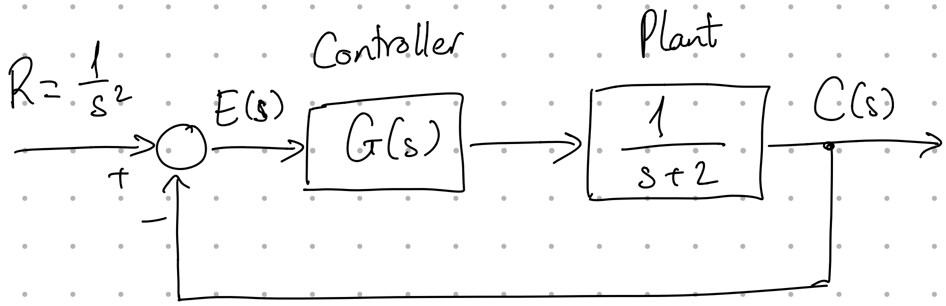
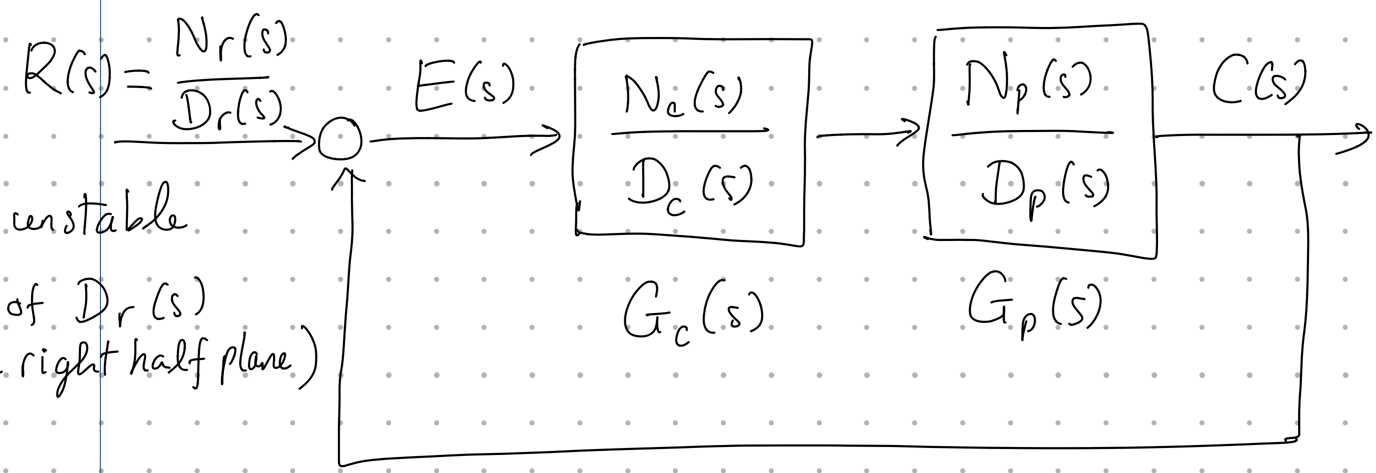


$$G(s) = \left(\frac{1}{s} + 1\right) \cdot \frac{1}{s} = \frac{s+1}{s^2}$$

$$E(s) = \frac{1}{1 + \frac{1}{s+2} \cdot \frac{s+1}{s^2}} \cdot \frac{1}{s^2}$$

$$= \frac{s^2(s+2)}{s^3 + 2s^2 + s + 1} \cdot \frac{1}{s^2}$$

$$= \frac{s+2}{s^3 + 2s^2 + s + 1}$$



$R(s)$ is unstable
(or roots of $D_r(s)$ are in the right half plane)

$$E(s) = \frac{D_p(s) \cdot D_c(s)}{D_p(s) \cdot D_c(s) + N_p(s) \cdot N_c(s)} \cdot \frac{N_r(s)}{D_r(s)}$$

plant : $\begin{cases} \dot{X} = AX + B_1 U + B_2 W \\ Y = CX \end{cases}$

Reference : $\begin{cases} \dot{x}_r = A_r x_r \\ r(t) = d_r \cdot x_r \end{cases}$

Target $\begin{cases} \text{(I) } Y \longmapsto r(t) \text{ when } t \longmapsto \infty \\ \text{(II) } X \text{ is bounded with bounded input} \\ \quad (\text{i.e. BIBO stable}) \end{cases}$

1. Reference is a step : ($R(s) = \frac{1}{s}$)

$$\begin{cases} \dot{x}_r = 0 \\ r(t) = x_r(t) \end{cases} \qquad \text{i.e : } \dot{r}(t) = 0$$

tracking error: $e(t) = -Y + r = -CX + r$

$\Rightarrow \dot{e} = -C\dot{X} + \dot{r} = -C\dot{X}$

Augmented state: $\bar{X} = \begin{bmatrix} \dot{X} \\ e \end{bmatrix}$

$$\dot{\bar{X}} = \begin{bmatrix} \ddot{X} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} \dot{X} \\ e \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} \dot{U} + \begin{bmatrix} B_2 \\ 0 \end{bmatrix} \dot{W}$$

suppose there is a gain K which make this system stable : (target II)

input control now is : $\dot{U} = K\bar{X} = K \begin{bmatrix} \dot{X} \\ e \end{bmatrix}$

$\Rightarrow U = K \begin{bmatrix} X \\ \int e \, dt \end{bmatrix} \Rightarrow$ satisfying target I

$\longrightarrow \frac{1}{s}$

2. Reference is a Ramp $\left(R(s) = \frac{M}{s^2} \right)$

$$\begin{cases} r(t) = M.t \\ \dot{r}(t) = M \end{cases} ; \quad M \in \mathbb{R}^+$$

the tracking error: $e = -Y + r = -CX + r$

$$\dot{e} = -C\dot{X} + \dot{r} = -C\dot{X} + M$$

$$\ddot{e} = -C\ddot{X}$$

Augmented state: $\bar{X} = \begin{bmatrix} \ddot{X} \\ \dot{e} \\ e \end{bmatrix}$

$$\dot{\bar{X}} = \begin{bmatrix} \dddot{X} \\ \ddot{e} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A & 0 & 0 \\ -C & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \ddot{X} \\ \dot{e} \\ e \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \\ 0 \end{bmatrix} \ddot{U} + \begin{bmatrix} B_2 \\ 0 \\ 0 \end{bmatrix} \ddot{W}$$

suppose there is a gain K which make this system stable :(target II)

$$\ddot{U} = K \begin{bmatrix} \ddot{X} \\ \dot{e} \\ e \end{bmatrix} \Rightarrow U = K \begin{bmatrix} X \\ \int e\,dt \\ \int\left(\int e\,dt\right) dt \end{bmatrix}$$

$\rightarrow \frac{1}{s^2}$

$\hookrightarrow$ satisfying target I

3. Higher order: repeat the same until:

$$\frac{d^m e}{d^m t} = -C \frac{d^m X}{d^m t}$$