Math Review 1-2

Intro.

1. distance → velocity → acceleration

$\searrow \frac{dx}{dt} \nearrow \searrow \frac{d^2x}{dt^2} \nearrow$

vel. → rate of chg of distance

accel. → rate of chg of velocity

momentum → force

$\searrow \frac{dp}{dt} \nearrow$

Force → rate of chg of momentum

2.) acceleration → velocity → displacement

integrals

$W = \int \vec{F} \cdot d\vec{r}$

3) $\frac{d}{dx}(ax^m + bx^k) = max^{m-1} + bkx^{k-1}$

a)

$\int ax^m + bx^k \, dx = \frac{a}{m} x^{m+1} + \frac{b}{k} x^{k+1} + C$

b) $\frac{d}{dt}(a\cos mt + b\sin kt)$

$= -am\sin mt + bk\cos kt$

$\int a\cos mt + b\sin kt \, dt = \frac{a}{m}\sin mt - \frac{b}{k}\cos kt + C$

c) $\frac{d}{dt}(ae^{mt} + be^{-kt})$

$= mae^{mt} - bke^{-kt}$

$\int ae^{mt} + be^{-kt} \, dt$

$= \frac{a}{m} e^{mt} - \frac{b}{k} e^{-kt} + C$

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vectors

1) $\vec{A} = (2, -3, 0)$ $\vec{B} = (-4, 2, 0)$

a) $\vec{A} + \vec{B} = -2\hat{i} - \hat{j}$

$|\vec{A} + \vec{B}| = \sqrt{(-2)^2 + (-1)^2}$ @ $\tan^{-1}\left(\frac{-1}{-2}\right)$

$= 2.23$ @ 27°

b) $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

$= (2+4)\hat{i} + (-3-2)\hat{j}$

$= 6\hat{i} - 5\hat{j}$

$|\vec{A} - \vec{B}| = \sqrt{(6^2) + (-5)^2}$ @ $\tan^{-1}\left(\frac{-5}{6}\right)$

$= 7.8$ @ -39.8°

c) $A + 3B = (2 + (3 \cdot -4))\hat{i} + (-3 + 2(3))\hat{j}$

$= -10\hat{i} + 3\hat{j}$

$|\vec{A} + 3\vec{B}| = \sqrt{(-10^2) + (3^2)}$ @ $\tan^{-1}\left(\frac{3}{-10}\right)$

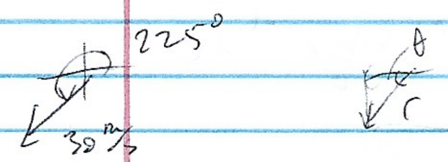
$= 10.4$ @ -17°

d) $A \cdot B = B \cdot A$

$\left(\sqrt{2^2 + 3^2}\right)\left(\sqrt{4^2 + 2^2}\right)\cos\left(\tan^{-1}\left(\frac{-3}{2}\right) - \tan^{-1}\left(\frac{2}{-4}\right)\right)$

$= 14$

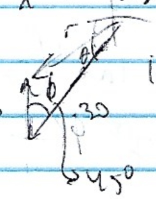
e) $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

2) 

$v_y = r\sin\theta =$ ~~30 sin(225)~~

$= 30\sin(225) = -21\, m/s\, \hat{j}$

$v_x = r\cos\theta = 30\cos(225) = -21\, m/s\, \hat{i}$

B) 

1) $r = \sqrt{A^2 + B^2 - 2AB\cos\phi}$

$= \sqrt{10^2 + 30^2 - 2(10)(30)\cos(45)}$

$= 24\, m/s$

2) $r = A\frac{\sin c}{\sin a} = B\frac{\sin c}{\sin B}$

3) $r = -11\hat{j} - 21\hat{i}$

$|r| = \sqrt{11^2 + 21^2} = 24\, m/s$

$\theta = \tan^{-1}\left(\frac{-11}{-21}\right) = 207°$

4) $\vec{r} = 24\, m/s$ @ 28° S of E

5) $\vec{r} = -21\hat{i} - 11\hat{j} + 0\hat{k}$

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kinematics

(2) A) It starts slowing down

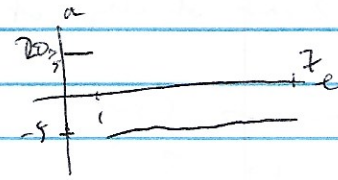
b) (0, 1): $\bar{v} = 10\, m/s$

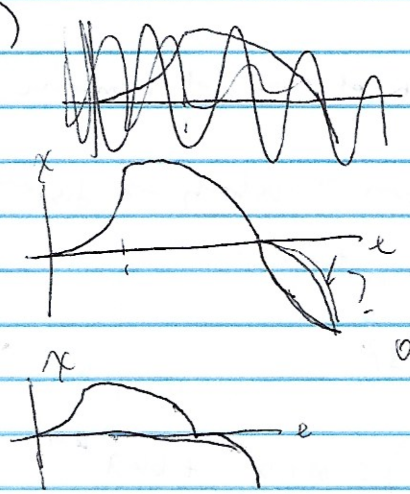
(1, 7): $\bar{v} = 5\, m/s$

c) $\Delta x = \int v\, dt$

$= (1)(20)(\frac{1}{2}) + (4)(20)(\frac{1}{2})$
$- (2)(10)(\frac{1}{2})$
$= 10 + 40 - 10 = \boxed{40\, m}$

d) $a = \frac{dv}{dt}$



e) 

