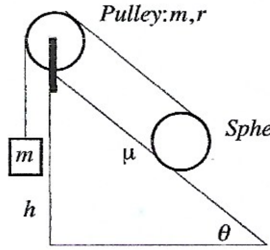
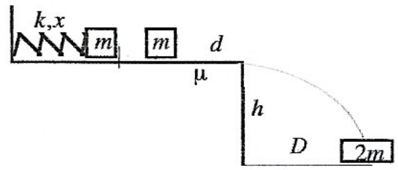
1.



In the set up above, the mass, the pulley, and the sphere are connected with a string as shown in the figure. The ***sphere*** starts from rest and rolls down the incline without slipping. Obtain the answers to the questions below in terms of the quantities given in the figure and the known physical quantities. For convenience, common moments of inertia are mr^2, $\frac{1}{2}mr^2$, $\frac{2}{5}mr^2$.

A. Draw a FBD for each object and show & label all forces.
B. Write down Newton's Law for each object for linear and rotational dynamics as applicable.
C. Obtain the linear acceleration of the sphere and the hanging mass.
D. About their center of mass, obtain the angular acceleration of
   1. the sphere
   2. the pulley
E. Obtain the tension in the string
   1. between the pulley and the mass
   2. between the pulley and the sphere
F. Obtain the net torque produced by the string on the pulley
G. Obtain the force of friction
H. Obtain the coeffecient of friction needed for the rolling without slipping for the sphere.
I. When the mass has moved a height h,
   1. Obtain the speed of the mass
   2. Obtain the linear speed of the sphere
   3. Obtain the angular speeds of the sphere and the pulley
   4. Obtain the linear momentum of the mass
   5. Obtain the linear momentum of the sphere
   6. Obtain the angular momenta of the sphere and the pulley.

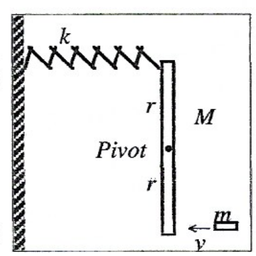
2.



The figure above shows a block of mass m attached to an ideal spring of constant k in equilibrium. The blocked is pushed to compress the spring by a displacement x. The block is released and collides with a second block of equal mass m at rest at the equilibrium position of the spring where the masses stick to each other, travel a distance D before flying of the edge of the table of heigh h and hit the ground at a distance D.

A. For this part of the question, assume $\mu = 0$. Obtain an equation between x and D in terms of the other given quantities.
B. Obtain the spring constant k in terms of all the other givens and variables.
C. What kind of experiment would you perform to obtain k experimentally with this set up. Be specific.
D. Now assume k is known and the surface starting at the point of collision (i.e. the part marked d in the figure) is no longer frictionless, i.e. we have a μ. We repeat the process described above. Obtain an equation for μ in terms of all the other givens and variables.
E. What kind of experiment would you perform to determine μ with this exact set up. Be specific.

3. One end of a spring is connected to a wall and the other to the end of a bar of length $L=2r$ as shown in the figure. The bar is pivoted through its center. A bullet is fired at the lower end of the bar and sticks to it. The system starts a simple harmonic motion. In terms of the given quantities and the known physical constants, answer the questons below.



A. Obtain the angular speed of the bullet-bar system the moment after the collision.
B. Obtain the force spring applies to the bar to stop the system after a small angular displacement θ.
C. Obtain the period of oscillations for a small angular displacement θ.

