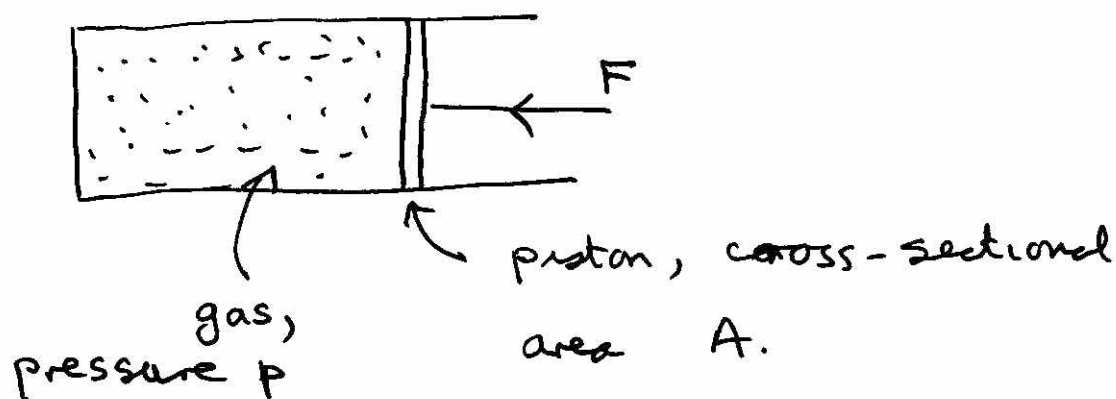


MPS Practice Problem Set 1

Solutions

1.



- To maintain the piston in position, the external force F on it must equal the force $p \cdot A$ of the gas on the piston,

$$F = p \cdot A$$

- If the piston is to move, the force F must be different from $p \cdot A$ — for compression of the gas, $F > p \cdot A$, for expansion of the gas, $F < p \cdot A$

- Consider compression first :

$$F = p \cdot A + \epsilon, \quad \epsilon > 0$$

If the piston moves inward a distance $dx > 0$, the work dW done on

the gas is

$$\begin{aligned} dW &= F \cdot dx \\ &= (p \cdot A + \epsilon) \cdot dx. \end{aligned}$$

For compression, volume decreases,

$$\Rightarrow dV = -A \cdot dx$$

$$\begin{aligned} \text{So } dW &= p \cdot A \cdot dx + \epsilon dx \\ &= p \cdot A \cdot \left(-\frac{dV}{A}\right) + \epsilon dx \\ &= -p \cdot dV + \epsilon dx \\ &> -p \cdot dV \end{aligned}$$

• Consider now expansion:

$$F = p \cdot A - \epsilon, \quad \epsilon > 0.$$

If the piston moves outward a distance $dx > 0$, it does work on the environment. So the work dW done on the gas is negative

$$\begin{aligned} dW &= -F \cdot dx \\ &= -(p \cdot A - \epsilon) \cdot dx \end{aligned}$$

For expansion, the volume increase,

$$\Rightarrow dV = A \cdot dx$$

$$\begin{aligned} \text{So } dW &= - (p \cdot A - \epsilon) dx \\ &= - p \cdot A \cdot dx + \epsilon \cdot dx \\ &= - p \cdot A \cdot \left(\frac{dV}{A} \right) + \epsilon \cdot dx \\ &= - p \cdot dV + \epsilon dx \\ &> - p \cdot dV \end{aligned}$$

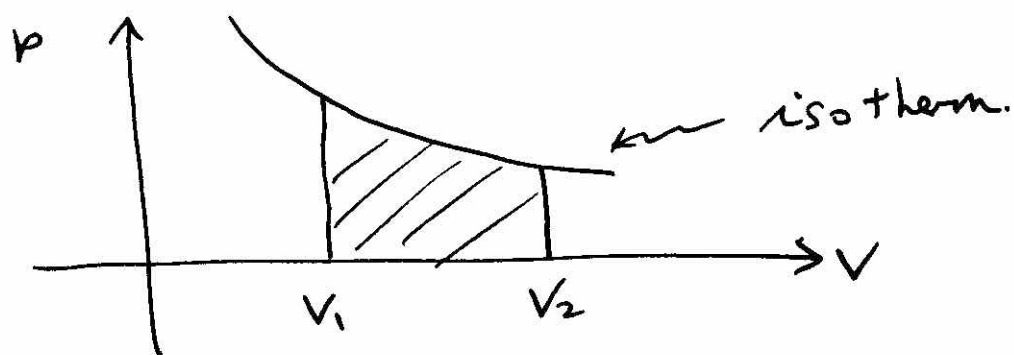
L⁴

2. For an ideal gas, we have the equation of state $p \cdot V = \nu R T$.

For an isothermal process, $p \cdot V = \underline{\text{constant}}$.

