# Delay-dependent Robust Stability Analysis of Power Systems with PID Controller

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***Abstract*—This technical report examines the robust stability of a power system, which is based on proportional-integral-derivative load frequency control and involves uncertain parameters and time delays. This report examines a power system model which is transformed into a closed-loop system with feedback control. The purpose of this report is to verify new augmented Lyapunov-Krasovskii (LK) functional to employ the new Bessel-Legendre inequality. The new Bessel-Legendre inequality is used to estimate the derivative of the functional to obtain a maximum lower bound. This report also verifies and provides a detailed description of stability criterion of the power system obtained by employing the LK functional and Bessel-Legendre inequality. Finally, this report validates the proposed method by applying it to practical numerical examples.**

***Index Terms*—Index Terms - To be modified later**

## I. Introduction

WITH the growing population, the demand for efficient power systems have increased drastically. Efficient power systems have the characteristics of high stability margin and less time delays. These characteristics are attainable if the operating frequency of the power system fluctuate within a small range of equilibrium value and the load frequency control (LFC) can control these fluctuations. During the process of data transmission, there is always a possiblity of occurance if time delays. These time delays occur randomly which makes it impossible to predict. Therefore, gaining insights on robust stability of power systems with time delays and uncertain parameters can help to overcome this problem.

The most research methods employed in time-delay systems are based on the Lyapunov direct time-domain method. Through the construction of the Lyapunov-Krasovskii (LK) functional, the stability of the system is analyzed with the aid of the Lyapunov stability theory, after which the stability criterion is derived. Finally, the stability margin of the system is obtained by employing the linear matrix inequality (LMI) toolbox.This report aims to improve the upper bound of the time delay of the system by proposing an augmented LK functional and using the Bessel-Legendre inequality to estimate the derivative of the functional. This study assumes that the forward channel of the controller possesses time delays and the inertia time constant of both the prime mover and speed governor may have uncertain parameters in the system. A time-varying delay power system model is established with uncertain parameters based on proportional-integral-derivative (PID) load frequency control. An appropriate LK functional is constructed, and the robust stability of the system is then analyzed using the Lyapunov stability theory. Finally, the robust stability criterion of the system is obtained by employing the Bessel-Legendre inequality discussed in Ref. [9], and the stability margin that the system can withstand is solved using an LMI toolbox. The advantages and effectiveness of the proposed method are demonstrated by comparing the results of the proposed method with those of previous methods under the two controller gains.

The variables used in this study are defined as follows: $R^n$ and $R^{n*m}$ denote n-dimensional vectors and $n * m$ dimensional matrices in the real number domain, respectively; $R^T$ and $R^-1$ represent the transpose and inverse of a matrix, respectively; I and 0 are identity and zero matrices, respectively; P should be greater than 0 means that the matrix P is symmetric and positive; $SymX = X + X^T$; * represents symmetric terms in a symmetric matrix; and diag() denotes a diagonal matrix.

## II. Concepts Needed

### A. Legendre Polynomials

A brief overview of Legendre polynomials obtained by solving Legendre ODE is provided below.

Legendre ODE is given as follows

$$(1-x^2)\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} - 2x\frac{\mathrm{d}f}{\mathrm{d}x} + k(k+1)y = 0 \tag{1}$$

Solution to ODE is obtained as follows:
Step 1: Convert Equation to the form shown below

$$\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} + p(x)\frac{\mathrm{d}y}{\mathrm{d}x} + q(x)y = G(x) \tag{2}$$

Therefore we have,

$$\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} - \frac{2x}{1-x^2}\frac{\mathrm{d}f}{\mathrm{d}x} + \frac{k(k+1)}{1-x^2}y = 0 \tag{3}$$

Step 2: Use power series solution about $x_0 = 0$
Therefore,

$$Y(x) = \sum_{n=0} \infty a_n x^n \tag{4}$$

$$Y'(x) = \sum_{\infty} n = 0 n a_n x^{n-1} \tag{5}$$

$$Y''(x) = \sum_{n=0}^{\infty} n(n-1)a_n x^(n-2) \tag{6}$$

Plug this back into the ODE to get the recurrence relation below

$$a_{n+2} = \frac{(n-k)(n+k+1)}{(n+2)(n+1)} \tag{7}$$

Solving recurrence relation

Odd Index solution

$$a_3 = \frac{a_1(1-k)(2+k)}{3!} \tag{8}$$
$$a_5 = \frac{a_1(k-3)(k-1)(k+2)(k+4)}{5!} \tag{9}$$
$$\tag{10}$$

