Варіант 19 СДУ

№1

$$A = \begin{pmatrix} 3 & -1 & 1 \\ 1 & 1 & 1 \\ 4 & -1 & 1 \end{pmatrix}$$

$$\det(A-\lambda I) = \begin{vmatrix} 3-\lambda & -1 & 1 \\ 1 & 1-\lambda & 1 \\ 4 & -1 & 1-\lambda \end{vmatrix}$$

$$\det(A-\lambda I) = (3-\lambda)\begin{vmatrix} 1-\lambda & 1 \\ -1 & 1-\lambda \end{vmatrix} - (-1)\begin{vmatrix} 1 & 1 \\ 4 & 1-\lambda \end{vmatrix} +$$

$$+ 1\begin{vmatrix} 1 & 1-\lambda \\ 4 & -1 \end{vmatrix}$$

$$\det(A-\lambda I) = (3-\lambda)((1-\lambda)^2-(-1)) + 1(1(1-\lambda)-4\cdot 1) +$$

$$+ 1(1\cdot(-1)-4(1-\lambda)) =$$

$$= (3-\lambda)(\lambda^2-2\lambda+2) - 3 + 4\lambda - 1 - 4 + 4\lambda -$$

$$= -\lambda^3 + 5\lambda^2 - 8\lambda + 4 = (\lambda-2)(-\lambda^2+3\lambda-2)$$

$$-\lambda^2 + 3\lambda - 2 = 0$$

$$\lambda_{2,3} = \frac{3 \pm \sqrt{13}}{2}$$

для $\lambda_1 = 2$:

$$(A-2I)\vec{v} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ 4 & -1 & -1 \end{pmatrix}\vec{v} = 0$$

$$\vec{v}_1 = \begin{pmatrix} \frac{2}{3} \\ \frac{5}{3} \\ 1 \end{pmatrix}$$

λ_2 :

$$\vec{v_2} = \begin{pmatrix} \frac{1}{6} - \frac{\sqrt{13}}{6} \\ \frac{1}{6} - \frac{\sqrt{13}}{6} \\ 1 \end{pmatrix}$$

λ_3:

$$\vec{v_3} = \begin{pmatrix} \frac{1}{6} + \frac{\sqrt{13}}{6} \\ \frac{1}{6} + \frac{\sqrt{13}}{6} \\ 1 \end{pmatrix}$$

$$\vec{r}(t) = C_1 e^{2t}\vec{v_1} + C_2 e^{\left(\frac{3}{2} - \frac{\sqrt{13}}{2}\right)t}\vec{v_2} + C_3 e^{\left(\frac{3}{2} + \frac{\sqrt{13}}{2}\right)t}\vec{v_3}$$

v2

$$A = \begin{pmatrix} 5 & -1 & 2 \\ -1 & 3 & -1 \\ -4 & 2 & -1 \end{pmatrix}$$

$$\det(A - \lambda I) = 0$$

$$\det(A - \lambda I) = \begin{vmatrix} 5-\lambda & -1 & 2 \\ -1 & 3-\lambda & -1 \\ -4 & 2 & -1-\lambda \end{vmatrix}$$

$$\det(A - \lambda I) = (5-\lambda)\begin{vmatrix} 3-\lambda & -1 \\ 2 & -1-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 \\ -4 & -1-\lambda \end{vmatrix} +$$

$$+2\begin{vmatrix} -1 & 3-\lambda \\ -4 & 2 \end{vmatrix}$$

$$\det(A - \lambda I) = (5-\lambda)((3-\lambda)(-1-\lambda) - (-1)\cdot 2) +$$

$$+\begin{vmatrix} -1 & -1 \\ -4 & -1-\lambda \end{vmatrix} + 2\begin{vmatrix} -1 & 3-\lambda \\ -4 & 2 \end{vmatrix}$$

