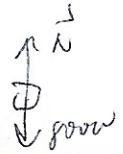
Newton's Laws

Block 25
Block 6
Plane + Block 31
Part 3
total 34



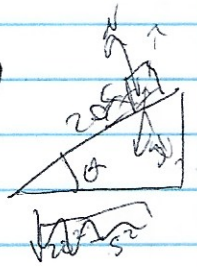
1) A 2) $\Sigma F = ma$

B

$N - mg = ma$

$N = 800N + \left(\frac{800}{10}\right)(5\frac{m}{s^2})$ ✓

$= 800 + 400 = 1200\ N$ (D)



3) $\Sigma F_y = mg\sin\theta$

$= (2kg)(10\frac{m}{s^2})\left(\sin\left(\frac{1}{20}\right)\right)$

$= 5N$ (A) ✓

4) $\Sigma F = F - \mu N = F - \mu mg = ma$

$a = \frac{F}{m} - \mu g$

$= \frac{(15N)}{2kg} - 0.4 \cdot 10$ ✓

$= 9 - 4 = 5\frac{m}{s^2}$ (C)

5)

$\Sigma F = mg\sin\theta - \mu mg\cos\theta$

$= ma$

$g\sin\theta - \mu g\cos\theta = a$ $\quad \theta = 30$

$a = g\sin\theta - 0.5\, g\cos\theta$

$= \frac{1}{2}g - \frac{1}{2}\cdot\frac{\sqrt{3}}{2}g$

$= g\left(\frac{1}{2} - \frac{\sqrt{3}}{4}\right)$

(A) ✓

6)

$\Sigma F = ma$

$\Sigma F_{Total} = Mg - mg = (m+M)a$

$a = g\left(\frac{M-m}{M+m}\right)$

(e) ✓

7) & 8) $\Sigma F = ma$ (D) ✓

9) $\Sigma F = F - \mu_s N$

$\mu_s N =$ [illegible]

$0.4 \cdot 100kg \cdot 9.8\frac{m}{s^2}$

$= 392\ N$

$344 < 392$ (D) ✓

10) [illegible] 11) II (A) ✓

(C) ✓ A ~ B

12) $F_c = \frac{mv^2}{r}$

$v = \sqrt{\frac{F_c}{m}} = \sqrt{\frac{50 \cdot 0.4}{5}}$

$= \sqrt{10}\frac{m}{s} = 3\frac{m}{s}$

(C) ✓ B

13) $F_c = F_f = \frac{mv^2}{r}$

$v = \sqrt{rg} = \sqrt{0.6 \cdot 10} = \sqrt{6}$

$= 2.4\frac{m}{s}$ (C) ✓

14) $T = 1\ s$ $\quad r = 5\omega$

$f = \frac{1}{T} = 1\ Hz$

$\omega = 2\pi f = 2\pi\ \frac{rad}{s}$

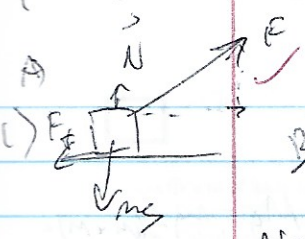
$a_c = \frac{v^2}{r} = \frac{r^2\omega^2}{r} = r\omega^2$

$= 5 \cdot 4\pi^2$ ✓

(d)

BL 4 K 1 BL 5 R 3

FRQ

1) a) 

b) $F_y = F\sin\theta$

$N = mg - F\sin\theta$ ✓

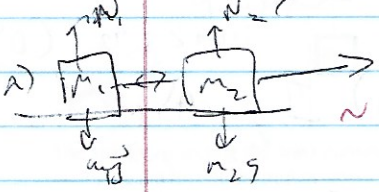
c) $\sum F_y = 0$ $\quad \sum F = F\cos\theta - F_f$

$= F\cos\theta - \mu(mg - F\sin\theta)$

$= F(\cos\theta + \sin\theta) - \mu mg$

$a = \frac{F}{m}(\sin\theta + \cos\theta) - \mu g$ ✓

d) max $\sin\theta + \cos\theta$: $\theta = 45°$ ~

2) a) 

~ draw both

b) $\sum F = m_{sys}\, a = F$

$F = (m_1 + m_2)\, a$

$a = \frac{F}{m_1 + m_2}$ ✓

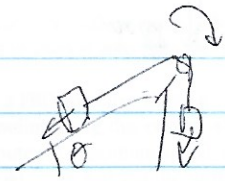
c) $a_1 = a_2$

$\sum F_1 = T_2 = m_1 a$

$\sum F_2 = F - T_2 = m_2 a$

$T = m_1 a = \frac{m_1 F}{m_1 + m_2}$ ✓

d) i) $a = \frac{F}{m_1 + m_2 + m}$ ✓ $\quad F_2 = ?$ ★

3) 

