f_{ck})_{cy}$	characteristic compressive (cylinder) strength of concrete, given by 0.80 times 28 days cube strength of concrete.
f_{sk}	characteristic strength of reinforcement
f_y	yield strength of steel
f_{ctm}	mean tensile strength of concrete
p_{ck}, p_y , p_{sk}	design strength of concrete, steel section and reinforcement respectively
h	height of element
h_n	depth of neutral axis from the middle line of the cross-section
I	second moment of area (with subscripts)
k	moment correction factor for second order effects
ℓ	buckling (or effective) length
L	length or span
M	moment (with subscripts)
P	axial force
M_p	plastic moment resistance of a cross-section
P_p	plastic resistance to compression of the cross section.
P_{pu}	plastic resistance to compression of the cross section with $\gamma_a = \gamma_c = \gamma_s = 1.0$
P_{cr}	elastic critical load of a column
P_c	axial resistance of concrete, $A_c\ p_{ck}$
t	thickness of element
Z_p	plastic section modulus

Greek letters

γ_f	partial safety factor for loads
γ	partial safety factor for materials (with subscripts)
γ^*_c	Reduction factor(1.35) used for reducing E_{cm} value
λ	slenderness ($\bar{\lambda}$ = non-dimensional slenderness)
ε	coefficient $\sqrt{250/f_y}$
α	imperfection factor
α_c	strength coefficient for concrete
χ	reduction factor buckling
χ_c	axial resistance ratio due to concrete, P_c/P_p
μ	moment resistance ratio

The subscripts to the above symbols are as follows:

a	structural steel
b	buckling
c	concrete
f	flange
k	characteristic value
s	reinforcement
w	web of steel section

Note-The subscript x, y denote the x-x and y-y axes of the section respectively. x-x denotes the major axes whilst y-y denotes the minor principal axes.

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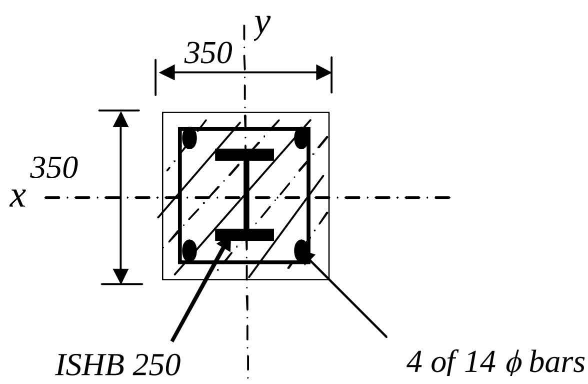
</table>

PROBLEM 1

Check the adequacy of the concrete encased composite section shown below for uni-axial bending.



4.1.1 DETAILS OF THE SECTION

Column dimension	*350 X 350 X 3000*
Concrete Grade	*M30*
Steel Section	*ISHB 250*
Steel Reinforcement	*4 Nos. of 14 mm dia bar, Fe415 grade*
Design Axial Load	*1500 kN*
Design bending moment about x-x axis	*180 kNm*
Design bending moment about y-y axis	*0 kNm*

Axial Load
P= 1500 kN

M_x = *180 kN*

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DESIGN CALCULATIONS:

4.1.2 LIST MATERIAL PROPERTIES

(1)Structural steel

Steel section ISHB 250
Nominal yield strength $f_y = 250\ N/mm^2$
Modulus of elasticity $E_a = 200\ kN/mm^2$

(2) Concrete

Concrete grade M30
Characteristic strength $(f_{ck})_{cu} = 30\ N/mm^2$
Secant modulus of elasticity for short term loading, $E_{cm} = 31220\ N/mm2$

(3) Reinforcing steel

Steel grade Fe 415
Characteristic strength $f_{sk} = 415 N/mm^2$
Modulus of elasticity $E_s = 200\ kN/mm^2$

(4) Partial safety factors

$\gamma_a = 1.15$
$\gamma_c = 1.5$
$\gamma_s = 1.15$

4.1.3 SECTION PROPERTIES OF THE GIVEN SECTION

(1) Steel section

$A_a = 6971\ mm^2$

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t_f = 9.7 mm; h = 250 mm ; t_w = 8.8 mm
$I_{ax} = 79.8 * 10^6$ mm^4
$I_{ay} = 20.1 * 10^6$ mm^4
$Z_{pax} = 699.8 * 10^3$ mm^3
$Z_{pay} = 307.6 * 10^3$ mm^3

(2) Reinforcing steel

4 bars of 14 mm dia, $A_s = 616$ mm^2

(3) Concrete

$$Ac = A_{gross} - A_a - A_s$$
$$= 350 * 350 - 6971 - 616$$
$$= 114913 \text{ mm}^2$$

4.1.4 DESIGN CHECKS

(1) Plastic resistance of the section

$$P_p = A_a f_y/\gamma_a + \alpha_c A_c (f_{ck})_{cy}/\gamma_c + A_s f_{sk}/\gamma_s$$

$$P_p = A_a f_y/\gamma_a + \alpha_c A_c (0.80 * (f_{ck})_{cu}) / \gamma_c + A_s f_{sk}/\gamma_s$$

$$= [6971 * 250/1.15 + 0.85 * 114913 * 25/1.5 + 616 * 415/1.15]/1000$$
$$= 3366 \text{ kN}$$

$P_p = 3366$ kN

(2) Effective elastic flexural stiffness of the section for short term loading

About the major axis

$$(EI)_{ex} = E_a I_{ax} + 0.8 E_{cd} I_{cx} + E_s I_{sx}$$

$$I_{ax} = 79.8 * 10^6 \text{ mm}^4$$

$E_{cd} = E_{cm}/\gamma^*_c = 31220/1.35 = 23125$ N/mm^2

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$I_{sx} = Ah^2$

$= 616 * [350/2-25-7]^2$

$= 12.6 * 10^6\ mm^4$

$I_{cx} = (350)^4/12 - [79.8 + 12.6] * 10^6$

$= 1158 * 10^6\ mm^4$

$(EI)_{ex} = 2.0 * 10^5 * 79.8 * 10^6 + 0.8 * 23125 * 1158 * 10^6 + 2.0 * 10^5 * 12.6 * 10^6$

$= 39.4 * 10^{12}\ N\ mm^2$

<u>About minor axis</u>

$(EI)_{ey} = 2.0 * 10^5 * 20.1 * 10^6 + 0.8 * 23125 * 1217.8 * 10^6 + 2.0 * 10^5 * 12.6 * 10^6$

$= 28.5 * 10^{12}\ N\ mm^2$

(3) Non dimensonal slenderness

$\bar{\lambda} = (P_{pu} / P_{cr})^{½}$

Value of P_{pu}:

$P_{pu} = A_a f_y + \alpha_c A_c (f_{ck})_{cy} + A_s f_{sk}$

$P_{pu} = A_a f_y + \alpha_c A_c * 0.80 * (f_{ck})_{cu} + A_s f_{sk}$

$= (6971 * 250 + 0.85 * 114913 * 25 + 415 * 616)/1000$

$= 4440\ kN$ — $P_{pu} = 4440\ kN$

$$(P_{cr})_x = \frac{\pi^2 (EI)_{ex}}{\ell^2}$$

$$= \frac{\pi^2 * 39.4 * 10^{12}}{(3000)^2} = 43207\ kN$$

$(P_{cr})_x = 43207\ kN$

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$$(P_{cr})_y = \frac{\pi^2 * 28.5 * 10^{12}}{(3000)^2} \quad = 31254\,kN$$

$(P_{cr})_y = 31254\ kN$

$\bar{\lambda}_x = (44.4 / 432.07)^{1/2} = 0.320$

$\bar{\lambda}_x = 0.320$

$\bar{\lambda}_y = (44.4 / 312.54)^{1/2} = 0.377$

$\bar{\lambda}_y = 0.377$

(4) Check for the effect of long term loading

The effect of long term loading can be neglected if anyone or both following conditions are satisfied:

- *Eccentricity, e given by*

  $e = M / P \geq 2$ *times the cross section dimension in the plane of bending considered.*

  $e_x = 180/1500$
  $= 0.12 < 2(0.35)$

  $e_y = 0$

- $\bar{\lambda} < 0.8$

Since condition (2) is satisfied, the influence of creep and shrinkage on the ultimate load need not be considered.

(5) Resistance of the composite column under axial compression

Design against axial compression is satisfied if following condition is satisfied:

$P < \chi P_p$

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Here,

$P = 1500\ kN$

$P_p = 3366\ kN$

and χ = *reduction factor for column buckling*

χ *values:*

<u>About major axis</u>

$\alpha_x = 0.34$

$\chi_x = 1 / \{\phi_x + (\phi_x^2 - \bar{\lambda}^2_x)^{½}\}$

$\phi_x = 0.5\ [1 + \alpha_x(\bar{\lambda}_x - 0.2) + \bar{\lambda}^2_x]$

$= 0.5\ [1 + 0.34(0.320 - 0.2) + (0.320)^2] = 0.572$

$\chi_x = 1 / \{0.572 + [(0.572)^2 - (0.326)^2]^{½}\}$

$= 0.956$

$\chi_x P_P > P$

$0.956 * 3366 = 3218\ kN > P\ (= 1500\ kN)$

<u>About minor axis</u>

$\alpha_y = 0.49$

$\phi_y = 0.5\ [1 + 0.49(0.377 - 0.2) + (0.377)^2]$
$= 0.61$

$\chi_y = 1 / \{0.61 + [(0.61)^2 - (0.377)^2]^{1/2}\}$
$= 0.918$

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$\chi_y P_P > P$

$0.918 * 3366 = 3090 kN > P (=1500 kN)$

$\therefore$ *The design is OK for axial compression.*

(6) Check for second order effects

Isolated non – sway columns need not be checked for second order effects if:

$P / P_{cr} \leq 0.1$ *for major axis bending*

$1500/43207 = 0.035 < 0.1$

$\therefore$ *Check for second order effects is not necessary*

(7) Resistance of the composite column under axial compression and uni-axial bending

Compressive resistance of concrete, $P_c = A_c\, p_{ck}$
$=1628\ kN$

Plastic section modulus of the reinforcement

$Z_{ps} = 4(\pi / 4 * 14^2) * (350/2-25-14/2)$
$= 88 * 10^3\ mm^3$

Plastic section modulus of the steel section

$Z_{pa} = 699.8 * 10^3\ mm^3$

Plastic section modulus of the concrete

$Z_{pc} = b_c h_c^2 / 4 - Z_{ps} - Z_{pa}$
$= (350)^3/4 - 88 * 10^3 - 699.8 * 10^3$
$= 9931 * 10^3\ mm^3$

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Check that the position of neutral axis is in the web

$$h_n = \frac{A_c p_{ck} - A'_s(2p_{sk} - p_{ck})}{2b_c p_{ck} + 2t_w(2p_y - p_{ck})}$$

$$= \frac{114913 * \frac{0.85 * 25}{1.5}}{2 * 350 * \frac{0.85*25}{1.5} + 2 * 8.8\left(2 * \frac{250}{1.15} - \frac{0.85*25}{1.5}\right)}$$

$$= 93.99\ mm < \left(h/2 - t_f\right) = \left(\frac{250}{2} - 9.7\right) = 115.3\,mm$$

The neutral axis is in the web.

A'_s= 0 as there is no reinforcement with in the region of the steel web

<u>Section modulus about neutral axis</u>

Z_{psn} =0 (As there is no reinforcement with in the region of $2h_n$ from the middle line of the cross section)

$Z_{pan} = t_w h_n^2 = 8.8 * (93.99)^2$
$= 77740.3\ mm^3$

$Z_{pcn} = b_c h_n^2 - Z_{psn} - Z_{pan}$
$= 350\,(93.99)^2 - 77740. = 3014.2 * 10^3\ mm^3$

Plastic moment resistance of section

$M_p = p_y\,(Z_{pa} - Z_{pan}) + 0.5\,p_{ck}\,(Z_{pc} - Z_{pcn}) + p_{sk}\,(Z_{ps} - Z_{psn})$

$= 217.4\,(699800 - 77740) + 0.5 * 0.85 * 25/1.5\,(9931000 - 3014200)$
$+ 361\,(88 * 1000)$

$= 216\ kNm$

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(8) Check of column resistance against combined compression and uni-axial bending

The design against combined compression and uni-axial bending is adequate if following condition is satisfied:

$M \leq 0.9\ \mu\ M_P$

M = 180 kNm

M_p =216 kNm

μ = moment resistance ratio

$= 1- \{(1 - \chi)\ \chi_d\}/\{(1 - \chi_c)\ \chi\}$

$= 1- \{(1 – 0.956)\ 0.446\}/\{(1 – 0.484)\ 0.956\}$

$= 0.960$

$\therefore M < 0.9\ \mu\ M_p$

$< 0.9\ (0.960) * (216)$

<187 kNm

Hence the composite column is acceptable and the check is satisfied.

$\chi_d = P / P_p$

$=1500/3366$

$=0.446$

$\chi_c = P_c/P_p$

$=1628 /3366$

$=0.484$

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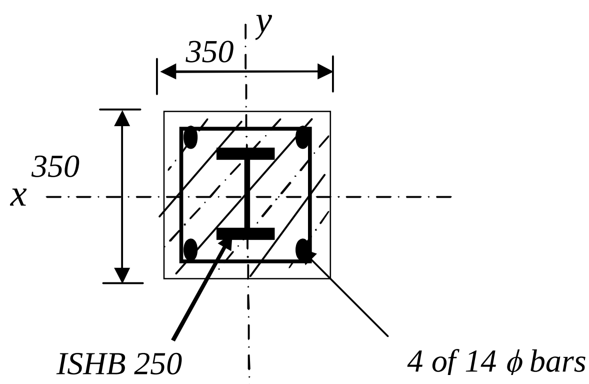
PROBLEM 2

Check the adequacy of the concrete encased composite section shown below for bi-axial bending



4.2.1 DETAILS OF THE SECTION

Column dimension	*350 X 350 X 3000*	
Concrete Grade	*M30*	
Steel Section	*ISHB 250*	
Steel Reinforcement	*Fe415*	
	4 Nos. of 14 mm dia bar	
Design Axial Load	*1500 kN*	*Axial Load = 1500kN*
Design bending moment about x-x axis	*180 kNm*	M_x *=180kNm*
Design bending moment about y-y axis	*120 kNm*	M_y *=120kNm*

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</table>

DESIGN CALCULATIONS:

4.2.2 LIST MATERIAL PROPERTIES

(1)Structural steel

Steel section ISHB 250
Nominal yield strength $f_{y} =$ *250 N/mm*^{2}
Modulus of elasticity E_a *= 200 kN/mm*^{2}

Concrete

Concrete grade M30
Characteristic strength $(f_{ck})_{cu}$ *=30 N/mm*^{2}
Secant modulus of elasticity for short term loading, E_{cm}*= 31220 N/mm*^{2}

Reinforcing steel

Steel grade Fe 415
Characteristic strength f_{sk} *= 415 N/mm*^{2}
Modulus of elasticity E_s *= 200 kN/mm*^{2}

Partial safety factors

$\gamma_a = 1.15$
$\gamma_c = 1.5$
$\gamma_s = 1.15$

4.2.3 LIST SECTION PROPERTIES OF THE GIVEN SECTION

(1) Steel section

$A_a = 6971\ mm^2$

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$t_f = 9.7$ mm
$h = 250$ mm
$t_w = 8.8$ mm
$I_{ax} = 79.8 * 10^6$ mm^4
$I_{ay} = 20.1 * 10^6$ mm^4
$Z_{px} = 699.8 * 10^3$ mm^3
$Z_{py} = 307.6 * 10^3$ mm^3

(2) Reinforcing steel

4 bars of 14 mm dia, $A_s = 616$ mm^2

(3)Concrete

$Ac = A_{gross} - A_a - A_s$
$= 350 * 350 - 6971 - 616$
$= 114913$ mm^2

4.2.4 DESIGN CHECKS

(1) Plastic resistance of the section

$$P_p = A_a\, f_y/\gamma_a + \alpha_c\, A_c\, (f_{ck})_{cy}/\gamma_c + A_s\, f_{sk}/\gamma_s$$

$$P_p = A_a\, f_y/\gamma_a + \alpha_c\, A_c\, 0.80*(f_{ck})_{cu}/\gamma_c + A_s\, f_{sk}/\gamma_s$$
$$= [6971 * 250/1.15 + 0.85 * 114913 * 25/1.5 + 616 * 415/1.15]/1000$$
$$= 3366 \text{ kN}$$

$P_p = 3366$ kN

(3) Effective elastic flexural stiffness of the section for short term loading

<u>About the major axis</u>

$$(EI)_{ex} = E_a I_{ax} + 0.8\, E_{cd} I_{cx} + E_s I_{sx}$$

$E_{cd} = E_{cm}/\gamma^*_c = 31220/1.35 = 23125$ N/mm^2

$I_{ax} = 79.8 * 10^6$ mm^4

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$I_{sx} = Ah^2$

$= 616 * [\ 350/2-25-7]^2$

$= 12.6 * 10^6\ mm^4$

$I_{cx} = (350)^4/12 - [\ 79.8 + 12.6] * 10^6$

$= 1158 * 10^6\ mm^4$

$(EI)_{ex} = 2.0 * 10^5 * 79.8 * 10^6 + 0.8 * 23125 * 1158 * 10^6 + 2.0 * 10^5 * 12.6 * 10^6$

$= 39.4 * 10^{12}\ N\ mm^2$

About minor axis

$(EI)_{ey} = 2.0 * 10^5 * 20.1 * 10^6 + 0.8 * 23125 * 1217.8 * 10^6 + 2.0 * 10^5 * 12.6 * 10^6$

$= 28.5 * 10^{12}\ N\ mm^2$

(4) Non dimensonal slenderness

$\bar{\lambda} = (P_{pu}/P_{cr})^{1/2}$

Value of P_{pu}:

$P_{pu} = A_a f_y + \alpha_c A_c (f_{ck})_{cy} + A_s f_{sk}$

$P_{pu} = A_a f_y + \alpha_c A_c\, 0.80 * (f_{ck})_{cu} + A_s f_{sk}$

$= (6971 * 250 + 0.85 * 114913 * 25 + 415 * 616)/1000$

$= 4440\ kN$ — $P_{pu} = 4440\ kN$

$$(P_{cr})_x = \frac{\pi^2 (EI)_{ex}}{\ell^2}$$

$$= \frac{\pi^2 * 39.4 * 10^{12}}{(3000)^2}$$

$= 43207\ kN$ — $(P_{cr})_x = 43207\ kN$

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$$(P_{cr})_y = \frac{\pi^2 * 28.5 * 10^{12}}{(3000)^2} = 31254kN$$

$$\bar{\lambda}_x = (44.4 / 432.07)^{½} = 0.320$$
$$\bar{\lambda}_y = (44.4/312.54)^{½} = 0.377$$

(5) Check for the effect of long term loading

The effect of long term loading can be neglected if anyone or both following conditions are satisfied:

- *Eccentricity, e given by*
  $e = M / P \geq 2$ *times the cross section dimension in the plane of bending considered)*
  $e_x = 180 /1500$
  $= 0.12 < 2(0.350)$
  $e_y = 120/1500$
  $= 0.08 < 2 (0 .350)$

- $\bar{\lambda} < 0.8$

Since condition (2) is satisfied, the influence of creep and shrinkage on the ultimate load need not be considered.

(6) Resistance of the composite column under axial compression

Design against axial compression is satisfied if following condition is satisfied:

$P < \chi P_p$

$(P_{cr})_y$ = 31254 kN

$\bar{\lambda}_x = 0.320$

$\bar{\lambda}_y = 0.377$

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Here,

$P = 1500\ kN$

$P_p = 3366\ kN$

and χ *= reduction factor for column buckling*

χ *values:*

<u>About major axis</u>

$\alpha_x = 0.34$

$\chi_x = 1 / \{\phi + (\phi^2 - \bar{\lambda}^2_x)^{½}\}$

$\phi_x = 0.5\,[1 + \alpha_x(\bar{\lambda}_x - 0.2) + \bar{\lambda}^2_x]$

$= 0.5\,[1 + 0.34(0.320-0.2) + (0.320)^2] = 0.572$

$\chi_x = 1 / \{0.572 + [(0.572)^2 - (0.320)^2]^{½}\}$

$= 0.956$

$\chi_x P_{Px} > P$

$0.956 * 3366 = 3218\ kN > P\ (=1500\ kN)$

<u>About minor axis</u>

$\alpha_y = 0.49$

$\phi_y = 0.5\,[1 + 0.49(0.377 - 0.2) + (0.377)^2]$

$= 0.61$

$\chi_y = 1 / \{0.61 + [(0.61)^2 - (0.377)^2]^{1/2}\}$

$= 0.918$

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$\chi_y P_{py} > P$

$0.918 * 3366 = 3090\ kN > P\ (=1500\ kN)$

$\therefore$ *The design is OK for axial compression.*

(7) Check for second order effects

Isolated non – sway columns need not be checked for second order effects if:

$P/(P_{cr})_x \leq 0.1$ *for major axis bending*

$1500/43207 = 0.035 \leq 0.1$

$P/(P_{cr})_y \leq 0.1$ *for minor axis bending*

$1500/31254 = 0.048 \leq 0.1$

$\therefore$ *Check for second order effects is not necessary*

(8) Resistance of the composite column under axial compression and bi-axial bending

Compressive resistance of concrete, $P_c = A_c\, p_{ck}$

$= 1628\ kN$

<u>About Major axis</u>

Plastic section modulus of the reinforcement

$Z_{ps} = 4(\pi/4 * 14^2) * (350/2 - 25 - 14/2)$

$= 88 * 10^3\ mm^3$

Plastic section modulus of the steel section

$Z_{pa} = 699.8 * 10^3\ mm^3$

Plastic section modulus of the concrete

$Z_{pc} = b_c h_c^2/4 - Z_{ps} - Z_{pa}$

$= (350)^3/4 - 88 * 10^3 - 699.765 * 10^3$

$= 9931 * 10^3\ mm^3$

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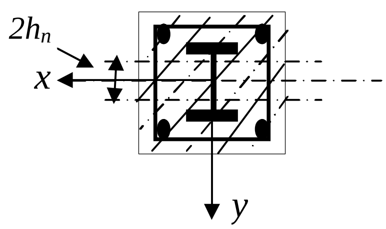
Check that the position of neutral axis is in the web



$$h_n = \frac{A_c p_{ck} - A'_s(2p_{sk} - p_{ck})}{2b_c p_{ck} + 2t_w(2p_y - p_{ck})}$$

$$= \frac{114913 * \frac{0.85 * 25}{1.5}}{2 * 350 * \frac{0.85 * 25}{1.5} + 2 * 8.8\left(2 * \frac{250}{1.15} - \frac{0.85 * 25}{1.5}\right)}$$

$$= 93.99\ mm \langle (h/2 - t_f) = \left(\frac{250}{2} - 9.7\right) = 115.3\ mm$$

The neutral axis is in the web

$A'_s = 0$ as there is no reinforcement with in the region of the steel web

<u>*Section modulus about neutral axis*</u>

Z_{psn} =0 (As there is no reinforcement with in the region of $2h_n$ from the middle line of the cross section)

$Z_{pan} = t_w h_n^2 = 8.8 * (93.99)^2$
$= 77740.3\ mm^3$

$Z_{pcn} = b_c h_n^2 - Z_{psn} - Z_{pan}$
$= 350\ (93.99)^2 - 77740.3$
$= 3014.2 * 10^3\ mm^3$

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Plastic moment resistance of section

$M_p = p_y\,(Z_{pa} - Z_{pan}) + 0.5\,p_{ck}\,(Z_{pc} - Z_{pcn}) + p_{sk}\,(Z_{ps} - Z_{psn})$

$= 217.4\,(699800 - 77740.3) + 0.5 * 0.85 * 25/1.5\,(9931000 - 3014200) + 361\,(88 * 1000)$

$= 216$ kNm

<u>About minor axis</u>

Plastic section modulus of the reinforcement

$Z_{ps} = 4(\pi / 4 * 14^2) * (350/2 - 25 - 14/2)$

$= 88 * 10^3\ mm^3$

Plastic section modulus of the steel section

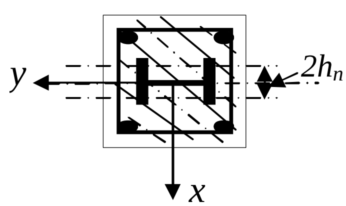
$Z_{pa} = 307.6 * 10^3\ mm^3$

Plastic section modulus of the concrete

$Z_{pc} = b_c h_c^2 / 4 - Z_{ps} - Z_{pa}$

$= (350)^3/4 - 88 * 10^3 - 307.6 * 10^3$

$= 10323 * 10^3\ mm^3$



$$h_n = \frac{A_c p_{ck} - A_s'(2p_{sk} - p_{ck}) + t_w(2t_f - h)(2p_y - p_{ck})}{2h_c p_{ck} + 4t_f(2p_y - p_{ck})}$$

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$$h_n = \frac{114913 * 14.2 + 8.8(2 * 9.7 - 250)(2 * 218 - 14.2)}{2 * 350 * 14.2 + 4 * 9.7\,(2 * 218 - 14.2)}$$

$$= 29.5\,mm \left({t_w}/{2} < h_n < {b}/{2} \right) = {8.8}/{2} < h_n < {250}/{2}$$

A'_s= 0 as there is no reinforcement with in the region of the steel web

<u>Section modulus about neutral axis</u>

Z_{psn} *=0 (As there is no reinforcement with in the region of $2h_n$ from the middle line of the cross section)*

$Z_{pan} = 2t_f\, h_n^2 + (h-2t_f)/4 * t_w^2$
$= 2(9.7)(29.5)^2 + [\{ 250-2(9.7)\} /4]*8.8^2$
$= 21.3*10^3\ mm^3$

$Z_{pcn} = h_c h_n^2 - Z_{psn} - Z_{pan}$
$= 350\ (29.5)^2 - 21.3*10^3$
$= 283.3 * 10^3\ mm^3$

$M_{py} = p_y\ (Z_{pa} - Z_{pan}) + 0.5\ p_{ck}\ (Z_{pc} - Z_{pcn}) + p_{sk}\ (Z_{ps} - Z_{psn})$

$= 217.4\ (307.589 - 21.3)*10^3 + 0.5 * 14.2 * (10323 - 283.3)*10^3 + 361\ (88 * 1000)$

$= 165\ kNm$

(9) Check of column resistance against combined compression and bi-axial bending

The design against combined compression and bi-axial bending is adequate if following conditions are satisfied:

(1) $M \le 0.9\ \mu\ M_P$

<u>About major axis</u>

$M_x = 180\ kNm$

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M_{px} =216 kNm

μ_x = moment resistance ratio

$= 1- \{(1 - \chi_x)\ \chi_d\}/\{(1 - \chi_c)\ \chi_x\}$

$= 1- \{(1 - 0.956)\ 0.446\}/\{(1 - 0.484)\ 0.956\}$

$= 0.960$

$\therefore M_x < 0.9\ \mu_x\ M_{px}$

$< 0.9\ (0.960) * (216)$

$= 187$ kNm

<u>About minor axis</u>

$M_y = 120$ kNm

$M_{py} = 165$ kNm

$\mu_y = 1- \{(1 - \chi_y)\ \chi_d\}/\{(1 - \chi_c)\ \chi_y\}$

$= 1- \{(1 - 0.918)\ 0.446\}/\{(1 - 0.448)\ 0.918\}$

$= 0.928$

$\therefore M_y < 0.9\ \mu_y\ M_{py}$

$< 0.9\ (0.928) * (165)$

<138 kNm (M_y=120 kN)

(2) $$\frac{M_x}{\mu_x M_{px}} + \frac{M_y}{\mu_y M_{py}} \le 1.0$$

$$\frac{180}{0.960*216} + \frac{120}{0.928*165} > 1.0$$

Since design check (2) is not satisfied, the composite column is not acceptable.

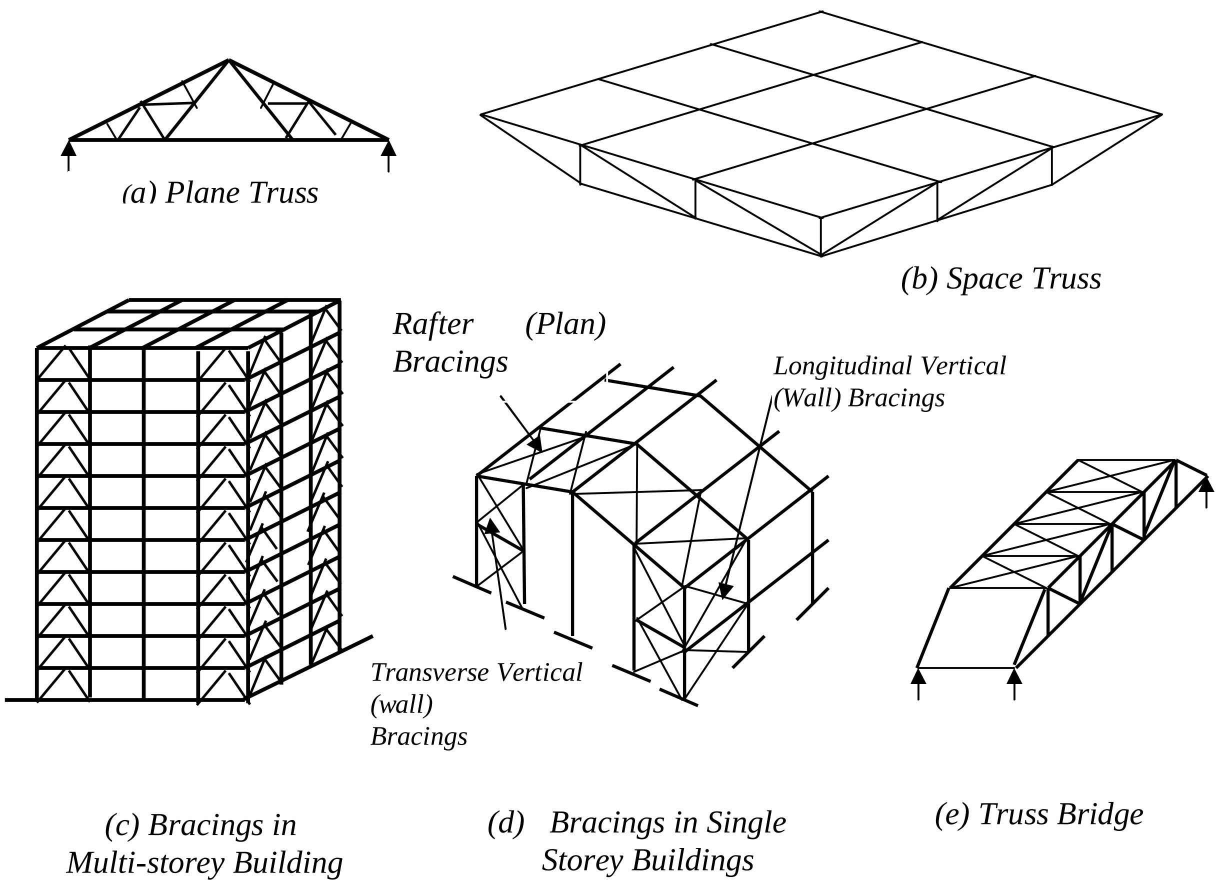
$\chi_d = P/P_p$
$=1500/3366$
$=0.446$

$\chi_c = P_c/P_p$
$=1628 /3366$
$=0.484$

27 TRUSSES

1.0 INTRODUCTION

Trusses are triangular frame works in which the members are subjected to essentially axial forces due to externally applied load. They may be plane trusses [Fig. 1(*a*)], wherein the external load and the members lie in the same plane or space trusses [Fig. 1(*b*)], in which members are oriented in three dimensions in space and loads may also act in any direction. Trusses are frequently used to span long lengths in the place of solid web girders and such trusses are also referred to as lattice girders.



(a) Plane Truss

(b) Space Truss

(c) Bracings in Multi-storey Building

(d) Bracings in Single Storey Buildings

(e) Truss Bridge

Fig. 1 Types of Trusses

Steel members subjected to axial forces are generally more efficient than members in flexure since the cross section is nearly uniformly stressed. Trusses, consisting of essentially axially loaded members, thus are very efficient in resisting external loads. They are extensively used, especially to span large gaps. Since truss systems consume relatively less material and more labour to fabricate, compared to other systems, they are particularly suited in the Indian context.

Trusses are used in roofs of single storey industrial buildings, long span floors and roofs of multistory buildings, to resist gravity loads [Figs. 1(*a*) and 1(*b*)]. Trusses are also used in multi-storey buildings and walls and horizontal planes of industrial buildings to resist lateral loads and give lateral stability [Figs. 1(*c*) and 1(*d*)]. Trusses are used in long span bridges to carry gravity loads and lateral loads [Fig. 1(*e*)].

Trusses often serve the action of the girder in transferring the gravity load over larger span, and are referred to also as lattice girders. Such lattice girders are usually deeper and much lighter than regular girders and hence are economical, particularly when repetitive fabrication is taken advantage of. These are used as flooring support systems in multi-storey buildings, within which depth all the ducts can be easily accommodated without increasing the depth of the ceiling.

Steel trusses can also be efficiently used along with concrete slabs in buildings and bridges by mobilising composite action between structural steel and concrete. In this chapter, initially, the details of structural steel trusses are discussed. Subsequently, the behaviour and design of structural steel - concrete composite trusses are discussed.

2.0 LOADS

The loads on trusses would depend upon the application for which the trusses are used. The loads may be static, as in the case of buildings, or dynamic, as in the case of bridges. These are briefly reviewed below.

2.1 Industrial Buildings

The roof trusses in industrial buildings are subjected to the following loads:

2.1.1 Dead load

Dead load on the roof trusses in single storey industrial buildings consists of dead load of claddings and dead load of purlins, self weight of the trusses in addition to the weight of bracings etc. Further, additional special dead loads such as truss supported hoist dead loads, special ducting and ventilator weight etc. could contribute to roof truss dead loads. As the clear span length (column free span length) increases, the self weight of the moment resisting gable frame increases drastically. In such cases roof trusses are more economical.

2.1.2 Live load

The live load on roof trusses consist of the gravitational load due to erection and servicing as well as dust load etc. and the intensity is taken as per IS:875-1987 (Reaffirmed 1992). Additional special live loads such as snow loads in very cold climates, crane live loads in trusses supporting monorails may have to be considered.

2.1.3 Wind load

Wind load on the roof trusses, unless the roof slope is too high, would be usually uplift force perpendicular to the roof, due to suction effect of the wind blowing over the roof. Hence the wind load on roof truss usually acts opposite to the gravity load, and its magnitude can be larger than gravity loads, causing reversal of forces in truss members.

The horizontal and vertical bracings employed in single and multi-storey buildings are also trusses [Fig. 1(*d*)], used primarily to resist wind and other lateral loads. These bracings minimize the differential deflection between the different frames due to crane surge in industrial buildings. They also provide lateral support to columns in small and tall buildings, thus increasing the buckling strength.

2.1.4 Earthquake load

Since earthquake load on a building depends on the mass of the building, earthquake loads usually do not govern the design of light industrial steel buildings. Wind loads usually govern. Howev er, in the case of industrial buildings with a large mass located at the roof, the earthquake load may govern the design. These loads are calculated as per IS:1893-1985.

2.2 Multi-Storey Buildings

The lateral load due to wind or earthquake may be resisted by vertical bracings acting as trusses. These bracings, properly designed, make these buildings very stiff in resisting lateral loads. Hence they are economical in the buildings of intermediate height ranges. In the case of earthquake loading, stiff buildings may attract larger inertia force and hence use of bracings may not be desirable.

2.3 Bridge Trusses

Trusses are used in bridges to transfer the gravity load of moving vehicles to supporting piers. Depending upon the site conditions and the span length of the bridge, the truss may be either through type or deck type. In the through type, the carriage way is supported at the bottom chord of trusses. In the deck type bridge, the carriage way is supported at the top chord of trusses. Usually, the structural framing supporting the carriage way is designed such that the loads from the carriage way are transferred to the nodal points of the vertical bridge trusses. More details of the trusses bridges are discussed in the chapter on bridges.

3.0 ANALYSIS OF TRUSSES

Generally truss members are assumed to be joined together so as to transfer only the axial forces and not moments and shears from one member to the adjacent members (they are regarded as being pinned joints). The loads are assumed to be acting only at the nodes of the trusses. The trusses may be provided over a single span, simply supported over the two end supports, in which case they are usually statically determinate. Such trusses can be analysed manually by the method of joints or by the method of sections. Computer programs are also available for the analysis of trusses. These programs are more useful in the case of multi-span indeterminate trusses, as well as in the case of trusses in which the joint rigidity has to be considered. The effect of joint rigidity is discussed later in greater detail.

From the analysis based on pinned joint assumption, one obtains only the axial forces in the different members of the trusses. However, in actual design, the members of the trusses are joined together by more than one bolt or by welding, either directly or through larger size end gussets. Further, some of the members, particularly chord members, may be continuous over many nodes. Generally such joints enforce not only compatibility of translation but also compatibility of rotation of members meeting at the joint. As a result, the members of the trusses experience bending moment in addition to axial force. This may not be negligible, particularly at the eaves points of pitched roof trusses, where the depth is small and in trusses with members having a smaller slenderness ratio (i.e. stocky members). Further, the loads may be applied in between the nodes of the trusses, causing bending of the members. Such stresses are referred to as secondary stresses. The secondary bending stresses can be caused also by the eccentric connection of members at the joints. The analysis of trusses for the secondary moments and hence the secondary stresses can be carried out by an indeterminate structural analysis, usually using computer software.

The magnitude of the secondary stresses due to joint rigidity depends upon the stiffness of the joint and the stiffness of the members meeting at the joint. Normally the secondary stresses in roof trusses may be disregarded, if the slenderness ratio of the chord members is greater than 50 and that of the web members is greater than 100. The secondary stresses cannot be neglected when they are induced due to application of loads on members in between nodes and when the members are joined eccentrically. Further the secondary stresses due to the rigidity of the joints cannot be disregarded in the case of bridge trusses due to the higher stiffness of the members and the effect of secondary stresses on fatigue strength of members. In bridge trusses, often misfit is designed into the fabrication of the joints to create prestress during fabrication opposite in nature to the secondary stresses and thus help improve the fatigue performance of the truss members at their joints.

3.0 CONFIGURATION OF TRUSSES

3.1 Pitched Roof Trusses

Most common types of roof trusses are pitched roof trusses wherein the top chord is provided with a slope in order to facilitate natural drainage of rainwater and clearance of dust/snow accumulation. These trusses have a greater depth at the mid-span. Due to this even though the overall bending effect is larger at mid-span, the chord member and web member stresses are smaller closer to the mid-span and larger closer to the supports. The typical span to maximum depth ratios of pitched roof trusses are in the range of 4 to 8, the larger ratio being economical in longer spans. Pitched roof trusses may have different configurations. In Pratt trusses [Fig. 2(*a*)] web members are arranged in such a way that under gravity load the longer diagonal members are under tension and the shorter vertical members experience compression. This allows for efficient design, since the short members are under compression. However, the wind uplift may cause reversal of stresses in these members and nullify this benefit. The converse of the Pratt is the Howe truss [Fig. 2(*b*)]. This is commonly used in light roofing so that the longer diagonals experience tension under reversal of stresses due to wind load.

Fink trusses [Fig. 2(*c*)] are used for longer spans having high pitch roof, since the web members in such truss are sub-divided to obtain shorter members.

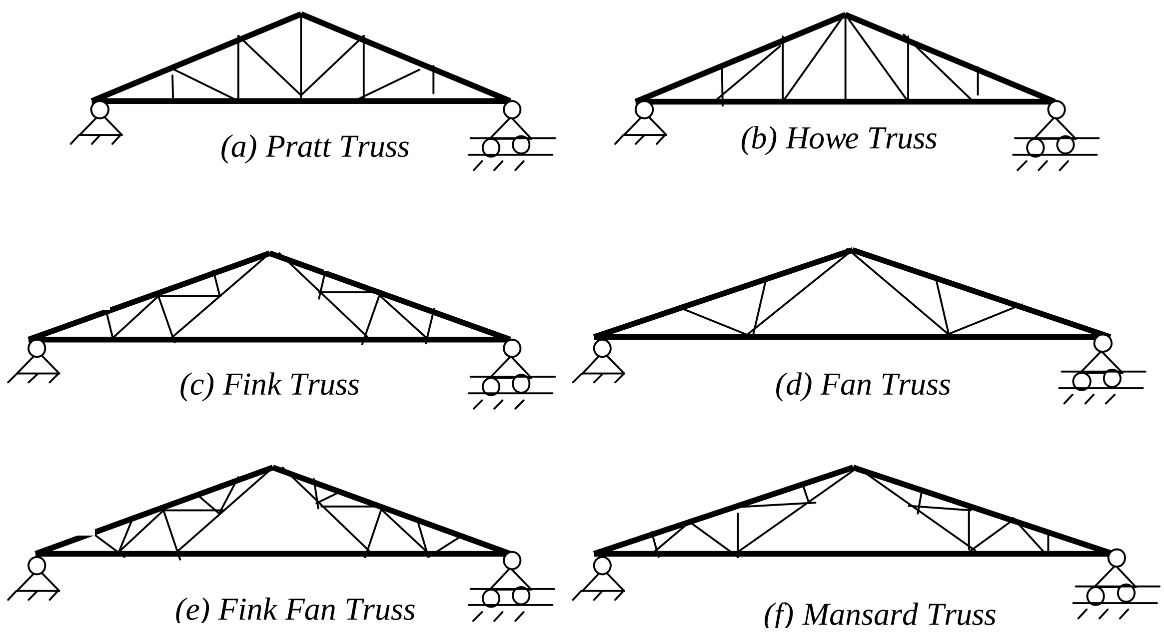


Fig. 2 Pitched Roof Trusses

Fan trusses [Fig. 2(*d*)] are used when the rafter members of the roof trusses have to be sub-divided into odd number of panels. A combination of fink and fan [Fig. 2(*e*)] can also be used to some advantage in some specific situations requiring appropriate number of panels.

Mansard trusses [Fig. 2(*f*)] are variation of fink trusses, which have shorter leading diagonals even in very long span trusses, unlike the fink and fan type trusses.

The economical span lengths of the pitched roof trusses, excluding the Mansard trusses, range from 6 m to 12 m. The Mansard trusses can be used in the span ranges of 12 m to 30 m.

3.2 Parallel Chord Trusses

The parallel chord trusses are used to support North Light roof trusses in industrial buildings as well as in intermediate span bridges. Parallel chord trusses are also used as pre-fabricated floor joists, beams and girders in multi-storey buildings [Fig. 3(*a*)]. Warren configuration is frequently used [Figs. 3(*b*)] in the case of parallel chord trusses. The advantage of parallel chord trusses is that they use webs of the same lengths and thus reduce fabrication costs for very long spans. Modified Warren is used with additional verticals, introduced in order to reduce the unsupported length of compression chord members. The saw tooth north light roofing systems use parallel chord lattice girders [Fig. 3(c)] to support the north light trusses and transfer the load to the end columns.

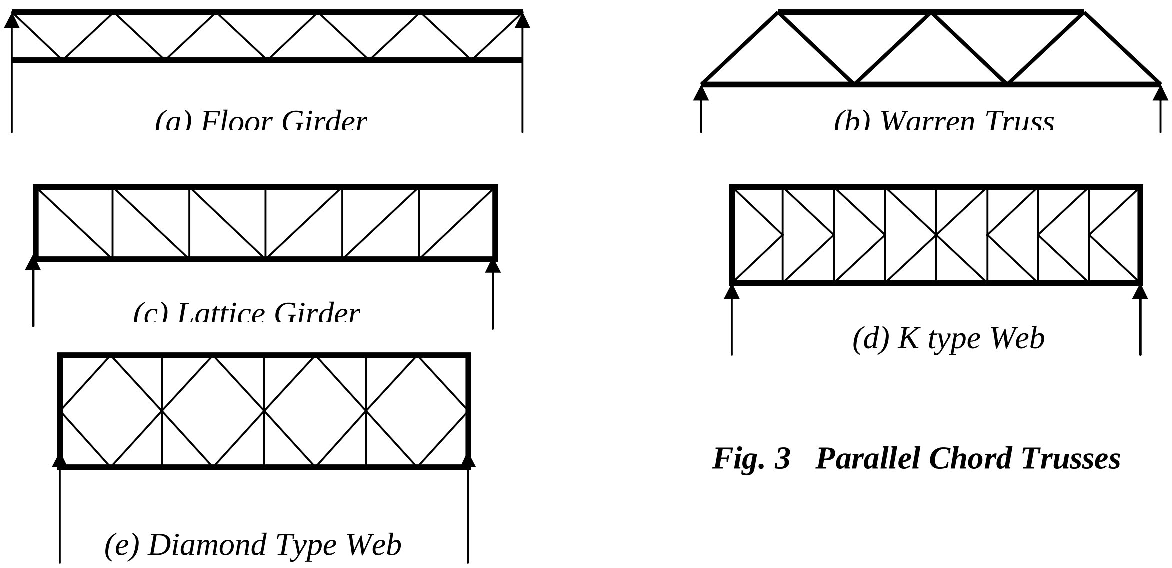


Fig. 3 Parallel Chord Trusses

The economical span to depth ratio of the parallel chord trusses is in the range of 12 to 24. The total span is subdivided into a number of panels such that the individual panel lengths are appropriate (6m to 9 m) for the stringer beams, transferring the carriage way load to the nodes of the trusses and the inclination of the web members are around 45 degrees. In the case of very deep and very shallow trusses it may become necessary to use K and diamond patterns for web members to achieve appropriate inclination of the web members. [Figs. 3(*d*), 3(*e*)]

3.3 Trapezoidal Trusses

In case of very long span length pitched roof, trusses having trapezoidal configuration, with depth at the ends are used [Fig. 4(*a*)]. This configuration reduces the axial forces in the chord members adjacent to the supports. The secondary bending effects in these members are also reduced. The trapezoidal configurations [Fig. 4(*b*)] having the sloping bottom chord can be economical in very long span trusses (spans > 30 m), since they tend to reduce the web member length and the chord members tend to have nearly constant forces over the span length. It has been found that bottom chord slope equal to nearly half as much as the rafter slope tends to give close to optimum design.

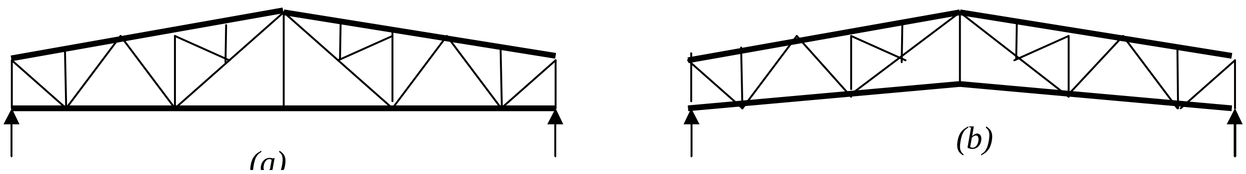
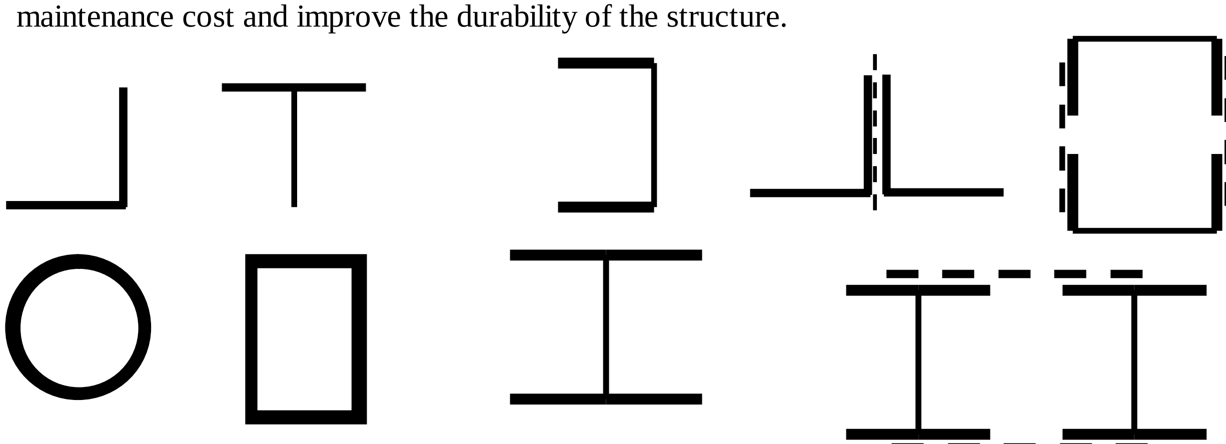


Fig 4. Trapezoidal Trusses

4.0 TRUSS MEMBERS

The members of trusses are made of either rolled steel sections or built-up sections depending upon the span length, intensity of loading, etc. Rolled steel angles, tee sections, hollow circular and rectangular structural tubes are used in the case of roof trusses in industrial buildings [Fig. 5(*a*)]. In long span roof trusses and short span bridges heavier rolled steel sections, such as channels, I sections are used [Fig. 5(*b*)]. Members built-up using I sections, channels, angles and plates are used in the case of long span bridge trusses [Fig. 5(*c*)]

Access to surface, for inspection, cleaning and repainting during service, are important considerations in the choice of the built-up member configuration. Surfaces exposed to the environments, but not accessible for maintenance are vulnerable to severe corrosion during life, thus reducing the durability of the structure. In highly corrosive environments fully closed welded box sections, and circular hollow sections are used to reduce the maintenance cost and improve the durability of the structure.

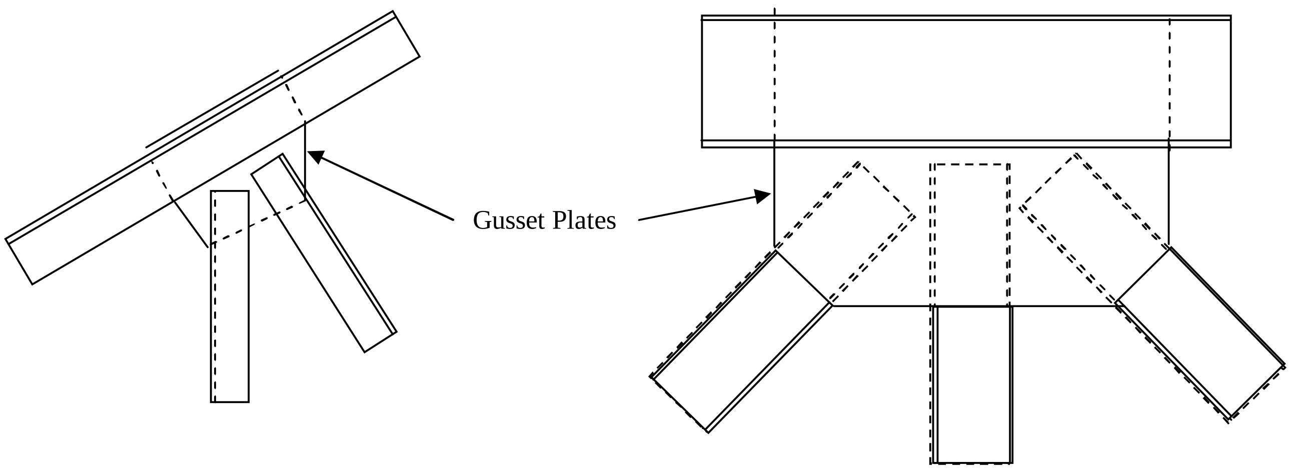


(a) Light Section *(b) Heavy Sections* *(c) Built-up Sections*

Fig. 5 Cross Sections of Truss Members

5.0 CONNECTIONS

Members of trusses can be joined by riveting, bolting or welding. Due to involved procedure and highly skilled labour requirement, riveting is not common these days, except in some railway bridges in India. In railway bridges riveting may be used due to fatigue considerations. Even in such bridges, due to recent developments, high strength friction grip (HSFG) bolting and welding have become more common. Shorter span trusses are usually fabricated in shops and can be completely welded and transported to site as one unit. Longer span trusses can be prefabricated in segments by welding in shop. These segments can be assembled by bolting or welding at site. This results in a much better quality of the fabricated structure. However, the higher cost of shop fabrication due to excise duty in contrast to lower field labour cost frequently favour field fabrication in India.



(a) Typical Roof Truss Joint *(b) Typical Bridge Truss Joint*

Fig. 6 Typical Truss Joints

If the rafter and tie members are T sections, angle diagonals can be directly connected to the web of T by welding or bolting. Frequently, the connections between the members of the truss cannot be made directly, due to inadequate space to accommodate the joint length. In such cases, gusset plates are used to accomplish such connections (Fig. 6). The size, shape and the thickness of the gusset plate depend upon the size of the member being joined, number and size of bolt or length of weld required, and the force to be transmitted. The thickness of the gusset is in the range of 8 mm to 12 mm in the case of roof trusses and it can be as high as 22 mm in the case of bridge trusses. The design of gussets is usually by rule of thumb. In short span (8 – 12 m) roof trusses, the member forces are smaller, hence the thickness of gussets are lesser (6 or 8 mm) and for longer span lengths

(> 30 m) the thickness of gussets are larger (12 mm). The design of gusset connections are discussed in a chapter on connections.

6.0 DESIGN OF TRUSSES

Factors that affect the design of members and the connections in trusses are discussed in this section.

6.1 Instability Considerations

While trusses are stiff in their plane they are very weak out of plane. In order to stabilize the trusses against out- of- plane buckling and to carry any accidental out of plane load, as well as lateral loads such as wind/earthquake loads, the trusses are to be properly braced out -of -plane. The instability of compression members, such as compression chord, which have a long unsupported length out- of-plane of the truss, may also require lateral bracing.

Compression members of the trusses have to be checked for their buckling strength about the critical axis of the member. This buckling may be in plane or out-of-plane of the truss or about an oblique axis as in the case of single angle sections. All the members of a roof truss usually do not reach their limit states of collapse simultaneously. Further, the connections between the members usually have certain rigidity. Depending on the restraint to the members under compression by the adjacent members and the rigidity of the joint, the effective length of the member for calculating the buckling strength may be less than the centre-to-centre length of the joints. The design codes suggest an effective length factor between 0.7 and 1.0 for the in-plane buckling of the member depending upon this restraint and 1.0 for the out of plane buckling.

In the case of roof trusses, a member normally under tension due to gravity loads (dead and live loads) may experience stress reversal into compression due to dead load and wind load combination. Similarly the web members of the bridge truss may undergo stress reversal during the passage of the moving loads on the deck. Such stress reversals and the instability due to the stress reversal should be considered in design.

The design standard (IS: 800) imposes restrictions on the maximum slenderness ratio, (ℓ/r), as given below:

Member type	Max ℓ/r limit
Members under compression under loads other than wind/ earthquake load	180
Tension members undergoing stress reversal due to loads other than wind load or seismic forces	180
Members normally under tension but may have to resist compression under wind load	250
Compression flange of a beam against lateral torsional buckling	300

Members designed only for tension even though they may experience stress reversal	350
Members always under tension (unless pre-tensioned to avoid sag)	400

These limits are imposed to ensure the following:

- Too slender a member is avoided which may be damaged during transportation and erection
- Members do not sag excessively under self-weight during service causing excessive deflection in truss.
- Compression members do not sag greater than 1/1000th of their length, which is beyond the imperfection limit assumed in the compressive strength calculation.

It is a common practice to specify a minimum angle size of 50 X 50 X 6 in the case of roof trusses.

7.0 ECONOMY OF TRUSSES

As already discussed trusses consume a lot less material compared to beams to span the same length and transfer moderate to heavy loads. However, the labour requirement for fabrication and erection of trusses is higher and hence the relative economy is dictated by different factors. In India these considerations are likely to favour the trusses even more because of the lower labour cost. In order to fully utilize the economy of the trusses the designers should ascertain the following:

- Method of fabrication and erection to be followed, facility for shop fabrication available, transportation restrictions, field assembly facilities.
- Preferred practices and past experience.
- Availability of materials and sections to be used in fabrication.
- Erection technique to be followed and erection stresses.
- Method of connection preferred by the contractor and client (bolting, welding or riveting).
- Choice of as rolled or fabricated sections.
- Simple design with maximum repetition and minimum inventory of material.

8.0 COMPOSITE TRUSSES

Trusses are efficient structural systems, since the members experience essentially axial forces and hence the materials are fully utilized. Steel as a structural material is equally strong both in tension and compression and hence steel trusses are more efficient. They tend to be economical to support loads over larger span lengths. However, the members in the compression chord of the simply supported steel truss (top chord) may prematurely buckle before the stresses reach the material strength. In this context the concrete slab acting in composite with the truss compression chord becomes useful (Fig. 6).

A reinforced concrete or composite deck floor is required in any case in building and other structures to provide a flat surface. Using it as a part of the compression member in truss system could be an economical proposition. Concrete has a lower strength compared with steel and hence requires larger cross section to sustain a given compression. Consequently, the concrete floor slab used as a part of the compression chord of the truss is less vulnerable to buckling failure. Further, concrete can more economically carry compression, whereas it is very weak in tension. In a composite truss system the relative merits of steel and concrete as construction materials are fully exploited. It is one of the most economical systems in longer span flooring construction. Thus composite truss systems are structurally efficient and economical.

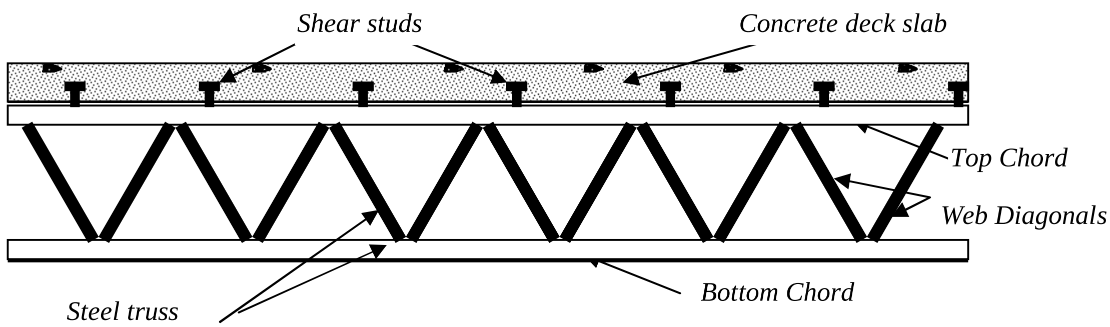


Fig. 6 Composite Truss

In multi-storey buildings, the composite truss systems also reduce the total height of the building, by accommodating the services (heating, ventilation, lighting and telecommunication ducts) within the depth of the truss, thus integrating structural, mechanical and electrical systems within in the floor space. This minimises the inter-floor height. Considering functional and structural efficiency and economy, it is only natural that composite steel-concrete trusses are a popular choice for long span and high-rise construction.

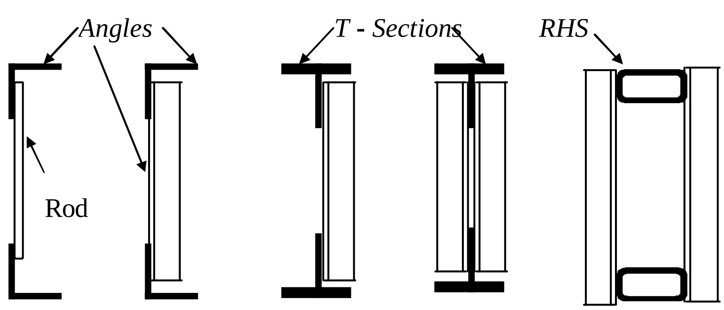


Fig. 7 Types of Truss Members

The composite truss usually consists of a parallel chord Warren truss, designed to resist, the superimposed gravity load in conjunction with the reinforced composite concrete deck slab attached to the truss through shear connectors (Fig. 6). The top and bottom chords of the truss may be made of angles, T sections or rolled steel structural tubes (Fig. 7). The web members, arranged in a Warren truss form, may be made of steel rods, single angle or double angle sections, or structural tubing welded to the chord members either directly (most common) or indirectly through gussets. The end nodes of the Warren truss are usually arranged to coincide with the reaction points of the orthogonal flooring member. The shear transfer between the steel truss and the concrete deck slab is mobilised usually using shear studs. The deck slab may consist of cast in place concrete, either over removable centering or left-in-place profiled sheeting. The profiles in the decking may run either parallel or perpendicular to the truss in the orthogonal floor system.

The application of composites in construction is a mature technology in developed countries, frequently chosen under competitive designs. Even in India, there are a few interesting applications of the composite truss construction. Many [1,2,3] have reviewed the progress of the technology since its inception.

Early applications relied essentially on the bond between concrete and steel to bring about composite action. The requirement for efficient shear transfer led to mechanical shear connectors in later applications. Angles, channels and several other proprietary shear connectors were explored. Shear studs (straight round rods with upset head) evolved as the standard, due to their ease of installation, labour reduction and cost efficiency.

The composite trusses consisting of readymade and made to order open-web joists/trusses with cast-in-place concrete slab are most common. Instead of removable shuttering, left-in-place permanent shuttering or steel profiled sheeting was subsequently used. These evolved into composite decking slabs, wherein the profiled sheeting in addition to serving as a shuttering for green concrete, also would act as tension reinforcement for hardened concrete. The shear studs are welded to the compression chord of the truss through the deck sheet, serving as a shear transfer unit both to the truss and profiled decking.

The World Trade Centre building in New York was one of the largest applications of the composite, open-web joist system. Subsequent developments used cold-formed specially shaped top chord members made of high strength steels. The profiled metal decking also provide lateral support to the compression chord member until the concrete hardened. In early 1970's competitive, efficient systems were developed with wide concrete ribs, requiring less number of shear studs. The volume of concrete in the deck was decreased and the sprayed-on fire protection requirements were also decreased through field tests.

While the earlier studies concentrated on ultimate strength evaluation, the recent studies have dealt with service load performance characteristics, such as creep and shrinkage effects of concrete on deflection, connection detailing, improving the performance of shear studs, slab crack control, member fatigue control, vibration and energy absorption characteristics, and trusses continuous over many spans.

8.1 Stud shear connectors

Stud shear connectors are commonly used to transfer shear between the steel compression strut and concrete deck slab. These studs are welded through the metal decking on to the compression chord of the truss in the case of composite deck slabs. The design of studs is treated in greater detail in the chapter on composite beams.

The stud diameter should be limited to 2.5 times the thickness of the part to which it is welded, in order to prevent the stud tearing out of the element. This could be a critical requirement in the case of composite trusses, because of the thin chord members that may be used.

8.2 Effective concrete slab

Due to shear lag, the entire width of the slab may not be fully stressed as per the simple beam theory and for the purpose of composite action an effective section of concrete is considered in stress, deflection and strength evaluation [4]. The equations for calculating the effective width of the slab is given in the chapter on composite beams.

8.3 Design considerations

8.3.1 Preliminary Design

For the preliminary design of a composite truss the following data is needed:

- The maximum bending moments and shear forces in the member
  - (a) at the construction stage (M_s, V_s),
  - (b) at the factored load acting at the limit state of collapse of the composite section (M_c, V_c).
- the concrete slab (regular or composite) sizes and
- the truss spacing.

<u>The following are the steps in the preliminary design:</u>

1. Decide on the depth of the truss girder.
   - The span to depth ratio of a simply supported composite truss is normally 15 to 20.
2. Develop the web member layout, usually using Warren configuration.
   - Use a slope of 30 degrees to horizontal to increase the opening and reduce the number of connections.
3. Design the top chord member.
   - Force in the top chord member at the construction load, R_t, is calculated from the corresponding moment, M_s, and the lever arm between the chord members (Fig. 8).
   - Size of the member is based on the member strength as governed by lateral buckling between the lateral supports to the top chord until the concrete hardens.

- A minimum width of 120 mm for the top chord is usually acceptable to support the decking in a stable manner during erection.
- Minimum of 8 mm thickness of the leg of the compression chord is required to weld the stud through the deck on to the leg.
- Vertical leg of the member should be adequate to directly weld the web members. Otherwise gusset may be required.
- Local bending should be considered in between the nodal points in case of loading between nodes

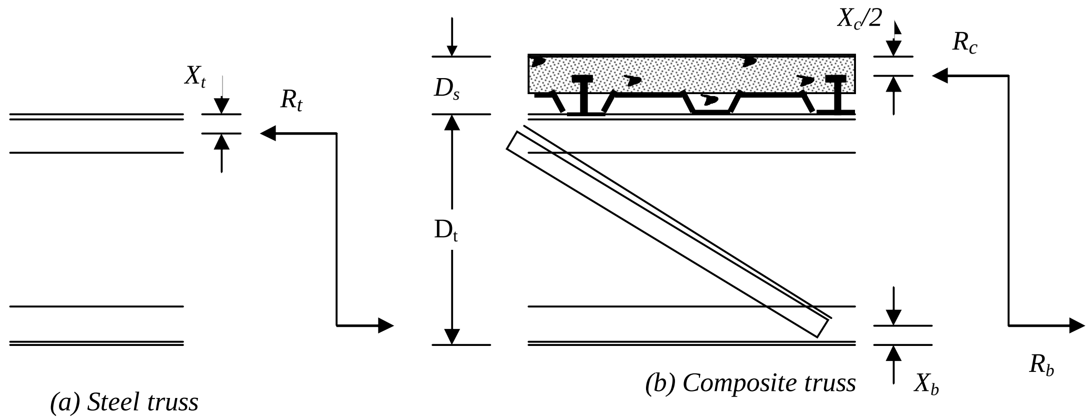


Fig. 8 Moment Capacity of Steel and Composite Trusses

1. Design the bottom chord member.
   - Calculate the tension in the bottom chord, R_b, at the factored load moment using the following equation.

$$R_b = M_c/(D_t + D_s - 0.5\, X_c - X_b) \qquad (4)$$

where $X_c = (D_s - D_p)\, R_b/R_c$, , D_p = Depth of the profile, R_b, R_t, R_c are the forces in the bottom chord, top chord of steel truss and the force in concrete slab, respectively.

   - Area of the bottom chord and the bottom chord member shape may be designed based on this force, R_b,, considering the yield strength of the member.

2. Check the slab capacity for the compression force at the limit state of collapse.
   - The slab capacity is given by
     $R_c = 0.45\, f_{ck}\, b_{eff}\, (D_s - D_p)$ (5)
     where f_{ck} = cube strength of concrete and b_{eff} is the effective width of the concrete slab acting integral with the truss.

3. Design the web member.
   - The maximum force in the web member is calculated by setting the vertical component of the member force equal to the maximum shear force in the truss.
   - The web member is designed to carry the force considering its yield strength in tension and buckling strength in compression.

8.3.2 Detailed Analysis and Design

The composite truss thus evolved may be analysed in an exact fashion using a more accurate truss analysis following either manual or computer method. The methods of modeling for computer analysis are presented in reference 3.

The composite truss should be checked for (a) limit state at construction load, (b) limit state at service load and (c) limit state of collapse.

a) The limit state at construction load

During the construction the truss has to carry all the superimposed loads until concrete sets. The top chord of the truss at this stage can fail either by reaching the material strength or lateral buckling strength, the lateral buckling being the more vulnerable mode of failure. In order to improve the lateral buckling strength, the top chord may be laterally braced in between supports either temporarily or permanently. In case the truss supports composite deck slab, once the profiled metal decking is attached to the top chord by the welding of studs to the top chord through the deck metal, it may be assumed to provide adequate lateral support to the compression chord.

Until the green concrete hardens, the steel section alone has to support all the dead weight and construction live load. Hence, the failure mode of the truss can be due to yielding / buckling (lateral or in-plane) of the top chord in the plane of truss due to compression, failure of the web member by yielding / buckling. In order to reduce the forces in members during this stage, propping of the truss from below at one or more points can be done.

b) The limit state at service load

Strength: Before the concrete hardens, the members of the truss experience forces due to its self-weight and the weight of composite deck profile, green concrete and reinforcements. The composite truss resists the loads applied after the concrete hardens (the super imposed dead load, and floor live loads). These loads cause axial forces in all the members due to truss action. Furthermore, the top chord is subjected to bending moment due to UDL / concentrated load between the nodes of the truss, which is resisted by the steel alone before concrete hardens and by the composite section after the concrete hardens.

In the allowable stress method, the members have to be checked for stresses at this service load to ensure adequate factor of safety in addition to deflection. If the construction is shored, then the stresses have to be calculated for the entire dead load acting on the composite section. If the construction is un-shored, the stresses due to the self weight including green concrete, sustained by steel section acting alone, have to be superposed on stresses due to super imposed dead load and live load acting on the composite member. In the case of cyclically loaded composite trusses, as in composite bridges, the stress range at the service load has to be calculated and checked for fatigue. Further, deflection at this service load is to be checked, as discussed in the following section.

Deflection: The deflection of the steel truss alone due to construction load has to be checked before concrete hardens and that of the composite truss for the full dead and service live load as given below.

At the time of concreting, the deflection of the truss system could cause ponding of concrete leading to a larger slab thickness while leveling concrete. In order to overcome this, pre-camber is specified for the truss, particularly in the unshored construction. If the calculated deflection of the steel truss alone under the construction load (dead load and construction live load) is less than 20mm no cambering is necessary. If the deflection is greater than 20mm camber is provided in the top chord of the truss to an extent slightly less than the calculated deflection. This is to account for moment restraint provide by even simple connections at the ends of the truss, the stiffness of the supporting member, non-hinged nature of the truss joints, all of which reduce the actual deflection to a value below the theoretical value.

The deflection under the full dead load and live load is calculated, considering the composite action under super imposed dead load and live load and simple steel truss action for dead load until the concrete hardens after accounting for camber given in the top chord. The deflection calculation should include the instantaneous deflection, creep effect and shrinkage effect. The shrinkage effect can be accounted for by calculating the deflection due to net restrained shrinkage strain of around 200 microns at the slab level. The creep deflection is calculated for the sustained load corresponding to the total dead load and sustained live load in the case of shored construction and only superimposed dead load and sustained live load in the case of unshored construction. For this purpose, the transformed area of concrete is calculated using the modular ratio corresponding to the creep modulus of concrete. The instantaneous deflection is calculated using the transformed section arrived at using the elastic modular ratio.

Span to depth ratio limitation can be effective to prevent excessive deflection and vibration under moving loads. The span to depth ratio of 20 for steel truss alone and 25 for the composite truss would be usually adequate for buildings. Slightly reduced values would be appropriate (15 to 20 respectively) in bridge trusses. The vibration control could be achieved by ensuring that any applied vibration frequency of any machinery is not close to the natural frequency of the composite flooring and ensuring the natural frequency is above 4 cycles per second. There is also a strong correlation between deflection control and vibration control, so much so that usually strict deflection control under loads would also ensure satisfactory vibration performance.

c) The limit state of collapse

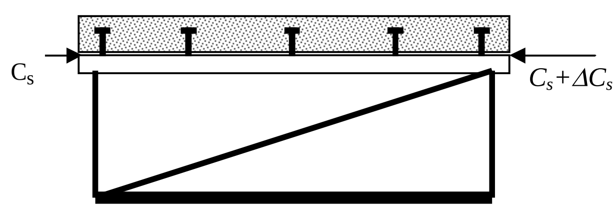
At the limit state of collapse the sequence of loading and the corresponding non-composite / composite member behaviour is immaterial. The composite member resists the total factored load. The different members of the composite truss are checked for their limit state of collapse under factored loads as given below:

- Ultimate tensile strength of bottom chord as governed by yield strength of the gross area or ultimate strength of net effective area.
- Ultimate tensile / compressive strength of the web members, depending upon the type of axial force under factored loading.
- Ultimate strength of the composite compression chord under combined bending (at nodes and in between nodes) due to load in between nodes and compression.

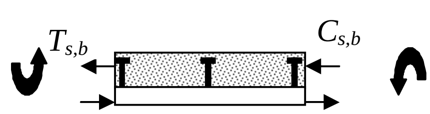
8.3.3 Design of Studs

The shear studs within a panel of a truss have to transfer the shear between the slab and top chord. This is due to overall composite truss action and the additional shear due to the bending of top chord between panel points, caused by the UDL/concentrated load between the panel points of the truss.

In the composite truss action, the forces in the composite top chord would be due to full load in the case of shored construction and due to super imposed dead and live load only in the case of un-shored construction. The unbalanced component of the compressive load on the concrete slab (ΔC_t) causes shear in the studs. The bending moments at the nodal point are calculated, only due to super imposed dead and live load. Due to this bending, the shear in the stud over half the span is calculated as (T_{sb} + C_{sb}) as shown in Fig. 9. The studs have to resist these combined forces due to local bending between nodes and overall truss action, at ultimate load, assuming the shear to be uniformly shared by the studs in the region.



(a) Due to Composite Truss



(b) Due to Composite Bending of Top Chord

Fig. 9 Forces in Studs at Limit State of

8.3.4 Partial Shear Connection

In the elastic range, the actual shear force in shear connections over the span length varies according to the variation of the shear diagram. At the ultimate load, redistribution of the shear force among shear connectors takes place due to the ductility of stud shear connector and the slip between the steel and concrete. Hence, the shear in the shear connectors in a shear span is assumed as uniform, at the ultimate load.

The shear connectors in bridges are spaced according to the elastic theory to avoid stress concentration and fatigue failure at service load and a limit of 55% of the shear stud capacity is imposed at the service load limit state. In buildings the shear connectors are spaced uniformly over the length.

The number of shear connectors as required by the elastic design may be very high. In such cases partial shear connection (50 – 70% of full shear connection) may be used. In such a case, the shear capacity of the shear connections and hence the effectiveness of the concrete in compression is to be reduced accordingly, some times leading to increase in the size of steel chord members. The use of partial shear connection also leads to slight increase in the service load deflection. However, considering the large area of concrete in compression, partial shear connections usually do not cause any appreciable changes in the final design.

8.3.5 Concrete Cracking

The deck slab may have a tendency to crack, especially at the interior supports of continuous composite beams. In order to minimise the cracking, the steel reinforcement is employed in the direction perpendicular to the potential cracks at supports.

8.3.6 Practical Considerations

- Ductile failure can be obtained, provided the design is governed by the ultimate strength of the tension chord member and the strength of top chord, web and stud connectors are large enough to preclude their premature failure.
- To facilitate stud welding, the top chord made of T or tubular section with a minimum width of 50mm is preferred, instead of smaller single or double angles.

8.4 Cost implications

The steel weight savings, the change of ratio in labour content to weight of the structure and the reduction in time for the completion of the work are three important factors that contribute to the cost reduction of the composite truss design. The project analysis division of the Canadian Institute of Steel Construction carried out a review of a number of design examples, covering steel framed buildings with braced steel core, gravity steel framing with concrete core(s). The total building costs including the deck slab and fire protection were considered. The results are tabulated in ref [2]. The summary of the findings of this study is as follows:

- The material savings in composite construction can be as high as 20 to 40 percent compared to non-composite steel construction, in the case of girder flooring. Further material savings of about 20 percent is possible if composite trusses are used instead of composite girders.

- The cost saving of composite girders is smaller (between 15 and 30%) compared to weight savings, due to the cost of studs and additional labour associated with composite construction. Further, cost savings of about 15% is possible by using composite trusses instead of composite girders.

In the case of composite construction in India, the difference between the percentage of weight saving and cost saving should be lesser due to the lower labour cost. Consequently the composite construction, particularly use of composite trusses in long span structure, could mean considerable economy as realised in U.K., New Zealand, South Africa, Australia and Singapore, in the past two decades.

9.0 SUMMARY

In this chapter, initially the behaviour and design of steel trusses were dealt with. Important aspects of truss systems such as the systems, their economy, their connections were discussed. Then the use of steel truss and reinforced concrete slab acting together as a composite truss was discussed in this chapter. After a brief introduction, the historical evolution of the system was discussed. The background information for the design of the composite trusses was presented. The economy of the system, particularly in the Indian context was evaluated. The discussions indicate that there is great potential for the use of the system in the Indian context.

10.0 REFERENCES

1. Anon, “Design of Composite Trusses”, Steel Construction Institute”, Ascot, 1992.

2. Anon “ Constructional Steel Design: An International Guide”, Elsevier, London, 1993.

3. Chien, E.Y.L. and Ritchie, J.K, “Composite Floor Systems – A Mature Design Option”, Journal of Constructional Steel Research, V. 25, 1993, pp107-139.

4. Vallenilla, C.R., and Bjorhovde, R., “Effective Width Criteria for Composite Beams”, Engineering Journal, AISC, Fourth Quarter, 1985, pp. 169-175.

<table>
<tr><td rowspan="4">Structural Steel Design Project

Calculation Sheet</td><td>Job No:</td><td>Sheet 1 of 13</td><td>Rev</td></tr>
<tr><td colspan="3">Job Title: ROOF TRUSS</td></tr>
<tr><td colspan="3">Worked Example - 1</td></tr>
<tr><td></td><td>Made by SSSR
Checked by PU</td><td>Date 9-2-2000
Date 16-08-00</td></tr>
</table>

PROBLEM 1:

Design a roof truss for an industrial building with 25 m span and 120 m long. The roofing is galvanized iron sheeting. The basic wind speed is 50 m/s and terrain is open industrial area and building is class A building. The building clear height at the eaves is 9 m.

Structural form:

For the purpose of this design example a trapezoidal truss is adopted with a roof slope of 1 to 5 and end depth of 1 m. For this span range the trapezoidal trusses would be normally efficient and economical.

Economical span to depth ratio is around 10.

Then, Span/depth = 25/3.5 = 7.1

Hence, depth is acceptable.

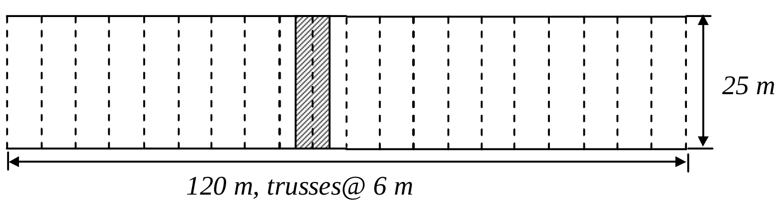
Truss spacing:

Truss spacing should be in the region of $1/4^{th}$ to $1/5^{th}$ of the span length.

For 6 m spacing,

Spacing/span = 6/25 = 1/4.17 (acceptable)

Then, number of bays = 120/6 = 20



Plan

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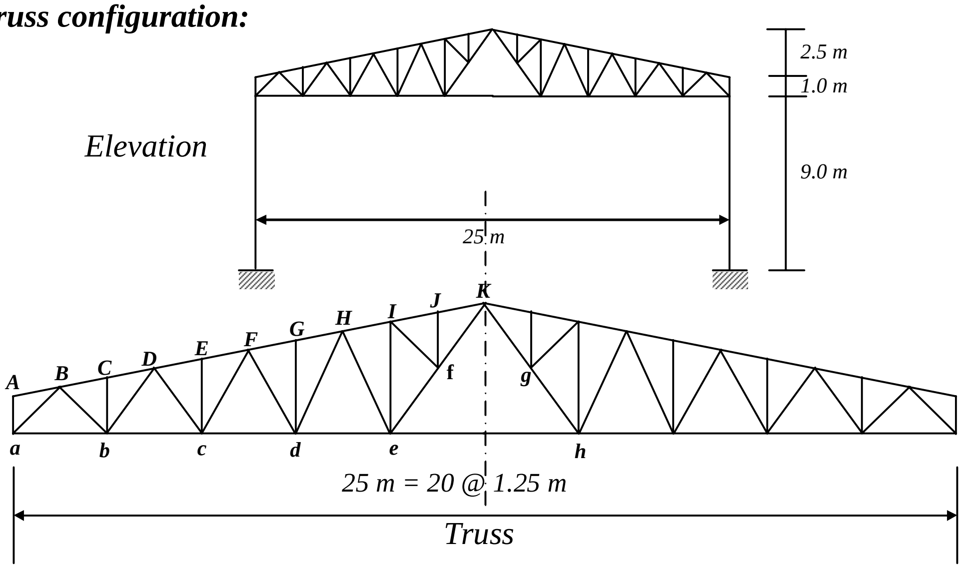
Truss configuration:



Loading:

kN/m^2

Dead load:	*GI sheeting*	*= 0.085*
	Fixings	*= 0.025*
	Services	*= 0.100*
	Total load	*= 0.210*

For 6 m bays,

Roof dead load	*= 0.21 * 25 * 6*	*= 31.5 kN*
Weight of purlin	*= 0.07 * 6 * 25*	*= 10.5 kN*
(Assuming 70 N/m^2)		
* *Self-weight of truss*	*= 0.133 * 6 * 25*	*= 20.0 kN*
Total dead load		*= 62.0 kN*

*[*For welded sheeted roof trusses, the self-weight is given approximately by*

$$w = (1/100)\,(5.37 + 0.053A)\ kN/m^2$$

$$= (5.37 + 0.053 * 6 * 25) = 0.133\ kN/m^2$$

Structural Steel Design Project Calculation Sheet	Job No:	Sheet *3 of 13*	Rev
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Dead Loads

Intermediate nodal dead load (W_1) = 62.0/20 = 3.1 kN

Dead load at end nodes (W_1 / 2) = 3.1/2 = 1.55 kN

(Acts vertically downwards at all nodes)

Wind load (IS: 875-1987):

Basic wind speed = 50 m/s

Wind load F on a roof truss by static wind method is given by

$F = (C_{pe} - C_{pi}) * A * p_d$

where, C_{pe}, C_{pi} are force co-efficient for exterior and interior of the building.

Value of C_{pi}:

Assume wall openings between 5-20% of wall area.

Then, $C_{pi} = \pm 0.5$

Value of C_{pe}:

Roof angle = $\alpha = \tan^{-1}\dfrac{1}{5} = 11.3^0$

Height of the building to eaves, h = 9 m

Lesser dimension of the building in plan, w = 25 m

Building height to width ratio is given by,

$$\frac{h}{w} = \frac{9}{25} = 0.36 < 0.5$$

Structural Steel Design Project **Calculation Sheet**	Job No:	Sheet *4 of 13*	Rev
	Job Title: *ROOF TRUSS*		
	Worked Example - 1		
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		Checked by *PU*	Date *16-08-00*

h/w	***Roof angle***	***Wind angle***		***Wind angle***	
	α	*Windward side*	*Leeward side*	*Windward side*	*Leeward side*
0.36	10^0	*- 1.2*	*- 0.4*	*- 0.8*	*- 0.8*
	20^0	*- 0.4*	*- 0.4*	*- 0.7*	*- 0.7*
	Here, $\alpha = 11.3^0$*, then by interpolation we get*				
	11.3^0	*- 1.1*	*- 0.4*	*- 0.79*	*- 0.79*

Risk Co-efficient, k_1 = *1.0*

(Assuming the industrial building as general building and its probable life about 50 years)

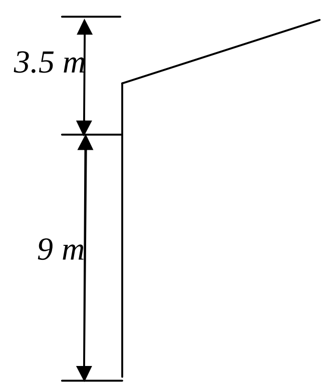
Terrain, height, structure size factor, k_2*:*

Roof elevation - 9 m to 12.5 m.

Height (m)	*Terrain category and class of building*
10	*0.91*
15	*0.97*

For 12.5 m, $k_2 = 0.94$

Assume, topography factor = $k_3 = 1.0$



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Wind pressure:

Total height of the building = 12.5 m

Basic wind speed, v_b = 50 m/s

Design wind speed v_Z *is given by,*

$v_Z = k_1 * k_2 * k_3 * v_b.$

$k_1 = 1.0$
$k_2 = 0.94$
$k_3 = 1.0$

$v_Z = 0.94 * 1 * 1 * 50 = 47$ m/s

Design wind pressure $(p_d) = 0.6\, v_Z^2 = 0.6 * (47)^2$
$= 1325$ N/m^2
$= 1.325$ kN/m^2

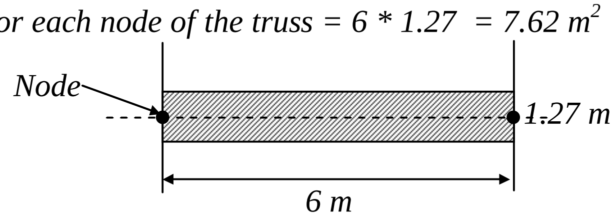
Tributary area for each node of the truss:

Length of each panel along sloping roof

$$= \frac{1.25}{\cos\ 11.3^0} = 1.27\ m \leq 1.4\ m$$

Spacing of trusses = 6m

Tributary area for each node of the truss $= 6 * 1.27 = 7.62$ m^2



Area

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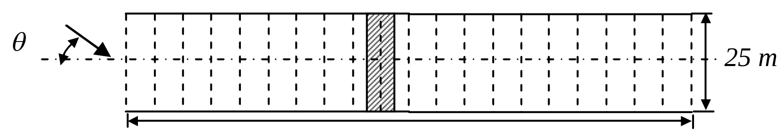
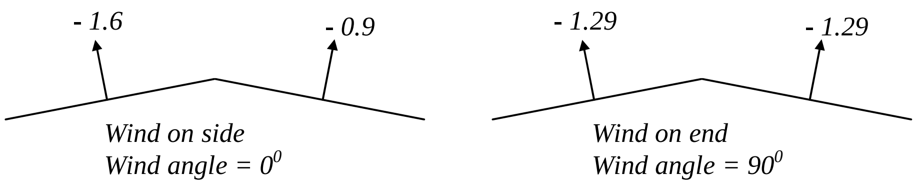
Wind load on roof truss:

Wind angle	Pressure co-efficient			$(C_{pe}-C_{pi})$		$A\ p_d$ (kN)	Wind load F (kN)	
	C_{pe}		C_{pi}	Wind ward	Lee ward		Wind ward	Lee ward
	Wind ward	Lee ward						
0^0	- 1.10	- 0.4	0.5 - 0.5	-1.6 - 0.6	- 0.9 0.1	10.1 10.1	- 16.2 - 6.1	- 9.1 1.0
90^0	- 0.79	- 0.79	0.5 - 0.5	- 1.29 - 0.29	- 1.29 - 0.29	10.1 10.1	- 13.0 - 2.9	- 13.0 - 2.9



Maximum $C_{pe} - C_{pi}$:



Critical wind loads to be considered for analysis:

Wind angle	Wind ward side (W_3)		Lee ward side (W_4)	
	Intermediate nodes W_3	End and apex nodes $W_3/2$	Intermediate nodes W_4	End and apex nodes $W_4/2$
0^0	- 16.2	- 8.1	- 9.1	- 4.55
90^0	- 13.0	- 6.5	- 13.0	- 6.5
***Loads in kN**				

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Imposed load:

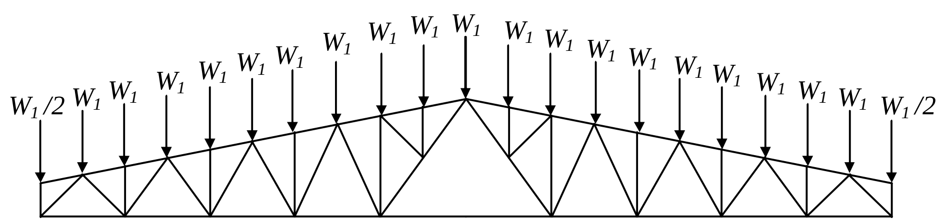
Live load = 0.35 kN/m² *[From IS: 875 – 1964]*

Load at intermediate nodes, W_2 $= 0.35 * 6 * 1.25$
$= 2.63\ kN$

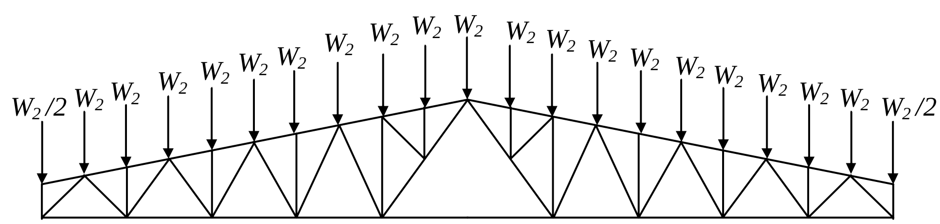
Load at intermediate nodes, $W_2 / 2$ $= 1.32\ kN$

(Acts vertically downwards)

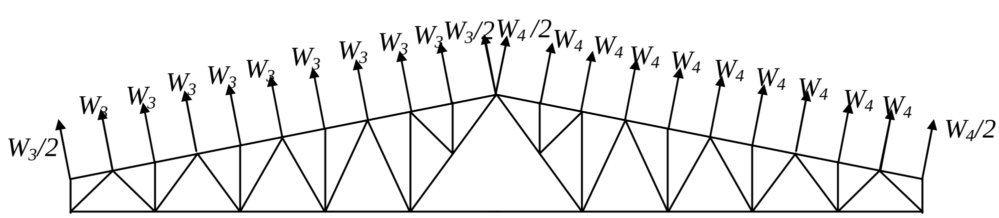
Loading pattern:



(a) Dead load



(b) Live load



(c) Wind load

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Forces in the members:

The truss has been modeled as a pin jointed plane truss and analysed using SAP90 software. The analysis results are tabulated below.
[See truss configuration for member ID]

Member	***Member Forces (kN)***			
	Dead load	***Live load***	***Wind on side***	***Wind on end***
A-B	*0*	*0*	*1.6*	*1.3*
B-C	*-47.4*	*-40.2*	*214.9*	*172.5*
C-D	*-47.4*	*-40.2*	*218.1*	*175.0*
D-E	*-63.2*	*-53.6*	*284.3*	*228.1*
E-F	*-63.2*	*-53.6*	*287.5*	*230.7*
F-G	*-66.4*	*-56.3*	*294.8*	*236.6*
G-H	*-66.4*	*-56.3*	*298*	*239.1*
H-I	*-63.2*	*-53.6*	*276*	*221.5*
I-J	*-64.5*	*-54.8*	*286.2*	*229.7*
J-K	*-64.5*	*-54.8*	*289.4*	*232.2*
a-A	*-1.6*	*-1.3*	*8.3*	*6.7*
a-B	*-41.6*	*-35.3*	*186.5*	*149.7*
a-b	*29.5*	*25*	*-131.8*	*-105.8*
b-B	*24.1*	*20.5*	*-104.8*	*-84.1*
b-C	*-3.1*	*-2.6*	*16.5*	*13.2*
b-D	*-17.1*	*-14.5*	*70.8*	*56.8*
b-c	*56.5*	*47.9*	*-247.1*	*-198.3*
c-D	*9.5*	*8.1*	*-35.4*	*-28.4*
c-E	*-3.1*	*-2.6*	*16.5*	*13.2*
c-F	*-5.3*	*-4.5*	*14*	*11.2*
c-d	*64.6*	*54.8*	*-274.5*	*-220.3*
d-F	*1*	*0.9*	*5.8*	*4.7*
d-G	*-3.1*	*-2.63*	*16.5*	*13.2*
d-H	*2.4*	*2*	*-23.7*	*-19.0*
d-e	*64.1*	*54.4*	*-262*	*-210.2*
e-H	*-5.1*	*-4.3*	*36.4*	*29.2*
e-I	*-4.6*	*-3.9*	*24.8*	*19.9*
e-f	*11.4*	*9.7*	*-71.1*	*-57.1*
e-h	*55.4*	*47*	*-205.5*	*-164.9*
f-I	*1.8*	*1.6*	*-9.7*	*-7.8*
f-J	*-3.1*	*-2.6*	*16.5*	*13.2*
f-K	*13.6*	*11.6*	*-83*	*-66.6*

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Load factors and combinations:

For dead + imposed

*1.5*DL + 1.5*LL*

For dead + wind

*1.5*DL + 1.5*LL*

or

*0.9*DL + 1.5*LL*

For dead + imposed + wind

Not critical as wind loads act in opposite direction to dead and imposed loads

Member Forces under Factored loads in kN:

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</table>

Member	*Member Design Forces (kN)*	
	DL + WL	***DL + LL***
A-B	2.4	0
B-C	251.25	-131.4
C-D	256.05	-131.4
D-E	331.65	-175.2
E-F	336.45	-175.2
F-G	342.6	-184.05
G-H	347.4	-184.05
H-I	319.2	-175.2
I-J	332.55	-178.95
J-K	337.35	-178.95
a-A	10.05	-4.35
a-B	217.35	-115.35
a-b	-153.45	81.75

Member	***DL + WL***	***DL + LL***
b-B	-121.05	66.9
b-C	20.1	-8.55
b-D	80.55	-47.4
b-c	-285.9	156.6
c-D	-38.85	26.4
c-E	20.1	-8.55
c-F	13.05	-14.7
c-d	-314.85	179.1
d-F	10.2	2.85
d-G	20.1	-8.595
d-H	-31.95	6.6
d-e	-296.85	177.75
e-H	46.95	-14.1
e-I	30.3	-12.75
e-f	-89.55	31.65
e-h	-225.15	153.6
f-I	-11.85	5.1
f-J	20.1	-8.55
f-K	-104.1	37.8

Top Chord Design:(G-H)

Maximum compressive force = 174.1 kN
Maximum tensile force = 357.4 kN

Trying ISNT 150 X 150 X 10 mm @ 0. 228 kN/ m

Sectional Properties:
Area of Cross section $= A_t = 2908\ mm^2$
Width of Section $= 2B = 150$ *mm*

<table>
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</table>

Section classification:

ε $=(250/f_y)^{0.5}$ $= (250/250)^{1/2} = 1.0$

Flange:

$B/T = 75/10 = 7.5 < 8.9\varepsilon$ (*Flange is plastic*)

Web:

$d/t = 140/10 = 14$ $[> 9.975\varepsilon$ and $< 19.95\varepsilon]$

(*Web is semi-compact*)

As no member in the section is slender, the full section is effective and there is no need to adopt reduction factor.

Maximum unrestrained length $= \ell_y$ $= 3810$ mm

(Assuming every two alternative nodes are restrained)

r_{yy} $= 30.3$ mm

λ_y $= 3810/30.3$ $= 125.7$

Then, σ_c $= 84.3$ N/mm^2

Hence, section is safe against axial compression

Axial tension capacity of the section = 2908 * 250/1.15 = 632 kN > 357.4 kN

Hence, section is safe in tension.

Bottom chord design:(c-d)

Maximum compressive force = 324.5 kN
Maximum tensile force = 169.4 kN [Try same section as top chord]
Axial tension capacity of the selected section = 2908 * 250/1.15 = 632 kN
Hence, section is safe in tension.

*Axial capacity = (84.3/1.10)*2908/1000 = 222.86 kN > 184.05 kN*

Maximum unrestrained length $= \ell_y$ $= 2500$ mm

(Assuming every node is restrained by longitudinal tie runner)

r_{yy} $= 30.3$m

λ_y $= 2500/30.3 = 82.5$

Then, σ_c $= 145.5$ N/mm^2

*Axial capacity = (145.5/1.15)*2908/1000 = 368 kN > 314.85 kN*

Hence, section is safe against axial compression also.

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Web member design:(b-B)

Maximum compressive force = 121.05 kN
Maximum tensile force = 66.9 kN

Try – ISA 80 X 80 X 8.0

$A = 1221\ mm^2$

$r_{xx} = 24.4\ mm$

$r_{uu} = 30.8\ mm$

Section classification:

$b/t = 80/8 = 10.0 < 14.0$

Hence, the section is not slender

Length of member $= (1250^2 + 1250^2)^{0.5} = 1767.5\ mm$

Slenderness ratio is taken as the greater of

$0.85 * 1767.5/24.4 = 61.6$

$1.0 * 1767/30.8 = 57.4$

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Then, σ_c $= 182.1\ N/mm^2$

Design compressive strength $= 1221 * (182.1/1.10)/1000$

$=202.13\ kN > 121.05\ kN$

Hence, safe in compression.

Tensile capacity of the section $= (250/1.10)*1221/1000$

$= 277.5\ kN > 66.9kN$

<u>Hence ISA 80 X 80 X 8.0 is adequate for the web member</u>

(The web members away from the support would have lesser axial force but longer and can be redesigned, if so desired)

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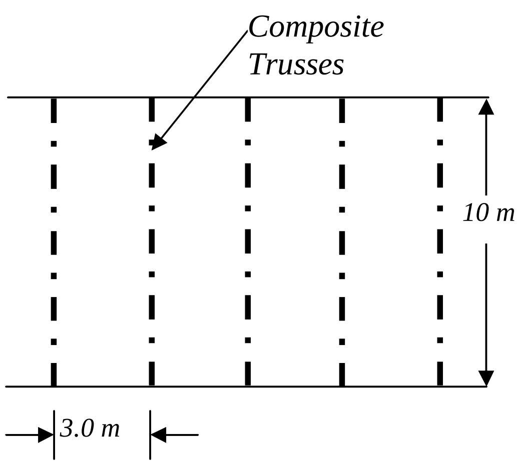
PROBLEM 2:

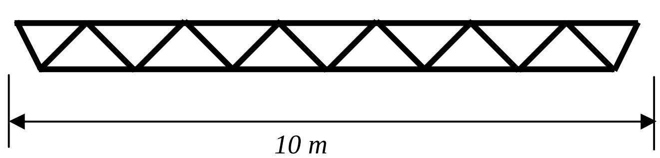
Design a composite truss of span 10.0 m with following data:

DATA:

Span $= \ell = 10.0$ *m*
Truss spacing $= 3.0$ *m*
Slab thickness $= D_s = 150$ *mm*
Profile depth $= D_p = 75.0$ *mm*
Self weight of deck slab $= 2.80\ kN/m^2$
Maximum laterally un-restrained length in top chord is 1.5 m.
Grade of concrete, M20 $= (f_{ck})_{cu} = 20$ *MPa*



FLOOR PLAN



TRUSS ELEVATION

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Loading:

	kN/m^2	Factored Load (kN/m^2)
Deck slab weight	*2.8*	*2.8*1.5 = 4.20*
Truss weight (assumed)	*0.4*	*0.4*1.5 = 0.60*
Ceiling, floor finish and Services	*1.0*	*1.0*1.5 = 1.5*
Construction Load	*1.0*	*1.0*1.5 = 1.5*
Superimposed live load	*5.0*	*5.0*1.5 = 7.5*

PRE-COMPOSITE STAGE:

Loading	kN/m^2	Factored Load (kN/m^2)
Deck slab weight	*2.8*	*2.8*1.5 = 4.20*
Truss weight	*0.4*	*0.4*1.5 = 0.60*
Construction load	*1.0*	*1.0*1.5 = 1.5*

Total factored load = *6.30* kN/m^2

Choose depth of truss = *Span/20* = *10000/20*
= *500 mm*

Total factored load = *6.30 * 3* = *18.9 kN/m*

Maximum bending moment = $w\ell^2/8$ = $18.9 * 10^2/8 = 240.98$ *kN-m*

Maximum shear = $w\ell/2$ = *18.9*10/2 = 94.50 kN*

Depth of truss (centre to centre distance of chords) = 0.5 m

Maximum axial compressive force in top chord = 240.98/0.5 =481.96 kN

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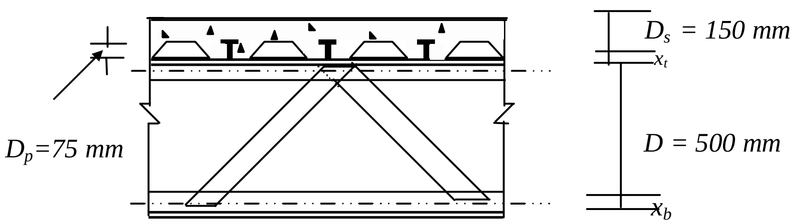
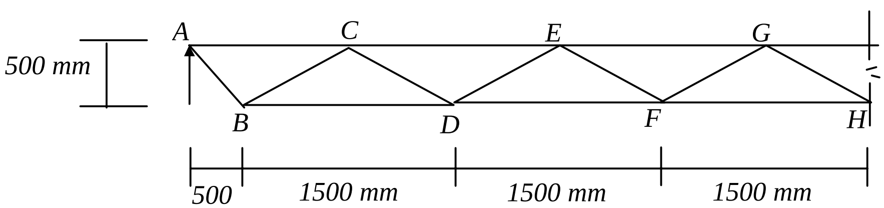


$D_t = 500 + x_t + x_b$

Truss configuration: *Choose the following truss configuration*

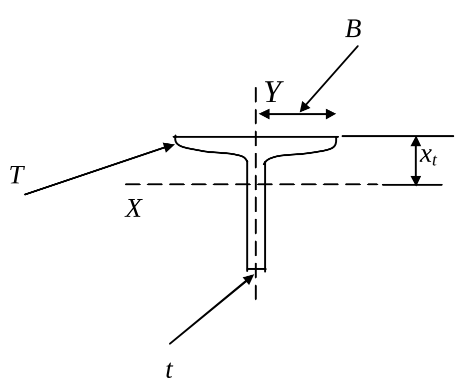


Top chord design:

Try ISNT 150 X 150 X 10 mm @ 0.228 kN/m

Sectional properties:

Area of cross-section $= A_t = 2908\ mm^2$
Depth of section $= 150$ *mm*
Width of section, b $= 2b^1 = 150$ *mm*
Thickness of flange $= T = 10.0$ *mm*
Thickness of web $= t = 10.0$ *mm*
Centre of gravity $= x_t = 39.5$ *mm*



$r_{xx} = 45.6$ *mm*
$r_{yy} = 30.3$ *mm*

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Section classification:

$\varepsilon = (250/f_y)^{0.5} = (250/250)^{1/2} = 1.0$

Flange:

$b^1/T = 75/10 = 7.5 < 8.9\varepsilon$ *Flange is plastic*

Web:

$d/t = 140/10 = 14\ (>9.98\varepsilon \text{ and } <19.95\varepsilon)$ *Web is semi-compact*

As no member in the section is slender, there is no need of adopting reduction factor (Yielding govern).

Given, maximum un-restrained length of top chord is 1.5 m during construction stage.

Maximum unrestrained length $= \ell_y = 1500$ mm

$\ell_x = 0.85*1500 = 1275$ mm

$r_{xx} = 45.6$ mm

$r_{yy} = 30.3$ mm

$\lambda_x = 1275/45.6 = 28$

$\lambda_y = 1500/30.3 = 49.5$

Then, $\sigma_c = 202.8$ N/mm^2 *[From Table - 3 of Chapter on axially compressed Columns]*

*Axial capacity = (202.8/1.15)*2908/1000 = 512.8 kN > 437.4 kN*

*Axial capacity = (202.8/1.1)*2908/1000 =536.13 kN > 481.96 kN*

Hence, section is safe against axial compression at construction stage.

[Other member design is governed by composite loading]

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COMPOSITE STATE:

	kN/m^2	*Factored Load* (kN/m^2)
Deck slab weight	*2.8*	*2.8*1.5 = 4.20*
Truss weight (assumed)	*0.4*	*0.4*1.5 = 0.60*
Ceiling, floor finish and Services	*1.0*	*1.0*1.5 = 1.5*
Superimposed live load	*5.0*	*5.0*1.5 = 7.5*

Total factored load $= (4.2+0.6+1.5+7.5)*3$

$= 13.8*3 = 41.40\ kN/m$

Maximum bending moment (M_c) $= w\ell^2/8 = 41.4*10^2/8 = 527.85\ kN\text{-}m$

Maximum shear $= w\ell/2 = 41.4*10/2 = 207\ kN$

Bottom chord design:

Force in bottom chord, $R_{b,req}$ *is given by: [See Fig. of the text]*

$R_{b,req}\{D + x_t + D_s-(D_s-D_p)/2\} = M_c$

[Assume NA is in the concrete slab]

$R_{b,req}\,(500+39.5+(150-37.5))/1000 = 527.85$

$R_{b,req}\,(652/1000) = 527.85\ kN\text{-}m$

$R_{b,req} = 527.85/0.652 = 809.59\ kN$

Area required $= 809.59*1000/(f_y/1.1)$

$= 809.59*1000/(250/1.1) = 3562.2\ mm^2$

Trial-1 *Trying ISHT 150 @ 0.294kN/m*

Sectional properties:

$A = 3742\ mm^2$; x_b = *Centre of gravity* $= 26.6\ mm$

Width of the section, $b = 2b_1 = 250\ mm$

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Axial tension capacity of the selected section (R_b):

R_b = (250/1.1)* 3742/1000 = 850.46kN > 809.59 kN

Hence, O.K.

Capacity of Composite Section in Compression:

Capacity of concrete slab, R_c, is given by

$R_c = 0.45\ (f_{ck})_{cu} * b_{eff} * (D_s - D_p)$

Effective width of the slab, b_{eff}: *[See the chapter Composite beams – II]*

$b_{eff} \leq \ell/4 \quad = 10000/4 = 2500$ mm

Therefore, b_{eff} = 2500 mm

R_c = 0.45*20*2500*75/1000 $\{f_{ck} = 20\ N/mm^2\}$
= 1687.5 kN > R_b *(tension governs)*

Neutral axis depth :

$x_c = (D_s - D_p)$*850.46/1687.5 = 75*850.46/1687.5 = 37.8 mm
D_t = 0.5+0.0266+0.0395 = 0.566 mm

Then, maximum moment it can carry

$M_{u,\ design}$ = 850.46(0.566+0.15-0.5*0.0378-0.0266)
= 570.23 kN-m > 527.85 kN-m

Hence, the slab and chord members are designed.

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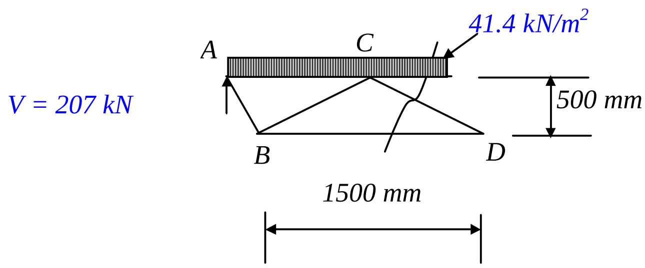
Web members:



F_{AB} = V(1.414) =207(1.414) = 292.7kN (tension)

F_{BC} = $(V-0.5*39.5)(500^2+750^2)^{0.5}/500$ =335.86kN (compression)

F_{CD} = $(V-2.0*39.5)(500^2+750^2)^{0.5}/500$ = 223.91kN (tension)

Hence, maximum tensile force in bracing members =292.7kN
Maximum compressive force in bracing members =335.86 kN

Design of tension members:

Trial gross area required $=292.7*10^3/(250/1.1) = 1287.88\ mm^2$

Trying 2 – ISA 70 X70 X6 .0 @ 0.126 kN/m

A_{gross} *provided* =2*806 $=1612\ mm^2$

Effective area:

(Assume, angle is welded to T- section)
$A_{net\ effective}$ $= 1612\ mm^2$
Axial tension capacity $= A_e*(f_y/\gamma_m)$
$= 1612*250/1.1$
$= 366.36\ kN > 292.7\ kN$

Hence, 2 – ISA 70 X 70 X 6.0 are adequate

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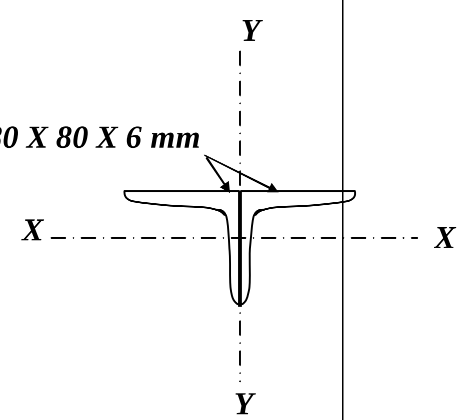
Design of compression member:



Maximum compressive load = 320 kN
Trying 2 – ISA 80 X 80 X 6.0 @ 0.146 kN/m
A = 1858 mm^2
r_{xx} = 24.6 mm
r_{uu} = 34.9 mm

Section classification:

$b/t = 80/6 = 13.3 < 15.75\varepsilon$

Hence, the section is not slender and no need to apply any reduction factor.

Slenderness ratio is taken as the greater of

Length of member $= (750^2 + 500^2)^{0.5} = 901$ mm
$\lambda_{xx} = 0.85 * 901/24.6 = 31.1$
$\lambda_{xx} = 1.0 * 901/34.9 = 25.8$

Design buckling strength $= \sigma_c = 231.2$ *Mpa*
[Table – 3 of chapter on axially compressed columns]

Design compressive strength $= 1858 * (231.2/1.1)/10^3 = 390.52$ kN > 335.86 kN

<u>Hence the 2 – ISA 80 X 80 X 6.0 are adequate for the web members</u>

(The web members away from the support would have lesser axial force and can be redesigned, if so desired. Preferably use the same section for all web members)

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Weight Schedule:

Description	*Section mm X mm X mm*	*Weight kN/m*	*Number*	*Length (m)*	*Total Length (m)*	*Weight kN*
Top Chord	*ISNT 150 X150X10*	*0.228*	*1*	*10.0*	*10.0*	*2.28*
Bottom Chord	*ISHT 150*	*0.294*	*1*	*10.0*	*10.0*	*2.94*
Bracing Members	*2-ISA 70 X 70 X 6*	*0.126*	*2*	*0.71*	*1.42*	*0.18*
Tension Members	*2-ISA 70 X 70 X 6*	*0.126*	*6*	*0.9*	*5.4*	*0.68*
Compression Members	*2-ISA 80 X 80 X 6*	*0.146*	*6*	*0.9*	*5.4*	*0.79*
						6.87
Allow 2 ½ % Extras						*0.17*
						7.04

Average weight per unit area of floor

$$= \frac{7.04}{10*3} = 0.23\ kN/m^2 < 0.4\ kN/m^2\ (Assumed)$$

Hence, O.K.

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Deflection:

Pre-composite stage:

The second moment of area of the steel truss, I_t can be calculated from the following equation.

$$I_t = \frac{A_b A_t}{(A_b + A_t)}\left[D_t - x_b - x_t\right]^2$$

Where,

A_b - *Cross-sectional area of bottom chord.*

A_t - *Cross-sectional area of top chord.*

In this problem,

A_b = $3742\ mm^2$

x_b = $26.6\ mm$

A_t = $2908\ mm^2$

x_t = $39.5\ mm$

D_t = $566\ mm$

$$I_t = \frac{3742 \times 2908}{(3742 + 2908)}\left[566 - 26.6 - 39.5\right]^2$$

$$= 409 \times 10^6\ mm^4$$

Loading:

	kN/m^2
Deck slab weight	*2.80*
Truss weight	*0.23*
Construction load	*1.00*

	4.03

Total Load = *4.03*3*10=121 kN*

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Deflection at pre composite state is given by

$$\delta_0 = (5*121*10000^3)/(384*200*409*10^6) = 19.3 \text{ mm}$$

Deflection at composite state due to dead load = $\delta_1 = (3.03/4.03)*19.3$
$= 14.5$ *mm*

[For composite stage construction load has to removed for calculating deflections]

Deflection - Composite stage:

The second moment of area, I_c*, of a composite truss can be calculated from the following equation*

$$I_c = \frac{A_b A_c / m}{\left(A_b + A_c / m\right)} \left[D_t + (D_s + D_p) / 2 - x_b\right]^2$$

Where,

A_c = *Cross-sectional area of the concrete in the effective breadth of slab*
$= (D_s - D_p)b_{eff}$
m = *modular ratio*

In this problem,

A_b = 3742 mm^2; $b_{eff} = 2500$ mm
A_c = $(150 - 75)* 2500 = 1875*10^2 \text{ mm}^2$
m = 15(light weight concrete)
D_t = 566 mm
x_b = 26.6 mm

$$I_c = \frac{3742 \times 1875 \times 10^2 / 15}{\left(3742 + 1875 \times 10^2 / 15\right)} \left[566 + \frac{225}{2} - 26.6\right]^2$$

$$= 1224 \times 10^6 \text{ mm}^4$$

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Loading:

Super Imposed load $= 5.0\ kN/m^2$

Total Load $= 5.0*3*10=150\ kN$

Deflection at composite state due to superimposed load is given by

$\delta_2 = (5*150*10000^3)/(384*200*1224*10^6) = 8.0\ mm$

10% allowance is given

Then, $\delta_2 = 8.8\ mm < \ell/360 = 10000/360 = 28\ mm$

Total deflection $= \delta_1 + \delta_2 = 14.5 + 8.8 = 23.3\ mm\ (\ell/429) < (\ell/325)$

Hence, design is O.K.

28 STEEL BEAMS WITH WEB OPENINGS

1.0 INTRODUCTION

The responsibility of a Structural Engineer lies in not merely designing the structure based on safety and serviceability considerations but he also has to consider the functional requirements based on the use to which the structure is intended. While designing a power plant structure or a multi-storeyed building, the traditional structural steel framing consists of beams and girders with solid webs. These hinder the provision of pipelines and air conditioning ducts required for satisfactory functioning for which the structure is put up. Very often, the service engineer who is on the scene long after the structural erection has been completed is required to fix air conditioning ducts in place. The re-routing of services (or increasing the floor height at the design stage for accommodating them) leads to additional cost and is generally unacceptable. The provision of beams with web openings has become an acceptable engineering practice, and eliminates the probability of a service engineer cutting holes subsequently in inappropriate locations.

Beams with web openings can be competitive in such cases, even though other alternatives to solid web beams such as stub girders, trusses etc are available. This form of construction maintains a smaller construction depth with placement of services within the girder depth, at the most appropriate locations.

The introduction of an opening in the web of the beam alters the stress distribution within the member and also influences its collapse behaviour. Thus, the efficient design of beams and plate girder sections with web openings has become one of the important considerations in modern structures.

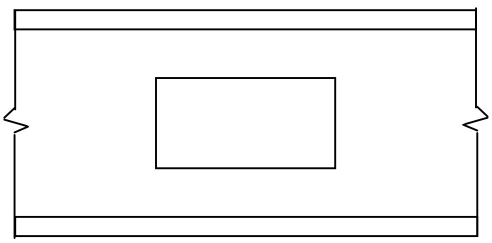
In this chapter, methods to evaluate the ultimate shear capacity of the beams and fabricated girders with circular or rectangular web openings are discussed. The methodology is based on the Von Mises yield criterion.

2.0 GUIDELINES FOR WEB OPENINGS AND STIFFENERS

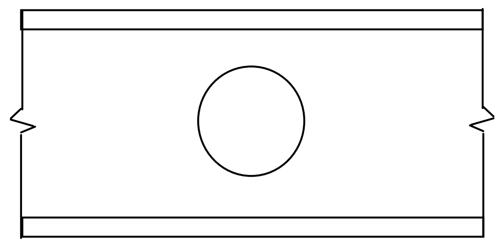
The shape of the web openings will depend upon the designer's choice and the purpose of the opening. There are no hard and fast rules to dictate the shapes of the openings. But, for designer's convenience, openings of regular shapes (such as circular or rectangular) are usually chosen. Introduction of openings in the web decreases stiffness of the beams resulting in larger deflections than the corresponding beams with solid webs. The strength of the beams with openings may be governed by the plastic deformations that occur due to both moment and shear at the openings.

The strength realised will depend on the interaction between the moment and shear. The moment capacity of the perforated beam will be reduced at the opening because of the reduction in the contribution of web to the moment capacity. This is not very significant, as usually the contribution of the web to the moment capacity is very small.

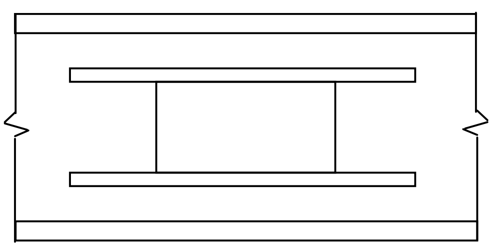
However, the reduction in shear capacity at the opening can be significant. Therefore the ultimate capacity under the action of moment and shear at the cross section where there is an opening will be less compared to that at the a normal cross section without opening. i.e. some strength is lost. To restore the strength lost, reinforcement along the periphery of the openings could be provided. As a general rule, we should avoid having openings in locations of high shear, nor should they be closely spaced. Common types of web openings with and without reinforcement are shown in Fig. 1. The following general design guidelines may be useful [Fig. 2 (*a*)].



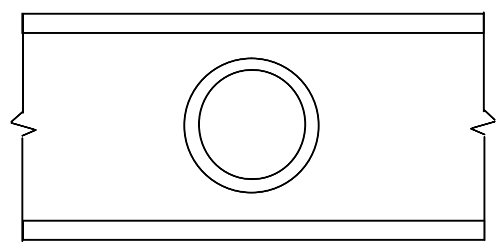
(b) Web with centrally placed rectangular hole



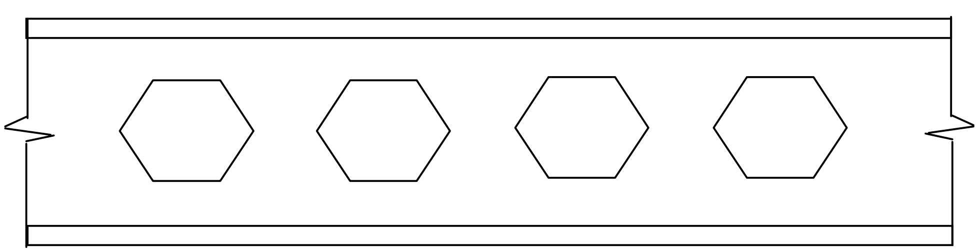
(b) Web with centrally placed circular hole



(c) Web with centrally placed rectangular hole with reinforcement

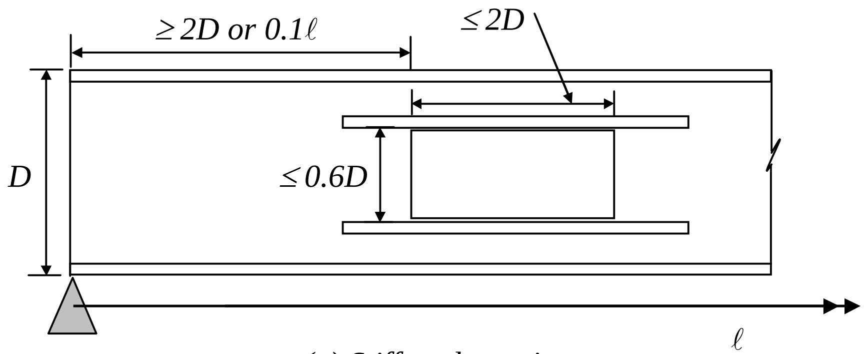


(d) Web with centrally placed circular hole with reinforcement

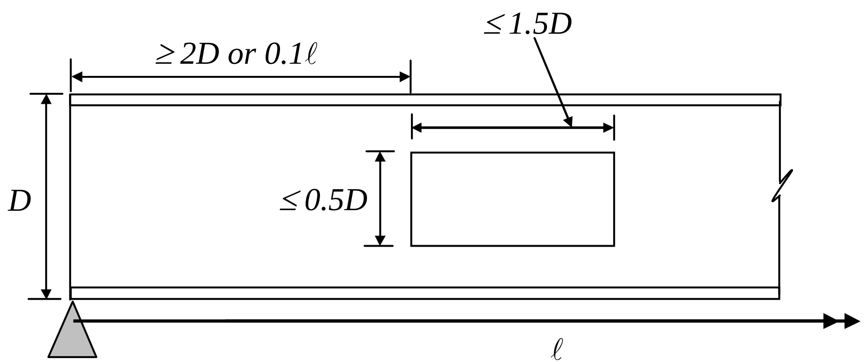


(e) Castellated beams

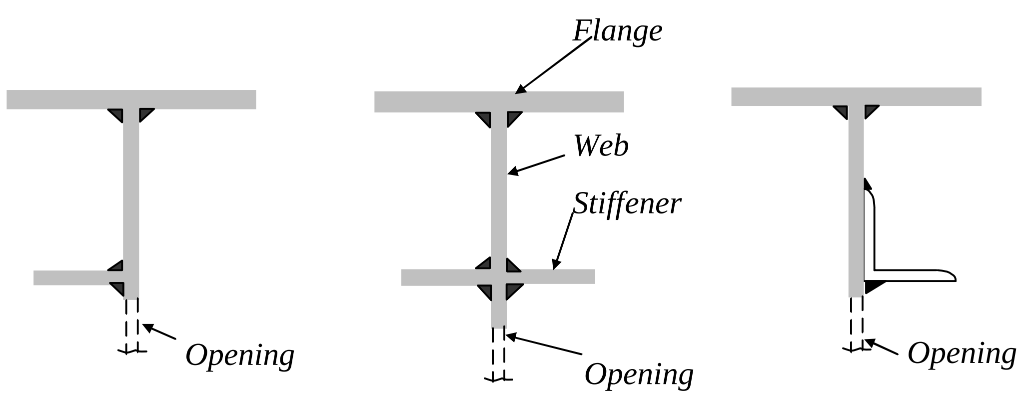
Fig.1 Common types of web openings



(a) Stiffened opening



(b) Un-Stiffened opening



(c) Various stiffening arrangements

Fig. 2 Guide lines for web holes and various stiffening arrangements

- The hole should be centrally placed in the web and eccentricity of the opening is avoided as far as possible.
- Unstiffened openings are not always appropriate, unless they are located in low shear and low bending moment regions.
- Web opening should be away from the support by at least twice the beam depth, *D* or *10%* of the span (ℓ), whichever is greater
- The best location for the opening is within the middle third of the span.
- Clear Spacing between the openings should not be less than beam depth, *D*.

- The best location for opening is where the shear force is the lowest.
- The diameter of circular openings is generally restricted to *0.5D*.
- Depth of rectangular openings should not be greater than *0.5D* and the length not greater than *1.5D* for un-stiffened openings. The clear spacing between such opening should be at least equal the longer dimension of the opening.
- The depth of the rectangular openings should not be greater than *0.6D* and the length not greater than *2D* for stiffened openings. The above rule regarding spacing applies.
- Corners of rectangular openings should be rounded
- Point loads should not be applied at less than *D* from side of the adjacent opening.
- If stiffeners are provided at the openings, the length of the welds should be sufficient to develop the full strength of the stiffener. Various types of stiffeners are shown in Fig. 2(*c*).
- If the above rules are followed, the additional deflection due to each opening may be taken as 3% of the mid-span deflection of the beam without the opening.

