Many Particle Systems – Practice Problem Set 4

These problems are not for assessment. However, it is recommended that you attempt them as practice for the test and exam.

1. A simple harmonic oscillator has energy eigenstates with energies

$$E_n = (n + \frac{1}{2})\hbar\omega, \quad n = 0, 1, 2, \cdots$$

   where ω is the angular frequency of the simple harmonic oscillator.

   Consider an isolated system consisting of four weakly interacting harmonic oscillators of angular frequency ω arranged in a line (isolated system means the total energy of the system of harmonic oscillators is fixed; weakly interacting means that energy can be exchanged between the harmonic oscillators).

   If the total energy of the closed system of four oscillators is $3\hbar\omega$, determine the probability that the first oscillator in the row is in a state with energy $\frac{1}{2}\hbar\omega$.

2. (a) For a system in contact with a heat reservoir, show that the mean energy of the system can be expressed in the form

$$\bar{E} = k_B T^2 \frac{\partial}{\partial T} \ln Z,$$

   where Z is the canonical partition function, $Z = \sum_n e^{-E_n/k_B T}$.
   Hint: start on the right hand side.

   (b) Consider a system of N identical particles, each of which has two energy eigentstates with energies 0 and ϵ. Assume the particles are in contact with a heat reservoir of temperature T and that the particles are *distinguishable*: for example, in a solid phase where each particle can be identified by its position.
   Give an expression for the canonical partition function of the system of N particles.

   (c) Compute the mean energy of the system.

   (d) Evaluate the mean energy in the limit $T \to 0$ and explain the result by a physical arguement.