$\lambda_1 = 2 \quad , \lambda_2 = 3$

gue $\lambda = 2$:

$$(A - 2I) = \begin{pmatrix} 3 & -1 & 2 \\ -1 & 1 & -1 \\ -4 & 2 & -3 \end{pmatrix}$$

$$\begin{cases} 3x - y + 2z = 0 \\ -x + y - z = 0 \\ -4x + 2y - 3z = 0 \end{cases}$$

$$\vec{v}_1 = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ 1 \end{pmatrix} \quad , \quad v_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$\lambda = 3$:

$$(A - 3I) = \begin{pmatrix} 2 & -1 & 2 \\ -1 & 0 & -1 \\ -4 & 2 & -4 \end{pmatrix}$$

$$\begin{cases} 2x - y + 2z = 0 \\ -x - z = 0 \\ -4x + 2y - 4z = 0 \end{cases}$$

$$\vec{v}_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$\vec{r}(t) = C_1 e^{2t} \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ 1 \end{pmatrix} + C_2 e^{2t} \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + C_3 e^{3t} \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

n3

$$A = \begin{pmatrix} 7 & -4 & 1 \\ 7 & -3 & 1 \\ 4 & -2 & 2 \end{pmatrix}$$

$$\det(A - \lambda I) = 0$$

$$\det(A - \lambda I) = \begin{vmatrix} 7-\lambda & -4 & 1 \\ 7 & -3-\lambda & 1 \\ 4 & -2 & 2-\lambda \end{vmatrix}$$

$$\det(A - \lambda I) = (7-\lambda)\begin{vmatrix} -3-\lambda & 1 \\ -2 & 2-\lambda \end{vmatrix} - (-4)\begin{vmatrix} 7 & 1 \\ 4 & 2-\lambda \end{vmatrix} +$$

$$+ 1\begin{vmatrix} 7 & -4 \\ 4 & -2 \end{vmatrix}$$

$$\begin{vmatrix} -3-\lambda & 1 \\ -2 & 2-\lambda \end{vmatrix} = (-3-\lambda)(2-\lambda) - (-2)\cdot 1 = \lambda^2 + \lambda - 4$$

$$\begin{vmatrix} 7 & 1 \\ 4 & 2-\lambda \end{vmatrix} = 7(2-\lambda) - 4\cdot 1 = -7\lambda + 10$$

$$\begin{vmatrix} 7 & -4 \\ 4 & -2 \end{vmatrix} = 7(-2) - (-4)\cdot 4 = -14 + 16 = 2$$

$$\det(A - \lambda I) = (7-\lambda)(\lambda^2 + \lambda - 4) + 4(-7\lambda + 10) + 2 =$$

$$= \cancel{(7-7\lambda} \; 7\lambda^2 + 7\lambda - 28 - \lambda^3 - \lambda^2 + 4\lambda =$$

$$= -\lambda^3 + 6\lambda^2 + 11\lambda + 14$$

$$-\lambda^3 + 6\lambda^2 + 11\lambda + 14 = 0$$

$$\lambda_1 = 2$$

$$(\lambda - 2)(-\lambda^2 + 4\lambda + 7)$$

$$\lambda_{2,3} = \frac{-4 \pm \sqrt{16 - 4(-7)}}{-2} = 2 \pm i$$

дне $\lambda_1 = 2$:

$$(A - 2I) = \begin{pmatrix} 5 & -4 & 1 \\ 7 & -5 & 1 \\ 4 & -2 & 0 \end{pmatrix}$$

$$\vec{v}_1 = \begin{pmatrix} \frac{1}{3} \\ \frac{2}{3} \\ 1 \end{pmatrix}$$

дне $\lambda = 2 - i$

$$(A - (2-i)I) = \begin{pmatrix} 5+i & -4 & 1 \\ 7 & -5+i & 1 \\ 4 & -2 & i \end{pmatrix}$$

$$\vec{v}_2 = \begin{pmatrix} \frac{1}{2} - \frac{i}{2} \\ 1 - \frac{i}{2} \\ 1 \end{pmatrix}$$