3.0 FORCE DISTRIBUTION AND FAILURE PATTERN AT WEB OPENINGS

3.1 Beams with perforated thick webs

In buildings, the depth to thickness ratio of the beam web is kept low (below 80). Such webs are not prone to local buckling in shear and are termed “thick webs”. On the other hand, in bridge structures the girder depths adopted are generally large and hence the plate girders are characterised by web slenderness ratios above 80. The behaviour of these plate girders has been elaborately discussed by Narayanan [4] and is not discussed here. However in building floor construction, stockier webs having slenderness ratio of 50 to 80 are more common and the discussion in the section is restricted to such beams with openings. Mathematical models for the design of thick webs containing openings have been developed by Redwood and his colleagues based on the plastic analysis of structures and is described below.

3.2 Basis of Analysis

A single rectangular opening in a traditional beam used in buildings is considered first [See Fig. 3(a)]. The hole is located a little bit above the Neutral axis for illustration. The hole may (or may not) be reinforced.

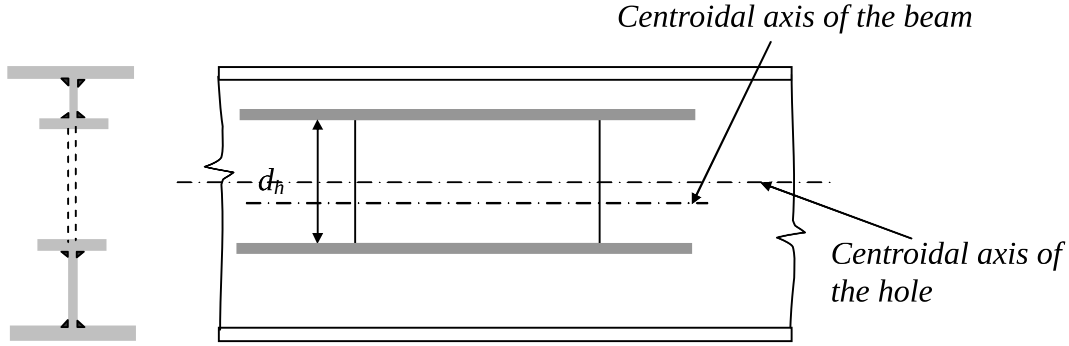


Fig. 3(a) Rectangular Hole in the web

The web of the beam is "thick" and is not prone to buckling in shear under the action of the loads, the collapse is likely to be initiated by the formation of four plastic hinges, near the four corners of the hole in the web above and below the openings. Note the location of four plastic hinges in Fig. 3(*b*). This is due to behaviour of the beam as a virendeel girder.

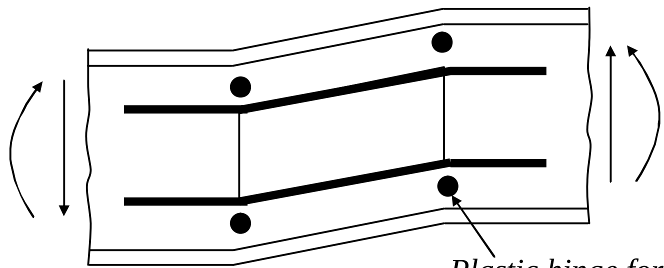


Fig. 3(b) Mode of failure

3.2.1 Force distribution and failure pattern

The forces acting at the ends of a rectangular opening are shown in the Fig. 4(*a*). For thick webs with a circular opening, Redwood proposed an equivalent effective size of rectangular opening as shown in Fig. 4(*b*). Note that R represents the radius of circular opening in Fig. 4(*b*). It is seen that the overall bending moment M is resisted by the compression T_2 in the top web plate, and tension T_1 in the bottom web plate forming a couple ($T_1 = T_2$ for equilibrium) acting at distance h apart, together with relatively small moments M_{t1} and M_{b1} acting in the top and bottom portions of the opening. These moments M_{t1} and M_{b1} are generated because of vierendeel action and hence called vierendeel moments. At the opening adjacent to lower moment section the shear V will be resisted by V_t shear in the web plate above the hole ("top web plate") and V_b shear in the web plate below the hole ("bottom web plate"). For equilibrium the following conditions have to be satisfied:

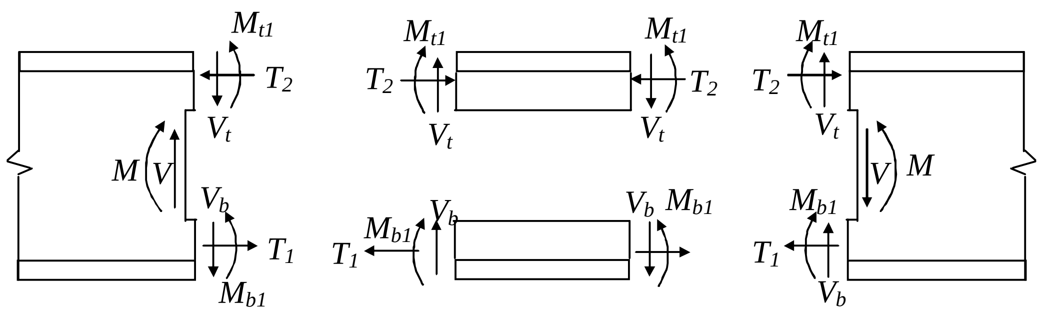
$$V = V_t + V_b$$

and $$M = (T_1 \text{ or } T_2) * h + M_{t1} + M_{b1}$$

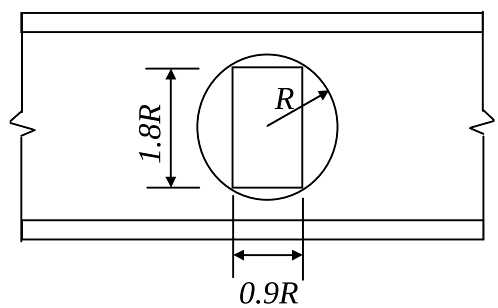
Inelastic shearing deformation in the web at the opening occurs under any combination of moment and shearing force. This shearing deformation is called as 'Vierendeel' deformation. This shearing deformation and the plastic hinge rotations near the corners of the opening at ultimate load, lead to a large relative deflection between ends of the opening.

At failure, all elements (i.e. the top flange, bottom flange and the web plates above and below openings) are subjected to high combined stresses caused by axial force and shear force from overall bending and local moments due to vierendeel action. The bottom web plate is likely to yield due to tension and top web plate above opening is susceptible to buckling/ yielding. The deformed shape of the beam caused by the moment and shear acting across the opening is shown in Fig. 4(c).

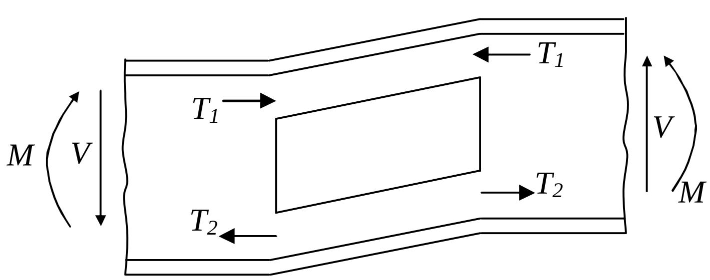
The Vierendeel moment across the opening is resisted by the plastic moment capacities of the sections. It is assumed that the upper and lower web sections resist the applied shear in proportion to square of their depths. Plastic moment capacities of the sections are reduced in the presence of this shear force and axial force.



(a) Force distribution in steel beam at opening



(b) Effective size of circular hole in the web



(c) A typical failure pattern

Fig. 4 Behaviour of steel beams with web holes

3.2.2 Web instability for beams with "thick" webs

The ultimate strength of the beam with a "thick web" is determined by plastic analysis without considering the effect of web buckling. This procedure is valid if effective depth of the web is limited to outstand proportion of compact sections i.e. $d_{te} \le 10t\varepsilon$ as plastic and compact sections are not vulnerable to local web buckling. Outstand proportions of the T-sections are given in Table 1.

Table 1 – Outstand proportions for T- sections

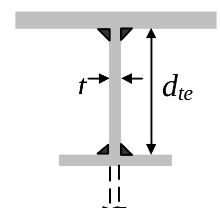
<table>
<tr><th rowspan="2">Type of element</th><th colspan="3">Class of section</th></tr>
<tr><th>Plastic</th><th>Compact</th><th>Semi-compact</th></tr>
<tr><td>Stems of T-sections</td><td>$d_{te}/t \le 8.9\varepsilon$</td><td>$d_{te}/t \le 10\varepsilon$</td><td>$d_{te}/t \le 20\varepsilon$</td></tr>
<tr><td colspan="4">Where, d_{te} – Effective depth of the T-sections
t – thickness of the T-sections
$\varepsilon = \sqrt{\dfrac{250}{f_y}}$ and f_y - yield stress of the steel

</td></tr>
</table>

Fig. 5 shows instability of web near opening in an un-stiffened girder, when the web is supported only on three sides as in T sections. By considering the variation of stress from compression to tension along the upper edge of the opening, the effective support of the flange and the continuity provided by the web adjacent to the opening, the effective depth of the web can be calculated using the equation given below:

$$d_{te} = \frac{d_t}{\sqrt{1 + \left(\dfrac{2d_t}{ka_h}\right)^2}} \qquad (1)$$

where, k - reflects the combined influence of the shape of the local bending moment diagram along the upper web-flange section, and the effect of continuity (See Table 2).

d_t - Web depth above the opening.

Table 2 – Values of k

Condition	Value of ***k***
The ratio of web stresses at either end of the opening is less than – *0.5*	0.5
The web opening is in uniform compression over its length	1.0

However, when the edges of the openings are un-stiffened, the semi-compact or slender sections are susceptible to local buckling of the web-flange section (T section) at the horizontal edge that is in compression due to the global bending action. The opening in such cases may be stiffened or only the elastic capacity of the web-flange section may be used. It is necessary to check for the compression zone stability of the large rectangular openings in high moment regions. This is carried out by treating it as an axially loaded column with effective length equal to that of the length of the opening. For unstiffened openings, this check is not necessary if the opening length is less than four times the depth of the T section under compression. Normally, instability of the vertical sides of the web opening will not occur in rolled sections except in high shear zones. But, fabricated beams are more susceptible to this form of the web instability.

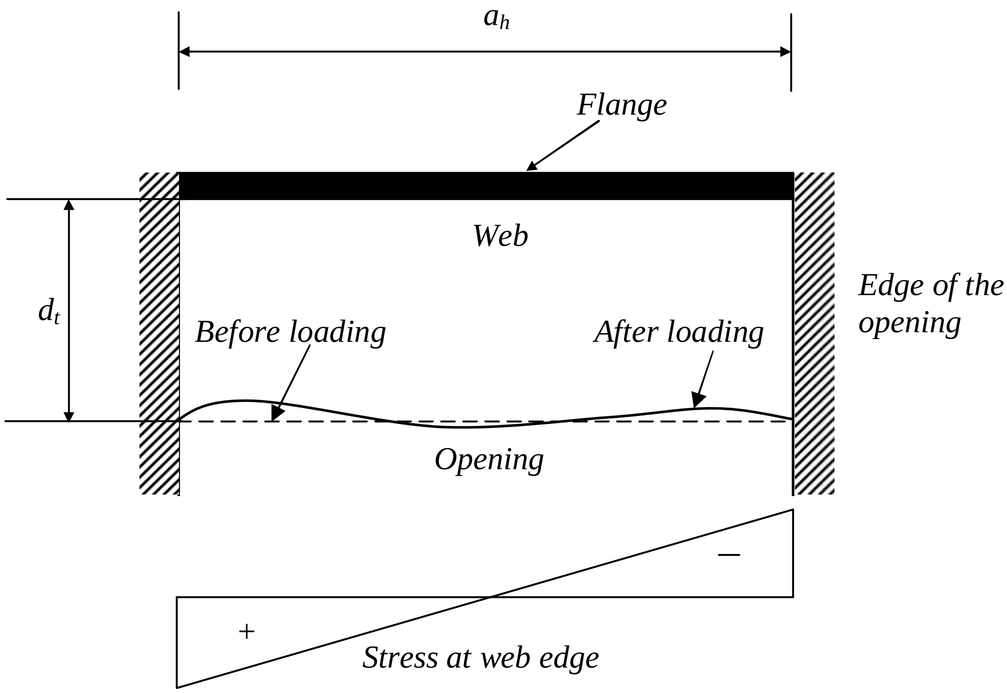


Fig. 5 Local buckling of un-stiffened web

3.2.3 Lateral torsional stability [2]

It is necessary to check for adequacy of the beam against lateral torsional buckling because the resistance at the opening should not govern the beam resistance. In this case while checking the stability, the effect of the opening is incorporated by multiplying the St. Venant's torsion constant, *J*, by

$$\left[1-\left(\frac{a_h}{\ell_u}\right)\left(\frac{d_h t-2A_r}{t(D+2B)}\right)\right]^2 \leq 1.0 \qquad 1(a)$$

where, ℓ_u - Unbraced length of the compression flange
a_h - Length of the opening
d_h - Depth of the opening
D - Overall depth of steel beam
B - Breadth of the flange of steel beam
t - Thickness of the web of the steel beam
A_r - Area of reinforcement provided at the opening

3.3 Analysis of beams with perforated thick webs

The unreinforced opening can be analysed reasonably well without using simplifying assumptions. From the point of view of economy, one may like to use unreinforced openings. If the reinforcement is required, the conservative assumptions made in the analysis of reinforced case increase the material requirement enormously. In addition, the cost is further increased due to welding. The analysis outlined here is conservative when compared with other more precise methods.

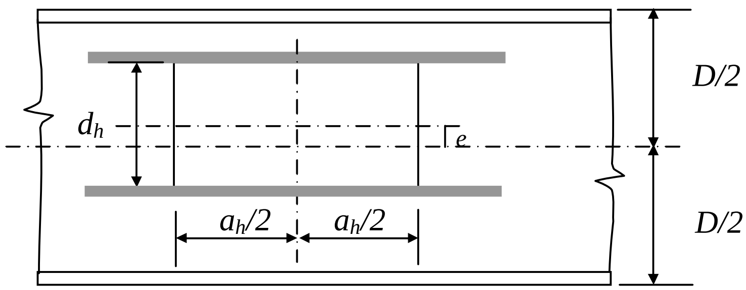


Fig. 6 Opening in beam with thick web

Fig. 6 shows an opening in a beam with a relatively thick web. The figure shows a web opening of length a_h and depth d_h. The opening is located at an eccentricity e with respect to centre line of the beam. The strength of the beam at the opening can be represented by a moment – shear interaction diagram as shown in Fig. 7. It may be noted that the shear capacity V_B of the section with opening is constant upto a factored moment of M_B. As the moment increases beyond M_B, the shear capacity would reduce because the compression due to moment and that due to shear make the compression diagonal to buckle. When the maximum moment capacity M_A is reached, the shear capacity becomes zero. For moments above M_B but less than M_A, shear capacity V may be obtained using the following non-linear interaction equation.

$$(V/V_B)^2 + [(M - M_B)/ (M_A - M_B)]^2 = 1.0$$

Conservatively, sometimes designers prefer to use a linear equation between the limits M_B and M_A. The shear capacity at M_B is V_B and the shear capacity at M_A is zero. This is shown by the dotted lines in the interaction diagram. If a beam is to be safe, the moment (M_f) and the shear (V_f) due to factored loading on the beam should be less than corresponding beam resistance given by moment-shear interaction curve. The factored shear V_f and moment M_f should be such that

$$V_f \leq V_B \qquad (2)$$

$$M_f \leq M_A - [M_A - M_B]\, V_f / V_B \qquad (3)$$

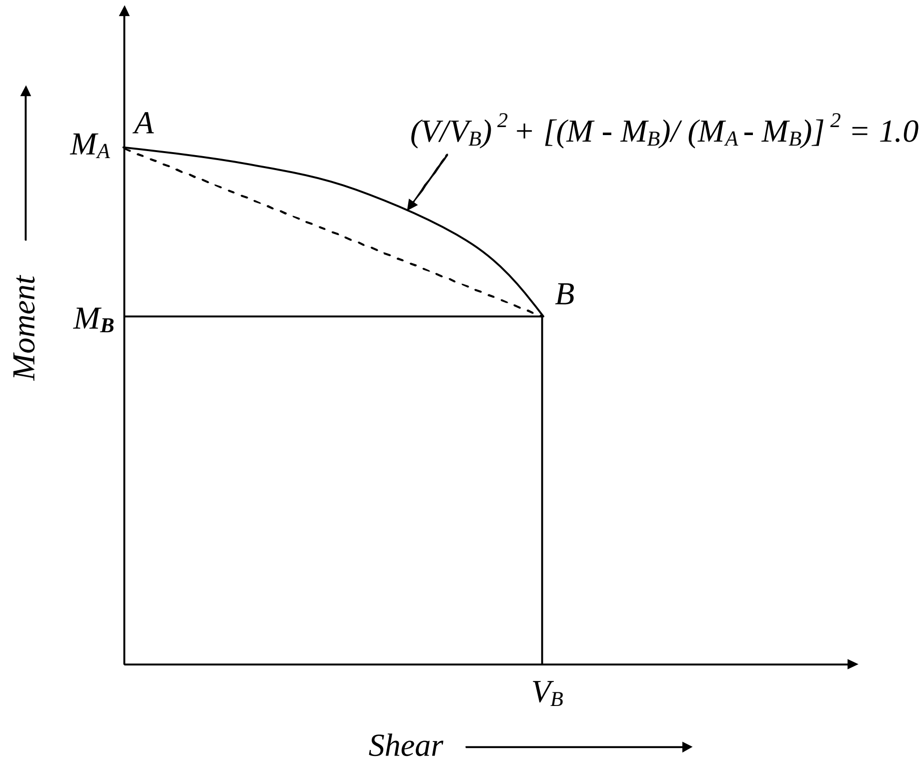


Fig.7 Interaction diagram

3.4 Unreinforced openings in beams with "thick" webs

Let us first examine the case of an unstiffened web opening. Redwood and his colleagues have proposed the following equations for evaluating M_A and M_B based on the plastic analysis described above. Due to the interaction of shear and moment, the capacity of the section gets reduced. The reduction in capacities (either moment or shear) at salient levels (i.e. at M_A, M_B and V_B) is expressed as a ratio of their plastic moment / plastic shear capacities as follows:

In the case of un-stiffened web openings, the salient points of the shear and moment interaction diagram (Fig. 7) can be written as:

$$\frac{M_A}{M_P} = 1 - \frac{\left\{ \frac{A_w}{4A_f} \left[\left(\frac{d_h}{D} \right)^2 + \left(\frac{4e}{D} \right) \left(\frac{d_h}{D} \right) \right] \right\}}{1 + \frac{A_w}{4A_f}} \tag{4}$$

$$\frac{M_B}{M_p} = \frac{\left[1 - \frac{1}{\sqrt{3}} \left[\frac{A_w}{A_f} \right] \left[\frac{a_h}{D} \right] \sqrt{\frac{\alpha_2}{1+\alpha_2}} \right]}{1 + \frac{A_w}{4A_f}} \tag{5}$$

$$\frac{V_B}{V_P} = \frac{1}{\sqrt{3}}\left[\frac{a_h}{D}\right]\left[\frac{\alpha_1}{\sqrt{1+\alpha_1}} + \frac{\alpha_2}{\sqrt{1+\alpha_2}}\right] \qquad (6)$$

$$\alpha_1 = \frac{3}{4}\left[\frac{D}{a_h}\right]^2\left[1 - \frac{d_h}{D} - \frac{2e}{D}\right]^2 \qquad (7)$$

$$\alpha_2 = \frac{3}{4}\left[\frac{D}{a_h}\right]^2\left[1 - \frac{d_h}{D} + \frac{2e}{D}\right]^2 \qquad (8)$$

where, M_p - Plastic moment capacity of the unperforated beam section.
V_p - Plastic shearing capacity of the unperforated beam section.
e - Eccentricity of opening and is taken as positive, whether the opening lies above or below the beam centre line.
A_f, A_w - Area of one flange and area of web of the steel beam section respectively
M_f, V_f - Factored moment and factored shear at opening centre line respectively.
D - Total depth of beam
a_h - Length of opening
d_h - Total depth of opening

The detailed derivation of the above equation may be followed from original source. [(2)]

3.5 Reinforced openings

If the opening is reinforced as shown in Fig. 6, then the eqs. (9) and (10) must be satisfied along with equations (2) and (3). In no case area of reinforcing bars A_r, should be greater than the area of one flange of steel beam section, to prevent inadvertent flange instability.

$$V_f \le V_p\left(1 - \frac{d_h}{D}\right) \qquad (9)$$

$$M_f \le M_P \qquad (10)$$

In this case the salient points of the interaction diagram (Fig. 7) are defined by

$$\left[\frac{M_A}{M_p}\right]_a = 1 + \frac{\left[\frac{A_r}{A_f}\left[\frac{d_h}{D}\right] - \frac{A_w}{4A_f}\left[\left[\frac{d_h}{D}\right]^2 + 4\left[\frac{d_h}{D}\right]\left[\frac{e}{D}\right] - 4\left[\frac{e}{D}\right]^2\right]\right]}{\left[1 + \frac{A_w}{4A_f}\right]} \quad \text{for } \frac{e}{D} \le \frac{A_r}{A_w} \qquad (11)$$

$$\left[\frac{M_A}{M_p}\right]_b = \frac{\left[\frac{M_A}{M_p}\right]_a - \frac{A_w}{A_f}\left[\frac{e}{D} - \frac{A_r}{A_w}\right]^2}{\left[1+\frac{A_w}{4A_f}\right]} \qquad for\ \frac{e}{D} > \frac{A_r}{A_w} \qquad (12)$$

$$\frac{M_B}{M_p} = \frac{1-\frac{A_r}{A_f}}{1+\frac{A_w}{4A_f}} \qquad (13)$$

$$\frac{V_B}{V_p} = 2\sqrt{3}\left(\frac{D}{a_h}\right)\left[\frac{A_r}{A_w}\right]\left[1-\frac{d_h}{D}\right] \qquad (14)$$

Substituting the values of M_A, M_B and V_B from eqs. (11) or (12), (13) and (14) into eqs. (2) and (3) leads to a quadratic equation. The solution of this quadratic equation gives the area of reinforcement required at the opening. This procedure can be used as a basis for design aids giving directly the required reinforcement area for a given loading.

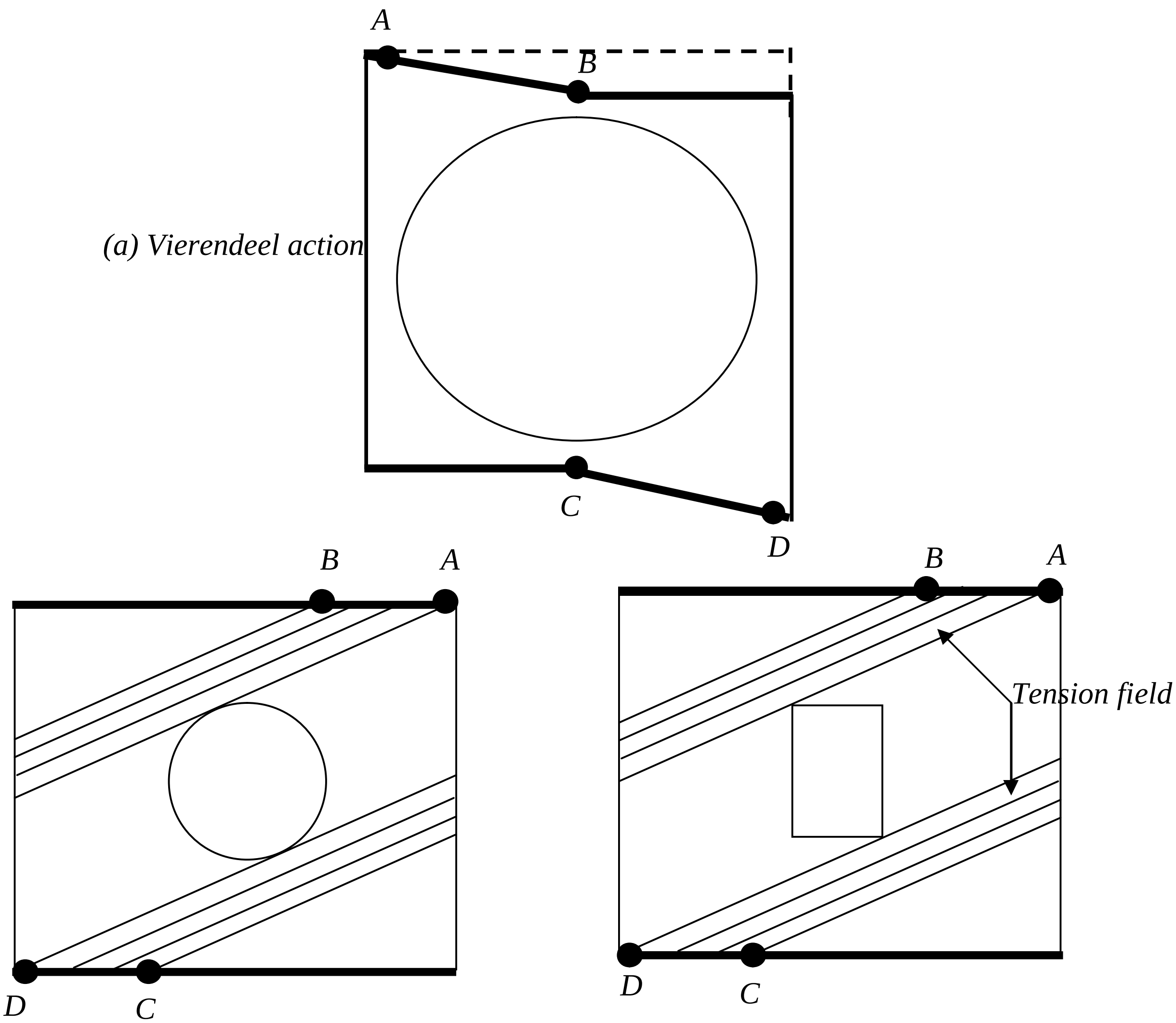
4.0 Plate girders with openings in the web

Typical slenderness values for these webs lie in the range of *150* to *250* and they invariably buckle prior to the actual collapse of girder. The following analysis is valid for plate girders in the practical range having depth to stiffener spacing greater than1.

The following is the summary of experimental studies [(4)]:

- In statically loaded plate girders containing circular and rectangular openings the girders having openings in the high shear zone failed at loads significantly lower than those that had openings in high moment zones.
- In case of circular openings, the ultimate shear capacity of webs dropped almost linearly with the increase in the diameter of the opening.
- The observed failure mechanism for plate girders with web openings is similar to that of plate girders with un-perforated webs discussed in the chapter on Plate girders - I, the only difference is in the position of hinges in the flanges under the action of diagonal tension field in the web.

Fig. 8(*a*) shows the position of hinges at the instant of failure in a plate girder when diameter of the web opening is nearly equal to the depth of the girder. In this case the internal two hinges are formed at the centre of flanges. Fig. 8(*b*), 8(*c*) shows the position of hinges (*A*, *B*, *C*, *D*) at the instant of failure in a plate girder with centrally placed small circular or rectangular openings. The internal plastic hinges (*B*, *C*) will form at the position of maximum bending moment, where the shear force is zero.



b) Failure mechanism in plate girders with centrally placed circular hole

(c) Failure mechanism in plate girders with centrally placed rectangular hole

Fig. 8 Plate girders with web holes

4.1 Analysis of Plate girders with web openings

The method adopted for evaluation of ultimate shear capacity of plate girders with web openings is similar to the method discussed in the chapter titled Plate Girders – I for un-perforated plate girders. Ultimate shear capacity can be obtained as the sum of following four contributions:

i. The reduced value of elastic critical load in the perforated web
ii. The load carried by the membrane tension in the post critical stage
iii. The load carried by the flanges
iv. The load carried by the reinforcement, if any

Elastic critical load of the web and load carried by membrane tension in the web are the two contributions affected by introduction of openings in the webs. The introduction of openings in the web decreases buckling resistance and hence elastic critical load is reduced. The amount of reduction actually depends upon the ratio of opening size to the width of the plate. Thus reduced buckling coefficient should be used in calculating elastic critical stress. The tension field in un-perforated web is developed predominantly along the diagonal band. Placement of web openings in this zone reduces the width of the Tension field and so causes significant drop in strength of the girder.

4.2 Approximate method

This section explains an approximate method to assess the ultimate capacity of a plate girder with the circular web opening without reinforcement. The method consists of linearly interpolating between the value of V_S for an unperforated web obtained from equation (6) of the- chapter on Plate girders-I and the Vierendeel load, V_V obtained as described below. The symbols used in the following equations are the same as in chapter on Plate girders -I unless otherwise specified.

If the diameter of the opening, D_h, covers the full depth, d, of the girder, the failure would be essentially due to Vierendeel mechanism as shown in Fig. 8(a). The corresponding collapse load, V_V, is given by

$$V_V = \frac{8M_p}{a} \tag{15}$$

where a is clear width of web plate between vertical stiffeners

Thus, for an opening diameter smaller than D_h, the ultimate shear capacity (V_{ult}) can be approximated by linear interpolation between the values of V_V and V_S.

$$V_{ult} = V_V + \left(\frac{V_S - V_V}{d}\right)(d - D_h) \tag{16}$$

where, d - depth of the web for the plate girder

4.3 Plate girder webs with unreinforced circular openings

The reduced value of elastic critical stress [$(q_{cr})_{red}$] due to web openings can be determined from classical stability theory if the boundary conditions of the web plate are known. In calculating $(q_{cr})_{red}$, the value of 'k' appropriate to a web fixed at its edges should be used. The reason for this is the relative stiffness of the flange in comparison with the web increases significantly when the opening is introduced in the web and the behaviour of the web plate will be closer to one having fixed supports at the flange web junction.

The critical shear stress for case without opening is given by

$$(q_{cr}) = k_s \frac{\pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t}{d}\right)^2 \tag{17}$$

However the effect of opening can be considered if reduced shear buckling co-efficient k is used in place of k_s, where

$$k = k_s\left(1-\frac{D_h}{d}\right) \tag{18a}$$

The shear buckling stress $(q_{cr})_{red}$ is thus obtained.

k_s is the shear buckling coefficient. For fixed edges, k_s is evaluated from

$$k_s = 8.98 + 5.6\left(\frac{d}{a}\right)^2 \quad \text{where } \frac{a}{d} \geq 1, \text{ i.e. for wide panels} \tag{18b}$$

$$k_S = 8.98\left(\frac{d}{a}\right)^2 + 5.6 \quad \text{where } \frac{a}{d} \leq 1, \text{ i.e. for webs with closely spaced transverse stiffeners} \tag{18c}$$

By using the virtual work method, the failure load can be computed from

$$V_{ult} = (q_{cr})_{red}\,.d.t + p_{yt}\,.t\sin^2\theta\left(d\cot\theta - a + c - D_h\,Co\sec\theta\right) + 4\frac{M_{pf}}{c} \tag{19}$$

where, a is clear width of web plate between vertical stiffeners,

c is distance between the hinges given by:

$$c = \frac{2}{\sin\theta}\sqrt{\frac{M_{pf}}{p_{yt}.t}} \tag{20}$$

Tensile membrane stress, p_{yt} can be calculated from the following equation,

$$\frac{p_{yt}}{p_{yw}} = \sqrt{\left(1-\frac{(q_{cr})_{red}}{q_{yw}}\right)^2\left(1-\frac{3}{4}\sin^2 2\theta\right)} - \frac{\sqrt{3}}{2}.\frac{(q_{cr})_{red}}{q_{yw}}\sin 2\theta \tag{21}$$

By a systematic set of parametric studies, Evans has established that θ is approximately equal to *2/3* of the inclination of diagonal of the web.

$$\theta \cong \frac{2}{3}\tan^{-1}\left(\frac{d}{a}\right) \qquad (22)$$

Notice that equation (19) is obtained by adding three quantities, namely web-buckling strength, post-buckling membrane strength of the web plate and the plastic moment capacity of the flange. This equation is valid only for openings having [$D_h \le d\cos\theta - a\sin\theta$]. This limitation is not highly restrictive as it includes openings of all practical proportions.

In this context, it must be noted that in order for the flanges to develop hinges without buckling locally under compression due to overall bending, the section used should be a "***plastic***" one. If flanges can not develop plastic hinges because they are compact, semi-compact or slender, this method of analysis is NOT applicable.

4.4 Plate girder webs with unreinforced rectangular openings (4)

Based on the research done by Narayanan and Der Avanessian on plate girders with rectangular and square openings, the reduced elastic critical shear stress can be evaluated by the following formula,

$$(q_{cr})_{red} = \frac{k_s \pi^2 E}{12(1-\nu^2)}\left(\frac{t}{d}\right)^2\left[1-\alpha_r\sqrt{\frac{A_o}{A}}\right] \qquad (23)$$

where, A - Total area of the plate including the opening

A_o - Area of the opening

k_s - Coefficient for shear buckling stress given by equations (18*b*) and (18*c*)

α_r - A coefficient, depending on the end conditions and has a value of *1.25* for clamped edges.

By using the virtual work method, the failure load can be computed from

$$V_{ult} = (q_{cr})_{red}\,.d.t + p_{yt}\,.t\sin^2\theta\left(d\cot\theta - a + c - \delta\,\mathrm{Cosec}\,\theta\right) + 4\frac{M_{pf}}{c} \qquad (24)$$

where, $\delta = \sqrt{(a_h^2 + d_h^2)}\sin(\alpha+\theta)$

a - Clear width of web plate between vertical stiffeners

d_h - Depth of the opening

a_h - Breadth of the opening

α - The angle of inclination of the diagonal of the opening, i.e., $\alpha = \tan^{-1}(d_h/a_h)$

Quantities p_{yt}, c and θ are evaluated from equations (20), (21) and (22), respectively. Eq. (24) covers all practical ranges of web openings and is valid for depths of openings given by $d_h < [d-(a+a_h)\tan\theta]$.

5.0 SUMMARY

This chapter emphasises the need for openings in the webs of girders. Two types of beams and girders are covered (i) beams with thick webs where web buckling is not a design criterion and (ii) plate girders with thin webs where web buckling is consideration. Various types of openings are discussed and guidelines are given wherever required for the convenience of designers. Failure phenomena are discussed and equations are given for the purpose of design. The method of analysis outlined in this chapter deals with the isolated openings such as those made for providing space for service ducts and other utilities. Analysis of castellated beams where openings occur at regular intervals and openings in the composite beam are not discussed in this chapter.

6.0 REFERENCES

1. Lawson. R.M, (1987): 'Design for openings in the webs of composite beams', CIRIA special publication 51, SCI publication 068.The Steel Construction Institute, Ascot.

2. Redwood. R. and Cho. S. H., (1993): 'Design of Steel and Composite beams with web openings', Journal of Constructional Steel Research, Volume 25, 23-41.

3. Redwood. R., (1983): 'Design of I-Beams with perforations', Beams and Beam Columns, Stability and Strength (Editor R. Narayanan), Applied Science publishers, London.

4. Narayanan. R., (1983): 'Ultimate shear capacity of Plate girders with openings in webs', Plated Structures, Stability and Strength Series, Elsevier Applied Science.

29 CONNECTION DESIGN – DESIGN REQUIREMENTS

1.0 INTRODUCTION

Steel sections are manufactured and shipped to some standard lengths, as governed by rolling, transportation and handling restrictions. However, most of the steel structural members used in structures have to span great lengths and enclose large three-dimensional spaces. Hence connections are necessary to synthesize such spatial structures from one- and two-dimensional elements and also to bring about stability of structures under different loads. Thus, connections are essential to create an integral steel structure using discrete linear and two-dimensional (plate) elements.

A structure is only as strong as its weakest link. Unless properly designed, the connections joining the members may be weaker than the members being joined. However, it is desirable to avoid connection failure before member failure for the following reasons:

- To achieve an economical design, usually it is important that the connections develop the full strength of the members.
- Usually connection failure is not as ductile as that of steel member failure. Hence it is desirable to avoid connection failure before the member failure.

Therefore, design of connections is an integral and important part of design of steel structures. They are also critical components of steel structures, since

- They have the potential for greater variability in behaviour and strength,
- They are more complex to design than members, and
- They are usually the most vulnerable components, failure of which may lead to the failure of the whole structure.

Thus designing for adequacy in strength, stiffness and ductility of connections will ensure deflection control during service load and larger deflection and ductile failure under over-load. Hence, a good understanding of the behaviour and design of joints and connections in steel structures is an important pre-requisite for any good design engineer. This chapter gives an overview of the design of connections in steel structures. The following five chapters deal with bolted and welded connections in greater detail.

2.0 COMPLEXITIES OF STEEL CONNECTIONS

Margins of safety of any design, in particular that of connection, involves uncertainty due to random nature of (a) the forces acting on the structure and (b) the actual strength of the joint designed.

The randomness of the loads has been discussed in an earlier chapter; that of the actual strength is due to the variability of the dimensions of the elements and that of the strength of constituent material as well as errors due to simplification in analysis and design.

The reasons for the high uncertainty and complexity of the connection are:

- Complexity of connection geometry
- Geometric imperfections
- Residual stresses and strains

2.1 Complexity of connection geometry

The geometry of connections is usually more complex than that of the members being joined (Fig.1). The stress analysis of the joint is complicated by the (locally) highly indeterminate nature of the joint, non-linear nature of the behaviour due to lack of fit, local yielding etc. and stress concentration due to discontinuity in elements around bolt holes and weld profiles.

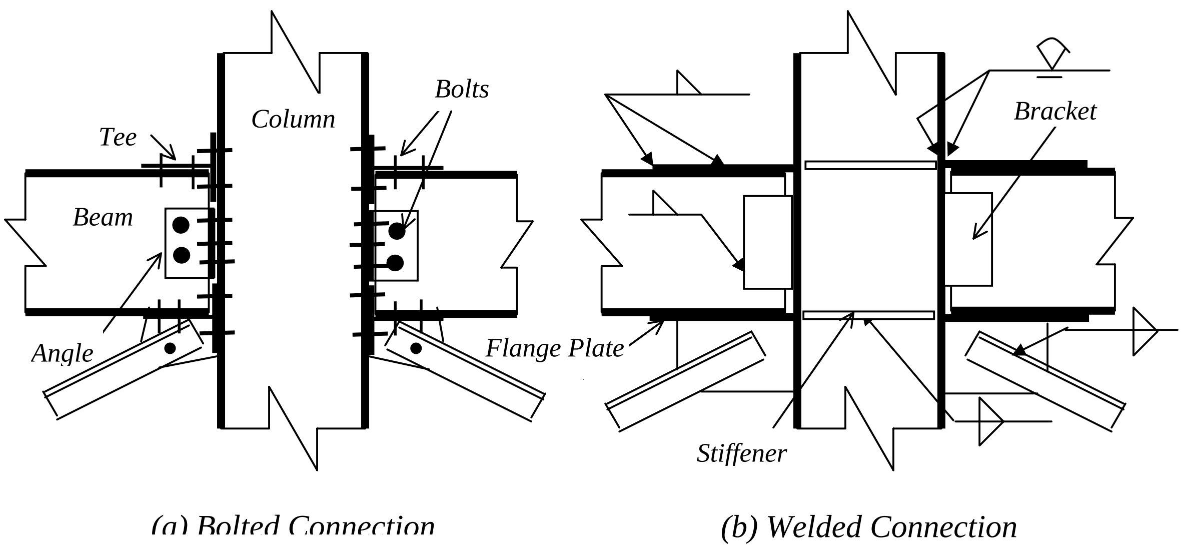


Fig. 1 Complex Beam to Column Connections

2.2 Geometric imperfections

The following factors contribute to the geometric imperfections in connection:

- Bow in the beam or column as rolled
- Lack of fit in black bolts in clearance holes
- Gaps in the connecting plate and the surface of the member to be connected to, due to fabrication errors, welding distortions, and tolerances allowed for ease of fabrication and erection

2.3 Residual Stresses and Strains

Residual stresses and strains are inherent features of steel joints due to differential cooling after the hot rolling, gas cutting and welding stages. The residual stresses cause premature local yielding and the residual strains cause distortions and lack of fit.

3.0 TYPES OF CONNECTIONS

Connections are normally made either by bolting or welding. Bolting is common in field connections, since it is simple and economical to make. Bolting is also regarded as being more appropriate in field connections from considerations of safety. However, welded connections, which are easier to make and are more efficient, are usually resorted to in shop fabrications.

3.1 Bolted Connections

Two types of bolts are used in bolted connection. The most common type is bearing bolts in clearance holes, often referred to as ordinary bolts or black bolts. They are popular since they are economical, both in terms of material and installation costs.

The force transfer mechanism under shear is as shown in Fig. 2(*a*). The force is transferred by bearing between the plate and bolts at the bolt holes. The bolts experience single or double shear depending upon the plate configuration. The failure may be either by shearing of the bolts or bearing of the plate and the bolt.

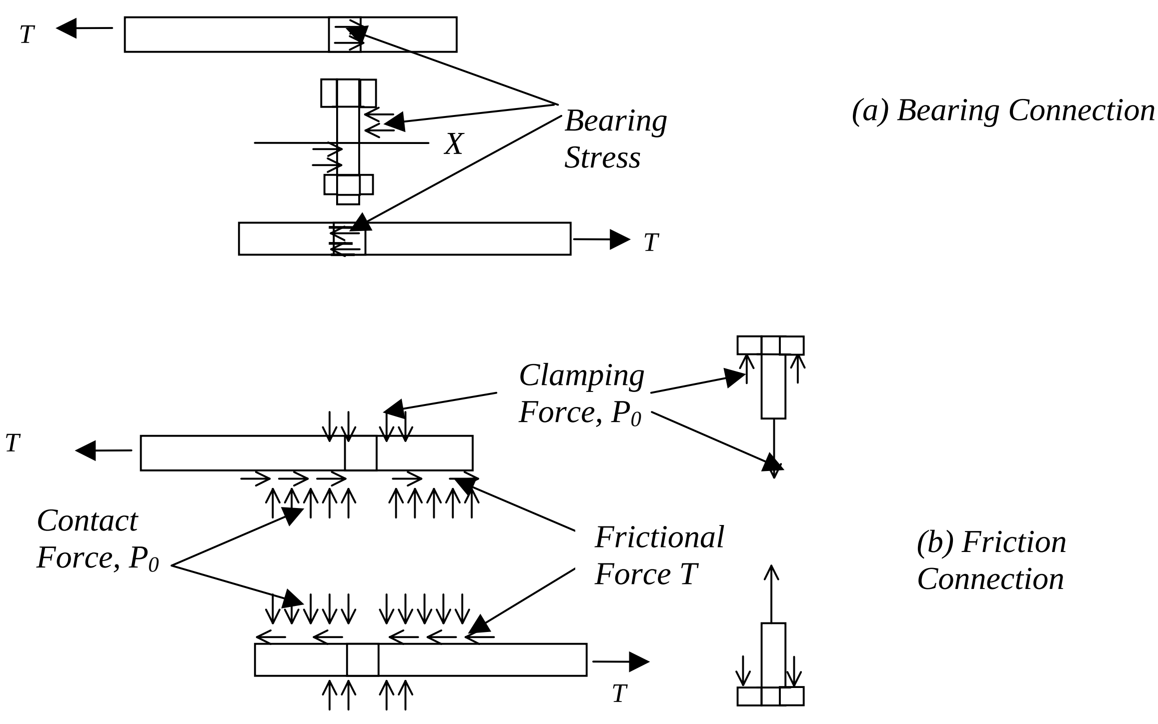


Fig. 2 Bolt Shear Transfer Mechanism

The main disadvantage of bearing type of bolted connections is that the elements undergo some slip even under a small shear, before being able to transfer force by bearing. This is due to clearance between the bolts and the holes. Such a slip causes increased flexibility in the lower ranges of load and unexpected joint behaviour in some situations. In such cases high strength friction grip (HSFG) bolts are used.

In HSFG bolted joints, high strength bolts (8G or 10K grade) are pre-tensioned against the plates to be bolted together, so that contact pressure is developed between the plates being joined [Fig. 2(*b*)]. When external shear force is applied, the frictional resistance to slip between the plates prevents their relative slip. These bolted joints achieve higher stiffness in shear because of frictional resistance between the contact surfaces. Only when the externally applied force exceeds the frictional resistance between the plates, the plates slip and the bolts bear against the bolt holes. Thus even after slip, there is a reserve strength due to bearing.

The HSFG bolts are expensive both from material and installation points of view. They require skilled labour and effective supervision. Due to their efficient force transfer mechanism they have become very popular recently. Moreover, their performance is superior under cyclic loading compared to other forms of jointing. This is discussed later.

3.2 Welded Connections

Welded connections are direct and efficient means of transferring forces from one member to the adjacent member. Welded connections are generally made by melting base metal from parts to be joined with weld metal, which upon cooling form the connection. The welded connections in a majority of the cases may be categorised as fillet weld or butt (or groove) welds as shown in Fig. 3.

Fillet welds, as shown in Fig. 3(*a*), are made against two surfaces of adjacent plates to join them together. The merits of the fillet welds are:

- no prior edge preparation is necessary,
- simple, fast and economical to make, and
- does not require very skilled labour.

The demerits of fillet welds are:

- not appropriate to transfer forces large in magnitude,
- poorer performance under fatigue loading, and
- less attractive in appearance.

Butt welds, as shown in Fig. 3(*b*), are made by butting plate surfaces against one another and filling the gap between contact surfaces with weld metal, in the process fusing the base metal also together. In order to ensure full penetration of the weld metal, normally the contact surfaces are cambered to obtain gap for the weld metal to flow easily.

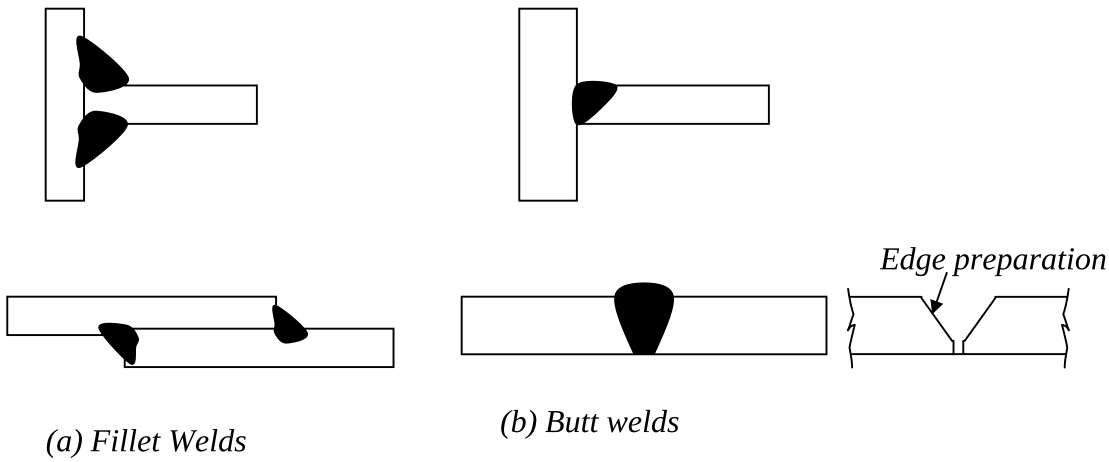


Fig. 3 Typical welded Connections

The merits of butt welds are:

- easily designed and fabricated to be as strong as the member,
- better fatigue characteristics, compared to fillet welds,
- better appearance, compared to fillet welds, and
- easy to detail and the length of the connection is considerably reduced.

The demerits of the butt welds are:

- more expensive than fillet welds because of the edge preparation required, and
- require more skilled manpower, than that required for filled welds.

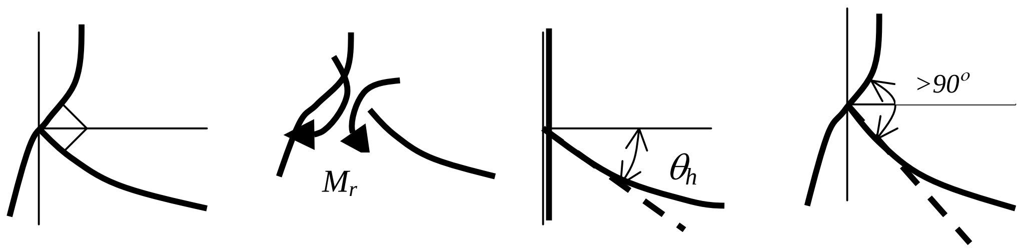
3.3 Riveted Joints

Riveted joints are very rare in modern steel construction practice. The behaviour and design of riveted connections are very similar to bearing type of bolted constructions. Since structural rivets are driven hot, the rivet shank expands to fill the hole while being driven. Hence, while calculating rivet strength, the hole diameter and not the nominal rivet diameter is used. Due to this, the slip in riveted joints is less than in bearing type of bolted joint. Further, in the process of cooling, the rivet shank length reduces, thereby causing some clamping force, as in HSFG.

Riveting has been traditionally limited to railway bridges in India. However, with the introduction of HSFG bolts, which are better suited under cyclic loading than rivets, their use is discontinued even in railway bridges in most countries.

3.4 Moment Resisting Connections

Moment resisting connections between beams and columns in multistoried buildings are very common. These connections may be made using bolting or welding. Depending upon the type of joining method and elements used to make the joint, the flexibility of the joint may vary from *hinged* to *rigid joint* condition. The moment at the joint, M, may vary between rigid joint moment, M_r [Fig. 4(a)], and zero value [Fig. 4(b)] and the relative rotation between members at the joint, θ, may vary between zero [Fig. 4(a)] and hinged joint rotation, θ_h [Fig. 4(b)].



(a) Rigid Joint *(b) Hinged Joint* *(c) Semi-rigid Joint*

Fig. 4 Types of Beam to Column Joints

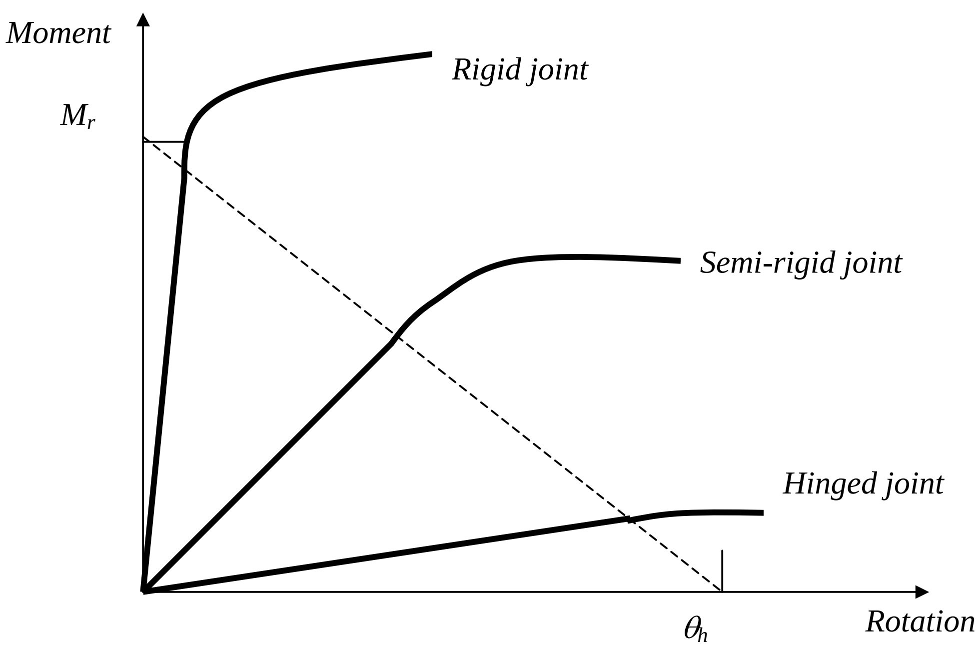


Fig. 5 Moment Versus Joint Rotation

In practice the joints are neither ideally hinged nor ideally rigid. In fact all the joints exhibit some relative rotation between members being joined [Fig. 4(*c*)]. This is due to the deformation of elements in the joint. The moment versus relative joint rotation of different types of connections is shown in Fig. 5. Any joint developing more than 90 % of the ideal rigid joint moment is classified as rigid and similarly any joint exhibiting less than 10 % of the ideal rigid joint moment is classified as hinged joint; and the joint developing moments and rotations in between are referred as semi-rigid. Based on test results and theoretical studies, moment rotation relationship for different standard connections exhibiting semi-rigid behaviour has been presented in literature.

4.0 CONNECTION DESIGN PHILOSOPHIES

Traditional methods of analysis of connection stresses were based on the following assumptions:

- Connected parts are rigid compared to connectors themselves and hence their deformations may be ignored
- Connectors behave in a linear-elastic manner until failure.
- Connectors have unlimited ductility.

However, in reality, connected parts such as end plates, angles etc. are flexible and deform even at low load levels. Further, their behaviour is highly non-linear due to slip, lack of fit, material non-linearity and residual stresses. Ductility of welds in some orientation with respect to direction of loads may be very limited, (eg. Transverse fillet welds)

Eventhough truss joints are assumed to be hinged the detailing using gusset plates and multiple fastener and welding does not represent hinged condition. However, in practice the secondary moment associated with such a rigid joint is disregarded unless the loading is cyclic.

The complexity and variability in strength of connections require a rational design philosophy to account for their behaviour. Keeping in view the large number of joints to be normally designed in a structure and the considerable variability in the design strength, any sophisticated analysis is neither desirable nor warranted. The design should ensure that equilibrium is satisfied, slenderness of the elements is consistent with the ductility demand and the deleterious effects of stress concentration on fatigue strength is considered in cyclically loaded structures. The following approach is consistent with connection design requirements in most general cases encountered in practice in statically loaded systems.

The steps to be followed in the proposed rational design approach are enumerated initially. These are illustrated using a simple framing angle connection between a beam and a column of a framed building designed to transfer a shear force of V, as shown in Fig. 6.

4.1 Steps in Transfer of Member Forces to Joints

Overall connection behaviour should be clearly understood in order to effectively and efficiently design connections following simple procedure, such as the one discussed below. To start with, the stress resultants (moment, shear, torsion, axial force etc.) transmitted by the members to be joined are to be determined. Normally analysis for forces is carried out using a model wherein members are represented by their centroidal line. Thus the calculated forces in the joints are at the intersection of centroidal line of members meeting at the joint. Therefore the effect of the size of the joint in reducing the design forces to correspond to that at the face of the joints, if substantial, has to be considered. The force resultants thus obtained should be replaced by an equivalent system of forces on the elements of the joint. In carrying out this replacement by an equivalent system of forces in the joint elements, the following are to be considered.

- The distribution of forces in the elements being connected is considered first. (For example, in the case of a beam, major proportion of the bending moment is carried by the flanges and the major proportion of shear force is carried by the web. Hence, the equivalent forces may be assumed to act on the corresponding elements at the interface).

- The equivalent system of forces should be consistent with the flexibility of the joint. For example plate elements are stiffer in resisting forces acting in their plane than in resisting forces normal to the plane. Hence most of the forces acting at a junction would be transferred to the plate in the plane of the force and little is transferred to a plate perpendicular to the force.

- Equivalent system of forces should be in equilibrium with the external force resultants and also in equilibrium with the joint as a whole.

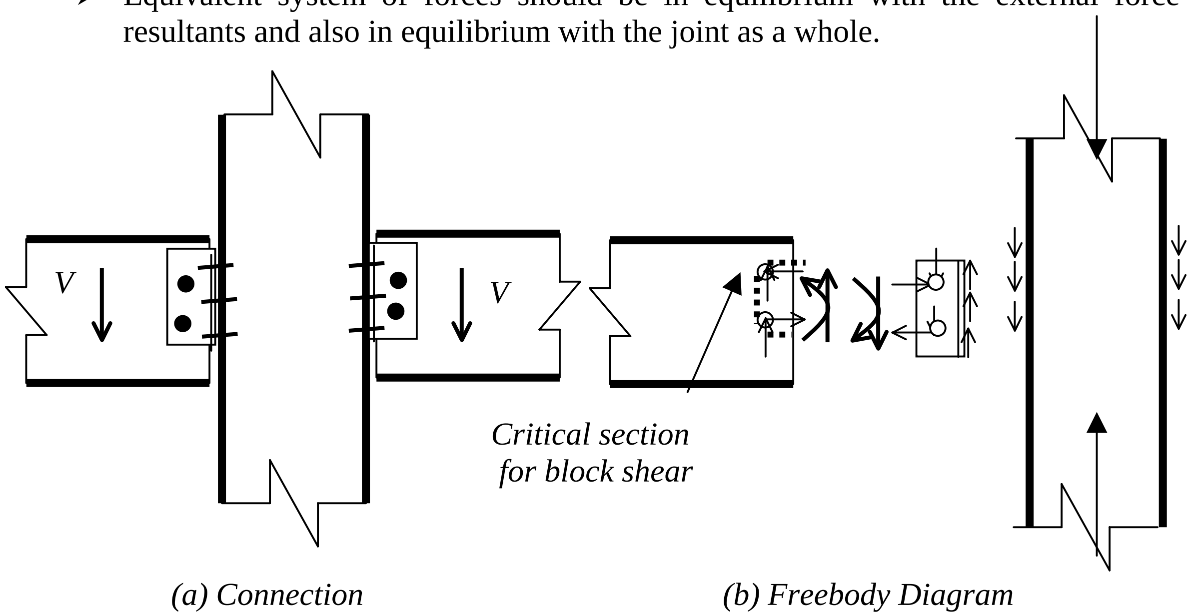


Fig. 6 Simple Framed Angle Shear Connection

In the framing angle joint shown in Fig. 6, the shear from the beam web acts eccentric with respect to reaction from the column flange causes couple. The framing angle leg connected to the column is weak in resisting any moment normal to the plane of the leg. Hence the moment at this flange connection may be assumed be negligible and only the shear force, *V*, may be assumed to be acting on the leg connected to the column flange. In the framing angle connection with the web of the beam, the forces act in the plane of the framing angle and in the plane of web of the beam. Hence both shear and the moment to equilibrate the couple due to eccentricity of shear in the framing angle can be resisted by this connection.

4.2 Determination of Force flow in the joint

Once the equivalent forces in the interface elements are obtained, the flow path of the forces through the elements in the joint is to be established by using equilibrium and simplifying assumptions regarding the force sharing, based on their relative stiffness as discussed earlier. At each stage, each element in the force flow path should be checked to ensure that they have

(a) adequate strength to withstand the force and
(b) adequate ductility to redistribute the forces to parallel elements in case of overload.

The strength and ductility evaluation is to be done for all component plates in the force path as well as all the joining elements such as bolts and welds.

As mentioned earlier the distribution of forces to different elements in the joints is complex due to highly indeterminate interaction of different element. Hence in practical joint design, the force flow analysis is based on simplifying assumptions with regard to sharing of forces. These assumptions may be at variance with the actual stresses in the elastic range. Hence it is important that adequate ductility is exhibited by all elements to redistribute the forces among alternate elements in case of over-load. This step in the framing angle joint example in Fig. 6(*a*) is illustrated in Fig. 6(*b*), in the form of free body diagram of all the elements and the force flow in the elements, while satisfying equilibrium.

Using these free body diagrams, the stresses/forces in the elements in the joint can be evaluated and compared with their respective strength, as given below:

- The bolts are assumed to share the shear force equally. Due to misfit and clearance between the bolts and the holes, in the elastic range, this need not be true. However, as long as the bolts behave in a ductile fashion, the assumption of equal sharing of shear by bolts is valid, before failure, due to plastification.

- The framing angle experiences shear and bending due to the eccentricity of the shear load. The section with holes corresponding to the bolts connecting framing angle to the beam web is the critical section, since this section experiences shear and moment. The "Strength of Materials" approach to calculate shear and bending stresses is not strictly appropriate here due to the deep beam nature of the bending behaviour of angle leg. Nevertheless, usually stresses in the framing angle are calculated based on "Strength of the Materials" concepts, due to very small value of these stresses. These stresses are usually very nominal and hence frequently need not be checked.

Bolts connecting framing angle with the beam web are subjected to the same shear force and moment in the angle legs. This is an eccentric bolted connection. The vertical shear and horizontal shear in the bolts due to the shear force and moment, respectively, are calculated and the resultant shear in the bolt is evaluated. This again is based on the rigid angle and flexible bolt assumption and the method of superposition. The maximum resultant shear force in the bolt has to be checked against the shear strength of the bolt.

The stresses in beam web and column flange can be checked at the location of bolt force transfer, by following block shear method at critical sections as shown in Fig. 6(*b*). Usually these stresses would be very nominal.

5.0 BEHAVIOUR OF ELEMENTS IN CONNECTIONS

Many local elements such as end plates, framing angles, stiffeners are used in a connection design. These elements on the load path have to perform the function of transferring forces imposed upon them. Frequently forces are distributed somewhat arbitrarily between parallel elements in the load path. In order to redistribute the loads as assumed and in order to avoid sudden failure, these elements have to behave in a ductile fashion in case of overloading.

5.1 Distribution of Forces in Elements

The joints are locally complex and theoretically exact calculation of element force/stress is a highly indeterminate analysis problem, making exact analysis of a joint impractical in day-to-day design. Theoretically exact analysis methods and experimental studies are used for research to develop a better understanding of the force flow and simplified connection design procedures. One often makes simplifying assumptions consistent with the internal behaviour of the elements and relies heavily on ductility to redistribute over-load on any element. This process requires a good understanding of the following:

- Free body diagram and equilibrium analysis of elements in the load transfer path,
- Relative stiffness of elements in the load transfer path, and
- Ductility demand on the elements and the consequent slenderness limitation.

The simplified analysis steps are illustrated through a few examples. Let us consider an interior beam to column moment resisting connection of a frame, as shown in Fig. 7. It is seen that shear and bending moment should be transferred from the beams to the column as shown. We know that a major portion of the bending moment in a beam is transferred through bending stresses in flanges and a major portion of the shear force in the beam is transferred through shear stress in the web, as shown. Equal and opposite forces act on the column flanges, as shown in Fig. 7.

The concentrated beam flange forces (*C* and *T*) have to be transferred as shear to the column web, since the column web plate is the stiff element in that plane in the load path. The transfer to the column web is through column flanges, which may cause excessive bending of column flanges and excessive bearing in the column web flange junction. In order to overcome this, we often use stiffener plates, S_1 and S_2 as shown.

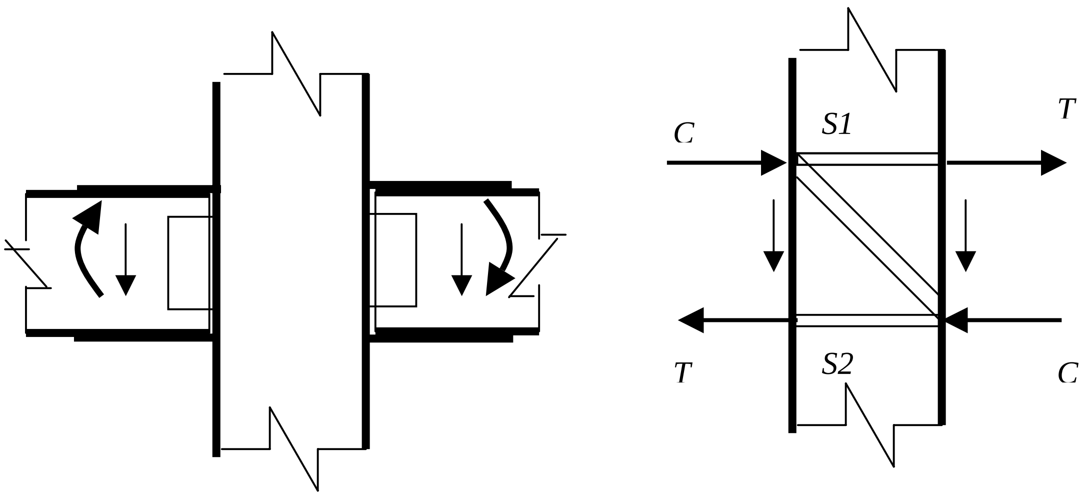


Fig. 7 Elements in Connections

The forces *T* and *C* may be either assumed to be fully transferred by the stiffeners provided or the balance force in excess of the bearing capacity of the web and bending capacity of the flange may be assumed as the design force in the stiffeners. The assumption made dictates the ductility requirement of the stiffener. If the entire force is assumed to be transferred by the stiffener, the actual force in the stiffener in the elastic range will be less than this and hence only semi-compact design requirement with regard to the b/t ratio has to be satisfied by the stiffener (see chapter on plate buckling), since it needs only to carry the load without local buckling.

If, however, the stiffener is designed for forces in excess of the capacity of the flange and web of the column, the design force on the stiffener is usually an underestimation of the actual force experienced by it in the elastic range. This is due to the higher rigidity of the stiffener compared to the column flanges. Consequently, the stiffener should deform plastically on over-loading so that the load on stiffener, in excess of what it has been designed for, can be redistributed. Hence stiffeners should not only sustain the force but also plastically deform (adequate ductility is needed) in order to redistribute the force and hence the slenderness of the stiffener should meet the compact plate element requirement (see the chapter on plate buckling).

The unbalanced beam moment transferred by the beam to the column at the junction causes shear (V = C+T in Fig. 7), locally at the joint in the column web. This may be in excess of shear capacity of the column web. Hence the column web may have to be locally thickened or provided with a diagonal stiffener, as shown in Fig. 7. Further, the welds between the stiffeners and the column flange should be sufficiently large so that they remain elastic during the plastic deformation of the stiffener, discussed earlier.

The shear from the beam is directly transferred to column B through column flanges, as an additional axial compression. Thus, all the elements in the force transfer path across the joint should be ensured to have adequate strength, stiffness and ductility, to perform the function based on rational simplifying assumptions.

6.0 COST OF CONNECTIONS

Usually cost of fabrication and erection constitute as high as 50% of the total cost of steel structures, per tonne of material used. Hence, designers of connections have a great responsibility in reducing the overall cost of steel structures.

Factors affecting design cost:

Important factors affecting connection design costs are discussed below:

- Connection design takes up a significant part of the overall design cost of steel structures and decisions made at this stage considerably influence the fabrication and erection costs.
- The connection designs should be done using simple and standard cases, so that using design tables, connections can be designed and detailed rapidly. Such tables considerably reduce repetitive calculations, improve accuracy and speedup fabrication.

Factors affecting fabrication/erection costs:

Important factors in improving productivity, decreasing cost of fabrication and erection of connection work are discussed below:

- Repetitive use of standard detail

The repetitive use of standard details spread the cost of learning, cost of setup, cost of templates etc. over a large number of products/components to be fabricated, thus reducing the cost and time required for fabrication. Special, complicated and precise fitting details should be avoided or minimized.

- Ease of joining

The detail should provide easy access to welding and bolting. The positioning of members should be simplified with temporary supports to facilitate quick release of the handling equipment, ease of adjustment and alignment and quick joining.

- Appropriate mix of automatic and manual fabrication

The productivity of numerically controlled automatic machineries (NC machines), and continuous submerged arc welding is very high compared to manual methods. The quality is usually superior. However, their setup costs are high. Hence, automatic fabrication methods are appropriate in large volume jobs. For example, a large number of framing angles can be cut and drilled to the same part detail using NC machines and long continuous fillet weld between plate girder web and flange can be done using an automatic submerged welding machine, economically. In the Indian market such machines are not widely available. Most fabrication shops still work with outdated equipment and require capital equipment infusement to bring about efficiency and economy in shop fabrication and erection.

Manual methods take less setup time and unit time costs are low, but productivity and quality are also low. Hence the manual methods are appropriate in fabricating a smaller number of elements or in shorter welds, such as web stiffener welding.

- Choice of connection method

Generally welded connections are more direct and more efficient, but require more elaborate preparation and machinery compared to bolted connection. This has generally led to the use of welding in shop and on ground field connections and the use of bolting at the erection connections.

There are exceptions to this general tendency. For example, if only a few angle trusses are to be fabricated, then pre-drilling of the members in shop, based on theoretical calculations of geometry of members and connection sizes and site assembly subsequently by bolting would be economical compared to laying the truss out and aligning the members appropriately and welding them together on ground. On the other hand, welded fabrication may be economical in the case of a large number of trusses fabricated to the same detail, wherein the higher cost incurred for templates, layout and welding are spread over the larger number of units to be fabricated.

- Choice of shop versus site fabrication

Shop fabrication is faster, cheaper, has better quality and higher productivity. In India, the cost advantage of shop fabrication is partly off set by differential excise duty rates between the shop and site fabricated components, as well as low productivity equipment and process used in shop practices.

Transportation cost also dictates the economy of shop fabrication. The transportation cost is governed by distance to be transported, weight and volume of component to be transported. Instead of transporting a very long girder from a shop, it can be shop fabricated in shorter segments and joined at field using bolting or welding, to achieve greater economy. Fittings such as framing angles can be pre-attached to one of the members being joined (say the web of beam) at shop using welding and connected at field to the other member by bolting.

- Other Factors

Difficult connection details cause difficulty in understanding and execution at site. This may lead to frustration, carelessness, poor quality connections, and also mistakes leading to delay, cost of repair and failure. Prefabricated units to be connected at site should be of nearly uniform weight so that handling capacity of the cranes is fully utilized, improving the productivity of the handling equipment available.

HSFG bolted connections involve higher material cost, more skilled labour, more complex equipment, higher level of inspection, when compared to ordinary bolts. Hence its use should be restricted to special situations such as high forces and fatigue environment. Otherwise, at site, black bolts in clearance holes are preferred. Usually, the same grade of bolt and only a few standard sizes should be used at site, in order to reduce complexity of erection, maintenance of inventory of different size bolts and mistakes in connection.

7.0 SUMMARY

Sound connection design is essential for safety and economy of steel structures. Economical connection designs mostly take into account practicalities of fabrication and erection. True behaviour of connections is complex, variable and very difficult to analyse exactly. However, the connection design should be simple and straightforward, based on a clear understanding of the load transfer path, the effect of stiffness of elements in the path on the force distributed to the elements in the connection and the effect of ductility on the connection behaviour. The detailing of connection should be simple and be based on repetitive use of standard practices to facilitate ease of fabrication and erection, thus accure speed and economy to the project.

8.0 REFERENCES

1. Dowling, P.J., Knowles, P.R., Graham, W. O. (1988), “Structural Steel Design”, Steel Construction Institute, Butterworths, London.
2. Fisher, J.W., and Struik, J.H.A., (1974), “guide to Design Criteria for Bolted and Riveted Joints”, Wiley, Chichester.
3. Owens, G.W., and Cheal, B.D., “Structural Steel work Connections”, Butterworths, 1989.

30 WELDS – STATIC AND FATIGUE STRENGTH – I

1.0 INTRODUCTION

The need for connections and the importance of connection designs in steel structures have been covered in an earlier chapter. It has been pointed out that steel sections are linear elements produced in certain convenient lengths due to constraints on manufacturing and transportation. Therefore connections are necessary to provide continuity, where required, as well as to create three-dimensional steel structures. One of the most efficient and possibly direct ways of providing connections in steel structures is by way of *welding*.

Welding is the process of joining two pieces of metal by creating a strong metallurgical bond between them by heating or pressure or both. It is distinguished from other forms of mechanical connections, such as riveting or bolting, which are formed by friction or mechanical interlocking. It is one of the oldest and reliable methods of jointing.

Welding was quite an art prevalent in ancient Greece to make bracelets. It was probably a forging process, where metals were heated and hammered together. Modern welding has been in existence since World War I. It was mainly used for repairing damaged ships. After 1919, the use of welding as a construction and fabrication method began to develop. Since then many improvements and developments have taken place. Today there are over 50 different welding processes, which can be used to join various metals and their alloys.

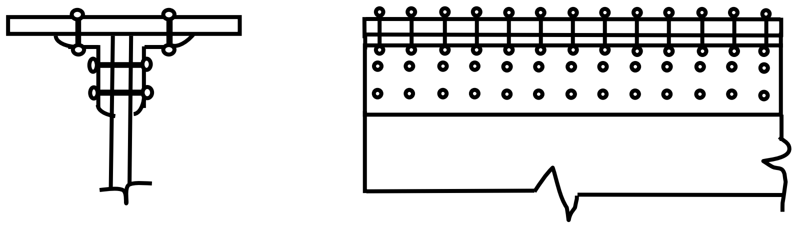
2.0 ADVANTAGES OF WELDING

Welding offers many advantages over bolting and riveting. Some of the advantages are listed in the following.

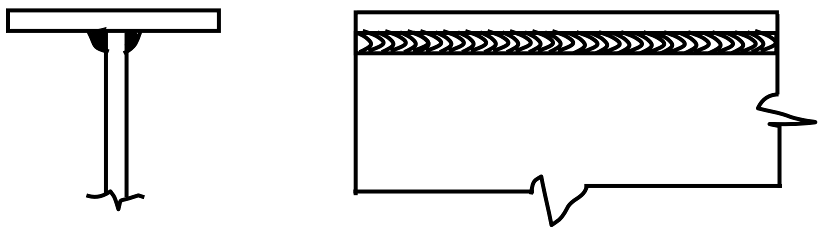
- Welding enables direct transfer of stress between members. Hence, the weight of the joint is minimum. Besides efficiency, design details are very simple. Less fabrication cost compared to other methods due to handling of fewer parts and elimination of operations like drilling, punching etc. The most striking advantage of welded structures is in the area of economy. Welded structures allow the elimination of a large percentage of the gusset and splice plates necessary for riveted or bolted structures. Time is saved in detailing, fabrication and field erection. In some bridge trusses it may be possible to save up to 15% of the steel weight by resorting to welding. Welding also requires considerably less labour for executing the work.

- Welding offers air tight and water tight joining of plates and hence ideal for oil storage tanks, ships etc.

- Welded structures usually have a neat appearance as against the cluttered surface of bolted or riveted connections. Fig. 1 shows a comparison of riveted plate girder and a welded plate girder. Further, welded connections offer the designer more freedom for innovation in his design concept. It enables him to use any cross section and the best configuration to transmit forces from one member to another.



(a) Bolted girder section



(a) Welded girder section

Fig.1 Comparison of appearance of riveted and welded plate girders

- The range of application of welding is very wide. For example, connection of a steel pipe column to other members can be made very easily by welding whereas it is virtually impossible by bolting or riveting. Welding is practicable even for complicated shapes of joints.

- There is no need for holes in members connected by welding except possibly for erection purposes. This has direct influence in the case of tension members as the problem of determining the minimum net section is eliminated. This also results in a member with a smaller cross section.

- Welded structures are more rigid compared to structures with riveted and bolted connections. The rigidity of welded structures is due to the direct connection of members by welding. In bolted or riveted structures, the connection is established through angles or plates, which deflect under loads, making the structure flexible.

- It is easier to make design changes and to correct mistakes during erection, if welding is used. It is also a simple procedure to strengthen the existing structures with welding.

- A truly continuous structure is formed by the process of fusing the members together. This gives the appearance of a one-piece construction. Generally welded joints are as strong or stronger than the base metal, thereby placing no restriction on the joints.

Due to this continuity advantage, a very large number of steel frames have been constructed all over the world.

- Stress concentration effect is considerably less in a welded connection. Some of the lesser important advantages of the welding processes are: relative silence of the process of welding and fewer safety precautions.

Some of the disadvantages of welding are:

- Welding process requires highly skilled manpower
- Experienced manpower is needed for inspection of welded connections. Also, non-destructive evaluation may have to be carried out to detect defects in welds
- Welded joints are highly prone to cracking under fatigue loading
- Costly equipment is essential to make welded connections
- Proper welding can not be done in the field environment
- Large residual stresses and distortion are developed in welded connections

In the earlier days, combination of bolting, riveting and welding was not practiced. Structures were completely welded, bolted or riveted. Presently both are used in a structure except that both connection techniques are not used in one and the same joint. The present trend is to use welding for workshop connections or splices, and high strength bolts for field joints.

3.0 FUNDAMENTALS OF WELDING

A welded joint is obtained when two clean surfaces are brought into contact with each other and either pressure or heat, or both are applied to obtain a bond. The tendency of atoms to bond is the fundamental basis of welding. The inter-diffusion between the materials that are joined is the underlying principle in all welding processes. The diffusion may take place in the liquid, solid or mixed state. In welding the metallic materials are joined by the formation of metallic bonds.

When two clean and flat surfaces are joined, the bonding takes place between surface atoms and a perfect connection is formed (Fig.2).

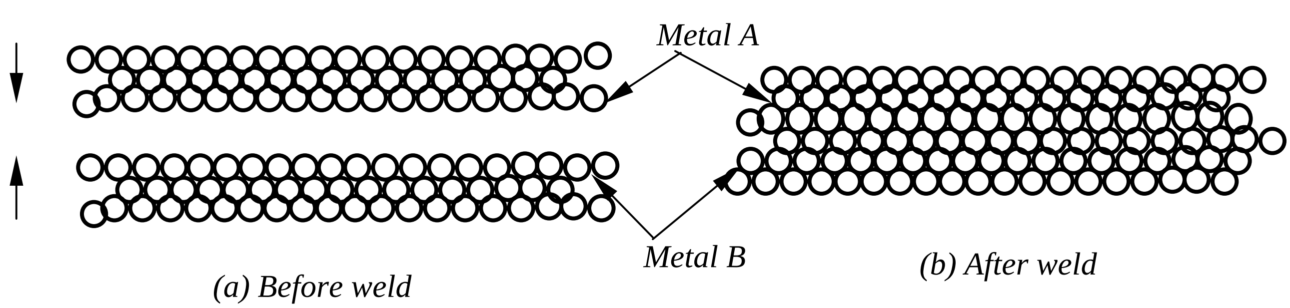


Fig.2 Two ideal surfaces brought together to form a weld

The efficiency obtained for the joint is 100% and its strength would be as much as that of the base metals. In practice however, it is very difficult to achieve a perfect joint; for, real surfaces are never smooth. When irregular surfaces are joined by welding, contact is established only at a few points in the surface, where atomic bonding occurs. Therefore the strength attained will be only a fraction of the full strength. Also, the irregular surface may not be very clean, being contaminated with adsorbed moisture, oxide film, grease layer etc. as shown in Fig 3(*a*).

In the welding of such surfaces, the contaminants have to be removed for the bonding of the surface atoms to take place. This can be accomplished by applying either heat or pressure. In practical welding, both heat and pressure are applied to get a good joint.

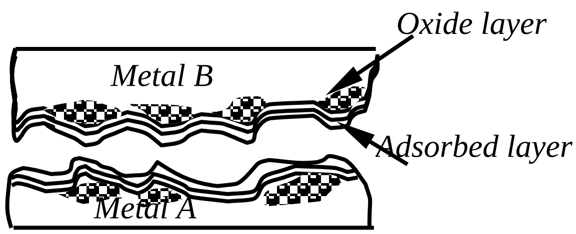


Fig.3 (a) Surface contaminants

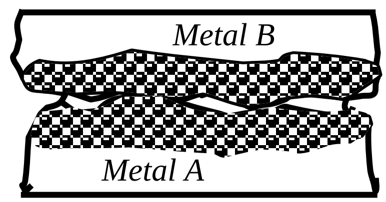


Fig. 3(b) Addition of filler material

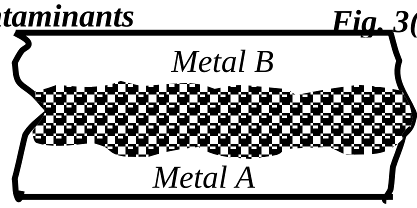


Fig.3(c) Near perfect weld

When heat is applied, the adsorbed layers are driven off, oxide films are broken and the yield strength of the base metals are lowered; with the application of pressure, plastic deformation takes place and brings into contact more atoms for welding. Heat application results in the melting of the metallic members, enabling atoms to come into contact by fluid flow and form bonds. Sometimes, a filler material, of the same type as the base material or compatible with it, is added to achieve the bond [Fig 3(*b*))]. When pressure is applied to form a joint it breaks the obstructing layers and sharp edges to bring the joining surfaces together for making the bond. Fig. 3(*c*) shows a weld formed by the application of heat and pressure.

As pointed out earlier, any welding process needs some form of energy, often heat, to connect the two materials. The relative amount of heat and pressure required to join two materials may vary considerably between two extreme cases in which either heat or pressure alone is applied. When heat alone is applied to make the joint, pressure is used merely to keep the joining members together. Examples of such a process are Gas Tungsten Arc Welding (GTAW), Shielded Metal Arc Welding (SMAW), Submerged Arc Welding (SAW) etc., which are explained later. On the other hand pressure alone is used to make the bonding by plastic deformation, examples being cold welding, roll welding, ultrasonic welding etc. There are other welding methods where both pressure and heat are employed, such as resistance welding, friction welding etc. The required heat is produced by a flame, an arc or resistance to an electric current. Electric arc is by far the most popular source of heat used in commercial welding practices.

4.0 BASIC WELDING PROCESSES

In general, gas and arc welding are employed; but, almost all structural welding is arc welding.

In gas welding a mixture of oxygen and some suitable gas is burned at the tip of a torch held in the welder's hand or by an automatic machine. Acetylene is the gas used in structural welding and the process is called oxyacetylene welding. The flame produced can be used both for cutting and welding of metals. Gas welding is a simple and inexpensive process. But, the process is slow compared to other means of welding. It is generally used for repair and maintenance work.

The most common welding processes, especially for structural steel, use electric energy as the heat source produced by the electric arc. In this process, the base metal and the welding rod are heated to the fusion temperature by an electric arc. The arc is a continuous spark formed when a large current at a low voltage is discharged between the electrode and the base metal through a thermally ionised gaseous column, called plasma. The resistance of the air or gas between the electrode and the objects being welded changes the electric energy into heat. A temperature of 3300^0 C to 5500^0 C is produced in the arc.

The welding rod is connected to one terminal of the current source and the object to be welded to the other. In arc welding, fusion takes place by the flow of material from the welding rod across the arc without pressure being applied.

4.1 Arc Welding Processes

Different processes of arc welding are explained in the following sections:

4.1.1 Shielded Metal Arc Welding (SMAW)

In Shielded Metal Arc Welding or SMAW (Fig. 4), heating is done by means of electric arc between a coated electrode and the material being joined. In case bare wire electrode (without coating) is employed, the molten metal gets exposed to atmosphere and combines chemically with oxygen and nitrogen forming defective welds. The electrode coating on the welding rod forms a gaseous shield that helps to exclude oxygen and stabilise the arc.

The coated electrode also deposits a slag in the molten metal, which because of its lesser density compared to the base metal, floats on the surface of the molten metal pool, shields it from atmosphere, and slows cooling. After cooling, the slag can be easily removed by hammering and wire brushing.

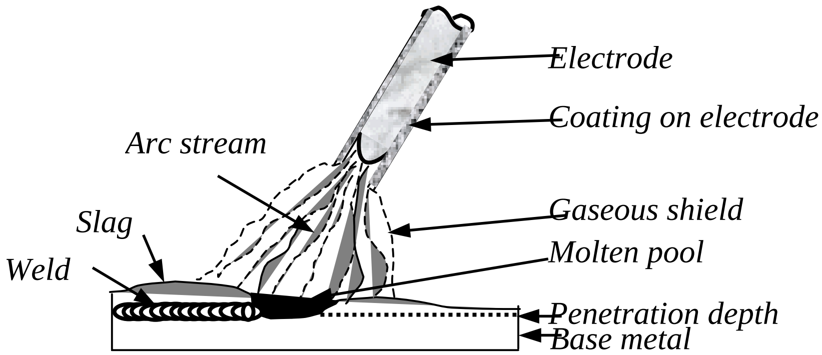


Fig.4. Shielded Metal Arc Welding (SMAW) process

The coating on the electrode thus

- Shields the arc from atmosphere
- Coats the molten metal pool against oxidation
- Stabilises the arc
- Shapes the molten metal by surface tension
- Provides alloying element to weld metal

The type of welding electrode used would decide the weld properties such as strength, ductility and corrosion resistance. The type to be used for a particular job depends upon the type of metal being welded, the amount of material to be added and the position of the work. The two general classes of electrodes are lightly coated and heavily coated. The heavily coated electrodes are normally used in structural welding. The resulting welds are stronger, more corrosion resistant and more ductile compared to welds produced by lightly coated electrodes. Usually the SMAW process is either automatic or semi-automatic.

4.1.2 Submerged Arc Welding (SAW)

In this arc welding process, the arc is not visible because it is covered by a blanket of fusible powdered flux. The bare metal electrode is deposited as a joining material. The flux, which is a special feature of the method, protects the weld pool against the atmosphere. The arc once started is at all times covered by the flux as shown in Fig. 5.

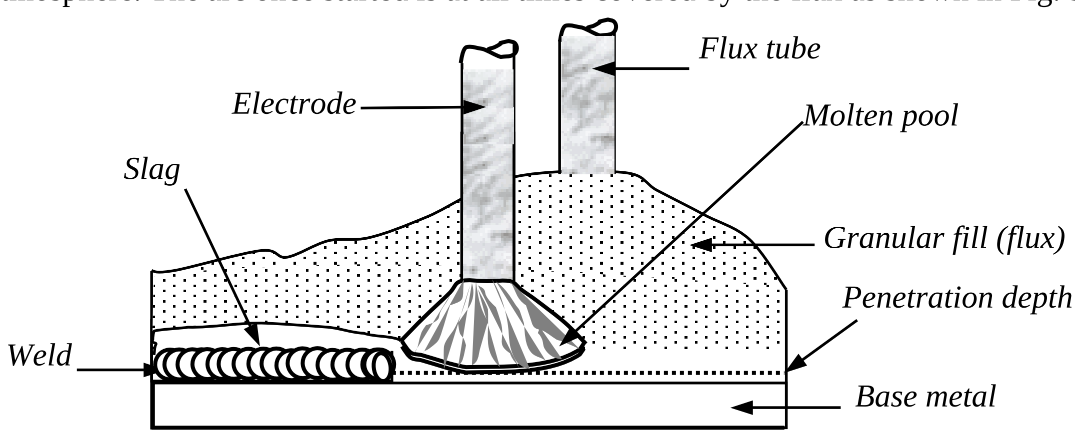


Fig.5. Submerged arc welding (SAW) process

The heat of the arc melts the electrode, the object to be welded, and part of the flux. The slag formed by the flux, which forms a coat over the solidified weld beam, may be removed by brushing. Welds made by submerged arc welding process have high quality, good ductility, high impact strength, high density and good corrosion - resistance. Their mechanical properties are as good as the base metal. Since more heat is input in this process, the penetration is deeper than the SMAW process. This is normally taken into account in the design.

4.1.3 Manual Metal Arc (MMA) Welding

This is a manually operated welding process and hence requires skill to produce good quality welds. The electrode is made up of a steel core wire (3.2 – 6.0 mm diameter) and the flux contains manganese and silicon as alloying elements. The electric arc melts the metallic object to be welded and the electrode. As the core wire metal melts and joins the weld pool, the electrode is moved to maintain the arc length. This is important as the arc length controls the width of the weld run. The flux also melts with steel core wire and forms the surface slag, which is removed after solidification.

Low capital cost and freedom of movement (up to 20 m from power supply) are the main advantages of MMA welding. It is well suited to structural and stainless steels. Its main disadvantage is that only a small volume of metal is deposited per electrode. This is not a problem for short welds, but for long welds this becomes a serious consideration.

4.1.4 Metal – Active Gas (MAG) welding

This is sometimes also referred to as Metal Inert Gas (MIG) welding. The arc and the weld pool are protected by an inert gas; the shielding gas often used is carbon dioxide or a mixture of oxygen and carbon dioxide. Flux is not necessary to shield the pool; however, occasionally a flux - cored electrode is used to produce slag. The arc length is maintained by the power supply unit. Though MAG welding is easier, more skill is required to establish the correct welding conditions. Using MAG welding, production is improved, as there is no need to deslag or change electrode. It is highly suitable for fillet - welded joints, such as beam to beam or stiffener to panel connections. Its disadvantage is due to restriction in movement due to equipment. This can be manual, semi-automatic or automatic process

4.1.5 Electroslag Welding Process

The method of Electroslag process (Fig. 6) used for vertical automatic welding is based on the heat produced by electrical current through molten slag. The electrode is immersed in the molten slag pool between the components to be welded and the copper moulding devices. As the melt is heated to a high temperature by current passing between the electrodes and the base metal, the electrical conductivity is increased. The slag pool temperature must exceed the melting points of the base and filler metals. Then the slag melts the faces of the connecting work and the electrode is immersed in the molten slag. The weld pool that forms when the molten base and filler metal collect at the bottom of the slag pool solidifies and forms the weld joining the faces of the members.

Electroslag welding is useful for joining thick sections in a vertical position. Single - pass welds can be made in any reasonable thickness of steel. Welding usually starts at the bottom of the joint and progresses towards the top of the vertical connection.

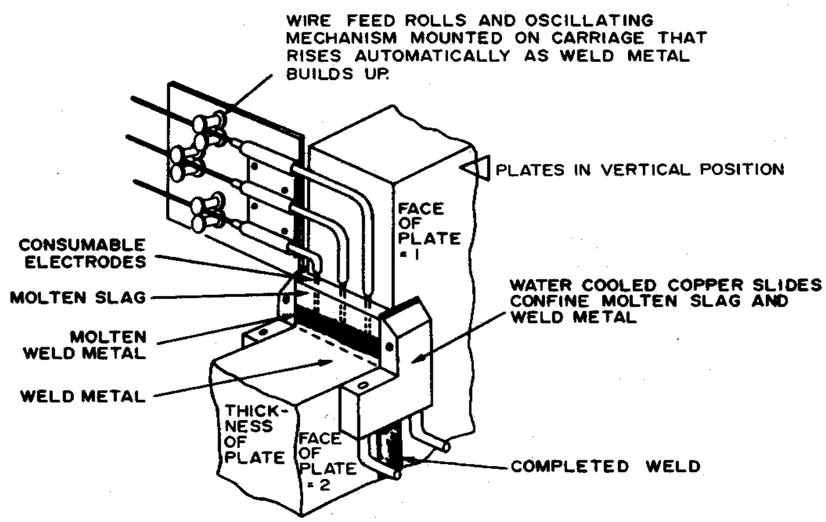


Fig. 6 Electroslag welding process

4.1.6 Stud Welding

The shear connectors that are used extensively in composite construction have to be attached to the steel beam by welding. Stud welding is an accurate and best method of attaching shear connectors with minimum distortion. The figure and details of a stud welding machine are given in Chapter 41 on Fabrication and Erection of Structural Steel Work.

Stud welding is another form of arc welding and is mechanised for the welding of studs to plane surfaces. The stud may be a plain bar with an upset head or threaded. The stud itself forms the electrode and is held in a chuck, which is connected to the power source. Initially, the stud is touched on the steel plate or section surface and the current is switched on. Soon the stud is moved away to establish the arc. A weld pool is formed and the stud end melts. Then the stud is forced into the steel plate automatically and the current supply is switched off. The molten metal is collected in an enclosure around the stud formed by a ceramic collar or ferrule. Thus the molten metal is formed into a fillet and is also protected from atmosphere by the ferrule.

4.2 Choice of Process

The choice of a particular process is made based on a number of parameters listed below:

- The location of the welding operation: In a protected place like a fabrication shop, SAW and MAG are best suited. For field conditions MMA is easier.
- Accuracy of setting up: SAW and MAG require good and accurate set-up.
- Penetration of the weld.

- Volume of weld to be deposited
- Access to joint: The welding plant and the welding torches have to be properly positioned during the welding operation. In easily accessible joints SAW or MAG is used, whereas in cramped locations MMA is preferred.
- Position of welding: SAW and MAG are not suitable for overhead positions. MMA is the best for overhead works.
- Steel composition: SAW and MAG do not generally develop HAZ (Heat Affected Zone) cracking. This offsets the disadvantage of MAG for site works.
- Comparative cost: cost of welding is calculated for unit length considering the duty cycles.

5.0 WELDING PROCEDURE

5.1 General

The term 'welding procedure' encompasses the complete operation of making a weld. Thus, it includes choice of electrode, edge preparation, preheat, welding parameters such as voltage, current, welding position, number of weld run to fill the groove and post weld treatments (e.g. grinding, heat treatment etc.). Establishment of such procedures helps to minimise the cost, achieve good impact properties, eliminates defects and controls distortion. Some of the important elements of weld procedure are elaborated below.

Environment: Weld procedure must account for actual site conditions. In cold regions, it may be necessary to heat steel up to 20%. The humid weather or condensation might help formation of porosity. Electrodes must be kept in dry condition. In moist / humid environments the electrodes may be kept in a warm container to avoid moisture entrapment in the flux coating.

Welding position: Vertical welding is slower compared to welding in the flat position. Overhead welding causes weld splutter and require special skills. It is better avoided.

Current: The current controls the heat input. A minimum current is required for fusing the plate and to keep the arc stable. Generally a high current is used to obtain quicker welding so as to reduce cost. It may not be possible to use maximum current always, a specific example being welding in the overhead position. The current limit for overhead use is 160A. Usually high current results in low impact properties. Further very high value of current may cause cracks in the Heat-Affected Zone (HAZ).

Shrinkage: While cooling after the welding operation, the hot metal in the welded region contracts causing the joint to shrink. But this contraction is prevented by the adjacent colder metal. This causes stress, sometimes even beyond yield stress, and causes plastic deformation. This also might cause distortion of the member. By following proper edge preparation and weld procedure, this can be minimised. After the plastic deformation a residual stress pattern is formed in the joint. Tensile stresses are formed in the weld metal and HAZ zones, whereas compression in the adjacent steel.

Pre heating: Hydrogen induced cracking (cold cracking, delayed cracking) is a serious problem affecting weldability. The degree of cracking occur due to the combined effects of four factors**:**

1. Brittle microstructure
2. Presence of hydrogen in weld metal
3. Tensile stresses in the weld area
4. Temperature range (-100^{o} C to 200^{o} C)

Pre heating of the weld area is the most effective and widely used method to prevent hydrogen induced cracking. Welding involves a cycle of sudden heating and cooling. By preheating the parent metal, the difference in temperature between the preheated temperature and the final temperature is reduced. This, in the cooling cycle, also helps to obtain a lower thermal gradient. As explained in the first chapter on 'Historical Development and Characters of Structural Steels', sudden cooling of steel results in a hard and less ductile material called martensite. The main function of preheating is to reduce the weld metal cooling rate so that transformation to martensite is reduced below a certain critical level. The slower cooling gives more time for hydrogen to diffuse out of the weld area and delays the development of maximum residual stresses. Gas torches, heat-treating furnaces or electric-resistance heaters are used in preheating the weld area.

5.2 Weldability of Steels

The term weldability is defined as the ability to obtain economic welds, which are good, crack - free and would meet all the requirements. Of great importance are the chemistry and the structure of the base metal and the weld metal. The effects of heating and cooling associated with fusion welding are experienced by the weld metal and the Heat Affected Zone (HAZ) of the base metal. The HAZ i.e. base metal surrounding the weld metal and the weld itself will have unduly varying hardness distribution across a weld. The hardness in steel depends upon the rate at which steel is cooled near the fusion zone; the hardness is maximum due to the higher temperature at that location. Further, these locations also have the maximum rate of cooling. Higher value of hardness leads to cracks in HAZ or in the weld. Cracks might be formed during or after the welding process.

Good design and standard welding procedure will minimise the cracking problem. Several features that affect weld cracking during the welding processes are

- Joint restraint that builds up high stress in the weld
- Bead shape (convex or concave)
- Carbon and alloy content of the base metal
- Cooling rate
- Hydrogen and nitrogen absorption

The cracks in HAZ are mainly caused by high carbon content, hydrogen enbrittlement and rate of cooling. For most steels, weld cracks become a problem as the thickness of the plates increases.

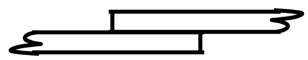
6.0 TYPES OF JOINTS AND WELDS

By means of welding, it is possible to make continuous, load bearing joints between the members of a structure. A variety of joints is used in structural steel work and they can be classified into four basic configurations as shown in Fig. 7.

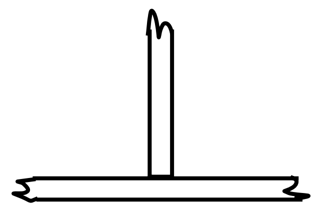
They are:

1. Lap joint
2. Tee joint
3. Butt joint, and
4. Corner joint

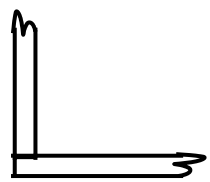
(a) Butt joint



(b) Lap joint



(c) Tee joint



(d) Corner joint

Fig. 7 Types of joints

For lap joints the ends of two members are overlapped, and for butt joints the two members are placed end to end. The T- joints form a Tee and in Corner joints, the ends are joined like the letter L. The common types of welds are shown in Fig. 8. Most common joints are made up of fillet weld and the groove weld. Plug and slot welds are not generally used in structural steel work. Fillet welds are suitable for lap joints and Tee joints and groove welds for butt and corner joints. Groove welds can be of complete penetration or incomplete penetration depending upon whether the penetration is complete through the thickness or partial. Generally a description of welded joints requires an indication of the type of both the joint and the weld.

Though fillet welds are weaker than groove welds, about 80% of the connections are made with fillet welds. The reason for the wider use of fillet welds is that in the case of fillet welds, when members are lapped over each other, large tolerances are allowed in erection. For groove welds, the members to be connected have to fit perfectly when they are lined up for welding. Further groove welding requires the shaping of the surfaces to be joined as shown in Fig. 9. To ensure full penetration and a sound weld, a backup plate is temporarily provided as shown in Fig. 9.

Welds are also classified according to their position into flat, horizontal, vertical and overhead (Fig. 10). Flat welds are the most economical to make while overhead welds are the most difficult and expensive.

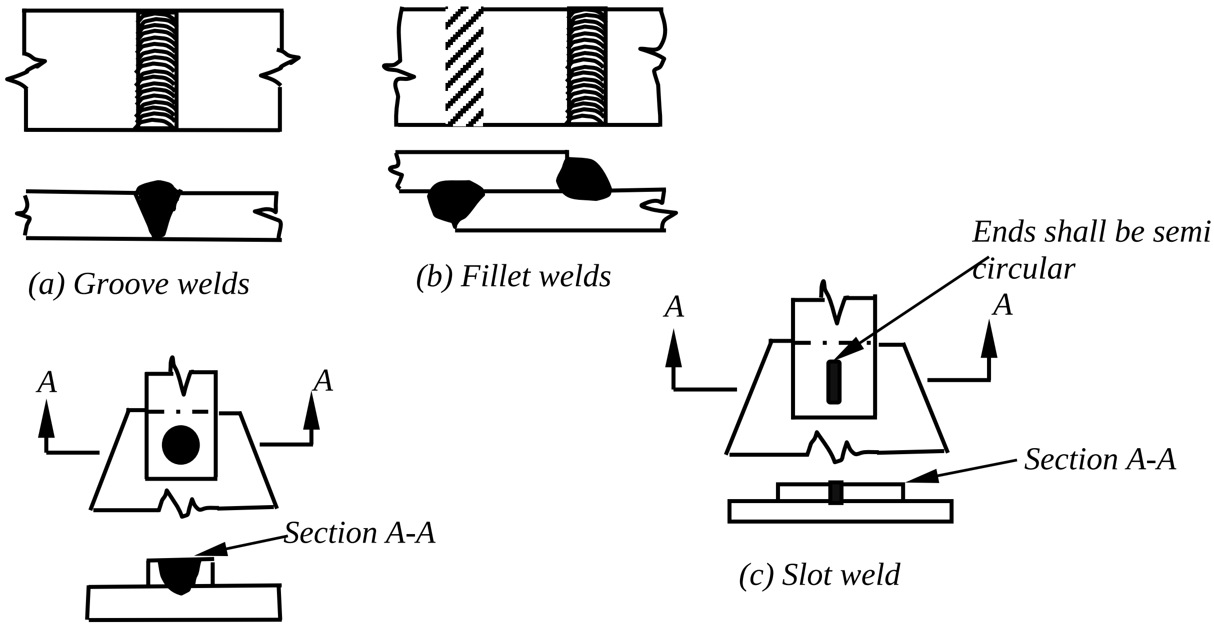


Fig.8 Common types of welds

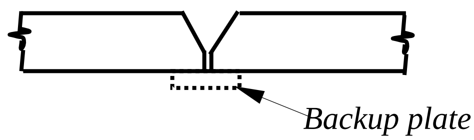


Fig. 9 Shaping of surface and backup plate

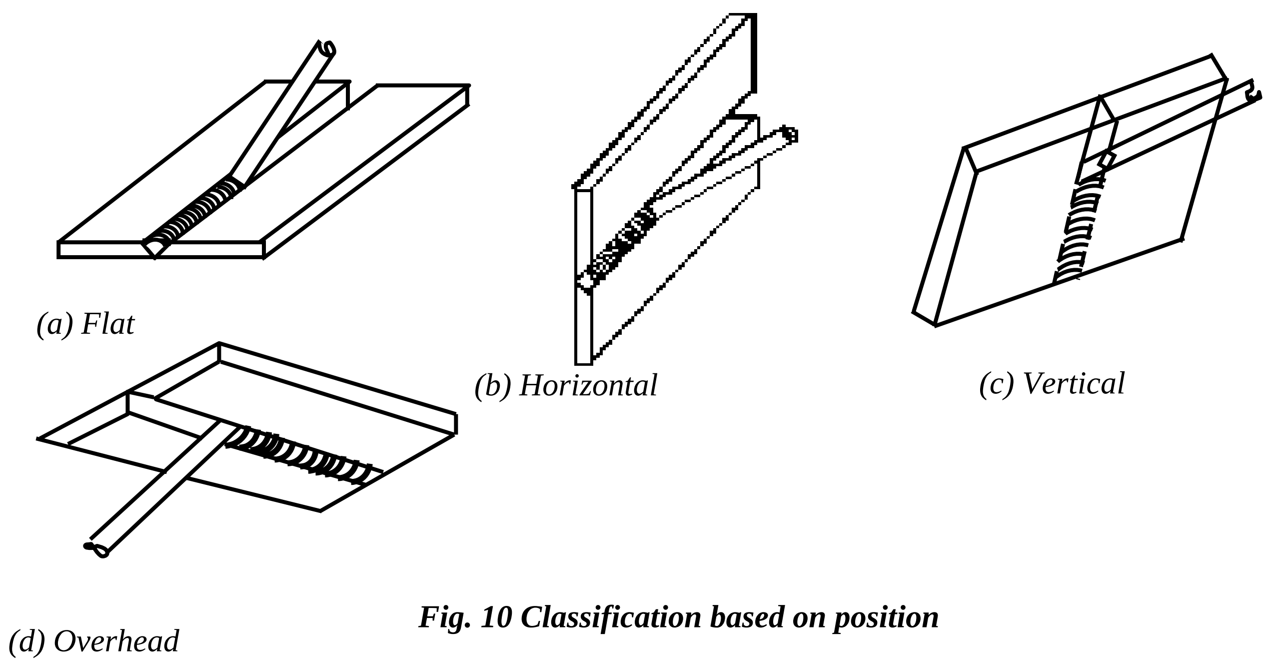


Fig. 10 Classification based on position

7.0 GROOVE WELDS

The main use of groove welds is to connect structural members, which are in the same plane. A few of the many different groove welds are shown in Fig. 11. There are many variations of groove welds and each is classified according to its particular shape. Each type of groove weld requires a specific edge preparation and is named accordingly. The proper selection of a particular type depends upon

- Size of the plate to be joined.
- Welding is by hand or automatic.
- Type of welding equipment.
- Whether both sides are accessible.
- Position of the weld.

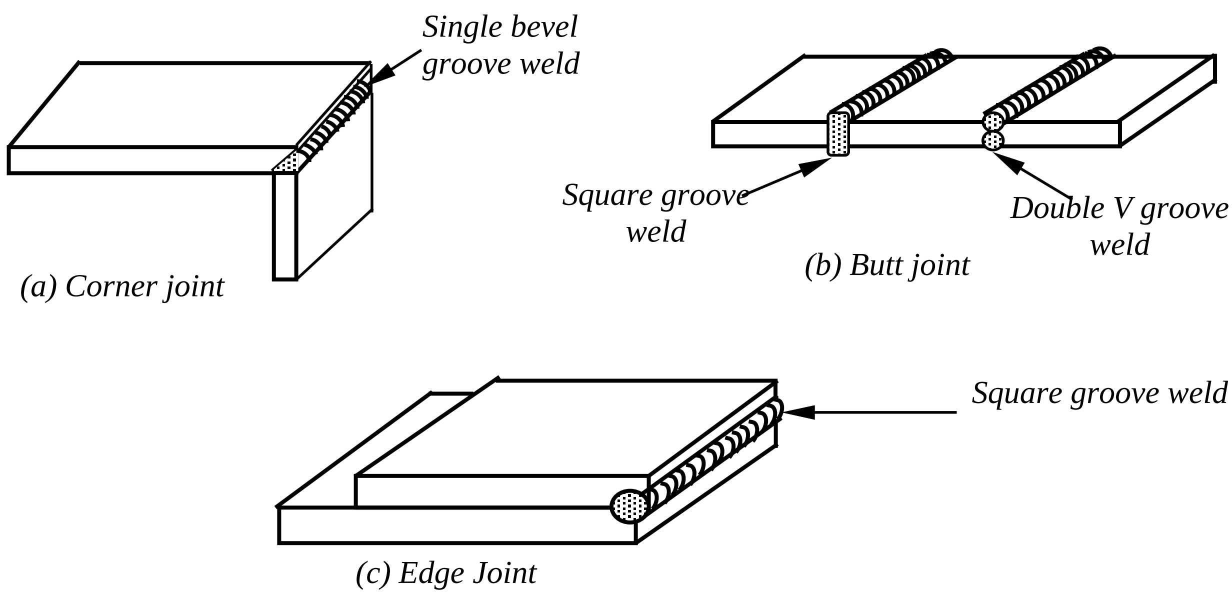


Fig.11 Typical connections with groove weld

The aim is to achieve the most economical weld of the requisite efficiency and strength. The butt weld whether of full penetration or partial penetration should attain the required strength of the joined parts. The size of the butt weld is defined by the thickness i.e. the thickness of the connected plate for complete penetration welds or the total depth of penetration for partial penetration welds.

Groove welds have high strength, high resistance to impact and cyclic stress. They are most direct joints and introduce least eccentricity in the joint. But their major disadvantages are: high residual stresses, necessity of edge preparation and proper aligning of the members in the field. Therefore, field butt joints are rarely used.

To minimise weld distortions and residual stresses, the heat input is minimised and hence the welding volume is minimised. This reduction in the volume of weld also reduces cost. Hence for thicker plates, double groove welds and U welds are generally used.

7.1 Edge Preparation for Butt Weld

Typical edge preparations are shown in Fig.12

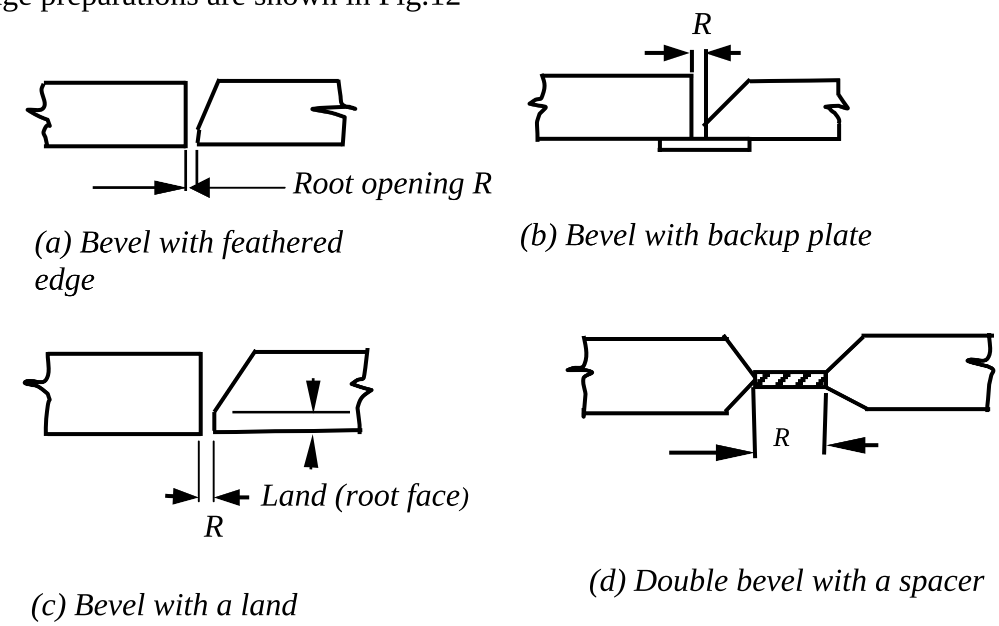


Fig.12 Typical edge preparation for butt weld

For a butt weld, the root opening, *R*, is the separation of the pieces being joined and is provided for the electrode to access the base of a joint. The smaller the root opening the greater the angle of the bevel. The depth by which the arc melts into the plate is called the depth of penetration [Fig.13 (*a*)]. Roughly, the penetration is about 1 mm per 100A and in manual welding the current is usually 150 – 200 A. Therefore, the mating edges of the

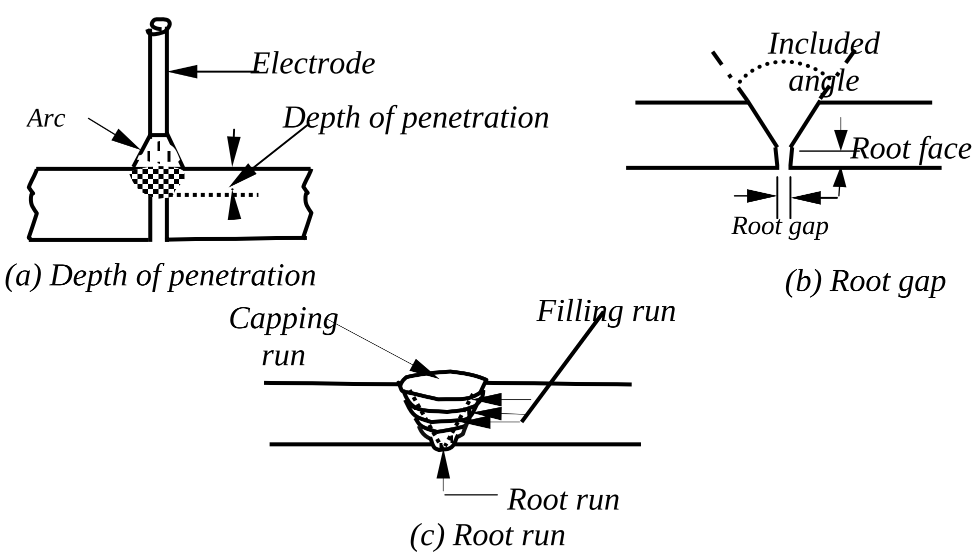


Fig.13 Groove weld details

plates must be cut back if through-thickness continuity is to be established. This groove is filled with the molten metal from the electrode. The first run that is deposited in the bottom of a groove is termed as the root run [Fig.13(*c*)]. For good penetration, the root faces must be melted. Simultaneously, the weld pool also must be controlled, preferably, by using a backing strip.

The choice of edge preparation depends on

1. Type of process
2. Position of welding
3. Access for arc and electrode
4. Volume of deposited weld metal
5. Cost of preparing edges
6. Shrinkage and distortion.

The square groove joint is used to connect thin material up to about 8 mm thick; for thicker material, single-vee groove and the double-vee groove welds have to be used.

8.0 FILLET WELDS

A typical fillet weld is shown in Fig.14 (*a*).

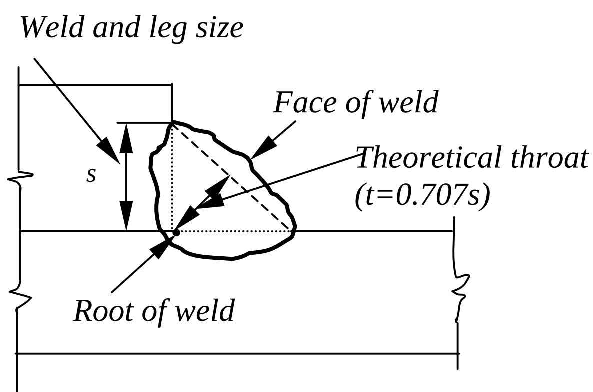


Fig. 14 (a) Typical fillet weld

Owing to their economy, ease of fabrication and adaptability, fillet welds are widely used. They require less precision in the fitting up because the plates being joined can be moved about more than the groove welds. Another advantage of fillet welds is that special preparation of edges, as required by groove welds, is not required. In a fillet weld the stress condition in the weld is quite different from that of the connected parts.

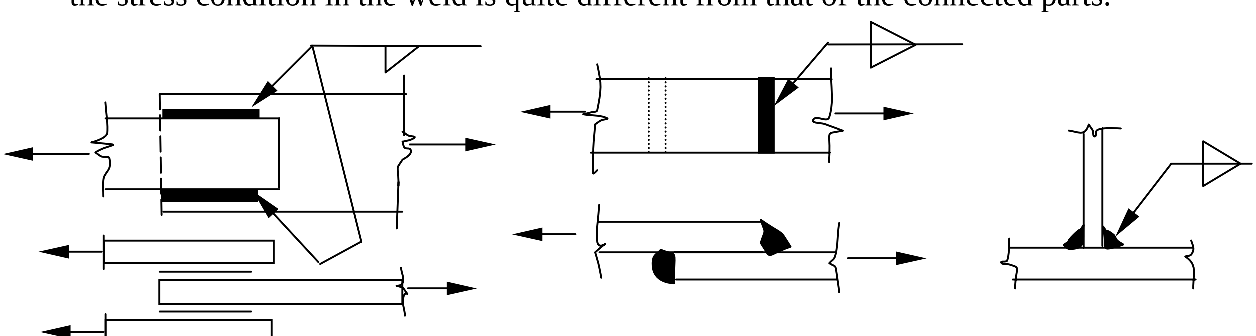


Fig.14 (b) Typical fillet weld connections

The size of a fillet weld is defined by the length of the two sides of the largest right triangle, which can be inscribed within the weld cross section. A major share of welds of this type has equal legs i.e. they form right isosceles triangle shown in Fig. 14(*c*). The typical fillet weld connections are shown in Fig. 14(*b*). The critical dimension of a fillet weld is its throat, the shortest distance from the root to the hypotenuse of the defining triangle shown in Fig. 14(*c*).

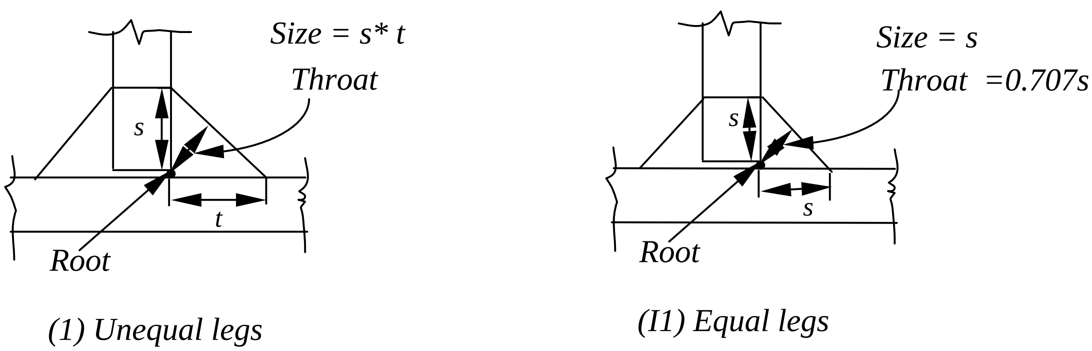


Fig.14 (c) Unequal and Equal legs fillet welds

The root of the weld is the point where the faces of the metallic members meet. The theoretical throat of a weld is the shortest distance from the root to the hypotenuse of the triangle. The throat area equals the theoretical throat distance times the length of the weld. Though a fillet weld is specified by defining the two sides of the inscribed triangle, its actual cross section will be quite complex. A fillet weld must penetrate the base metal and the interface of the weld is either concave or convex [Fig .15(*a*)&(*b*)].

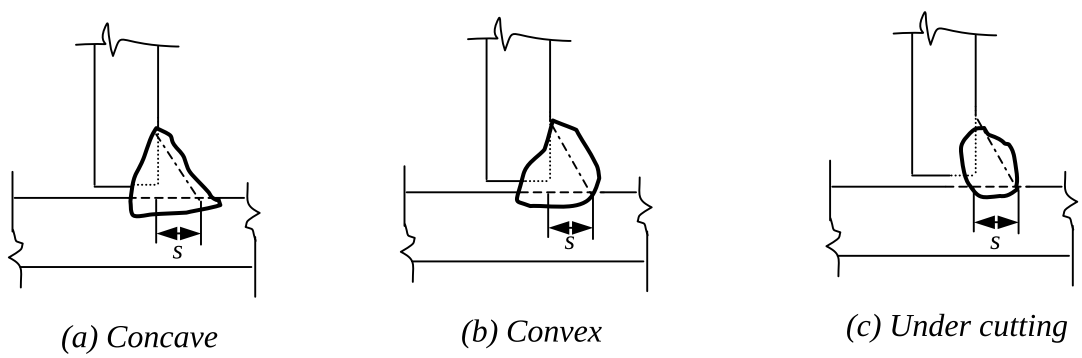


Fig.15 Cross section of fillet weld

The concave shape of free surface provides a smoother transition between the connected parts and hence causes less stress concentration than a convex surface. But it is more vulnerable to shrinkage and cracking than the convex surface and has a much reduced throat area to transfer stresses. On the other hand, convex shapes provide extra weld metal or reinforcement for the throat. But while making a convex surface there is always the possibility of causing undercut at the edges, which undermines the strength of the joint [Fig. 15(*c*)]. The stress concentration is higher in convex welds than in concave welds. It is generally recommended that for statically loaded structures, a slightly convex shape is preferable, while for fatigue – prone structures, concave surface is desirable.

Large welds are invariably made up of a number of layers or passes. For reasons of economy, it is desirable to choose weld sizes that can be made in a single pass. Large welds scan be made in a single pass by an automatic machine, though manually, 8 *mm* fillet is the largest single-pass layer.

9.0 FACTORS AFFECTING THE QUALITY OF WELDED CONNECTIONS

A good weld is obtained from a combination of many factors, from the design of the weld to the welding operation. Even a well-designed weld may not give a strong connection if it is not properly made. Therefore, a structural engineer must be aware of the various factors that affect the quality of the weld. Some of those factors are explained in the following.

9.1 Proper Electrodes, Welding Apparatus and Procedures

Depending on the grade of steel and its thickness, appropriate electrode with suitable diameter has to be selected. The size of the electrode is chosen based on the size of the weld to be made and also on the output of the welding apparatus. It is important that the welding apparatus is capable of delivering enough current for the size of the electrode. Since the output of welding apparatus can be controlled within limits, an electrode of small size may also be used.

In metal–arc welding, the metal is deposited by electromagnetic shield and not by gravity. Therefore the welder is not limited to horizontal or flat welding positions. It is better to avoid overhead welding as the controlling of the process is very difficult and requires a highly skilled welder. In the field, it may not be possible to avoid overhead welding fully; so adequate care must be taken in specifying and making such a weld.

9.2 Welding Sequence

Sequence of welding plays a key role in obtaining a satisfactory welded fabrication. The smallest weld size that can fulfil the requirements is the hallmark of a good weld designer. It is always advisable to weld away from a point of restraint; welding of a joint should start from the centreline and proceed towards the free end. The principle of doubling–up method is employed for a single run fillet weld on either side of the vertical member (Fig. 16).

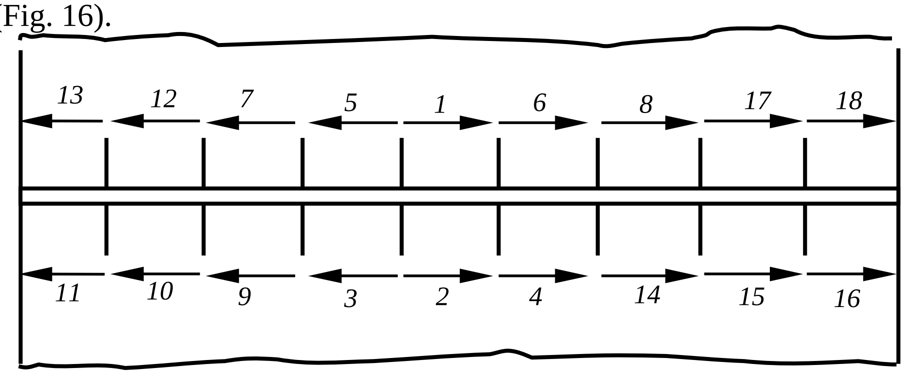


Fig.16 Sequence of welding fillet welds on either side of a vertical member (doubling-up method)

This eliminates transverse angular distortion of a Tee joint. The planned wandering method of welding, shown in Fig.17 is used for butt welded joints with two operators.

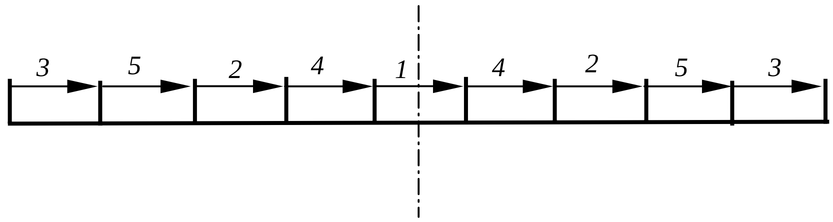


Fig.17 Planned Wandering Method for Welding Long Butt Joint with Two Operators

Another alternative is the stepback method (Fig. 18) in which the welding is carried out in an evenly balanced manner about the centerline of a joint.

