NAME:

12.

1. Determine if the sequences below converge. If they do find the limits $n \to \infty$

   A. $\dfrac{\sin n}{n}$ B. ne^{-n}

2.

   A. Show that if $\lim_{n\to\infty} a_{2n} = L$ and $\lim_{n\to\infty} a_{2n+1} = L$, $\{a_n\}$ is convergent and $\lim_{n\to\infty} a_n = L$

   B. If $a_1 = 1$ and $a_{n+1} = 1 + \dfrac{1}{1+a_n}$, show that $\lim_{n\to\infty} a_n = \sqrt{2}$.

   Hint: Use part A.

3.

   A. Calculate $\sum_{n=0}^{\infty} ar^n$

   B. Under what conditions is this series convergent and what does it converge to?

4. For what values of p is the sum $\sum_{n=1}^{\infty} \dfrac{1}{n^p}$ convergent?

5. Estimate $\sum_{n=1}^{\infty} \dfrac{1}{n^2}$ using the sum of the first 9 terms, then approximating the rest.

6. Determine if the series below are convergent

   A. $\sum_{n=0}^{\infty} \dfrac{1}{2^n + 1}$

   B. $\sum_{n=1}^{\infty} \dfrac{1}{n^2 + n}$

7. For what value of x is the series $\sum_{n=1}^{\infty} n!x^n$ convergent?

8. Obtain the radius of convergenceand interval of convergence for $\sum_{n=1}^{\infty} \dfrac{(-a)^n x^n}{\sqrt{n+1}}$.

9. Obtain a power series representation for ln(1-x).

10. Write down the Taylor and Maclaurin series of the function *f(x)*.

11. Obtain the Taylor series of e^x at a and 0.

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Ch. 11 Test

1) a) $\frac{\sin n}{n}$ n^{th} term test

$\lim_{n\to\infty} \frac{\sin n}{n} \overset{\text{L'Hopital}}{=} \frac{\cos n}{1} \neq 0 \to$ diverges

b) $ne^{-n} = \frac{n}{e^n}$

$\lim_{n\to\infty} \frac{n}{e^n} = \frac{1}{e^n} = 0$ ☆

2) a) $\lim_{n\to\infty} a_{2n} = L$, $\lim_{n\to\infty} a_{2n+1} = L$

$\{a_n\}$ converges & $\lim_{n\to\infty} a_n = L$

if a_{2n} & a_{2n+1} ... ? ☆

b) ? ☆

3) a) $\sum_{n=0} ar^n = \frac{a}{1-r}$ ☆

b) converges if $|r|<1$ to $\frac{a}{1-r}$

4) $\sum_{n=1}^{\infty} \frac{1}{n^p} \to$ p-series, $\boxed{p>1}$

5) $\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \frac{1}{36} + \frac{1}{49} + \frac{1}{64} + \frac{1}{81} + \dots$

$= 1.5397 \dots \to 1.6?$ $1.75?$

6) a) dir. comp. $\sum \frac{1}{2^n} = (\frac{1}{2})^n \to$ geo, conv.

$\sum \frac{1}{2^n+1}$ conv. $\frac{1}{2^n+1} < \frac{1}{2^n}$

b) $\frac{1}{n^2} \to$ p-series $\to$ conv.

$\frac{1}{n^2+n} < \frac{1}{n^2}$ $\sum \frac{1}{n^2+n}$ conv.

7) $\sum_{n=1}^{\infty} n!\,x^n$ $\lim_{n\to\infty} \left| \frac{(n+1)!\,x^{n+1}}{n!\,x^n} \right| < 1$

$= \lim_{n\to\infty} |(n+1)x| \geq 1$

Can only converge if $x = 0$

8) $\lim_{n\to\infty} \left| \frac{(-a)^{n+1} x^{n+1}}{\sqrt{n+2}} \cdot \frac{\sqrt{n+1}}{(-a)^n x^n} \right|$

$= \lim_{n\to\infty} \left| \frac{-a \cdot x \cdot \sqrt{n+1}}{\sqrt{n+2}} \right| < 1$

$|-ax| < 1$

$-ax < 1$

$ax \leq 1$

$ax > -1$ $-1 < ax < 1$

$-\frac{1}{a} < x < \frac{1}{a}$

$\boxed{R = \frac{2}{a}}$ at $x = -\frac{1}{a}$, $\frac{(-a)^n \cdot (-\frac{1}{a})^n}{\sqrt{n+1}}$

$= \frac{1}{\sqrt{n+1}}$ $\frac{1}{n^{\frac{1}{2}}} \to$ div.

at $x = \frac{1}{a}$ $(-a)^n \cdot (\frac{1}{a})^n$

$\frac{(-1)^n}{\sqrt{n+1}}$ alt. series conv.

IOC $\boxed{-\frac{1}{a} < x \leq \frac{1}{a}}$

9) $\ln(1-x)$ ☆

10) Taylor $\frac{(x-a)^n \cdot f^{(n)}(a)}{n!}$

$\sum_{n=0}^{\infty} \frac{f^{(n)}(a) \cdot (x-a)^n}{n!}$

Maclaurin: $\sum_{n=0}^{\infty} \frac{f^{(n)}(0) \cdot (x)^n}{n!}$

1c) e^x $f(a) = e^a, e^a, e^a \dots$

$f(0) = e^0 = 1, 1, 1, 1 \dots$

at $c=a$:

$$e^x = \frac{(1)\cdot e^a}{0! = 1} + \frac{(x-a)\cdot e^a}{1} + \frac{(x-a)^2 e^a}{2!}$$

$$= e^a + (x-a)e^a + \frac{(x-a)^2 e^a}{2} + \frac{(x-a)^3 e^a}{6} + \dots$$

at $c=0$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \dots$$

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1) b) 2) a) b) 3) a) 9)

1) ne^{-n}, if $\lim_{n\to\infty} ne^{-n}$ exists, the series conv

$\lim_{n\to\infty} \frac{n}{e^n} \to$ l'Hopital's rule $\lim_{n\to\infty} \frac{1}{e^n} = 0$ conv.

2) ?

3) $\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + ar^3 + \dots = S_n + ar^n$

$rS_n = ar + ar^2 + \dots ar^{n+1}$

$= ar + ar^n + ar^n$

$S_n - rS_n = a - ar^n$

$S_n(1-r) = a - ar^n$

$$S_n = \frac{a(1-r^n)}{1-r}$$

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9) $\frac{d}{dx}\ln(1-x) = -\frac{1}{1-x} = \frac{1}{x-1}$

$= -\sum_{n=0}^{\infty} x^n$ $\quad \ln(1-x) = \int\left(-\sum_{n=0}^{\infty} x^n\right)dx$

$= -C - \sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1}$