Delay-dependent Robust Stability Analysis of Power Systems with PID Controller

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***Abstract*—This technical report examines the robust stability of a power system, which is based on proportional-integral-derivative load frequency control and involves uncertain parameters and time delays. This report examines a power system model which is transformed into a closed-loop system with feedback control. The purpose of this report is to verify new augmented Lyapunov-Krasovskii (LK) functional to employ the new Bessel-Legendre inequality. The new Bessel-Legendre inequality is used to estimate the derivative of the functional to obtain a maximum lower bound. This report also verifies and provides a detailed description of stability criterion of the power system obtained by employing the LK functional and Bessel-Legendre inequality. Finally, this report validates the proposed method by applying it to practical numerical examples.**

I. Introduction

WITH the growing population, the demand for efficient power systems have increased drastically. Efficient power systems have the characteristics of high stability margin and less time delays. These characteristics are attainable if the operating frequency of the power system fluctuate within a small range of equilibrium value and the load frequency control (LFC) can control these fluctuations. During the process of data transmission, there is always a possiblity of occurance if time delays. These time delays occur randomly which makes it impossible to predict. Therefore, gaining insights on robust stability of power systems with time delays and uncertain parameters can help to overcome this problem.

The most research methods employed in time-delay systems are based on the Lyapunov direct time-domain method. Through the construction of the Lyapunov-Krasovskii (LK) functional, the stability of the system is analyzed with the aid of the Lyapunov stability theory, after which the stability criterion is derived. Finally, the stability margin of the system is obtained by employing the linear matrix inequality (LMI) toolbox.This report aims to improve the upper bound of the time delay of the system by proposing an augmented LK functional and using the Bessel-Legendre inequality to estimate the derivative of the functional. This study assumes that the forward channel of the controller possesses time delays and the inertia time constant of both the prime mover and speed governor may have uncertain parameters in the system. A time-varying delay power system model is established with uncertain parameters based on proportional-integral-derivative (PID) load frequency control. An appropriate LK functional is constructed, and the robust stability of the system is then analyzed using the Lyapunov stability theory. Finally, the robust stability criterion of the system is obtained by employing the Bessel-Legendre inequality discussed in Ref. [9], and the stability margin that the system can withstand is solved using an LMI toolbox. The advantages and effectiveness of the proposed method are demonstrated by comparing the results of the proposed method with those of previous methods under the two controller gains.

The variables used in this study are defined as follows: R^n and R^{n*m} denote n-dimensional vectors and $n * m$ dimensional matrices in the real number domain, respectively; R^T and R^-1 represent the transpose and inverse of a matrix, respectively; I and 0 are identity and zero matrices, respectively; P should be greater than 0 means that the matrix P is symmetric and positive; $SymX = X + X^T$; * represents symmetric terms in a symmetric matrix; and diag() denotes a diagonal matrix.

II. Concepts Needed

A. Legendre Polynomials

A brief overview of Legendre polynomials obtained by solving Legendre ODE is provided below.

Legendre ODE is given as follows

$$(1-x^2)\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} - 2x\frac{\mathrm{d}f}{\mathrm{d}x} + k(k+1)y = 0 \tag{1}$$

Solution to ODE is obtained as follows:
Step 1: Convert Equation to the form shown below

$$\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} + p(x)\frac{\mathrm{d}y}{\mathrm{d}x} + q(x)y = G(x) \tag{2}$$

Therefore we have,

$$\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} - \frac{2x}{1-x^2}\frac{\mathrm{d}f}{\mathrm{d}x} + \frac{k(k+1)}{1-x^2}y = 0 \tag{3}$$

Step 2: Use power series solution about $x_0 = 0$
Therefore,

$$Y(x) = \sum_{n=0} \infty a_n x^n \tag{4}$$

$$Y'(x) = \sum_{\infty} n = 0 n a_n x^{n-1} \tag{5}$$

$$Y''(x) = \sum_{n=0}^{\infty} n(n-1) a_n x^(n-2) \tag{6}$$

Plug this back into the ODE to get the recurrence relation below

$$a_{n+2} = \frac{(n-k)(n+k+1)}{(n+2)(n+1)} \tag{7}$$

Solving recurrence relation

Odd Index solution

$$a_3 = \frac{a_1(1-k)(2+k)}{3!} \tag{8}$$

$$a_5 = \frac{a_1(k-3)(k-1)(k+2)(k+4)}{5!} \tag{9}$$

$$\tag{10}$$

Polynomial Soln for k(even or odd)
Set a_k to calculate a_{k-2}, a_{k-4}

$$a_n = \frac{(n-k)(n+k+1)}{(n+2)(n+1)} a_n \tag{11}$$

replace n by k-2

$$a_{k-2k} = \frac{-1(k)(k-1)}{2(2k-1)} \tag{12}$$

$$a_k = \frac{-1(2k-2)!}{2^k(k-1)!(k-2)!} \tag{13}$$

Similarly,

$$a_{k-4} = \frac{(-1)^2(2k-4)!}{2(2^k)(k-2)!(k-4)!} \tag{14}$$

Therefore the general formula

$$a_{k-2m} = \frac{-1^m(2k-m)!}{(m!)(2^k)(k-m)!(k-2m)!} \tag{15}$$

Therefore

$$P_k(x) = \sum_{m=0}^{k/2} a_{k-2m} x^{k-2m} \tag{16}$$

Properties
Legendre Polynomials are orthogonal

$$(2n-1)xP_(n-1)(x) = nP(x) + (n-1)P_(n-2)(s)n = 2, 3... \tag{17}$$

B. Bessle Legendre Inequalities

$$\int_{-h}^{0} x(u)Rx(u)du \geq \frac{1}{h} \begin{bmatrix} \omega_0 \\ . \\ . \\ . \\ \omega_n \end{bmatrix} R_N \begin{bmatrix} \omega_0 \\ . \\ . \\ . \\ \omega_n \end{bmatrix} \tag{18}$$

where R is symmetric positive definite matrix and

$$R_N = diag(R, 3R, ..., (2N+1)R) \tag{19}$$

$$\omega_k = \int_{-h}^{0} P_k(u)x(u)du, \tag{20}$$

for all natural numbers k and x is a square integrable function from an open interval,I to R^n