~ draw both axes

A) $\sum F = m_2 g - m_1 g\sin\theta = (m_1 + m_2)\, a$

(i)

$a = \frac{m_2 g - m_1 g\sin\theta}{m_1 + m_2}$ ✓

$m_2 g - m_1 g\sin\theta$ must be positive

$m_2 g = m_1 g\sin\theta$

$\sin\theta = \frac{m_2}{m_1}$

$\theta \geq \sin^{-1}\left(\frac{m_2}{m_1}\right)$ ✓

(ii) $\sum F = m_2 g - m_1 g\sin\theta = 0$

$\sin\theta = \frac{m_1}{m_2}$ ~

B) $m_2 g - m_1 g\sin\theta - \mu m_1 g\cos\theta = (m_1 + m_2)\, a$

(i) $a = \frac{m_2 g - m_1 g\sin\theta - \mu m_1 g\cos\theta}{m_1 + m_2}$ ✓

$m_2 g = m_1 g\sin\theta + \mu m_1 g\cos\theta$

$(\sin\theta + \mu\cos\theta) = \frac{m_2}{m_1}$ ✓

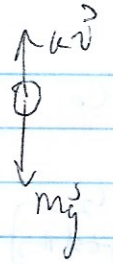
(ii) $m_2 g - m_1 g\sin\theta - \mu m_1 g\cos\theta = 0$

$m_1(g\sin\theta + \mu g\cos\theta) = m_2 g$

$\sin\theta + \mu\cos\theta = \frac{m_2}{m_1}$

4) $F = kv$

A)



✓

B) $\sum F = ma = mg - k\vec{v}$

$a = g - \frac{kv}{m}$ ✓

c) $mg = kv$

$v = \frac{mg}{k}$ ✓

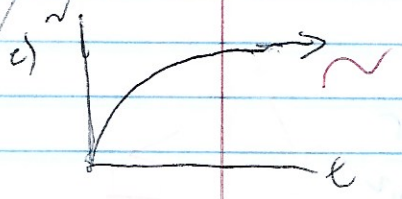
d) $ma = mg - kv$

$m \cdot \frac{dv}{dt} = mg - kv$

$\frac{dv}{dt} = g - \frac{kv}{m}$

$m \cdot \frac{dv}{dt} + kv = mg$

e)



~

f) $\frac{dx}{dt} = v$

d) $m\,dv = (mg - \frac{kv}{m})dt$

$mv = mgt - kvt$

$v(m + kt) = mgt$

$v = \frac{mgt}{m+kt}$ ~

3

f) $\frac{mgt}{m+kt} = \frac{dx}{dt}$

$\frac{mgt}{m+kt}\,dt = dx$

$x = \int \frac{mgt}{m+kt}\,dt$

$u = m + kt$

$\frac{du}{dt} = k$

$dt = \frac{du}{k}$

$= \int \frac{mgt}{u} \cdot \frac{du}{k}$

$= \frac{mg}{k} \int \frac{t}{u}\,du$

$= \frac{mg}{k} \int \frac{\frac{u-m}{k}}{u}\,du$

$= \frac{mg}{k^2} \int \frac{u-m}{u}\,du$

$= \frac{mg}{k^2} \int 1 - \frac{m}{u}\,du$

$= \frac{mg}{k^2}\left(u - m\ln|u|\right)$

$= \frac{mg}{k^2}\left(k + m - m\ln|k+m|\right)$

$= \frac{mg}{k^2}\left(kt + m - m\ln|kt+m|\right)$

$= \frac{mgt}{k} + \frac{mg}{k^2}\left(m - m\ln|kt+m|\right)$

$= \frac{mgt}{k} - \frac{mg}{k^2}\left(m\ln|kt+m|\right)$

$= \frac{mgt}{k} - \frac{mg}{k^2} \cdot m \ln|kt+m|$

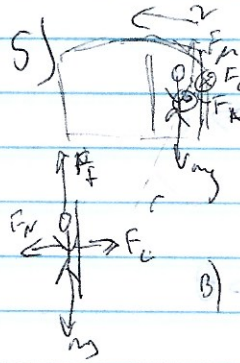
$= \frac{mgt}{k} - \frac{m^2 g}{k^2}\ln|kt+m|$

$= \int \frac{k}{mg} - \frac{1}{gt}\,dt$

$= \frac{kt}{mg} - \frac{1}{g}\int \frac{1}{t}\,dt$

$x = \frac{kt}{mg} - \frac{1}{g}\ln|t|$ ~

5)



b/c the reference frame is non inertial, centrifugal force is included.

