Uncertainty in number of quanta.

Suppose the field is initially in a coherent state, i.e. the probability of measuring n photons at $t=0$ is:

$$P_n(0) = \frac{\langle n\rangle^n}{n!} e^{-\langle n\rangle}$$ where $\langle n\rangle$ is the average photon number $\left(\sum_{n=0}^{\infty} \frac{\langle n\rangle^n}{n!} e^{-\langle n\rangle} = e^{\langle n\rangle} e^{-\langle n\rangle} = 1\right)$

=> If the atom is initially in the ground state:

$$|\Psi(0)\rangle = \sum_{n=0}^{\infty} \left(\frac{\langle n\rangle^n}{n!}\right)^{\frac{1}{2}} e^{-\frac{\langle n\rangle}{2}} |g,n\rangle$$

$$|\Psi(t)\rangle = \hat{U}(t)|\Psi(0)\rangle$$ Assuming resonance

$$= \sum_{n=0}^{\infty} \left(\frac{\langle n\rangle^n}{n!}\right)^{\frac{1}{2}} e^{-\frac{\langle n\rangle}{2}} \left(-i\sin(\lambda\sqrt{n}\,t)|e,n-1\rangle + \cos(\lambda\sqrt{n}\,t)|g,n\rangle\right)$$

=> The probability of measuring excited state:

$$P_e(t) = \left(\sum_{n=0}^{\infty} \left(\frac{\langle n\rangle^n}{n!}\right)^{\frac{1}{2}} e^{-\frac{\langle n\rangle}{2}} \sin(\lambda\sqrt{n}\,t)\right)^2$$