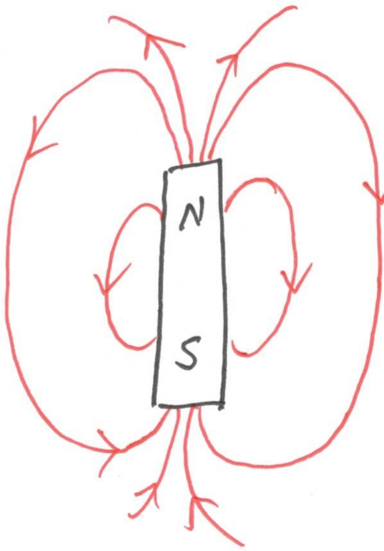
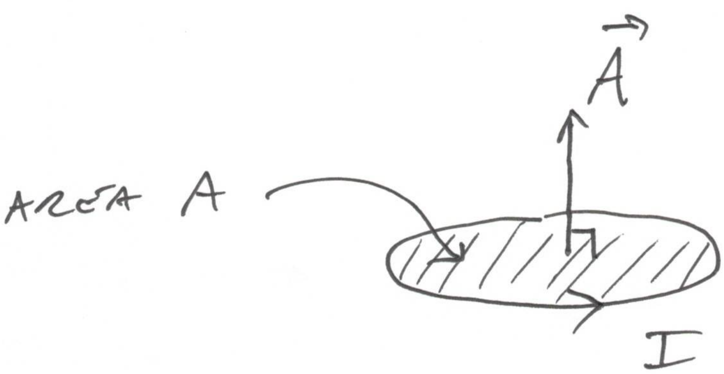
- A <u>MAGNETIC DIPOLE MOMENT</u> (MDM) $\vec{\mu}$ IS — ROUGHLY SPEAKING — SOMETHING WITH A MAGNETIC FIELD OF A BAR MAGNET



- WE SHOULD THINK OF A MDM AS A LITTLE LOOP OF CURRENT I WITH AN AREA A



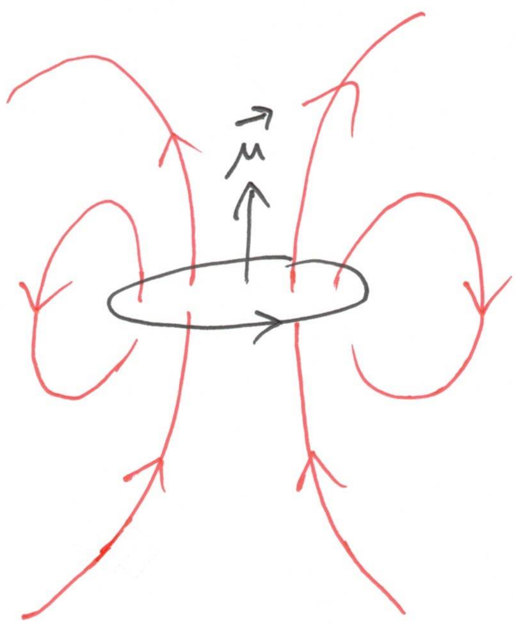
$\vec{A}$ — AREA VECTOR

THE AREA VECTOR $\vec{A}$ HAS A MAGNITUDE

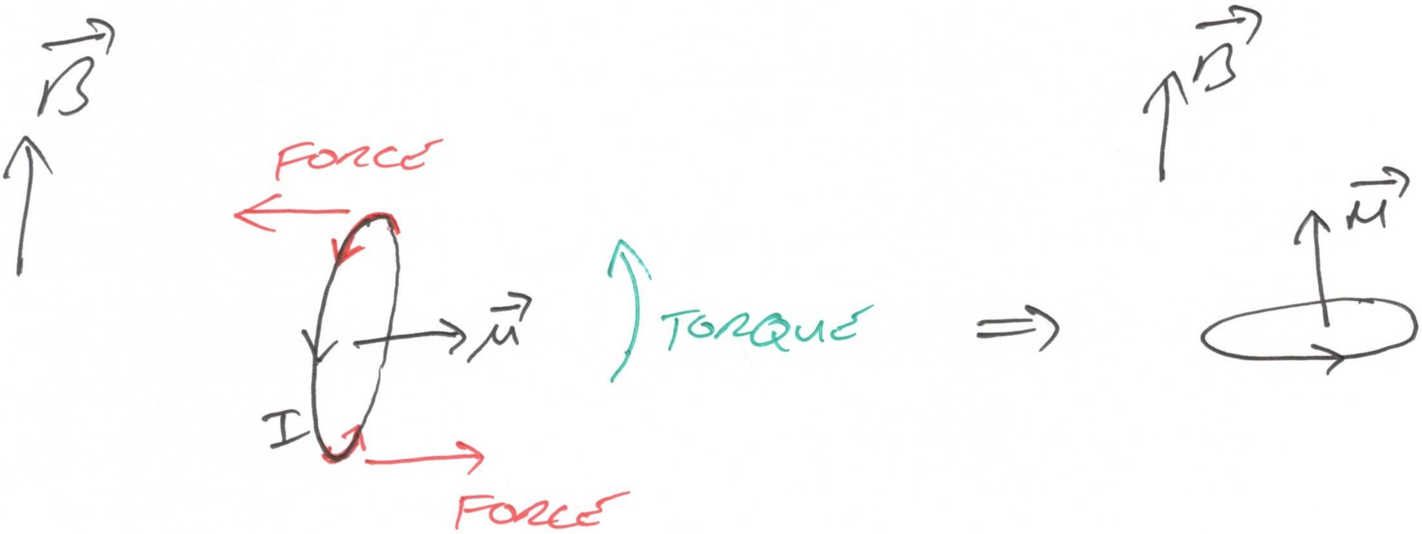
A = AREA OF CURRENT LOOP

AND DIRECTION $\perp$ TO PLANE OF THE LOOP. THEN

$$\vec{\mu} := I\vec{A}$$



- WHEN PLACED IN AN EXTERNAL MAGNETIC FIELD $\vec{B}$ THE MDM EXPERIENCES A TORQUE TENDING TO ALIGN $\vec{\mu}$ WITH $\vec{B}$



