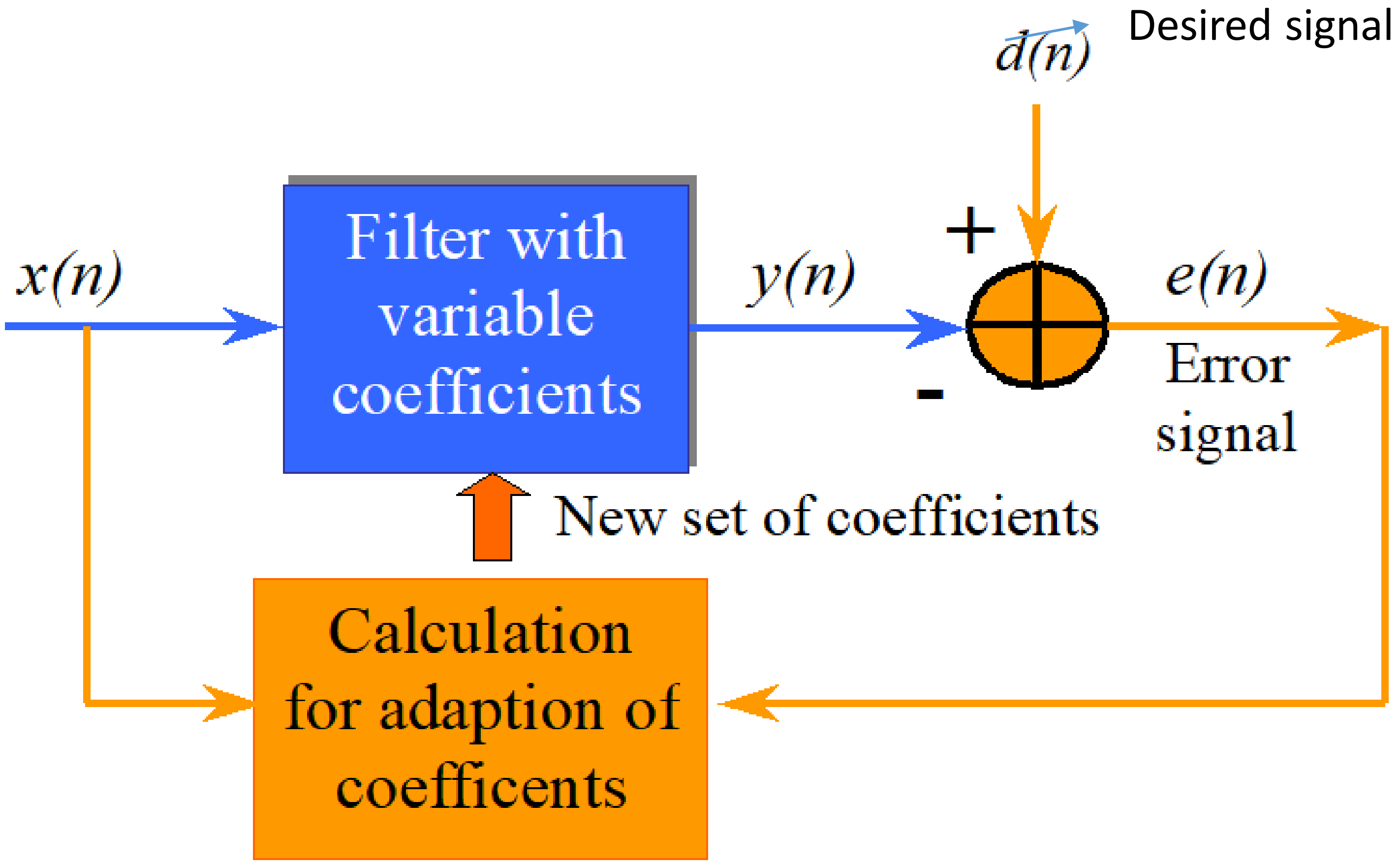
Experiment 5



$$\mathbf{x}[n] = \left[x[n] \quad x[n-1] \quad . \quad x[n-m+1]\right]^T$$

$$\mathbf{h} = \left[h[0] \quad h[1] \quad . \quad h[m+1]\right]^T$$

Where, $y[n] = \mathbf{x}[n]^T \mathbf{h}$

To minimize, $J(\mathbf{h}) = E\{[d[n] - y[n]]^2\}$

i.e. $J(\mathbf{h}) = \sigma_d^2 - \mathbf{p}^T\mathbf{h} - \mathbf{h}^T\mathbf{p} + \mathbf{h}^T\mathbf{R}\mathbf{h}$

