Determining gravitational acceleration using a simple pendulum

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Summary

This report describes how to determine gravitational acceleration g using a simple pendulum composed of a brass bob and a nylon thread. The pendulum period T is measured for different values of the pendulum length L. The experiments demonstrate a clear linear relationship between T^2 and L which corresponds with the theoretical description of the pendulum motion. The slope of a linear fit of T^2 and L gives $g = 9.78 \pm 0.05$ m/s^2. The value found for g is in agreement with the value cited in the literature. The fit intercept value indicates that there is a systematic error in determining L. This is due to a cavity inside the pendulum's brass bob, which was unknown at the start of the experiment. This finding suggests using a simple pendulum to detect whether an object is compositionally non-homogeneous.
Improvements for following research can be made by using longer pendulums or electronic measurement systems to increase the accuracy of the time measurements.

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1 Introduction

Not everyone realises that a simple pendulum (a mass placed at the other end of a fixed wire) is a simple device which can be used to determine the gravitational acceleration or as proof of the rotation of the earth [1]. This gravitational acceleration can differ per location. In Helsinki (60° NB), the gravitational acceleration is equal to 9.825 m/s^2, and in Kuala Lumpur (3° NB) 9.776 m/s^2 [2].

In this research project, the gravitational acceleration in Delft will be determined using a pendulum composed of a brass bob and a nylon thread. Furthermore, this research will investigate the accuracy and certainty of using a pendulum for determining the gravitational acceleration.

The theory in this report is based on a simple pendulum [3], which assumes the mass at the end of the wire to be a point mass and the wire to have no mass. Furthermore, in the method the pendulum period (the time required to complete one swing) is measured for different values of pendulum length. The value of g and the associated uncertainty can then be determined by applying a least squares fit of the results of the experiments to a theoretical expression for the pendulum motion. The fit can also provide information on any systematic errors.

The experimental method is set out in Chapter 2 of this report, followed by the results in Chapter 3. A discussion of the results is contained in Chapter 4 and, finally, the conclusions of this research project are set out in Chapter 5.

This experiment is part of the Introduction to Experimentation laboratory course in the Bachelor's in Applied Physics at Delft University of Technology.

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Experimental Method

This experiment is based on simple pendulum theory [1]. This theory describes the pendulum motion of a point mass suspended at the end of a massless string. The simple pendulum is therefore also known as a 'mathematical pendulum'. A schematic representation of the pendulum is provided in Figure 2.1.

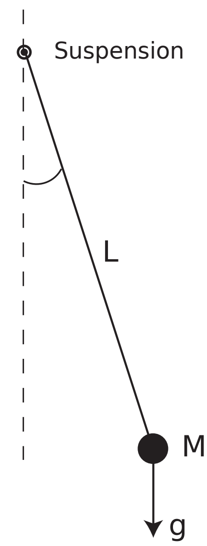

Figure 2.1: Schematic representation of a simple pendulum. L is the length of the pendulum, M is the mass and θ is the angular displacement. g is the gravitational acceleration.

L is the length of the pendulum measured from the point of suspension to the centre of gravity of the mass M. θ is the angular displacement and g is the gravitational acceleration.

The period T of the simple pendulum is given by

$$T = 2\pi\sqrt{\frac{L}{g}}, \tag{2.1}$$

which is valid for small angles (approximately $\theta < 15°$).

To approximate the theoretical model as closely as possible in practice, a brass sphere is used for the point mass with an approximate spherical shape with mass ($m = 182.5 \pm 0.1$ g). The average value for its diameter is 43.6 ± 0.5 mm. The string is a nylon thread with a diameter of 0.4 mm. The mass of the thread equals 0.14 ± 0.01 g/m, which means that compared to the mass of the brass sphere, the thread's influence on the position of the centre of mass is negligible. The centre of mass of the pendulum must therefore coincide to a high degree with that of the sphere, and therefore the mathematical pendulum approximation is valid for this pendulum.

For accuracy, multiple measurements of the period T will be done for different values of the length L of the pendulum. This will also give us a good indication on the validity of this method and deviations from the theory. Gravitational acceleration g can be derived by plotting T^2 as a function of L and applying a least squares fit. Equation 2.1 will be written as:

$$T^2 = \frac{4\pi^2}{g} L \tag{2.2}$$

To reduce measurement error in the period T, the time of ten periods is measured (T_{10}). The error in the time measurements is primarily due to differences in human reaction times in starting and stopping the digital stopwatch. A reasonable estimate of human reaction time is 0.05 s. A plausible estimate for the error of 10 measurements will at most be $1/10^{\text{th}}$ of the error of a single measurement. The error in T^2 is then calculated via the calculus approach in formula 2.3.

$$u(T^2) = \left(\frac{\partial T^2}{\partial T}\right) u(T) = 2Tu(T) \tag{2.3}$$

In which $u(x)$ is the error of variable x. Calculating T from the literature value of g and equation (2.1) gives T=1 s. This leads to $u(T^2) \approx 0.01\,\text{s}^2$ which is a relative error of approximately 1%.

