Q1.1

Q1.1 Given for point P, $x_1 = x_2 = [0, 0, 1]^T$

By the fundamental matrix relation,

$$\tilde{x}_2^T F \hat{x}_1 = 0$$

Expanded

$$[0\ 0\ 1]\begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0 \quad \therefore$$

Solving, we get

$\boxed{F_{33} = 0}$ as desired.

Q1.2

Q1.2 Given translation is parallel to the x-axis, we can denote the translation matrix

$$t = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} = \begin{bmatrix} t_x \\ 0 \\ 0 \end{bmatrix}$$

The cross product matrix can be written as

$$t_\times = \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix}$$

Pure Rotation means

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Essential Matrix

$$E = t_\times R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix}$$

Epipolar Lines are then

$$l_1^T = \tilde{x}_2^T E = [x_2 \ y_2 \ 1] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix} = [0 \ t_x \ -t_x y_2]$$

$$l_2^T = \tilde{x}_1^T E = [x_1 \ y_1 \ 1] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix} = [0 \ -t_x \ t_x y_1]$$

$\therefore$ Equation for line 1 is

$$t_x y - t_x y_2 = 0$$

Equation for line 2 is

$$t_x y - t_x y_1 = 0$$

Both are parallel to the x-axis, as desired #

Q1.3

Q1.3 Let $[u, v, w]^T$ be the coordinate of the object in 3D

$[x_i\ y_i]^T$ be position at time i

$$\begin{bmatrix}x_1\\y_1\\1\end{bmatrix} = K\left(R_1\begin{bmatrix}u\\v\\w\end{bmatrix}+t_1\right)$$

$$\begin{bmatrix}u\\v\\w\end{bmatrix} = R^{-1}\left(K^{-1}\begin{bmatrix}x_1\\y_1\\1\end{bmatrix}-t_1\right) \quad \text{by rearranging}$$

$$= R_1^T K^{-1}\begin{bmatrix}x_1\\y_1\\1\end{bmatrix} - R_1^T t_1$$

$$\begin{bmatrix}x_2\\y_2\\1\end{bmatrix} = K\left(R_2\begin{bmatrix}u\\v\\w\end{bmatrix}+t_2\right)$$

$$= K\left(R_2\left(R_1^T K^{-1}\begin{bmatrix}x_1\\y_1\\1\end{bmatrix} - R_1^T t_1\right) + t_2\right)$$

$$= K R_2 R_1^T K^{-1}\begin{bmatrix}x_1\\y_1\\1\end{bmatrix} - K R_2 R_1^T t_1 + K t_2$$

$\therefore\ R_{Rel} = K R_2 R_1^T K^{-1}$

$t_{Rel} = -K R_2 R_1^T t_1 + K t_2$

$E = t_{Rel} \times R_{Rel}$

$F = (K^{-1})^T E K^{-1}$

$= (K^{-1})^T (t_{Rel} \times R_{Rel}) K^{-1}$

Q1.4

Q1.4 Given all points on object are equal distance to mirror, transformation is pure translation

Thus, $R_{rel} = I$

$t_{rel} = [t_x, t_y, t_z]$

From above, $F = (K^{-1})^T (t_{rel} \times R_{rel}) (K^{-1})$

$$= (K^{-1})^T \begin{bmatrix} 0 & t_z & -t_y \\ -t_z & 0 & t_x \\ t_y & -t_x & 0 \end{bmatrix} K^{-1}$$

vs $$F^T = (K^{-1})^T \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_z \\ -t_y & t_x & 0 \end{bmatrix} K^{-1}$$