C. Lyapunov-Krasovskii Functional

Lyapunov-Krasovskii Functional is obtained from the Lyapunov theory which is used to check the stability of a system. The LK functional derivied from the Lyapunov theory thia power system is given below:

$$V(x_t) = \eta^T(t)P\eta(t) + \int_t -h^t \theta^T(t,s)Q\theta(t,s)ds$$
$$+ \int_{-h}^{0} \int_{t+\theta}^{t} \frac{\mathrm{d}x^T(s)}{\mathrm{d}t} R \frac{\mathrm{d}x^T}{\mathrm{d}t} ds d\theta$$

D. Prime mover

Prime mover in power provides the necessary mechanical input to the steam turbine. It converts power into mechanical input.

E. Speed governor

Speed governor in control system is used to measure or regulate the speed of the steam turbine.

III. Controllability and Stability Analysis

A. Controllability

The above linear time variant power system is controllable only if the matrix

$$\begin{bmatrix} B & AB & AB^2 \end{bmatrix} \text{preserves rank} \tag{21}$$

where t_0 is the initial time
t_f is the final time Checking the condition for controllability: We have

$$B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_G} \end{bmatrix} A = \begin{bmatrix} \frac{D}{M} & \frac{D}{M} & 0 \\ 0 & \frac{-1}{T_T} & \frac{1}{T_T} \\ \frac{-1}{T_G R} & 0 & \frac{-1}{T_G} \end{bmatrix} \tag{22}$$

$$AB = \begin{bmatrix} 0 \\ \frac{1}{T_G T_T} \\ \frac{1}{T_G^2} \end{bmatrix} A^2B = \begin{bmatrix} \frac{D}{MT_G T_T} \\ \frac{T_G+T_T}{T_G^2 T_T^2} \\ \frac{1}{T_G^2} \end{bmatrix} \tag{23}$$

$$\begin{bmatrix} B & AB & AB^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{D}{MT_G T_T} \\ 0 & \frac{1}{T_G T_T} & \frac{T_G+T_T}{T_G^2 T_T^2} \\ \frac{1}{T_G} & \frac{1}{T_G^2} & \frac{1}{T_G^2} \end{bmatrix} \tag{24}$$

$$\det \begin{bmatrix} B & AB & AB^2 \end{bmatrix} = \frac{-D}{MT_G^3 T_T^2} \neq 0 \tag{25}$$

Therefore the the matrix $[B \quad AB \quad A^2B]$ preserves rank. Hence, the power system is controllable which simultaneously implies that the system is stabilizable.

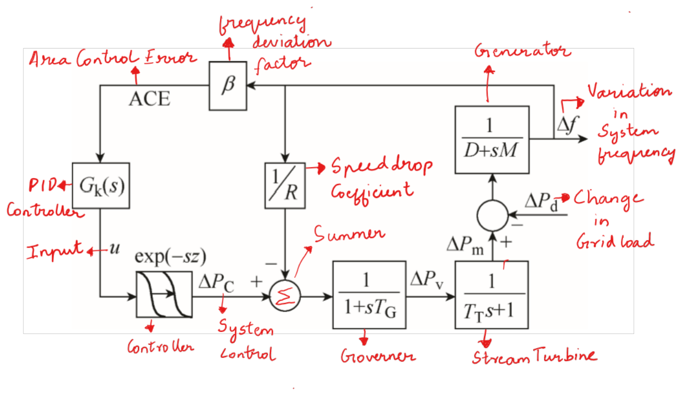

Fig. 1. System

IV. System Model and Derivations

The system above controls the frequency of the load(LFC) to some desired output.

Whenever there is a change in grid load, the system adjust the frequency for the new load by doing the following.

1) Generator generates a new frequency
2) Frequency deviation factor(β) is used to get the error
3) Error is fed into PID to handle frequency variation
4) A control signal is generated to control the governor
5) Output is fed into Governor(acts as an actuator) to control valve
6) Valve is opened based on signal
7) Variation in valve opening causes a desired power change

From the figure above, the equations of motion can be represented as

$$\frac{\mathrm{d}\Delta f}{\mathrm{d}t} = \frac{-D}{M}\Delta f + \frac{D}{M}\Delta P_m - \frac{1}{M}\Delta P_d \tag{26}$$

$$\tag{27}$$

$$\frac{\mathrm{d}\Delta P_m}{\mathrm{d}t} = \frac{-\Delta P_m}{T_T} + \frac{P_v}{T_T} \tag{28}$$

$$\tag{29}$$

$$\frac{\mathrm{d}\Delta P_v}{\mathrm{d}t} = \frac{-1}{T_G R}\Delta f - \frac{1}{T_G}\Delta P_v \tag{30}$$

$$\tag{31}$$

$$ACE(t) = \Delta f \beta \tag{32}$$

This can be represented in state space equation as

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = Ax_o(t) + Bu(t-h(t)) + F(\omega t) \tag{33}$$

$$y_o(t) = Cx_o(t) \tag{34}$$

where

$$x_o(t) = [\Delta f \quad \Delta P_m \quad \Delta P_v] \tag{35}$$

$$y_o(t) = ACE \tag{36}$$

$$A = \begin{bmatrix} \frac{D}{M} & \frac{D}{M} & 0 \\ 0 & \frac{-1}{T_T} & \frac{1}{T_T} \\ \frac{-1}{T_G R} & 0 & \frac{-1}{T_G} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_G} \end{bmatrix} \tag{37}$$

