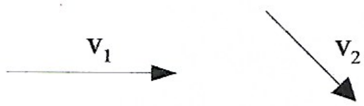
CH 3,4

Questions 1-2.

I. Force	II. Mass	III. Velocity	IV. Acceleration
V. Distance	VI. Area	VII. Speed	VIII. Displacement

1. Which of the quantities above are scalars?
2. Which one of the quantities above are vectors?
3. At time $t = t_1$, the velocity of an object is given by the vector v_1 and at a later time $t = t_2$ the object's velocity is given by the vector v_2



What is the direction of the object's acceleration between times t_1 and t_2?

4. Which of the following is/are true? Justify your answer.
   - I. If an object's acceleration is constant, then it must move in a straight line.
   - II. If an object's acceleration is zero, then its speed must remain constant.
   - III. If an object's speed remains constant, then its acceleration must be zero.

Next 5 Questions A basketball is thrown straight upward on Earth with an initial velocity of 25 $\frac{m}{s}$.

5. What is its acceleration at the highest point?
6. What is its velocity at the highest point?
7. When does it reach the highest point?
8. With respect to the initial position, what is the highest point?
9. When does it returns to its initial position?
10. How long, approximately, would it take a car, starting from rest and accelerating uniformly in a straight line at $5\frac{m}{s^2}$ to cover a distance of 200m?
11. A rock is dropped off a cliff and strikes the ground with an impact velocity of $30\frac{m}{s}$. How high is the cliff approximately?
12. A stone is thrown *horizontally* with initial speed of 10 $\frac{m}{s}$ from a bridge. If the air resistance is ignored, how long would it take the stone to strike the water 80 m below the bridge?

Next 8 Questions A soccer ball is kicked on the ground with an initial velocity of 20 m/s at a 53° angle with respect to the horizontal. Obtain the following

13. The moment after it is kicked, its acceleration
14. The moment after it is kicked, the magnitude of its horizontal velocity
15. The moment after it is kicked, the magnitude of its vertical velocity
16. At the maximum height its acceleration
17. At the maximum height the magnitude of its velocity
18. The moment before it hits the ground, its acceleration
19. The magnitude of its impact velocity with the ground
20. The impact angle with the ground with respect to the horizontal
21. What is the direction of velocity in a circular motion with constant speed?
22. What is the direction of acceleration in a circular motion with constant speed

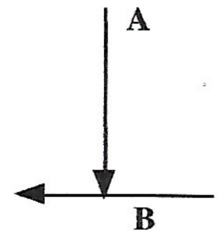
Next 3 Questions A particle is moving at the constant speed of 15 m/s in a circle of radius 4m on a flat horizontal plane.

23. Obtain the magnitude of the angular velocity.
24. Obtain the magnitude of the centripetal acceleration.
25. Obtain the magnitude of the tangential acceleration.

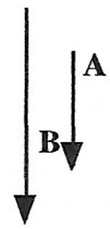
Next 6 Questions A bicycle wheel of negligible mass of radius r starts rolling from a rest position with an angular acceleration α. After a time interval t, in terms of α, r, and t, obtain the following quantities

26. Angular displacement θ of a point on the wheel
27. Linear displacement x of a point on the wheel
28. Angular velocity ω of a point on the wheel
29. Linear velocity v of a point on the wheel
30. Tangential acceleration a_t of a point on the wheel
31. Centripetal acceleration a_c of a point on the wheel

Obtain **AxB** for the given vectors



32.



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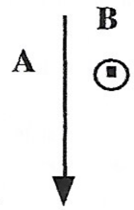
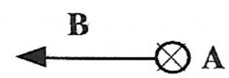
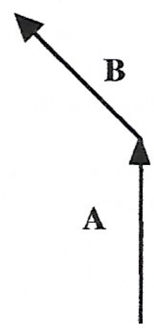


34.

35.





36.

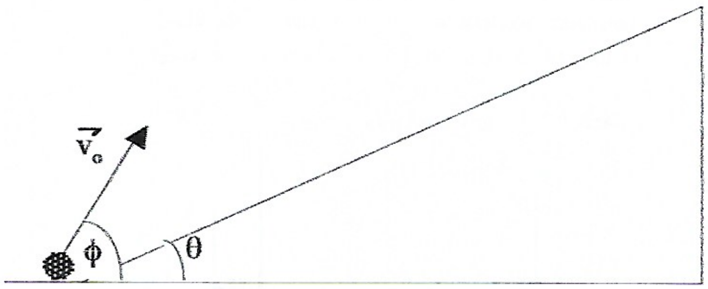
37. *An alien visiting our planet jumps straight upward with an initial velocity of 20 m/s at time t=0 at the north ridge of the grand canyon. On the way down hshe just misses the edge and continues to fall down where there is a rock 100 m below the edge of the ridge.*

