Many Particle Systems – Practice Problem Set 2

These problems are not for assessment. However, it is recommended that you attempt them as practice for the test and exam.

1. For N particles of an ideal monatomic ideal gas,

$$E = \frac{3}{2} N k_B T$$

and

$$S = N k_B \left(\frac{3}{2} \ln T + \ln V + \kappa \right),$$

where κ is a constant.

   (a) Express the entropy in terms of its "natural" variables E, V and N.

   (b) Determine the three equations of state for the ideal gas that follow from partial derivatives of the entropy with respect to E, V and N.

2. The Gibbs free energy is defined as

$$G = E - TS + pV.$$

   (a) Prove that the Gibbs free energy is naturally a function of the variables T, p and $\vec{N}$, in the sense that the change in the Gibbs free energy in an infinitesimal quasistatic change can be shown to depend on the change in these variables using conservation of energy (which is applicable to all thermodynamic systems).

   (b) Determine the three equations of state relating partial derivatives of the $G(T, p, \vec{N})$ to S, V and the chemical potentials μ_i.

3. The entropy of a gas of photons (e.g. the thermal radiation that fills a darkened room as a result of the temperature of the walls) as a function of its internal energy E and volume V is

$$S(E, V) = \frac{4}{3} a \, V^{\frac{1}{4}} \, E^{\frac{3}{4}},$$

where a is a constant. Note that the entropy is independent of the number N of photons in the gas.

   (a) Verify Stefan-Boltzmann law (which states the energy density of a photon gas is proportional to T^4).

(b) Determine the pressure of the gas of photons as a function of its temperature T (this is the radiation pressure exerted by the photon gas).

(c) Prove that the free energy of a photon gas at temperature T occupying a volume V is given by

$$F = -\frac{a^4}{3} V T^4,$$

(independent of the number of photons in the gas).

(d) Determine an equation of state for the gas involving pressure and temperature by taking a partial derivative of the free energy, and show that the result is consistent with part (b).

(e) Express the product pV in terms of the energy of the gas.

(f) Show that for an adiabatic compression of a photon gas (i.e. no heat gain or loss by the gas),

$$pV^{4/3} = \text{const.}$$

(g) Show that the Gibbs free energy of a photon gas is zero.

4. The entropy of a monatomic gas of particles confined to move in a two dimensional surface of area A is given by

$$S = Nk_B \left(\ln \left(\frac{mAE}{2\pi\hbar^2 N^2} \right) + 2 \right),$$

where m is the mass of a particle, N is the number of particles, and E is the energy of the gas. Determine the temperature of the gas, and explain how the result relates to the equipartition theorem.

5. Consider the statement of conservation of energy

$$dE = TdS - p\,dV + \sum_i \mu_i dN_i.$$

(a) What are the "natural variables" for the total energy of a system?

(b) Write down the general form of three equations of state in terms of partial derivatives of the energy.

Note: these are not particularly useful in practice as it is difficult to control the entropy of a system experimentally.