MPC for 2D-Quadrotor with suspended load

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***Abstract*—**

I. INTRODUCTION

II. DYNAMICS

A. Quadrotor Dynamics

Following equations present the dynamics for the quadrotor shown in Figure 1

$$\dot{\mathbf{x}} = \frac{d}{dt}\left(\begin{bmatrix} y \\ z \\ \phi \\ \dot{y} \\ \dot{z} \\ \dot{\phi} \end{bmatrix}\right) = \begin{bmatrix} \dot{y} \\ \dot{z} \\ \dot{\phi} \\ 0 \\ -g \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\frac{\sin\phi}{m_Q} & 0 \\ \frac{\cos\phi}{m_Q} & 0 \\ 0 & J_Q \end{bmatrix} \begin{bmatrix} f \\ M \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} f \\ M \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -l_Q & l_Q \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \tag{2}$$

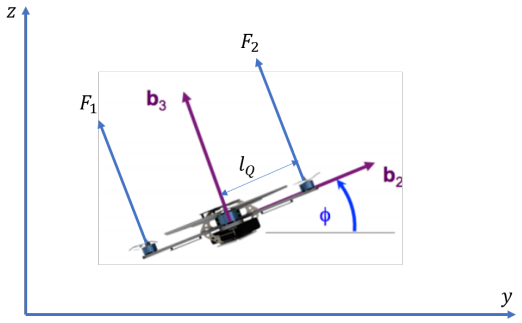

Fig. 1. Quadrotor

B. Quadrotor w/ Load Dynamics

variable	description
y_L & z_L	load position
ϕ_L	load angle wrt z
l	cable length
m_L	load mass
$y_Q = yL - l\sin\phi_L$ & $z_Q = z_L + l\cos\phi_L$	quad position
ϕ_Q	quad angle wrt y
m_Q	quad mass

*This work is in Partial Fulfillment of the Requirements for ME-C231 course project

$$x_L = \begin{bmatrix} y_L \\ z_L \end{bmatrix}, \quad e_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{3}$$

$$p = \begin{bmatrix} \sin(\phi_L) \\ -\cos(\phi_L) \end{bmatrix}, \quad R = \begin{bmatrix} \cos(\phi_Q) & -\sin(\phi_Q) \\ \sin(\phi_Q) & \cos(\phi_Q) \end{bmatrix} \tag{4}$$

$$(m_Q + m_L)(\dot{v}_L + g e_3) = (f\cos(\phi_Q - \phi_L) - m_Q l \dot{\phi}_L^2)p \tag{5}$$

$$m_Q l \ddot{\phi}_L = \sin(\phi_Q - \phi_L) \tag{6}$$

$$J_Q \ddot{\phi}_Q = M \tag{7}$$

$$\mathbf{X} = \begin{bmatrix} y_L & z_L & \phi_L & \phi_Q & \dot{y}_L & \dot{z}_L & \dot{\phi}_L & \dot{\phi}_Q \end{bmatrix}^T \tag{8}$$

$$\dot{\mathbf{X}} = \underbrace{\begin{bmatrix} \dot{y}_L \\ \dot{z}_L \\ \dot{\phi}_L \\ \dot{\phi}_Q \\ -\frac{m_Q}{m_Q+m_L} l \dot{\phi}_L^2 \sin\phi_L \\ \frac{m_Q}{m_Q+m_L} l \dot{\phi}_L^2 \cos\phi_L - g \\ \frac{1}{m_Q}\sin(\phi_Q - \phi_L) \\ 0 \end{bmatrix}}_{f} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{\sin\phi_L}{m_Q+m_L}\cos(\phi_Q - \phi_L) & 0 \\ \frac{-\cos\phi_L}{m_Q+m_L}\cos(\phi_Q - \phi_L) & 0 \\ 0 & 0 \\ 0 & \frac{1}{J_Q} \end{bmatrix}}_{g} \begin{bmatrix} f \\ M \end{bmatrix} \tag{9}$$

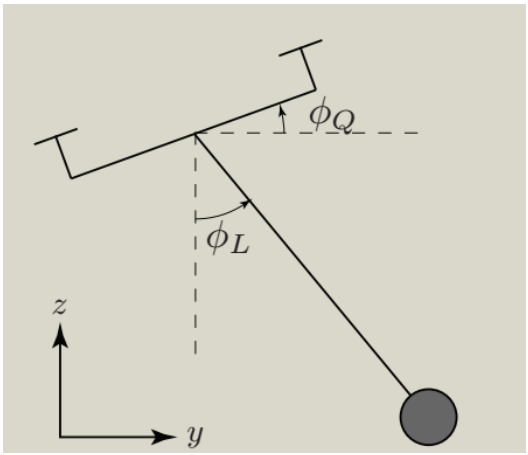

Fig. 2. Quadrotor with Suspended Load