a. When does hshe reach the maximum height?
b. What is the maximum height hshe reaches?
c. What is hihers velocity at the max height?
d. What is hihers acceleration at the max height?
e. When does hshe get back to hihers initial location?
f. What is hihers velocity at when hshe reaches hihers original height?
g. When does hshe hit the rock 100m below the ridge?
h. What is hihers velocity the moment before hshe hits the rock?
i. What is hihers location at time t=3.5s?
j. What is hihers location when hihers velocity is 10 m/sand hshe is falling down.?

38. A bird is flying 30 $\frac{m}{s}$ at an angle 60° West of North with respect to ground. There is a strong wind blowing 20 $\frac{m}{s}$ at an angle 30° West of North.

a. Draw a figure representing the situation (do your best to draw it to a scale).
b. What is the bird's air speed? In other words what would be the bird's velocity with respect to ground if there were no wind?

39. An object is at the base of an inclined surface which is at an angle θ with respect to the horizontal x-axis as shown in the figure. The object is given an initial velocity with magnitude v_o at and angle ϕ with respect to the horizontal x-axis.



a. Draw a figure showing the projectile's path.

b. Obtain v_{ox} in terms of v_o and ϕ.

Obtain v_{oy} in terms of v_o and ϕ.

c. Obtain the horizontal distance the object will cover in time t in terms of v_o, ϕ, g, t.

Obtain the vertical distance the object will cover in time t in terms of v_o, ϕ, g, t.

d. Obtain the (x,y) coordinates of the point the object will hit on the inclined surface in terms the distance r from the initial position in terms of r and θ.

e. Use your answer to part (d) to obtain y in terms of x.

f. Use your answer to part (c) to obtain y in terms of x by eliminating t.

g. Use your answers to parts (e) and (f) to obtain x in terms of v_o, θ, ϕ, g and to obtain y in terms of v_o, θ, ϕ, g.

h. Use your answer to part (g) to obtain r, the distance along the inclined surface in terms of v_o, θ, ϕ, g.

i. State how you would proceed to check your answer for correctness.

j. State how you would proceed to find ϕ which gives the maximum range.

Black	49
Blue	6
Red	5
Total	63

$\Big\rangle$ 60 expected

Ch. 3 & 4 John Yang

1) II, V, VII +1

2) I, III, IV, VI, VIII +1

3) $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$ $\longrightarrow v_1$ $\searrow v_2$

$a \swarrow$, since the object had to move in a circle to change direction. the $\vec{a}$ vector points towards the inside of the circle., not the right approach... ~

4) I. not true; the object can be moving in a circle w/ constant centripetal acceleration.

II. true - $a = \frac{\Delta v}{\Delta t}$, so if the speed changes a will have a non zero value.

III false - The object can be moving in a circle with a constant tangential speed while accelerating. +1

5) $v_1 = +25 \frac{m}{s}$

a is always $\boxed{-9.8 \frac{m}{s^2}}$ +1

6) $v_{highest} = \boxed{0 \frac{m}{s}}$ +1

7) $\cancel{v_f}^0 = v_0 + at$

$t = \frac{-v_0}{a} = -(25 \frac{m}{s})/(-9.8 \frac{m}{s^2}) = \boxed{2.6 s}$ +1

8) $\cancel{v_f}^0 = v_0^2 + 2a\Delta x$

$\Delta x = \frac{-v_0^2}{2a} = -(25 \frac{m}{s})^2 / 2(-9.8 \frac{m}{s^2}) = \boxed{32 m}$ +1

9) $t_{total} = 2t_{up} = 2(2.6 s) = \boxed{5.2 s}$ +1

10) $a = 5 \frac{m}{s^2}$, $\Delta x = 200 m$

$\Delta x = \cancel{v_0 t}^0 + \frac{1}{2}at^2$

$t = \sqrt{\frac{2\Delta x}{a}} = \sqrt{2(200 m)/5 \frac{m}{s^2}} = \boxed{8.9 s}$ +1

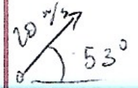
11) $v_f^2 = \cancel{v_0}^0 + 2a\Delta x$

$\Delta x = v_f^2/2a = (30 \frac{m}{s})^2 / 2(9.8 \frac{m}{s^2}) = \boxed{46 m}$ +1

