Ch. 16 Examples

1) a) $F = \frac{k|q_1||q_2|}{r^2} = \frac{(9\times10^9 \frac{N\cdot m^2}{C^2})(\cancel{4}2\times10^{-6}C)(5.5\times10^{-6}C)}{(0.15m)^2}$

$= 2.8 N$

b) $\rightarrow F_E$ $\quad F_E \leftarrow$

2) a) $F = \frac{kqQ}{r^2} = \frac{(9\times10^9 \frac{N\cdot m^2}{C^2})(2.0\times10^{-6}C)(3\times10^{-6}C)}{(1.25m)^2}$

$= 0.0346\ N$ towards each other

b) it's an action/reaction pair

3) $F_1 = \frac{k|q_1||q_2|}{r_1^2}\hat{j} = \frac{(9e9 \frac{Nm^2}{C^2})(2e{-6}C)(4e{-6}C)}{(0.10m)^2}\hat{j}$

$= 7.2 N \hat{j}$

$F_2 = \frac{k|q_1||q_2|}{r^2}(-\hat{i}) = -\frac{(9e9)(2e{-6})(3e{-6})}{(.15m)^2}\hat{i}$

$= -2.4 N \hat{i}$

$F_{total} = \sqrt{(7.2)^2 + (2.4)^2} = 7.6 N$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{7.2N}{-2.4N}\right) = 72^\circ$

72° θ

$\theta = 180 - 72 = \boxed{108^\circ}$

4) $\frac{1.0\times10^{-6}C}{1.602\times10^{-19}C} = \boxed{6.24\times10^{12}}$

5) $F_e = \frac{k\,q\cdot q}{r^2}$ $\quad F_g = \frac{G\,m\cdot m}{r^2}$

$\frac{F_g}{F_e} = \frac{G\cdot m\cdot m}{\cancel{r^2}} \cdot \frac{\cancel{r^2}}{k\cdot q\cdot q}$

$= \frac{G\cdot m\cdot m}{k\cdot q\cdot q} = \frac{(6.67\times10^{-11}\frac{N\cdot m^2}{kg^2})(9.11\times10^{-31}kg)^2}{(9.0\times10^9\frac{N\cdot m^2}{C^2})(1.6\times10^{-19})^2}$

$= \boxed{2.40\times10^{-43}}$

6) a) $F_e = \frac{k|q||q|}{r^2} = \frac{(9\times10^9 \frac{Nm^2}{C^2})(1.602\times10^{-19}C)^2}{(5.29\times10^{-11}m)^2}$

$= \boxed{8.25\times10^{-8} N}$

$F_c = Ma_c$ $\quad a_c = \frac{F_c}{m} = \frac{8.25\times10^{-8}N}{9.11\times10^{-31}kg}$

$= \boxed{9.02\times10^{22} m/s^2}$

7) $F = qE = (1.602\times10^{-19}C)(125 \frac{N}{C}) = 2\times10^{-17} N$

Must be the direction of the field

(+) test charge

8) a) $E = \frac{kq}{r^2} = \frac{(9.0\times10^9 \frac{Nm^2}{C^2})(1.602\times10^{-19}C)}{(5.29\times10^{-11}m)^2}\hat{r}$

b) $= \boxed{5.15\times10^{11} \frac{N}{C}}$

9) $E_1 = \frac{kQ}{r} = \frac{(9.0\times10^9 \frac{N\cdot m^2}{C^2})(2.0\times10^{-6}C)}{(0.07m)^2} = 3.67\times10^6 \frac{N}{C}\ -\hat{j}$

$E_2 = \frac{kQ}{r} = \frac{(9.0\times10^9 \frac{Nm^2}{C^2})(-2.0\times10^{-6}C)}{(0.05m)^2} = 7.2\times10^6 \frac{N}{C}\ -\hat{i}$