Polynomial Soln for k(even or odd)
Set $a_k$ to calculate $a_{k-2}, a_{k-4}$

$$a_n = \frac{(n-k)(n+k+1)}{(n+2)(n+1)} a_n \tag{11}$$

replace n by k-2

$$a_{k-2k} = \frac{-1(k)(k-1)}{2(2k-1)} \tag{12}$$
$$a_k = \frac{-1(2k-2)!}{2^k(k-1)!(k-2)!} \tag{13}$$

Similarly,

$$a_{k-4} = \frac{(-1)^2(2k-4)!}{2(2^k)(k-2)!(k-4)!} \tag{14}$$

Therefore the general formula

$$a_{k-2m} = \frac{-1^m(2k-m)!}{(m!)(2^k)(k-m)!(k-2m)!} \tag{15}$$

Therefore

$$P_k(x) = \sum_{m=0}^{k/2} \tag{16}$$

Properties
Legendre Polynomials are orthogonal

$$(2n-1)xP_(n-1)(x) = nP(x) + (n-1)P_(n-2)(s)n = 2, 3... \tag{17}$$

### B. Bessle Legendre Inequalities

$$\int_{-h}^{0} x(u)Rx(u)du \geq \frac{1}{h} \begin{bmatrix} \omega_0 \\ . \\ . \\ . \\ \omega_n \end{bmatrix} R_N \begin{bmatrix} \omega_0 \\ . \\ . \\ . \\ \omega_n \end{bmatrix} \tag{18}$$

where R is symmetric positive definite matrix and

$$R_N = diag(R, 3R, ..., (2N+1)R) \tag{19}$$
$$\omega_k = \int_{-h}^{0} P_k(u)x(u)du, \tag{20}$$

for all natural numbers k and x is a square integrable function from an open interval,I to $R^n$

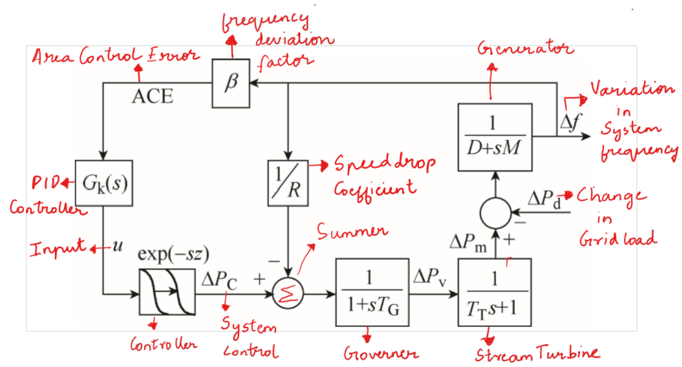


Fig. 1. System

### C. Lyapunov-Krasovskii Functional

### D. Prime mover

### E. Speed governor

## III. System Model and Derivations

The system above controls the frequency of the load(LFC) to some desired output.

Whenever there is a change in grid load, the system adjust the frequency for the new load by doing the following.

1) Generator generates a new frequency
2) Frequency deviation factor($\beta$) is used to get the error
3) Error is fed into PID to handle frequency variation
4) A control signal is generated to control the governor
5) Output is fed into Governor(acts as an actuator) to control valve
6) Valve is opened based on signal
7) Variation in valve opening causes a desired power change

From the figure above, the equations of motion can be represented as

$$\frac{\mathrm{d}\Delta f}{\mathrm{d}t} = \frac{-D}{M}\Delta f + \frac{D}{M}\Delta P_m - \frac{1}{M}\Delta P_d \tag{21}$$
$$\tag{22}$$
$$\frac{\mathrm{d}\Delta P_m}{\mathrm{d}t} = \frac{-\Delta P_m}{T_T} + \frac{P_v}{T_T} \tag{23}$$
$$\tag{24}$$
$$\frac{\mathrm{d}\Delta P_v}{\mathrm{d}t} = \frac{-1}{T_G R}\Delta f - \frac{1}{T_G}\Delta P_v \tag{25}$$
$$\tag{26}$$
$$ACE(t) = \Delta f \beta \tag{27}$$