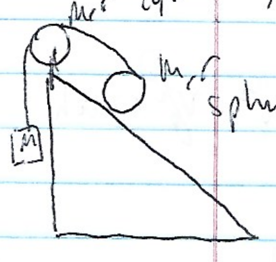
Revised Cumulative 1

Points 38

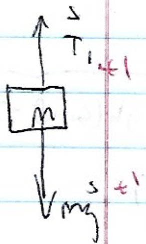
Total 70

1) m, r cylinder, $I = \frac{1}{2}mr^2$

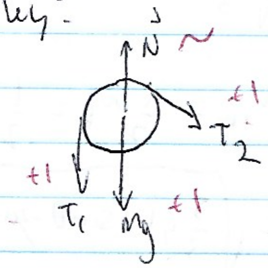
m, r sphere, $I = \frac{2}{5}mr^2$



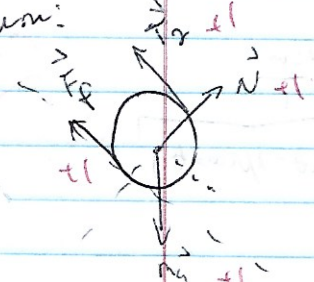
a) mass:



pulley:



sphere:



b) $\sum \vec{F} = ma_b$

mass: $\vec{T}_1 + m\vec{g} = ma_b$

$T_1 - mg = ma_b$ +1

signage

cylinder:

$\sum F = ma = 0$

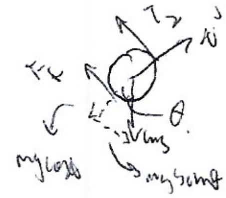
$N - mg = 0$

$\sum \tau = I\alpha$

$rT_2 - rT_1 = I\alpha$ +1

$\cancel{r}T_2 - \cancel{r}T_1 = \frac{1}{2}m\cancel{r^2} \cdot \frac{a_b}{\cancel{r}}$

$T_2 - T_1 = \frac{1}{2}ma_b$



Sphere: $\sum F = ma$

$\sum F_{\parallel} = ma_{cm}$ $\quad \sum F_{\perp} = 0$

$mg\sin\theta - F_f - T_2 = ma_{cm}$ +1

$N - mg\cos\theta = 0$

$F_f = \mu N$

$= \mu mg\cos\theta$

$\sum \tau = I\alpha$

$rF_f - rT_2 = I\alpha$

$v_{block} = v_{string} = 2v_{cm}$ $\quad a_b = 2a_{cm}$

$\cancel{r}F_f - \cancel{r}T_2 = \frac{2}{5}m\cancel{r^2} \cdot \frac{a_{cm}}{\cancel{r}}$

$F_f - T_2 = \frac{2}{5}ma_{cm}$

c) $T_1 = mg + ma_b$ (1) +1

$T_2 = \frac{1}{2}ma_b + T_1$ (2) +1

$T_2 = mg\sin\theta - \mu mg\cos\theta - ma_{cm}$ (3) ~

$T_2 = \mu mg\cos\theta - \frac{2}{5}ma_{cm}$ (4) ~

$a_b = 2a_{cm}$ (5)

$mg\sin\theta - \mu mg\cos\theta - ma_{cm} = \mu mg\cos\theta - \frac{2}{5}ma_{cm}$

$\frac{3}{5}\cancel{m}a_{cm} = \cancel{m}g\sin\theta - 2\mu\cancel{m}g\cos\theta$

$\frac{3}{5}a_{cm} = g(\sin\theta - 2\mu\cos\theta)$

$\boxed{a_{cm} = \frac{5}{3}g(\sin\theta - 2\mu\cos\theta)}$ ~

$a_b = 2a_{cm} = \boxed{\frac{10}{3}g(\sin\theta - 2\mu\cos\theta)}$ ~

11

3

d) $\alpha = \frac{a}{r}$

1) sphere: $\cancel{\alpha = \frac{a_s}{r}}$

$\cancel{= \frac{5g}{3r}(\sin\theta - 2\mu\cos\theta)}$

~~2) cylinder:~~ $\cancel{\alpha = \frac{a}{r}}$

$\alpha = \frac{a_{cm}}{r} = \frac{a_{cm}}{r} = \frac{5g}{3r}(\sin\theta - 2\mu\cos\theta)$ +1 ← sub in original eqn for a

