Ch. 2 Linear Systems

§ 2.1 Intro to Systems of Linear Eqns

Ex. 1 N/A Ex. 2 N/A Ex. 3 N/A

Ex. 4 a) $x - y = 1$
$x + y = 3$
$2x = 4 \quad x = 2$
$y = x - 1 = 1 \quad (2, 1)$

b) $2x - 2y = 4$ → with infinitely many solutions
$2x - 2y = 4$

c) no solution

Ex. 5 $x - y - z = 2$
$y + 3z = 5$
$5z = 10$
$z = 2 \quad y + 6 = 5$
$y = -1$
$x + 1 - 2 = 2$
$x = 3 \quad (3, -1, 2)$

Ex. 6 $x - y - z = 2$
$3x - 3y + 2z = 16$
$2x - y + z = 9$

$2x - 2y - 2z = 4$
$2x - y + z = 9$

$y + 3z = 5$
$y = 5 - 3z$

$3x - 15 + 9z + 2z = 16$
$11z + 3x = 31$
$x = 2 + y + z$
$11z + 6 + 3y + 3z = 11z + 3z + 6 + 15 - 9z = 31$
$5z = 10$
$z = 2$
$y = 5 - 6 = -1$
$x = 2 + 2 - 1 = 3 \quad [3, -1, 2]$

using a matrix:

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 2 \\ 3 & -3 & 2 & 16 \\ 2 & -1 & 1 & 9 \end{array}\right] = \left[\begin{array}{ccc|c} 3 & -3 & -3 & 6 \\ 3 & -3 & 2 & 16 \\ 2 & -1 & 1 & 9 \end{array}\right]$$

$$\hookrightarrow \left[\begin{array}{ccc|c} 1 & -1 & -1 & 2 \\ 0 & 0 & 5 & 10 \\ 2 & -1 & 1 & 9 \end{array}\right] = \left[\begin{array}{ccc|c} 2 & -2 & -2 & 4 \\ 0 & 0 & 5 & 10 \\ 2 & -1 & 1 & 9 \end{array}\right]$$

$$\hookrightarrow \left[\begin{array}{ccc|c} 1 & -1 & -1 & 2 \\ 0 & 0 & 5 & 10 \\ 0 & +1 & 3 & 5 \end{array}\right]$$

$$\hookrightarrow \left[\begin{array}{ccc|c} +1 & -1 & -1 & 2 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 5 & 10 \end{array}\right]$$

$$\hookrightarrow \left[\begin{array}{ccc|c} 1 & -1 & -1 & 2 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$z = 2 \quad y + 3z = 5$
$y = -1$
$x - y - z = 2$
$x + 1 - 2 = 2$
$x = 3 \quad [3, -1, 2]$

§ 2.2 direct methods for solving linear systems

Ex. 2, 7 N/A

Ex. 2, 8
$2x + 4y = 1$
$-y = 2$
$y = -2$
$2x - 8 = 1$
$2x = 9$
$x = 9/2$

$x + 0y = 1$
$0 + y = 5$
$0x + 0y = 4$
NO solution

$x + y + 2z = 1$

$z = 3$

$0x + 0y + 0z = 0$

$z = 3 \quad x + y = -5$

multiple Solutions

$0x + 2y + 0z + w + v = 3$

$-z + w + 2v = 2$

$4v = 0$

$0x + 0y + 0z + 0w + 0v = 5$ no solution

Ex. 9

$$\begin{bmatrix} 1 & 2 & -4 & -4 & 5 \\ 2 & 4 & 0 & 0 & 2 \\ 2 & 3 & 2 & 1 & 5 \\ -1 & 1 & 3 & 6 & 5 \end{bmatrix}$$

$R_2 - 2R_1$

$$\begin{bmatrix} 1 & 2 & -4 & -4 & 5 \\ 0 & 0 & 8 & 8 & -8 \\ 2 & 3 & 2 & 1 & 5 \\ -1 & 1 & 3 & 6 & 5 \end{bmatrix}$$

