MPS PRACTICE PROBLEM SET 7

SOLUTIONS

1(a) Number of helium atoms per unit volume

$$= 10^9 \text{ kg m}^{-3} \times \frac{1 \text{ helium atom}}{7 \times 10^{-27} \text{ kg}}$$

$$= 1.4 \times 10^{35} \text{ helium atoms per m}^3.$$

Each helium atom is completely ionized, and so contributes two electrons to the Fermi gas

$\Rightarrow$ density of electron gas $= 2.8 \times 10^{35}$ electrons/m^3

(b) From lecture

$$\varepsilon_F = \frac{\hbar^2}{2m}\left(3\pi^2 \frac{\bar{N}}{V}\right)^{2/3}.$$

From (a), $\dfrac{\bar{N}}{V} = 2.8 \times 10^{35} \text{ m}^{-3}$

$m =$ electron mass $= 9.1 \times 10^{-31}$ kg

$\hbar = \dfrac{h}{2\pi} = \dfrac{1}{2\pi} \times 6.6 \times 10^{-34}$ J s.

$$\Rightarrow \varepsilon_F = \frac{1}{4\pi^2} \times \frac{(6.6 \times 10^{-34} \text{ kg m}^2 \text{ s}^{-1})^2}{2 \times 9.1 \times 10^{-31} \text{ kg}} \times (3\pi^2)^{2/3}$$

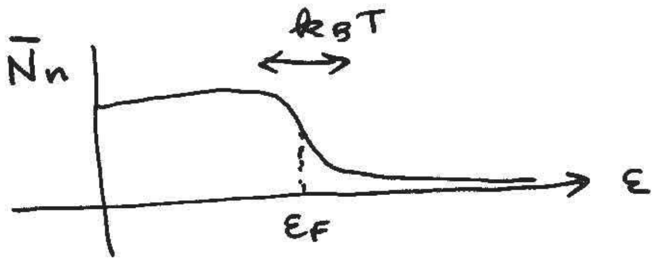
$$\times (2.8 \times 10^{35} \text{ m}^{-3})^{2/3}$$

$= 2.5 \times 10^{-14}\ kg\ m^2\ s^{-2}$

$= 2.5 \times 10^{-14}\ J \times \dfrac{1\ eV}{1.6 \times 10^{-19}\ J}$

$= 1.6 \times 10^5\ eV$

(c)



The electron gas is degenerate if $k_B T << \varepsilon_F$.

From (b), $\varepsilon_F \simeq 1.6 \times 10^5\ eV$.

$k_B T = 1.4 \times 10^{-23}\ J\ K^{-1} \times 10^7\ K$

$= 1.4 \times 10^{-16}\ J \times \dfrac{1\ eV}{1.6 \times 10^{-19}\ J}$

$= 8.8 \times 10^2\ eV$

$<< \varepsilon_F$.

So the electron gas can be treated as being degenerate i.e its properties will be similar to those for an electron gas at $T = 0$.

(d) $\frac{GM^2}{R^4}$ ~ electron gas pressure

$$\sim \varepsilon_F^{5/2} \quad \text{from lecture notes}$$

$$\sim \left(\frac{\bar{N}}{V}\right)^{5/3}$$

Since $V = R^3$, and the number of electrons $\bar{N}$ is twice the mass of the star divided by the mass of a helium atom

$$\frac{GM^2}{R^4} \sim \left(\frac{M}{R^3}\right)^{5/3} = \frac{M^{5/3}}{R^5}$$

$$\Rightarrow \quad M^{5/3} - 2 \sim R^{5-4}$$

$$\Rightarrow \quad M^{-1/3} \sim R$$

(e) Density $\rho = \frac{\text{mass}}{\text{volume}}$

$$= \frac{M}{\frac{4}{3}\pi R^3}$$

$$\sim \frac{M}{M^{-1}}$$

$$= M^2$$