Assignment - HW 8

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4.66 Derive Skewness of Brownian motion.

$$E[S_t^3] = e^{3\sigma^2 t}$$

$$Skew[S_t] = \frac{1}{\sigma_t^3}\left[e^{3\sigma^2 t} - 3e^{\sigma^2 t} + 2\right]$$

Ans)

$$E[(S_t - E(S_t))^3]$$

Since $E[S_t] = 1$ (from above)

$$= E[(S_t - 1)^3]$$

$$= E[S_t^3 - 1 - 3S_t^2 + 3S_t]$$

$$= E[S_t^3] - E[1] - E[3S_t^2] + 3E[S_t]$$

$$= e^{3\sigma^2 t} - 1 - 3e^{\sigma^2 t} + 3$$

$$= e^{3\sigma^2 t} - 3e^{\sigma^2 t} + 2$$

$$\therefore Skew[S_t] = \frac{1}{\sigma^3 t}\left[e^{3\sigma^2 t} - 3e^{\sigma^2 t} + 2\right]$$

4.67 $Skew[S_t] = \frac{1}{\sigma_t^3}\left[1 + 3\sigma^2 t - 3 - 3\sigma^2 t + 2\right] \to 0$

Ans) $e^x \approx 1 + x$

where $x << 1$

therefore $skew(S_t)$

$$\to \frac{1}{\sigma^2 t}\left(1 + 3\sigma^2 t - 3(1 + \sigma^2 t) + 2\right)$$

$$= \frac{1}{\sigma^3 t} \times 0$$

$$= 0$$

(hence proved)

4.68 $E[S_t^4] = e^{6\sigma^2 t}$

$$Kurt[S_t] = \frac{1}{\sigma_t^4}\left[e^{6\sigma^2 t} - 4e^{3\sigma^2 t} + 6e^{\sigma^2 t} - 3\right]$$

Ans)

$$E[(S_t - E(S_t))^4]$$

$$= E[(S_t - 1)^4]$$

$= E\left[S_t^4 - 4S_t^3 + 6S_t^2 - 4S_t + 1\right]$

$= e^{6\sigma^2 t} - 4e^{3\sigma^2 t} + 6e^{\sigma^2 t} - 4 + 1$

$= e^{6\sigma^2 t} - 4e^{3\sigma^2 t} + 6e^{\sigma^2 t} - 3$

hence proved.

4.69 $\quad \text{Kurt}[S_t] = \frac{1}{\sigma_t^4}\left[1 + 6\sigma^2 t - 4 - 12\sigma^2 t + 6 + 6\sigma^2 t - 3\right] \to 0$

$= \frac{1}{\sigma^4 t}\left(1 + 6\sigma^2 t - 4(1 + 3\sigma^3 t) + 6(1 + \sigma^3 t) - 3\right)$

$= \frac{1}{\sigma^4(t)} \times 0$ (from above)

$= 0$ hence proved.