STAT MECH FOR IDENTICAL PARTICLES

- FOR N IDENTICAL DISTINGUISHABLE PARTICLES (MAXWELL-BOLTZMANN STATS) WE LABEL THE SYSTEM'S ENTIRE STATE BY EACH OF THE INDIVIDUAL PARTICLE'S STATES.

eg SUPPOSE EACH PARTICLE CAN BE IN AN ENERGY E'STATE LABELLED BY

$$n = 1, 2, 3, \ldots$$

SINGLE PARTICLE QUANTUM N°

THEN THE STATE OF THE SYSTEM IS LABELLED BY AN N-VECTOR

$$\vec{n} = (n_1, n_2, n_3, \ldots, n_N)$$

WHERE

n_i — THE STATE OF PARTICLE i

$(n_i = 1, 2, 3, \ldots)$

TO CONSTRUCT THE CANONICAL PARTITION FUNCTION Z WE MUST SUM OVER ALL $\vec{n}$.

THIS IS NOT USEFUL FOR <u>IDENTICAL INDISTINGUISHABLE</u> (I.I.) PARTICLES — IT OVERCOUNTS THE No° OF SYSTEM STATES.

- FOR I. I. PARTICLES (OF WHICH THERE ARE TWO KINDS $\begin{cases} \text{BOSONS} \\ \text{FERMIONS} \end{cases}$)

TO LABEL THE SYSTEM'S STATES WE INSTEAD USE OCCUPATION No°S WHICH TELL US HOW MANY PARTICLES ARE IN EACH SINGLE PARTICLE STATE.

EXAMPLE: BOSE - EINSTEIN STATS

CONSIDER $N=2$ BOSONS (A & A) EACH WITH THREE ACCESSIBLE SINGLE PARTICLE STATES $n=1$, $n=2$, $n=3$.

THE TOTAL NO° OF STATES OF THE 2 PARTICLE SYSTEM IS:

	$n=1$	$n=2$	$n=3$	OCCUPATION No°s
STATE 1	AA	—	—	(2,0,0)
STATE 2	—	AA	—	(0,2,0)
STATE 3	—	—	AA	(0,0,2)
STATE 4	A	A	—	(1,1,0)
STATE 5	A	—	A	(1,0,1)
STATE 6	—	A	A	(0,1,1)

HERE WE HAVE INTRODUCED OCCUPATION No°s

$$(N_1, N_2, N_3)$$

N_n = No° OF PARTICLES IN STATE n

NOTE: $N_1 + N_2 + N_3 = N = 2$

THERE IS A 1 − 1 CORRESPONDANCE BETWEEN STATES OF THE SYSTEM & OCCUPATION No°S.

EXAMPLE: FERMI − DIRAC STATS

FOR $N=2$ FERMIONS (A & A), EACH WITH 3 SINGLE PARTICLE STATES

$n=1$, $n=2$, $n=3$

THE TOTAL No° OF STATES OF THE 2 I.I. PARTICLE SYSTEM IS:

	$n=1$	$n=2$	$n=3$	OCCUPATION No°S
STATE 1	A	A	−	(1, 1, 0)
STATE 2	A	−	A	(1, 0, 1)
STATE 3	−	A	A	(0, 1, 1)

IN GENERAL : FOR N I.I. PARTICLES

THE SYSTEM'S STATE IS SPECIFIED

BY A SET OF INTEGERS

$$(N_1, N_2, N_3, \ldots)$$ — OCCUPATION No°s

WITH N_n = No° OF PARTICLES IN SINGLE PARTICLE STATE n

SUBJECT TO $$\sum_n N_n = N$$

[FOR F.D. STATS $N_n = 0$ OR 1]

THE ENERGY OF THE STATE

$$(N_1, N_2, N_3, \ldots)$$

IS

$$E_{STATE} = N_1 \varepsilon_1 + N_2 \varepsilon_2 + \cdots$$

ε_n — ENERGY OF SINGLE PARTICLE STATE n

THE PARTITION FUNCTION IS

$$Z = \sum_{STATES} e^{-E_{STATE}/k_B T}$$

$$= \sum_{(N_1, N_2, \dots)} e^{-(N_1 \varepsilon_1 + N_2 \varepsilon_2 + \cdots)/k_B T}$$

$$= \left(\sum_{N_1} e^{-N_1 \varepsilon_1 / k_B T} \right) \left(\sum_{N_2} e^{-N_2 \varepsilon_2 / k_B T} \right) \cdots$$

BUT THIS IS NOT SO EASY TO EVALUATE SINCE THE SUMS OVER EACH N_n ARE NOT UNRESTRICTED, BUT SUBJECT TO $\sum_n N_n = N$.