Upsampling

$$x[n] = x_c(nT)$$

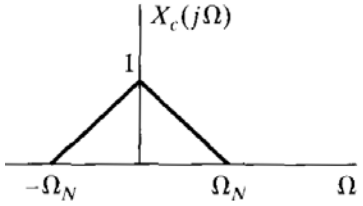
Expander

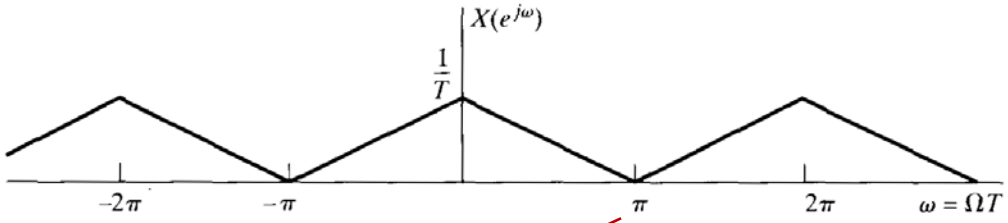
$$x_e[n] = \begin{cases} x[n/L], & n = 0, \pm L, \pm 2L, \ldots, \\ 0, & \text{otherwise}, \end{cases}$$

$$x_e[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n - kL].$$

$$X_e(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \left(\sum_{k=-\infty}^{\infty} x[k]\delta[n-kL] \right) e^{-j\omega n}$$

$$= \sum_{k=-\infty}^{\infty} x[k]e^{-j\omega Lk} = X(e^{j\omega L}).$$





ω is replaced by ωL $\quad\quad$ $\omega = \Omega T'$