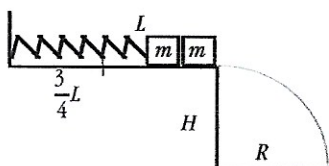


### Simple Harmonic Motion

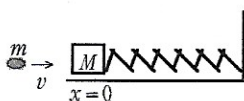
1. Which statements are characteristics of SHM?  
 I. The acceleration is constant  
 II. The restoring force is proportional to the displacement  
 III. The frequency is independent of the amplitude  
 A. II only      B. I and II only      C. I and III only  
 D. II and III only      E. I, II, and III
2. A block attached to an ideal spring undergoes simple harmonic motion. The acceleration of the block has its maximum magnitude at the point where  
 A. the speed is maximum  
 B. the potential energy is minimum  
 C. the speed is minimum  
 D. the restoring force is minimum  
 E. the kinetic energy is the maximum
3. A block attached to an ideal spring undergoes simple harmonic motion about its equilibrium position ( $x=0$ ) with amplitude  $A$ . What fraction of the total energy is in the form of kinetic energy when the block is at  $x=A/2$ ?  
 A.  $1/3$     B.  $3/8$     C.  $1/2$     D.  $2/3$     E.  $3/4$     F.
4. A student measures the maximum speed of a block undergoing simple harmonic oscillations of amplitude  $A$  on the end of an ideal spring. If the block is replaced by one with twice the mass but the amplitude of its oscillations remains the same, then the maximum speed of the block will  
 A. decrease by a factor of 4      B. decrease by a factor of 2  
 C. decrease by a factor of  $\sqrt{2}$     D. remain the same  
 E. increase by a factor of 2
5. A spring-block simple harmonic oscillator is set up so that the oscillations are vertical. The period of the motion is  $T$ . If the spring and block are taken to the surface of the Moon, where the gravitational acceleration is  $1/6$  of its value here, then the vertical oscillations will have a period of  
 A.  $T/6$     B.  $7T/3$     C.  $T/\sqrt{6}$     D.  $T$     E.  $T\sqrt{6}$     F.
6. Blocks of various masses attached to a spring of spring constant  $k$ , then the frequencies of the harmonic motions are measured. If  $f^2$  vs  $1/m$  is plotted, what will be the slope of the straight line?  
 A.  $4\pi^2 / k^2$       B.  $4\pi^2 / k$       C.  $4\pi^2 k$   
 D.  $k / (4\pi^2)$       E.  $k^2 / (4\pi^2)$       F.
7. A block of mass  $m=4$  kg on a frictionless, horizontal table is attached to one end of spring of force constant  $k=400$  N/m and undergoes simple harmonic oscillations about its equilibrium position ( $x=0$ ) with amplitude  $A=6$  cm. If the block is at  $x=6$  cm at time  $t=0$  then which of the following equations (with  $x$  in cm and  $t$  in s) gives the block's position as a function of time  
 A.  $x=6\sin(10t+\pi/2)$     B.  $x=6\sin(10\pi t+\pi/2)$     C.  $x=6\sin(10\pi t-\pi/2)$   
 D.  $x=6\sin(10t)$       E.  $x=6\sin(10t-\pi/2)$
8. A block attached to an ideal spring undergoes simple harmonic motion about its equilibrium position with amplitude  $A$  and angular frequency  $\omega$ . What is the maximum magnitude of the block's acceleration?  
 A.  $A\omega$     B.  $A^2\omega$     C.  $A\omega^2$     D.  $A/\omega$     E.  $A/\omega^2$     F.
9. A simple pendulum swings about the vertical equilibrium position with a maximum angular displacement of  $5^\circ$  and period  $T$ . If the same pendulum is given a maximum angular displacement of  $10^\circ$ , which of the following best gives the period of the oscillations?  
 A.  $T/2$     B.  $T/\sqrt{2}$     C.  $T$     D.  $T\sqrt{2}$     E.  $2T$
10. A simple pendulum of length  $L$  and mass  $m$  swings about the vertical equilibrium with a maximum angular displacement  $\theta_{\max}$ . What is the tension in the connecting rod when the pendulum's angular displacement is  $\theta = \theta_{\max}$ ?  
 A.  $mg\sin\theta_{\max}$     B.  $mg\cos\theta_{\max}$     C.  $mgL\sin\theta_{\max}$   
 D.  $mgL\cos\theta_{\max}$     E.  $mgL(1-\cos\theta_{\max})$



1.