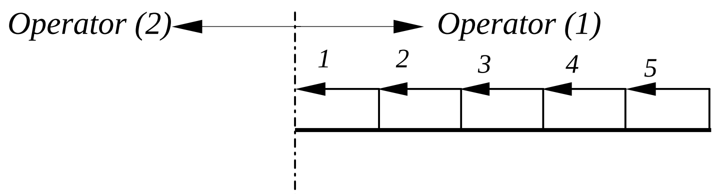


Fig.18 Step Back Method for Welding Long Joint With Two Operators

10.0 RESIDUAL STRESSES AND WELD DISTORTION

10.1 Residual Stresses

In any fusion welding process, the first step is to heat the surfaces to be joined. This heating process is fast and restricted to a narrow portion, where the joint is to be made. The region, which is heated, expands, whereas the rest of the metal resists the expansion. As a consequence compressive stresses are induced near the joint during the heating. After the welding operation is over, the region of intense heating starts cooling. The contraction, which should result, is prevented because the two separate pieces, which were originally free to expand, have now been joined and are not free to contract. Further, the temperature distribution of the weldment would be vastly different from that of the two separate pieces. The net result is that tensile stresses are developed near the weld and compressive stresses away from it. This is shown in Fig. 19. Thus in any welding process involving rapid heating and cooling, residual stresses would always be present. The amount of the residual stresses would vary depending up on the joint restraint, geometry, thermal properties of the material, welding process employed etc. If the parts to be joined are free to expand and contract then the residual stresses would be minimum. However the presence of external restraints would prevent free expansion and contraction resulting in large levels of stresses. In general, residual stresses are increased when increased number of passes is made. The thermal expansion coefficient and thermal conductivity influence the residual stress level. Poor thermal conductivity and high thermal expansion coefficient would significantly increase the residual stresses in a weldment.

Yield strength of material is the upper limit for the residual stress. If the stresses exceed the yield strength, which is normal in welding, the material yields leading to permanent deformation or change of shape. This permanent deformation caused by stresses resulting from thermal cycles is called distortion. This is explained in the next section.

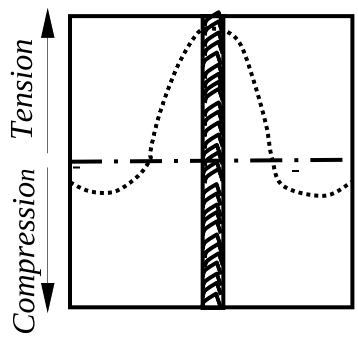


Fig.19 Longitudinal residual stress due to weld

10.2 Distortion in Weldments

In a welding operation, due to heating and cooling of the weld metal and base metal regions near the weld, thermal strains are induced in them. The strains thus produced may be accompanied by plasticity. The stresses due to these strains combine and react to produce internal forces causing bending, buckling and rotation. The displacement arising out of these forces are called distortion.

Three basic dimensional changes that occur during the welding process cause distortion in fabricated structures:

1. Transverse shrinkage perpendicular to the weld line.
2. Longitudinal shrinkage parallel to the weld line.
3. Angular distortion (rotation around the weld line).

These dimensional changes are classified according to their appearance into the following

(a) Transverse shrinkage: the shrinkage is perpendicular to the weld line
(b) Angular change: Non-uniform thermal distortion in the thicker direction causes the angular change close to the weld line, otherwise known as transverse distortion.
(c) Rotational distortion: angular distortion in the plane of the plate due to thermal expansion.
(d) Longitudinal shrinkage: shrinkage in the direction of the weld line.

(e) Longitudinal bending distortion: distortion in plane through the weld line and perpendicular to the plate.
(f) Buckling distortion: thermal compressive stresses cause instability in thin plates.

Some of the above are illustrated in Fig. 20.

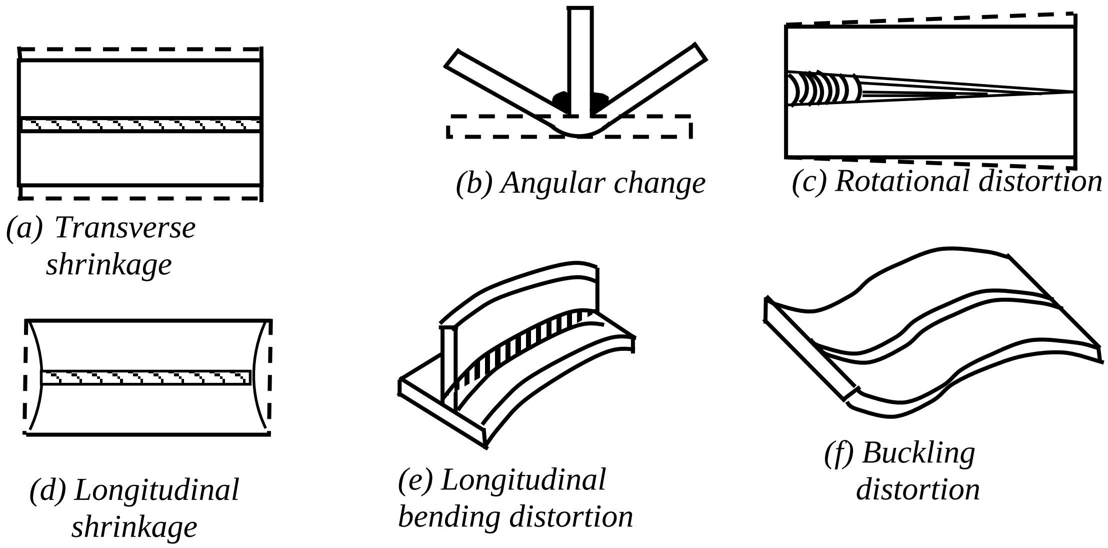


Fig. 20 Weld distortion

The three main approaches to overcome the problems of distortion are:

(1) Minimising the distortion using carefully controlled welding procedures.
(2) Developing standards for acceptable distortion limits.
(3) Using appropriate techniques to remove distortion.

A judicious combination of the above approaches would result in making distortion free welds:

The first approach is to use a welding process that would produce no shrinkage or distortion. However there is no process that would completely eliminate distortion. Close attention should be paid to factors such as welding sequence, degree of restraint, welding condition, joint details, the preheat etc. that contribute to weld distortion.

As distortion is unavoidable, it would be prudent to set up rational standards for acceptable distortion, considering the reliability, economic value and fabrication cost of the structure.

If distortion exceeds the acceptable limits, then the distortion has to be removed with minimum damage to the structure. Distortion may occur even during the service due to overloading or impact. Some of the techniques used for removal of distortion are: (a) flame heating at selected spots and cooling with water, and (b) hammering while it is being heated or by applying a local force to cause counter distortions.

11.0 WELD SYMBOLS

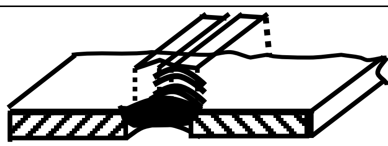
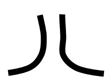
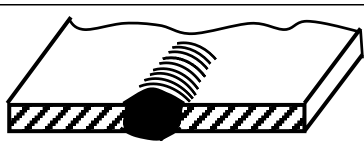
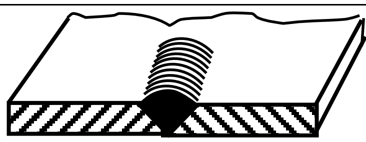
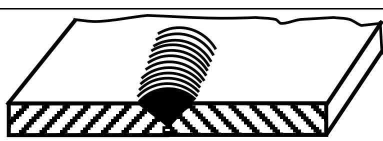
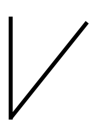
Welding will become a powerful engineering tool when the information required for welding is provided by the designers to the operators. The information concerning type, size, position, welding process etc. of the welds in welded joints is conveyed by standard symbols in drawings. Usage of standard symbols by all designers and fabricators would help avoid confusion and misunderstanding. The symbolic representation gives clearly all necessary indications regarding the specific weld to be achieved.

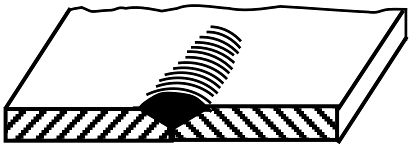
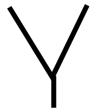
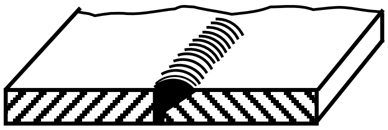
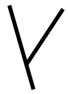
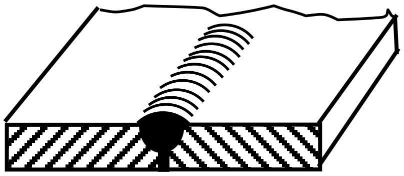
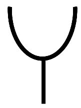
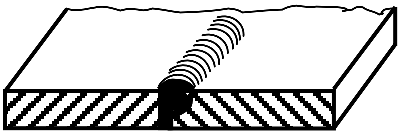
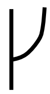
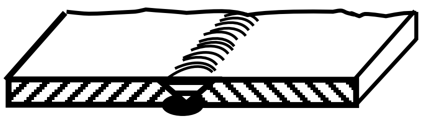
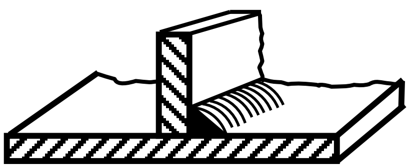
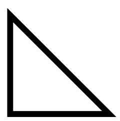
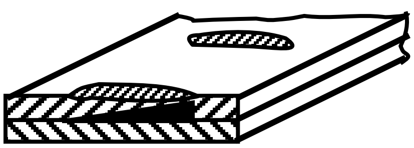
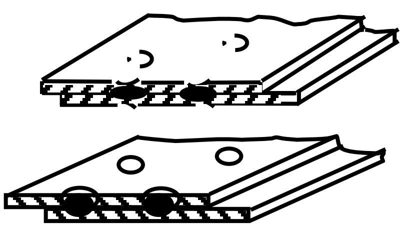
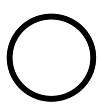
The symbolic representation includes elementary symbols along with a) supplementary symbol, b) a means of showing dimensions, or c) some complementary indications. IS: 813-1986, "Scheme Of Symbols for Welding" gives all the details of weld representation in drawings.

Elementary symbols represent the various categories of the weld and look similar to the shape of the weld to be made. Combination of elementary symbols may also be used, when required. Elementary symbols are shown in Table 1.

Supplementary symbols characterise the external surface of the weld and they complete the elementary symbols. Supplementary symbols are shown in Table 2. Combinations of elementary and supplementary symbols are given in Table 3. The weld locations are defined by specifying, a) position of the arrow line, b) position of the reference line, and c) the position of the symbol. More details of weld representation may be obtained from IS 813-1986.

Table 1. Elementary Symbols

Illustration(Fig.)	**Symbol**	**Description**
		Butt weld between plates with raised edges*(the raised edges being melted down completely)
		Square butt weld
		Single-V butt weld
		Single-bevel butt weld

		Single – V butt weld with broad root face
		Single – bevel butt weld with broad root face
		Single – U butt weld (parallel or sloping sides)
		Single – J butt joint
		Backing run; back or backing weld
		Fillet weld
		Plug weld; plug or slot weld
		Spot weld

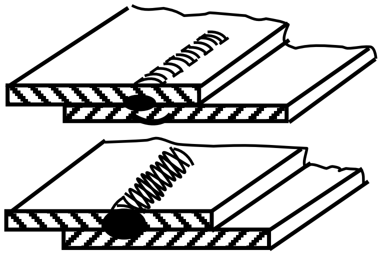
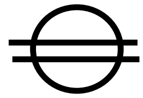
		Seam weld

Table 2. Supplementary Symbols

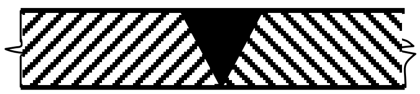
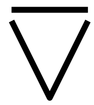
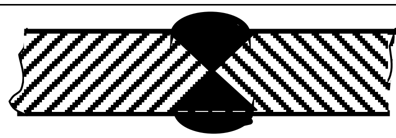
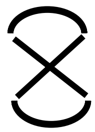
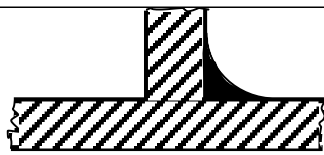
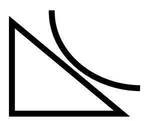
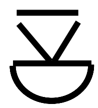
		Flat (flush) single – V butt weld
		Convex double – V butt weld
		Concave fillet weld
		Flat (flush) single – V butt with flat (flush) backing run

Table 3 Combination of Elementary and Supplementary Symbols

Shape Of Weld Surface	symbol
(a) flat (usually finished flush)	—
(b) convex	
(c) concave	

11.1 Position of symbols in drawings

Apart from the symbols as covered earlier, the method of representation (Fig. 21) also include the following:

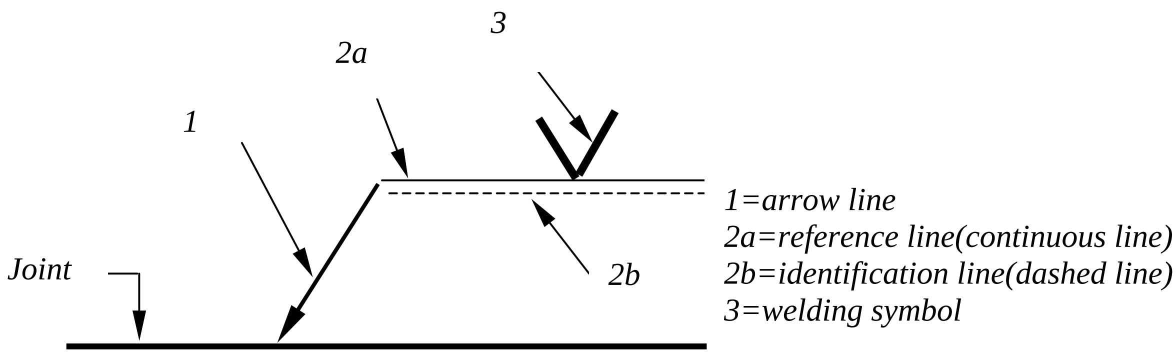


Fig. 21 Method of Representation

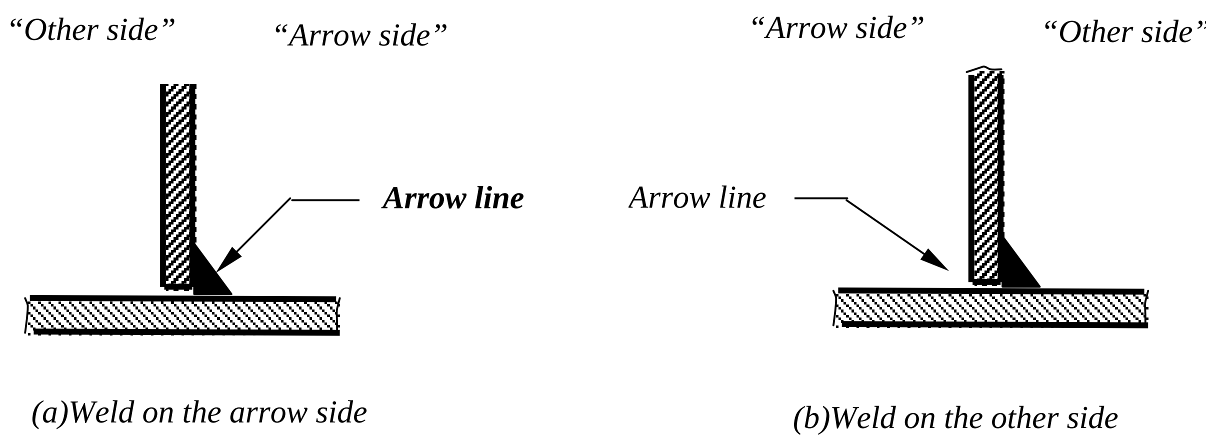


Fig. 22 T-Joint with one fillet weld

- An arrow line for each joint
- A dual reference line, consisting of two parallel lines, one continuous and the other dashed.
- A certain number of dimensions and conventional signs

The location of welds is classified on the drawings by specifying:

- Position of the arrow line
- Position of the reference line and
- The position of the symbol

The relation between arrow line and the joint, shown in Fig. 22 (a, b) explain the meaning of the terms 'arrow side' of the joint and ' other side' of the joint. The position of arrow line with respect to the weld has no special significance. The arrow line joins one end of the continuous reference line such that it forms an angle with it and shall be completed by an arrowhead or a dot. The reference line is a straight line drawn parallel to the bottom edge of the drawing.

The symbol is placed either above or beneath the reference line. The symbol is placed on the continuous side of the reference line if the weld is on the other side of the joint; the symbol is placed on the dashed line side

12.0 DEFECTS IN WELDS

If good welding methods and procedures are not followed a number of defects may be developed causing discontinuities within the weld. Some of the important defects are described in the following.

12.1 Incomplete fusion

Complete fusion may not take place, if the mating surfaces are not properly cleaned of all coatings such as mill scales, slag, oxides etc. This defect may also be caused by in-sufficient current, because of which the base metal does not melt properly. Rapid rate of welding also leads to improper fusion. The different types of incomplete fusion are shown in Fig.23.

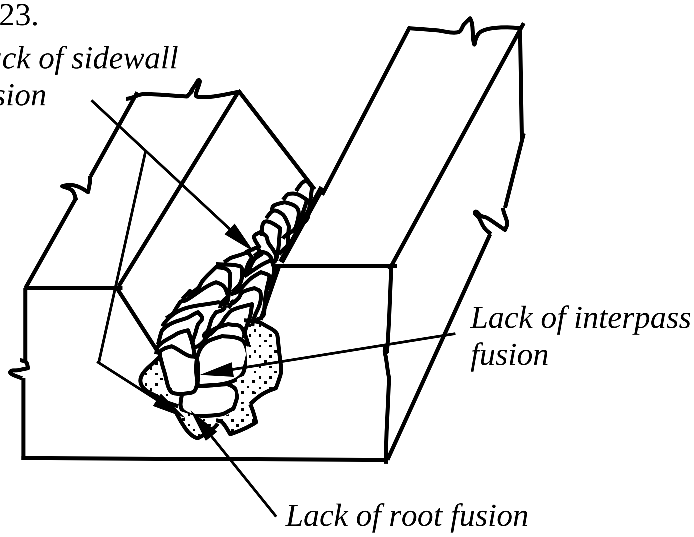


Fig.23 Lack of fusion (or) incomplete fusion

12.2 Porosity

Porosity is formed when a number of gas pockets or voids are trapped during the cooling process. Use of excessively high current and longer arc length are the reasons for this type of defect. Porosity may occur in two ways: Either dispersed through the weld or as a large pocket at the root near to the backup plate in a groove weld. Improper welding procedures and careless use of backup plates result in porosity in groove welds.

12.3 Inadequate penetration

In certain instances, partial penetration may be adequate. However when the weld penetration is less than that specified, it is termed as inadequate penetration. This type of defect, primarily occurring in groove welds, is due to insufficient groove angles, very large electrodes, inadequate weld current, larger welding rates, or insufficient gaps at the root of welds. The defect can be avoided by means of backup plates. Inadequate penetration is shown in Fig.24.

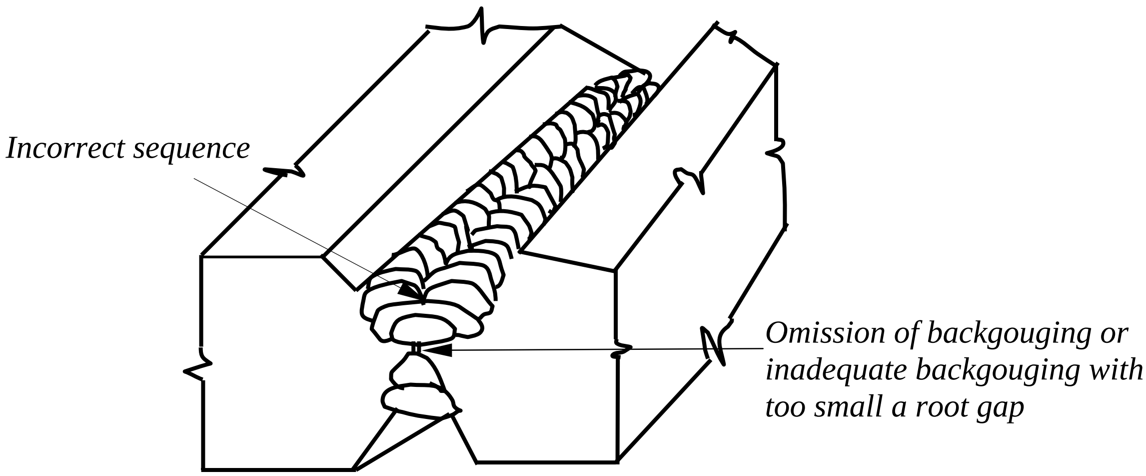


Fig.24 Lack of penetration

12.4 Under cutting

This type of defect is formed due to the use of excessive current or an excessively long arc. A portion of the metal is burnt away reducing the thickness of the joint at the edge of the weld. The defect is detected easily by visual inspection and repaired easily by depositing additional weld material. Examples of under cutting are shown in Fig.25.

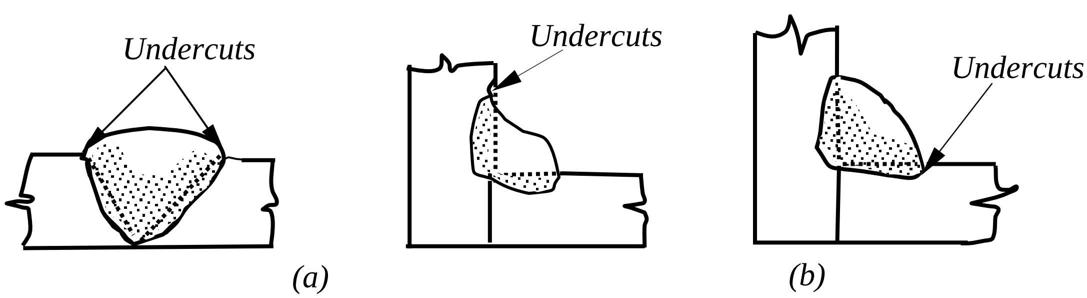


Fig. 25 Typical examples of undercut defect. (a) Wide and curved, (b) Narrow and crack like

12.5 Slag inclusion

Slag is formed in the welding process due to the chemical reaction of the melted electrode coating. It normally consists of metal oxides and other compounds. Since it has less density than the molten weld metal the slag usually floats on the surface. On cooling, this is removed by the welder. But, if the cooling is rapid, the slag may get trapped before it can rise to the surface. When several passes of weld are made to achieve the desired weld size, the slag that forms between each process must be removed completely. The main reason for slag inclusion is due to the failure to remove the slag fully between runs. Overhead welds are also susceptible to slag inclusion and hence adequate care should be taken. Slag inclusion is shown in Fig. 26.

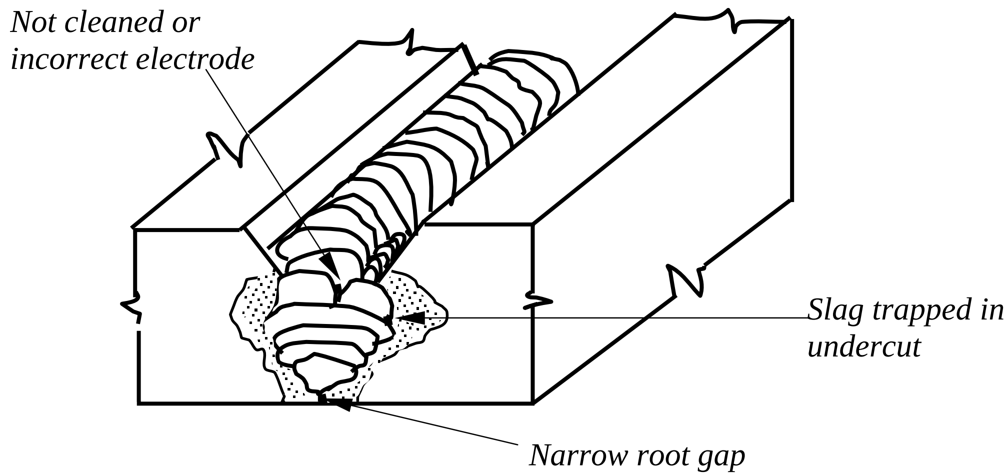


Fig.26 Slag inclusion

12.6 Cracks

Cracks are by far the most severe form of weld defects. Cracks occur in the form of breaks in the weld metal. They are the result of internal stresses and form either longitudinally or transversely to the line of weld. Cracks may extend from the welded metal into the base metal. They may also be completely in the base metal very near to the weld in HAZ.

Cracks may occur either in the hot or cold form. Hot cracks are formed as the weld begins to solidify. Uniform heating and slower cooling will prevent hot cracks. Cold cracks, which occur at room temperature, run parallel to but under the weld in the base metal. Using certain special electrodes and proper preheating and post heating, cold cracking can be reduced.

12.7 Lamellar Tearing

Lamellar tearing is a type of cracking that occurs in the base metal beneath the weld. It is caused by the combined effects of high, localised stresses from weld contraction and poor through - thickness ductility in the steel. The tearing is started by the separation of the interface between inclusions and metal (also known as delamination) or by fracture of an inclusion itself. The cracks grown by the joining of the delamination in the same plane or by the shear steps, which join the cracks in different planes. This results in a characteristic step-like appearance for lamellar tearing. Lamellar tear is shown in Fig. 27.

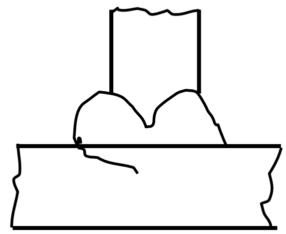


Fig. 27 Lamellar tear

The influencing factors in lamellar tearing are:

- Amount of Non-metallic inclusion and their orientation.
- Magnitude of induced normal stresses normal to the plate surface.

The presence of inclusions reduces the ductility of steel in the through - thickness direction because the bond between the inclusion and steel is much weaker than steel itself. The magnitude of stresses induced depends on the joint design, the imposed degree of restraint on the joint, plate thickness, size of the weld and orientation of the weld. Restrained corner or T-joints are most susceptible to lamellar tearing, as the through - thickness contraction stresses are high. Butt welds rarely experience lamellar tear. Thick plate, high restraint (rigid clamping) and large weld beads all contribute to residual stresses and the chances of tearing. Hydrogen also increases the vulnerability to lamellar tearing. Hence preheat is beneficial to reduce the tendency to lamellar tearing. It is also beneficial to use low hydrogen consumables.

The following precautions may be taken to reduce lamellar tearing.

- Using plate material with improved through – thickness properties.
- Designing the joint with minimum through-thickness stresses.
- Using lower strength welding consumables.
- In special cases, the plate may be ground to a level below where lamellar tearing is expected. The area can be provided with weld metal and the attachment weld can be made.

13.0 WELD DEFECT ACCEPTANCE LEVELS

13.1 General

Weld defects acceptance levels are closely related to the available methods of Non - Destructive Examination (NDE).

Previously, radiography was the best technique available for inspection of welds. Then, acceptance conditions were in terms of maximum slag inclusion and porosity levels. Presence of cracks was not acceptable and repairs were absolutely essential. The use of ultrasonic technique has made it possible to detect smaller cracks, when compared to radiography. Thus, welds, which passed radiographic inspection, required extensive repair by the new method of inspection. The method of fracture mechanics has made it possible to assess the potential of cracks to cause serious damage and thus to come up with tolerable defect sizes. Slag inclusion and porosity may not be particularly deleterious defects unless fatigue type of loading is anticipated. By fracture mechanics approach, it has been established that the cracks detected by the ultrasonic methods are far smaller than those that affect the safety of the structure. Uses of fracture mechanics method has proved that tolerable defect sizes are large. This would result in cost saving in terms of repair and reduce the level of NDE inspection.

13.2 Accepted Criteria for Welded Joints

In general the following weld defects detected during inspection are acceptable for structures.

- For joints welded from both the sides, incomplete penetration with thickness up to 5% of the parent metal thickness, but not exceeding 2 mm and the length more than 500 mm can be accepted. The aggregate length of flaw shall not be more than 200 mm per meter length of the joint. Incomplete penetration and cracks are not allowed at or near the end or beginning of a joint.
- For joints welded from one side without backing strip, incomplete penetration with thickness up to 15% of parent metal thickness but not exceeding 3 mm at the root is allowed.
- Slag inclusion located along the weld as a chain or unbroken line is allowed if their aggregate length does not exceed 200 mm per meter of weld length. Size of the slag may also be considered.
- Total of isolated gas pores and slag inclusion shall not exceed 5 in number per square centimetre of the weld.
- Total of incomplete penetration, slag inclusion on pores located separately or as a chain shall not exceed 10% of metal thickness but not greater than 2 mm when welding is done from both the sides and 15% of metal thickness, but not greater than 3 mm when welding is done from one side.
- For metal thickness up to 10 mm, undercuts shall not be more than 0.5 mm. For metal thickness more than 10 mm, undercuts shall not be greater than 1 mm.

Incomplete weld, molten metal flow, pits and cracks shall not be allowed.

14.0 WELDING INSPECTION

There are essentially three steps to be followed to ensure good welding; they are:

1 Establishing good welding procedures
2 Use of pre-qualified welders
3 Availability of competent inspectors in shop and field

It is essential that welded joints are thoroughly examined and defects are detected so that any possible distress could be averted. There are several non-destructive testing methods to check the quality of welds. They are explained in the following.

14.1 Visual inspection

Visual inspection by a competent person will give a good indication of the quality of welds; but may not be able to gauge the sub surface condition of the welds. An experienced welder, by visual inspection, would be able to know whether satisfactory fusion and penetration are obtained. He will be able to recognise good welds by their shape, size and general appearance. In a good weld, the metal should be nearly its original colour after it has cooled. In case of over heating, it will give a rusty appearance. There are several scales and gauges to check the size and shape of welds.

Methods of determining the internal soundness of a weld are described in the following section.

14.2 Liquid Penetrants

In this method, a type of dye is spread over the weld surface. This dye penetrates into the surface cracks of the weld. After the penetration of the dye, any excess material is removed and a powdery developer is sprayed to draw the dye out of the cracks. Then, the outline of the cracks can be seen with naked eye. In some cases, fluorescent dyes are used for improved visibility of the cracks.

14.3 Magnetic Particles

The weld that is inspected is ‘magnetised’ electrically. Cracks, which are present at or near the surface, would cause North and South poles to form on each side of the cracks. Dry iron filings are then kept on the weld. They form patterns when they cling to cracks. From the patterns, the location of cracks, their size and shapes are established.

14.4 Ultrasonic Testing

By means of the ultrasonic equipment, sound waves are sent through one side of the material and they are reflected from the opposite side. These reflections are indicated in a cathode ray tube. Any defect in the weld will alter the time of the sound transmission. By the help of the picture in the tube, flaws can be detected and their severity can be judged.

14.5 Radiography

This is an expensive method and can be used to check the welds in important structures. Portable X-ray machines along with radium or radioactive cobalt would give excellent pictures. This method is reliable for butt welds, but is not satisfactory for fillet welds due to difficulty in interpreting pictures. Another drawback of the method is the radioactive danger. Much care has to be taken while carrying out this inspection to protect the workers on the job.

A properly welded connection is usually much stronger (1.5 to 2 times) than the strength of the members being connected. The reasons for the extra strength are: electrode wire is made up of premium steel, the metal is melted electrically and the cooling rate is rapid. Due to these factors, the weld strength is always higher than required by the design.

15.0 CONCLUSION

In this chapter, fundamentals of welding, details of the various welding processes, types of welds, common weld defects and weld inspection have been covered. Advantages of welding over other forms of connection such as bolting and riveting are explained in detail. Design of various types of welded connections in steel structures is explained in next chapter.

16.0 REFERENCES

1. Morrow H.W., "Elements of Steel Design" ", Prentice Hall, Inc. / Englewood Cliffs, New Jersey-1987.
2. Dowling P.J., Knowles P.R. and Owens G.W., Structural Steel Design, The Steel Construction Institute, Butterworths, London, 1988.
3. McCormac J.C., "Structural Steel Design"(Third Edition)., Harper and Row, Publishers, New York, 1981.
4. IS: 813 – 1986. "Scheme Of Symbols For Welding", Bureau of Indian Standards New Delhi.

31 WELDS- STATIC AND FATIGUE STRENGTH – II

1.0 INTRODUCTION

In the previous chapter, a detailed account of various welding processes, types of welds, advantages of welded connections etc. were presented. It was seen that welded connections are continuous and more rigid when compared to bolted connections. It was also pointed out that fillet welds and butt welds constitute respectively 80% and 15% of all welds in the construction industry; the balance 5% is made up by plug, slot and spot resistance welds.

In this chapter, the behaviour and design of welded connections under various static loading conditions is considered. A typical connection design process is initiated with the design, which is followed by the welding operation and, concludes with inspection.

2.0 CONNECTION DESIGN

In the design of connections, due attention must be paid to the flow of the force through the connection. The transfer of forces should occur smoothly, without causing any stress concentration or cracks. The connections can be either concentric or eccentric. In concentric connections, the forces acting on the connections will essentially be axial in nature, whereas in eccentric connections, the axial forces will be coupled with bending or torsion. These types of connections are described in the following.

2.1 Concentric connections

Static strength of a welded joint depends upon the following factors

- Type and size of the weld
- Manner of welding, and
- Type of electrode used.

A primary responsibility of a designer is to select the type and size of the weld. A number of varieties of welds are available. When it is properly chosen with the correct electrode, it develops full strength of the parent material. The chosen type of weld should develop minimal residual stresses and distortions.

As stated in the introduction, butt and fillet welds are the usual forms of welds in practical building construction. Butt welds are used at an edge-to-edge junction or a tee junction. A butt weld connection is made by bringing the plates to be joined face to face edgewise and then filling the cavity formed by edge preparation or by just penetrating the unprepared junction. Butt welds can be either full penetration or partial penetration.

Partial penetration butt welds may be used for static loading, if reduced strength is acceptable. On the other hand, a fillet weld is made away from the edges of the abutting plates. The joint is formed by welding the members in an overlapped position or by using a secondary joining material. The main advantage of a fillet weld is that the requirements of alignment and tolerance are less rigorous when compared to butt welds. Fillet welding could be applied for lap joints, tee joints and corner joints. A detailed description of these two types of welds and their design requirements are presented in the following.

2.2 Butt welds

Full penetration butt welds are formed when the parts are connected together within the thickness of the parent metal. For thin parts, it is possible to achieve full penetration of the weld. For thicker parts, edge preparation may have to be done to achieve the welding, which has been discussed in the previous chapter.

There are nine different types of butt joints: square, single V, double V, single U, double U, single J, double J, single bevel and double bevel. They are shown in Fig. 1. In order to qualify for a full penetration weld, there are certain conditions to be satisfied while making the welds. The more important ones are given below:

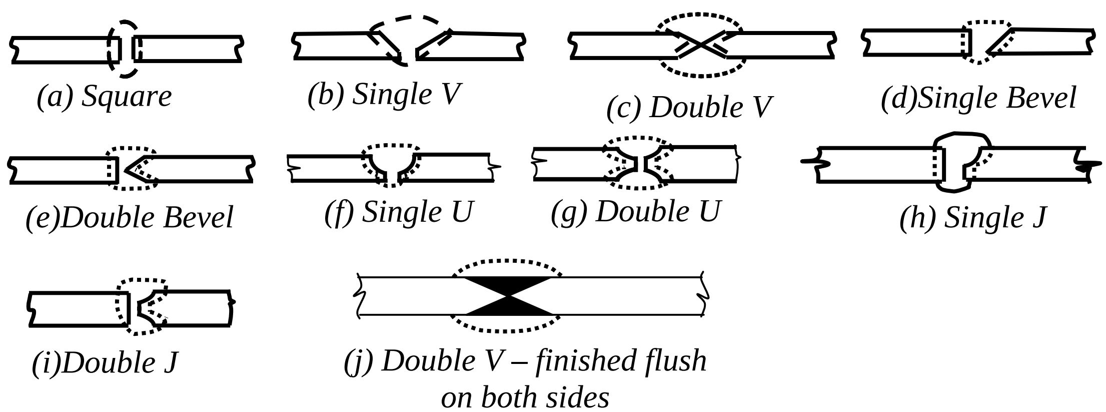


Fig. 1 Different types of butt joints

2.2.1 Static behaviour of butt welds

For butt welds the most critical form of loading is tension applied in the transverse direction (Fig. 2). It has been observed from tests conducted on tensile coupons containing a full penetration butt weld normal to the applied load that the welded joint had higher strength than the parent metal itself. During the application of the load, the welded portion and the HAZ (Heat Affected Zone of the parent metal) have less transverse contraction compared to the parent metal. The yield stress of the weld metal and the parent metal in the HAZ region was found to be much higher than the parent metal. The increase in yield stress in the HAZ is due to the quenching effect associated with rapid cooling after deposition of the weld.

The yield stress of the weld metal is also raised due to the quenching effect. The metal alloys constituting the electrode contribute to the increase in yield stress. These alloys penetrate the parent metal influencing its mechanical properties.

Due to the lower yield stress and higher transverse contraction in the parent metal, it experiences a higher true stress. As a consequence, the failure of the coupon always occurs away from the weld. The higher strength achieved by the welded joint adversely affects its toughness and ductility properties. These negative effects can be minimised by choosing proper electrodes.

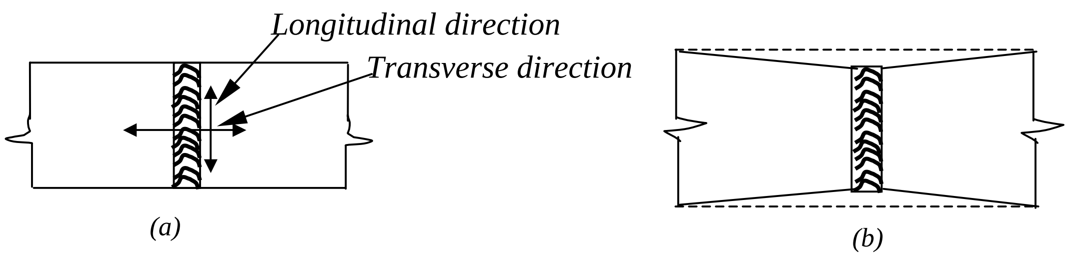


Fig.2 (a) Load applied in transverse direction (b) longitudinal shrinkage restraint

Partial penetration welds, shown in Fig. 3, differ in two ways from the full penetration welds: the reduction in cross section and the uncertainty of the weld root quality. Firstly, there is a reduction in the cross section at the joint resulting in overloading and severe plastic straining. Further, the weld root quality cannot be inspected and they cannot be repaired as may be done for full penetration welds.

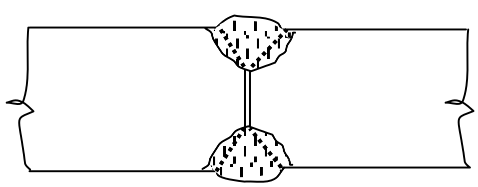


Fig.3 Partial penetration weld

2.2.2 Design

The butt weld is normally designed for direct tension or compression. However, a provision is made to protect it from shear. Design strength value is often taken the same as the parent metal strength. For design purposes, the effective area of the butt-welded connection is taken as the effective length of the weld times the throat size. Effective length of the butt weld is taken as the length of the continuous full size weld. The throat size is specified by the effective throat thickness. For a full penetration butt weld, the throat dimension is usually assumed as the thickness of the thinner part of the connection. Even though a butt weld may be reinforced on both sides to ensure full cross-sectional areas, its effect is neglected while estimating the throat dimensions. Such reinforcements often have a negative effect, producing stress concentration, especially under cyclic loads.

Generally speaking, partial penetration welds must be avoided. Partial penetration groove welds are used in non-critical details, so as to avoid back-gouging. If they are considered essential, they should be designed with care. Some codes of practice do not recommend their use in tension. Others specify that they be designed in the same way as fillet welds. This is because the load transfer is not smooth and efficient with partial penetration welds. The effective throat thickness of a partial penetration weld is taken as the minimum thickness of the weld metal common to the parts joined, excluding reinforcement. For stress calculation, a maximum value of reduced effective throat thickness equal to 5/8 of the thickness of the thinner part joined must be used. The unwelded portion in partial penetration butt welds, welded from both sides, shall not be greater than ¼ thickness of the thinner part joined, and should be in the central portion.

If the stresses are uniform across the throat thickness, the average stress concept may be applied to determine its strength. Connections with partial penetration welds with welding on only one side is generally avoided under tensile load due to the eccentric loading involved. Otherwise, the eccentricity effects should be considered in the design.

Unsealed butt welds of V, U, J and bevel types and incomplete penetration butt welds should not be used for highly stressed joints and joints subjected to dynamic and alternating loads. Intermittent butt welds are used to resist shear only and the effective length should not be less than four times the longitudinal space between the effective length of welds nor more than 16 times the thinner part. They are not to be used in locations subjected to dynamic or alternating stresses. Some modern codes do not allow intermittent welds in bridge structures.

For butt welding parts with unequal cross sections, say unequal width, or thickness, the dimensions of the wider or thicker part should be reduced at the butt joint to those of the smaller part. This is applicable in cases where the difference in thickness exceeds 25 % of the thickness of the thinner part or 3.0 mm, whichever is greater. The slope provided at the joint for the thicker part should not be steeper than one in five [Figs.4 (a) &(b)]. In instances, where this is not practicable, the weld metal is built up at the junction equal to a thickness which is at least 25 % greater than the thinner part or equal to the dimension of the thicker part [Fig. 4(*c*)]. Where reduction of the wider part is not possible, the ends of the weld shall be returned to ensure full throat thickness.

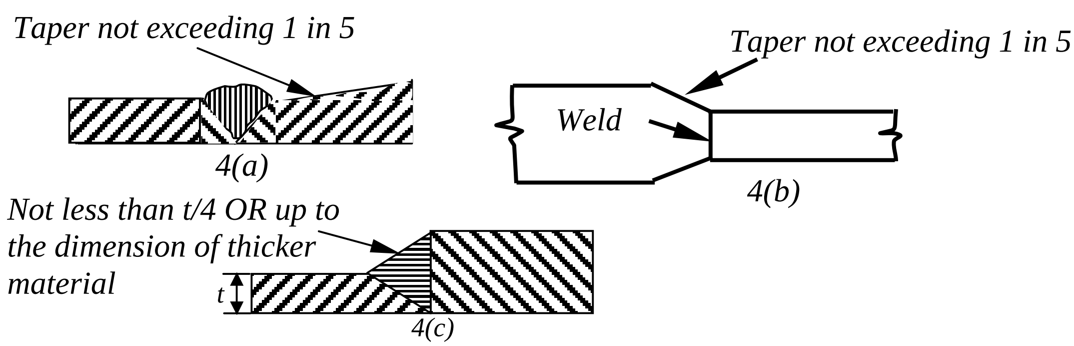


Fig. 4 Butt welding of members with (a)&(b) unequal thickness (c) unequal width

Permissible stresses for butt welds are assumed same as for the parent metal with a thickness equal to the throat thickness. For field welds, the permissible stresses in shear and tension may be reduced to 80% of the above value.

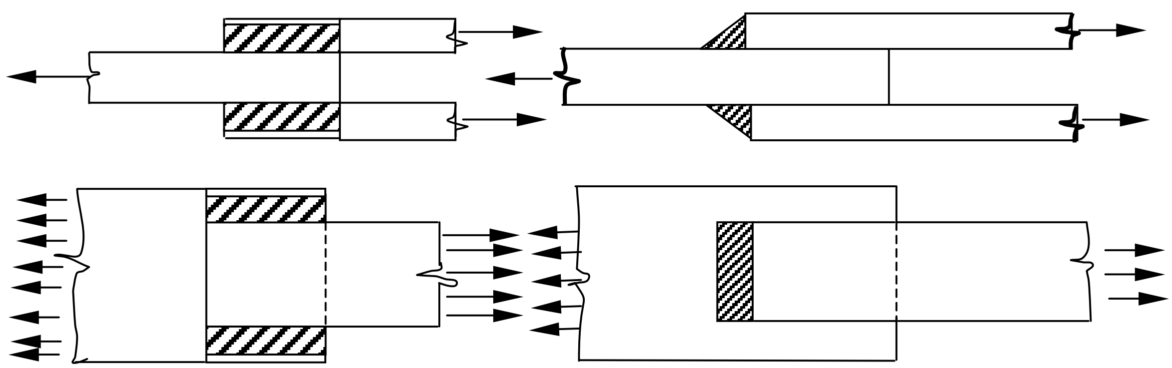
2.3 Fillet welds

2.3.1 General

These are generally used for making lap joint splices and other connections where the connecting parts lap over each other. Though a fillet weld may be subjected to direct stresses, it is weaker in shear and therefore the latter is the main design consideration.

2.3.2 Behaviour

Fillet welds are broadly classified into side fillets and end fillets (Fig. 5). When a connection with end fillet is loaded in tension, the weld develops high strength and the stress developed in the weld is equal to the value of the weld metal. But the ductility is minimal. On the other hand, when a specimen with side weld is loaded, the load axis is parallel to the weld axis. The weld is subjected to shear and the weld shear strength is limited to just about half the weld metal tensile strength. But ductility is considerably improved. For intermediate weld positions, the value of strength and ductility show intermediate values.



(a) *(b)*

Fig.5 Fillet (a) side welds and (b) end welds

Actual distribution of stresses in a fillet weld is very complex. A rigorous analysis of weld behaviour has not been possible so far. Multiaxial stress state, variation in yield stress, residual stresses and strain hardening effects are some of the factors, which complicate the analysis.

In many cases, it is possible to use the simplified approach of average stresses in the weld throat (Fig. 6). In order to apply this method, it is important to establish equilibrium with the applied load. Studies conducted on fillet welds have shown that the fillet weld shape is very important for end fillet welds. For equal leg lengths, making the direction of applied tension nearly parallel to the throat leads to a large reduction in strength. The optimum weld shape recommended is to provide shear leg equal to □3 times the tension leg. A small variation in the side fillet connections has negligible effect on strength. In general, fillet welds are stronger in compression than in tension.

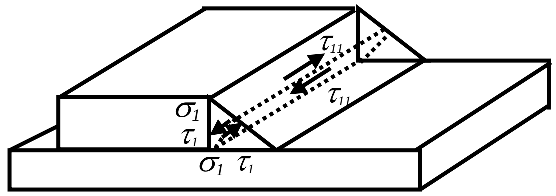


Fig.6 Average stress in the weld throat

2.3.3 Design

A simple approach to design is to assume uniform fillet weld strength in all directions and to specify a certain throat stress value. The average throat thickness is obtained by dividing the applied loads summed up in vectorial form per unit length by the throat size. Alternatively, design strength can be different with direction of the load vector. This method is limited in usage to cases of pure shear, tension or compression (Fig.7). It cannot be used in cases where the load vector direction varies around weld group. For the simple method, the stress is taken as the vector sum of the force components acting in the weld divided by the throat area.

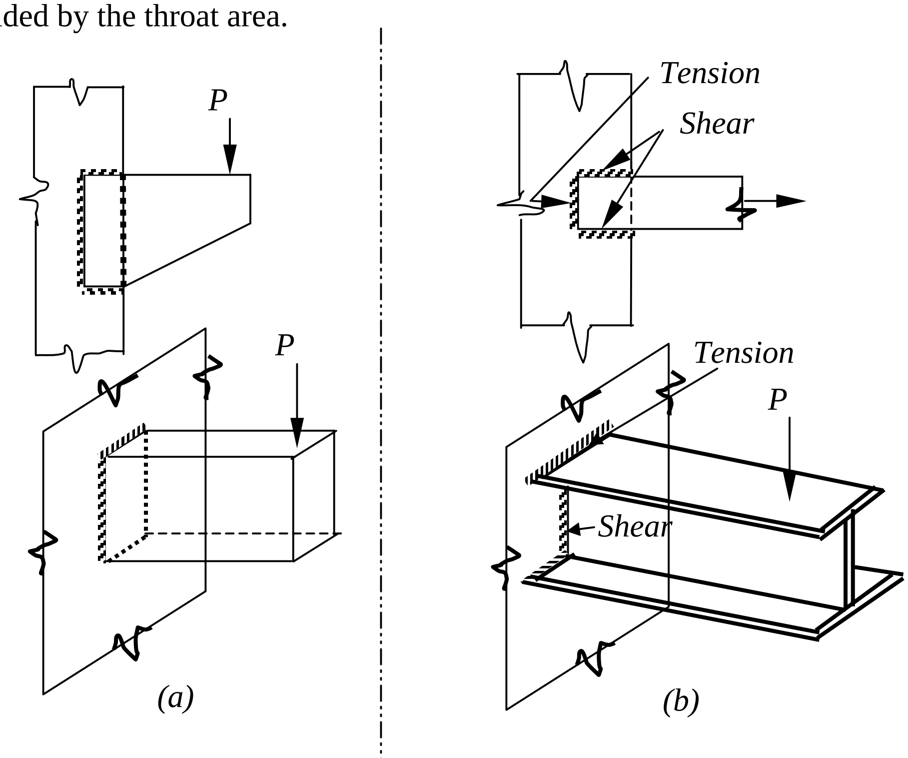


Fig.7 (a) connections with simple weld design,(b) connections with direction- dependent weld design

The size of a normal fillet should be taken as the minimum leg size (Fig. 8).

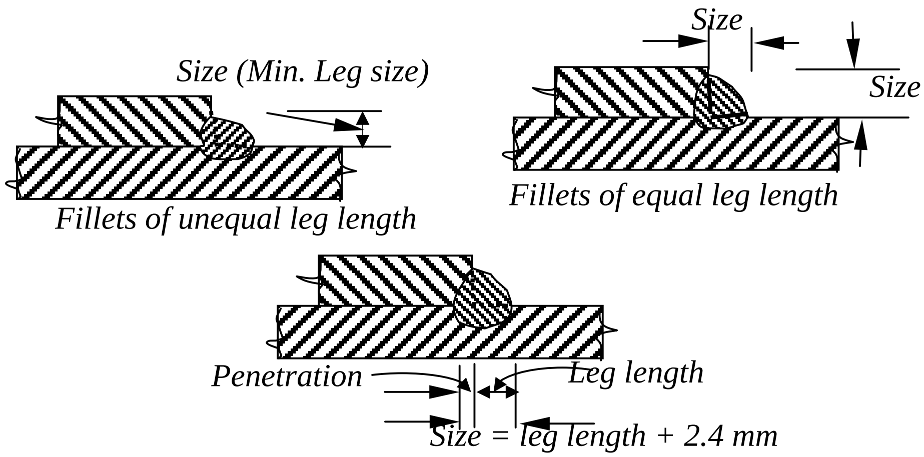


Fig. 8 Sizes of fillet welds

For a deep penetration weld, the depth of penetration should be a minimum of 2.4 mm. Then the size of the weld is minimum leg length plus 2.4 mm. The size of a fillet weld should not be less than 3 mm or more than the thickness of the thinner part joined. Minimum size requirement of fillet welds is given below in Table 1. *Effective throat thickness* should not be less than 3 mm and should not exceed 0.7 t and 1.0 t under special circumstances, where 't' is the thickness of thinner part.

Table 1 Minimum size of first run or of a single run fillet weld

Thickness of thicker part		**Minimum size (mm)**
Over (mm)	Up to and including (mm)	
-	10	3
10	20	5
20	32	6
32	50	8 (First run) 10 (Minimum size of fillet)

For stress calculations, the effective throat thickness should be taken as K times fillet size, where K is a constant. Values of K for different angles between tension fusion faces are given in Table 2. Fillet welds are normally used for connecting parts whose fusion faces form angles between 60° and 120°. The actual length is taken as the length having the effective length plus twice the weld size. Minimum effective length should not be less than four times the weld size. When a fillet weld is provided to square edge of a part, the weld size should be at least 1.5 mm less than the edge thickness [Fig. 9(*a*)]. For the rounded toe of a rolled section, the weld size should not exceed 3/4 thickness of the section at the toe [Fig. 9(*b*)].

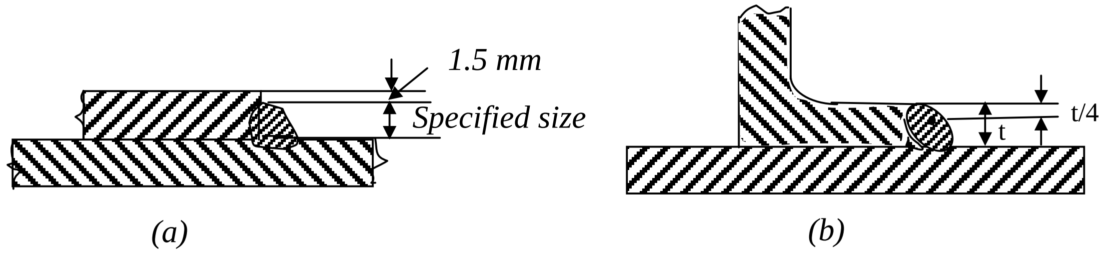


Fig.9 (a) fillet welds on square edge of plate, (b) fillet welds on round toe of rolled section

Table 2. Value of K for different angles between fusion faces

Angle between fusion faces	60° - 90°	91°-100°	101°-106°	107°-113°	114°-120°
Constant K	0.70	0.65	0.60	0.55	0.50

Intermittent fillet welds may be provided where the strength required is less than that can be developed by a continuous fillet weld of the smallest allowable size for the parts joined. The length of intermediate welds should not be less than 4 times the weld size with a minimum of 40 mm. The clear spacing between the effective lengths of the intermittent welds should be less than or equal to 12 times the thickness of the thinner member in compression and 16 times in tension; in no case the length should exceed 20 cm. Chain intermittent welding is better than staggered intermittent welding. Intermittent fillet welds are not used in main members exposed to weather. For lap joints, the overlap should not be less than five times the thickness of the thinner part. For fillet welds to be used in slots and holes, the dimension of the slot or hole should comply with the following limits:

a) The width or diameter should not be less than three times the thickness or 25 mm whichever is greater
b) Corners at the enclosed ends or slots should be rounded with a radius not less than 1.5 times the thickness or 12 mm whichever is greater, and
c) The distance between the edge of the part and the edge of the slot or hole, or between adjacent slots or holes, should be not less than twice the thickness and not less than 25 mm for the holes.

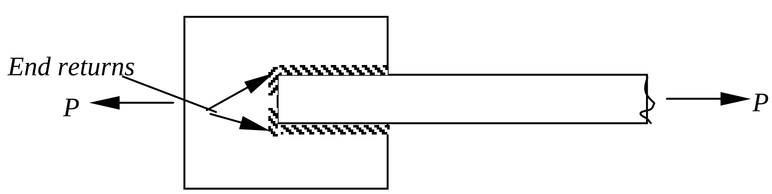


Fig. 10 End returns

The effective area of a plug weld is assumed as the nominal area of the whole in the plane of the *faying* surface. Plug welds are not designed to carry stresses. If two or more of the general types of weld (butt, fillet, plug or slots) are combined in a single joint, the effective capacity of each has to be calculated separately with reference to the axis of the group to determine the capacity of the welds.

The high stress concentration at ends of welds is minimised by providing welds around the ends as shown in Fig. 10. These are called *end returns*. Most designers neglect end returns in the effective length calculation of the weld. End returns are invariably provided for welded joints that are subject to eccentricity, impact or stress reversals. The end returns are provided for a distance not less than twice the size of the weld.

2.4 Slot Welds

In certain instances, the lengths available for the normal longitudinal fillet welds may not be sufficient to resist the loads. In such a situation, the required strength may be built up by welding along the back of the channel at the edge of the plate if sufficient space is available. This is shown in Fig. 11(a). Another way of developing the required strength is by providing slot or plug welds. Slot and plug welds [Fig. 11(b)] are generally used along with fillet welds in lap joints. On certain occasions, plug welds are used to fill the holes that are temporarily made for erection bolts for beam and column connections. However, their strength may not be considered in the overall strength of the joint.

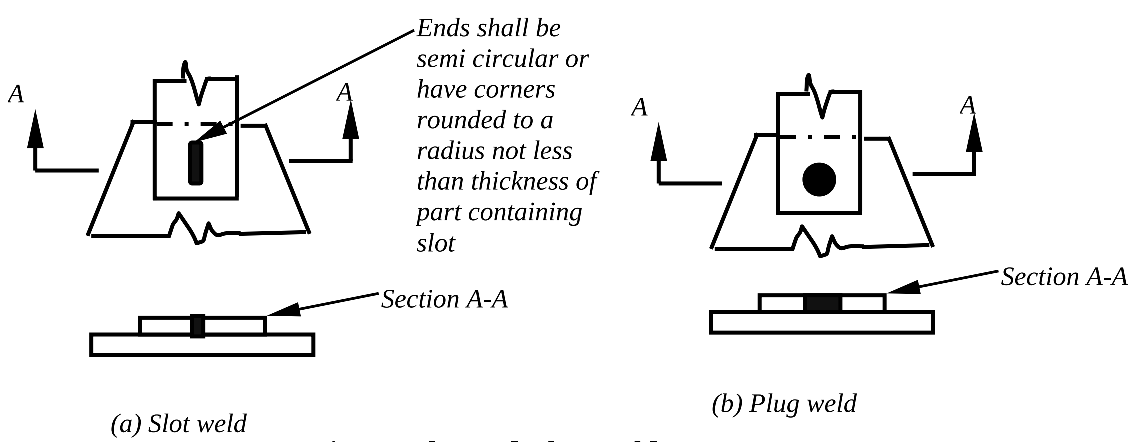


Fig. 11 Slot and Plug welds

The limitations given in specifications for the maximum sizes of plug and slot welds are necessary to avoid large shrinkage, which might be caused around these welds when they exceed the specified sizes. The strength of a plug or slot weld is calculated by considering the allowable stress and its nominal area in the shearing plane. This area is usually referred to as the faying surface and is equal to the area of contact at the base of the slot or plug. The length of the slot weld can be obtained from the following relationship:

$$L = \frac{Load}{(width)\ allowable\ stress} \tag{1}$$

3.0 ECCENTRIC JOINTS

In some cases, eccentric loads may be applied to fillet welds causing either shear and torsion or shear and bending in the welds. Examples of such loading are shown in Fig. 12. These two common cases are treated in this section.

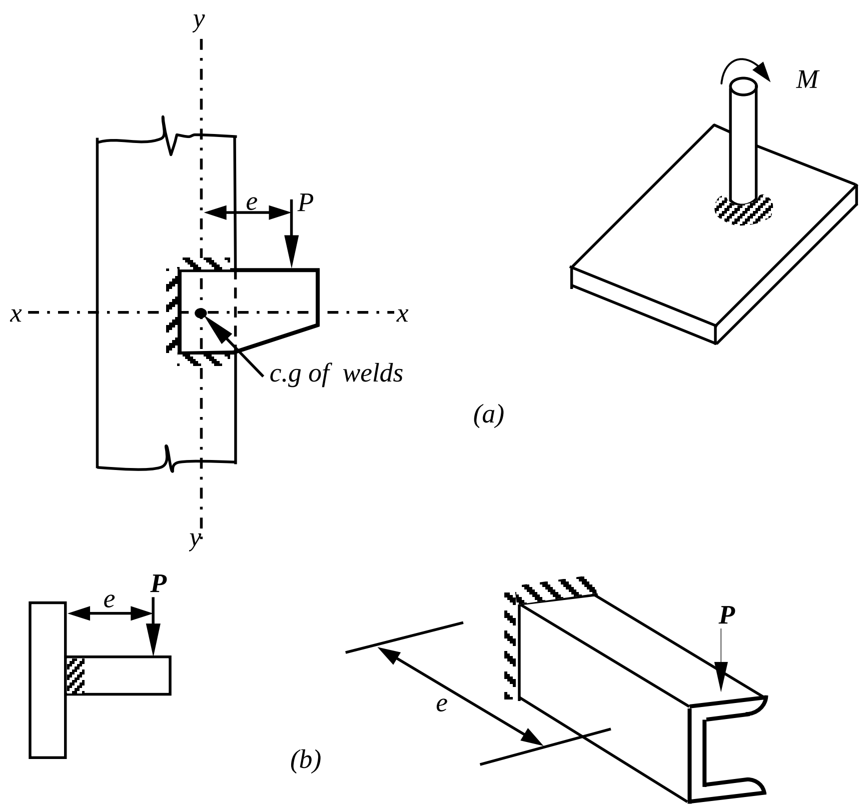


Fig. 12 (a) Welds subjected to shear and torsion, (b) Welds subjected to shear and bending

3.1 Shear and torsion

Considering the welded bracket shown in Fig. 12(a), an assumption is made to the effect that the parts being joined are completely rigid and hence all the deformations occur in the weld. As seen from the figure, the weld is subjected to a combination of shear and torsion. The force caused by torsion is determined using the formula

$$F = T.s/J = (Moment / Polar\ moment\ of\ inertia) \quad (2)$$

where, T is the tension, s is the distance from the centre of gravity of the weld to the point under consideration, and J is the polar moment of inertia of the weld. For convenience, the force can be decomposed into its vertical and horizontal components:

$$F_h = Tv/J \quad and \quad f_v = Th/J \quad (3)$$

where, v and h denote the vertical and horizontal components of the distance *s*. The stress due to shear force is calculated by the following expression

$$\tau = R/L \tag{4}$$

where, τ is the shearing stress and *R* is the reaction and *L* is the total length of the weld. While designing a weld subjected to combined shear and torsion, it is a usual practice to assume a unit size weld and compute the stresses on a weld of unit length. From the maximum weld force per unit length the required size of the fillet weld can be calculated.

3.2 Shear and bending

Welds, which are subjected to combined shear and bending are shown in Fig. 12(*b*). It is a common practice to treat the variation of shear stress as uniform if the welds are short. But, if the bending stress is calculated by the flexure formula, the shear stress variation for vertical welds will be parabolic with a maximum value equal to 1.5 times the average value. These bending and shear stress variations are shown in Fig. 13.

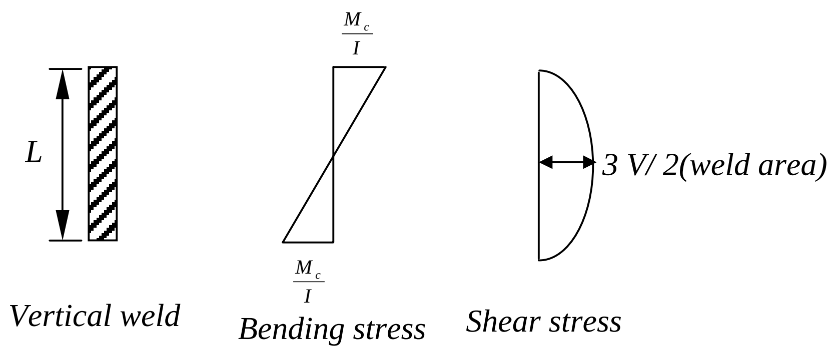


Fig. 13 Variation of bending and shear stress

It may be observed here that the locations of maximum bending and shearing stresses are not the same. Hence, for design purposes the stresses need not be combined at a point. It is generally satisfactory if the weld is designed to withstand the maximum bending stress and the maximum shear stress separately. If the welds used are as shown in Fig. 14, it can be safely assumed that the web welds would carry all the of the shear and the flange welds all of the moment.

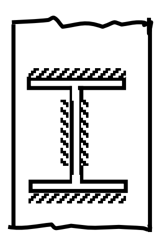


Fig.14 Weld provision for carrying shear and moment

4.0 TRUSS CONNECTIONS

Connections are very important in structural steelwork design. Therefore, the type of connections has to be finalized at the conceptual stage of the design to achieve maximum economy. This is especially true in the case of trusses. Efficient sections for the members alone do not result in economy unless suitable and economic connections are also designed.

Fabrication cost of connections would depend on the following factors.

- Precise cutting to length of sections(minimization of wasted length)
- Requirement of weld preparation
- Requirement of close control on weld root gaps
- Need for stiffening of the connection
- Chosen weld type

If some of the rigours in connections can be avoided, their cost can be reduced contributing to overall economy. Thus, for the economy of the entire truss, close attention has to be paid to connections.

4.1 Planar trusses

In a conventional riveted or bolted trusswork, gusset plates are generally provided at the connections. In a welded truss, it may be possible to omit the gusset plates. Tees or angles with unequal legs are normally used for the top and bottom chord members. The web member angles may be welded directly to the vertical sides of the chord members (Fig. 15).

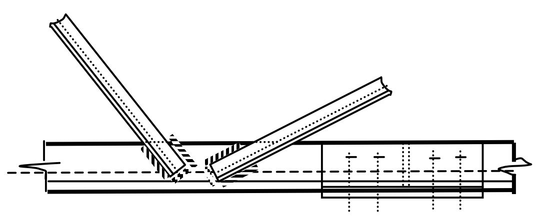


Fig.15 Direct connection of web members

When the trusses are very long, they may have to be fabricated in parts in fabrication shops. Such parts are assembled at site by bolting. Welded connection at the apex of a roof truss is shown in Fig. 16.

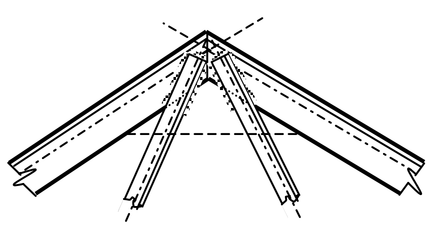


Fig.16 connection at the apex of a roof truss

The two rafters may either be butt welded together or a small plate is introduced at the connection facilitating fillet welding. If Tees are used for the rafters, the web members can be welded directly to the stalk of the Tees. Splices are sometimes provided in welded trusses for the purpose of transportation.

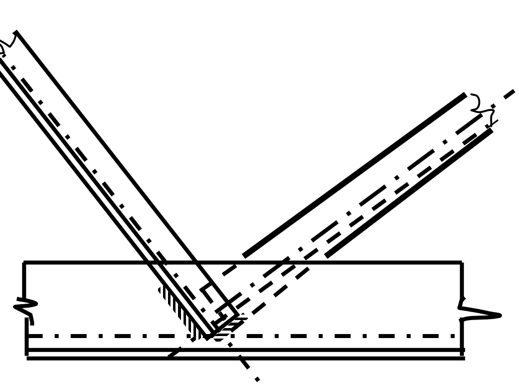


Fig. 17 typical welded truss connection

It is often possible to forego the use of gusset plates by joining the members eccentrically. This provision makes the trusses economical by making the connection simple; the resulting penalty on the member size is marginal. Alternative method to avoid eccentricity is to join the web members on opposite sides of the truss [Fig. 17]. A typical eaves level connection is shown in Fig. 18. For low-pitch roofs, this would produce a long connection. To avoid such a connection, often the connection is truncated and the resulting eccentricity is accounted for in the design.

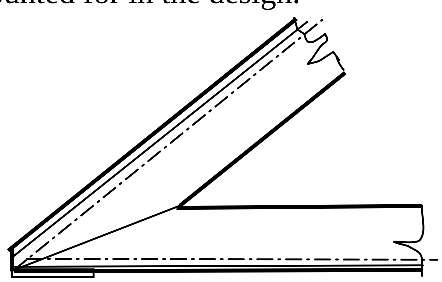


Fig.18 Eaves connection

4.2 Eccentricities in truss connections

Eccentricity in truss connections can occur due to two reasons: 1) element centroidal axes not intersecting at a point, and 2) connection centroid not coinciding with the element centroid.

Figure 19 shows the two cases of eccentricity. All the members joining the connection resist the moment caused by the eccentricity, *Pe*. It is distributed between them in proportion to their bending stiffness per unit length (*I/l*). Each member connecting to the joint should be designed to withstand the axial forces and its share of bending moment. If the connection centroids do not coincide with the member axes, they will be subjected to additional moments.

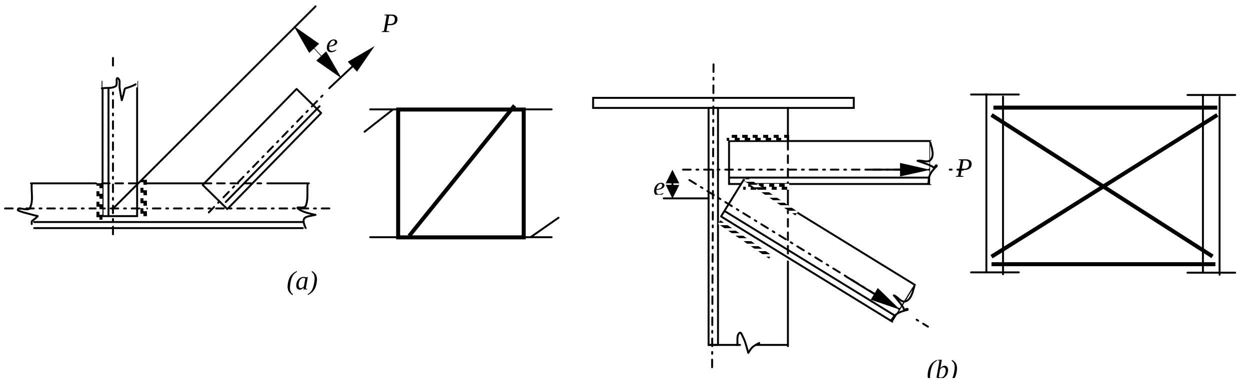


Fig.19 Eccentricities in truss connections (a) Pratt truss, (b) cross bracing between plate girders

A simple way to consider both moments due to eccentricity is to design for axial load and then use interaction diagrams for weld groups subject to combined loading to determine the increase in weld size required to include the moments. It is often observed that there is minimal reduction in axial capacity. This could be comfortably absorbed in the factor of safety usually assumed for connection design.

5.0 PORTAL FRAME CONNECTIONS

5.1 General

Portal frames are widely used for steel industrial buildings. They are used in various spans and heights. However, the frame spacing is often limited to the range between 4.5 m and 7.5 m. Lot of optimisation studies of frame connections have been carried out both in theory and practice and the results of the study have been incorporated in the design of connections.

5.2 Connection locations

Connections in frames are provided at the eaves and apex locations. Based on different types of analysis (elastic, plastic etc.), it is known that critical moment generally occurs at the eaves. An elastic analysis with rolled prismatic sections does not provide economy due to the poor utilisation of the rafter capacity. A plastic analysis leads to redistribution of moments and generally a lighter section for the rafter. However, design of connection has to provide the requisite rotation capacity. Due to the difficulty in meeting this requirement, haunched sections are sometimes introduced. The added benefit of using lighter sections for rafters lead to considerable economy. Presently, non-prismatic members, fabricated by automatic means provide large economy; they also possess the required strength.

5.3 Connection at eaves and apex

Figs. 20 and 21 show two principal ways of transferring moments around the corner. For ease of understanding, the moments are replaced by a pair of forces in the tension and compression flanges of the beam and column. In the first case, equilibrium is achieved by shear on the corner panel. If a diagonal stiffener is added, then the overall equilibrium is obtained by the system of forces, shown in Fig. 21.

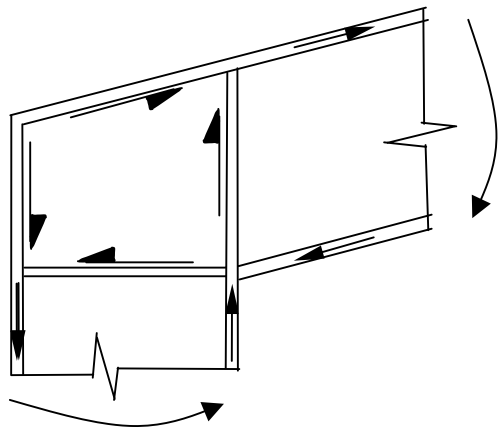


Fig.20 Moment transmission at corner using a shear panel

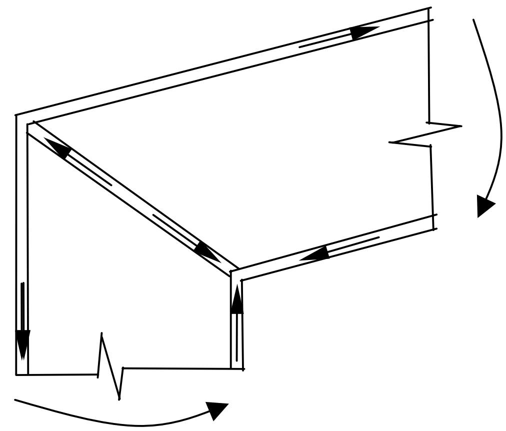


Fig.21 Moment transmission at corner using a diagonal stiffener

Various ways of providing welded eaves connection are shown in Fig. 22.

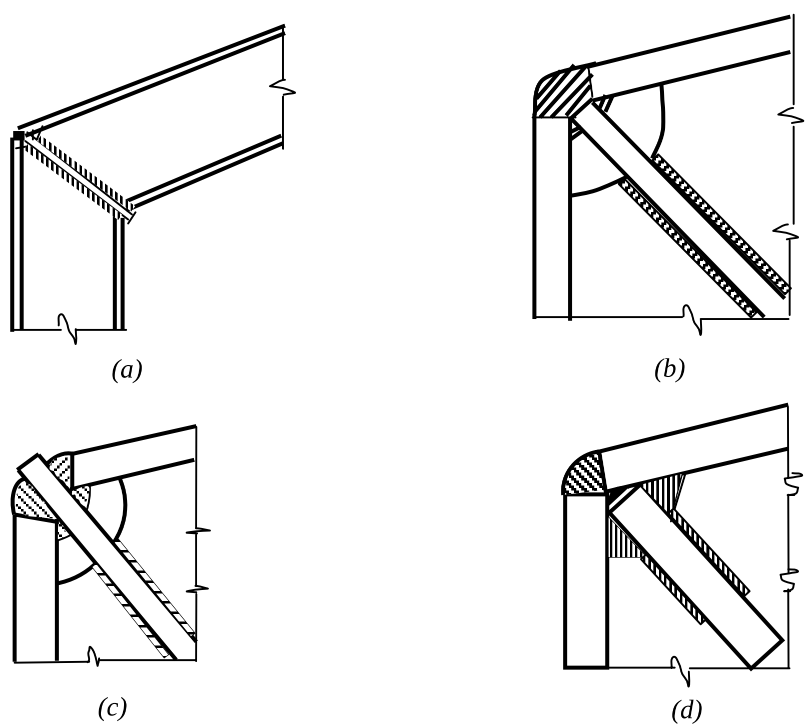


Fig.22 Various eaves connections of portal frames

The most efficient method is shown in Fig. 22(*a*). Here, a diagonal stiffening element, 10-16 mm thickness, is provided. The butt weld configuration for the tension flange is shown in Fig. 22(*b*). For thicker dimension plates, the weld configuration in Fig. 22(*c*) may be used. Figure 22(*d*) shows a welded connection where the butt joint is made initially and then the stiffeners are added.

Welded apex connections are made very similar to eaves connections and the various considerations are also same. Various butt- and fillet- welded connections are shown in Fig. 23.

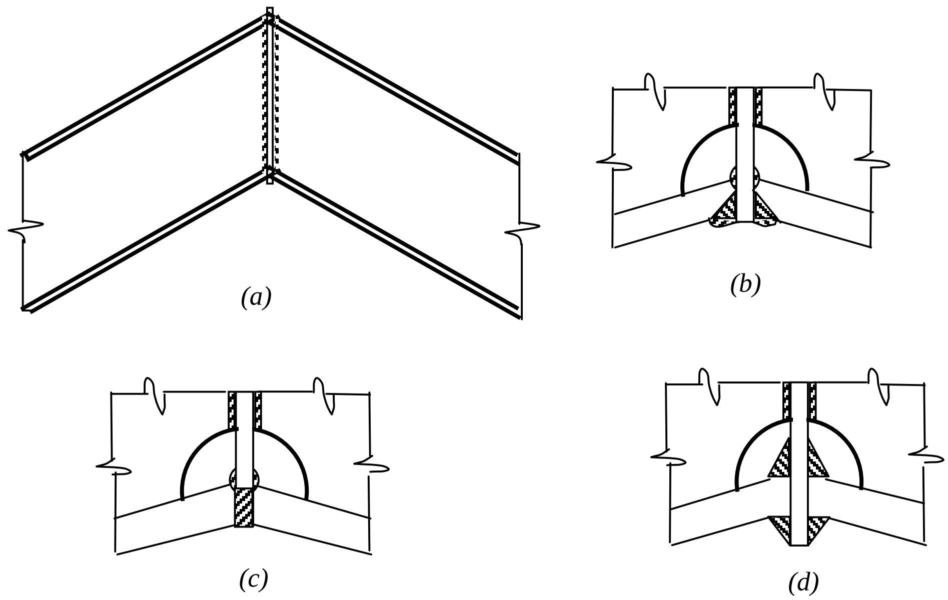


Fig. 23 Welded apex connections. (a) General arrangement (b) – (d) different approaches to the tension flange connection

6.0 COLUMN AND BEAM SPLICES

6.1 General

The need for splices arises due to the reason that structural sections are available only in specific lengths. Splices are provided, as far as possible, at locations away from critical sections. In the cases of beams and columns, the section at which the bending moment is minimum is chosen for locating splices. For beams, the section at splice location should have a higher capacity than the shear force at the location. If the member being spliced is subject to instability, such as in a column, the splice is necessarily to be located close to a point of restraint. If, for any reason, the splice needs to be provided away from a restraint, it may require special consideration.

6.2 Column splice

If a compression member is loaded concentrically, then theoretically that member does not need splicing. Compression will be transmitted from a top member to the bottom one by direct bearing. Column sections could be kept one on top of another and they function satisfactorily. However, this ideal case cannot be adopted in practice due to various reasons. Firstly the load is never truly axial; secondly the bearing surfaces of adjacent sections, however well prepared, are not perfect. Also, a real column is subjected occasionally to either laterally applied loads or eccentric loads. Hence, adjacent members of the column should be definitely connected.

6.2.1 Butt welded splices

Full penetration welds can be provided; however, it is economical to make the column ends flush with each other and apply partial penetration welds [Fig. 24(*a*)]. Direct bearing will transfer most of the load if the ends are faced.

6.2.2 Welded splice plate joint

It is possible to make use of welded splice plates for making column splices. They are used at column ends, which are faced for bearing [Fig. 24(b)].

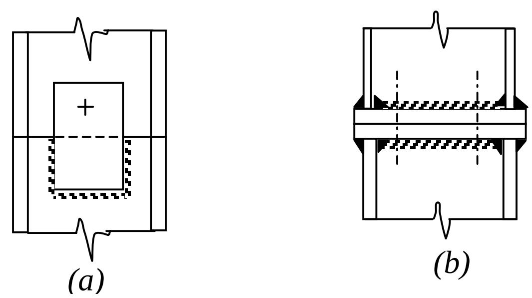


Fig. 24 (a) butt-welded column splices, (b) welded splice plate connection

6.3 Beam and girder splices

Rolled beams are fabricated in one piece and they do not generally need splicing. But, in very long beams, splices may be required. In such cases, splices are located in regions of the beam where the bending moment is low. The structural requirements of these regions are also not very stringent. Requirements for rolled beams are simpler compared to those for plate girders.

Splicing of webs is very common in plate girders. This may become necessary due to several reasons. Few of them are: 1) the required length of plate is not available, 2) the girder may be cambered at the splice, and 3) the thickness of the girder may be varied. When splicing is done in the shop, the web can be spliced independently of the flanges. In other words, flanges need not be spliced at the point of web splicing.

In practice, there are two main methods for the design of beam splices. In one case, the splice is designed to resist the calculated moment and shear at the point of splice. The capacity of the designed splice may be much less than that of the full capacity of the section. In the second case, the splice may be designed to develop full resistance of the section in both shear and flexure, though at the location of the splice, the actual moment and shear will be much lower.

6.3.1 Types of beam splice

6.3.1.1 Butt-welded connections

The simplest, but the most expensive, form of connection is by connecting the elements with full-strength butt welds. Due to the high cost, its use is limited to cases where aesthetics are important. Proper arrangement is necessary for temporary support during welding. The staggered form as shown in Fig. 25 is often recommended. For good alignment, welds are omitted for a short distance on either side of the connection. They are completed after the main welding is over. In order to minimise distortion effects due to transverse weld shrinkage, flange welds are done before web welds.

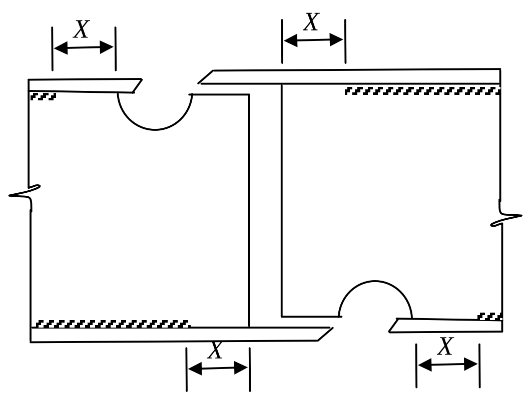


Fig.25 Staggered form of arrangement for temporary support

6.3.1.2 Welded splice plate connection

For lightly loaded beams, splice plates can be used for connections (Fig. 26). This would overcome the necessity of good alignment for butt-welded connections. But provision of double splice plates for this method of connection is difficult and hence limited to light beams.

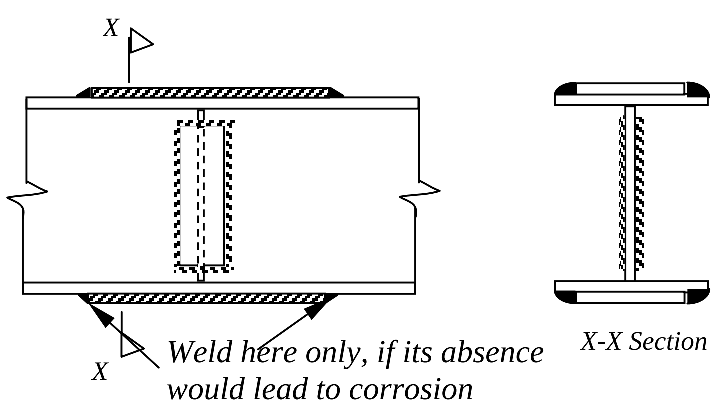


Fig.26 Welded splice plate beam connection

6.4 Hybrid connections

In this form of connection with splice plates (Fig. 27), the plates are attached to half of the splice by welding (mostly done in fabricating shops) and field bolting makes the other half of the connection. HSFG bolts are generally used for this connection to provide good stiffness. Since only single plates can be used, this type of connection has to be restricted to light beams. As both welding and drilling of holes are necessary at the fabrication shops, this type of connection is not very common.

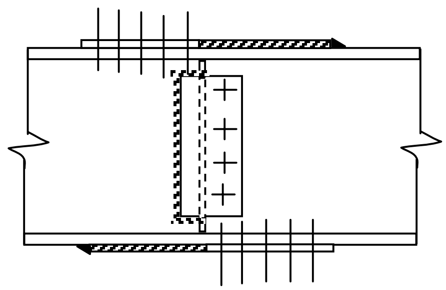


Fig.27 Hybrid connection

7.0 WELDED BEAM-TO-BEAM AND BEAM-TO-COULMN CONNECTIONS

This section deals with beam-to-beam and beam-to-column connections commonly used in steel buildings.

7.1 Types of beam connections

According to their rotational characteristics under load, beam-to-column connections are broadly divided into three classes: simple, semirigid and rigid. A connection, which is free to rotate and has no moment resistance, is called a simple connection. On the other hand, a rigid connection has complete moment resistance and does not rotate at all. A semirigid type connection has moment resistance, whose value falls in between the simple and rigid types.

In reality, connections are never completely rigid or flexible. They are always in between these two extreme cases. The classification of connections, as presented above, are made on the basis of a percentage of moment developed to complete moment resistance. An approximate rule for classification is:

- 0-20% moment resistance – simple connections
- 20-90% ,, – semirigid connections
- > 90% ,, – rigid connections

Rotational characteristics of connections cannot be obtained theoretically; experimental studies are conducted and moment-rotation relationship curves are plotted for each type of connection. Each of the three types of connections is briefly described in the following.

7.1.1 Simple connections

These are very flexible and under loads, the beam tends to rotate freely downwards by a large amount. Although this assumption of rotation is generally made, there is some amount of moment resistance, which is neglected. They are assumed to resist shear only. Examples of flexible connections are shown in Fig. 28. A common practice is to shop-weld the web angles to the beam web and field-bolt them to the column.

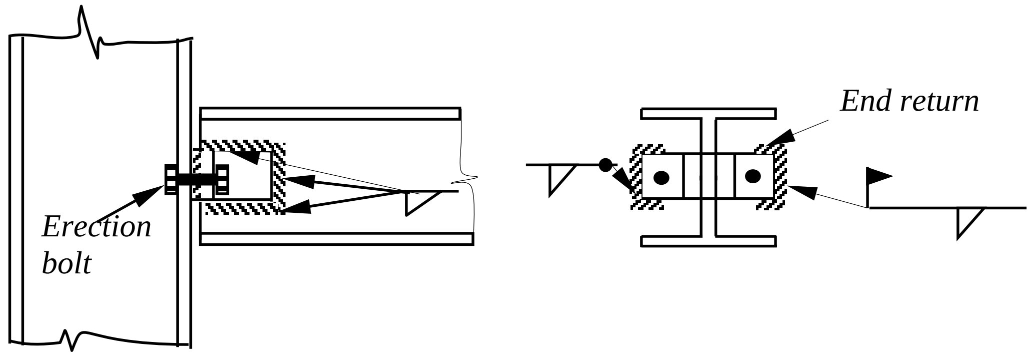


Fig. 28 Framed simple connection

7.1.2 Semirigid connections

These types of connection offer considerable resistance to end rotation and hence develop end moments (Fig. 29). In design practice, the designer assumes that the connections are either fully rigid or fully flexible in order to simplify the analysis. If the designer were to consider the actual nature of connection, namely semirigid, he could bring about considerable amount of moment reduction as shown in Fig. 30. As shown in the figure, the beam with different percentages of rigidity is loaded uniformly.

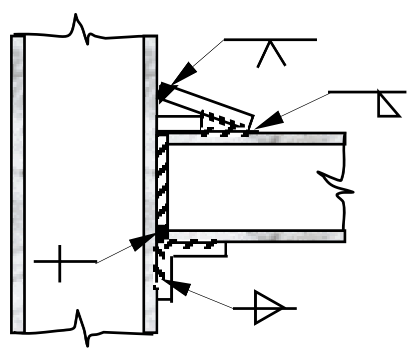


Fig. 29 Welded semirigid connection

Though most of the connections are semirigid, no moment reduction is assumed in calculations. Two main reasons for doing so are:

- it is very difficult to show how much moment reduction a connection is capable of providing, and
- There is no easy method of analysis to consider the varying percentages of moment restraint.

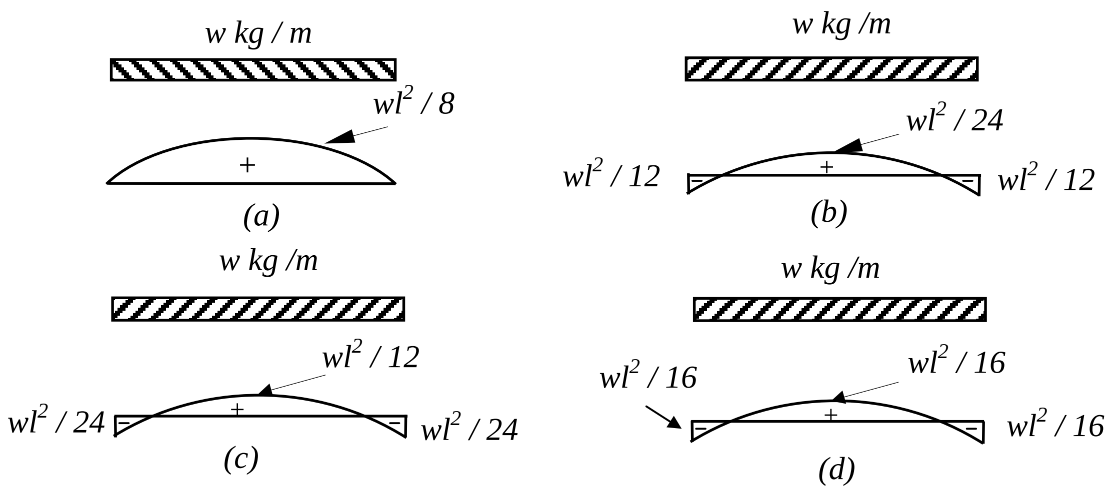


Fig.30 (a) simple connections(0%) (b) rigid connections (100%) (c) semi rigid connections (50%) (d) semirigid connections (75%)

7.1.3 Rigid connections

Rigid connections do not allow rotation at the beam ends and hence transfer the moment fully of a fixed end. This type of connection is usually provided in tall buildings, for which wind resistance is developed by the continuity of members provided by the rigidity of the connection. A welded rigid joint is shown in Fig. 31.

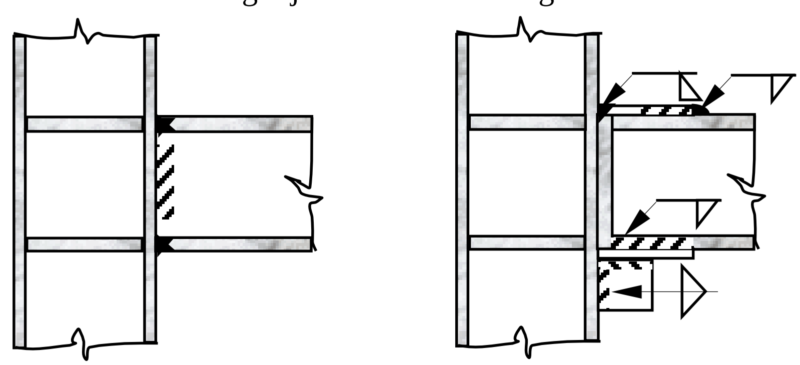


Fig.31 Welded moment - resisting connection

7.2 Types of welded beam connections

There are several methods of making welded connections between beams and girders, and between beam and columns. Some of them are:

- Web angles
- Beam seats
- Stiffened beam seats, and
- Moment resistant connections

While the first three are designed as simple connections with only shear transfer (known as shear connections) the last category transfers moment also (referred to as moment connections)

For designing a connection, a designer has to clearly understand the force paths in the structure and their transmission through the connections. Two of the very common examples are:

- Bending forces are resisted mostly by beam flanges and to transfer these forces the welds must be provided at the beam flanges
- To transfer shear forces in the beam, which occur primarily in the webs, the weld has to be positioned on the webs.

While designing welded connections, the designer has an option to weld the member directly to the other member without using connecting plates or angles. Direct connection of members is often difficult due to lack of proper fit in the field. Therefore, members are connected using connecting plates or angles, if necessary by clamping. Further, direct connections may not offer flexibility necessary in shear connections.

7.2.1 Welded web angles

Beams, which are connected to either columns or girders by means of web angles (Fig.32) are assumed to have simple end supports. These web angle connections are designed to transmit shear only and do not provide any resistance to rotation. Generally, these angles are shop-welded to the beam and then field-connected to the columns or girders with high strength bolts. The distance by which the angles project out from the beam web (usually about *12 mm*) is known as setback.

Erection bolts are used to erect these beams and often they are located at the bottom of the angle (Fig. 32) so that they induce least rigidity to the angles. For some reason, if they are provided at the top, they can be removed after the erection of the beam.

In order to achieve simply supported end conditions, the beam ends should be capable of rotating; to facilitate this only thin angles (e.g ISA 90×90×6), which can deflect easily are used for making the connection. Generally, *100 mm* size legs are used for connecting the beam and the leg size at the column or girder side is a little longer. The length of the angle is usually kept equal to the beam depth, which must also be sufficient for welding. The weld size will be 1 to 3 mm smaller than the web angle thickness. The required maximum weld length is the beam depth minus its width.

Though it is often assumed that the web angles are subject to only shear force, which is equal to the end reaction, it is not so. There will be also moment due to eccentricity of the reaction forces with respect to both shop and field welds, as shown in Fig. 33.

Due to this rotation effect, the field welds cause web angles to press against the beam web at the top and tear apart at the bottom, thus inducing horizontal shear in the fillet weld. General practice is to assume the neutral axis is located at the distance *1/10 L* from the top of the angle. The horizontal shear at *1/10* height is taken as zero and maximum at the bottom of the angle.

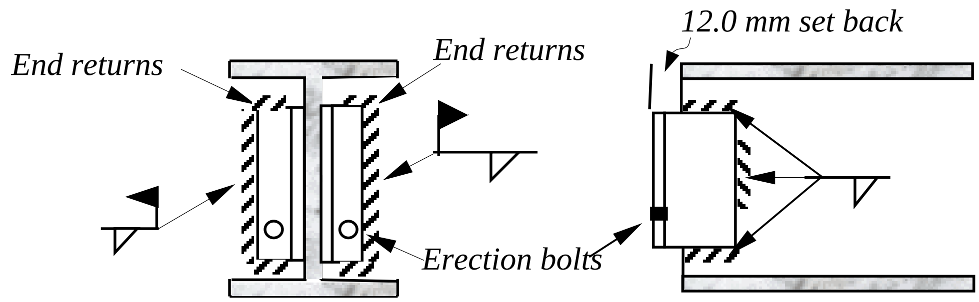


Fig. 32 Welded web angles

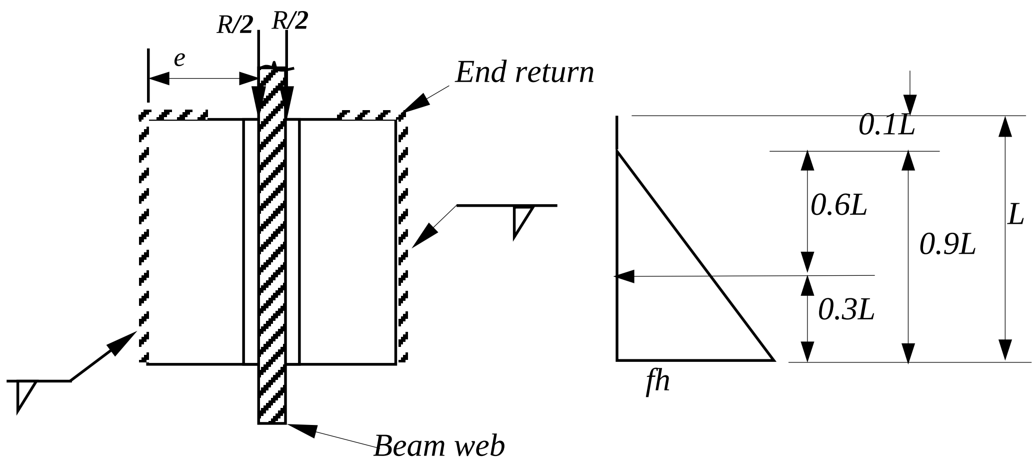


Fig. 33 eccentricity of reaction forces

The top pressure acts at the *1/10 th* point from the top and the centre of gravity of the horizontal shear is located at *0.7 L* from the top of the angle. Equating the moment produced by these forces and the externally applied moment, the value of the horizontal shear f_h can be determined.

$$0.5 \times 0.9\, L \times f_h \times 0.6\, L = R/2 \times e \qquad (5)$$

$$f_h = Re/0.54\, L^2 \qquad (6)$$

The vertical shear stress f_s in the weld will be *R/2* divided by the height of the weld. The weld size can be determined after calculating the maximum stress on the weld by the equation

$$F_r^2 = f_h^2 + f_s^2 \qquad (7)$$

7.2.2 Welded seated beam connections

This is a flexible type of connection (Fig. 34). The beam seat is normally shop-welded to the column and after seating the beam properly it is field-welded or bolted. Seat angles, also known as shelf angles, are provided with bolt holes for erection purposes. Normally, an angle is welded at the top of the beam also to provide the required lateral support; no load is carried by this angle. Flexible angles (say 100 x 100 x 6 mm) are provided at the top to allow the beam to rotate under the imposed load.

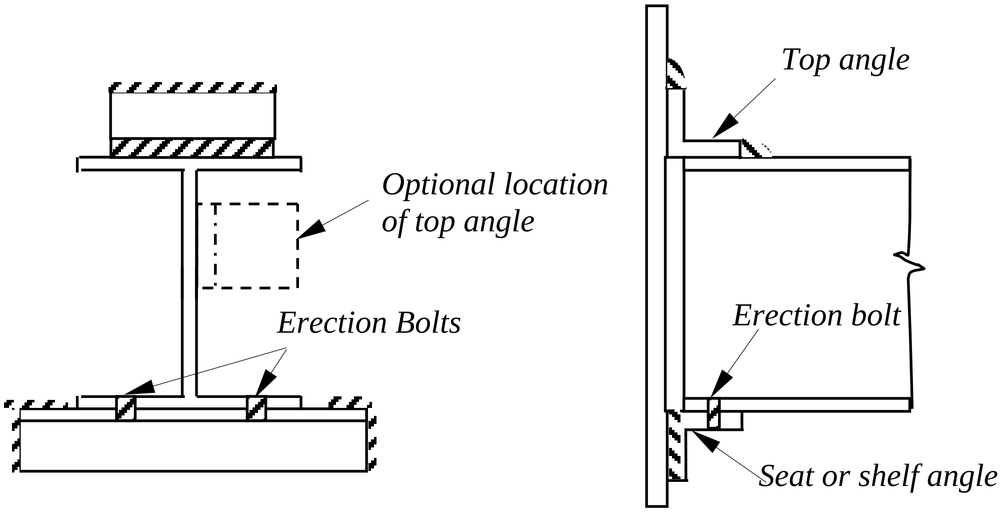


Fig. 34 Welded seated- beam connections

For relatively light loads, only the two vertical ends of the seat angle need be welded. For allowing the beam to rotate, the top angle is welded only on its toes. The bearing length normally provided is 75 to 100 mm. The horizontal leg of the angle is subjected to a moment due to the beam reaction. The critical section for bending is assumed to be at the toe of the fillet weld, which is approximately t+ 10 mm from the back of the vertical leg. The length of the vertical leg can be calculated from the weld size required. This is often done by trial and error after assuming an initial depth of the weld. The stress variation in the vertical welds is normally assumed such that the neutral axis occurs at mid depth.

7.2.3 Welded stiffened beam seat connections

For relatively heavy beam connections (above 200 kN to 250 kN), stiffened beam seats are used. Stiffened beam seats are generally made of Tee sections or plates welded to the shape of a Tee (Fig. 35). The stiffened beam seat is designed to provide sufficient bearing length considering the web crippling of the beam. As the beam is loaded and it rotates, the centre of gravity of the beam reaction moves away towards the outer edge of the seat. As an approximation, the centre of gravity of the reaction is assumed at the centre of the bearing length. The stems of the Tee sections are provided a thickness equal to the web thickness of the beam. The depth of the stem is arrived at depending upon the weld length required. The flange of the beam seat is kept a little wider than the beam flange to provide simple field welds. A minimum of two weld size extra width may be provided on either side of the beam flange.

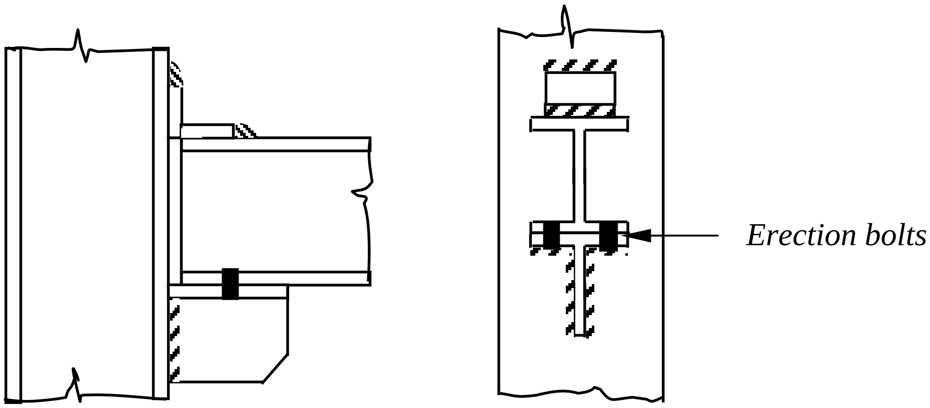


Fig. 35 Stiffened beam seat connection

The weld for the seat must be designed to resist the applied shear and the bending moment. The vertical welds being very close to each other cannot resist flexure. Therefore, the bottom side of the flange is welded to a distance of ¼ to ½ of the web depth. These horizontal welds significantly improve the resistance of the connection to twisting. No weld can be provided at the top of the flange because of the possible interference with beam.

7.2.4 Moment-resistant connections

Fig.36 shows a moment-resistant connection. Such connections are used for fully continuous structures, where the connections are designed to resist full moments. The efficiency of a welded connection is fully utilized in this type of connection. Firstly, the negative moments at the supports tend to reduce the positive moments at midspan resulting in usage of smaller size members. Secondly, in case of overloading of structures, the plastic redistribution is possible in continuous structures.

For the connection in Fig. 36(*a*), the tensile force (caused by hogging moments) at the top flange of the beam is transferred to the top flange plate by fillet welds and from the plates to the column by groove welds. The top plate is often provided a taper for ease in welding [Fig. 36(*b*)]. Under gravity loads, the force on the welds will be tension T in the top flange and compression C in the bottom flange. In case lateral forces are considered, the welds must be designed to resist both tension and compression. While providing

moment-resisting welded connections, it is usual to butt weld the beam flanges flush with the column at one end and connect the beam at the other end with the connection details as described above.

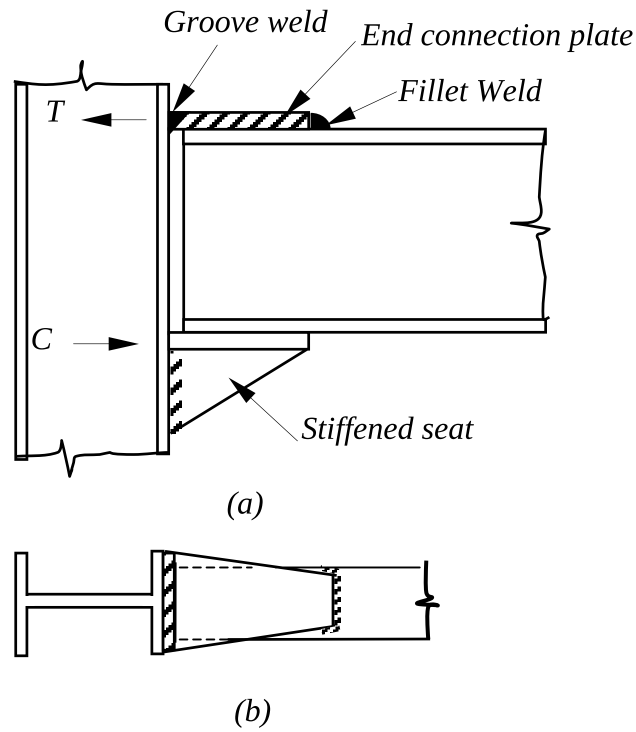


Fig.36 (a)&(b) Moment resisting connections

Two possibilities may arise whereby the moment resistance of the connection maybe reduced. The bending of the column at the connection point would reduce the moment resistance. The top connection plate while pulling away from the column tries to bend the column flange [Fig.37 (*a*)] and the middle part of the weld is overstressed. For these reasons, the column flange is normally stiffened with plates opposite to the beam flange [Fig. 37(*b*)]. The connection between different parts of a built-up beam or plate girder should be proper in order to meet to the demand.

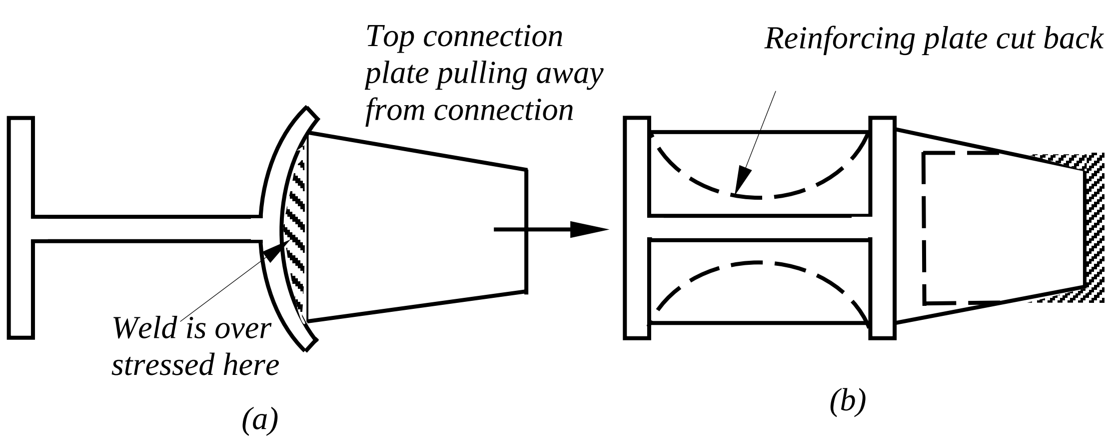


Fig. 37 (a) Overstressing of the weld, (b) Column flange Stiffened with plates

8.0 INSPECTION

As pointed out in a previous chapter, it is vital that all welded connections must be inspected to ensure that they conform to specifications. Good welding procedures can be formed from the relevant codal provisions and the guidelines from the manufacturers of welding supplies and equipment. The recommended procedures, if followed, would ensure sound welds.

The inspection and control should start just from the start of welding and continue through the welding procedure. If essential, a pretest of the connection should be done to ensure required performance.

9.0 CONCLUSIONS

In this chapter, behaviour and design of major types of welding, namely butt and fillet are explained in detail. The necessity and methods of providing beam and column splices are described. Truss connections are presented. Types of beam-to-beam and beam-to-column welded connections are described. Practical methods of providing welded connections are also presented. Worked examples are included for clarity.

10.0 REFERENCES

1. Owens G.W., Knowles P.R. and Dowling P.J (Editors), "Steel Designers' Manual" (Fifth Ed.), Blackwell Scientific Publications, London, 1992
2. Owens G. W. and Cheal B.D., "Structural Steelwork Connections", Butterworths, London, 1989
3. McCormac J.C., "Structural Steel Design" (Third Ed.), Harper & Row Publishers, New York, 1981
4. Salmon C.G. and Johnson J.E., " Steel structures-Design and Behaviour" (Third Ed.), Harper Collins Publishers, New York, 1990.
5. IS:816-1969, "Code of Practice for use of metal arc welding for general construction in mild steel, Bureau of Indian Standards.

32 WELD - STATIC AND FATIGUE STRENGTH -III

1.0 INTRODUCTION

A component or a structure, which can withstand a single application of load, may fracture if the same load is applied a large number of times. This type of failure is classified as fatigue fracture. Thus, fatigue failure can be defined as the number of cycles or the time taken to attain a pre-defined failure criterion. A more precise definition of fatigue is given as the process of progressive localised permanent structural change occurring in a material subjected to conditions, which produce fluctuating stresses and strains at some points and which may culminate in cracks or complete rupture after sufficient number of fluctuations. Hence, fatigue phenomenon is experienced by structures, which are subjected to moving loads, such as bridges and crane girders, or structures subjected to cyclic loads such as offshore platform structures and machinery supporting structures.

Fatigue as a failure mechanism was identified initially in the rolling stocks and tracks of railways and subsequently in railway bridges. Recently, fatigue problem has been experienced in highway bridges, with a few failures of bridges. In many instances, due to timely repair measures, catastrophic collapse of structures due to fatigue has been avoided.

The lower the stress ranges i.e., the difference between the alternating maximum and minimum stresses, the larger the number of cycles the structure can withstand before the occurrence of fracture. In the case of fatigue fracture of engineering structures, the following are the two main types of fatigue loading.

- High–cycle low–stress fatigue.
- Low–cycle high–stress fatigue.

In a typical high-cycle fatigue problem, the endurance limit of the material after millions of cycles of load application is of concern, whereas in low-cycle fatigue, fracture before 10^5 cycles is the consideration. In high-cycle fatigue problems, the critical portion of the structure is subjected to frequent repeated loads, such as welded tubular joints in steel offshore platform structures subjected to wave loading. In such areas, several million (100 million) cycles are achieved during the lifetime of the offshore structure (about 25 years). An example of low fatigue fracture is the hull structure of a ship. When structural components are exposed to corrosion environment, such as seawater, the synergistic effect of corrosion and fatigue, known as corrosion fatigue becomes a serious problem.

Since fatigue failure evaluation is influenced by a number of uncertainties, an accurate prediction of fatigue life is difficult even for a very simple detail. Fatigue failure prediction is difficult in structural components due to the following uncertain features:

- The effect of environment in which the structure is functioning.
- Difficulty in accurate calculation of internal stresses developed due to external forces at critical locations in the structure
- The time to failure of the structure.

Two basic approaches for fatigue life assessment of structural components are: 1) the $S - N$ method and 2) the method of fracture mechanics. The $S - N$ method of life prediction is based on empirically derived relationships between applied stress ranges (S) and number of cycles of load application (N). The fracture mechanics approach takes into account the crack growth rate of an existing defect as it propagates under the cyclic loading.

In this chapter, topics such as characteristics of fatigue, methods of evaluation of fatigue life, improvement of fatigue resistance, fatigue-resistant design etc. have been covered.

2.0 CHARACTERISTICS OF FATIGUE FRACTURE

Structural materials undergo mechanical changes when subjected to cyclic stresses and trigger many engineering failures due to fatigue. Poor design and fabrication are the prime reasons for the failures. A fatigue failure occurs as a result of various mechanisms, which take place in three stages during the life of a structure. As a result of cyclic loading a microscopic defect initiates, then propagates in a gradual manner, resulting finally in an unstable fracture. Cracking originates mostly on the surface at a point of stress concentration – a hole, notch, keyway, scratch, weld bead, sharp fillet etc. Crack initiation may occur, occasionally, at an interior point such as a defect in a weld. In most welded steel structures the crack initiation phase does not exist as crack–like weld defects are invariably present in them. Thus the fatigue life of a connection containing welds is entirely due to crack growth. Final failure usually occurs in a tension region when the reduced section is no more sufficient to carry the peak load.

Many repetitions of the stresses - of the order of millions – may be required for complete rupture. Similarly, the time required for final collapse may be short or many years in some cases. The maximum stress at the fracture location would be well below the value obtained under static loading. It is very difficult to detect a fatigue crack even up to the point of failure. Since there is very little plastic deformation around the crack, there is no evidence of the presence of crack, repeated through large deformation.

A small crack initiated grows slowly with the repetition of stress cycles. A fatigue crack is said to be transgranular i.e., its grows within grains rather than along the grain boundaries. As the crack propagates, the cross-section reduces and the stress on the reduced cross section increases. As a result, there will be an increase in the rate of crack propagation. The final rupture occurs when the remaining area is no longer sufficient to support the applied load.

The above features of fracture due to fatigue can be seen on the fracture surface. The fracture surface may be either crystalline or fibrous depending upon whether the fracture is brittle or ductile. In the close neighbourhood of the crack's origin, the fractured surface has a smooth, silky appearance, which is produced by rubbing of the surface as the crack propagates (Fig.1). The smooth region grows progressively into a rougher texture as the distance from the origin increases. An examination of this surface would reveal the presence of concentric rings or beach markings around the fracture nucleus and radial lines emanating from it.

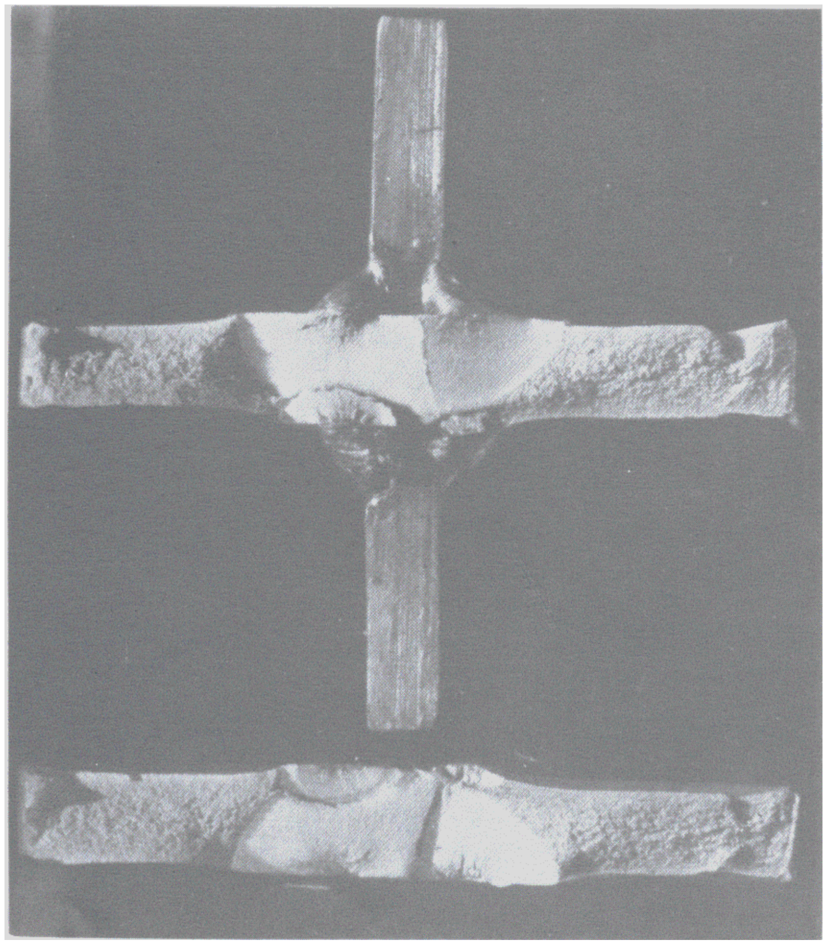
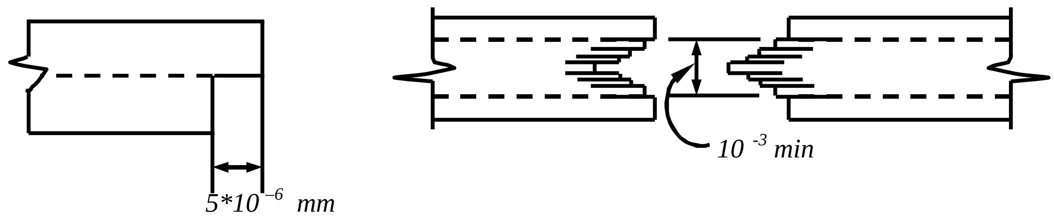


Fig.1 Fractured surface of a specimen

3.0 THE MECHANISM OF FATIGUE

Presently there is no rational theory available for fatigue failure prediction relating stresses and material properties. This is due to the complex mechanism involved in the fatigue process. The mechanism of fatigue is explained briefly in the following.

Due to stress concentration effects, the stress in a localised region in a structural element may attain the value needed for plastic flow. The nominal stress or the stress without concentration effects may be below the proportional limit. At this stage, slip might occur in an unfavorably oriented crystallographic plane due to excessive shear stress on the plane. This might be a fine slip i.e., a slip of order 10^{-6}*mm* below adjacent region of the crystal [Fig.2 (*a*)]. The reversal of stress at this time might partially set right the disorientation. Repetition of the stress cycle and the resulting back and forth slip on closely spaced parallel planes will cause slip band to develop [Fig.2 (*b*)]. This would form a notch. A microscopic crack may form because of the stress raising effect of the notch or the notch itself becomes deeper. Once the crack has formed, the process is further intensified.



(a) Fine slip ***(b) Coarse bond produced by alternating slip***

Fig. 2 Fatigue mechanism

4.0 FACTORS INFLUENCING FATIGUE BEHAVIOR

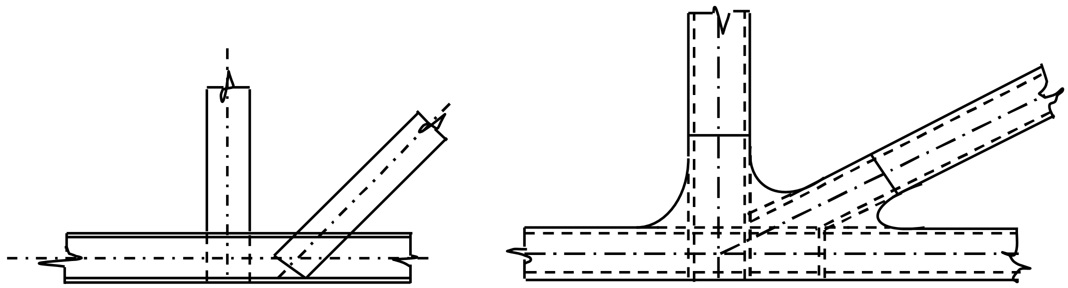
The fatigue behavior of various types of structures, members and connections is affected by a large number of factors, many of which may produce interrelated effects. The parameters that influence fatigue behavior are: stress range, material, stress concentration, rate of cyclic loading, residual stresses, size, geometry, environment, temperature, and previous stress history. These are explained briefly in the following.

4.1 Stress range

The most important factor governing the rate of crack growth is the stress range in the vicinity of the crack tip. Hence in a fatigue design, the stress concentration effect has to be reduced and the stress range has to be realistically estimated. The importance of stress concentration is brought out in detail in the following section.

4.2 Stress concentration

The geometry and the consequent stress concentration have a large impact on fatigue lives of structural members and their connections with other members. Stress distribution is generally different from that adopted in design mainly due to stress concentration. Such points of stress concentration under cyclic loading undergo reduction in strength, often leading to fracture. The importance of stress concentration is illustrated in Fig.3. In Fig.3 (*a*), typical detail of a connection in an industrial roof structure is shown. Here the design is mainly governed by static strength. The local stress at the connection could be 5 to 10 times the average stress calculated by the simple theory for static design. The structure, by way of local yielding, accommodates safely the discontinuities and stress concentration. Fig.3 (b) shows a fatigue-sensitive bridge structure connection detail.



(a) Industrial roof structure (b) Bridge structure

Fig. 3 Typical connection details

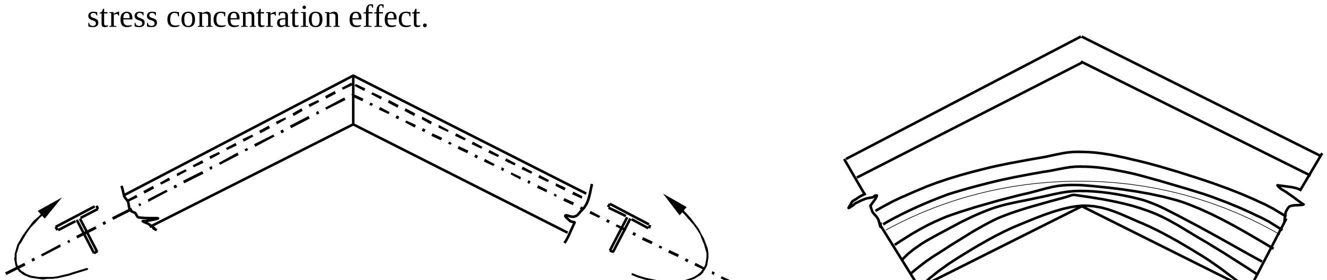
Here, stress concentration has to be reduced by careful design. Detailed design considering both primary axial stress and secondary bending stress has to be performed. Weld profiling should be smoothened to minimize local discontinuities. It is appropriate now to consider three levels of stress concentration.

4.2.1 Structural action

In the static analysis of a structure, elastic analysis is carried out based on compatibility concepts. The relative deformation between neighboring elements is often ignored. These local deformations develop additional strains and stresses. Secondary members, whose effects are often ignored in the static analysis, develop stresses due to the relative deformation in neighbouring elements. These additional stresses cause stress concentration. In the roof connection shown in Fig.3 (*a*), a simple static design would ignore the incompatibility that is caused by the restraint to the end rotation of the individual elements. The resulting bending stresses may be of similar magnitude as axial stresses in a truss with larger size sections. Therefore, in a fatigue design these bending stresses must be considered.

4.2.2 Macroscopic stress concentration

This type of stress concentration arises due to geometric interruptions to stress flow. The smooth flow of stress trajectories are modified due to changes in cross section, notches, holes and other discontinuities. Fig. 4 shows a discontinuous structure with the attendant stress concentration effect.

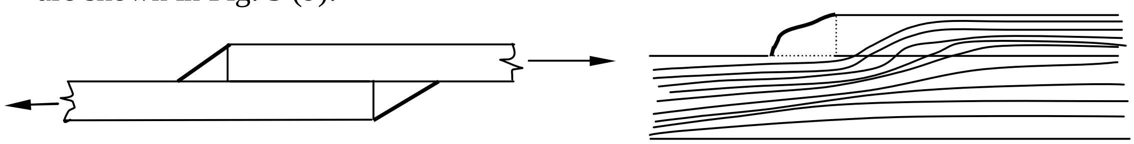


(a) Beam arrangement ***(b) Stress flow at change in direction***

Fig. 4 Bending stresses in a discontinuous beam

4.2.3 Local geometric stress concentration factor

This level of stress concentration is related to crack tip effects and other microscopic defect. They occur generally within a weld or Heat Affected Zone (HAZ) [Fig. 5(*a*)]. Due to the sharpness of these defects, the stress concentration effect will be mostly localized. The stress trajectories through welds are affected by the structural action of the weld, by its geometry, surface roughness and by the relative position of the joined elements. These are shown in Fig. 5 (*b*).



(a) Fillet weld arrangement ***(b) Tensile stress flow***

Fig. 5 Stress concentration at the toe of a fillet weld

4.3 Frequency of Cyclic Loading

The frequency of loading does not influence fatigue strength significantly when the applied stress range is low and frequency is less than 50 Hz. But when the stress range is high capable of producing plastic deformation with each cycle of loading (low cycle fatigue) an increase in frequency produces an increase in apparent fatigue strength.

4.4 Residual stresses

The effect of residual stress varies considerably, depending upon the material, state and magnitude of residual and applied stresses. The effect of compressive residual stress generally is to increase the fatigue resistance for lower levels of stress. But for higher levels of stress close to yielding, its effect is negligible. The residual tensile stresses do not affect fatigue resistance except in cases where residual tension reduces the stress range in cyclic loading.

4.5 Size

In the case of small members subjected to flexure, the fatigue resistance increases due to the small size and the resulting increase in strain gradient. For large flexural members and axially loaded members the size effect is very small.

4.6 Material

In general the fatigue resistance of structural steel is proportional to its ultimate strength. Tests have shown that under ideal conditions, fatigue limit is approximately 50% of ultimate stress. However, other factors may considerably alter this fatigue limit.

