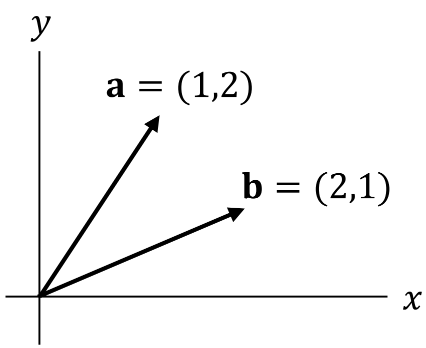
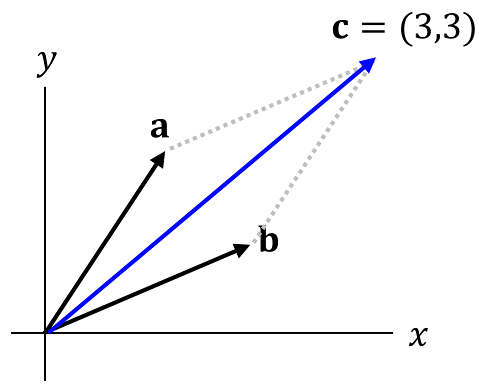
Vector

- A vector has **magnitude** and **direction**
- It is written in **bold**



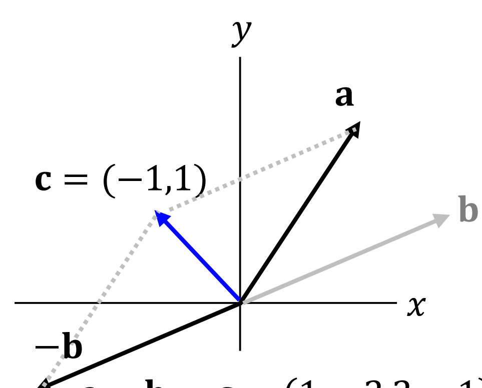
$$\mathbf{a} = (x, y)$$

Vector addition



$$\mathbf{a} + \mathbf{b} = \mathbf{c} = (1+2, 2+1) = (3,3)$$

Vector subtraction

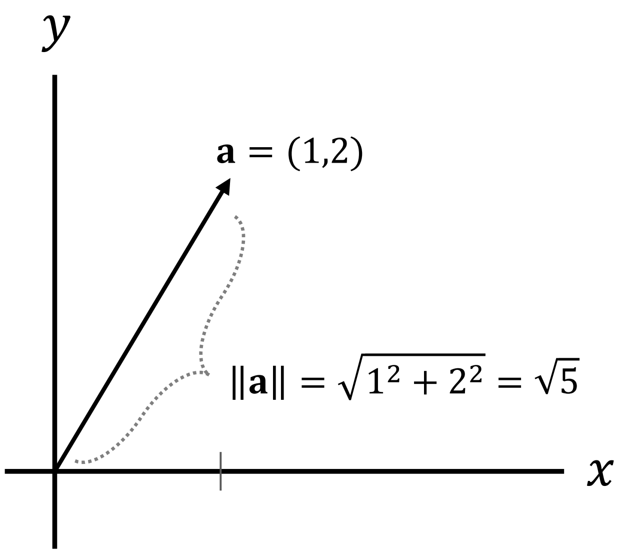


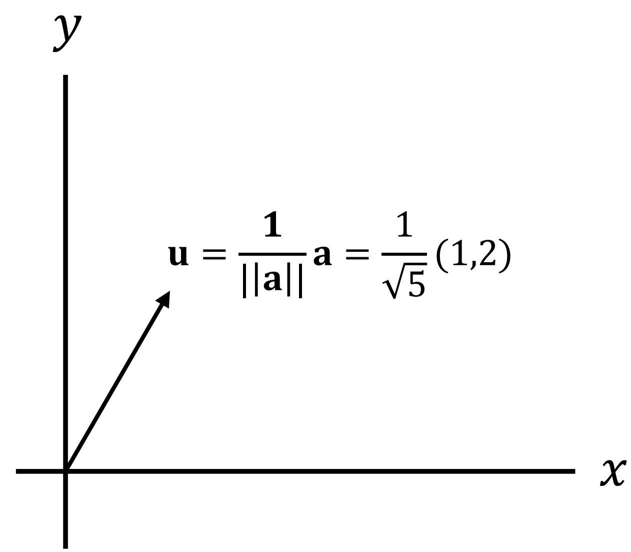
$$\mathbf{a} - \mathbf{b} = \mathbf{c} = (1-2, 2-1) = (-1,1)$$

