CHAPTER 2 MC

1. A. complete circle

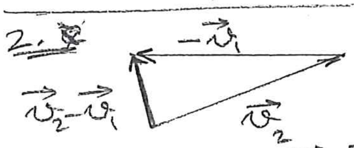
displacement = 0

distance = $2\pi r$

$$\text{speed} = \frac{2\pi r}{T} = v$$

$$a = \frac{v^2}{r}$$

2.



$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

Δt is positive. Therefore, $\vec{a}$ is in the same direction as $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$

3. C I is false. Under the effect of gravity a projectile can follow a a paraboli trajectory.

II is true. $\vec{a} = 0 \Rightarrow \Delta\vec{v} = 0$ & / or v = constant.

III is false: $\vec{a} \neq 0$ if $\vec{v}$ changes direction even if v is constant.

4. C throughout the flight $\vec{a} = g\downarrow$

5. A $\Delta r = v_0 t + \frac{1}{2}at^2 = 0 + \frac{1}{2}at^2$

$$t = \sqrt{\frac{2\Delta r}{a}} \Rightarrow t = 9s.$$

6. D Using kinematics expressions, derive $v^2 = v_0^2 + 2a\Delta r$ or use $\Delta KE = W$

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = mad$$

$$\rightarrow \Delta r = \frac{v^2}{2a} = \frac{v^2}{2g} \approx 45m$$

7. C $\Delta y = v_{oy}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}gt^2$

$$t = \sqrt{\frac{2\Delta y}{g}} = 4s$$

8. 3 Time to reach the max height

$$\vec{v}_y = \vec{v}_{oy} + gt_{mh} = 0$$

$$t_{mh} = \frac{v_{oy}}{g}$$

Total flight time $t_t = 2t_{mh}$

$$t_t = 2t_{mh} = 2\frac{v_{oy}}{g} = 2\frac{1}{g}v_0 \sin\theta$$

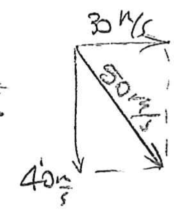
$$= 2\frac{1}{10\frac{m}{s^2}} \cdot 10\frac{m}{s} \sin 30° = 1s$$

9. C $t = 4s \Rightarrow \Delta v_y = gt = 40\frac{m}{s}$

$v_{oy} = 0 \Rightarrow v_y = 40\frac{m}{s}$ @ $t = 4s$

$v_x = v_{ox}$ constant.

$$v = \sqrt{v_x^2 + v_y^2} = 50\frac{m}{s}$$



10. E $\vec{a} = g\downarrow$

On the way up, speed decreases

On the way down, speed increases as time elapses.

CHAPTER 2 OE

1\. a @ $t=1s$, $\vec{v}$ starts to decrease as $\vec{a}$ (the slope) changes from +ve to −ve.

b $\vec{v}_{av} = \frac{1}{2}(\vec{v}_{t=0} + \vec{v}_{t=1s}) = \frac{1}{2}(0 + \vec{20}\,\frac{m}{s}) = \vec{10}\,\frac{m}{s}$

$\vec{v}_{av} = \frac{1}{2}(\vec{v}_{t=1s} + \vec{v}_{t=5s}) = \frac{1}{2}(\vec{20}\,\frac{m}{s} + 0) = \vec{10}\,\frac{m}{s}$

The two v_{av}'s are the same. $\vec{v}_{av} = \frac{1}{2}(\vec{v}_1 + \vec{v}_2)$ is valid only when $\vec{a}$ = constant.

c Area under the curve: displacement

$t=0 \to 5s$: $\Delta r_{5s} = \frac{1}{2}(5s - 0s)(20\frac{m}{s} - 0) = 50m$

$t=5s \to 7s$: $\Delta r_{7s-5s} = -\frac{1}{2}(7s - 5s)(10\frac{m}{s} - 0) = -10m$

Total displacement $= \Delta r = 50m + (-10m) = 40m$

d the slope.

$t = 0s \to 1s$ $\vec{a} = \frac{\vec{20}\frac{m}{s} - 0}{1s - 0} = \vec{20}\,\frac{m}{s^2}$

$t = 1s \to 7s$ $\vec{a} = \frac{\vec{-10}\frac{m}{s} - \vec{20}\frac{m}{s}}{7s - 1s} = \vec{-5}\,\frac{m}{s^2}$

e $r(t) = \int v(t)\,dt$

$$v(t) = \begin{cases} 20t & 0 \le t \le 1 \\ -5t + 25 & 1 \le t \le 7 \end{cases}$$

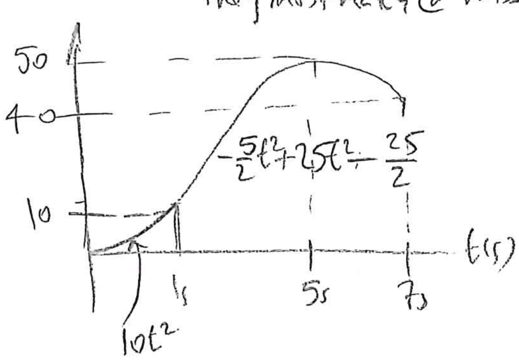
slope

to obtain $v = 20\frac{m}{s}$ @ $t=1s$.

