Many Particle Systems – Practice Problem Set 5

These problems are not for assessment. However, it is recommended that you attempt them as practice for the test and exam.

1. A system consists of two identical particles, each of which can occupy two single particle quantum states with energies ϵ and 2ϵ respectively. Suppose the system of two particles is in contact with a heat reservoir at temperature T. Give an expression for the canonical partition function if:

   (a) the particles are distinguishable;

   (b) the particles are indistinguishable and we approximate using modified Maxwell-Boltzmann statistics;

   (c) the particles are indistinguishable and obey Bose-Einstein statistics;

   (d) the particles are indistinguishable and obey Fermi-Dirac statistics.

   (e) Compute the mean energy of the system in cases (c) and (d) above.

   (f) Compute the entropy of the system in case (d) above.

   (g) Provide a physical interpretation for the result in (f).

2. (a) Consider a system which has two energy eigenstates with energies ϵ_1 and ϵ_2, with $\epsilon_2 > \epsilon_1$. Suppose the system is placed in contact with a heat reservoir at temperature T. Give an expression for the mean energy of the system.

   (b) Explain why we can consider the case $\frac{(\epsilon_2 - \epsilon_1)}{k_B T} \ll 1$ to be a "high temperature" limit, and the case $\frac{(\epsilon_2 - \epsilon_1)}{k_B T} \gg 1$ a "low temperature limit".

   (c) Determine the high and low temperature limits of the mean energy. Note: $e^x \approx 1 + x$ for $|x| << 1$.

   (d) Give an explanation for the results you obtain in (c).