B) $F_n = \frac{mv^2}{r}$ $\sum F = mg = 0$

$\mu N - mg = 0$

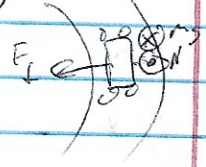
$\mu \frac{mv^2}{r} = \mu mg$

$v = \sqrt{\frac{rg}{\mu}}$

~ μ_{max} ~ solve for μ

c) mass doesn't matter here. ✓

6) a)

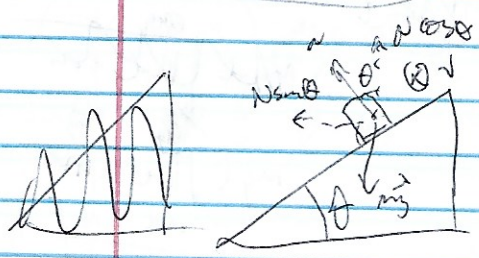


b) $\frac{mv^2}{r} = \mu mg$

$v = \sqrt{\frac{\mu mgr}{m}}$

$\boxed{v = \sqrt{\mu rg}}$ ✓

c)



d) $\frac{mv^2}{r} = N\sin\theta$

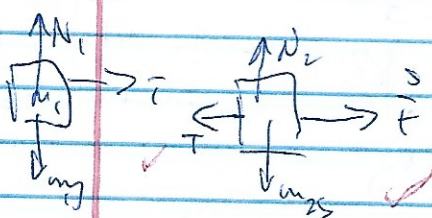
$N\cos\theta = mg$ $\frac{mv^2}{r} = \frac{mg}{\cos\theta}\sin\theta$

$N = \frac{mg}{\cos\theta}$

$\boxed{v = \sqrt{rg\tan\theta}}$

too low θ 3

2) a)



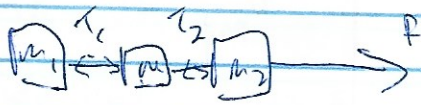
b) maximize $\sin\theta\cos\theta$

$\frac{d}{d\theta}\sin\theta\cos\theta = 0$

$\cos^2\theta - \sin^2\theta = 0$

$\cos^2\theta = \sin^2\theta$ $\theta = 45°$

2) d) is equiv. to



$F = ma$

$qE = ma$

$T_1 + T_2 - F = ma$ ✓

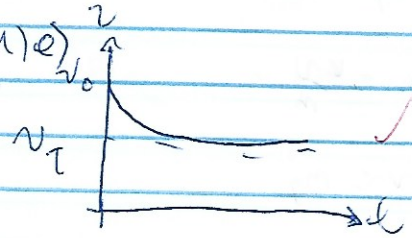
3) A) (if) $\sum F = m_2 g - m_1 g\sin\theta = 0$

$m_1 g\sin\theta = m_2 g$

$\sin\theta = \frac{m_2}{m_1}$

$\theta \leq \sin^{-1}\left(\frac{m_2}{m_1}\right)$ ✓

4) a)



✓

5) b) $\frac{mv^2}{r} = g$

$\mu = \frac{gr}{v^2}$ ✓ $>$ $=$

c) d) $\frac{v^2}{r} = g\tan\theta$

$\tan\theta = \frac{v^2}{rg}$

$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$ ✓

2

4

1d) maximum $\cos\theta + \mu \sin\theta$

$\frac{d}{d\theta}(\cos\theta + \mu \sin\theta) = 0$

$-\sin\theta + \mu\cos\theta = 0$

$\mu = \frac{\sin\theta}{\cos\theta} = \tan\theta$ ✓

g, μ, h

2) b) $a = \frac{F}{m_1 + m_2 + m}$

$F + F_1 - F_2 = m_1 + m_2 \, a$

$F_2 - F_1 = F - (m_1 + m_2) a$

$= F\left(\frac{m_2}{m_1 + m_2 + m}\right)$ ✓

4) c) $a = \frac{dv}{dt}$ $\frac{dv}{dt} = \frac{k}{m}\left(v - g\frac{m}{k}\right)$

$\frac{dv}{v - g\frac{m}{k}} = -\frac{k}{m} dt$

$\ln\left|v + g\frac{m}{k}\right| = -\frac{k}{m} t + C$

$v - g\frac{m}{k} = e^{C} e^{-\frac{k}{m}t}$

(a) $t = 0, v = v_0$; $e^{C} = v_0 - g\frac{m}{k}$

$v(t) = \frac{m}{k} g\left(v_0 - \frac{k}{k} g\right) e^{\frac{k}{m}t}$ ✓