$R_3 + 2R_4$

$$\begin{bmatrix} 1 & 2 & -4 & -4 & 5 \\ 0 & 0 & 8 & 8 & -8 \\ 0 & 5 & 8 & 13 & 15 \\ -1 & 1 & 3 & 6 & 5 \end{bmatrix}$$

$R_4 + R_1$, $\frac{1}{8}R_2$

$$\begin{bmatrix} 1 & 2 & -4 & -4 & 5 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 5 & 8 & 13 & 15 \\ 0 & 3 & -1 & 2 & 10 \end{bmatrix}$$

$R_2 \leftrightarrow R_4$

$$\begin{bmatrix} 1 & 2 & -4 & -4 & 5 \\ 0 & 3 & -1 & 2 & 10 \\ 0 & 5 & 8 & 13 & 15 \\ 0 & 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_3 - \frac{5}{3}R_2$

$$\begin{bmatrix} 1 & 2 & -4 & -4 & 5 \\ 0 & 3 & -1 & 2 & 10 \\ 0 & 0 & \frac{29}{3} & \frac{29}{3} & -\frac{5}{3} \\ 0 & 0 & 1 & 1 & -1 \end{bmatrix}$$

$3R_3$

$$\begin{bmatrix} 1 & 2 & -4 & -4 & 5 \\ 0 & 3 & -1 & 2 & 10 \\ 0 & 0 & 29 & 29 & -5 \\ 0 & 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_3 - 29R_4$

$$\begin{bmatrix} 1 & 2 & -4 & -4 & 5 \\ 0 & 3 & -1 & 2 & 10 \\ 0 & 0 & 0 & 0 & 24 \\ 0 & 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_4 + R_2$

$$\begin{bmatrix} 1 & 2 & -4 & -4 & 5 \\ 0 & 3 & -1 & 2 & 10 \\ 0 & 0 & 0 & 0 & 24 \\ 0 & 0 & 0 & 3 & 9 \end{bmatrix}$$

$\frac{1}{3}R_4$

$R_4 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 2 & -4 & -4 & 5 \\ 0 & 3 & -1 & 2 & 10 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 24 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -4 & -4 & 5 \\ 2 & 4 & 0 & 0 & 2 \\ 2 & 3 & 2 & 1 & 5 \\ -1 & 1 & 3 & 6 & 5 \end{bmatrix} \downarrow \rightarrow \begin{bmatrix} 1 & 2 & -4 & -4 & 5 \\ 0 & 0 & 8 & 8 & -8 \\ 0 & -1 & 10 & 9 & -5 \\ 0 & 3 & -1 & 2 & 10 \end{bmatrix}$$

$R_2 - 2R_1$

$R_3 - 2R_1$

$R_4 + R_1$

$R_4 + 3R_3$

$R_4 \leftrightarrow R_2$

$\frac{1}{8}R_4$

$$\begin{bmatrix} 1 & 2 & -4 & -4 & 5 \\ 0 & 0 & 29 & 29 & -5 \\ 0 & -1 & 10 & 9 & -5 \\ 0 & 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_2 - 29R_4$

$$\begin{bmatrix} 1 & 2 & -4 & -4 & 5 \\ 0 & 0 & 0 & 0 & 24 \\ 0 & -1 & 10 & 9 & -5 \\ 0 & 0 & 1 & 1 & -1 \end{bmatrix}$$

reduced rows:

$$\begin{bmatrix} 1 & 2 & -4 & -4 & 5 \\ 0 & -1 & 10 & 9 & -5 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 24 \end{bmatrix}$$

$R_2 \leftrightarrow R_4$

$$\begin{bmatrix} 1 & 2 & -4 & -4 & 5 \\ 0 & 0 & 1 & 1 & -1 \end{bmatrix}$$

Ex. 10

$$\begin{bmatrix} 0 & 2 & 3 & 8 \\ 2 & 3 & 1 & 5 \\ 1 & -1 & -2 & -5 \end{bmatrix} \quad R_1 \leftrightarrow R_3, \quad R_2 - 2R_1$$

$$\begin{bmatrix} 1 & -1 & -2 & -5 \\ 0 & 5 & 5 & 15 \\ 0 & 2 & 3 & 8 \end{bmatrix} \quad \frac{1}{5}R_2, \quad R_3 - 2R_2$$

$$\begin{bmatrix} 1 & -1 & -2 & -5 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$x_3 = 2$

