1. Multiple choices: Choose all the correct answers (You don’t need to state your reason) (3×6=18‘)

(1) A static magnetic field exists in the space with a right-handed Cartesian coordinates (直角坐标系). A small current unit $Id\vec{l}$ is used to test the magnetic field. When $d\vec{l}$ points to the “+x” direction, the magnetic force $\vec{F} = 0$. When $d\vec{l}$ points to the “+y” direction, the magnetic force $\vec{F}$ points to the “-z” direction.

A. $\vec{B}$ points to the “+x” direction.

B. $\vec{B}$ points to the “-x” direction.

C. $\vec{B}$ points to the “+z” direction.

D. $\vec{B}$ points to the “-z” direction.

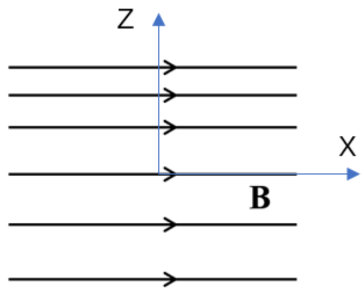
(2) **In magnetostatic case**, if the magnetic field lines are parallel straight lines in the space along the x-axis, then:

A. $\vec{B}$ must be uniform along x if there’s no current.

B. $\vec{B}$ must be uniform along z if there’s no current.

C. $\vec{B}$ must be uniform along x even if there’re currents.

D. $\vec{B}$ must be uniform along z even if there’re currents.



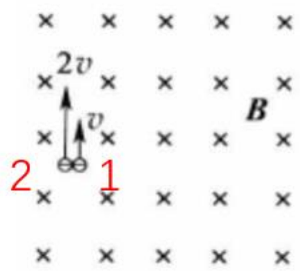
(3) The space is filled with a static uniform magnetic field. Two electrons are released with different initial speeds $v_1 = v$ and $v_2 = 2v$, as the following figure shows.

A. The magnitudes of the Lorentzian force $F_2 = 2F_1$

B. The angular frequency $\omega_2 = 2\omega_1$

C. The period (Time for one cycle) $T_2 = 2T_1$

D. The radius of the trail $R_2 = 2R_1$



(4) A round wire loop moves in a static uniform magnetic field. In which of the following conditions there may be emf (电动势) on the loop?

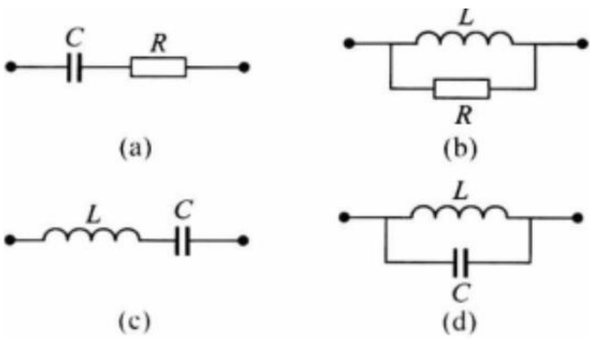
A. The loop moves parallel to the field without rotation.

B. The loop moves perpendicular to the field without rotation.

C. The loop rotates with an axis parallel to the field.

D. The loop rotates with an axis perpendicular to the field.

(5) Which of the following circuit can block both the low frequency $(\omega \to 0)$ and the high frequency $(\omega \to \infty)$ signals?



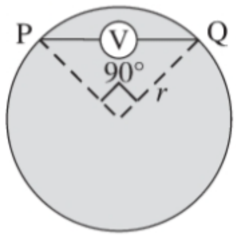
(6) A circular wire loop (radius r, uniform resistance R) encloses a region with a uniform magnetic field $\vec{B}$ perpendicular to its plane. The field increases linearly with time ($B = \alpha t$). An ideal voltmeter(电压表) with negligible size connects P and Q in a straight segment. Then the voltmeter possibly reads:

A. $\pi\alpha r^2/4$

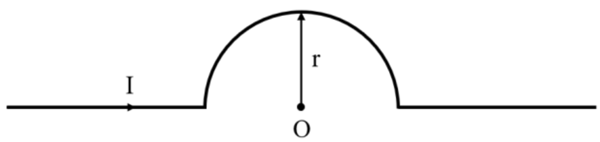
B. $-3\pi\alpha r^2/4$

C. 0

D. $\alpha r^2/2$



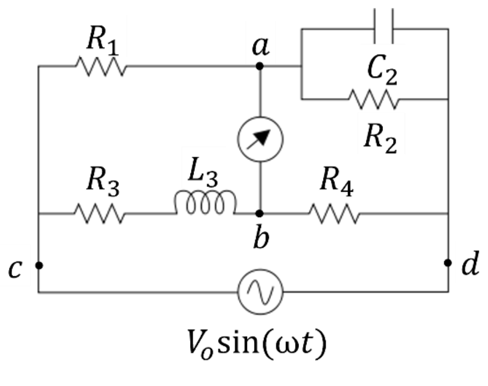
2. As shown in the figure, an infinitely long wire with a current *I* has a part bent to a semi-circle with a radius r. Calculate the magnetic field **B** at the center O.（8'）



