Chapter 13 Notes - MC

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Contents

13 Vectors and the Geometry of Space

13.1 Three-Dimensional Coordinate Systems

- Coordinates - (x, y, z)
- 3D space is split into octants
- Projections - easiest way to visualize is that the object/shape/line/point is in a glass box. If you look at the box from the chosen plane or angle and trace a 2D outline of it, it is the projection.
- The Cartesian product $\mathbb{R}\times\mathbb{R}\times\mathbb{R} = \{(x, y, z)|x, y, z \in \mathbb{R}\}$ is the set of all ordered triples of real numbers, denoted by $\mathbb{R}^3$
- Distance formula in three dimensions:

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - Z_1)^2}$$

- Sphere - set of all points in 3D space a certain distance from the center.
  - Sphere with center $C(h, k, l)$ and radius r is given by:

$$(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$$

  - Sphere at the origin is given by:

$$x^2 + y^2 + z^2 = r^2$$

13.2 Vectors

- Vectors - values with magnitudes and directions
- vectors are expressed in boldface and/or with an arrow over the letter: $\mathbf{a}$, $\vec{a}$, $\vec{\mathbf{a}}$
- the magnitude of a vector is expressed: $|\mathbf{a}|$

- Vector addition - head to tail, take the magnitude of the resultant vector from the beginning of the first vector to the end of the second vector
- Scalar multiplication - multiply a vector by a scalar: the magnitude is changed by a factor of c but the direction stays the same.

13.3 The Dot Product

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13.4 The Cross Product

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13.5 Equations of Lines and Planes

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13.6 Cylinders and Quadric Surfaces

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