

## Homework 1

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written

**Due: 4:00pm on Friday, September 23, 2016**

This homework has two parts: a written section and a programming section.

### Written (50 pts)

For the written section of this assignment, type up your answers and submit a computer based document to us. You can submit MS Word doc files, pdf files, or txt files.

1. (10 pts) Weiss, Exercise 2.1

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Order the following functions by growth rate:  $N$ ,  $\sqrt{N}$ ,  $N^{1.5}$ ,  $N^2$ ,  $N \log N$ ,  $N \log \log N$ ,  $N \log^2 N$ ,  $N \log(N^2)$ ,  $2/N$ ,  $2^N$ ,  $2^{N/2}$ , 37,  $N^2 \log N$ ,  $N^3$ . Indicate which functions grow at the same rate.

a) Increasing order of function by growth rate:

$$2/N < 37 < \sqrt{N} < N < N \log \log N < N \log N < N \log(N^2) < N \log^2 N < N^{1.5} < N^2 < N^2 \log N < N^3 < 2^{N/2} < 2^N$$

b) Functions that grow at the same rate:

i) The time complexity of the function  $N \log N$  is  $O(N \log N)$ .

The time complexity of the function  $N \log(N^2)$  is:

$$\begin{aligned} N \log(N^2) &= N(2 \log N) \\ &= 2N \log N \\ &= O(N \log N) \end{aligned}$$

$\text{growth}(N \log(N^2)) = \text{growth}(N \log N)$

**Therefore,  $N \log N$  and  $N \log(N^2)$  have the same growth rate  $O(N \log N)$ .**

2. (10 pts) Weiss, Exercise 2.6

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In a recent court case, a judge cited a city for contempt and ordered a fine of \$2 for the first day. Each subsequent day, until the city followed the judge's order, the fine was squared (that is, the fine progressed as follows: \$2, \$4, \$16, \$256, \$65, 536, . . .).

a. What would be the fine on day  $N$ ?

Day	Fine (\$)	Pattern: $2^n$
1	2	$2^1 = 2^{2^0}$
2	4	$2^2 = 2^{2^1}$
3	16	$2^4 = 2^{2^2}$
4	256	$2^8 = 2^{2^3}$
5	65536	$2^{16} = 2^{2^4}$
...	...	...
$N$	...	$2^{2^{N-1}}$

**Therefore, on day  $N$ , the fine would be  $2^{2^{N-1}}$  dollars.**

- b. How many days would it take the fine to reach D dollars? (A Big-Oh answer will do.)

$$\begin{aligned}
 D &= 2^{2^{N-1}} \\
 \log_2 D &= \log_2 2^{2^{N-1}} \\
 \log D &= 2^{N-1} \\
 \log(\log D) &= \log(2^{N-1}) \\
 \log(\log D) &= N-1 \\
 \log(\log D) + 1 &= N
 \end{aligned}$$

Therefore,  $N = \log \log D + 1$  and the Big-Oh notation for  $2^{2^{N-1}}$  is  $O(\log \log N)$ .

3. (10 pts) Give an analysis of the Big-Oh running time for each of the following program fragments:

a.

```

int sum = 0;
for ( int i = 0; i < 23; i ++)
    for ( int j = 0; j < n ; j ++)
        sum = sum + 1;
    
```

The first for loop runs 23-number times. The inner for loop runs for each outer for loop iteration. Then, the for loop runs n-number of times... i.e 1 -> O(n), 2 -> O(n)...25 -> O(n). Therefore, the total complexity of the given code is :  $O(23 * n)$ . After eliminating the constant value, the complexity of the given code is  $O(n)$ .

$O(N)$

b.

```

int sum = 0;
for ( int i = 0; i < n ; i ++)
    for ( int k = i ; k < n ; k ++)
        sum = sum + 1;
    
```

The first for loop runs n-number times. The inner for loop runs for each outer for loop iteration. The inner for loop runs n-number of times...i.e 1 -> O(n), 2 -> O(n). Therefore, the complexity of the given code is  $O(n^2)$ .

$O(N^2)$

c.

```

public int foo(int n, int k) {
    if(n<=k)
        return 1;
    else
        return foo(n/k,k) + 1;
}
    
```

The recursive relation of n compares with k every time. Then, the second iteration n is divide by k times. The value is divided by k. Therefore, the recursive function will call n number of times. The time complexity is  $O(\log N)$ .

