Differentiationor the Rate of Change

$$\frac{d}{dt}t^{n}=nt^{n-1}$$

$$\frac{d}{dt}\left(f(t)g(t)\right)=\left(\frac{d}{dt}f(t)\right)g(t))+f(t)\left(\frac{d}{dt}g(t)\right)$$

$$\frac{d}{dt}\left(f(t)+g(t)\right)=\left(\frac{d}{dt}f(t)+\frac{d}{dt}g(t)\right)$$

Integration or the Sum

$$\int t^{n}dt=\frac{t^{n+1}}{n+1}+Ct$$

$$\int\left(f(t)+g(t)\right)dt=\int f(t)dt+\int g(t)dt$$

$$\int udv=uv-\int vdu$$

This is a result of the chain rule of differentiation above

$d(uv)=(du)v+vdu$. Integrate $\int d(uv)=\int(du)v+\int vdu$

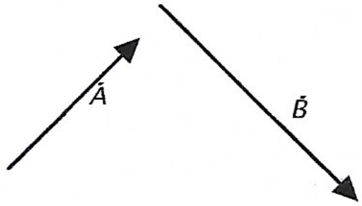
Questions and Problems

Make a list of physical quantities which are the derivative of another physical quantity. With respect to what variable each one is a rate of change?

1. Make a list of physical quantities which are the derivative of another physical quantity. With respect to what variable each one is a rate of change?
2. Make a list of physical quantities which are the integral of another physical quantity. With respect to what variable each one is an integral/sum?
3. Obtain the derivatives and integrals of
   A. $ax^{m}+bx^{k}$
   B. $a\cos mt+b\sin kt$
   C. $ae^{mt}+be^{-kt}$

Vectors

Consider two vectors **A** and**B**:

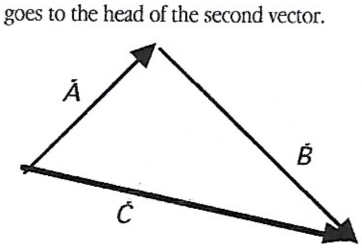


Addition of Vectors:

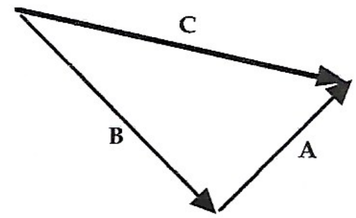
$\mathbf{A}+\mathbf{B}=\mathbf{C}$

$\mathbf{B}+\mathbf{A}=\mathbf{C}$

Head-to-Tail and the resultant starts from the tail of the first vector and goes to the head of the second vector.



Introduction



Notice in both cases the size and the direction of the resultant vector **C** remain the same, i.e. independent of the order of addition.

Subtraction of Vectors:

There are two ways of subtracting vectors.

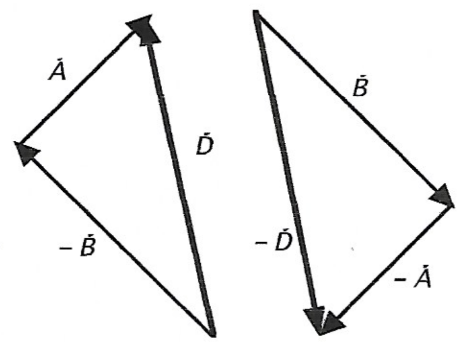
Method 1

a. Reverse the direction of the vector to be subtracted
b. Follow the methods of addition

$\mathbf{A}-\mathbf{B}=\mathbf{D}$

$\mathbf{B}-\mathbf{A}=-\mathbf{D}$

Head-to-Tail and the resultant starts from the tail of the first vector and goes to the head of the second vector.

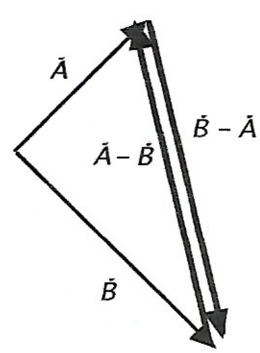


Notice that even though the size of the resultant is the same, its direction has changed as it should.

Method 2: Put the vectors tail to tail

A–B = **D** is the vector that starts at the tip of **B** and ends at the tip of **A**.

