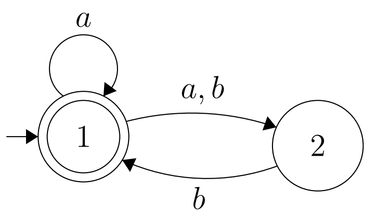
CITS2211 Discrete Structures
Week 11 Exercises – Regular expressions and regular languages & PDAs

2022

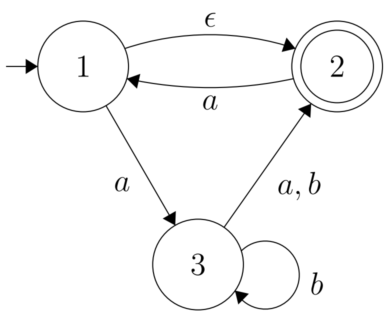
Topics: Regular Expressions, Regular Languages, The pumping lemma for regular languages, Context-free languages, Context-free grammars, Push-down automata

1 FSM Revision

1. *[Source: Sipser 1.16]* Convert the following nondeterministic FSM (NFSM) to an equivalent deterministic finite automata (DFSM).



2. *[Source: Sipser 1.16]* Convert the following nondeterministic FSM (NFSM) to an equivalent deterministic finite automata (DFSM).



2 Regular expressions and languages

1. Prove that if A is a regular set with alphabet I, then the language defined by taking the set difference $I^* - A$ is also regular.

2. Simplify the following regular expression as much as possible

$$(((\mathtt{a}^*)^*)^*)^*(\epsilon + \mathtt{b})\mathtt{c}(\mathtt{c} + (\epsilon + \epsilon))^*$$

Explain your reasons for each simplification step.

3 PDAs

1. State the Pumping Lemma for Regular Languages. Write out your answer in a way that helps you to remember the lemma.

2. Use the Pumping Lemma for Regular Languages to prove that the language of all binary strings that have equal numbers of 0s and 1s is *not* regular.

3. Describe the error in the following "proof" that 0^*1^* is *not* a regular language. Note that there is an error because 0^*1^* *is* a regular language.

   The proof is by contradiction. Assume that $L = 0^*1^*$ is regular and p is the pumping length for L given by the pumping lemma. Choose w to be the string 0^p1^p. You know that $w \in L$ but w can not be pumped, since any $xyyz$ will have more 0s than 1s. Thus you have a contradiction so 0^*1^* is not regular.

4. For the language $L = \{a^i b^j c^k \mid i, j, k \geq 0 \wedge (i = 1 \rightarrow j = k)\}$

   (a) show that L is not regular
   Hint: Try to use Kleene's theorem and the pigeonhole principle instead of the pumping lemma in this case.

   (b) show that $w = a^i b^j c^k$ satisfies the pumping lemma conditions (for some i,j,k).
   Challenge: You can show that all words $w = a^i b^j c^k \in L$ with $|w| > 2$ satisfy the pumping lemma conditions.

   (c) explain why parts a) and b) do not contradict the pumping lemma

5. Describe a grammar that generates all binary strings that have equal numbers of 0s and 1s.

6. Design a pushdown automata (PDA) and draw the state machine diagram for the language of all binary strings that have equal numbers of 0s and 1s.

7. Define a grammar that generates all binary strings with more 0s than 1s.

8. Design a pushdown automata (PDA) and draw the state machine diagram for the language of all binary strings with more 0s than 1s.