$\cancel{t\frac{10g}{3r}(\sin\theta - 2\mu\cos\theta)}$

2) cylinder: $\alpha = \frac{a_b}{r} = \frac{10g}{3r}(\sin\theta - 2\mu\cos\theta)$

e) $T_1 = mg + ma_b$ +1 (pulley & mass)

$= m\left(\frac{10}{3}g(\sin\theta - 2\mu\cos\theta) + g\right)$

consistency points +1

$= mg\left(\frac{10}{3}(\sin\theta - 2\mu\cos\theta) + 1\right)$

$T_2 = \frac{1}{2}ma_b + T_1$ +1 (sphere & pulley)

$= m\left(\frac{5}{3}g(\sin\theta - 2\mu\cos\theta) + g\cdot\frac{10}{3}(\sin\theta - 2\mu\cos\theta)\right)$

$= 5mg(\sin\theta - 2\mu\cos\theta)$ +1 consistency points

f) $\tau_{string} = 5mgr(\sin\theta - 2\mu\cos\theta)$ ~

g) $f_f = \mu N = \mu mg\cos\theta$ ~

h) $mg\sin\theta - \mu mg\cos\theta = 0$ ~

$\boxed{\mu = \tan\theta}$

5

(i) $\Delta y = h$

1) $v_f = v_0 + at \qquad v_f^2 = \cancel{v_0^2} + 2ah$

$v_f = \sqrt{2ah}$

$= \sqrt{\frac{20}{3}gh(\sin\theta - 2\mu\cos\theta)}$ ~

2) $v_{cm} = \frac{1}{2}v_b = \frac{1}{2}\sqrt{\frac{20}{3}gh(\sin\theta - 2\mu\cos\theta)}$

$= \sqrt{\frac{5}{3}gh(\sin\theta - 2\mu\cos\theta)}$ ~

3) sphere: $\omega = \frac{v_{cm}}{r}$ consistency points

$= \frac{1}{r}\sqrt{\frac{5}{3}gh(\sin\theta - 2\mu\cos\theta)}$ +1

pulley: $\omega = \frac{v_b}{r}$

$= \frac{1}{r}\sqrt{\frac{20}{3}gh(\sin\theta - 2\mu\cos\theta)}$

4) $p = mv = m\sqrt{\frac{20}{3}gh(\sin\theta - 2\mu\cos\theta)}$ +1

5) $p = mv = m\sqrt{\frac{5}{3}gh(\sin\theta - 2\mu\cos\theta)}$ +1

6) sphere: $L = I\omega = \frac{2}{5}mr^2\omega$

$= \frac{2}{5}mr\sqrt{\frac{5}{3}gh(\sin\theta - 2\mu\cos\theta)}$ +1

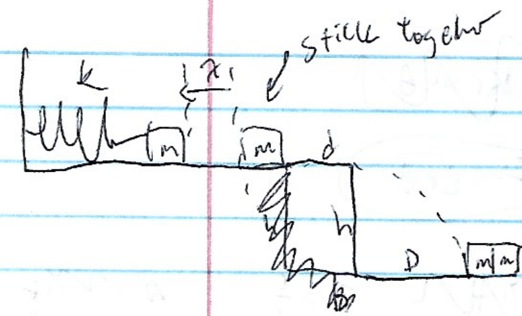
pulley: $L = I\omega = \frac{1}{2}mr^2\omega$

$= \frac{1}{2}mr\sqrt{\frac{20}{3}gh(\sin\theta - 2\mu\cos\theta)}$

$= mr\sqrt{\frac{5}{3}gh(\sin\theta - 2\mu\cos\theta)}$ +1

5

2)



A) $\frac{1}{2}kx^2 = \frac{1}{2}mv_b^2$ (1) +1

$mv_b = 2mv_a$ (2) +1

$v_a = v_x$ $\quad D = v_x t$

$\cancel{v_{Ay} = v_{0y} + gt}$

$h = \cancel{v_{0y}t} + \frac{1}{2}gt^2$

$t = \sqrt{\frac{2h}{g}}$ +1

$D = v_x\sqrt{\frac{2h}{g}}$ (3).