$x_2 + 2 = 3$

$x_2 = 1$

$x_1 - 1 - 4 = -5$

$x_1 = 0$

$[0, 1, 2]$

write as column vector $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$

Ex. 11

$$\begin{bmatrix} 1 & -1 & -1 & 2 & 1 \\ 2 & -2 & -1 & 3 & 3 \\ -1 & 1 & -1 & 0 & -3 \end{bmatrix} \quad R_2 - 2R_1, \quad R_3 + R_1$$

$$\begin{bmatrix} 1 & -1 & -1 & 2 & 1 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & -2 & 2 & -2 \end{bmatrix} \quad R_3 + 2R_2$$

$$\begin{bmatrix} 1 & -1 & -1 & 2 & 1 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$y - z = 1 \qquad y = z + 1$

$w - x - y + 2z = 1$

infinitely many solutions

parametrize the equations

$w - x - z - 1 + 2z = 1$

$w = x - z + 2 \qquad x = s \qquad z = t$

$y = z + 1$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} s - t + 2 \\ s \\ t + 1 \\ t \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} s + \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} t$$

Ex. 12

$$\begin{bmatrix} 1 & -1 & 2 & 3 \\ 1 & 2 & -1 & -1 \\ 0 & 2 & -2 & 1 \end{bmatrix} \quad R_2 - R_1$$

$$\begin{bmatrix} 1 & -1 & 2 & 3 \\ 0 & 3 & -3 & -6 \\ 0 & 2 & -2 & 1 \end{bmatrix} \quad \frac{1}{3}R_2, \quad R_3 - 2R_2 \qquad \begin{bmatrix} 1 & -1 & 2 & 3 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

no solutions

Ex. 13

$$\begin{bmatrix} 1 & -1 & -1 & 2 & 1 \\ 2 & -2 & -1 & 3 & 3 \\ -1 & 1 & -1 & 0 & -3 \end{bmatrix} \xrightarrow{\text{REF}} \begin{bmatrix} 1 & -1 & -1 & 2 & 1 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 + R_2$

$$\begin{bmatrix} 1 & -1 & 0 & 1 & 2 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \checkmark$$

$w \quad x \quad y \quad z \quad c$

$y - z = 1 \qquad y = z + 1 \qquad z = t$

$w - x + z = 2 \qquad w = 2 + x - z \qquad x = s$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} s + \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} t$$

Ex. 14

$x + 2y - z = 3$

$2x + 3y + z = 1$

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 3 & 1 & 1 \end{bmatrix} \quad R_2 - 2R_1$$

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -1 & 3 & -5 \end{bmatrix} \quad R_1 + 2R_2 \qquad \begin{bmatrix} 1 & 0 & 5 & -7 \\ 0 & -1 & 3 & -5 \end{bmatrix}$$

$$-R_2 \begin{bmatrix} 1 & 0 & 5 & -7 \\ 0 & 1 & -3 & 5 \end{bmatrix} \qquad x + 5z = -7 \qquad z = t$$

$y - 3z = 5$

$x = -7 - 5t$

$y = 5 + 3t$

$z = t$

Ex. 15 $\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ ①

$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} + t \begin{bmatrix} 3 \\ -1 \\ -1 \end{bmatrix}$ ②

② → usar a d.ds. parâmetro

$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} + s \begin{bmatrix} 3 \\ -1 \\ -1 \end{bmatrix}$

$P + tu = q + sv$

$\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} - \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} = s \begin{bmatrix} 3 \\ -1 \\ -1 \end{bmatrix} - t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$3s - t = 1$ $\quad s = 2 - t$