(assuming it starts at x=0)

$v = \frac{dx}{dt}$

$dx = v\, dt$

$x = vt$

$v = at = \frac{dx}{dt}$

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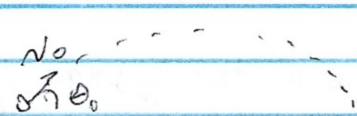
(3) 

$v_{0y} = v_0 \sin\theta_0$ $v_{0x} = v_0\cos\theta_x$

~~$v_{0y} + 2a_y h_{max}$~~

$\cancel{v_y^2} = v_{0y}^2 + 2a_y h_{max}$

$-v_{oy}^2 = 2a_y h_{max} = 2g h_{max}$

$$h_{max} = \frac{-v_0^2 \sin^2\theta_0}{2g}$$

b) $R = v_{0x} t$ $v_{fy} = v_{oy} + g t_{1/2}$

$$t_{1/2} = \frac{-v_{oy}}{g}$$

$$t = \frac{-2v_{oy}}{g} = \frac{-2v_0 \sin\theta}{g}$$

$$R = v_0\cos\theta_0 \cdot \frac{-2v_0\sin\theta}{g}$$

$$= \frac{-2v_0^2 \sin\theta\cos\theta}{g}$$

c) $R = \frac{-2v_0^2 \sin\theta\cos\theta}{g}$

maximize $\sin\theta\cos\theta$ $0<\theta<90$

~~$\frac{d}{d\theta}\sin\theta\cos\theta = $~~ $-\sin^2\theta + \cos^2\theta$

this $\sin\theta\cos\theta$ is greatest when $\theta = 45$

D) $h = v_{oy}t + \frac{1}{2}a_y t^2$

$v_0\sin\theta\, t + \frac{1}{2} g t^2 - h = 0$

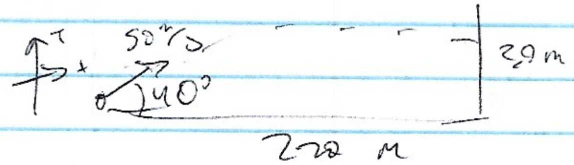
$\cancel{t = \frac{-v_0\sin\theta \pm \sqrt{v_0^2\sin^2\theta + 2h(\sin g)}}{g}}$

$\frac{1}{2} g t^2 + v_0 \sin\theta\, t - h = 0$

$$t = \frac{-v_0\sin\theta \pm \sqrt{v_0^2\sin^2\theta + 2gh}}{g}$$

(4)

A)



from (13), $\cancel{R = \frac{-2v_0^2 \sin\theta\cos\theta}{g}}$

$\cancel{= \frac{-2(50\,m/s)^2 \sin 40 \cos 40}{-9.8\,m/s^2}}$

$h = v_{oy}t + \frac{1}{2}a_y t^2$

$v_f^2 = v_0^2 + 2a\Delta x$

$v_{fy} = \sqrt{v_{oy}^2 + 2g\Delta y}$

b) $v_x t = \Delta x = R$

$$t = \frac{R}{v_{0x}} = \frac{R}{v_0\cos\theta} = \frac{220\,m}{50\,m/s \cos 40}$$

$= 5.7\,s$

a) $h_y = v_{oy}t + \frac{1}{2}a_y t^2$

$= (50\,m/s \sin 40)(5.7\,s) + \frac{1}{2}(-9.8\,m/s^2)(5.7\,s)^2$

$= 24.0\,m$ → $h_{wall} = 30\,m$ → stair

c)

$\frac{1}{2}a_y t^2 + v_{oy}t - 30\,m = 0$

d) $h_y(t) = v_{oy}t + \frac{1}{2}a_y t^2 = 30\,m$

$$t = \frac{-v_{oy} \pm \sqrt{v_{oy}^2 + 4 \cdot 30\,m \cdot \frac{1}{2}a_y}}{a_y}$$

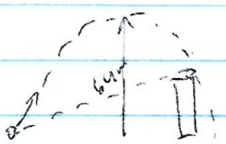
$$= \frac{-(50\sin 40) \pm \sqrt{(50\sin 40)^2 + 4(30)(\frac{1}{2})(-9.8\,m/s^2)}}{-9.8\,m/s^2}$$