$v_a = \frac{1}{2}v_b$ $\quad v_b^2 = \frac{k}{m}x^2$

$v_b = x\sqrt{\frac{k}{m}}$ +1

$v_x = v_a = \frac{1}{2}x\sqrt{\frac{k}{m}}$ +1

$D = \frac{1}{2}x\sqrt{\frac{k}{m}}\sqrt{\frac{2h}{g}}$

$\boxed{D = \frac{1}{2}x\sqrt{\frac{2hk}{mg}}}$ +1

b) $D^2 = \frac{1}{4}x^2\left(\frac{2hk}{mg}\right)$

$4D^2 mg = 2x^2hk$

$2D^2mg = x^2hk$

$\boxed{k = \frac{2D^2mg}{x^2h}}$ +1

7

c) Set m in a SHM on the table with the spring k.

$T_s = 2\pi\sqrt{\frac{m}{k}}$

measure the period of oscillation for a variety of masses.

$\frac{T^2}{4\pi^2} = \frac{m}{k}$ $\quad y = mx$

$\frac{4\pi^2}{T^2} = \frac{k}{m}$

plot $\frac{4\pi^2}{T^2}$ vs. $\frac{1}{m}$; find the slope to get k.

+1

d) $\frac{1}{2}kx^2 = \frac{1}{2}mv_b^2$

$mv_b = 2mv_a$

$v_a = \frac{1}{2}v_b = \frac{1}{2}x\sqrt{\frac{k}{m}}$

$\frac{1}{2}mv_a^2 = \frac{1}{2}mv_f^2 + \mu 2mgd$

$\frac{1}{2}mv_f^2 = \frac{1}{2}m \cdot \frac{1}{4}x^2 \cdot \frac{k}{m} - 2\mu mgd$

$D = v_f\sqrt{\frac{2h}{g}}$

$v_f = D\sqrt{\frac{g}{2h}}$ +1

$\frac{1}{2}mD^2 \cdot \frac{g}{2h} = \frac{1}{8}mx^2\frac{k}{m} - 2\mu mgd$

$\mu mgd = \frac{1}{16}mx^2\frac{k}{m} - \frac{1}{4}mD^2 \cdot \frac{g}{2h}$

$\mu gd = \frac{x^2k}{16m} - \frac{gD^2}{8} = \frac{1}{16}\left(\frac{kx^2}{m} - \frac{1}{2}\frac{gD^2}{h}\right)$

$\mu = \frac{1}{16gd}\left(\frac{kx^2}{m} - \frac{1}{2}\cdot\frac{gD^2}{h}\right)$

2

e) perform the experiment with known x, k, m, d, and D.

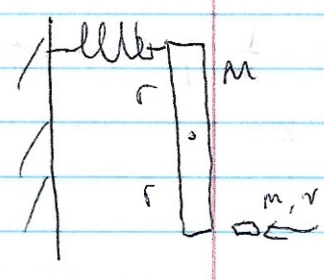
—~

or,

perform the experiment such that the two blocks don't leave the table. measure width d to calculate the coefficient of friction.

~

3)



$I_{bar} = 2 \cdot \frac{1}{3} M r^2 = \frac{1}{3} M r^2$

$L = rp = I\omega$

$rmv = \left(\frac{1}{3} M r^2 + m r^2\right) \omega_f$ +1

$\omega_f = \frac{rmv}{r^2\left(\frac{1}{3}M + m\right)} = \frac{mv}{r\left(\frac{1}{3}M + m\right)}$

$\frac{1}{3}M + m = \frac{M + 3m}{3}$

$\omega_f = \frac{mv}{r} \cdot \frac{3}{M + 3m}$ $\left(\frac{1}{3}M + m\right) = \frac{M + 3m}{3}$

