PR 6 RM

Black 12 Red 7
Blue 4
Black + Blue 16 Total 23

1) $v = r\omega = 0.06\,m \times \frac{5\,rev}{s} \cdot \frac{2\pi\,rad}{rev}$

$= 1.9\,m/s$ (b) ✓

2) $\Delta\theta = \bar{\omega} t$

$s = r\theta = r\omega t$

$= (1.9\,m/s)(40\,m)(60\,s)$

$= 4560\,m$ (e) ✓

3) $a_c = \frac{v^2}{r}$ $\omega = \frac{v}{r}$

$v = r\omega$

$a_c = \frac{r^2\omega^2}{r}$

$= r\omega^2$

$\omega = \sqrt{\frac{a_c}{r}} = \sqrt{\frac{9\,m/s^2}{0.25\,m}} = 6\,rad/s$ (c) ✓

4) $\Delta\theta = \cancel{\omega_0 t}^{0} + \frac{1}{2}\alpha t^2$

$\omega_f = \omega_0 + \alpha t$

$\alpha = \frac{2\Delta\theta}{t^2} = \frac{2(60\,rad)}{(10\,s)^2}$

$= 1.2\,rad/s^2$ (c) ✓

5) $\tau = r \times F = 0.2\,m \cdot 20\,N = 4\,N\cdot m$ (d) ✓

6) $\theta = 150°$

$\tau = rF \sin 150$

$= 0.8\,m \cdot 0.5\,kg \cdot 9.8\,m/s^2 \sin 150$ ✓

$= 1.96\,N\cdot m$ (d)

7) 50 pivot 10

30N 10N Mg

(7) $30N \cdot 0.5\,m = 10M \cdot 0.3\,m$

$M = \frac{30N \cdot 0.5\,m}{0.3\,m \cdot 10} = 5\,kg$ (b) ✓

(8) $I_m = 4mL^2$

$I_{\parallel} = \frac{4}{3}mL^2$

$I_{\perp} = 2mL^2$

$I_{total} = \frac{22}{3}mL^2$ (b)?

rods are massless; $I = 4mL^2$ (a) ✓

9) $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v^2}{R^2}\right)$

$mgh = \frac{1}{2}mv^2 + \frac{1}{5}mv^2$

$mgh = \frac{7}{10}mv^2$

$v = \sqrt{\frac{10}{7}gh}$ (A) ✓

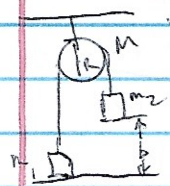
10) $\frac{1}{2}I\omega^2 = \frac{1}{2}2I\omega_f^2$

$\omega_f^2 = \frac{1}{2}\omega^2$

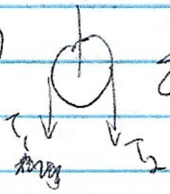
$\omega_f = \frac{1}{\sqrt{2}}\omega$ (c) ✓ B

6 3 1R

FR

1)  

α, a, v

A) 

$\Sigma\tau = T_2R - T_1R = I\alpha$

$= \frac{1}{2}MR^2 \cdot \frac{a}{R}$

$= \frac{1}{2}MRa$

$\Sigma F_2 = m_2g - T_2 = m_2a$ $\quad\Sigma F_1 = T_1 - m_1g = m_1a$

$T_2 = m_2g - m_2a = m_2(g-a)$

$T_1 = m_1a - m_1g = m_1(a-g)$

$m_2R(g-a) - m_1R(a-g) = \frac{1}{2}MRa$

$m_2R(g-a) + m_1R(g-a) = \frac{1}{2}MRa$

$(g-a)(m_2R + m_1R) = \frac{1}{2}MRa$

$\frac{1}{2}Ma = (g-a)(m_2+m_1)$

$\frac{M}{2(m_1+m_2)}a = g-a$

$a\left(\frac{M}{2(m_1+m_2)} + 1\right) = g$

$a\left(\frac{M+2(m_1+m_2)}{2(m_1+m_2)}\right) = g$

$a = \frac{2g(m_1+m_2)}{M+2(m_1+m_2)}$

$\Delta x = \cancel{v_0t} + \frac{1}{2}at^2$

$v_f^2 = \cancel{v_0^2} + 2a\Delta x$

$v_f = \sqrt{2ab}$

$= \sqrt{\frac{2\cdot 2bg(m_1+m_2)}{M+2(m_1+m_2)}}$

$= 2\sqrt{\frac{bg(m_1+m_2)}{M+2(m_1+m_2)}}$ ~

b) $\alpha = \frac{a}{r} = \frac{2g(m_1+m_2)}{r(M+2(m_1+m_2))}$ ~ $\quad a = r\alpha$

