

MPC for 2D-Quadrotor with suspended load

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Abstract—

I. INTRODUCTION

II. DYNAMICS

A. Quadrotor Dynamics

Following equations present the dynamics for the quadrotor shown in Figure 1

$$\dot{\mathbf{x}} = \frac{d}{dt} \begin{bmatrix} y \\ z \\ \phi \\ \dot{y} \\ \dot{z} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \dot{y} \\ \dot{z} \\ \dot{\phi} \\ 0 \\ -g \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\frac{\sin \phi}{m_Q} & 0 \\ \frac{\cos \phi}{m_Q} & 0 \\ 0 & J_Q \end{bmatrix} \begin{bmatrix} f \\ M \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} f \\ M \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -l_Q & l_Q \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \quad (2)$$

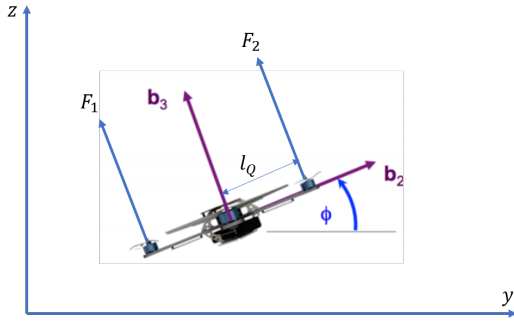


Fig. 1. Quadrotor

B. Quadrotor w/ Load Dynamics

variable	description
y_L & z_L	load position
ϕ_L	load angle wrt z
l	cable length
m_L	load mass
$y_Q = y_L - l \sin \phi_L$ & $z_Q = z_L + l \cos \phi_L$	quad position
ϕ_Q	quad angle wrt y
m_Q	quad mass

$$x_L = \begin{bmatrix} y_L \\ z_L \end{bmatrix}, \quad e_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (3)$$

$$p = \begin{bmatrix} \sin(\phi_L) \\ -\cos(\phi_L) \end{bmatrix}, \quad R = \begin{bmatrix} \cos(\phi_Q) & -\sin(\phi_Q) \\ \sin(\phi_Q) & \cos(\phi_Q) \end{bmatrix} \quad (4)$$

$$(m_Q + m_L)(\dot{v}_L + g e_3) = (-f \cos(\phi_Q - \phi_L) - m_Q l \dot{\phi}_L^2) p \quad (5)$$

$$m_Q l \ddot{\phi}_L = \sin(\phi_Q - \phi_L) \quad (6)$$

$$J_Q \ddot{\phi}_Q = M \quad (7)$$

$$\mathbf{X} = [y_L \quad z_L \quad \phi_L \quad \phi_Q \quad \dot{y}_L \quad \dot{z}_L \quad \dot{\phi}_L \quad \dot{\phi}_Q]^T \quad (8)$$

$$\dot{\mathbf{X}} = \underbrace{\begin{bmatrix} \dot{y}_L \\ \dot{z}_L \\ \dot{\phi}_L \\ \dot{\phi}_Q \\ -\frac{m_Q}{m_Q + m_L} l \dot{\phi}_L^2 \sin \phi_L \\ \frac{m_Q}{m_Q + m_L} l \dot{\phi}_L^2 \cos \phi_L - g \\ \frac{1}{m_Q} \sin(\phi_Q - \phi_L) \\ 0 \end{bmatrix}}_f + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{-\sin \phi_L}{m_Q + m_L} \cos(\phi_Q - \phi_L) \\ \frac{\cos \phi_L}{m_Q + m_L} \cos(\phi_Q - \phi_L) \\ 0 \\ 0 \\ 0 \\ \frac{1}{J_Q} \end{bmatrix}}_g \begin{bmatrix} f \\ M \end{bmatrix} \quad (9)$$

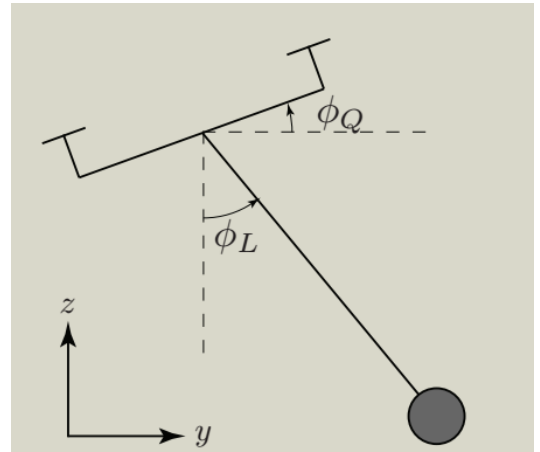


Fig. 2. Quadrotor with Suspended Load