$E_{total} = \sqrt{(3.67e6)^2 + (7.2e6)^2} = \boxed{8.08\times10^6 \frac{N}{C}}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = 27 + 180^\circ = \boxed{207^\circ}$

$F = qE = (-3.0\times10^{-6}C)(8.08\times10^{6}\frac{N}{C})$

$= \boxed{24.2 N}$

10) $\vec{p} = |Q|\vec{d} = |Q|d$

from pt. P, $r_1 = z - \frac{d}{2}$

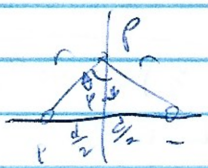
$E_1 = \frac{kQ_1}{r_1^2} = \frac{k(+|Q|)}{(z-\frac{d}{2})^2}$

$E_2 = \frac{kQ_2}{r_2^2} = \frac{k(-|Q|)}{(z+\frac{d}{2})^2}$

$E_{total} = k|Q|\left(\frac{1}{(z-\frac{d}{2})^2} - \frac{1}{(z+\frac{d}{2})^2}\right)$

When $z \gg d$, $\vec{E} \approx \frac{k2p}{z^3}\hat{k}$

11) $E_1 = \frac{k(-|Q|)}{r^2}$ $E_2 = \frac{k(+|Q|)}{r^2}$



$r^2 = y^2 + \frac{d^2}{4}$

$E_1 = \frac{k|Q|}{(y^2+\frac{d^2}{4})}(\cos\theta\hat{j} - \sin\theta\hat{k})$

$E_2 = \frac{k|Q|}{(y^2+\frac{d^2}{4})}(-\hat{j}\cos\theta - \hat{k}\sin\theta)$

$\vec{E} = \frac{kQ}{(y^2+\frac{d^2}{4})}(-2\sin\theta\hat{k})$

$\sin\theta = \frac{d}{2\sqrt{y^2+\frac{d^2}{4}}}$

$E = \frac{-kQd}{(y^2+\frac{d^2}{4})^{3/2}}\hat{k}$

$= \frac{-kp}{(y^2+\frac{d^2}{4})^{3/2}}\hat{k}$

when $y \gg d$ $\vec{E} \approx \frac{-kp}{y^3}\hat{k}$

12) $E = \frac{-kp}{y^3} = \frac{-(9.0\times10^9 \frac{Nm^2}{C^2})(6.0\times10^{-30} Cm)}{(1\times10^{-7}m)^3}$

$= \boxed{5.4\times10^7 \frac{N}{C}}$

13) $\tau = \vec{r}\times\vec{F}$ $\tau_1 = \vec{r}_1 \times |Q|\vec{E}$

$= \frac{\vec{d}}{2}\times|Q|\vec{E}$

$\tau_2 = \vec{r}_2 \times |Q|\vec{E}$

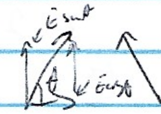
$= -\frac{\vec{d}}{2}\times -|Q|\vec{E} = \frac{\vec{d}}{2}\times|Q|\vec{E}$

$\tau = \tau_1 + \tau_2 = |Q|\vec{d}\times\vec{E}$

$= \vec{p}\times\vec{E}$

14) Symmetry

each differential charge creates a field



that gets cancelled horizontally by symmetry. Only the vertical component matters.

~~charge ... linear charge density~~

$E = \frac{kQ}{r^2}$ $E_{summary} = \frac{kQ}{r^2}\sin\theta = \frac{z}{r}$

$r^2 = z^2 + R^2$

$E = \frac{kQ}{r^2}\cdot\frac{z}{r} = \frac{kQz}{r^3}$

$= \boxed{\frac{kQz}{(R^2+z^2)^{3/2}}}$

15) the sum result from ↑ integrated over each little ring?