$-s - t = -2$ $\quad 3(2-t) - t = 1$

$-s - t = -2$ $\quad 6 - 6t - t = 1$

$-7t = -5$

$t = \frac{5}{7}$

$s = \frac{9}{7}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + \frac{5}{7} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 12/7 \\ 5/7 \\ -2/7 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4/4 \\ 5/4 \\ 1/4 \end{bmatrix}$

Ex. 16 $\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$

$R_2 - R_1$ $\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$ $-2 = 1$

$R_3 - R_2$ $\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 2 & 2 \end{bmatrix}$ $-1 = 2$

$\frac{1}{2} R_3$ $\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

$R_1 - 2R_2$ $\begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

$x_3 = 1$

$x_2 = 2$

$x_1 + 1 = 2$

$x_1 = 1$ $\quad \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

Ex. 17 $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix}$

$R_2 - R_1$ $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$ Resultou $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$

$R_4 - R_1$

$R_1 - R_2$ $\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$

$x_3 + x_4 = 0$

$x_2 + x_3 = 0$

$x_1 + x_4 = 1$

From prev:

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$R_5 + R_4$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 + R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$ rref

$x_1 + x_4 = 1 \qquad x_4 = t$

$x_2 + x_4 = 0$

$x_3 + x_4 = 0$

$x_1 + t = 1 \to x_1 = t + 1$

$x_2 + t = 0 \qquad x_2 = t$

$x_3 + t = 0 \qquad x_3 = t$

$x_4 = t$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$t = 0$ or $t = 1$

$$\begin{bmatrix} 1+1 \\ 0+1 \\ 0+1 \\ 0+1 \end{bmatrix} \text{ or } \begin{bmatrix} 1+0 \\ 0+0 \\ 0+0 \\ 0+0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

§2.3 Spanning sets & linear independence

Ex 18

a) $x\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} + y\begin{bmatrix} 4 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$$\hookrightarrow \left[\begin{array}{cc|c} 1 & 4 & 1 \\ 0 & 1 & 2 \\ 3 & -3 & 3 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 4 & 1 \\ 0 & 1 & 2 \\ 1 & -1 & 1 \end{array}\right]$$

$$\hookrightarrow \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right] \qquad x = 3 \quad y = 2$$

$$3\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} + 2\begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

b) $\left[\begin{array}{cc|c} 1 & -1 & 2 \\ 0 & 1 & 3 \\ 3 & -3 & 4 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 3 \\ 3 & -3 & 4 \end{array}\right]$

$$\hookrightarrow \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 3 \\ 0 & -3 & -11 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & -2 \end{array}\right]$$ no solution

Ex. 19

Show that $x\begin{bmatrix} 2 \\ -1 \end{bmatrix} + y\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$

$$\left[\begin{array}{cc|c} 2 & 1 & a \\ -1 & 3 & b \end{array}\right] = \left[\begin{array}{cc|c} 1 & -3 & -b \\ 2 & 1 & a \end{array}\right]$$

$$= \left[\begin{array}{cc|c} 1 & -3 & -b \\ 0 & 7 & a+2b \end{array}\right] = \left[\begin{array}{cc|c} 1 & -3 & -b \\ 0 & 1 & \left(\frac{a+2b}{7}\right) \end{array}\right]$$

$$= \left[\begin{array}{cc|c} 1 & 0 & -b+\frac{3}{7}(a+2b) \\ 0 & 1 & \frac{a+2b}{7} \end{array}\right]$$

$$\therefore \left(-b+\frac{3}{7}(a+2b)\right)\begin{bmatrix} 2 \\ -1 \end{bmatrix} + \left(\frac{a+2b}{7}\right)\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

Ex. 20 N/A

Ex. 21 $\text{Span}\left(\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}\right)$

Thus Of it was plane: $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \text{Span}\left(\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}\right)$

$$s\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} + t\begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \to \left[\begin{array}{cc|c} 1 & -1 & x \\ 0 & 1 & y \\ 3 & -3 & z \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & -1 & x \\ 0 & 1 & y \\ 3 & -3 & z \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & -1 & x \\ 0 & 1 & y \\ 0 & 0 & z-3x \end{array}\right] \xrightarrow{R_1+R_2} \left[\begin{array}{cc|c} 1 & 0 & x+y \\ 0 & 1 & y \\ 0 & 0 & z-3x \end{array}\right]$$