Basic mathematics

Unit Vector

- A Vector has a **magnitude of 1**
- Only the direction of the unit vector is important





Unit vector $\mathbf{u}$ has the same direction with $\mathbf{a}$ but its magnitude is ‘1’

Basic mathematics

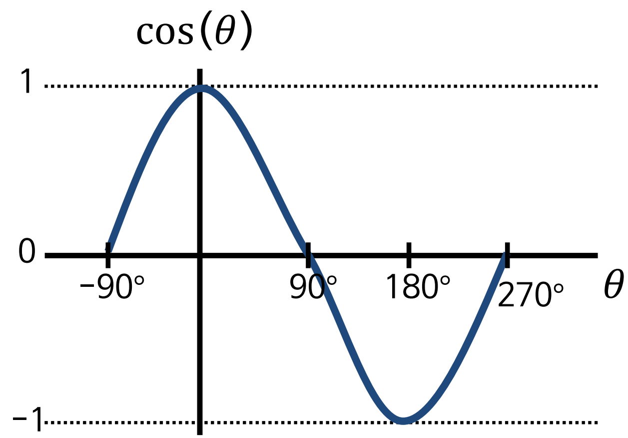
Inner Vector

- The inner product formula is

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2$$

$$\mathbf{a} = (a_1, a_2)$$

$$\mathbf{b} = (b_1, b_2)$$

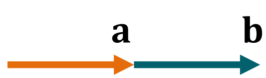


- Different form is
  (please note that $\cos\theta$)

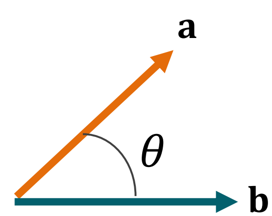
$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \|\mathbf{b}\| \cos\theta$$

$$\frac{\mathbf{a}}{\|\mathbf{a}\|} \cdot \frac{\mathbf{b}}{\|\mathbf{b}\|} = \cos\theta$$

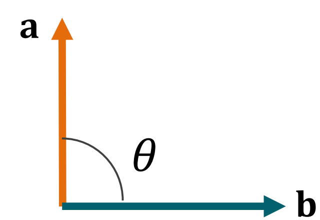
If the two vectors are unit vectors



$$\theta = 0$$



$$0 < \theta < \frac{\pi}{2}$$

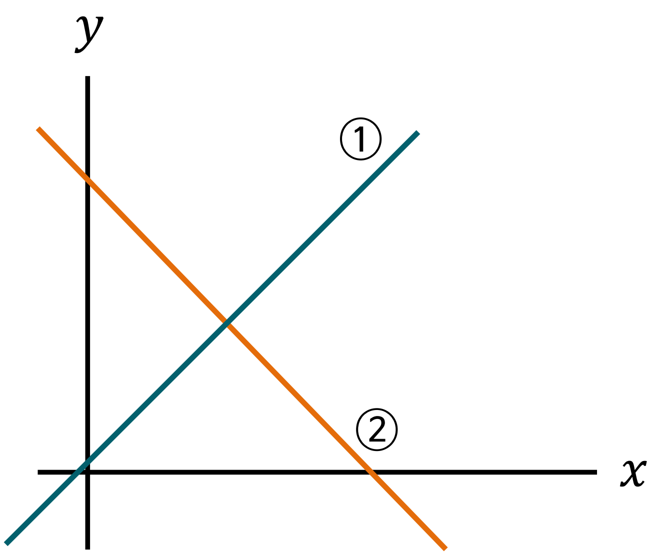


$$\theta = \frac{\pi}{2}$$

Basic mathematics

Matrix

- A set of vectors
- A set of equations



Euclidean (2-dimensional) space

<u>Different lines in Euclidean space</u>

$$x - y = 0 \quad \cdots\cdots \quad ①$$

$$x + y = 1 \quad \cdots\cdots \quad ②$$

$$\underbrace{\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}}_{\mathbf{A}} \underbrace{\begin{pmatrix} x \\ y \end{pmatrix}}_{\mathbf{x}} = \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{\mathbf{b}}$$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