The fluge shows a block of mass  $m$  attached to an ideal spring of constant  $k$  and length  $L$  in equilibrium. The block is pushed to compress spring  $3/4$  of  $L$ . The block is released and collides with a second block of mass  $m$  at rest at the equilibrium position of the spring where the spring stretches back to its original length  $L$ . During the collision, half the original KE is lost to heat, the other half is equally distributed between the two masses. The second block leaves the table after the collisions and lands at a distance  $R$  from the edge of the table.

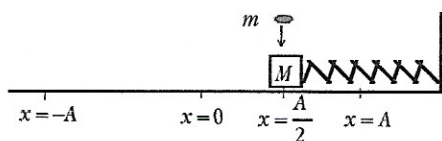
- What is the acceleration of block 1 at the moment it is released from its initial position in terms of  $k$ ,  $L$ , and  $m$ ?
- If  $\vec{v}_1$  is the velocity of block 1 just before impact, show that the velocity of block 1 just after impact is  $\vec{v}_1/2$ .
- Determine the amplitude of the oscillations of block 1 and block 2 has left the table in terms of  $L$ .
- Determine the period of the oscillations of block 1 after the collision in terms of  $T_0$  the period of oscillations that block 1 would have had if it had not collided with block 2.
- Find an expression for  $R$  in terms of  $H$ ,  $k$ ,  $L$ ,  $m$ , and  $g$ .



2.

A bullet of mass  $m$  is fired horizontally with speed  $v$  into a block of mass  $M$  initially at rest, at the end of an ideal spring on a frictionless table. At the moment the bullet hits the mass, the spring is in its unstretched length  $L$ . The bullet is embedded in the block and SHO ensues.

- Determine the speed of the block immediately after the impact.
- Determine the amplitude of the ensuing oscillations.
- Compute the frequency of the oscillations.
- Derive an equation which gives the position of the block as a function of time.



3.

A block of mass  $M$  oscillates with amplitude  $A$  on a frictionless horizontal table, connected to an ideal spring of constant  $k$ . The period of its oscillations is  $T$ . At the moment when the block is at position  $x=A/2$  and moving to the right, a ball of clay of mass  $m$  is dropped from above and lands on the block.

- What is the velocity of the block just before the clay hits?
- What is the velocity of the block just after the clay hits?
- What is the new period of the oscillations of the block?
- What is the new amplitude of the oscillations in terms of  $A$ ,  $k$ ,  $M$ , and  $m$ ?
- Would the answer to part C be different if the clay had landed on the block when it was at a different position? Justify your answer.
- Would the answer to part D be different if the clay had landed on the block when it was at a different position? Justify your answer.

- An object of mass  $M$  and moment of inertia  $I$  swings around a fixed point  $P$ . The object's c.m. is at a distance  $d$  from  $P$ .
  - Calculate the torque produced by the weight of the object when the c.m. of the object is at angle  $\theta$ .
  - For small  $\theta$ , show that the torque is approximately  $\tau = -k\theta$ .

A SHO satisfies the equation  $\frac{d^2z}{dt^2} = -bz$  where  $z$  is the displacement from the equilibrium. The period of its oscillations is  $T = \frac{2\pi}{\sqrt{b}}$ .