$z-3x=0$, all y

Ex. 22 N/A

Ex. 23

a) $\begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ — independent; $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$ cannot be multiplied by 0 to get $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$

b) $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$$c_1\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + c_3\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$R_2 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$R_3 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right] \quad \frac{1}{2}R_3 \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$R_2 + R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \quad R_1 - R_3 \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$c_1 = 0 \quad c_2 = 0 \quad c_3 = 0$

$\therefore$ they are linearly independent

c) $c_1\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + c_2\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} + c_3\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right]$$

$R_2 + R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right]$$

$-R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right] \quad R_3 + R_2 \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$c_1 - c_3 = 0 \quad c_1 = c_3$

$c_2 - c_3 = 0 \quad c_2 = c_3 = c_1 = 1 \Rightarrow$ dependent

note that: $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

d) $c_1\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + c_2\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} + c_3\begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & 1 & 4 & 0 \\ 0 & -1 & 2 & 0 \end{array}\right] \quad R_2 - 2R_1 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & -1 & 2 & 0 \end{array}\right]$$

$R_3 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad -R_2 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$R_1 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

multiple solutions $\Rightarrow$ dependent

Ex. 24 N/A Ex. 25 N/A

Ex. 26 N/A

§2-4 Applications

Ex. 27

$2x_1 + 2x_2 + 4x_3 = 2300$

$x_1 + 2x_2 = 800$

$x_1 + 3x_2 + x_4 = 1500$

$$\left[\begin{array}{cccc|c} 2 & 2 & 4 & 0 & 2300 \\ 1 & 2 & 0 & 0 & 800 \\ 1 & 3 & 0 & 1 & 1500 \end{array}\right] \quad R_2 \leftrightarrow R_1$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 800 \\ 2 & 2 & 4 & 0 & 2300 \\ 1 & 3 & 0 & 1 & 1500 \end{array}\right]$$

$$\begin{matrix} R_2 - 2R_1 \\ R_3 - R_1 \end{matrix} \left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 800 \\ 0 & -2 & 4 & 0 & 700 \\ 0 & 1 & 1 & 0 & 700 \end{array}\right]$$

$$\begin{matrix} \div -2R_2 \\ R_1 + 2R_2 \end{matrix} \left[\begin{array}{cccc|c} 1 & 0 & 4 & 0 & 1500 \\ 0 & 1 & -2 & 0 & -350 \\ 0 & 1 & 1 & 0 & 700 \end{array}\right]$$

$$R_3 - R_2 \left[\begin{array}{cccc|c} 1 & 0 & 4 & 0 & 1500 \\ 0 & 1 & -2 & 0 & -350 \\ 0 & 0 & 3 & 0 & 1050 \end{array}\right]$$

$$\div 3 \left[\begin{array}{cccc|c} 1 & 0 & 4 & 0 & 1500 \\ 0 & 1 & -2 & 0 & -350 \\ 0 & 0 & 1 & 0 & 350 \end{array}\right]$$

$$R_2 + 2R_3 \left[\begin{array}{cccc|c} 1 & 0 & 4 & 0 & 1500 \\ 0 & 1 & 0 & 0 & 350 \\ 0 & 0 & 1 & 0 & 350 \end{array}\right]$$

$$R_1 - 4R_3 \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 100 \\ 0 & 1 & 0 & 0 & 350 \\ 0 & 0 & 1 & 0 & 350 \end{array}\right]$$

Strain I $\begin{bmatrix} 100 \\ 350 \\ 350 \end{bmatrix}$

Strain II

Strain III

Ex. 28

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1500 \\ 1 & 2 & 3 & 3000 \\ 1 & 3 & 5 & 4500 \end{array}\right]$$

$$\begin{matrix} R_2 - R_1 \\ R_3 - R_1 \end{matrix} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1500 \\ 0 & 1 & 2 & 1500 \\ 0 & 2 & 4 & 3000 \end{array}\right]$$

$$R_3 - 2R_2 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1500 \\ 0 & 1 & 2 & 1500 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{matrix} R_1 - R_2 \\ \text{R.R.E.F.} \end{matrix} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 1500 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 = x_3$

