An examination of the high-order dynamic interactions underlying multi-dimensional timeseries data

Lucy L. W. Owen[1], Paxton C. Fitzpatrick[1], Kirsten Ziman[1],
Andrew C. Heusser[1,2], Stephen F. Satterthwaite[1], Thomas Hao Chang[1,3],
Qiang Liu[1,4], and Jeremy R. Manning[1,†]

[1]Department of Psychological and Brain Sciences,
Dartmouth College, Hanover, NH
[2]Akili Interactive, Inc, Boston, MA
[3]Amazon.com, Seattle, WA
[4]Department of Computer Science,
University of Texas at Austin, Austin, TX
[†]Address correspondence to jeremy.r.manning@dartmouth.edu

October 15, 2018

Abstract

Most complex systems reflect dynamic interations between myriad evolving components (e.g., interacting molecules, interacting brain systems, interacting individuals within a social network or ecological system, coordinated components within a mechanical or digital device, etc.). Despite that these interactions are central to the full system's behavior (e.g., removing a component from the full system can change the entire system's behavior), dynamic interactions cannot typically be directly measured. Rather, the interactions must be inferred through their hypothesized role in guiding the dynamics of system components. Here we use a model-based approach to inferring dynamic interactions from timeseries data. In addition to examining first-order interactions (e.g., between pairs of components) we also examine higher-order interactions (e.g., that characterize mirrored structure in the patterns of interaction dynamics displayed by different subsets of components). We apply our approach to two datasets. First, we use a synthetic dataset, for which the underlying dynamic interactions are known, to show that our model recovers those ground-truth dynamic interactions. We also apply our model to a neuroimaging dataset and show that the high-order dynamic interactions exhibited by brain data vary meaningfully as a function of the cognitive "richness" of the stimulus people are experiencing.

Introduction

The dynamics of the observable universe are meaningful in three respects. First, the behaviors of the *atomic units* that exhibit those dynamics are highly interrelated. The actions of one unit typically have implications for one or more other units. In other words, there is non-trivial *correlational structure* defining how different units interact with and relate to each other. Second, that correlational structure is *hierarchical* in the sense that it exists on many spatiotemporal scales. The way one group of units interacts may relate to how another group of units interact, and the interactions between those groups may exhibit some rich structure. Third, the structure at each level of this correlational hierarchy changes from moment to moment, reflecting the "behavior" of the full system.

These three properties (rich correlations, hierarchical organization, and dynamics) are major hallmarks of many complex systems. For example, within a single cell, the cellular components interact at many spatiotemporal scales, and those interactions change according to what that single cell is doing. Within a single human brain, the individual neurons interact within each brain structure, and the structures interact to form complex networks. The interactions at each scale vary according to the functions our brains are carrying out. And within social groups, interactions at different scales (e.g., between individuals, family units, communities, etc.) vary over time according to changing goals and external constraints.

Although many systems exhibit rich dynamic correlations at many scales, a major challenge to studying such patterns is that typically neither the correlations nor the hierarchical organizations of those correlations may be directly observed. Rather, these fundamental properties must be inferred indirectly by examining the observable parts of the system– e.g., the behaviors of the individual atomic units of that system. In the *Methods* section, we propose a series of mathematical operations that may be used to recover dynamic correlations at a range of scales (i.e., orders of interaction). In the *Results* section, we demonstrate how our approach may be applied to multi-dimensional timeseries data: a synthetic dataset where the underlying dynamic correlations are known (we use this dataset to validate our approach), and a neuroimaging dataset comprising data collected as participants listened to a story (Simony et al., 2016). In different experimental conditions in the neuroimaging study, participants listened to altered versions of the story that varied in cognitive richness: the intact story (fully engaging), a scrambled version of the story where the paragraphs were presented in a randomized order (moderately engaging), a second scrambled condition where the words were presented in a random order (minimally engaging), and a "rest" condition where the participants did not listen to any version of the story (control condition). We use the neuroimaging dataset to examine how higher-order structure in brain data varies as a function of the cognitive richness of the stimulus.

Methods

There are two basic steps to our approach. In the first step, we take a number-of-timepoints (T) by number-of-features (F) *matrix of observations* ($\mathbf{X}$) and we return a T by $\frac{F^2-F}{2}$ *matrix of dynamic correlations* ($\mathbf{Y}$). Here $\mathbf{Y_0}$ describes, at each moment, how all of the features (columns of $\mathbf{X}$) are inferred to be interacting. (Since the interactions are assumed to be non-recurrent and symmetric, only the upper triangle of the full correlation matrix is computed.) In the second step, we project $\mathbf{Y_0}$ onto an F-dimensional space, resulting in a new T by F matrix $\mathbf{Y_1}$. Note that $\mathbf{Y_1}$ contains information about the correlation dynamics present in $\mathbf{X}$, but represented in a compressed number of dimensions. By repeatedly applying these two steps in sequence, we can examine and explore higher order dynamic correlations in $\mathbf{X}$.

$$\text{corr}_t(x, y) = \frac{\sum_{t=1}^{T} (x_t - \bar{x}_t)(y_t - \bar{y}_t)}{\sqrt{\sum_{t=1}^{T} \sigma_{x_t}^2 \sigma_{y_t}^2}}, \text{where} \tag{1}$$

$$\bar{a}_t = \sum_{t=1}^{T} w(t)_i a_i \tag{2}$$

$$\sigma_{a_t}^2 = \sum_{t=1}^{T} (a_t - \bar{a}_t)^2 \tag{3}$$

$$w(t) = \mathcal{N}(1 \ldots T \mid t, \sigma_{\mathcal{N}}) \tag{4}$$

Results

Discussion

Concluding remarks

Acknowledgements

We acknowledge useful discussions with Luke Chang, Hany Farid, Matthijs van der Meer, and Emily Whitaker. Our work was supported in part by NSF EPSCoR Award Number 1632738 to J.R.M. and by a sub-award of DARPA RAM Cooperative Agreement N66001-14-2-4-032 to J.R.M. The content is solely the responsibility of the authors and does not necessarily represent the official views of our supporting organizations.

References

Simony, E., Honey, C. J., Chen, J., & Hasson, U. (2016). Uncovering stimulus-locked network dynamics during narrative comprehension. *Nature Communications*, *7*(12141), 1–13.