AND IT HAS A <u>MAGNETIC POTENTIAL ENERGY</u>

OF

$$\boxed{V_{mag} = -\vec{\mu} \cdot \vec{B}}$$

- MINIMUM VALUE IF $\vec{\mu}$ & $\vec{B}$ ALIGN $(-\mu B)$
- MAXIMUM VALUE IF $\vec{\mu}$ & $\vec{B}$ ANTI-ALIGN $(+\mu B)$

ALIGN : $\uparrow \vec{\mu} \;\; \uparrow \vec{B}$

ANTI-ALIGN : $\downarrow \vec{\mu} \;\; \uparrow \vec{B}$

THE MDM "WANTS" A MINIMUM ENERGY

& SO WANTS TO ALIGN WITH $\vec{B}$.

- SPIN - $\frac{1}{2}$ PARTICLES HAVE A MDM $\vec{\mu}$ RELATED TO THEIR SPIN — AND THE RULES OF QUANTUM MECHANICS MEAN THAT THE COMPONENT OF $\vec{\mu}$ ALONG ANY AXIS CAN TAKE ONLY TWO VALUES, $\pm\mu$ (ie IT IS <u>QUANTISED</u>).

EXAMPLE: CONSIDER A 1D ARRAY OF FOUR SPIN-$\frac{1}{2}$ ATOMS IN AN EXTERNAL MAGNETIC FIELD $\vec{B}$.

THE ENERGY OF EACH ATOM IS

$\vec{\mu}$ & $\vec{B}$ ALIGNED

$$\varepsilon_{\pm} = \mp \mu B$$

$\vec{\mu}$ & $\vec{B}$ ANTI-ALIGNED

A POSSIBLE STATE FOR THIS SYSTEM IS

PARTICLE STATE σ_i:	$\sigma_1 = +$	$\sigma_2 = +$	$\sigma_3 = -$	$\sigma_4 = +$
ENERGY OF PARTICLE:	$\varepsilon_+ = -\mu B$	$\varepsilon_+ = -\mu B$	$\varepsilon_- = \mu B$	$\varepsilon_+ = -\mu B$

THE STATE OF THIS SYSTEM IS

$$\vec{n} = (\sigma_1, \sigma_2, \sigma_3, \sigma_4)$$

$$= (+, +, -, +)$$

THE TOTAL ENERGY OF THIS STATE IS

$$E_{\vec{n}} = \varepsilon_{\sigma_1} + \varepsilon_{\sigma_2} + \varepsilon_{\sigma_3} + \varepsilon_{\sigma_4}$$

$$= \varepsilon_+ + \varepsilon_+ + \varepsilon_- + \varepsilon_+$$

$$= -\mu B - \mu B + \mu B - \mu B$$

$$= -2\mu B$$

- FOR N SPIN-$\frac{1}{2}$ ATOMS IN A 1D ARRAY IN AN EXTERNAL $\vec{B}$ FIELD

LOWEST ENERGY STATE ("GROUND STATE")

$$\vec{n} = (+, +, +, \ldots, +) \qquad \text{(ALL +)}$$

WITH ENERGY $\quad E_{\vec{n}} = -N\mu B$

NEXT LOWEST ENERGY STATE ("1ST EXCITED STATE")

$\vec{n}$ HAS ONE $-$ve SIGN & ALL OTHERS $+$ve. THERE ARE N SUCH STATES.

NEXT LOWEST ENERGY STATE ("2nd EXCITED STATE")

$\vec{n}$ HAS TWO $-$ve SIGNS & ALL OTHERS $+$ve. THERE ARE $\binom{N}{2}$ SUCH STATES

• FOR SUCH AN N ATOM ARRAY OF SPIN-$\frac{1}{2}$ ATOMS IN CONTACT WITH A HEAT RESERVOIR AT TEMPERATURE T

$$Z = \sum_{\vec{n}} e^{-E_{\vec{n}}/k_B T} \qquad \vec{n} = (\sigma_1, \sigma_2, \ldots, \sigma_N)$$

$$= \sum_{\sigma_1 = \pm} \sum_{\sigma_2 = \pm} \cdots \sum_{\sigma_N = \pm} e^{-(\varepsilon_{\sigma_1} + \varepsilon_{\sigma_2} + \cdots + \varepsilon_{\sigma_N})/k_B T}$$

$$= \left(\sum_{\sigma_1 = \pm} e^{-\varepsilon_{\sigma_1}/k_B T} \right) \times \cdots \times \left(\sum_{\sigma_N = \pm} e^{-\varepsilon_{\sigma_N}/k_B T} \right)$$

$$= \left(\sum_{\sigma = \pm} e^{-\varepsilon_\sigma / k_B T} \right)^N$$

$$= (Z_1)^N$$

WITH $$\boxed{Z_1 = e^{-\varepsilon_+/k_BT} + e^{-\varepsilon_-/k_BT}}$$

SO $$\boxed{Z = (Z_1)^N}$$ — BOLTZMANN STATISTICS

Z – CANONICAL PARTITION FUNCTION FOR THE SYSTEM

Z_1 – CANONICAL PARTITION FUNCTION FOR JUST A SINGLE ATOM

BOLTZMANN STATISTICS APPLIES TO <u>IDENTICAL BUT DISTINGUISHABLE</u> PARTICLES.