$x_2 = 1500 - 2x_3$

$x_3 = t \quad x_1 = t \quad x_2 = 1500 - 2t$

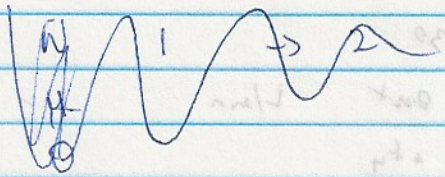
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} t \\ 1500 - 2t \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ 1500 \\ 0 \end{bmatrix} + t\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$t \geq 0, \quad 1500 - 2t \geq 0$

$t \leq 750$

Ex. 29

$NH_3 + O_2 \rightarrow N_2 + H_2O$



$x_1 NH_3 + x_2 O_2 \rightarrow x_3 N_2 + x_4 H_2O$

N: $x_1 = 2x_3$

H: $3x_1 = 2x_4$

O: $2x_2 = x_4$

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 3 & 0 & 0 & -2 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{array}\right]$$

$R_2 - 3R_1$

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & 0 & 6 & -2 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{array}\right]$$

$\frac{1}{6} R_2$

$\frac{1}{2} R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{3} & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & 0 \end{array}\right]$$

$R_1 + 2R_2$

$R_2 \leftrightarrow R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{2}{3} & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{1}{3} & 0 \end{array}\right]$$

$x_1 = \frac{2}{3} x_4$

$x_2 = \frac{1}{2} x_4$ → LCD 6

$x_3 = \frac{1}{3} x_4$

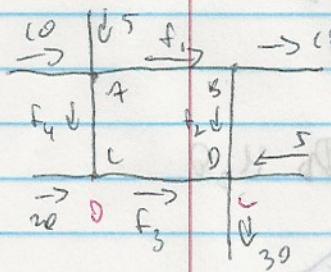
$x_4 = 6$

$x_1 = 4 \quad x_3 = 2$

$x_2 = 3 \quad x_4 = 6$

$4NH_3 + 3O_2 \rightarrow 2N_2 + 6H_2O$

Ex. 30



flow in = flow out L/min

are A: $10 + 5 = f_1 + f_4$

B: $f_1 = f_2 + 10$

C: $f_4 + 20 = f_3$

D: $f_2 + f_3 + 5 = 30$

$f_1 + f_4 = 15$

$f_1 - f_2 = 10$

$f_3 - f_4 = 20$

$f_2 + f_3 = 25$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 15 \\ 1 & -1 & 0 & 0 & 10 \\ 0 & 1 & 0 & -1 & 20 \\ 0 & 1 & 1 & 0 & 25 \end{array}\right]$$

$R_2 - R_1$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 15 \\ 0 & -1 & 0 & -1 & -5 \\ 0 & 1 & 0 & -1 & 20 \\ 0 & 1 & 1 & 0 & 25 \end{array}\right]$$

$R_3 + R_2$

$R_4 + R_2$

$-R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 15 \\ 0 & 1 & 0 & 1 & 5 \\ 0 & 0 & 0 & -2 & 15 \\ 0 & 0 & 1 & -1 & 20 \end{array}\right]$$

$-\frac{1}{2} R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 15 \\ 0 & 1 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 & -7.5 \\ 0 & 0 & 1 & -1 & 20 \end{array}\right]$$

$R_1 - R_3$

$R_2 - R_3$

$R_4 \leftrightarrow R_3$

$R_3 + R_4$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 22.5 \\ 0 & 1 & 0 & 0 & 12.5 \\ 0 & 0 & 1 & 0 & 12.5 \\ 0 & 0 & 0 & 1 & -7.5 \end{array}\right]$$