Basic mathematics

Matrix

- Matrix addition

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{pmatrix}$$

- Matrix multiplication

Multiplication between a matrix and a vector

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} a_{11}b_1 + a_{12}b_2 \\ a_{21}b_1 + a_{22}b_2 \end{pmatrix}$$

Scalar multiplication

$$k \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \end{pmatrix}$$

Multiplication between two matrices

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{pmatrix}$$

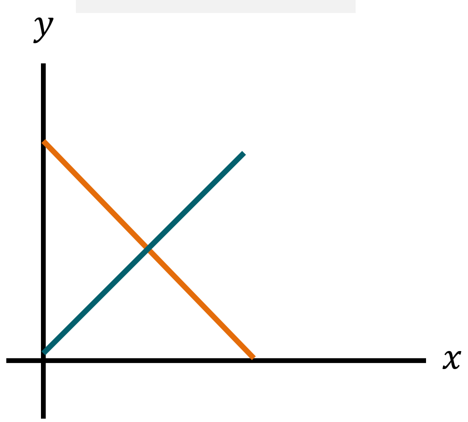
Basic mathematics

Matrix

- A condition that two equations will have a unique solution
  - if and only if determinant ≠ 0
  - otherwise, no solution or infinite solutions

Different slopes of the lines

$\det(\mathbf{A}) \neq 0$

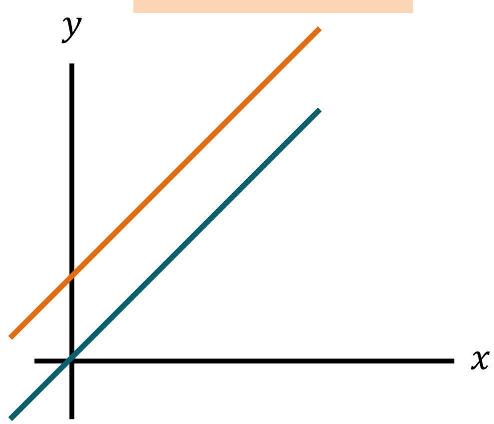


$x - y = 0$

$x + y = 0$

Same slopes of the lines

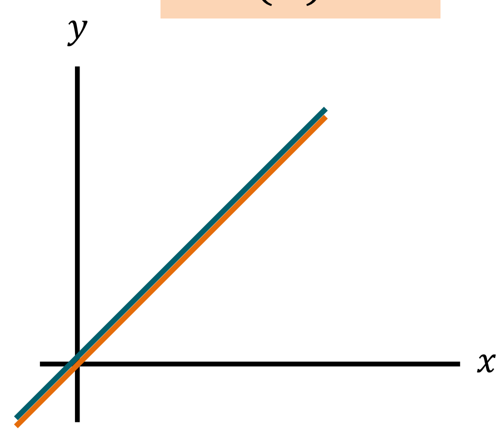
$\det(\mathbf{A}) = 0$



$x + y = 0$

$x + y = 1$

$\det(\mathbf{A}) = 0$



$x + y = 0$

$x + y = 0$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \qquad \det(\mathbf{A}) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

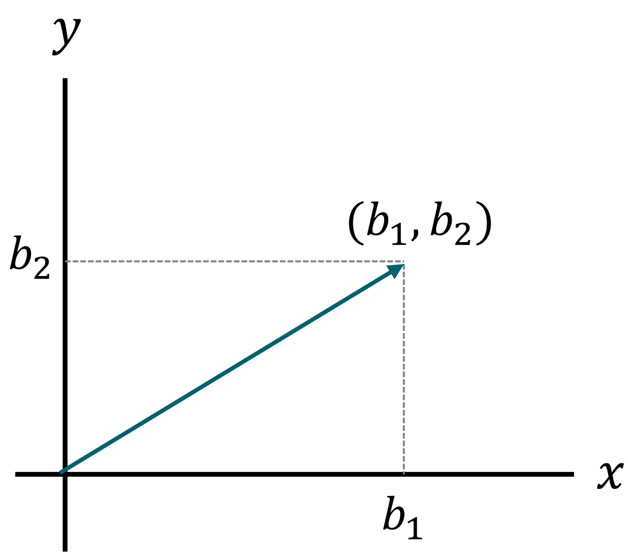
Matrix

- unit matrix $\mathbf{I}$ and Inverse Matrix $\mathbf{A}^{-1}$

Unit matrix

$$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \mathbf{Ix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