$$\Rightarrow r(t) = \begin{cases} 10t^2 & 0 \le t \le 1 \\ -\frac{1}{2}5t^2 + 25t - \frac{1}{2}25, & 1 \le t \le 7 \end{cases}$$

Notice $10t^2\,(t=1s) = \left(-\frac{1}{2}5t^2 + 25t - \frac{1}{2}25\right)(t=1s)$

they must match @ $t=1s$



2\. a $h_{max} \Rightarrow \vec{v}_y = \vec{v}_{0y} + \vec{g}t = 0$

$t = \frac{v_{0y}}{g}$

$\Delta y = v_{0y}t - \frac{1}{2}gt^2 \Rightarrow h = \frac{v_{0y}^2}{2g} = \frac{v_0^2\sin^2\theta_0}{2g}$

or you can use $\Delta y = v_{y\,av.} \cdot t$

$\Delta y = \left(\frac{0 + v_{0y}}{2}\right) \cdot \frac{v_{0y}}{g} = \frac{v_0^2\sin^2\theta_0}{2g}$

b. $t_{total} = 2t = 2\frac{v_{0y}}{g} = \frac{2v_0\sin\theta_0}{g}$

$\Delta x = v_{0x}t_{total} = \frac{(v_0\cos\theta_0)(2v_0\sin\theta_0)}{g}$

$\Delta x = \frac{2}{g}v_0^2\sin\theta_0\cos\theta_0 = \frac{v_0^2\sin 2\theta_0}{g}$

c $\Delta x = v_{0x}t$

$R = v_{0x}t_{total} = \frac{v_0^2\sin 2\theta_0}{g}$

R is maximum @ $\sin 2\theta_0 = 1$

$2\theta_0 = 90° \Rightarrow \theta_0 = 45°$

d $v_{0y}t - \frac{1}{2}gt^2 = h$

$\frac{1}{2}gt^2 - v_{0y}t + h = 0$

$t = \frac{v_{0y} \pm \sqrt{v_{0y}^2 - 2gh}}{g}$

$t_1 = \frac{1}{g}\left(v_{0y} - \sqrt{v_{0y}^2 - 2gh}\right)$

$t_2 = \frac{1}{g}\left(v_{0y} + \sqrt{v_{0y}^2 - 2gh}\right)$

$\Delta t = t_2 - t_1 = \frac{2}{g}\sqrt{v_{0y}^2 - 2gh}$

3 a Use kinematics expressions, to derive the range expression. Then substitute

$$R = \frac{v_0^2 \sin 2\theta_0}{g} = \frac{50\frac{m}{s^2}\sin(2\cdot 45^\circ)}{10\frac{m}{s^2}}$$

$R = 250m. > 220m$

The cannonball reaches the wall.

$h = 30m$ has to be at a point where the cannonball can clear it.

$X = 220m$ $t = ?$

$$x = v_{0x}t \Rightarrow t = \frac{x}{v_0\cos\theta_0}$$

Substitute in y:

$$y = v_{0y}t - \frac{1}{2}gt^2 = v_0\sin\theta_0 \frac{x}{v_0\cos\theta_0} - \frac{g}{2}\left(\frac{x}{v_0\cos\theta_0}\right)^2$$

$$y = x\tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0} = 23m < 30m$$

where $x = 220m$, $g \approx 10\frac{m}{s^2}$, $\theta_0 = 40^\circ$, $v_0 = 50\frac{m}{s^2}$

b From part a: $t = \frac{x}{v_0\cos\theta_0} = \frac{220m}{50\frac{m}{s}\cos 40^\circ} = 5.7$

c From part a $h = 23m$

d From part a $y = x\tan\theta_0 - \frac{gx^2}{2v_0^2}(1+\tan^2\theta_0)$

or $$\frac{gx^2}{2v_0^2}(\tan\theta_0)^2 - x\tan\theta_0 + \left(y + \frac{gx^2}{2v_0^2}\right) = 0$$

$x = 220m$ & $y = 30m$ gives

$$(94.864)(\tan\theta_0)^2 - 220\tan\theta_0 + 124.864 = 0$$

The quadratic formula (which you can derive) gives

$$\tan\theta_0 = \frac{220 \pm \sqrt{(220)^2 - 4(94.9)(124.9)}}{2\cdot(94.9)}$$

$= 0.99, 1.33$

$\theta_0 \simeq 44.7^\circ, 53.0^\circ$

4. a $v(t) = \int a(t)\,dt$

$= \int 6t\,dt$

$= 3t^2 + v_0$

$v(t) = 3t^2 + 2$

$v(t) = 14$ & solve for t.

$3t^2 + 2 = 14 \Rightarrow t = \pm 2s$

since $t > 0$, only $t = 2s$ is valid.

b. $x(t) = \int v(t)\,dt$

$= \int (3t^2 + 2)\,dt$

$= t^3 + 2t + x_0$

$x(t=0) = 4m \Rightarrow x_0 = 4m$

$\Rightarrow x(t) = t^3 + 2t + 4$

$t = 3s$

$x(t = 3s) = 37m.$