3. The following figure shows an AC electric circuit oscillating with an angular frequency ω. This circuit is known as a Maxwell Inductance Capacitance Bridge (Maxwell Bridge). Derive

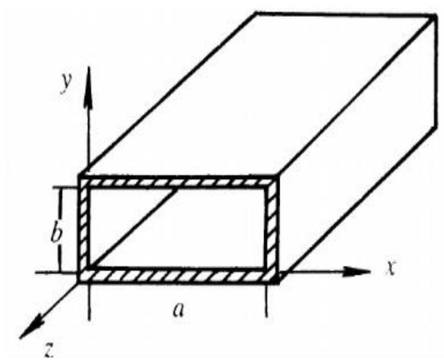
(1) The relation between R_1, C_2, R_2, R_3, L_3, and R_4 when the bridge is at balance (i.e. U_{ab}=0). (4')

(2) The impedance of the bridge (电路阻抗) Z_{cd} when the bridge is at balance. (4')

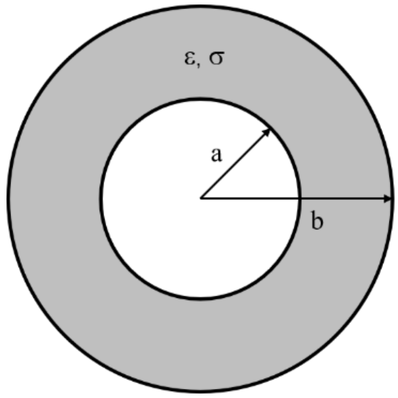


4. A TE (transverse electric) wave is propagating through a rectangular waveguide in z-direction. The size of the waveguide is as shown in the following figure.

(1) Write down the boundary conditions of E and B in this particular case. (3')
(2) Find the cut-off frequency. (2')
(3) Can a TEM (transverse electromagnetic) wave propagate inside this rectangular waveguide? Explain why. (3')



5. A concentric spherical capacitor (with an inner radius a and an outer radius b) is filled with some leaky dielectric material. After it is charged up to a total charge Q_0, the capacitor is disconnected from the source, so charges start to leak out slowly.
(1) Calculate the free current density. (4')
(2) Calculate the displacement current density. (4')
(3) Calculate the magnetic field between the two plates in this case. (4')



6. Suppose we have a resonator metal box with side lengths (L_1, L_2, L_3) and with vacuum inside.
(1) It is known that the electric field is sinusoidal in all three directions. Write down the electric field directly based on the boundary conditions. Calculate the magnetic field **B** according to the Maxwell equations, and verify it also satisfies the boundary conditions. (6')
(2) Suppose $L_1 = L_2 = L_3 = 1\ cm$, estimate the number of standing wave modes with frequencies between 1.1×10^{15}~$1.2 \times 10^{15} Hz$. (The speed of light c=3× 10^8m/s) (6')

7. An infinite cylinder of radius R carries a uniform surface charge σ. We propose to set it spinning around its axis at a final angular frequency ω. How much work per unit length will this take? Do it in two ways, and compare your answers:
(1) Find the magnetic field and the induced electric field (in the quasi static approximation),

inside and outside the cylinder, in terms of ω, dω/dt, and r (the distance from the axis). (4')

(2) Calculate the torque you must exert, and from that obtain the work done per unit length. (4')

(3) Calculate the energy stored in the resulting magnetic field directly. (4')

8. (1) Directly write down the divergence and the curl of $\vec{A}$ with respect to $\vec{B}$ under the Coulomb gauge. (2')

   (2) Directly write down the divergence and the curl of $\vec{B}$ with respect to $\vec{j}$ in the static case. (2')

   (3) Calculate $\vec{A}$ for an infinitely long solenoid (radius R, current I, loops per unit length n) under the Coulomb gauge. (6')

9. A waveguide's cross section is a "right angle isosceles triangle" (等腰直角三角形). The wall of the waveguide is perfect conductor and inside is vacuum. Try to calculate the possible propagation modes, and the electric field, magnetic field, and cut off frequency of each mode. (12')