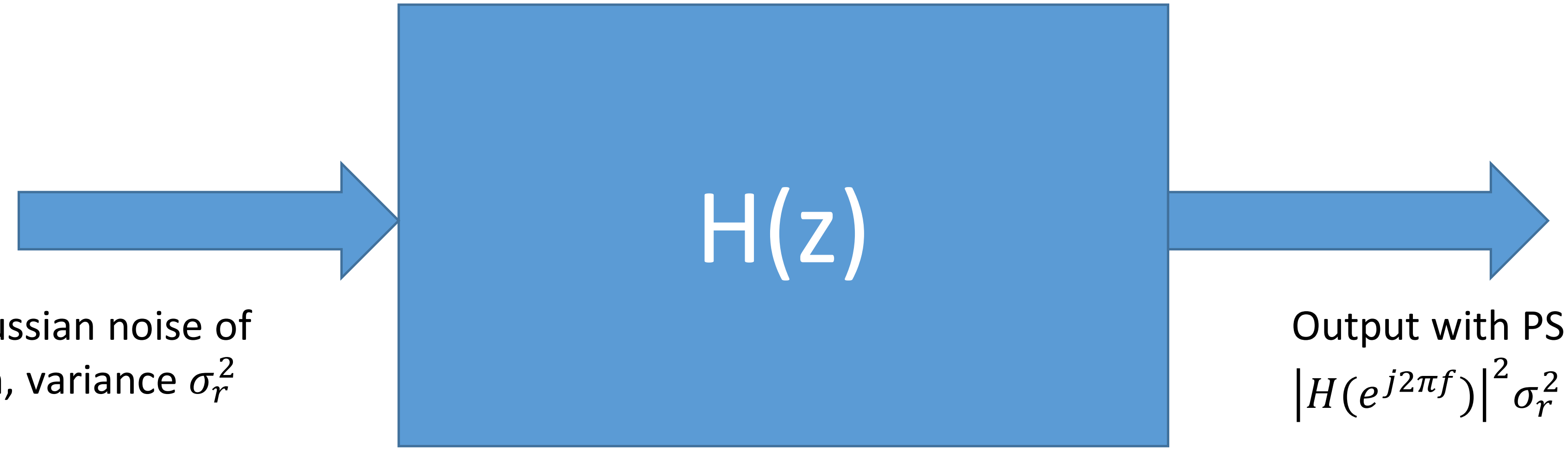
DSP Lab

Experiment 04

Generating what is to be estimated



White Gaussian noise of zero mean, variance σ_r^2

Output with PSD $\left|H(e^{j2\pi f})\right|^2\sigma_r^2$

Classical Periodogram

- PSD of a random sequence

$$P_{xx}(f) = \lim_{M\to\infty}\left[\frac{1}{2M+1}E\left\{\left|\sum_{n=-M}^{M}x(n)e^{-j2\pi fn}\right|^2\right\}\right]$$

- Estimation by classical method

$$\hat{P}_{xx}(f) = \frac{1}{M}\left|\sum_{n=0}^{M-1}x(n)e^{-j2\pi fn}\right|^2$$

$$\mathrm{Bias}[\hat{\theta}] = E_{P(x|\theta)}[\hat{\theta}] - \theta \longrightarrow$$ 0 as M increases

$$\mathrm{var}(\hat{\theta}) = E[(\hat{\theta} - E(\hat{\theta}))^2]$$ does not

Welch’s Method

$$P_{xx}^{W}(f)=\frac{1}{L}\sum_{i=0}^{L-1}\widetilde{P}_{xx}^{(i)}(f)$$

$$\hat{P}_{xx}^{(i)}(f)=\frac{1}{MU}\left|\sum_{n=0}^{M-1}x_i(n)w(n)e^{-j2\pi fn}\right|^2 \quad i=0,1,\ldots,L-1$$

$$U=\frac{1}{M}\sum_{n=0}^{M-1}w^2(n)$$

1. Divide $x(n)$ into L (Typically, L = 8) overlapping blocks, each block of length M with D samples common between two successive blocks as,

$$x_i(n) = x(n+iD) \quad n = 0,1,\ldots,M-1$$
$$i = 0,1,\ldots,L-1$$

For the sake of simplicity you may however take $D=0$ first, which means no overlapping. Later, you can observe the effect of overlapping by choosing different values of D (Typically, 50% overlapping is used).

2. Obtain the estimated PSD of block i as,

$$\hat{P}_{xx}^{(i)}(f)=\frac{1}{MU}\left|\sum_{n=0}^{M-1}x_i(n)w(n)e^{-j2\pi fn}\right|^2 \qquad i=0,1,\ldots,L-1$$

where $w(n)$ is the window function of length M (usually, a Hamming window) and U is a normalization factor for the power in the window function defined as,

$$U=\frac{1}{M}\sum_{n=0}^{M-1}w^2(n)$$

- Note that the DTFT at step 4 is to be computed by FFT algorithm and can be obtained only at discrete frequencies. You may zero pad each block to N_0. In that case the frequencies at which the PSD is obtained is $f=k/N_0$ $k=0,1,\ldots,(N_0-1)$.

3. Obtain Welch spectrum estimate by the average of these modified periodogram, that is,

$$P_{xx}^{W}(f)=\frac{1}{L}\sum_{i=0}^{L-1}\widetilde{P}_{xx}^{(i)}(f)$$

4. Plot the estimated PSD and compare it with the known PSD $\left|H\left(e^{j2\pi f}\right)\right|^2\sigma_r^2$ (fig.1).

5. Try with different transfer function $H(z)$.

Given in instruction sheet for a specific H(z)