

Se nos da la siguiente funcion de transferencia

$$G_0 = \frac{3}{s^2 + 2s + 1}$$

Se usa la siguiente formula de retroalimentación:

$$Feedback = \frac{G_0}{1 + G_0 * H_0}$$

1) Se hace el Feedback:

$$\frac{\frac{3}{s^2 + 2s + 1}}{1 + \frac{3}{s^2 + 2s + 1} * 1} = \frac{\frac{3}{s^2 + 2s + 1}}{\frac{s^2 + 2s + 1}{s^2 + 2s + 1} + \frac{3}{s^2 + 2s + 1}} = \frac{3}{s^2 + 2s + 4}$$

$$= \left(\frac{3}{4}\right) \left(\frac{4}{s^2 + 2s + 4}\right)$$

Se usa la formula:

$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \left(\frac{3}{4}\right) \left(\frac{4}{s^2 + 2s + 4}\right)$$

2) Se calcula ω_n^2

$$\omega_n^2 = 4 = \sqrt{\omega_n^2} = \sqrt{4} = \omega_n = 2$$

3) Se calcula ζ

$$\zeta\omega_n = 1 = \zeta 2 = 1 = \zeta = \frac{1}{2}$$

4) Se calcula Ceros y Polos

Ceros=

$$Z = \emptyset$$

Polos=

$$\alpha = \zeta\omega_n = \frac{1}{2} * 2 = \alpha = 1$$

$$\omega = \omega_n \sqrt{1 - (\zeta)^2} = \omega = 2 \sqrt{1 - \left(\frac{1}{2}\right)^2} \quad \omega = \sqrt{3}$$

Segun la formula general

$$s = -\zeta \omega \pm \sqrt{\omega^2 (\zeta^2 - 1)}$$

Los polos quedan

$$P_1 = -1 + \sqrt{3}$$

$$P_2 = -1 - \sqrt{3}$$

5) Verificación en Octave

```
octave:10> num=[3];
octave:11> den=[1 2 1];
octave:12> G=tf(num,den)
```

Transfer function 'G' from input 'u1' to output ...

$$y1: \frac{3}{s^2 + 2s + 1}$$

Continuous-time model.

```
octave:13> H=tf([1],[1])
```

Transfer function 'H' from input 'u1' to output ...

$$y1: 1$$

Continuous-time model.

```
octave:14> feedback(G,H)
```

Transfer function 'ans' from input 'u1' to output ...

$$y1: \frac{3}{s^2 + 2s + 4}$$

Continuous-time model.

```
octave:15> [z,p,k]=tf2zp([3],[1,2,4])
```

```
z = [](0x1)
```

```
p =
```

```
    -1.0000 + 1.7321i
```

```
    -1.0000 - 1.7321i
```

```
k = 3
```