дне $\lambda = 2 + i$

$$\vec{v}_3 = \begin{pmatrix} \frac{1}{2} + \frac{i}{2} \\ 1 + \frac{i}{2} \\ 1 \end{pmatrix}$$

$$\vec{r}(t) = C_1 e^{2t} \begin{pmatrix} \frac{1}{3} \\ \frac{2}{3} \\ 1 \end{pmatrix} + C_2 e^{(2-i)t} \begin{pmatrix} \frac{1}{2} - i \\ \cancel{1} - \frac{i}{2} \\ 1 \end{pmatrix} + C_3 e^{(2+i)t} \begin{pmatrix} \frac{1}{2} + i \\ 1 + \frac{i}{2} \\ 1 \end{pmatrix}$$

№ 4

$$\begin{cases} \dot{x} = y + \cos 2t - 2\sin 2t \\ \dot{y} = -x + 2y + 3\cos 2t + 2\sin 2t \end{cases}$$

$$\begin{cases} \dot{x} = y \\ \dot{y} = -x + 2y \end{cases}$$

$$\ddot{x} = \dot{y}$$

$$\dot{y} = -x + 2\dot{x}$$

$$\ddot{x} = -x + 2\dot{x}$$

$$r^2 - 2r + 1 = 0 \Rightarrow (r-1)^2 = 0$$

$$r = 1$$

$$\begin{cases} x_{одн}(t) = (C_1 + C_2 t) e^t \\ y_{одн}(t) = (C_1 + C_2 + C_2 t) e^t \end{cases}$$

$$x_{част}(t) = A\cos 2t + B\sin 2t$$

$$y_{частн}(x) = C\cos 2t + D\sin 2t$$

$$\dot{x} = -2A\sin 2t + 2B\cos 2t$$

$$y = C\cos 2t + D\sin 2t$$

$$-2A\sin 2t + 2B\cos 2t = C\cos 2t + D\sin 2t$$

$$\cos 2t: \quad 2B = C + 1$$

$$\sin 2t: \quad -2A = D - 2$$

$$\dot{y} = -2C\sin 2t + 2D\cos 2t$$

$$-x + 2y = -(A\cos 2t + B\sin 2t) + 2C\cos 2t + 2D\sin 2t$$

$-2C\sin 2t+2D\cos 2t=-(A\cos 2t+B\sin 2t)+$
$+2C\cos 2t+2D\sin 2t+3\cos 2t+2\sin 2t$

$\cos 2t:\quad 2D=-A+2C+3$

$\sin 2t:\quad -2C=-B+2D+2$

$$\begin{cases} 2B=C+1 \\ -2A=D-2 \\ 2D=-A+2C+3 \\ -2C=-B+2D+2 \end{cases}$$

$C=2B-1$

$A=-\dfrac{D-2}{2}$

$2D=-A+2(2B-1)+3$

$-2(2B-1)=-B+2D+2$

$A=0,\ B=2,\ C=3,\ D=0$

$x_{част}(t)=2\sin 2t$

$y_{част}(t)=3\cos 2t$

Общее решение:

$x(t)=x_{одн}(t)+x_{част}(t)=(C_1+C_2t)e^t+2\sin 2t$

$y(t)=y_{одн}(t)+y_{част}(t)=(C_1+C_2+C_2t)e^t+3\cos 2t$

№5

$$\begin{cases} \dot{x} = x - 2y \\ \dot{y} = x - y + \frac{1}{2\sin t} \end{cases}$$

