Bipartite Matching and Max Flow Proof

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Let W be set of vertices denoting workers.

Let J be set of vertices denoting jobs

Clearly, $|W| = n$ & $|J| = m$

Let E be set of edges mapping each worker to the job(s) that they are willing to do.

i.e. $(w, j) \in E \Rightarrow$ Worker w is willing to do job j

Consider $V = W \cup J \cup \{s, t\}$ where s, t are source, sink vertices.

Consider additional edges A mapping source to each worker and each job to sink.

i.e. $A = \left[\bigcup_{w \in W} (s, w)\right] \cup \left[\bigcup_{j \in J} (j, t)\right]$

Now, consider the network N having graph $G(V, E \cup A)$, source s and sink t, with each edge having capacity 1.

Clearly, a matching of cardinality λ from W to J induces a flow of λ in N.

Also, a flow of λ in N implies a matching of cardinality λ from W to J.

Therefore, the maximum bipartite matching problem is a reduction of the max-flow problem.

Q.E.D.