Appendix A. BENCHMARK DEFINITION AND COMPARISON OF VARIABLE SELECTION METHODS

The low-fidelity inputs introduced in Sect. 3.1 are defined as follows:

$f_{\mathtt{M}}^{(1)}(x) = 30x - 10,$

$f_{\mathtt{M}}^{(2)}(x) = \dfrac{(6x-2)^2 \sin(12x-4)}{2} + 10x - 10 + 3H(x - \dfrac{1}{2}),$

$f_{\mathtt{M}}^{(3)}(x) = f_{\mathtt{M}}^{(2)}(x) + 4\epsilon - 2,$

$f_{\mathtt{M}}^{(4)}(x) = f_{\mathtt{M}}^{(2)}(x) - \dfrac{1}{10}\tilde{\epsilon},$

$f_{\mathtt{M}}^{(5)}(x) = f_{\mathtt{M}}^{(2)}(x) + 2\epsilon - \dfrac{8}{100}\tilde{\epsilon} - 1,$

$f_{\mathtt{M}}^{(6)}(x) = 8\epsilon - 4,$

where ϵ is standard Gaussian noise and $\tilde{\epsilon}$ is cumulative noise.

Moreover, we report a comparison of our method with alternative input selection approaches based on the Correlation Coefficient (CC) and Mutual Information (MI) methods, mentioned in Sec. 2.2. In Tab. A.1, these quantities are given for the joint set of MF- and BO-training samples, together with the inputs selected by our method.

It is observed that, based on the CC, one would select $y\mathtt{M}^{(5)}$ as input. Based on the MI, one would correctly select $y\mathtt{M}^{(2)}$ as the first input for the model, but it would not be possible to identify that $y_{\mathtt{M}}^{(1)}$ contains the remaining useful information for the HF signal estimation.

Table A.1. Correlation Coefficient (CC) and Mutual Information for the MF- and BO-training sets, and the input hierarchy selected by our method on the benchmark test case.

Var.	CC$(\cdot, y_{\mathtt{HF}})$ [%]	Mutual Information [nat] $y_{\mathtt{HF}}$	$y_{\mathtt{LF}}^{(1)}$	$y_{\mathtt{LF}}^{(2)}$	$y_{\mathtt{LF}}^{(3)}$	$y_{\mathtt{LF}}^{(4)}$	$y_{\mathtt{LF}}^{(5)}$	Input Hier.
$y_{\mathtt{HF}}$	100.0	**1.42**	0.42	0.65	0.54	0.58	0.52	-
$y_{\mathtt{LF}}^{(1)}$	74.01	0.43	**1.32**	0.79	0.74	0.94	0.15	2
$y_{\mathtt{LF}}^{(2)}$	94.73	0.65	0.79	**1.42**	0.96	1.12	0.45	1
$y_{\mathtt{LF}}^{(3)}$	93.65	0.54	0.74	0.96	**1.42**	1.06	0.26	-
$y_{\mathtt{LF}}^{(4)}$	89.50	0.58	0.96	1.12	1.06	**1.42**	0.33	-
$y_{\mathtt{LF}}^{(5)}$	97.24	0.52	0.15	0.45	0.26	0.33	**1.42**	-
$y_{\mathtt{LF}}^{(6)}$	19.23	0.0	0.25	0.27	0.3	0.29	0.0	-