12) $\Delta x = \cancel{v_0 t}^0 + \frac{1}{2}at^2$

$t = \sqrt{\frac{2\Delta x}{a}} = \sqrt{2(80 m)/9.8 \frac{m}{s^2}} = \boxed{4 s}$ +1

11



13) $a = \boxed{-9.8\,\frac{m}{s}\hat{j}}$, always +1

14) $v_x = v\cos\theta = (20\,\frac{m}{s})\cos 53 = \boxed{12.\,\frac{m}{s}\hat{i}}$ +1

15) $v_y = v\sin\theta = (20\,\frac{m}{s})\sin 53 = \boxed{16\,\frac{m}{s}\hat{j}}$ +1

16) $a = \boxed{-9.8\,\frac{m}{s}\hat{j}}$, always +1

17) $\Delta y_{max} - v_y = 0$; $v_{total} = v_x = \boxed{12\,\frac{m}{s}\hat{i}}$ +1

18) $a = \boxed{-9.8\,\frac{m}{s}\hat{j}}$, always +1

19) $v_0 = v_x + v_y$; $v_f = v_x - v_y = (12\hat{i} - 16\hat{j})\,\frac{m}{s} = \boxed{20\,\frac{m}{s}}$ +1

20) $\theta_f = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-16\,m/s}{12\,m/s}\right) = -53° = \boxed{53° \text{ below horiz.}}$ +1

21) Tangent to the circle +1

22) towards the center +1

23) $v = r\omega$ $\quad \omega = \frac{v}{r} = (15\,\frac{m}{s})/4m = \boxed{3.8\,\frac{rad}{s}}$ +1

24) $a_c = \frac{v^2}{r} = (15\,\frac{m}{s})^2/4m = \boxed{56\,m/s^2}$ +1

25) $\boxed{a_t = 0}$ since v_t does not change. +1

26) $a = r\alpha$ $\quad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

$\theta = \int \omega\, dt$ $\quad \omega = \int \alpha\, dt = \alpha t + C_1$ $\quad \omega_0 = 0$; $C_1 = 0$

$\theta = \int \alpha t\, dt = \frac{1}{2}\alpha t^2 + C_2$ $\quad \theta_0 = 0$; $C_2 = 0$

$\boxed{\theta = \frac{1}{2}\alpha t^2}$ +1

27) $x = r\theta = \boxed{\frac{1}{2}\alpha r t^2}$ +1

28) $\omega = \int \alpha\, dt = \alpha t + [\omega_0 = 0]$

$\boxed{\omega = \alpha t}$ +1

29) $v = r\omega = \boxed{r\alpha t}$ +1

30) $\boxed{a_t = r\alpha}$ +1

31) $a_c = \frac{v_t^2}{r} = \frac{(r\omega)^2}{r} = \boxed{r\omega^2}$ express in terms of α, r, t

32) $A \times B = \boxed{|A||B|, \text{ into the page}}$ +1

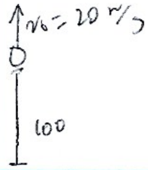
33) $A \times B = \boxed{0}$ +1

34) $A \times B = \boxed{|A||B|, \text{ to the left}}$ +1

35) $A \times B = \boxed{|A||B|, \text{ towards the top of the page}}$ +1

36) $A \times B = \boxed{|A||B|\sin\theta, \text{ out of the page}}$ +1

73



37) a) $\cancel{v_f^0} = v_0 + at$

$t = -\frac{v_0}{a} = -(20\ m/s)/-9.8\ m/s^2 = \boxed{2.0\ s}$ +1

b) $\Delta y = v_0 t + \frac{1}{2}at^2 = (20\ m/s)(2.0\ s) + \frac{1}{2}(-9.8\ m/s^2)(2.0\ s)^2$

$= \boxed{20.4\ m}$ above starting pt, or $\boxed{120\ m}$ above rock

+1

c) $v_f = \boxed{0\ m/s}$ +1

d) $a = \boxed{-9.8\ m/s^2}$, always +1

more parts than in answer key

e) $t_{back} = 2t_{up} = 2(2.0\ s) = \boxed{4.0\ s}$ ✓

f) $v_{back} = -v_0 = \boxed{-20\ m/s}$ ✓

g) $\Delta x = v_0 t + \frac{1}{2}at^2$

$-100\ m = (20\ m/s)t + \frac{1}{2}(-9.8\ m/s^2)t^2$ +5 pts

$-4.9t^2 + 20t + 100 = 0$ ✓

$t = \frac{-20 \pm \sqrt{20^2 + 4 \times 4.9 \times 100}}{-9.8} = \boxed{7.0\ s}$ ✓