An overview of the variables used in determining the length of the pendulum is given in Figure 2.2. In practice, the distance between the point of suspension and the mounting hook on the bob (L_{thread}) is measured with a tape measure. The total length includes the length of the mounting hook and the radius of the bob: $L = L_{\text{thread}} + L_{\text{hook}} + R_{\text{bob}}$. $L_{\text{hook}} = 6.1 \pm 0.1$ mm and $R_{\text{bob}} = 21.8 \pm 0.3$ mm. L_{hook} and R_{bob} are measured using a slide gauge. Here it is assumed that the brass bob is highly spherical. The error in the measurement of L_{thread} is 1 mm for $L < 0.25$ m and 2 mm where $0.25 < L < 0.9$ m. Since these errors are considerably greater than the errors in L_{hook} and R_{bob}, a realistic estimate of the error in the length L is 2 mm where $0.25 < L < 0.9$ m and 1 mm for $L < 0.25$ m.

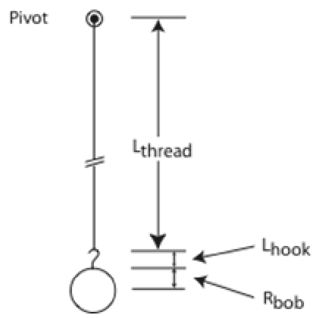

Figure 2.2: Overview of the variables involved in determining the length of the pendulum. L_{thread} is the length of the thread between the point of suspension and the mounting hook on the bob, L_{hook} is the length of the hook and R_{bob} is the radius of the sphere.

The error in L is approximately 1% and the error in T^2 appears to be the same. Both errors must therefore be taken into account when determining the weights, which are required for a least squares fit. The method used for this is described in the appendix.

3

Results

The results of the measurements of the period T for different pendulum lengths and the associated errors are given in Table 3.1. In order to plot T^2 against L, the values for T^2 are also included along with the associated errors.

Table 3.1: Measurement results for the period of a simple pendulum for different pendulum lengths. The variable of T^2 and the associated error is also included for further analysis. The columns containing $u(x)$, give the errors in x. $u(T) = 0.005\,\mathrm{s}$.

L_{thread} (m)	T_{10} (s)	L (m)	$u(L)$ (m)	T (s)	$T^2(\mathrm{s}^2)$	$u(T^2)(\mathrm{s}^2)$
0.106	7.52	0.134	0.001	0.752	0.566	0.008
0.206	9.88	0.234	0.001	0.988	0.98	0.01
0.303	11.70	0.331	0.002	1.170	1.37	0.01
0.414	13.42	0.442	0.002	1.342	1.80	0.01
0.521	14.94	0.549	0.002	1.494	2.32	0.01
0.605	16.04	0.633	0.002	1.604	2.57	0.02
0.703	17.29	0.731	0.002	1.729	2.99	0.02
0.820	18.64	0.848	0.002	1.864	3.47	0.02

To get a first indication of the validity, figure 3.1 shows the measured values of T plotted against L, compared with the theoretical relationship, where $g = 9.81\,\mathrm{m/s^2}$.