The general relation between fatigue limit and ultimate stress is given below

$$F_1 = 140+0.25\, F_u \qquad (1)$$

where F_1 = fatigue limit for zero to maximum tensile loading in *Mpa*

F_u = ultimate stress of the material in *MPa*

5.0 EFFECT OF FATIGUE LOADING ON STRUCTURAL MEMBERS AND WELD CONNECTIONS

Fatigue behaviour of axially loaded tension and compression members are quite different. Compressive cyclic loading will not generally produce fatigue failure, except when tensile stresses are present. The fatigue behavior of tensile members is generally decided by their connections, except where the member itself has points of stress concentration. Properly detailed and fabricated flexural members have a higher fatigue resistance. Welded details such as cover plate, splices, and stiffeners reduce the fatigue resistance significantly.

Welds invariably contain small crack-like defects; hence crack initiation stage does not exist. Only the number of cycles for the crack to grow to the point of unstable fracture constitutes the fatigue life. Residual stresses of yield stress level are always present in the vicinity of welds. Therefore stress cycling is always from the yield stress downwards and fatigue life is a function of stress range only. Fatigue life varies with the type of weld details due to the varying nature of the defects in the different details.

Welded connections have comparatively lower fatigue resistance, if they are not properly detailed and fabricated. The most important factor that influences the fatigue resistance of welded joint is the geometry and the resulting stress concentration effects. Fatigue failure at a welded joint may occur in any one or combination of the following.

- Failure in the deposited metals from the porosity or slag inclusions and defect locations, which act as stress concentration points.
- Failure in the line of fusion, due to lack of proper fusion or microscopic cracks.
- Failure in the heat affected zone due to crystalline change in the base metal.
- Failure at the toe edge of the weld, which is a stress concentrated point due to the joint design, weld contour undercut, etc.

The importance of smooth stress flow through a joint has been emphasized elsewhere. Butt-welded joints, where the stress flow is smooth, have much better fatigue resistance compared to fillet welded joints. The fatigue strength of a butt-welded connection may be further improved by finishing the weld flush with the surface of the plate by grinding.

6.0 FATIGUE ANALYSIS

6.1 General

Generally, structures are designed for strength and deflection criteria. In the case of structures prone to fatigue effects, a fatigue analysis is carried out to ensure that the fatigue life of the structure is more than the intended life of the structure. Fatigue analysis involves determination of nominal stresses in structural members, stress concentration factors at critical points and safe number of stress reversals before the onset of failure.

The two main approaches to fatigue life determination, namely, the $S - N$ curve method and the fracture mechanics method are explained in this section.

6.2 S – N Curves

Fatigue data is commonly presented in the form of an $S - N$ curve, where the cyclic stress range (S) is plotted against the number of cycles to failure (N). Various parameters of interest in fatigue are shown in Fig. 6. A typical $S - N$ curve is shown in Fig. 7.

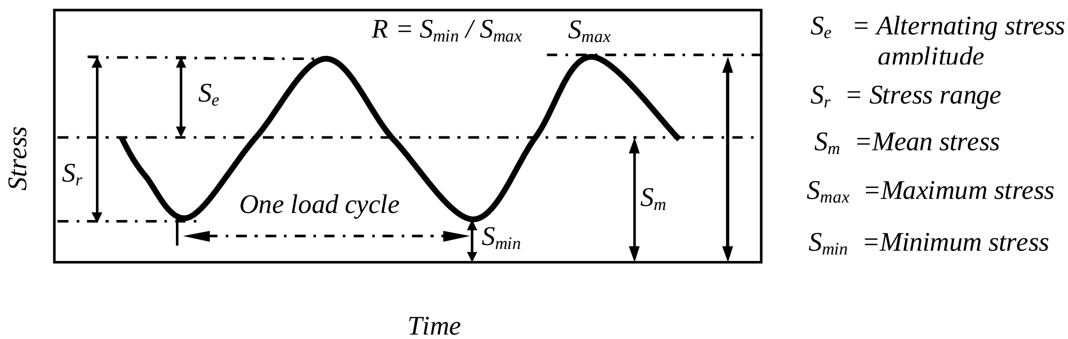


Fig. 6 Various parameters in fatigue

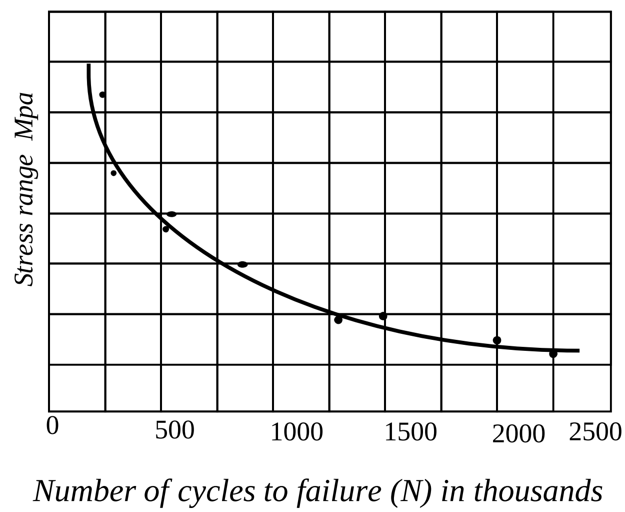
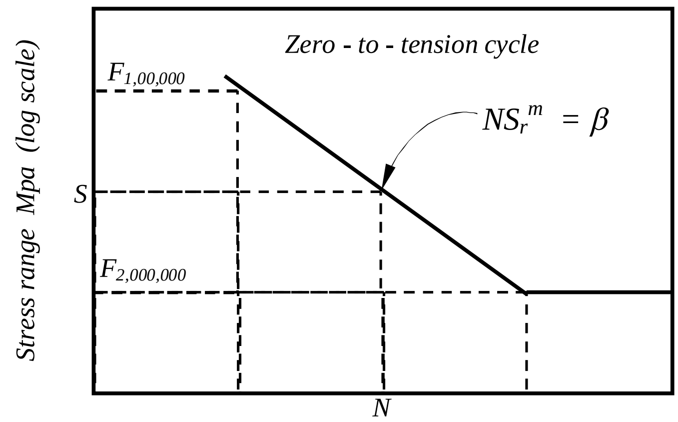


Fig.7 S – N Curve (Wohler curve)

In order to determine the fatigue strength of a welded joint configuration, under a given load condition, it is necessary to test a series of similar specimens. Each of the specimens is subjected to constant amplitude loading and the number of loading cycles required to produce failure in each specimen is recorded. The relationship between the applied stress, S and the number of cycles to failure, N, is obtained. Logarithmic scales are commonly used for both axes, namely, $LogS - LogN$ (Fig.8). Because $LogS - LogN$ relationship for many materials is approximately linear, most fatigue data are presented as log – log relationships.



$F_{1,00,000}$ = fatigue strength for 100,000 Cycles
$F_{2000,000}$ = fatigue strength for 200,000 Cycles

Fig. 8 S – N Curve presented on a log-log scale

For welded joints the relationship between fatigue life and applied stress range is linear over a wide range of stress and takes the form

$$N\,S_r^{\,m} = \beta \qquad (2)$$

where N = number of cycles to failure, S_r = applied stress range and m, β = constants depending upon the joint type.

Using such a relationship, the fatigue strength can be calculated over the range covered by the sloping line for any selected number of cycles of the same type of stress cycle, if the slope of the line and one point on the line are known. Only one type of stress cycle is represented by each $S – N$ curve. For understanding the general behaviour of a joint, it is necessary to develop $S –N$ curve for each type of stress cycle.

The data for the various $S – N$ curves are summarized in a Goodman's Diagram. Fig. 9 shows a Goodman's diagram for an as-rolled plate. It provides a complete representation of the effects of various types of stress cycles, from static tension, zero to tension and complete reversal.

In the Fig.9, the range of stress (maximum stress to minimum stress) is indicated by the vertical distance between the two sloping lines. At the extreme left side, the stress range is complete reversal from a compressive stress to a numerically equal tensile stress. At the extreme right side, the maximum stress line intersects the minimum stress line at the level of ultimate strength, and the stress range is zero (this representing the static yield stress). At the point where the minimum stress line intersects the zero stress line, the maximum stress represents fatigue strength under a pulsating load (zero to tension).

In any welded joint there are at least five locations at which fatigue cracks may initiate. These are at the weld toe in each of the two points joined, at the two ends and in the weld itself. Each is classified separately.

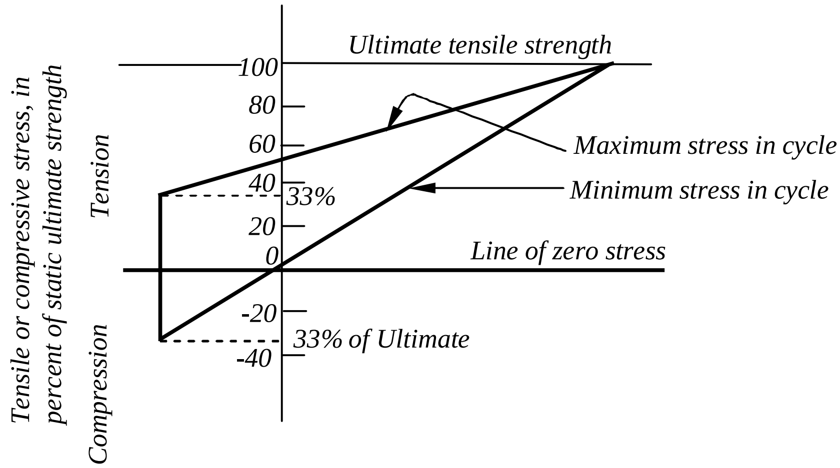


Fig.9 Goodman diagram – a composite representation of the effects of various types of stress cycles on fatigue life

6.3 Variable Amplitude Loading

So far, fatigue loading has been considered as a single fluctuating load of constant amplitude producing constant stress range. In practice, it is common for structures to be subjected to more than one type of loading and each type of loading may vary in intensity, i.e. they may be subjected to a loading spectrum of varying amplitudes or random vibrations. In situations where variable stress history is encountered, the sequence is broken into a stress range spectrum as shown in Fig. 10. In order to do this, cycle counting methods, such as Reservoir Method (for short stress histories by hand calculations) and the Rainflow Method (for analysing long stress histories by computer) are employed.

For structures subjected to variable amplitude of stresses, fatigue life has to be estimated by calculating the damage due to each band of the stress spectrum and summing them. The damage due to each band is ***n / N,*** where n is the number of cycles in the stress range during the design life and N is the endurance limit over that stress range. Palmgren – Miner rule is invoked to determine the failure of the joint. It states that to prevent failure the damage done by all the stress ranges must not exceed unity

$$n_1 / N_1 + n_2 / N_2 + n_3 / N_3 + \text{-------------------} + n_n / N_n \leq 1.0 \qquad (3)$$

In the case of variable amplitude loading, even non – propagating stress ranges (below fatigue limit) will also cause damage. The cracks formed under higher amplitude stress ranges will continue to propagate because the enhanced stress range at the crack tip (due to stress concentration) will be sufficient to continue the propagation even for very nominal stress ranges.

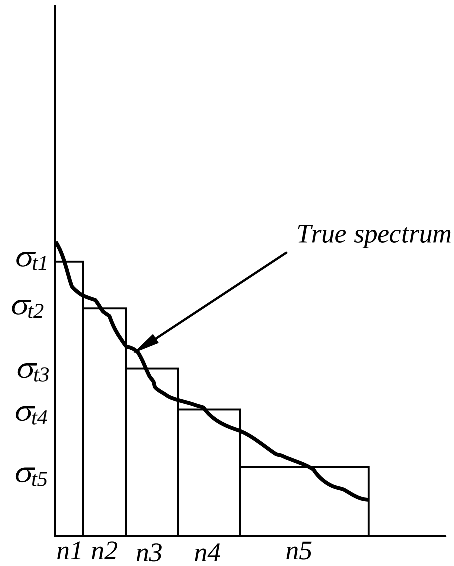


Fig. 10 Stress range spectrum

6.4 Fracture Mechanics Analysis

Fracture mechanics discipline, dealing with the behavior of cracks in materials and structures, has recently been employed for fatigue life assessment. Here the presence of defects or cracks is explicitly taken into account. A parameter is defined to represent the state of stress at the tip of the crack, called the stress intensity factor (K). This is represented in the following form

$$K = Y\,\sigma\,(\pi\, a)^{1/2} \tag{4}$$

where, Y = geometric correction factor
σ = nominal stress
a = crack length.

Fatigue life assessment by fracture mechanics is based on the observed relationship between the stress intensity factor range (ΔK) and rate of growth of fatigue cracks $da\,/\,dN$. This approach was proposed by Paris and Erdogan in a power law form for metallic materials in the Journal of Basic Engineering (1963), and is summarised below:

$$\frac{da}{dN} = C(\Delta K)^m \tag{5}$$

where $\frac{da}{dN}$ = crack extension per cycle

C, m = crack growth constants, and

$(\Delta K) = K_{max} - K_{min}$ are the maximum and minimum stress intensity factors respectively in each cycle. The exponent *m* has a value in the range of 3-4. Since the crack growth rate is related to (ΔK) raised to the exponent, determination of (ΔK) becomes crucial for meaningful crack growth prediction. Values of *C* and m can be determined by conducting tests on the materials being used. Typical values of *C* and m for ferrite steel are:

$m = 3$

$C = 3*10^{-13}$ for normal environment up to 100^0 C.

$C = 3*10^{-12}$ for aggressive environment up to 200^0 C.

If a typical da / dN versus (ΔK) relationship is plotted on a log-log scale, sigmoidal curve is obtained as shown in Fig.11. Below a threshold value of stress intensity factor range $(\Delta K)_{th}$, crack growth does not occur. For intermediate values of (ΔK) the growth rate is approximately linear.

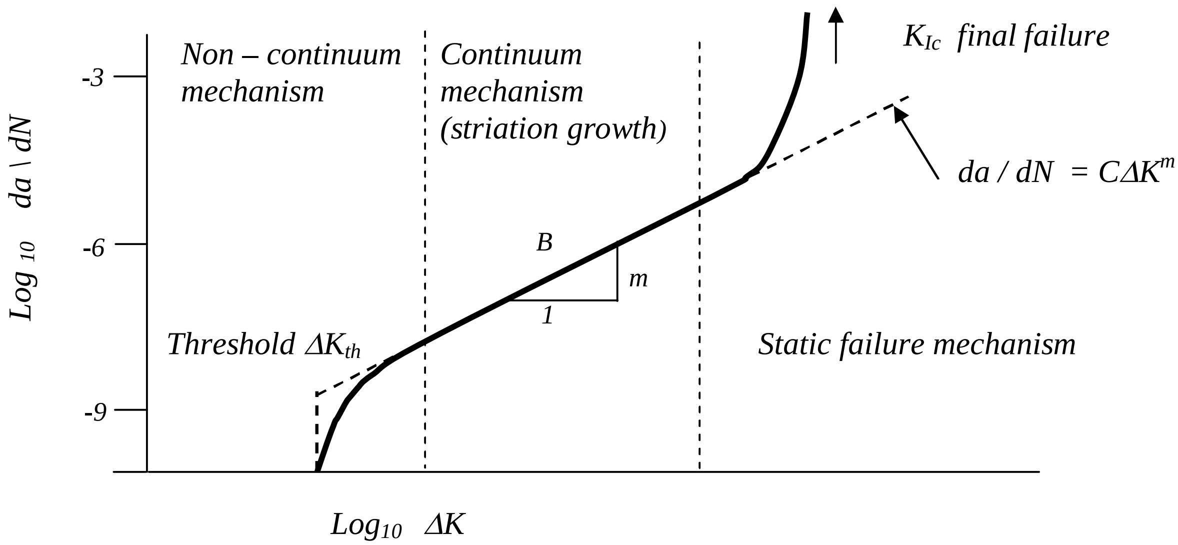


Fig. 11 Schematic representation of crack growth

For a crack at the toe of a welded joint, stress intensity factor range can be expressed as

$$(\Delta K) = M_k \, Y \, \Delta S_r \sqrt{\pi a} \tag{6}$$

where, M_k = A factor representing stress concentration effect depending up on crack size, plate thickness, joint geometry and loading.

Y = Correction factor, which depends up on the crack size, shape and loading.

ΔS_r = Applied stress range.

a = Crack depth.

The fatigue strength of the joint can be predicted by substituting Eqn. (6) in Eqn. (2). Rearranging and integrating we get

$$\int_{a_i}^{a_f} \frac{d_a}{\left[M_k \, Y\sqrt{\pi\, a}\right]^a} = C \Delta S_r^{\,m} N \qquad (7)$$

where, a_i = initial crack depth and a_f = the final crack depth corresponding to failure. Hence if a welded joint contains a crack, the above equation can be used to predict its fatigue life, considering that it consists of crack growth from the initial size, which is assumed as known.

7.0 INDIAN STANDARD PRACTICE

7.1 General

Indian standard IS: 1024 – 1979 “Code of Practice for Use of Welding in Bridges and Structures Subjected to Dynamic Loading” covers the use of metal arc welding in bridges and structures subjected to fatigue loading. It has presented guidelines for the design of structures under fatigue environment.

7.2 Loading

Working stress should be reduced to allow for the effects of fatigue. Allowance for fatigue should be made for combination of stresses due to dead load, live load, and impact load including secondary stresses due to eccentricity of connection etc. Elements of a structure may be subjected to a very large variety of stress cycles both in range and magnitude. Each element of the structure must be designed for the number of cycles of different magnitudes of stress to which the element is to be subjected during the expected life of the structure. Since the fatigue strength of a welded structure depends upon the type and location of joint details, these should be decided in advance in order to arrive at the permissible stresses under repeated loading. For the purpose of determining the allowable stresses the details are classified into seven classes. IS:1024-1979 gives values of allowable number of stress cycles for different values of stress ranges for the various classes of construction details which are described below.

7.3 Classes of Welded Construction Details

Class A

(1) Members fabricated with continuous full penetration longitudinal or transverse butt welds with the reinforcement dressed flush with the plate surface are classed as A. It is a requirement that the weld should be proved free from defects by non-destructive examination. Further, the members should not have exposed gas cut edges.

(2) Welds should be dressed flush by machining or grinding, or both, which should be finished in the direction parallel to the direction of applied stress.

Class B

(1) Members fabricated with the continuous longitudinal butt welds with full penetration made with either submerged or gas shielded metal arc automatic process but with no intermediate start-stop positions within the weld length are classed B.

(2) Members fabricated with continuous longitudinal fillet welds made with either submerged or gas shielded metal arc automatic process but with no intermediate start-stop positions within the weld length are also classed B.

Class C

Members fabricated with continuous longitudinal butt welds including fabricated beams with full web penetration of the web to flange welds, with start-stop position within the length of the weld are classed C.

Class D

(1) Members fabricated with full penetration transverse butt welds made in the shop by manual welding with electrodes other than deep penetration electrode, providing that all runs are made in the down hand position and that there is no undercutting are classed D. This does not include welds made on a backing strip if the backing strip is left in the position.

(2) Members fabricated with full penetration transverse butt welds, other than those in (1), and having the weld reinforcement dressed flush and with no undercutting.

(3) Members with continuous longitudinal fillet welds with start-stop positions within the length of the weld.

Class E

(1) Members fabricated with transverse butt welds, other than those mentioned in class D(2), or with transverse butt welds made on a backing strip are termed class E.
(2) Members fabricated with full penetration cruciform butt welds (Fig. 12) are also classed E.

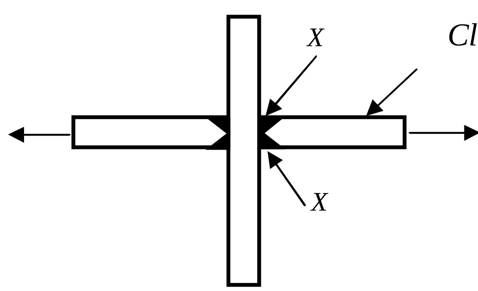


Fig.12 Class E fulls penetration cruciform butt weld.

Class F

The following categories are classed F:

(1) Members with T type full penetration butt welds (Fig.13).
(2) Members with intermittent longitudinal or transverse non-load carrying fillet or butt welds, except for the details covered in class G (Fig.13).
(3) Members connected by transverse load carrying fillet welds.
(4) Members with stud connectors.

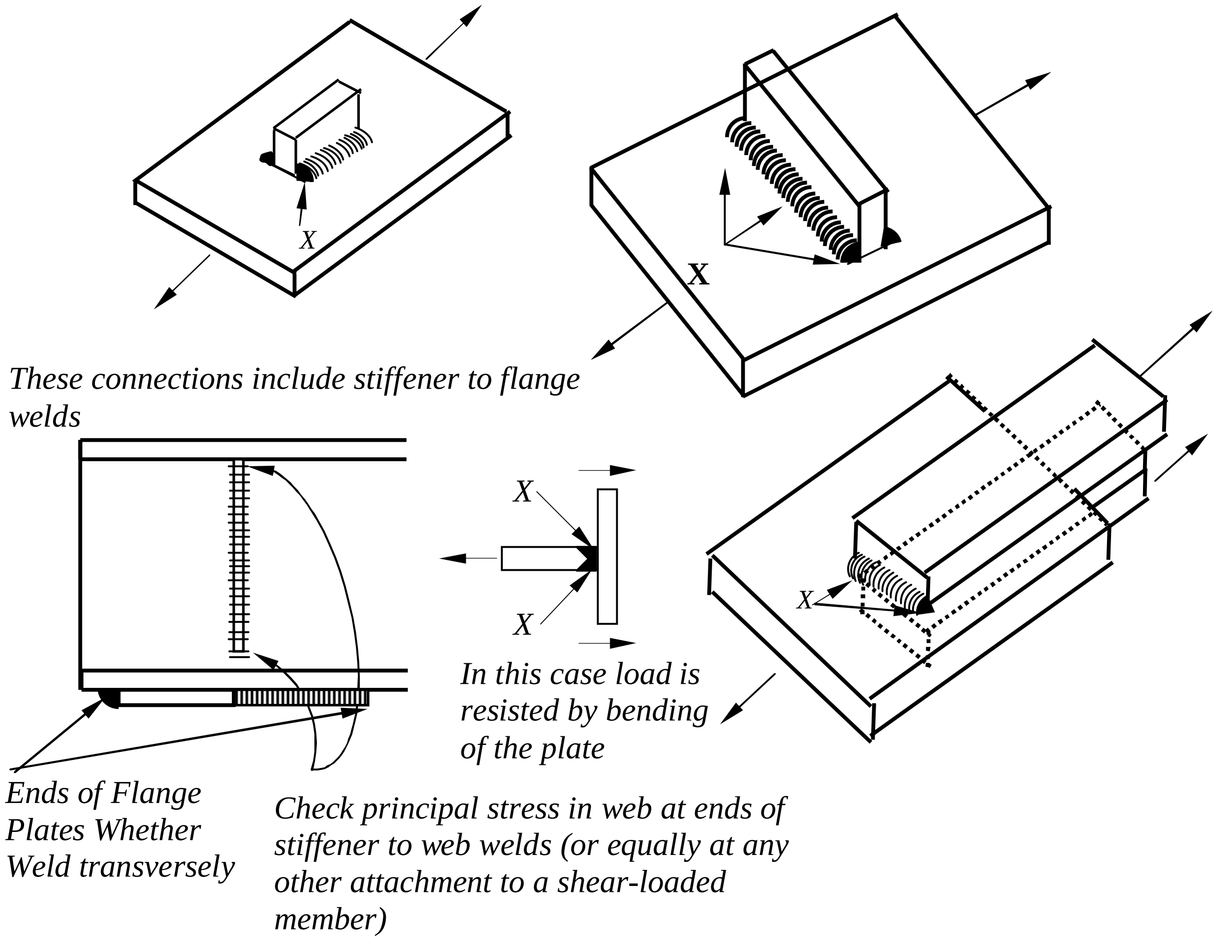


Fig. 13 Typical class F weld details

Class G

The following categories are classed G

(1) Members connected by longitudinal load carrying fillet welds [Fig. 14(*a*)].
(2) Members connected by load carrying cruciform fillet welds [Fig. 14(*b*)].
(3) Members with intermittent non-load carrying fillet or butt welded attachments on or adjacent to their edges [Fig. 14(*c*)].

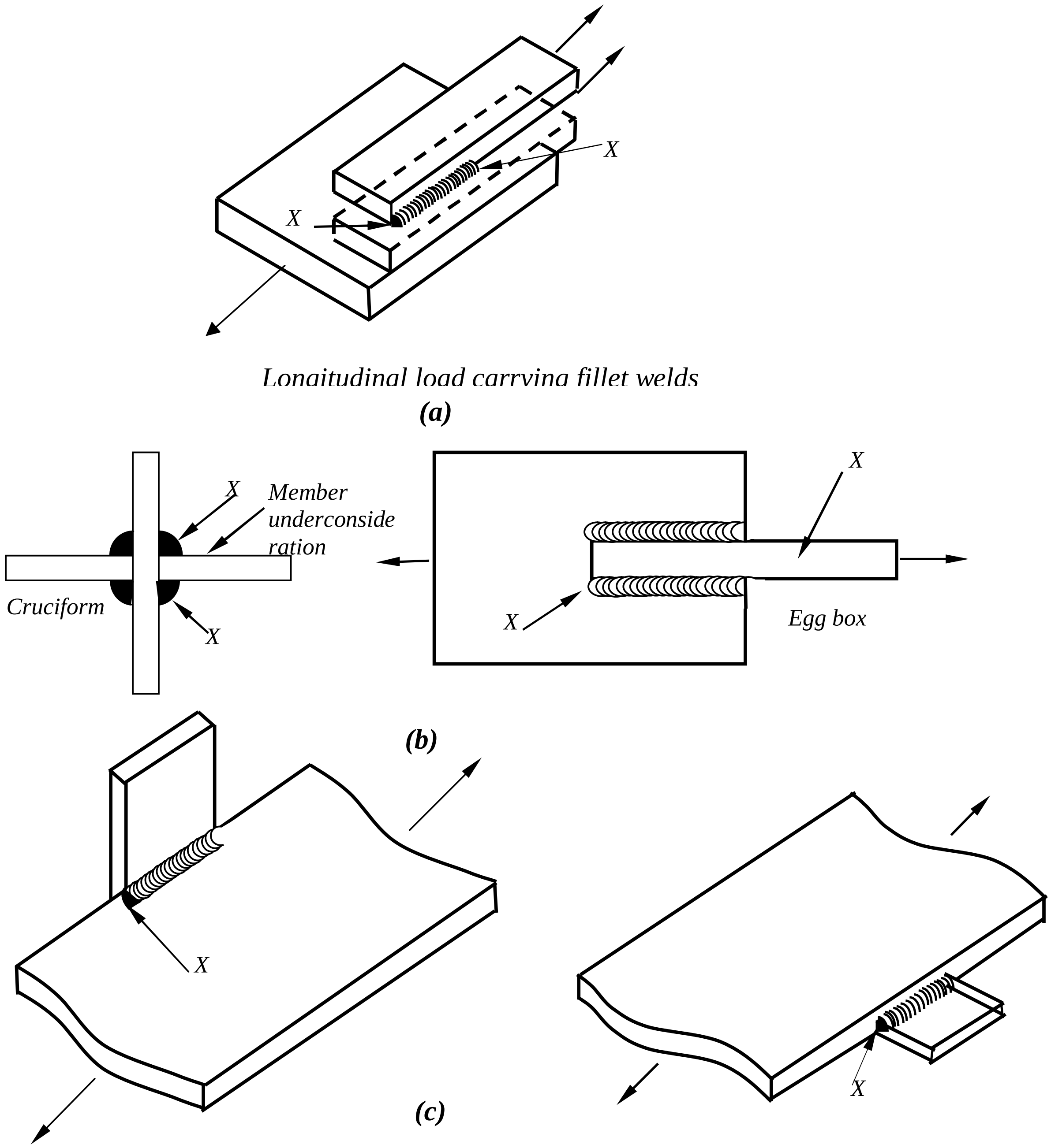


Fig. 14 Typical Class G weld details

For each of the stress ranges, the maximum allowable number of cycles N_1, N_2 ……N_n should be determined from the tables given in the IS Code 1024 - 1979.

Considering the expected number of cycles for each stress level as n_1, n_2 ……n_n, the element should be so designed such that

$$n_1/N_1 + n_2/N_2 + n_3/N_3 + \text{-------------------} + n_n/N_n > 1.0 \qquad (8)$$

Permissible stress in welds

Permissible stresses have been specified for butt and fillet welds. Butt and fillet welds are treated, as parent metal with thickness equal to throat thickness and the stresses should not exceed those in the parent metal. For fillet welds, the permissible stress in the fillet welds should not exceed the permissible values given in IS: 1024-1979 and reproduced here.

For combined shear and bending stresses, the equivalent stress f_e is given by:

$$f_e = \sqrt{\left(f_{bt}^2 + 3\ f_q^2\right)} \ \text{or}\ \sqrt{\left(f_{bc}^2 + 3\ f_q^2\right)} \qquad (9)$$

f_{bt} = tensile bending stress.
f_{bc} = compressive bending stress.
f_q = shear stress.

Table-1 Permissible shear stresses

Steel conforming to	*Permissible shear stress in Mpa*
IS: 226 – 1975 *IS: 2062 – 1969*	*108*
IS: 961 - 1975	*131*

For combined shear, bearing and bending stresses, equivalent stress is given by:

$$f_e = \sqrt{f_{bt}^2 + f_b^2 + f_{bt}\,f_b + 3\ f_q^2} \ \text{or}\ \sqrt{\left(f_{bc}^2 + f_b^2 - f_{bc}\,f_b + 3f_q^2\right)} \qquad (10)$$

The equivalent stress f_e should not exceed $0.9F_y$ where F_y is the yield strength of the steel. The value of f_e for various steels and yield strengths are given in Table 2.

Table 2 Equivalent stress, f_e

Steel conforming to	***F_y (Mpa)***	***f_e (Mpa)***
IS: 226-1975 and IS:2062-1969	*230*	*215*
	240	*225*
IS: 961-1975	*250*	*230*
	280	*245*
	330	*295*
	340	*310*
	350	*330*

8.0 IMPROVEMENT OF FATIGUE STRENGTH AND REMEDIAL TECHNIQUES

The fatigue performance of a connection can be improved by using weld improvement techniques. Presently, there are no practical design rules for the improvement techniques. It is recommended that no advantage be taken of these improvements in the initial design. The designer may fall back upon their contribution in case of severity of design fatigue condition or unsatisfactory service behaviour. Though steels are manufactured with excellent weldability, the low fatigue strength of a welded connection is mainly due to the short crack initiation period. Crack initiation life can be extended by

- Reducing the stress concentration of the weld.
- Removing crack-like defects at the weld toe.
- Reducing tensile welding residual stress or introducing compressive stresses.

The various methods of improvements can be classified into:

- *Weld geometry improvement* by grinding, weld dressing and profile control.
- *Residual stress reduction* by peening and thermal stress relief

Much of the current information on weld improvements has been derived from tests on small-scale specimens. In real structures, there will be large residual stresses that may affect fatigue life. Whereas peak stresses develop at the weld toe of a small joint, in a large multi-pass joint, peak stress may occur in any of the several beads and cracks initiate anywhere in this region. Brief details of some of the weld improvement techniques are summarized below.

8.1 Weld toe grinding

In cases where fatigue cracks tend to grow from the toe of welds in single-pass welding or at the junction between beads in multi-pass welding, 30% improvement in fatigue strength can be achieved by grinding the weld toe for the former and the entire region for the latter. A factor of 2.2 can be taken for the fatigue life. The purpose of grinding is to remove the small cracks that are invariably present and act as initiators for future fatigue crack growth. These small sharp cracks are generally 0.5mm deep. The treatment should produce a smooth concave surface to the weld toe with a depth of at least 0.5mm below the bottom of any visible undercut. In fillet-weld connections, the application of grinding should not result in any reduction in the throat area designed. Where fatigue crack may grow from the weld root (such as in a partial penetration weld) there will be no improvement in the fatigue life using this technique.

8.2 Weld dressing and profiling

The profile of a fillet weld is smoothened by dressing. This method is used where the design is very fatigue–sensitive and the costs of repair of fatigue damage would be very high. The technique is used especially in welded steel offshore platforms.

8.3 Weld toe remelting

Weld toe remelting has been found to increase the fatigue strength. This is achieved by providing a low contact angle in the transition between the plate and the weld. This also removes slag inclusions and undercuts at the toe.

Weld toe remelting either by TIG or Plasma Arc Dressing involves remelting the weld toe region with a torch held at angle of 50^0 or 90^0 to the plate. No fillet material is used in the process. The difference between the TIG and Plasma Arc Dressing is in the higher heat input required by the latter.

8.4 Hammer peening

Peening is the application of repeated hammering, often with a round-headed punch or hammer, to produce local yielding of the material. The hammering is applied to the weld toe or other locations, where fatigue cracks are likely to initiate. It has the effect of reducing the local residual stresses. Peening reduces the mean stress and therefore improves fatigue life. The improved fatigue properties are due to;

- Introduction of high compressive residual stresses.
- Flattening of crack-like defects at the toe.
- An improved toe profile.

Weld improvement techniques improve the fatigue life of weldments considerably.

8.5 Repairs to cracked welds

Repair requirements for cracked welds would be different for different circumstances and cannot be generalized. But the following general comments may be kept in mind while planning the repair of welds.

- Cracks, which may be present act as stress raisers.
- The repaired weld at the field location, done in harsh environment, might contain defects and will have a lower fatigue life.
- The repaired weld should be to a revised detail having a better fatigue classification than the original one.
- Additional stiffening or reinforcing should be provided to reduce the stress range on the detail where the crack has occurred.

9.0 FATIGUE-RESISTANT DESIGN

Bridges, offshore structures, towers and cranes are some of the major structures, which require consideration for fatigue design. The adverse effects of fatigue in these complex structures has been observed extensively and today there are efficient analytical tools available to calculate their fatigue lives. Fatigue life of a structure can be predicted with accuracy, if the loading sequences on the structure for the whole of its life is known fairly well; then the specific fatigue-critical locations can be identified and detailed inspections carried out.

In general, for welded structures subjected to fatigue loading, particular attention must be paid to welded joints. Fatigue and brittle fractures can start at discontinuities of shape, notches and cracks, which help develop high local stress. Fatigue life can be considerably increased by avoiding local peak stresses by good design and detailing. A good design should also take into account the fabrication procedure to be adopted for the structure.

Repairs of in-service structures are quite expensive and may even require closing down of the facility. Proper attention is required when providing secondary attachments to main steelwork. These attachment details are usually not given attention during the design as they are not very complicated. A number of failures due to fatigue crack growth has been observed from weld attachment in offshore structures. General suggestions are listed below for guidance while designing a welded structure with respect to fatigue strength.

- Adopt butt or single and double bevel butt welds in preference to fillet welds.
- Use double-sided in preference to single sided fillet welds.
- Aim to place weld, particularly toe, root and weld end in area of low stress.
- Avoid details that produce severe stress concentration or poor stress distribution.
- Provide gradual transitions in sections and avoid reentrant notch like corners.
- Avoid abrupt changes of section or stiffness of members or components.
- Avoid points so as to eliminate eccentricities or reduce them to a minimum.
- Avoid making attachments on parts subjected to severe fatigue loading. If attachments in such locations are unavoidable, the weld profile should merge smoothly into the parent metal.
- Use continuous rather than intermittent welds.
- Avoid details that introduce localized constraints.
- During fabrication, carry out necessary inspection to ensure proper deposition of welds.
- Provide suitable inspection during the fabrication and erection of structures.
- Intersection of welds should be avoided.
- Edge preparation for butt welding should be designed with a view of using minimum weld metal so as to minimize warping and residual stress build up.
- Ask for pre and post heating, if necessary, to relieve the build up residual stresses.
- Fillet welds carrying longitudinal shear should not be larger in size than necessary from design consideration.
- Deep penetration fillet welds should be used in preference to normal fillet welds.
- Structures subjected to fatigue loading especially in critical locations should be regularly inspected for the presence of fatigue cracks and when such cracks are discovered, immediate steps should be taken to prevent their further propagation into the structure.
- Any repair measures taken should be designed to avoid introduction of more severe fatigue condition.
- Provide multiple load path and / or structural redundancy in the structure to avoid overall collapse of the structure due to failure of one element in the structure in fatigue.
- Provide crack arresting features in the design at critical locations to avoid propagation of cracks into the entire structure.

10.0 CONCLUSIONS

In this chapter, various factors affecting fatigue behaviour of welded connections have been explained. The nature of fatigue in welded connections and its critical importance are emphasised. Methods of evaluating fatigue lives of welded connections are described. Techniques for improvement in fatigue performance are presented. Fatigue-resistant design and Indian Standard codal provisions are also included.

11.0 REFERENCES

1. Masubuchi K., 'Analysis of welded structures', Pergammon Press, New York, 1980.
2. Owens G.W. and Cheal B.D., 'Structural Steelwork Connections', Butterworths, London, 1989.
3. Owens G.W, Knowles P.R. and Dowling P.J, Steel Designers' Manual, (Fifth Edn.) Blackwell Scientific Publications, London, 1992.
4. IS:1024-1979, 'Code of Practice for Use of Welding in Bridges and Structures Subjected to Dynamic Loading', Bureau of Indian Standards.
5. Paris P. and Erdogan F., 'A critical analysis of crack propagation laws', Journal of Basic Engineering, Dec. 1963, pp. 528-534.

33 BOLTED CONNECTIONS – I

1.0 INTRODUCTION

Connections form an important part of any structure and are designed more conservatively than members. This is because, connections are more complex than members to analyse, and the discrepancy between analysis and actual behaviour is large. Further, in case of overloading, we prefer the failure confined to an individual member rather than in connections, which could affect many members.

Connections account for more than half the cost of structural steelwork and so their design and detailing are of primary importance for the economy of the structure.

The type of connection designed has an influence on member design and so must be decided even prior to the design of the structural system and design of members. For example, in the design of bolted tension members, the net area is calculated assuming a suitable number and diameter of bolts based on experience. Therefore, it is necessary to verify the net area after designing the connection. Similarly in the analysis of frames, the member forces are determined by assuming the connections to be pinned, rigid, or semi-rigid, as the actual behaviour cannot be precisely defined.

Just as members are classified as bending members or axially loaded members depending on the dominant force/moment resisted, connections are also classified into idealised types while designing. But the actual behaviour of the connection may be different and this point should always be kept in mind so that the connection designed does not differ significantly from the intended type. Take for example, the connection of an axially loaded truss member at a joint. If the truss is assumed to be pin jointed, then the member should ideally be connected by means of a single pin or bolt. However, in practice, if the pin or bolt diameter works out to be larger than that possible, more than one bolt will be used. The truss can then be considered pin-jointed only if the bending due to self-weight or other superimposed loads is negligible. Note that the connection behaviour will also influence the calculation of the effective length for the buckling analysis of struts.

The connections provided in steel structures can be classified as 1) riveted 2) bolted and 3) welded connections. Riveted connections were once very popular and are still used in some cases but will gradually be replaced by bolted connections. This is due to the low strength of rivets, higher installation costs and the inherent inefficiency of the connection. Welded connections have the advantage that no holes need to be drilled in the member and consequently have higher efficiencies. However, welding in the field may be difficult, costly, and time consuming. Welded connections are also susceptible to failure by cracking under repeated
cyclic loads due to fatigue which may be due to working loads such as trains passing over a bridge (high-cycle fatigue) or earthquakes (low-cycle fatigue). A special type of bolted connection using High Strength Friction Grip (HSFG)

bolts has been found to perform better under such conditions than the conventional black bolts used to resist predominantly static loading. Bolted connections are also easy to inspect and replace. The choice of using a particular type of connection is entirely that of the designer and he should take his decision based on a good understanding of the connection behaviour, economy and speed of construction.

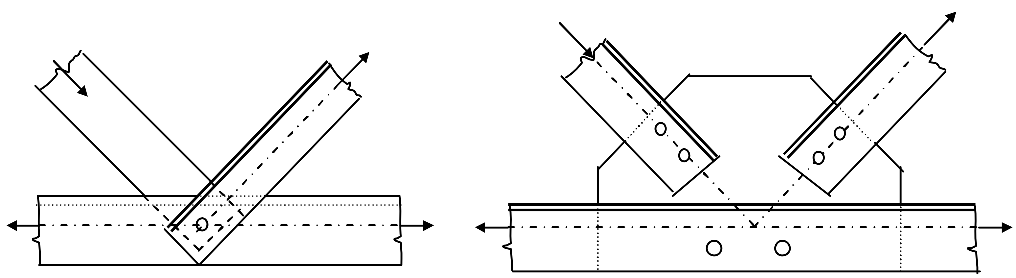
In this chapter, the different types of bolts and bolted connections used in steel structures are introduced. The scope of the present chapter is limited to bolted connections used in tension and compression members as well as in hangers. Bolted connections, which resist moments and connections between structural members, will be covered in the next chapter.

2.0 BOLTED CONNECTIONS

Connections can also be classified in the following ways:

(a) Classification based on the type of resultant force transferred: The bolted connections are referred to as concentric connections (force transfer in tension and compression member), eccentric connections (in reaction transferring brackets) or moment resisting connections (in beam to column connections in frames).

Ideal concentric connections should have only one bolt passing through all the members meeting at a joint [Fig. 1(*a*)]. However, in practice, this is not usually possible and so it is only ensured that the centroidal axes of the members meet at one point [See Fig. 1(*b*)].

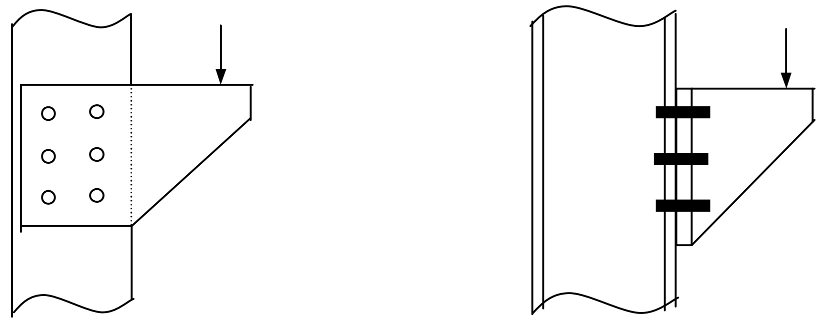


(a) *(b)*

Fig. 1 Concentric Connections

The Moment connections are more complex to analyse compared to the above two types and are shown in Fig. 2(*a*) and Fig. 2(*b*). The connection in Fig. 2(*a*) is also known as bracket connection and the resistance is only through shear in the bolts.

The connection shown in Fig. 2(*b*) is often found in moment resisting frames where the beam moment is transferred to the column. The connection is also used at the base of the column where a base plate is connected to the foundation by means of anchor bolts. In this connection, the bolts are subjected to a combination of shear and axial tension. Moment resisting connections will be dealt with in the next chapter.

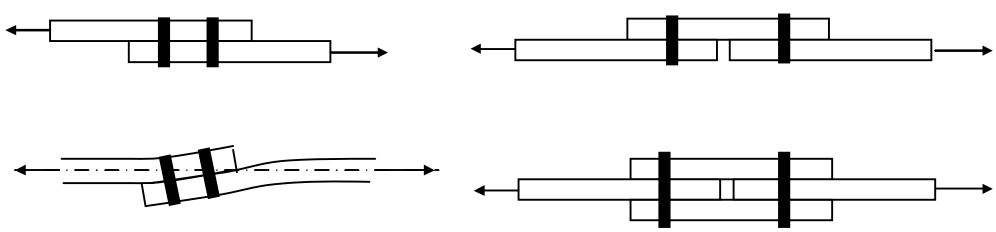


(a) *(b)*

Fig. 2 Moment Connections

(b) Classification based on the type of force experienced by the bolts: The bolted connections can also be classified based on geometry and loading conditions into three types namely, shear connections, tension connections and combined shear and tension connections.

Typical shear connections occur as a *lap* or a *butt* joint used in the tension members [See Fig. 3]. While the lap joint has a tendency to bend so that the forces tend to become collinear, the butt joint requires *cover plates*. Since the load acts in the plane of the plates, the load transmission at the joint will ultimately be through shearing forces in the bolts.
In the case of lap joint or a single cover plate butt joint, there is only one shearing plane, and so the bolts are said to be in *single shear*. In the case of double cover butt joint, there are two shearing planes and so the bolts will be in *double shear*. It should be noted that the single cover type butt joint is nothing but lap joints in series and also bends so that the centre of the cover plate becomes collinear with the forces.



(a) Lap Connection *(b) Butt Connection*

Fig. 3 Shear Connections

A hanger connection is shown in Fig. 4(*a*). In this connection, load transmission is by pure tension in the bolts. In the connection shown in Fig. 4(*b*), the bolts are subjected to both tension and shear.

(c) Classification based on force transfer mechanism by bolts: The bolted connections are classified as bearing type (bolts bear against the holes to transfer the force) or friction

type (force transfer between the plates due to the clamping force generated by the pre–tensioning of the bolts). The force transfer in either case is discussed in more detail later.

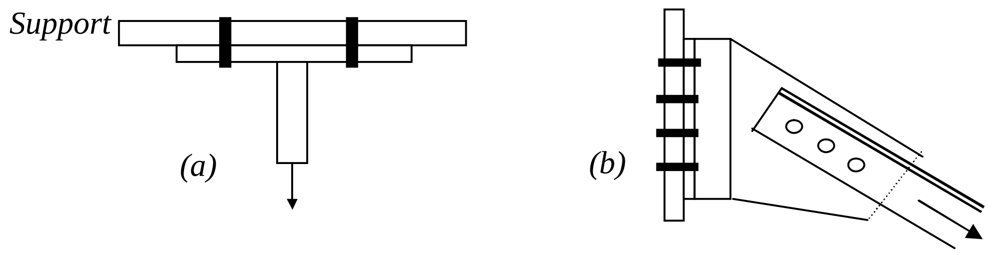


Fig. 4 (a) Tension Connection (b) Tension plus Shear Connection

3.0 BOLTS AND BOLTING

Bolts used in steel structures are of three types: 1) Black Bolts, 2) Turned and Fitted Bolts and 3) High Strength Friction Grip (HSFG) Bolts.

The International Standards Organisation designation for bolts, also followed in India, is given by Grade *x.y*. In this nomenclature, *x* indicates one-tenth of the minimum ultimate tensile strength of the bolt in kgf/mm^2 and the second number, *y*, indicates one-tenth of the ratio of the yield stress to ultimate stress, expressed as a percentage. Thus, for example, grade 4.6 bolt will have a minimum ultimate strength 40 kgf/mm^2 (392 Mpa) and minimum yield strength of 0.6 times 40, which is 24 kgf/mm^2 (235 Mpa).

Black bolts are unfinished and are made of mild steel and are usually of Grade 4.6. Black bolts have adequate strength and ductility when used properly; but while tightening the nut snug tight ("Snug tight" is defined as the tightness that exists when all plies in a joint are in firm contact) will twist off easily if tightened too much. Turned –and- fitted bolts have uniform shanks and are inserted in close tolerance drilled holes and made snug tight by box spanners. The diameter of the hole is about 1.5 to 2.0 mm larger than the bolt diameter for ease in fitting. High strength black bolts (grade 8.8) may also be used in connections in which the bolts are tightened snug fit.

In these *bearing type of connections,* the plates are in firm contact but may slip under loading until the hole surface bears against the bolt .The load transmitted from plate to bolt is therefore by bearing and the bolt is in shear. Under dynamic loads, the nuts are liable to become loose and so these bolts are not allowed for use under such loading. In situations where small slips can cause significant effects as in beam splices, black bolts are not preferred. However, due to the lower cost of the bolt and its installation, black bolts are quite popular in simple structures subjected to static loading.

Turned and fitted bolts are available from grade 4.6 to grade 8.8. For the higher grades there is no definite yield point and so 0.2% proof stress is used.

High Strength Friction Grip bolts (HSFG) provide extremely efficient connections and perform well under fluctuating/fatigue load conditions. These bolts should be tightened to their proof loads and require hardened washers to distribute the load under the bolt heads. The washers are usually tapered when used on rolled steel sections. The tension in the bolt ensures that no slip takes place under working conditions and so the load transmission from plate to the bolt is through friction and not by bearing. However, under ultimate load, the friction may be overcome leading to a slip and so bearing will govern the design.

HSFG bolts are made from quenched and tempered alloy steels with grades from 8.8 to 10.9. The most common are, the so-called, general grade of 8.8 and have medium carbon content, which makes them less ductile. The 10.9 grade have a much higher tensile strength, but lower ductility and the margin between the 0.2% yield strength and the ultimate strength is also lower.

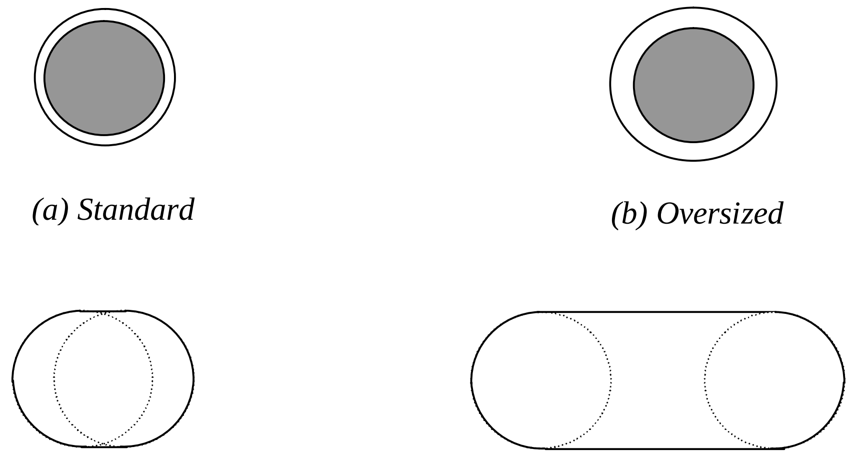
The tightening of HSFG bolts can be done by either of the following methods:

1. Turn-of-nut tightening method: In this method the bolts are first made snug tight and then turned by specific amounts (usually either half or three-fourth turns) to induce tension equal to the proof load.
2. Calibrated wrench tightening method: In this method the bolts are tightened by a wrench calibrated to produce the required tension.
3. Alternate design bolt installation: In this method special bolts are used which indicate the bolt tension. Presently such bolts are not available in India.
4. Direct tension indicator method: In this method special washers with protrusions are used [Fig. 5(*b*)]. As the bolt is tightened, these protrusions are compressed and the gap produced by them gets reduced in proportion to the load. This gap is measured by means of a feeler gauge, consisting of small bits of steel plates of varying thickness, which can be inserted into the gap.

Fig. 5 Tightening of HSFG bolts

Since HSFG bolts under working loads, do not rely on resistance from bearing, holes larger than usual can be provided to ease erection and take care of lack-of-fit. Typical hole types that can be used are standard, extra large and short or long slotted. These are shown in Fig. 6. However the type of hole will govern the strength of the connection.

Holes must also satisfy pitch and edge/end distance criteria. A minimum pitch is usually specified for accommodating the spanner and to limit adverse interaction between the bearing stresses on neighbouring bolts. Maximum pitch criteria takes care of buckling of the plies under compressive loads.



(a) Standard *(b) Oversized*

(c)Short Slot *(d) Long slot*

Fig. 6 Hole types for HSFG bolts

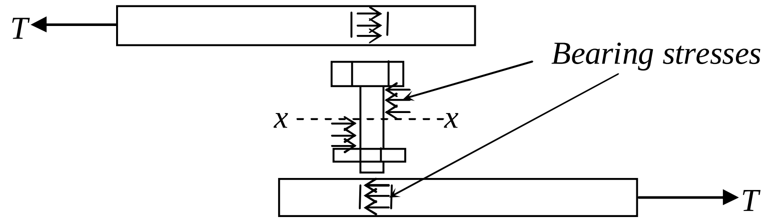
4.0 FORCE TRANSFER MECHANISM

In this section the force transfer mechanisms of bearing and friction type of bolted connections are described. This would help in identifying the modes of failure discussed in the next section.

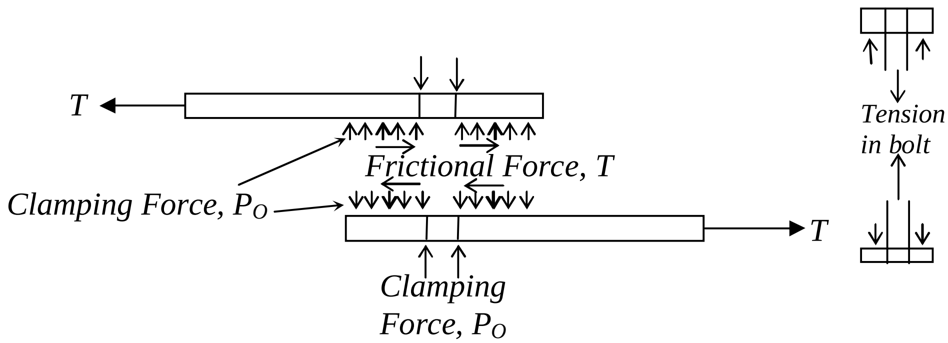
4.1 Force transfer by shear in bolts

Fig. 7(*a*) shows the free body diagram of the shear force transfer in bearing type of bolted connection. It is seen that tension in one plate is equilibrated by the bearing stress between the bolt and the hole in the plate. Since there is a clearance between the bolt and the hole in which it is fitted, the bearing stress is mobilised only after the plates slip relative to one another and start bearing on the bolt .The section *x-x* in the bolt is critical section for shear. Since it is a lap joint there is only one critical section in shear (single shear) in the bolt .In the case of butt splices there would be two critical sections in the bolt in shear (double shear), corresponding to the two cover plates.

The free body diagram of an HSFG connection is shown in Fig. 7(*b*). It can be seen that the pretension in the bolt causes clamping forces between the plates even before the external load is applied. When the external load is applied, the tendency of two plates to slip against one another is resisted by the friction between the plates. The frictional resistance is equal to the coefficient of friction multiplied by the normal clamping force between the plates. Until the externally applied force exceeds this frictional resistance the relative slip between the plates is prevented. The HSFG connections are designed such that under service load the force does not exceed the frictional resistance so that the relative slip is avoided during service. When the external force exceeds the frictional resistance the plates slip until the bolts come into contact with the plate and start bearing against the hole. Beyond this point the external force is resisted by the combined action of the frictional resistance and the bearing resistance.



(a) Bearing Connection



(b) Friction Connection

Fig. 7 Bolt Shear Transfer – Free Body Diagram

4.2 Transfer of tension by bolts

The free body diagram of the tension transfer in a bearing type of bolted connection is shown in Fig. 8(*a*). The variation of bolt tension due to externally applied tension is shown in Fig. 8(*c*). It is seen that before any external tension is applied, the force in the bolt is almost zero, since the bolts are only snug tight. As the external tension is increased it is equilibrated by the increase in bolt tension. Failure is reached due to large elongation when the root of the bolt starts yielding. Depending on the relative flexibility of the plate and the bolt, sometimes the opening of the joint may be accompanied by prying action [Fig. 8(*d*)].

The free body diagram of an HSFG bolted connection is shown in Fig. 8(*b*). It is seen that even before any external load is applied, the force in the bolt is equal to proof load. Correspondingly there is a clamping force between the plates in contact. When the external load is applied, part of the load (nearly 10%) of the load is equilibrated by the increase in the bolt force. The balance of the force is equilibrated by the reduction in contact between the plates. This process continues and the contact between the plates is maintained until the contact force due to pre-tensioning is reduced to zero by the externally applied load. Normally, the design is done such that the externally applied tension does not exceed this level. After the external force exceeds this level, the behaviour of the bolt under tension is essentially the same as that in a bearing type of joint.

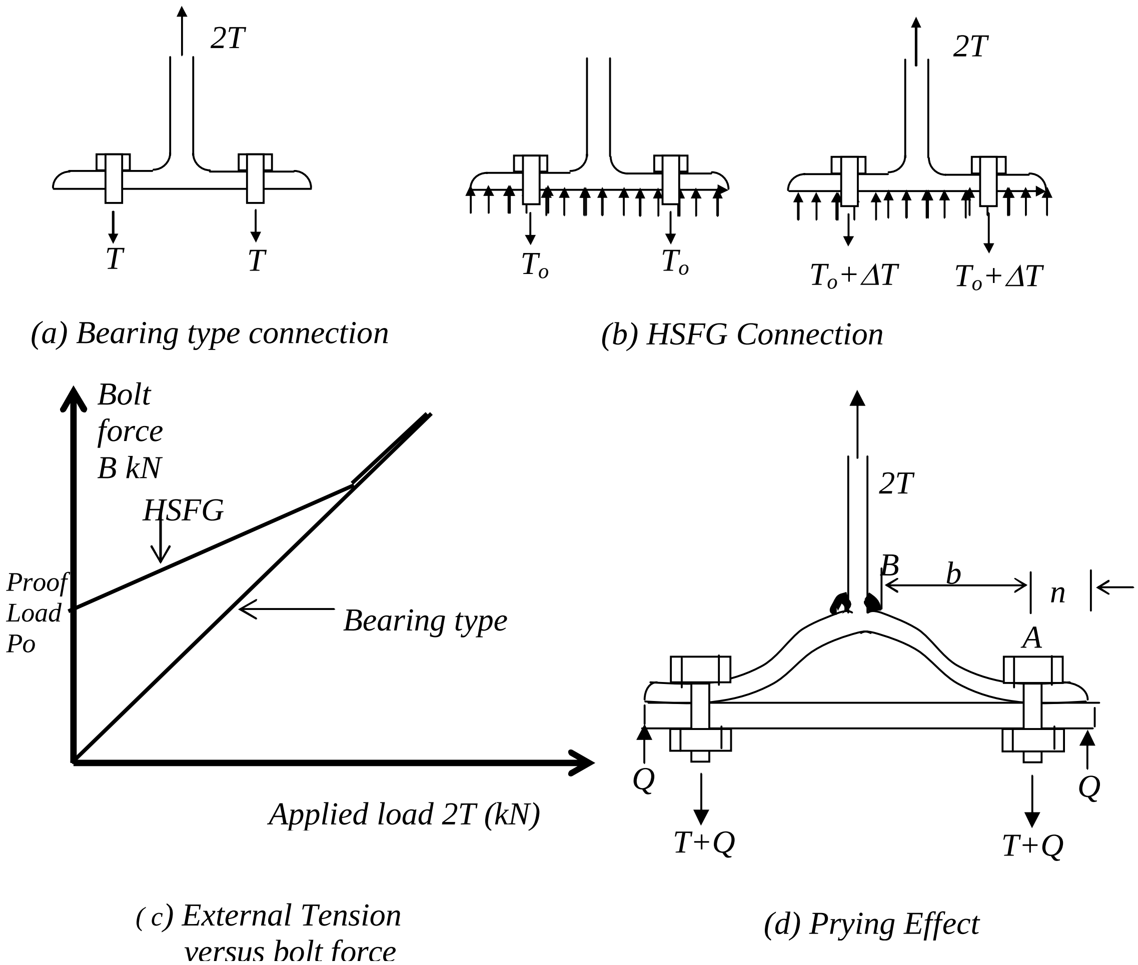
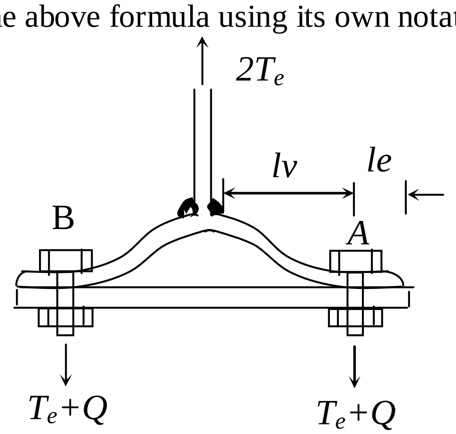


Fig. 8 Bolts under tension and prying effect

The design formula for minimum prying force is given by (Owens and Cheal, 1989)

$$Q = \frac{b}{2n}\left[T - \frac{\beta\gamma p_o w t^4}{27nb^2}\right] \qquad (1)$$

IS: 800 has laid down the above formula using its own notations as given below



As per IS:800 the above fig will replace Fig. 8. (d)

$$Q = \frac{l_v}{2\,l_e}\left[T_e - \frac{\beta\gamma\, f_o b_e t^4}{27 l_e l_v^2}\right]$$

where, *b* is the distance from the bolt centreline to the toe of the fillet weld or to half the root radius for a rolled section (see Fig. 8d); *n* = distance between prying force and bolt centreline and is the minimum of, either the end distance, or the value given by eq.(2); β = 2 for non pre-loaded bolt; γ = 1.5 for limit state design; *w* = the effective width of flange per pair of bolts; p_o = the proof stress in consistent units and t is the thickness of the end plate.

$$n = 1.1t\sqrt{\frac{\beta \cdot p_o}{f_y}} \qquad (2)$$

Even if the bolts are strong enough to carry the additional prying forces, the plate can fail by developing a mechanism with yield lines at the centreline of the bolt and at the distance l_v from it. Therefore, the minimum thickness of the end plate (*t*), to avoid yielding of the plate, can be obtained by equating the moment in the plate at the bolt centreline (point A) and at the distance l_v from it, to the plastic moment capacity of the plate M_p. Thus,

$$M_A = Q.l_e;\; M_B = T_e.l_v - Q.l_e \qquad (3)$$

$$M_A = M_B = \frac{T_e.l_v}{2} = M_p \qquad (4)$$

taking M_p *as*

$$M_p = \frac{f_y}{1.10}\frac{wt^2}{4} \qquad (5)$$

the minimum thickness for the end plate can be obtained as

$$t_{\min} = \sqrt{\frac{1.10 \times 4 \times M_p}{f_y \times w}} \qquad (6)$$

The corresponding prying force can then be obtained as $Q = M_p / l_e$. If the total force in the bolt $(T_e + Q)$ exceeds the tensile capacity of the bolt, then the thickness of the end plate will have to be increased.

5.0 FAILURE OF CONNECTIONS

5.1 Connections in shear

The failure of connections with bearing bolts in shear involves either bolt failure or the failure of the connected plates. In the case of HSFG bolts, however, it may simply be a slip between the connected plates. In this section, the failure modes are described along with the codal provisions for design and detailing shear connections.

5.1.1 Bearing bolts

In connections made with bearing type of bolts, the behaviour is linear until i) yielding takes place at the net section of the plate under combined tension and flexure or ii) shearing takes place at the bolt shear plane or iii) failure of bolt takes place in bearing, iv) failure of plate takes place in bearing and v) block shear failure occurs. Of these the first is discussed in the chapter on tension members while the last is described separately in section 5.4. The remaining three is described below.

1. Shear capacity of Bolts as per IS 800: The design strength of the bolt, V_{dsb}, as governed shear strength is given by

$V_{dsb} = V_{nsb} / \gamma_{mb}$

V_{nsb} = nominal shear capacity of a bolt, calculated as follows:

$$V_{nsb} = \frac{f_u}{\sqrt{3}}\left(n_n A_{nb} + n_s A_{sb}\right)$$

f_u = ultimate tensile strength of a bolt

n_n = number of shear planes with threads intercepting the shear plane

n_s = number of shear planes without threads intercepting the shear plane

A_{sb} = nominal plain shank area of the bolt

A_{nb} = net shear area of the bolt at threads, may be taken as the area corresponding to root diameter at the thread

γ_{mb} =Partial safety factor for bolted connection with bearing type bolts

The underlying assumption behind the design of bolted connections, namely that all bolts carry equal load is not true in some cases as mentioned below.

Long Joints:

When the length of the joint, l_j, of a splice or end connection in a compression or tension element containing more than two bolts (i.e. the distance between the first and last rows of bolts in the joint, measured in the direction of the load transfer) exceeds 15d in the direction of load, the nominal shear capacity V_{ns}, shall be reduced by the factor, β_{lj}, given by

$\beta_{lj} = 1.075 - l_j / (200\, d)$ but $0.75 \leq \beta_{lj} \leq 1.0$

$= 1.075 - 0.005(l_j / d)$

d= nominal diameter of the fastener

It shall be kept in mind that this provision does not apply when the distribution of shear over the length of joint is uniform as in the connection of web of a section to the flanges.

2. Bearing strength of Bolts as per IS 800: The design bearing strength of a bolt on any plate, V_{dpb}, as governed by bearing is given by

$V_{dpb} = V_{npb} / \gamma_{mb}$

V_{npb} = nominal bearing strength of a bolt, calculated as follows:

$V_{npb} = 2.5\, k_b\, d\, t\, f'_u$

k_b is smaller of $\frac{3}{3d_0}; \frac{p}{3d_0} - 0.25; \frac{f_{ub}}{f_u}; 1.0$

e, p = end and pitch distances of the fastener along bearing direction
d_0 = diameter of the hole
f'_u = smaller of f_{ub}, f_u
f_{ub}, f_u = ultimate tensile stress of the bolt and the ultimate tensile stress of the plate, respectively
d = nominal diameter of the bolt
t = summation of the thicknesses of the connected plates experiencing bearing stress in the same direction, or, if the bolts are countersunk, the thickness of the plate minus one half of the depth of countersinking

In the direction normal to the slots in slotted holes the bearing resistance of bolts in holes other than standard clearance holes is reduced by multiplying the bearing resistance obtained (i.e., V_{npb}), by 0.7 for over size & short slotted holes or 0.5 for long slotted holes.

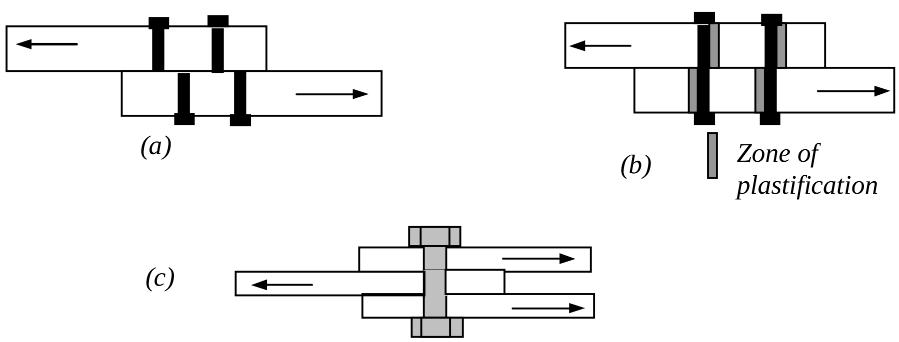


Fig. 9 Types of failures in a shear connection (a) Shearing of bolts (b) Bearing failure of plate (c) Bearing failure of bolt

3. Tensile capacity of Bolts as per IS 800: A bolt subjected to a factored tensile force (T_b) shall satisfy

$T_b \leq T_{db}$ *Where,* $T_{db} = T_{nb} / \gamma_{mb}$ and T_{nb} is the nominal tensile capacity of the bolt, given as: $T_{nb} = 0.90\, f_{ub}\, A_n < f_{yb}\, A_{sb}\, (\gamma_{mb} / \gamma_{m0})$

f_{ub} is the ultimate tensile stress of the bolt, f_{yb} is the yield stress of the bolt, A_n is the net tensile stress area. For bolts where the tensile stress area is not defined, A_n is taken as the area at the bottom of the threads and A_{sb} is the shank area of the bolt

4. Bolt Subjected to Combined Shear and Tension – A bolt required to resist both design shear force (V_{sd}) and design tensile force (T_b) at the same time shall satisfy

$$\left(\frac{V_{sb}}{V_{db}}\right)^2 + \left(\frac{T_b}{T_{db}}\right)^2 \leq 1.0$$

V_{sb} is the factored shear force acting on the bolt and V_{db} is the design shear capacity (Section ***5.1.1***). Similarly, T_b is the factored tensile force acting on the bolt and T_{db} is the design tension capacity (Section ***5.1.1***)

5.1.2 HSFG bolts

HSFG bolts will come into bearing only after slip takes place. Therefore if slip is critical (i.e. if slip cannot be allowed) then one has to calculate the slip resistance, which will govern the design. However, if slip is not critical, and limit state method is used then bearing failure can occur at the Limit State of collapse and needs to be checked. Even in the Limit State method, since HSFG bolts are designed to withstand working loads without slipping, the slip resistance needs to be checked anyway as a Serviceability Limit State.

1. Slip Resistance as per IS: 800: Slip resistance per bolt is given by

$V_{sf} \leq V_{dsf}$
$V_{dsf} = V_{nsf} / \gamma_{mf}$
V_{nsf} = nominal shear capacity of a bolt as governed by slip for friction type connection, and is given as:
$V_{nsf} = \mu_f . n_e . K_h . F_o$

μ_f is the coefficient of friction (slip factor) as specified in Table 4 ($\mu_f \leq 0.55$), n_e is the number of effective interfaces offering frictional resistance to slip, K_h is equal to 1.0 for fasteners in clearance holes, 0.85 for fasteners in oversized short slotted holes & for fasteners in long slotted holes loaded perpendicular to the slot and 0.7 for fasteners in long slotted holes loaded parallel to the slot

γ_{mf} is equal to 1.10 (if slip resistance is designed at service load), 1.25 (if slip resistance is designed at ultimate load), F_o is the minimum bolt tension (proof load) at installation and may be taken as $0.8A_{sb}.f_o$,

A_{sb} is the shank area of the bolt in tension and f_o is the proof stress (= 0.70 $f_u b$)

Table 4 : Typical Average Values of Coefficient of Friction (μ_f)

Treatment of surface	Coefficient of friction *(μf)*
Surfaces not treated	0.20
Surfaces blasted with short or grit with any loose rust removed, no	0.50
Surfaces blasted with shot or grit and hot-dip galvanized	0.10
Surfaces blasted with shot or grit and spray-metallized with zinc (thickness 50-70μm)	0.25
Surface s blasted with shot or grit and painted with ethyl zinc silicate coat (thickness 30-60 μm)	0.30
Sand blasted surface, after light rusting	0.52
Surfaces blasted with shot or grit and painted with ethyl zinc silicate coat (thickness 60-80 μm)	0.30
Surfaces blasted with shot or grit and painted with alkali zinc silicate coat	0.30
Surface blasted with shot or grit and spray metallized with aluminium (thickness > 50 μm)	0.50
Clean mill scale	0.33
Sand blasted surface	0.48
Red lead painted surface	0.10

5.2 Tension Failure

In a tension or hanger connection, the applied load produces tension in the bolts. If the attached plate is allowed to deform, additional tensile forces called prying forces are developed in the bolts as shown in Fig. 8. The prying forces can be kept small by using a thick plate or by limiting the distance between the bolt and the plate edge. Black bolts and turned and fitted bolts have sufficient ductility which takes care of prying forces simply by an increase in the bolt strain under constant yield stress. Tensile stresses recommended by BS 5950 for grade 4.6 and grade 8.8 bolts are 195 and 450 N/mm^2 respectively. However, HSFG bolts which are pre-tensioned and which have less ductility are susceptible to failure and so are normally designed to take only 0.9 times their proof load.

A friction bolt subjected to a factored tension force (T_f) shall satisfy

$T_f < T_{df}$ where, $T_{df} = T_{nf} / \gamma_{mb}$

T_{nf} is the nominal tensile strength of the friction bolt, calculated as follows:
$T_{nf} = 0.9 f_{ub} A_n \leq f_{yb}\ A_{sb}(\gamma_{m1}/ \gamma_{m0})$

f_{ub} is the ultimate tensile stress of the bolt, A_n is the net tensile stress area as specified in IS: 1367. For bolts where the tensile stress area is not defined, A_n shall be taken as the area at the root of the threads and A_{sb} is the shank area of the bolt.

5.3 Combined Shear and tension failure

In the case of black bolts subjected to combined action of shear and tension the following relation has to be satisfied.

$$\left(\frac{V_{sf}}{V_{df}}\right)^2 + \left(\frac{T_f}{T_{df}}\right)^2 \leq 1.0$$

V_{sf} is the applied factored shear at design load, V_{df} is the design shear strength, T_f is the externally applied factored tension at design load and T_{df} is the design tension strength

5.4 Block shear

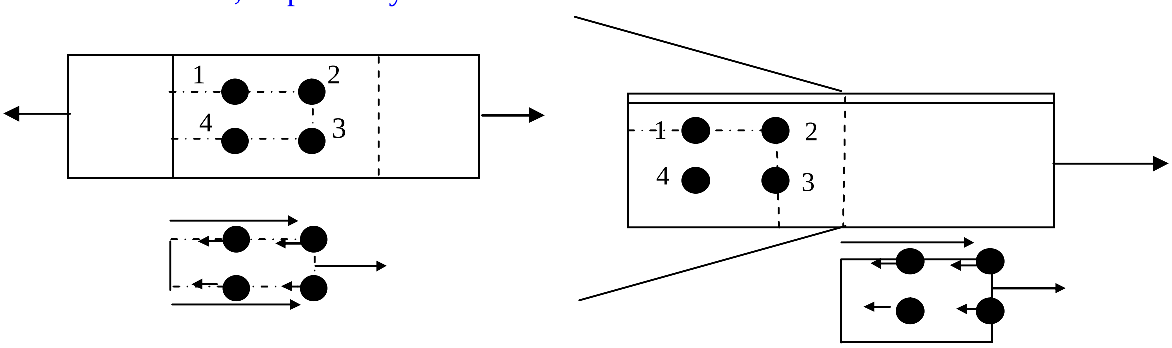
Failure by block shear occurs when a portion of the member tears out in a combination of tension and shear. The equations given for block shear in the chapter on Tension Members are repeated here. The strength as governed by block shear is the minimum of

$$T_{db} = \left(A_{vg}\, f_y/(\sqrt{3}\,\gamma_{m0}) + 0.9A_{tn}\, f_u/\gamma_{m1} \right)$$

or

$$T_{db} = \left(0.9A_{vn}\, f_u/(\sqrt{3}\,\gamma_{m1}) + A_{tg}\, f_y/\gamma_{m0} \right)$$

$A_{vg,}$ and A_{vn} are the minimum gross and net area respectively in shear along bolt line parallel to external force, respectively (1-2 and 3-4 as shown in Fig 6.3a and 1-2 as shown in Fig 6.3b). $A_{tg,}$ and Atn are the minimum gross and net area respectively in tension from the bolt hole to the toe of the angle, end bolt line, perpendicular to the line of force, respectively (2-3 as shown in Fig 6.3b). f_u and f_y are the ultimate and yield stress of the material, respectively.



(a) Plate (b) Angle

FIG 6.3 BLOCK SHEAR FAILURE

Check for block shear should be carried out when using high strength bolts with minimum pitch and edge distances and in coped sections.

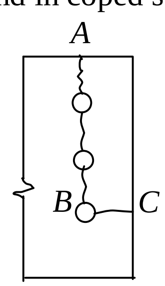


Fig. 11 Block Shear

6.0 SUMMARY

Different types of bolted connections were described and classified. The bearing and friction grip bolts were introduced and their installation procedures described. The force transfer mechanisms were explained and the failure modes and corresponding strength calculations were given. This will help in the design of simple bolted connections as in the worked examples.

7.0 REFERENCES

1. Owens. G.W and Cheal. B.D., (1989): 'Structural Steelwork Connections', Butterworths.

2. Owens G.W and Knowles P.R., (1994): 'Steel Designers Manual', The Steel Construction Institute, Blackwell Scientific Publications, ELBS 5th edition.

3. Geschwindner L.F. et al (1994): 'Load and Resistance Factor Design of Steel Structures', Prentice Hall, Englewood Cliffs, New Jersey.

Structural Steel Design Project **Calculation Sheet**	Job No:	Sheet ***1 of 1***	Rev
	Job Title: ***Bolted Connections***		
	Worked Example – 1 Lap joint		
		Made by *SRSK*	Date *15-07-00*
		Checked by *RN*	Date

Remarks

***Design Example 1**: Design a Lap joint between plates 65 × 8 as shown in Fig. E1 below so as to transmit a factored load of 45 kN using black bolts of 16mm diameter and grade 4.6. The plates are made of steel of grade ST-42-S.*

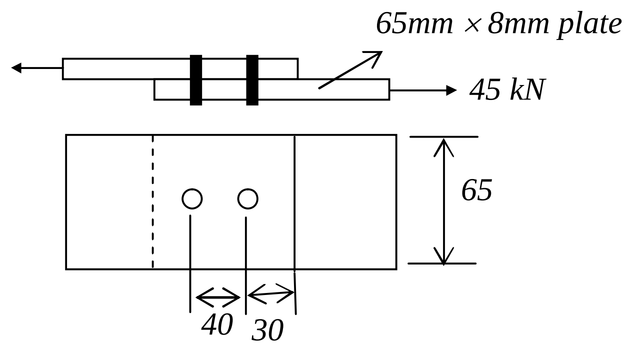


Fig. E1

Solution:

1) Strength Calculations:

Nominal diameter of bolt d= 16 mm,

hole diameter D = 16 +1.5 = 17.5 mm

Shear Area of one bolt $A_s = 0.8A = 0.8 \times 201.06 = 161\ mm^2$

Shear strength of each bolt $= p_s A_s = 160 \times 161 \times 10^{-3} = 25.76\ kN$ — Eq. (7)

Since p_{bb} *for bolt is greater than* p_{bs} *of plate, plate will fail in bearing before the bolt.*

Bearing strength of plate $= p_{bs}\, d\, t = 418 \times 16 \times 8 \times 10^{-3} = 53.5\ kN$ — Eq. (9)

Therefore, bolt value = 25.76 kN

No. of bolts required = 45/25.76 = 1.75 say 2 bolts

2)Detailing:

Minimum pitch = 2.5 d = 40 mm

Minimum edge distance = 1.4 D = 24.5 mm say 25 mm

Provide 2 bolts as shown.

Structural Steel Design Project	Job No:	Sheet ***1 of 1***	Rev
	Job Title: ***Bolted Connections***		
	Worked Example – 2 Hanger joint		
		Made by SRSK	Date*15-7-00*
Calculation Sheet		Checked by RN	Date

***Design Example 2**: Design a hanger joint along with an end plate to carry a downward load of 330 kN. Use end plate size 240 mm × 160 mm and appropriate thickness and M25 HSFG bolts (2 nos).* — *Remarks*

Solution

Assume 10mm fillet weld between the hanger plate and the end plate

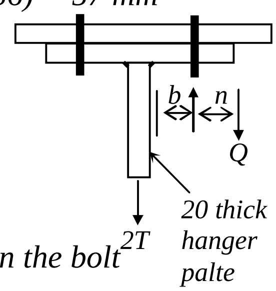
Distance from center line of bolt to toe of fillet weld b = 60 mm

1) For minimum thickness design, M = Tb/2 = 165 × 60/2 = 4950 N-m

$$\therefore t_{\min} = \sqrt{\frac{1.10\times 4\times 4950\times 10^3}{236\times 160}} = 24.02 \text{ say } 25\, mm$$

2) Check for prying forces

distance'n' from center line of bolt to prying force is the minimum of edge distance or $1.1t\sqrt{(\beta p_o/f_y)}$ = 1.1 x 25 $\sqrt{(2\times 510/236)}$ = 57 mm Eq. (6)



∴ n = 40 mm

prying force = M/n = 4950/40= 123.75 kN

bolt load = 165+123.75=288.75 kN

tension capacity of 25 mm dia HSFG bolt = 0.9Po Eq. (2) Table 3

=0.9×195.6=176 kN << 288.75 unsafe !

3) In order to reduce the load on bolt to a value less than the bolt capacity, a thicker end plate will have to be used.

Allowable prying force Q = 176-165 = 11 kN

Trying a 40 mm thick end plate gives n = 40 mm as before

Moment at toe of weld = Tb-Qn = 165 × 60 – 11 × 40 = 9460 N-m

Moment capacity = $(236/1.15)(160 \times 40^2/4) \times 10^{-3}$

= 13134 N-m > 9460 OK

Minimum prying force Eq. (3)

$$= \frac{b}{2n}\left[T - \frac{\beta\gamma p_o w t^4}{27nb^2}\right] = \frac{60}{2\times 40}\left[165 - \frac{2\times 1.5\times 0.512\times 160\times 40^4}{27\times 40\times 60^2}\right]$$

Eq. (1)

$$Q = \frac{l_v}{2\,l_e}\left[T_e - \frac{\beta\,\gamma\, f_o\, b_e\, t^4}{27\, l_e\, l_v^2}\right]$$

= 2.4 kN < 11 kN ∴ safe!

Therefore, 40 mm end plate needs to be used to avoid significant prying action.

34 BOLTED CONNECTIONS – II

1.0 INTRODUCTION

Connections become complex when they have to transmit axial and shear forces in addition to bending moments, between structural members oriented in different directions. A variety of components such as angle cleats, stiffeners and end plates are used to transfer and also disperse the loads from one member to another. In particular, bolted connections pose additional problems because they employ discrete rather than continuous load paths to effect the transfer. This is in addition to the complex behaviour of the bolts themselves. Attempts to develop complex design procedures which can produce more economical and yet safe connections are rendered futile by the variety of connection configurations possible and more importantly, the variability of behaviour due to practical limitations in fabrication and erection. Therefore the most rational philosophy for design would be to base it on simple analysis and use higher load factors for increased safety. However, it is easier said than done and is fraught with pitfalls unless one develops a certain insight into the behaviour.

In structural design, it is a common tendency to follow a tradition, which has produced satisfactory designs in the past so that one need not worry about having overlooked some important aspect of behaviour. This tendency has crystallised into some standard connection types for which simplified analysis procedures can be used with great advantage. The flange and web angle connection shown in Fig. 1(*a*) is an example of a typical beam-to-column moment connection. It is important for the novice to become familiar with such connection types and their advantages/disadvantages, which are described in the subsequent sections of this chapter.

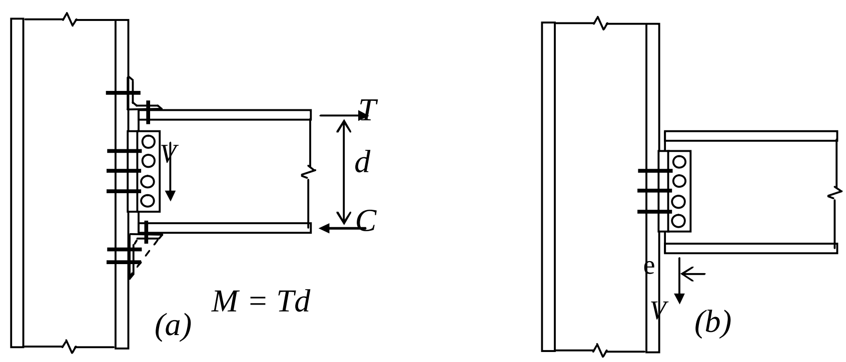


Fig. 1 Standard Connections (a) moment connection (b) simple connection

It is also equally important to remember the assumptions made in traditional analysis so that the elements of the connection are proportioned appropriately. For example, with reference to the connection shown in Fig. 1(*a*), the angles are assumed to be rigid compared to the bolts. However the top angle will have a tendency to open out while the bottom will have a tendency to close unless sufficiently thick angles are used. Similarly, the connector behaviour is assumed to be linearly elastic, whereas in reality, if bearing bolts are used, due to the hole size being larger than the bolt shank some slip is likely to take place. This slip may be adequate to release the end moment and make the beam behave as a simply supported one. Therefore it may be advantageous to go for HSFG bolts, which will behave linearly at least at working loads thereby ensuring serviceability of the connection.

Another simplification commonly used is that the distribution of forces is arrived at, by assuming idealised load paths. In the simple connection effected through a pair of short web cleats [Fig. 1(*b*)], it is assumed that the bolts connecting the beam web and the angles resist only the shear that is transferred. However, if the length of the web cleats is comparable to the depth of the beam, additional shear forces are likely to arise in these bolts due to the eccentricity between the bolt line and the column face and should be considered in design. Further, if the web cleats are unduly stiff, they will satisfy equilibrium but the required rotation for the beam end to act as simply supported may not be possible. Therefore it is important to build ductility into the system by keeping the angles as thin as possible. The aim of the present chapter is to point out these aspects, which will lead to good connection designs.

2.0 ANALYSIS OF BOLT GROUPS

In general, any group of bolts resisting a moment can be classified into either of two cases depending on whether the moment is acting in the shear plane or in a plane perpendicular to it. Both cases are described in this section.

2.1 Combined Shear and Moment in Plane

Consider an eccentric connection carrying a load of P as shown in Fig. 2. The basic assumptions in the analysis are (1) deformations of plate elements are negligible, (2) the total shear is assumed to be shared equally by all bolts and (3) the equivalent moment at the geometric centre (point *O* in Fig. 2) of the bolt group, causes shear in any bolt proportional to the distance of the bolt from the point *O* acting perpendicular to the line joining the bolt centre to point *O* (radius vector).

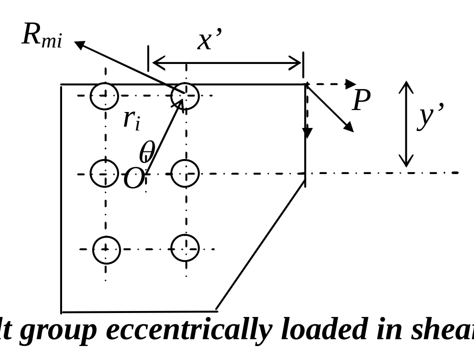


Fig. 2 Bolt group eccentrically loaded in shear

Resolving the applied force P into its components P_x and P_y in x and y-directions respectively and denoting the corresponding force on any bolt i to these shear components by R_{xi} and R_{yi} and applying the equilibrium conditions we get the following:

$$R_{xi} = P_x/n \text{ and } R_{yi} = P_y/n \tag{1}$$

where n is the total number of bolts in the bolt group and R_{xi} and R_{yi} act in directions opposite to P_x and P_y respectively.

The moment of force P about the centre of the bolt group (point O) is given by

$$M = P_x\, y' + P_y\, x' \tag{2}$$

where x' and y' denote the coordinates of the point of application of the force P with respect to the point O. The force in bolt i, denoted by R_{mi}, due to the moment M is proportional to its distance from point O, r_i ,and perpendicular to it.

$$R_{mi} = k\, r_i \tag{3}$$

where, k is the constant of proportionality. The moment of R_{mi} about point O is

$$M_i = k\, r_i^2 \tag{4}$$

Therefore the total moment of resistance of the bolt group is given by

$$MR = \Sigma\, k\, r_i^2 = k\, \Sigma\, r_i^2 \tag{5}$$

For moment equilibrium, the moment of resistance should equal the applied moment and so k can be obtained as $k = M/\Sigma\, r_i^2$, which gives R_{mi} as

$$R_{mi} = M\, r_i / \Sigma\, r_i^2 \tag{6}$$

Total shear force in the bolt R_i is the vector sum of R_{xi}, R_{yi} and R_{mi}

$$R_i = \sqrt{\left[\left(R_{xi} + R_{mi}\cos\theta_i\right)^2 + \left(R_{yi} + R_{mi}\sin\theta_i\right)^2\right]} \tag{7}$$

After substituting for R_{xi}, R_{yi} and R_{mi} from equations (1) and (6) in (7), using $\cos\theta_i = x_i/r_i$ and $\sin\theta_i = y_i/r_i$ and simplifying we get

$$R_i = \sqrt{\left\{\left[\frac{P_x}{n} + \frac{My_i}{\sum(x_i^2 + y_i^2)}\right]^2 + \left[\frac{P_y}{n} + \frac{Mx_i}{\sum(x_i^2 + y_i^2)}\right]^2\right\}} \tag{8}$$

The x_i and y_i co-ordinates should reflect the positive and negative values of the bolt location as appropriate.

2.2 Combined Shear and Moment out-of-plane

In the connection shown in Fig. 3, the bolts are subjected to combined shear and tension. The neutral axis may be assumed to be at a distance of one-sixth of the depth d above the bottom flange of the beam so as to account for the greater area in the compressed portions of the connection per unit depth.

The nominal tensile force in the bolts can be calculated assuming it to be proportional to the distance of the bolt from the neutral axis l_i in Fig. 3. If there exists a hard spot on the compressive load path such as a column web stiffener on the other side of the lower beam flange, the compressive force may be assumed to be acting at the mid-depth of the hard spot as shown in Fig. 3c. In such a case, the nominal tensile force in the bolts can be calculated in proportion to the distance of the bolt from the compressive force $(l_i = L_i)$.

$T_i = kl_i$ where k = constant (9)

$M = \Sigma T_i L_i = k \Sigma l_i L_i$ (10)

$T_i = Ml_i / \Sigma l_i L_i$ (11)

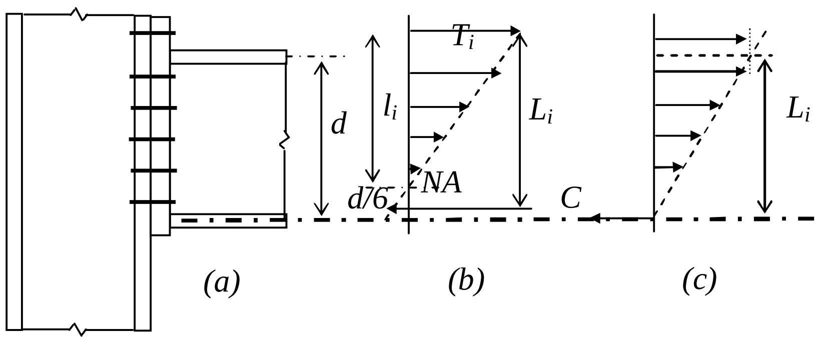


Fig. 3 Bolt group resisting out-of-plane moment

In the case of extended end plate connections, the top portion of the plate behaves as a T-stub symmetric about the tension flange. For calculating the bolt tensions in the rows immediately above and below the tension flange, l_i can be taken as the distance of the tension flange from the neutral axis to the line of action of the compressive force, as the case may be. If the end plate is thin, prying tension is likely to arise in addition to the nominal bolt tension calculated as above.

The shear can be assumed to share equally by all the bolts in the connection. Therefore, the top bolts will have to be checked for combined shear and tension as explained in the previous chapter on Bolted Connections I.

3.0 BEAM AND COLUMN SPLICES

It is often required to join structural members along their length due to the available length of sections being limited and also due to transportation and erection constraints. Such joints are called splices. Splices have to be designed so as to transmit all the member forces and at the same time provide sufficient stiffness and ease in erection. Splices are usually located away from critical sections. In members subjected to instability, the splice should be preferably located near the point of lateral restraint else the splice may have to be designed for additional forces arising due to instability effects.

3.1 Beam Splices

Beam Splices typically resist large bending moments and shear forces. If a rolled section beam splice is located away from the point of maximum moment, it is usually assumed that the flange splice carries all the moment and the web splice carries the shear. Such an assumption simplifies the splice design considerably. Where such simplification is not possible, as in the case of a plate girder, the total moment is divided between the flange and the web in accordance with the stress distribution. The web connection is then designed to resist its share of moment and shear.

A typical bolted splice plate connection is shown in Fig. 4 (*a*). To avoid deformation associated with slip before bearing in bearing bolts, HSFG bolts should be used. Usually double-splice plates are more economical because they require less number of bolts. However, for rolled steel sections with flange widths less than 200 mm, single splice plates may be used in the flange. End-plate connections may also be used as beam splices [Fig. 4(*b*)] although they are more flexible.

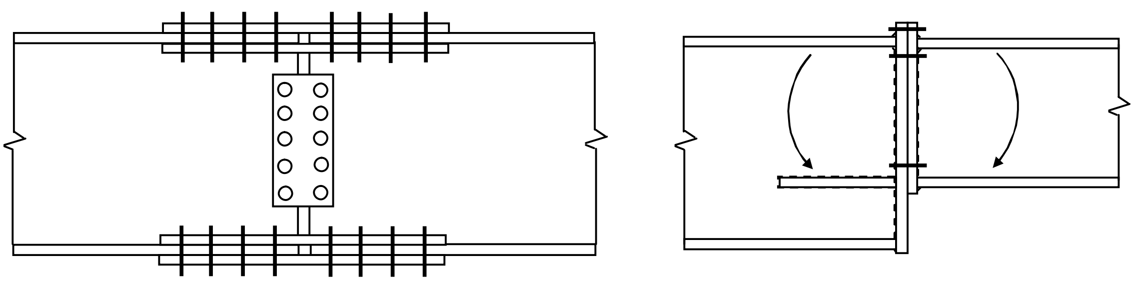


Fig. 4 Bolted Beam Splice: (a) Conventional Splice (b) End-plate Splice

3.2 Column Splice

Column splices can be of two types. In the bearing type, the faces of the two columns are prepared to butt against each other and thus transmit the load by physical bearing. In such cases only a nominal connection needs to be provided to keep the columns aligned. However, this type of splice cannot be used if the column sections are not prepared by grinding, if the columns are of different sizes, if the column carries moment or if continuity is required. In such cases, HSFG bolts will have to be used and the cost of splice increases. When connecting columns of different sizes, end plates or packing plates should be provided similar to the beam splice shown in Fig. 4(*b*).

4.0 BEAM-TO-COLUMN CONNECTIONS

Beam-to-column connections can be classified as simple, semi-rigid and rigid depending on the amount of moment transfer taking place between the beam to the column.

Simple connections are assumed to transfer only shear at some nominal eccentricity. Therefore such connections can be used only in non-sway frames where the lateral loads are resisted by some alternative arrangement such as bracings or shear walls. Simple connections are typically used in frames up to about five storeys in height, where strength rather than stiffness govern the design. Some typical details adopted for simple connections are shown in Fig. 5.

The clip and seating angle connection [Fig. 5(*a*)] is economical when automatic saw and drill lines are available. An important point in design is to check end bearing for possible adverse combination of tolerances. In the case of unstiffened seating angles, the bolts connecting it to the column may be designed for shear only assuming the seating angle to be relatively flexible. If the angle is stiff or if it is stiffened in some way then the bolted connection should be designed for the moment arising due to the eccentricity between the centre of the bearing length and the column face in addition to shear. The clip angle does not contribute to the shear resistance because it is flexible and opens out but it is required to stabilise the beam against torsional instability by providing lateral support to compression flange.

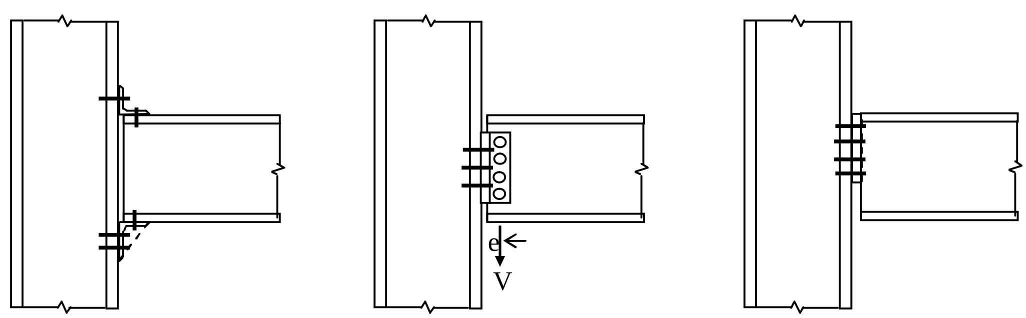


Fig. 5 Simple beam-to-column connections (a) Clip and seating angle (b) Web cleats (c) Curtailed end plate

The connection using a pair of web cleats, referred to as framing angles, [Fig. 5(*b*)] is also commonly employed to transfer shear from the beam to the column. Here again, if the depth of the web cleat is less than about 0.6 times that of the beam web, then the bolts need to be designed only for the shear force. Otherwise by assuming pure shear transfer at the column face, the bolts connecting the cleats to the beam web should be designed for the moment due to eccentricity.

The end plate connection [Fig. 5(*c*)] eliminates the need to drill holes in the beam. A deep end plate would prevent beam end rotation and thereby end up transferring significant moment to the column. Therefore the depth of the end plate should be limited to that required for shear transfer. However adequate welding should be provided between end plate and beam web. To ensure significant deformation of the end plate before bolt fracture, the thickness of the end plate should be less than one-half of the bolt diameter for Grade 8.8 bolts and one-third of the bolt diameter for Grade 4.6 bolts.

Rigid connections transfer significant moments to the columns and are assumed to undergo negligible deformations. Rigid connections are necessary in sway frames for stability and also contribute in resisting lateral loads. In high-rise and slender structures, stiffness requirements may warrant the use of rigid connections. Examples of rigid connections are shown in Fig. 6.

Using angles or T-sections to connect beam flanges to the column as shown in Fig. 1 is not economical due to the large number of bolts required. Further, these connections require HSFG bolts for rigidity. Therefore extended end-plate connections have become the popular method for rigid connections. It is fairly easy to transfer about 0.7 to 0.8 times the yield moment capacity of the beam using these connections. Column web stiffening will normally be required and the bolts at the bottom are for preventing the springing action. These bolts can however be used for shear transfer. In the case of deep beams connected to relatively slender columns a haunched connection as shown in Fig. 6c may be adopted. Additional column web stiffeners may also be required in the form of diagonal stiffeners [Fig. 6(*b*)] or web plates [Fig. 6(*c*)]. The general design method including prying action is explained in the worked example at the end of this chapter.

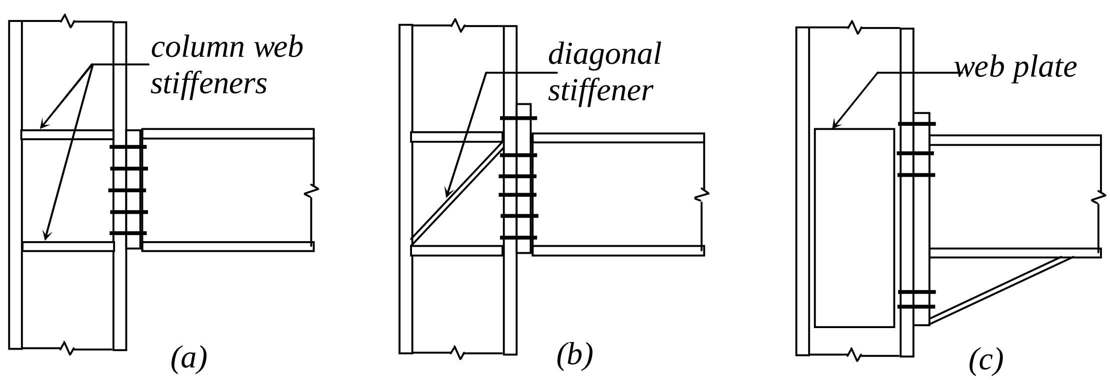


Fig. 6 Rigid beam-to-column connections (a) Short end plate (b) Extended end plate (c) Haunched

Semi-rigid connections fall between the two types mentioned above. The fact that most simple connections do have some degree of rotational rigidity was recognised and efforts to utilise it led to the development of the semi-rigid connections. They are used in conjunction with other lateral load resisting systems for increased safety and performance. Use of semi-rigid connections makes the analysis somewhat difficult but leads to economy in member designs. The analysis of semi-rigid connections is usually done by assuming linear rotational springs at the supports or by advanced analysis methods, which account for non-linear moment-rotation characteristics.

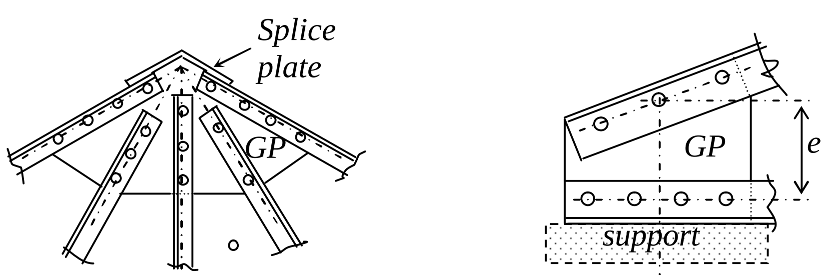
5.0 BEAMS-TO-BEAM CONNECTION

Beam to beam connections are similar to beam to column connections. Sometimes rigid connections may be provided for moment continuity between secondary beams. In such cases if the primary beam is torsionally flexible, then the torsion transferred to it may be ignored. Typically in simple connections, only web cleats are used because the web of the main beam cannot take a seating angle. For coplanar top flanges, the top flange of the secondary beam may have to be coped and checked for block shear in the design calculations. This is further illustrated in a worked example at the end of this chapter.

6.0 TRUSS CONNECTIONS

Truss connections form a high proportion of the total truss cost. Therefore it may not always be economical to select member sections, which are efficient but cannot be connected economically. Trusses may be single plane trusses in which the members are connected on the same side of the gusset plates or double plane trusses in which the members are connected on both sides of the gusset plates.

It may not always be possible to design connection in which the centroidal axes of the member sections are coincident [Fig. 7(*a*)]. Small eccentricities may be unavoidable and the gusset plates should be strong enough to resist or transmit forces arising in such cases without buckling (Fig. 7b). The bolts should also be designed to resist moments arising due to in-plane eccentricities. If out-of-plane instability is foreseen, use splice plates for continuity of out-of-plane stiffness (Fig. 7a).



(a) Apex Connection *(b) Support connection*

Fig. 7 Truss Connections

7.0 FATIGUE BEHAVIOUR

Fatigue is a phenomenon, which leads to the initiation and growth of cracks in a structure under fluctuating stresses even below the yield stress of the material. The cracks usually initiate from points where stress concentrations occur. Therefore, it is important to ensure that stress concentrations are kept to a minimum in structures subjected to fluctuating stresses. Possible ways of doing this in bolted connections are by using gusset plates of proper shape, drilling holes more accurately by matching the plates to be connected and using HSFG bolts instead of bearing type bolts. Another aspect, which has a profound effect on the fatigue performance, is the range of stress fluctuations and reversal of stress. By using HSFG bolts, which are pretensioned, stress reversals can be avoided thereby improving the fatigue performance. However, due to the bearing of the bolt head on the plies, these bolts exhibit *fretting corrosion* at the edges of the regions of high bearing pressure.

Fatigue design is usually carried out by means of an S-N curve, which is a plot of the stress range S versus the number of cycles to failure N on a log-log scale. As the stress range decreases, the number of cycles to failure rapidly increases and below a certain level known as the endurance limit, the connection will be able to withstand a sufficiently large number of cycles. The S-N curve is plotted for various types of connections and used for design. Tension connections using HSFG perform extremely well under fatigue. Shear connections using HSFG are better than most welded connections while shear connections using black bolts are inferior to welded connections.

8.0 SUMMARY

The issues involved in the design of connections were described so that the designer can get an insight into the behaviour of various types of connections. Simple analysis methods for bolt groups resisting in-plane and out-of-plane moments were described. Beam and column splices as well as various types of beam-to-column connections were described and their general behaviour as well as points to be kept in mind during their design were explained. In addition, Beam-to-beam and truss connections were also described. Lastly, the topic of fatigue design was touched upon. Finer points in design will be clarified by means of the worked examples at the end of the chapter.

35 PLASTIC ANALYSIS

1.0 INTRODUCTION

The elastic design method, also termed as *allowable stress method (or Working stress method)*, is a conventional method of design based on the elastic properties of steel. This method of design limits the structural usefulness of the material upto a certain allowable stress, which is well below the elastic limit. The stresses due to working loads do not exceed the specified allowable stresses, which are obtained by applying an adequate factor of safety to the yield stress of steel. The elastic design does not take into account the strength of the material beyond the elastic stress. Therefore the structure designed according to this method will be heavier than that designed by plastic methods, but in many cases, elastic design will also require less stability bracing.

In the method of plastic design of a structure, the ultimate load rather than the yield stress is regarded as the design criterion. The term *plastic* has occurred due to the fact that the ultimate load is found from the strength of steel in the plastic range. This method is also known as *method of load factor design* or *ultimate load design*. The strength of steel beyond the yield stress is fully utilised in this method. This method is rapid and provides a rational approach for the analysis of the structure. This method also provides striking economy as regards the weight of steel since the sections designed by this method are smaller in size than those designed by the method of elastic design. Plastic design method has its main application in the analysis and design of statically indeterminate framed structures.

2.0 BASIS OF PLASTIC THEORY

2.1 Ductility of Steel

Structural steel is characterised by its capacity to withstand considerable deformation beyond first yield, without fracture. During the process of 'yielding' the steel deforms under a constant and uniform stress known as 'yield stress'. This property of steel, known as *ductility*, is utilised in plastic design methods.

Fig. 1 shows the idealised stress-strain relationship for structural mild steel when it is subjected to direct tension. Elastic straining of the material is represented by line *OA*. *AB* represents yielding of the material when the stress remains constant, and is equal to the yield stress, f_y. The strain occurring in the material during yielding remains after the load has been removed and is called the plastic strain and this strain is at least ten times as large as the elastic strain, ε_y at yield point.

When subjected to compression, the stress-strain characteristics of various grades of structural steel are largely similar to Fig. 1 and display the same property of yield. The major difference is in the strain hardening range where there is no drop in stress after a peak value. This characteristic is known as ***ductility of steel.***

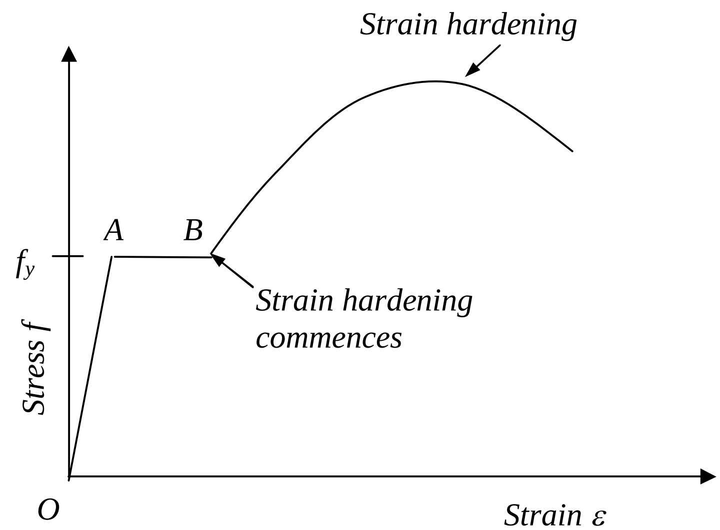


Fig. 1 Idealised stress – strain curve for steel in tension

2.2 Theoretical Basis

As an incremental load is applied to a beam, the cross-section with greatest bending moment will eventually reach the yield moment. Elsewhere the structure is elastic and the 'peak' moment values are less than yield. As load is incremented, a zone of yielding develops at the first critical section, but due to ductility of steel, the moment at that section remains about constant. The structure, therefore, calls upon its less heavily stressed portions to carry the increase in load. Eventually the zones of yielding are formed at other sections until the moment capacity has been exhausted at all necessary critical sections. After reaching the maximum load value, the structure would simply deform at constant load. Thus it is a design based upon the ultimate load-carrying capacity (maximum strength) of the structure. This ultimate load is computed from a knowledge of the strength of steel in the plastic range and hence the name 'plastic'.

2.3 Perfectly Plastic Materials

The stress-strain curve for a perfectly plastic material upto strain hardening is shown in Fig. 2. Perfectly plastic materials follow Hook's law upto the limit of proportionality. The slopes of stress-strain diagrams in compression and tension i.e. the values of Young's modulus of elasticity of the material, are equal. Also the values of yield stresses in tension and compression are equal. The strains upto the strain hardening in tension and compression are also equal. The stress strain curves show horizontal plateau both in tension and compression. Such materials are known as ***perfectly plastic materials.***

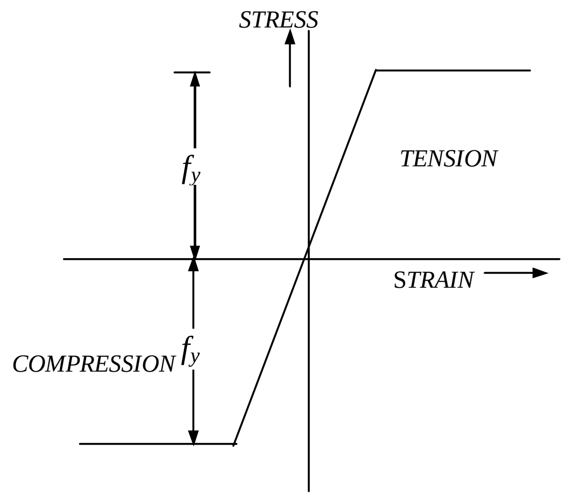


Fig. 2 Stress - Strain Curve for perfectly plastic materials

2.4 Fully Plastic Moment of a Section

The fully plastic moment M_p, of a section is defined as the maximum moment of resistance of a fully plasticized or yielded cross-section. The assumptions used for finding the plastic moment of a section are:

(i) The material obeys Hooke's law until the stress reaches the upper yield value; on further straining, the stress drops to the lower yield value and thereafter remains constant.

(ii) The yield stresses and the modulus of elasticity have the same value in compression as in tension.

(iii) The material is homogeneous and isotropic in both the elastic and plastic states.

(iv) The plane transverse sections (the sections perpendicular to the longitudinal axis of the beam) remain plane and normal to the longitudinal axis after bending, the effect of shear being neglected.

(v) There is no resultant axial force on the beam.

(vi) The cross section of the beam is symmetrical about an axis through its centroid parallel to plane of bending.

(vii) Every layer of the material is free to expand and contract longitudinally and laterally under the stress as if separated from the other layers.

In order to find out the fully plastic moment of a yielded section of a beam as shown in Fig. 3, we employ the force equilibrium equation, namely the total force in compression and the total force in tension over that section are equal.

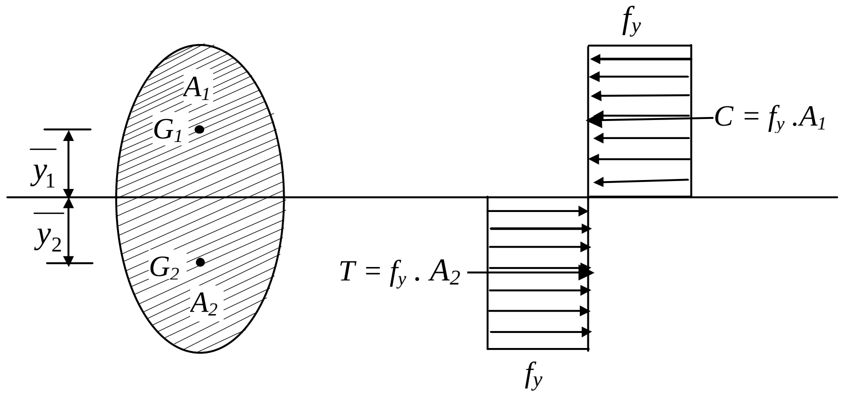


Fig. 3

$$\text{Total compression},\ C = \text{Total tension},\ T$$

$$f_y . A_1 = f_y . A_2$$
$$\therefore \quad A_1 = A_2$$
$$A = A_1 + A_2$$
$$A_1 = A_2 = A/2$$

Plastic Moment of resistance, $$M_p = f_y \cdot A_1 \cdot \overline{y_1} + f_y \cdot A_2 \cdot \overline{y_2}$$
$$= f_y \cdot \frac{A}{2} \cdot \left(\overline{y_1} + \overline{y_2}\right)$$
$$= f_y \cdot Z_p \tag{1}$$

where Z_p , the plastic modulus of the section $= \frac{A}{2} \cdot \left(\overline{y_1} + \overline{y_2}\right)$

The ***plastic modulus*** of a completely yielded section is defined as the combined statical moment of the cross-sectional areas above and below the neutral axis or equal area axis. It is the resisting modulus of a completely plasticised section.

3.0 BENDING OF BEAMS SYMMETRICAL ABOUT BOTH AXES

The bending of a symmetrical beam subjected to a gradually increasing moment is considered first. The fibres of the beam across the cross section are stressed in tension or compression according to their position relative to the neutral axis and are strained in accordance with Fig. 1.

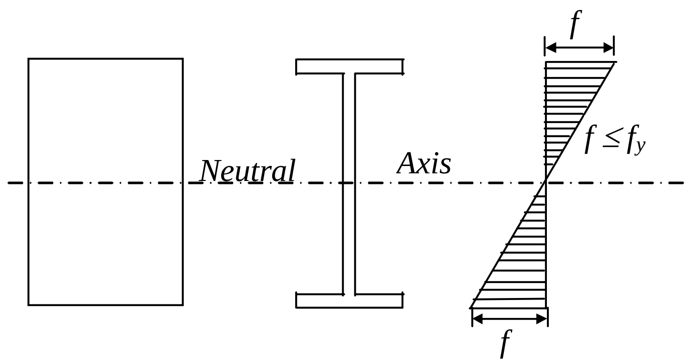


Fig. 4 Elastic stresses in beams

While the beam remains entirely elastic the stress in every fibre is proportional to its strain and to its distance from the neutral axis. The stress *(f)* in the extreme fibres cannot exceed f_y. (see Fig. 4)

When the beam is subjected to a moment slightly greater than that, which first produces yield in the extreme fibres, it does not fail. Instead the outer fibres yield at constant stress (f_y) while the fibres nearer to the neutral axis sustain increased elastic stresses. Fig. 5 shows the stress distribution for beams subjected to such moments.

Such beams are said to be 'partially plastic' and those portions of their cross-sections, which have reached the yield stress, are described as 'plastic zones'.

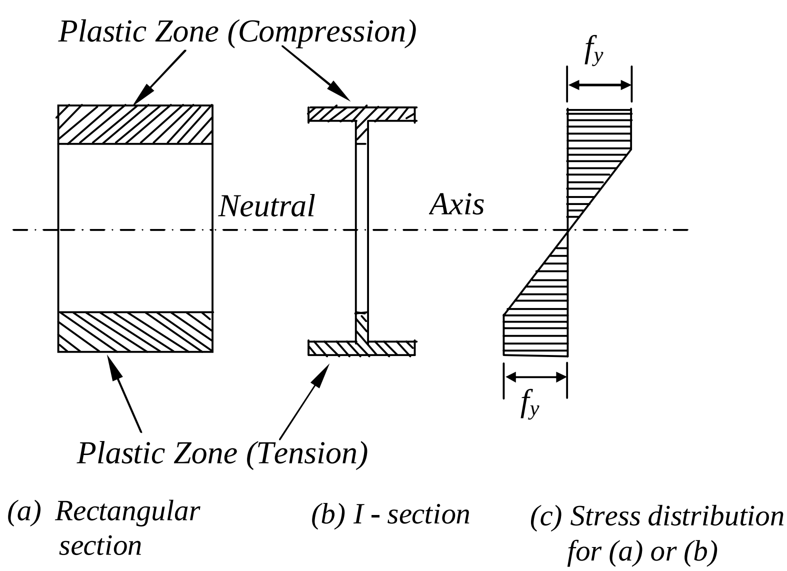
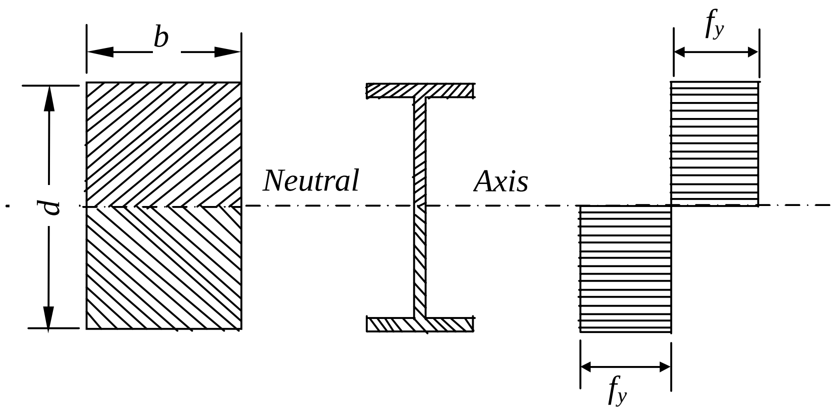


Fig. 5 Stresses in partially plastic beams

The depths of the plastic zones depend upon the magnitude of the applied moment. As the moment is increased, the plastic zones increase in depth, and, it is assumed that plastic yielding can occur at yield stress *(f_y)* resulting in two stress blocks, one zone yielding in tension and one in compression. Fig. 6 represents the stress distribution in beams stressed to this stage. The plastic zones occupy the whole of the cross section, and are described as being 'fully plastic'. When the cross section of a member is fully plastic under a bending moment, any attempt to increase this moment will cause the member to act as if hinged at the neutral axis. This is referred to as a ***plastic hinge.***

The bending moment producing a plastic hinge is called the full ***plastic moment*** and is denoted by 'M_p'. Note that a plastic hinge carries a constant moment, M_P.



(a) Rectangular section *(b) I - section* *(c) Stress distribution for (a) or (b)*

Fig. 6 Stresses in fully plastic beams

4.0 GENERAL REQUIREMENTS FOR UTILISING PLASTIC DESIGN CONCEPTS

Generally codes (such as IS 800, BS 5950) allow the use of plastic design only where loading is predominantly static and fatigue is not a design criterion.

For example, in order to allow this high level of strain, *BS 5950* prescribes the following restrictions on the properties of the stress-strain curve for steels used in plastically designed structures (clause 5.3.3).

1. The yield plateau (horizontal portion of the curve) is greater than 6 times the yield strain.
2. The ultimate tensile strength must be more than 1.2 times the yield strength.
3. The elongation on a standard gauge length is not less than 15%.

These limitations are intended to ensure that there is a sufficiently long plastic plateau to enable a hinge to form and that the steel will not experience a premature strain hardening.

4.1 Shape Factor

As described previously there will be two stress blocks, one in tension, the other in compression, both of which will be at yield stress. For equilibrium of the cross section, the areas in compression and tension must be equal. For a rectangular cross section, the elastic moment is given by,

$$M = \frac{bd^2}{6} f_y \tag{2.a}$$

The plastic moment is obtained from,

$$M_p = 2.b.\frac{d}{2}\cdot\frac{d}{4}\cdot f_y = \frac{bd^2}{4} f_y \tag{2.b}$$

Here the plastic moment M_p is about 1.5 times greater than the elastic moment capacity. In developing this moment, there is a large straining in the extreme fibres together with large rotations and deflection. This behaviour may be plotted as a moment-rotation curve. Curves for various cross sections are shown in Fig. 7.

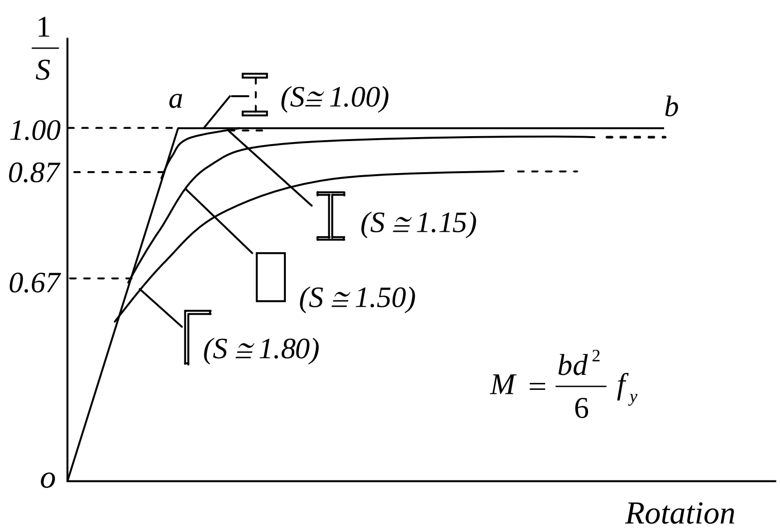


Fig.7 Moment – rotation curves

The ratio of the plastic modulus *(Z_p)* to the elastic modulus *(Z)* is known as the shape factor *(S)* and will govern the point in the moment-rotation curve when non-linearity starts. For the theoretically ideal section in bending i.e. two flange plates connected by a web of insignificant thickness, this will have a value of 1. When the material at the centre of the section is increased, the value of *S* increases. For a universal beam the value is about 1.15 increasing to 1.5 for a rectangle.

5.0 PLASTIC HINGES

In deciding the manner in which a beam may fail it is desirable to understand the concept of how plastic hinges form where the beam is fully plastic.

At the plastic hinge an infinitely large rotation can occur under a constant moment equal to the plastic moment of the section. Plastic hinge is defined as a yielded zone due to bending in a structural member at which an infinite rotation can take place at a constant plastic moment M_p of the section. The number of hinges necessary for failure does not vary for a particular structure subject to a given loading condition, although a part of a structure may fail independently by the formation of a smaller number of hinges. The member or structure behaves in the manner of a hinged mechanism and in doing so adjacent hinges rotate in opposite directions.

Theoretically, the plastic hinges are assumed to form at points at which plastic rotations occur. Thus the length of a plastic hinge is considered as zero.

The values of moment, at the adjacent section of the yield zone are more than the yield moment upto a certain length ΔL, of the structural member. This length ΔL, is known as the hinged length. The hinged length depends upon the type of loading and the geometry of the cross-section of the structural member. The region of hinged length is known as ***region of yield*** or ***plasticity.***

5.1 Hinged Length of a Simply Supported Beam with Central Concentrated Load

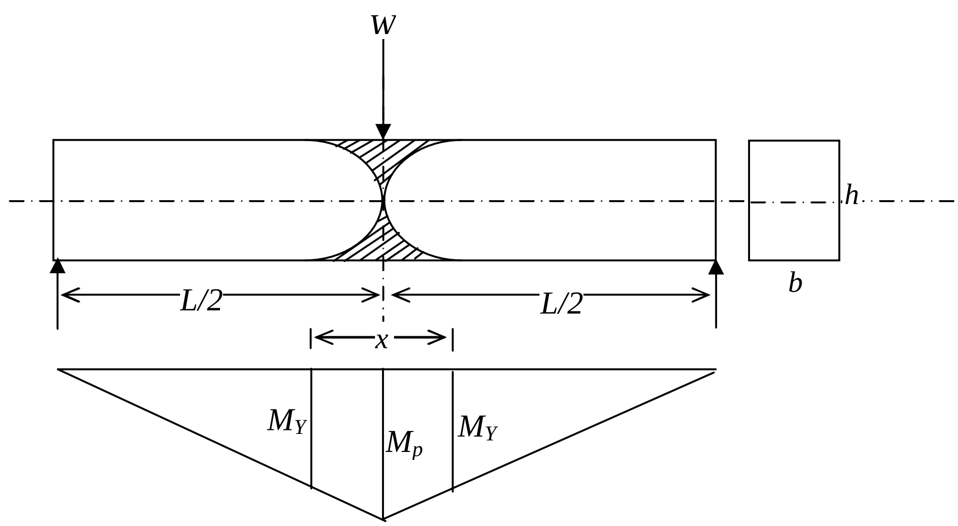


Fig. 8

In a simply supported beam with central concentrated load, the maximum bending moment occurs at the centre of the beam. As the load is increased gradually, this moment reaches the fully plastic moment of the section M_p and a plastic hinge is formed at the centre.

$$M_p = \frac{W\,l}{4}$$

$$= f_y \cdot \frac{bh^2}{4} \qquad \left(\because\ Z_p = \frac{bh^2}{4}\right)$$

$$M_y = f_y \cdot \frac{bh^2}{6} = \left(f_y \cdot \frac{bh^2}{4}\right)\frac{2}{3}$$

$$\therefore \quad M_y = \frac{2}{3} M_p$$

Let x (= ΔL) be the length of plasticity zone.