- Use this information to obtain the period of oscillations for our object.
- Revise your answer in C for a uniform bar of mass  $M$  and length  $L$  ( $I = \frac{1}{3}ML^2$ )



# PR8 SHM

Blau 16

hul 10

Blau 2

tom 28

B+B 18

MC) 1) II & III (d) ✓

2) (c) ✓

$$3) \frac{1}{2} k A^2 = \frac{1}{2} k \left(\frac{A}{2}\right)^2 + \frac{1}{2} m v^2$$

$$= \frac{1}{8} k A^2 + \frac{1}{2} m v^2$$

$$k A^2 = \frac{1}{4} k A^2 + m v^2$$

$$\frac{3}{4} k A^2 = m v^2 \quad (e) \checkmark$$

$$4) \frac{1}{2} k A^2 \rightarrow 16 \text{ m (d)}$$

$$\frac{1}{2} k A^2 = \frac{1}{2} (k m) v^2$$

half speed  
(b) (c) x

$$5) T = 2\pi \sqrt{\frac{m}{k}} \quad (d) \checkmark$$

$$6) f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$f^2 = \frac{1}{4\pi^2} \cdot \frac{k}{m} \quad \text{slope} \sim \frac{k}{m} \quad (d) \checkmark$$

7) ? (A)

$$8) F = kx = ma$$

$$a = \frac{k}{m} x = \frac{k}{m} A$$

$$f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$\omega = 2\pi f = \sqrt{\frac{k}{m}}$$

$$A \omega^2 \quad (C) \text{ (d)}$$

(A)

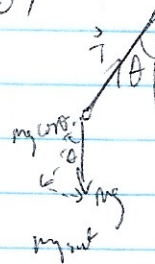
5

hul 10

9) ~~Amplitude~~

amplitude decreases more if the  $\angle$  is small (c) ✓

(c)



$$T = mg \cos \theta_{\max} \quad (b) \checkmark$$

FR)

$$1) \text{ For A) } \sum \vec{F} = \vec{F}_k + \vec{F}_g = m \vec{a}$$

$$= \frac{1}{2} k \left(\frac{1}{4} L\right)^2 = m a$$

$$kx = ma$$

$$\frac{1}{4} k L = m a$$

$$a = \frac{k L}{4 m} \quad \checkmark$$

$$b) \frac{1}{2} \left(\frac{1}{2} m v_1^2\right) = \frac{1}{2} m v_2^2 + \frac{1}{2} k x^2$$

$$v_2^2 = v_1^2 - \frac{1}{2} v_1^2 \quad v_2 = \frac{1}{2} v_1$$

$$2 v_2^2 = \frac{1}{2} v_1^2$$

$$v_2^2 = \sqrt{\frac{v_1^2}{4}} = \frac{1}{2} v_1 \quad \checkmark$$

$$c) \frac{1}{2} m (v_1)^2 = \frac{1}{2} k A^2$$

$$\frac{1}{2} (P D_{s_1}) = \frac{1}{2} k A^2 \quad \sqrt{\frac{1}{2}}$$

$$\frac{1}{2} \left(\frac{1}{2} k \left(\frac{1}{4} L\right)^2\right) = \frac{1}{2} k A^2$$

$$\frac{1}{32} L^2 = A^2$$

$$A = \frac{L}{\sqrt{32}}$$

$$A = \frac{1}{4\sqrt{2}} L = \left(\frac{\sqrt{2}}{8}\right) L \quad \checkmark$$

$$d) T = 2\pi \sqrt{\frac{m}{k}} \quad T = T_0 \quad \checkmark$$

$$e) \frac{1}{2} \left( \frac{1}{2} k(x)^2 \right) = mgh \quad \text{and} \quad \frac{1}{2} m v_a^2 = \frac{1}{2} m v^2$$