c) $S = r\theta = b$

$\theta = \frac{b}{r}$ ✓

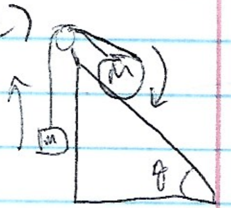
d) $b = \cancel{v_0t} + \frac{1}{2}at^2$

$t = \sqrt{\frac{2b}{a}}$

$= \sqrt{\frac{\cancel{2}b(M+2(m_1+m_2))}{\cancel{2}g(m_1+m_2)}}$

$= \sqrt{\frac{b(M+2(m_1+m_2))}{g(m_1+m_2)}}$ ~

almost!!

2) 

$v = R\omega$

$s = r\theta$

$s = r\theta$ $\quad \theta = 2\pi$

$C = 2\pi r$

$\Delta x = \bar{v} t$

$2\pi r = v_{cm} t$

$v_{cm} = \frac{2\pi r}{t}$

$f = \frac{1}{t}$; $\omega = 2\pi f$,

$\frac{2\pi}{t} = \omega$

$v_{cm} = R\omega$ ✓

b) 

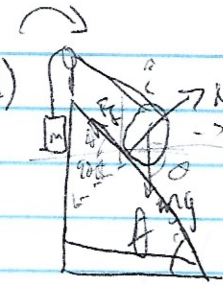
$v = r\omega$

has rotational ω_r as well as orbital ω_o ; $\omega_o = \omega_r$

$v_{top} = r\omega = 2R\omega$ ✓

$= 2 v_{cm}$

c) $a_m = a_{cm}$ (system) ~

d) 

$\Sigma \tau = I\alpha = \mu N - T$

$= \mu Mg \cos(90 - \theta)$

$= \mu Mg \cos\theta - T$

$= \frac{1}{2} MR^2 \cdot \frac{a}{R}$

$= \frac{1}{2} MRa$

$\Sigma F = m_{total} a = Mg\sin\theta - mg - F_f$

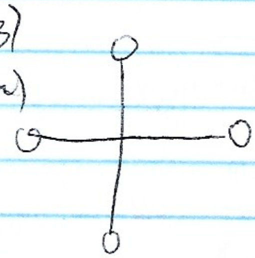
$= (m+M)a$

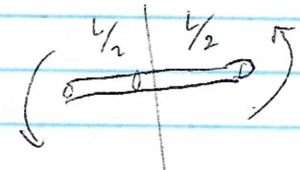
$\cancel{a = \frac{Mg\sin\theta - mg}{m+M} = g\frac{M\sin\theta - m}{m+M}}$

$Mg\sin\theta - mg - \mu Mg\cos\theta = (m+M)a$

$a = g \cdot \frac{M\sin\theta - \mu M\cos\theta - m}{m+M}$ ~

e) $a_{block} = a =$

3) a) 



$\lambda = \frac{m}{L}$

$dm = \lambda dL$

$dr = dL$ $\quad I = mr^2$

$= 2\int_0^{L/2} \lambda L^2 dL$?