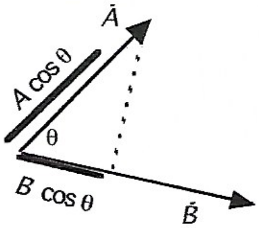
B – **A**= **–D** is the vector that starts at the tip of **A** and ends at the tip of**B**.



Multiplication of vectors:

I. Scalar (Dot) Product: Obtain the projection of one vector onto another, then multiply the magnitudes. The result of a scalar product of two vectors is a scalar.

$\vec{A}.\vec{B} = AB\cos\theta$



I. Vector (Cross) Product

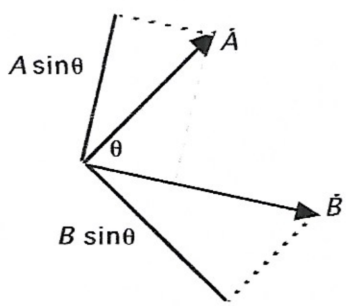
Magnitude: Obtain the component of one vector perpendicular to the other, then multiply them.

Direction: Use the right hand rule

1. Point your index finger in the direction of the first vector
2. Point you middle finger in the direction of the second vector
3. Your thumb show the direction of the product

The result of a vector product of two vectors is a vector. Notice that the magnitude of the vector product is equal to the area of the parallelogram these two vectors represent.

$\vec{A}\times\vec{B} = AB\sin\theta \;\; \overrightarrow{RHR},\;\; \vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

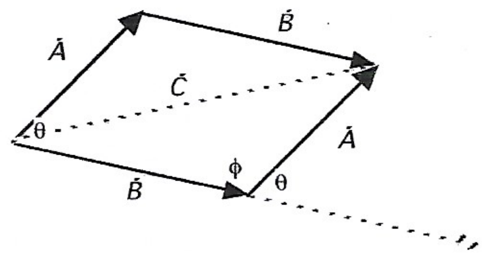


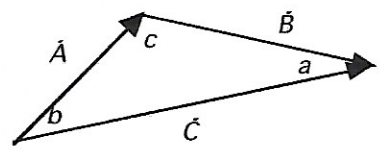
The Law of Cosines:

$\vec{A}-\vec{B}=\vec{D}$

$\vec{D}.\vec{D}=(\vec{A}-\vec{B}).(\vec{A}-\vec{B})=\vec{A}.\vec{A}+\vec{B}.\vec{B}-2\vec{A}.\vec{B}$

$D^2 = A^2 + B^2 = 2AB\cos\phi$





$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Vectors

1. Given $\vec{A} = (2,-3,0)$ and $\vec{B} = (-4,2,0)$, compute
   - A. $\vec{A}+\vec{B},\ |\vec{A}+\vec{B}|$
   - B. $\vec{A}-\vec{B},\ |\vec{A}-\vec{B}|$
   - C. $\vec{A}+3\vec{B},\ |\vec{A}+3\vec{B}|$
   - D. $\vec{A}\bullet\vec{B},\ \vec{B}\bullet\vec{A}$
   - E. $\vec{A}\times\vec{B},\ \vec{B}\times\vec{A}$

2. A bird is flying due southwest 30 m/s.
   - A. Obtain the north and east components of the velocity vector.
   - B. Now there is a wind blowing due north 10 m/s. Obtain the bird's velocity with respect to ground using
     1. the law of sines
     2. the law of cosines
     3. components.
     4. Give the final velocity vector in terms of its magnitude and direction with respect to East.
     5. Give the final velocity vector in terms of x, y, z components.

Solutions

1. $\vec{A} = (A_x, A_y, A_z)$

$\vec{A}+\vec{B} = (A_x+B_x,\ A_y+B_y,\ A_z+B_z)$

$\vec{A}-\vec{B} = (A_x-B_x,\ A_y-B_y,\ A_z-B_z)$

$\vec{A}\bullet\vec{B} = A_xB_x + A_yB_y + A_zB_z$

$\vec{A}\times\vec{B} = \vec{i}\,(A_yB_z - A_zB_y) + \vec{j}(A_zB_x - A_xB_z) + \vec{k}(A_xB_y - A_yB_x)$

$|\vec{A}| = \sqrt{A_x^{\,2} + A_y^{\,2} + A_z^{\,2}}$.