$= \frac{3mv}{r(M + 3m)}$ +1

b)

for small θ, $\sin\theta \approx \theta$

$x = r\theta \approx r\sin\theta$

2

$\cancel{F = kx = kr\sin\theta}$

$F = -kr\theta$ +1

c) $T = \frac{1}{f}$ $\omega = 2\pi f$ $f = \frac{\omega}{2\pi}$

$T = \frac{2\pi}{\omega}$

$\cancel{T = \frac{2\pi r(2M + 3m)}{3mv}}$

$\frac{1}{2} I \omega_{max}^2 = \frac{1}{2} k r^2 \sin^2\theta$

$\sum \tau = I\alpha$

$-r(kr\theta) = I\alpha = I\frac{d^2\theta}{dt^2}$ +1

$-kr^2\theta = \left(\frac{1}{3}Mr^2 + mr^2\right)\frac{d^2\theta}{dt^2}$

$-k\theta = \left(\frac{1}{3}M + m\right)\frac{d^2\theta}{dt^2}$

$\frac{d^2\theta}{dt^2} + \left(+\frac{k\theta}{\frac{1}{3}M + m}\right) = 0$

$\omega^2 = +\frac{k}{M + 3m}$ $\left[\frac{d^2x}{dt^2} + \omega^2 x = 0\right]$

$\omega = \sqrt{\frac{3k}{M + 3m}}$

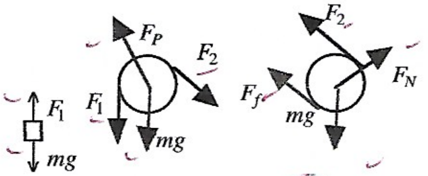
$T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{M + 3m}{3k}}}$ +1

3

total 70

1. *I believe I have some algebraic mistakes here and there. Follow with caution.*

A. *[10 pts] 1 pt for each vector drawn and labeled correctly. Either direction for F_f is acceptable.*



10

B. Mass: $ma = mg - F_1$ [1 pt]

Pulley: $I_P\alpha_P = r(F_1 - F_2)$ [1 pt]

Sphere:

About Center of Mass: $I_S\alpha_S = r(F_2 - F_f)$ [1 pt]

About Contact Point:

$(I_S + mr^2)\alpha_S = 2rF_2 - rmg\sin\theta$ [1 pt]

Linear Motion: $I_S\alpha_S = r(F_2 - F_f)$ [1 pt]

$M_{sph}a = F_2 + F_f - mg\sin\theta$

$0 = F_N - mg\cos\theta$

C. Lets reorganize the equations and use rolling without slipping $\alpha r = a$. [1 pt] Since F_f reacts to F_2, we cannot be sure how large it exactly is in $F_f \le \mu F_N$. Therefore, we will use the equations without F_f.

$ma = mg - F_1$ [1 pt]

$\frac{1}{2}mr^2\alpha_P = r(F_1 - F_2)$ [1 pt]

$\Rightarrow \frac{1}{2}ma = F_1 - F_2$ [1 pt]

$(\frac{2}{5}mr^2 + mr^2)\alpha_S = 2rF_2 - rmg\sin\theta$ [1 pt] 10

$\Rightarrow \frac{7}{10}ma = F_2 - \frac{1}{2}mg\sin\theta$ [1 pt]

Eliminating F_1 and F_2 gives

$\left(1 + \frac{1}{2} + \frac{7}{10}\right)a = g\left(1 - \frac{1}{2}\sin\theta\right)$ and

$a = g\,\frac{10}{22}\left(1 - \frac{1}{2}\sin\theta\right)$ [1 pt]

D. $\alpha = \frac{a}{r} = \frac{g}{r}\frac{10}{22}\left(1 - \frac{1}{2}\sin\theta\right)$ [1 pt]

E.

