Ch. 10 Notes

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1 Spin vs. Orbital Motion

- **Spin** - rotational motion of an object/system about an axis through its cm
- **Orbital motion** - object/system moving in space w/ respect to a reference frame
  - does not need to be circular
- point particle can represent cm

2 Orbital Angular Momentum

- Orbital angular momentum: $\vec{L} = \vec{r} \times \vec{p}$, units $\frac{kg \cdot m^2}{s}$
  - with respect to a certain point from which $\vec{r}$ is measured

3 Circular Orbital Motion of a Single Particle

- $\vec{L}$ and $\vec{\omega}$ - perpendicular to plane of motion; use grabbing rhr
- moment of inertia I for a particle: $I = mr^2$
- $\vec{L} = I\vec{\omega}$
- torque: $\vec{\tau} = \vec{r} \times \vec{F}$, units $N \cdot m$, NOT J
  - forces that go through cm have 0 torque
- Use grabbing rhr for torque
- $\Sigma\vec{\tau} = \dfrac{d\vec{L}}{dt} = I\vec{\alpha}$ (constant I)

4 Noncircular Orbital Motion

- Central forces (act along line connecting two particles) have 0 torque; $\vec{L}$ is constant
- cannot use $I\vec{\alpha}$ if $\vec{\omega}$ is changing
- noncircular motion; I is not constant, must use $\Sigma\vec{\tau} = \dfrac{d\vec{L}}{dt}$

5 Rigid Bodies and Symmetry Axes

- rigid bodies - nondeformable, maintain shape
- symmetry axis - line for any point particle at $\vec{r}_{i\perp}$, another point particle at $-\vec{r}_{i\perp}$. $\vec{r}$ needs to be $\perp$ with the axis of symmetry as O

6 Spin Angular Momentum of a Rigid Body

- all particles have orbital motion with the same $\vec{\omega}$
- assumptions:
  - CM of the system is at rest
  - axis of rotation is the same as the axis of symmetry
- moment of inertia of a rigid body:
$$I_{CM} = \sum_i m_i r_{i\perp}^2$$
- $L_{spin} = I_{CM}\vec{\omega}_{spin}$

7 Time rate of change of spin angular momentum

- no internal torque; total torque on a system only by external forces
-
$$\Sigma\vec{\tau}_{ext} = I_{CM}\frac{d\vec{\omega}}{dt}$$
  - If I_{CM} is constant,
$$\Sigma\vec{\tau}_{ext} = \frac{d\vec{L}}{dt} = I_{CM}\vec{\alpha}$$

8 Moments of Inertia

- Point Particle: $I = mr^2$
- Collection of point particles: $I_{CM} = \sum_i m_i r_{i\perp}^2$
- rigid bodies:
$$I = \int_{object} r_\perp^2 dm$$
  - Thin cylindrical hoop: $I = mR^2$
  - solid cylinder: $I = \frac{1}{2}mR^2$
  - thick cylindrical loop: $I = \frac{1}{2}m(R_1^2 + R_2^2)$, where R_2 is total radius and R_1 is inner radius
  - Rect. Plate: $I = \frac{1}{12}m(a^2 + b^2)$, where a and b are length and width
  - Long thin rod: $I = \frac{1}{12}ml^2$, about center of rod lengthwise
  - Sphere: $I = \frac{2}{5}mR^2$
  - Thin spherical shell: $I = \frac{2}{3}mR^2$

9 Kinetic energy of a spinning system

- $$KE_r = \frac{1}{2}I\omega^2$$

10 Spin distorts the shape of the earth

- earth is oblate because it is spinning
- earth distorts shape to provide centripetal acceleration for masses at surface (?)

11 Precession of a rapidly spinning top

- forces on a top: weight, normal force and static friction; normal force and static friction have no torque since $r = 0$
- $\tau = \frac{dL}{dt} = mgr\sin\theta$
- top not spinning; falls over due to torque from weight
- spinning top: precesses in a circular motion since $\vec{L}$ is along the symmetry axis of the top; torque changes the direction of L rather than magnitude
- $\omega_{precess} = \frac{mgr}{L_{spin}}$
- $\vec{L}_{total} = \vec{L}_orbit + \vec{L}_spin$
- $L_{orbit} << L_{spin}$

12 Precession of spinning Earth

- think of earth like a spinning top
- earth tilted at 23.5°
- precessional period 25,785 years

13 Simultaneous spin and orbital motion

- $KE_{total} = KE_t + KE_r = KE_r + \frac{1}{2}mv_{cm}^2$
- $L_{total} = L_{spin} + L_{orbit} = I_{CM}\vec{\omega}_{spin} + mr_{\perp}^2\vec{\omega}_{orbit}$

14 Synchronous rotation and the parallel axis theorem

- system in synchronous rotation if $\vec{\omega}_{spin} = \vec{\omega}_{orbit}$
  - two vectors are parallel
  - spin angular speed = orbital angular speed
- parallel axis theorem, $I = I_{CM} + md^2$, applies to any rigid body in synchronous rotation

15 Rolling without slipping

- special case of synchronous motion
- conditions: relationships between...
  - Distance cm travels and corresponding angle through which the system rotates about the symmetry axis through cm
  - speed of cm and angular speed of rotation
  - magnitude of acceleration of cm and magnitude of angular acceleration of system
- cm moves $s = R\theta$ as the system rotates an angle θ in radians
- $v_{cm} = R\omega$
- $a_{cm} = R\alpha$
- $KE_{total} = \frac{1}{2}I\omega^2$ for rolling without slipping; includes both rotational and translational

16 Wheels

- mechanical advantage - $\frac{r}{R}$, where r is the smaller radius of an inner hub and R is the radius of the entire wheel
- mechanical advantage reduces force while doing the same amount of work
- rolling something on wheels takes less force than dragging it due to mechanical advantage of the wheel

17 total angular momentum and torque

- If a reference point P:
  - is in an inertial reference frame, or
  - is the cm of the system, or
  - has an acceleration parallel or antiparallel to a vector to the cm
- then time rate of change of angular momentum of the system about P = total torque about P;

$$\frac{dL_P}{dt} = \vec{\tau}_p$$

18 Conservation of angular momentum

- If total torque on a system is 0, then angular momentum is conserved; $\vec{L}_{total}$ of the system is a constant vector

19 Conditions for static equilibrium

- two conditions for static equilibrium:
  - total force on system = 0;
  - total torque on system = 0
- therefore, a and α are both 0