Many Particle Systems – Practice Problem Set 7

These problems are not for assessment. However, it is recommended that you attempt them as practice for the test and exam.

1. When a *main sequence* star exhausts its hydrogen fuel in the nuclear reactions which power it, there is insufficient radiation and gas pressure to prevent the star collapsing under the influence of gravity. The star consists mainly of helium (the fusion product of hydrogen), which is completely ionized in the collapse. The collapse is eventually stabilized by the pressure of the ionized electrons, which form a Fermi gas. The resulting state is called a *white dwarf.*

   (a) The density of a white dwarf is of the order of 10^9 kg m^{-3}. Given that the mass of a helium atom is 7×10^{-27} kg, determine the number density of the electron gas inside the white dwarf (assume the helium atoms are fully ionized i.e. two electrons per atom are liberated to make up the electron gas).

   (b) Determine the Fermi energy for the electron gas.

   (c) The temperature of a young white dwarf is of order 10^7 K. Determine whether the electron gas in the white dwarf star can be treated as being degenerate i.e. whether the distribution function for the mean occupation number of the energy eigenstates for an electron differs significantly from that at zero temperature. Explain your reasoning.

   (d) The size of a white dwarf is determined by the requirement that the electron pressure must balance the inward gravitational force. This requires a pressure of the order of GM^2/R^4, where M is the mass of the white dwarf and R is its radius (*very* crudely, this comes from GM/R^2, the acceleration due to gravity at the surface, times the mass M, giving a measure of the force at the surface, divided by the area $4\pi R^2$ of the surface, to give the pressure at the surface). By equating this pressure with the pressure of the electron gas, show that R is proportional to $M^{-1/3}$ for a white dwarf.

   (e) Show that the density of a white dwarf depends on the square of its mass.