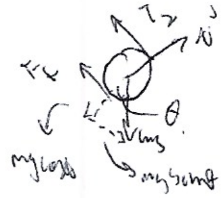
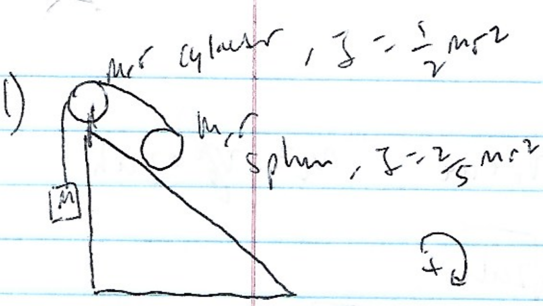
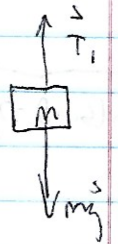
Revised Cumulative 1



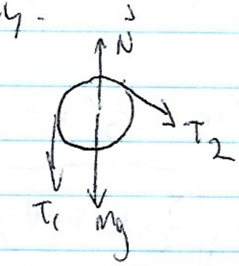
1)



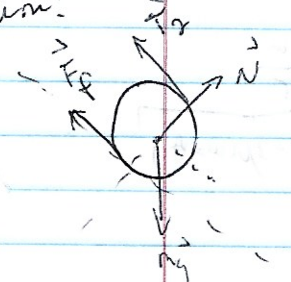
a) mass:



pulley:



Sphere:



b) $\sum \vec{F} = ma_b$

mass: $\vec{T}_1 + m\vec{g} = ma_b$

$T_1 - mg = ma_b$

cylinder:

$\sum F = ma_{cm} = 0$

$N - mg = 0$

$\sum \tau = I\alpha$

$rT_2 - rT_1 = I\alpha$

$\cancel{r}T_2 - \cancel{r}T_1 = \frac{1}{2}m\cancel{r^2} \cdot \frac{a_b}{\cancel{r}}$

$T_2 - T_1 = \frac{1}{2}ma_b$

Sphere: $\sum F = ma$

$\sum F_{\parallel} = ma_{cm}$ $\quad \sum F_{\perp} = 0$

$mg\sin\theta - F_f - T_2 = ma_{cm}$

$N - mg\cos\theta = 0$ $\quad F_f = \mu N = \mu mg\cos\theta$

$\sum \tau = I\alpha$

$rF_f - rT_2 = I\alpha$

$v_{block} = v_{string} = 2v_{cm}$ $\quad a_b = 2a_{cm}$

$\cancel{r}F_f - \cancel{r}T_2 = \frac{2}{5}m\cancel{r^2} \cdot \frac{a_{cm}}{\cancel{r}}$

$F_f - T_2 = \frac{2}{5}ma_{cm}$

c) $T_1 = mg + ma_b$ (1)

$T_2 = \frac{1}{2}ma_b + T_1$ (2)

$T_2 = mg\sin\theta - \mu mg\cos\theta - ma_{cm}$ (3)

$T_2 = \mu mg\cos\theta - \frac{2}{5}ma_{cm}$ (4)

$a_b = 2a_{cm}$ (5)

$mg\sin\theta - \mu mg\cos\theta - ma_{cm} = \mu mg\cos\theta - \frac{2}{5}ma_{cm}$

$\frac{3}{5}\cancel{m}a_{cm} = \cancel{m}g\sin\theta - 2\mu \cancel{m}g\cos\theta$

$\frac{3}{5}a_{cm} = g(\sin\theta - 2\mu\cos\theta)$

$\boxed{a_{cm} = \frac{5}{3}g(\sin\theta - 2\mu\cos\theta)}$

$a_b = 2a_{cm} = \boxed{\frac{10}{3}g(\sin\theta - 2\mu\cos\theta)}$

d) $\alpha = \frac{a}{r}$

1) sphere: $\cancel{\alpha = \frac{a_{cm}}{r}}$

$\cancel{= \frac{5g}{3r}(\sin\theta - 2\mu\cos\theta)}$

$\cancel{2)\ \text{cylinder: } \alpha = \frac{a}{r}}$

$\alpha = \frac{a_{cm}}{r} = \frac{a_{cm}}{r} = \frac{5g}{3r}(\sin\theta - 2\mu\cos\theta)$