$$\frac{1}{2} m v_a^2 = \frac{1}{8} k \left( \frac{1}{4} L \right)^2 - mgh$$

$$\frac{1}{2} m v_a^2 = \frac{1}{8} \cdot k \frac{1}{16} L^2 - mgh$$

$$m v_a^2 = \frac{1}{64} k L^2 - mgh$$

$$v_a = \sqrt{\frac{k L^2}{64 m} - gh}$$

$$v = \sqrt{\frac{k L^2}{4 m}}$$

$$v_{fy}^2 = v_y^2 + 2gh$$

$$v_f = v_0 + gt$$

$$\sqrt{2gh} = gt$$

$$t = \sqrt{\frac{2h}{g}}$$

AK

$$R = \bar{v} t = \sqrt{\frac{k L^2}{4 m} \cdot \frac{2h}{g}}$$

$$= \sqrt{\frac{k L^2 h}{32 m g}}$$

$$= \frac{\sqrt{k}}{\sqrt{32}} \frac{L \sqrt{h}}{\sqrt{m g}}$$

$$2) \quad \vec{m} \vec{v} \rightarrow \text{spring}$$

$$A) \quad m v = (m+M) v_a$$

$$v_a = \frac{m v}{m+M} \quad \checkmark$$

$$B) \quad \frac{1}{2} (m+M) v_a^2 = \frac{1}{2} k A^2$$

$$A^2 = \frac{m+M}{k} \cdot \frac{m^2 v^2}{(m+M)^2}$$

$$= \frac{m^2 v^2}{k(m+M)}$$

$$A = \frac{m v}{\sqrt{k(m+M)}} \quad \checkmark$$

$$A = \frac{m v}{\sqrt{k(m+M)}} \quad \checkmark$$

$$c) \quad f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{k}{m+M}} \quad \checkmark$$

$$d) \quad \text{max?} \quad \sim$$

$$3) \quad \frac{1}{2} k A^2 = \frac{1}{2} m v^2 + \frac{1}{2} k \left( \frac{A}{2} \right)^2$$

$$\frac{1}{2} k A^2 = \frac{1}{2} m v^2 + \frac{1}{8} k A^2$$

$$k A^2 - \frac{1}{4} k A^2 = m v^2 \quad \sim \frac{3}{4} k A^2$$

$$v = \sqrt{\frac{3 k A^2}{4 m}} \quad \checkmark$$

$$b) \quad \frac{1}{2} (m+M) v^2 = \frac{1}{2} k \left( \frac{A}{2} \right)^2 = \frac{1}{8} k A^2$$

$$v^2 (m+M) = k A^2 - k \left( \frac{A}{2} \right)^2$$

$$v^2 = \frac{3 k A^2}{4 (m+M)}$$

$$v = \sqrt{\frac{3 k A^2}{4 (m+M)}} \quad \sim$$



$$c) T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{(m+M)}{k}} \quad \checkmark$$

$$d) \frac{1}{2}(m+M)v^2 + \frac{1}{2}kA^2 = \frac{1}{2}kA_f^2$$

$$A_f^2 = A^2 + \frac{m+M}{k} v^2$$

$$A_f = A + v \sqrt{\frac{m+M}{k}} \quad \checkmark$$

e) no, it only depends on the mass  $\checkmark$

f) No, energy is conserved  $\sim$

u) ?

$$b = \sin(\phi)$$

$$\sin(\phi) = 1$$

$$\phi = \frac{\pi}{2} \quad (A) \quad \checkmark$$

$$8) \text{ Answer } \textcircled{a} x = A$$

$$F = kA = ma$$

$$a = \frac{k}{m} A$$

$$\omega = \sqrt{\frac{k}{m}} = A\omega^2 \quad (c) \quad \sim$$

$$1) c) K_{1a} \rightarrow U_{sa} \rightarrow \frac{1}{2}mv_{1a}^2 = \frac{1}{2}kA_a^2$$

$$A_a^2 = \frac{mv_{1a}^2}{k}$$

$$U_{sb} = \frac{1}{2}k\left(-\frac{L}{a}\right)^2; \quad K_{1b} = \frac{1}{2}mv_{1b}^2$$

$$mv_{1b}^2 = \frac{1}{4}kL^2 \quad A_a = \frac{1}{8}L; \quad v_{1b} = \sqrt{\frac{k}{m}} \frac{L}{4}$$

$$e) H = \frac{1}{2}gt^2 \quad t = \sqrt{\frac{2H}{g}} \quad v_{1b} = \sqrt{\frac{k}{m}} \frac{L}{4}$$

$$R = v_{0x}t = \frac{1}{2}v_{1b} \sqrt{\frac{2H}{g}} = \left[ \frac{L}{8} \sqrt{\frac{2kH}{mg}} \right] \quad \checkmark$$

$$2) d) x = A \sin(\omega t + \phi)$$

$$\phi = 0 \quad x = 0 \quad t = 0$$

$$x = \frac{mv}{\sqrt{k(m+M)}} \sin\left(\sqrt{\frac{k}{m+M}} t\right) \quad \checkmark$$

