MPS PRACTICE PROBLEM SET 3

SOLUTIONS

1. THE FOUR POSSIBLE ENERGY E'STATES OF THE SYSTEM ARE

$$E_1 = -\varepsilon$$

$$E_2 = 0$$

$$E_3 = 0$$

$$E_4 = +\varepsilon$$

WHERE $\varepsilon = 1 \times 10^{-20}\,\mathrm{J}$

FOR A SYSTEM IN CONTACT WITH A HEAT RESERVOIR AT TEMPERATURE T, THE PROBABILITY IT WILL BE FOUND IN AN ENERGY E'STATE IS

$$P_n^{(eq)} = \frac{e^{-E_n/k_BT}}{Z}$$

WHERE n LABELS THE ACCESSIBLE ENERGY E'STATES, E_n IS THE ENERGY OF THE E'STATE, AND

$$Z = \sum_n e^{-E_n/k_BT}$$

IS THE CANONICAL PARTITION FUNCTION.

IN THIS CASE

$$Z = e^{-E_1/k_BT} + e^{-E_2/k_BT} + e^{-E_3/k_BT} + e^{-E_4/k_BT}$$

$$= e^{\varepsilon/k_BT} + e^{0/k_BT} + e^{0/k_BT} + e^{-\varepsilon/k_BT}$$

$$= 2 + e^{-\varepsilon/k_BT} + e^{+\varepsilon/k_BT}$$

THERE ARE TWO ENERGY E'STATES WITH ENERGY 0 J, EACH OF WHICH HAS A PROBABILITY TO BE OCCUPIED OF

$$\rho_2^{(eq)} = \rho_3^{(eq)} = \frac{e^{0/k_BT}}{Z}$$

$$= \frac{1}{2 + e^{-\varepsilon/k_BT} + e^{+\varepsilon/k_BT}}$$

SO THE PROBABILITY OF OCCUPATION OF AN ENERGY EIGENSTATE WITH ENERGY ZERO IS

$$2 \times \rho_2^{(eq)} = \frac{2}{2 + e^{-\varepsilon/k_BT} + e^{+\varepsilon/k_BT}}$$

USING

$$\varepsilon = 1 \times 10^{-20}\ \mathrm{J}$$

$$k_B = 1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}}$$

$$T = 200\ \mathrm{K}$$

$$\frac{\varepsilon}{k_BT} = 3.62$$

WE FIND THE PROBABILITY OF OCCUPYING EACH OF THE 4 STATES IS (LISTING ALL 4 JUST FOR COMPLETENESS)

$$P_1^{(eq)} = \frac{e^{+\varepsilon/k_BT}}{Z} = 0.949$$

$$P_2^{(eq)} = P_3^{(eq)} = \frac{1}{Z} = 0.0253$$

$$P_4^{(eq)} = \frac{e^{-\varepsilon/k_BT}}{Z} = 6.76 \times 10^{-4}$$

SO THE PROBABILITY OF OCCUPYING A STATE WITH ZERO ENERGY IS

$$2 \times P_2^{(eq)} = 2 \times 0.0253 = \underline{\underline{0.0507}}$$

2. (a) THIS IS THE SAME AS QUESTION 1 WITH "SYSTEM" REPLACED BY "PARTICLE" AND $\mathcal{E}$ REPLACED BY a. (THAT IS, NOW OUR SYSTEM IS JUST A SINGLE PARTICLE). SO THE PROBABILITY THAT A STATE WITH ZERO ENERGY IS OCCUPIED IS

$$\frac{2}{2 + e^{-a/k_BT} + e^{a/k_BT}} .$$

(b) EACH OF THE PARTICLES A & B CAN BE IN STATES WITH ENERGIES

$$E_1 = -a$$

$$E_2 = 0$$

$$E_3 = 0$$

$$E_4 = +a$$

IF THE TOTAL ENERGY OF THE TWO PARTICLES SYSTEM IS $+a$, THEN THE TOTAL SYSTEM CAN BE IN ANY OF THE FOLLOWING STATES:

	A	B
<u>STATE 1</u>:	E_4	E_2
<u>STATE 2</u>:	E_4	E_3
<u>STATE 3</u>:	E_2	E_4
<u>STATE 4</u>:	E_3	E_4

FOR EXAMPLE, THE <u>STATE 1</u> FOR THE SYSTEM HAS PARTICLE A IN THE ENERGY E' STATE $E_4 = +a$, & PARTICLE B IN THE ENERGY E' STATE $E_2 = 0$.

SINCE THE SYSTEM IS ISOLATED, EACH OF THE STATES OF THE ENTIRE SYSTEM STATE 1, STATE 2, STATE 3 AND STATE 4 ARE EQUALLY LIKELY TO BE OCCUPIED.

$\Longrightarrow$ THE PROBABILITY FOR THE PARTICLE A TO BE IN THE ENERGY ε' STATE WITH ENERGY $+a$ (ie FOR A TO BE IN THE STATE WITH ENERGY E_4) IS

$$\frac{2}{4} = \frac{1}{2}$$

[ie THERE ARE TWO OF FOUR STATES FOR THE SYSTEM IN WHICH A HAS ENERGY $+a$: STATE 1 & STATE 2.]