This can be represented in state space equation as

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = Ax_o(t) + Bu(t-h(t)) + F(\omega t) \tag{28}$$
$$y_o(t) = Cx_o(t) \tag{29}$$

where

$$x_o(t) = [\Delta f \quad \Delta P_m \quad \Delta P_v] \quad (30)$$
$$y_o(t) = ACE \quad (31)$$

$$A = \begin{bmatrix} \frac{D}{M} & \frac{D}{M} & 0 \\ 0 & \frac{-1}{T_T} & \frac{1}{T_T} \\ \frac{-1}{T_G R} & 0 & \frac{-1}{T_G} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_G} \end{bmatrix} \quad (32)$$

$$(33)$$

$$F = \begin{bmatrix} \frac{1}{M} & 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} \beta & 0 & 0 \end{bmatrix} \quad (34)$$

$$w(t) = \Delta P_d \quad u(t - h(t)) = \Delta P_c(t) \quad (35)$$

Meaning of Symbols

$\Delta f->$ Variation in system frequency (36)

$\Delta P_m->$ Variation in Mechanical Power (37)

$\Delta P_v->$ Variation in Control Valve Opening (38)

$ACE(t)->$ Area Controller Error (39)

$h(t)->$ Delay Stability Margin (40)

$u(t-h(t))->$ Input with Delay (41)

$\Delta P_d->$ Changes in Grid load (42)

$\Delta P_c->$ Changes in System Control Signals (43)

$M->$ Moment of Inertia (44)

$D->$ Damping Coefficient of generator (45)

$T_T->$ Inertia time of the Generator of Steam Turbine (46)

$T_G->$ Inertia time of the Governor of the unit (47)

$R->$ Speed Drop coefficient of the Governor (48)

$\beta->$ frequency Deviation Factor (49)

The PID Controller handles the Area Controller Error(ACE) and produces a feedback using the equation stated as follows

$$u(t) = K_p ACE(t) + K_I \int ACE(t)dt + K_D \frac{\mathrm{d}ACE(t)}{\mathrm{d}dt} \quad (50)$$

where

$K_I->$ Integral gain (51)

$K_P->$ Proportional gain (52)

$K_D->$ Derivative gain (53)

The virtual state and output variables can be writen as follows

$$\bar{x}_0(t) = \begin{bmatrix} x_0^T(t) & \int y_0^T(t)dt \end{bmatrix}^T \quad (54)$$

$$\bar{y}_0(t) = \begin{bmatrix} y_0^T(t) & \int y_0^T(t)dt & \frac{\mathrm{d}y_0^T(t)}{\mathrm{d}t} \end{bmatrix}^T \quad (55)$$

Because CB = 0, we have the transformation

$$\frac{\mathrm{d}\bar{x}}{\mathrm{d}t} = \bar{A}\bar{x}_0(t) + \bar{B}u(t - h(t)) + \bar{F}\omega(t) \quad (56)$$
$$\bar{y}_0(t) = \bar{C}\bar{x}_0(t) + \bar{D}_w\omega(t) \quad (57)$$
$$u(t) = -K\bar{y}_0(t) \quad (58)$$

$$\frac{\mathrm{d}\bar{x}_0(t)}{\mathrm{d}t} = \bar{A}\bar{x}_0(t) + \bar{A}_d\bar{x}_0(t - h(t)) + \bar{B}_w\omega(t) \quad (59)$$

$$(60)$$

where $\bar{A}_d = -\bar{B}K\bar{C}$ ,$\bar{B}_w = \bar{F} - \bar{B}K\bar{D}_w$

Consider an equilibrium point (point for which $\frac{\mathrm{d}\bar{x}_0(t)}{\mathrm{d}t} = 0$) at $\bar{x}_0^*(t)$ Therefore,

$$0 = \bar{A}\bar{x}_0^*(t) + \bar{A}_d\bar{x}_0^*(t - h(t)) + \bar{B}_w\omega(t) \quad (61)$$

Perfoming State Transformation where $x(t) = \bar{x}(t) - \bar{x}_0^*(t)$, we have