$$\tag{38}$$

$$F = \begin{bmatrix} \frac{1}{M} & 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} \beta & 0 & 0 \end{bmatrix} \tag{39}$$

$$w(t) = \Delta P_d \quad u(t-h(t)) = \Delta P_c(t) \tag{40}$$

Meaning of Symbols

$$\Delta f -> \text{Variation in system frequency} \tag{41}$$
$$\Delta P_m -> \text{Variation in Mechanical Power} \tag{42}$$
$$\Delta P_v -> \text{Variation in Control Valve Opening} \tag{43}$$
$$ACE(t) -> \text{Area Controller Error} \tag{44}$$
$$h(t) -> \text{Delay Stability Margin} \tag{45}$$
$$u(t-h(t)) -> \text{Input with Delay} \tag{46}$$
$$\Delta P_d -> \text{Changes in Grid load} \tag{47}$$
$$\Delta P_c -> \text{Changes in System Control Signals} \tag{48}$$
$$M -> \text{Moment of Inertia} \tag{49}$$
$$D -> \text{Damping Coefficient of generator} \tag{50}$$
$$T_T -> \text{Inertia time of the Generator of Steam Turbine} \tag{51}$$
$$T_G -> \text{Inertia time of the Governor of the unit} \tag{52}$$
$$R -> \text{Speed Drop coefficient of the Governor} \tag{53}$$
$$\beta -> \text{frequency Deviation Factor} \tag{54}$$

The PID Controller handles the Area Controller Error(ACE) and produces a feedback using the equation stated as follows

$$u(t) = K_p ACE(t) + K_I \int ACE(t)dt + K_D \frac{\mathrm{d}ACE(t)}{\mathrm{d}dt} \tag{55}$$

where

$$K_I -> \text{Integral gain} \tag{56}$$
$$K_P -> \text{Proportional gain} \tag{57}$$
$$K_D -> \text{Derivative gain} \tag{58}$$

The virtual state and output variables can be writen as follows

$$\bar{x}_0(t) = \begin{bmatrix} x_0^T(t) & \int y_0^T(t)dt \end{bmatrix}^T \tag{59}$$

$$\bar{y}_0(t) = \begin{bmatrix} y_0^T(t) & \int y_0^T(t)dt & \frac{\mathrm{d}y_0^T(t)}{\mathrm{d}t} \end{bmatrix}^T \tag{60}$$

Because CB = 0, we have the transformation

$$\frac{\mathrm{d}\bar{x}}{\mathrm{d}t} = \bar{A}\bar{x}_0(t) + \bar{B}u(t-h(t)) + \bar{F}\omega(t) \tag{61}$$

$$\bar{y}_0(t) = \bar{C}\bar{x}_0(t) + \bar{D}_w\omega(t) \tag{62}$$

$$u(t) = -K\bar{y}_0(t) \tag{63}$$

$$\frac{\mathrm{d}\bar{x}_0(t)}{\mathrm{d}t} = \bar{A}\bar{x}_0(t) + \bar{A}_d\bar{x}_0(t-h(t)) + \bar{B}_w\omega(t) \tag{64}$$

$$\tag{65}$$

where $\bar{A}_d = -\bar{B}K\bar{C}$,$\bar{B}_w = \bar{F} - \bar{B}K\bar{D}_w$

Consider an equilibrium point (point for which $\frac{\mathrm{d}\bar{x}_0(t)}{\mathrm{d}t} = 0$) at $\bar{x}_0^*(t)$ Therefore,

$$0 = \bar{A}\bar{x}_0^*(t) + \bar{A}_d\bar{x}_0^*(t-h(t)) + \bar{B}_w\omega(t) \tag{66}$$

Perfoming State Transformation where $x(t) = \bar{x}(t) - \bar{x}_0^*(t)$, we have

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \bar{A}x(t) + \bar{A}_d x(t-h(t)) \tag{67}$$

where

$$\bar{A} = \begin{bmatrix} \frac{-D}{M} & \frac{1}{M} & 0 & 0 \\ 0 & \frac{-1}{T_T} & \frac{1}{T_T} & 0 \\ \frac{-1}{T_G R} & 0 & \frac{-1}{T_G} & 0 \\ \beta & 0 & 0 & 0 \end{bmatrix} \tag{68}$$

$$\bar{A}_d = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{\beta D K_D}{M T_G} - \frac{\beta K_P}{T_G} & \frac{-\beta K_D}{M T_G} & 0 & \frac{-K_1}{T_G} \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{69}$$

Since the parameters of the power system experience some disturbance, this is accounted for by the following:
let α be the disturbance deviation for prime mover
let β be the disturbance deviation for Governor

Because the values of the deviations are extremely small, we use the percentage error. Therefore,the new value of T_T due to error change can be accounted for as

$$T_{Ta}\epsilon[(1-\alpha\%)T_T, (1+\alpha\%)T_T] \tag{70}$$

$$T_{Ga}\epsilon[(1-\gamma\%)T_G, (1+\gamma\%)T_G] \tag{71}$$

$$T_{Ta} -> \text{Inertia time constant for prime mover} \tag{72}$$

$$T_{Ga} -> \text{Inertia time contant for Governor} \tag{73}$$

Given the values above, T_T can be expressed as

$$T_T = \frac{1+\gamma\alpha\%}{(1-\alpha\%)(1+\alpha\%)} T_T a \tag{74}$$

where $\gamma\epsilon[-1 \quad 1]$ is a constant

We can however write the above equation in a more general form by letting γ be function on the same interval. Therefore we write in a more compact form as follows

$$\frac{1}{T_{Ta}} = \frac{\alpha_1}{T_T} + f_1(t)\frac{\alpha_2}{T_T} \tag{75}$$

$$\frac{1}{T_{Ga}} = \frac{\gamma_1}{T_G} + f_2(t)\frac{\gamma_2}{T_G} \tag{76}$$

where

$$f_1(t), f_2(t)\epsilon[-1, 1] \tag{77}$$

$$\alpha_1 = \frac{1}{(1-\alpha)(1+\alpha\%)}, \quad \alpha_2 = \alpha\%\alpha_1 \tag{78}$$

$$\gamma_1 = \frac{1}{(1-\gamma)(1+\gamma\%)}, \quad \gamma_2 = \gamma\%\gamma \tag{79}$$