Each row indicates the axis of the space



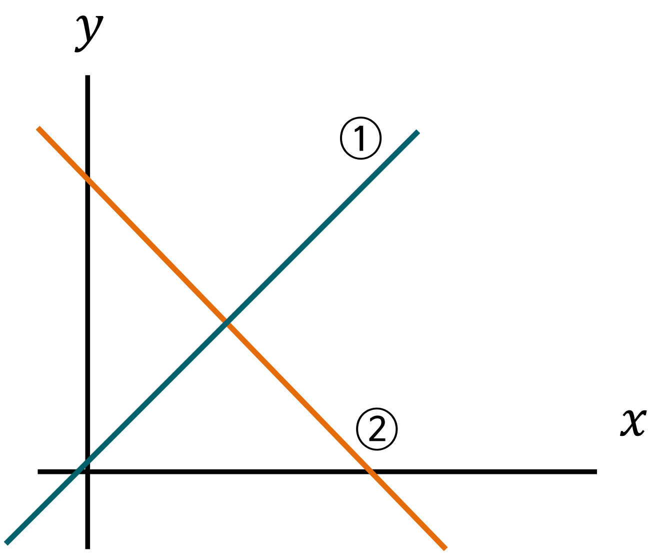
Inverse matrix

$$\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \qquad \mathbf{A}^{-1} = \frac{1}{\det(\mathbf{A})} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$\mathbf{AA}^{-1} = \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Matrix

- Find solutions of linear systems
- Fundamentals of machine learning



Euclidean space

Different lines in Euclidean space

$$x - y = 0 \quad \cdots\cdots \quad ①$$
$$x + y = 1 \quad \cdots\cdots \quad ②$$

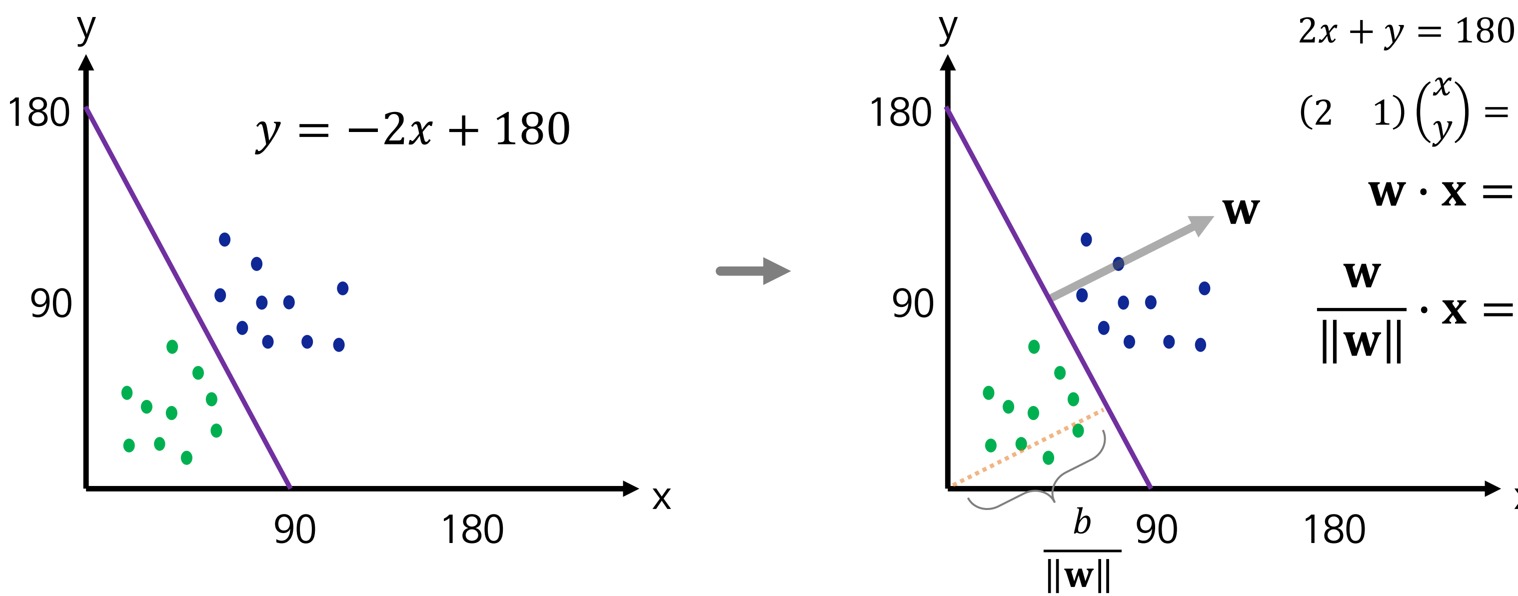
$$\underbrace{\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}}_{\mathbf{A}} \underbrace{\begin{pmatrix} x \\ y \end{pmatrix}}_{\mathbf{x}} = \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{\mathbf{b}}$$