$$3) b) M\vec{v} = (m+M)\vec{v}_a$$

$$v_a = \frac{A}{(m+M)} \sqrt{\frac{2kH}{A}} \quad \checkmark$$

R4

$$F. Q. 1) c) \frac{1}{2}m(v_{1a})^2 = \frac{1}{2}kA^2$$

$$PE_{s1} = \frac{1}{2}kA^2$$

$$\frac{1}{2}\left(\frac{1}{2}k\left(\frac{1}{a}L\right)^2\right) = \frac{1}{2}kA^2$$

$$A = \frac{1}{8}L \quad \sim$$

$$4) \frac{1}{2}mv^2 = \frac{1}{2}kA^2$$

$$v = A \sqrt{\frac{k}{m}}$$

$$\frac{1}{\sqrt{2}} \quad (c) \quad \checkmark$$

$$7) x = A \sin(\omega t + \phi)$$

$$\omega = 2\pi f = \frac{2\pi}{T} = \frac{2\pi}{\frac{1}{\sqrt{\frac{k}{m}}}} = 2\pi \sqrt{\frac{k}{m}} = 10 \quad \checkmark$$

$$3) d) E_a = \frac{1}{2} k A^2$$

$$K_a + U_s = \frac{1}{2} (M+m) v_a^2 + \frac{1}{2} k \left( \frac{A}{2} \right)^2$$

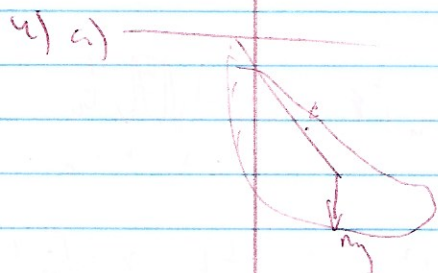
$$E_a = \frac{1}{2} k A_a^2 = K_a + U_s$$

$$\frac{1}{2} (M+m) \left( \frac{A}{M+m} \sqrt{\frac{3kM}{4}} \right)^2 + \frac{1}{2} k \left( \frac{A}{2} \right)^2$$

$$= \frac{1}{2} k A_a^2$$

$$A_a = \frac{A}{2} \sqrt{\frac{3Mm}{M+m}}$$

f) Yes; long. of motion ↓



$$\tau = Mg d \sin \theta$$

b)  $\sin \theta \approx \theta$   $\tau = Mg d \theta$

c)  $\tau = I \ddot{\alpha}$

$$dMg\theta = I \ddot{\alpha} = I \frac{d^2 \theta}{dt^2}$$

$$\frac{d^2 \theta}{dt^2} = - \left( \frac{dMg}{I} \right) \theta = -b\theta$$

$$T = \frac{2\pi}{\sqrt{b}} = \frac{2\pi}{\sqrt{b}} = 2\pi \sqrt{\frac{I}{dMg}}$$

d)  $I = \frac{1}{2} M L^2$  ;  $d = \frac{L}{2}$

$$T = 2\pi \sqrt{\frac{2L}{3g}} \quad \checkmark \quad R6$$



# CHAPTER 8 MC

1.  $\underline{D}$   $ma = kx$   $a$  is not constant

2.  $\underline{C}$   $a = \frac{kx}{m}$   $a$  is max @  $\pm x_{\max}$

3.  $\underline{E}$  C.o.E:  $K + U_s = \text{const.}$

@  $x_{\max}$   $U_s = \frac{1}{2}kA^2$  &  $K = 0$

@  $x = 0$   $K = \frac{1}{2}mv_{\max}^2 = \frac{1}{2}kA^2$   
from C.o.E.