h) $v_f = v_0 + at = (20\ m/s) + (-9.8\ m/s^2)(7.0\ s) = \cancel{-48.6\ m/s}$

$\boxed{-49\ m/s}$ ✓

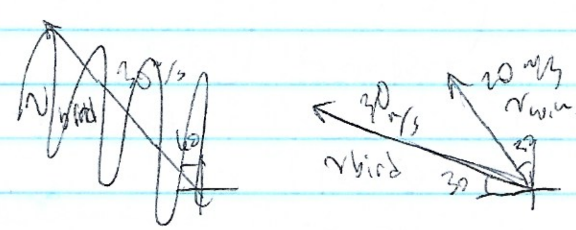
i) $x_f = v_0 t + \frac{1}{2}at^2 + x_0 = (20\ m/s)(3.5\ s) + (\frac{1}{2})(-9.8\ m/s^2)(3.5\ s)^2 + 100\ m$

$= \boxed{110\ m}$ above the rock, $\boxed{10\ m}$ above original location ✓

j) $v_f^2 = v_0^2 + 2a\Delta x$

$\Delta x = \frac{v_f^2 - v_0^2}{2a} = \frac{(10\ m/s)^2 - (20\ m/s)^2}{2(-9.8\ m/s^2)} = \boxed{15\ m}$ above ledge, $\boxed{115\ m}$ above bottom ✓

38) a)



$v_{bird} = 30\ m/s$ @ $150°$

~

b) $v_{BA} = v_{BG} - v_{GA} = v_{BG} + v_{AG} = (30\ m/s \cos 150 + 20\ m/s \cos 120)\hat{i}$

$+ (30\ m/s \sin 150 + 20\ m/s \sin 120)\hat{j}$

$= -36\hat{i} + 32\hat{j}\ m/s$

$= 48\ m/s$ @ $138°$

$= \boxed{48\ m/s @ 48° \text{ W of N}}$ ~

9

a)

+1

39) b) $v_{0x} = v_0 \cos\phi$ +1

$v_{0y} = v_0 \sin\phi$

c) $\Delta x = v_0 t + \cancel{\frac{1}{2}at^2}$ $\quad a_x = 0$

$\Delta x = v_{0x} t = \boxed{v_0 \cos\phi\, t}$ +1

$\Delta y = v_{0y} t + \frac{1}{2} g t^2$

$= \boxed{v_0 \sin\phi\, t + \frac{1}{2} g t^2}$ (signage) ~

d) $\sin\theta = \frac{y}{r}$ $\quad \cos\theta = \frac{x}{r}$

$y = r\sin\theta\ ;\ x = r\cos\theta\ ;\ (r\cos\theta, r\sin\theta)$ +1

e) $\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{r\sin\theta}{x}$ $\quad r\sin\theta = x\tan\theta$

$\boxed{y = x\tan\theta}$ +1

f) $y = v_0 \sin\phi\, t + \frac{1}{2} g t^2$

$x = v_0 \cos\phi\, t$ $\quad t = \frac{x}{v_0 \cos\phi}$

$\cancel{y = \frac{v_0 \sin\theta\, x}{v_0 \cos\theta} + \frac{1}{2} g \left(\frac{x^2}{v_0^2\cos^2\phi}\right) = x\tan\theta + \frac{g x^2}{2 v_0^2 \cos^2\phi}}$

$\cancel{g)\ x = \frac{y}{\tan\theta}\quad y = \left(\frac{y}{\tan\theta}\right)\tan\theta + \frac{g y^2}{2 v_0^2 \cos^2\phi \tan^2\theta}}$

$\cancel{y = y + \frac{g y^2}{2 v_0^2 \cos^2\phi \tan^2\theta}}$

f) $y = v_0 \sin\phi \left(\frac{x}{v_0 \cos\phi}\right) + \frac{g}{2}\left(\frac{x^2}{v_0^2 \cos^2\phi}\right) = \boxed{x\tan\phi + \frac{g x^2}{2 v_0^2 \cos^2\phi}}$ +1 ~

g) $\cancel{x = \frac{y}{\tan\theta}\quad y = \frac{y \tan\phi}{\tan\theta} + \frac{g y^2}{2 v_0^2 \cos^2\phi \tan^2\theta}}$

$x\tan\theta = x\tan\phi + \frac{g x^2}{2 v_0^2 \cos^2\phi}$

$\frac{g x^2}{2 v_0^2 \cos^2\phi} + x(\tan\phi - \tan\theta) = 0$

$x\left(x\left(\frac{g}{2 v_0^2 \cos^2\phi}\right) + (\tan\phi - \tan\theta)\right) = 0$