$f_1 = 22.5$ Lpm, →

$f_2 = 12.5$ Lpm, ↓

$f_3 = 12.5$ Lpm, →

$f_4 = 7.5$ Lpm, ↑

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 15 \\ 1 & -1 & 0 & 0 & 10 \\ 0 & 1 & 1 & 0 & 25 \\ 0 & 0 & 1 & -1 & 20 \end{array}\right]$$

$f_1 = 15 - t$

$f_2 = 5 - t$

$f_3 = 20 + t$

$f_4 = t$

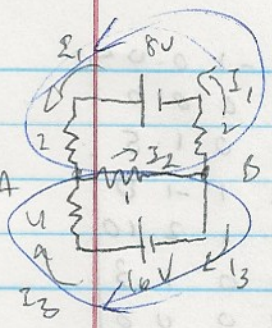
$10 \leq f_1 \leq 15$

$0 \leq f_2 \leq 5$

$20 \leq f_3 \leq 25$

$0 \leq f_4 \leq 5$

Ex-31



need 3 eqns

at A: $I_1 + I_3 = I_2$

$V = IR$

$R = \frac{V}{I}$

Circuit 1: $2I_1 + 2I_1 + I_2 = 8V$

Circuit 2: $4I_3 + I_2 = 16V$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 4 & 1 & 0 & 8 \\ 0 & 1 & 4 & 16 \end{array}\right]$$

$R_2 - 4R_1$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 5 & -4 & 8 \\ 0 & 1 & 4 & 16 \end{array}\right]$$

$R_2 - 5R_3$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 0 & -24 & -72 \\ 0 & 1 & 4 & 16 \end{array}\right]$$

$-\frac{1}{24} R_2$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 4 & 16 \end{array}\right]$$

$R_3 - 4R_2$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 4 \end{array}\right]$$

$R_1 + R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 4 \end{array}\right]$$

$R_1 - R_2$

$R_2 \leftrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$I_1 = 1A \quad I_2 = 4A \quad I_3 = 3A$

Ex-32

Wheatstone bridge

at node A: $I_t = I_1 + I_4$ — need 6 eqns

B: $I_1 = I_2 + I_3$

C: $I_4 + I_3 = I_5$

D: $I_2 + I_5 = I_t$

} current rule

Voltage rule $V = IR$

$10V = I_4 + 2I_5$

$10V = 2I_1 + I_2$

$10V = 2I_1 + 2I_3 + 2I_5$

	I_t	I_1	I_2	I_3	I_4	I_5	C
1	1	−1	0	0	−1	0	0
2	0	1	−1	−1	0	0	0
3	0	0	0	1	1	−1	0
4	−1	0	1	0	0	1	0
5	0	0	0	0	1	2	10
6	0	2	1	0	0	0	10
7	0	2	0	2	0	2	10

$R_4 + R_1$

1	−1	0	0	−1	0	0	
2	0	1	−1	−1	0	0	
	0	1	0	1	0	0	
3	0	0	0	1	−1	0	
4	0	−1	1	0	−1	1	0
5	0	0	0	0	1	2	10
6	0	2	1	0	0	0	10
7	0	2	0	2	0	2	10

$R_4 + R_2$

$R_6 - 2R_2$

$R_7 - 2R_2$

$R_4 + R_2$, $R_6 - 2R_2$, $R_7 - 2R_2$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 10 \\ 0 & 0 & 3 & 2 & 0 & 0 & 10 \\ 0 & 0 & 2 & 4 & 0 & 2 & 10 \end{bmatrix}$$

$\frac{1}{2}R_7$, $R_7 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 & 5 \\ 0 & 0 & 0 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 10 \\ 0 & 0 & 3 & 2 & 0 & 0 & 10 \\ 0 & 0 & 0 & 1 & 1 & -1 & 0 \end{bmatrix}$$

$R_6 - 3R_3$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 & 5 \\ 0 & 0 & 0 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 10 \\ 0 & 0 & 0 & -4 & 0 & -3 & -5 \\ 0 & 0 & 0 & 1 & 1 & -1 & 0 \end{bmatrix}$$