$= -F$ #

$\therefore F$ is a skew-symmetric matrix

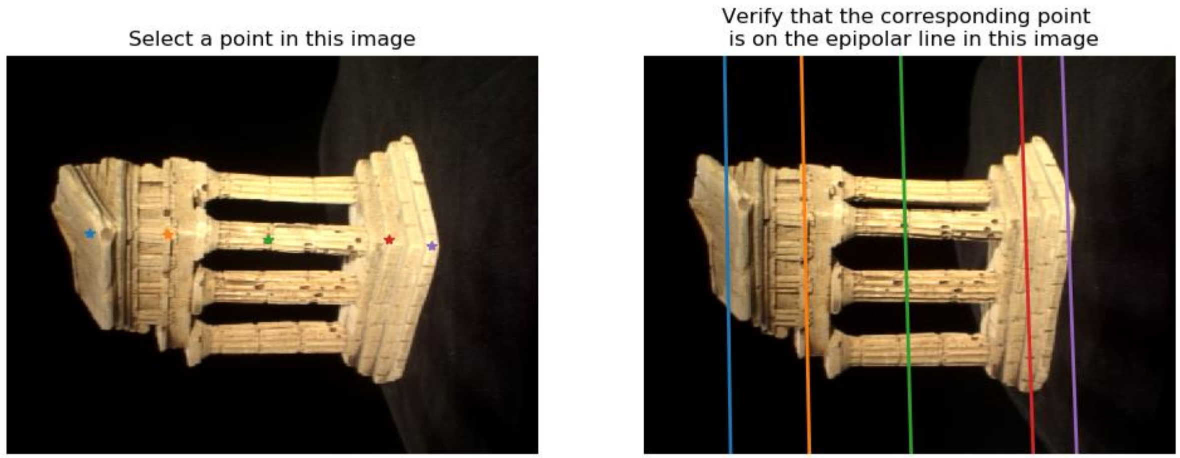
Q 2.1

F8 =

[[9.78833286e-10 -1.32135929e-07 1.12585666e-03]

 [-5.73843315e-08 2.96800276e-09 -1.17611996e-05]

[-1.08269003e-03 3.04846703e-05 -4.47032655e-03]]



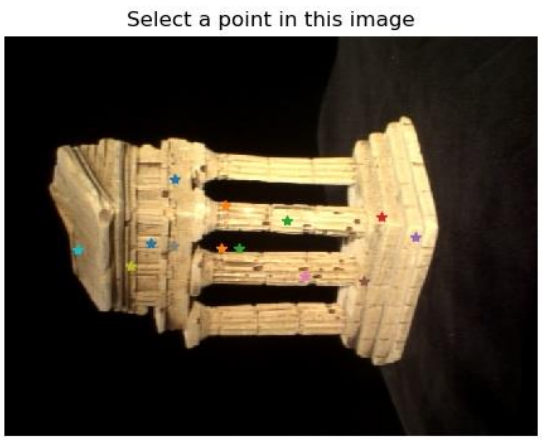
Q 2.2

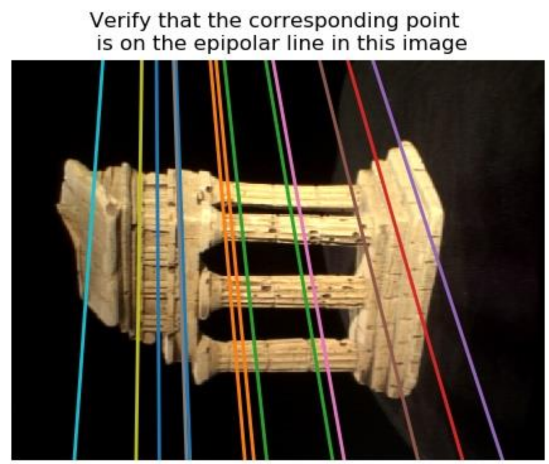
F7 =

[[-1.94078149e-07 7.96477251e-07 6.97294189e-04]

  [-7.90268545e-07 -9.25318810e-09 1.34406658e-04]

  [-6.14775991e-04 -1.40855499e-04 -6.59754289e-03]]





Q 3.1

Using F8 from eight point algorithm, K1 & K2 from intrinsics.npz

E=

```
[[ 2.26268684e-03 -3.06552495e-01  1.66260633e+00]
 [-1.33130407e-01  6.91061098e-03 -4.33003420e-02]
 [-1.66721070e+00 -1.33210351e-02 -6.72186431e-04]]
```