$K + U_s = \frac{1}{2}kA^2$

$K + \frac{1}{2}k\left(\frac{A}{2}\right)^2 = \frac{1}{2}kA^2 \Rightarrow K = \frac{3}{8}kA^2$

$\Rightarrow \frac{K}{E} = \frac{\frac{3}{8}kA^2}{\frac{1}{2}kA^2} = \frac{3}{4}$

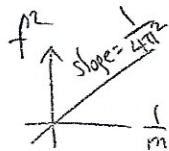
4.  $\underline{C}$   $\frac{1}{2}mv_{\max}^2 = \frac{1}{2}kA^2 \Rightarrow v_{\max} = \sqrt{\frac{k}{m}}A$

$m \rightarrow 2m \Rightarrow v_{\max} \rightarrow \frac{1}{\sqrt{2}}v_{\max}$

5.  $\underline{D}$   $T = 2\pi\sqrt{\frac{m}{k}}$  & independent of  $g$ .

6.  $\underline{D}$   $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$f^2 = \frac{1}{4\pi^2} \frac{k}{m}$



7.  $\underline{A}$   $\omega = 2\pi f = \sqrt{\frac{k}{m}} = \sqrt{\frac{400 \text{ N/m}}{4 \text{ kg}}} = \frac{10}{s}$

$x = A \sin(\omega t + \phi_0) = 6 \sin(10t + \phi)$

$x = 6 \text{ cm}$  @  $t = 0 \Rightarrow \phi_0 = \frac{\pi}{2}$

8.  $\underline{A}$   $x = A \sin(\omega t + \phi)$

$v = \frac{dx}{dt} = \omega A \cos(\omega t + \phi_0)$

$v_{\max} = \omega A$

9.  $\underline{C}$  Small angular displacement  $\rightarrow$   
 $T$  is independent of amplitude

10.  $\underline{B}$   $ma_c = F_T - mg \cos \theta$   
 $a_c = \frac{v^2}{L} = 0 \Rightarrow F_T = mg \cos \theta$   
 $a_t = g \sin \theta$

OE

1.  $\underline{A}$  The spring is compressed to  $\frac{3}{4}L$ .  
 $\therefore x = -\frac{1}{4}L$  with respect to the equilibrium position.

$\vec{F}_s = m\vec{a}$ ,  $\vec{a} = -\frac{k(-\frac{1}{4}L)}{m} = \frac{kL}{4m}$

1b) C.o.P:  $m\vec{v}_{1b} = m\vec{v}_{1a} + m\vec{v}_{2a}$

C.o.E  $KE_{1b} + KE_{2b} + KE_{\text{loss}} = KE_{1a} + KE_{2a}$

$\frac{1}{2}mv_{1b}^2 - \frac{1}{2}(mv_{1b}^2) = \frac{1}{2}mv_{1a}^2 + \frac{1}{2}mv_{2a}^2$

(1)  $v_{1b} = v_{1a} + v_{2a}$  C.o.P

(2)  $v_{1b}^2 = 2v_{1a}^2 + 2v_{2a}^2$  C.o.E

$2 \times (1)^2 \Rightarrow 2v_{1b}^2 = 2v_{1a}^2 + 4v_{1a}v_{2a} + 2v_{2a}^2$  (3)

Subtract (2) from this

$v_{1b}^2 = 4v_{1a}v_{2a}$  (4)

Substitute (4) in this

$v_{1a}^2 + v_{2a}^2 + 2v_{1a}v_{2a} = 4v_{1a}v_{2a}$

$v_{1a}^2 + v_{2a}^2 - 2v_{1a}v_{2a} = 0$

$(v_{1a} - v_{2a})^2 = 0$

$v_{1a} = v_{2a}$

Use in (1) & (4) to get

$v_{1a} = v_{2a} = \frac{1}{2}v_{1b}$

(c)  $K_{1a} \rightarrow U_{2a} \Rightarrow \frac{1}{2}mv_{1a}^2 = \frac{1}{2}kA_a^2$

Substitute  $v_{1a}$ :  $A_a^2 = \frac{mv_{1a}^2}{4k}$

$U_{2b} = \frac{1}{2}k\left(-\frac{1}{4}L\right)^2$  &  $K_{1b} = \frac{1}{2}mv_{1b}^2$