• For a quasi static process, the work done by the gas in expanding from volume V_1 to volume $V_2 > V_1$ is

$$W = \int_{V_1}^{V_2} p \cdot dV = \text{"area under the } p\text{-}V \text{ curve"}$$



i.e. $W = \int_{V_1}^{V_2} \frac{\nu R T}{V} dV$

$$= \nu R T \int_{V_1}^{V_2} \frac{dV}{V}$$

since $T = \text{constant}$

$$= \nu R T [\ln V]_{V_1}^{V_2}$$

$$= \nu R T (\ln V_2 - \ln V_1)$$

$$= \nu R T \ln \frac{V_2}{V_1}$$

Here, $V_1 = 5 \text{ l}$
 and $p_1 V_1 = 5 \text{ l} \times 10^7 \text{ Pa}$

So for an isothermal expansion

$$p_2 V_2 = p_1 V_1 = 5 \text{ l} \times 10^7 \text{ Pa}$$

$$\begin{aligned} \Rightarrow V_2 &= \frac{5 \text{ l} \times 10^7 \text{ Pa}}{p_2} \\ &= \frac{5 \text{ l} \times 10^7 \text{ Pa}}{10^5 \text{ Pa}} \\ &= 500 \text{ l} \end{aligned}$$

$$\text{So } W = nRT \ln \frac{500 \text{ l}}{5 \text{ l}}$$

For $T = 300 \text{ K}$,

$$\underline{W = 2.3 \times 10^5 \text{ J}}$$

• To work out the heat released in burning the hydrogen, we need to compute the number of moles:

$$n = \frac{p_1 V_1}{RT} = 20.1 \text{ moles.}$$

$$\begin{aligned} \Rightarrow \text{Heat released on combustion} \\ &= 2.9 \times 10^5 \text{ J/mole} \times 20.1 \text{ moles} \\ &= 5.8 \times 10^6 \text{ J.} \end{aligned}$$

This is about 25 times the amount of

energy provided by the isothermal expansion. Given that the hydrogen used in the isothermal expansion gets to "live another day", it is a fairly efficient source of work.

3.

ν_1	ν_2
T_1	T_2
V_1	V_2

• The internal energy of the system is initially

$$E_i = E_1 + E_2 = \nu_1 C_V T_1 + \nu_2 C_V T_2$$

• The final state has $(\nu_1 + \nu_2)$ moles of gas at some final temperature T

$$\Rightarrow E_f = (\nu_1 + \nu_2) C_V T$$

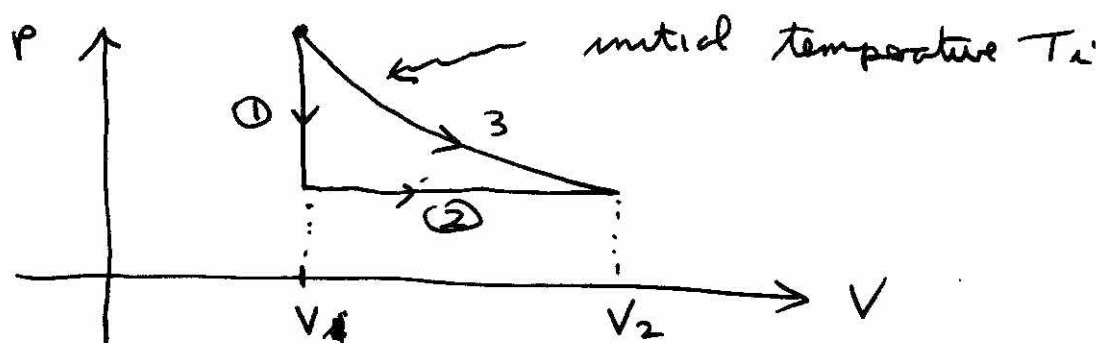
• Conservation of energy

$$\Rightarrow E_i = E_f$$

$$\Rightarrow C_V (\nu_1 T_1 + \nu_2 T_2) = (\nu_1 + \nu_2) C_V T$$

$$\Rightarrow T = \frac{(\nu_1 T_1 + \nu_2 T_2)}{(\nu_1 + \nu_2)}$$

4.



Let T_f denote the temperature at the end of process ①.

• Given: for an infinitesimal quasi-static process for an ideal gas

$$dQ = \nu C_V dT + p \cdot dV$$

Then $ds = \frac{dQ}{T}$

$$= \nu C_V \frac{dT}{T} + \frac{p}{T} dV$$

$$= \nu C_V \frac{dT}{T} + \nu R \frac{dV}{V}$$

using $pV = \nu RT$

For process ①, $dV = 0$, initial

temperature T_i , final temperature T_f

$$\begin{aligned} \Rightarrow \int_1 ds &= \nu C_V \int_{T_i}^{T_f} \frac{dT}{T} \\ &= \nu C_V [\ln T]_{T_i}^{T_f} \\ &= \nu C_V \ln \frac{T_f}{T_i} \end{aligned}$$