$O(\log N)$

4. (10 pts) Weiss, Exercise 2.11

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An algorithm takes 0.5 ms for input size 100. How long will it take for input size 500 if the running time is the following (assume low-order terms are negligible):

a. linear

The run time of linear function:  $T(N) = O(N)$

$N_1 = 100$

$T(N_1) = 0.5 \text{ ms}$

$N_2 = 500$

$$\begin{aligned} T(N_2) &= (N_2 / N_1) \times T(N_1) \\ &= (500 / 100) \times 0.5 \text{ ms} \\ &= 5 \times 0.5 \text{ ms} \\ &= \mathbf{2.5 \text{ ms}} \end{aligned}$$

**Therefore, the time it takes to complete an algorithm of size 500 is 2.5 ms.**

b.  $O(N \log N)$

The running time of  $N \log N$  function is  $T(N) = O(N \log N)$ .

$N_1 = 100$

$T(N_1) = 0.5 \text{ ms}$

$N_2 = 500$

$$\begin{aligned} T(N_2) &= ((N_2 \log N_2) / (N_1 \log N_1)) \times T(N_1) \\ &= ((500 \times \log_2 500) / (100 \times \log_2 100)) \times 0.5 \\ &= 5 \times ((\log_2 500) / (\log_2 100)) \times 0.5 \\ &= 2.5 \times ((\log_2 500) / (\log_2 10^2)) \\ &= 2.5 \times ((\log_2 500) / (2 \times \log_2 10)) \\ &= (2.5 / 2) \times (\log_{10} 500) && \text{** } (\log_c(a) / \log_c(b)) = \log_b a \\ &= 1.25 \times (\log_{10} 500) \\ &= 1.25 \times 2.6989 \\ &= \mathbf{3.3737 \text{ ms}} \end{aligned}$$

**Therefore, the time it takes to complete an algorithm of size 500 is 3.374 ms.**

c. quadratic

The running time of quadratic function is  $T(N) = O(N^2)$ .

$N_1 = 100$

$T(N_1) = 0.5 \text{ ms}$

$$N_2 = 500$$

$$\begin{aligned}T(N_2) &= ((N_2^2) / (N_1^2)) \times T(N_1) \\&= ((500^2) / (100^2)) \times 0.5 \text{ ms} \\&= 25 \times 0.5 \text{ ms} \\&= \mathbf{12.5 \text{ ms}}\end{aligned}$$

Therefore, the time it takes to complete an algorithm of size 500 is 12.5 ms.

d. cubic

The running time of quadratic function is  $T(N) = O(N^3)$ .

$$N_1 = 100$$

$$T(N_1) = 0.5 \text{ ms}$$

$$N_2 = 500$$

$$\begin{aligned}T(N_2) &= ((N_2^3) / (N_1^3)) \times T(N_1) \\&= ((500^3) / (100^3)) \times 0.5 \text{ ms} \\&= 125 \times 0.5 \text{ ms} \\&= \mathbf{62.5 \text{ ms}}\end{aligned}$$

Therefore, the time it takes to complete an algorithm of size 500 is 62.5 ms.

5. (10 pts) Weiss, Exercise 2.15

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Give an efficient algorithm to determine if there exists an integer  $i$  such that  $A_i = i$  in an array of integers  $A_1 < A_2 < A_3 < \dots < A_N$ . What is the running time of your algorithm?

Solution:

Let  $c$  be an integer between 1 and  $N$ .

If  $A_c < c$ , then  $A_i < i$ , for  $i = 1$  to  $(c - 1)$ .

If  $A_c > c$ , then  $A_i > i$ , for  $i = (c + 1)$  to  $N$

Algorithm:

- Check if the middle element satisfies  $A_i = i$ , and if it does the answer is yes.
- If  $A_i < i$  we can apply the same strategy to the subarray to the right of the middle element.
- If  $A_i > i$  we can apply the same strategy to the subarray to the left of the middle element.

$\text{left} = 0$ ,  $\text{right} = a.\text{length}$

```
if (a[0] > 1)
    return -1;
```

```
while ( left <= right ) {
```

```
    int mid = (left + right)/2;
```

```
if a[mid] = mid + 1  
    return (mid+1);  
  
else if (a[mid] > (mid+1))  
    right = mid - 1;  
  
else (a[mid] < (mid+1))  
    left = mid + 1;  
}  
return -1;
```

**Runtime Analysis:**

The problem is halved in size at each step, for a constant amount of work.  
Therefore, the run time of the algorithm is O (log N).