$$\left.\begin{aligned} \mathbf{Ax} &= \mathbf{b} \\ \mathbf{A}^{-1}\mathbf{Ax} &= \mathbf{A}^{-1}\mathbf{b} \\ \mathbf{Ix} &= \mathbf{A}^{-1}\mathbf{b} \\ \mathbf{x} &= \mathbf{A}^{-1}\mathbf{b} \end{aligned}\right\}$$

Find a solution of linear system based on matrix operations

Vectors / Matrices in machine learning (classification)

- Training model and data are represented in vectors
  - To train a model, we utilize matrix and vector operations



$$2x + y = 180$$

$$\begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 180 \end{pmatrix}$$

$$\mathbf{w} \cdot \mathbf{x} = b$$

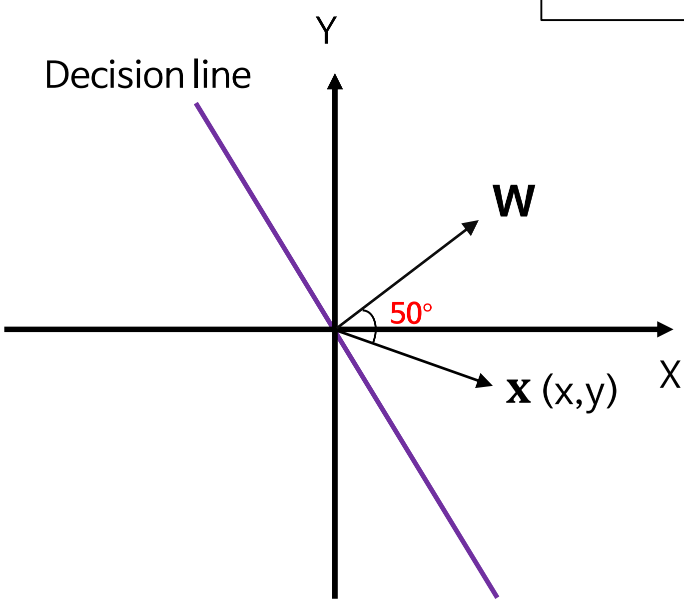
$$\frac{\mathbf{w}}{||\mathbf{w}||} \cdot \mathbf{x} = \frac{b}{||\mathbf{w}||}$$

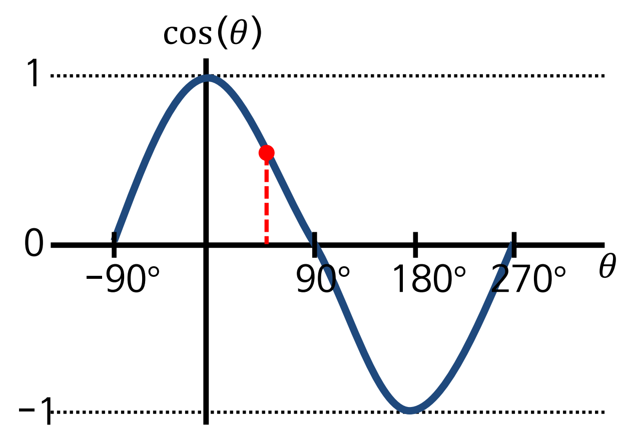
Vectors / Matrices in machine learning (classification)