$\Rightarrow mv_{1b}^2 = \frac{1}{16}kL^2$  Substitute in  $A_a^2$

$\Rightarrow A_a = \frac{1}{8}L$ ,  $v_{1b} = \sqrt{\frac{k}{m}} \frac{L}{4}$

(d)  $T$  is a fn of  $k$  & mass  $\Rightarrow T_a = T_b$

(e) Slides off the table w/  $\frac{1}{2}v_{1b}$ ,  $v_{2b} = 0$

$H = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2H}{g}}$ ,  $v_{1b} = \sqrt{\frac{k}{m}} \frac{L}{4}$

$R = v_{1b}t = \frac{1}{2}v_{1b}\sqrt{\frac{2H}{g}} = \frac{1}{8}\sqrt{\frac{kH}{mg}}$

2 a C.o.P:  $m\vec{v} = (m+M)v_a$

$$v_a = \frac{m}{m+M} v$$

b C.o.E:  $K = U$   
 $\frac{1}{2}(m+M)\left(\frac{mv}{m+M}\right)^2 = \frac{1}{2}kA^2$

$$A = \frac{mv}{\sqrt{k(m+M)}}$$

c mass =  $m+M$   $\omega = \sqrt{\frac{k}{\text{mass}}}$   
 $f = \frac{1}{2\pi} \sqrt{\frac{k}{m+M}}$   $\omega = \sqrt{\frac{k}{m+M}}$

d  $x = A \sin(\omega t + \phi_0)$

@  $t=0$ ,  $x=0 \Rightarrow \phi_0 = 0$

$$x = \frac{mv}{\sqrt{k(m+M)}} \sin\left(\sqrt{\frac{k}{m+M}} t\right)$$

3 a C.o.E  $K + U = E$

$$\frac{1}{2}Mv^2 + \frac{1}{2}k\left(\frac{A}{2}\right)^2 = \frac{1}{2}kA^2$$

$$v = A \sqrt{\frac{3k}{4M}}$$

b C.o.P:  $M\vec{v} = (M+m)\vec{v}_a$

$$v_a = \frac{A}{(M+m)} \sqrt{\frac{3kM}{4}}$$

c  $T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{M+m}{k}}$

d  $E_a = \frac{1}{2}kA_a^2$

C.o.E:  $K_a + U_s = \frac{1}{2}(M+m)v_a^2 + \frac{1}{2}k\left(\frac{A}{2}\right)^2$

$$E_a = \frac{1}{2}kA_a^2$$

$K_a + U_s = E_a$

$$\frac{1}{2}(M+m)\left(\frac{A}{(M+m)} \sqrt{\frac{3kM}{4}}\right)^2 + \frac{1}{2}k\left(\frac{A}{2}\right)^2 = \frac{1}{2}kA_a^2$$

Do the algebraic manipulations to get  $A_a = \frac{A}{2} \sqrt{\frac{m+4M}{m+M}}$

c  $T$  is a fn of  $k$  & mass only  
 $\Rightarrow$  No.

f Yes. If the clay lands when  $x=A$ ,  $v_a=0 \Rightarrow$  No change in the block's speed.

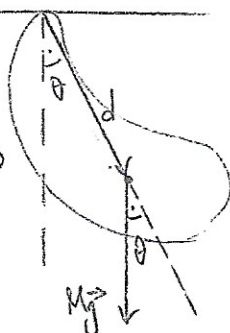
$$\Rightarrow \Delta KE = 0, \Delta E = 0$$

$$\Rightarrow A = \sqrt{\frac{2E}{k}}$$

4 a

$\vec{r} \times \vec{F}$

$\vec{\tau} = Mg d \sin \theta$



b  $\sin \theta$

$\vec{\tau} = Mg d \theta$

c  $\vec{\tau} = I \alpha$

$$dMg \theta = I \alpha = I \frac{d^2 \theta}{dt^2}$$

$$\frac{d^2 \theta}{dt^2} = -\left(\frac{dMg}{I}\right) \theta = -\frac{b}{I} \theta$$

↑ similar to  $\omega$

$$T = \frac{2\pi}{\omega} \rightarrow T = \frac{2\pi}{\sqrt{b}}$$

$$T = 2\pi \sqrt{\frac{I}{dMg}}$$

d  $I = \frac{1}{3}ML^2$ ,  $d = \frac{L}{2}$

$$T = 2\pi \sqrt{\frac{2L}{3g}}$$