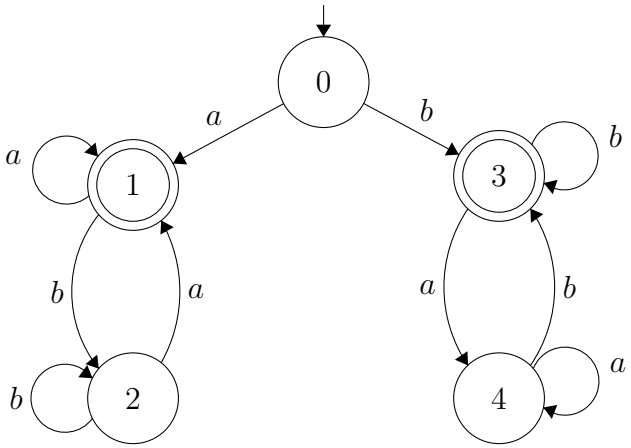
CITS2211 Discrete Structures
Week 10 Exercises – Finite State Machines and Regular Expressions

October 2022

1 Finite State Machines

1. For the following FSM, M, document its parts by listing each of the following:
   a) alphabet b) states c) starting state d) accepting states e) transitions.
   List three strings (from Σ^*) that M accepts.
   List three strings (from Σ^*) that M does not accept.



2. Design deterministic FSMs to recognise each of the following languages:

   (a) Words over the alphabet $\{a, b, c\}$ that contain no c.

   (b) Words over the alphabet $\{a, b\}$ such that the last two symbols are the same.

(c) Words over the alphabet $\{a, b\}$ of the form $(ab)^n$ for $n > 0$ (that is ab repeated n times).

(d) Words over the alphabet $\{a, b\}$ that contain exactly 2 bs.

3. Define a divisible by 3 checker as a machine that accepts a string from the alphabet $\{0, 1\}$ if and only if the *sum of its digits* is divisible by 3. (Hint: The sum of the empty string ϵ is zero.)
   Design an FSM to implement a divisible by 3 checker.

4. Design a *nondeterministic* FSM (NFSM) to recognise any binary string that ends in 01. Hint: you only need 3 states.

5. Convert your NFSM from the previous question into a deterministic FSM for the same language.

6. Devise an FSM to solve the farmer, wolf, goat and cabbage problem and find the solutions. This problem involves a farmer, a wolf, a goat and a cabbage all on one side of a river. There is a boat but the farmer can carry only one passenger at a time. The farmer wants to get them all to the other side of the river. However, left alone, the wolf will eat the goat, and the goat will eat the cabbage. How do they all get to the other side?

   (The cabbage counts as a passenger!)

7. *Source: Sipser 1.12*

   Let $D = \{w \mid w$ contains an even number of as and
   an odd number of bs and
   does not contain the substring ab $\}$

   Give an FSM over the alphabet $\Sigma = \{a, b\}$ with 5 states that recognises D.

   Hint: describe D more simply.

2 Regular expressions and languages

1. Give a regular expression for the following sets:

   a) the set of all binary strings (i.e. strings of 0s and 1s) beginning with 1 and ending with 1
   b) the set of all strings of 0s and 1s containing exactly two 1s
   c) the set of all strings of 0s and 1s having an odd number of 1s

d) the set of all strings of 0s and 1s containing at least one 0
e) the set of all strings of 0s and 1s where each 0 is followed by two 1s
f) the set of all strings of 0s and 1s containing exactly three 0s

2. Find a regular expression for the language L consisting of all strings over $\{0, 1\}$ with no consecutive zeros (that is, any string containing 00 is *not* in the language).

3. Does the string 01110111 belong to the regular set $(1^*01)^*(11 + 0^*)$? Justify your answer.

4. Does the string 011100101 belong to the regular set $01^*10^*(11^*0)^*$? Justify your answer.