$\boxed{x = \frac{g(\tan\theta - \tan\phi)}{2 v_0^2 \cos^2\phi}}$ ~ 6

g) ...cont $y = x\tan\theta$

$$= \left(\frac{g(\tan\theta - \tan\phi)}{2v_0^2\cos^2\phi}\right)\tan\theta$$

$$= \boxed{\frac{g(\tan^2\theta - \tan\theta\tan\phi)}{2v_0^2\cos^2\phi}}$$ ~

h) $\sin\theta = \frac{x}{r}$ $\quad r = \frac{x}{\sin\theta} = \boxed{\frac{g(\tan\theta - \tan\phi)}{2v_0^2\cos^2\phi\sin\theta}}$ ~

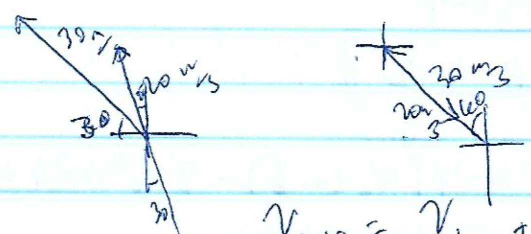
i) Substitute these equations into previous ones and see is they are correct. No. ~

j) use answer from (g) → $x = \frac{g(\tan\theta - \tan\phi)}{2v_0^2\cos^2\phi}$

g, θ, v_0 are all constants

solve for ϕ that gives the largest value of x ~ No.

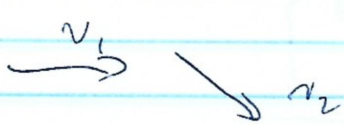
38)



$v_{mr} = v_{bg} + v_{gm} = v_{bg} - v_{mg}$

$= (30\ m/s - 20\ m/s)$ @ $150°$ No.

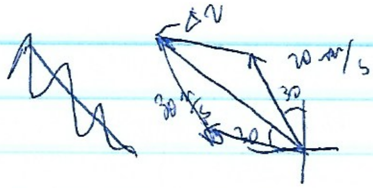
$= \boxed{10\ m/s @ 60° \text{ West } N}$ ~

3) 

$a = \frac{\Delta v}{\Delta t}$  Δv → dir. of acceleration +1

1, blue

38) a)



$v_{bird} = 30\ \text{m/s} @ 150° = (-26\hat{i} + 15\hat{j})\ \text{m/s}$

$v_{wind} = 20\ \text{m/s} @ 120° = (40\hat{i} + 17\hat{j})\ \text{m/s}$

b) $v_{air} = \Delta v = v_{bird} - v_{wind}$

$= (-26\hat{i} + 10\hat{i}) + (15 - 17)\hat{j}\ \text{m/s}$

$= -16\hat{i} - 2\hat{j}\ \text{m/s} = \boxed{16\ \text{m/s} @ 187°}$ angle?

$(\sqrt{x^2+y^2})$ +1 $(\tan^{-1}(\frac{y}{x}))$

39) c) $\Delta y = v_{0y} t + \frac{1}{2} a_y t^2 = v_{0y} t - \frac{1}{2} g t^2$

$\boxed{y = v_0 \sin\phi\, t - \frac{1}{2} g t^2}$ +1

39) f) $x = v_0 \cos\phi\, t$

$t = \dfrac{x}{v_0 \cos\phi}$

$$\boxed{y = \frac{v_0 \sin\phi\, x}{\cos\phi} - \frac{g x^2}{2 v_0^2 \cos^2\phi}} = \boxed{v_0 \tan\phi\, x - \frac{g x^2}{2 v_0^2 \cos^2\phi}}$$

g) $y = x \tan\theta$

$x \tan\theta = v_0 \tan\phi\, x - \dfrac{g x^2}{2 v_0^2 \cos^2\phi}$ +1

$x(v_0 \tan\phi - \tan\theta) = \dfrac{g x^2}{2 v_0^2 \cos^2\phi}$

$$\boxed{x = \frac{2 v_0^2 \cos^2\phi\, (v_0 \tan\phi - \tan\theta)}{g}}$$ +1

$y = x \tan\theta = \frac{1}{g}(2 v_0^2 \cos^2\phi\, (v_0 \tan\phi) - 2 v_0^2 \cos^2\phi \tan\theta) \tan\theta$

$$\boxed{y = \frac{1}{g}(2 v_0^3 \sin\phi - 2 v_0^2 \sin^2\theta)}$$ +1

h) $r = \dfrac{x}{\sin\theta} = \boxed{\dfrac{2 v_0^3 \sin\phi - 2 v_0^2 \sin^2\theta}{g \sin\theta}}$ ~