Therefore the new equation including delay is

$$\frac{\mathrm{d}x}{\mathrm{d}t} = (A_0 + \Delta A_0)x(t) + (A_d + \Delta A_d)(x(t-h(t))), \tag{80}$$

where

$$[\Delta A_0 \quad \Delta A_d] = HF(t)[E_1 \quad E_2] \tag{81}$$

$$H = I \tag{82}$$

$$F(t) = diag\{0, f_1(t), f_2(t), 0\} \tag{83}$$

$$A_0 = \begin{bmatrix} \frac{-D}{M} & \frac{1}{M} & 0 & 0 \\ 0 & \frac{-\alpha_1}{T_T} & \frac{\alpha}{T_T} & 0 \\ -\frac{\gamma}{T_G R} & 0 & -\frac{\gamma_1}{T_G} & 0 \\ \beta & 0 & 0 & 0 \end{bmatrix} \tag{84}$$

$$A_d = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{\gamma_1 \beta D K_D}{M T_G} - \frac{\gamma_1 \beta K_p}{T_G} & 0 & -\frac{\gamma_1 \beta K_D}{M T_G} & -\frac{\gamma_1 K_1}{T_G} \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{85}$$

$$E_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{\alpha_2}{T_1} & \alpha_2 T_T & 0 \\ -\frac{\gamma_2}{T_G R} & 0 & \frac{-\gamma_2}{T_G} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{86}$$

$$E_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{\gamma_2 \beta D K_D}{M T_G} - \frac{\gamma_2 \beta K_p}{T_G} & 0 & -\frac{\gamma_2 \beta K_D}{M T_G} & -\frac{\gamma_2 K_1}{T_G} \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{87}$$

A. lemma1

let ω be a differentiable functional:$[a, b] -> R^n$. For a real symmetric matrix $Z > 0$ and any real matrix M with appropriate dimensions, the followin inequality holds:

$$-\int_a^b \frac{\mathrm{d}w^T}{\mathrm{d}s} Z \frac{\mathrm{d}w^T}{\mathrm{d}s} ds \leq \xi^T[Sym\{\Pi^T M\} + (b-a)M^T \widetilde{Z} M]\xi \tag{88}$$

where

$$\widetilde{Z} = diag\{Z, \quad 3Z, \quad 5Z\} \tag{89}$$

$$\eta = [\omega(b) \quad \omega \quad (a) \quad \gamma_1 \quad \gamma_2]^T \tag{90}$$

$$\gamma_1 = \int_a^b \frac{\omega(s)ds}{b-a} \tag{91}$$

$$\gamma_2 = \int_a^b \frac{\omega(s)(b-s)ds}{(b-a)^2} \tag{92}$$

$$H = \begin{bmatrix} I & -I & 0 & 0 \\ I & I & -2I & 0 \\ I & -I & -6I & 12I \end{bmatrix} \tag{93}$$

B. lemma2

For any real matrices $Z = Z^T, H$ and E with appropriate dimensions, if the inequality $Z + HFE + E^T F^T H^T < 0$ is true, for anymore F that satisfies $F^T F \leq I$, we can obtain the following inequality when a scalar $\lambda > 0$ exists $Z + \lambda HH^T + \lambda^{-1} E^T E < 0$

Proof

Consider $H^T = FE$ and $\lambda = 1$, therefore we have $Z + HH^T + E^T(F^T F)E < 0$. Since $F^T F < I$, it implies $Z + HH^T + E^T E < 0$

Notation To be Used

$$\theta_0(s) = [\frac{\mathrm{d}x^T(s)}{\mathrm{d}t} x^T(s)]^T \quad \widetilde{h}(t) = h - h(t) \tag{94}$$

$$\tag{95}$$

$$v_i = \int_{t-h(t)}^{t} \frac{(t-s)^{i-1} x^T(s)}{h^i(t)} ds \tag{96}$$

$$\tag{97}$$

$$\omega_i = \int_{t-h}^{t-h(t)} \frac{[(t-h(t)-s)]^{i-1} x^T(s)}{\widetilde{h}^i(t)} ds \quad i = 1, 2 \tag{98}$$

$$\tag{99}$$

$$\varkappa_1 = \int_{t-h}^{t} x^T(s) ds \quad \varkappa_2 = \int_{t-h}^{t} (t-s) x^T(s) ds \tag{100}$$

$$\tag{101}$$

$$\eta_0(t) = [x^T(t) \quad x^T(t-h)]^T] \tag{102}$$

$$\tag{103}$$

$$\eta_1(t) = [x^T(t) \quad x^T(t-h(t)) \quad x^T(t-h)]^T \tag{104}$$

$$\tag{105}$$

$$\eta_2(t) = [v_1 \quad v_2 \quad \omega_1 \quad \omega_2]^T \tag{106}$$

$$\tag{107}$$

$$\eta_3(t) = [h(t)v_1 \quad h(t)v_2 \quad \widetilde{h}(t)\omega_1 \quad \widetilde{h}(t)\omega_2]^T \tag{108}$$

$$\tag{109}$$

$$\eta_4(t) = [\frac{\mathrm{d}x^T}{\mathrm{d}t}(t-h) \quad \frac{\mathrm{d}x^T}{\mathrm{d}t}]^T \tag{110}$$

$$\tag{111}$$

$$\eta(t) = [\eta_0^T(t) \quad \varkappa_1 \quad \varkappa_2]^T \tag{112}$$

$$\tag{113}$$

$$\theta(t,s) = [\theta_0^T(s) \quad \eta_0^T(t) \quad \int_{t-h}^{s} x^T(\theta) d\theta]^T \tag{114}$$

