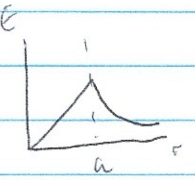
Cumulative 2

1) $E = \begin{cases} \alpha r \hat{r}, & r \le a \quad (1)\ \text{region I} \\ \frac{\beta}{r^2}\hat{r}, & r > a \quad (2)\ \text{region II} \end{cases}$

a) $\alpha r = \frac{\beta}{r^2}$ when $r = a$

$\alpha a = \frac{\beta}{a^2}$ $\qquad \boxed{\alpha = \frac{\beta}{a^3}}$



b) region I: $E = -\frac{dV}{dr}$

$\cancel{+E\,dV = dr}$

$\cancel{dV = -\frac{dr}{E}}$ $\qquad V = -\int E \cdot dr$

$\cancel{V = -\int E}$

r≤a $\quad V = -\alpha \int \vec{r}\, dr$

$\cancel{\Big|_0^r}$ $\quad = -\alpha \cdot \frac{1}{2} r^2 + C$

$C = 0 \quad r = 0\ E = 0\ V = 0$

$= \boxed{-\frac{\alpha}{2} r^2}$

~~Region II: $V = -\int E \cdot dr$~~

(−) negative potential means

(+) charges are ~~more~~ attracted towards the center and (−) charges are attracted towards $\hat{r}$

region II: $V = -\int_a^r E \cdot dr \qquad r^{-2}$

$= -\beta \int_a^r \frac{dr}{r^2}$

$= \frac{\beta}{r}\Big|_a^r$

$= \frac{\beta}{r} - \frac{\beta}{a} = \boxed{\beta\left(\frac{1}{r} - \frac{1}{a}\right)}$

c) region I: continuous distribution of charge equally distributed throughout the sphere

uniformly charged sphere

$E = \frac{kQ}{R^3}\vec{r} \qquad \alpha = \frac{kQ}{R^3} \qquad R = a$

$\sigma = \frac{Q}{\text{Volume}} \qquad Q = \frac{\alpha a^3}{k}$

$\text{Volume} = \frac{4}{3}\pi r^3$

$\sigma = \frac{\alpha a^3}{k} \cdot \frac{3}{4} \cdot \frac{1}{\pi a^3} = \boxed{\frac{3\alpha}{4\pi k}}$

d) $\Sigma F = ma = F_e = (e)E$

$= -e\alpha\vec{r} = ma$

$\boxed{a = \frac{-e\alpha r}{m}\hat{r}}$

towards the center

e) v_{max} is when $r = 0$

$\frac{1}{2} m v_{max}^2 = qV_0$

$\frac{1}{2} m v^2 = (-e)\left(-\frac{\alpha a^2}{2}\right)$

$\cancel{\frac{1}{2}} m v^2 = \frac{e\alpha a^2}{\cancel{2}}$

$v^2 = \frac{e\alpha a^2}{m} \qquad \boxed{v = \sqrt{\frac{e\alpha a^2}{m}}}$

f) the electron executes SHM from a to −a through the center, in the radial direction.

g) yes, it gets all the way to the other side b/c of ~~[illegible]~~ CoE a)

2) $\vec{E}$ is $\perp$ to equipotential lines.

E is radially outward or inward from the origin

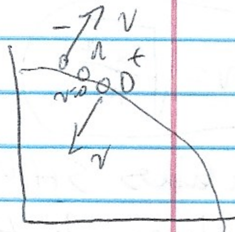
because voltage increases as r increases, V is $\frac{PE}{q}$, so (+) charges would have more potential energy the farther they get, so they would be attracted toward the origin. therefore, <u>the $\vec{E}$ points radially towards the origin.</u>

at (C), $V = -Er$

$E = \frac{V}{r} = \frac{20V}{2m} = \boxed{10 \frac{N}{C}}$

at (D), $E = \frac{V}{r} = \frac{30V}{3m} = \boxed{10 \frac{N}{C}}$

both towards the origin...

b) 

for the electron, it will move radially away from the center

the neutron will stay in place.

the proton will move towards the center.

c) $\Delta V = 10V$

(1) $KE = -\Delta PE = q\Delta V = 10V(10^{-19}C)$

$= \boxed{10^{-18} J}$

for both electron & proton

$KE_n = 0$

(2) electron: $KE = \frac{1}{2}mv^2$

$v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2(10^{-18}J)}{10^{-30}kg}}$

$= \boxed{1.414 \times 10^6 \ m/s}$

proton: $v = \sqrt{\frac{2KE}{m}}$

$= \sqrt{\frac{2(10^{-18}J)}{10^{-27}kg}} = \boxed{4.47 \times 10^4 \ m/s}$

$v_n = 0$

3) $B = \begin{cases} \beta r \hat{\theta} & r \le r_0 \\ \frac{\lambda}{r}\hat{\theta}, & r > r_0 \end{cases}$

a) when $r = r_0$, $\beta r = \frac{\lambda}{r}$

$\beta = \frac{\lambda}{r^2}$

b) at $r = r_0$, $F = qvB\sin\theta$ ($\sin\theta = 1$)

$= q v_0 \beta r_0$

v_0 is towards the center

$= -e v_0 \beta r_0 = ma$

$a = \frac{-e v_0 \beta r_0}{m}$

perpendicular to the velocity

c) $v_{max} = v_0$?

d) the electron takes an elliptical orbit within the inner region

e) $I = \frac{dQ}{dt}$ $\frac{dr}{dt} = v$

4) a) grav. field points towards the origin.

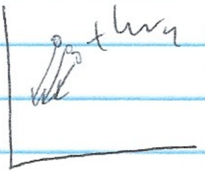
at (c): $V = \frac{PE}{m}$ $g = \frac{V}{r}$

$= \frac{20\ J/kg}{2\ m}$

$= 10\ m/s^2$

at P: $g = \frac{V}{r} = \frac{30\ J/kg}{3\ m} = 10\ m/s^2$

both towards center.

b)  they will all move towards the origin.

c) $\Delta KE = -\Delta PE = m\Delta V = 10\ J/kg \cdot m$

(1) electron: $KE = (10\ J/kg)(10^{-30}\ kg)$

$= 10^{-29}\ J$

proton and neutron: $KE = (10\ J/kg)(10^{-27}\ kg)$

$= 10^{-26}\ J$

(2) electron: $v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2(10^{-29}\ J)}{10^{-30}\ kg}}$

$= 4.47\ m/s$

proton & neutron: $v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2(10^{-26}\ J)}{10^{-27}\ kg}}$

$= 4.47\ m/s$