$I_{total} = \frac{4}{3} ML^2 + 4mL^2$ ~

$L = r \times p = rp\sin\theta$

2

$L_0 = L_f$

$I\omega_0 = I\omega_f$

$I\cancel{\omega_0} + L m_b v = (I + m_b L^2)\,\omega_f$

$$\omega_f = \frac{m_b v L}{\frac{4}{3}ML^2 + 4mL^2 + m_b L^2} = \frac{m_b v}{\frac{4}{3}ML + 4mL + m_b L}$$

c) $v = r\omega = \dfrac{m_b v L^2}{\frac{4}{3}ML^2 + 4mL^2 + m_b L^2}$

$$= \frac{m_b v}{\frac{4}{3}M + 4m + m_b}$$

d) $KE_f = \frac{1}{2} I \omega_f^2$

$$= \frac{1}{2} I \cdot \frac{(m_b v L)^2}{I^2}$$

$$= \frac{(m_b v L)^2}{2\left(\frac{4}{3}ML^2 + 4mL^2 + m_b L^2\right)}$$

$$= \frac{m_b^2 v^2}{2\left(\frac{4}{3}M + 4m + m_b\right)}$$

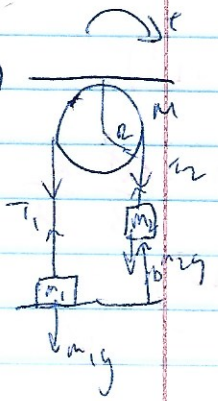
$$= \frac{m_b^2 v^2}{\frac{8}{3}M + 4m + m_b}$$

$KE_0 = \frac{1}{2} m_b v_b^2$

$$\frac{KE_f}{KE_0} = \frac{m_b^{\cancel{2}} \cancel{v_b^2}}{\frac{8}{3}M + 4m + m_b} \cdot \frac{2}{\cancel{m_b} \cancel{v_b^2}}$$

$$= \frac{2 m_b}{\frac{8}{3}M + 4m + m_b}$$

1) a)



assume 0 normal force

$\sum \tau = I\alpha = \frac{1}{2}MR^2\frac{a}{R} = T_2R - T_1R$

$\frac{1}{2}MRa = T_2R - T_1R$

$\sum F_2 = m_2a = m_2g - T_2 = m_2a$

$\sum F_1 = m_1a = T_1 - m_1g = m_1a$

$T_1 = m_1a + m_1g$

$T_2 = m_2g - m_2a$

$(m_2g - m_2a)R - (m_1a + m_1g)R = \frac{1}{2}MRa$

$\frac{1}{2}Ma = m_2g - m_2a - m_1a - m_1g$

$a\left(\frac{1}{2}M + m_2 + m_1\right) = m_2g - m_1g$

$a = g\left(\frac{m_2 - m_1}{\frac{1}{2}M + m_2 + m_1}\right)$

$\cancel{= 2g\left(\frac{m_2 - m_1}{M + 2m_2 + 2m_1}\right)}$

A) $v_f^2 = \cancel{v_0^2} + 2ab$

$v_f = \sqrt{2ab}$

$= \sqrt{2gb\left(\frac{m_2 - m_1}{\frac{1}{2}M + m_2 + m_1}\right)}$

1

B) $v = r\omega$ $\quad \omega = \frac{v}{R} = \frac{1}{R}\sqrt{2gb\left(\frac{m_2 - m_1}{\frac{1}{2}M + m_1 + m_2}\right)}$

d) $v_f = \cancel{v_0} + at$ $\quad \Delta x = \cancel{v_0 t} + \frac{1}{2}at^2$

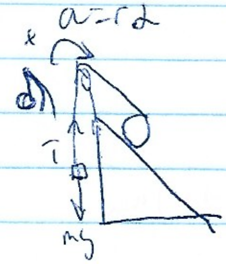
$2b = at^2$ $\quad t^2 = \frac{2b}{a}$

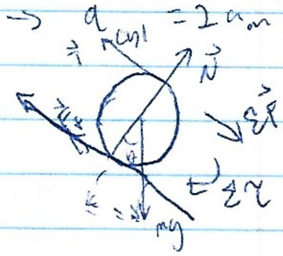
$t = \sqrt{\frac{2b}{a}} = \sqrt{\frac{2b\left(\frac{1}{2}M + m_1 + m_2\right)}{g(m_2 - m_1)}}$

2) c), d), e)

c) b/c the pivot point is on the perimeter,

$a = r\alpha \rightarrow a_{cyl} = 2a_{cm}$? why?