$$\tag{115}$$

$$\xi(t) = [\eta_1^T(t) \quad \eta_2^T(t) \quad \eta_3^T(t) \quad \eta_4^T]^T \tag{116}$$

$$\tag{117}$$

$$e_i = [O_{n(i-1)n} \quad I_n \quad O_{n(13-i)n}] \quad i = 1, 2, ..., 13 \tag{118}$$

$$\tag{119}$$

$$\Pi_0 = [h(t)(e_9^T + e_{10}^T) + \widetilde{h}(t) e_{11}^T]^T \tag{120}$$

$$\Pi_1 = [e_1^T \quad e_3^T \quad e_8^T + e_{10}^T \quad \Pi_0^T]^T \tag{121}$$

$$\tag{122}$$

$$\Pi_2 = [e_{13}^T \quad e_1^T - e_3^T \quad e_{12}^T \quad e_8^T + e_{10}^T \quad - he_3^T]^T \tag{123}$$

$$\tag{124}$$

$$\Pi_3 = [e_{13}^T \quad e_1^T \quad e_1^T \quad e_3^T + e_8^T + e_{10}^T]^T \tag{125}$$

$$\tag{126}$$

$$\Pi_4 = [e_{12}^T \quad e_3^T \quad e_1^T \quad e_3^T \quad 0]^T \tag{127}$$

$$\tag{128}$$

$$\Pi_5 = [e_1^T - e_3^T \quad e_8^T + e_{10}^T \quad he_1^T \quad he_3^T \quad \Pi_0^T]^T \tag{129}$$

$$\tag{130}$$

$$\Pi_6 = [0 \quad 0 \quad e_{13}^T \quad e_{12}^T \quad - e_3^T]^T \tag{131}$$

$$\tag{132}$$

$$\Pi_{7a} = [e_1^T \quad e_2^T \quad e_4^T \quad e_5^T]^T \tag{133}$$

$$\tag{134}$$

$$\Pi_7 = [\Pi \quad \Pi_{7a}] \tag{135}$$

$$\tag{136}$$

$$\Pi_{8a} = [e_2^T \quad e_3^T \quad e_6^T \quad e_7^T]^T \tag{137}$$

$$\tag{138}$$

$$\Pi_8 = [\Pi \quad \Pi_{8a}] \tag{139}$$

$$\tag{140}$$

$$\Pi_{9a} = [e_8^T - h(t)e_4^T \quad e_9^T - h(t)e_5^T]^T \tag{141}$$

$$\tag{142}$$

$$\Pi_{9b} = [e_{10}^T - \widetilde{h}(t)e_6^T \quad e_{11}^T - \widetilde{h}(t)e_7^T]^T \tag{143}$$

$$\tag{144}$$

$$\Pi_9 = [\Pi_{9a}^T \quad \Pi_{9b}^T]^T \tag{145}$$

$$\tag{146}$$

$$\Pi_{10} = [e_{13}^T X_1 + e_1^T X_2 + e_2^T X_3]^T \tag{147}$$

$$\tag{148}$$

$$\Pi_{11} = [e_1^T A_0^T + e_2^T A_d^T - e_{13}^T]^T \tag{149}$$

$$\tag{150}$$

$$\theta_1 = [\Pi_{10}^T \Pi]^T \tag{151}$$

$$\tag{152}$$

$$\theta_2 = [e_1^T E_1^T + e_2^T E_2^T]^T \tag{153}$$

$$\tag{154}$$

$$\Phi_1 = Sym\{\Pi_1^T P \Pi_2 + \Pi_5^T Q \Pi_6\} \tag{155}$$

$$\tag{156}$$

$$+ \Pi_3^T Q \Pi_3 - \Pi_4^T Q \Pi_4 + h \frac{\mathrm{d}x^T(t)}{\mathrm{d}t} R \frac{\mathrm{d}x}{\mathrm{d}t} \tag{157}$$

$$\tag{158}$$

$$\Phi_2 = Sym\{\Pi_{10}^T \Pi_{11} + N\Pi_9 + \Pi_7^T Y_1 + \Pi_8^T Y_2\} \tag{159}$$

$$\tag{160}$$

$$\Phi_3 = h(t) Y_1^T \widetilde{R}^{-1} Y_1 + \widetilde{h}(t) Y_2^T \widetilde{R}^{-1} Y_2 \tag{161}$$

$$\widetilde{R} = diag\{R \quad 3R \quad 5R\}$$

V. ROBUST STABILITY

Theorem1: For a scalar $\lambda > 0$, if there exist real symmetric matrices $P(\in R^{4n*4n}) > 0$, $Q(\in R^{5n*5n}) > 0$,$R(\in R^{n*n}) >$

0 any real matrices Y_1, Y_2, X_1, X_2, X_3 and N with appropriate dimensions, the system is stable if the following LMIs and are satisfied for $0 \leqslant h(t) \leqslant h$

$$\begin{bmatrix} \phi(0) & \theta_1^T & hY_2^T \\ * & -\lambda I & 0 \\ * & * & -h\bar{R} \end{bmatrix} < 0 \tag{162}$$

$$\begin{bmatrix} \phi(h) & \theta_1^T & hY_1^T \\ * & -\lambda I & 0 \\ * & * & -h\bar{R} \end{bmatrix} < 0 \tag{163}$$

$$\tag{164}$$

where $\phi(h(t)) = \phi_{+}\phi_2 + \lambda\theta_2^T\theta_2$

proof From lyapunov theory, an LK functional can be constructed as follows

$$V(x_t) = \eta^T(t)P\eta(t) + \int_t -h^t\theta^T(t,s)Q\theta(t,s)ds \tag{165}$$

$$+\int_{-h}^{0}\int_{t+\theta}^{t}\frac{\mathrm{d}x^T(s)}{\mathrm{d}t}R\frac{\mathrm{d}x^T}{\mathrm{d}t}dsd\theta \tag{166}$$

when $P > 0$ $Q > 0$ and $R > 0$, the LK functional is positive. Therefore,

$$\frac{\mathrm{d}V(x_t)}{\mathrm{d}t} = \xi^T(t)\Phi_1\xi(t) - \int_{t-h}^{t}\frac{\mathrm{d}x^T(s)}{\mathrm{d}t}R\frac{\mathrm{d}x(s)}{\mathrm{d}s}ds \tag{167}$$