From the bending moment diagram shown in Fig. 8

$$\frac{M_y}{\frac{L}{2} - \frac{x}{2}} = \frac{M_p}{\frac{L}{2}}$$

$$\frac{M_y}{M_p} = \frac{\frac{L}{2} - \frac{x}{2}}{\frac{L}{2}}$$

$$\frac{M_y}{M_p} = 1 - \frac{x}{L}$$

$$(L - x)M_p = L \cdot M_y$$

$$(L - x)M_p = L \cdot \frac{2}{3} \cdot M_p$$

$$x = \frac{1}{3} L \tag{3}$$

Therefore the hinged length of the plasticity zone is equal to one-third of the span in this case.

6.0 FUNDAMENTAL CONDITIONS FOR PLASTIC ANALYSIS

(i) ***Mechanism condition:*** *The ultimate or collapse load is reached when a mechanism is formed. The number of plastic hinges developed should be just sufficient to form a mechanism.*

(ii) ***Equilibrium condition*** : $\Sigma F_x = 0, \Sigma F_y = 0, \Sigma M_{xy} = 0$

(iii) ***Plastic moment condition:*** *The bending moment at any section of the structure should not be more than the fully plastic moment of the section.*

6.1 Mechanism

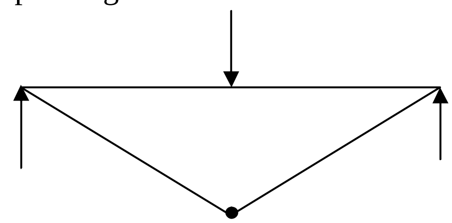
When a system of loads is applied to an elastic body, it will deform and will show a resistance against deformation. Such a body is known as a *structure*. On the other hand if no resistance is set up against deformation in the body, then it is known as a *mechanism*.

Various types of independent mechanisms are

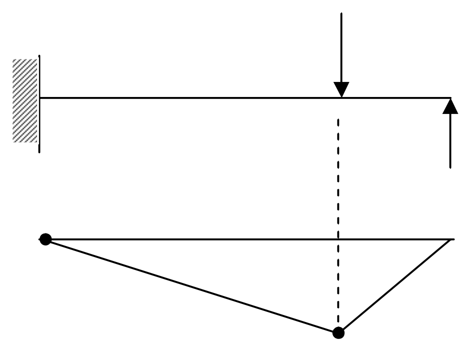
6.1.1 Beam Mechanism

Fig. 9 sketches three simple structures and the corresponding mechanisms.

(a) A simply supported beam has to form one plastic hinge at the point of maximum bending moment.
Redundancy, $r = 0$



(b) A propped cantilever requires two hinges to form a mechanism.
Redundancy, $r = 1$
No. of plastic hinges formed,
$= r + 1 = 2$



(c) A fixed beam requires three hinges to form a mechanism.
Redundancy, $r = 2$
No. of plastic hinges $= 2 + 1 = 3$

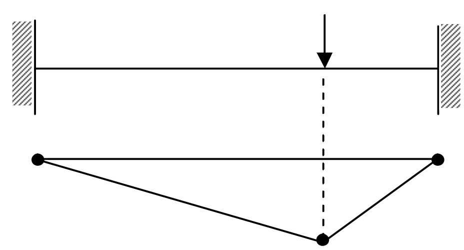
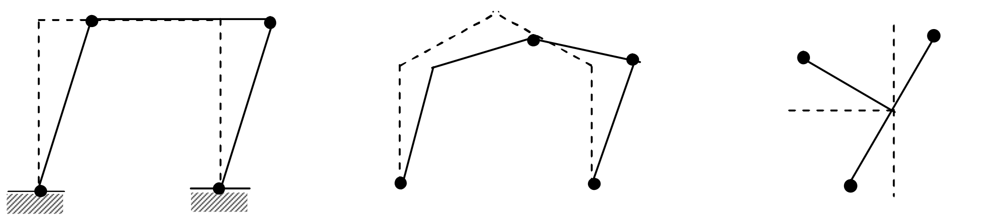


Fig. 9

From the above examples, it is seen that the number of hinges needed to form a mechanism equals the statical redundancy of the structure plus one.

6.1.2 Panel or Sway Mechanism

Fig. 10 (A) shows a panel or sway mechanism for a portal frame fixed at both ends.



(A) Panel Mechanism *(B) Gable Mechanism* *(C) Joint Mechanism*

Fig. 10

6.1.3 Gable Mechanism

Fig. 10 (B) shows the gable mechanism for a gable structure fixed at both the supports.

6.1.4 Joint Mechanism

Fig. 10 (C) shows a joint mechanism. It occurs at a joint where more than two structural members meet.

6.1.5 Combined Mechanism

Various combinations of independent mechanisms can be made depending upon whether the frame is made of strong beam and weak column combination or strong column and weak beam combination. The one shown in Fig.11 is a combination of a beam and sway mechanism. Failure is triggered by formation of hinges at the bases of the columns and the weak beam developing two hinges. This is illustrated by the right hinge being shown on the beam, in a position slightly away from the joint.

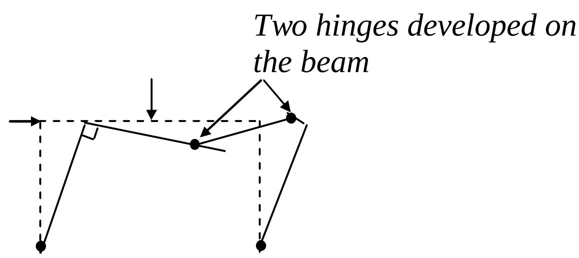


Fig. 11 Combined Mechanism

6.2 LOAD FACTOR AND THEOREMS OF PLASTIC COLLAPSE

Plastic analysis of structures is governed by three theorems, which are detailed in this section.

The load factor at rigid plastic collapse *(λ_p)* is defined as the lowest multiple of the design loads which will cause the whole structure, or any part of it to become a mechanism.

In a limit state approach, the designer is seeking to ensure that at the appropriate factored loads the structure will not fail. Thus the rigid plastic load factor λ_p must not be less than unity.

The number of independent mechanisms *(n)* is related to the number of possible plastic hinge locations *(h)* and the number of degree of redundancy *(r)* of the frame by the equation.

$$n = h - r \tag{4}$$

The three theorems of plastic collapse are given below for reference.

6.2.1 Lower Bound or Static Theorem

A load factor (λ_s) computed on the basis of an arbitrarily assumed bending moment diagram which is in equilibrium with the applied loads and where the fully plastic moment of resistance is nowhere exceeded will always be less than or at best equal to the load factor at rigid plastic collapse, (λ_p).

λ_p is the highest value of λ_s which can be found.

6.2.2 Upper Bound or Kinematic Theorem

A load factor (λ_k) computed on the basis of an arbitrarily assumed mechanism will always be greater than, or at best equal to the load factor at rigid plastic collapse (λ_p)

λ_p is the lowest value of λ_k which can be found.

6.2.3 Uniqueness Theorem

If both the above criteria are satisfied, then the resulting load factor corresponds to its value at rigid plastic collapse (λ_p).

7.0 RIGID PLASTIC ANALYSIS

As the plastic deformations at collapse are considerably larger than elastic ones, it is assumed that the frame remains rigid between supports and hinge positions i.e. all plastic rotation occurs at the plastic hinges.

Considering a simply supported beam subjected to a point load at midspan, the maximum strain will take place at the centre of the span where a plastic hinge will be formed at yield of full section. The remainder of the beam will remain straight, thus the entire energy will be absorbed by the rotation of the plastic hinge. (See Fig. 12)

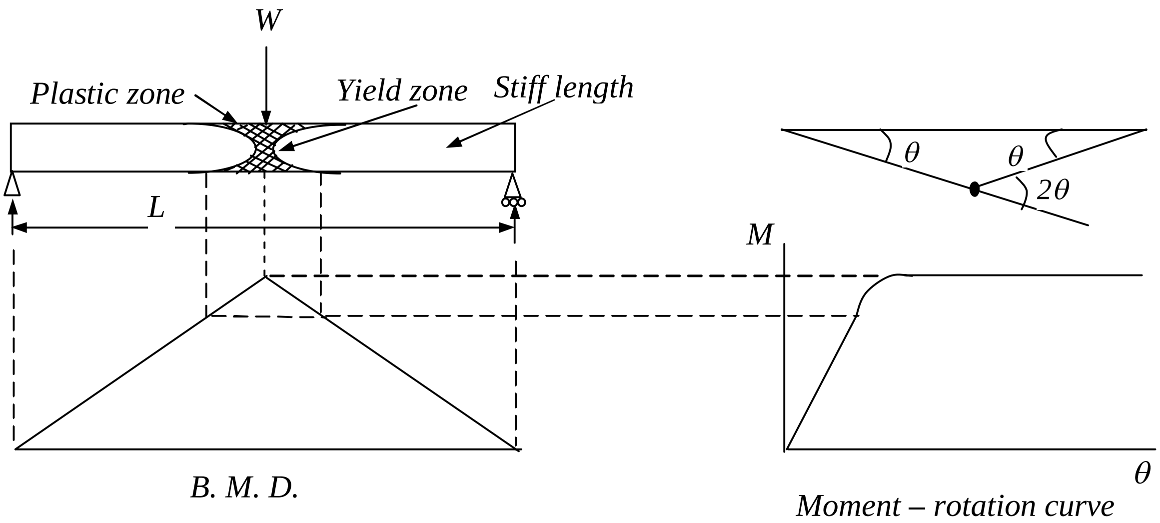


Fig. 12 Simply supported beam at plastic hinge stage

Considering a centrally loaded simply supported beam at the instant of plastic collapse (see Fig. 12)

Workdone at the plastic hinge $= M_p 2\theta$ (5a)

Workdone by the displacement of the load = $W\left(\frac{L}{2}.\theta\right)$ (5b)

At collapse, these two must be equal

$$2Mp.\theta = W\left(\frac{L}{2}.\theta\right)$$

$$M_p = \frac{WL}{4} \tag{6}$$

The moment at collapse of an encastre beam with a uniform load is similarly worked out from Fig. 13. It should be noted that three hinges are required to be formed at *A*, *B* and *C* just before collapse.

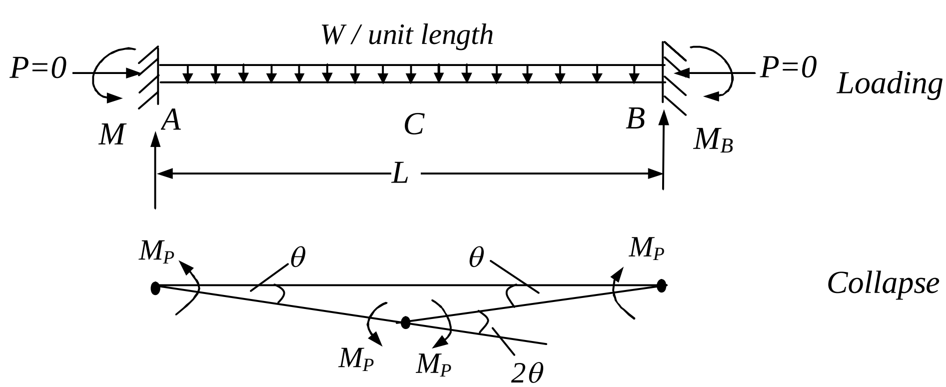


Fig. 13 Encastre Beam

Workdone at the three plastic hinges $= M_p(\theta + 2\theta + \theta) = 4M_p\theta$ (7.a)

Workdone by the displacement of the load $= W/L \,.\, L/2 \,.\, L/2 \,.\, \theta$ (7.b)

$$\frac{WL}{4}\theta = 4M_p\theta$$

$$WL = 16 M_p \quad (8)$$

$$M_p = \frac{WL}{16} \quad (9)$$

In other words the load causing plastic collapse of a section of known value of M_p is given by eqn. (8). All the three hinges at *A*, *B* and *C* will have a plastic moment of M_p as given in eqn. (9).

7.1 Continuous Beams

Consider next the three span continuous beam of uniform section throughout (constant M_p) as shown in Fig. 14(a). Here a conventional approach is more laborious but the collapse load may be readily determined by consideration of the collapse patterns. Each pattern represents the conversion of each of the three spans into mechanism.

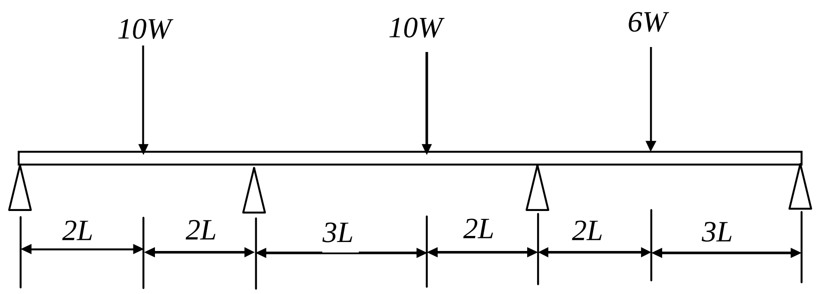


Fig. 14 (a)

Collapse pattern 1:

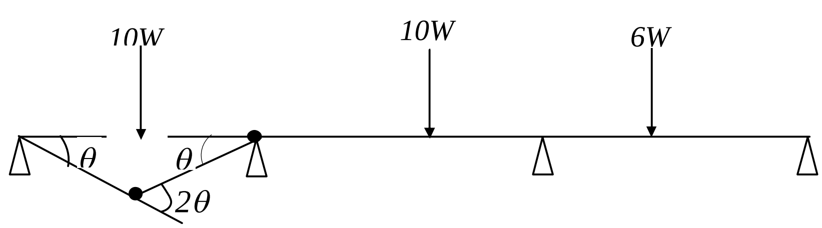


Fig. 14(b)

Work done in hinges $= M_p(2\theta + \theta) = 3M_p\theta$

Work done by loads = $10\,W(\,2L\theta\,)$ = $20WL\theta$

Collapse load, W_c = $3\,M_p\,/\,20L$ = $0.15\,M_p\,/\,L$

<u>Collapse pattern 2:</u>

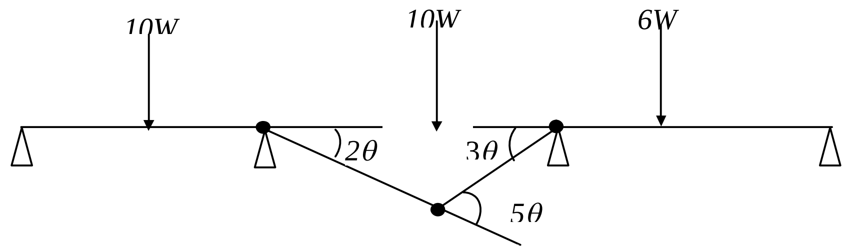


Fig. 14 (c)

Work done in plastic hinges = $M_p\,(\,2\theta\,+5\theta+3\theta\,)$ = $10\,M_p\,\theta$

Work done by loads = $10W(\,6L\theta\,)$ = $60\,WL\theta$

Collapse load, W_c = $10\,M_p\,/\,60\,L$ = $0.17\,M_p\,/\,L$

<u>Collapse pattern 3:</u>

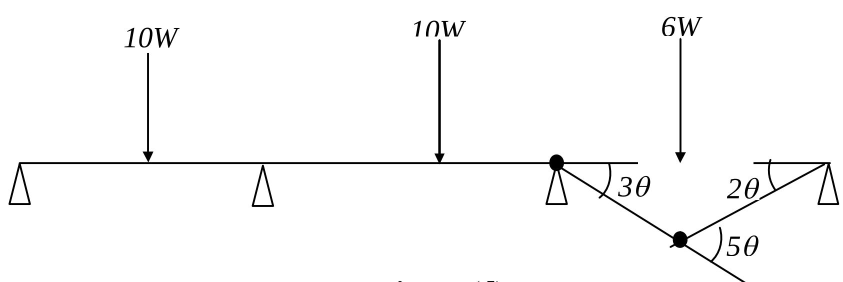


Fig. 14(d)

Work done in hinges = $M_p\,(\,3\theta\,+5\theta\,)$ = $8\,M_p\,\theta$

Work done by loads = $6W\,(6L\theta\,)$ = $36\,WL\theta$

Collapse load, W_c = $8\,M_p\,/\,36L$ = $0.22\,M_p\,/\,L$

Thus collapse will occur in the mode of Fig. 14 (b) when $W_c\;=\;0.15\,M_p\,/\,L$.

7.2 Mechanism Method

In the mechanism or kinematic method of plastic analysis, various plastic failure mechanisms are evaluated. The plastic collapse loads corresponding to various failure mechanisms are obtained by equating the internal work at the plastic hinges to the external work by loads during the virtual displacement. This requires evaluation of displacements and plastic hinge rotations.

For gabled frames and other such frames, the kinematics of collapse is somewhat complex. It is convenient to use the instantaneous centres of rotation of the rigid elements of the frame to evaluate displacements corresponding to different mechanisms. In this, properties of rotations of a rigid body during an infinitesimally small angle θ are assumed as follows (see Fig.15):

(i) Any point *P* will move by distance $r\theta$ to point *P'* normal to the radius vector OP for length *r*, due to the rotation of the rigid body by an angle θ about *O*.

(ii) The work done by a force *F* due to the rotation of the rigid body about *O* by an angle θ is given by $F*x*\theta$, where *x* is the shortest (perpendicular distance) between vector F and the centre of rotation *O*.

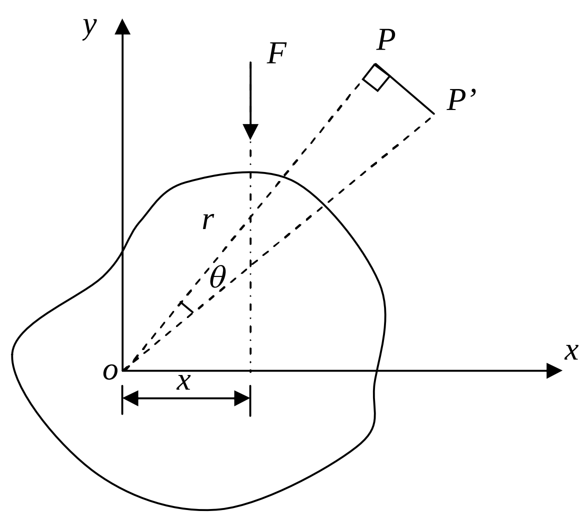


Fig.15 Rigid body rotation

Consider, for example the structure shown in Fig 16. Let us consider the plastic mechanism by formation of plastic hinges *A*, *C*, *F* and *G*. Let the virtual rotation of the member *FG* be θ about plastic hinge *G*. Point *F* moves, normal to line *FG* to *F'* due to rotation about *G*. This would cause a part of the structure *ABC* to rotate about point *A* and point *C* would move to *C'*. Since point *F* moves to *F'* the instantaneous centre of rotation of segment *CDF* would be along the line *FG*. Similarly since point *C* would move normal to line *AC*, the instantaneous centre of rotation of element *CDF* should also be along the line *AC*. Thus we can locate the instantaneous centre which will be the point of intersection of line *AC* and *GF*, obtained by extending them to meet at *I*.

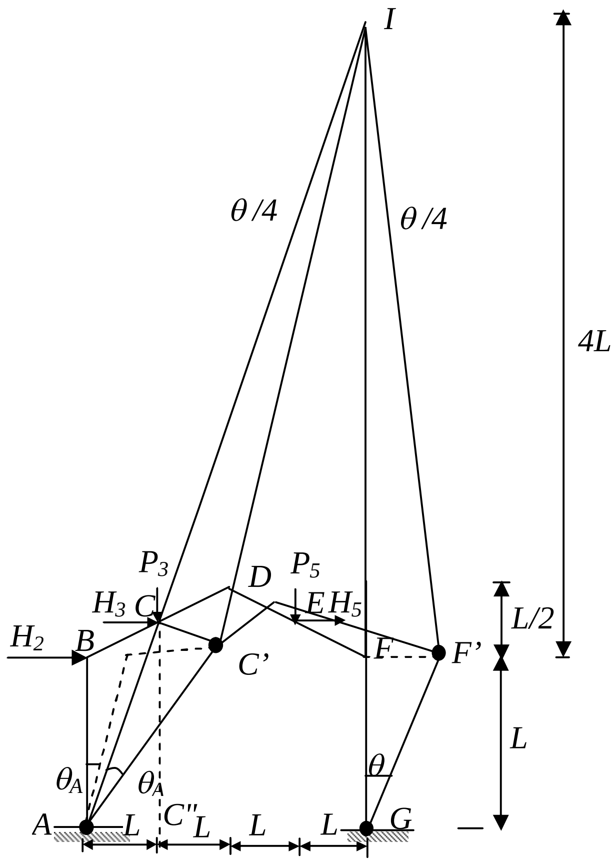


Fig. 16

Let us find the rotation of element *CDF* about instantaneous centre of rotation I,

Let < FGF' =θ

From FGF' we get FF' = $L*\theta$

To find the location of I, consider similar triangles ACC" and IAG,

$$\frac{IG}{AG} = \frac{CC''}{AC''}$$

$$\frac{x + L + L/2}{4L} = \frac{L + \frac{L}{4}}{L}$$

$$x + \frac{3L}{2} = 4L + L$$

$$x = \frac{7L}{2}$$

Similarly from IFF'

$(x + L/2)\theta_I$ =FF' =L*θ

$(7L/2+L/2)\ \theta_I = L\theta$

$$\theta_I = \frac{\theta}{4}$$

Similarly from *ICC'* and *ACC'*

$$\frac{\theta}{4} * 3L = \theta_A L$$

$$\theta_A = \frac{3\theta}{4}$$

The displacements of loads in the direction of application of loads are as follows:

<u>Displacement of horizontal loads</u>

For H_2; $\Delta_2 = \dfrac{3\theta}{4} L$

For H_3; $\Delta_3 = \dfrac{3\theta}{4}\left(L + \dfrac{L}{4}\right) = \dfrac{15\theta}{16} L$

For H_5; $\Delta_5 = \dfrac{\theta}{4}\left(4L - \dfrac{L}{4}\right) = \dfrac{15\theta}{16} L$

<u>Displacement of vertical loads</u>

For P_3; $\Delta'_2 = \dfrac{3\theta}{4} L$

For P_5; $\Delta'_5 = \dfrac{\theta}{4} L$

The rotations at the plastic hinges are as follows:

$$\theta_A = \frac{3\theta}{4}$$

$$\theta_C = \left(\frac{3\theta}{4} + \frac{\theta}{4}\right) = \theta$$

$$\theta_F = \left(\frac{\theta}{4} + \theta\right) = \frac{5\theta}{4}$$

$$\theta_G = \theta$$

With these information the virtual work equation can be written.

7.3 Rectangular Portal Framework and Interaction Diagrams

The same principle is applicable to frames as indicated in Fig. 17(a) where a portal frame with constant plastic moment of resistance M_p throughout is subjected to two independent loads H and V.

This frame may distort in more than one mode. There are basic independent modes for the portal frame, the pure sway of Fig. 17 (b) and a beam collapse as indicated in Fig. 17 (c). There is now however the possibility of the modes combining as shown in Fig. 17(d).

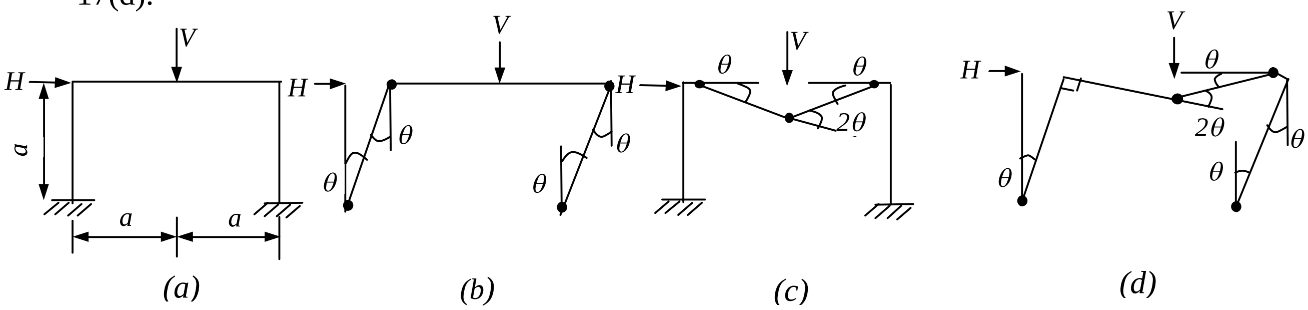


Fig.17 Possible Failure Mechanisms

From Fig. 17(b)

Work done in hinges = $4\, M_p\theta$
Work done by loads = $Ha\theta$
At incipient collapse $Ha / M_p = 4$ (10)

From Fig. 17 (c)

Work done in hinges = $4\, M_p\theta$
Work done by loads = $Va\theta$
At incipient collapse = $Va / M_p = 4$ (11)

From Fig. 17(d)

Work done in hinges = $6\, M_p\theta$
Work done by loads = $Ha\theta + Va\theta$
At incipient collapse $Ha / M_p + Va / M_p = 6$ (12)

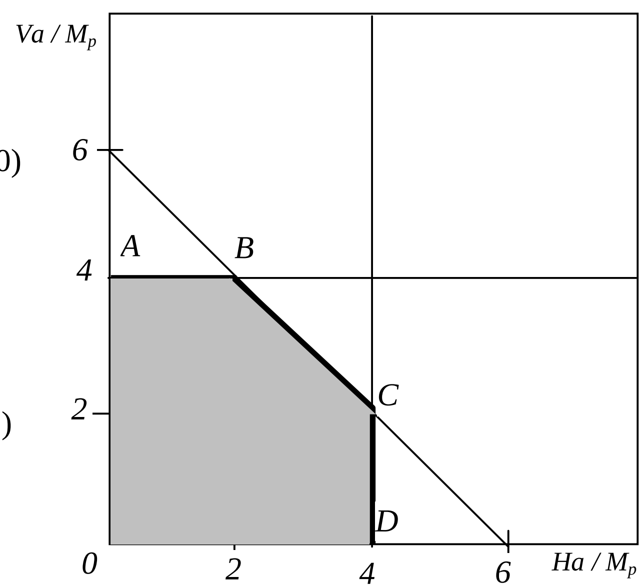


Fig. 18

The resulting equations, which represent the collapse criteria, are plotted on the interaction diagram of Fig. 18. Since any line radiating from the origin represents proportional loading, the first mechanism line intersected represents failure. The failure condition is therefore the line *ABCD* and any load condition within the area *OABCD* is therefore safe.

7.4 Frames not of Constant Section Throughout

Let us suppose however that the beam had an enhanced value of fully plastic moment of $2M_p$. The possible modes of collapse are unaltered but wherever a hinge forms at the beam/stanchion joint, it will occur in the weaker member - in this case it will be at the stanchion. For clarity it is customary to draw the hinge location just away from the joint as indicated in Fig. 19, but in the ensuing geometric computations it is assumed that its location is at the joint. The previous calculation is then modified as follows:

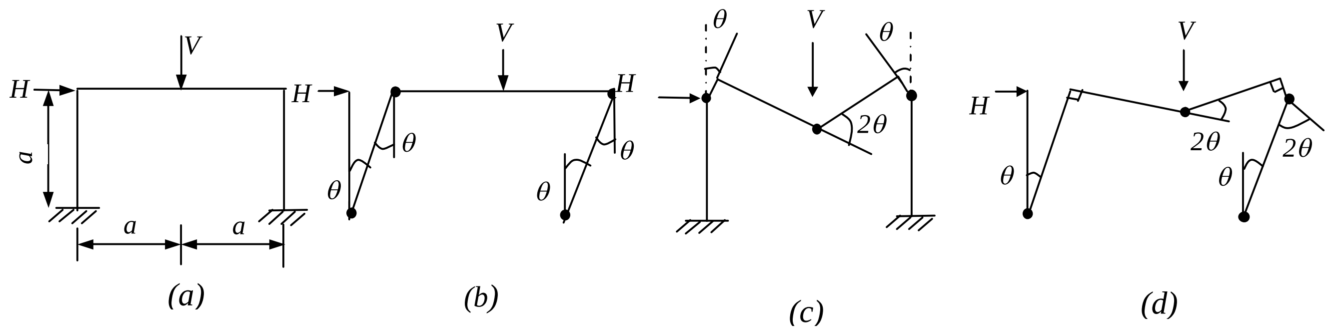


Fig.19

From Fig. 19 (b)

Work done in hinges = $4\ M_p\theta$
Work done by loads = $Ha\theta$
At incipient collapse $Ha\ /\ M_p = 4$ (13)

From Fig. 19 (c)

Work done in hinges = $6\ M_p\theta$
Work done by loads = $Va\theta$
At incipient collapse $Va\ /\ M_p = 6$ (14)

From Fig.19 (d)

Work done in hinges = $8\ M_p\theta$
Work done by loads = $Ha\theta + Va\theta$
At incipient collapse $Ha\ /\ M_p + Va\ /\ M_p = 8$ (15)

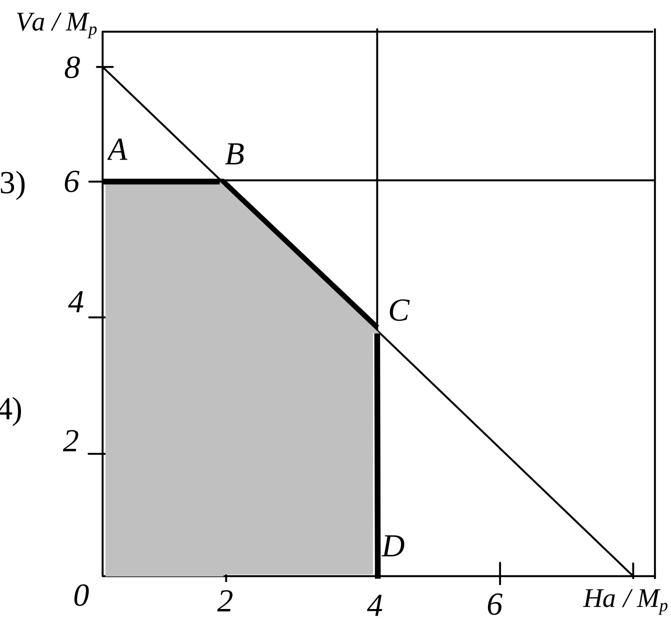


Fig. 20

The interaction diagram then becomes as shown in Fig. 20.

8.0 STABILITY

For plastically designed frames three stability criteria have to be considered for ensuring the safety of the frame. These are

1. General Frame Stability.
2. Local Buckling Criterion.
3. Restraints.

8.1General Frame Stability

Under loading, all structures move. In some cases this movement is small compared to the frame dimensions and the designer does not need to consider these any further. In other cases, the movement of the structure will be sufficient to cause the factor of safety to drop by a significant amount (for more details readers may wish to refer to *BS:5950 Part 1*, clauses 5.1.3, 5.5.3.2 and 5.5.3.3). In these cases the designer will need to take this drop in the load carrying capacity into account in checking the structure.

8.2 Local Buckling Criterion

At the location of a plastic hinge, there is a considerable strain, and at ultimate load this can reach several times the yield strain. Under these conditions it is essential that the section does not buckle locally, or the moment capacity will drop considerably. In order to ensure that the sections remain stable, limiting values are provided for flange outstands and web depth ratios. In no circumstances should sections <u>not</u> complying with the plastic section classification limits given in the code be used in locations where there are plastic hinges; otherwise there is a real risk of a premature reduction in the moment capacity of the member at the hinge location.

The limits for the sizing of flanges and webs are discussed in another chapter on " Local Buckling and Section Classification".

8.3 Restraints

In order to ensure that the plastic hinge position does not become a source of premature failure during the rotation, torsional restraint should be provided at the plastic hinge locations. These are discussed in the next chapter, which covers the design requirements in detail.

9.0 EFFECT OF AXIAL LOAD AND SHEAR

If a member is subjected to the combined action of bending moment and axial force, the plastic moment capacity will be reduced.

The presence of an axial load implies that the sum of the tension and compression forces in the section is not zero (Fig. 21). This means that the neutral axis moves away from the equal area axis providing an additional area in tension or compression depending on the type of axial load.

Consider a rectangular member of width *b* and depth *d* subjected to an axial compressive force *P* together with a moment *M* in the vertical plane (Fig. 21).
The values of *M* and *P* are increased at a constant value of *M/P* until the fully plastic stage is attained, then the values of *M* and *P* become:

$$M_{pa} = 0.25\, f_y\, b\, (d^2 - 4y^2) \tag{16}$$

$$P = 2y \times b\, f_y \tag{17}$$

where f_y = yield stress
y = distance from the neutral axis to the stress change for
M_p without axial force, $M_p = f_y bd^2/4$ (18)

If axial force acts alone $-P_y = f_y bd$ (19)
at the fully plastic state.

From equations (16) to (19) the interaction equation can be obtained:

$$M_x/M_p = 1 - P^2/P_y \tag{20}$$

The presence of shear forces will also reduce the moment capacity.

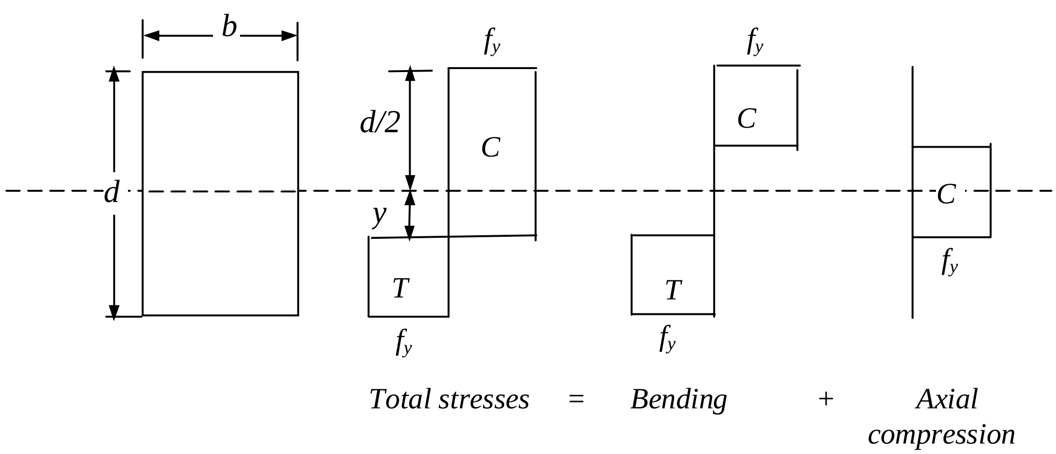


Fig. 21 Effect of axial force on plastic moment capacity

10.0 PLASTIC ANALYSIS FOR MORE THAN ONE CONDITION OF LOADING

When more than one condition of loading can be applied to a beam or structure, it may not always be obvious which is critical. It is necessary then to perform separate calculations, one for each loading condition, the section being determined by the solution requiring the largest plastic moment.

Unlike the elastic method of design in which moments produced by different loading systems can be added together, plastic moments obtained by different loading systems cannot be combined, i.e. the plastic moment calculated for a given set of loads is only valid for that loading condition. This is because the 'Principle of Superposition' becomes invalid when parts of the structure have yielded.

11.0 CONCLUDING REMARKS

Basic concepts on Plastic Analysis have been discussed in this chapter and the methods of computation of ultimate load causing plastic collapse have been outlined. Theorems of plastic collapse and alternative patterns of hinge formation triggering plastic collapse have been discussed. Worked examples illustrating plastic methods of analysis have been provided.

12.0 REFERENCES

1. Clarke, A. B. and Coverman, S. H. Structural Steelwork, Limit state design, Chapman and Hall Ltd, London, 1987.
2. Horne, M. R. Plastic Theory of Structures, Pergamon Press Ltd, Oxford, 1979.
3. Introduction to Steelwork Design to BS 5950: Part 1, The Steel Construction Institute, 1988
4. Owens G.W., Knowles P.R : "Steel Designers Manual", The Steel Construction Institute, Ascot, England, 1994

Structural Steel Design Project	Job No.	Sheet 1 of 2	Rev.
	Job title: ***PLASTIC ANALYSIS***		
	Worked Example. ***1***		
		Made by ***RSP***	Date ***April 2000***
CALCULATION SHEET		Checked by ***RN***	Date ***April 2000***

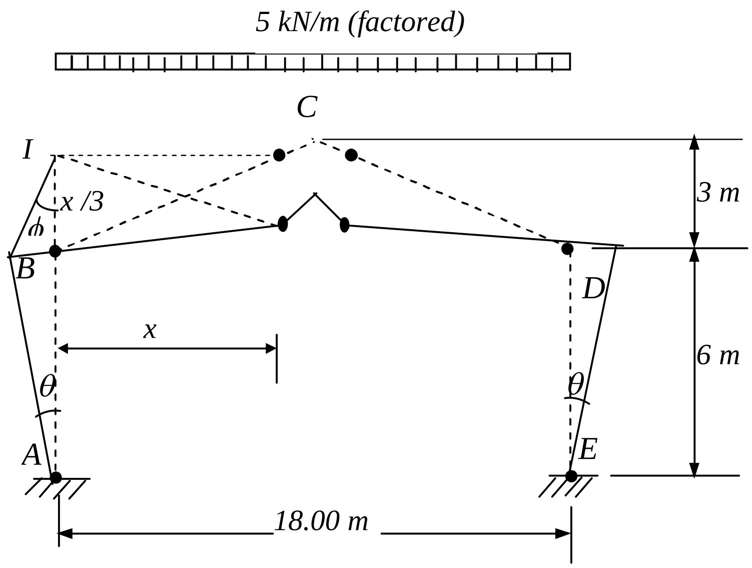


Fig. 22 Portal with fixed base and symmetrical vertical loading

Determine the M_p required for a symmetric single bay pitched portal frame with a factored UDL of 5 kN /m by instantaneous centre method.

In the case of gable frames, computation of the geometrical relationship of the displacement in the direction of the load as the structure moves through the mechanism may become somewhat tedious. In such cases the method of instantaneous centres may be used. This method is discussed below along with its use for solving practical problems.

In the following problem, when the structure moves under loading, the point B will move in a direction perpendicular to line AB . Then its centre of rotation should be along line AB extended. The point C will move vertically downwards and its centre of rotation should lie in a horizontal line. The point I satisfies both the conditions. Thus I is the centre of rotation of the member BC . The rotation at both the column bases is taken as θ. Assume that the hinges will form in the rafter at a distance of x from B and very close to roof apex.

<table>
<tr><td rowspan="4">Structural Steel Design Project

CALCULATION SHEET</td><td>Job No.</td><td>Sheet 2 of 2</td><td colspan="2">Rev.</td></tr>
<tr><td colspan="4">Job title: PLASTIC ANALYSIS</td></tr>
<tr><td colspan="4">Worked Example. 1</td></tr>
<tr><td rowspan="2"></td><td>Made by RSP
Checked by RN</td><td colspan="2">Date April 2000
Date April 2000</td></tr>
</table>

Data (as shown in Fig. 20).

Frame centres	*5.0 m*
Span of portal	*18.0 m*
Eves height	*6.0 m*
Eves to ridge height	*3.0 m*
Purlin spacing	*1.5 m*

Solution:

$$(x/3)\,\phi = 6\theta$$

$$\therefore \phi = 18\,\theta/x$$

$$M_p\,(\theta + 18\,\theta/x + \theta + 18\,\theta/x) = 18\,\theta/x\,[5x^2/2 + (9-x)\,5x]$$

$$M_p = \frac{9\left(-2.5x^2 + 45x\right)}{\left(18 + x\right)}$$

For maximum value of M_p, $\dfrac{dM_p}{dx} = 0$

$$(18 + x)(-5x + 45) - (-2.5x^2 + 45x) = 0$$

$$-2.5x^2 - 90x - 810 = 0$$

$$x = 7.5 \text{ m}$$

Substituting in eqn. for M_p, M_p = <u>69.5 kNm</u>

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Find M_p for the portal frame with electrically operated travelling crane as shown in Fig. 21 by 'Reactant moment diagram' method. The roof pitch is 30°. Neglect the effect of wind acting vertically on the roof.

Horizontal wind pressure is = 1 kN/m^2
γ_f *= 1.2 for the combined effects of wind, crane, dead load and live load.*

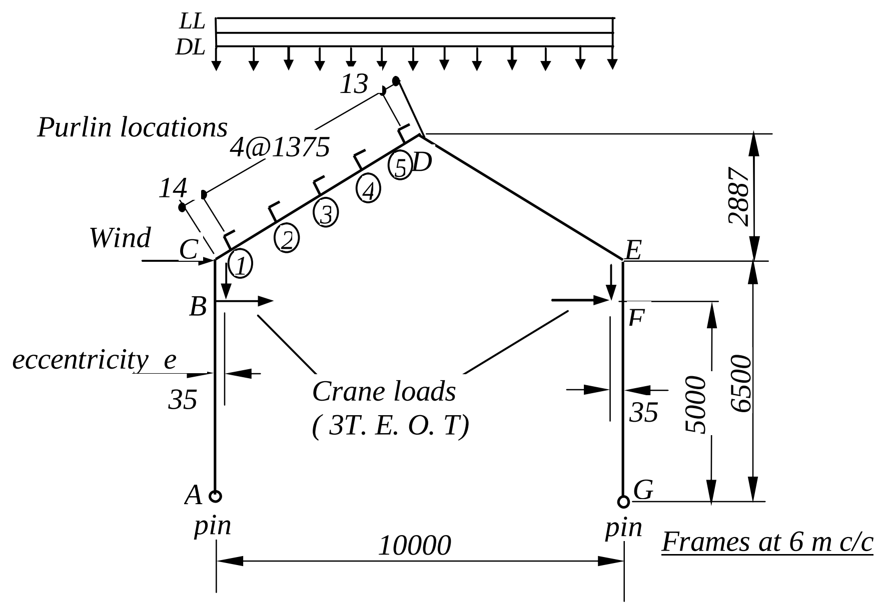

Fig. 23 Summary of typical loads and dimensions for Example 2

PRELIMINARY CALCULATIONS

(1) Forces due to dead load and live load on roof

Superimposed load = 0.6 kN/m^2
Dead load = 0.5 kN/m^2
Total load = $(0.6 + 0.5) \times 1.2 \times 6$ = 7.92 kN/m

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(2) Crane loading

3 ton capacity crane, 9.3 m span.

Horizontal crane loading

This may be shared between each side of the portal, based on the assumption that the crane wheels are flanged, and in effect share the load between the two rails. Check that the crane wheels are flanged when the vendor is selected, or place entire horizontal crane load at point B for a more onerous case.

Vertical crane loading

Maximum wheel load = 26.5 kN (2 wheels)

Minimum wheel load = 7.25 kN (2 wheels)

Maximum reaction at column due to loaded crane $= 2 \times 26.5 = 53$ *kN*

Minimum reaction at column due to loaded crane $= 2 \times 7.25 = 14.5$ *kN*

Moment due to vertical crane loading (unfactored)

Moment at B $= 53 \times 0.35 \times 1.2 = 22.3$ *kNm*

Moment at F $= 14.5 \times 0.35 \times 1.2 = 6.1$ *kNm*

Load on the crane bracket is 350 mm eccentric from column centre line.

Transverse crane loading

Transverse load due to crab and load $= 0.1\,(6.0 + 3.0) = 3.6$ *kN*

Shared between points B and F, i.e. 1.8 kN each.

Moment due to transverse crane loading

Moment at B $= 1.8 \times 5 \times 1.2 = 10.8$ *kNm*

Split the frame at the apex, then it can be treated as two cantilevers.

Total roof load = 7.92 kN/m

Load at purlin point 5 $= \left(113 + \dfrac{1191}{2}\right)\dfrac{7.92}{1000} = 5.61$ *kN*

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Load at purlin point (4) $= \left(\frac{1191}{2} + \frac{1191}{2}\right)\frac{7.92}{1000} = 9.43\ kN$

Moment at purlin point (3) $= \left[9.43 \times 1191 + 5.61 \times (1191 \times 2)\right]\frac{1}{1000}$

$= 24.59\ kNm$

<u>Moment at A</u>

Moment due to roof load $= 7.92 \times 5 \times 2.5 = 99\ kNm$

Moment due to wind load on ABC $= (1 \times 6 \times 6.5)\frac{6.5}{2} \times 1.2 = 152.1\ kNm$

Moment due to vertical crane loading $= 53 \times 0.35 \times 1.2 = 22.3\ kNm$

Moment due to transverse crane load $= 1.8 \times 1.2 \times 5 = 10.8\ kNm$

Total $= 284.2\ kNm$

<u>Moment at G</u>

Moment due to roof load $= 99\ kNm$

Moment due to vertical crane loading $= 6.1\ kNm$

Moment due to transverse crane load $= 10.8\ kNm$

Total $= 115.9\ kNm$

Summary of the reactant moment diagram method for the portal frame is shown in Fig. 22(a) and 22(b). For solution by calculation, let purlins be numbered 1 to 'n' from roof to apex. Put moment for each purlin point into the reactant moment diagram equations and solve for successive purlin points. The largest value of M_p *found by this method is the design case.*

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For this design the equations for M_p are:

At A : $284.2 - m - 9.387\,R - 5\,S = 0$

At point 3 : $24.59 - m - 1.42\,R - 2.495\,S = -M_p$

At E : $99 - m - 2.887\,R + 5\,S = +M_p$

At G : $115.9 - m - 9.387\,R + 5\,S = 0$

Using the method and equations illustrated in Fig. 22(b) these equations can be solved simultaneously (or by matrix) to give R = 16.2 kN, S = 16.83 kN, m = 48 kN, and
$M_p = 88.4$ *kNm.*

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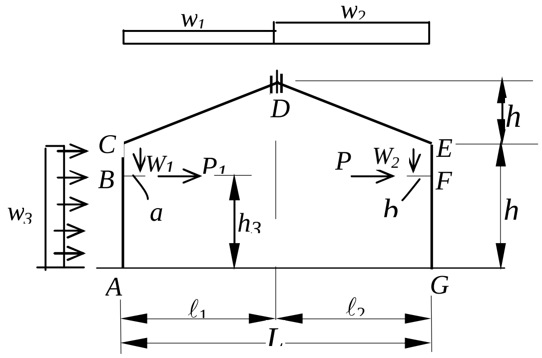
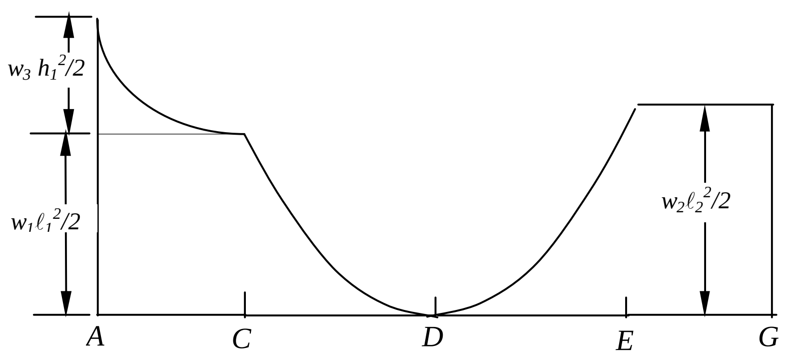
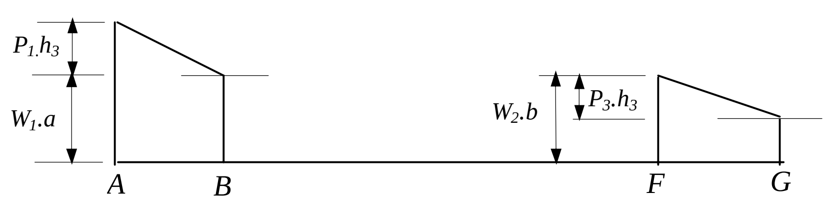


Loading and geometry



Free moment diagram for UDLs



Free moment diagram due to crane loads

Fig. 24(a) Loadings and free moment diagrams for the portal frame

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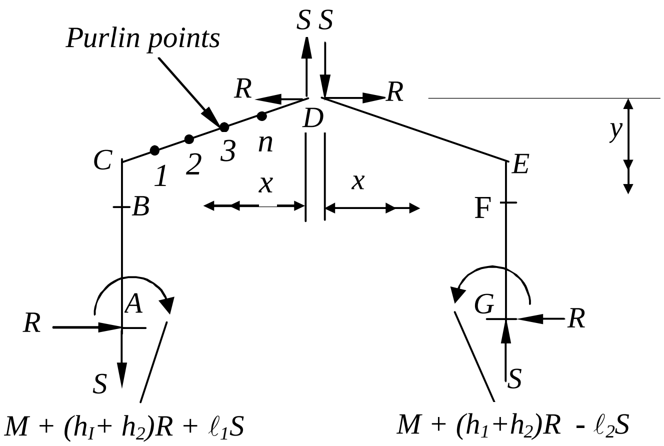
For the purpose of analysis, frame is notionally split at apex, and treated as two cantilevers



Redundant reactants

<u>*Equations for fixed bases*</u>

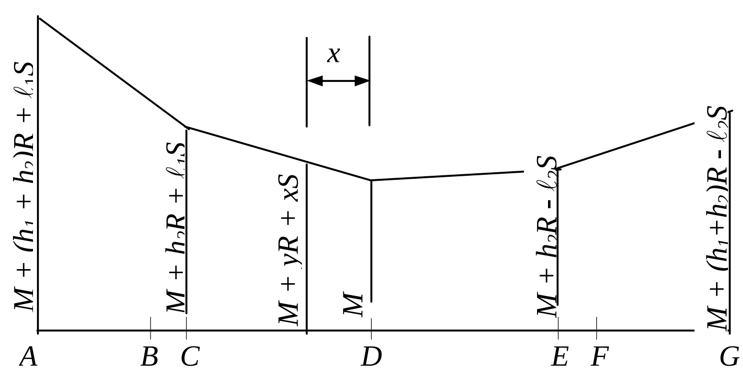
at C ; Free M – M – $h_2R - \ell_1S = +M_p$

at n ; Free M – M – $yR - xS = -M_p$

at E ; Free M – M – $h_2R + \ell_2S = +M_p$

at G ; Free M – M – $(h_1+h_2)R + \ell_2S = -M_p$

Equations for pinned bases are as for fixed bases but with moment at the bases set to zero. Put the factored values of free moment into the equations and solve simultaneously or by matrix. Solve for successive purlin points, thus finding the largest value of M_p



Redundant reactant moment diagram

Fig. 24(b) Summary of the reactant moment diagram method for a portal frame

36 PORTAL FRAMES

1.0 INTRODUCTION

The basic structural form of portal frames was developed during the Second World War, driven by the need to achieve the low - cost building envelope. Now they are the most commonly used structural forms for single-storey industrial structures. They are constructed mainly using hot-rolled sections, supporting the roofing and side cladding via cold-formed purlins and sheeting rails. With a better understanding of the structural behaviour of slender plate elements under combined bending moment, axial load and shear force, many fabricators now offer a structural frame fabricated from plate elements. These frames are composed of tapered stanchions and rafters in order to provide an economic structural solution for single-storey buildings. Portal frames of lattice members made of angles or tubes are also common, especially in the case of longer spans.

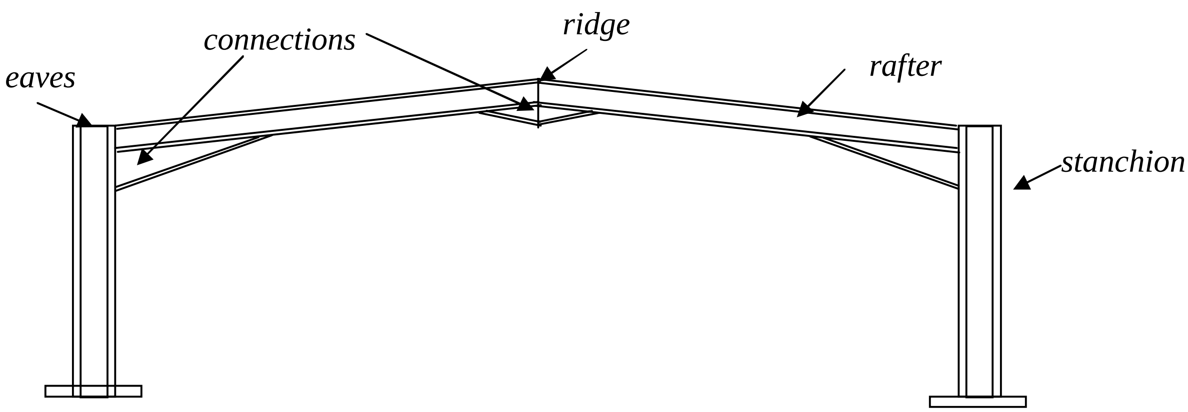
The slopes of rafters in the gable portal frames (Fig.1) vary in the range of *1* in *10* to *1* in *3* depending upon the type of sheeting and its seam impermeability. With the advent of new cladding systems, it is possible to achieve roof slopes as low as 1^0. But in such cases, frame deflections must be carefully controlled and the large horizontal thrusts that occur at the base should be accounted for. Generally, the centre-to-centre distance between frames is of the order *6 to 7.5 m,* with eaves height ranging from *6 -15 m*. Normally, larger spacing of frames is used in the case of taller buildings, from the point of economy. Moment-resisting connections are to be provided at the eaves and crown to resist lateral and gravity loadings. The stanchion bases may behave as either pinned or fixed, depending upon rotational restraint provided by the foundation and the connection detail between the stanchion and foundations. The foundation restraint depends on the type of foundation and modulus of the sub-grade. Frames with pinned bases are heavier than those having fixity at the bases. However, frames with fixed base may require a more expensive foundation.

For the design of portal frames, plastic methods of analysis are mainly used, which allows the engineer to analyse frames easily and design it economically. The basis of the plastic analysis method is the need to determine the load that can be applied to the frame so that the failure of the frame occurs as a mechanism by the formation of a number of plastic hinges within the frame. The various methods of plastic analysis are discussed in an earlier chapter. In describing the plastic methods of structural analysis, certain assumptions were made with regard to the effect of axial force, shear, buckling etc. Unless attention is given to such factors, the frame may fail prematurely due to local, or stanchion or rafter buckling, prior to plastic collapse.

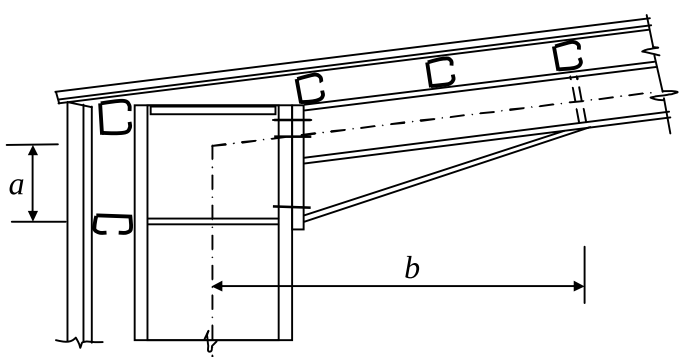
In the analysis, the problem is to find the ultimate load of a given structure with known plastic moment values of its members. But in design, the problem is reversed. Given a certain set of loads, the problem is to select suitable members.

2.0 HAUNCHED PORTAL FRAMES

The most common form of portal frame used in the construction industry is the pinned-base frame with different rafter and column member size and with haunches at both the eaves and apex connections (Fig.1). These two important design features of the modern portal frame have been developed over a number of years, from practical and economic considerations.



(a) Haunched portal frame



(b) Eaves detail

Fig. 1 Typical gable frame

Due to transportation requirements, field joints are introduced at suitable positions. As a result, connections are usually located at positions of high moment, i.e. at the interface of the column and rafter members (at the eaves) and also between the rafter members at the apex (ridge) (See Fig.1). It is very difficult to develop sufficient moment capacity at these connections by providing 'tension' bolts located solely within the small depth of the rafter section. Therefore the lever arm of the bolt group is usually increased by haunching the rafter members at the joints. This addition increases the section strength.

Although a short length of the haunch is enough to produce an adequate lever arm for the bolt group, haunch is usually extended along the rafter and column adequately to reduce the maximum moments in the uniform portion of the rafter and columns and hence reduce the size of these members. But due to this there will be a corresponding increase in the moment in the column and at the column-haunch-rafter interface. This allows the use of smaller rafter member compared to column member. The resulting solution usually proves to be economical, because the total length of the rafter is usually greater than the total length of the column members. The saving in weight is usually sufficient to offset the additional cost of haunch.

The haunched frame may be designed in a manner similar to that of an unhaunched frame, the only difference being that the hinges, which were assumed to be at nodes, are forced away from the actual column-rafter junction to the ends of the haunches. Provided the haunch regions remain elastic, hinges can develop at their ends. The haunch must be capable of resisting the bending moment, axial thrust and shear force transferred by the joining members. The common practice is to make the haunch at the connection interface approximately twice the depth of the basic rafter section, so that the haunch may be fabricated from the same basic section.

3.0 GENERAL DESIGN PROCEDURE

The steps in the plastic design of portals, according to SP: 6(6) – 1972, are given below:

a) Determine possible loading conditions.
b) Compute the factored design load combination(s).
c) Estimate the plastic moment ratios of frame members.
d) Analyse the frame for each loading condition and calculate the maximum required plastic moment capacity, M_p
e) Select the section, and
f) Check the design according to new IS: 800.

The design commences with determination of possible loading conditions, in which decisions such as, whether to treat the distributed loads as such or to consider them as equivalent concentrated loads, are to be made. It is often convenient to deal with equivalent concentrated loads in computer aided and plastic analysis methods.

In step (b), the loads determined in (a) are multiplied by the appropriate load factors to assure the needed margin of safety. This load factor is selected in such a way that the real factor of safety for any structure is at least as great as that decided upon by the designer. The load factors to be used for various load combinations are presented in an earlier chapter on Limit states method.

The step (c) is to make an assumption regarding the ratio of the plastic moment capacities of the column and rafter, the frame members. Optimum plastic design methods present a direct way of arriving at these ratios, so as to obtain an optimum value of this ratio. The following simpler procedure may be adopted for arriving at the ratio.

(i) Determine the absolute plastic moment value for separate loading conditions.

(Assume that all joints are fixed against rotation, but the frame is free to sway). For beams, solve the beam mechanism equation and for columns, solve the panel (sway) mechanism equation. These are done for all loading combinations. The moments thus obtained are the absolute minimum plastic moment values. The actual section moment will be greater than or at least equal to these values.

(ii) Now select plastic moment ratios using the following guidelines.

- At joints establish equilibrium.
- For beams use the ratio determined in step (i)
- For columns use the corner connection moments M_p *(Col)* = M_p *(beam)*

In the step (d) each loading condition is analysed by a plastic analysis method for arriving at the minimum required M_p. Based on this moment, select the appropriate sections in step (e). The step (f) is to check the design according to secondary design considerations discussed in the following sections (IS: 800).

4.0 DESIGN CONSIDERATIONS

The 'simple plastic theory' neglects the effects of axial force, shear and buckling on the member strength. So checks must be carried out for the following factors.

a) Reductions in the plastic moment due to the effect of axial force and shear force.
b) Instability due to local buckling, lateral buckling and column buckling.
c) Brittle fracture.
d) Deflection at service loads.

In addition, proper design of connections is needed in order that the plastic moments can be developed at the plastic hinge locations.

4.1 Combined Axial Force and Bending Moment

Under combined axial force and bending moment section strength as governed by material failure and member strength as governed by buckling failure have to be checked as given below.

4.1.1 Section Strength

4.1.1.1 Plastic and Compact Sections ─ In the design of members subjected to combined axial force (tension or compression) and bending moment, the following should be satisfied

$$\left(\frac{M_y}{M_{ndy}}\right)^{\alpha_1} + \left(\frac{M_z}{M_{ndz}}\right)^{\alpha_2} \leq 1.0$$

Conservatively, the following equation may be used under combined axial force and bending moment

$$\frac{N}{N_d} + \frac{M_y}{M_{dy}} + \frac{M_z}{M_{dz}} \leq 1.0$$

where

M_y, M_z = factored applied moments about the minor and major axis of the cross section, respectively

M_{ndy}, M_{ndz} = design reduced flexural strength under combined axial force and the respective uniaxial moment acting alone, (**4.1.1.2**)

N = factored applied axial force (Tension T or Compression F)

N_d = design strength in tension (T_d) or in compression

$N_d = A_g f_y / \gamma_{m0}$

M_{dy}, M_{dz} = design strength under corresponding moment acting alone

A_g = gross area of the cross section

α_1, α_2 = constants as given in Table 4.1

n = N / N_d

TABLE. 4.1 CONSTANTS α_1 AND α_2

(Section 4.1.1.1)

Section	**α_1**	**α_2**
I and Channel	$5n \geq 1$	2
Circular tubes	2	2
Rectangular tubes	$1.66/(1-1.13n^2) \leq 6$	$1.66/(1-1.13n^2) \leq 6$
Solid rectangles	$1.73+1.8n^3$	$1.73+1.8\,n^3$

4.1.1.2 For plastic and compact sections without bolts holes, the following approximations may be used.

a) *Plates*

$$M_{nd} = M_d\,(1-n^2)$$

b) *Welded I or H sections*

$$M_{ndz} = M_{dz}\,(1-n)\,/\,(1-0.5a) \leq M_{dz}$$

$$M_{ndy} = M_{dy}\left[1-\left(\frac{n-a}{1-a}\right)^2\right] \leq M_{dy}$$

where $n = N / N_d$ and $a = (A - 2\,b\,t_f) / A \leq 0.5$

c) *For standard I or H sections*

$$M_{ndz} = 1.11\,M_{dz}\,(1-n) \leq M_{dz}$$

for $n \leq 0.2$ $\quad M_{ndy} = M_{dy}$

for $n > 0.2$ $\quad M_{ndy} = 1.56\, M_{dy}\,(1-n)\,(n+0.6)$

d) *For Rectangular Hollow sections and Welded Box sections* – When the section is symmetric about both axes and without bolt holes:

$$M_{ndz} = M_{dz}\,(1-n)\,/\,(1-0.5a_w) \le M_{dz}$$

$$M_{ndy} = M_{dy}\,(1-n)\,/\,(1-0.5a_f) \le M_{dy}$$

where

$$a_w = (A - 2\,b\,t_f)\,/\,A \quad \le 0.5$$

$$a_f = (A - 2\,h\,t_w)\,/\,A \quad \le 0.5$$

e) *Circular Hollow Tubes without Bolt Holes*

$$M_{nd} = 1.04\, M_d\,(1-n^{1.7}) \le M_d$$

4.1.1.3 Semi-compact section – In the absence of high shear force (**4.2.1**) semi-compact section design is satisfactory under combined axial force and bending, if the maximum longitudinal stress under combined axial force and bending, f_x, satisfies the following criteria.

$$f_x \le f_y / \gamma_{m0}$$

For cross section without holes, the above criteria reduces to

$$\frac{N}{N_d} + \frac{M_y}{M_{dy}} + \frac{M_z}{M_{dz}} \le 1.0$$

where

N_d, M_{dy}, M_{dz} are defined in **4.1.1.1**

4.1.2 Overall Member Strength

Members subjected to combined axial force and bending moment shall be checked for overall buckling failure as given in this section.

4.1.2.1 Bending and Axial Tension – The reduced effective moment, M_{eff}, under tension and bending calculated as given below, should not exceed the bending strength due to lateral torsional buckling, M_d,

$$M_{eff} = \left\lfloor M - \psi\; T\, Z_{ec} / A \right\rfloor \quad \le M_d$$

where

M, T = factored applied moment and tension, respectively

A = area of cross section

Z_{ec} = elastic section modulus of the section with respect to extreme compression fibre

ψ = 0.8 if T and M can vary independently

= 1.0 otherwise.

4.1.2.2 Bending and Axial Compression – Members subjected to combined axial compression and biaxial bending shall satisfy the following interaction relationship.

$$\frac{P}{P_d} + \frac{K_y M_y}{M_{dy}} + \frac{K_z M_z}{M_{dz}} \le 1.0$$

where

K_y, K_z = moment amplification factor about minor and major axis respectively

P = factored applied axial compression

M_y, M_z = maximum factored applied bending moments about y and z-axis of the member, respectively.

P_d, M_{dy}, M_{dz} = design strength under axial compression, bending about y and z-axis respectively, as governed by overall buckling as given below:

a) The design compression strength is the smallest of the minor axis (P_{dy}) and major axis (P_{dz}) buckling strength.

b) Design bending Strength M_{dz} about major axis as given below

$$M_{dz} = M_d$$

where

M_d = design flexural strength about z axis when lateral torsional buckling is prevented and where lateral torsional buckling governs

i) For flexural buckling failure

$$K_z = 1 - \frac{\mu_z P}{P_{dz}} \leq 1.5$$

where

μ_z is larger of μ_{LT} and μ_{fz} as given below:

$$\mu_{LT} = 0.15\lambda_y \beta_{MLT} - 0.15 \qquad \leq 0.90$$

$$\mu_{fz} = \lambda_z (2\beta_{Mz} - 4) + \frac{Z_{pz} - Z_{ez}}{Z_{ez}} \qquad \leq 0.90$$

$$K_y = 1 - \frac{\mu_y P}{P_{dy}}$$

$$\mu_y = \lambda_y (2\beta_{My} - 4) + \left[\frac{Z_{py} - Z_{ey}}{Z_{ey}}\right] \leq 0.9$$

β_{My}, β_{Mz}, β_{MLT} = equivalent uniform moment factor obtained from Table 4.2, according to the shape of the bending moment diagram between lateral bracing points in the appropriate plane of bending

P_{dy}, P_{dz} = design compressive strength as governed by flexural buckling about the respective axis

λ_y, λ_z = non-dimensional slenderness ratio about the respective axis

c) Design Bending Strength about Minor axis

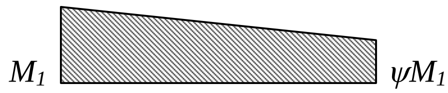
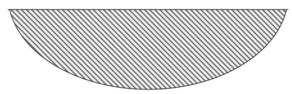
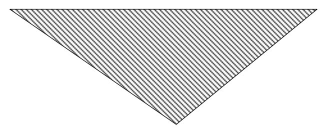
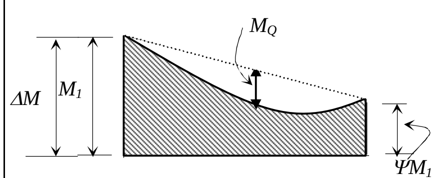
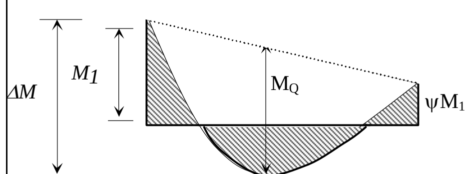
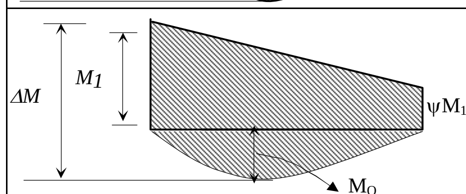
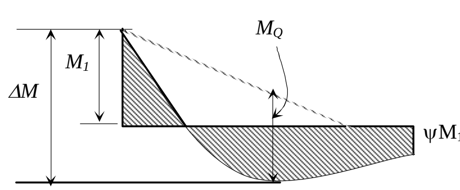
$$M_{dy} = M_d$$

where

M_d = design flexural strength about y-axis calculated using plastic section modulus for plastic and compact sections and elastic section modulus for semi-compact sections.

TABLE 4.2 EQUIVALENT UNIFORM MOMENT FACTOR

(Section 4.1.2.2)

Particulars	***BMD***	β_M
Due to end moments		$1.8-0.7\psi$
Moment due to lateral loads		1.3
		1.4
Moment due to lateral loads and end moments		$1.8-0.7\psi+\frac{M_Q}{\Delta_M}(0.7\psi-0.5)$
		$M_Q=\lvert M_{max}\rvert$ due to lateral load alone
		$\Delta M=\lvert M_{max}\rvert$ (same curvature)
		$\Delta M=\lvert M_{max}\rvert+\lvert M_{min}\rvert$ (reverse curvature)

4.2 Combined Shear and Bending

4.2.1 No reduction in moment capacity of the section is necessary as long as the cross section is not subjected to high shear force (factored value of applied shear force is less than or equal to 60 percent of the shear strength of the section), the moment capacity may be taken as, $M_{d,}$ (**Cl. 8.2 of IS:800**) without any reduction.

4.2.2 When the factored value of the applied shear force is high (exceeds the limit in **4.2.1**), the factored moment of the section should be less than the moment capacity of the section under higher shear force, M_{dv}, calculated as given below:

a) Plastic or Compact Section

$$M_{dv} = M_d - \beta \left(M_d - M_{fd}\right) \leq 1.2\, z_e\, f_y / \gamma_{m0}$$

where

$$\beta = \left(2\,V/V_d - 1\right)^2$$

M_d = plastic design moment of the whole section disregarding high shear force effect considering web buckling effects

V = factored applied shear force as governed by web yielding or web buckling.

V_d = design shear strength as governed by web yielding or web buckling

M_{fd} = plastic design strength of the area of the cross section excluding the shear area, considering partial safety factor γ_{m0}

b) Semi-compact Section

$$M_{dv} = Z_e\, f_y / \gamma_{m0}$$

where Z_e = elastic section modulus of the whole section

4.3 Local Buckling of Flanges and Webs

If the plates of which the cross section is made are not stocky enough, they may be subject to local buckling either before or soon after the first plastic moment is reached. Due to this, the moment capacity of the section would drop off and the rotation capacity would be inadequate to ensure formation of complete failure mechanism. Therefore, in order to ensure adequate rotation at M_p values and to avoid premature plastic buckling, the compression elements should have restriction on the width-thickness ratios. The variables representing dimensions of typical sections are indicated in Fig. 2.

According to new IS: 800 the projection of flange or other compression element beyond

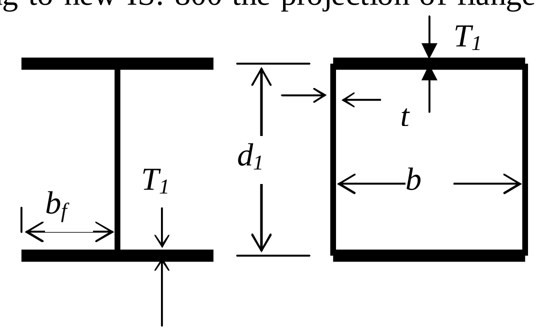


Fig. 2 Plate Elements in Steel Sections

its outermost point of attachment shall not exceed the value given below:

TABLE 4.3 LIMITING WIDTH TO THICKNESS RATIOS

Compression element			Ratio	Class of Section		
				Class 1 Plastic	Class 2 Compact	Class 3 Semi-Compact
Outstanding element of compression flange	Rolled section		b/t_f	9.4ε	10.5ε	15.7ε
	Welded section		b/t_f	8.4ε	9.4ε	13.6ε
Internal element of compression flange	Compression due to bending		b/t_f	29.3ε	33.5ε	42ε
	Axial compression		b/t_f	Not applicable		
Web of an I-H-or box section [c]	Neutral axis at mid-depth		d/t_w	83.9ε	104.8ε	125.9ε
	Generally	If r_1 is negative:	d/t_w	$\frac{84\varepsilon}{1+r_1}$ but $\geq 42\varepsilon$	$\frac{104.8\varepsilon}{1+r_1}$	$\frac{125.9\varepsilon}{1+2\,r_2}$ but $\geq 42\varepsilon$
		If r_1 is positive:	d/t_w		$\frac{104.8\varepsilon}{1+1.5r_1}$ but $\geq 42\varepsilon$	
	Axial compression		d/t_w	Not applicable		
Web of a channel			d/t_w	42ε	42ε	42ε
Angle, compression due to bending (Both criteria should be satisfied)			b/t d/t	9.4ε $9.4\ \varepsilon$	10.5ε 10.5ε	15.7ε 15.7ε
Single angle, or double angles with the components separated, axial compression (All three criteria should be satisfied)			b/t d/t $(b+d)/t$	Not applicable		15.7ε 15.7ε 25ε
Outstanding leg of an angle in contact back-to-back in a double angle member			d/t	9.4ε	10.5ε	15.7ε
Outstanding leg of an angle with its back in continuous contact with another component						
Circular tube subjected to moment or axial compression	CHS or built by welding		D/t	$44\varepsilon^2$	$55\varepsilon^2$	$88\varepsilon^2$
Stem of a T-section, rolled or cut from a rolled I-or H-section			D/t_f	8.4ε	9.4ε	18.9ε

Note 1: Section having elements which exceeds semi-compact limits are to be taken as slender cross sections

Note2: $\varepsilon=(250/f_y)^{1/2}$

Note 3: Check webs for shear buckling in accordance with **cl. 8.4.2 IS:800** *when d/t > 67 ε. Where, b is the width of the element may be taken as clear distance between lateral supports or between lateral support and free edge, as appropriate, t is the thickness of element, d is the depth of the web, D mean diameter of the element,*

Note 4: Different elements of a cross-section can be in different classes. In such cases the section is classified based on the least favourable classification.

Note 5: The stress ratio r_1 and r_2 *are defined as*

$$r_1=\frac{\text{actual average axial compressive stress}}{\text{design compressive stress of web alone}}\ ,\quad r_2=\frac{\text{actual average axial compressive stress}}{\text{design compressive stress of overall section}}$$

4.4 Laterally Unsupported Beams

Beam experiencing bending about major axis and not restrained against lateral buckling of the compression flange may fail by lateral torsional buckling before material failure. Effect of lateral torsional buckling on flexural strength also needs to be considered unless $\lambda_{LT} \leq 0.4$

where, λ_{LT} = non-dimensional slenderness ratio for lateral torsional buckling as given below

The design bending strength of laterally unsupported beam as governed by lateral torsional buckling is given by: $M_d = \beta_b Z_p f_{bd}$

where, β_b = 1.0 for plastic and compact sections

= Z_e/Z_p for semi-compact sections

Z_p, Z_e = plastic section modulus and elastic section modulus with respect to extreme compression fibre.

f_{bd} = design bending compressive stress, obtained as given below

$$f_{bd} = \chi_{LT} f_y / \gamma_{m0}$$

where, χ_{LT} = reduction factor to account for lateral torisonal buckling given by:

$$\chi_{LT} = \frac{1}{\left\{\phi_{LT} + \left[\phi_{LT}^2 - \lambda_{LT}^2\right]^{0.5}\right\}} \leq 1.0$$

$$\text{in which } \phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\lambda_{LT} - 0.2\right) + \lambda_{LT}{}^2\right]$$

The values of imperfection factor, α_{Lt}, for lateral torisonal buckling of beams is given by:

where

α_{LT} = 0.21 for rolled section

α_{LT} = 0.49 for welded section

The non-dimensional slenderness ratio, λ_{LT}, is given by

$$\lambda_{LT} = \sqrt{\beta_b Z_p f_y / M_{cr}}$$

$$= \sqrt{f_y / f_{cr,b}}$$

where, M_{cr}= elastic critical moment calculated as per **4.1**

$f_{cr,b}$ = extreme fibre compressive elastic lateral buckling moment

4.4.1 *Elastic Lateral Torsional Buckling Moment* – The elastic lateral buckling moment, M_{cr}, can be determined as:

$$M_{cr} = \sqrt{\left\{\left(\frac{\pi^2 EI_y}{(KL)^2}\right)\left[GI_t + \frac{\pi^2 EI_w}{(KL)^2}\right]\right\}}$$

The following simplified conservative equation may be used in the case of prismatic members made of standard rolled I-sections and welded doubly symmetric I-sections, for calculating the elastic lateral buckling moment, $M_{cr.}$

$$M_{cr} = \frac{\beta_{LT}\pi^2 EI_y h}{2(KL)^2}\left[1+\frac{1}{20}\left[\frac{KL/r_y}{h/t_f}\right]^2\right]^{0.5} = \beta_b Z_p f_{cr,b}$$

where

I_t = torsional constant
I_w = warping constant
I_y = moment of inertia about the weak axis
r_y = radius of gyration of the section about the weak axis
KL = effective laterally unsupported length of the member (**4.2**)
h = overall depth of the section
t_f = thickness of the flange
β_{LT} = 1.20 for plastic and compact sections with $t_f / t_w \leq 2.0$
= 1.00 for semi-compact sections or sections with $t_f / t_w > 2.0$

4.4.2 Effective Length of Compression Flanges

4.4.2.1 For simply supported beams and girders of span length, L, where no lateral restraint to the compression flanges is provided, but where each end of the beam is restrained against torsion, the effective length KL of the compression flanges shall be taken as follows:

a) With ends of compression flanges unrestrained against lateral bending (that is, free to rotate in plan) at the bearings $KL = L$

b) With ends of compression flanges partially restrained
$KL = 0.85 \times L$ against lateral bending (that is, not fully free to rotate in plan at the bearings)

c) With ends of compression flanges fully restrained against lateral bending (that is, rotation in plan at the bearings completely restrained) $KL = 0.7 \times L$

Restraint against torsion can be provided by:

i) web or flange cleats, or
ii) bearing stiffeners acting in conjunction with the bearing of the beam, or
iii) lateral end frames or to the external supports to the ends of the compression flanges, or
iv) their being built into walls

Where the ends of the beam are not restrained against torsion or where the load is applied to the compression flange and both the load and flange are free to move laterally the above values of the effective length shall be increased by 20 percent.

The end restraint element shall be capable of safely resisting in addition to wind and other applied external forces, a horizontal force acting at the bearing in the plane in a direction normal to the axis of compression flange of the beam at the level of the centroid of the flange and having a value equal to not less than *2.5* percent of the maximum compressive force occurring in the flange.

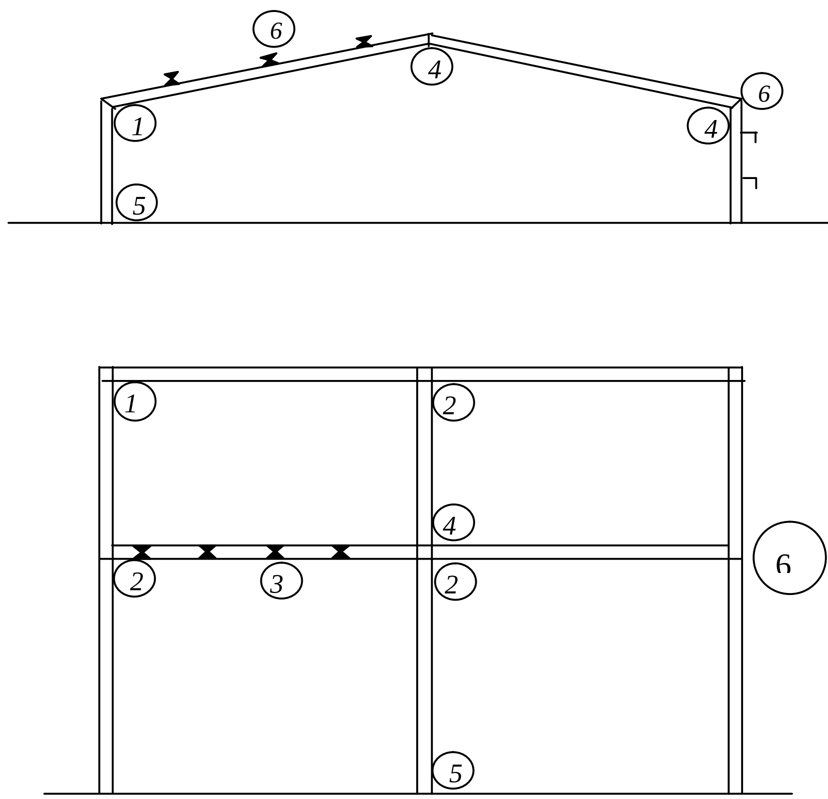
4.5 Connections

In a portal frame, points of maximum moments usually occur at connections. Further, at corners the connections must accomplish the direction of forces change. Therefore, the design of connections must assure that they are capable of developing and maintaining the required moment until the frame fails by forming a mechanism.

The various types of connections that might be encountered in steel frame structures are shown in Fig.*3*.

There are four principal requirements, in design of a connection

a) *Strength* - The connection should be designed in such a way that the plastic moment (M_p) of the members (or the weaker of the two members) will be developed. For straight connections the critical or 'hinge' section is assumed at point H in Fig. 4 (a). For haunched connections, the critical sections are assumed at R_1 and R_2, [Fig. 4 (b)].



1. Corner | *4. Column Splice*
2. Beam - column | *5. Column Base*
3. Beam- Girder | *6. Miscellaneous*

Fig. 3 Types of Connections in Buildings Frames According to Their Function

b) *Stiffness* - Average unit rotation of the connecting region should not exceed that of an equivalent length of the beam being joined. The equivalent length is the length of the connection or haunch measured along the frame line. Thus in Fig. 4(a).

$\Delta L = r_1 + r_2$

This requirement reduces to the following

$$\theta_h \leq \frac{M_p}{EI} . \Delta L$$

where θ_h is the joint rotation.

The above equation states that the change in angle between sections R_1 and R_2 as computed shall not be greater than the curvature (rotation per unit of length) times the equivalent length of the knee.

c) *Rotation Capacity* – The plastic rotation capacity at the connection hinge is adequate to assure that all necessary plastic hinges will form in the structure to enable failure mechanism and hence all connections should be proportioned to develop adequate rotation at plastic hinges.

d) *Economy* - Extra connecting materials and labour required to achieve the connection should be kept to a minimum.

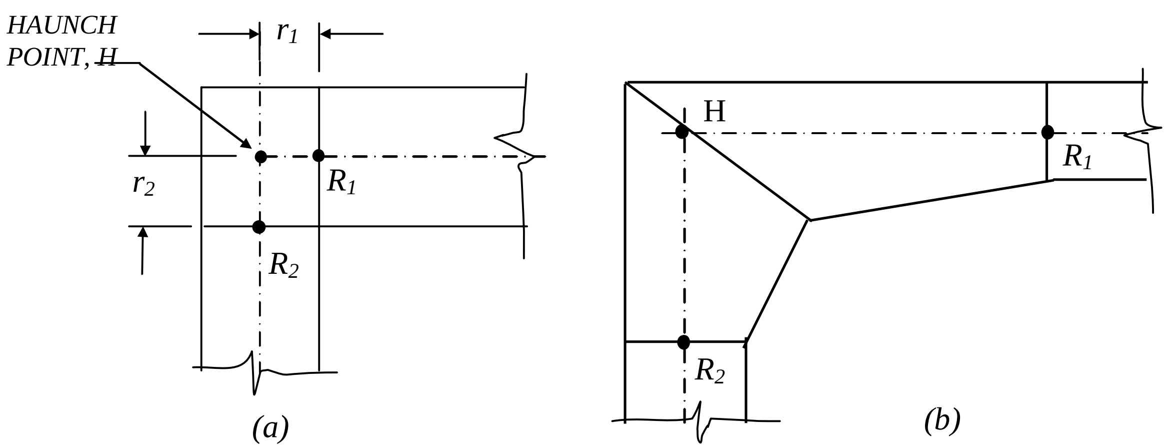


Fig.4 Designation of Critical Sections in Straight and Haunched Sections

4.6.1 Straight Corner Connections:

In the case of unstiffened corner connections, (Fig. 5) the design objective is to prevent yielding of the web due to shear force. For this the moment at which the yielding commences due to shear force, $M_h\ (\tau)$, given by Eq.(a), should not be less than the plastic moment, M_p.

Using the maximum shear stress yield condition $\tau_y = f_y / \sqrt{3}$ and assuming that the shear stress is uniformly distributed in the knee web, and that the flange carries all of the flexural stress (Fig. 6), we can get the value of $M_h\ (\tau)$ as

$$M_{h(\tau)} = \frac{td_1d_2}{\sqrt{3}} f_y \tag{a}$$

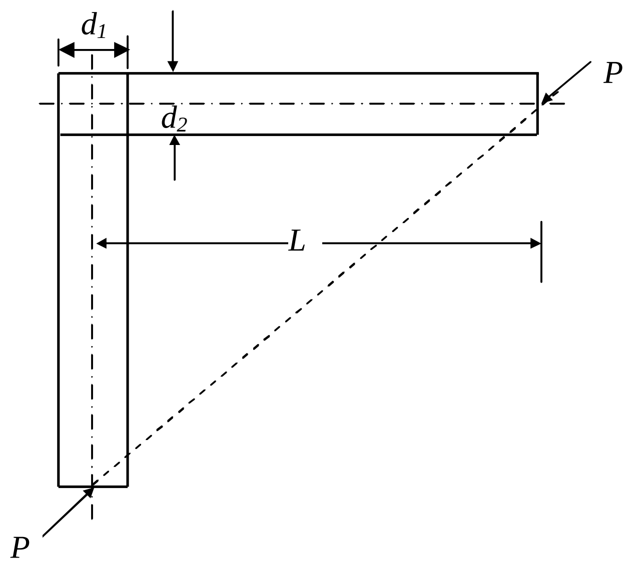


Fig. 5 Idealised Loading on Straight Corner Connection

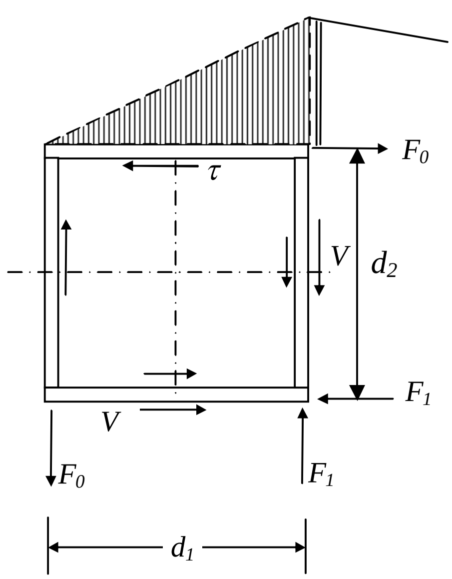


Fig.6 Forces and Stresses Assumed to act on Unstiffened Straight Corner Connection

Relating this to $M_p = f_y Z$ to obtain the required web thickness given by:

$$t_w \geq \sqrt{3}\frac{Z}{d_1 d_2}$$

where Z is the smaller of the plastic section modulus of the members meeting at the joint.

If the knee web is deficient in resisting the shear force, a diagonal stiffener may be used. (Fig. 7). Then the force F_0 is made up of two parts, a force carried by the web in shear and a force transmitted at the end by the diagonal stiffener. i.e., $F_o = F_{web} + F_{stiffener}$.

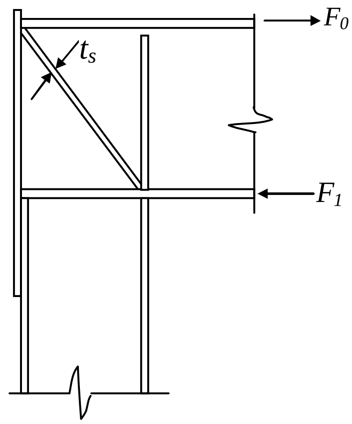


Fig. 7 Stiffened Corner Joint

When both web and diagonal stiffener have reached the yield condition

$$F_o = \frac{f_y\, t\, d_1}{\sqrt{3}} + f_y . b_s . t_s \frac{d_1}{\sqrt{d_1^2 + d_2^2}}$$

where b_s and t_s are the sum of the width and the thickness of the diagonal stiffeners provided on both the sides of the web.

The available moment capacity of this connection type is thus given by:

$$M_h = f_y d_2 \left[t\frac{d_1}{\sqrt{3}} + b_s t_s \frac{d_1}{\sqrt{{d_1}^2 + {d_2}^2}} \right]$$

The required thickness of diagonal stiffeners in corner connections, t_s that would ensure the moment as governed by shear resistance of the corner (M_h) which is greater than the plastic moment capacity $M_p = Z\ f_y$ is obtained from

$$t_s \geq \left[\frac{Z}{d_1 d_2} - \frac{t_w}{\sqrt{3}} \right] \frac{\sqrt{d_1^2 + d_2^2}}{b_s}$$

4.6.2 Haunched Connections

Some of the typical haunched connections are shown in Fig. *8*. Haunched connections are to be proportioned to develop plastic moment at the junction between the rolled steel section and the haunch. In order to force formation of hinge at the end of a tapered haunch (Fig. *8*), make the flange thickness in the haunch, to be *50* percent greater than that of section joined. Check the shear resistance of the web to ensure M_p governs the strength.

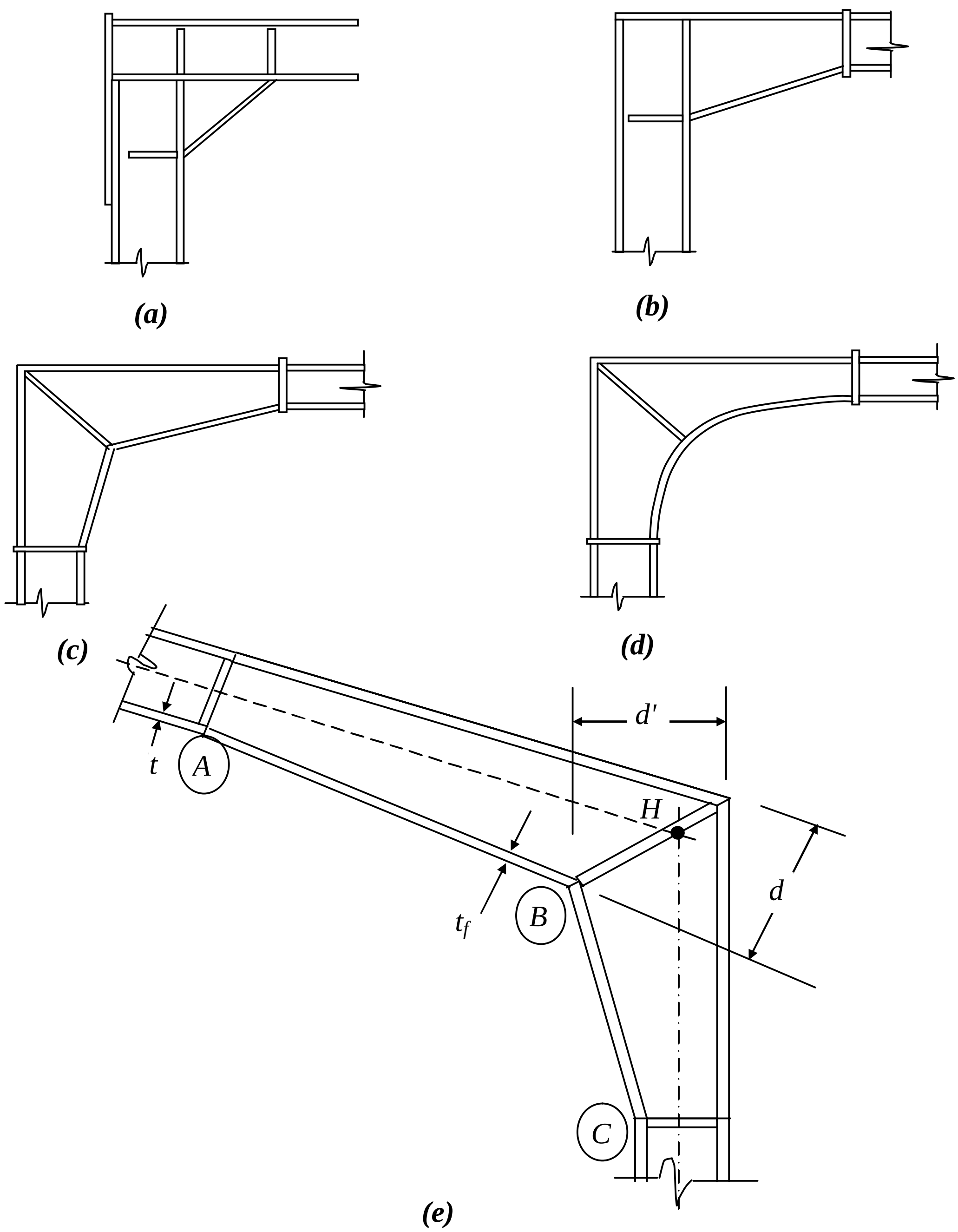


Fig. 8 Typical Haunched Corner connections

4.6.3 Interior Beam to Column Connections

Typical interior beam-column connections are shown in Fig. 9.

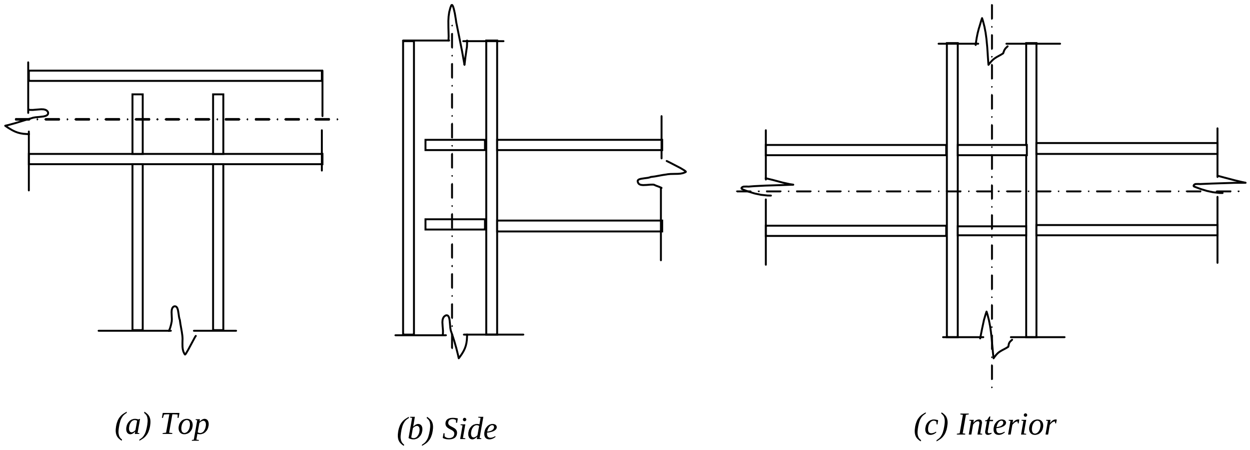


Fig.9 Beam to column connections of (a) Top, (b) Side, and (c) Interior Type

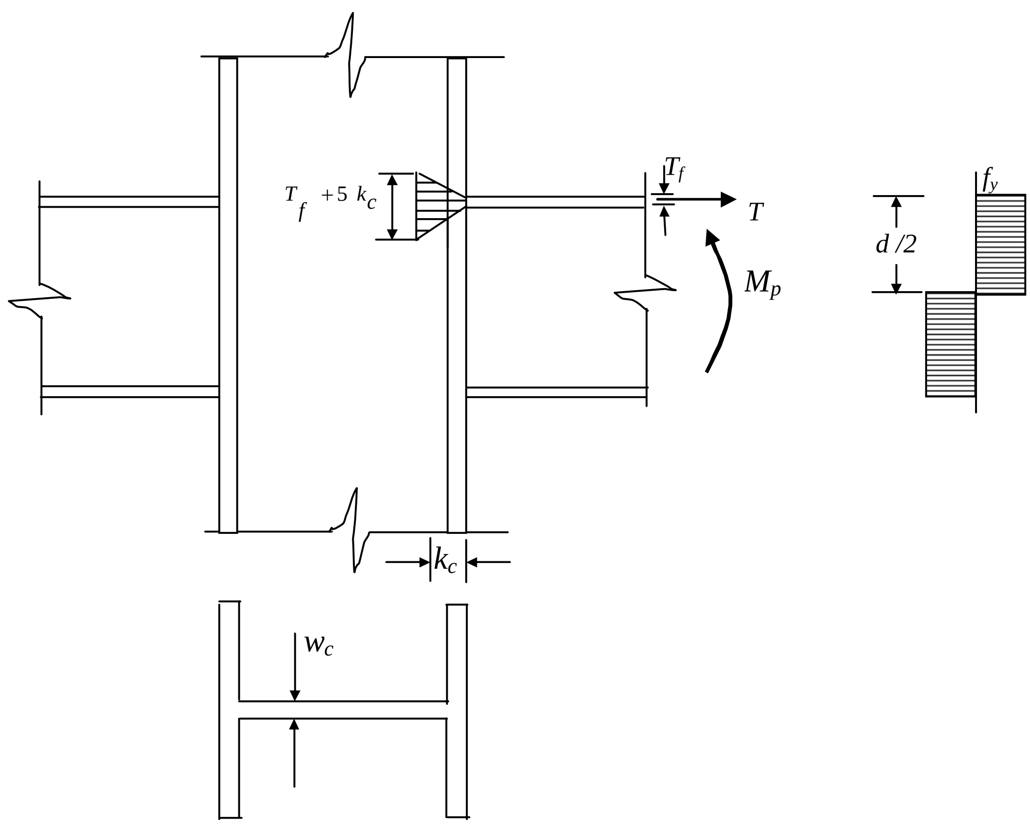


Fig. 10 Assumed stress Distribution in beam column connection with no stiffeners

The function of the 'Top' and the 'Interior' connections is to transmit moment from the left to the right beam, the column carrying any unbalanced moment. The 'side' connection transmits beam moment to upper and lower columns. The beam - column connection should have sufficient stiffening material so that it can transmit the desired moment (usually the plastic moment M_p) without the shear strength of the corner governing the design.

In an unstiffened beam-to-column connection, the concentrated force, T_f, from the beam flange, which the column web can sustain, is given by the following equation (Fig.12)

The reaction width is equal to the column web thickness, w_c.

$$f_y \, . \, A_{fb} = \lfloor t_{wc} \left(T_{fb} + 5k_c \right) \rfloor \left(f_y \right)$$

where A_{fb}, T_{fb} are area and thickness of the beam flange, t_{wc} is the thickness of the column web and k_c is the distance to the roof of the column web. Thus,

$$t_{wc} = \frac{A_{fb}}{T_{fb} + 5k_c}$$

which gives the required minimum column web thickness to develop the plastic moment in the beam, without stiffening the corner.

If the column web thickness does not meet the requirement for preventing the column web from buckling, stiffeners may be provided (Fig.11). In that case, the plastic moment (M_p) will be acting at the end of the beam, and the thrust T from the beam flange should be balanced by the strength of the web (T_w) and of the stiffener plate (T_s) or

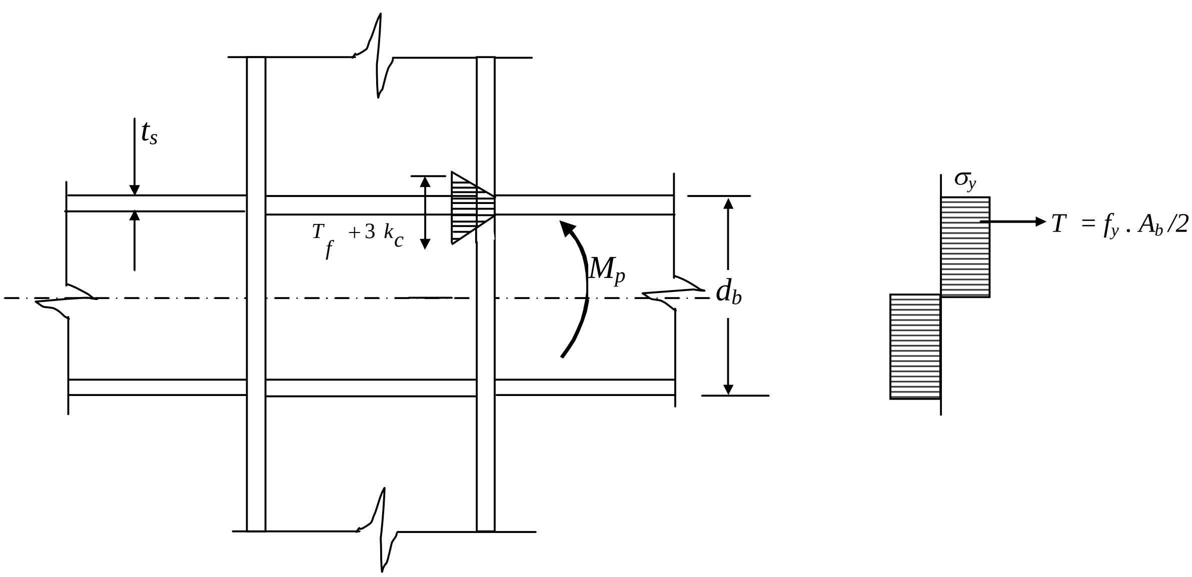


Fig. 11 Assumed Stress Distribution in Beam to Column Connection with flange type Stiffener

$T = T_w + T_s$

where T_w = force resisted by the column web $= f_y t_{wc}(T_f + 5k_c)$

T_s = force resisted by stiffener plate $= f_y.t_s.b_s$ and

$T = f_y \,.\, A_{fb}$

If 'flange' stiffeners are used for reinforcement, their required thickness of the stiffener is given by:

$$t_s = \lfloor A_{fb} - t_{wc}\left(T_f + 5k_c\right)\rfloor / 2b$$

where b is the width of the stiffener on each side of the web.

5.0 STRUCTURAL DUCTILITY

Ordinary structural grade steel for bridges and buildings may be used with modifications, when needed, to ensure weldability and toughness at lowest service temperature.

Fabrication processes should be such as to promote ductility. Sheared edges and punched holes in tension flanges are not permitted. Punched and reamed holes for connecting devices would be permitted if the reaming removes the cold-worked material.

In design, triaxial states of tensile stress set up by geometrical restraints should be avoided.

6.0 SUMMARY

The analysis, design of members and connections in steel portal frames encountered in single storey industrial buildings was discussed. Example problem is illustrated in the appendix to this chapter.

7.0 REFERENCES

1. IS800: Code of practice for use of structural steel in general building construction.
2. SP:6 (6) – 1972, "Handbook for Structural Engineers – Application of Plastic Theory in Design of Steel Structures".

37 MULTI-STOREY BUILDINGS - I

1.0 INTRODUCTION

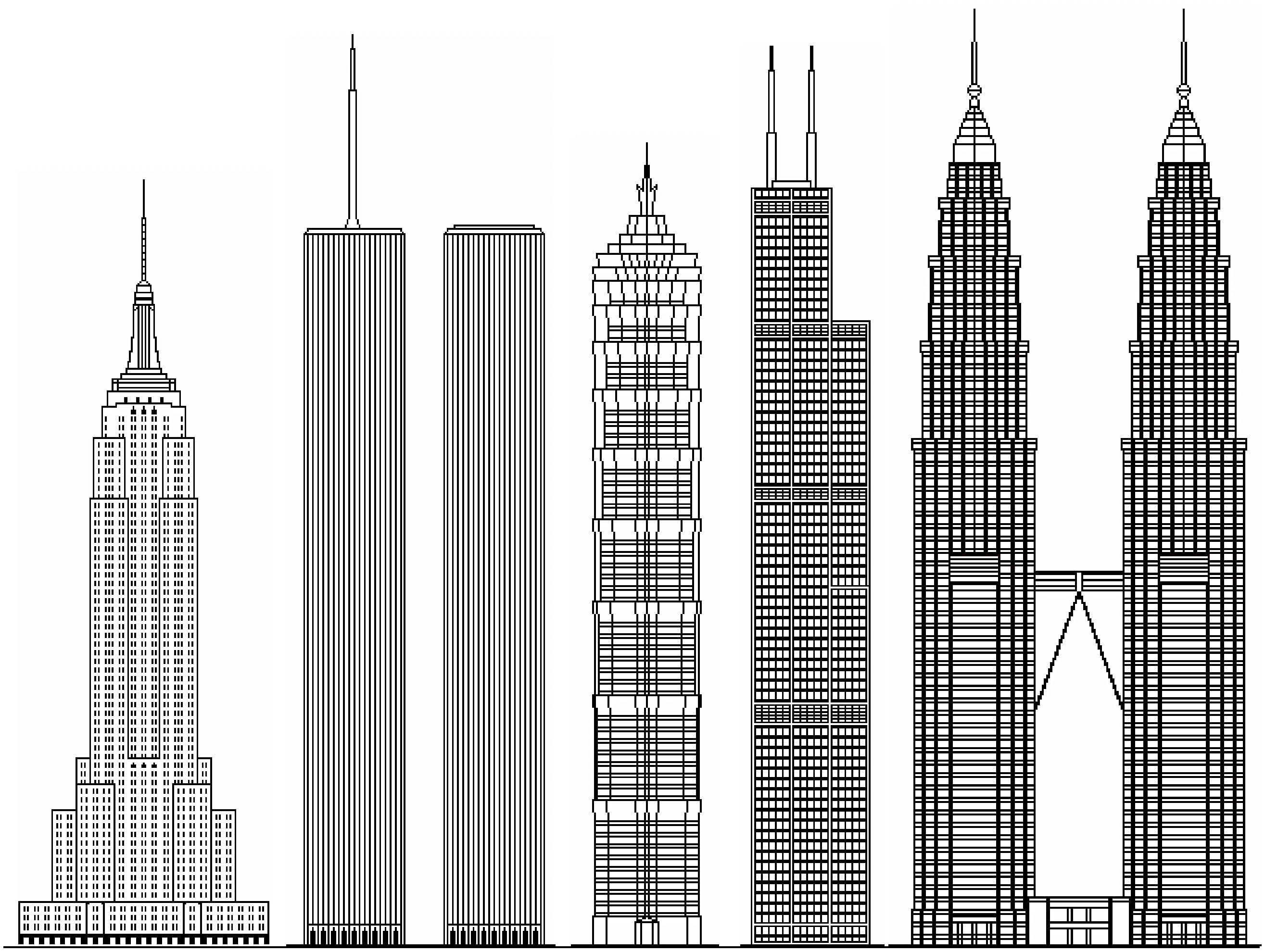
The tallness of a building is relative and can not be defined in absolute terms either in relation to height or the number of stories. But, from a structural engineer's point of view the tall building or multi-storeyed building can be defined as one that, by virtue of its height, is affected by lateral forces due to wind or earthquake or both to an extent that they play an important role in the structural design. Tall structures have fascinated mankind from the beginning of civilisation. The Egyptian Pyramids, one among the seven wonders of world, constructed in 2600 B.C. are among such ancient tall structures. Such structures were constructed for defence and to show pride of the population in their civilisation. The growth in modern multi-storeyed building construction, which began in late nineteenth century, is intended largely for commercial and residential purposes.

The development of the high-rise building has followed the growth of the city closely. The process of urbanisation, that started with the age of industrialisation, is still in progress in developing countries like India. Industrialisation causes migration of people to urban centres where job opportunities are significant. The land available for buildings to accommodate this migration is becoming scarce, resulting in rapid increase in the cost of land. Thus, developers have looked to the sky to make their profits. The result is multi-storeyed buildings, as they provide a large floor area in a relatively small area of land in urban centres.

The construction of multi-storeyed buildings is dependent on available materials, the level of construction technology and the availability of services such as elevators necessary for the use in the building. In ancient Rome, people used to build multi-storeyed structures with wood. For those buildings built after the Great Fire of Rome, Nero used brick and a form of concrete material for construction. Wood lacked strength for buildings of more than five stories and was more susceptible to fire hazard. But, the buildings constructed with brick and masonry occupied a large space for their walls. Technology responded to these drawbacks of construction materials with the development of high strength and structurally more efficient materials like wrought iron and then subsequently steel. These new materials resulted in construction of skyscrapers of the order of *120* storeys such as Petronas Towers, Sears Tower, World Trade Centre, Empire State Building etc. all over the world [Fig. 1]. In contrast, the tallest building in India is *35* storeys in reinforced concrete, Hotel Oberoi Sheraton (*116 m*). Even though in the last two decades a number of multi-storeyed buildings have been constructed in India, the tall building technology is at its infancy in India, particularly in structural steel.

In developed countries a very large percentage of multi-storeyed buildings are built with steel where as steel is hardly used in construction of multi-storeyed frames in India even though it has proved to be a better material than reinforced concrete. For example, over

90% of the new multi-storeyed buildings in London are built of steel or steel-composite framed construction. Buildings in the *100*-storey range are invariably erected with steel or steel-concrete composites in the West. A look at world-class high-rise steel-framed buildings constructed in various parts of world shown in Fig. 1 may inspire one to become a structural engineer of such a class of structures.



(a) Empire State Building (381 m) *(b) World Trade Centre (415 m) (417 m)* *(c) Jin Mao Building (421m)* *(d) Sears Tower (443 m)* *(e) Petronas Towers (452 m)*

Fig. 1 World's tallest buildings

The use of steel in multi-storey building construction results in many advantages for the builder and the user. The reasons for using steel frames in the construction of multi-storey buildings are listed below:

- Steel frames are faster to erect compared with reinforced concrete frames. The availability of the building in a shorter period of time results in economic advantages to the owner due to shorter period of deployment of capital, without return. For example, at the time the steel-framed Empire State Building was completed, the tallest reinforced concrete building, the exchange building in Seattle, had attained a height of only *23* storeys.

- In comparison with concrete construction, steel frames are significantly lighter. This results in very much reduced loads on foundations.
- The elements of framework are usually prefabricated in the factory under effective quality control thus enabling a better product.
- This form of construction results in much reduced time on site activities, plant, materials and labour, causing little disruption to normal life of the community, unlike wet concrete construction process.
- The use of steel makes possible the creation of large, column-free internal spaces. This is of particular advantage for open-plan offices and large auditoria and concert halls.
- The use of steel frame when compared with R.C. frame results in sufficient extra space to accommodate all service conduits without significant loss in head room.
- Subsequent alterations or strengthening of floors are relatively easy in steel frames compared with concrete frames.
- The framework is not susceptible to delays due to slow strength gain, as in concrete construction.
- The material handling capacity required at site in steel construction is less than prefabricated concrete construction.
- Steel structure occupies lesser percentage of floor area in multi-storeyed buildings.
- The steel frame construction is more suitable to withstand lateral loads caused by wind or earthquake.

This chapter deals with the anatomy of multi-storey buildings; the different loads to be considered and various structural systems adopted in such steel-framed multi-storey buildings.

2.0 ANATOMY OF MULTI-STOREY BUILDINGS

The vertical or gravity load carrying system of a multi-storey steel-framed building comprises a system of vertical columns interconnected by horizontal beams, which supports the floors and roofing. The resistance to lateral loads is provided by diagonal bracing or shear walls or rigid frame action between the beams and columns. Thus, the components of a typical steel-framed structure are:

- Beams
- Columns
- Floors
- Bracing Systems
- Connections

2.1 Beam-and-Column Construction

This is often called as "skeleton construction". The floor slabs, partitions, exterior walls etc. are all supported by a framework of steel beams and columns. This type of skeleton structure can be erected easily leading to very tall buildings.

In such a beam and column construction, the frame usually consists of columns spaced *6 -10 m* apart, with beams and girders framed into them from both directions at each floor level. An example of skeleton construction is shown in Fig. 2.

Generally columns used in the framework are hot-rolled I-sections or concrete encased steel columns. They give unobstructed access for beam connections through either the flange or the web. Where the loading requirements exceed the capacity of available section, additional plates are welded to the section.

The selection of beam sections depends upon the span, loading and limitations on overall depth from headroom considerations. Simple beams with precast floors or composite metal deck floors are likely to be the most economical for smaller spans. For larger spans, plate-girders or plated-beams are used.

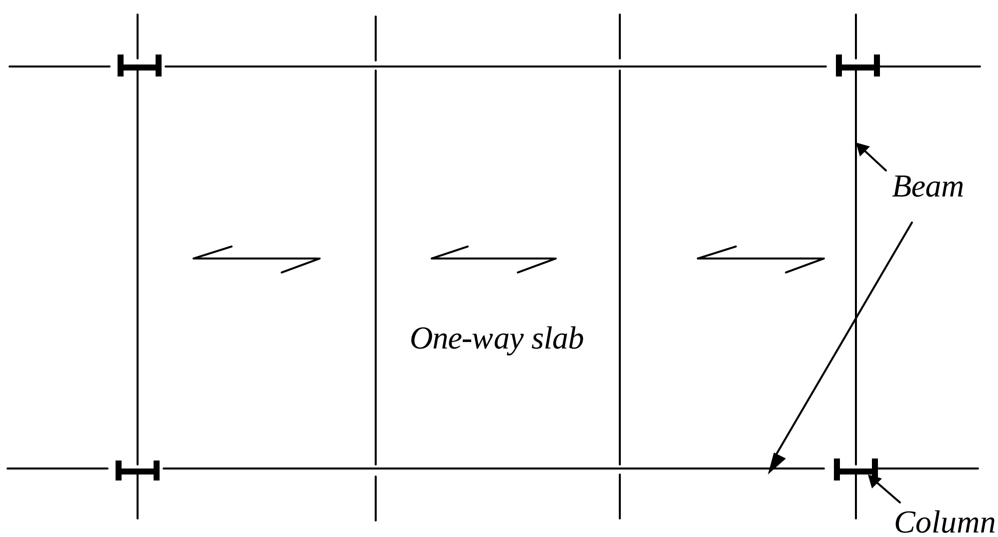


Fig. 2 Beam – and – column construction

2.2 Common types of floor system

The selection of an appropriate flooring in a steel-framed building depends on various factors like the loads to be supported, span length, fire resistance desired, sound and heat transmission, the likely dead weight of the floor, the facilities needed for locating the services, appearance, maintenance required, time required to construct, available depth for the floor etc. The different types of floors used in steel-framed buildings are as follows:

- Concrete slabs supported by open-web joists
- One-way and two-way reinforced concrete slabs supported on steel beams
- Concrete slab and steel beam composite floors
- Profiled decking floors
- Precast concrete slab floors.

2.2.1. Concrete slabs supported with open-web joists

This is one of the most common types of floor slabs used for steel frame buildings in U.S. Steel forms or decks are usually attached to the joists by welding and concrete slabs are poured on top. This is one of the lightest types of concrete floors. For structures with light loading, this type is economical. A sketch of an open-web joist floor is shown in Fig. 3

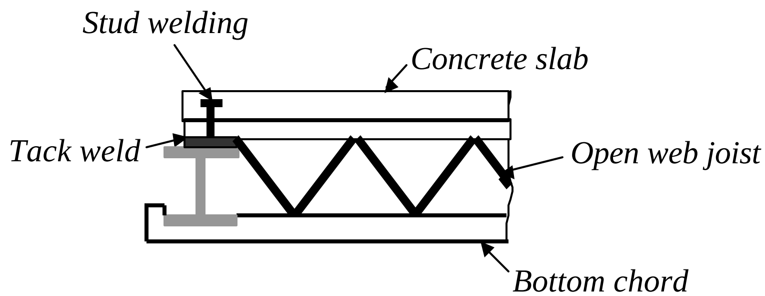


Fig. 3 Open- web joists

2.2.2. One-way and two-way reinforced concrete slabs.

These are much heavier than most of the newer light weight floor systems and they take more time to construct, thus negating the advantage of speed inherent in steel construction. This floor system is adopted for heavy loads. One way slabs are used when the longitudinal span is two or more times the short span. In one-way slabs, the short span direction is the direction in which loads get transferred from slab to the beams. Hence the main reinforcing bars are provided along this direction. However, temperature, shrinkage and distribution steel is provided along the longer direction.

The two-way concrete slab is used when aspect ratio of the slab supported along all four edges i.e. longitudinal span/transverse span is less than *2*. The main reinforcement runs in both the directions. A typical cross-section of a one-way slab floor with supporting steel beams is shown in Fig. 4

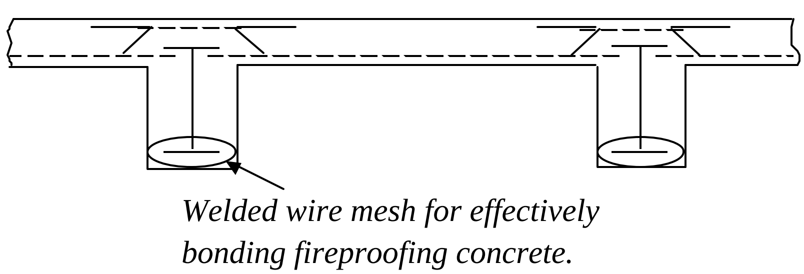
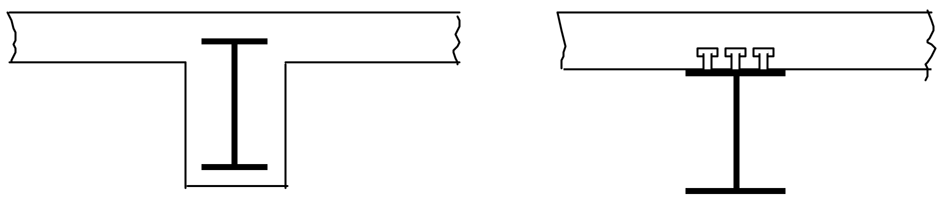


Fig. 4 Cross section of one-way slab floor

2.2.3. Composite floors with a reinforced concrete slab and steel beams

Composite floors have steel beams bonded with concrete slab in such a way that both of them act as a unit in resisting the total loads. The sizes of steel beams are significantly smaller in composite floors, because the slab acts as an integral part of the beam in compression. The composite floors require less steel tonnage in the structure and also result in reduction of total floor depth. These advantages are achieved by utilising the compressive strength of concrete by keeping all or nearly all of the concrete in compression and at the same time utilises a large percentage of the steel in tension.

The types of composite floor systems normally employed are shown in Fig. 5



(a) Steel beam encased in concrete (Rarely used nowadays)

(b) Steel beam acting composite with concrete slab using shear connectors

Fig. 5 Composite floors

2.2.4. Profiled - decking floors

In the last three decades, a new form of composite floor construction, consisting of profiled and formed steel decking with a concrete topping has become popular for office and apartment buildings. The structural behaviour is the same as that of reinforced concrete slab with steel sheeting acting as centring until concrete hardens and as the tension reinforcement after concrete hardens. It is popular where the loads are not very heavy. The advantages of steel-decking floors are given below:

(i) They do not need form work
(ii) The lightweight concrete is used resulting in reduced dead weight
(iii) The decking distributes shrinkage strains, thus prevents serious cracking
(iv) The decking stabilises the beam against lateral buckling, until the concrete hardens
(v) The cells in decking are convenient for locating services.

More details of composite construction using profiled decking floors are provided in chapters on Composite floors. Typical cross-section of steel decking floor is shown in Fig. 6

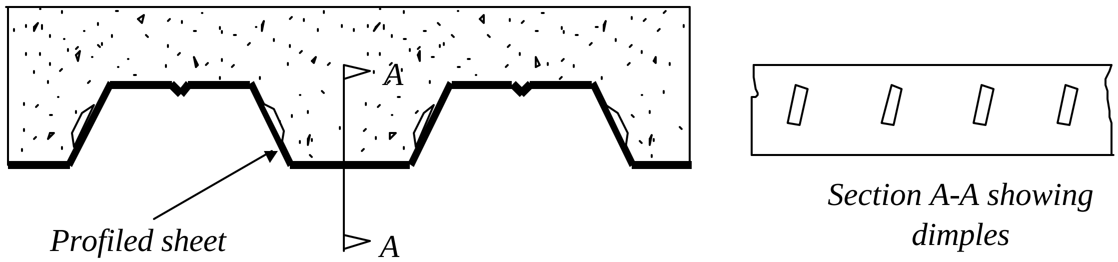


Fig. 6 Composite floor system using profiled sheets

2.2.5. Precast concrete floors

Precast concrete floors offer speedy erection and require only minimal formwork. Light-weight aggregates are generally used in the concrete, making the elements light and easy to handle. Typical precast concrete floor slab sections are shown in Fig.7. It is necessary to use cast in place mortar topping of *25 to 50 mm* before installing other floor coverings. Larger capacity handling machines are required for this type of construction when compared with those required for profiled decking. Usually prestressing of the precast element is required.

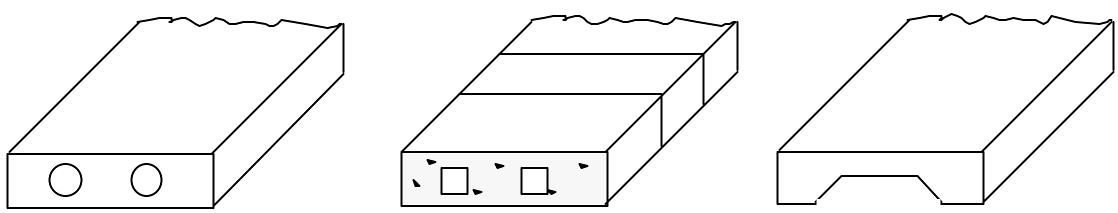


Fig. 7 Precast concrete floor slabs

2.3. Lateral load resisting systems

2.3.1. Lateral forces

Lateral forces due to wind or seismic loading must be considered for tall buildings along with gravity forces. Very often the design of tall buildings is governed by lateral load resistance requirement in conjunction with gravity load. High wind pressures on the sides of tall buildings produce base shear and overturning moments. These forces cause horizontal deflection in a multi-storey building. This horizontal deflection at the top of a building is called *drift*. The drift is measured by *drift index*, D/h, where, D is the horizontal deflection at top of the building and h is the height of the building. Lateral drift of a typical moment resisting frame is shown in Fig. 8.