1. $ma = mg - F_1 \Rightarrow F_1 = m(g - a)$ [1 pt]

$\Rightarrow F_1 = m\left(g - g\frac{10}{22}\left(1 - \frac{1}{2}\sin\theta\right)\right)$ [1 pt]

$\Rightarrow F_1 = mg\left(\frac{6}{11} + \frac{5}{22}\sin\theta\right)$ [1 pt]

2. $\frac{1}{2}ma = F_1 - F_2 \Rightarrow F_2 = F_1 - \frac{1}{2}ma$

[1 pt]

$F_2 = m(g - a) - \frac{1}{2}ma = m\left(g - \frac{3}{2}a\right)$ d

$\Rightarrow F_2 = mg\left(1 - \frac{3}{2}\frac{10}{22}\left(1 - \frac{1}{2}\sin\theta\right)\right)$

[1 pt]

$\Rightarrow F_2 = mg\left(\frac{7}{22} + \frac{15}{44}\sin\theta\right)$ [1 pt]

-8

F. $\tau_P = r(F_1 - F_2) = \frac{1}{2}mr^2\alpha_P$ [1 pt]

$\tau_P = mrg\frac{5}{22}\left(1 - \frac{1}{2}\sin\theta\right)$ [1 pt]

G. Use either $I_S\alpha_S = r(F_2 - F_f)$ or

$(I_S + mr^2)\alpha_S = 2rF_2 - rmg\sin\theta$

$F_f = F_2 - \frac{I_S\alpha_S}{r} = m\left(g - \frac{3}{2}a\right) - \frac{2}{5}ma$ [1 pt]

$F_f = mg\left(1 - \frac{19}{10}\frac{10}{22}\left(1 - \frac{1}{2}\sin\theta\right)\right)$ [1 pt]

$F_f = mg\left(\frac{3}{22} + \frac{19}{44}\sin\theta\right)$ [1 pt]

H. $F_f \le \mu F_N$

$\mu \le \frac{F_N}{F_f} = \frac{\cos\theta}{\left(\frac{3}{22} - \frac{19}{44}\sin\theta\right)}$ [1 pt]

I. Use CoE. When the mass is displaced by h, the sphere moves along the surface by h and its height changes by $h\sin\theta$ [1 pt]

$PE_m = KE_m + KE_{Pul} + KE_{Sp} + PE_{Sp}$ [1 pt]

$mgh = \frac{1}{2}mv^2 + \left(\frac{1}{2}I_P\omega^2\right)$

$+\left(\frac{1}{2}I_S\omega^2 + \frac{1}{2}mv^2\right) + mgh\sin\theta$ [1 pt]

1. $gh(1 - \sin\theta) = \frac{29}{20}v^2$

$v = \sqrt{\frac{20}{29}gh(1 - \sin\theta)}$ [1 pt]

2. $v = \sqrt{\frac{20}{29}gh(1 - \sin\theta)}$ [1 pt] — 10

3. $\omega = \frac{1}{r}\sqrt{\frac{20}{29}gh(1 - \sin\theta)}$ [1 pt]

4. $p_m = mv = m\sqrt{\frac{20}{29}gh(1 - \sin\theta)}$ [1 pt]

5. $p_S = m_S v = m\sqrt{\frac{20}{29}gh(1 - \sin\theta)}$ [1 pt]

6. $L_P = I_P\omega = \frac{1}{2}mr\sqrt{\frac{20}{29}gh(1 - \sin\theta)}$

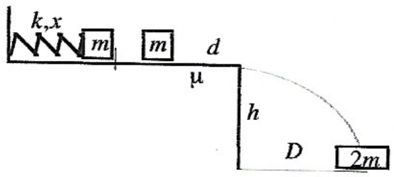
[1 pt]

$L_S = I_S\omega = \frac{2}{5}mr\sqrt{\frac{20}{29}gh(1 - \sin\theta)}$

[1 pt]

-5

2.