$\cancel{t\ \frac{10g}{3r}(\sin\theta - 2\mu\cos\theta)}$

2) cylinder: $\alpha = \frac{a_b}{r} = \frac{10g}{3r}(\sin\theta - 2\mu\cos\theta)$

e) $T_1 = mg + ma_b$ (pulley & mass)

$= m\left(\frac{10}{3}g(\sin\theta - 2\mu\cos\theta) + g\right)$

$= mg\left(\frac{10}{3}(\sin\theta - 2\mu\cos\theta) + 1\right)$

$T_2 = \frac{1}{2}ma_b + T_1$ (sphere & pulley)

$= m\left(\frac{5}{3}g(\sin\theta - 2\mu\cos\theta) + g\cdot\frac{10}{3}(\sin\theta - 2\mu\cos\theta)\right)$

$= 5mg(\sin\theta - 2\mu\cos\theta)$

f) $\tau_{string} = 5mgr(\sin\theta - 2\mu\cos\theta)$

g) $f_k = \mu N = \mu mg\cos\theta$

h) $mg\sin\theta - \mu mg\cos\theta = 0$

$\boxed{\mu = \tan\theta}$

(i) $\Delta y = h$

1) $v_f = v_0 + at$ $\quad v_f^2 = \cancel{v_0^2} + 2ah$

$v_f = \sqrt{2ah}$

$= \sqrt{\frac{20}{3}gh(\sin\theta - 2\mu\cos\theta)}$

2) $v_{cm} = \frac{1}{2}v_b = \frac{1}{2}\sqrt{\frac{20}{3}gh(\sin\theta - 2\mu\cos\theta)}$

$= \sqrt{\frac{5}{3}gh(\sin\theta - 2\mu\cos\theta)}$

3) sphere: $\omega = \frac{v_{cm}}{r}$

$= \frac{1}{r}\sqrt{\frac{5}{3}gh(\sin\theta - 2\mu\cos\theta)}$

pulley: $\omega = \frac{v_b}{r}$

$= \frac{1}{r}\sqrt{\frac{20}{3}gh(\sin\theta - 2\mu\cos\theta)}$

4) $p = mv = m\sqrt{\frac{20}{3}gh(\sin\theta - 2\mu\cos\theta)}$

5) $p = mv = m\sqrt{\frac{5}{3}gh(\sin\theta - 2\mu\cos\theta)}$

6) sphere: $L = I\omega = \frac{2}{5}mr^2\omega$

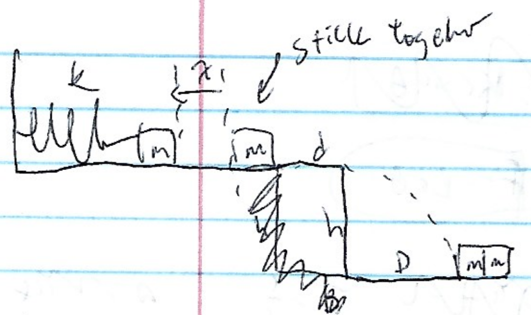
$= \frac{2}{5}mr\sqrt{\frac{5}{3}gh(\sin\theta - 2\mu\cos\theta)}$

pulley: $L = I\omega = \frac{1}{2}mr^2\omega$

$= \frac{1}{2}mr\sqrt{\frac{20}{3}gh(\sin\theta - 2\mu\cos\theta)}$

$= mr\sqrt{\frac{5}{3}gh(\sin\theta - 2\mu\cos\theta)}$

2)



A) $\frac{1}{2}kx^2 = \frac{1}{2}mv_b^2$ (1)

$mv_b = 2mv_a$ (2)

$v_a = v_x$ $\quad D = v_x t$

$\cancel{v_{fy} = v_{0y} + gt}$

$h = \cancel{v_{0y}t} + \frac{1}{2}gt^2$

$t = \sqrt{\frac{2h}{g}}$

$D = v_x\sqrt{\frac{2h}{g}}$ (3)

$v_a = \frac{1}{2}v_b$ $\quad v_b^2 = \frac{k}{m}x^2$

$v_b = x\sqrt{\frac{k}{m}}$

$v_x = v_a = \frac{1}{2}x\sqrt{\frac{k}{m}}$

$D = \frac{1}{2}x\sqrt{\frac{k}{m}}\sqrt{\frac{2h}{g}}$

$\boxed{D = \frac{1}{2}x\sqrt{\frac{2hk}{mg}}}$

b) $D^2 = \frac{1}{4}x^2\left(\frac{2hk}{mg}\right)$

$\cancel{4}^{2}D^2mg = \cancel{2}x^2hk$

$2D^2mg = x^2hk$

$\boxed{k = \frac{2D^2mg}{x^2h}}$

c) Set m in a SHM on the table with the spring k.