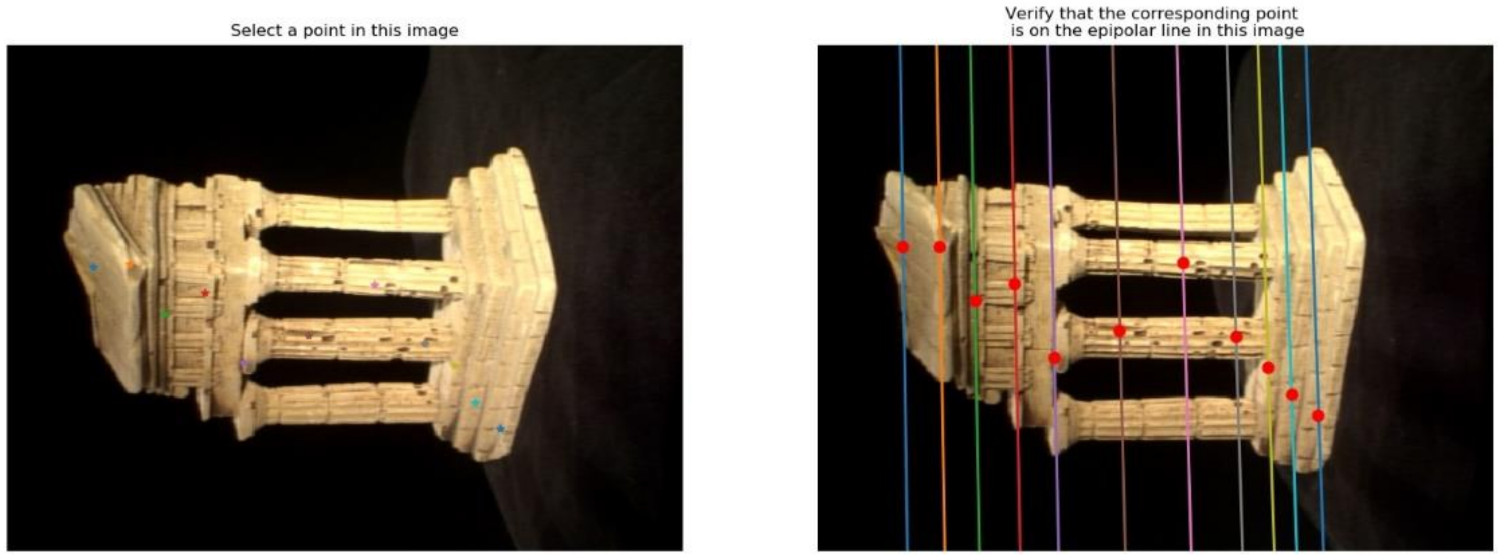
Q 3.2

Q3.2

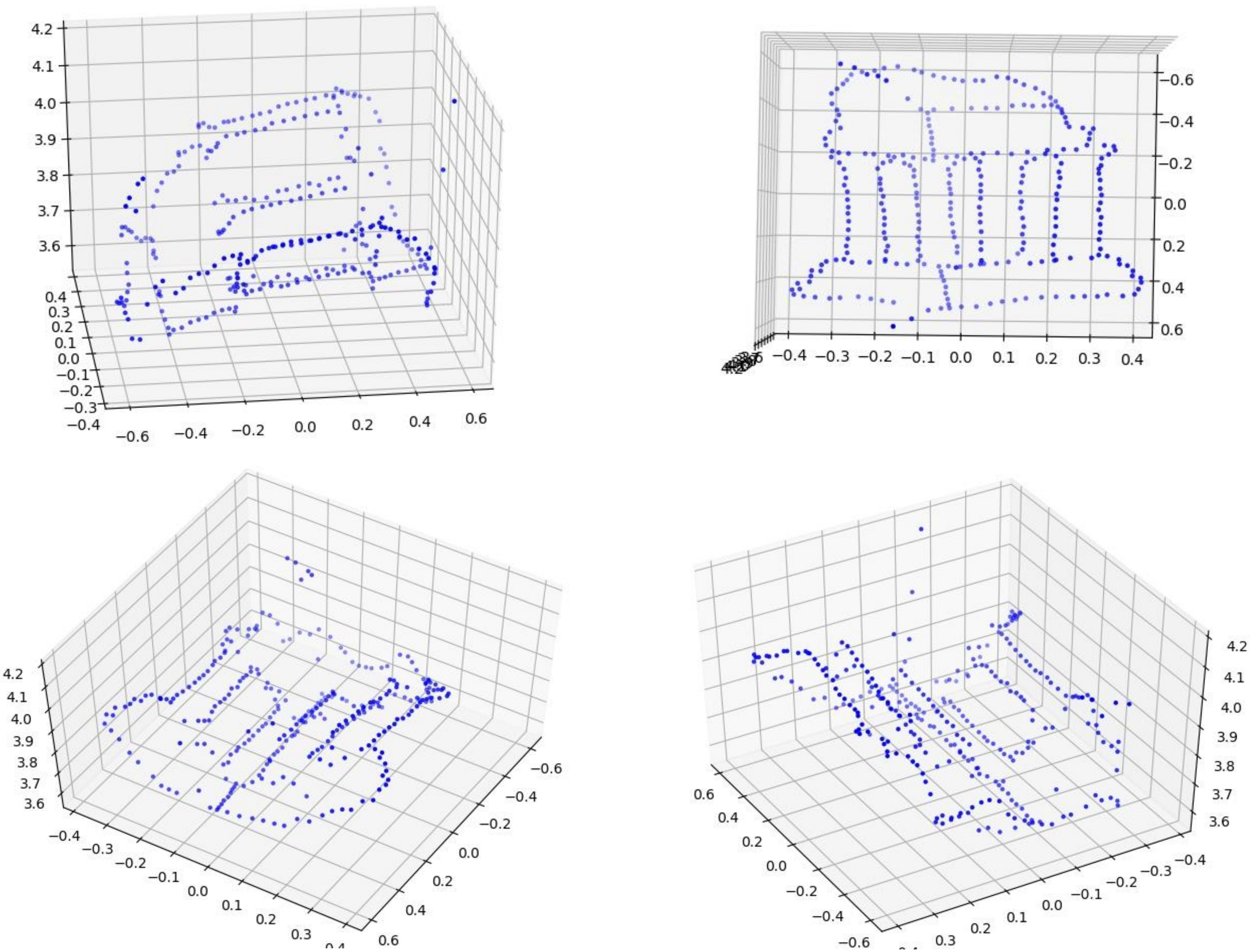
$$A_i w_i = 0$$

$$A_i = \begin{bmatrix} x_1[i] \times C1[2,:] - C1[0,:] \\ y_1[i] \times C1[2,:] - C1[1,:] \\ x_2[i] \times C2[2,:] - C2[0,:] \\ y_2[i] \times C2[2,:] - C2[1,:] \end{bmatrix}$$

Q 4.1


Select a point in this image
Verify that the corresponding point
is on the epipolar line in this image

Q 4.2



4.2 4.1 4.0 3.9 3.8 3.7 3.6
0.4 0.3 0.2 0.1 0.0 −0.1 −0.2 −0.3 −0.4
−0.6 −0.4 −0.2 0.0 0.2 0.4 0.6
−0.6 −0.4 −0.2 0.0 0.2 0.4 0.6
−0.4 −0.3 −0.2 −0.1 0.0 0.1 0.2 0.3 0.4
4.2 4.1 4.0 3.9 3.8 3.7 3.6
−0.4 −0.3 −0.2 −0.1 0.0 0.1 0.2 0.3
−0.6 −0.4 −0.2 0.0 0.2 0.4 0.6
4.2 4.1 4.0 3.9 3.8 3.7 3.6
0.6 0.4 0.2 0.0 −0.2 −0.4 −0.6
0.3 0.2 0.1 0.0 −0.1 −0.2 −0.3 −0.4