Where

$$\mathbf{R} = E\{\mathbf{x}[n]\mathbf{x}[n]^T\}$$

$$\mathbf{p} = E\{\mathbf{x}[n]d[n]\}$$

Steepest descent:

$$\mathbf{h}[n+1] = \mathbf{h}[n] - \frac{1}{2}\mu\frac{\partial J(\mathbf{h}[n])}{\partial \mathbf{h}[n]}$$

Where

$$\frac{\partial J(\mathbf{h})}{\partial \mathbf{h}} = -2\mathbf{p} + 2\mathbf{R}\mathbf{h}$$

$$\mathbf{h}[n+1] = \mathbf{h}[n] + \mu(\mathbf{p} - \mathbf{R}\mathbf{h}[n])$$

But **R** and **p** are not known.

Approximation: $\hat{\mathbf{R}}[n] = \mathbf{x}[n]\mathbf{x}[n]^T$

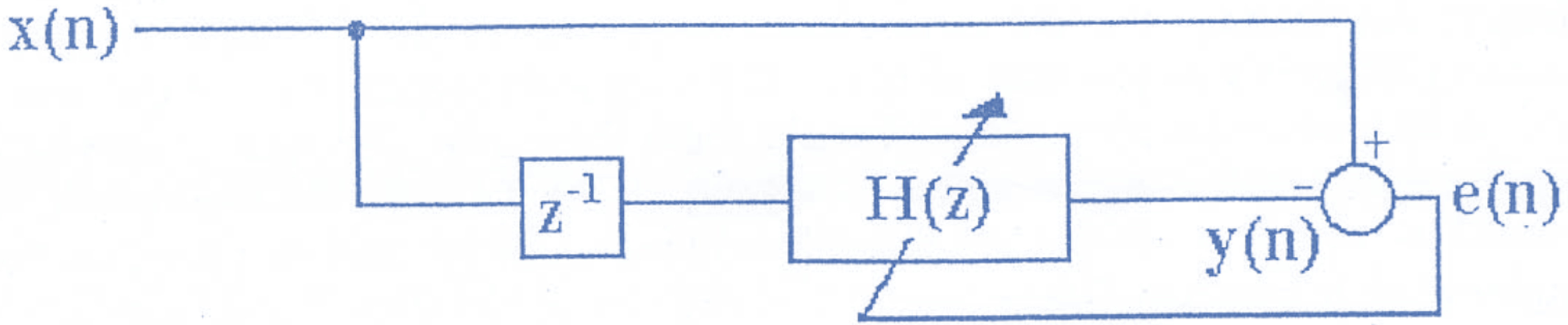
$$\hat{\mathbf{p}}[n] = \mathbf{x}[n]d[n]$$

$$\mathbf{h}[n+1] = \mathbf{h}[n] + \mu\mathbf{x}[n](d[n] - \mathbf{x}[n]^T\mathbf{h}[n])$$

$$e[n] = (d[n] - \mathbf{x}[n]^T\mathbf{h}[n]) = (d[n] - y[n])$$

$$\mathbf{h}[n+1] = \mathbf{h}[n] + \mu\mathbf{x}[n]e[n]$$

1. Take a sinusoidal message waveform $m(t) = A\sin(2\pi \times F_0 t)$ (Take $A = 2$ and F_0=1 kHz).
2. Add white Gaussian noise $n(t)$ of zero mean and unity variance to $m(t)$ and obtain the input signal $x(t)$.
3. Sample it properly to generate the discrete input signal $x(n)$.
4. Pass $x(n)$ through the system as shown below (Fig.2)



