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Q1.1

Q1.1 Want to minimize $||I_{t+1}(x+p) - I_t(x)||^2$

Taylor Expansion $\approx ||I_{t+1}(x') + \frac{\delta I_{t+1}(x')}{\delta x'^T} \frac{\delta W(x;p)}{\delta p^T} \Delta p - I_t(x)||^2$

Given $\Delta p = \underset{\Delta p}{\arg\min} ||A\Delta p - b||^2$

② 
$$\boxed{\begin{aligned} A &= \frac{\delta I_{t+1}(x')}{\delta x'^T} \frac{\delta W(x;p)}{\delta p^T} \\ b &= I_t(x) - I_{t+1}(x') \end{aligned}}$$

Since $\Delta p$ has only translation components for $x$ & $y$,

① 
$$\boxed{\frac{\delta W(x;p)}{\delta p^T} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$$

③ Differentiating $||A\Delta p - b||^2$ to find $\underset{\Delta p}{\arg\min}$, we get

$$2(A\Delta p - b)^T A = 0$$

$\boxed{\therefore A^T A \text{ is invertible}}$ in order to obtain unique solution for $\Delta p$

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Q1.3





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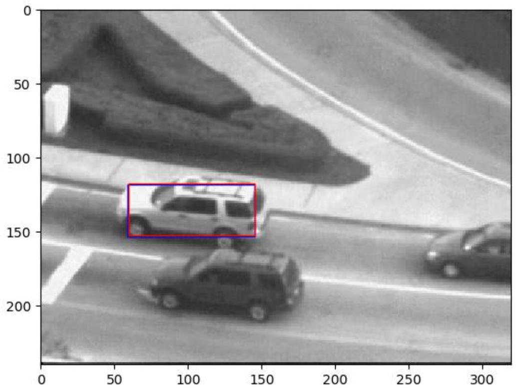
Q1.4



Blue: With Template Correction

Red: Without Template Correction

Q 2.1

Q2.1 $I_{t+1}(x) = I_t(x) + \sum_{k=1}^{K} w_k B_k(x) \Rightarrow \sum_{k=1}^{K} w_k B_k(x) + I_{t+1}(x) - I_t(x)$

$B_k(x)\left(w_1 B_1(x) + w_2 B_2(x) + \ldots + w_K B_K(x)\right) = \left(I_{t+1}(x) - I_t(x)\right) B_k(x)$

Using orthogonality

$w_1 0 + \ldots + w_k \|B_k(x)\|^2 + \ldots + w_K 0 = B_k(x)\left(I_{t+1}(x) - I_t(x)\right)$

Rearranging:

$$w_k = \frac{B_k(x)}{\|B_k(x)\|^2}\left(I_{t+1}(x) - I_t(x)\right)$$

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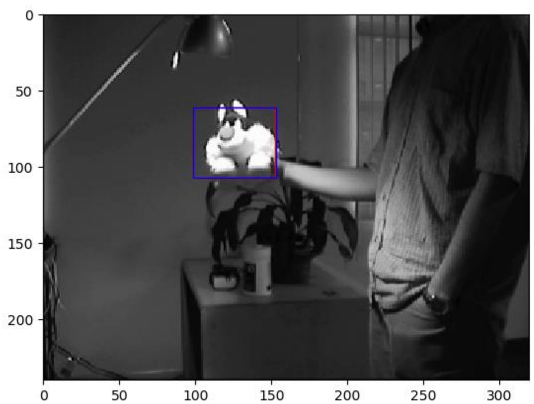
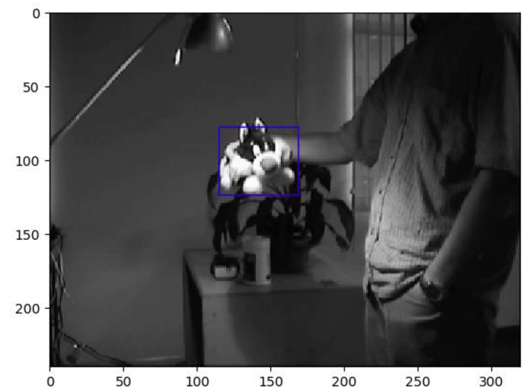
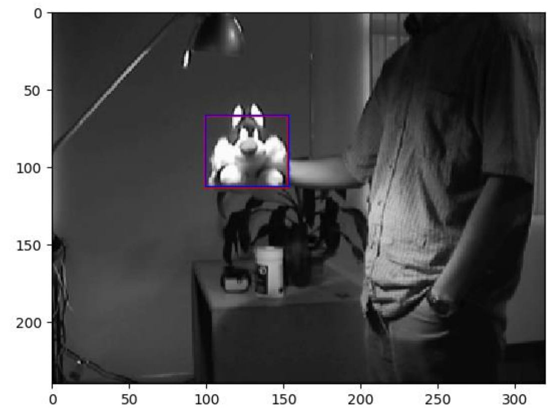
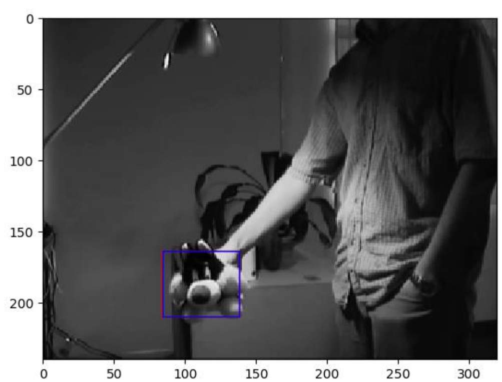
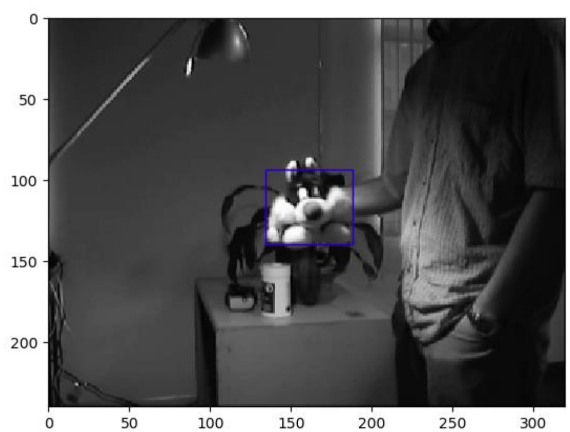
Q 2.3