31) $a_c = r\omega^2 = \boxed{r \alpha^2 t^2}$ $[\omega = \alpha t]$ +1

A 5

34) f) $\frac{y}{x} = \tan\phi - \frac{gx}{2v_0^2\cos^2\phi}$ +1

g) $x = \frac{2}{g} v_0^2 (\tan\phi - \tan\theta)\cos^2\phi$

$y = \frac{2}{g} v_0^2 (\tan\phi - \tan\theta)\tan\theta\cos^2\phi$

h) $r = \frac{2v_0^2\cos\phi\sin(\phi-\theta)}{g\cos^2\theta}$ +2

i) use $\theta = 0$ to find r if ground is level. +1

j) derive r w/respect to ϕ to get

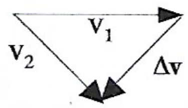
$$\cos\phi_0\cos(\phi_0-\theta) - \sin\phi_0\sin(\phi_0-\theta) = 0$$ +1

5

EACH PART IS ONE POINT UNLESS OTHERWISE STATED.

Questions 1-2.

1. **Scalars**: Mass, Distance, Speed 1 pt
2. **Vectors**: Force, Velocity, Acceleration, Areas, Displacement 1 pt
3.



1 pt

Since $\mathbf{a}=\frac{\Delta \mathbf{v}}{\Delta t}$, $\Delta\mathbf{v}$ gives the direction of the acceleration.

4. If an object's acceleration is constant, then it must move in a straight line. *Not correct. Counter example: Gravity near Earth.*
If an object's acceleration is zero, then its speed must remain constant. *Correct since* $\mathbf{a}=\frac{\Delta \mathbf{v}}{\Delta t}$ *requires* $\Delta\mathbf{v}=0$. *Both the direction and the magnitude of velocity must remain the same.*
If an object's speed remains constant, then its acceleration must be zero. *Not correct. Counter example: circular motion with constant speed.*
1 pt

5. $\vec a = g\downarrow$ 1 pt
6. $\vec v_{\max h}=0$ since it is thrown straight up. 1 pt
7. $\vec v_{\max h}=0=\vec v_o-\vec g t=25-10t$, $t=2.5s$ 1 pt
8. $\Delta h=v_{av}t$ or $\Delta h=v_o t-\frac{1}{2}gt^2$. Both give $31.25m$ 1 pt
9. $t=2\times 2.5s=5s$ 1 pt
10. $\vec d=\vec v_o t+\frac{1}{2}\vec a t^2$, $200m=0+\frac{1}{2}5\frac{m}{s^2}t^2$, $t=\sqrt{80}s\approx 9s$ 1 pt 10
11. $\vec v=\vec v_o+\vec g t$, $30\frac{m}{s}=10\frac{m}{s^2}t$, $t=3s$ 1 pt

$\Delta\vec h=\vec v_o t+\frac{1}{2}\vec g t^2$, $\vec h=0+\frac{1}{2}10\frac{m}{s^2}\downarrow(3s)^2$, $\vec h=45m\downarrow$ 1 pt

12. Horizontal motion has no effect on the vertical.

$80m\downarrow=0+\frac{1}{2}10\frac{m}{s^2}\downarrow t^2$, $t=4s$ 1 pt

13. $\vec g=10\frac{m}{s^2}\downarrow$ 1 pt
14. $v_{ox}=20\frac{m}{s}\cos 53=12\frac{m}{s}$ 1 pt
15. $v_{oy}=20\frac{m}{s}\sin 53=16\frac{m}{s}$ 1 pt
16. $\vec g=10\frac{m}{s^2}\downarrow$ 1 pt
17. Since $v_y=0$ at the maximum height, $\vec v=\vec v_x+\vec v_y=\vec v_{ox}=12\frac{m}{s}\rightarrow$ 1 pt
18. $\vec g=10\frac{m}{s^2}\downarrow$ 1 pt
19. $\vec v=\vec v_x+\vec v_y=v_{ox}\rightarrow + v_{oy}\downarrow$. However, the magnitude does not change $v=\sqrt{v_{ox}^2+v_{oy}^2}=20\frac{m}{s}$ 1 pt