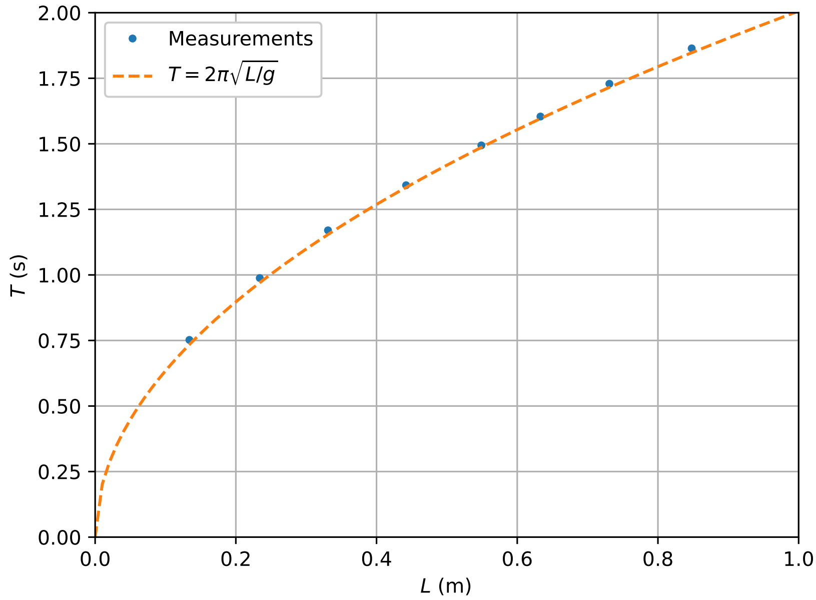

Figure 3.1: The period T of a simple pendulum plotted against the length L of the pendulum. The points drawn are the measurements and the bars give the respective errors. The line is drawn using equation (2.1) with $g = 9.81\,\mathrm{m/s^2}$.

The points indicate errors in L and T. However, the errors are barely visible on the scale of figure 3.1. The continuous line represents the theoretical value of T where $g = 9.81$ m/s^2 (the value of gravitational acceleration in the Netherlands). At first sight, there appears to be a fairly high degree of agreement between the measurements and the theoretical value. When we examine this in greater detail, we see that the experimental values are consistently slightly higher.

To find g from these measurements, T^2 is plotted against L in figure 3.2. Deviations from the trend can be identified more easily.

Figure 3.2 clearly shows a linear relationship between T^2 and L. Again, as seen in 3.1, the measurements are consistently slightly higher than the expected value.

To conduct a quantitative analysis, a linear weighted least squares fit is applied to the measurements ($T = aL + b$). The method for calculating the weights is described in the appendix. The output of the fit is $a = 4.04 \pm 0.02$ s^2/m and $b = 0.03 \pm 0.01$ s^2.

The straight line corresponding to these numbers is plotted in Figure 3.2 (solid line). The value of the slope gives $g = 9.78 \pm 0.05$ m/s^2. The errors indicated are the standard errors (1σ). The measurement points marked with error flags lie just above and below the line. It is noteworthy that the intercept with the T^2-axis (i.e. b) has a value which is clearly greater than the distance from the individual measurement points to the line.

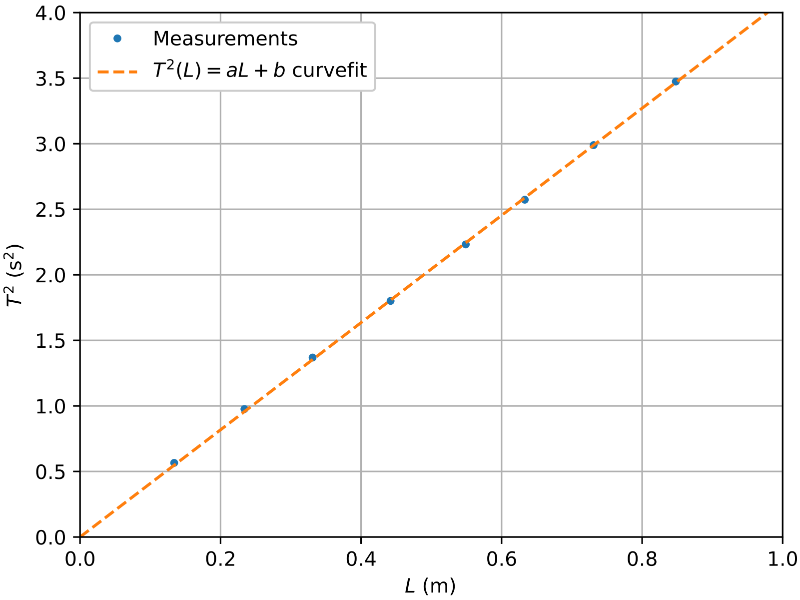

Figure 3.2: The squared period T^2 plotted against the length L of a simple pendulum. The solid line represents the linear least squares fit for the measurement points with slope $(4.04 \pm 0.02)\,\mathrm{s}^2/\mathrm{m}$ and offset $(0.03 \pm 0.01)\,\mathrm{s}^2$. The slope gives $g = 9.78 \pm 0.05\ \mathrm{m/s}^2$.

4

Discussion

The average gravitational acceleration in the Netherlands is equal to 9.81 m/s^2. Because $|9.75-9.81| < 2\sqrt{0.05^2}$, the found value for the gravitational acceleration ($9.75 \pm 0.05\,\mathrm{m/s^2}$) is not contradictory with the literature value.