- Training model and data are represented in vectors
  - To train a model, we utilize matrix and vector operations

$$\mathbf{w} \cdot \mathbf{x} = 0$$

$$\boxed{\mathbf{w} \cdot \mathbf{x} = \|\mathbf{w}\|\|\mathbf{x}\|\cos\theta}$$



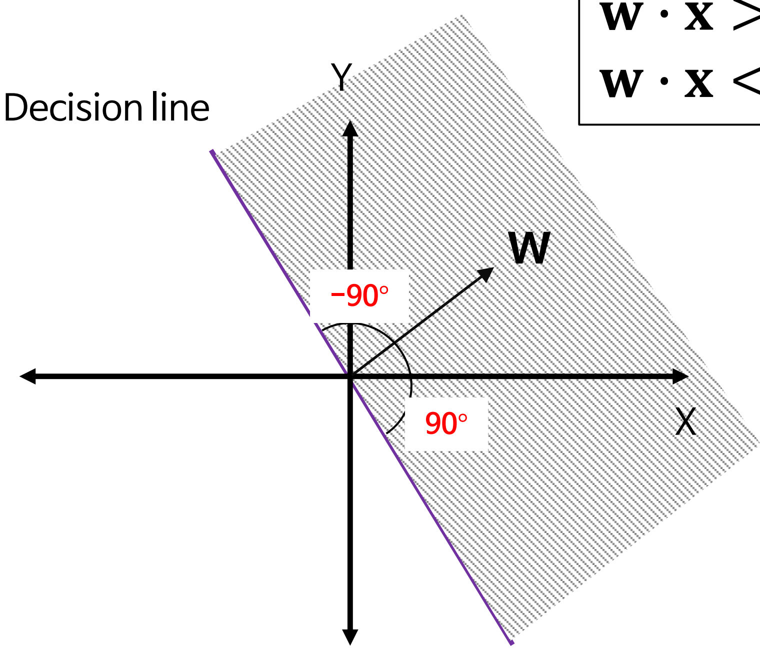


Basic mathematics

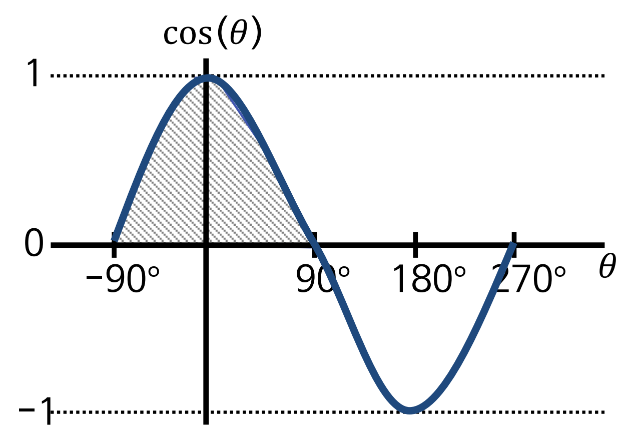
Vectors / Matrices in machine learning (classification)

- Training model and data are represented in vectors
  - To train a model, we utilize matrix and vector operations
    : determine the label of samples based on vector inner product

$$\mathbf{w} \cdot \mathbf{x} = 0$$



$\mathbf{w} \cdot \mathbf{x} > 0$	Angle between two vectors −90~90
$\mathbf{w} \cdot \mathbf{x} < 0$	Angle between two vectors 90~270