For process ②: $dp=0$

The gas starts at temperature T_f and finishes at temperature T_i

$$\Rightarrow \int_2 ds = \nu C_v \int_{T_f}^{T_i} \frac{dT}{T} + \nu R \int_{V_1}^{V_2} \frac{dV}{V}$$

$$= \nu C_v \ln \frac{T_i}{T_f} + \nu R \ln \frac{V_2}{V_1}$$

$$\Sigma_0 (\Delta S)_1 + (\Delta S)_2$$

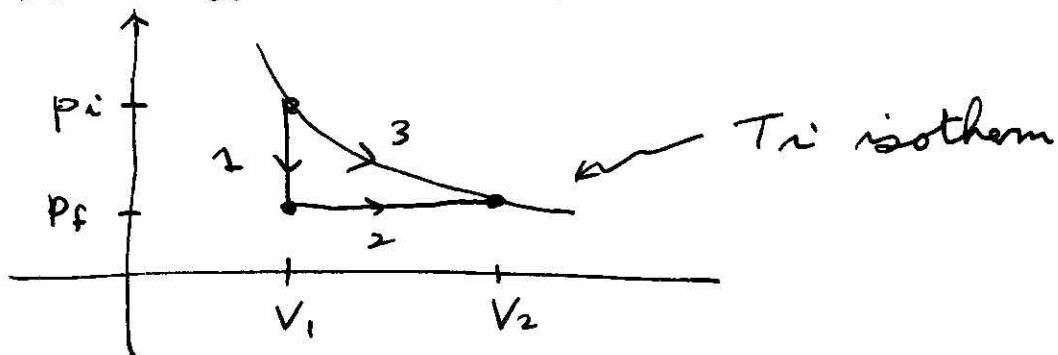
$$= \nu C_v \ln \frac{T_f}{T_i} + \nu C_v \ln \frac{T_i}{T_f} + \nu R \ln \frac{V_2}{V_1}$$

$$= \nu C_v \ln \frac{T_f}{T_i} - \nu C_v \ln \frac{T_f}{T_i} + \nu R \ln \frac{V_2}{V_1}$$

$$= \nu R \ln \frac{V_2}{V_1},$$

which is the same as $(\Delta S)_3$ computed in lectures.

(Note: we did not have to compute T_f , but we could have:



Using the ideal gas equation of state for the start and end states in process (2),

$$P_f V_1 = \nu R T_f$$

$$P_f V_2 = \nu R T_i$$

$$\Rightarrow \frac{T_f}{T_i} = \frac{V_1}{V_2}$$

$$\Rightarrow T_f = \frac{V_1}{V_2} T_i$$

5. From lectures, for a monatomic ideal gas,

$$S = \nu R \left(\frac{3}{2} \ln T + \ln V + \kappa \right)$$

(where κ is a constant)

$$\text{and } E = \frac{3}{2} \nu R T.$$

(a)

Let V_1 be the initial volume, V_2 the final volume.

$$\begin{aligned} \Delta S &= S_2 - S_1 \\ &= \nu R \left(\frac{3}{2} \ln T_2 + \ln V_2 + \kappa \right) \\ &\quad - \nu R \left(\frac{3}{2} \ln T_1 + \ln V_1 + \kappa \right) \\ &= \frac{3}{2} \nu R (\ln T_2 - \ln T_1) \\ &\quad + \nu R (\ln V_2 - \ln V_1) \\ &= \frac{3}{2} \nu R \ln \frac{T_2}{T_1} + \nu R \ln \frac{V_2}{V_1}. \end{aligned}$$

But the pressure p of the gas is constant during the process

$$\Rightarrow p V_1 = \nu R T_1, \quad p V_2 = \nu R T_2.$$

Taking the ratio of these expressions

$$\frac{V_2}{V_1} = \frac{T_2}{T_1}$$

$$\Rightarrow \Delta S = \frac{3}{2} \nu R \ln \frac{T_2}{T_1} + \nu R \ln \frac{T_2}{T_1}$$

$$= \frac{5}{2} \nu R \ln \frac{T_2}{T_1}$$

(b) $ds = \frac{dQ}{T}$ for a quasistatic process.

$$dQ = dE - dW$$

$$= \frac{3}{2} \nu R dT + p dV$$

$$= \frac{3}{2} \nu R dT + d(pV)$$

since $p = \text{constant}$

$$= \frac{3}{2} \nu R dT + d(\nu R T)$$

$$= \frac{3}{2} \nu R dT + \nu R dT$$

$$= \frac{5}{2} \nu R dT$$

$$\Rightarrow ds = \frac{dQ}{T} = \frac{5}{2} \nu R \frac{dT}{T}$$

$$\Rightarrow \Delta S = \int_1^2 ds = \frac{5}{2} \nu R \int_{T_1}^{T_2} \frac{dT}{T}$$

$$= \frac{5}{2} \nu R [\ln T]_{T_1}^{T_2}$$

$$= \frac{5}{2} \nu R (\ln T_2 - \ln T_1)$$

$$= \frac{5}{2} \nu R \ln \frac{T_2}{T_1}$$

which agrees with (a)

(c) Entropy is a well-defined quantity for the initial and final states, so the change in entropy as we go from the initial to the final state is always

$$\Delta S = S_{\text{final}} - S_{\text{initial}},$$

regardless of the process.