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20. $\tan^{-1}\left(\frac{-v_y}{v_x}\right)=53°$ below the horizontal. 1 pt
21. Tangent to the path. 1 pt
22. Toward the center. 1 pt
23. $\omega=\frac{v}{r}=\frac{15}{4}=3.75\ rad/s$ 1 pt
24. $a_c=\omega^2 r=\frac{v^2}{r}=\frac{15^2}{4}=56.25\frac{m}{s^2}$ 1 pt
25. $a_t=0$ since the speed remains constant. 1 pt
26. $\theta=\theta_o+\omega_o t+\frac{1}{2}\alpha t^2=\frac{1}{2}\alpha t^2$ 1 pt
27. $s=\theta r=\frac{1}{2}r\alpha t^2$ 1 pt
28. $\omega=\omega_o+\alpha t=\alpha t$ 1 pt
29. $v=r\omega=r\alpha t$ 1 pt
30. $a_t=r\frac{d\omega}{dt}=r\alpha$ 1 pt
31. $a_c=\omega^2 r=r\alpha^2 t^2$ 1 pt
32. $\otimes$ 1 pt
33. 0 1 pt
34. $\leftarrow$ 1 pt
35. $\uparrow$ 1 pt
36. $\bullet$ 1 pt
37.

A. $\vec v_{\max h}=\vec v_o+\vec g t \Rightarrow 0=20\frac{m}{s}\uparrow+10\frac{m}{s^2}\downarrow t\Rightarrow t=2s$ 1 pt

B. $\vec h=\vec h_o+\vec v_{av}t=0+\left(\frac{20\uparrow+0}{2}\right)\frac{m}{s}\times 2s$ or

$\vec h=\vec h_o+\vec v_o t+\frac{1}{2}\vec g t^2$

$=0+20\frac{m}{s}\uparrow 2s+\frac{1}{2}10\frac{m}{s^2}\downarrow(2s)^2=20m\uparrow$ 1 pt

C. 0 1 pt 40

D. Moving under the effect of gravity: $\vec g=10\frac{m}{s^2}\downarrow$. 1 pt

E. It takes 2s to go up and 2s to come down. Therefore, the velocity back at the original height will have the same magnitude as the initial velocity but in the opposite direction: $\vec v=20\frac{m}{s}\downarrow$. You can also use 1 pt

$\vec v_{back}=\vec v_o+\vec g t=20\frac{m}{s}\uparrow+10\frac{m}{s^2}\downarrow 4s=20\frac{m}{s}\downarrow$.

F. $\vec h-\vec h_o=\vec v_o t+\frac{1}{2}\vec g t^2\Rightarrow 100m\downarrow=20\frac{m}{s}\uparrow t+\frac{1}{2}10\frac{m}{s^2}\downarrow t^2=0$

$\Rightarrow 5t^2-20t-100=0$ 1 pt

$t=\frac{-(-20)\pm\sqrt{(-20)^2-4(5)(-100)}}{2(5)}=\frac{20\pm\sqrt{2400}}{10}$ 1 pt

$=(2\pm\sqrt{24})s\approx 7s,-3s$

Since we are going forward in time, t=7s is the correct answer.

G. $\vec{v}_{impact} = \vec{v}_0 + \vec{g}t = 20\frac{m}{s}\uparrow + 10\frac{m}{s^2}\downarrow 7s = 50\frac{m}{s}\downarrow$ 1 pt

$$\vec{b} - \vec{b}_0 = \vec{v}_0 t + \frac{1}{2}\vec{g}t^2$$

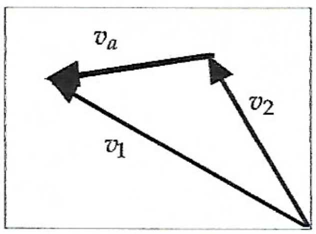
$$= 20\frac{m}{s}\uparrow 3.5s + \frac{1}{2}10\frac{m}{s^2}\downarrow (3.5s)^2 = 8.75m\uparrow$$ 1 pt

$$\vec{v} = \vec{v}_0 + \vec{g}t \Rightarrow 10\frac{m}{s}\downarrow = 20\frac{m}{s}\uparrow + 10\frac{m}{s^2}\downarrow t \Rightarrow t = 3s$$

Use $\vec{b} - \vec{b}_0 = \vec{v}_{av}t = 0 + \left(\frac{20\uparrow + 10\downarrow}{2}\right)\frac{m}{s} \times 3s$ or

$$\vec{b} - \vec{b}_0 = \vec{v}_0 t + \frac{1}{2}\vec{g}t^2$$

$$= 20\frac{m}{s}\uparrow 3s + \frac{1}{2}10\frac{m}{s^2}\downarrow (3s)^2 = 15m\uparrow$$ 1 pt



38.