$= -34.3\,s$ ~~-300~~

d) if it barely clears the wall,

If $\theta = 45°$, $h_{max} = \frac{-v_0^2 (\sin\theta)^2}{2g}$

$= \frac{-(50 \, m/s)^2 (\sin^2(45°))}{2(-9.8 \, m/s^2)}$



$= 64 \, m$

if $h_{max} = 30 \, m$ $\theta = 29°$

If $\theta = 29°$, $R = \frac{-2v_0^2 \sin\theta\cos\theta}{g}$

$= 216 \, m \rightarrow$ hits the ground early

If $\theta = 45$, $R = 255$

$\frac{1}{2} a_y t^2 + v_{0y} t - 30 \, m = 0$

$-4.9 t^2 + 50 \sin\theta \, t - 30 \, m = 0$

$v_{fy}^2 = v_{0y}^2 + 2a\Delta y$

$\Delta x = v_{0x} t = v_0 \cos\theta \, t$

$= 50 \cos\theta \, t = 220 \, m$

$t = \frac{220 \, m}{50\cos\theta}$

$-4.9 \left(\frac{220 \, m}{50\cos\theta}\right)^2 + 50 \cdot \frac{220 \, m \cdot \sin\theta}{50\cos\theta} = 30$

$\sec^2\theta(-94.9) + 220\tan\theta = 30$

$\theta = 44.8, \ 53.0$

1) uniform circular motion

II, I (B)

2)

(E)

3) I - False; changes direction

II - Speed or velocity? true

III - False; constant mag

(C)

4) (C) - always g

5) $\Delta x = \cancel{v_0 t} + \frac{1}{2} a t^2$

$t = \sqrt{\frac{2\Delta x}{a}} = \sqrt{\frac{2(200 \, m)}{5 \, m/s^2}}$

$= 8.9 \, s$ (A)

6) $v_f = \cancel{v_0} + at$ $\quad v_f^2 = v_0^2 + 2a\Delta x$

$\Delta x = \frac{2v_f^2}{a} = \frac{2(30 \, m/s)^2}{(9.8 \, m/s^2)}$

$= 184 \, m$

7) $v_f = \cancel{v_{0y}} + at$

$v_f^2 = v_0^2 + 2a\Delta y$

$t = \frac{v_{fy}}{a} = \frac{\sqrt{2ay}}{a} = \frac{\sqrt{2(9.8 \, m/s^2)(80 \, m)}}{9.8 \, m/s^2}$

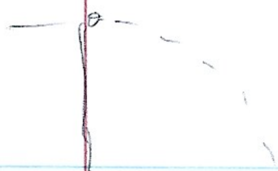
$= 4 \, s$ (C)

8) $v_{fy} = v_{0y} + at$

$t_{up} = \frac{-v_{0y}}{a} = \frac{-v_0 \sin\theta}{g}$

$t_{total} = \frac{-2v_0 \sin\theta}{g} = \frac{-2(10 \, m/s)\sin 30}{-9.8 \, m/s^2}$

$= 1.0 \, s$ (b)



9) $v_{fx} = v_{0x} = 30\,\frac{m}{s}$

$t = 4s$

$v_{fy} = \cancel{v_{0y}} + a_y t$

$v_{fy} = (9.8\,\frac{m}{s^2})(4s) = 39.2\,\frac{m}{s}$

$v_{wht} = \sqrt{39.2^2 + 30^2} = 50\,\frac{m}{s}$ (C)

10)

R

A) False
B) false
C) false
D) false
E) true

11) $x(t) = 7 + 10t - 6t^2$

$v = \frac{dx}{dt}$

$v(t) = x'(t) = 10 - 12t$

$v(4) = 10 - 12(4)$

$= -38\,\frac{m}{s}$ (E)?

15) $a(t) = 6t\,\frac{m}{s^2}$

$v_0 = 2\,\frac{m}{s}$ $x_0 = 4m$

A) $v(t) = \int a(t)\,dt$

$= \int 6t\,dt = 3t^2 + C$

$C = v(0) = 2\,\frac{m}{s}$

$v(t) = 3t^2 + 2\,\frac{m}{s}$

$3t^2 + 2 = 14\,\frac{m}{s}$

$t^2 = 4$ $\boxed{t = 2s}$

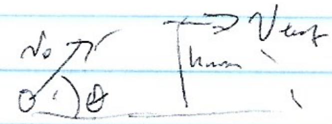
B) $x(t) = \int v(t)\,dt = \int 3t^2 + 2\,dt$

$= t^3 + 2t + C$ $C = x(0) = 4$

$x(t) = t^3 + 2t + 4$

$x(3) = 27 + 6 + 4 = \boxed{37m}$

16)



R

A) (1)
B) (1)
C) (4), (5)
D) (1)