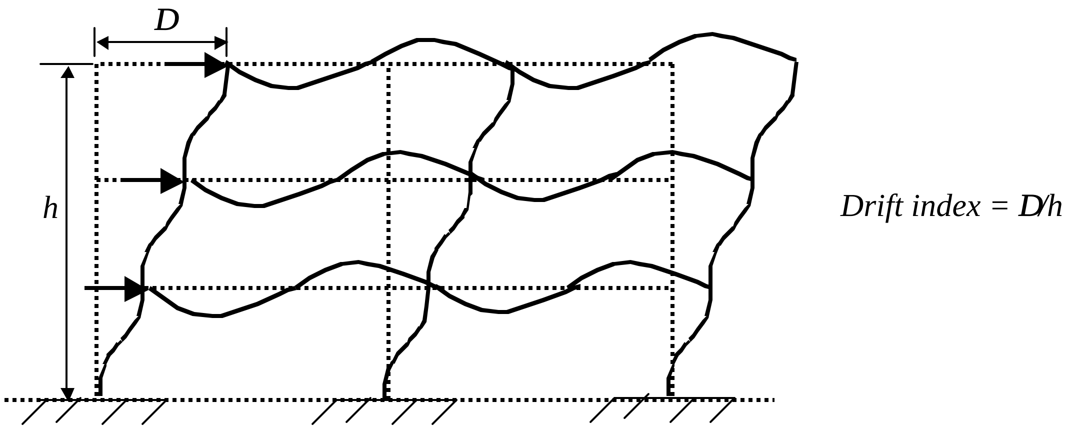


Fig. 8 Lateral drift

The usual practice in the design of multi-storey steel buildings is to provide a structure with sufficient lateral stiffness to keep the drift index between approximately *0.0015* and *0.0030* of the total height. Normally, the provision of lateral stiffness requires about *5* to *10%* of extra steel. The extra steel is used for bracing systems as described in the next section. The IS code require drift index to be not more than *0.002* of total height.

2.3.2 Lateral loading systems

A multi-storey building with no lateral bracing is shown in Fig. 9(*a*). When the beams and columns shown are connected with simple beam connections, the frame would have practically no resistance to the lateral forces and become geometrically unstable. The frame would laterally deflect as shown in Fig. 9(*b*) even under a small lateral load.

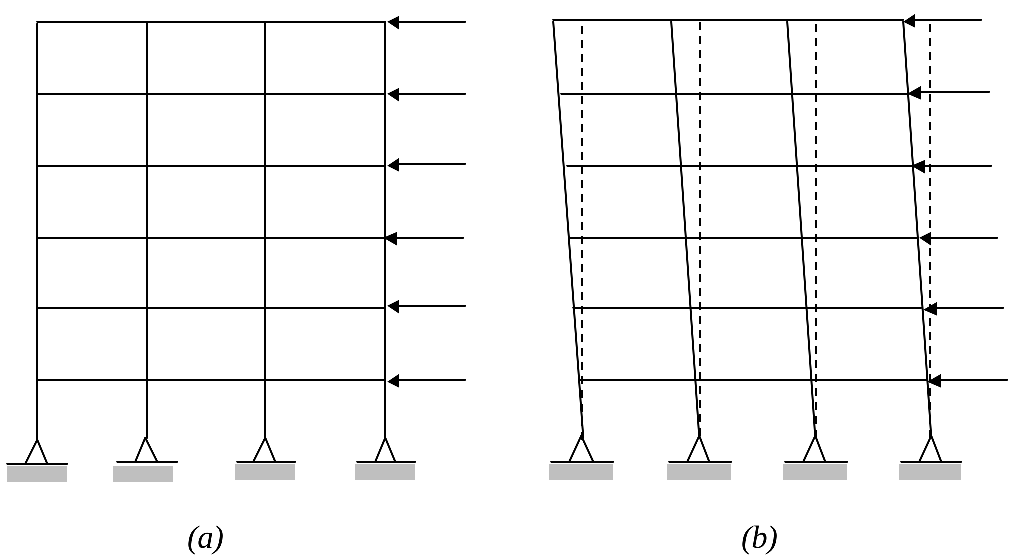


Fig. 9 Multi-storey frame without lateral bracing

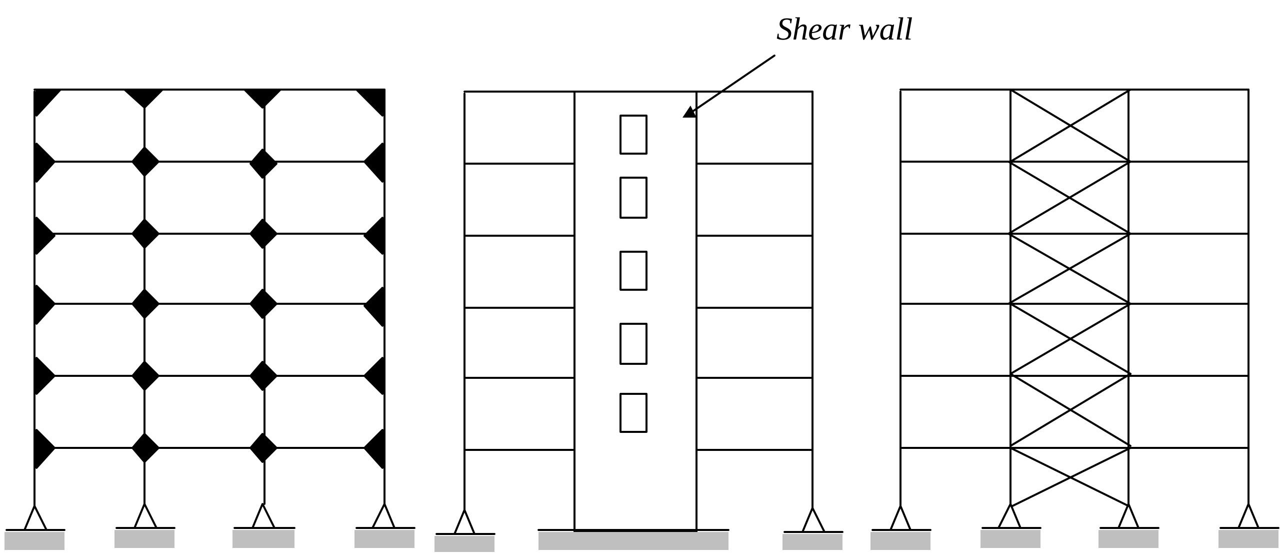
One of the following three types structural systems is used to resist the lateral loads and limit the drift within acceptable range mentioned above:

- Rigid Frames
- Shear walls.
- Braced frames

Combinations of these systems and certain other advanced forms are also used for very tall buildings. The advanced structural forms are discussed in section 3.0.

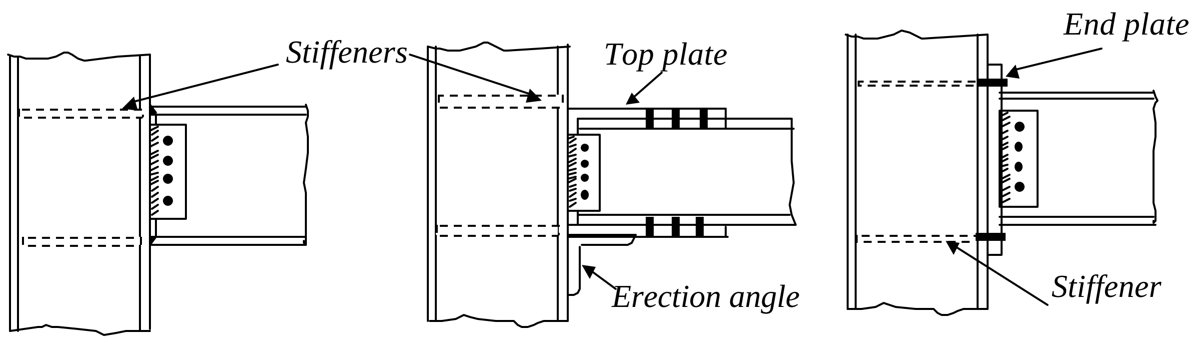
Rigid Frames

Rigidly jointed frames or sway-frames are those with moment resisting connections between beams and columns. A typical rigid frame is shown in Fig. 10(*a*). It may be used economically to provide lateral load resistance for low-rise buildings. Generally, it is less stiff than other systems. However, moment resisting connections may be necessary in locations where loads are applied eccentrically with respect to centre line of the columns. Three types of commonly employed moment resisting connections are shown in Fig. 11. The connection shown in Fig. 11(*a*) and 11(*c*) are more economical. However, the moment-rotation performance of the connection shown in Fig. 11(*b*) is likely to be superior to that of either Fig. 11(*a*) or Fig. 11(*c*).



(a) Moment resisting frames *(b) Shear wall frames* *(c) Braced frames*

Fig. 10 Lateral load resisting systems



(b) *Shop welded and field bolted connection* (a) *Field welded and field bolted connection* (c)*End plated connection*

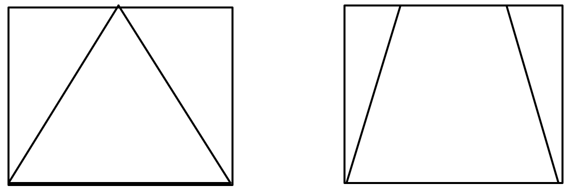
Fig. 11 Moment resistant connections

Shear Walls

The lateral loads are assumed to be concentrated at the floor levels. The rigid floors spread these forces to the columns or walls in the building. Lateral forces are particularly large in case of tall buildings or when seismic forces are considered. Specially designed reinforced concrete walls parallel to the directions of load are used to resist a large part of the lateral loads caused by wind or earthquakes by acting as deep cantilever beams fixed at foundation. These elements are called as *shear walls*. Frequently buildings have interior concrete core walls around the elevator, stair and service wells. Such walls may be considered as shear walls. The advantages of shear walls are (i) they are very rigid in their own plane and hence are effective in limiting deflections and (ii) they act as fire compartment walls. However, for low and medium rise buildings, the construction of shear walls takes more time and is less precise in dimensions than steelwork. Generally, reinforced concrete walls possess sufficient strength and stiffness to resist the lateral loading. Shear walls have lesser ductility and may not meet the energy required under severe earthquake. A typical framed structure braced with core wall is shown in Fig. 10(*b*).

Braced frames

To resist the lateral deflections, the simplest method from a theoretical standpoint is the intersection of full diagonal bracing or *X*-bracing as shown in Fig. 10(*c*). The *X*-bracing system works well for *20* to *60* storey height, but it does not give room for openings such as doors and windows. To provide more flexibility for the placing of windows and doors, the *K*-bracing system shown in Fig. 12(*a*) is preferred instead of *X*- bracing system. If, we need to provide larger openings, it is not possible with *K*-bracing system; we can use the full-storey knee bracing system shown in Fig. 12(*b*). Knee bracing is an eccentric bracing that is found to be efficient in energy dissipation during earthquake loads by forming plastic hinge in beam at the point of their intersection of the bracings with the beam.



(a) K- brace *(b) Knee brace*

Fig. 12 Alternative bracing systems

2.4 Connections

The most important aspect of structural steel work for buildings is the design of connections between individual frame components. Depending upon the structural behaviour, the connections can be classified as following:

Simple connections - The connection is detailed to allow the beam end to rotate freely and the beam behaves as a simply supported beam. Such a connection transfers shear and axial forces between the connecting members but does not transfer bending moment.

Rigid connections - The connection is detailed to ensure a monolithic joint such that the angle between beam and column before deformation remains the same even after deformation. Such a connection transfers shear, axial force and bending moment from the beam to the column.

Semi-rigid connections - Due to flexibility of the joint some relative rotation between the beam and column occurs. When this is substantial, the joints are designed as semi-rigid. These connections are designed to transmit the full shear force and a fraction of the rigid joint bending moment across the joint. The analysis of frames with such joints is complex and their application is dependent on development of joint characteristics based on experimental evidence. The procedures for analysis and design should be simple so that they can be easily adopted for manual computation in design offices. With the advent of computers, it is possible to account for flexibility of joints in the frame analysis by adopting suitable computer packages. Recent advances in research on the topic have led to results that can be used in practical analysis and design of semi-rigid connections.

3.0 ADVANCED STRUCTURAL FORMS

The bracing systems discussed so far are not efficient for buildings taller than *60* stories. This section introduces more advanced types of structural forms that are adopted in steel-framed multi-storeyed buildings larger than *60* storey high. Common types of advanced structural forms are:

3.1 Framed -Tube Structures

The framed tube is one of the most significant modern developments in high-rise structural form. The frames consist of closely spaced columns, *2 - 4 m* between centres, joined by deep girders. The idea is to create a tube that will act like a continuous perforated chimney or stack. The lateral resistance of framed tube structures is provided by very stiff moment resisting frames that form a tube around the perimeter of the building. The gravity loading is shared between the tube and interior columns. This structural form offers an efficient, easily constructed structure appropriate for buildings having *40* to*100* storeys.

When lateral loads act, the perimeter frames aligned in the direction of loads act as the webs of the massive tube cantilever and those normal to the direction of the loading act as the flanges. Even though framed tube is a structurally efficient form, flange frames tend to suffer from shear lag. This results in the mid face flange columns being less stressed than the corner columns and therefore not contributing to their full potential lateral strength. Aesthetically, the tube looks like the grid-like façade as small windowed and is repetitious and hence use of prefabrication in steel makes the construction faster. A typical framed tube is shown in Fig. 13(*a*).

3.2 Braced tube structures

Further improvements of the tubular system can be made by cross bracing the frame with *X*-bracing over many stories, as illustrated in Fig. 13(*b*). This arrangement was first used in a steel structure, in Chicago's John Hancock Building, in 1969.

As the diagonals of a braced tube are connected to the columns at each intersection, they virtually eliminate the effects of shear lag in both the flange and web frames. As a result the structure behaves under lateral loads more like a braced frame reducing bending in the members of the frames. Hence, the spacing of the columns can be increased and the depth of the girders will be less, thereby allowing large size windows than in the conventional framed tube structures.

In the braced tube structure, the braces transfer axial load from the more highly stressed columns to the less highly stressed columns and eliminates differences between load stresses in the columns.

3.3 Tube-in-Tube Structures

This is a type of framed tube consisting of an outer-framed tube together with an internal elevator and service core. The inner tube may consist of braced frames. The outer and inner tubes act jointly in resisting both gravity and lateral loading in steel-framed buildings. However, the outer tube usually plays a dominant role because of its much greater structural depth. This type of structures is also called as Hull (Outer tube) and Core (Inner tube) structures. A typical Tube-in-Tube structure is shown in Fig. 14.

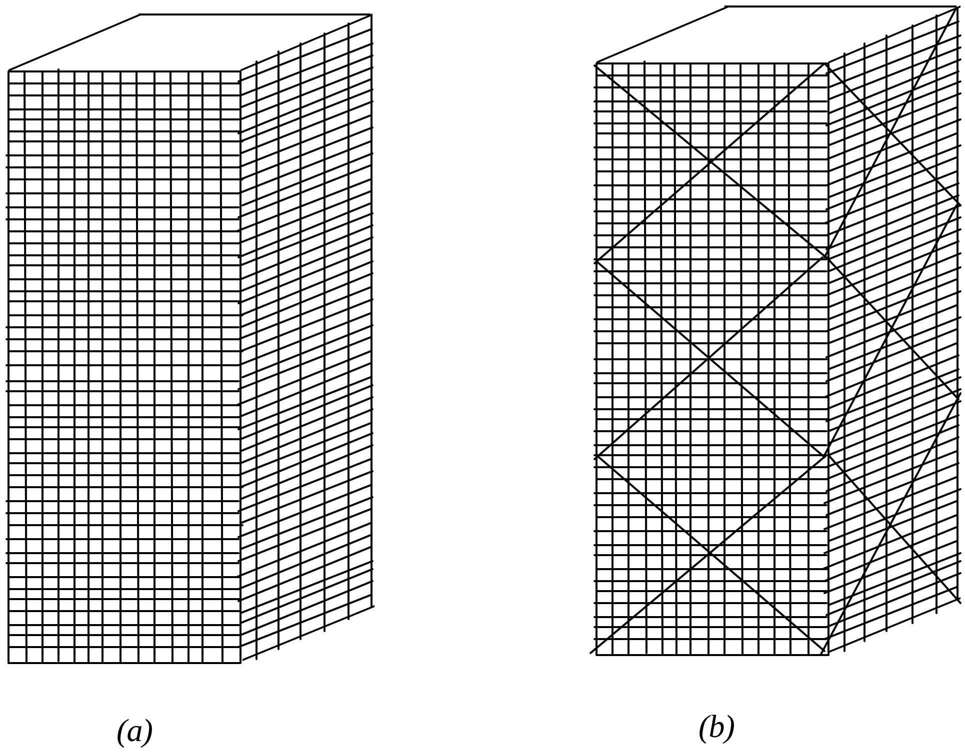


Fig. 13 (a) Framed tube (b) Braced framed tube

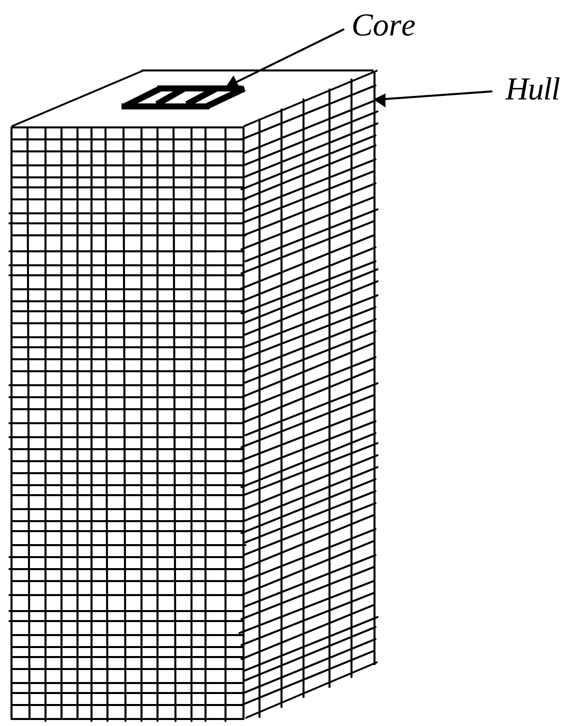


Fig. 14 Tube-in-Tube frame

3.4 Bundled Tube

The bundled tube system can be visualised as an assemblage of individual tubes resulting in multiple cell tube. The increase in stiffness is apparent. The system allows for the greatest height and the most floor area. This structural form was used in the Sears Tower in Chicago. Fig. 1(*d*) shows bundled tubes in the Sears Tower. In this system, introduction of the internal webs greatly reduces the shear lag in the flanges. Hence, their columns are more evenly stressed than in the single tube structure and their contribution to the lateral stiffness is greater.

4.0 LOADING

Loading on tall buildings is different from low-rise buildings in many ways such as large accumulation of gravity loads on the floors from top to bottom, increased significance of wind loading and greater importance of dynamic effects. Thus, multi-storeyed structures need correct assessment of loads for safe and economical design. Excepting dead loads, the assessment of loads can not be done accurately. Live loads can be anticipated approximately from a combination of experience and the previous field observations. But, wind and earthquake loads are random in nature. It is difficult to predict them exactly. These are estimated based on probabilistic approach. The following discussion describes the influence of the most common kinds of loads on multi-storeyed structures.

4.1 Gravity loads

Dead loads due the weight of every element within the structure and live loads that are acting on the structure when in service constitute gravity loads. The dead loads are calculated from the member sizes and estimated material densities. Live loads prescribed by codes are empirical and conservative based on experience and accepted practice. The equivalent minimum loads for office and residential buildings are specified in Table - 1.

Table – 1 Live load magnitudes [IS: 875 - 1987 Part -II]

Occupancy classification	**Uniformly distributed load (kN/m^2)**	**Concentrated load (kN)**
Office buildings		
• Offices and Staff rooms	2.5	2.7
• Class rooms	3.0	2.7
• Corridors, Store rooms and Reading rooms	4.0	4.5
Residential buildings		
• Apartments	2.0	1.8
• Restaurants	4.0	2.7
• Corridors	3.0	4.5

A floor should be designed for the most adverse effect of uniformly distributed load and concentrated load over *0.3 m* by *0.3 m* as specified in Table-1, but they should not be considered to act simultaneously. All other structural elements such as beams and columns are designed for the corresponding uniformly distributed loads on floors.

Reduction in imposed load may be made in designing columns, load bearing walls etc., if there is no specific load like plant or machinery on the floor. This is allowed to account for improbability of total loading being applied over larger areas. The supporting members of the roof of the multi-storeyed building is designed for *100%* of uniformly distributed load; further reductions of *10%* for each successive floor down to a minimum of *50%* of uniformly distributed load is done. The live load at floor level can be reduced in the design of beams, girders or trusses by *5%* for each $50m^2$ area supported, subject to a maximum reduction of *25%*. In case the reduced load of a lower floor is less than the reduced load of an upper floor, then the reduced load of the upper floor should be adopted in the lower floor also.

4.2 Wind load

The wind loading is the most important factor that determines the design of tall buildings over *10* storeys, where storey height approximately lies between *2.7 – 3.0 m*. Buildings of up to *10* storeys, designed for gravity loading can accommodate wind loading without any additional steel for lateral system. Usually, buildings taller than *10* storeys would generally require additional steel for lateral system. This is due to the fact that wind loading on a tall building acts over a very large building surface, with greater intensity at the greater heights and with a larger moment arm about the base. So, the additional steel required for wind resistance increases non-linearly with height as shown in Fig. 15.

As shown in Fig. 15 the lateral stiffness of the building is a more important consideration than its strength for multi-storeyed structures. Wind has become a major load for the designer of multi-storeyed buildings. Prediction of wind loading in precise scientific terms may not be possible, as it is influenced by many factors such as the form of terrain, the shape, slenderness, the solidarity ratio of building and the arrangement of adjacent buildings. The appropriate design wind loads are estimated based on two approaches. Static approach is one, which assumes the building to be a fixed rigid body in the wind. This method is suitable for buildings of normal height, slenderness, or susceptible to vibration in the wind. The other approach is the dynamic approach. This is adopted for exceptionally tall, slender, or vibration prone buildings. Sometimes wind sensitive tall buildings will have to be designed for interference effects caused by the environment in which the building stands. The loading due to these interference effects is best ascertained using wind tunnel modelled structures in the laboratory.

However, in the Indian context, where the tallest multi-storeyed building is only *35* storey high, multi-storeyed buildings do not suffer wind-induced oscillation and generally do not require to be examined for the dynamic effects. For detailed information on evaluating wind load, the reader is referred to IS: 875 - 1987 (Part - III).

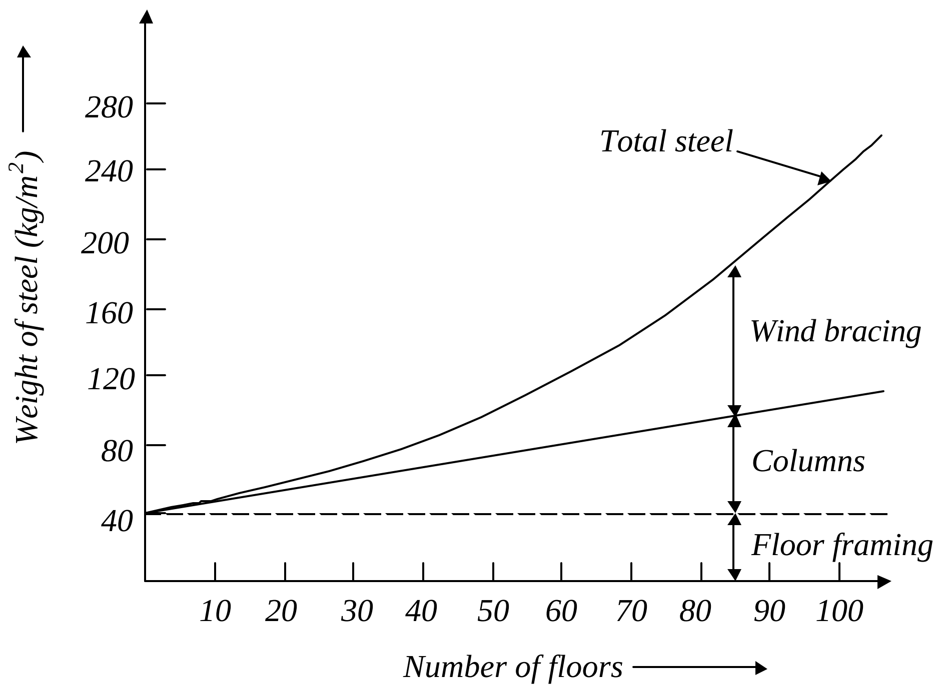


Fig. 15 Weight of steel in multi-storeyed buildings

4.3 Earthquake load

Seismic motion consists of horizontal and vertical ground motions, with the vertical motion usually having a much smaller magnitude. Further, factor of safety provided against gravity loads usually can accommodate additional forces due to vertical acceleration due to earthquakes. So, the horizontal motion of the ground causes the most significant effect on the structure by shaking the foundation back and forth. The mass of building resists this motion by setting up inertia forces throughout the structure.

The magnitude of the horizontal shear force F shown in Fig. 16 depends on the mass of the building M, the acceleration of the ground a, and the nature of the structure. If a building and the foundation were rigid, it would have the same acceleration as the ground as given by Newton's second law of motion, i.e. $F = Ma$. However, in practice all buildings are flexible to some degree. For a structure that deforms slightly, thereby absorbing some energy, the force will be less than the product of mass and acceleration [Fig. 16(*b*)]. But, a very flexible structure will be subject to a much larger force under repetitive ground motion [Fig. 16(*c*)]. This shows the magnitude of the lateral force on a building is not only dependent on acceleration of the ground but it will also depend on the type of the structure. As an inertia problem, the dynamic response of the building plays a large part in influencing and in estimating the effective loading on the structure. The earthquake load is estimated by Seismic co-efficient method or Response spectrum method. The later takes account of dynamic characteristics of structure along with ground motion. For detailed information on evaluating earthquake load, reader is referred to IS: 1893 - 1984 and the chapter on Earth quake resistant design.

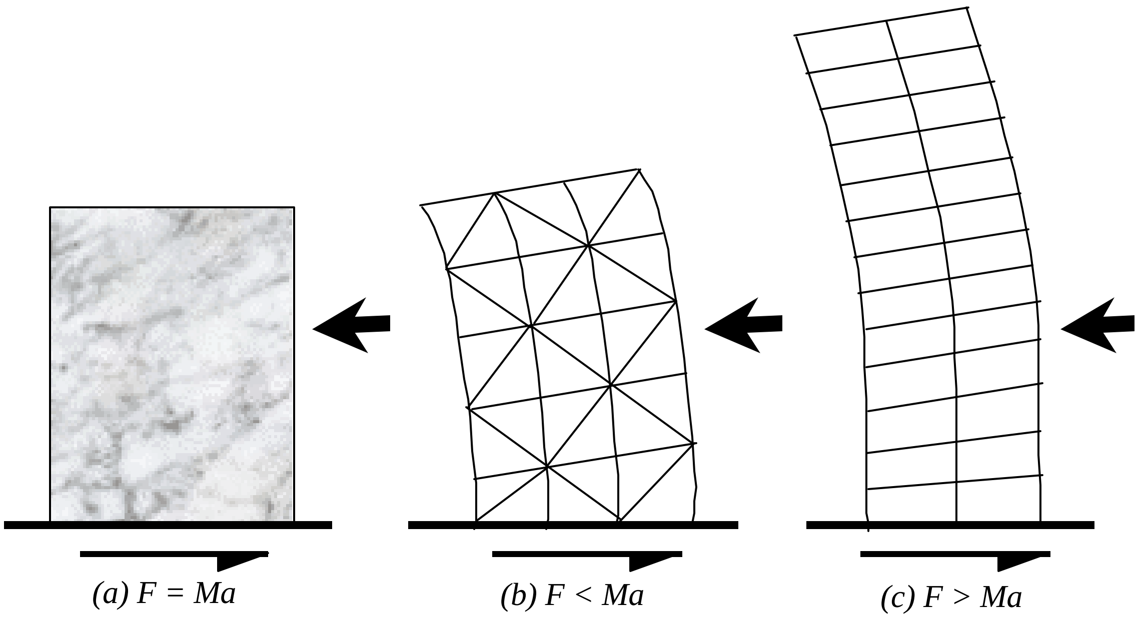


Fig. 16 Force developed by earthquake

5.0 SUMMARY

Pride seems to be the prime motivation for the construction of ancient tall structures such as the pyramids of Egypt, the Mayan temples of Mexico and the Kutub Minar of India. Industrialisation and urbanisation have led to the evolution of modern tall buildings for residential and commercial purposes. Significant advances in the design and construction of high-rise buildings have occurred in recent years. This has been possible on account of developments in the use of new materials, construction techniques or forms of service. This chapter mainly concentrated with the evolution, anatomy and different types of tall structural systems and loadings. Meeting the design challenges are described in conceptual way.

6.0 REFERENCES

1. McCormac. C. J., (1994): "Structural Steel Design", Harper Collins College Publishers.
2. Owens. G. W. and Knowles. P., (1994) "Steel Designers Manual", The Steel Construction Institute, ELBS Blackwell Scientific Publishers, London.
3. Taranath. S. B., (1984): "Structural analysis and design of tall buildings", McGraw-Hill Book Company
4. Schuller. W., (1976): "High-rise building structures", John Wiley & Sons
5. Smith. B. S., and Coull. A., (1991): "Tall building structures: Analysis and Design", John Wiley & Sons.
6. IS: 875 - 1987 (Parts - I to V), Indian Code of Practice for evaluating loads excepting earthquake load.
7. IS: 1893 - 1984, Indian Code of Practice for evaluating earthquake load.

38 MULTI-STOREY BUILDINGS - II

1.0 INTRODUCTION

Modern design offices are generally equipped with a wide variety of structural analysis software programs, invariably based on the stiffness matrix method. These Finite Element Analysis packages such as MSC/NASTRAN, SAP - 90, STAAD etc., give more accurate results compared with approximate methods, but they involve significant computational effort and therefore cost. They are generally preferred for complex structures. The importance of approximate hand methods for the analysis of forces and deflections in multi-storeyed frames can not be ignored; they have served the Structural Engineer well for many decades and are still useful for preliminary analysis and checking. This chapter describes various approximate methods to analyse simple and rigid multi-storeyed frames.

2.0 BRACED FRAMES - METHODS OF ANALYSIS FOR LATERAL LOADS

In this section, simple hand methods for the analysis of statically determinate or certain low-redundant braced structures is reviewed.

2.1 Member Force Analysis

Analysis of the forces in a statically determinate triangulated braced frame can be made by the method of sections. For instance, consider a typical single-diagonal braced pin-jointed panel as shown in Fig. 1. When this bent is subjected to an external shear Q_i in i-th storey and external moments M_i and M_{i-1} at floors i and $i-1$, respectively, the force in the brace can be found by considering the horizontal equilibrium of the free body above section XX, thus,

$$F_{BC} \cos\theta = Q_i$$

Hence,

$$F_{BC} = Q_i / \cos\theta$$

The axial force F_{BD} in the column BD is found by considering moment equilibrium of the upper free body about C, thus

$$F_{BD} * \ell = M_{i-1}$$

Hence,

$$F_{BD} = M_{i-1} / \ell$$

Similarly the force F_{AC} in column AC is obtained from the moment equilibrium of the upper free body about B. It is given by

$F_{AC} = M_i / \ell$

This procedure can be repeated for the members in each storey of the frame. The member forces in more complex braced frames such as knee-braced, X-braced and K-braced frames can also be obtained by taking horizontal sections.

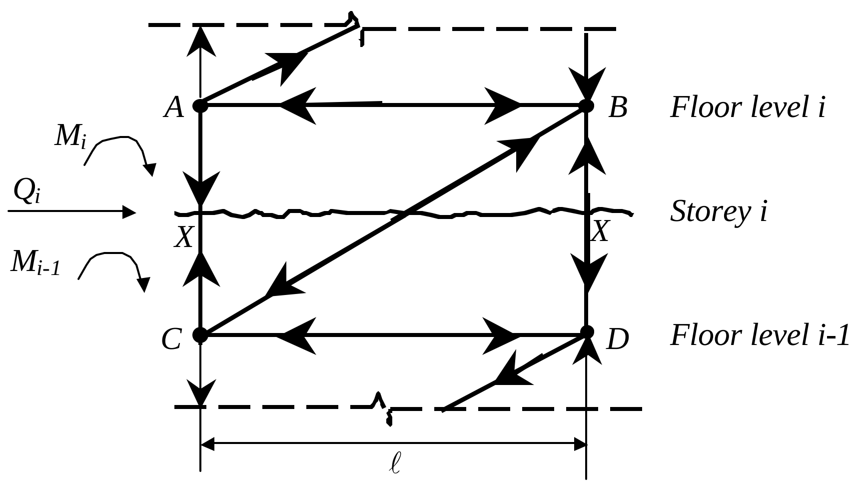


Fig. 1 Single diagonal braced panel

2.2 Drift Analysis

Drift in building frames is a result of flexural and shear mode contributions, due to the column axial deformations and to the diagonal and girder deformations, respectively. In low rise braced structures, the shear mode displacements are the most significant and, will largely determine the lateral stiffness of the structure. In medium to high rise structures, the higher axial forces and deformations in the columns, and the accumulation of their effects over a greater height, cause the flexural component of displacement to become dominant.

The storey drift in a braced frame reaches a maximum value at or close to the top of the structure and is strongly influenced by the flexural component of deflection. This is because the inclination of the structure caused by the flexural component accumulates up the structure, while the storey shear component diminishes toward the top.

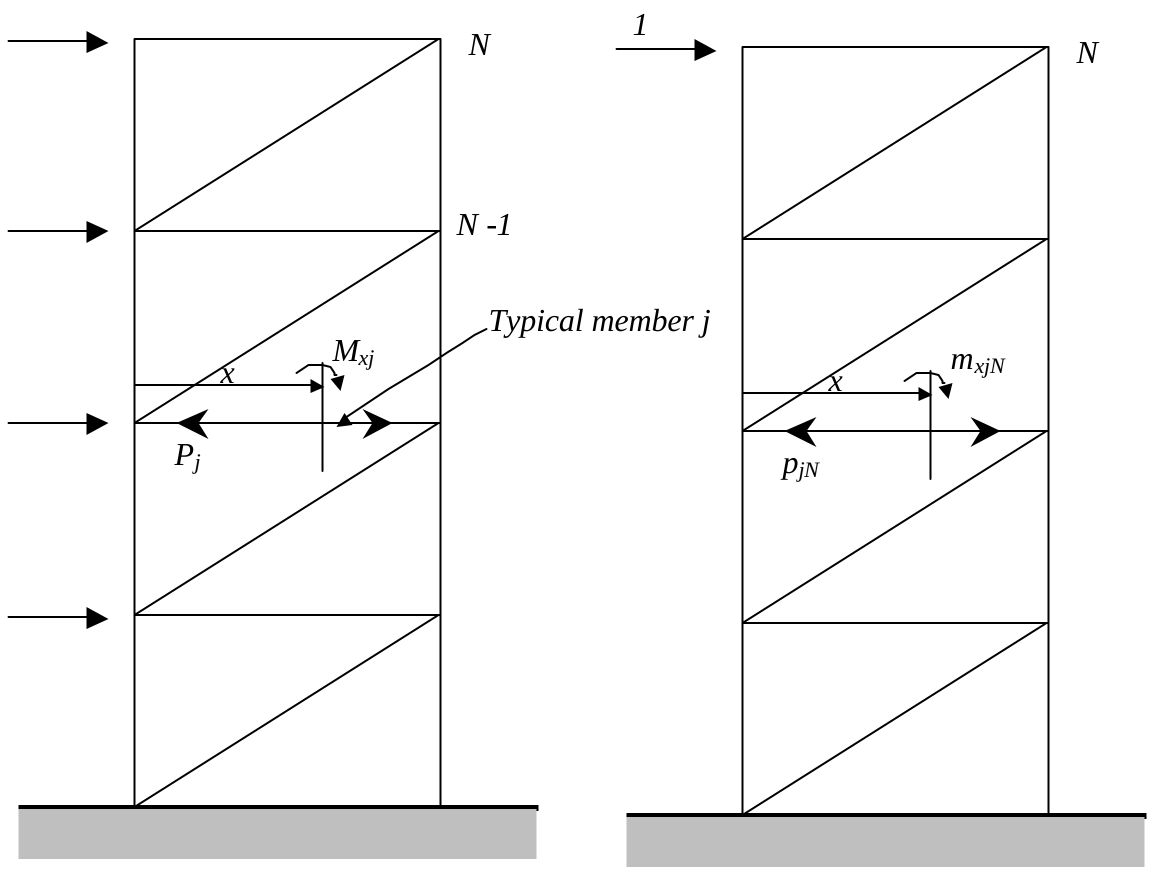
Hand analysis for drift allows the drift contributions of the individual frame members to be seen, thereby providing guidance as to which members should be increased in size to effectively reduce an excessive total drift or storey drift. The following section explains a method for hand evaluation of drift.

2.2.1 Virtual work drift analysis

In this method, a force analysis of the structure is carried out for design lateral loads in order to determine the axial force P_j in each member j and the bending moment M_{xj} at sections x along those members subjected to bending [See Fig. 2(*a*)]. A second force analysis is then carried out with the structure subjected to only a unit imaginary or "dummy" lateral load at the level N whose drift is required [Fig. 2(*b*)] to give the axial force p_{jN} , and moment m_{xjN} at section x in the bending members. The resulting horizontal deflection at N is then given by

$$D_N = S\, p_{jN}\left(\frac{P\ell}{EA}\right)_j + S\int_0^{\ell_j} m_{xjN}\left(\frac{M_x}{EI}\right)_j dx \qquad (1)$$

where, ℓ_j - length of member j
A_j - sectional area of member j
E - elastic modulus
I_j - moment of inertia of member j.



(a) (b)

Fig. 2 Member forces in a typical braced frame

This method is exact and can easily be systematised by tabulation. A good assessment of the deflected configuration, the total drift, and the storey drifts can be obtained by plotting the deflection diagram from the deflections at just three or four equally spaced points up the height of the structure. It requires one design load force analysis and three or four “dummy” unit load analyses.

3.0 ANALYSIS OF FRAMES WITH MOMENT-RESISTING JOINTS FOR LATERAL LOADS

Multi-storey building frames subjected to lateral loads are statically indeterminate and exact analysis by hand calculation takes much time and effort. Using simplifying assumptions, approximate analyses of these frames yield good estimate of member forces in the frame, which can be used for checking the member sizes. The following methods can be employed for lateral load analysis of rigidly jointed frames.

- The Portal method.
- The Cantilever method
- The Factor method

The portal method and the cantilever method yield good results only when the height of a building is approximately more than five times its least lateral dimension. Either classical techniques such as slope deflection or moment distribution methods or computer methods using stiffness or flexibility matrices can be used if a more exact result is desired.

3.1 The Portal Method

This method is satisfactory for buildings up to *25* stories, hence is the most commonly used approximate method for analysing tall buildings. The following are the simplifying assumptions made in the portal method:

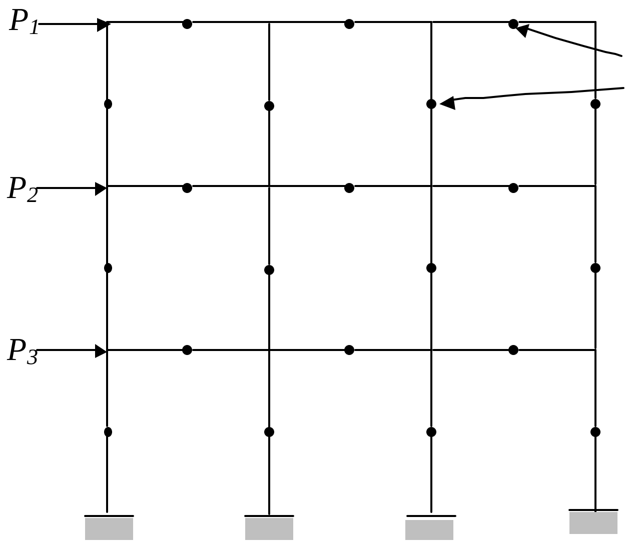
1. A point of contraflexure occurs at the centre of each beam.
2. A point of contraflexure occurs at the centre of each column.
3. The total horizontal shear at each storey is distributed between the columns of that storey in such a way that each interior column carries twice the shear carried by each exterior column.

The above assumptions convert the indeterminate multi-storey frame to a determinate structure. The steps involved in the analysis of the frame are detailed below:

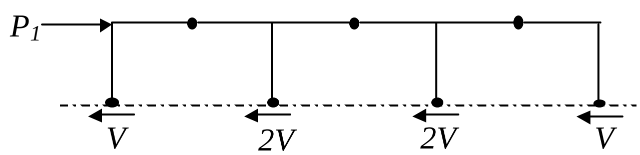
1. The horizontal shears on each level are distributed between the columns of that floor according to assumption (3).
2. The moment in each column is equal to the column shear multiplied by half the column height according to assumption (2).
3. The girder moments are determined by applying moment equilibrium equation to the joints: by noting that the sum of the girder moments at any joint equals the sum of the

column moments at that joint. These calculations are easily made by starting at the upper left joint and working joint by joint across to the right end.

4. The shear in each girder is equal to its moment divided by half the girder length. This is according to assumption (1).
5. Finally, the column axial forces are determined by summing up the beam shears and other axial forces at each joint. These calculations again are easily made by working from left to right and from the top floor down.



(a) A multi-storey frame showing points of contraflexure



(a) Top storey of the frame shown in (a) showing horizontal shear distribution in the columns of top storey

Fig. 3 A multi-storey frame subjected to wind loading

Assumptions of the Portal method of analysis are diagrammatically shown in Fig. 3 and method of analysis is illustrated in worked example - *1*

3.2 The Cantilever Method

This method gives good results for high-narrow buildings compared to those from the Portal method and it may be used satisfactorily for buildings of *25* to *35* storeys tall. It is not as popular as the portal method.

The simplifying assumptions made in the cantilever method are:

1. A point of contraflexure occurs at the centre of each beam
2. A point of contraflexure occurs at the centre of each column.
3. The axial force in each column of a storey is proportional to the horizontal distance of the column from the centre of gravity of all the columns of the storey under consideration.

The steps involved in the application of this method are:

1. The centre of gravity of columns is located by taking moment of areas of all the columns and dividing by sum of the areas of columns.
2. A lateral force P acting at the top storey of building frame is shown in Fig. 4(*a*). The axial forces in the columns are represented by F_1, F_2, F_3 and F_4 and the columns are at a distance of x_1, x_2 , x_3 and x_4 from the centroidal axis respectively as shown in Fig. 4(*b*).

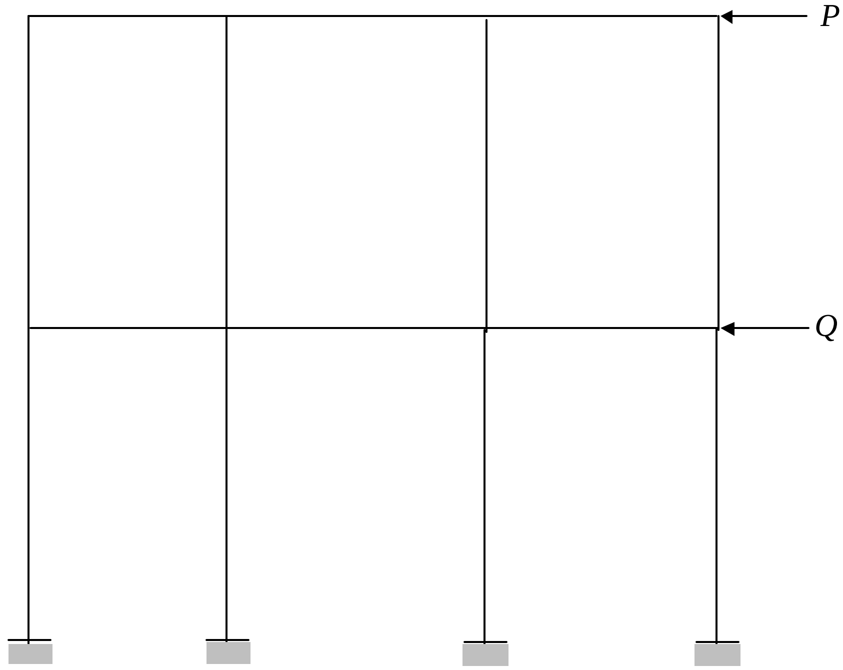


Fig. 4(a) Typical frame

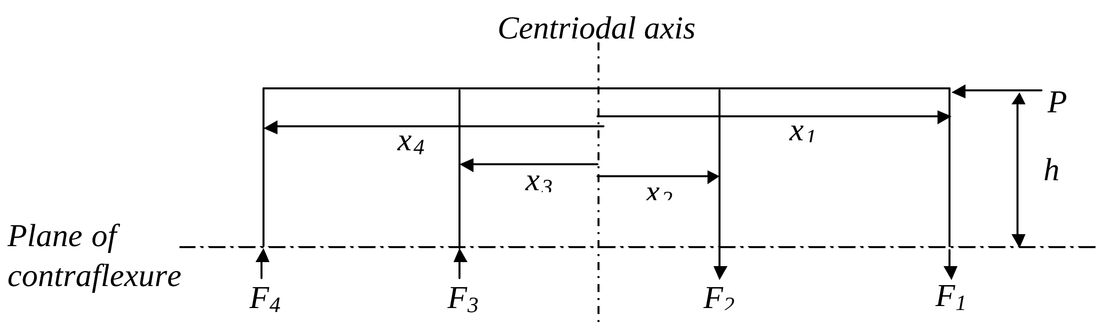


Fig. 4(b) Top storey of the above frame above plane of contraflexure

By taking the moments about the centre of gravity of columns of the storey,

$$P\,h - F_1x_1 - F_2x_2 - F_3x_3 - F_4x_4 = 0$$

The axial force in one column may be assumed as F and the axial forces of remaining columns can be expressed in terms of F using assumption (3).

3. The beam shears are determined joint by joint from the column axial forces.
4. The beam moments are determined by multiplying the shear in the beam by half the span of beam according to assumption (1).
5. The column moments are found joint by joint from the beam moments.
6. The column shears are obtained by dividing the column moments by the half-column heights using assumption (2)

The cantilever method analysis is illustrated in worked example - *2*.

3.3 The Factor Method

The factor method is more accurate than either the portal method or the cantilever method. The portal method and cantilever method depend on assumed location of hinges and column shears whereas the factor method is based on assumptions regarding the elastic action of the structure. For the application of Factor method, the relative stiffness ($k = I/\ell$), for each beam and column should be known or assumed, where, I is the moment of inertia of cross section and ℓ is the length of the member.

The application of the factor method involves the following steps:

1. The girder factor g, is determined for each joint from the following expression.

$$g = \frac{\sum k_c}{\sum k}$$

where, $S\,k_c$ - Sum of relative stiffnesses of the column members meeting at that joint.

$S\,k$ - Sum of relative stiffnesses of all the members meeting at that joint.

Each value of girder factor is written at the near end of the girder meeting at the joint.

2. The column factor c, is found for each joint from the following expression

$$c = 1 - g$$

Each value of column factor c is written at the near end of each column meeting at the joint. The column factor for the column fixed at the base is one.

At each end of every member, there will be factors from step 1 or step 2. To these factors, half the values of those at the other end of the same member are added.

3. The sum obtained as per step 2 is multiplied by the relative stiffness of the respective members. This product is termed as column moment factor C, for the columns and the girder moment factor G, for girders.

4. *Calculation of column end moments for a typical member ij* - The column moment factors [C values] give approximate relative values of column end moments. The sum of column end moments is equal to horizontal shear of the storey multiplied by storey height. Column end moments are evaluated by using the following equation,

$M_{ij} = C_{ij} A$

where, M_{ij} - moment at end i of the ij column
C_{ij} - column moment factor at end i of column ij
A - storey constant given by

$$A = \left(\frac{Total\ horizontal\ shear\ of\ storey \times Height\ of\ the\ storey}{Sum\ of\ the\ column\ end\ moment\ factors\ of\ the\ storey} \right)$$

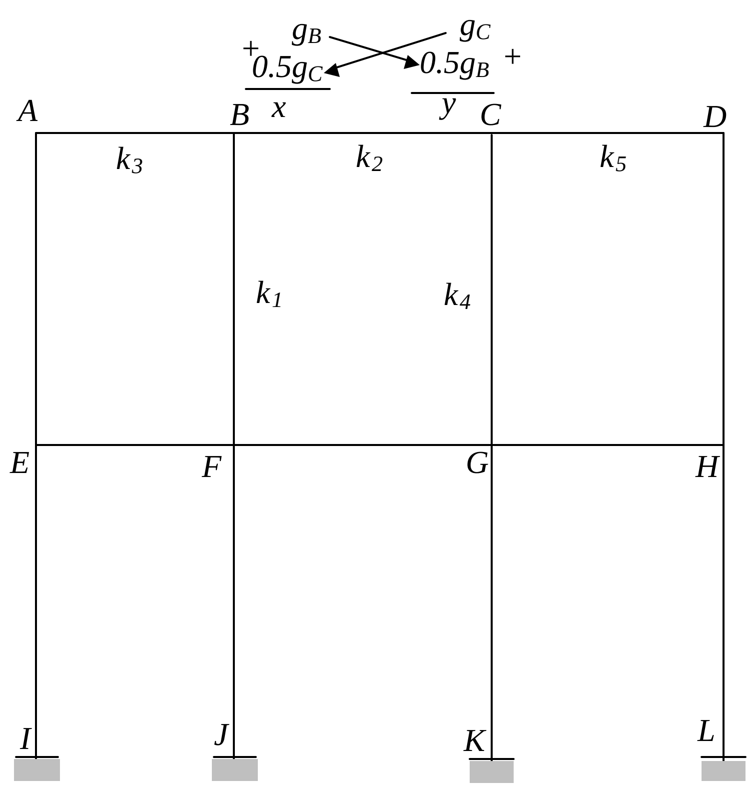


Fig. 5 Typical frame

5. *Calculation of beam end moments* - The girder moment factors [G values] give the approximate relative beam end moments. The sum of beam end moments at a joint is equal to the sum of column end moments at that joint. Beam end moments can be worked out by using following equation,

$M_{ij} = G_{ij} B$

where, M_{ij} - moment at end i of the ij beam

G_{ij} - girder moment factor at end i of beam ij
B - joint constant given by

$$B = \left(\frac{Sum\ of\ column\ moments\ at\ the\ joint}{Sum\ of\ the\ girder\ end\ moment\ factors\ of\ that\ joint} \right)$$

Illustration of calculation of G values:

Consider the joints B and C in the frame shown in Fig. 5.

Joint B: $g_B = k_1 /(k_1 + k_2 + k_3)$

$c_B = 1 - g_B$

Joint C: $g_C = k_4 /(k_2 + k_4 + k_5)$

$c_C = 1 - g_C$

As shown in Fig. 5, we should obtain values like x and y at each end of the beam and column. Thereafter we multiply them with respective k values to get the column or girder moment factors. Here, $G_{BC} = x\ k_2$ and $G_{CB} = y\ k_2$. Similarly we calculate all other moment factors. The detailed factor method of analysis is illustrated in the worked example -*3*.

4.0 ANALYSIS OF BUILDINGS FOR GRAVITY LOADS

As discussed in previous chapter, building frames may be of three types, namely, simple framing, semi-rigid framing and rigid framing. Generally, the beams and girders of upper floors may very well be designed on the basis of simple beam moments, while those of lower floors may be designed as continuous members with moment resisting connections.

4.1 Simple Framing

If a simple framing is used, the design of beams is quite simple because they can be considered as simply supported. In such cases, shears and moments can be determined by statics. The gravity loads applied to the columns are relatively easy to estimate, but the column moments may be a little more difficult to find out. The column moments occur due to uneven distribution and unequal magnitude of live load. If the beam reactions are equal on each side of interior column, then there will be no column moment. If the reactions are unequal, the moment produced in the column will be equal to the difference between reactions multiplied by eccentricity of the beam reaction with respect to column centre line.

4.2 Semi Rigid Framing

The analysis of semi-rigid building frames is complex. The semi-rigid frames are designed by using special techniques developed based on experimental evidence. This will be discussed in a later chapter.

4.3 Rigid Framing

Rigid frame buildings are analysed by one of the approximate methods to make an estimate of member sizes before going to exact methods such as slope-deflection or moment-distribution method. If the ends of each girder are assumed to be completely fixed, the bending moments due to uniform loads are as shown in full lines of Fig. 6(*a*). If the ends of beam are connected by simple connection, then the moment diagram for uniformly distributed load is shown in Fig. 6(*b*). In reality, a moment somewhere between the two extremes will occur which is represented by dotted line in Fig. 6(*a*). A reasonable procedure is to assume fixed end moment in the range of $w\ell^2/10$, where ℓ is clear span and w is magnitude of uniformly distributed load.

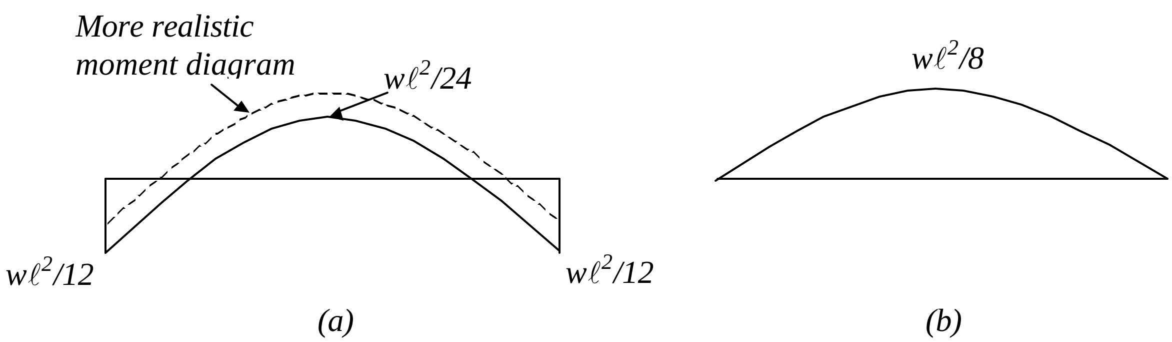


Fig. 6 (a) Fixed beam (b) Simply - supported beam bending moment diagrams

4.3.1 Analysis of structural frames for gravity loads - (according to IS: 456 - 1978)

The following assumptions are made for arrangement of live load for the analysis of frames:

a) Consideration is limited to combination of:
   1. Design dead load on all spans with full design live load on two adjacent spans and
   2. Design dead load on all spans with full design live load on alternate spans.

b) When design live load does not exceed three-fourths of the design dead load, the load arrangement of design dead load and design live load on all the spans can be used.

Unless more exact estimates are made, for beams of uniform cross-section which support substantially uniformly distributed loads over three or more spans which do not differ by more than *15%* of the longest, the bending moments and shear forces used for design is obtained using the coefficients given in Table 1 and Table 2 respectively. For moments at supports where two unequal spans meet or in cases where the spans are not equally loaded, the average of the two values for the hogging moment at the support may be used for design.

Where coefficients given in Table 1 are used for calculation of bending moments, redistribution of moments is not permitted.

Table 1: Bending moment coefficients

TYPE OF LOAD	**SPAN MOMENTS**		**SUPPORT MOMENTS**	
	Near middle span	At middle of interior span	At support next to the end support	At other interior supports
Dead load + Imposed load (fixed)	+ 1/12	+1/24	- 1/10	- 1/12
Imposed load (not fixed)	+1/10	+1/12	- 1/9	- 1/9
For obtaining the bending moment, the coefficient is multiplied by the total design load and effective span.				

Table 2: Shear force coefficients

TYPE OF LOAD	**At end support**	**At support next to the end support**		**At all other interior supports**
		Outer side	Inner side	
Dead load + Imposed load(fixed)	0.40	0.60	0.55	0.50
Imposed load(not fixed)	0.45	0.60	0.60	0.60
For obtaining the shear force, the coefficient is multiplied by the total design load				

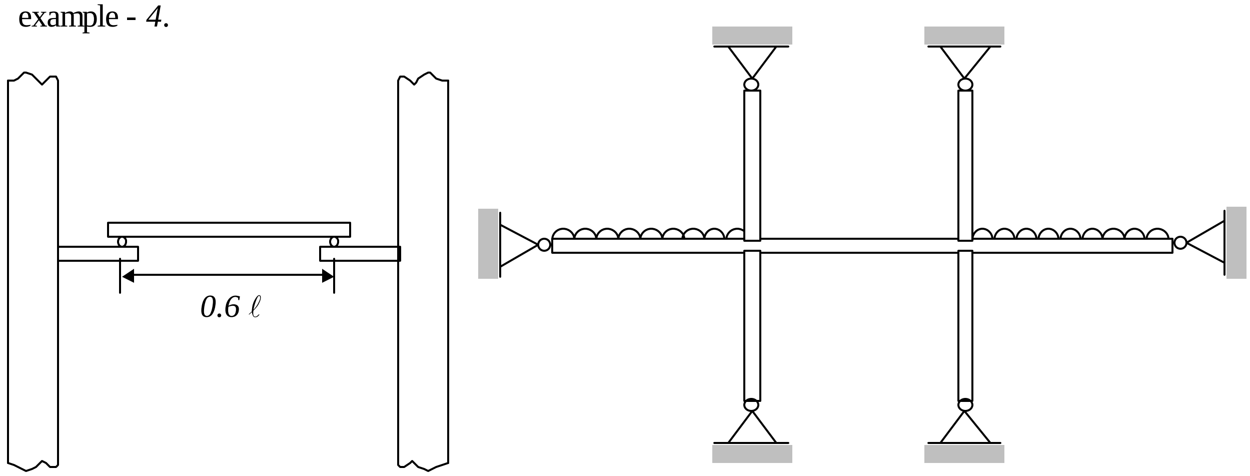
4.3.2 Substitute frame method

Rigid frame high-rise buildings are highly redundant structures. The analysis of such frames by conventional methods such as moment distribution method or Kane's method is very lengthy and time consuming. Thus, approximate methods (such as two cycled moment distribution method) are adopted for the analysis of rigid frames under gravity loading, one of such methods is Substitute Frame Method.

Substitute frame method is a short version of moment distribution method. Only two cycles are carried out in the analysis and also only a part of frame is considered for analysing the moments and shears in the beams and columns. The assumptions for this method are given below:

1) Moments transferred from one floor to another floor are small. Hence, the moments for each floor are separately calculated.
2) Each floor will be taken as connected to columns above and below with their far ends fixed.

If the columns are very stiff, no rotation will occur at both ends of a beam and the point of contraflexure will be at about $0.2\ \ell$. The actual beam can be replaced by a simply – supported beam of span $0.6\ \ell$ as shown in Fig. 7(*a*). If, the columns are flexible, then all the beams can be considered as simply supported of span ℓ as the beam – column joint will rotate like a hinge, an approximate model for middle floor beam is shown in Fig. 7(*b*). An approximate method of analysis for gravity loads is illustrated in worked example - *4*.



(a) Stiff column frame

(b) Flexible column frames

Fig. 7 Substitute approximate models for analysis of frames

4.4 Drift in Rigid Frames

The lateral displacement of rigid frames subjected to horizontal loads is due to the following three modes:

- Girder Flexure
- Column Flexure
- Axial deformation of columns

The sum of the storey drifts from the base upward gives the drift at any level and the storey drifts can be calculated from summing up the contributions of all the three modes discussed earlier in that particular storey. If the total drift or storey drift exceeds the limiting value then member sizes should be increased to avoid excessive drift.

5.0 COMPUTER ANALYSIS OF RIGID FRAMES

Although the approximate methods described earlier have served structural engineers well for decades, they have now been superseded by computer analysis packages. Computer analysis is more accurate, and better able to analyse complex structures. A typical model of the rigid frame consists of an assembly of beam-type elements to represent both the beams and columns of the frame. The columns are assigned their principal inertia and sectional areas. The beams are assigned with their horizontal axis inertia and their sectional areas are also assigned to make them effectively rigid. Torsional stiffnesses and shear deformations of the columns and beams are neglected.

Some analysis programs include the option of considering the slab to be rigid in plane, and some have the option of including P-Delta effects. If a rigid slab option is not available, the effect can be simulated by interconnecting all vertical elements by a horizontal frame at each floor, adding fictitious beams where necessary, assuming the beams to be effectively rigid axially and in flexure in the horizontal plane.

6.0 SUMMARY

In this chapter short cut methods for approximate analyses of simple braced frame as well as for frames with moment resisting joints are described and illustrative worked examples appended. Simplified analyses of building frames with gravity loads as well as frames with lateral loads have been discussed. More accurate methods making use of flexibility or stiffness matrices are generally incorporated in sophisticated software in many design offices.

7.0 REFERENCES

1. McCormac. C. J., (1994): "Structural Steel Design", Harper Collins College Publishers.
2. Owens. G. W. and Knowles. P., (1994) "Steel Designers Manual", The Steel Construction Institute, ELBS Blackwell Scientific Publishers, London.
3. Taranath. S. B., (1984): "Structural analysis and design of tall buildings", McGraw-Hill Book Company
4. Schuller. W., (1976): "High-rise building structures", John Wiley & Sons
5. Smith. B. S., and Coull. A., (1991): "Tall building structures: Analysis and Design", John Wiley & Sons.

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Multi-Storey Buildings – III

1.0 Introduction

In the earlier two chapters, the analysis and design of the multi-storey building frames were illustrated based on the assumptions that all members meeting at a particular joint of the structure undergo the same amount of rotation and hence the name *'rigid framed structures'*. In other words, the joints are assumed to be "rigid", and there is no relative rotation of one member with respect to the other. In fact, this has been the main underlying assumption in most of our frame analysis. At the other extreme, we assume the joints to be hinged in the case of truss structures. Thus, at the supports of steel structures, it is assumed that either ideally fixed or ideally pinned conditions exist. In reality, many "rigid" connections in steel structures permit a certain amount of rotation to take place within the connections, and most "pinned" connections offer a small amount of restraint against rotation. Thus, if a more accurate analysis of such structures is desired, it is necessary to consider the connections as being flexible or semi – rigid.

2.0 Connection flexibility in steel frames

To illustrate the connection flexibility in steel frames, let us consider the two-storey steel frame structure shown in Fig.1a. The beam *BC* is connected to the supporting columns by connections which may be carried out in several ways.

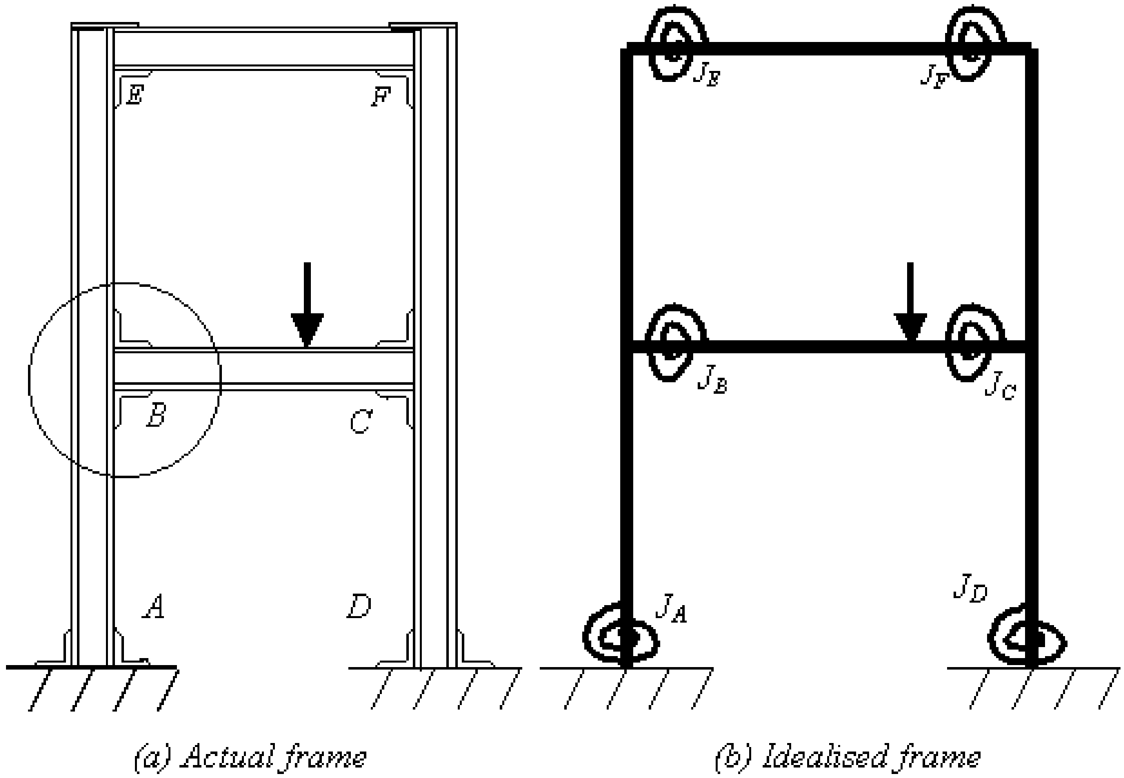
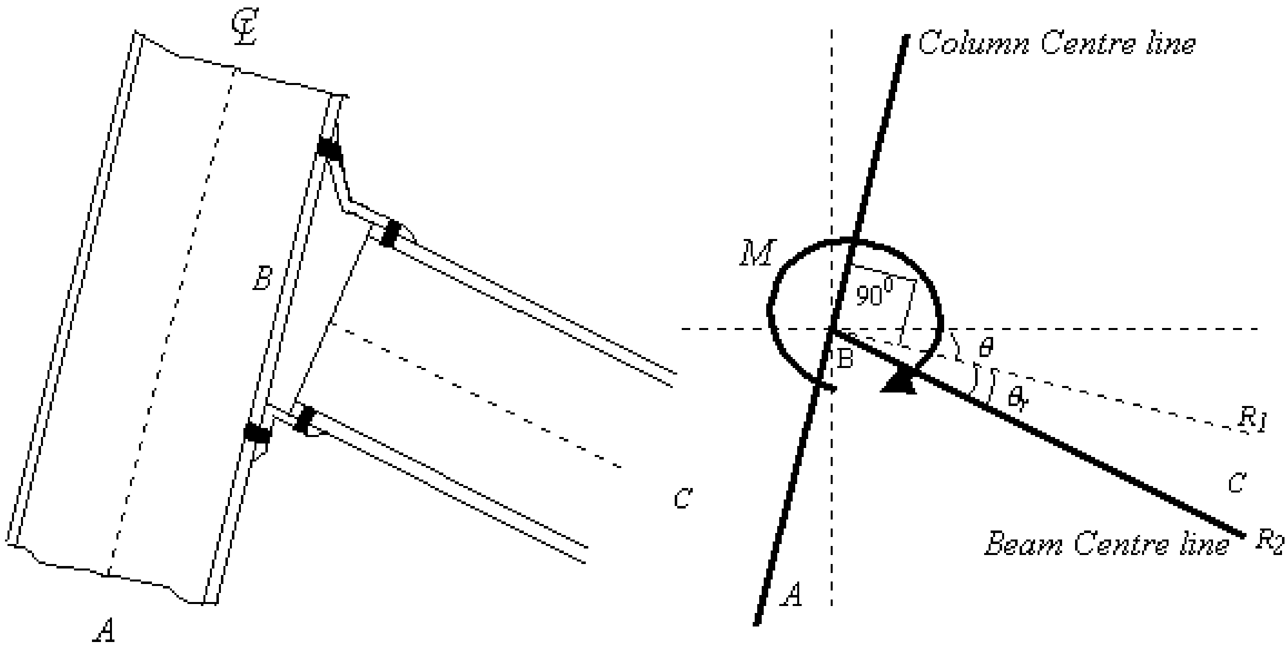


Fig. 1 Steel frame connections and their modelling

For example, the ends of the beam may be welded directly to the column flanges, or by using angles attached to the top and bottom of the beam, or framing angles may be used on the web of the beam. Regardless of the manner of connection, there will be a certain amount of flexibility in connections due to the deformations of the connection components and the flanges of the column. For this illustration let us assume that the beam column joint at *B* in Fig.1(a) is made up of using '*top angle and seat angle*' connection. To understand the connection flexiblity, let us focus our attention at the deformation of joint *B*, due to application of load. The deformation of the joint *B*, to an exaggerated scale, is shown in Fig.2(a). From this figure it is inferred that as the moment to be transferred increases, the connection angles are likely to deform.



(a) Deformed shape of joint B
(Exaggerated scale)

(b) Centre line diagram of joint B

Fig.2 Connection flexibility of beam-column joint

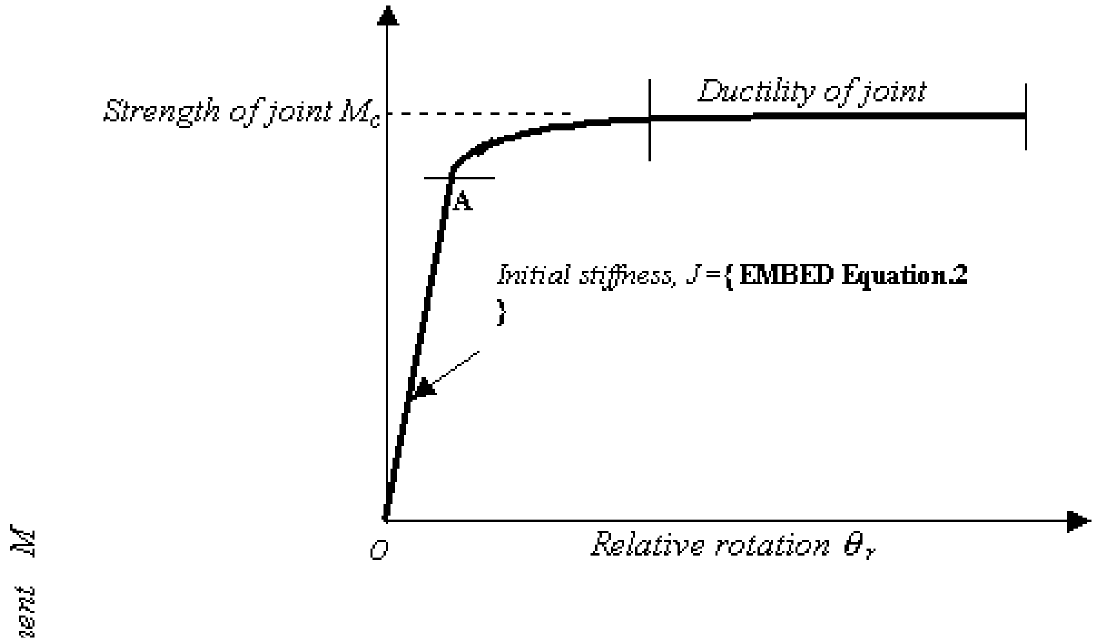
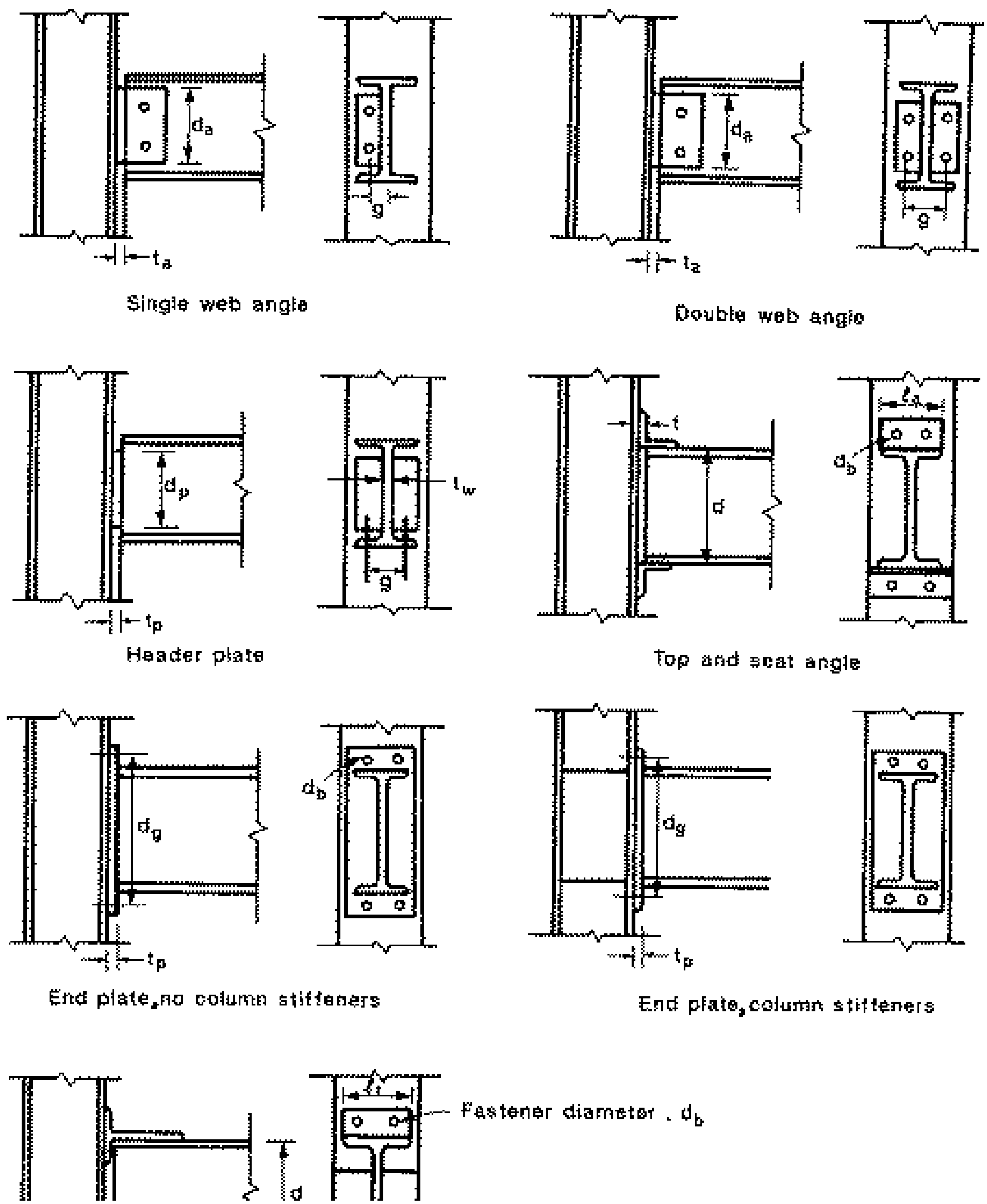


Fig.3 Joint characteristics (M-θ relationship)

Due to this connection deformation, the beam *BC* will rotate through a larger clockwise angle than the column *BA*. From Fig.2(b), we infer that if the beam column joint were to be perfectly rigid, the beam *BC* would have rotated along the line BR_1 which is orthogonal to the deformed column centre line. Instead, the beam has rotated to the position along BR_2 . This means that the beam has rotated an extra angle q_r relatively to the column, called the '*relative angle of rotation*'. It is obvious that the rotation component q_r is due to the connection flexibility. Hence if one wants to consider connection flexibility in the analysis, the relation between the applied moment '*M*' at the joint and the relative rotation q_r becomes very important. Fig.3 shows a typical *M* vs q_r relation observed in flexible connections. Initially the connection behaves nearly elastically and the curve *OA* is nearly a straightline with a slope $J=M/q_r$, which represents the rotational spring constant of the connection and is called the *joint modulus*. On further loading, the joint begins to deform inelastically and the angle of rotation increases rapidly. The connection stiffness decreases as the load increases and it is characterised by the M-q_r curve becoming flatter and flatter as it asymptotically approaches the plastic moment capacity M_c of the connection. Due to inherent ductility in the connection components, usually there would be considerable amount of ductility in the joints. However at normal working loads, the behavior of the connections of most structures can be approximated by a straightline such as *OA*. For future discussions of this chapter we would assume that connections behave linearly and their stiffnesses could be represented by their joint modulus '*J*'. In such a case, we can idealise the steel frame in Fig.1(a) as composed of members with an elastic rotational springs located at connections joining beam and column. Such an idealised frame is shown in Fig.1(b). For clarity in drawing the sketches, the springs are located at a small distance from the corresponding joints of the structure. For example, the hinge and rotational spring representing the connection at joint *B* are located at a small distance from the theoritical intersection of members *BA* and *BC* in Fig.1(b). In calculations it will be assumed that this distance is equal to zero, although the hinge and spring are still considered to be a part of beam *BC*.

3.0 Moment – rotation characteristics of structural steel connections

The various types of structural steel connections that are commonly used in practice are shown in Fig.4. Depending on the flexibility (or inversely the stiffness of the connections) the various type connections can be classified into flexible or stiff connections. The schematic classification of these connections has been presented in Fig. 5(a). For ease of design these connections are better classified as rigid (in which the rigid elastic assumption is valid), semi-rigid (in which connection flexibility is to be taken into account) and pinned connections (in which no moment is assumed to be transferred across the joint). As evident from the complexity of connections shown in Fig.4, it is almost impossible to develop analytical expressions for calculating the stiffness of these connections. Hence, connection characteristics are mostly determined using experiments. Based on these experiments, analytical expressions are prescribed for design in the form of empirical equations. To get the empirical equations, numerous results of investigations of semi-rigid connections are put into a data bank of M- q_r curves. Then curve-fitting methods are used on the experimental data to develop appropriate M-q_r curves for design. There are several curve-fitting techniques used.



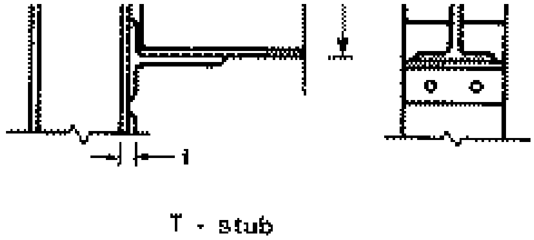


Fig.4 Some typical structural steel connections
(Chen W.F and Lui E.M., "Stability Design of Steel Frames", CRC Press, 1991

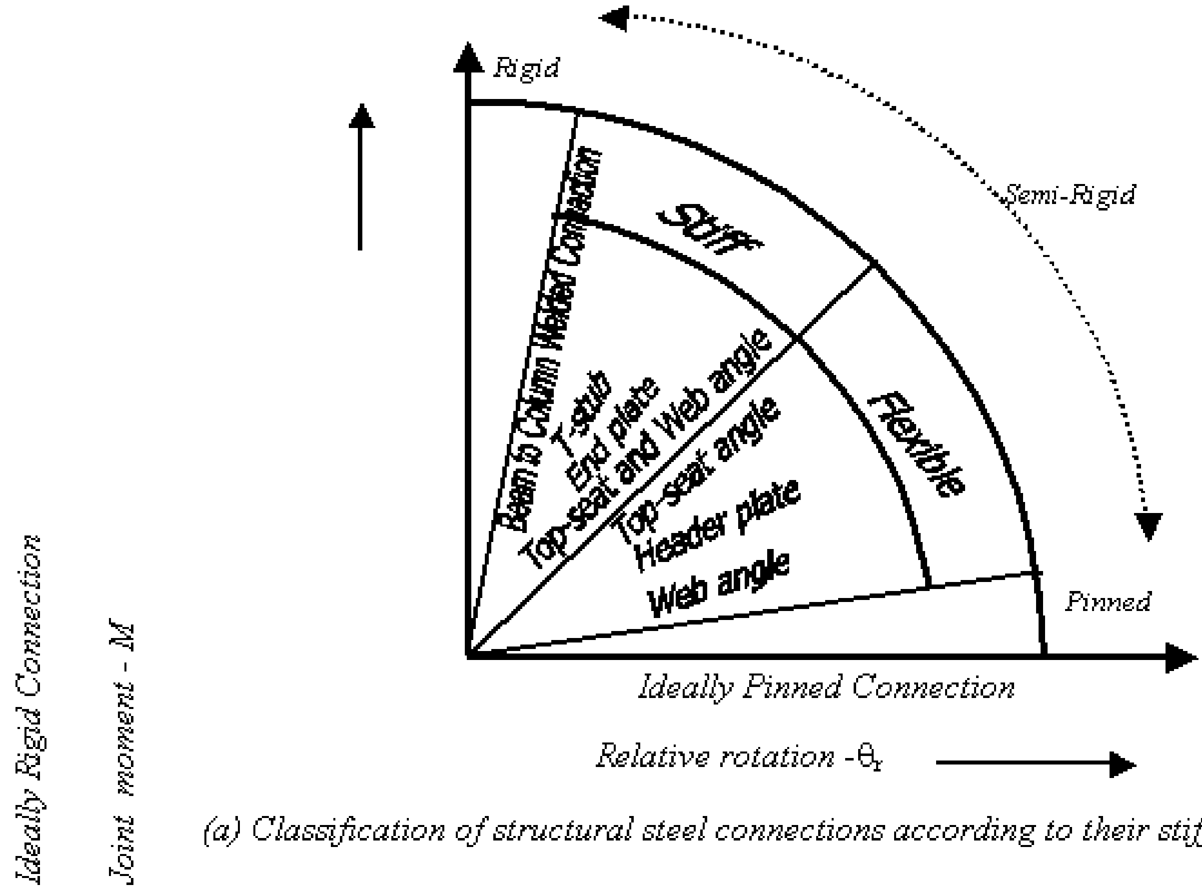
They can be broadly classified as:

· B-spline models

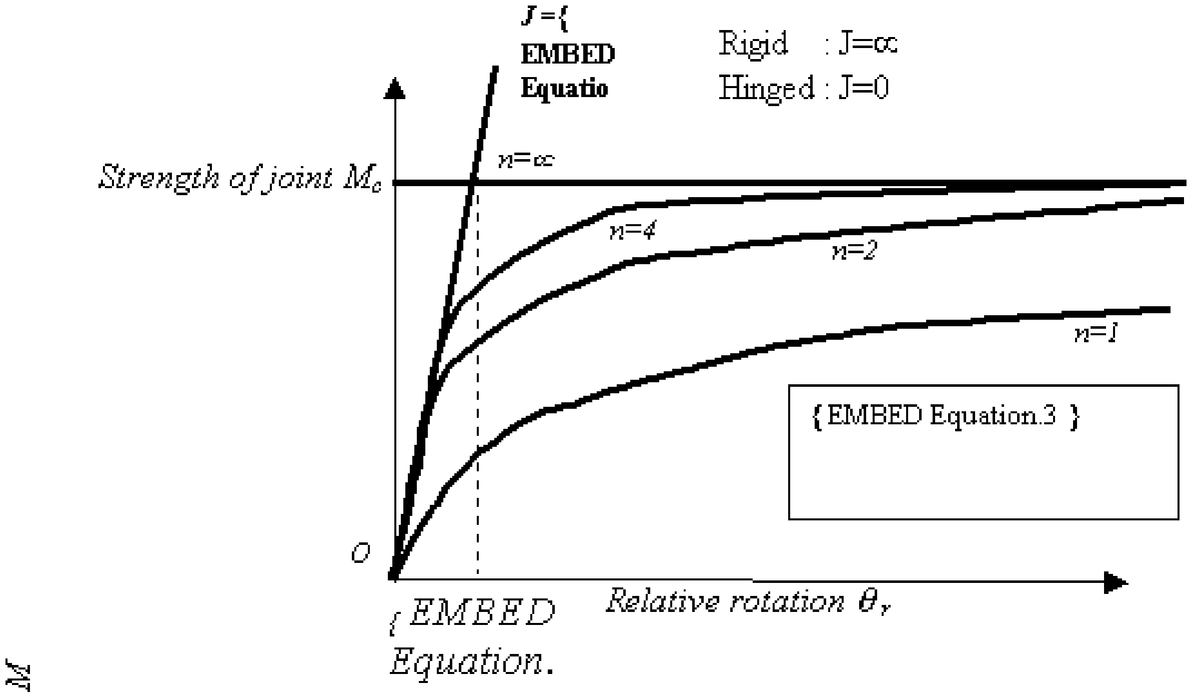
· Polynomial models

· Exponential models and

· Power models.



(a) Classification of structural steel connections according to their stiffness



(b) Three parameter model for M- θ_r relationship

Fig.5 Connection Stiffness and their representation

One such popular model is the Kishi and Chen (1990) three-parameter power model as shown in Fig.5 (b). The experimental data are fitted into a curve using three parameters such as *J*, M_C and *n*. By suitably adjusting the value of M_c and '*n*', a family of M-q_r curves could be generated. However, for the subsequent part of our discussion we are interested in the linear connection behaviour, and hence only the connection modulus '*J*' alone is of interest to us. It is to be noted that in the case of nonlinear analysis, all the parameters are needed.

4.0 Derivation of basic equationS for the analysis of semi-rigid steel frames using moment distribution method

In this section, we would see as to how semi-rigid or flexibly connected steel frames could be analysed using the popular "Moment Distribution Method (MDM)".Since MDM has been well documented in engineering text books, the fundamentals of MDM would not be repeated here. The following discussions are based on the assumption that the reader has prior knowledge of MDM.

As we have seen earlier, semi-rigid steel frames could be idealised as bare steel frames with connections modelled as flexural springs as in Fig.1(b). Hence, it is apparent that to model the connection flexibility using MDM, the first step in the analysis is the determination of moment distribution factors for a beam (which are based on connection stiffnesses) with a spring at one end or springs at both ends. Firstly we would see individual members having flexible connections at one or both ends and later we would consider the entire steel frame to be composed of these individual members. When there is a flexible connection at each end of the beam, the stiffness and carry-over factors can be derived from a consideration of the beam shown in Fig.6.

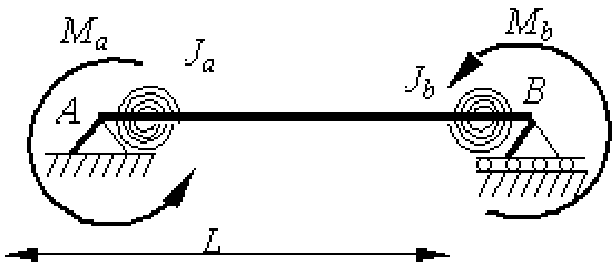


Fig.6 Beam with connection springs at both ends

The beam is simply supported at the ends and the joint modulii are J_a and J_b at ends *A* and *B*, respectively. Under the action of moments M_a and M_b acting at the ends, the ends of the beam will rotate through small angles θ_a and θ_b, which are assumed positive in the same directions as the positive end moments as shown in Fig 6. The angles of rotation at the ends of the portion of the beam between the springs could be written as

$$\frac{M_a L}{3EI} - \frac{M_b L}{6EI} \text{ and } -\frac{M_a L}{6EI} + \frac{M_b L}{3EI} \qquad (1)$$

Due to rotations of the elastic springs representing the connections, the ends of the beam rotate through additional angles equal to

$$\frac{M_a}{J_a} \text{ and } \frac{M_b}{J_b} \qquad (2)$$

Hence the total angles of rotation of the ends of the beam in Fig.6 is given by

$$\theta_a = \frac{M_a L}{3EI} - \frac{M_b L}{6EI} + \frac{M_a}{J_a} \qquad \text{(a)}$$

$$\theta_b = -\frac{M_a L}{6EI} + \frac{M_b L}{3EI} + \frac{M_b}{J_b} \qquad \text{(b)} \qquad (3)$$

The above equations are fundamental in nature using which we can calculate the stiffness and carry-over factors of any member with flexible connection at it ends.

4.1 Members with far end fixed and flexible connections at both ends

If the far end of the beam *AB* is fixed (Fig.7), the angle of rotation at end *B* is zero. The carry-over factor from end *A* to the end *B* can be found by solving Eq.3.(b) for the ratio of M_b to M_a; we get

$$COF_{ab} = \frac{1}{2\left(1 + \frac{3EI}{LJ_b}\right)} \tag{4}$$

Introducing a factor called *'j'*

$$j = \frac{EI}{LJ} \tag{5}$$

where *'j'* is a dimensionless quantity called the *joint factor*. Hence Eq.4 becomes

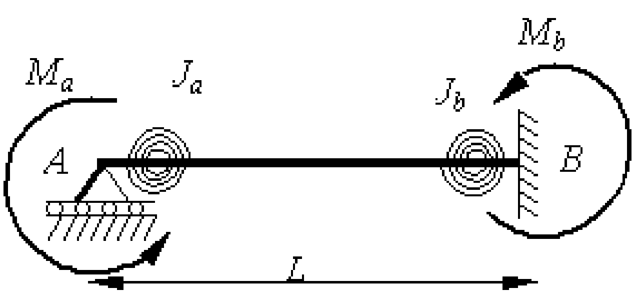


Fig.7 Beam with far end fixed

$$COF_{ab} = \frac{1}{2(1 + 3j_b)} \tag{6}$$

On the other hand, if the connection at the far end of the beam is rigid instead of flexible, it represents a rigid connection and it is equivalent to a joint with an infinitely large joint modulus *J*. The corresponding value of the joint factor '*j*' is zero (see Eq.5) and when this value is substituted into Eq.6 the carry-over factor becomes 0.5, a well known factor for the carry over moment in case far end is fixed.

4.2 Members with far end pinned and flexible connections at both ends

Similarly, if the connection at the far end *B* is completely flexible and offers no restraint against rotation, the value of *J* is zero, the joint factor *j* becomes infinite, and Eq.6 gives a carry-over factor of zero which corresponds to a beam with the far end pinned. In such a case, the rotational stiffness of the beam is obtained from Eqs.3 (a) by substituting for M_b its expression in terms of M_a and solving for the ratio M_a/θ_a which is the rotational stiffness. This manipulation gives the fundamental formula

$$K_{ab} = \frac{4EI}{L}\frac{1+3j_b}{1+4(j_a+3j_aj_b+j_b)} \qquad (7)$$

4.3 Members with extremes of joint stiffness

In special cases such as connections at both the ends are rigid *(i.e.* $j_a=j_b=0$*), Eq.7* reduces to the well known results of $K_{ab}=4EI/L$ for a beam with the far end fixed. When the connections at *A* is rigid ($j_a=0$) and the connection at *B* offers no restraint against rotation ($j_b=\infty$) the result is $K_{ab}=3EI/L$, again a standard stiffness value for a beam with far end pinned.

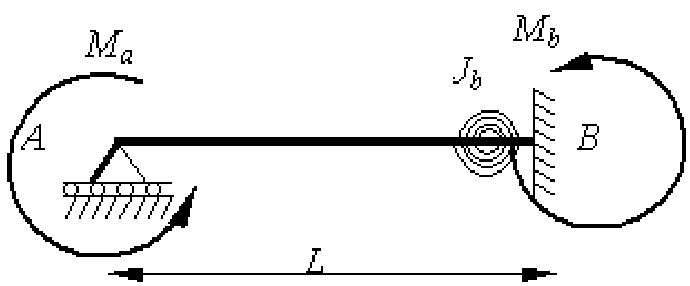


Fig.8 Beam with flexible connection only at the far end

When the connections at the ends of the beam are identical ($j_a=j_b=j$) the stiffness of the beam is

$$K_{ab} = \frac{4EI}{L}\frac{1+3j}{(1+2j)(1+6j)} \qquad (8)$$

4.4 Beam with flexible connection at one end and far end fixed

Sometimes it is quiet common to have a flexible connection only at one end of the member. The case, as shown in Fig.8, has a flexible connection at the far end only.

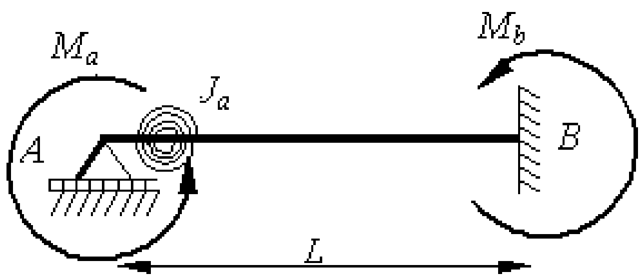


Fig.9 Beam with flexible connection at near end and far end fixed

This can be considered as a special case of the beam in which the spring at the near end A of the member has an infinitely large joint modulus ($j_a=\infty$). Thus, by making use of Eq.6 and Eq.7, the carry-over and stiffness factors can be written as

$$COF_{ab} = \frac{1}{2(1+3j_b)} \qquad (9)$$

$$K_{ab} = \frac{4EI}{L}\frac{1+3j_b}{1+4j_b} \qquad (10)$$

The case in which the flexible connection is located at the near end of the member and the far end is fixed (shown in Fig.9) can be obtained from Eq.6 and Eq.7 by substituting $j_b=0$.

Thus the carry-over and stiffness factors for this case become

$$COF_{ab} = 1/2 \qquad (11)$$

$$K_{ab} = \frac{4\ EI}{L}\frac{1}{1+4\ j_a} \qquad (12)$$

4.5 Beam with flexible connection at near end and far end pinned

When the far end of the beam is simply supported instead of fixed (Fig.10) the carry-over factor is zero and the stiffness factor is

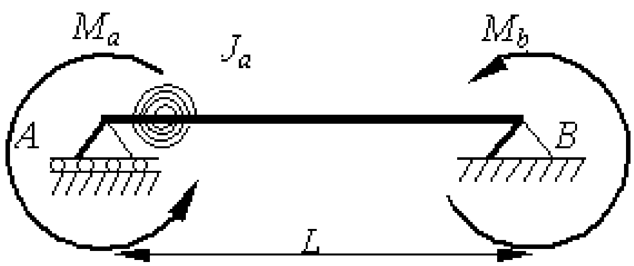


Fig.10 Beam with flexible connection at near end and far end pinned

$$K_{ab} = \frac{3EI}{L}\frac{1}{1+3j_a} \qquad 13)$$

4.6 Fixed end moments for beams with flexible connections

In the earlier sections we discussed the two important parameters for MDM, namely the stiffness and carry over factors. Another important parameter for the MDM is the 'fixed end moments'. A fixed – end beam carrying a uniform load of intensity 'w' is shown in Fig.11 It is assumed that the flexible connections at the ends of the beam are identical and have a joint modulus equal to *J*.

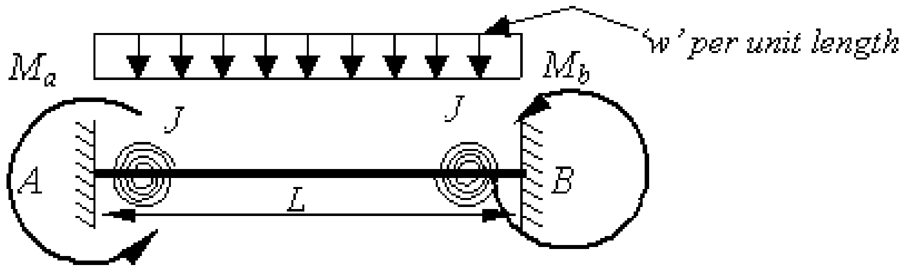


Fig.11 Beam with UDL and flexible connections at both ends

Hence the beam is symmetrical and the fixed – end moments are numerically equal but opposite in sign ($M_b = -M_a$). By suitable manipulations it could be shown that the fixed end moments are given by

$$M_a = -M_b = \frac{wL^2}{12}\frac{1}{1+2j} \qquad (14)$$

The above equations for fixed end moments are derived based on the assumption that the connection modulus $J_a = J_b = J$. But in actual practice this need not be the case. Hence considering any end of the beam to be flexible (End '*B*' is assumed to be flexible in Eq.15&16) we can show the fixed end moments to be

$$M_a = \frac{wL^2}{12}\frac{(1+6j_b)}{(1+4j_b)} \qquad (15)$$

$$M_b = \frac{wL^2}{12}\frac{-1}{1+4j_b} \qquad (16)$$

For a case where J_a ¹ J_b , the fixed end moments could be obtained by simple algebraic addition using eqn.15&16 by suitably substituting J_a and J_b values. The fixed end moment caused by a concentrated load P acting at a distance '*kl*' from the left end of the beam (Fig.12) can be shown to be

$$M_a = PLk(1-k)\frac{1+4j-k(1+2j)}{(1+2j)(1+6j)} \qquad (17)$$

$$M_b = -PLk(1-k)\frac{2j+k(1+2j)}{(1+2j)(1+6j)} \qquad (18)$$

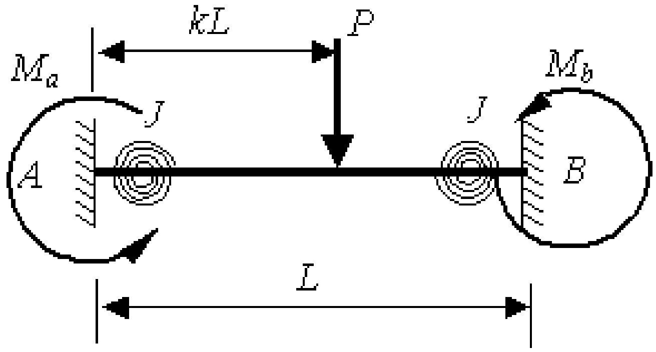


Fig.12 Beam with Concentrated load and flexible connections at both ends

When the connections are rigid *(j=0)* these expressions reduce to the usual formulas for fixed end moments. As before, considering the joint modulii at the ends of the member different, we get the fixed end moments for a flexibly connected beam under concentrated load as,

$$M_a = \frac{PkL(1-k)}{2(1+3j_a)}\left((2-k) - \frac{(2(1+k)(1+3j_a) - (2-k))}{3(1+4j_b)(1+3j_a)}\right) \qquad (19)$$

$$M_b = -\frac{PkL(1-k)}{3(1+4j_b)(1+3j_a)}(2(1+k)(1+3j_a) - (2-k)) \qquad (20)$$

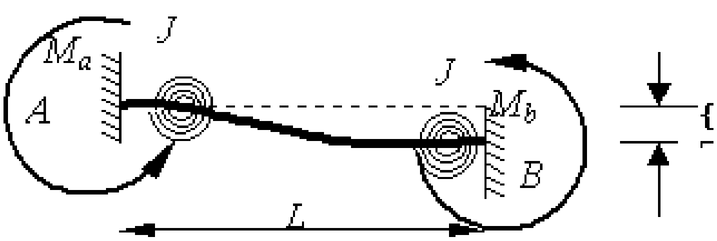


Fig.13 Member with sway deflection

4.7 Joint Translation in sway frames

In the case of sway frames, the member ends also experience lateral displacement. Fixed end moment formulae for beams in which one end is displaced laterally with respect to the other can be obtained without difficulty. For example, if both ends of the beam are fixed as in Fig.13, the fixed end moments could be shown to be

$$M_a = M_b = \frac{6EI\Delta}{L^2}\frac{1}{1+6j} \qquad (21)$$

which reduce to $M_a = M_b = \frac{6EI\Delta}{L^2}$ when the connections are rigid ($j=0$). If there is flexibility at only one end (Fig.14)of the beam, the stiffness and carry–over factors are not the same at each end of the beam; but must be obtained from separate expressions. The carry – over and the stiffness factors at end A are

$$COF_{ab} = \frac{1}{2} \qquad K_{ab} = \frac{4EI}{L}\frac{1}{1+4j} \qquad (22)$$

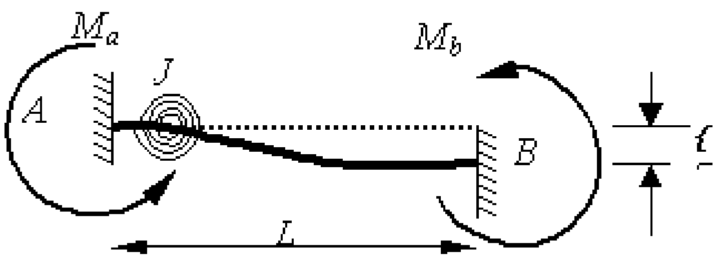


Fig.14 Member with sway deflection and flexible connection at one end

The corresponding quantities at end B are

$$COF_{ab} = \frac{1}{2(1+3j)} \qquad K_{ab} = \frac{4EI}{L}\frac{1+3j}{1+4j} \qquad (23)$$

Using the above four expressions the fixed – end moments could be calculated as

$$M_a = \frac{6EI\Delta}{L^2}\frac{1}{1+4j} \qquad M_b = \frac{6EI\Delta}{L^2}\frac{1+2j}{1+4j} \qquad (24)$$

For a case where $J_a \neq J_b$, the fixed end moments could be obtained by simple algebraic addition using eqn.24. As a final case, it is assumed that there is a flexible connection at one end of the beam and that the other end of the beam is simply supported (Fig.15). The moment M_a at the fixed end of the beam is equal to the moment which is required to rotate that end of the beam through an angle Δ / l. This moment is equal to the stiffness factor for the beam with the far end simply supported times Δ / l; therefore

$$M_a = \frac{3EI\Delta}{L^2}\frac{1}{1+3j_a} \qquad (25)$$

To summarise the above sections, it was demonstrated as to how the important parameters such as stiffness, carryover factor and fixed end moments could be derived from principles of mechanics. Once these basic expressions are available, the entire semi-rigid multi-storey steel frame could be considered as made up of these basic components.

5.0 Analysis of semi-rigid steel frames

Using the expressions presented above we would solve some problems to understand the analysis of frames with semi-rigid connections. Let us take an example of a continuous beam as shown in Fig.16.

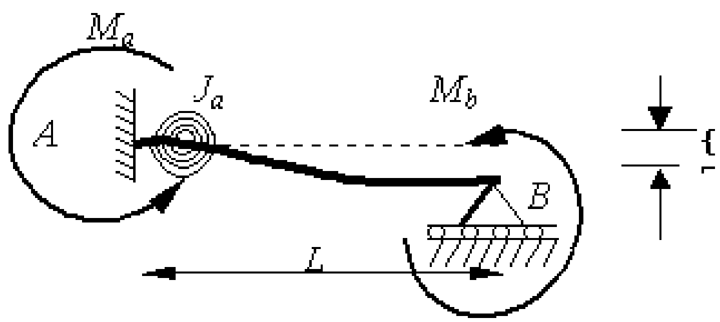


Fig.15 Member with sway deflection with far end pinned

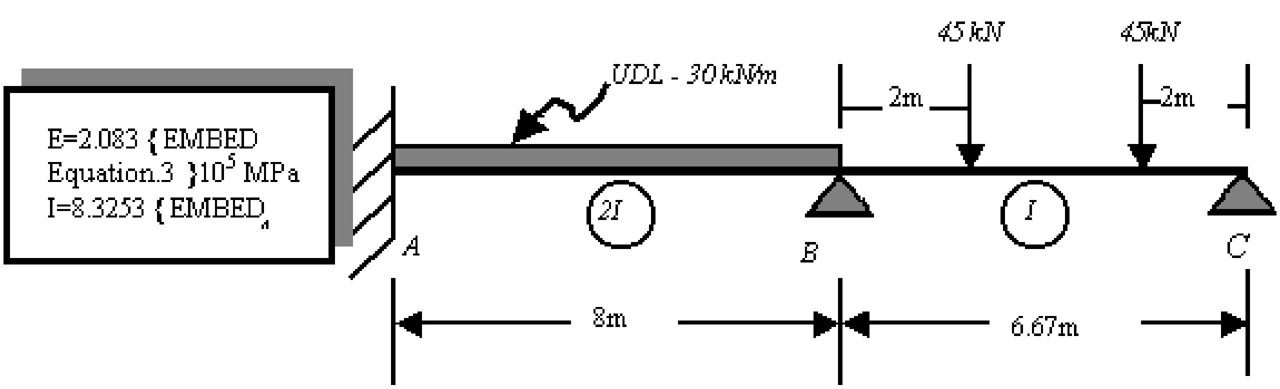


Fig.16 Continuous beam with rigid connections

Table 1

	End	*AB*	*BA*	*BC*	*CB*
	DF		0.625	0.375	
	COF		0.5	0.5	0.5
	FEM	+160.000	-160.000	+63.014	-63.014
Iteration 1	***Balance B***		+60.616	+36.370	
	Carry over	30.308			+18.185
	Balance C				+44.829
	Carry over			22.415	
Iteration 2	***Balance B***		-14.010	-8.406	
	Carry over	-7.005			-4.203
	Balance C				+4.203
	Carry over			+2.102	
Iteration 3	***Balance B***		-1.314	-0.788	
	Carry over	-0.657			-0.394
	Balance C				+0.394
	Final moments (app.)	**+182.646**	**-114.708**	**+114.708**	**0.000**

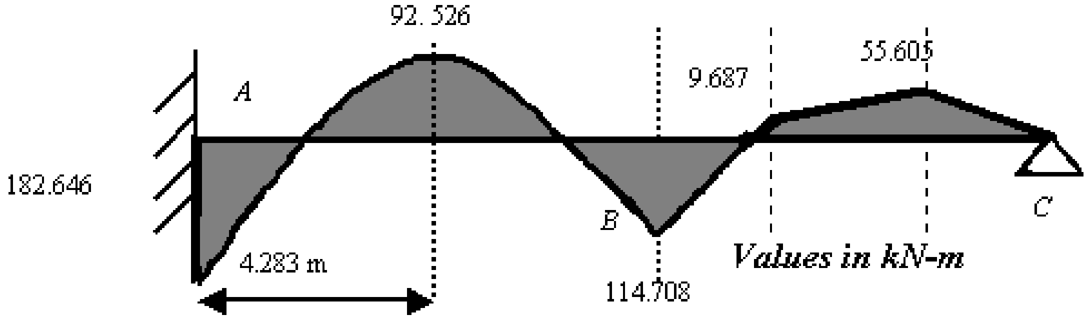


Fig.17 Bending Moment Diagram (rigid case)

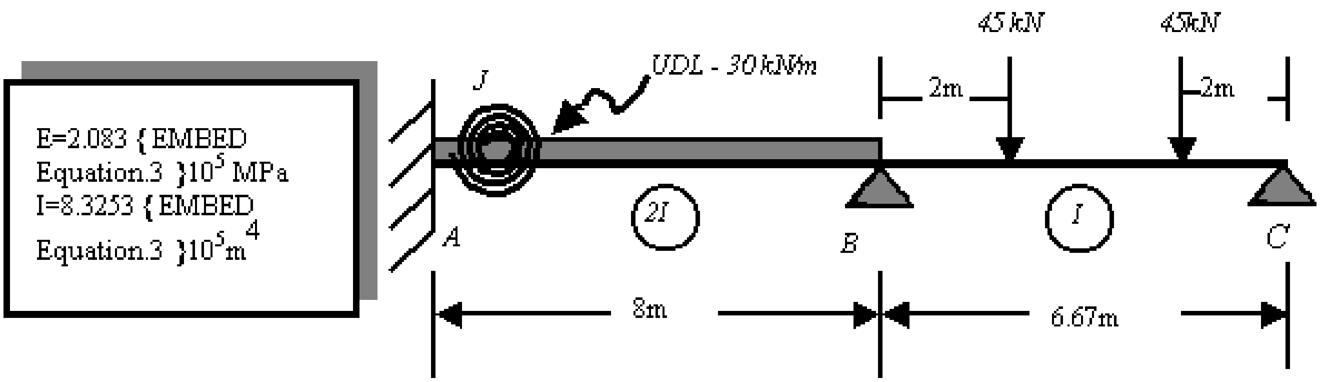


Fig.18 Continuous beam with flexible connection

At the first instance, let us assume that the support at A is rigid and accordingly we would work out the stiffness of joints, distribution factors and carry over factors. We obtain distribution factors $D_{BA} = 0.625$ and $D_{BC} = 0.375$ based on stiffnesses $K_{BA}, K_{BC.}$ Since the connections are assumed perfectly rigid, half the moment induced at *B* and *C* would be carried over to the adjuscent joint. Regarding the hinged node *C,* there are two ways to handle.

Table 2

	End	AB	BA	BC	CB
	DF		0.606	0.394	
	COF		0.377	0.5	0.5
	FEM	+111.421	-184.290	+63.014	-63.014
Iteration 1	***Balance B***		+73.493	+47.783	
	Carry over	+27.707			+23.892
	Balance C				+39.123
	Carry over			+19.562	
Iteration 2	***Balance B***		-11.855	-7.707	
	Carry over	-4.469			-3.854
	Balance C				+3.854
	Carry over			+1.927	
Iteration 3	***Balance B***		-1.168	-0.759	
	Carry over	-0.440			-0.380
	Balance C				+0.380
	Final moments (app.)	**+134.219**	**-123.820**	**+123.820**	**0.000**

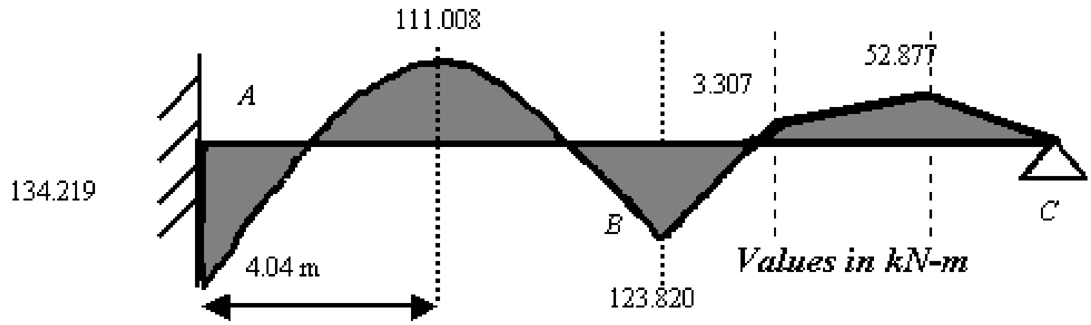


Fig.19 Bending Moment Diagram (flexible case)

Firstly we can get the stiffness K_{BC} considering the far end C is hinged and obtain $K_{BC}=3EI/L$ and fixed end moment M_{CB} is set to zero. Alternatively C could be considered as rigid, and subsequently we can balance C to zero and carry over the moments to B. The later method is adopted in the present example. The MDM is presented in Table 1 for the rigid case. To start with, all the nodes are assumed to be locked. First we unlock node B. An unbalanced moment of –96.986 kN-m appears which is balanced by distributing it at node B. Now because of the appearance of the new balancing moments half the moment is carried over to adjacent end. This is done in the carry over column as shown in Table 1. This introduces unbalancing moments at node *C*, which is then balanced and moments carried over. Now we have completed one cycle. Similarly we can repeat this exercise until two consequent change in moment at any node is within an acceptably small value. However in the present example only three iterations are shown in Table 1. We see from the results (Fig.17)that the ratio of negative support moment at *A* to the positive span moment in *AB* is 1.97. We shall consider the same example but assume that the connection at *A* is flexible and the connection stiffness *J*=40000 kN-m/rad. The problem is shown in Fig.18 and the procedure is presented in Table 2. From Table 2 we observe that the connection flexiblity affects several parameters. Firstly the stiffness of a particular joint gets reduced if the far end connection is flexible. We see that the stiffness K_{BA} is reduced and hence gets only 0.606 time the connection moment as against 0.625 in the rigid case. The Remainder of the moment is distributed to the other members connected to it. Similarly we also see from Table 2, that the moment carried over to the far end gets reduced because of the connection flexiblity. Another important observation is that the fixed end moment is reduced at the end where the connection flexiblity occurs leaving a increased share of the end moment to the rigid end. Hence we see(Fig.19) that fixed end moment M_{AB} is reduced to 134.219 kN-m from 186.646kN-m in the rigid case. At the same time M_{BA} increased from 114.708 kN-m to 123.820 kN-m. The final end moments are presented in Table 2.

We see that the , in the span *AB*, ratio of negative moment at *A* to the maximum positive span moment is brough down to 1.209. The design bending moment in the span *AB* has reduced by 36%. The reduced design moment is one of the main advantages of semi-rigid steel frames. In the chapter on "Welds- Static and Fatigue Strength –II', the effect on connection flexiblity on the moment redistribution is well explained. Since steel beams are equally good both in compression and tension, we see that there is a better utilisation of the material of the beam for carrying the load in flexure. If we see the chapter on 'Plastic Analysis', this is exactly what we are trying to achieve. In an ideal situation we could get a ratio of the negative bending moment to positive span moment as 1.0.

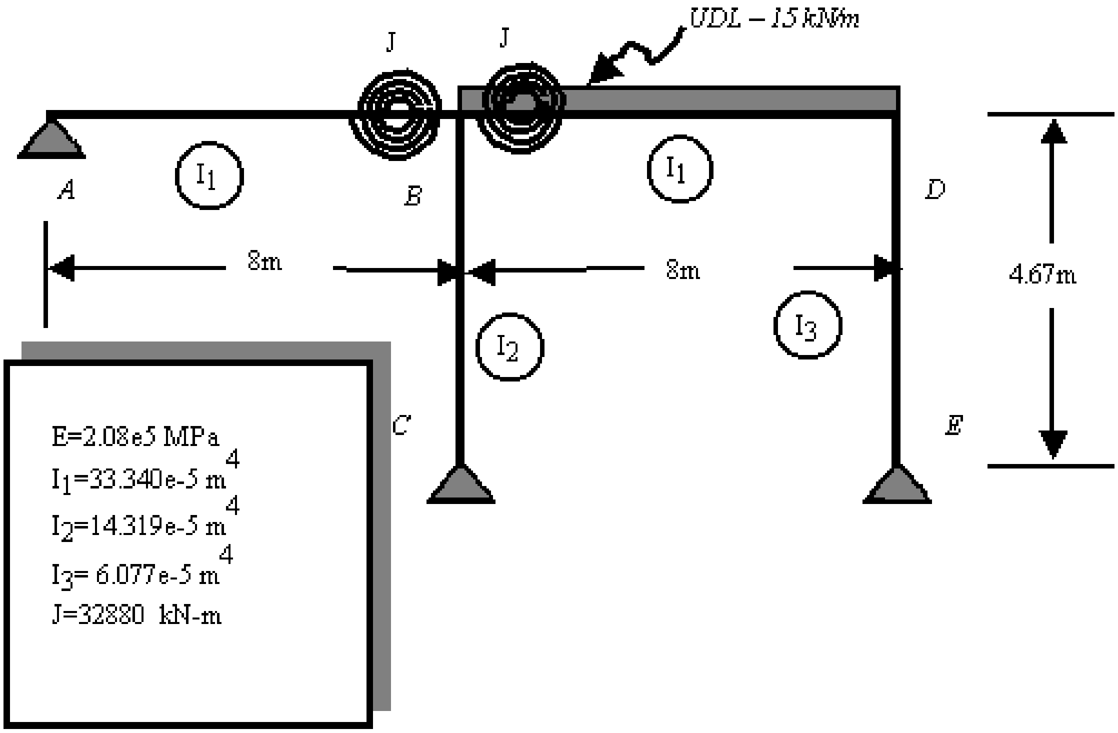


Fig.20 Single storey steel frame

Table 3

Rigid Connection								
End	AB	BA	BC	BD	CB	DB	DE	ED

DF		0.366	0.269	0.366		0.762	0.238	
COF	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
FEM	0.0	0.0	0.0	80.0	0.0	-80.0	0.0	0.0
Final end Moments	0.0	-40.16	-29.55	69.71	0.0	-20.21	20.21	0.0
Flexible connection								
End	AB	BA	BC	BD	CB	DB	DE	ED
DF		0.284	0.432	0.284		0.762	0.238	
COF	0.278	0.500	0.500	0.500	0.500	0.278	0.500	0.500
FEM	0.0	0.0	0.0	38.740	0.0	-100.6	0.0	0.0
Final end Moments	0.0	-18.11	-23.96	42.07	0.0	-23.55	23.55	0.0

Another example of a single storey frame is provided as an illustration as shown in Fig.20. The distribution and carryover factors for the rigid and semi-rigid case are presented in Table 3. One can observe the change in the fixed end moments in Table.3. The problem could be solved manually or by using the program presented in the Appendix. From the final end moments it is observed that the maximum moment has been brought down to 42.07 kN-m from 69.71 kN-m. We also observe that connection flexibility results in redistribution of moments and a better utilisation of the beam material.

The procedure explained above could be extended to any multi storey steel frame. However as more number of storeys and bays are considered, the hand computation of MDM becomes very laborious. Nevertheless, the MDM could be programmed as computer software. The ideal solution for semi-rigid analysis of steel frames is the Finite Element Method (FEM) as it provides greater flexibility in modelling. Since treatment of FEM is outside the scope of this chapter, it will suffice to know that FEM could be used very effectively for both linear and non-linear analysis of semi-rigid steel frames.

6.0 Semi-rigid design of frames

Many of the codes of practice allow the use of semi-rigid design methods for steel frames. IS:800(1984) also allows the semi-rigid design methods provided some rational analysis procedures are used. However the code does not elaborate any further. For example, BS:5950 Part –1 allows semi-rigid design stating that (Clause 2.12.4) " The moment and rotation capacity of the joint should be based on experimental evidence which may permit some limited plasticity provided that the ultimate tensile capacity of the fasteners is not the failure criterion". Euro Code (EC3) also allows the semi-rigid design methods and the main provisions could be summarised as follows:

· Moment –rotation behaviour shall be based on theory supported by experiments.

· The real behaviour may be represented by a rotational spring.

· The actual behaviour is generally nonlinear. However, an appropriate design curve may be derived from a more precise model by adapting linear approximations such that the whole curve lies below the accurate curve as shown in Fig.21.

· Three properties are defined in the M-q_r characteristics

· Maximum moment of resistance (M_C)

· Rotational stiffness (the secant stiffness $J=M/q_r$

· The rotation capacity q_c

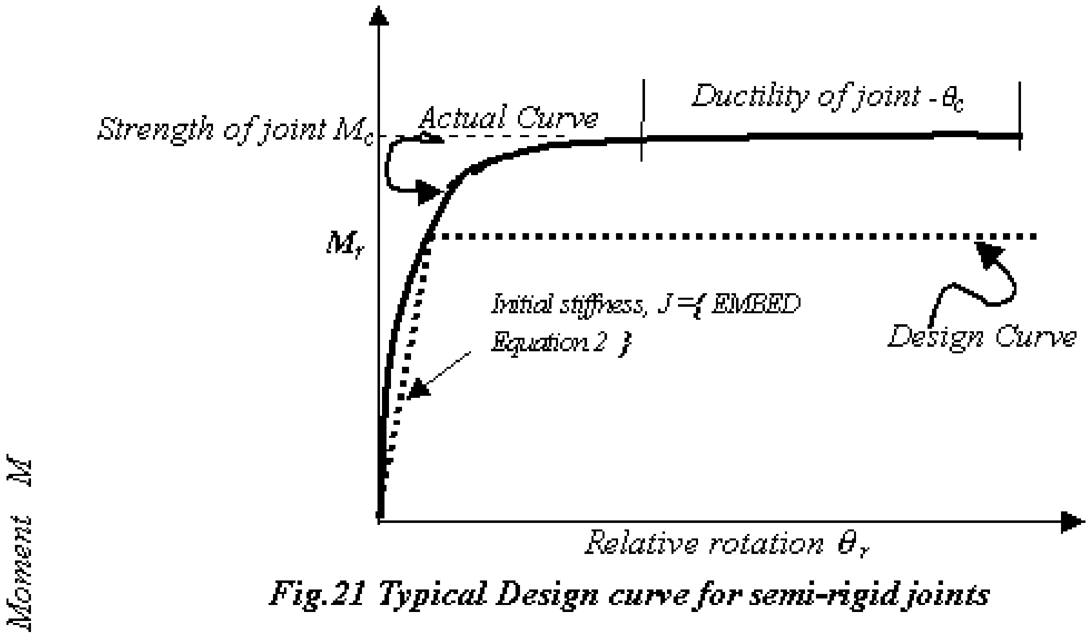


Fig.21 Typical Design curve for semi-rigid joints

In the design of components such as beams, columns and beam columns the procedure is the same as in the rigid elastic design of multi storey frames. Only in the case of columns and beam columns, the effective lengths of members have to be ascertained using alignment charts which considers connection flexibility or by an elaborate instability analysis.

7.0 Computer program "FLEXIFRAME" for the semi-rigid analysis of steel frames

A FORTRAN computer program "FLEXIFRAME" has been written to incorporate the derived flexibility equations using Moment Distribution Method (MDM) derived in this chapter. The computer implementation of the MDM results in the Gauss – Seidel iteration method. The program is capable of analysing non-sway steel frames with flexible connections. However with little modifications, the program could be extended to the analysis of sway frames. The computer program FLEXIFRAME has been presented in Appendix. The input details of the program have also been given in Appendix. The reader is encouraged to try out various problems of multi-storey semi-rigid steel frames to understand the effect of connection flexibility using the computer program.

8.0 Summary

In this chapter, the fundamentals of connection flexibility in steel frames are described. The stiffness equations for the semi-rigid analysis of steel frames using the popular moment distribution method are derived. Example problems, which use the derived stiffness equations, have been presented. The fundamental differences between the behaviour of fully rigid and semi-rigid frames have been brought out. The importance of experimental evaluation of the connection stiffness has also been described. Finally a brief outline of the design procedures has been presented.

9.0 References

1. Gere J.M., “Moment distribution”, D. Van Nostrand Co. Inc, NY, (1963)

2. Chen W.F. and Lui E.M., “Stability design of steel frames”, CRC Press Inc., (1991).

3. Cornelius T. “ Techniques in buildings and bridges”, Gordon and Breach Int. series, Vol.11,(1999)

4. Kishi N. and Chen W.F.,” Moment –rotation relations of semi-rigid connections with angles”, Journal of Structural Engineering, ASCE, 116(7), 1813-1834, (1990).