$$\begin{cases} \dot{x} = x - 2y \\ \dot{y} = x - y \end{cases}$$

$$\ddot{x} = \dot{x} - 2\dot{y}$$

$$\ddot{x} = \dot{x} - 2(x - y)$$

из первого уравнения $y = \frac{\dot{x} - x}{2}$

$$\ddot{x} = \dot{x} - 2x + (\dot{x} - x) = 2\dot{x} - 3x$$

$$r^2 - 2r + 3 = 0$$

$$r = 1 \pm i\sqrt{2}$$

$$x_{одн}(t) = e^{t}(C_1 \cos(\sqrt{2}t) + C_2 \sin(\sqrt{2}t))$$

$$y_{одн}(t) = \frac{\dot{x} - x}{2}$$

$$x(t) = e^{t}(U_1(t)\cos(\sqrt{2}t) + U_2(t)\sin(\sqrt{2}t))$$

$$y(t) = e^{t}(V_1(t)\cos(\sqrt{2}t) + V_2(t)\sin(\sqrt{2}t))$$

для $\dot{x}$:

$$(U_1(t)\cos(\sqrt{2}t) + U_2(t)\sin(\sqrt{2}t) - 2V_1(t)\cos(\sqrt{2}t) - 2V_2(t)\sin(\sqrt{2}t))e^{t} = (-\sqrt{2}U_1(t)\sin(\sqrt{2}t) + U_1(t)\cos(\sqrt{2}t) + U_2(t)\sin(\sqrt{2}t) + \sqrt{2}U_2(t)\cos(\sqrt{2}t) + \sin(\sqrt{2}t)\dot{U}_2(t) + \cos(\sqrt{2}t)\dot{U}_1(t))e^{t}$$

где $\dot{y}$:

$$\Big(U_1(t)\cos(\sqrt{2}t) + U_2(t)\sin(\sqrt{2}t) - V_1(t)\cos(\sqrt{2}t) - V_2(t)\sin(\sqrt{2}t) + \frac{1}{2\sin t}\Big)e^t = \Big(-\sqrt{2}V_1(t)\sin(\sqrt{2}t) + V_1(t)\cos(\sqrt{2}t) + V_2(t)\sin(\sqrt{2}t) + \sqrt{2}V_2(t)\cos(\sqrt{2}t) + \sin(\sqrt{2}t)\dot{V}_2(t) + \cos(\sqrt{2}t)\dot{V}_1(t)\Big)e^t$$

где $\dot{x}$:

$\cos(2t)$: $U_1(t) - 2V_1(t) = U_1(t) + 2U_2(t) + \dot{U}_1(t)$

$\sin(2t)$: $U_2(t) - 2V_2(t) = -2U_1(t) + U_2(t) + \dot{U}_2(t)$

где $\dot{y}$:

$\cos(2t)$: $U_1(t) - V_1(t) = V_1(t) + 2V_2(t) + \dot{V}_1(t$

$\sin(2t)$: $U_2(t) - V_2(t) + \frac{1}{2\sin t} = -2V_1(t) + V_2(t) + \dot{V}_2(t)$

$$U_1 - 2V_1 = U_1 + 2U_2 + \dot{U}_1$$
$$U_2 - 2V_2 = -2U_1 + U_2 + \dot{U}_2$$
$$U_1 - V_1 = V_1 + 2V_2 + \dot{V}_1$$
$$U_2 - V_2 + \frac{1}{2\sin t} = -2V_1 + V_2 + \dot{V}_2$$

$$\begin{cases} \dot{U}_1(t) = -2U_2(t) - 2V_1(t) \\ \dot{U}_2(t) = 2U_1(t) - 2V_2(t) \\ \dot{V}_1(t) = U_1(t) - 2V_1(t) - 2V_2(t) \\ \dot{V}_2(t) = U_2(t) + 2V_1(t) - 2V_2(t) + \frac{1}{2\sin t} \end{cases}$$

$$x(t) = e^{t}\left(U_1(t)\cos(\sqrt{2}t) + U_2(t)\sin(\sqrt{2}t)\right)$$

$$y(t) = e^{t}\left(V_1(t)\cos(\sqrt{2}t) + V_2(t)\sin(\sqrt{2}t)\right)$$