# Chapter 7 Notes - LA

John Yang

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## 7 Distance and Approximation

### 7.1 Inner Product Spaces

- An inner product on a vector space $V$ is an operation that assigns to every pair of vectors $\mathbf{u}$ and $\mathbf{v}$ in $V$ a real number $\langle \mathbf{u}, \mathbf{v} \rangle$ such that the following properties hold for all vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ in $V$ and all scalars $c$:
  - $\langle \mathbf{u}, \mathbf{v} \rangle = \langle \mathbf{v}, \mathbf{u} \rangle$
  - $\langle \mathbf{u}, \mathbf{v} + \mathbf{w} \rangle = \langle \mathbf{u}, \mathbf{v} \rangle + \langle \mathbf{u}, \mathbf{w} \rangle$
  - $\langle c\mathbf{u}, \mathbf{v} \rangle = c\langle \mathbf{u}, \mathbf{v} \rangle$
  - $\langle \mathbf{u}, \mathbf{u} \rangle \geq 0$ and $\langle \mathbf{u}, \mathbf{u} \rangle = 0$ IFF $\mathbf{u} = \mathbf{0}$
- A vector space with an inner product is called an inner product space.
- Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors in an inner product space $V$ and let $c$ be a scalar.
  - $\langle \mathbf{u} + \mathbf{v}, \mathbf{w} \rangle = \langle \mathbf{u}, \mathbf{w} \rangle + \langle \mathbf{v} + \mathbf{w} \rangle$
  - $\langle \mathbf{u}, c\mathbf{v} \rangle = c\langle \mathbf{u}, \mathbf{v} \rangle$
  - $\langle \mathbf{u}, \mathbf{0} \rangle = \langle \mathbf{0}, \mathbf{v} \rangle = 0$
- Let $\mathbf{u}$ and $\mathbf{v}$ be vectors in an inner product space $V$.
  - The length (or norm) of $\mathbf{v}$ is $||\mathbf{v}|| = \sqrt{\langle \mathbf{v}, \mathbf{v} \rangle}$.
  - The distance between $\mathbf{u}$ and $\mathbf{v}$ is $d(\mathbf{u}, \mathbf{v}) = ||\mathbf{u} - \mathbf{v}||$
  - $\mathbf{u}$ and $\mathbf{v}$ are orthogonal if $\langle \mathbf{u}, \mathbf{v} \rangle = 0$.
- Pythagoras' Theorem: Let $\mathbf{u}$ and $\mathbf{v}$ be vectors in an inner product space $V$. Then $\mathbf{u}$ and $\mathbf{v}$ are orthogonal IFF
$$||\mathbf{u} + \mathbf{v}||^2 = ||\mathbf{u}||^2 + ||\mathbf{v}||^2$$
- The Cauchy-Schwarz Inequality: Let $\mathbf{u}$ and $\mathbf{v}$ be vectors in an inner product space $V$.

## 7.2 Norms and Distance Functions

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## 7.3 Least Squares Approximation

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## 7.4 The Singular Value Decomposition

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## 7.5 Applications

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