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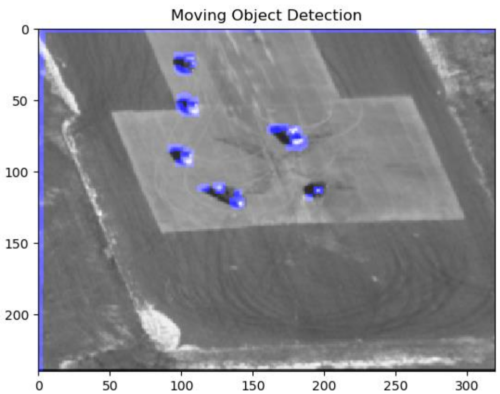
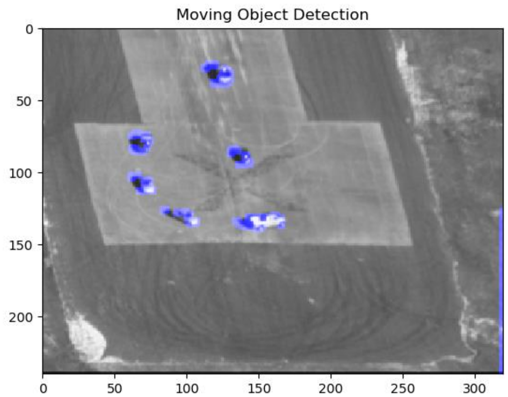
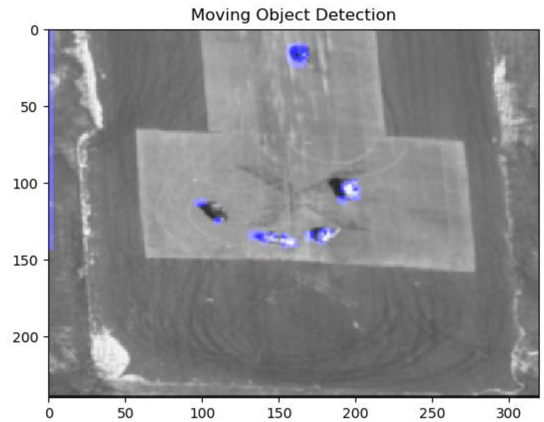
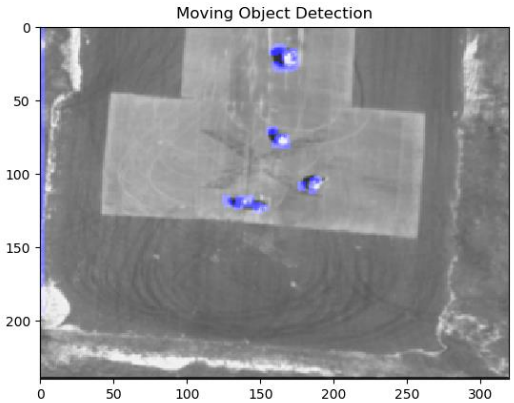
Q 3.3



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Q 4.1

The inverse compositional approach is more computationally efficient because it precomputes A and $(A^T A)^{-1} A^T$ for use until delta p converges, but for the classical approach, you need to calculate A every iteration, which is computationally more intensive.