In Figure 3.1, where T vs. L is plotted, the measurement points consistently lie above the theoretical curve at a distance of $2u(T)$. In Figure 3.2, T^2 vs. L, the same measurement points are evenly distributed above and below the fitted line. This line intersects all of the measurement points when the uncertainties are accounted for, indicating that the error estimate is consistent with the measurement results. It is noteworthy that, in Figure 3.2, in contrast to the theoretical line, the fitted line does not pass through the origin. This points to a systematic error in L.

It appears that the values provided for L are consistently too low. This is why the points in Figure 3.1 lie above the theoretical curve and the line in Figure 3.2 does not pass through the origin. The size of the systematic error in L can be estimated by calculating the intersection of the line with the L-axis. Using the data from the fit, this calculation gives: $\Delta L_{syst} = 7.5$ mm. This value is quite high and cannot be due to the practical difficulties with measuring L. These lead to a (random) error in the range 1 – 2 mm. A possible explanation for this is that the metal bob's centre of mass is not in the centre of the sphere. The calculation for the mass of a brass bob with the same diameter as that of the bob used here gives: 370 g, which is more than twice the mass of the bob we used (182 g). This demonstrates that the bob is not solid. This conclusion could be proven retrospectively by removing the mounting hook from the bob, revealing that it is indeed hollow. However, it was not possible to determine the shape and position of the cavity inside the bob with sufficient accuracy. It does appear highly probable that the hollow bob's centre of mass is not located at its geometric centre but approximately 1/3 of the radius lower.

The systematic error thus identified suggests a method for demonstrating the non-homogeneity of an object: suspend the object from a cord and study the pendulum motion.

The relative error in g is 0.5%. The errors in the variables for T^2 and L are 1%. By applying the fit procedure, the gravitational acceleration g can effectively be considered as an average value with a standard error that is one factor $\sqrt{N}$ lower than the individual errors (N is the number of measurements, here $N = 8$).

To determine g more accurately, it is recommended to use a significantly longer pendulum. In this report, L varies between 0.1 and 0.9 m. In the case of a long L, we can expect to see a lower relative error in L and T.

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Conclusion

The gravitational acceleration, measured using a simple pendulum composed of a brass bob and a nylon thread, is $g = 9.78 \pm 0.05$ m/s^2. This is in excellent agreement with the value cited in the literature. This result is obtained by measuring pendulum period as a function of pendulum length. The relationship observed between the period and the length agrees to a high degree with the theoretical description. A systematic error in determining the pendulum length discovered using this method suggests using a simple pendulum to detect non-homogeneous composition of an object. To obtain results with a smaller margin of error, it is recommended to use a longer pendulum (> 1 m).

Bibliography

[1] Foucault pendulum. http://en.wikipedia.org/wiki/Foucault_pendulum, 2018. geraadpleegd 9 januari 2018.

[2] Gravity of Earth. http://en.wikipedia.org/wiki/Gravity_of_Earth, 2018. geraadpleegd 24 januari 2018.

[3] Richard Wolfson. *Essential University Physics*. Pearson Education,Inc., Addison Wesley, San Francisco,, first edition, 2007.

A

Appendix

Weights for least squares fit for comparable errors in x and y.

When applying a least squares fit, we may also provide a weight for a given measurement point (x,y). For example, if the error in the y-value for a given measurement point is significantly greater than in the x-value (and we assume that the errors are normally distributed), then

$$w = \frac{1}{u(y)^2} \tag{A.1}$$

is taken for the weight w.

If the errors in the x- and y-values are comparable, it is more difficult to determine the weights. In this report, we followed the procedure below:

First, the error in the x-value is 'converted' into an error in the y-value; subsequently, the result is quadratically added to the original error in the y-value. If the theoretical relationship between y and x is expressed as $y = y(x)$, then an error in x, that is $u(x)$, is converted to:

$$u_{new} = \frac{dy}{dx} u(x) \tag{A.2}$$

This variable is quadratically added to $u(y)$:

$$u_{tot}^2 = u(y)^2 + \left(\frac{dy}{dx} u(x)\right)^2 \tag{A.3}$$

The total weigth then equals $w = 1/u_{tot}^2$.

For the simple pendulum, this gives: $T^2 = 4\pi^2 L/g$. In this case the total error is:

$$u_{tot}^2 = u(T^2)^2 + \frac{(4\pi^2)^2}{g^2} u(L)^2 \tag{A.4}$$

To find u_{tot}^2, a value is required for g, which is precisely the focus of this research project. A good estimate of g can already be obtained by applying a least squares fit without weights.