<u>Appendix</u>

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```
c   A computer program to analyse non-sway semi-rigid steel frames
    Program Flexiframe
    parameter (nsize=50)
    character *12 inpf,outf
    character *87 tit
    real xlen(50),mi(nsize),jm(nsize,2),jstiffa,jstiffb,ja,jb,kval
    integer cvity(nsize,5)
    common/loads/udlval(100),nconc,p(10),a(10),b(10)
    dimension nconnect(nsize),ie(nsize,2),distf(nsize,5)
    dimension var(nsize,nsize),cof(nsize,nsize),fem(nsize,nsize)
    dimension fimom(nsize,nsize),ibc(nsize,2),stiff(nsize,nsize)
    write(*,*)'enter input file name'
    read(*,'(a\)')inpf
    write(*,*)'enter output file name'
    read(*,'(a\)')outf
    open(10,file=inpf)
    open(11,file=outf)
c    title
    read(10,'(a)')tit
        write(11,'(a)')tit
c    general data
    read(10,*)nmem,nnode,ymod,niter
    write(11,*)nmem,nnode,ymod,niter
```

```
c     nodal data
      do 10 i=1,nnode
         read(10,*)m,nconnect(m)
         write(11,*)m,nconnect(m)
         read(10,*)(cvity(m,j),j=1,nconnect(i))
         write(11,*)(cvity(m,j),j=1,nconnect(i))
10    continue
c     memeber data
      do 20 i=1,nmem
         read(10,*)m,xlen(m),mi(m),ie(m,1),ie(m,2),jm(m,1),jm(m,2)
     .           ,ibc(m,1),ibc(m,2)
         write(11,*)m,xlen(m),mi(m),ie(m,1),ie(m,2),jm(m,1),jm(m,2)
     .           ,ibc(m,1),ibc(m,2)
         n1=ie(m,1)
         n2=ie(m,2)
c        initialise the fixed end moments
         fem(n1,n2)=0.0
        fem(n2,n1)=0.0
         jstiffa=jm(m,1)
         jstiffb=jm(m,2)
         ja= (ymod*mi(m)) / (xlen(m)*jstiffa)
         jb= (ymod*mi(m)) / (xlen(m)*jstiffb)
c        to determine the stiffness values
         st=(4.*ymod*mi(m)) / xlen(m)
         xnum=1. + 3.* jb
         den =1. + 4.*(ja+3.*ja*jb + jb)
         stiff(n1,n2)=st*(xnum/den)
         xnum=1. + 3.* ja
         den =1. + 4.*(ja+3.*ja*jb + jb)
         stiff(n2,n1)=st*(xnum/den)
c        carry over factor
         cof(n1,n2)=0.5 * ( 1./ (1.+ 3.*jb))
         cof(n2,n1)=0.5 * ( 1./ (1.+ 3.*ja))
c        load data
```

```
        read(10,*)udlval(i),nconc
        write(11,*)udlval(i),nconc
        fixm=(udlval(i)*xlen(m)*xlen(m)) / 12.
       denudl=(3.+12.*ja+12.*jb+36.*ja*jb)
        fem(n1,n2)=        fixm * ((3.*(1.+6.*jb))/ denudl)
        fem(n2,n1)= (-1.0)* fixm * ((3.*(1.+6.*ja))/ denudl)
        do 30 j=1,nconc
          read(10,*)p(j),a(j),b(j)
          write(11,*)p(j),a(j),b(j)
          kval=a(j)/xlen(m)
          ylen=xlen(m)
              pval=p(j)
          call femconc(kval,ylen,ja,jb,pval,fema,femb)
          fem(n1,n2)=fem(n1,n2) + fema
          fem(n2,n1)=fem(n2,n1) + femb
30       continue
20    continue

c     compute distribution facors
      do 40 k=1,nmem
        n1=ie(k,1)
        n2=ie(k,2)
c        for 'i' node
c        sum stiffness of members meeting at 'i' node
        stsum=0.0
        do 60 j=1,nconnect(n1)
          stsum=stsum + stiff(n1,cvity(n1,j))
60       continue
        distf(n1,n2)=(-1.0)*(cof(n1,n2)*stiff(n1,n2) / stsum)
c        for 'j' node
c        sum stiffness of members meeting at a point
          stsum=0.0
          do 80 j=1,nconnect(n2)
            stsum=stsum + stiff(n2,cvity(n2,j))
```

```
80          continue
          distf(n2,n1)=(-1.0)*(cof(n2,n1)*stiff(n2,n1) / stsum)
40    continue
c     initialise var
      do 81 i=1,nmem
        n1=ie(i,1)
        n2=ie(i,2)
        var(n1,n2)=0.0
        var(n2,n1)=0.0
81    continue
c     the main Gauss - Seidel iterartion starts here
      do 90 i=1,niter
        do 100 j=1,nmem
          n1=ie(j,1)
          n2=ie(j,2)
          if(ibc(j,1) .ne. 1)then
c            sum of the fixed end moments at 'i' node
            m1=nconnect(n1)
            sumfix=0.0
            do 110 k=1,m1
              sumfix=sumfix + fem(n1,cvity(n1,k))
110           continue
c            to find the sum of 'var' meeting at 'i' node
            sumvar=0.0
            do 120 k=1,nconnect(n1)
              sumvar=sumvar + var(cvity(n1,k),n1)
120           continue
            var(n1,n2)=distf(n1,n2) * (sumfix + sumvar)
          endif
          if(ibc(j,2) .ne. 1)then
c            sum of the fixed end moments at 'j' node
            m1=nconnect(n2)
            sumfix=0.0
            do 111 k=1,m1
```

```
            sumfix=sumfix + fem(n2,cvity(n2,k))
111         continue
c           to find the sum of 'var' meeting at 'j' node
          sumvar=0.0
          do 121 k=1,nconnect(n2)
             sumvar=sumvar + var(cvity(n2,k),n2)
121         continue
          var(n2,n1)=distf(n2,n1) * (sumfix + sumvar)
        endif
100     continue
90    continue


c     computation of final moments
      write(*,*)'***** Final support moments *****'
          write(11,*)'***** Final support moments *****'
      write(*,*)'Member no:    I-node moment         J-node moment'
      write(11,*)'Member no:    I-node moment         J-node moment'
      do 130 i=1,nmem
        n1=ie(i,1)
        n2=ie(i,2)
        fimom(n1,n2)=fem(n1,n2) + (var(n1,n2)/cof(n1,n2)) + var(n2,n1)
        fimom(n2,n1)=fem(n2,n1) + (var(n2,n1)/cof(n2,n1)) + var(n1,n2)
999         continue
        write(*,888)i,fimom(n1,n2),fimom(n2,n1)
        write(11,888)i,fimom(n1,n2),fimom(n2,n1)
130   continue
888   format(i3,10x,f15.4,7x,f15.4)
      stop
      end
c     ------------------------------------------------
      subroutine femconc(kval,xlen,ja,jb,pval,fema,femb)
c     ------------------------------------------------
      real kval,num,ja,jb
      xnum=pval*kval*xlen*(1.-kval)
```