$R_7 \leftrightarrow 4R_4$, $R_7 + R_4$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 & 5 \\ 0 & 0 & 0 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 10 \\ 0 & 0 & 0 & 0 & 4 & -7 & -5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_6 - 4R_5$, $-R_4$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 10 \\ 0 & 0 & 0 & 0 & 0 & -15 & -45 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$-\frac{1}{15}R_6$, $R_1 + R_2$

$$\begin{bmatrix} 1 & 0 & -1 & -1 & -1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 10 \\ 0 & 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 + R_3$, $R_2 + R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & -1 & 0 & 5 \\ 0 & 1 & 0 & 1 & 0 & 1 & 5 \\ 0 & 0 & 1 & 2 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 10 \\ 0 & 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 - 2R_4$

$R_2 - R_4$, $R_1 - R_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 2 & 5 \\ 0 & 1 & 0 & 0 & -1 & 2 & 5 \\ 0 & 0 & 1 & 0 & -2 & 3 & 5 \\ 0 & 0 & 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 10 \\ 0 & 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 + 2R_5$, $R_2 + R_5$, $R_3 + 2R_5$, $R_4 - R_5$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 5 & 20 \\ 0 & 1 & 0 & 0 & 0 & 4 & 15 \\ 0 & 0 & 1 & 0 & 0 & 7 & 25 \\ 0 & 0 & 0 & 1 & 0 & -3 & -10 \\ 0 & 0 & 0 & 0 & 1 & 2 & 10 \\ 0 & 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 - 5R_6$, $R_2 - 4R_6$, $R_3 - 7R_6$, $R_4 + 3R_6$, $R_5 - 2R_6$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} I_6 \\ I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 4 \\ -1 \\ 4 \\ 3 \end{bmatrix}$$

Ex-33 N/A

Ex-34 N/A

Ex-35 $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

only 2 choices, on or off $\rightarrow \mathbb{Z}_2$

Create a vector for each switch:

$a = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ $b = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ $c = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$ $d = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$ $e = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$

$x_1 a + x_2 b + x_3 c + x_4 d + x_5 e = f$

where f is the final configuration:

$$\begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{array}\right]$$

$R_2 + R_1$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{array}\right]$$

$R_4 + R_2$, $R_2 \leftrightarrow R_3$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{array}\right]$$

$R_5 + R_4$, $R_1 + R_2$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$R_1 + R_3$, $R_2 + R_3$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \leftarrow$$

$R_1 + R_4$, $R_2 + R_4$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

After $x_5 = t$

$x_1 + x_5 = 1$ $\quad x_1 = t + 1$

$x_2 + x_5 = 0$ $\quad x_2 = t$

$x_3 = 1$ $\quad x_3 = 1$

$x_4 + x_5 = 1$ $\quad x_4 = t + 1$

$x_5 = t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = t \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$t = 0$ or $t = 1$

when $t=0$: $\begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$ $\quad t=1$: $\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$

$f = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{array}\right]$$

$R_2 + R_1$, $R_3 + R_1$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{array}\right]$$

$R_3 + R_2$, $R_4 + R_2$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{array}\right]$$

$R_4 + R_3$, $R_5 + R_3$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$R_5 + R_4$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

no solution

Ex. 34

$f = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ in $\mathbb{Z}_3$ $a = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $c = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 2 \end{array}\right]$$

$R_2 + R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \end{array}\right]$$

$R_2 \leftrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$R_1 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$R_1 - 2R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & +2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

§2.5 Iterative methods for Solving Linear Sys.

Ex. 37 N/A

Ex. 38 $\left[\begin{array}{cc|c} 1 & -1 & 1 \\ 2 & 1 & 5 \end{array}\right]$

$x_1 - x_2 = 1$ $\quad x_1 = 1 + x_2$

$2x_1 + x_2 = 5$ $\quad x_2 = 5 - 2x_1$

	0					
x_1	0	1	4	-3	12	14
x_2	0	3	-3	11	-19	43

$\Rightarrow$ diverges

no solution

Solution is actually $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

Ex. 34 N/A