$\sigma = \frac{Q}{\pi R^2}$ bunch of little diff. radii, r of area $2\pi r dr$

$dq = (2\pi r dr)\sigma$

$dE = \frac{k\,dq}{r^2}\sin\theta$

same result from above,

$dE = \frac{k\,dq}{(z^2+r^2)^{3/2}} = \frac{k(2\pi r dr)\sigma}{(z^2+r^2)^{3/2}}$

$E = 2kz\pi\sigma\int\frac{r\,dr}{(z^2+r^2)^{3/2}}\hat{k}$

$= kz2\pi\sigma\left(\frac{-1}{\sqrt{(z^2+r^2)}}\right)\Big|_0^R\hat{k}$

$= \frac{k2Q}{R^2}\left(1 - \frac{z}{\sqrt{(z^2+R^2)}}\right)\hat{k}$

20) 

$\cancel{PE_e = qV}$ E is uniform
$F_e = qE$

$\sum F = ma$
$qE = ma$
$a = \frac{qE}{m}$

$v_f^2 = \cancel{v_0^2} + 2ad$
$v_f^2 = \frac{2qEd}{m}$
$v_f = \sqrt{\frac{2qEd}{m}}$

$v = -\sqrt{\frac{2qEd}{m}}$ if using vectors

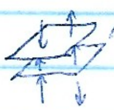
16) From 15), $E = \frac{k2Q}{R^2}\left(1 - \frac{z}{\sqrt{(z^2+R^2)}}\right)\hat{r}$

but as R approaches ∞, $\frac{z}{\sqrt{z^2+R^2}}$ becomes very small.

$E = k2\pi\sigma\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)\hat{r}$

$\approx 2\pi k\sigma = \frac{\sigma}{2\varepsilon_0}$

17) $E_1 = \frac{\sigma}{2\varepsilon_0}$ $E_2 = -\frac{\sigma}{2\varepsilon_0}$



In the middle, $E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$
$= \frac{\sigma}{\varepsilon_0}$

outside, the forces cancel out.

18) when $r < R$, $E = 0$ since field ~~charge~~ inside a conducting sphere is always 0

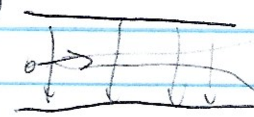
when $r > R$, $F_e = \frac{kQ}{r^2}\hat{r}$

19) ~~the~~ when $r < R$, $E = \frac{kQ_{enc}}{r^2}$

$Q_{enc} = Q \cdot \frac{r}{R}$ $E = \frac{kQ}{R \cdot r}\hat{r}$?

$E = \frac{kQ_{enc}}{R^2} = \frac{kQr}{R^3}\hat{r}$

when $r > R$, $E = \frac{kQ}{r^2}\hat{r}$

21) 

$\sum F_x = ma = 0$
$\sum F_y = F_e = ma$
$qE = ma$
$a_y = \frac{qE}{m}$

$\Delta y = \cancel{v_{0y}t} + \frac{1}{2}a_y t^2$
~~$v_f^2 = v_{0y}^2 + 2a_y\Delta y$~~

$\Delta y = \frac{1}{2} \cdot \frac{qE}{m} \cdot t^2$

$v_{0x}t = \Delta x = l$
$t = \frac{l}{v_{0x}}$

$\Delta y = \frac{1}{2} \cdot \frac{qE}{m} \cdot \frac{l^2}{v_0^2}$

$\Delta y = \frac{qEl^2}{2mv_0^2}$

$F = -qE$,
$\Delta y = -\frac{qEl^2}{2mv_0^2}$

22) $\Phi = \frac{Q_{enc}}{\varepsilon_0}$

a) $\Phi = \frac{(2\mu C - 2\mu C)}{\varepsilon_0} = 0$

b) $\Phi = \frac{2\mu C}{\varepsilon_0} = \frac{2\times10^{-6}C}{8.85\times10^{-12}F/m} = 2.3\times10^5 \frac{C \cdot F}{m}$

c) $\Phi = \frac{-2\mu C}{\varepsilon_0} = -2.3\times10^5 \frac{C \cdot F}{m}$

d) $\Phi = \frac{0}{\varepsilon_0} = 0$