$a_b = 2a_{cm}$ $\quad \sum \tau = I\alpha = \frac{1}{2}MR^2\frac{a_{cm}}{R}$

$\cancel{R}\mu Mg\cos\theta - T\cancel{R} = \frac{1}{2}M\cancel{R}a_{cm}$

$\sum F = ma_b$

$-mg + T = ma_b$

$T - mg = m(2a_{cm})$ $\quad \mu Mg\cos\theta - T = \frac{1}{2}Ma_{cm}$

$T = \mu Mg\cos\theta - \frac{1}{2}Ma_{cm}$

$T = 2ma_{cm} + mg = \mu Mg\cos\theta - \frac{1}{2}Ma_{cm}$

$2ma_{cm} + \frac{1}{2}Ma_{cm} = \mu Mg\cos\theta - mg$

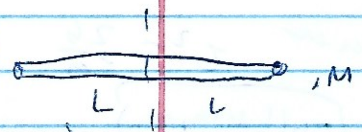
$a_{cm}\left(2m + \frac{1}{2}M\right) = \mu Mg\cos\theta - mg$ 3

$a_{cm}(2m+\frac{1}{2}M) = \mu M g\cos\theta - mg$

$a_{cm} = g\left(\frac{\mu M\cos\theta - m}{2m+\frac{1}{2}M}\right)$ ~

c) $a_b = 2a_{cm}$

$= 2g\left(\frac{\mu M\cos\theta - m}{2m+\frac{1}{2}M}\right)$ ~

3) a) 

??? ~

b) d)

---

2) b) $T - mg = ma_b = 2ma_{cm}$

$Mg\sin\theta - F_f - T = Ma_{cm}$

$RF_f - RT = I\alpha$

$RF_f - RF_T = \frac{1}{2}MR^2\frac{a_{cm}}{R}$

$F_f - T = \frac{1}{2}Ma_{cm}$

$Mg\sin\theta - 2T = \frac{3}{2}Ma_{cm}$

$2T - 2mg = 4ma_{cm}$

$Mg\sin\theta - 2mg = \left(\frac{3M}{2}+4m\right)a_{cm}$

$a_{cm} = \left(\frac{M\sin\theta - 2m}{\frac{3}{2}M+4m}\right)g$ ✓

c) $a_b = 2a_{cm} = 2\left(\frac{M\sin\theta - 2m}{\frac{3}{2}M+4m}\right)g$ ✓

3) a) 

$2L, M$

linear density $\frac{M}{2L} = \lambda$ $\quad dm = \lambda\,dx$

$I = \int r^2\,dm = \int_{-L}^{L} x^2\lambda\,dx = \lambda\int_{-L}^{L} x^2\,dx$

$= \lambda\left[\frac{1}{3}x^3\right]_{-L}^{L} = \frac{\lambda}{3}(L^3+L^3)$

$= \frac{2L^3}{3}\lambda = \frac{2L^3}{3}\cdot\frac{M}{2L}$

$= \boxed{\frac{1}{3}ML^2}$ ✓

b) $\vec{l} = \vec{r}\times\vec{p}$ $\quad l = I\omega$

$l = rp_\perp = rm_bv_\perp = rm_bv\cos\theta$

$= Lm_bv\cos\theta$

$Lm_bv\cos\theta = \left(4mL^2 + 2\left(\frac{1}{3}ML^2\right) + m_bL^2\right)\omega_f$

$\omega_f = \frac{m_bv\cos\theta}{L(4m+\frac{2}{3}M+m_b)}$ ✓

c) $v = R\omega$ $\quad v_f = L\omega_f$

$v_f = \left(\frac{m_bv\cos\theta}{4m+\frac{2}{3}M+m_b}\right)$ ✓

d) $\frac{KE_f}{KE_0} = \frac{\frac{1}{2}I\omega_f^2}{\frac{1}{2}m_bv^2}$

$= \frac{\left(4mL^2+\frac{2}{3}ML^2+m_bL^2\right)\left(\frac{m_bv\cos\theta}{L(4m+\frac{2}{3}M+m_b)}\right)^2}{m_bv^2}$

$= \boxed{\frac{m_b\cos^2\theta}{4m+\frac{2}{3}M+m_b}}$ ✓