$T_s = 2\pi\sqrt{\frac{m}{k}}$

measure the period of oscillation for a variety of masses.

$\frac{T^2}{4\pi^2} = \frac{m}{k}$ $\quad y = mx$

~~plot~~ $\frac{4\pi^2}{T^2} = \frac{k}{m}$

plot $\frac{4\pi^2}{T^2}$ vs. $\frac{1}{m}$; find the slope to get k.

d) $\frac{1}{2}kx^2 = \frac{1}{2}mv_b^2$

$mv_b = 2mv_a$

$v_a = \frac{1}{2}v_b = \frac{1}{2}x\sqrt{\frac{k}{m}}$

$\frac{1}{2}mv_a^2 = \frac{1}{2}mv_f^2 + \mu 2mgd$

$\frac{1}{2}mv_f^2 = \frac{1}{2}m\cdot\frac{1}{4}x^2\cdot\frac{k}{m} - 2\mu mgd$

$D = v_f\sqrt{\frac{2h}{g}}$

$v_f = D\sqrt{\frac{g}{2h}}$

$\frac{1}{2}mD^2\cdot\frac{g}{2h} = \frac{1}{8}mx^2\frac{k}{m} - 2\mu mgd$

$\mu mgd = \frac{1}{16}\cancel{m}x^2\frac{k}{\cancel{m}} - \frac{1}{4}mD^2\cdot\frac{g}{2h}$

$\mu gd = \frac{x^2k}{16m} - \frac{gD^2}{8} = \frac{1}{16}\left(\frac{kx^2}{m} - \frac{1}{2}\frac{gD^2}{h}\right)$

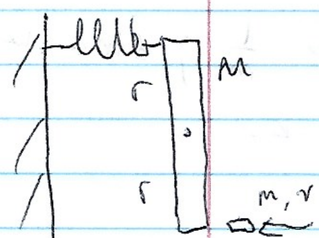
$\mu = \frac{1}{16gd}\left(\frac{kx^2}{m} - \frac{1}{2}\cdot\frac{gD^2}{h}\right)$

e) perform the experiment with known x, k, m, d, and D.

or,

perform the experiment such that the two blocks don't leave the table. measure both d to calculate the coefficient of friction.

3)



$$I_{bar} = 2 \cdot \frac{1}{3} \frac{M}{2} r^2 = \frac{1}{3} M r^2$$

$$L = rp = I\omega$$

$$rmv = \left(\frac{1}{3} M r^2 + m r^2\right)\omega_f$$

$$\omega_f = \frac{rmv}{r^2\left(\frac{1}{3}M + m\right)} = \frac{mv}{r\left(\frac{1}{3}M+m\right)}$$

$$\frac{1}{3}M + m = \frac{M + 3m}{3}$$

$$\omega_f = \frac{mv}{r} \cdot \frac{3}{M+3m}$$

$$= \frac{3mv}{r(M+3m)}$$

b)

for small θ, $\sin\theta \approx \theta$

$x = r\theta \approx r\sin\theta$

$$\boxed{F = -kx = -kr\sin\theta}$$

$$\boxed{F = -kr\theta}$$

c) $T = \frac{1}{f}$ $\quad \omega = 2\pi f$ $\quad f = \frac{\omega}{2\pi}$

$$T = \frac{2\pi}{\omega}$$

$$\cancel{T = \frac{2\pi r(2M+3m)}{3mv}}$$

$$\frac{1}{2} I \omega_{max}^2 = \frac{1}{2} k r^2 \sin^2\theta$$

$$\Sigma\tau = I\alpha$$

$$-r(kr\theta) = I\alpha = I\frac{d^2\theta}{dt^2}$$

$$-kr^2\theta = \left(\frac{1}{3}Mr^2 + mr^2\right)\frac{d^2\theta}{dt^2}$$

$$-k\theta = \left(\frac{1}{3}M + m\right)\frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} + \left(+\frac{k\theta}{\frac{1}{3}M+m}\right) = 0$$

$$\left[\frac{d^2x}{dt^2} + \omega^2 x = 0\right]$$

$$\omega^2 = +\frac{k}{M+3m}$$

$$\omega = \sqrt{\frac{3k}{M+3m}}$$

$$T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{M+3m}{3k}}}$$