Therefore, with $\vec{A} = (2,-3,0)$ and $\vec{B} = (-4,2,0)$,

A. $\vec{A}+\vec{B} = (-2,-1,0)$, $|\vec{A}+\vec{B}| = \sqrt{5}$

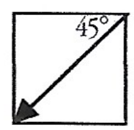
B. $\vec{A}-\vec{B} = (6,-5,0)$, $|\vec{A}-\vec{B}| = \sqrt{61}$

C. $\vec{A}+3\vec{B} = (-10,3,0)$, $|\vec{A}+3\vec{B}| = \sqrt{109}$

D. $\vec{A}\bullet\vec{B} = -14$, $\vec{B}\bullet\vec{A} = -14$

E. $\vec{A}\times\vec{B} = (0,0,-8)$, $\vec{B}\times\vec{A} = (0,0,8)$

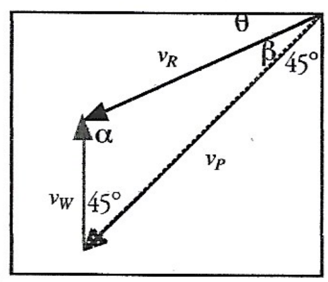
2.



$v_E = \frac{30}{\sqrt{2}}\frac{m}{s}(-\hat{East})$

$v_N = \frac{30}{\sqrt{2}}\frac{m}{s}(-\hat{North})$

A.



B.

1. $\frac{v_E}{\sin\beta} = \frac{v_P}{\sin\alpha} = \frac{v_R}{\sin 45}$

2. $v_R^{\,2} = v_P^{\,2} + v_W^{\,2} - 2v_Pv_W\cos 45°$

$v_R^{\,2} = 900 + 100 - 2\bullet 30\bullet 10\bullet\frac{1}{\sqrt{2}} = 575.7\frac{m^2}{s^2}$

$v_R = 24\frac{m}{s}$, $\sin\beta = \frac{v_W}{v_R}\sin 45 = 0.236$

$\beta = 14°$ *with respect to East*

$\theta_E = 211°$, $\theta = 31°$, $\theta_E = 210°$ 2 *sd.*

3. $\vec{v}_{RE} = \frac{30}{\sqrt{2}}\frac{m}{s}(-\hat{East}) = 21.2\frac{m}{s}(-\hat{East})$

$\vec{v}_{RN} = \left(\frac{30}{\sqrt{2}} - 10\right)\frac{m}{s}(-\hat{North}) = 11.2\frac{m}{s}(-\hat{North})$

$\tan\theta = \frac{v_{RN}}{v_{RE}} = \frac{-11}{-21}$

$\theta = 27.6°$, $\theta_E = 208° \approx 210°$ 2 *sd.*

4. $\vec{v}_R = 24\frac{m}{s} @ 210°$ *w / East*

5. $\vec{v}_R = \left(-21\frac{m}{s},\ -11\frac{m}{s},\ 0\right)$

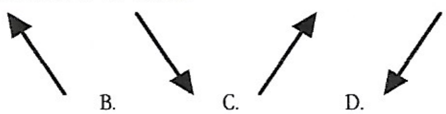
Kinematics

1. An object moving with constant speed travels once around a circular path. Which statement(s) below describe(s) this motion correctly?
I. The displacement is zero
II. The average speed is zero
III. The acceleration is zero
A. I only B. I and II only C. I and III only
D. III only E.. II and III only F.

2. At time $t = t_1$, an object's velocity is given by the vector $\bar{v}_1$

At a later time $t = t_2$, the object's velocity is given by $\bar{v}_2$

Which of the vectors below best describe the direction of the acceleration of the object?



A. B. C. D.

E.

F.

3. Which statement(s) is(are) true?
I. If an object's acceleration is constant, then it must move in a straigth line.
II. If an object's acceleration is zero, then its speedmust remain constant.
III. If an object's speed remains constant, then its acceleration must be zero.
A. I and II only B. I and III only C. II only
D. III only E.. II and III only F.

4. A baseball is thrown straight upward. What is the ball's acceleration at its highest point?
A. 0 B. g/2↓ C. g↓ D. g/2↑ E. g↑ F.

5. How long would it take a car, starting from rest and accelerating uniformly in a straight line at $5\,m/s^2$, to cover a distance of 200 m?
A. 9 s B. 11 s C. 12 s D. 16 s E. 20 s F.