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \bar{A}x(t) + \bar{A}_d x(t - h(t)) \quad (62)$$

where

$$\bar{A} = \begin{bmatrix} \frac{-D}{M} & \frac{1}{M} & 0 & 0 \\ 0 & \frac{-1}{T_T} & \frac{1}{T_T} & 0 \\ \frac{-1}{T_G R} & 0 & \frac{-1}{T_G} & 0 \\ \beta & 0 & 0 & 0 \end{bmatrix} \quad (63)$$

$$\bar{A}_d = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{\beta D K_D}{M T_G} - \frac{\beta K_P}{T_G} & \frac{-\beta K_D}{M T_G} & 0 & \frac{-K_1}{T_G} \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (64)$$

Since the parameters of the power system experience some disturbance, this is accounted for by the following:
let $\alpha$ be the disturbance deviation for prime mover
let $\beta$ be the disturbance deviation for Governor

Because the values of the deviations are extremely small, we use the percentage error. Therefore,the new value of $T_T$ due to error change can be accounted for as

$$T_{Ta}\epsilon[(1 - \alpha\%)T_T, (1 + \alpha\%)T_T] \quad (65)$$
$$T_{Ga}\epsilon[(1 - \gamma\%)T_G, (1 + \gamma\%)T_G] \quad (66)$$

$T_{Ta}->$ Inertia time constant for prime mover (67)

$T_{Ga}->$ Inertia time contant for Governor (68)

Given the values above, $T_T$ can be expressed as

$$T_T = \frac{1 + \gamma\alpha\%}{(1 - \alpha\%)(1 + \alpha\%)} T_T a \quad (69)$$

where $\gamma\epsilon[-1 \quad 1]$ is a constant

We can however write the above equation in a more general form by letting $\gamma$ be function on the same interval. Therefore we write in a more compact form as follows

$$\frac{1}{T_{Ta}} = \frac{\alpha_1}{T_T} + f_1(t)\frac{\alpha_2}{T_T} \tag{70}$$

$$\frac{1}{T_{Ga}} = \frac{\gamma_1}{T_G} + f_2(t)\frac{\gamma_2}{T_G} \tag{71}$$

where

$$f_1(t), f_2(t) \epsilon [-1, 1] \tag{72}$$

$$\alpha_1 = \frac{1}{(1-\alpha)(1+\alpha\%)}, \quad \alpha_2 = \alpha\%\alpha_1 \tag{73}$$

$$\gamma_1 = \frac{1}{(1-\gamma)(1+\gamma\%)}, \quad \gamma_2 = \gamma\%\gamma \tag{74}$$

Therefore the new equation including delay is

$$\frac{\mathrm{d}x}{\mathrm{d}t} = (A_0 + \Delta A_0)x(t) + (A_d + \Delta A_d)(x(t - h(t))), \tag{75}$$

where

$$[\Delta A_0 \quad \Delta A_d] = HF(t)[E_1 \quad E_2] \tag{76}$$

$$H = I \tag{77}$$

$$F(t) = diag\{0, f_1(t), f_2(t), 0\} \tag{78}$$

$$A_0 = \begin{bmatrix} \frac{-D}{M} & \frac{1}{M} & 0 & 0 \\ 0 & \frac{-\alpha_1}{T_T} & \frac{\alpha}{T_T} & 0 \\ -\frac{\gamma}{T_G R} & 0- & \frac{\gamma_1}{T_G} & 0 \\ \beta & 0 & 0 & 0 \end{bmatrix} \tag{79}$$

$$A_d = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{\gamma_1 \beta D K_D}{M T_G} - \frac{\gamma_1 \beta K_p}{T_G} & 0- & \frac{\gamma_1 \beta K_D}{M T_G} & -\frac{\gamma_1 K_1}{T_G} \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{80}$$

$$E_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{\alpha_2}{T_1} & \alpha_2 T_T & 0 \\ -\frac{\gamma_2}{T_G R} & 0 & \frac{-\gamma_2}{T_G} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{81}$$

$$E_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{\gamma_2 \beta D K_D}{M T_G} - \frac{\gamma_2 \beta K_p}{T_G} & 0- & \frac{\gamma_2 \beta K_D}{M T_G} & -\frac{\gamma_2 K_1}{T_G} \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{82}$$

## IV. Robust Stability

## V. Results

## VI. Discussion and Summary

## Acknowledgment