AVERAGE NUMBER OF PARTICLES IN CONTACT WITH A HEAT AND PARTICLE RESERVOIR

- THE NUMBER OF PARTICLES IS NOT FIXED, BUT

$$\boxed{\bar{N} = \sum_m N_m P_m^{(eq)}} \quad \text{IS FIXED}$$

- $$\boxed{P_m^{(eq)} = \frac{e^{-(E_m - \mu N_m)/k_B T}}{\tilde{Z}(T,V,\mu)}}$$

WHERE E_m IS THE ENERGY OF THE SYSTEM'S QUANTUM STATE m, AND N_m IS THE No OF PARTICLES IN THE SYSTEM IN STATE m

$$\tilde{Z}(T,V,\mu) = \sum_m e^{-(E_m - \mu N_m)/k_B T}$$

↑

GRAND CANONICAL PARTITION FUNCTION.

- $$\bar{N} = \sum_m N_m P_m^{(eq)} = \frac{1}{\tilde{Z}(T,V,\mu)} \sum_m N_m e^{-(E_m - \mu N_m)/k_B T}$$

NOTE:

$$\frac{\partial}{\partial \mu} \ln \tilde{Z}(T,V,\mu) = \frac{1}{\tilde{Z}} \sum_m \frac{N_m}{k_B T} e^{-(E_m - \mu N_m)/k_B T}$$

$$= \frac{1}{k_B T} \bar{N}$$

So $$\boxed{\bar{N} = k_B T \frac{\partial}{\partial \mu} \left(\ln \tilde{Z}(T,V,\mu) \right)}$$

[NOTE: $$\bar{N} = - \frac{\partial \Phi(T,V,\mu)}{\partial \mu}$$

WITH $$\Phi(T,V,\mu) = - k_B T \ln \tilde{Z}(T,V,\mu)$$]

- FOR BOSE-EINSTEIN STATISTICS (IDENTICAL INDISTINGUISHABLE BOSONS, NO PAULI EXCLUSION PRINCIPLE):

$$\ln \tilde{Z}_{BE}(T,V,\mu) = \ln \left[\frac{1}{\left(1 - e^{-(\varepsilon_1 - \mu)/k_B T}\right)} \times \frac{1}{\left(1 - e^{-(\varepsilon_2 - \mu)/k_B T}\right)} \times \cdots \right]$$

$$\Rightarrow \ln \tilde{Z}_{BE} = -\ln\left(1 - e^{-(\varepsilon_1-\mu)/k_BT}\right) - \ln\left(1 - e^{-(\varepsilon_2-\mu)/k_BT}\right) - \ldots$$

$$= -\sum_{i} \ln\left(1 - e^{-(\varepsilon_i-\mu)/k_BT}\right)$$

SUM OVER ALL SINGLE PARTICLE STATES i WITH ENERGIES ε_i

$$\bar{N}_{BE} = k_BT \frac{\partial}{\partial \mu} \ln \tilde{Z}_{BE}$$

$$= k_BT \sum_i \frac{(-1)\left(\frac{1}{k_BT}\right) e^{-(\varepsilon_i-\mu)/k_BT}}{1 - e^{-(\varepsilon_i-\mu)/k_BT}}$$

$$= \sum_i \frac{e^{-(\varepsilon_i-\mu)/k_BT}}{1 - e^{-(\varepsilon_i-\mu)/k_BT}} \times \frac{e^{(\varepsilon_i-\mu)/k_BT}}{e^{(\varepsilon_i-\mu)/k_BT}}$$

$$\boxed{\bar{N}_{BE} = \sum_i \frac{1}{e^{(\varepsilon_i-\mu)/k_BT} - 1}}$$

OR

$$\boxed{(\bar{N}_{BE})_i = \frac{1}{e^{(\varepsilon_i - \mu)/k_B T} - 1}}$$

MEAN OCCUPANCY OF THE i^{th} SINGLE PARTICLE ENERGY E'STATE IN A <u>BOSE</u> GAS.

ie $(\bar{N}_{BE})_1$ = AVERAGE No. OF PARTICLES IN THE GROUND STATE

- FOR IDENTICAL INDISTINGUISHABLE FERMIONS (PAULI EXCLUSION PRINCIPLE APPLIES)

$$\ln \tilde{Z}_{FD}(T,V,\mu) = \ln\left[\left(1+e^{-(\varepsilon_1-\mu)/k_BT}\right)\left(1+e^{-(\varepsilon_2-\mu)/k_BT}\right)\times\ldots\right]$$

$$= \sum_{i} \ln\left(1+e^{-(\varepsilon_i-\mu)/k_BT}\right)$$

SUM OVER SINGLE PARTICLE ENERGY E'STATES

$$\bar{N}_{FD} = k_B T \frac{\partial}{\partial \mu} \ln \tilde{Z}_{FD}$$

$$= k_B T \sum_i \frac{\left(\frac{1}{k_B T}\right) e^{-(\varepsilon_i - \mu)/k_B T}}{1 + e^{-(\varepsilon_i - \mu)/k_B T}}$$

$$\boxed{\bar{N}_{FD} = \sum_i \frac{1}{e^{(\varepsilon_i - \mu)/k_B T} + 1}}$$

$$\Rightarrow \boxed{(\bar{N}_{FD})_i = \frac{1}{e^{(\varepsilon_i - \mu)/k_B T} + 1}}$$

MEAN No. OF PARTICLES IN THE i^{th} SINGLE PARTICLE ENERGY LEVEL.

NOTE:

$$\boxed{(\bar{N}_{FD})_i = \frac{1}{\left(\text{POSITIVE No. OR ZERO}\right) + 1} \leq 1}$$

THE FORMULA "KNOWS ABOUT THE PAULI EXCLUSION PRINCIPLE".