5. Adapt the filter coefficients as

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu\,\mathbf{x}(n)\,e(n)$$

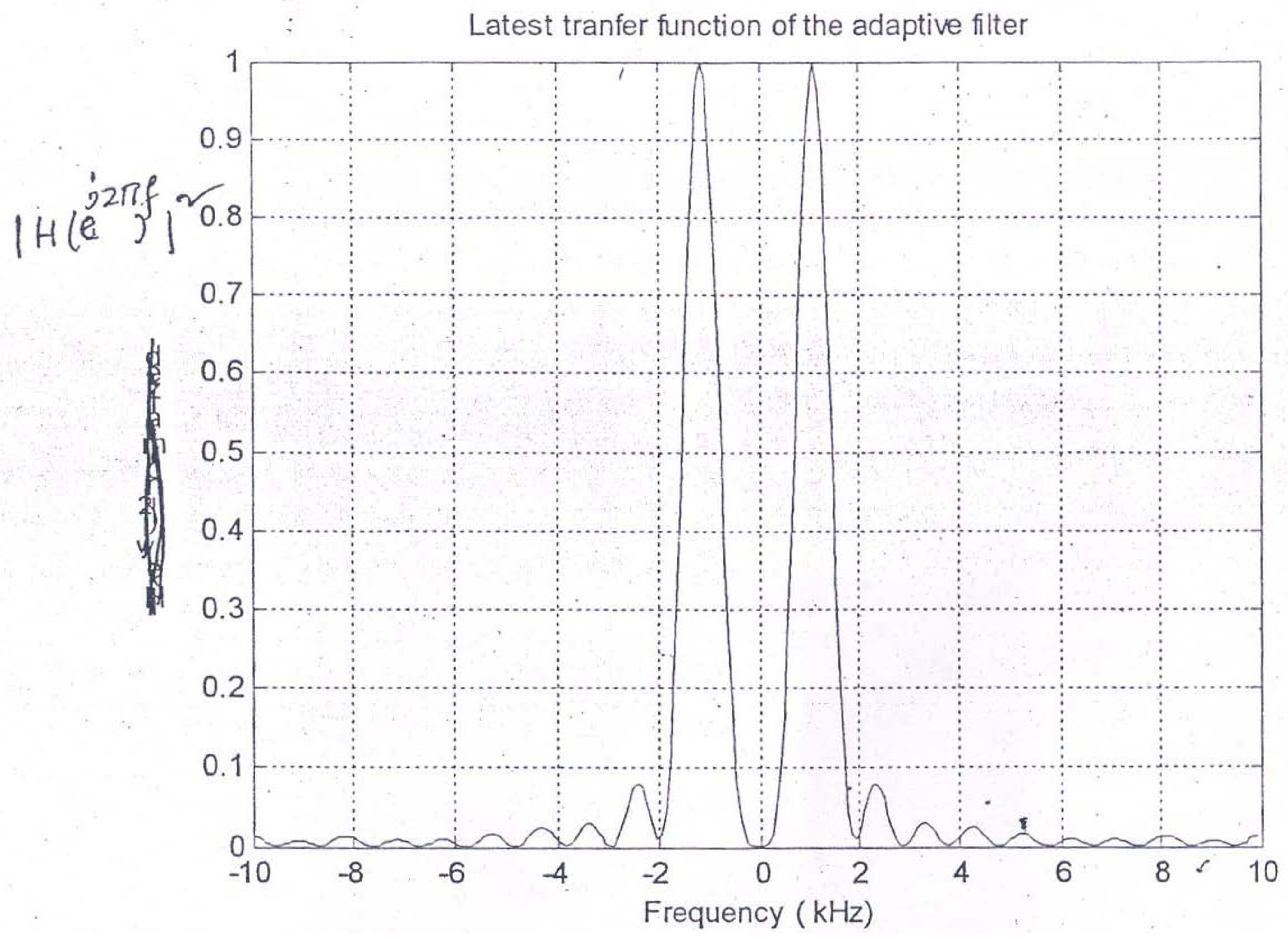
Take $\mu = 10^{-4}$.

6. Continue the iteration in step 5. until the relative change $\dfrac{\left\|\mathbf{w}(n+1) - \mathbf{w}(n)\right\|^2}{\left\|\mathbf{w}(n)\right\|^2} < \varepsilon'$

Take $\varepsilon' = 10^{-3}$.

7. Plot the latest transfer function $\left|H\left(e^{j2\pi f}\right)\right|^2$ of the filter (Fig.3).

8. Repeat the above with F_0=2, 3, 10 kHz and observe the change in $\left|H\left(e^{j2\pi f}\right)\right|^2$.



Latest tranfer function of the adaptive filter
Frequency (kHz)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-10
-8
-6
-4
-2
0
2
4
6
8
10
$|H(e^{j2\pi f})|^2$