Then

$$-\int_{t-h}^{t-h(t)}\frac{\mathrm{d}x^T(s)}{\mathrm{d}s}R\frac{\mathrm{d}x(s)}{\mathrm{d}s}ds = \tag{168}$$

$$-\int_{t-h}^{t-h(t)}\frac{\mathrm{d}x^T(s)}{\mathrm{d}s}R\frac{\mathrm{d}x(s)}{\mathrm{d}s}ds - \int_{t-h(t)}^{t}\frac{\mathrm{d}x^T(s)}{\mathrm{d}s}R\frac{\mathrm{d}x(s)}{\mathrm{d}s}ds \tag{169}$$

On applying Lemma 1, we can obtain

$$-\int_{t-h(t)}^{t}\frac{\mathrm{d}x^T(s)}{\mathrm{d}s}R\frac{\mathrm{d}x(s)}{\mathrm{d}s}ds \leqslant \tag{170}$$

$$\xi^T(t)(Sym\Pi_7^TY_1 + h(t)Y_1^T\tilde{\mathrm{R}}^{-1}Y_1)\xi(t) \tag{171}$$

$$-\int_{t-h}^{t-h(t)}\frac{\mathrm{d}x^T(s)}{\mathrm{d}s}R\frac{\mathrm{d}x(s)}{\mathrm{d}s}ds \leqslant \tag{172}$$

$$\xi^T(t)(Sym\Pi_8^TY_2 + \tilde{\mathrm{h}}(\mathrm{t})Y_2^T\tilde{\mathrm{R}}^{-1}Y_2)\xi(t) \tag{173}$$

For any real matrix N with compatible dimensions, the following equation holds

$$2\xi^T(t)N\Pi_9\xi(t) = 0 \tag{174}$$

Considering any real matrices i X (i=1, 2, 3) with compatible dimensions, the following equation holds

$$0 = \begin{bmatrix} -\dfrac{\mathrm{d}x(t)}{\mathrm{d}t} + (A_0 + \Delta A_0)x(t) \\ +(A_d + \Delta A_d)x^T(t - h(t)) \end{bmatrix} * \tag{175}$$

$$2\left[\frac{\mathrm{d}x^T(t)}{\mathrm{d}t}X_1 + x^T(t)X_2 + x^T(t - h(t))X_3\right] \tag{176}$$

From

$$[\Delta A_0 \quad \Delta A_d] = HF(t)[E_1 \quad E_2] \tag{177}$$

, we can have formula (22) transformed into

$$\xi^T(t)(Sym\Pi_{10}^T\Pi_{11} + \theta_1^TF(t)\theta_2)\xi(t) = 0 \tag{178}$$

Combining formulas (17)-(23), we can easily obtain

$$\frac{\mathrm{d}V(x_t)}{\mathrm{d}t} \leqslant \xi^T(t)\bar{\Phi}\xi(t) \tag{179}$$

where

$$\bar{\Phi} = \Phi_0 + \theta_1^TF(t)\theta_2 + \theta_2^T(t)F(t)\theta_1 \tag{180}$$

If

$$\bar{\Phi} = \Phi_0 + \theta_1^TF(t)\theta_2 + \theta_2^T(t)F(t)\theta_1 < 0 \tag{181}$$

is true, and by employing Lemma 2 for a scalar λ is greater than 0, we can obtain

$$\Phi_0 + \lambda^-1\theta_1^T\theta_1 + \lambda\theta_2^T\theta_2 < 0 \tag{182}$$

The inequality (108) is equivalent to the inequalities (88) and (89) by applying the Schur theorem. Therefore, if the inequalities (88) and (89) are true, we can hold

$$\frac{\mathrm{d}V(t)}{\mathrm{d}t} < 0 \tag{183}$$

. Then, the system (80) is stable from Lyapunov stability theory.

Now, we can assume that the time-delay is constant, that is

$$\frac{\mathrm{d}h(t)}{\mathrm{d}t} = 0 \tag{184}$$

The system (80) is then transformed into the following system

$$\frac{\mathrm{d}x}{\mathrm{d}t} = (A_0 + \Delta A_0)x(t) + (A_d + \Delta A_d)(x(t) - \bar{h})), \tag{185}$$

where $\bar{h}$ represents a constant time-delay that satisfies

$$0 \leqslant \bar{h} \leqslant h \tag{186}$$

, and other parameters are the same as system (80). We choose the LK functional to be

$$\bar{V}(t) = \bar{\eta}^T(t)\bar{P}\bar{\eta}(t) + \int_{t-h}^{t}\bar{\theta}^T(t,s)\bar{Q}\bar{\theta}(t,s)ds \tag{187}$$

$$+\int_{-h}^{0}\int_{t+\theta}^{t}\frac{\mathrm{d}x}{\mathrm{d}s}^T(s)\bar{R}\frac{\mathrm{d}x}{\mathrm{d}s}(s)dsd\theta \tag{188}$$

where $\bar{\eta}(t) = [\bar{e}_1^T \quad \chi_1 \quad \chi_2]^T$

$$\bar{\theta}(t,s) = \begin{bmatrix} \dfrac{\mathrm{d}x}{\mathrm{d}s}^T(s) \\ \eta_0^T(t) \\ \int_{t-h}^{s}x^T(\theta)d\theta \end{bmatrix} \tag{189}$$

$$\bar{\xi}(t) = \begin{bmatrix} \eta_0^T(t) \\ \frac{\chi_1}{h} \\ \frac{\chi_1}{h^2} \\ \frac{\mathrm{d}x}{\mathrm{d}t}^T(t-h) \\ \frac{\mathrm{d}x}{\mathrm{d}t}^T(t) \end{bmatrix} \tag{190}$$

$$\bar{e}_i = \begin{bmatrix} 0_n * (i-1)n & I_n & 0_n * (6-i)n \end{bmatrix} i = 1, 2, ..., 6 \tag{191}$$