```
den=(1.+4.*ja+4.*jb+12.*ja*jb)
num=1.+4.*jb -kval*(1.+2.*jb)
 fema=(xnum*num)/den
 xnum=(-1.0)*pval*kval*xlen*(1.- kval)
 num=2.*ja + kval*(1.+2.*ja)
 femb= xnum*num/den
 return
 end
```

Input to Flexiframe

-

Card set No 1: nmem –number of members in the steel frame

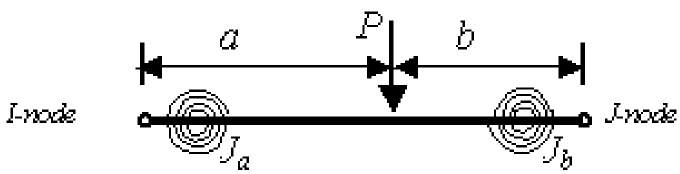
nnode –number of nodes in the steel frame

ymod -Youngs Modulus

niter -Number of moment distribution iterations

Card set No.2: For every node

Node number, number of nodes connected to that particular node



Node numbers connected

Card set No.3: For every member

Node number, Length, Moment of inertia, I-node,J-node, Ja value, Jb value,

Displacement code for I-node,Displacement code for J-node

Disp. Code -1 – joint is fixed

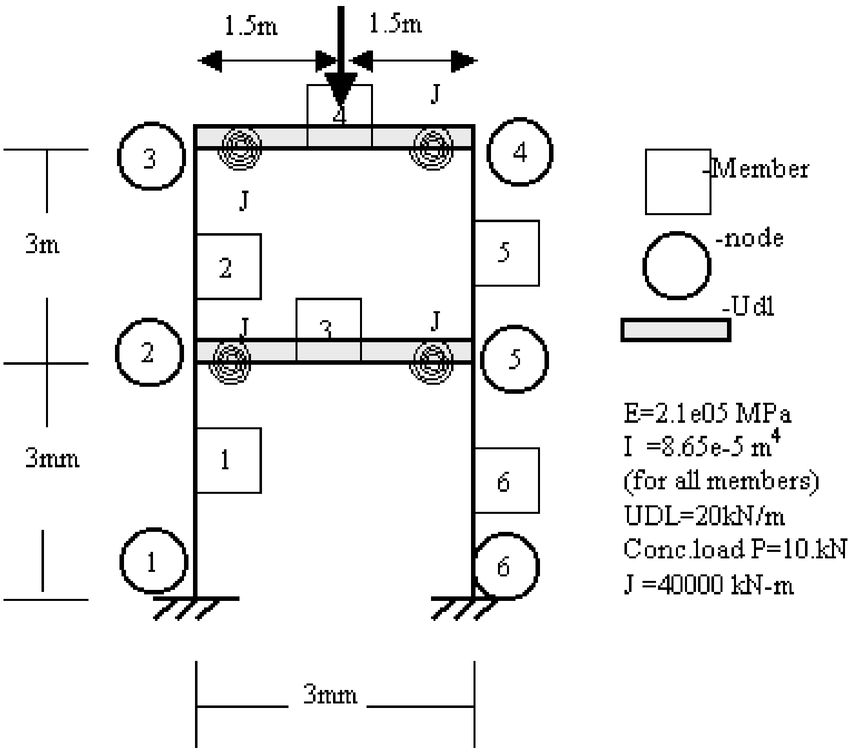
Disp. Code –0 - joint is pinned or it can rotate

UDL value, number of concentrated loads

For number of concentrated loads

Load value, a –distance, b-distance

Example Problem:



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Input data for example Problem:

data for example problem (All units in kN -m)

6,6,2.1e05,40

1,1

2

2,3

1,3,5

3,2

2,4

4,2

3,5

5,3

2,4,6

6,1

5

1, 3.,8.65e-5,1,2,1.e20,1.e20,1,0

0. 0

2, 3.,8.65e-5,2,3,1.e20,1.e20,0,0

0. 0

3, 3.,8.65e-5,2,5,40000.,40000.,0,0

20.0 0

4, 3.,8.65e-5,3,4, 40000., 40000.,0,0

20.0 1

10. 1.5 1.5

5, 3.,8.65e-5,4,5,1.e20,1.e20,0,0

0. 0

6, 3.,8.65e-5,5,6,1.e20,1.e20,0,1

0.0 0

-

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40 MULTI – STOREY BUILDINGS – IV

1.0 INTRODUCTION

Historically monuments such as the pyramids of Egypt, Taj Mahal in India, the Temples of Greece, the Viaduct of Rome were all built principally with masonry using some form of stone or moulded bricks. Since the introduction of Besemer's process in 1856 several tall buildings have been built using steel. The *381 m* Empire State Building, the Twin Towers of the World Trade Centre and the Sears Tower in Chicago completed in 1974 have clearly established the suitability of steel frame construction for Tall Buildings.

The innovations in lateral load resisting systems (such as introduction of frame-wall, framed tube, belt truss with outrigger, tube in tube and bundled tube systems) to cater to different storey heights and environmental requirements based on susceptibility of structures to either wind or seismic effects, have made it possible to build tallest buildings in the world using steel frames. The advancements in computer techniques and the interaction of Computer Aided Design and Computer Aided Manufacture are likely to have their impact on improved fabrication and erection techniques in reaching even taller structures using steel frames in the foreseeable future.

When we build such tall structures it becomes necessary to consider some of the effects such as the effect of lateral deflection, Δ on gravity loading, P which are normally ignored in the design of building frames of normal height (say three or four storeys).

A building frame deflects under lateral load. The columns of tall buildings carry large axial loads. A building frame, which deflects under lateral load, is further forced to undergo additional deflection because of the eccentricity of gravity load from the centre of gravity of the column due to the deflected shape. These two effects of large axial loads P in the columns combined with significant lateral deflection Δ needs careful consideration in the design of tall multi-storey buildings. The combined effect of the large axial loads P and lateral deflection Δ give rise to the destabilising P-Δ effect. However, in frames that are only a few storeys high, this effect is negligible and hence ignored in the analysis. It is therefore necessary to classify frames based on the relative importance P-Δ effects for the purpose of evaluating design forces.

2.0 CLASSIFICATION OF FRAMES

A frame in which sway Δ is prevented is called a "non-sway" frame. However, there are some frames, which may sway only by a small amount since the magnitude of sway in such frame is small it will have only a negligible P-Δ effect. Such frames are also classified as "non-sway" frames. Therefore, to define the non-sway frame precisely, its lateral stiffness is used as the criteria irrespective of whether it is braced or not. For such frames lateral stiffness is provided by one of the following:

(i) rigidity of the joints.
(ii) provision of bracing system.
(iii) connecting the frame to a braced frame, shear core, shear wall or a lift well.

The inter storey deflection (s) (i.e. the difference in deflection of top and bottom end of a column in that storey) is used to quantify the lateral stiffness of the frame. The meaning of inter storey deflection (s) is shown in Fig. 1(c). Fig.1 (a) shows a typical multi-storey frame subjected to factored (dead + live) load. To ascertain the stiffness of the frame, it is analysed when subjected to assumed forces of magnitude *0.5%* of factored (dead + live) load applied laterally on the frame at each floor level as shown in Fig.1 (b) for getting the inter storey deflection (s_i) for the i^{th} storey. Note that the lateral loads are applied without the presence of dead and live loads. The maximum s_i for any storey is taken as a measure of the frame stiffness.

For a frame to be of the non-sway type the maximum inter storey deflection permitted in any storey is generally taken as follows:

$$s_i = \frac{h_i}{4000} \text{ for bare frames}$$

$$= \frac{h_i}{2000} \text{ for frames with cladding}$$

where h_i is the height of the i^{th} storey.

3.0 IDEALISATION OF MATERIAL BEHAVIOUR FOR ANALYSIS OF FRAMES

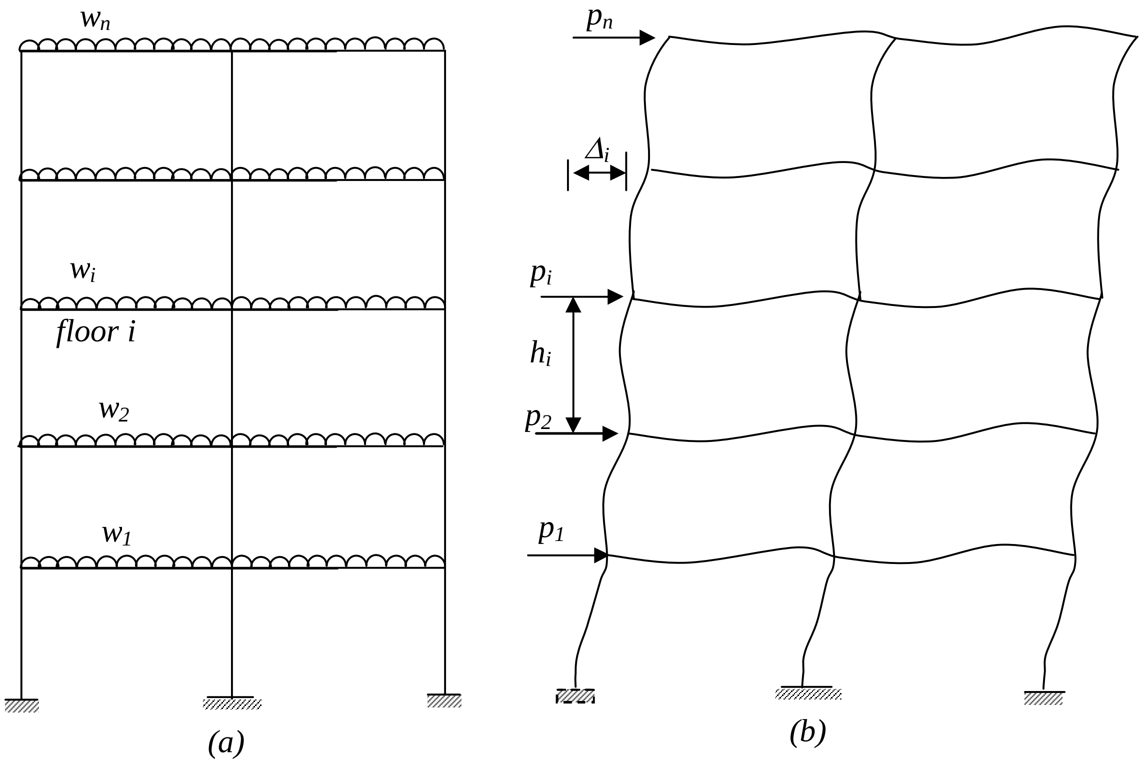
The strength and stability of a rigid jointed frame is examined based on material stress – strain idealisation of its true behaviour.

3.1 Elastic Behaviour

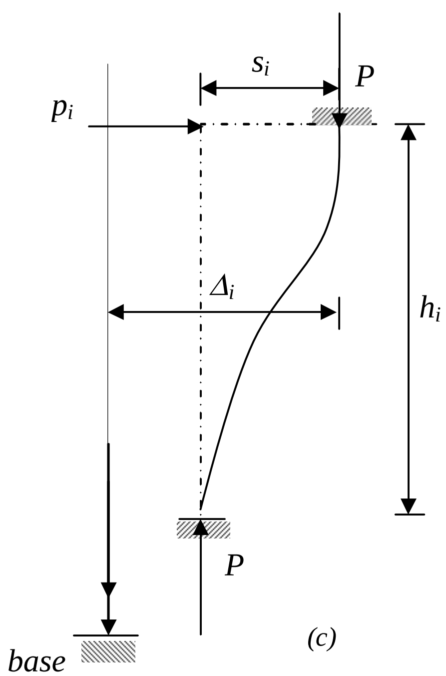
Fig.2 (a) shows idealised elastic stress–strain behaviour of typical steel for most structures where deformations are not large enough to change the equilibrium equations, this idealised elastic behaviour has been the basis of analysis. This analysis is invalid both in the non-linear range of material behaviour (material non-linearity) or when deformations are so large that the change the line of action of the force requires study of the equilibrium under deformed shape.

3.2 Elastic – Plastic Behaviour

Fig 2(b) shows a more realistic elastic-plastic idealisation. This elastic–plastic idealisation can be used to obtain ultimate load. Within the range of admissible deformations, this idealisation is sufficiently accurate though it does not consider strain-hardening effects. This has been successfully used by researchers for obtaining full range of behaviour using a realistic mathematical model of the structure. However, it is found to be time consuming and unsuitable for design office work.



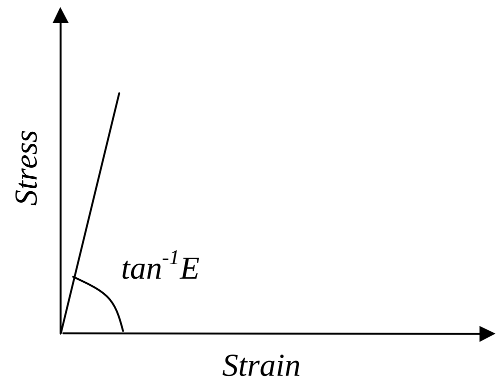
$w_I = \gamma_f (w_{di} + w_{li})$
w_{di} = *dead load on* i^{th} *floor*
w_{li} = *live load on* i^{th} *floor*
w_I = *factored load on* i^{th} *floor*



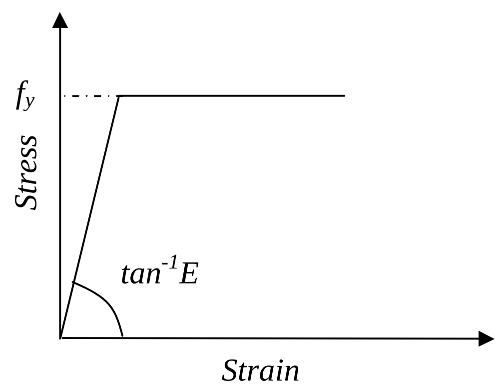
Δ_i = *Total* i^{th} *storey deflection*
S_i = *Interstorey deflection for* i^{th} *floor*
P =*Column Load*
p_i =*Assumed lateral load 0.5% of factored load*

$$= \frac{0.5}{100} * W_i$$

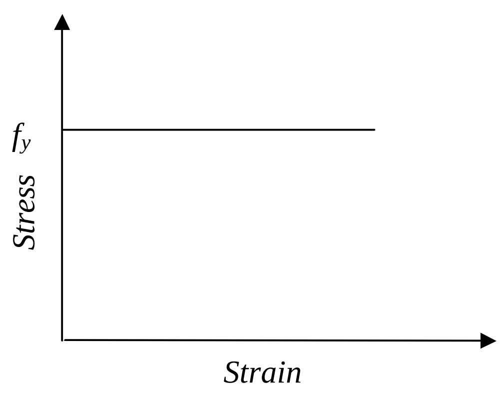
Fig.1 Approximate calculation of frame stiffness for classification of frames



(*a*) *Elastic behaviour*



(*b*) *Elastic-Plastic behaviour*



(*c*) *Rigid-Plastic behaviour*

Fig2: Idealisation of Material Behaviour curve

3.3 Rigid Plastic Behaviour

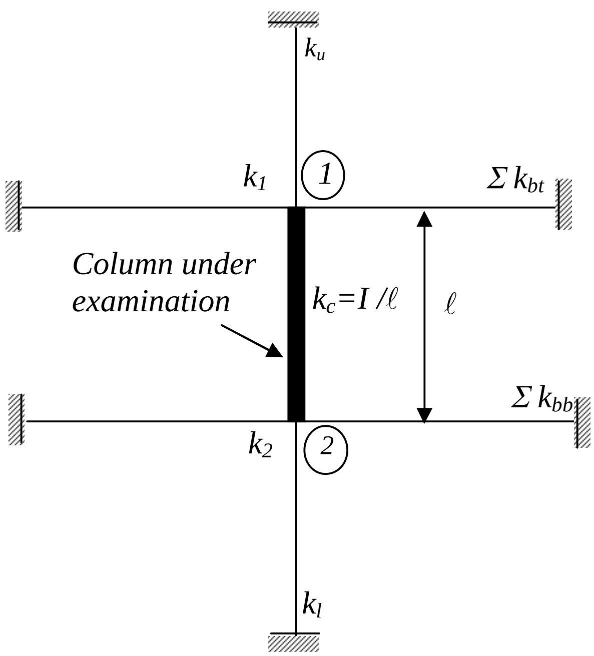
In order to have a realistic estimate of collapse load based on mechanism of failure, rigid-plastic stress-strain idealisation shown in Fig. 2(*c*) has been used. This gives acceptable results for non-sway frames. In tall multi-storey frames, the sway deflections affect the equilibrium equations. Thus mere consideration of rigid-plastic idealisation grossly over estimates the collapse load.

In estimating the realistic collapse load, it has been shown by Horne that the results based on perfectly elastic as well as rigid-plastic idealisations can be combined to give acceptable estimate of actual collapse load for the cases of sway frames. Therefore, it is necessary to study the behaviour of the frame under these idealised material behaviour conditions.

4.0 EFFECTIVE LENGTH OF COLUMNS

4.1 Limited Frame Method

The behaviour of a column under compression is largely controlled by its effective length. A number of idealised end conditions such as pinned, fixed, partially fixed, free and supported on rollers, etc., are used in text books to describe the restraint at the two ends of a column. In multi-storey buildings, columns are continuous and beam members frame into them at floor levels connected rigidly. These columns become a part of either a non-sway or sway frame.



$$k_1 = \frac{k_c + k_u}{k_c + k_u + \sum k_{bt}}$$

$$k_2 = \frac{k_c + k_l}{k_c + k_l + \sum k_{bb}}$$

Fig.3 Limited Substitute Frame

The column, which is a part of the multi-storey non-sway frame, can be idealised to be a part of a limited subframe shown in Fig.3. Let ℓ_e be the effective length of the column, ℓ the actual length between floor beams. The effective length factor for the column is defined as $k = \ell_e/\ell$

In the figure k_u and k_l are relative stiffness I/ℓ values for upper and lower column respectively. Σk_{bt} and Σk_{bb} are the sum of I/ℓ values for beams framing into the column under examination at the top and bottom respectively. The joint restraint coefficient k_n for the column at the top and bottom is obtained from

$$k_n = \frac{Column\ stiffness\ of\ columns\ meeting\ at\ the\ joint}{Total\ stiffness\ of\ all\ members\ meeting\ at\ joint\ n}$$

In Fig.3, k_1 and k_2 represents the joint stiffness of the column *1-2* at the end *1* and *2* respectively.

4.2 Effective length for Non-sway ($k_3 = \infty$) and sway k_3=0 frames

Based on the work of Wood, the value of relative end restraints k_1 and k_2 can be obtained from Contour Plot reproduced in Fig 4 (*b*) and Fig. 5 (*b*) for the non-sway frame shown in Fig.4 (*a*) and for sway frame shown in Fig.5 (*a*). In the case of non-sway frame, stability criteria considered are rotations that take place at top and bottom end of the column for the elastic critical load using stability functions.

However, in the case of sway frames, [Fig.5 (*a*)] in addition to rotations, the effect of lateral deflection has been considered. Subsequently, it was shown by Wood that the plots in Fig.4(*b*) and Fig.5(*b*) can also be used when the columns at the top and (or) bottom are continuous over stories provided that the joint stiffness at top and bottom are correctly accounted for.

The effective length factor for the column $k = \ell_e/\ell$ for non-sway frames lie in the range of "*0.5* to *1.0*". For sway frames the range increases to "*1* to ∞ " indicating clearly the contribution of lateral sway to instability.

<u>Example *1:*</u>

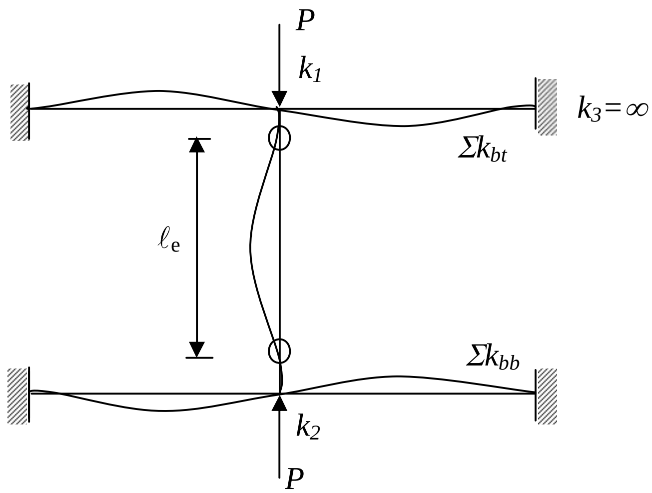
As an example, let us examine the case of a column with $k_1 = \dfrac{k_{co\ell}}{\sum k_{bt} + k_{topcol}} = 0.5$

and $k_2 = \dfrac{k_{co\ell}}{\Sigma k_{bb} + k_{botco\ell}} = 0.6$ for a non-sway frame. The effective length factor k for $k_1 = 0.5$ and $k_2 = 0.6$ and $k_3 = \infty$ (non-sway) from Fig.4(*b*) is *0.72*. Therefore the effective length of the column $\ell_e = 0.72\ \ell$.

For a sway frame ($k_3 = 0$), the effective length factor from Fig. 5(*b*) becomes $k = 1.55$ leading to the effective length of column $\ell_e = 1.55\ \ell$.

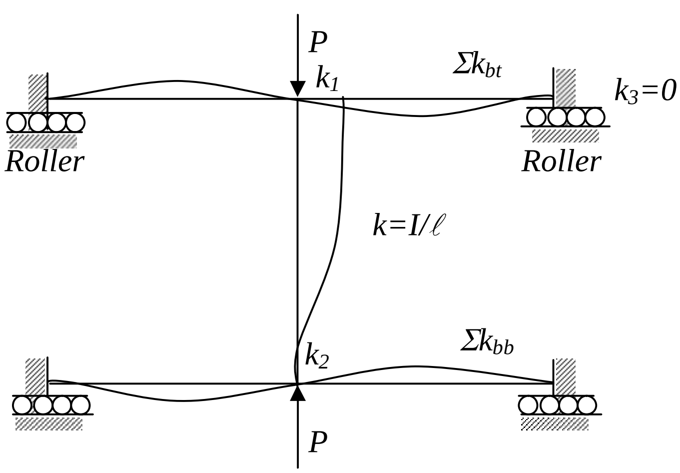
This shows the importance of considering sway for multi-storey columns forming a part of a frame in which $k_2 = \infty$

4.3 Effective length of insufficiently restrained columns in the frames



k_1 =Distribution coefficient at top
Σk_{bt} =Sum of beam stiffness I/ℓ at top
Σk_{bb}=Sum of beam stiffness I/ℓ at bottom
k_2 =Distribution coefficient at bottom
k_3 =Bracing stiffness
=∞ (since braced)

Fig.4(a) Non sway frame ($k_3=\infty$)



k_3 =Bracing stiffness
=0 (since full sway permitted)

Fig.5(a) Sway frame ($k_3=0$)

While using the charts given in Fig.4 (*b*) and Fig.5 (*b*), following limitations should be considered.

(i) When a member is either not present or not firmly connected to the frame, it should be considered to have zero stiffness.

(ii) If a framing member carries nearly full moment (*90%* of its capacity) it will not provide resistance for preventing the column from buckling when plastic hinges have formed. For such beams, stiffness should be taken as zero.

(iii) If the column under question itself carries full moment (*90%* of its capacity) it will develop flexural hinge at top and bottom and as such its effective length should be taken as ℓ.

(iv) When the column is attached to the foundation, a rational value of k at the bottom should be chosen (i.e. $k=1$ if pinned, *0.9* if not rigidly connected and *0.5* if rigidly connected with transverse beams).

The above cases highlight the importance of rotational continuity being distributed by either plasticity or partial release due to practical foundation problems which are likely to reduce the restraint at the ends of the column.

4.4 Effective length consideration when the frame is partially braced

Neither the column considered in Fig.4 (*a*) with full restraint nor the column considered in Fig.5 (*a*) with no restraint can be applied to a case of a frame partially restrained by filler walls in between the framing members. These panel walls partially inhibit sway. In such cases, the effective length will depend on the relative stiffness of bracing system provided.

The relative stiffness of the bracing system to that of the frame is designated as k_3.

BS 5950 gives a method of computing the relative stiffness of the frame based on computed values of the stiffnesses of columns and that of the panels in that storey. This expression given in Equation 1 can be shown to be based on elastic stiffness contribution of the panel to that of the frame:

$$k_3 = \frac{h^2 \Sigma S_p}{80 E \Sigma k_c} \quad but \leq 2 \tag{1}$$

where,

h = Storey height

ΣS_p = Sum of the spring stiffness calculated as horizontal force required to produce unit horizontal deflection of the panel in the storey in which the column is located.

E = Modulus of Elasticity of Column

Σk_c = Sum of the stiffness of all columns in that storey represented by their I/ℓ values.

The spring stiffness S_p in equation (1) can be conveniently obtained from the unit load method as given in equation (2)

$$S_p = \frac{0.6(h/b)}{\left[1+\left(h/b\right)^2\right]^2} t E_p \tag{2}$$

where

h = storey height
b = width of panel
t = thickness of panel
E_p = Modulus of Elasticity of panel

Fig. 6 and Fig.7 show the charts for computing effective length ratios for sway bracing stiffness of $k_3 = 1$ and $k_3 = 2$ respectively. Thus, effective length factor for a column being a part of the frame with $k_3 = 1$ as well as $k_3 = 2$ can be determined using these charts. These charts are intended to account for the effect of partial sway bracing.

The actual effective length factor for the partial sway bracing case for a particular case of bracing stiffness k_3 determined from equation (1) is determined by interpolating the k values obtained for $k_3 = 0$ [Fig 5 (*b*)], $k_3 = 1$ (Fig 6) and $k_3 = 2$ (Fig.7).

Example *2:*

As an example, take the case of $k_1 = 0.5$, $k_2 = 0.6$ for which effective length factor when no bracing is provided was shown to be *1.55*. From Fig.6, for $k_3 = 1$ effective length for $k_1 = 0.5$ and $k_2 = 0.6$ is $1.44\ \ell$. From Fig.7 for $k_3 = 2$ effective length for $k_1 = 0.5$ and $k_2 = 0.6$ is $1.255\ \ell$. If $k_3 = 1.5$ (relative stiffness of bracing to the frame) then the value of effective length factor k will be $\frac{1.44 + 1.255}{2} = 1.3975 \approx 1.4$. Thus the effective length of the column with partial restraint of $k_3 = 1.5$ is $1.4\ \ell$. For this example the effective length factor k for various stiffness of framing system is as shown in Table 1.

Table 1: Effective length factor for the example frame

S.No.	Conditions for lateral restraint	k_1	k_2	k_3	k
1.	Non-sway	*0.5*	*0.6*	∞	*0.72*
2.	Sway	*0.5*	*0.6*	*0*	*1.55*
3.	Partial restraint by panel walls	*0.5*	*0.6*	*1.5*	*1.40*

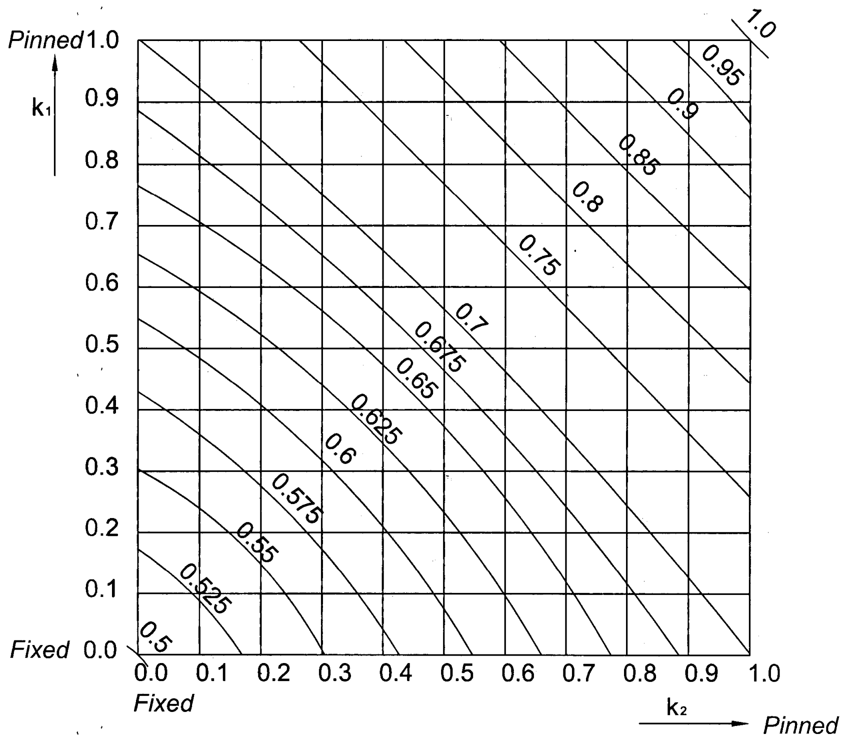


Fig.4 (b) Effective Length ratio ℓ_e/ℓ for a column in a rigid- jointed frame braced against sidesway for $k_3=\infty$

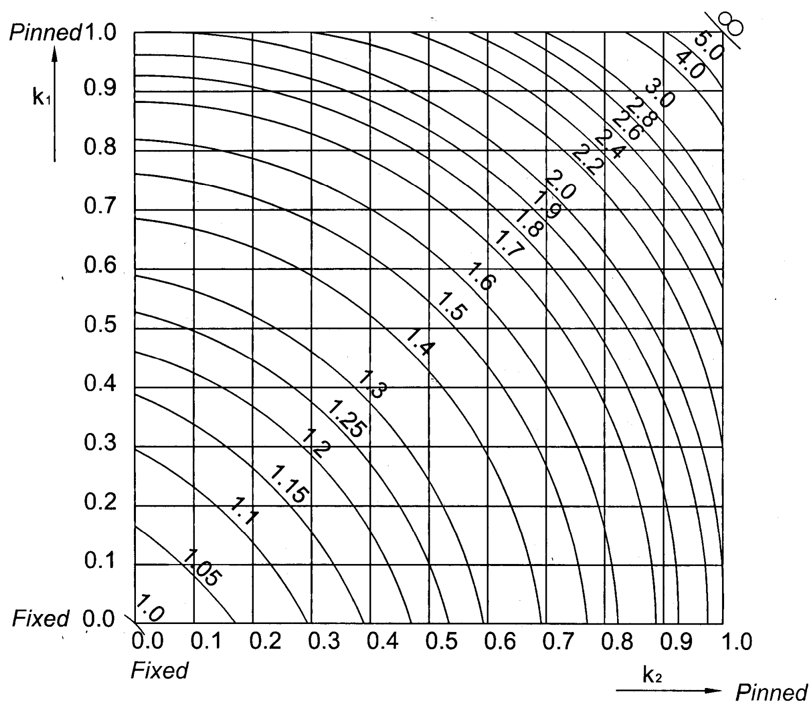


Fig.5 (b) Effective Length ratio ℓ_e/ℓ for a column in a rigid- jointed frame with unrestricted sidesway for $k_3=0$

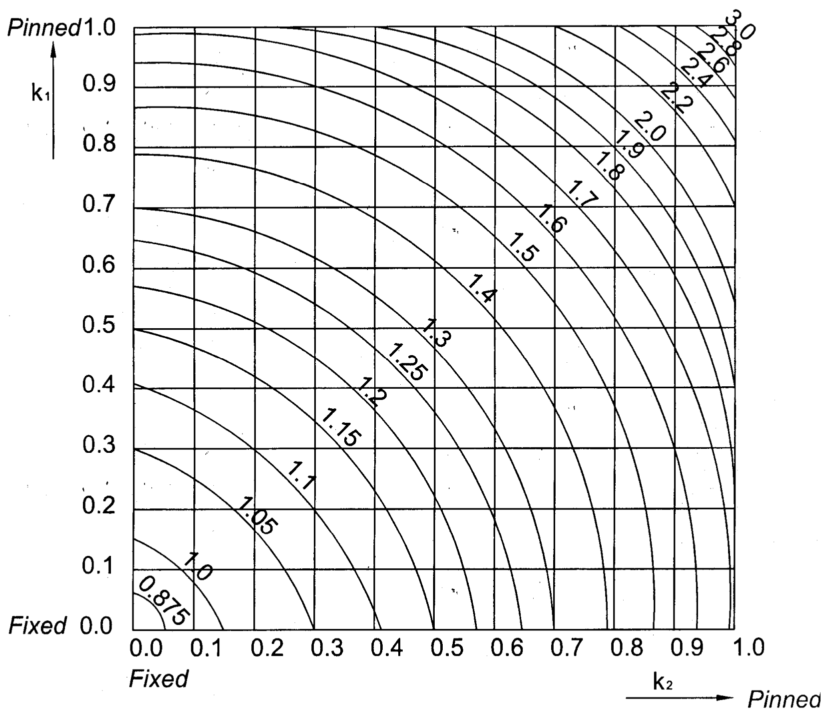


Fig.6 Effective Length ratio ℓ_e/ℓ for a column in a rigid- jointed frame with partial sway bracing of relative stiffness k_3=1

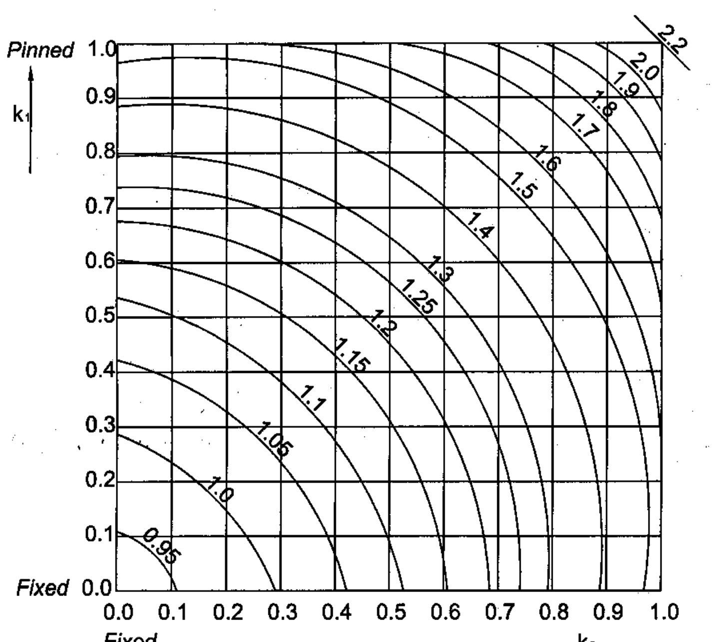


Fig.7 Effective Length ratio ℓ_e/ℓ for a column in a rigid-jointed frame with partial sway bracing of relative stiffness k_3=2

4.5 Consideration of realistic beam stiffness based on buckling mode

It is assumed that the ends of the beam away from the column end under consideration is fully restrained. This assumption is realistic (as shown by Wood) and acceptable because about 48 to 60 percent of the width of slabs are available for stiffening beams and for carrying the fixed end moments of loaded beams. However, this assumption is not appropriate for base frames which are not integral with concrete floor and hence the value I/ℓ used for such floors should be modified taking into account the critical buckling mode at failure.

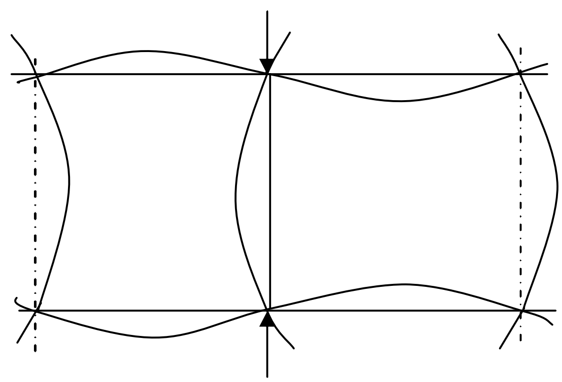


Fig.8 Critical Buckling Mode of a Braced Frame

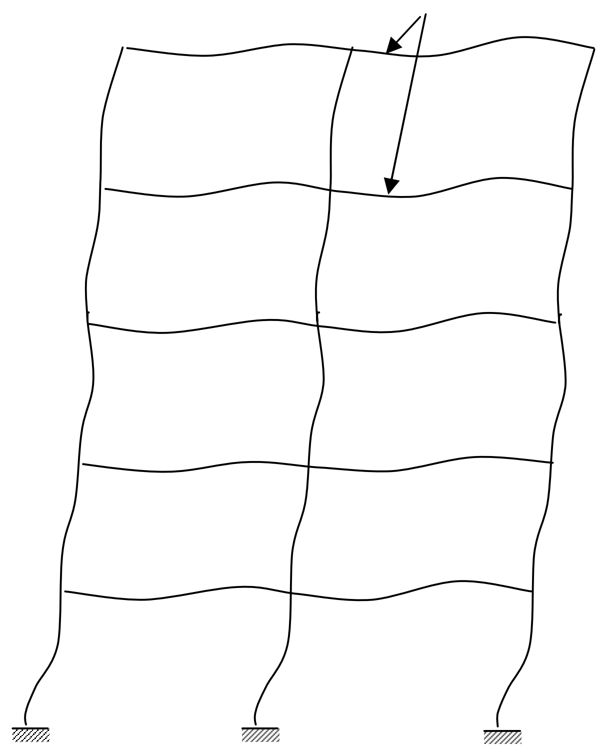


Fig.9 Critical Buckling Mode for an Unbraced Frame

For a non-sway frame, the beams are bent into single curvature as shown in Fig.8. For this case, the beam stiffness is $0.5\ I/\ell$.

In the case of a sway frame, the bending mode will have double curvature as shown in Fig.9. The beam stiffness in this case is $1.5\ I/\ell$. The effective length obtained for the column using this assumption is appropriate. A more exact value can be obtained from the consideration of frame instability discussed later.

It is assumed that the beam members are not subjected to axial forces. In case they are subjected to axial forces, the limited frame method can still be used, provided the frame is a non-sway one and proper care is taken to use reduced stiffness for beams based on the level of axial load carried by it, to its elastic buckling load P_{cr}.

4.6 Method for Determining Effective Length of Columns in Frames according to IS: 800

4.6.1 Method for Determining Effective Length of Columns in Frames

4.6.1.1 In the absence of a more exact analysis, the effective length of columns in framed structures may be obtained by multiplying the actual length of the column between the centres of laterally supporting members (beams) with the effective length factor *K*, calculated by using the equations given below, provided the connection between beam and column is rigid type.

a) Non-sway Frames (Braced Frame)

A frame is designated as non-sway frame if the relative displacement between the two adjacent floors is restrained by bracings or shear walls. The effective length factor, *K*, of column in non-sway frames is given by

$$K = \frac{\left[1 + 0.145(\beta_1 + \beta_2) - 0.265\beta_1\beta_2\right]}{\left[2 - 0.364(\beta_1 + \beta_2) - 0.247\beta_1\beta_2\right]}$$

b) Sway frames (Moment Resisting Frames)

The effective length factor, *K*, of column in sway frames is given by

$$K = \left[\frac{1 - 0.2(\beta_1 + \beta_2) - 0.12\beta_1\beta_2}{1 - 0.8(\beta_1 + \beta_2) + 0.6\beta_1\beta_2}\right]^{0.5}$$

where

β_1, β_2 are given, $\beta = \dfrac{\sum K_c}{\sum K_c + \sum K_b}$

K_c, K_b = effective flexural stiffness of the columns and beams meeting at the joint at the ends of the columns and rigidly connected at the joints, and those are calculated by

$K = C\,(I/L)$

I = moment of inertia of the member about an axis perpendicular to the plan of the frame

L = length of the member equal to centre to centre distance of the intersecting member

C = correction factor as shown in Table 4.6.1

TABLE 4.6.1

Far end condition	Correction Factor C	
	Braced frame	**Unbraced frame**
Pinned	$1.5(1-\bar{n})$	$1.5(1-\bar{n})$
Rigidly connected to column	$1.0(1-\bar{n})$	$1.0(1-0.2\bar{n})$
Fixed	$2.0(1-0.4\bar{n})$	$0.67(1-0.4\bar{n})$

Note: $\bar{n} = \dfrac{P}{P_e}$

where P_e = elastic buckling load

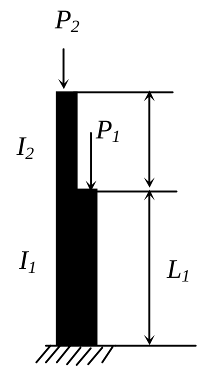
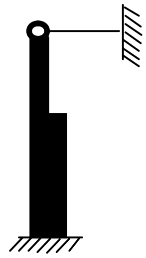
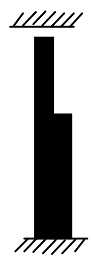
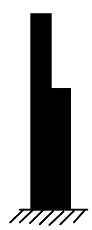
P = applied load

4.6.2 Method for determining Effective Length for Stepped Columns

4.6.2.1 Single Stepped Columns – Effective length in the plane of steeping (bending about axis *z-z*) for bottom and top parts for single stepped column shall be taken as given in Table E.2 of IS: 800.

TABLE E.2 EFFECTIVE LENGTH OF SINGLED STEPPED COLUMNS

(Appendix E.1.1)

Sl. No.	**Degree of End Restraint**	**Sketch**	**Effective Length Coefficients**	**Column Parameters for all Cases**
a)	Effectively held in position and restrained against rotation at both ends		$K_1 = \sqrt{\frac{K_{12}{}^2 + K_{11}{}^2(\alpha - 1)}{\alpha}}$ $K_2 = \frac{K_1}{C_1} \leq 3$ where K_{12} and K_{11} are to be taken as per Table E.3 of IS: 800	 $\alpha = \frac{P_2}{P_1 + P_2}$ $C_1 = \frac{L_2}{L_1}\sqrt{\frac{I_1}{I_2 \alpha}}$ $\frac{i_2}{i_1} = \frac{I_2}{L_2} \times \frac{L_1}{I_1}$ Effective length of bottom part of column in plane of stepping = $K_1 L_1$ Effective length of top part of column in plane of stepping = $K_2 L_2$
b)	Effectively held in position at both ends and restrained against rotation at bottom end only		$K_1 = \sqrt{\frac{K_{12}{}^2 + K_{11}{}^2(\alpha - 1)}{\alpha}}$ $K_2 = \frac{K_1}{C_1} \leq 3$ where K_{12} and K_{11} are to be taken as per Table E.4 of IS: 800	
c)	Effectively held in position and restrained against rotation at bottom end, and top end held against rotation but not held in position		K_1 to be taken as per Table E.5 of IS: 800 $K_2 = \frac{K_1}{C_1} \leq 3$	
d)	Effectively held in position and restrained against rotation at bottom end, and top end neither held against rotation nor held in position		K_1 to be taken as per Table E.6 of IS: 800 $K_2 = \frac{K_1}{C_1} \leq 3$	

5.0 A SIMPLIFIED SWAY METHOD

This is one of the approximate methods recommended by BS 5950 for elastic design of sway frames. In this method, the effect of instability of the column on bending moments and deflection is considered by appropriately increasing their magnitude (magnifying) by a factor $\frac{1}{1 - \frac{P}{P_{cr}}}$ where P is the current load level and P_{cr} is the load required to cause

instability. This method has been tested for different ratios of moments acting at top and bottom of the column. If we designate this moment ratio as m_o (smaller end moment / larger end moment) the magnification factor due to instability for different ratios of m_o is shown (by Wood) as in Fig.10.
If $\lambda_{cr} = P_{cr} / P$ design, then the amplification factor will be

$$MF = \left(\frac{1}{1 - \frac{1}{\lambda_{cr}}} \right) = \left(\frac{\lambda_{cr}}{\lambda_{cr} - 1} \right) \qquad (3)$$

The influence of frame instability on elastic response is shown in Fig.11. BS 5950 in the simplified sway method requires that all moments obtained by elastic analysis due to horizontal forces be increased by this magnification factor. Since the effects of instability are incorporated by moment magnifier method, the effective length of the column is kept as actual length of the column itself.

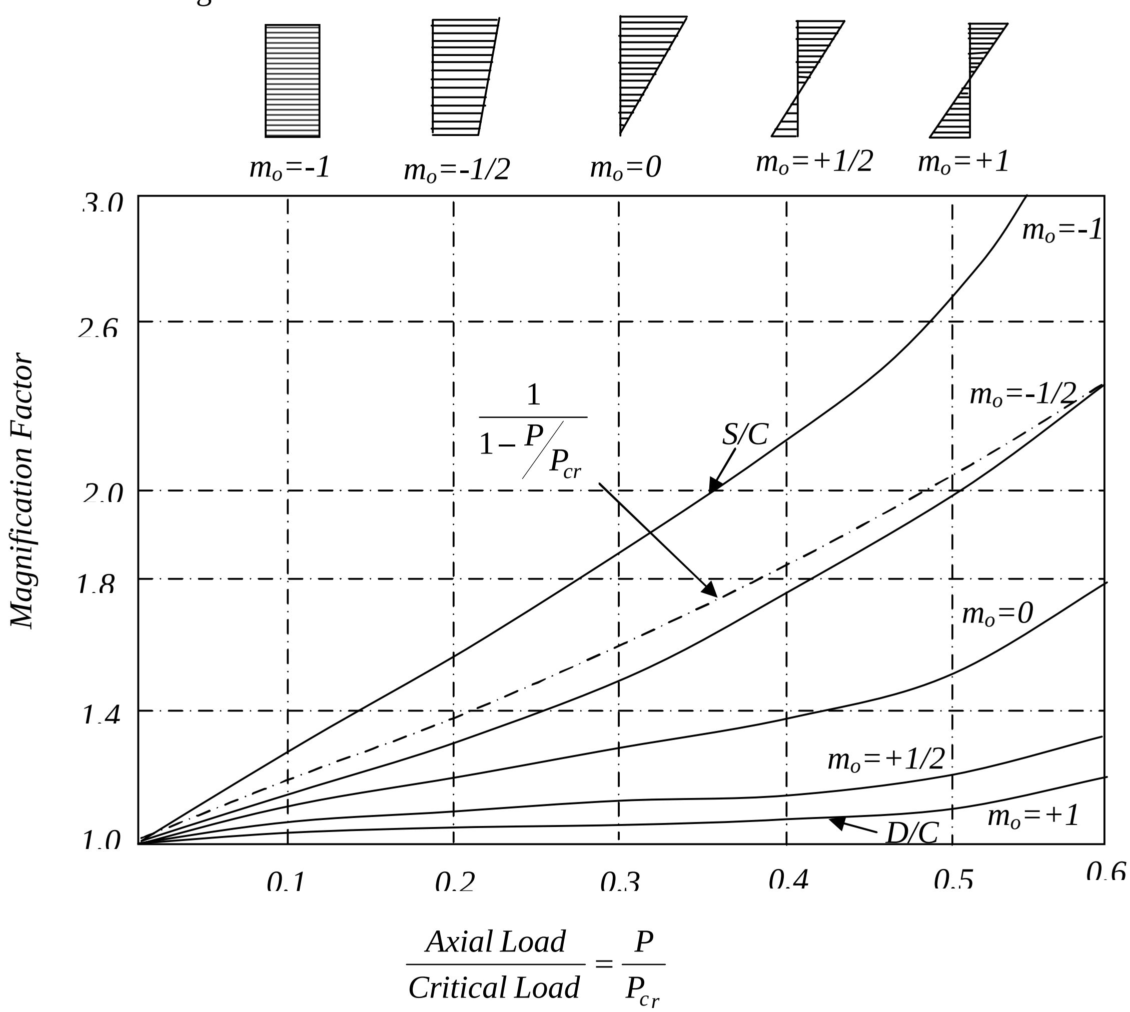
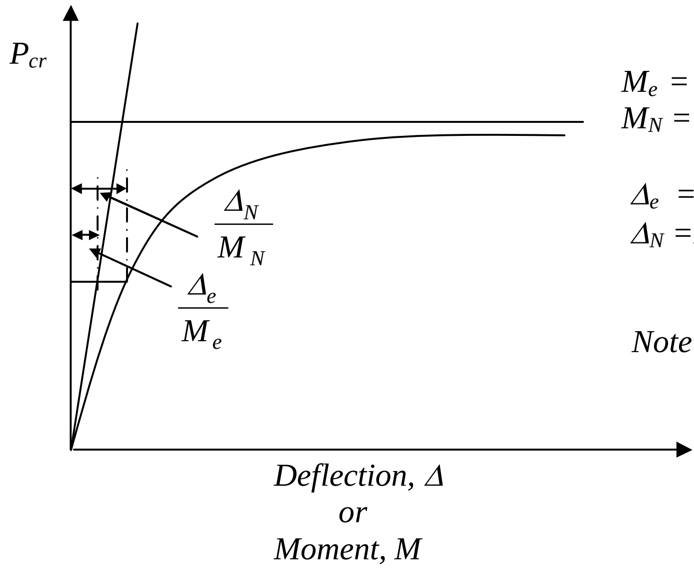


Fig. 10 Magnification of Moment due to axial load (non- sway)

S/C - Single curvature bending
D/C - Double curvature bending

$$m_o = \frac{Smaller\ end\ moment}{Larger\ end\ moment}$$



M_e = Linear elastic moment
M_N = Non Linear elastic moment including instability
Δ_e =linear elastic response
Δ_N =Non-Linear elastic response including instability

$$Note: \frac{\lambda_{cr}}{\lambda_{cr} - \mathbf{1}} = \frac{\Delta_N}{\Delta_e} = \frac{M_N}{M_e} = MF$$

Fig. 11 Response (magnified) due to elastic instability

6.0 ELASTIC DESIGN OF MULTISTOREY RIGID FRAMES

6.1General

The elastic design as per BS 5950 is made for factored loads when the deflections are small. The deflections should generally be limited to *span/200*. In these cases, deflections do not cause any significant instability. The design of beams and columns are made using substitute frames for gravity loading described earlier. For horizontal loading it is necessary to consider entire frame. One of the approximate methods described earlier can be used. Even when elastic design is used, moment redistribution to the extent of *10%* can be made provided compact or plastic sections are used and minor axis column moments are not reduced while maintaining equilibrium.

6.2Non - sway frames

For gravity loading non-sway frames are analysed either using full frame or using substitute frame. The effective length of columns is obtained as described earlier in Section 4.0 taking them as braced. For load cases involving horizontal load, pattern vertical loading is not considered and the entire frame is analysed.

6.3Sway Frames

The frames, which exceed the non-sway limit as specified in Section 2.0 are designed considering sway.

As a first step, the frame is analysed for vertical gravity loading considering also pattern loading as a non-sway frame using effective length of columns applicable to those braced against sidesway.
Next, the effects of sway is considered under all combination of loading, considering vertical loading effects on sway, the notional lateral load as described in Section 2.0 is

applied at each storey level and one of the following two design methods is adopted to get the final design forces.

(i) Simplified Design Method

The side sway is allowed. The effective length as explained in Section 4.0 using limited frame method is used and the design forces are obtained.

(ii) Amplified Sway Method

The bending moments due to lateral loads are magnified by moment magnification factor $\left(\dfrac{\lambda_{cr}}{\lambda_{cr}-1}\right)$ as explained in Section 5.0 and the final design forces are obtained. Since the moments have been magnified the effective length of the column is assumed as actual length of column (i.e. $\ell_{eff} = \ell$).

7.0 STABILITY CONSIDERATIONS OF SWAY FRAME UNDER ELASTIC-PLASTIC FAILURE LOADS.

7.1Elastic Critical Conditions

In a normal elastic frame, the deflection function $F(\Delta)$ of the frame is proportional to the deflection $f(\Delta)$ of the frame under unit load. Thus

$$F(\Delta) = \lambda f(\Delta) \tag{4}$$

where λ is the load factor.

The axial forces in the column are proportionate to applied loading. These axial forces introduce the instability effects. It is necessary to compute the reduction in stiffness of the columns as they approach the critical loads. At certain critical load factors $\lambda_{c1} \le \lambda_{c2} \le \lambda_{c3}$ (the eigen values), the stiffnesses vanish leading to large deflections. These correspond to critical modes at those load factors. Eigen vectors at the corresponding loads are represented by deflection function $f(\Delta_1)$, $f(\Delta_2)$, $f(\Delta_3)$ etc.

Using the orthogonal property of the mode shapes, we can express the deflection as

$$f(\Delta) = a_1 f(\Delta_1) + a_2 f(\Delta_2) + a_3 f(\Delta_3) + \ldots \tag{5}$$

where a_1, a_2, and a_3 are participation factors for each mode.

When instability effects are considered the resulting deflection accommodating non-linear effect can be expressed as

$$f(\Delta) = \left(\frac{\lambda_{c1}}{\lambda_{c1} - \lambda}\right) a_1 \, f(\Delta_1) + \left(\frac{\lambda_{c2}}{\lambda_{c2} - \lambda}\right) a_2 \, f(\Delta_2) + \left(\frac{\lambda_{c3}}{\lambda_{c3} - \lambda}\right) a_3 \, f(\Delta_3) + \ldots\ldots\ldots \tag{6}$$

Deflections approach very large values as λ approaches λ_{c1}, λ_{c2} etc. When deflections become large, it is not acceptable to express them in terms of eigen vectors and the deflection pattern will change the member forces in the columns. For deflections within practical limits, equation (6) is applicable.

It is necessary to find the lowest critical load because it shows the onset of elastic critical condition. The elastic critical load factor λ_{cr} of the frame is the ratio by which each of the factored loads will have to be increased to cause elastic instability.

This load factor is also required to be used in the approximate method for evaluating elastic-plastic failure loads. The deflection method given in Appendix F of BS 5950 Part 1: 1985 is an approximate method based on the work of Horne to arrive at a reasonable estimate of elastic buckling load λ_{cr}. This method is described below:

Consider the rigid frame shown in Fig.1 (*a*) and the analysis performed as indicated in Section 2.0 under lateral loads whose magnitude is *0.5%* of the factored dead and live loads as shown in Fig. 1 (*b*). The sway index of the typical i^{th} storey is

$$\phi_i = \frac{s_i}{h_i} \tag{7}$$

Note that s_i is the i^{th} storey inter storey displacement. Thus the values of $\phi_1, \phi_2, \phi_3, \ldots\ldots \phi_i \ldots\ldots \phi_n$ for all storeys are computed. If ϕ_{max} is the maximum of all ϕ_i values, then the elastic critical load factor is

$$\lambda_{cr} = \frac{1}{200\,\phi_{max}} \tag{8}$$

Horne has shown that the above expression gives an approximate lower bound to the elastic critical load.

7.2 Deteriorated Critical Load

The stability of a structure depends on the equilibrium state with reference to the potential energy U. A structure with small deformation will have a typical load–deflection curve as indicated by curve *XYZ* in Fig.12 (*b*). The effect of load due to lateral deflection in these structures are not significant. The points X, Y and Z represent three different states of stability of the frame shown in Fig.12 (*a*). The potential energy U is the sum of the potential energy of loads U_w and the elastic strain energy stored U_e.

Thus

$$U = U_w + U_e \tag{9}$$

The condition of stability of the frame can be assessed based on whether the first partial derivative with respect to deflection is greater than zero, less than zero or equal to zero. When it is greater than zero the system is stable. When it is equal to zero the system is neutral i.e. more displacement will not change the system. When it is less than zero the system is unstable i.e. a small change will cause collapse.

For equilibrium

$$\frac{\partial U}{\partial \Delta} = 0 \tag{10}$$

On the rising part, i.e., at point *X*,

$$\frac{\partial^2 U}{\partial \Delta^2} > 0 \ (Stable) \tag{11}$$

On the falling part, i.e. at *Z*

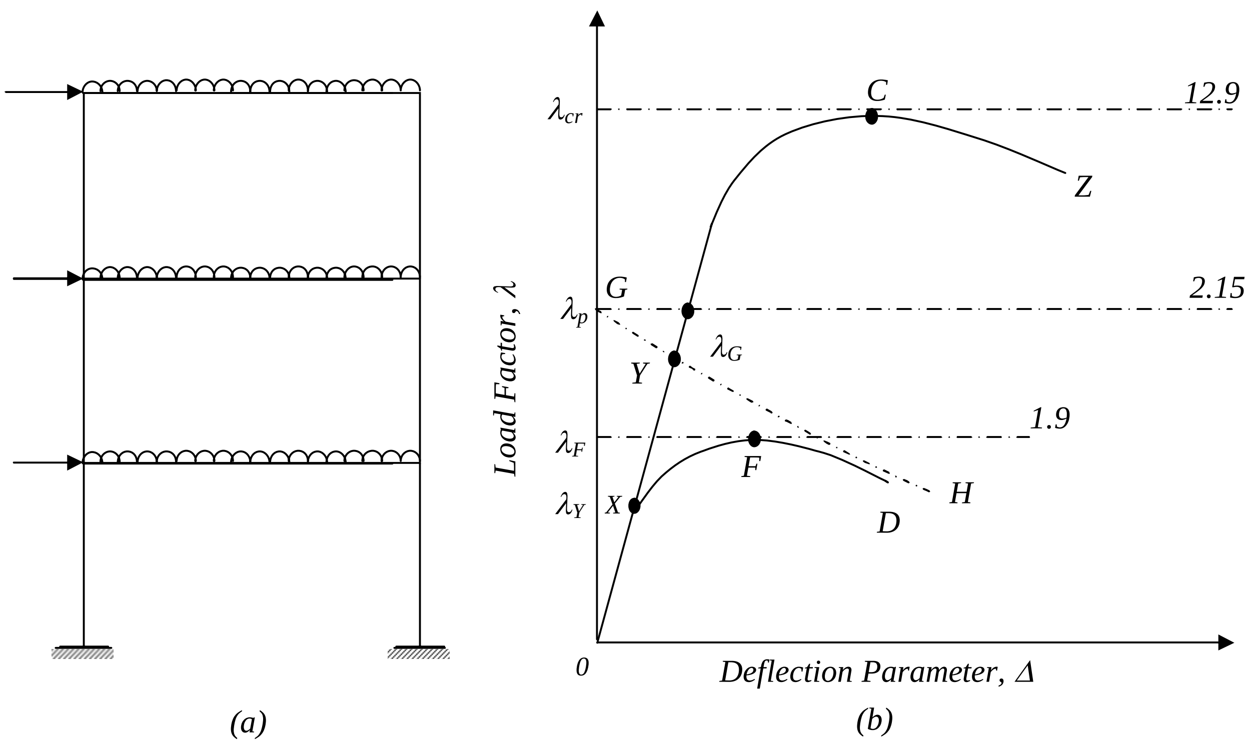


Fig.12 Load-deflection curve for an elastic-plastic structure compared with an elastic structure

$$\frac{\partial^2 U}{\partial \Delta^2} < 0 \ (Unstable) \tag{12}$$

and at *C*

$$\frac{\partial^2 U}{\partial \Delta^2} = 0 \ (neutral) \quad (13)$$

The above explanation is valid for an elastic system undergoing instability problem.

Consider the load deflection curve *OXFD* in Fig.12 (*b*) for a typical elastic-plastic non-linear structure system. This should include U_p the energy absorbed in plastic deformation. Now the total energy U_N is

$$U_N = U_w + U_e + U_p \quad (14)$$

For equilibrium

$$\frac{\partial U_N}{\partial \Delta} = \mathbf{0} \text{ Valid for all points on OXFD} \quad (15)$$

In the plastic zone $\frac{\partial^2 U_p}{\partial \Delta^2} = 0$ since the stress is constant

$$\frac{\partial^2 U_N}{\partial \Delta^2} > 0 \quad \text{upto } \lambda_F \text{ is reached} \quad (16)$$

$$\frac{\partial^2 U_N}{\partial \Delta^2} = 0 \quad \text{at } \lambda_F \text{ i.e at point } F \quad (17)$$

$$\frac{\partial^2 U_N}{\partial \Delta^2} < \mathbf{0} \quad \text{beyond } F \text{ in the falling branch of } FD \quad (18)$$

The condition at failure

$$\frac{\partial^2 (U_w + V_e)}{\partial U^2} = 0 \quad (19)$$

From the above it is clear that failure criteria for elastic-plastic structure is similar to elastic structure with plastically deforming parts eliminated i.e. the plastically deforming parts contribution becomes zero. The elastic portion between plastic hinges will still be contributing to the energy. The structure with the eliminated parts is termed “deteriorated or depleted”. The critical load obtained under this depleted or deteriorated structure is known as deteriorated critical load.

The curve *OXC* represents the behaviour of ideally elastic frame.

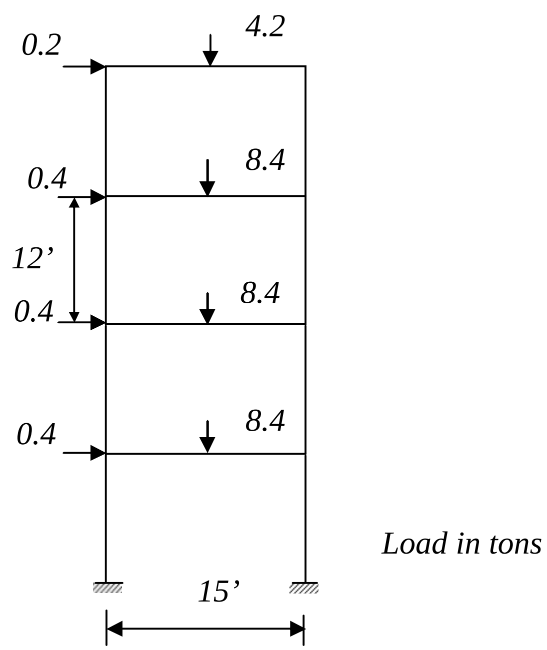


Fig.13 Frame analysed by Wood

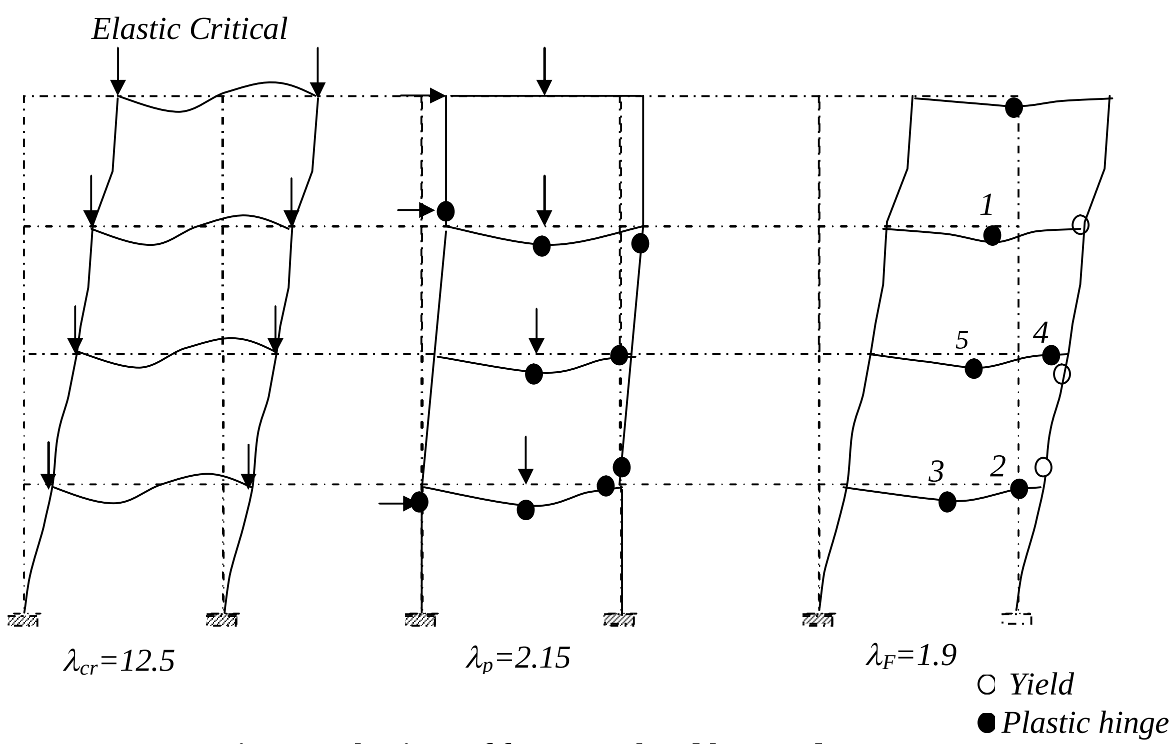


Fig. 14 Behaviour of frame analysed by Wood

The following are identified with respect to “deteriorated” critical load condition.

λ_{cr} = elastic critical load factor
λ_p = rigid plastic critical load factor
λ_G = rigid plastic critical load considering members between hinges formed.
λ_F = deteriorated critical load factor without the energy component of these parts which are plastically deforming
λ_y = load factor at on set of yield.

Wood analysed the frame shown in Fig.13. The values he obtained λ_{cr} , λ_p and λ_f the corresponding behaviour of the example frame is reproduced in Fig.14. The deterioration of critical load λ_D for the partially plastified structure is shown in Fig.15. None of the deteriorated structures correspond exactly to the actual structure with hinges at failure [Fig.14(*c*)] λ_F obtained at elastic plastic failure is *1.90*. Though ten hinges are required for the rigid plastic collapse of the frame, nearly two hinges are sufficient to reduce the elastic critical load by nearly *50%*.

Such a complete analysis as discussed above is required for a realistic estimate of deteriorated critical load. In the absence of sophisticated Computer Programme to carry out such an analysis, a simplified method is required for considering the deteriorated critical load for use by designers. Such an empirical approach proposed by Merchant Rankine Wood Equation is discussed in the next section.

8.0 SIMPLIFIED EMPIRICAL APPROACH USING MERCHANT – RANKINE – WOOD EQUATION

An examination of Fig.12 reveals that λ_{cr}, the elastic critical value is too high and cannot be reached. If rigid plastic behaviour is assumed the critical load is represented by the drooping curve *GH* descending from λ_p the rigid plastic load factor. Merchant suggested that realistic failure load λ_F can be expressed as a function of λ_p and λ_{cr}. According to original Merchant Rankine Equation.

$$\frac{1}{\lambda_F} = \frac{1}{\lambda_p} + \frac{1}{\lambda_{cr}} \tag{20}$$

This is shown in Fig.16 in which $\frac{\lambda_F}{\lambda_p}$ is plotted vertically against $\frac{\lambda_{MR}}{\lambda_{cr}}$. If we call the failure load as Merchant – Rankine load λ_{MR} then

$$\lambda_{MR} = \frac{\lambda_p \lambda_{cr}}{\lambda_p + \lambda_{cr}} \tag{21}$$

Wood suggested a modification of Merchant Rankine load considering strain- hardening and restraint provided by cladding if

$$\frac{\lambda_{cr}}{\lambda_p} > 10; \text{ then } \lambda_F = \lambda_p \quad (22)$$

and

$$\lambda_F = \lambda_{MRW}$$
$$= \frac{\lambda_p \lambda_{cr}}{\lambda_p + 0.9\lambda_{cr}}$$
$$= \frac{\lambda_p}{0.9 + \frac{\lambda_p}{\lambda_{cr}}} \quad (23)$$

when $10 > \frac{\lambda_{cr}}{\lambda_p} > 4$; then

Consider stocky structures i.e.

with $\frac{\lambda_{cr}}{\lambda_p} > 10$ or $\lambda_{cr} > 10$ and $\lambda_p > 1$

ensures that structures have adequate strength. For slender structures

$$10 > \frac{\lambda_{cr}}{\lambda_p} > 4 \quad (24)$$

or

$4.6 < \lambda_{cr} < 10$ as used in BS 5950 the values of $\lambda_p \geq \frac{0.9\lambda_{cr}}{(\lambda_{cr} - 1)}$

and $\lambda_{cr} > 4.6$ (25)

This is applicable to clad frames in which no account has been taken of cladding.

These equations are modified for unclad frames or frames where stiffness of cladding is considered as indicated below:

$\lambda_{cr} \geq 5.75$ or (26)

$$5.75 \leq \lambda_{cr} < 20 \quad (27)$$

$$\lambda_p \geq \frac{9.5\lambda_{cr}}{(\lambda_{cr} - 1)} \quad (28)$$

when

$\lambda_{cr} \geq 20$; use $\lambda_p \geq 1$ (30)

Thus the method involves finding λ_{cr}, the elastic critical load and λ_p the rigid plastic critical load and then appropriate equation satisfied based on whether the frame is a clad one or otherwise.

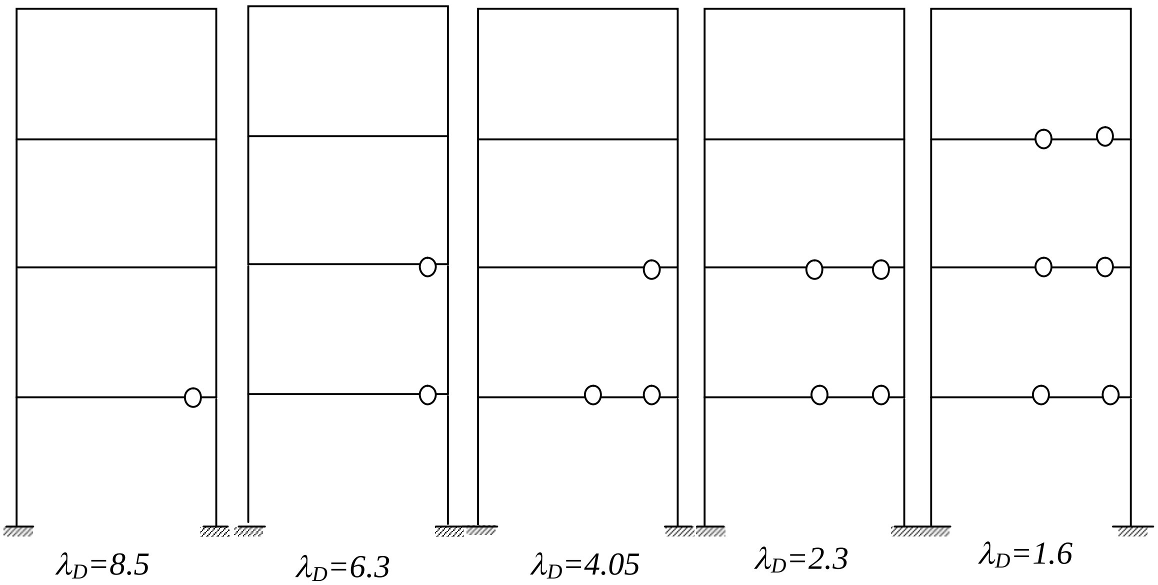
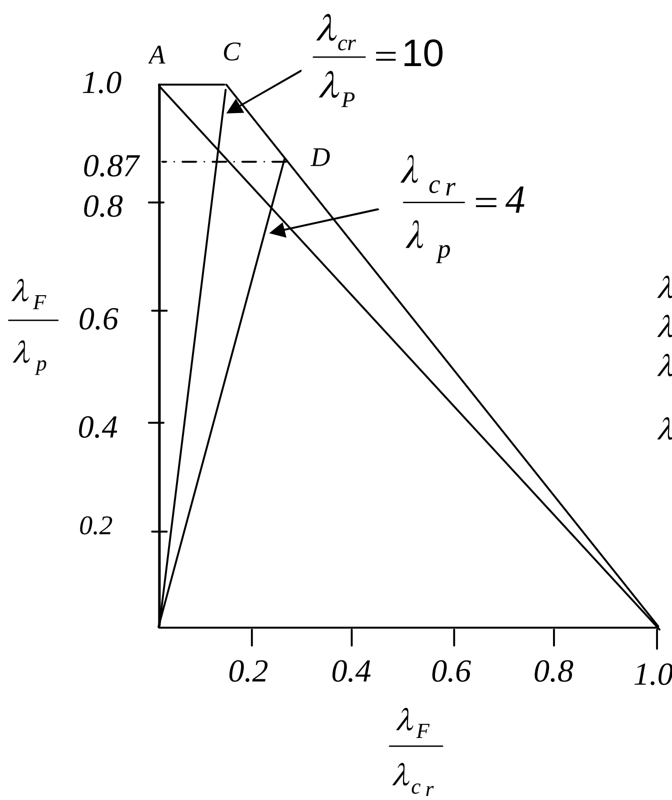


Fig. 15 Deteriorated Critical Loads of Frame analysed by Wood



λ_F= Failure Load
λ_p =Rigid plastic collapse load
λ_c=Elastic critical load

$$\lambda_F = \lambda_{MR} = \frac{\lambda_p \lambda_{cr}}{\lambda_p + \lambda_{cr}}$$

(Merchant Rankine)

Fig. 16 Merchant - Rankine (modified Load)

9.0 PLASTIC DESIGN OF MULTISTOREY RIGID FRAMES

Plastic design of frames can be used for frames, which are effectively braced against out of plane sway.

9.1 Non Sway Frame

The frame should be braced against lateral sway such that it can be classified as a non-sway frame as per the condition explained in Section 2.0. However, while considering the sway, against lateral loads, the bending stiffness of the frame should be ignored, as its buckling resistance will not be available to prevent sidesway when the frame reaches its plastic capacity.

9.2 Sway Frames

Either of the following two methods is used:

a) Rigorous Analysis: A full elastic-plastic sway analysis is performed where proper allowance is made for frame instability effects as indicated in Section 7.2.

b) Simplified Empirical Approach: A simplified frame stability check, as given in Section 7.3, is made using Merchant–Rankine–Wood Equation provided the following conditions are satisfied.

   (i) The beam side-sway mechanism with hinges in all beam ends and at base of columns should be applicable. There should not be other hinges in the column, which may lead to premature failure.
   (ii) The column in the ground floor should be designed to remain within elastic limit.
   (iii) Under the combination of unfactored load and notional horizontal load to simulate sway (wind force not included), forces and moments in the frame should be within elastic limit.

10.0 SUMMARY

In this chapter the behaviour of multistorey frames under lateral loads is described. Elastic design of multistorey rigid frames using simplified design method as well as amplified sway method have been included. Stability consideration of sway frames under Elastic-Plastic failure loads have been included. Finally plastic design of multistorey frame using simplified approach has been is presented.

11.0 REFERENCES

1. Wood. R.H, "Effective Length of Columns in Multi-storey Building", The Structural Engineer, Vol.52, No.7-9, July – Sept. 1974.

2. Kirby. P.A, and Nethercot. D.A, "Design for Structural Stability", SCI/(Constrado) Monograph 1979.

3. Kirby. P.A, "Continuous Multi-storey Frames" – Lecture 11, Introduction of Steel Work Design to BS 5950 Part 1, SCI publication 069, 1988.

4. Horne. M.R; "Frame instability and the Plastic Design of Rigid Frames", Chapter 1, Structural Engineer 53, 242 (1975).

41 FABRICATION AND ERECTION OF STRUCTURAL STEELWORK

1.0 INTRODUCTION

The steel-framed building derives most of its competitive advantage from the virtues of prefabricated components, which can be assembled speedily at site. Unlike concreting, which is usually a wet process conducted at site, steel is produced and subsequently fabricated within a controlled environment. This ensures high quality, manufacture offsite with improved precision and enhanced speed of construction at site.

The efficiency of fabrication and erection in structural steelwork dictates the success of any project involving steel-intensive construction. Current practices of fabrication and erection of steel structures in India are generally antiquated and inefficient. Perhaps, this inadequate infrastructure for fabrication is unable to support a large growth of steel construction. In India, the fabrication and erection of structural steelwork has been out of the purview of the structural designer. Nevertheless, in the future emerging situation, the entire steel chain, i.e. the producer, client, designer, fabricator and contractor should be able to interact with each other and improve their efficiency and productivity for the success of the project involving structural steelwork. Hence it becomes imperative that structural designers also must acquaint themselves with all the aspects of the structural steel work including the "fabrication and erection," and that is the subject matter of the present chapter to briefly introduce good fabrication and erection practices.

2.0 FABRICATION PROCEDURE

Structural steel fabrication can be carried out in shop or at the construction site. Fabrication of steelwork carried out in shops is precise and of assured quality, whereas field fabrication is comparatively of inferior in quality. In India construction site fabrication is most common even in large projects due to inexpensive field labour, high cost of transportation, difficulty in the transportation of large members, higher excise duty on products from shop. Beneficial taxation for site work is a major financial incentive for site fabrication. The methods followed in site fabrication are similar but the level of sophistication of equipment at site and environmental control would be usually less. The skill of personnel at site also tends to be inferior and hence the quality of finished product tends to be relatively inferior. However, shop fabrication is efficient in terms of cost, time and quality.

Structural steel passes through various operations during the course of its fabrication. Generally, the sequence of activities in fabricating shops is as shown in Table1. The sequence and importance of shop operations will vary depending on the type of fabrication required. All these activities are explained briefly in the subsequent parts of the section.

Table 1: Sequence of activities in fabricating shops

S.No.	Sequence of Operation
1.	Surface cleaning
2.	Cutting and machining
3.	Punching and drilling
4.	Straightening, bending and rolling
5.	Fitting and reaming
6.	Fastening (bolting, riveting and welding)
7.	Finishing
8.	Quality control
9.	Surface treatment
10.	Transportation

2.1 Surface cleaning

Structural sections from the rolling mills may require surface cleaning to remove mill scale prior to fabrication and painting.

Hand preparation, such as wire brushing, does not normally conform to the requirements of modern paint or surface protection system. However in some applications manual cleaning is used and depending on the quality of the cleaned surface they are categorised into Grade St-2 and Grade St-3.

Blast cleaning is the accepted way of carrying out surface preparation in a well-run fabrication shop. Abrasive particles are projected on to the surface of the steel at high speed by either compressed air or centrifugal impeller to remove rust and roughen the surface before applying the coating. By using shot or slag grits, both of which have an angular profile, surface oxides are removed and a rougher surface is obtained to provide an adequate key for metal spraying or special paint. Depending upon the increase in the quality of the cleaned surface, the blast cleaning is categorised into Grade – Sa2, Grade – Sa2½ and Grade Sa- 3.

Flame cleaning is another method of surface cleaning. In this method the surface is cleaned using an oxy-acetylene torch which works on the principle of differential thermal expansion between steel and mill scale. In another method called ‘ the steel piece is immersed in a suitable acid and the scale and rust are removed.

2.2 Cutting and Machining

Following surface preparation, cutting to length is always the first process to be carried out, and this is done by any of the following methods.

2.2.1 Shearing and cropping

Sections can be cut to length or width by cropping or shearing using hydraulic shears. Heavy sections or long plates can be shaped and cut to length by specialist plate shears. For smaller plates and sections, machines featuring a range of shearing knives, which can accept the differing section shapes, are available.

2.2.2 Flame Cutting or Burning

In this method, the steel is heated locally by a pressurised mixture of oxygen and a combustible gas such as propane, which passes through a ring of small holes in a cutting nozzle. The heat is focussed on to a very narrow band and the steel melts at 1500^0 C when a jet of high-pressure oxygen is released through a separate hole in the centre of the nozzle to blast away the molten metal in globules. The desired cuts are obtained quickly by this process. However due to a rapid thermal cycle of heating and cooling, residual stresses and distortion are induced and hence structural sections that are fabricated using flame cutting are treated specially in the design of structural steelwork.

2.2.3 Arc Plasma Cutting

In this method, the cutting energy is produced electrically by heating a gas in an electric arc produced between a tungsten electrode and the workpiece. This ionises the gas, enabling it to conduct an electric current. The high-velocity plasma jet melts the metal of the work piece. The cut produced by plasma jet is very clean and its quality can be improved by using a water injection arc plasma torch. Plasma cutting can be used on thicknesses upto about *150 mm* but the process is very slow.

2.2.4 Cold Sawing

When a section cannot be cut to length by cropping or shearing, then it is normally sawn. All saws for structural applications are mechanical and feature some degree of computer control. There are three forms of mechanical saw - circular, band and hack. The circular saw has a blade rotating in a vertical plane, which can cut either downwards or upwards, though the former is more common. Band saws have less capacity. Sections greater than *600 mm X 600 mm* cannot be sawn using band saws. The saw blade is a continuous metal edged, with cutting teeth, which is driven by an electric motor. Hack saws are mechanically driven reciprocating saws. They have normal format blades carried in a heavy duty hack saw frame. They have more productivity than band saws.

2.3 Punching and Drilling

Most fabrication shops have a range of machines, which can form holes for connections in structural steelwork. The traditional drilling machine is the radial drill, a manually operated machine, which drills individual holes in structural steelwork. But this method has become too slow for primary line production. Therefore, larger fabricators have installed NC (Numerically Controlled) tooling, which registers and drills in response to keyed in data. These can drill many holes in flanges and webs of rolled steel sections simultaneously. It is also possible to punch holes, and this is particularly useful where square holes are specified such as anchor plates for foundation bolts. While this method is faster compared to drilling, punching creates distortion and material strain hardening around the holes, which increase with material thickness. Its use is currently restricted to smaller thickness plates. In order to reduce the effect of strain hardening and the consequent reduction in ductility of material around punched holes, smaller size (*2 mm* to *4 mm* lesser than final size) holes are punched and subsequently reamed to the desired size.

2.4 Straightening, Bending and Rolling

Rolled steel may get distorted after rolling due to cooling process. Further during transportation and handling operations, materials may bend or may even undergo distortion. This may also occur during punching operation. Therefore before attempting further fabrication the material should be straightened. In current practice, either rolls or gag presses are used to straighten structural shapes.

Gag press is generally used for straightening beams, channels, angles, and heavy bars. This machine has a horizontal plunger or ram that applies pressure at points along the bend to bring it into alignment. Long plates, which are cambered out of alignment longitudinally, are frequently straightened by rollers. They are passed through a series of rollers that bend them back and forth with progressively diminishing deformation.

Misalignments in structural shapes are sometimes corrected by spot or pattern heating. When heat is applied to a small area of steel, the larger unheated portion of the surrounding material prevents expansion. Upon cooling, the subsequent shrinkage produces a shortening of the member, thus pulling it back into alignment. This method is commonly employed to remove buckles in girder webs between stiffeners and to straighten members. It is frequently used to produce camber in rolled beams. A press brake is used to form angular bends in wide sheets and plates to produce cold formed steel members.

2.5 Fitting and Reaming

Before final assembly, the component parts of a member are fitted-up temporarily with rivets, bolts or small amount of welds. The fitting-up operation includes attachment of previously omitted splice plates and other fittings and the correction of minor defects found by the inspector.

In riveted or bolted work, especially when done manually, some holes in the connecting material may not always be in perfect alignment and small amount of reaming may be required to permit insertion of fasteners. In this operation, the holes are punched, *4 to 6 mm* smaller than final size, then after the pieces are assembled, the holes are reamed by electric or pneumatic reamers to the correct diameter, to produce well matched holes.

2.6 Fastening Methods

The strength of the entire structure depends upon the proper use of fastening methods. There are three methods of fastening namely bolting, riveting and welding. A few decades back, it was a common practice to assemble components in the workshop using bolts or rivets. Nowadays welding is the most common method of shop fabrication of steel structures. In addition to being simple to fabricate, welded connection considerably reduce the size of the joint and the additional fixtures and plates. However, there is still a demand for structural members to be bolted arising from a requirement to avoid welding because of the service conditions of the member under consideration. These may be low temperature performance criteria, the need to avoid welding stresses and distortion or the requirement for the component to be taken apart during service e.g. bolts in crane rails or bolted crane rails.

2.7 Finishing

Structural members whose ends must transmit loads by bearing against one another are usually finished to a smooth even surface. Finishing is performed by sawing, milling or other suitable means. Several types of sawing machines are available, which produce very satisfactory finished cuts. One type of milling machine employs a movable head fitted with one or more high-speed carbide tipped rotary cutters. The head moves over a bed, which securely holds the work piece in proper alignment during finishing operation.

Bridge specifications require that sheared edges of plates over a certain thickness be edge planed. This is done to remove jagged flame cut edges and the residual stresses at the edges. In this operation, the plate is clamped to the bed of milling machine or a planer. The cutting head moves along the edge of the plate, planing it to a neat and smooth finish.

The term finish or mill is used on detail drawings to describe any operation that requires steel to be finished to a smooth even surface by milling, planing, sawing or other machines.

2.8 Surface Treatment

Structural steelwork is protected against corrosion by applying metal or paint coating in the shop or at site.

2.8.1 Metal Coatings

The corrosion protection afforded by metallic coating largely depends upon the surface preparation, the choice of coating and its thickness. It is not greatly influenced by the method of application. Commonly used methods of applying metal coating to steel surfaces are hot-dip galvanising, metal spraying, and electroplating. Electroplating is generally used for fittings and other small items.

Galvanising is the most common method of applying a metal coating to structural steelwork. In this method, the cleaned and fluxed steel is dipped in molten zinc at a temperature of about 450^0C. The steel reacts with molten zinc to form a series of zinc or iron alloys on its surface. As the steel workpiece is removed, a layer of relatively pure zinc is deposited on top of the alloy layers. For most applications galvanised steel does not require painting.

An alternative method of applying metallic coating to structural steelwork is by metal spraying of either zinc or aluminium. The metal, in powder or wire form, is fed through a special spray gun containing a heat source, which can be either an oxy-gas flame or an electric arc. Molten globules of the metal are blown by a compressive jet on to the previously blast cleaned steel surface. No alloying occurs and the coating, which is produced, consists of porous overlapping platelets of metal. The pores are subsequently sealed, either by applying a thin organic coating which soaks into the surface, or by allowing the metal coating to weather, when corrosion products block the pores.

2.8.2 Paint Coatings

Painting is the principal method of protecting structural steelwork from corrosion. Paints are usually applied one coat on top of another, each coat having a specific function or use.

The primer is applied directly on to the cleaned steel surface. Its purpose is to wet the surface and to provide good adhesion for subsequently applied coats. Primers for steel surfaces are also usually required to provide corrosion inhibition. They are usually classified according to the main corrosion-inhibitive pigments used in their formulation, e.g. zinc phosphate, zinc chromate, red lead, and metallic-zinc. Each of these inhibitive pigments can be incorporated into a range of binder resins e.g. zinc phosphate alkyd primers, zinc phosphate epoxy primers, zinc phosphate chlorinated-rubber primers.

The intermediate coats (or undercoats) are applied to build the total film thickness of the system. This may involve application of several coats. The finishing coats provide the first-line defence against the environment and also determine the final appearance in terms of gloss, colour etc. They also provide UV protection in exposed condition. Intermediate coats and finishing coats are usually classified according to their binders, e.g. vinyl finishes, urethane finishes.

The various superimposed coats within a painting system have, of course, to be compatible with one another. They may be all of the same generic type or may be different, e.g. chlor-rubber base intermediate coats that form a film by solvent evaporation and no oxidative process, may be applied on to an epoxy primer that forms a film by an oxidative process which involves absorption of oxygen from the atmosphere. However, as a first precaution, all paints within a system should normally be obtained from the same manufacturer. The reader may refer to *IS:487(1985)* to know more about the surface treatment using paints.

Detailed treatment of corrosion protection systems will be found in the Chapter on 'Corrosion, fire protection and fatigue considerations of steel

2.9 Welded connections

Welding is used extensively for joining metals together and there is no doubt that it has been a most significant factor in the phenomenal growth of many industries. The different terminology used in welds are explained in *IS:812(1957)*.
A welded joint is made by fusing (melting) the steel plates or sections along the line of joint. The metal melted from each member of the joint unites in a pool of molten metal, which bridges the interface. As the pool cools, molten metal at the fusion boundary solidifies, forming a solid bond with the parent metal. When solidification completes, there is a continuity of metal through the joint.

There are five welding process regularly employed namely:

(i) Shielded Metal Arc Welding (SMAW)
(ii) Submerged-Arc Welding (SAW)
(iii) Manual Metal-Arc welding (MMA)
(iv) Metal-Active Gas welding (MAG)
(v) Stud welding

All these methods of welding has been described with illustrations, in the chapter on 'Welds - Static and Fatigue Strength - I'. Nevertheless, for the sake of completeness, these methods are briefly enumerated below.

2.9.1 Methods of welding

(1) Shielded Metal Arc Welding (SMAW)

This is basically a semi-automated or fully automated welding procedure. The type of welding electrode used would decide the weld properties. Since this welding is carried out under controlled condition, the weld quality is normally good.

(2) Submerged-Arc welding (SAW)

This is fully mechanised process in which the welding head is moved along the joint by a gantry, boom or tractor. The electrode is a bare wire, which is advanced by a motor. Here again, since the welding is carried out in controlled conditions, better quality welds are obtained.

(3) Manual Metal-Arc welding (MMA)

This is the most widely used arc welding process and appears to be advantageous for labour intensive Indian construction practices. As it is manually operated it requires considerable skill to produce good quality welds. Hence in the case of MMA, stringent quality control and quality assurance procedures are needed. In India, the Welding Research Institute, BHEL, Trichy, Tamil Nadu, conducts periodical courses for welders and weld inspection personnel. Welders who are employed in actual fabrication are, infact, graded according to their training and skills acquired.

(4) Metal-Active Gas welding (MAG)

This process is sometimes referred to as Metal-Inert Gas (MIG) welding. It is also manually operated. A gas that does not react with molten steel shields the arc and the weld pool. This protection ensures that a sound weld is produced free from contamination-induced cracks and porosity. Nevertheless, this procedure also depends on the skills of the welder.

(5) Stud welding

This is an arc welding process and is extensively used for fixing stud shear connectors to beam in the composite construction. The equipment consists of gun hand tool (Fig.1(a) and 1(b)), D.C. power source, auxiliary contractor and controller. The stud is mounted into the chuck of the hand tool and conical tip of the stud is held in contact with the work piece by the pressure of a spring on the chuck. As soon as the current is switched on, the stud is moved away automatically to establish an arc. When a weld pool has been formed and the end of the stud is melted the latter is automatically forced into the steel plate and the current is switched off. The molten metal, which is expelled from the interface, is formed into a fillet by a ceramic collar or ferrule, which is placed around the stud at the beginning of the operation. The ferrule also provides sufficient protection against atmospheric contamination (Figs. 1(a) and 1(b)).

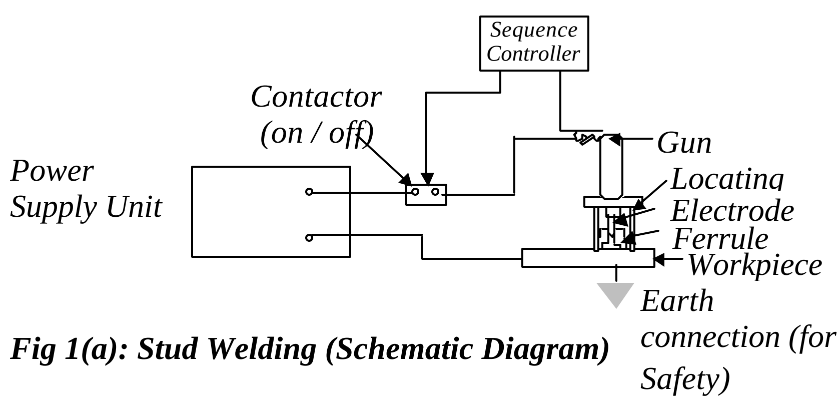
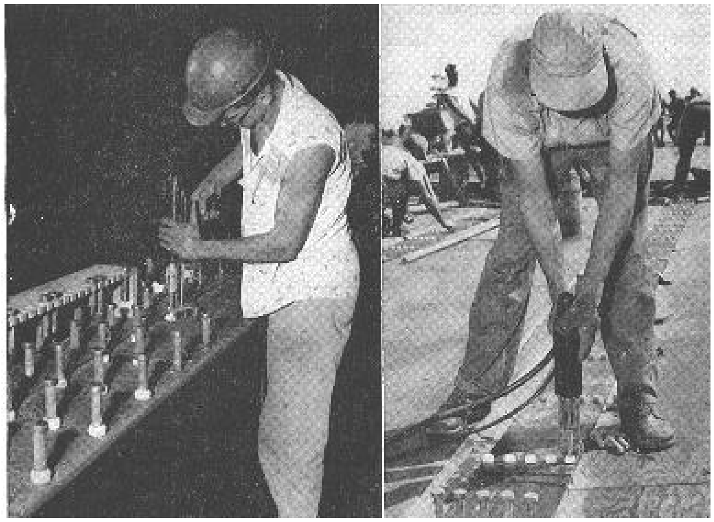


Fig 1(a): Stud Welding (Schematic Diagram)

This process offers an accurate and fast method for attaching shear connectors, etc with the minimum distortion. While it requires some skill to set up the weld parameters (voltage, current, arc time and force), the operation of equipment is relatively straight forward.



(a) Shop welding (b) Site welding

Fig.1 (b) Stud Welding on composite beam

3.0 RESIDUAL WELDING STRESSES AND DISTORTION

3.1 Residual welding stresses

When a weld such as a butt weld is completed and begins to cool the hot weld and parent metal contracts longitudinally. The surrounding cold parent metal resists this contraction so that the weld is subjected to a tensile stress. This is balanced by the compressive stresses induced in the cold regions of the parent plate. These self-equilibrating forces introduce residual stresses both in the longitudinal and transverse direction. These stresses can even reach yield stress. Hence, the fabricator should adopt good fabrication practices that reduce the detrimental effect of residual stresses.

3.2 Residual distortions due to welding

3.2.1 Butt welds

Fig. 2 shows a typical angular rotation of the plates due to a single V butt weld. This occurs because the major part of the weld is to one side of the neutral axis of the plate. This induces greater contractile stresses on that side. A double V or double U butt weld preparation reduces this distortion.

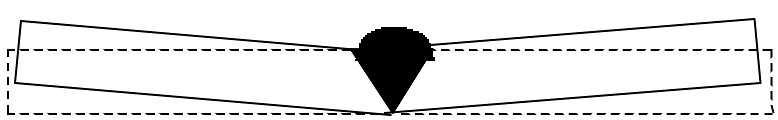


Fig 2: Angular distortion of butt weld

The welding sequence for double preparation has an important influence on the resultant distortion. If a few weld runs are first made on one side, and the plate turned over and then the same number of runs are made on the second side (i.e., sequential welding), a 'balanced' weld will be produced with little distortion. This will not, of course, be possible in situations where rotation of the plate is impracticable such as a plate, which is part of a large fabrication.

One aspect of butt-welding that should be noted is where back gouging is necessary to produce a full penetration weld. This can lead to distortion because the back gouging will produce bigger weld on the second side about the neutral axis of the plate. Such distortion can be reduced using an unsymmetrical weld section. Single V butt welds may produce cusping as shown in Fig.3 if the overall plate is restrained. This can be reduced by using a double V butt weld.

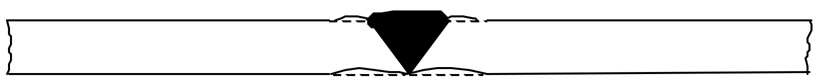


Fig 3: Cusping due to transverse butt weld

3.2.2 Fillet welds

In single and double fillets, shrinkage across the throat area can lead to distortion as shown in Fig.4. The distortion caused by a double fillet weld is important in box or plate girder webs where stiffeners are attached to only one side of the web. The use of a thicker plate can reduce the fillet weld angular distortion due to increased stiffness.

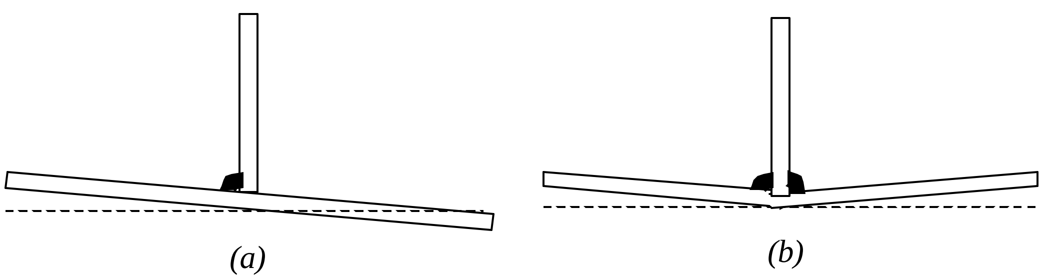
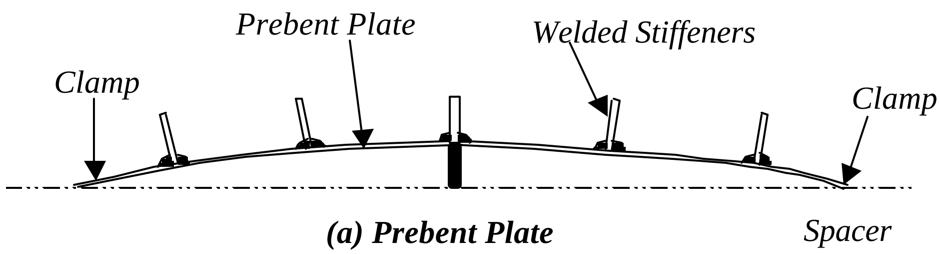


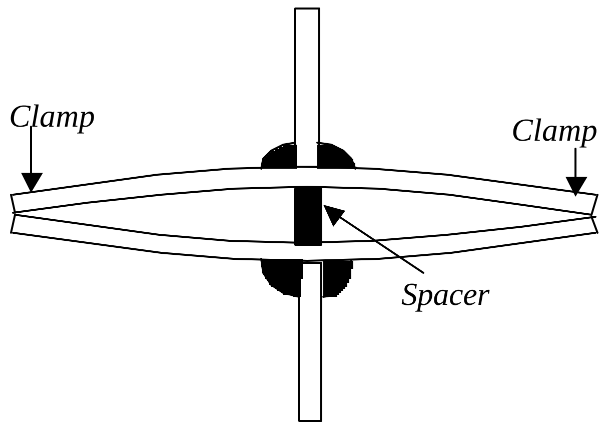
Fig 4: Angular distortion of fillet welds

3.3 Control of distortion

Some distortion from welding is due to transverse and longitudinal contraction of weldments. Adopting suitable methods that can resist contraction can control the distortion. Weld distortion of a flat plate with a series of stiffeners on one side can be countered by elastically prebending the plates. In a similar manner two T sections can be welded, prebent back to back, to prevent final curvature in the web plate. Presetting the flange plate at an angle initially as shown in Fig.5 and Fig.6 may reduce the angular rotation due to a single fillet.



(a) Prebent Plate



(b) Prebent T's

Fig 5: Prebending

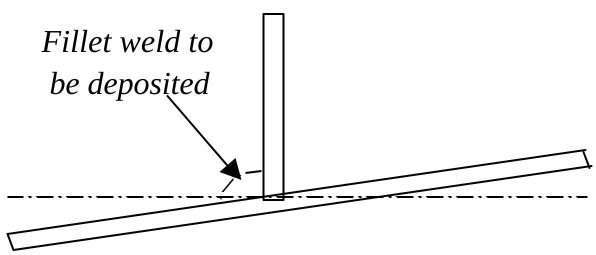


Fig 6: Preset for fillet weld

Sometimes both presetting and prebending may be required, e.g. in plate girder fabrication where the web to flange welds are made automatically. When the welds are made manually, it is customary to put the stiffeners into the girder before the web/flange welds are made; in this way the square profile of the web to flange is maintained. Where automatic welding is employed the stiffeners cannot be put first since they would impede the progress of the automatic machine; in this presetting of the flange plates may be required. Welding should preferably be started at the centre of the fabrication and all succeeding welds from the centre outwards. This allows contraction to occur in the free condition. If the welding sequence is not chosen correctly, locked up stresses at either end of a welded portion can lead to uncorrectable distortions.

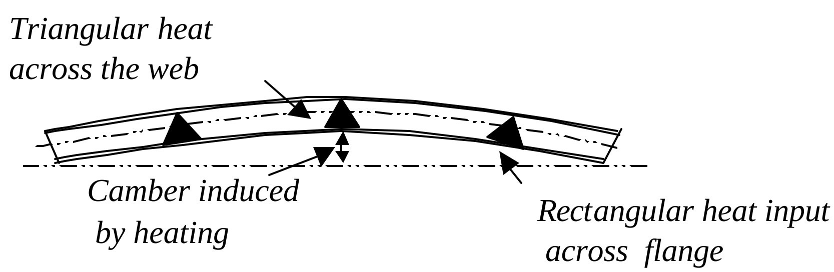
Restraint procedure to reduce the effect of weld distortion should be carefully planned otherwise it can lead to solidification cracking.

3.4 Methods of correcting distortion

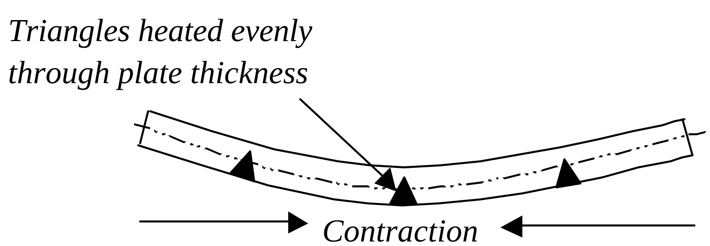
In general, there are two methods available to correct distortion namely:

(a) applying force and (b) heating

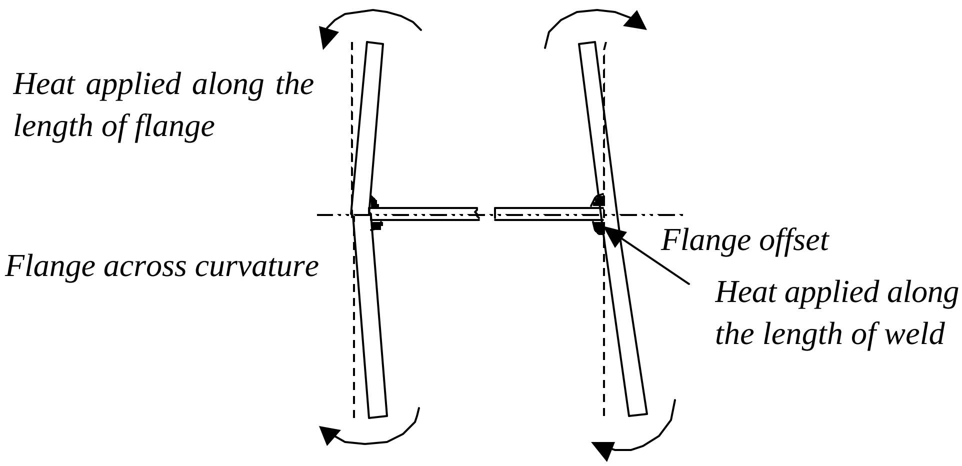
Light sections can be corrected by applying force such as by hydraulic presses and local jacking or wedging. While heavier structures will require heating to apply stresses to reduce or eliminate the distortion. The effect of heating is similar to that of welding in which distortion results from the induced stresses. An area of steelwork will expand when heated but this expansion will be constrained by the surrounding cold unheated area, causing a plastic upset. On cooling, the area contracts and the element then becomes shorter, this principle can be used to correct or induce any curvature. The heat must be evenly applied right through the material, if not, unwanted curvature may occur in the plan of the section. Fig.*7* shows some of the methods to induce and correct distortion. Fig.*7(c)* shows how it can be applied to a H section in which a camber is required. Rectangular heating across the bottom flange will shorten it compared with the top flange and hence induce camber. Since the shortening of the flange in the heated areas may tend to buckle the web adjacent to the flange, the heat is also applied to the web in a triangular manner such that the most affected part of the web contracts with the flange. In a similar manner a cambered plate may be straightened by applying triangular heating with the bases of the triangles parallel to the plate edges to be shortened. When the plate cools the heated edge will shorten and so reduce the camber. For panels in box girder webs, spot heating as shown in Fig.*7(d)* may be employed to reduce the concavity produced by the welding around the panel perimeter. Each spot contracts on cooling and induces a local plate shrinkage within the panel boundary and so reduces the dish. If the heat applied and the web panel thickness are such that there is a large temperature difference between the surfaces of the plate at each spot heat, then the resultant contraction on the hotter surface will produce a greater correction of the dish.



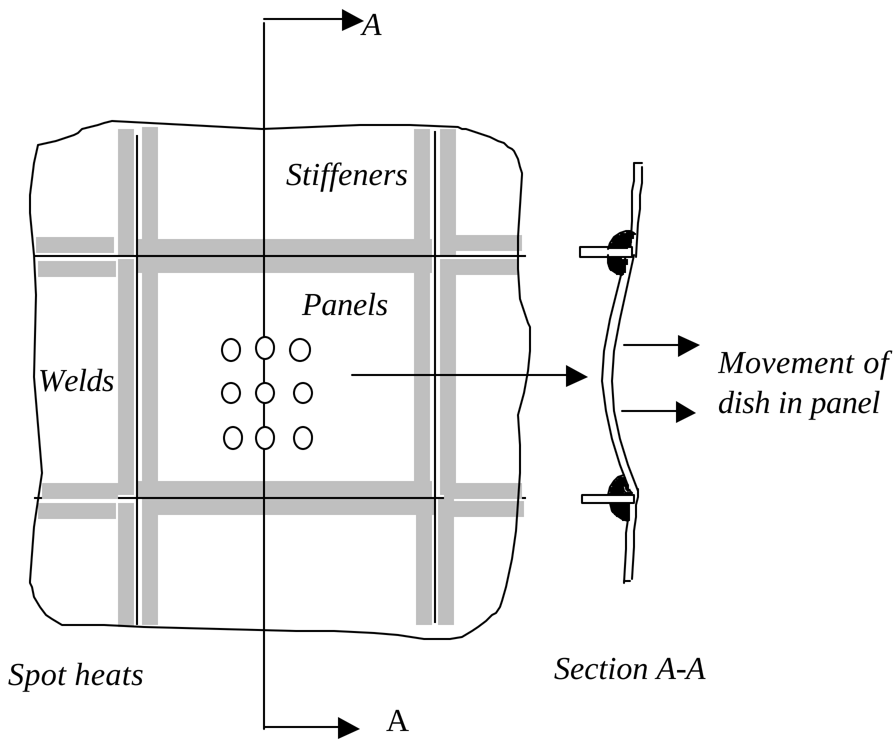
7(a) Camber of beam by heating



7(b) Cambered Plate



7(c) Distorted flanges



7(d) Spot Heating of dished Panel

Fig 7. Methods of correction of distortion

3.5 Defects in welds

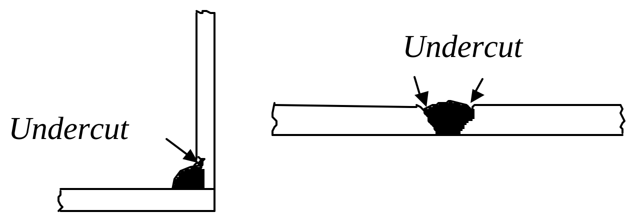
Faulty welding procedure can lead to defects in the welds, thereby reducing the strength of the weld.
Fig.*8* shows some of the common defects in welds. Some of these are:

(i) Undercut

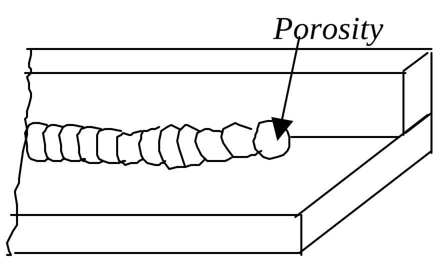
(ii) Porosity
(iii) Incomplete Penetration
(iv) Lack of side wall fusion
(v) Slag inclusions
(vi) cracks

All these weld defects are discussed in the chapter on 'Weld – Static and Fatigue strength – I'. It should be emphasised that a 'theoretical 100% error free' weld is not achievable in practice. While good quality welds are the priority of welders and weld inspectors, minor defects do normally creep in. Hence these defects are assessed during a weld inspection.

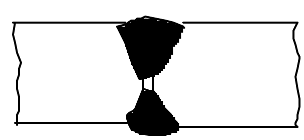
If the defects are within acceptable limits, they are accepted. If not, alternative measures of rectification may have to be carried out. Table *2* shows nature of some of the defects and their acceptability limits.



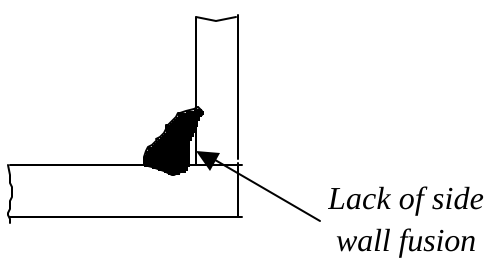
(a) Undercut



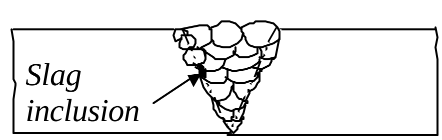
(b) Porosity



(c) Lack of Penetration



(d) Lack of side wall fusion



(e) Slag inclusion

Fig8: Defects in welds

Table 2: Nature of defects and their acceptability limits.

Nature of Defect	Acceptance Norms	Disposition
1. Crack, Lack of Fusion	Not accepted	Confirm by Magnetic Particle Inspection, repair and retest.
2. Crater	Not accepted	Fill by weld deposit.
3. Undercut	Upto *0.8 mm* accepted	Fill and grind smooth.
4. Porosity for butt or fillet welds	One pore of dia. < *2.4 mm* every *100 mm* length is permitted. However pores of dia. > *2.4 mm* not accepted	To be repaired.

4.0 QUALITY CONTROL IN FABRICATION

Quality assurance during fabrication assumes utmost importance in ensuring that the completed structure behaves in the manner envisaged during design stage. Any deviation from these design considerations as reflected in detail drawings may introduce additional stresses to the structure and affect its strength and durability. This section discusses the relevant aspects in fabrication and erection, which need to be considered to achieve the desired quality.

4.1 Fabrication

A fabricator's work starts from the point of procurement of raw materials including fasteners and ends with the dispatch of the fabricated items to site for erection.

In order to ensure that the fabrication can be carried out in accordance with the drawings, it is necessary that inspection and checking is carried out in accordance with an agreed Quality Assurance Plan (QAP). The plan should elaborate on checks and inspections of the raw materials and also of the components as they are fabricated, joined etc.

During the last two decades, fabrication activities have increased steadily in yards adjacent to work. In the absence of controlled environment (as in an organised workshop), the quality of workmanship of such fabrication is likely to suffer. It has, therefore, become all the more important to motivate the fabricators to appreciate the usefulness of Quality Assurance Plans and introduce the system in all their works and at site as well.

4.1.1 Imperfections in Fabrication

Structural steelwork cannot be fabricated to exact dimensions and some degree of imperfection is bound to occur during fabrication process. The limits of various imperfections are spelt out in the specifications. In the design, these are accounted by adopting a factor of safety for material. However, in some components an increase of imperfection beyond these limits may lead to reduction in the strength and durability of the structure e.g., imperfections on the straightness of the individual flanges of a rolled beam or a fabricated girder results in the reduction of strength of the girder due to lateral torsional buckling which may cause an overall bow in the girder. This, in turn, may generate twisting moments at the supports.

As a rule all columns and struts should be checked for straightness on completion of fabrication. Also, all rolled and fabricated girders should be checked over a distance in the longitudinal direction equal to the depth of the section in the region and points of concentrated load.

4.1.2 Making holes

Excessive cold working of structural steel can cause reduction in ductility, embrittlement and cracking. Punching holes is a cold-working operation and can, therefore, cause brittle fracture. This becomes critical for the durability of structures subjected to fluctuation of stresses such as railway bridges and crane girders. Under cyclic loading fatigue cracks can initiate from such punched holes. In such cases, holes for bolts may be formed either by drilling or by punching undersize holes followed by reaming to desired size. Drilling is preferable to punching, because it reduces the chances of brittle fracture. Studies show that punching may produce short cracks extending radially from the hole, thereby enhancing the possibility of initiating brittle fracture at the hole when the member is loaded. Even in statically loaded structures the maximum thickness of plates in which holes can be punched is restricted.

4.1.3 Shop assembly and camber check

For important structures particularly for bridges, it is necessary to have the fabricated units temporarily assembled at the place of fabrication before these are dispatched to site for erection. During this operation, the overall dimensions of the structure including alignment, squareness, camber etc. should be confirmed. Inadequate or erroneous camber, in fact, introduces huge secondary stresses in the members instead of eliminating these as originally desired. Shop assembly also ensures that the open holes drilled in various units are within tolerable limits.

4.1.4 Welded joints

As presented in the previous sections, welded joints are very important as far as the quality control of the joints is concerned. It is well known that joints are the last straw of strength in structural steelwork. Any poor quality weld would detrimentally affect the joint and in turn affect the performance of the whole structure itself. Hence welded joints need thorough inspection during and after the fabrication. Different methods of Non-Destructive Testing (NDT) and evaluation of welds are available. The NDT procedures are elaborated in the chapter on 'Welds Static and Fatigue Strength – I'. Depending upon the severity of service loading, the QAP may call for the level of NDT to be used. Guidance could be obtained from *IS:822(1970)* for the inspection of the welded joints.

5.0 ERECTION

5.1 General

Erection of steel structures is the process by which the fabricated structural members are assembled together to form the skeletal structure. The erection is normally carried out by the erection contractor. Generally the steps that are involved in the erection of steel structures are shown in Table 3. The erection process requires considerable planning in terms of material delivery, material handling, member assembly and member connection. Proper planning of material delivery would minimise storage requirement and additional handling from the site storage, particularly heavy items. Erection of structural steel work could be made safe and accurate if temporary support, falsework, staging etc. are erected. Before erection the fabricated materials should be verified at site with respect to mark numbers, key plan and shipping list. The structural components received for erection should be stacked in such a way that erection sequence is not affected due to improper storing. Care also should be taken so that steel structural components should not come in contact with earth or accumulated water. Stacking of the structures should be done in such a way that, erection marks and mark numbers on the components are visible easily and handling do not become difficult. From the earlier discussion it should emphasised that safe transportation of fabricated items to the site, their proper storage and subsequent handling are the pivotal processes for the success of fabrication of structural steel work.

Table 3: Sequence of Activities during Erection

S.No.	Sequence of Operation
1.	Receiving material from the shop and temporarily stacking them, if necessary.
2.	Lifting and placing the member and temporarily holding in place.
3.	Temporarily bracing the system to ensure stability during erection.
4.	Aligning and permanently connecting the members by bolting or welding.
5.	Connecting cladding to the steel structural skeleton.
6.	Application of a final coat of painting.

Guidance for handling and storage for material shall be obtained from *IS: 7969(1975*). The fabrication at shop or site should be so planned that units to be handled weigh nearly the same. The erection drawing should reach the site of construction well in advance to plan the erection sequence and material handling. Erection should be carried out with the help of maximum possible mechanisation. Normally anyone or more of the material handling systems, such as tower crane; crane mounted on rails, crawling crane, pneumatic tire mounted crane, and derrick crane may be used for handling the material. Details of the above said erection equipments can be found in any standard textbooks on construction equipment.

A variety of methods can be employed for the erection of a structure. Normally, the selection of the method is influenced by the type of the structure, site conditions, equipment, quality of skilled labour, etc. available to the erector.

However, regardless of the method adopted the main aim during erection is the safety and preservation of the stability of the structure at all times. Most structures which collapse do so during erection and these failures are very often due to a lack of understanding on someone's part of what another has assumed about the erection procedure. Hence, it is emphasised that as far as strength and stability of the components during erection are concerned, they must satisfy the provision of *IS: 800(1984)*. For the guidance on general fabrication and erection of structural steel work, Chapter 11 of IS:800 (1984) must be followed. As far as safety is concerned guidance could be obtained from Indian safety code for structural steelwork *IS:7205(1974)*. Before the commencement of the erection, all the erection equipment tools, shackles, ropes etc. should be tested for their load carrying capacity. Such tests if needed may be repeated at intermediate stages also.

5.2 Bracings

During the entire erection period, the steelwork should be securely bolted or otherwise fastened and braced to take care of the stresses from erection equipment or the loads carried during erection. In addition to this, adequate provisions to resist lateral forces and wind loads during erection should also be made according to local conditions.

Normally bracings are built into all types of structures to give them a capability to withstand horizontal forces produced by wind, temperature and the movements of crane and other plant in and on the building. Bracings can be permanent or temporary.

Temporary bracings required at some stages of the work must have properly designed connections and should be specifically referred to in the erection method statement.

The decision on sequence of erection such as which member should be erected first for providing initial stability to the structure or whether temporary bracings should be used for this purpose should be taken at an early stage of planning of the erection process. Fig.*9* illustrates this point. As permanent bracings have been provided in AB, bay erection should logically start from AB bay to give stability and ensure proper alignment of the erected structure. In case, for some reason erection has to start from DE bay, it would be necessary to provide temporary bracings in this bay. The bracing system should be retained till the permanent bracings are fixed in the AB bay. Any mis-alignment at initial stage will impair the performance, of the structure when completed. Early or unauthorised removal of temporary bracings is a common cause of collapse in a partially completed frame.

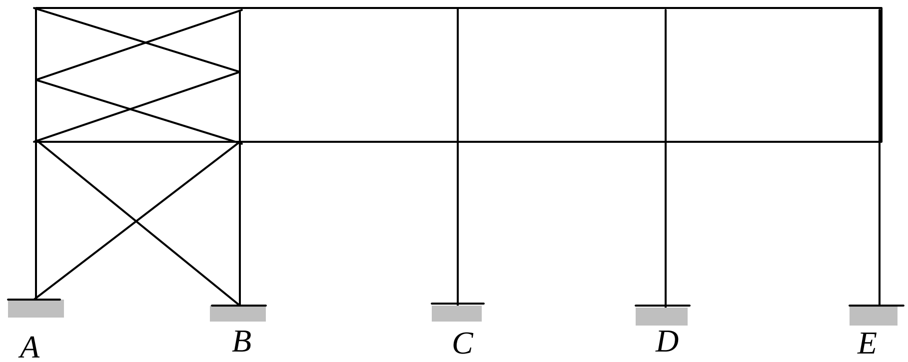


Fig 9. Bracing System

Having considered the need for installing temporary bracings and the need to postpone fixing permanent bracings, consideration should be given to the overall economy of retaining the temporary bracings and perhaps leaving out the permanent bracings. It is a costly and potentially dangerous business to go back into a structure solely in order to take out temporary members, or to insert components that had to be left out temporarily.

5.3 Maintaining tolerances

The best way of erecting a structure within the acceptable tolerance limits is to make sure that accuracy is achieved from the very beginning of the job.

Table 4: Maximum permissible tolerance in erected steel structures

S.No	Description	Tolerance in(mm)
1.	**Columns:** Out of plumbness of column axis from true vertical axis	
(i)	Heights upto *30 m*	$\pm\ell/1000$ or ± 25 whichever is less
(ii)	Heights over *30 m*	$\pm\ell/1200$ or ± 35 whichever is less
2.	**Trusses:** Lateral shift in location of truss from its true vertical position	± 10
3.	**Crane girders and ribs:** Shift in plane of alignment with respect to true axis of crane rail.	± 5
4.	**Chimney and towers:** Out of plumbness (vertically from true vertical axis)	*1/1000* of the height of the chimney or tower

Thus quality control must start from the setting out of the foundations and the holding down bolts. This operation is often done at a stage when site conditions are disorderly and most untidy and the environment appears to be incongruous to accuracy. However, inaccuracies in marking the centrelines and the levels of foundations allowed at this stage are likely to cause misfit in the connections and misalignment of the structure leading to secondary stresses in the members. In such areas corrective measure must be taken by way of locally modifying some of the components so as to eliminate the mismatch. Table 4 shows some typical tolerances that are accepted in structural steel work.

5.4 Joints

Most steel structures are fabricated by either bolting or welding in the shop and bolting or welding in the field. Durability of a structure largely depends on the quality of the joints made at site.

In bolted connections, care should be taken to ensure that all parts intended to be bolted together should be in contact over the whole surface and the surfaces should be thoroughly cleaned and painted with specified primer paint and the two matching plates or sections secured together while the paint is still wet by service bolts. After erection, the joint should be made by filling not less than *50%* of the holes with bolts. The service bolts are to be tightened. The holes that need enlargement to admit bolts or rivets should be reamed only after carefully examining the extent of the inaccuracy and the effect on the soundness of the structure. Such holes must not be formed by gas cutting process. The contact surfaces in HSFG connection if painted will develop lesser friction and this should have been accounted for in design. The fundamentals of HSFG connections are elaborated in the chapter on Bolted connections.

For connections to be done by welding, the components should be securely held in position to ensure alignment, camber etc., before welding is commenced.

In the case of field assembly using bolts the number of washers for the permanent bolts should not be more than two (and not less than one) for the nuts and one for the bolt head. It is desirable to use wooden rams and mallet to force the members in position so as to protect steelwork from injury and shock. It should also be ensured that the bolts project through the nut by atleast one thread. In the case of field assembly by welding almost all the precautions needed for shop welding may be followed. In the case of High Strength Friction Grip (HSFG) bolts the material surfaces should be absolutely free from grease, lubricant, dust, rust etc. and shall be thoroughly cleaned before assembling. The nuts should be pretensioned by a torque–wrench or by the turn of the nut method with the help of pneumatic wrench/lever. After tightening the bolt heads, nuts and edges of the mating, surfaces should be sealed with a coat of paint to obviate entry of moisture. In the case of connections such as base plate they must be aligned and levelled using wedges/ shims and subsequently filled by grouting.

6.0 SUMMARY

In this chapter the importance of fabrication and erection in structural steelwork is underlined. The various tasks involved in fabrication are discussed. The joints, which are important components in structural steelwork are explained, with a special emphasis on welded joints. Some aspects of erection of steel structures are also presented. Thus an overall view of the fabrication and erection of structural steelwork is covered in this chapter.

REFERENCES

1. Pratt.J.L., " Introduction to the Welding of Structural Steelwork", 3rd revised edition, The Steel Construction Institute, 1989.

2. Owens G.W and Knowles P., 'Steel Designers manual', 5th Edition, The Steel Construction Institute, ELBS Blackwell Scientific Publishers, London 1994.

3. Ghoshal.A., " Quality Assurance and Training for Fabrication and Erection of Durable Steel Structures", Journal of the Indian National Group of the International Association for Bridge and Structural Engineering, vol.30, n1, Feb. 2000.

4. *IS:812-1957* Glossary of terms relating to welding and cutting of metals.

5. *IS:822-1970* Code of procedure for inspection of welds.

6. *IS:7205-1974* Safety code for erection of structural steelwork.

7. *IS: 7969-1975* Safety code for handling and storage of building materials.

8. *IS:*487-1985 Brush, Paint and Varnish (I) oval ferrule bound (ii) round ferrule bound (Third revision).

9. *IS 800- (1984)* Code of practice for general construction in steel, Chapter 11.

42 LEARNING FROM FAILURES: CASE STUDIES

1.0 INTRODUCTION

In pre-industrial societies, once a craft-based technique or thumb-rule for design was judged adequate for building an artefact, it was not considered necessary to develop it any further. The methods of design of buildings in those societies changed very slowly over time. Nevertheless, medieval society was indeed developing although at a relatively slow pace, leading eventually to the construction and erection of large and visible structures. Generally, these buildings symbolised the greatness or valour of a particular emperor or the glory of a particular God or religion. The impressive temples built by the great Chola or Pandia Kings in South India or the great Gothic Churches and Cathedrals in Europe (particularly in Italy) are excellent examples, which are impressive even by today's standards. The enhanced functional requirements of such buildings have continued to challenge the designers and technological pressures have continued to grow. For example, there has been an increasing demand to achieve the longest possible spans and the greatest possible heights in most prestigious buildings. In their desire to meet their clients' or patrons' needs, the designers did sometimes stray beyond the limits of contemporary technology and buildings and cathedrals collapsed as a consequence. This was the case with Beauvais Cathedral, which – when built - was considered to be the most daring achievement in Gothic Architecture. When its roof collapsed in *1284,* its restoration consisted of using tie rods of iron to hold the Gothic arches together, suggesting that the original designers had clearly over-reached themselves in the design of arches. (As is well known, arches are mainly compression structures, and develop horizontal thrust under purely vertical loads. We need sturdy supports to resist these thrusts. Clearly, there was design error in this case).

New developments in design are often the direct consequence of lessons learnt from previous failures, which are caused when the designers went too far beyond the state-of-the art or the contractors did not implement the design intent in the construction.

The development of scientific methods and reasoning, which started in the *17th* century, led to the ability to predict the forces to which a structure might be subjected. This led to the ability to validate structural designs – at least to some extent – in advance of construction. The process of industrialisation of societies also ensured the production of new materials whose properties could be predicted (unlike the natural materials - like stone - which they replaced). This combined with increase in knowledge and development of new materials actually led to the occurrence of more failures, principally as a result of enhanced demand for many types of novel structures for which there were no historical precedents, (for example, railway bridges).

2.0 THE NEED FOR FORENSIC STUDIES

Post mortem is an exact science. By employing it, we can establish the illness, which caused the death of the patient with a high degree of certainty. Many advances in Medical Sciences have been made possible by a systematic compilation of the results of post-mortem studies.

Engineering Designers, on the other hand, have been reluctant to reflect openly upon the causes of design failures, thus denying themselves and the profession an opportunity to understand the limitations of the particular design concept and improve the methodology. For example, by 1840 the British Engineers had simply abandoned the design development of suspension bridges, following the collapse of Menai Strait Bridge and suspension structures at Brighton Pier. All these failed in high winds, due to inadequate stiffening of the decks, a deficiency not recognised by the designers at the time.

Rather than interpreting the failures as an indictment of the form chosen, a contemporary American Engineer John Roebling collected case studies and established the forces - not hitherto considered - which must be designed against in order to build a successful suspension bridge. This resuscitated the suspension bridge technology. The famous English bridge-builder, Robert Stephenson, whose design of a Trussed Girder for Dee Bridge failed because of a very low factor of safety, was no doubt embarrassed but was candid enough to admit that ***" nothing was so instructive to the younger members of the profession, as the records of the accidents in large works and the means employed in repairing the damage"***. There were indeed plenty of bridge failures both in the U.S. and in Great Britain during the latter half of *19th* century and much discussion of the catastrophic failures did, in fact, take place. These influenced the design development of a number of new forms of the bridges. The cantilever bridge across the Firth of Forth (the Forth Bridge) designed by Benjamin Baker is a good example of this new development and was adopted by several bridge builders the world over. An editorial titled " The
neering News (*1887*) noted that ***" …. There is no Engineer who, if he will look back upon the past and be honest with himself will not find that his most valuable and most effective instruction has come from his own failings…... Structures which fail are the only ones which are really instructive, for those which stand do not in themselves reveal whether they are well designed or so overly designed as to be wasteful of materials and resources…. The natural impulse of those who are in anyway responsible for failures….is to keep the matter as quiet as possible ….something not difficult to do in cases where there is no great catastrophe or loss of life".***

It is clear that much can be learned through the failure of a structure rather than a study of structures, which are successful. The proper appreciation of the causes of failure helps us to refocus on our conceptual understanding of structural behaviour. We could then assess our analytical models, which are essential for successful design practice, and help us to exercise proper engineering judgement.

Many design decisions are inevitably based on engineering judgement, which does not merely come from an understanding of theory or a powerful command of computational tools. Even extensive design experience in an academic context can only provide limited perspectives in engineering judgement. Most fruitful lessons in engineering judgement are obtained from the case histories of failures, which point invariably to examples of bad judgement; these, naturally, provide guideposts for negotiating around the pitfalls in conceptual design. They also offer invaluable insights into the potential trip-wires in early attempts at innovative design and construction. In many cases, important new principles of engineering science may be brought out in the study of failure case studies.

Some structural failures are caused due to:

(1) Poor communication between the various design professionals involved, e.g. engineers involved in conceptual design and those involved in the supervision of execution of works.

(2) poor communication between the fabricators and erectors.

(3) Bad workmanship, which is often the result of failure to communicate the design decisions to the persons, involved in executing them.

(4) Compromises in professional ethics and failure to appreciate the responsibility of the profession to the community at large could also result in catastrophic failures.

Other common causes of structural failure are summarised below:

- lack of appropriate professional design and construction experience, especially when novel structures are needed.
- complexity of codes and specifications leading to misinterpretation and misapplication.
- unwarranted belief in calculations and in specified extreme loads and properties.
- inadequate preparation and review of contract and shop drawings.
- poor training of field inspectors.
- compressed design and/or construction time.

In this chapter some case studies of failure are presented. In each case study, the possible learning points, technical aspects and ethical implications are also discussed.

3.0 POOR CONCEPTUAL DESIGN

3.1 Tacoma Narrows Bridge

The destruction of the Tacoma Narrows Bridge by aerodynamic forces subsequently revolutionised the thinking of structural engineers, on how wind loading could affect large slender structures. This is a good example of errors in Conceptual Design.

In *1940*, Tacoma Narrows Bridge was opened across Tacoma Narrows in Washington State. On Nov *7, 1940,* with a wind speed of about *60 km/h* (well below the design wind speed), the bridge began twisting and oscillating violently. As a result the tie down cables intended to stiffen the bridge snapped, causing the entire structure to crash into the river below.

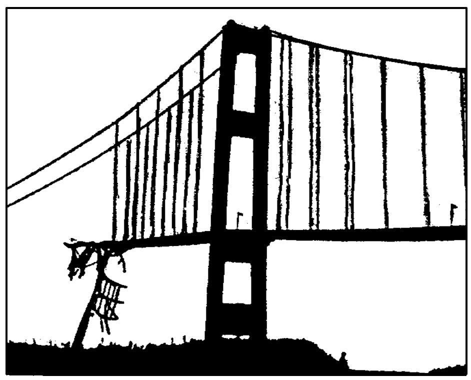


Fig. 1: Tacoma Narrows Bridge

Investigations (subsequent to the collapse) showed that the excessive vertical and torsional oscillations (which occurred prior to failure) were the result of extraordinary degree of flexibility of the structure and its relatively small capacity to absorb the dynamic forces. The deck was too narrow for the span and thus its torsional rigidity was inadequate. The plate girders, which were provided for stiffening, had insufficient flexural rigidity and little torsional rigidity. Their elevation caused wind vortices above and below the deck in moderate and steady winds. From the day bridge was opened very substantial horizontal and vertical movements of the deck in waveforms were noticeable even in moderate wind and high traffic.

The failure was indeed caused by a lack of proper understanding of aerodynamic forces and knowledge of torsional rigidity in the whole profession. It was not realised by the designers that the aerodynamic forces (which had proven disastrous in the past to much higher and shorter flexible suspension bridges) would affect a structure of such magnitude as the Tacoma Narrows Bridge, despite the fact that its flexibility was greatly in excess of that of any other long span suspension bridge.

It is clearly dangerous to exceed the design paradigm without fully understanding the forces one is dealing with and the limitations of applicability of current design concepts.

3.2 Millennium Bridge at London

This *320 m* span Aluminium and Stainless Steel Bridge across the River Thames in London was opened on *10* June *2000* amidst a lot of fanfare. It is the first river crossing to be built in London, after Tower Bridge (completed in *1884*) and links St. Paul's Cathedral (in the North Bank) and the new Tate Modern and Globe Theatre (in the South Bank).

In many ways it is an unusual structure. Sir Norman Foster, a famous British Architect claimed to have designed it in association with an Artist, Sir Antony Caro and the Engineers were Ove Arup and Partners, a distinguished firm of consulting Engineers. From the start, Foster emphasised the innovative nature of its design. The objective was " to push the suspension bridge technology as far as possible, to create a uniquely thin bridge profile, forming a slender blade across the River Thames". Jonathan Duffy, a BBC commentator remarked " It sounds great and on paper, probably looked sublime, but often reality is the harshest judge of cutting edge Architects". The bridge was made of Aluminium decking and stiffened by suspension cables in the horizontal plane. No attempt was made to stiffen it in the vertical plane.

During the first weekend (*10-11* June *2000*), some *160,000* persons crossed the bridge essentially because of its novelty. As people began to cross, it became apparent that the bridge was swaying several inches from side to side. The transient population on the bridge swayed drunkenly as they walked in synchrony, as if choreographed. The bridge was indeed wobbling dangerously over very deep waters. Many felt sea-sick while crossing. It was obvious that the bridge was not adequately stiffened to resist gravity loading. An American visitor remarked that " the design of the bridge looks as flimsy as some of the rope bridges seen in Indiana Jones films....".

The bridge had to be closed to traffic after having been open only for two days. The Engineers/Designers are hoping to install dampers (similar to shock absorbers) to reduce the oscillations to a minimum (acceptable) level.

This case study illustrates the dangers of over confidence. The designers had extrapolated the established Technology into untested (and dangerous) situations. It is true that dozens (if not hundreds) of Bridges have been built all over the world. Nevertheless it remains the case that all the suspension bridges (as indeed all the structures) should be adequate both with respect to "strength" as well as "stiffness".

4.0 DESIGN INADEQUACY

Cleddau Bridge, Milford Haven, (UK)

The failure of three box girder bridges during erection in *1970* in quick succession revealed the need for a radical re-examination of the prevailing design methodology for Thin Plated Structures and their erection.

On *2* June *1970*, Cleddau Bridge in Milford Haven failed during its erection by cantilevering segments of the span, out from the piers. The bridge was designed as a single continuous box girder of welded steel. The span that collapsed was the second one on the south side. The boxes were fabricated in sections and moved over the previously built sections, aligned in place and welded. The collapse occurred when the last section of box for the second span was being moved out along the cantilever. This section slid forward down the cantilever buckled, at the support and collapsed into the river (Fig *2*), killing four men, including the site-engineer.

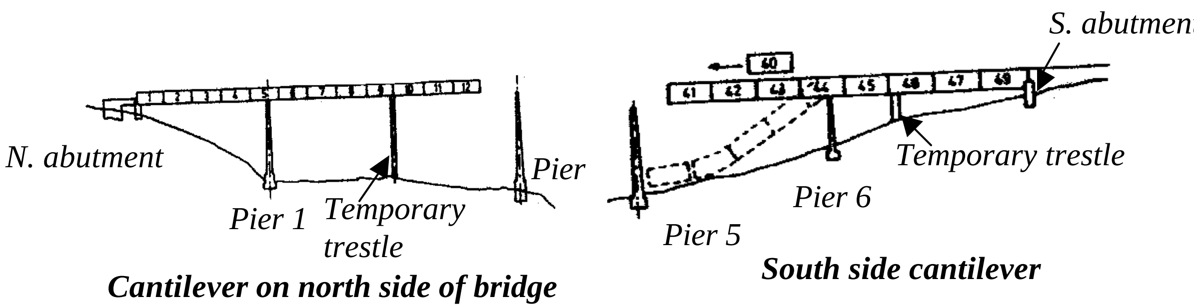


Fig. 2: Failure of Milford Haven Bridge

Investigation of collapse showed that the collapse was due to the buckling of the diaphragm at the support (i.e., at the root of the second span being erected). The diaphragm was torn away from the sloping web near the bottom. This caused reduction in the lever arm between flanges resisting negative bending moment at the support. The tendency of the bottom flange to buckle was inevitably increased by the reduction of the distance between the flanges, as this increased the force needed in each flange to carry the moment with the reduced lever arm.

The support diaphragm was, in effect, a transverse plate girder, which carried heavy loads from the webs of the plate girder at its extreme ends and was supported by the bearings as shown in Fig *3*. It was therefore subjected to a hogging bending moment and a large vertical shear force. The shear of the transverse girder and diffusion of the point load from the bearings were compounded with the effects of inclination of the webs of the main bridge girder. These produced an additional horizontal compression and out-of-plane bending effects caused by bearing eccentricity.

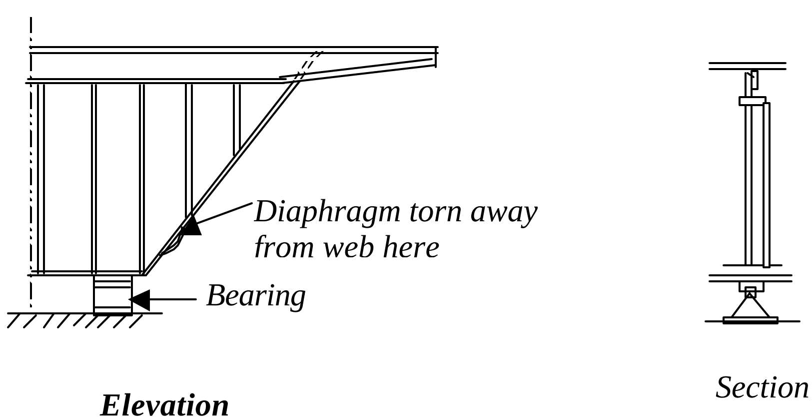


Fig. 3: Diaphragm over Pier6 of Milford Haven Bridge

The total load transmitted by the diaphragm to the bearings just before collapse was computed as *9700 kN*. This load would not have caused any problem provided the diaphragm was designed to carry it. Allowing for likely values of distortion and residual stress, the calculated design strength, using design rules that were drafted subsequently, was found to be as low as *5000 kN*. Thus, the failure was essentially due to design inadequacy.

5.0 POOR COMMUNICATION BETWEEN THE DESIGNER AND THE FABRICATOR

Hyatt Regency Walkway Collapses

The case study presented here focuses on the professional responsibilities of structural engineers as they assume overall responsibility for their designs. It also focuses on the need for a uniform understanding of the means by which specific responsibilities are communicated between the members of project team.

On *7th* July *1981*, a dance was being held in the lobby of the Hyatt Regency Hotel, Kansas City. As spectators gathered on suspended walkways above the dance floor, the support gave way and the upper walkway fell on the lower walkway, and the two fell onto the crowded dance floor, killing *114* people and injuring over *200*.

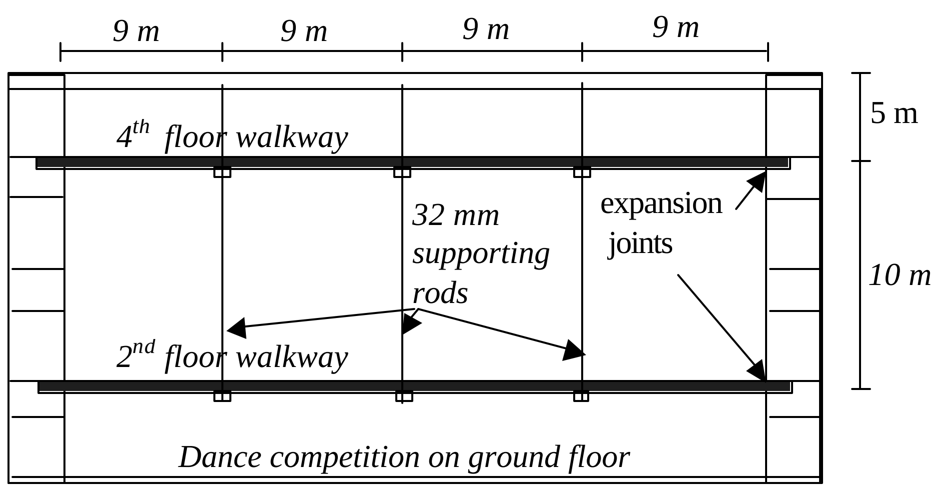


Fig. 4: Kansas City Hyatt Hotel: arrangement of walkways

The two walkways were supported above one another and suspended from the ceiling by hanger rods as shown in Fig *4*. The walkways were supported on box beams, which were made of two steel channels, welded together.

In the original design a single rod supported the two walkways as shown in Fig. *5(a)*. But the originally designed hanger detail for the two walkways was altered at the time of fabrication as shown in Fig. *5(b)*. The second floor walkway was suspended from the fourth one as shown. As a result, the connection between the fourth floor cross beam and the hanger supported double the load originally intended as shown in Fig *6(b)*. Examination of the box beams supporting the upper walkway after the collapse showed that the upper hanger rod had pulled through the beam. The beam design was also unsatisfactory, and this condition was aggravated by the increased load on the nut. The nut pulled through the box beam as shown in Fig. 7.

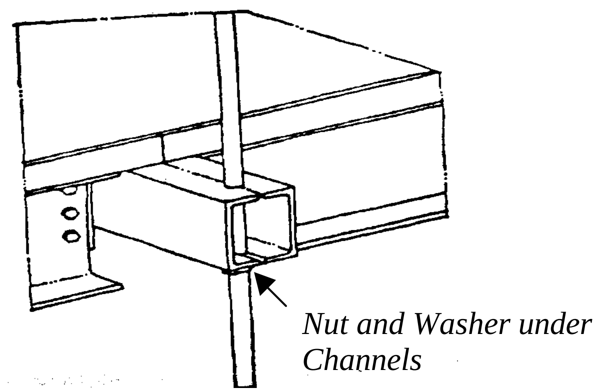


Fig. 5(a): Hyatt Regency Hanger Details As-Designed

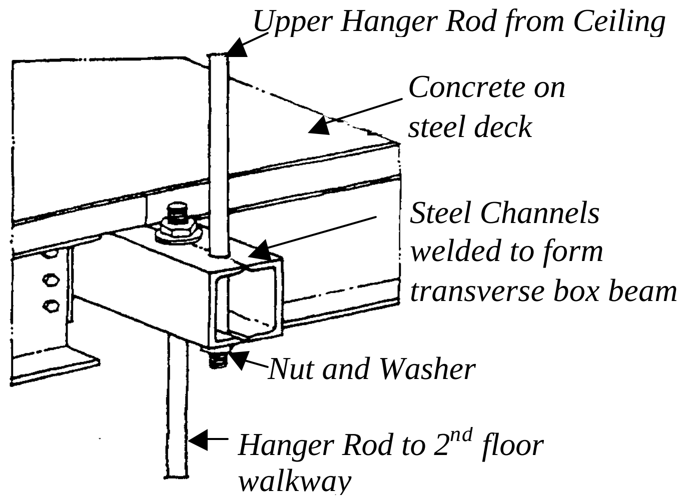


Fig. 5(b): Hyatt Regency Hanger Details As-Built

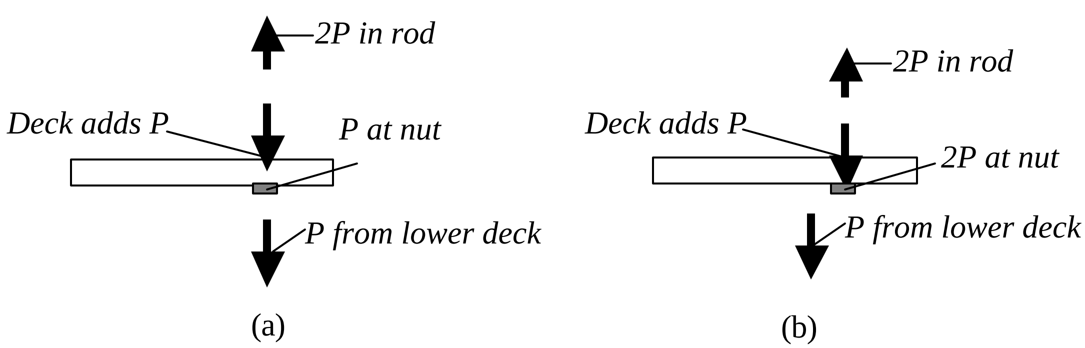


Fig. 6: Free-Body Diagram (a) As Designed (b) As Built

After the investigations, it was found that the steel fabricator who built the hanger detail requested a change in detail from the originally designed detail. The engineer approved it without checking the calculations. This accident occurred due to the carelessness of the engineer concerned as he failed to understand the importance of the details he had changed. It also illustrates the importance of understanding the force flow in the joints and that of what is often considered as minor detail.

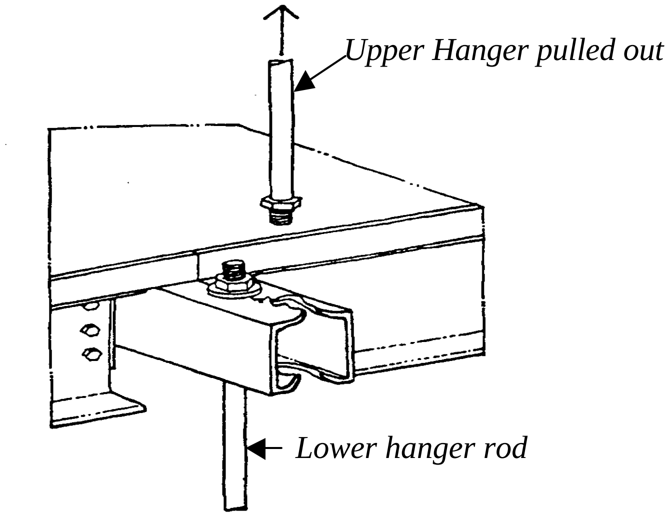


Fig. 7: Pulled -Out Rod at Fourth-Floor Box Beam

6.0 POOR DETAILING

King's Bridge, Melbourne

The next case study is an example of poor detailing compounded by poor communication and a lack of necessary inspection.

Kings Bridge in Melbourne is one of the relatively few examples of failure in service. It was opened in *1961*, but only *15* months later, on *10th* July *1962*, it failed when a *45*-ton vehicle was passing over it. Collapse was only prevented by a wall, which had been built to enclose the space under the affected span.

The superstructure consisted of many spans in which each carriageway was supported by four steel plate girders spanning *30 m*, and topped with a R.C.C deck slab as shown in Figs. *8(a)* and *8(b).* Each plate girder bottom flange was supplemented by an additional cover plate in the region of high bending moment. The cover plate was attached to the flange by a continuous *5 mm*-fillet weld all around (see Fig. *9)*

An investigation into the possible cause of failure indicated that the failure was due to brittle fracture and many other spans of the bridge were in danger of similar failure. Cracks were found in the main tension flange plate of the affected span under seven of the eight transverse fillet welds. One crack had extended such that tension flange was completely severed, and the crack had extended halfway up the web.

Investigation also revealed that difficulties were experienced during welding. Special care to avoid unnecessary restraints during welding was not taken, despite the specifications. The longitudinal welds were made before the transverse welds. As a result there was a complete restraint against contraction, when transverse welds were made. Moreover, transverse welds were made in three stages. In some instances cracks were caused in the main flange plate by the first run and later covered up by a subsequent one. In many other cases a crack was caused by the last run and later covered up by priming paint before the girders left the factory. The penetration of later paint coats into the cracks showed that they had often extended further before the bridge was opened for traffic.

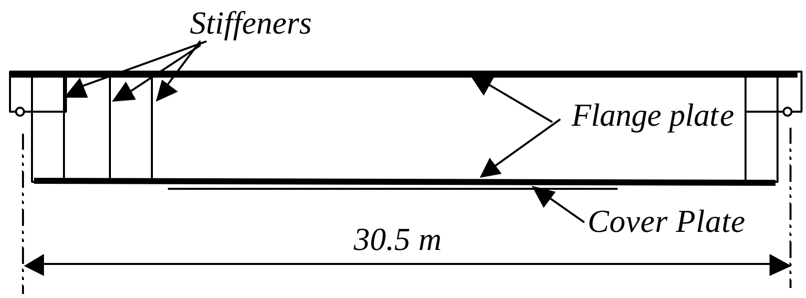


Fig. 8(a) Girder Elevation

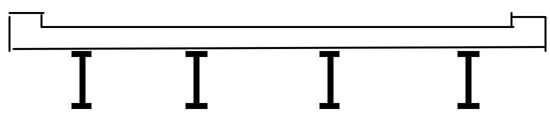


Fig. 8(b) Girder Cross section

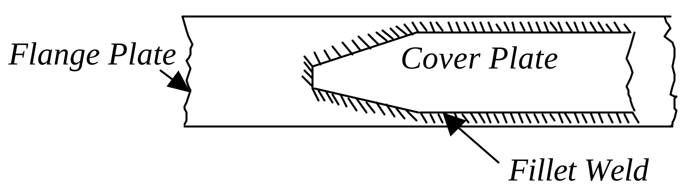


Fig. 9: fillet weld

The results of investigations clearly indicated that, the failure of King's Bridge was due to carelessness of those who fabricated the girders as well as those who inspected the bridge. It was also found that the most likely and most dangerous cracks were regularly missed by inspectors, who had carefully got the less harmful longitudinal cracks cut out and repaired.

7.0 POOR JUDGEMENT

Quebec Bridge Failure

The following case study is intended to show how, the errors of the judgement of the engineers could lead to the failure of the structure and loss of so many lives.

On *29* August *1907*, the partially constructed south cantilever arm of the Quebec Bridge in Canada collapsed killing *75* workmen due to the grave error made in assuming the dead load for the calculations. Even when this error was subsequently noticed the designer chose to ignore it, relying on the margin of safety inherent in his design.

The bridge was intended to carry rail traffic across the St. Lawrence River at Quebec. It was designed and built under the supervision of Theodore Cooper, doyen of American bridge builders in the late *19th* century. The bridge consisted of giant truss cantilevers on two main piers, with a suspended span in the middle.

Two compression chords (made of lattice construction) in the south cantilever arm failed by the shearing of their lattice rivets. As the distress spread through the entire superstructure, the nineteen thousand tons of the south anchor, the cantilever arms and the partially completed centre span thundered down onto banks of the St.Lawrence River and into the water the bridge had been designed to cross.

Investigation report on the major events leading to the accident is summarised below.

The bridge was put for a ‘ design and construct ‘ contract although the original specification of T. Cooper was followed. Originally the bridge was designed for a span of *1600 ft*. Later, the span was increased to *1800 ft*, considering both engineering and expenditure. For this Cooper had provided modified specifications that would allow for high unit stresses. Accordingly, the design calculations were revised but due to an oversight the added dead weight in the increased span was not included in the calculations and fabrication of steelwork began.

After placing the first steelwork on site, Cooper realised that the weights of the fabricated components were not corresponding to previously estimated dead loads and that the working stresses were in fact *7* to *10%* greater than that allowed by specification. But Cooper decided that the increase in stresses was safe and permitted work to continue.

During construction, Cooper was once informed that some problems were encountered in riveting the bottom chord splices of south anchor arm on account of their faced ends not matching. But Cooper instructed that the work should continue, as it was not a serious matter.

When work on the central, suspended span proceeded, the rapidly increasing stresses (and the consequent buckles) on the compression members became intolerable. Later the end details of the compression chords began to buckle. The buckles started developing in an alarming fashion leading to the collapse of the structure.

Thus the bridge, subject to hasty design decisions, came to an untimely end. The court of enquiry found a number of factors, which had contributed to the accident. Among these were the unusually high permissible stresses allowed in the specification and the lack of communication between consultant, designers and the site management. It was recognised, however, that these were factors, which only served to aggravate the main cause of failure, which was that the designer had failed to provide the main compressive load bearing members with adequate strength. In the subsequent enquiry and investigations it became clear that the lacing system and the splice joints of the compression members were not able to resist the effects of the buckling tendency of the compression members

8.0 POOR INSPECTION AND MAINTENANCE

<u>Silver Bridge Collapse</u>

Silver bridge collapse is considered to be one of the failures that had been very influential. It led to the approval of the *1968* National Bridge Inspection Standards by the U.S. Congress. Built to specifications, this American Suspension Bridge was completed in *1928* and failed in *1967*. The cause of failure was a fracture in an eyebar link resulting from a crack which had grown through stress, corrosion and corrosion fatigue.

A brief report on the causes of failure is summarised below.

On Dec *15, 1967* Silver Bridge, considered to be first eyebar suspension bridge in the United States, collapsed without warning into the Ohio River. The bridge was spanning Ohio River between Point Pleasant, West Virginia, and Gallipolis, Ohio. The collapse occurred when the bridge was crowded with heavy traffic resulting in the loss of *46* lives and nine injuries.

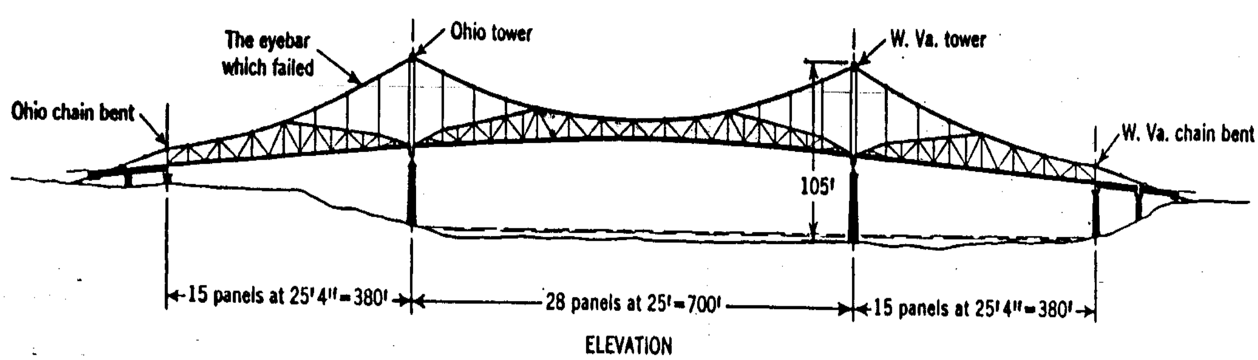


Fig. 10 a: Silver Bridge at Point Pleasant

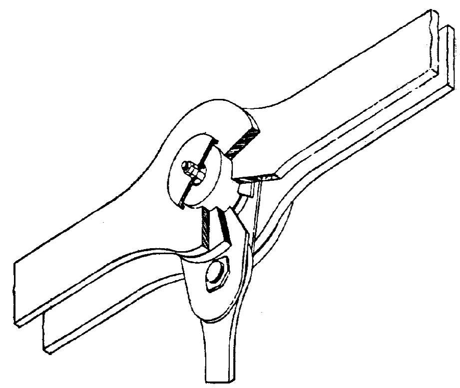


Fig. 10 b: Detail of Eyebar Chain Joint

A thorough investigation revealed that the collapse of the bridge was caused by the failure of the eyebar at the first panel point west of Ohio tower as shown in Fig.*10*.

At the beginning, the first joint of the eyebar, west of the Ohio tower came apart. As a result of the separation of the joint and the failure of the eyebar, the Ohio tower fell eastward. The collapse continued eastward, causing the West Virginia tower to fall eastward. Thus once the continuity of the suspension system was severed at first panel point west of Ohio tower, the unbalanced forces on each side of that joint caused the bridge to totally disintegrate.

Investigations showed that there were two main elements in the design and construction of the chain that caused the failure - extremely high tensile stresses and corrosion on the inside of the eyebar.

Moreover the chain was composed of two bars, which meant that the breaking of one bar would inevitably result in total instantaneous collapse of the entire bridge. It was also found that the factors of safety for the eyebar design were too low compared to the requirements of the original design. No consideration was given in the design to secondary stresses arising from

- inaccuracies in the manufacture of the bars.
- stresses created by unbalanced loads.
- unequal distribution of the total stress between the two eyebars.
- lack of complete free movement around the pins.

Another undesirable feature of the design was that the eye, where the pin fits, was elongated *3 mm* in a horizontal direction for ease of erection. This detail created an air space where corrosion could develop undetected and unabated. The inspection or lubrication of the inside of the head of the eyebar in the Silver Bridge was impossible without dismantling the joint

Thus the combination of high tensile stresses and corrosion caused a crack on the inside of the eyebar, under the pin at a location of a manufacturing flaw about *6 mm* in size.

The tragedy of the Silver Bridge did not go unnoticed and unrecognised. Its collapse created a huge uproar in the United States. The first major benefit was that it led to the approval of the *1968* National Bridge Inspection Standards by the U.S. Congress (Systematic Bridge Inspection and Evaluation). Another major benefit arising out of the Silver Bridge tragedy is the attention paid to eyebar trusses and details. In particular, tension members composed of two eyebars became suspect and required special attention. Such lower chords were strengthened or replaced. A third benefit was the attention given to all connections: floor beams to trusses, stringers to floor beams, trusses to bearings, and so on. It became necessary to inspect these details with great care.

9.0 POOR CONSTRUCTION

Cracking in suspended floors of a school building.

The next case study (*The Structural Engineer, 1994)* concerns the cracking of a slab caused by the constructor not paying attention to the requirements of the Serviceability Limit State. Although this example concerns a R.C.C slab, it is regarded as important for structural engineers involved in Steel Design as frequently his designs incorporate R.C slab as a component in Composite Construction.

In the school building reported herein, cracks were noticed in the suspended floors. All the cracks were found on the top surfaces of the one-way slabs, on each side of, and parallel to, the beams that were supporting them.

In the course of investigation, surface crack widths were measured. A covermeter survey was carried out near the cracks. In addition, the slab was pierced at a number of locations in order to supplement the covermeter survey and also to measure the slab and plaster thickness. The range of measurements and a comparison with the specifications in the structural drawings are shown in Table *1*.

Table 1 Suspended floor slab parameters

	Specification	Measured range
Slab thickness (mm)	100	83-106
Reinforcement spacing (top) (mm)	225	235-400
Effective depth of top steel (mm)	75	35-55

Table *1*, clearly indicates that poor workmanship was responsible for the cracking observed. There were adverse deviations from the specifications in slab thickness, effective depth, as well as reinforcement spacing.

10.0 POOR CONSTRUCTION PRACTICES

Roof Truss Collapse

The next case study is taken from the *Journal of Performance of Constructed Facilities, ASCE (1992)*. In this example the designer of record did not have any field inspection responsibilities. Construction was left in the hands of contractors who, whether experienced or not, used "customary" installation techniques that left the trusses inadequately braced.

The roof of a shopping centre (consisting of several timber roof trusses) collapsed after two days of snow and rain. Most of the trusses on one side of the centre beam had collapsed and the top of the load-bearing wall had been pushed out. The centre beam was undamaged and undeflected. Investigations showed that the building had been in service for six years. The structure was a rectangular building consisting of *3.7m* high concrete block bearing walls and a wood truss supported system. A steel beam supported by steel pipe columns was installed in the centre of the building running along the longer dimension. The truss system consisted of two monopitch trusses placed peak to peak forming a conventional "A" shaped roof. The pair of trusses spanning between the sidewall and the centre beam, rested on the top flange of the beam but weren't connected. The roof system acted as two independent halves. And the building was subdivided into several stores by non-load bearing partition walls. It was found that the trusses that were still standing on the affected side of the beam had no lateral bracings and none of the internal diagonals had any bracing. The lateral bracings were provided only for the vertical members of the trusses at the beam bearing. The first diagonal members in compression were found to be out of plane by several centimetres. They had failed as load bearing members. An analysis showed that these members, when unbraced, exceeded the allowable length to depth ratio for in plane compression. They were not able to withstand the requisite snow loading. Thus the trusses had been left unbraced and understrength at the completion of construction.

Shop drawings for the trusses produced by its manufacturers showed that two lateral members were required for the first diagonal and one brace was required for the second diagonal. This information was either never furnished to the installer or ignored by the installer. During investigation it was also revealed that though the manufacturers had developed handling and bracing recommendations, many truss installers ignored these guidelines.

11.0 HIGH ETHICAL STANDARDS AND TIMELY ACTION PREVENT A FAILURE

The fifty nine storey crisis

The next case study is an example of the high ethical standards and professionalism characteristic of a competent engineer involved in areas of safety and welfare of the public.

The Citicorp centre, a fifty nine-storey tower in Manhattan, New York designed by William J. Le Meassurier, would have faced a major disaster if a serious error in its design had not been detected in time. He acknowledged the errors done by his team, prepared new plans and got all the necessary changes put into effect to avert a possible disaster.

The Citicorp centre was the seventh tallest building in the world at that time. The tower had twenty five thousand individual steel jointed elements behind its aluminium skin. It was supported on four massive two hundred and seventy eight meter high columns, which were positioned at the centre of each side allowing the building corners to cantilever twenty two metres out. Its wind bracing system consisted of forty-eight braces (in six tiers of eight), arrayed like giant chevrons. A tuned mass damper was also provided to dampen the wind-induced vibrations (due to the heavy mass of the damper, the severity of the vibrations would be reduced).

The trouble started when Le Meassurier learned that the wind braces designed by his team were not checked for diagonal winds, which would result in a forty percent increase in strain in four out of the eight chevrons. Moreover despite the welded joints specified, bolted joints were provided by the contractor as the welded joints were considered to be expensive and stronger than necessary. But if the bracing system was sensitive to diagonal winds, so were the joints that held it together. The joints must be strong enough to resist the moment, which was the difference between the overturning moment caused by wind forces, and the resisting moment provided by the weight of the building.

At any given level of a building, the value of compression would remain constant. Even if the wind blows harder, the structure would not get heavier. Thus immense leverage could result from higher wind forces. In the Citicorp tower, the *40%* increase in stress produced by diagonal winds caused a hundred and sixty percent increase in stress on the bolts at some levels of the building. The assumption of *40%* increase in stress from diagonal winds was theoretically correct, but it would go higher in reality, when the storm lashed at the building. This fact was completely disregarded by his design team. The weakest joint was discovered at the thirtieth floor and if that one gave way, catastrophic failure of the whole structure would have resulted.

The statistical probability of occurrence of a storm was found to be one in every sixteen years. This was further reduced to one in fifty five years if the tuned mass damper (which had been installed) was taken into account. But this machine required electric current, which might fail as soon as a major storm hits.

Le Meassurier learnt of these design faults after the building was completed and handed over. Nevertheless, he acknowledged these errors because keeping silent would mean risking people's lives. So he brought these errors to the notice of the owners of the building and persuaded them to invest in his newly prepared rectification scheme. Since the bolted joints were readily accessible, the new proposal was to strengthen the joints, which were weak. All the weak joints were reinforced by welding heavy steel plates over them.

His honesty, courage, adherence to ethical and social responsibility during this ordeal remains a testimony to the high ideals of a true professional.

12.0 INDIAN EXPERIENCE

All the Case Studies reported so far in this chapter have been compiled from published reports and journals from UK and US.

The culture of reporting failures and the lessons learnt from them has not yet developed in India. In many cases the reluctance of the Engineers concerned is also due to the fear of potential legal action and resulting claims.

12.1 Improper design leads to heavy restoration

The next case study is an example of errors committed in design due to inexperience and wrong assumptions.

The case study concerns a factory building near Nellore, India. The building was of size *25.7 m X 52.5 m*. The roof was made up of steel Pratt trusses supported on concrete columns. The entire truss was exposed except for the bottom chord members, which were embedded in concrete slab.

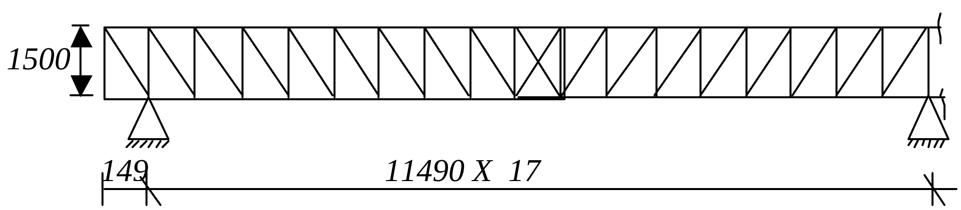


Fig. 11: Elevation of Truss

After curing the concrete slab, when the scaffoldings were removed, the deflection of the roof was found to be *100 mm (>L/325)*. In order to find out the cause of these disproportionate deflections, the truss was reanalysed. The configuration of the truss is shown in Fig. *11*. From the analysis, it was found that most of the members of the roof truss were not safe. After performing several analyses, it was concluded that in the original design, the designer might have miscalculated the loads. In India, industrial buildings were normally covered by asbestos cement sheetings. Hence the original designer of the truss, due to his inexperience, might have considered the truss to support Asbestos sheeting, instead of heavier concrete slab. The analysis considering only AC sheet roofing confirmed that all the members of the roof would be safe for the reduced loading.

By the time the investigation started, the expensive machinery, which were to be housed inside the building had arrived and were under installation at various parts of the building. Hence the repair of the roof had to be done without affecting the work of installing these machineries. Hence it was decided to strengthen the top and bottom flanges of the top chord members by welding extra plates on them. It was also decided to weld angles inside the web members of the truss. Since the bottom chord member was inside the concrete slab, it was not possible to add anything to the bottom chords.

It was also recommended to provide temporary supports to the truss at *1/3* point for welding extra plates and angles. But the contractor did not provide the temporary supports and welded the extra plates and angles. This resulted in the buckling of the web of the top chord members. The web members in one or two trusses had also buckled. Hence after careful consideration it was decided to reduce the span of the truss which would eventually reduce the forces in the members. After consultation with the manager of the plant, the locations of these intermediate columns were fixed and then the work was carried out.

12.2 Restoration of a factory building

The next case study deals with another design error made by the designer due to his overconfidence.

A factory building located at about *100 km* from Bombay collapsed during a windstorm in *1994*. The building was built using cold-formed channel members. The layout of the building is shown in Fig.*12* and the elevation in Fig.*13*.

The structure was provided with column bracings in every sixth bay. However no gable end bracings were provided. Extra columns were provided at the gable end to support the cladding. The structure was covered with asbestos roofing and all the sides were covered with asbestos sheet cladding. The structure was designed to support a *4t* gantry in each bay. A crane bracket supported by the column (see Fig.*13*) supported the gantry.

In order to understand the failure of the structure, the original design was examined. The main causes of failure were found to be,

(i) wind loads were not estimated properly as per *IS:875*.
(ii) column and rafter sections were found to be inadequate to resist the load; they did not even satisfy the main ℓ/r ratio specified in the Code.
(iii) during erection, the bracings were not connected properly to the main members.

For the restoration of the structure, the designer was asked to use the same sections and produce a design, which would not increase the cost of the project considerably. Hence same channels were used; but their spacing was altered to form a box section with diagonal bracings of channel section. The span and bay width were kept the same. This arrangement increased the moment of inertia of the section along the frame and perpendicular to the frame. The rigidity of the structure increased considerably in both the directions and the bending stress was found to be well within the allowable range of stresses.

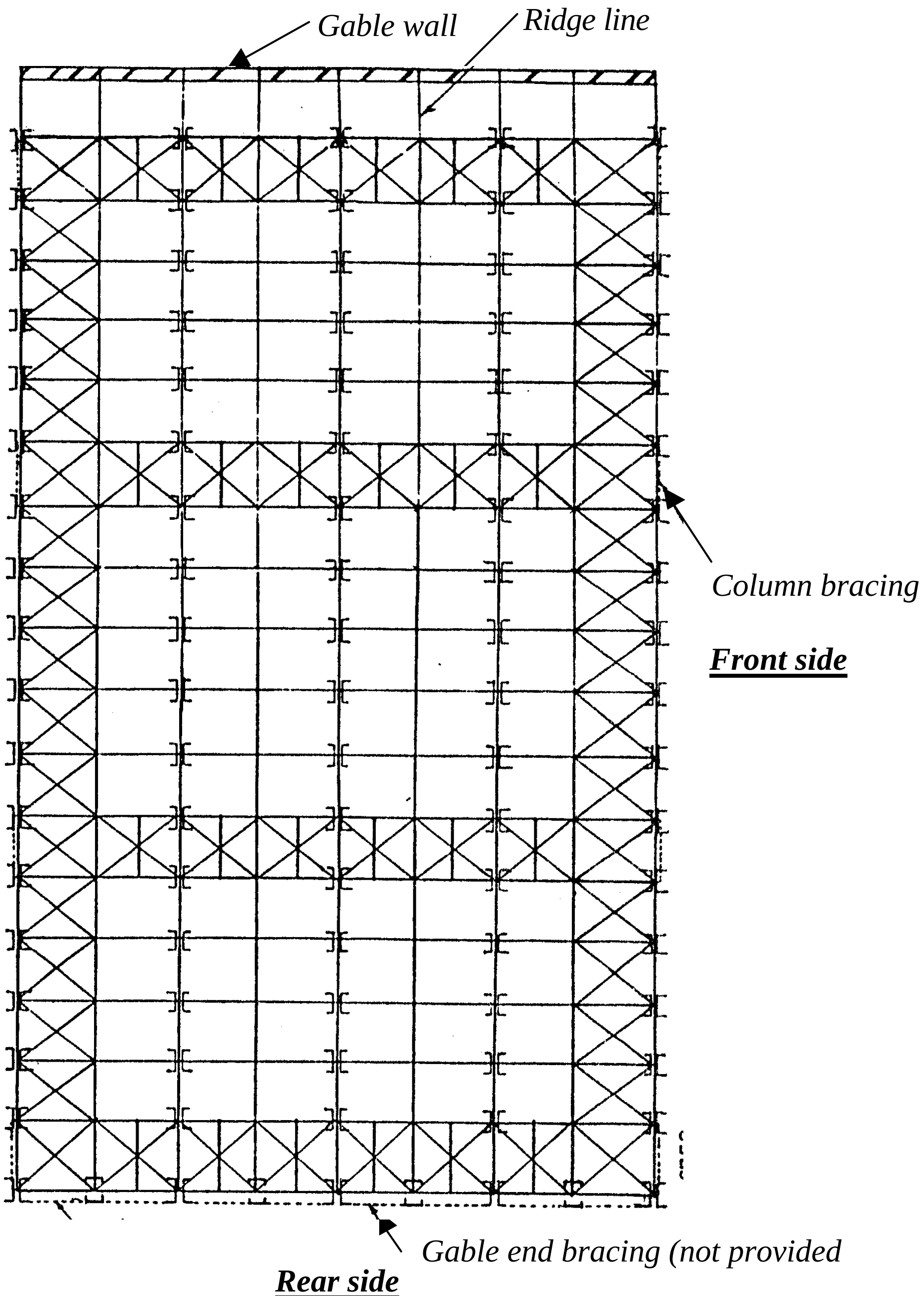


Fig. 12: Layout plan of the factory building

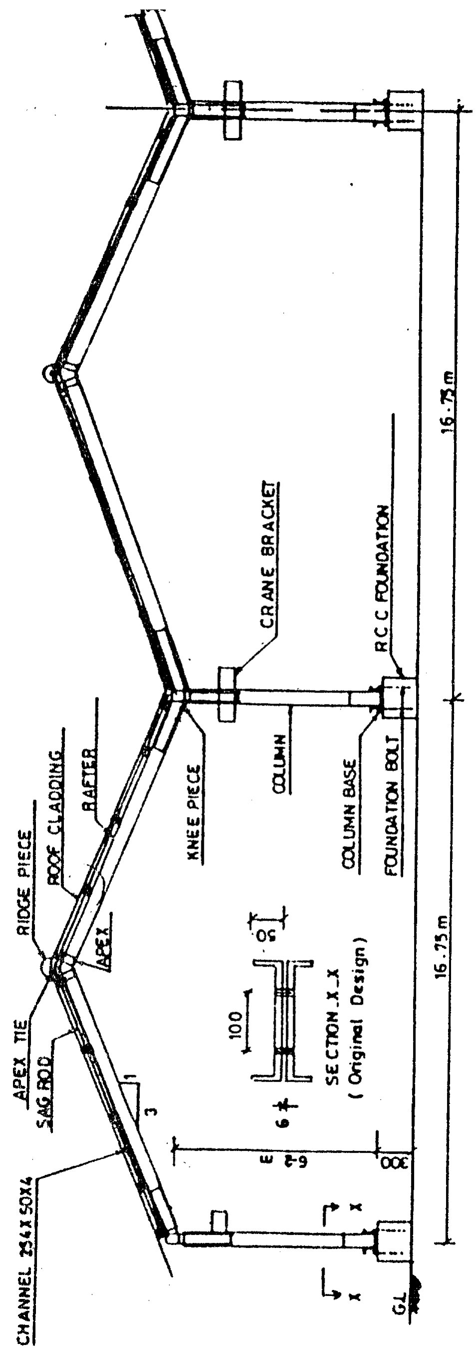


Fig. 13: Cross section of collapsed frame

12.3 Improper detailing results in delayed commissioning

In the next case study, wrong detailing adopted by a designer resulted in delayed commissioning of the project.

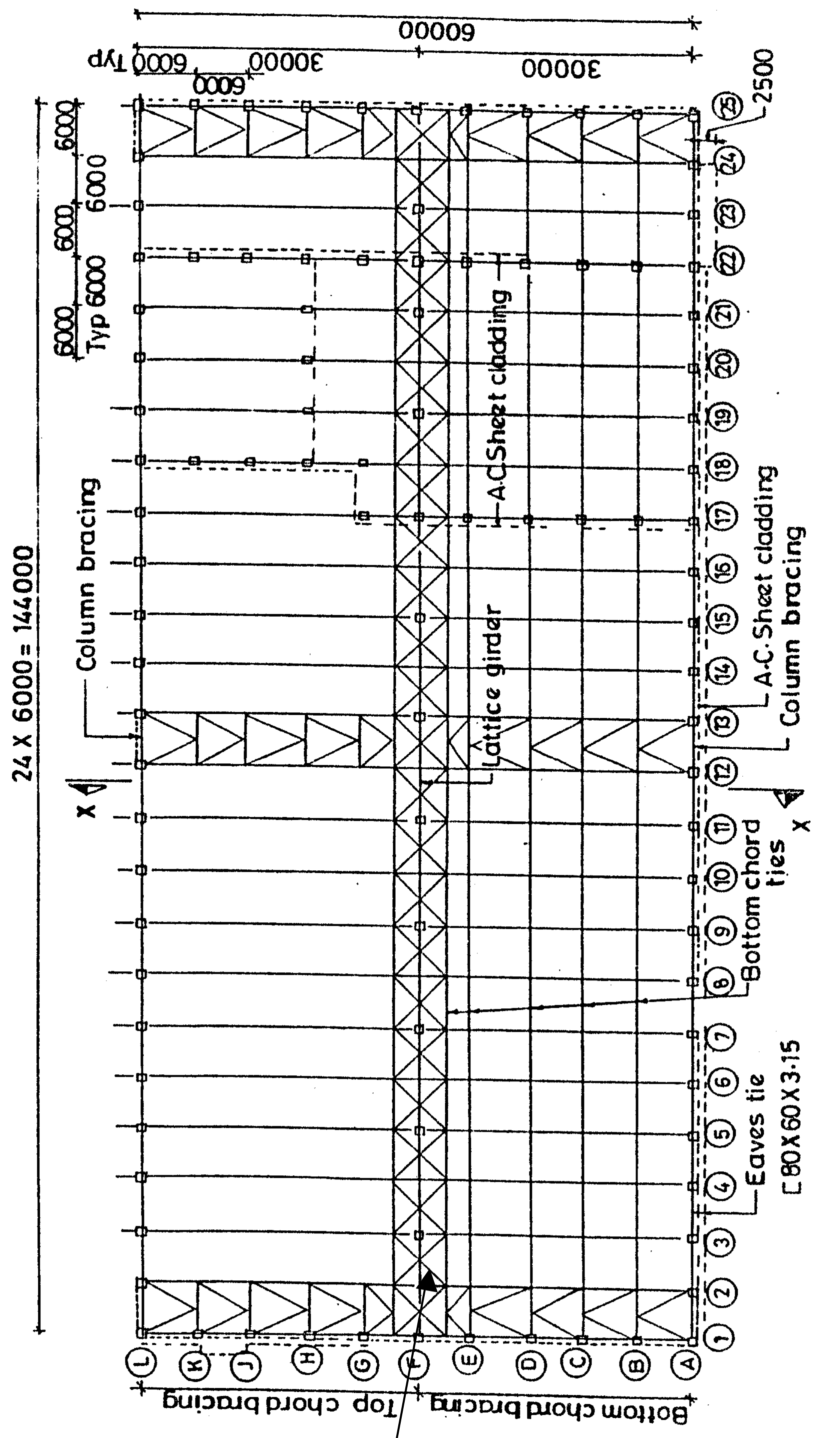
A *144 m X 60 m* factory building was constructed at Cochin, India. The plan is shown in Fig.*14*. The building was made up of portal frames spanning *60 m* and placed at *6 m* intervals. The portal frames were supported alternately on columns and on lattice girder that was placed longitudinally at the mid-span. The portals and columns were made up of four angles, which were laced to form a box section. The lattice girder and the central column were made up of *4* channels, laced to form a compound section. The sides of the building were also covered by cladding from *1.5 m* above G.L.

After the erection of portal frames and placing of asbestos sheets, some problems were encountered. The purlins and side cladding girts got twisted. This resulted in the cracking of some AC sheets. Some columns (especially those supporting the partitions) were not straight and gave a buckled column appearance. Most of the side cladding girts were sagging.

A careful investigation showed that the purlins were not detailed properly and were placed in the wrong orientation, which resulted in the torsion of the purlin sections. This is explained in Fig. *15*. Moreover the detailer had given a connection detail as shown in Fig.*16 (a)* to connect the purlins at rafter points as against the correct detail in Fig.*16 (b)*. Due to this the purlins got twisted till the tip of the purlins rested on the rafter section.

By this time all the machinery of the plant had arrived and the erection of these was in progress. It was also a costly proposition to remove all the AC sheets, correct the detailing errors by refabricating the joints and relaying the AC sheets. Hence after several rounds of discussions, it was decided to replace only the cracked AC sheets and to adopt a temporary solution as shown in Fig.*17* which would arrest further twisting of the purlin.

The sagging of the side cladding girts was due to the fact that the sag rods were not anchored by providing diagonal sag rods at the ends. This was rectified. The other problem was due to the fact that the fabricator was not experienced in cold rolled steel sections. Since these sections were flexible and made of thin sections, the fabricators simply bent the columns and fixed them at the required place. These mistakes were also rectified. However, these corrective measures delayed the starting of the plant production by about six months.



Lattice girder support

Fig.14: Plan view of the structure

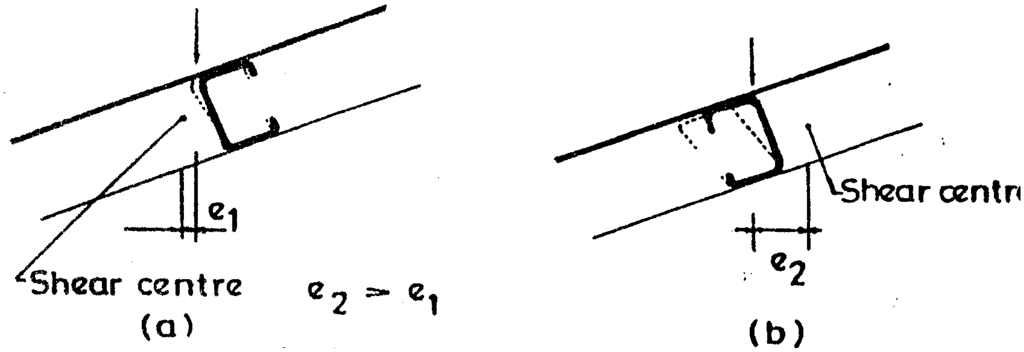
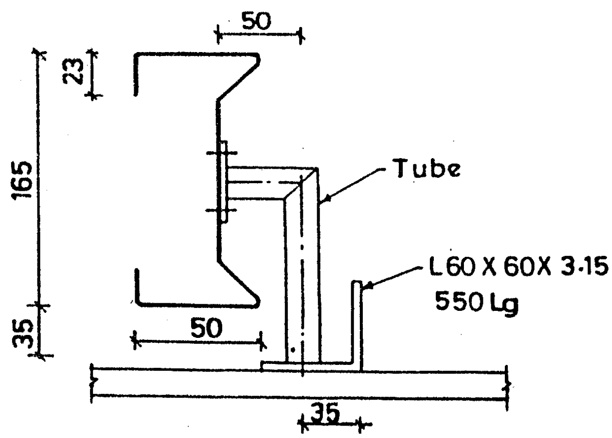
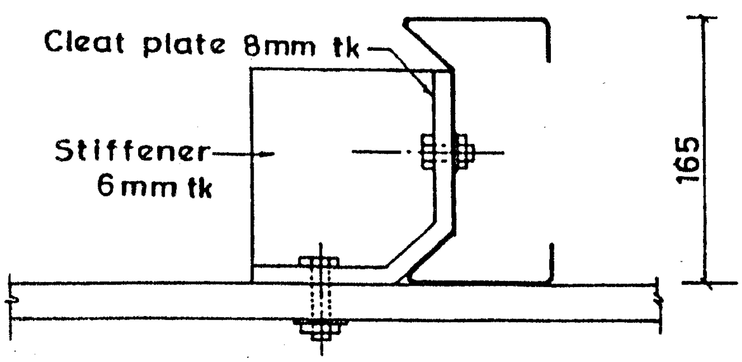


Fig. 15: Torsion of purlins



(a) Connection detail as adopted at site



(b) Correct connection detail

Fig.16: Adopted and correct seating detail of purlin

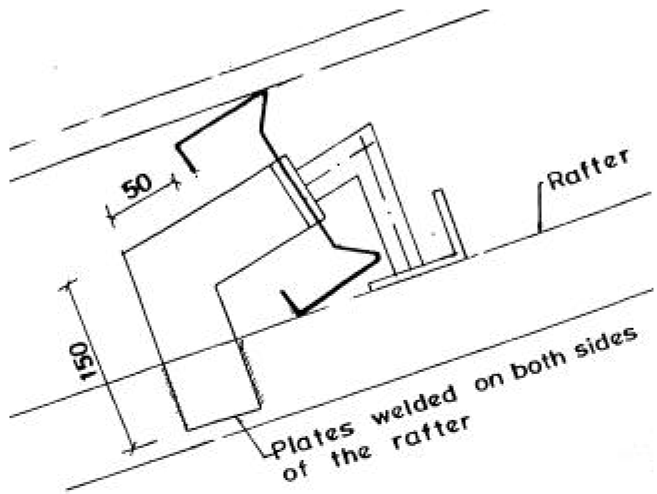


Fig.17: Solution adopted to hold the purlin in place

13.0 Lessons learnt from the Gujarat Earthquake of 26 January 2001

Over 30,000 people are reported to have died in the earthquake of magnitude 7.9 (on the Richter Scale), which hit parts of Northwest Gujarat on Republic Day 2001. Thousands of people - who had led respectable life-styles till then - have seen their life-savings vanish and their lives irrevocably destroyed. Do we have to accept this human suffering and carnage with fatalism and detachment? Or can we ***protect*** our buildings by careful designs and thereby save lives?

Earthquakes in California of even larger magnitudes have not resulted in losses in life of this magnitude, because the buildings in that State are required to comply with the State's Earthquake-resistant Design Codes. Indeed, when an earthquake of similar magnitude hit Seattle, U.S.A. on 1 March 2001 ***there was not even a single loss of life and only a few persons were injured, none seriously***. This is the result of the extensive retrofitting that was carried out in the city during the 1970's. ***The Central Public Works Department has claimed that none of their buildings in the Kutch area had suffered any damage during the earthquake due to their sound structural designing.*** Clearly, the technology exists to ***protect*** our buildings and ***prevent*** loss of lives and - equally clearly - the building designs in Gujarat have not been subjected to checks on their structural adequacy and on their safety by qualified Structural Engineers and Soil Engineers. (IS 13920 pertains to "Ductile Detailing of Reinforced Structures subjected to Seismic Forces" and IS4326, to "Earthquake resistant Design and Construction of Buildings".) Clearly, these Code-prescribed checks were not insisted on before the appropriate authorities approved the designs for construction. Thousands of these buildings collapsed subsequently, despite the availability of design guidance.

There is an irrational reluctance to use Steel Structures among many professionals in India largely due to misinformation, lack of confidence, or inexperience. Steel is inherently ductile; steel structural components, when stretched or elongated under overload, do not fail or collapse. On the other hand, concrete is a fracture-sensitive material, which cracks under tensile forces. As a material, concrete is inherently unsuitable to sustain overloads or repeated loads caused by earthquakes nevertheless reinforced concrete was used as the preferred material of choice in practically every building. The only steel-concrete composite multi-storeyed building under construction in Ahmedabad suffered no damage due to the earthquake.

The ignorance of currently available technology is compounded by the willingness of the Indian Builders and clients to accept shoddy and primitive construction, particularly in concrete structures. There is very little quality control of concretes used in Indian buildings. There are endemic problems such as low cement content, poor quality reinforcing-steel, inadequate concrete cover to reinforcing steel and non-existent site supervision; all of these will need to be remedied systematically to prevent repetition of this disaster.

The following is a partial list of inadequacies and infractions, identified by professional engineers who visited Ahmedabad after the disaster:

- Most buildings that had a planning approval for (Ground floor plus 4 levels) had a further floor added illegally; buildings with approval for (Ground floor plus 10 levels) had two further floors added illegally. Swimming pools and/or roof gardens (on soils spread to a depth of one metre over the roof) were added features of some of these luxury buildings. Obviously they were unsafe and triggered the collapse.

- Columns loaded from above were terminated at the free end of a cantilever at the second floor level. There was no provision for transferring these loads on to the foundations.

- Most buildings that collapsed were built on stilts, with the ground floor being used for car parking. The flexible columns at the ground floor level failed rapidly during the earthquake and initiated the progressive collapse of these buildings. This type of failure could have been prevented by concrete infill walls or suitably designed bracings to the ground floor columns.

- The falling concrete debris from collapsed structural components caused substantial loss of life during the earthquake. ***It is essential that the structural integrity of the building be maintained even if the individual members had failed***. Each building should be effectively tied together at each principal floor and roof level in both directions. Reinforcing bars in concrete floors should be effectively anchored to the beams at its edges, so that these floors would function as edge-supported membranes, rather than fall down on the floor below, thereby causing damage.

- In many buildings, only the lift shaft was saved, and the rest of the building collapsed around it, as the former was not effectively connected to the latter. Well-designed lift shafts would effectively function as core walls and provide the much needed stability in multi-storey buildings.

- Buildings that are unsymmetrical in plan will be subjected to unexpected twisting which would cause substantial damage. Re-entrant corners should be avoided. It is sensible to split such plans into rectangles, with a crumple zone (or construction joint) in-between.

- The structural framing system chosen should invariably be of the "strong column

- ***All buildings should invariably be designed to prevent collapse and loss of life under the most severe earthquake it is likely to be subject to within its design life.*** All structural components should be designed with adequate ductility, which would allow large plastic deformations to develop, without significant loss of strength or structural integrity.

The lessons from this experience and loss of life must be an eye-opener for all building professionals. Sound engineering principles should never be compromised and there is no room for complacency, when it comes to safety.

14.0 SUMMARY

In recent years, case studies have come to be recognised as a source of understanding our present state of technology and its limitations. Much improvement of our design concepts has been possible from a study of failures; these provide an invaluable source of information about design limitations. Design is a process of the anticipation of failure, and as such the more knowledgeable the designer is about failures, the more reliable his designs will be.

This chapter provides examples of failures due to design error, construction error and communication gap among the team members having different responsibilities.

REFERENCES

1. Petroski, H. " *Failure as a Source of Engineering Judgement: Case of John Roebling*", Journal of Performance of Constructed Facilities, ASCE, vol 7, n2, Feb. 1993, 46-58.

2. Kagan, H. A., " *Common Causes of Collapse of Metal-Plate-Connected Wood Roof Trusses*", Journal of Performance of Constructed Facilities, vol. 7, n4, Nov, 1993, pp. 225-233.

3. Morgenstern, J., " *The Fifty- Nine-Story Crisis*", Journal of Professional issues in Engineering Education and Practice, vol. 123, n1, Jan.,19 97, pp. 23-29.

4. Delatte, N. J., " *Failure Case Studies and Ethics in Engineering mechanics Courses*", Journal of Professional issues in Engineering Education and Practice, vol. 123, n1, Jan.,1997, pp. 111-116.

5. Lichtenstein, A. G., " *The Siver Bridge Collapse Recounted*", Journal of Performance of Constructed Facilities, vol. 7, n4, Nov, 1993, pp. 249-261.

6. Dias, W. P. S., " *Structural Failures and Design Philosophy*", The Structural Engineer, vol. 72, n2, 18 Jan., 1994, pp. 25-29.

7. Petroski, H., " *Case Histories and the Study of Structural Failures*", Structural Engineering International, April 1995, pp. 250-254.

8 Banset, E. A., " Communication Failure in Hyatt Regency Disaster ", Journal of Professional issues in Engineering Education and Practice**,** vol.115, n3, July 1989, 273-288.

9. Subramaniam, N., " *Rehabilitation of Steel Structures- Some Case Studies*", Advanced Course on " Design of Steel Skeletal Structures", Dec 16-18, 1998, SERC, Madras.

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43 STEEL BRIDGES - I

1.0 INTRODUCTION

As discussed in earlier chapters the main advantages of structural steel over other construction materials are its strength and ductility. It has a higher strength to cost ratio in tension and a slightly lower strength to cost ratio in compression when compared with concrete. The stiffness to weight ratio of steel is much higher than that of concrete. Thus, structural steel is an efficient and economic material in bridges. Structural steel has been the natural solution for long span bridges since 1890, when the Firth of Forth cantilever bridge, the world's major steel bridge at that time was completed. Steel is indeed suitable for most span ranges, but particularly for longer spans. Howrah Bridge, also known as Rabindra Setu, is to be looked at as an early classical steel bridge in India. This cantilever bridge was built in 1943. It is *97 m* high and *705 m* long. This engineering marvel is still serving the nation, deriding all the myths that people have about steel. [See Fig. 1]

Fig. 1 Howrah bridge

The following are some of the advantages of steel bridges that have contributed to their popularity in Europe and in many other developed countries.

- They could carry heavier loads over longer spans with minimum dead weight, leading to smaller foundations.
- Steel has the advantage where speed of construction is vital, as many elements can be prefabricated and erected at site.

- In urban environment with traffic congestion and limited working space, steel bridges can be constructed with minimum disruption to the community.
- Greater efficiency than concrete structures is invariably achieved in resisting seismic forces and blast loading.
- The life of steel bridges is longer than that of concrete bridges.
- Due to shallow construction depth, steel bridges offer slender appearance, which make them aesthetically attractive. The reduced depth also contributes to the reduced cost of embankments.
- All these frequently leads to low life cycle costs in steel bridges

In India there are many engineers who feel that corrosion is a problem in steel bridges, but in reality it is not so. Corrosion in steel bridges can be effectively minimised by employing newly developed paints and special types of steel. These techniques are followed in Europe and other developed countries. These have been discussed in chapter 2.

2.0 STEEL USED IN BRIDGES

Steel used for bridges may be grouped into the following three categories:

(i) *Carbon Steel:* This is the cheapest steel available for structural users where stiffness is more important than the strength. Indian steels have yield stress values up to 250 N/mm^2 and can be easily welded. The steel conforming to IS: 2062 - 1969, the American ASTM A36, the British grades 40 and Euronorm 25 grades 235 and 275 steels belong to this category.

(ii) *High strength steels:* They derive their higher strength and other required properties from the addition of alloying elements. The steel conforming to IS: 961 - 1975, British grade 50, American ASTM A572 and Euronorm 155 grade 360 steels belong to this category. Another variety of steel in this category is produced with enhanced resistance to atmospheric corrosion. These are called '*weathering*' steels in Europe, in America they conform to ASTM A588 and have various trade names like *' cor-ten'*.

(iii) *Heat-treated carbon steels:* These are steels with the highest strength. They derive their enhanced strength from some form of heat-treatment after rolling namely normalisation or quenching and tempering.

The physical properties of structural steel such as strength, ductility, brittle fracture, weldability, weather resistance etc., are important factors for its use in bridge construction. These properties depend on the alloying elements, the amount of carbon, cooling rate of the steel and the mechanical deformation of the steel. The detailed discussion of physical properties of structural steel is presented in earlier chapter.

3.0 CLASSIFICATION OF STEEL BRIDGES

Steel bridges are classified according to

- the type of traffic carried
- the type of main structural system
- the position of the carriage way relative to the main structural system

These are briefly discussed in this section.

3.1 Classification based on type of traffic carried

Bridges are classified as

- Highway or road bridges
- Railway or rail bridges
- Road - cum - rail bridges

3.2 Classification based on the main structural system

Many different types of structural systems are used in bridges depending upon the span, carriageway width and types of traffic. Classification, according to make up of main load carrying system, is as follows:

(i) *Girder bridges* - Flexure or bending between vertical supports is the main structural action in this type. Girder bridges may be either solid web girders or truss girders or box girders. Plate girder bridges are adopted for simply supported spans less than *50 m* and box girders for continuous spans upto *250 m*. Cross sections of a typical plate girder and box girder bridges are shown in Fig. 2(*a*) and Fig. 2(*b*) respectively. Truss bridges [See Fig. 2(*c*)] are suitable for the span range of *30 m* to *375 m*. Cantilever bridges have been built with success with main spans of *300 m* to *550 m*. In the next chapter girder bridges are discussed in detail. They may be further, sub-divided into simple spans, continuous spans and suspended-and-cantilevered spans, as illustrated in Fig. 3.

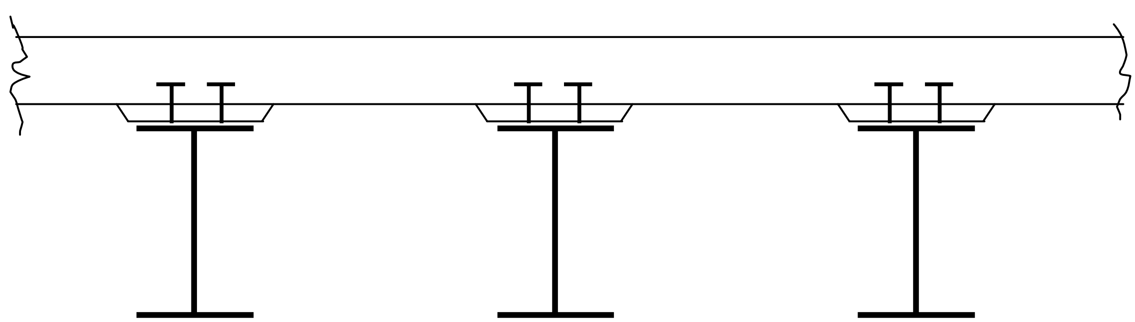


Fig. 2(a) Plate girder bridge section

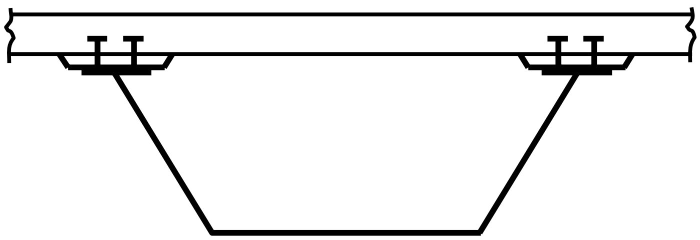


Fig. 2(b) Box girder Bridge section

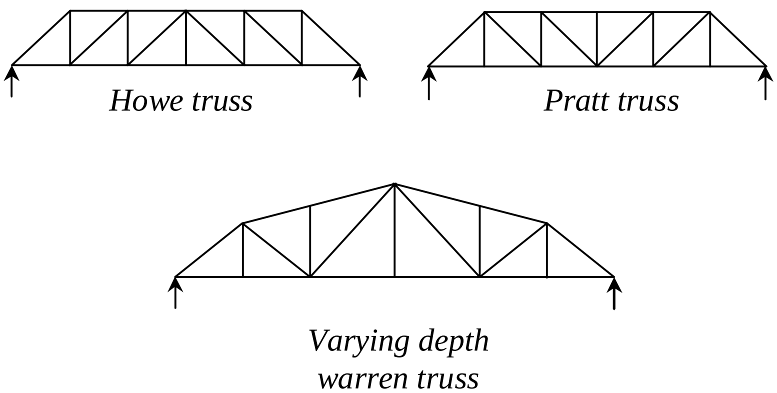


Fig. 2(c) Some of the trusses used in steel bridges

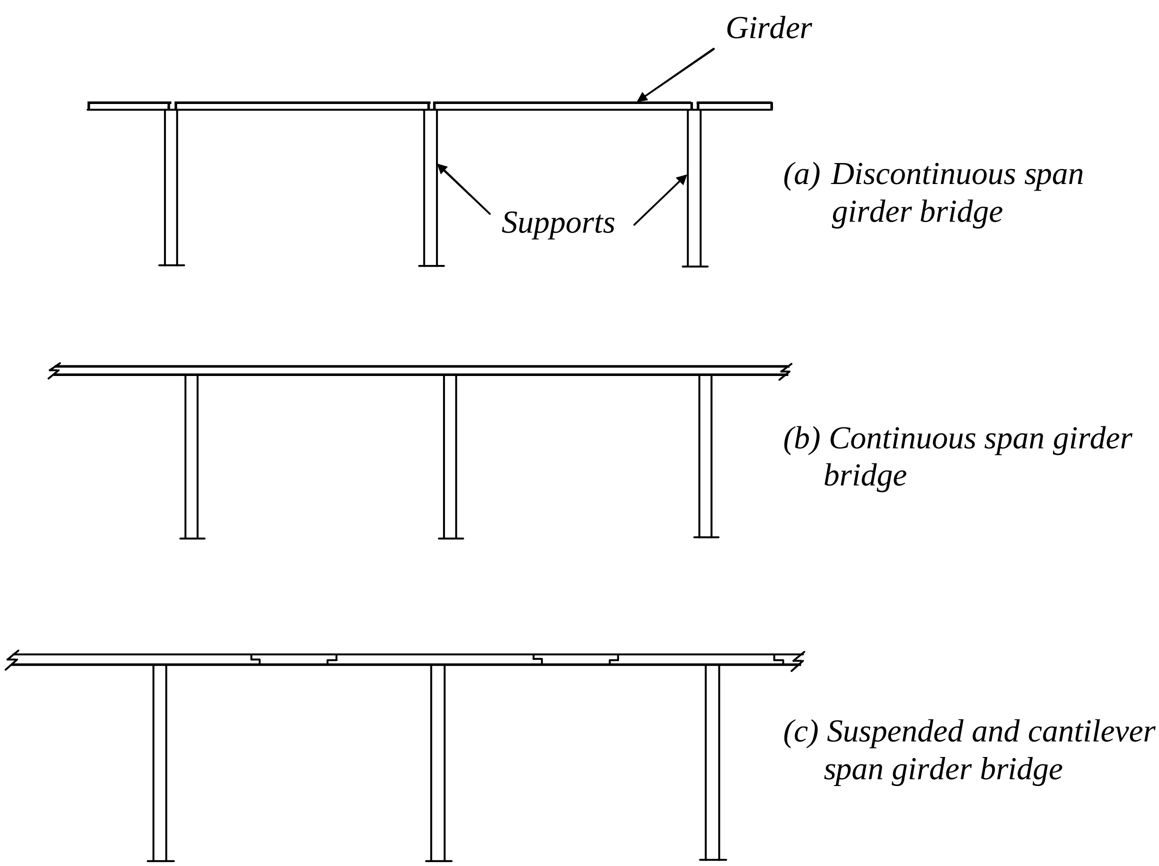


Fig. 3 Typical girder bridges

(ii) *Rigid frame bridges* - In this type, the longitudinal girders are made structurally continuous with the vertical or inclined supporting member by means of moment carrying joints [Fig. 4]. Flexure with some axial force is the main forces in the members in this type. Rigid frame bridges are suitable in the span range of *25 m* to *200 m.*

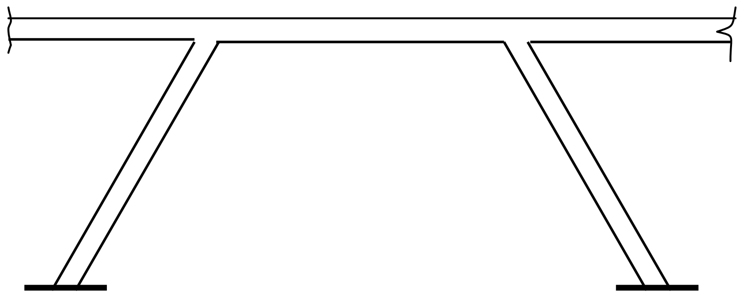


Fig. 4 Typical rigid frame bridge

(iii) *Arch bridges* - The loads are transferred to the foundations by arches acting as the main structural element. Axial compression in arch rib is the main force, combined with some bending. Arch bridges are competitive in span range of *200 m* to *500 m*. Examples of arch bridges are shown in Fig. 5.

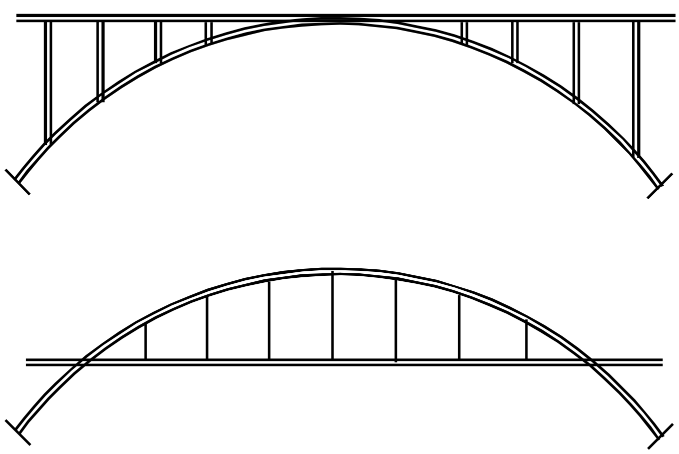


Fig. 5 Typical arch bridges

(iv) *Cable stayed bridges* - Cables in the vertical or near vertical planes support the main longitudinal girders. These cables are hung from one or more tall towers, and are usually anchored at the bottom to the girders. Cable stayed bridges are economical when the span is about *150 m* to *700 m*. Layout of cable stayed bridges are shown in Fig. 6.

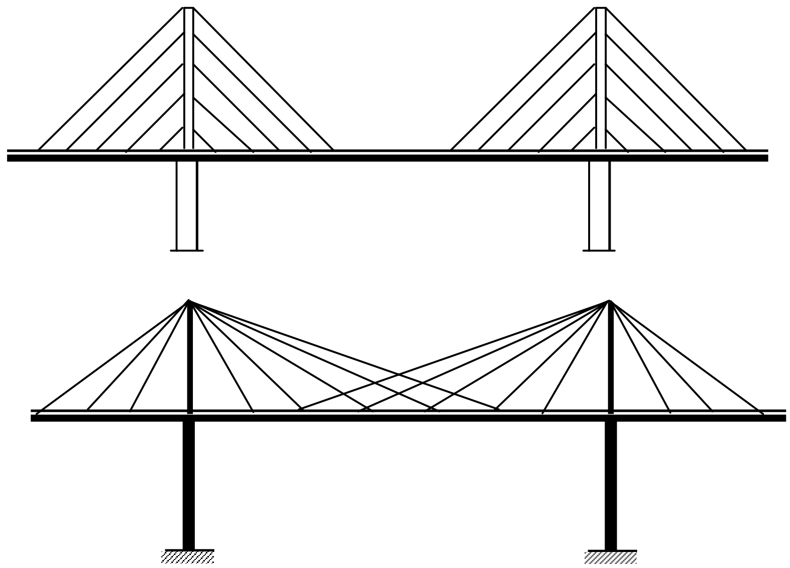


Fig. 6 Layout of cable stayed bridges

(v) *Suspension bridges* - The bridge deck is suspended from cables stretched over the gap to be bridged, anchored to the ground at two ends and passing over tall towers erected at or near the two edges of the gap. Currently, the suspension bridge is best solution for long span bridges. Fig. 7 shows a typical suspension bridge. Fig. 8 shows normal span range of different bridge types.

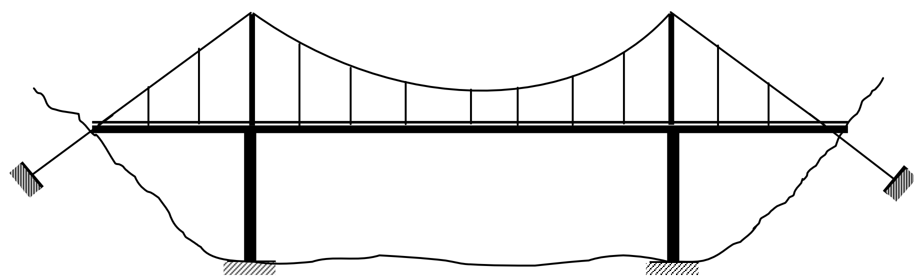


Fig. 7 Suspension bridge

3.3 Classification based on the position of carriageway

The bridges may be of the "deck type", "through type" or "semi-through type". These are described below with respect to truss bridges:

(i) *Deck Type Bridge* - The carriageway rests on the top of the main load carrying members. In the deck type plate girder bridge, the roadway or railway is placed on the top flanges. In the deck type truss girder bridge, the roadway or railway is placed at the top chord level as shown in Fig. 9(*a*).

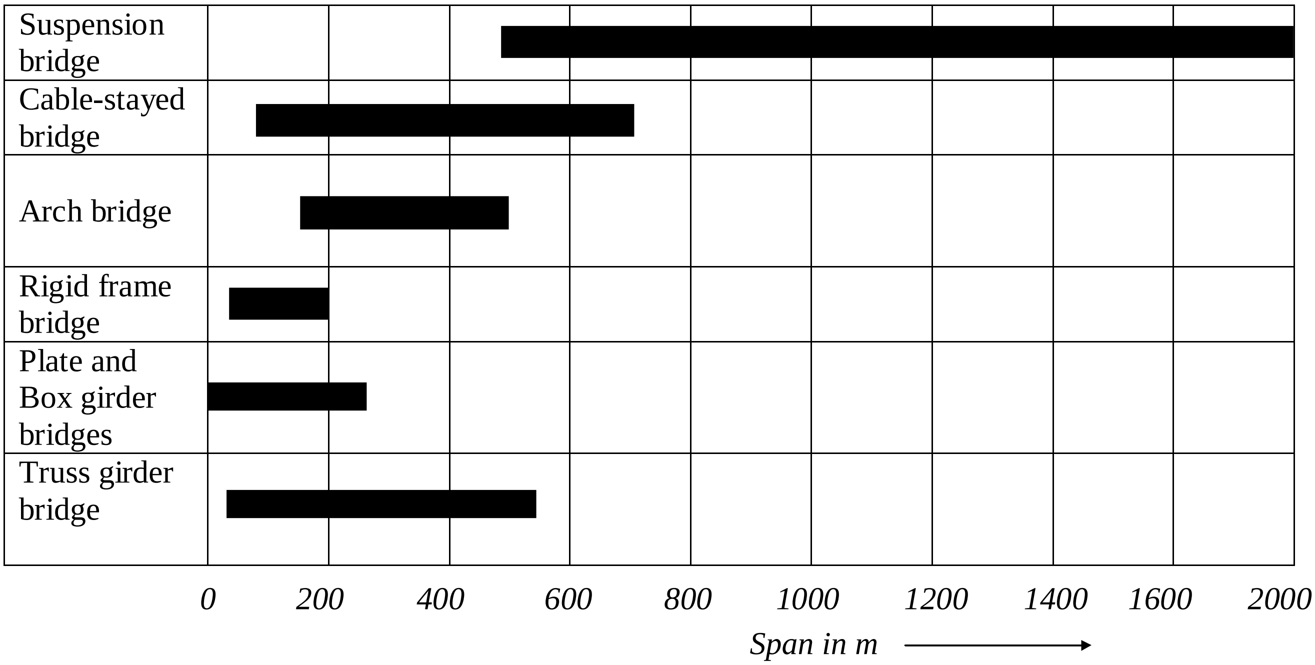


Fig. 8 Normal span ranges of bridge system

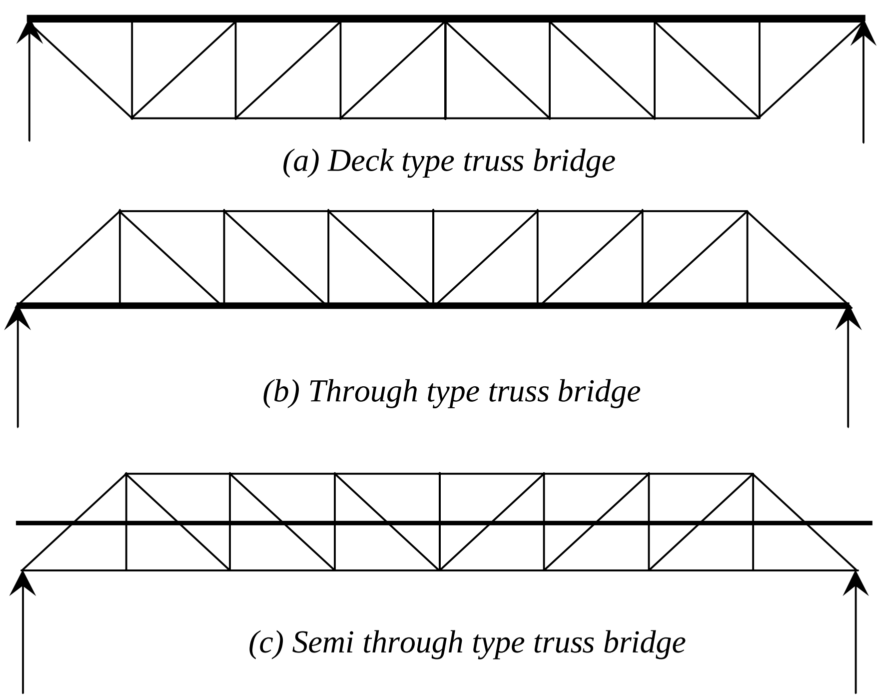


Fig.9 Typical deck, through and semi- through type truss bridges

(ii) *Through Type Bridge* - The carriageway rests at the bottom level of the main load carrying members [Fig. 9(*b*)]. In the through type plate girder bridge, the roadway or railway is placed at the level of bottom flanges. In the through type truss girder bridge, the roadway or railway is placed at the bottom chord level.

The bracing of the top flange or lateral support of the top chord under compression is also required.

(iii) *Semi through Type Bridge* - The deck lies in between the top and the bottom of the main load carrying members. The bracing of the top flange or top chord under compression is not done and part of the load carrying system project above the floor level as shown in Fig. 9(*c*). The lateral restraint in the system is obtained usually by the U-frame action of the verticals and cross beam acting together.

4.0 LOADS ON BRIDGES

The following are the various loads to be considered for the purpose of computing stresses, wherever they are applicable.

- Dead load
- Live load
- Impact load
- Longitudinal force
- Thermal force
- Wind load
- Seismic load
- Racking force
- Forces due to curvature.
- Forces on parapets
- Frictional resistance of expansion bearings
- Erection forces

Dead load **–** The dead load is the weight of the structure and any permanent load fixed thereon. The dead load is initially assumed and checked after design is completed.

Live load **–** Bridge design standards specify the design loads, which are meant to reflect the worst loading that can be caused on the bridge by traffic, permitted and expected to pass over it. In India, the Railway Board specifies the standard design loadings for railway bridges in bridge rules. For the *highway bridges,* the Indian Road Congress has specified standard design loadings in IRC section II. The following few pages brief about the loadings to be considered. For more details, the reader is referred to the particular standard.

Railway bridges: Railway bridges including combined rail and road bridges are to be designed for railway standard loading given in bridge rules. The standards of loading are given for:

- Broad gauge - Main line and branch line
- Metre gauge - Main line, branch line and Standard C
- Narrow gauge - H class, A class main line and B class branch line

The actual loads consist of axle load from engine and bogies. The actual standard loads have been expressed in bridge rules as *equivalent uniformly distributed loads* (EUDL) in tables to simplify the analysis. These equivalent UDL values depend upon the span length. However, in case of rigid frame, cantilever and suspension bridges, it is necessary for the designer to proceed from the basic wheel loads. In order to have a uniform gauge throughout the country, it is advantageous to design railway bridges to Broad gauge main line standard loading. The EUDLs for bending moment and shear force for broad gauge main line loading can be obtained by the following formulae, which have been obtained from regression analysis:

For bending moment:

$$\text{EUDL in kN} = 317.97 + 70.83\ell + 0.0188\ell^2 \geq 449.2 \text{ kN} \qquad (1)$$

For shear force:

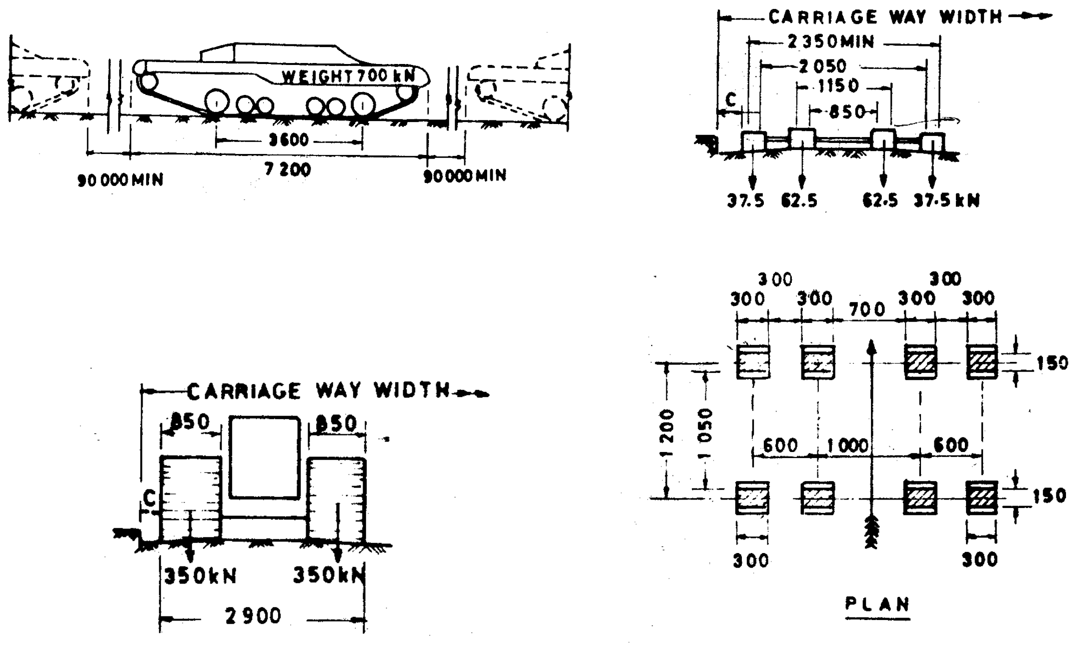
$$\text{EUDL in kN} = 435.58 + 75.15\ell + 0.0002\ell^2 \geq 449.2 \text{ kN} \qquad (2)$$

Note that, ℓ is the effective span for bending moment and the loaded length for the maximum effect in the member under consideration for shear. 'ℓ ' should be in metres. The formulae given here are not applicable for spans less than or equal to 8 m with ballast cushion. For the other standard design loading the reader can refer to Bridge rules.

Highway bridges: In India, highway bridges are designed in accordance with IRC bridge code. IRC: 6 - 1966 – Section II gives the specifications for the various loads and stresses to be considered in bridge design. There are three types of standard loadings for which the bridges are designed namely, IRC class AA loading, IRC class A loading and IRC class B loading.

IRC class AA loading consists of either a tracked vehicle of 70 tonnes or a wheeled vehicle of 40 tonnes with dimensions as shown in Fig. 10. The units in the figure are mm for length and tonnes for load. Normally, bridges on national highways and state highways are designed for these loadings. Bridges designed for class AA should be checked for IRC class A loading also, since under certain conditions, larger stresses may be obtained under class A loading. Sometimes class 70 R loading given in the Appendix - I of IRC: 6 - 1966 - Section II can be used for IRC class AA loading. Class 70R loading is not discussed further here.

Class A loading consists of a wheel load train composed of a driving vehicle and two trailers of specified axle spacings. This loading is normally adopted on all roads on which permanent bridges are constructed. Class B loading is adopted for temporary structures and for bridges in specified areas. For class A and class B loadings, reader is referred to IRC: 6 - 1966 – Section II.



(a) Tracked vehicle *(a) Wheeled vehicle*

Fig. 10 IRC AA loading

Foot Bridges and Foot path on Bridges – The live load due to pedestrian traffic should be treated as uniformly distributed over the pathway. For the design of foot bridges or foot paths on railway bridges, the live load including dynamic effects should be taken as 5.0 kN/m^2 of the foot-path area. For the design of foot-path on a road bridges or road-rail bridges, the live load including dynamic effects may be taken as 4.25 kN/m^2 except that, where crowd loading is likely, this may be increased to 5.0 kN/m^2.

The live load on foot path for the purpose of designing the main girders has to be taken as follows according to bridge rules:

(i) For effective spans of 7.5 m or less - 4.25 kN/m^2

(ii) The intensity of load be reduced linearly from 4.25 kN/m^2 for a span of 7.5 m to 3.0 kN/m^2 for a span of 30 m.

(iii) For effective spans over 30 m, the UDL may be calculated as given below:

$$P = \frac{1}{100}\left(13.3 + \frac{400}{\ell}\right)\left(\frac{17 - W}{1.4}\right) kN/m^2 \qquad (3)$$

where, P = Live load in kN/m^2

ℓ = Effective span of the bridge in m.

W = Width of the foot path in m.

Where foot-paths are provided on a combined rail-road bridge, the load on foot-path for purpose of designing the main girders should be taken as 2.0 kN/m^2.

Impact load – The dynamic effect caused due to vertical oscillation and periodical shifting of the live load from one wheel to another when the locomotive is moving is known as impact load. The impact load is determined as a product of impact factor, *I*, and the live load. The impact factors are specified by different authorities for different types of bridges. The impact factors for different bridges for different types of moving loads are given in the table 1. Fig. 11 shows impact percentage curve for highway bridges for class AA loading. Note that, in the above table ℓ is loaded length in m and B is spacing of main girders in m.

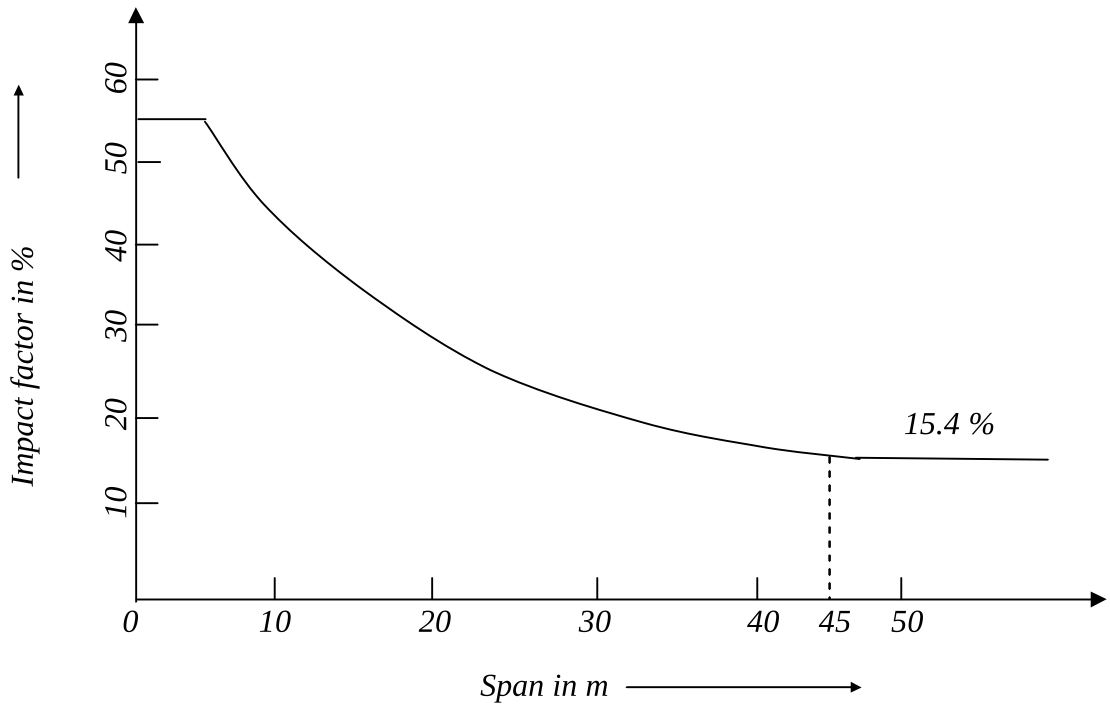


Fig. 11 Impact percentage curve for highway bridges for IRC Class A and IRC Class B Loadings

Longitudinal Forces – Longitudinal forces are set up between vehicles and bridge deck when the former accelerate or brake. The magnitude of the force *F*, is given by

$$F = \frac{W}{g}\frac{\delta V}{\delta t} \tag{4}$$

where, W – weight of the vehicle
g – acceleration due to gravity
δV – change in velocity in time δt

This loading is taken to act at a level *1.20 m* above the road surface. No increase in vertical force for dynamic effect should be made along with longitudinal forces. The possibility of more than one vehicle braking at the same time on a multi-lane bridge should also be considered.

Table 1: Impact factors for different bridges

BRIDGE	LOADING		IMPACT FACTOR *(I)*
Railway bridges according to bridge rules	Broad gauge and Meter gauge	(a) Single track	$\frac{20}{14+\ell} \leq 1.0$
		(b) Main girder of double track with two girders	$0.72\times\frac{20}{14+\ell} \leq 0.72$
		(c) Intermediate main girder of multiple track spans	$0.60\times\frac{20}{14+\ell} \leq 0.60$
		(d) Outside main girders of multiple track spans	Specified in (a) or (b) whichever applies
		(e) Cross girders carrying two or more tracks	$0.72\times\frac{20}{14+\ell} \leq 0.72$
	Broad gauge	Rails with ordinary fish plate joints and supported directly on sleepers or transverse steel troughing	$\frac{7.32}{B+5.49}$
	Meter gauge		$\frac{5.49}{B+4.27}$
	Narrow gauge		$\frac{9.5}{91.5+\ell}$
Highway bridges according to IRC regulations	IRC class AA loading	(i) Spans less than 9 m. (a) Tracked vehicle (b) Wheeled vehicle	0.25 for spans up to 5 m and linearly reducing to 0.10 to spans of 9 m 0.25
		(ii) Spans 9 m or more (a) Tracked vehicle (b) Wheeled vehicle	0.10 0.25 for spans up to 23 m and in accordance with the curve indicated in Fig .11 for spans in excess of 23 m
	IRC class A loading and IRC class B loading	Spans between 3 m and 45 m	$\frac{9}{13.5+\ell}$ In accordance with the curve indicated in Fig .11 for all spans
Foot bridges			No separate impact allowance is made

Thermal forces – The free expansion or contraction of a structure due to changes in temperature may be restrained by its form of construction. Where any portion of the structure is not free to expand or contract under the variation of temperature, allowance should be made for the stresses resulting from this condition. The coefficient of thermal expansion or contraction for steel is $11.7 \times 10^{-6} /^{0} C$

Wind load – Wind load on a bridge may act

- Horizontally, transverse to the direction of span
- Horizontally, along the direction of span
- Vertically upwards, causing uplift
- Wind load on vehicles

Wind load effect is not generally significant in short-span bridges; for medium spans, the design of sub-structure is affected by wind loading; the super structure design is affected by wind only in long spans. For the purpose of the design, wind loadings are adopted from the maps and tables given in IS: 875 (Part III). A wind load of 2.40 kN/m^2 is adopted for the unloaded span of the railway, highway and footbridges. In case of structures with opening the effect of drag around edges of members has to be considered.

Racking Force – This is a lateral force produced due to the lateral movement of rolling stocks in railway bridges. Lateral bracing of the loaded deck of railway spans shall be designed to resist, in addition to the wind and centrifugal loads, a lateral load due to racking force of 6.0 kN/m treated as moving load. This lateral load need not be taken into account when calculating stresses in chords or flanges of main girders.

Forces on Parapets - Railings or parapets shall have a minimum height above the adjacent roadway or footway surface of 1.0 m less one half the horizontal width of the top rail or top of the parapet. They shall be designed to resist a lateral horizontal force and a vertical force each of 1.50 kN/m applied simultaneously at the top of the railing or parapet.

Seismic load – If a bridge is situated in an earthquake prone region, the earthquake or seismic forces are given due consideration in structural design. Earthquakes cause vertical and horizontal forces in the structure that will be proportional to the weight of the structure. Both horizontal and vertical components have to be taken into account for design of bridge structures. IS: 1893 – 1984 may be referred to for the actual design loads.

Forces Due to Curvature - When a track or traffic lane on a bridge is curved allowance for centrifugal action of the moving load should be made in designing the members of the bridge. All the tracks and lanes on the structure being considered are assumed as occupied by the moving load.

This force is given by the following formula:

$$C = \frac{W V^2}{12.7 R} \tag{5}$$

where, C - Centrifugal force in kN/m
W - Equivalent distributed live load in kN/m
V - Maximum speed in km/hour
R - Radius of curvature in m

***Erection forces* –** There are different techniques that are used for construction of railway bridges, such as launching, pushing, cantilever method, lift and place. In composite construction the composite action is mobilised only after concrete hardens and prior to that steel section has to carry dead and construction live loads. Depending upon the technique adopted the stresses in the members of the bridge structure would vary. Such erection stresses should be accounted for in design. This may be critical, especially in the case of erection technologies used in large span bridges.

5.0 LOAD COMBINATIONS

Stresses for design should be calculated for the most sever combinations of loads and forces. Four load combinations are generally considered important for checking for adequacy of the bridge. These are given in Table 2 and are also specified in IS 1915 - 1961.

Table 2: Load combinations

S.No.	Load combination	Loads
1	Stresses due to normal loads	Dead load, live load, impact load and centrifugal force
2	Stresses due to normal loads + occasional loads	Normal load as in (1) + wind load, other lateral loads, longitudinal forces and temperature stresses
3	Stresses due to loads during erection	-
4	Stresses due to normal loads + occasional loads + Extra-ordinary loads like seismic excluding wind load	Loads as in (2) + with seismic load instead of wind

6.0 ANALYSIS OF GIRDER BRIDGES

As discussed above, bridge decks are required to support both static and moving loads. Each element of a bridge must be designed for the most severe conditions that can possibly be developed in that member. Live loads should be placed in such a way that they will produce the most severe conditions. The critical positions of live loads will not be the same for every member. A useful method for determining the most severe condition of loading is by using “influence lines”.

An influence line represents some internal force such as shear force, bending moment etc. at a particular section or in a given member of girder, as a unit load moves over the span. The ordinate of influence line represents the value of that function when the unit load is at that particular point on the structure. Influence lines provide a systematic procedure for determining how the force (or a moment or shear) in a given part of a structure varies as the applied load moves about on the structure. Influence lines of responses of statically determinate structures consist only of straight lines whereas this is not true of indeterminate structures. It may be noted that a shear or bending moment diagram shows the variation of shear or moment across an entire structure for loads fixed in one position. On the other hand an influence line for shear or moment shows the variation of that response at one particular section in the structure caused by the movement of a unit load from one end of the structure to the other. In the following section, influence lines only for statically determinate structures are discussed.

6.1 Influence lines for beams and plate girders

Fig. 12(*a*) shows the influence line for shear at a section in a simply supported beam. It is assumed that positive shear occurs when the sum of the transverse forces to the left of a section is in the upward direction or when the sum of the forces to the right of the section is downward.

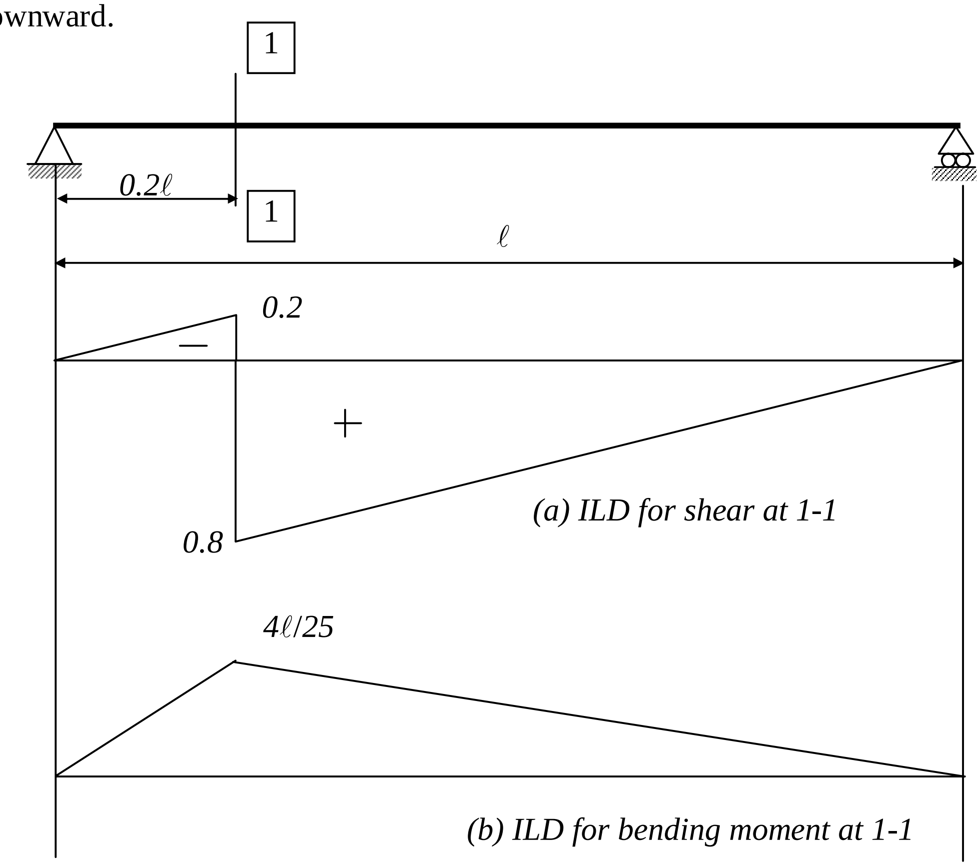


Fig. 12 Influence lines for shear and bending moment

A unit force is placed at various locations and the shear force at sections 1-1 is obtained for each position of the unit load. These values give the ordinates of influence line with which the influence line diagram for shear force at sections 1-1 can be constructed. Note that the slope of the influence line for shear on the left of the section is equal to the slope of the influence line on the right of the section. This information is useful in drawing shear force influence line in all cases.

Influence line for bending moment at the same section 1-1 of the simple beam is shown in Fig. 12(*b*). For a section, when the sum of the moments of all the forces to the left is clockwise or when the sum to the right is counter-clockwise, the moment is taken as positive. The values of bending moment at sections 1-1 are obtained for various positions of unit load and influence line is plotted. The detailed calculation of ordinates of influence lines is illustrated for members of the truss girder in the following section.

6.2 Influence lines for truss girders

Influence lines for support reactions and member forces for truss may be constructed in the same manner as those for beams. They are useful to determine the maximum force that may act on the truss members. The truss shown in Fig. 13 is considered for illustrating the construction of influence lines for trusses.

The member forces in U_3U_4, U_3L_4 and L_3L_4 are determined by passing a section *X-X* and considering the equilibrium of the free body diagram of one of the truss segments.

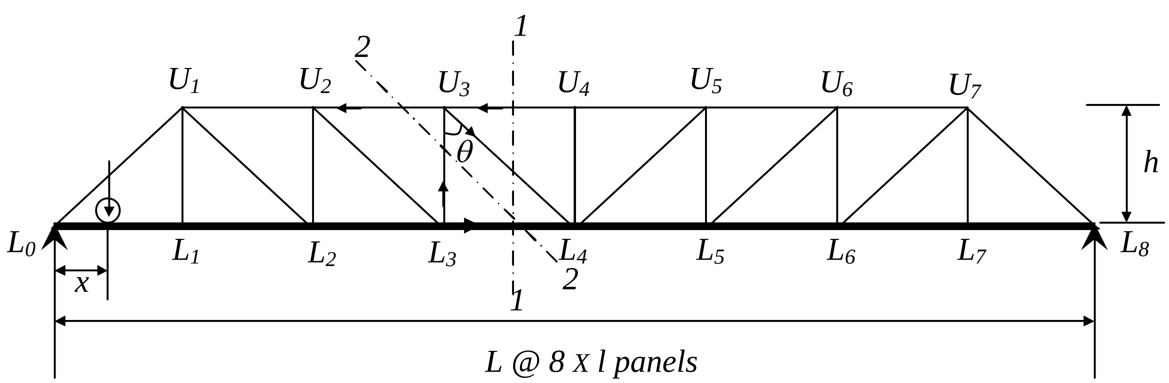


Fig.13 A typical truss

6.2.1 Influence line diagram for member U_3U_4 (Top chord member) [Fig. 14(a)]

Consider a section *1-1* and assume unit-rolling load is at a distance *x* from L_0. Then, from equilibrium considerations reactions at L_8 and L_0 are determined. The reactions are:

Reaction at L_8 = $\left(\dfrac{x}{L}\right)$

Reaction at L_0 = $\left(1-\dfrac{x}{L}\right)$

Consider the left-hand side of the section and take moments about L_4 by assuming appropriate directions for the forces in the members.

When unit load is in between L_0 and L_4:

$$\sum M_{L4} = 0$$

$$U_3U_4 \times h - \frac{x}{L} \times 4l \quad = 0$$

$$U_3U_4 = \frac{x}{h}\frac{4l}{L} = 0.5\,\frac{x}{h}$$

When unit load is in between L_4 and L_8:

Then, there will not be rolling unit load in the left-hand side section.

$$U_3U_4 = \frac{4l}{h}\left(1 - \frac{x}{L}\right)$$

Note that the influence diagram gives force in the member U_3U_4 directly, due to the unit load.

6.2.2 Influence line diagram for member U_3L_4 (Inclined member)[Fig 14(b)]

Again consider the left-hand side of the section 1-1, and use the equilibrium equation for vertical forces i.e. $\sum V = 0$ where, V represents the vertical force.

When unit load is in between L_0 and L_3:

$$\frac{x}{L} + U_3\, L_4\, Cos\theta = 0$$

$$\Rightarrow U_3L_4 = \frac{-x}{Lcos\theta}$$

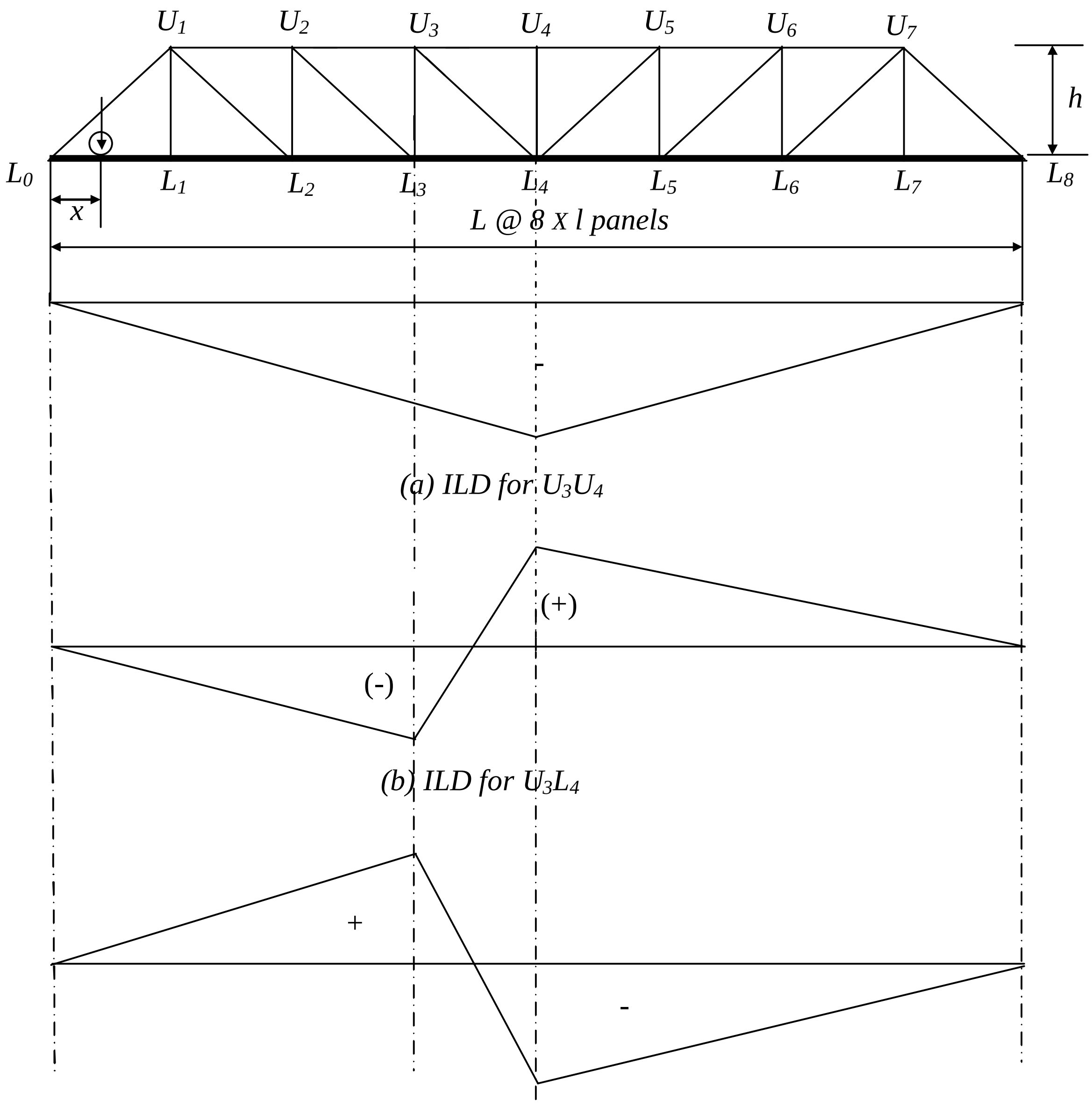
$$where,\ \theta = \tan^{-1}\left(\frac{\ell}{h}\right)$$

When unit load is in between L_4 and L_8:

$$U_3L_4 = \frac{1}{Cos\theta}\left(1 - \frac{x}{L}\right)$$

When unit load is in between L_3 and L_4:

Since the variation of force in member U_3L_4 is linear as the unit load moves from L_3 to L_4 joining the ordinates of influence line at L_3 and L_4 by a straight line gives the influence line diagram in that zone. Note that, U_3L_4 represents the force in that member.



(c) ILD for U_3L_3

Fig. 14 Typical shapes of influence lines

6.2.3 Influence line diagram for U_3L_3 (Vertical member) [Fig. 14(c)]

Consider the left-hand side of the section *2-2* shown in Fig. 13 for illustrating the construction of influence line for vertical member.

When unit load is in between L_0 and L_3:

By considering the equilibrium equation on the section left hand side of axis 2-2.

$$U_3 L_3 - \frac{x}{L} = 0$$

$$\Rightarrow U_3 L_3 = \frac{x}{L}$$

When unit load is in between L_4 and L_8:

$$U_3 L_3 = -\left(1 - \frac{x}{L}\right)$$

When unit load is in between L_3 and L_4:

Joining the ordinates of influence line at L_3 and L_4 by a straight line gives the influence line diagram between L_3 and L_4.U_3L_3 represents the force in that member.

Similarly influence line diagrams can be drawn for all other members. Typical shapes of influence line diagrams for the members discussed are shown in Fig. 14. The design force in the member is obtained in the following manner. In this chapter, compressive forces are considered negative and tensile forces are positive.

Case (1): If the loading is Railway loading (UDL)

- Influence line diagram for force is drawn for that member
- The algebraic sum of areas of influence line under loaded length multiplied by magnitude of uniformly distributed load gives the design force.

Case (2): If the loading is Highway loading (Concentrated loading)

- Influence line diagram for force is drawn for that member
- The algebraic sum of the respective ordinates of influence line at the concentrated load location multiplied by concentrated loads gives design load of that member
- The series of concentrated loads are arranged in such a way that the maximum value of the desired member force is obtained.

7.0 SUMMARY

After brief introduction, the steel used in bridges and its properties were discussed. The broad classification of bridges was mentioned and various loads to be considered in designing railway and highway bridges in India were discussed. Finally analysis of girder bridges was discussed using influence line diagrams.

8.0 REFERENCES

1. Owens. G.W., Knowles. P.R., Dowling. P.J. (1994): Steel Designers' Manual, Fifth edition, Blackwell Scientific Publications.

2. Chatterjee. S. (1991): The Design of Modern Steel Bridges, First edition, BSP Professional books.

3. Demetrios. E.T. (1994): Design, Rehabilitation and Maintenance of Modern Highway Bridges, McGraw-Hill Publishers.

4. Victor. D.J. (1973): Essentials of Bridge Engineering, Oxford and IBH Publishers.

5. IRC: 6 - 1966 – Section II, Indian Standard for loads and stresses on Highway Bridges.

6. Bridge rules - 1982, Specifications for Indian Railway loading.