# Chapter 1 Notes - LA

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## 1 Vectors

### 1.1 The Geometry and Algebra of Vectors

- A vector is a directed line segment that corresponds to a displacement from one point A to another point B.
- Column vectors and row vectors are different ways to express the same thing:

$$[3, 2] = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

- The point is that components of vectors are ordered.
- Two vectors are equal if they have the same magnitude and direction. Two vectors can still be equal if they have different initial and terminal points.
- Standard position of a vector - when the initial point is at the origin.
- Sum $\mathbf{u} + \mathbf{v} = [u_1 + v_1, u_2 + v_2]$
- Place vectors from head to tail.
- Scalar multiples: $c\mathbf{v} = [cv_1, cv_2]$ aka scaling a vector
- Subtraction is just adding the negative.
- Properties of vectors in $\mathbb{R}^n$: let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors in $\mathbb{R}^n$ and let $c$ and $d$ be scalars. Then:
  - $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
  - $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$
  - $\mathbf{u} + \mathbf{0} = \mathbf{u}$
  - $\mathbf{u} + (-\mathbf{u}) = 0$
  - $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$
  - $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$

- $c(d\mathbf{u}) = (cd)\mathbf{u}$
- $1\mathbf{u} = \mathbf{u}$

- A vector $\mathbf{v}$ is a linear combination of vectors $\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_k$ if there are scalars $c_1, c_2, \cdots, c_k$ such that $\mathbf{v} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_k\mathbf{v}_k$. Those scalars are called the coefficients of the linear combination.
- Binary vectors - the components are either 0 or 1.
- Modulus function - divide by a given number and you're left with the remainder.

## 1.2 Length and Angle: the Dot Product

- dot product: If

$$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} \quad \text{and} \quad \mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

then the dot product of $\mathbf{u} \cdot \mathbf{v}$ of $\mathbf{u}$ and $\mathbf{v}$ is defined by

$$\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + \cdots + u_nv_n$$

- properties of dot product: let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors in $\mathbb{R}^n$ and let $c$ be a scalar. Then:
  - $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$
  - $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$
  - $(c\mathbf{u}) \cdot \mathbf{v} = c(\mathbf{u} \cdot \mathbf{v})$
  - $\mathbf{u} \cdot \mathbf{u} \geq \mathbf{0}$ and $\mathbf{u} \cdot \mathbf{u} = 0$ IFF $\mathbf{u} = \mathbf{0}$
  - Length or norm of a vector $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$ in $\mathbb{R}^n$ is the nonnegative scalar $||\mathbf{v}||$ defined by

$$||\mathbf{v}|| = \sqrt{\mathbf{v} \cdot \mathbf{v}} = \sqrt{v_1^2 + v_2^2 + \cdots + v_n^2}$$

- Normalizing a vector means finding the unit vector.
- Cauchy-Schwarz Inequality: For all vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$,

$$|\mathbf{u} \cdot \mathbf{v}| \leq ||\mathbf{u}||||\mathbf{v}||$$

- Triangle inequality: for all vectors $\mathbf{u}$ and $\mathbf{v}$ and $\mathbb{R}^n$,

$$||\mathbf{u} + \mathbf{v}|| \leq ||\mathbf{u}|| + ||\mathbf{v}||$$

- Distance between two vectors is defined by

$$d(\mathbf{u}, \mathbf{v}) = ||\mathbf{u} - \mathbf{v}||$$

## 1.3 Lines and Planes

## 1.4 Applications