On applying the proposed method, we can derive the following criterion for this functional:

Corollary 1: For a scalar $\bar{\lambda} > 0$, if there exist real symmetric matrices $\bar{P} \in R^{4n*4n}) > 0$, $\bar{Q}(\in R^{4n*4n}) > 0$, $\bar{R}(\in R^{n*n}) > 0$, and any real matrices $\bar{Y}, \bar{X}_1, \bar{X}_2, and \bar{X}_3$. with appropriate dimensions, the system (111) is stable if the following LMI (121) is satisfied $0 \leqslant \bar{h} \leqslant h$

$$\begin{bmatrix} \bar{\Phi} & \bar{\theta}_1^T & h\bar{Y}_1^T \\ * & -\lambda I & 0 \\ * & * & -h\bar{R} \end{bmatrix} < 0 \tag{192}$$

where

$$\bar{R}1 = diag[\bar{R} \quad 3\bar{R} \quad 5\bar{R}] \quad \bar{\Phi} = \bar{\Phi}_1 + \bar{\Phi}_2 \tag{193}$$

$$\tag{194}$$

$$\bar{\Phi}_1 = Sym[\bar{\Pi}_1^T \bar{P} \bar{\Pi}_2 + \bar{\Pi}_5^T \bar{Q} \bar{\Pi}_6 + \bar{\Pi}_7^T \bar{Y} + \bar{\Pi}_8^T \bar{\Pi}_9] \tag{195}$$

$$\tag{196}$$

$$\bar{\Phi}_2 = \bar{\Pi}_3^T \bar{Q} \bar{\Pi}_3 - \bar{\Pi}_4^T \bar{Q} \bar{\Pi}_4 + h \frac{\mathrm{d}x}{\mathrm{d}t}^T(t) \bar{R} \frac{\mathrm{d}x}{\mathrm{d}t}(t) + \bar{\lambda} \theta_2^T \theta_2 \tag{197}$$

$$\bar{\Pi}_1 = [\bar{e}_1^T \quad h\bar{e}_3^T \quad h^2\bar{e}_4^T]^T \tag{198}$$

$$\tag{199}$$

$$\bar{\Pi}_2 = [\bar{e}_6^T \quad \bar{e}_1^T \quad -\bar{e}_2^T \quad h\bar{e}_3^T \quad -h\bar{e}_2^T]^T \tag{200}$$

$$\tag{201}$$

$$\bar{\Pi}_3 = [\bar{e}_6^T \quad \bar{e}_1^T \quad \bar{e}_2^T \quad h\bar{e}_3^T]^T \tag{202}$$

$$\tag{203}$$

$$\bar{\Pi}_4 = [\bar{e}_5^T \quad \bar{e}_1^T \quad \bar{e}_2^T \quad 0]^T \tag{204}$$

$$\tag{205}$$

$$\bar{\Pi}_5 = [\bar{e}_1^T \quad -\bar{e}_2^T \quad h\bar{e}_1^T \quad h\bar{e}_2^T \quad h^2\bar{e}_4^T]^T \tag{206}$$

$$\tag{207}$$

$$\bar{\Pi}_6 = [0 \quad \bar{e}_6^T \quad \bar{e}_5^T \quad -\bar{e}_2^T]^T \tag{208}$$

$$\tag{209}$$

$$\bar{\Pi}_{7a} = [\bar{e}_1^T \quad \bar{e}_2^T \quad \bar{e}_3^T \quad \bar{e}_4^T]^T \tag{210}$$

$$\tag{211}$$

$$\bar{\Pi}_7 = \Pi\bar{\Pi}_{7a} \tag{212}$$

$$\tag{213}$$

$$\bar{\Pi}_8 = [\bar{e}_6^T \bar{X}_1 + \bar{e}_1^T \bar{X}_2]^T + \bar{e}_2^T \bar{X}_3 \tag{214}$$

$$\tag{215}$$

$$\bar{\Pi}_9 = [\bar{e}_1^T \bar{A}_0^T + \bar{e}_2^T \bar{A}_d^T - \bar{e}_6^T]^T \tag{216}$$

The remaining elements and the calculation process are the same as those of Theorem 1.

VI. Case Analysis

1) Analysed the given power system by deriving the state space equation and performed controllability analysis on the system.
2) Analysed the delay in the power systems and tried to increase the upper bound of time delay stability margin.
3) Studied the effect of introducing feedback in the form of PID controller in the output to eliminate area control error.
4) Derived the new LK functional for the power system using Lyapunov theory.