$\vec{v}_g = \vec{v}_a + \vec{v}_w$, $\vec{v}_{gx} = \vec{v}_{ax} + \vec{v}_{wx}$, $\vec{v}_{gy} = \vec{v}_{ay} + \vec{v}_{wy}$

$v_1 \sin\theta_1 \leftarrow = \vec{v}_{ax} + v_2 \sin\theta_2 \leftarrow$

$\vec{v}_{ax} = (v_1 \sin\theta_1 - v_2 \sin\theta_2)\leftarrow$ cosθ, 1 pt

$v_1 \cos\theta_1 \uparrow = \vec{v}_{ay} + v_2 \cos\theta_2 \uparrow$

$\vec{v}_{ay} = (v_1 \cos\theta_1 + v_2 \cos\theta_2)\uparrow$ sinθ 1 pt

$\tan\theta = [(v_1 \cos\theta_1 + v_2 \cos\theta_2)/(v_1 \sin\theta_1 - v_2 \sin\theta_2)]$

$$v_a = \sqrt{v_{ax}^2 + v_{ay}^2}$$

$$= \sqrt{(v_1 \sin\theta_1 - v_2 \sin\theta_2)^2 + (v_1 \cos\theta_1 + v_2 \cos\theta_2)^2}$$

$$= \sqrt{v_1^2 + v_2^2 - 2v_1 v_2 \cos(\theta_1 - \theta_2)}$$

The last line canbe obtained using the law of cosine. 1 pt

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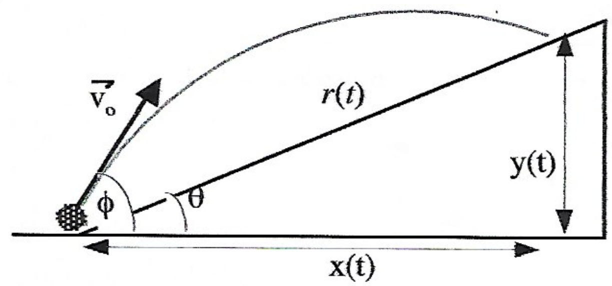
39.

a.



1 pt

b. $v_{ox} = v_o \cos\phi$, $v_{oy} = v_o \sin\phi$ 1 pt

c. $x = v_{ox}t = v_o t\cos\phi$, 1 pt

$$y = v_{oy}t - \frac{1}{2}gt^2 = v_o t\sin\phi - \frac{1}{2}gt^2$$ 1 pt

d. $x = r\cos\theta$, $y = r\sin\theta$ 1 pt

e. $\frac{y}{x} = \tan\theta$, $y = x\tan\theta$ 1 pt

f. $t = \frac{x}{v_o\cos\phi}$, $y = x\tan\phi - \frac{1}{2}g\left(\frac{x}{v_o\cos\phi}\right)^2$ 1 pt

$$\frac{y}{x} = \tan\phi - \frac{gx}{2v_o^2\cos^2\phi}$$

g. $\frac{y}{x} \Rightarrow \tan\theta = \tan\phi - \frac{gx}{2v_o^2\cos^2\phi}$ 1 pt

$$x = \frac{2}{g}v_o^2(\tan\phi - \tan\theta)\cos^2\phi$$

$$y = \frac{2}{g}v_o^2(\tan\phi - \tan\theta)\tan\theta\cos^2\phi$$ 1 pt

h. $r = \sqrt{x^2 + y^2} = x\sqrt{1 + \frac{y^2}{x^2}}$ 1 pt

$$r = \frac{2}{g}v_o^2(\tan\phi - \tan\theta)\cos^2\phi\sqrt{1 + \tan^2\theta}$$

$$r = \frac{2}{g}v_o^2(\tan\phi - \tan\theta)\frac{\cos^2\phi}{\cos\theta}$$

$$= \frac{2}{g}v_o^2\left(\frac{\sin\phi\cos\theta - \sin\theta\cos\phi}{\cos\theta\cos\phi}\right)\frac{\cos^2\phi}{\cos\theta}$$ 1 pt

$$r = \frac{2v_o^2\cos\phi\sin(\phi - \theta)}{g\cos^2\theta}$$

i. $\theta = 0$ should give the same r that we would find for level ground:

$r = \frac{2v_o^2\cos\phi\sin\phi}{g}$ which is precisely so. 1 pt

j. $\left.\frac{dr}{d\phi}\right]_{\phi_o} = \frac{2v_o^2}{g\cos^2\theta}\frac{d}{d\phi}[\cos\phi\sin(\phi - \theta)] = 0$ 1 pt

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$$\cos\phi_o\cos(\phi_o - \theta) - \sin\phi_o\sin(\phi_o - \theta) = 0$$