6. A rock is dropped off a cliff and strikes the ground with an impact velocity of 30 m/s. How high is the cliff?
A. 15 m B. 20 m C. 30 m D. 45 m E. 60 m F.

7. A stone is thwon horizontally with an initial speed of 10 m/s from a bridge. If air reistance could be ignored, how long would it take the stone to strike the water 80 m below?
A. 1 s B. 2 s C. 4 s D. 6 s E. 8 s F.

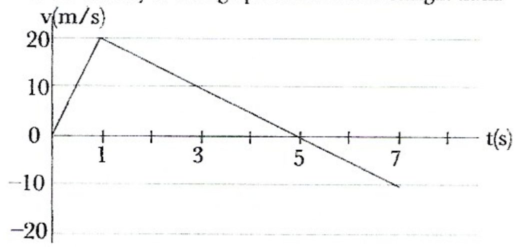
8. A soccer ball, at rest on the ground, is kicked with an initial velocity of 10 m/s at a launch angle of 30°. Calculate its total flight time, assuming negligible air resistance.
A. 0.5 s B. 1 s C. 1.7 s D. 2 s E. 4 s F.

9. A stone is thrown horizontally with an inital speed of 30 m/s from a bridge. Find the stone's total speed when it enters the water 4 s later, assuming negligible air resistance.
A. 30 m/s B. 40 m/s C. 50 m/s D. 60 m/s E. 70 m/s F.

10. Which of the statements below is true concerning the motionof an ideal projectile launched at an angle of 45° with the horizontal?
A. The acceleration vector points in the opposite to the velocity vector on the way up and in the same directionas the velocity vector on the way down.
B. The speed at the top of trajectory is zero
C. The object's speed remains constant during the flight
D. The horizontal component of the velcity decreases on the way up and increases on the way down
E. The component of the velocity decreases on the way up and increases on the way down

11. The position of an object moving in a straight line is given by $x = 7 + 10t - 6t^2$ in meters with t in seconds. What is the object's velocity at 4 s?
A. $-8m/s$ B. $-62m/s$ C. $298m/s$ D $-278m/s$ E._

12. Below is the velocity vs time graph for a car on a straight track.



A. What happened to the car at t= 1 s?
B. What is the car's average velocity t=0 s to 1s and t=1 s to 5s?
C. What is the car's displacement from t=0 s to t=7 s?
D. Plot the acceleration vs time graph for the car.
E. Plot the car's positionwith respect to the starting point.

13. A projectile is launched with an initial velocity v_o at an angle θ_o with the horizontal and moves in a parabolic trajectory under the effect of constant gravity.
A. Calculate h_{max}
B. Calculate the horizontal range R
C. Obtain θ_o which maximizes R
D. Calculate the elapsed time from the initial position to an arbitrary height $h < h_{max}$ on the way up and on the way down

14. A cannonball is shot with an initial speed of 50 m/s at a launch angle of 40° toward a catle wall 220 m away. The height of the wall is 30 m. Air friction is negligible.
A. Show that the cannonball strikes the castle wall.
B. How long will it take for the cannonball to strike the wall?
C. At what height above the firing point will the canonball strike?
D. Assuming that neither the location of the cannon nor the launch speed can be varied, determine the launch angles of the cannonball to barely clear the top of the wall.

15. A particle moves along a straight axis in such a way that its acceleratio at time t is given by the equation $a(t) = 6t(m/s^2)$. If the particle's initial velocity is 2 *m/s* and its initial position 4 *m* from the origin, determine
A. the time the particle reaches 14 *m/s*
B. the position of the particle at t=3 *s*.

16. Consider a projectile fired from the ground level at an angle with the horizontal and returns back to the ground level. Which statements are true about the time of flight, the velocity, the acceleration, the kinetic energy of the projectile?
A. The amount of time to reach the maximum height vs the amount of time to come down from maximum height to the ground level
1. the same
2. shorter to reach maximum height than to come down
3. longer to reach maximum height than to come down
B. The launch speed vs the impact on return to ground
1. the same
2. larger launch speed than the impact
3. smaller launch speed than the impact
C. The acceleration
1. the same
2. larger on the way up
3. smaller on the way up
4. g vertical, zero horizontal
5. constant along the path
6. changes along the path
D. The kinetic energy
1. the same at launch as at impact
2. larger at launch
3. smaller at launch