T_T	T_G	R	D	M
0.3	0.1	0.05	1.0	10

Fig. 2. System Parameters

Consider the two controller parameters

$$K_1 : [K_P \quad K_I \quad K_D] = [-0.1000, \quad 0.0668, \quad 0.0531] \tag{217}$$

$$K_2 : [K_P \quad K_I \quad K_D] = [-0.4036, \quad 0.6356, \quad 0.1832] \tag{218}$$

When we set $\alpha = 2$ and $\gamma \in [0, 4]$,the random time-delay stability margins in different disturbances of different methods are as listed in Table 2. Figure 2 and Figure 3 intuitively show

γ	Controller K_1		Controller K_2	
	Ref. [12]	Theorem 1	Ref. [12]	Theorem 1
0	2.777 2	3.386 7	0.607 5	0.627 2
0.5	2.579 7	3.152 5	0.580 9	0.601 7
1.0	2.067 5	2.510 8	0.492 8	0.533 7
1.5	1.477 9	1.632 3	0.357 4	0.392 1
2.0	1.024 1	1.068 6	0.265 2	0.288 8
2.5	0.712 2	0.736 9	0.197 5	0.213 0
3.0	0.481 5	0.494 1	0.141 1	0.150 6
3.5	0.285 4	0.293 7	0.083 4	0.087 9
4.0	0.033 0	0.033 0	—	—

Fig. 3. System Stability Margin for Different methods

the random time-delay stability margins of the perturbed state obtained by Theorem 1 under different controllers. Through Table 2, Figure 4 and Figure 5, the following conclusions can be clearly obtained:

1) When the same method is used, the usage of different controllers gives rise to significantly different time-delay stability margins.
2) The system time-delay stability margin obtained by the proposed method is obviously superior to that in Ref. [12].

To further verify the correctness of the proposed method, Corollary 1 is applied to calculate the constant time-delay stability margin that the system can withstand when the controller is K2, corresponding to 1.290 6 s.

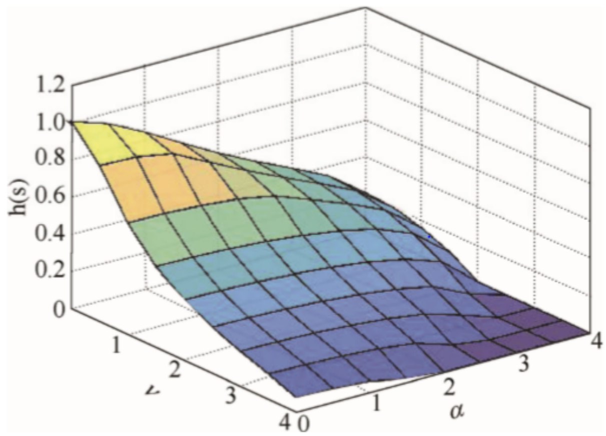

Fig. 4. System Stability Margin for controller K1

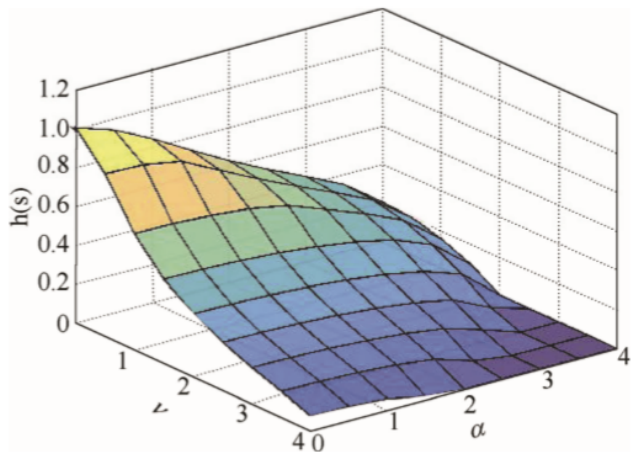

Fig. 5. System Stability Margin for controller K2

If we assume the load in the region increases by 0.01 pu at 10 s, i.e. $\Delta P_d = 0.01$ pu , then the response curve of the system with different time delays can be derived as shown in Figure 6.

If the load increases at the 10 s, the following frequency deviation response is obtained, as shown in Fig. 4. When the time delay is not considered, the frequency deviation converts to zero by the primary frequency modulation of the speed regulating system and the secondary frequency modulation of LFC, and the grid frequency returns to the regulated value. When the system time delay is 1.29 s, the response time of the system frequency deviation increases, which indicates that the existence of a time delay has an impact on the system stability, although it tends to be stable. When the time delay of the system is 1.30 s, the system diverges and is no longer stable. Therefore, it can be concluded that the time-delay stability margin that the system can withstand is within the interval [1.29 s, 1.30 s], and the stability margin 1.290 6 s obtained by the proposed method is within this interval, indicating the correctness of the proposed method.

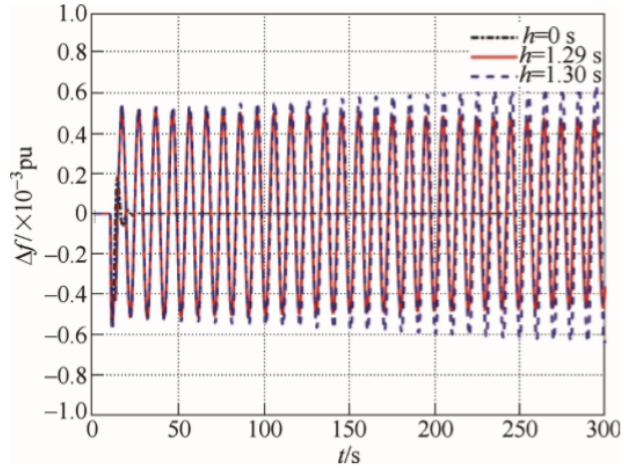

Fig. 6. Frequency deviation response of different time delays

VII. Conclusion

The time-delay robust stability of a power system with uncertain parameters with PID load frequency control was analyzed in this study. New LK functional derived from the Lyapunov theory was put to test by comparing the results with the existing method used to obtain stability margin.

VIII. Summary

Concepts learned

1) Knowledge on Power Systems - Superficial understanding on power systems
2) Controls with Power - Concepts for LTI was applied for controls component(PID)
3) LK functional -We didn't finish digesting the paper on LK functional sent to us by Prof Sesan as we were really pressed for time

IX. Discussion

We made an attempt to use the LMI toolbox, but due to time constraints and complexity of the equation, the full verification of the above model was not possible.

Acknowledgment

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Appendix A
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