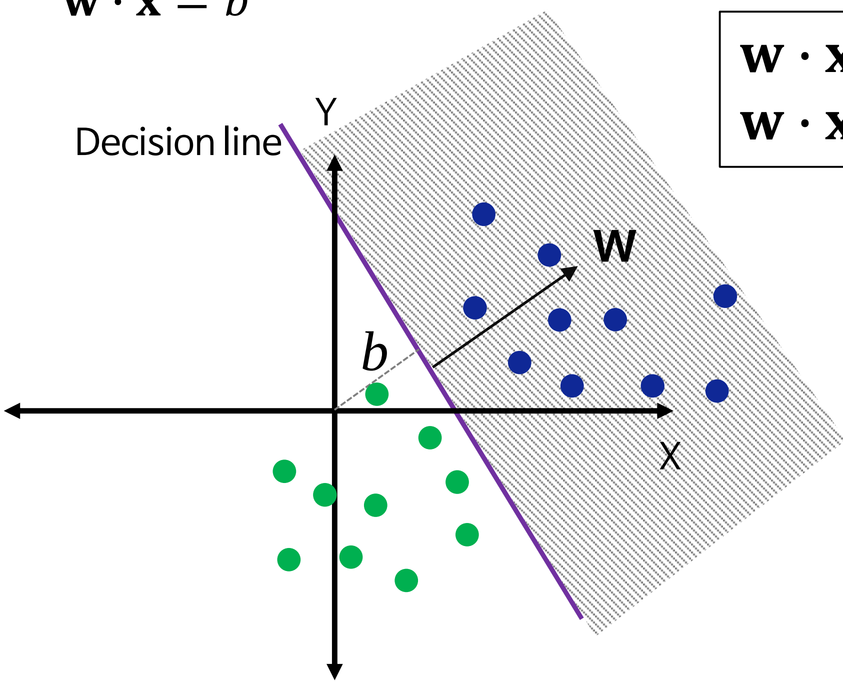


Basic mathematics

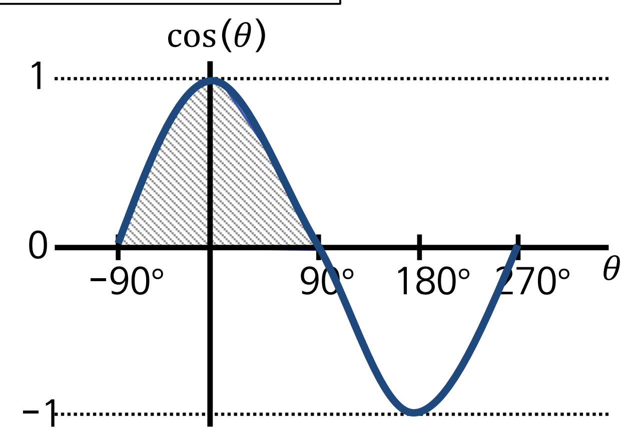
Vectors / Matrices in machine learning (classification)

- Training model and data are represented in vectors
  - To train a model, we utilize matrix and vector operations
    : determine the label of samples based on vector inner product
    additionally, when we consider bias (b) then,

$$\mathbf{w} \cdot \mathbf{x} = b$$

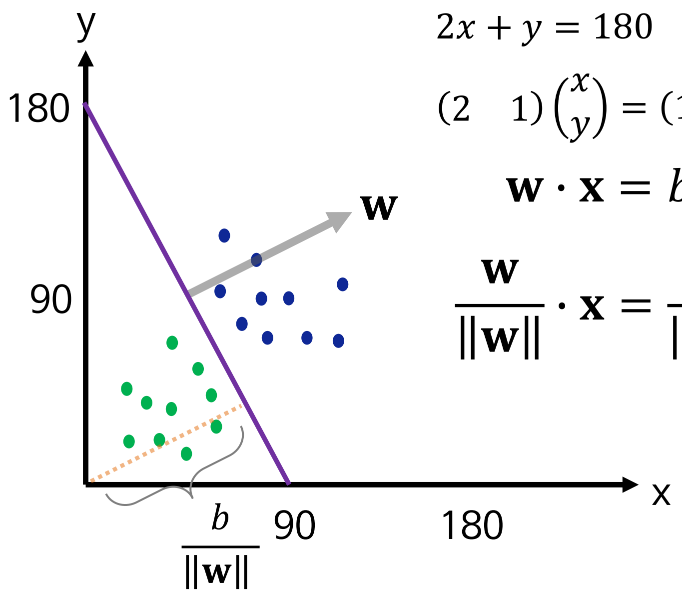


$\mathbf{w} \cdot \mathbf{x} > \mathrm{b}$	Then	class1
$\mathbf{w} \cdot \mathbf{x} < \mathrm{b}$	Then	class2



Vectors / Matrices in machine learning (classification)

- Training model and data are represented in vectors
  - These are fundamentals for machine learning
  - When programming, matrix/vector operation **is very simple and advantageous**



$$2x + y = 180$$

$$\begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = (180)$$

$$\mathbf{w} \cdot \mathbf{x} = b$$

$$\frac{\mathbf{w}}{\|\mathbf{w}\|} \cdot \mathbf{x} = \frac{b}{\|\mathbf{w}\|}$$

Generalized learning parameters

$\mathbf{w} = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 & 1 \end{pmatrix}$: weight

$b = \frac{180}{\sqrt{5}}$: bias