Ch. 22 Notes

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21 Sinusoidal AC Circuit Analysis

21.1 Representations of a complex variable

- Rectangular, Polar, Exponential

$$z = x + iy = r\angle\theta = re^{i\theta}$$

where

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$\tan\theta = \frac{\mathrm{Im}}{\mathrm{Re}} = \frac{y}{x}$$

$$z = r\angle\theta = r\cos\theta + ir\sin\theta = re^{i\theta}$$

21.2 Arithmetic Operations with Complex Variables

- Adding and subtracting: Add real and add imaginary separately
- Multiplication:

$$z_1 z_2 = r_1 r_2 e^{i(\theta_1+\theta_2)}$$

$$z_1 z_2 = (r_1\angle\theta_1)(r_2\angle\theta_2) = r_1 r_2\angle(\theta_1+\theta_2)$$

$$z_1 z_2 = (x_1+iy_1)(x_2+iy_2)$$

$$= (x_1x_2 - y_1y_2) + i(x_1y_2 + x_2y_1)$$

- Division:

$$\frac{z_1}{z_2} = \frac{r_1 e^{i\theta_1}}{r_2 e^{i\theta_2}} = \frac{r_1}{r_2}\angle(\theta_1-\theta_2)$$

$$\frac{z_1}{z_2} = \frac{r_1\angle\theta_1}{r_2\angle\theta_2} = \frac{r_1}{r_2}\angle(\theta_1-\theta_2)$$

- Complex conjugates:

$$z = x + iy$$

$$z^* = x - iy$$

21.3 Complex potential differences and currents: Phasors

- Sinusoidally oscillating potential differences and currents:

$$V(t) = V_0\cos(\omega t+\theta)$$

$$I(t) = I_0\cos(\omega t+\phi)$$

21.4 The Potential difference and current Phasors for resistors, Inductors, and Capacitors

- Resistor:

$$V = IR$$

$$V(t) = RI(t)$$

- Inductor:

$$V = L\frac{dI}{dt}$$

- Capacitor:

$$C = \frac{Q}{V}$$

$$I = C\frac{dV}{dt}$$

- Impedances:

$$V = IZ$$

  - Resistor:

$$Z_R = R$$

  - Inductor:

$$Z_L = i\omega L$$

  - Capacitor:

$$Z_C = \frac{1}{i\omega C}$$

- Impedance is measured in Ohms. Impedances for capacitors and inductors are imaginary numbers. Impedance is not a phasor.

21.5 Series and parallel combinations of impedances

- Impedances in series combine like resistors in series
- Impedances in parallel combine like resistors in parallel

21.6 Complex independent AC voltage sources

- Complex voltage sources:

$$V_{\text{source}}(t) = V_0 \cos(\omega t) + iV_0 \sin(\omega t)$$

$$V_{\text{source}}(t) = V_0 e^{i(\omega t)}$$

$$V_{\text{source}}(t) = V_0 \angle(\omega t)$$

21.7 Power absorbed by circuit elements in AC Circuits

- Average power absorbed by a circuit element:

$$\langle P \rangle = \frac{1}{2} V_0 I_0 \cos\beta$$

where

$$\beta = \theta - \phi$$

- $\cos\beta$ is the power factor.
- Peak values divided by $\sqrt{2}$ are known as effective values of potential difference and current, also known as rms values.

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$$

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$$

$$\langle P \rangle = V_{\text{rms}} I_{\text{rms}} \cos\beta$$

- Multimeters read rms values and not peak values
- For resistors, $\cos\beta = 1$
- For capacitors, the average power is 0 because $\beta = -\frac{\pi}{2}$
- For inductors, the average power is 0 because $\beta = \frac{\pi}{2}$

21.8 A Filter circuit

- Filter circuits let certain frequencies pass relatively unimpeded and filter out or eliminate one or another range of frequencies.

21.9 A Series RLC Circuit

- For fixed L and C, the numerical value of the resistance affects the shape of the graph of $\langle P \rangle$ vs. ω. The smaller the resistance R, the more sharply peaked the curve.