A. CoE in the first part: $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$ [1 pt]

$$v = x\sqrt{\frac{k}{m}} \text{ [1 pt]}$$

CoP in thesecond part: $mv = (m+m)v_a$ [1 pt]

$$v_a = \frac{x}{2}\sqrt{\frac{k}{m}} \text{ [1 pt]}$$

Then use kinematics:

$$h = \frac{1}{2}gt^2,\ t = \sqrt{\frac{2h}{g}} \text{ [1 pt]}$$

$$D = v_a t = x\sqrt{\frac{hk}{2mg}} \text{ or } D^2 = \frac{hk}{2mg}x^2 \text{ [1 pt]}$$

B. $k = \frac{2mg}{h}\frac{D^2}{x^2}$ [1 pt]

C. Change x, measure D [1 pt]

plot D^2 vs $\frac{h}{2mg}x^2$ (or D vs x) [1 pt]

The slope is k (or similar constant). [1 pt] — 10

There are other graphing options.

D. We have to subtract the energy lost to friction after the masses collide and stick to each other.

$$\frac{1}{2}(2m)(v_a)^2 - \mu mgd = \frac{1}{2}mv_f^2 \text{ [1 pt]}$$

$$\frac{1}{2}(2m)\left(x\sqrt{\frac{k}{m}}\right)^2 - \mu mgd = \frac{1}{2}mv_f^2 \text{ [1 pt]}$$

$$\mu = \frac{1}{gd}\left(\left(x\sqrt{\frac{k}{m}}\right)^2 - \frac{1}{2}v_f^2\right) \text{ [1 pt]}$$

$$D = v_f t \Rightarrow v_f = \frac{D}{t} = D\sqrt{\frac{g}{2h}} \text{ [1 pt]}$$

$$\mu = \frac{1}{gd}\left(\left(x\sqrt{\frac{k}{m}}\right)^2 - \frac{1}{2}\left(D\sqrt{\frac{g}{2h}}\right)^2\right) \text{ [1 pt]}$$

E. Change x, measure D,[1 pt] plot

$$\frac{1}{gd}\left(x\sqrt{\frac{k}{m}}\right)^2 \text{ vs } \frac{1}{2}\frac{1}{gd}\left(D\sqrt{\frac{g}{2h}}\right)^2 \text{ [1 pt]}$$

x-y intercepts give μ. [1 pt]

There are other graphing options.

— 7

3.

A. Use Co**L** at the moment of impact about the pivot point:

$$mvr = I\omega = \left(\frac{1}{12}M(2r)^2 + mr^2\right)\omega \text{ [1 pt]}$$

$$\omega = \frac{mv}{\left(\frac{1}{3}M + m\right)r} \text{ [1 pt]}$$

B. $F = kx = kr\theta$ [1 pt]

$$\frac{\omega}{2}t = \theta,\ t = \frac{2\theta}{\omega} \text{ [1 pt]}$$

$$\tau = I\alpha = I\frac{\omega - 0}{t} = I\frac{\omega^2}{2\theta} \text{ [1 pt]}$$

$$rF = \tau = \left(\frac{1}{3}M + m\right)\frac{r^2\omega^2}{2\theta} \text{ [1 pt]}$$

$$F = \left(\frac{1}{3}M + m\right)\frac{r\omega^2}{2\theta} = F_s = kr\theta \text{ [1 pt]}$$

C. $F = kx = kr\theta$

$$\tau = I\alpha = \left(\frac{1}{3}M + m\right)r^2\frac{d^2\theta}{dt^2} = rkr\theta \text{ [1 pt]}$$

$$\frac{d^2\theta}{dt^2} = \frac{k}{\left(\frac{1}{3}M + m\right)}\theta = \left(\frac{2\pi}{T}\right)^2\theta \text{ [1 pt]}$$

$$T = 2\pi\sqrt{\frac{\left(\frac{1}{3}M + m\right)}{k}} \text{ [1 pt]}$$

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