

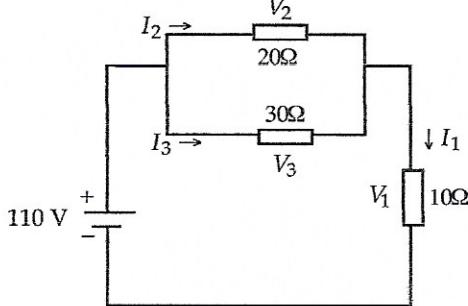
FORM A

- In terms of conservation laws, what is the physical meaning of Kirchhoff's Junction Rule (KJR)?
  - Conservation of momentum
  - Conservation of angular momentum
  - Conservation of energy
  - Conservation of charge
  - None of these
- In terms of conservation laws, what is the physical meaning of Kirchhoff's Loop Rule (KLR)?
  - Conservation of momentum
  - Conservation of angular momentum
  - Conservation of energy
  - Conservation of charge
  - None of these
- What is the equivalent resistance of the combination of the resistors given below?
 

A.  $5.45\Omega$  B.  $10\Omega$  C.  $15\Omega$  D.  $22\Omega$  E.  $60\Omega$
- What is the equivalent resistance of the combination of the resistors given below?
 

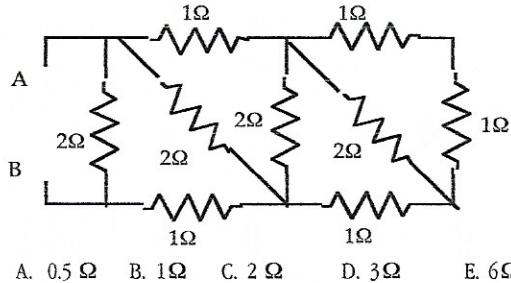
A.  $5.45\Omega$  B.  $10\Omega$  C.  $15\Omega$  D.  $22\Omega$  E.  $60\Omega$

Questions 5-18 Use the circuit below to answer the questions.



- How many junctions are there in this circuit?
  - 1
  - 2
  - 3
  - 4
  - 5
- How many independent loops are there in this circuit?
  - 1
  - 2
  - 3
  - 4
  - 5
- What is the equivalent resistance of this circuit?
  - $5.45\Omega$
  - $10\Omega$
  - $15\Omega$
  - $22\Omega$
  - $60\Omega$
- What is the magnitude of the current  $I_1$ ?
  - 11 A
  - 5 A
  - 3 A
  - 2 A
  - 0 A
- What is the magnitude of the voltage  $V_1$ ?
  - 110 V
  - 80 V
  - 60 V
  - 50 V
  - 0 V
- What is the magnitude of the voltage  $V_2$ ?
  - 110 V
  - 80 V
  - 60 V
  - 50 V
  - 0 V
- What is the magnitude of the voltage  $V_3$ ?
  - Same as  $V_2$
  - Same as  $V_1$
  - 110 V
  - 40 V
  - $30V$
- What is the magnitude of the current  $I_2$ ?
  - 11 A
  - 5 A
  - 3 A
  - 2 A
  - 0 A
- What is the magnitude of the current  $I_3$ ?
  - 11 A
  - 5 A
  - 3 A
  - 2 A
  - 0 A
- How much power is produced by the battery?
  - 550 W
  - 250 W
  - 180 W
  - 120 W
  - 0 W

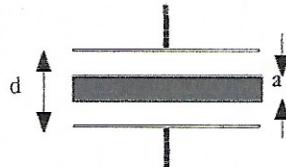
- How much power is consumed by the  $10\Omega$  resistor?
  - 550 W
  - 250 W
  - 180 W
  - 120 W
  - 0 W
- How much power is consumed by the  $20\Omega$  resistor?
  - 550 W
  - 250 W
  - 180 W
  - 120 W
  - 0 W
- How much power is consumed by the  $30\Omega$  resistor?
  - 550 W
  - 250 W
  - 180 W
  - 120 W
  - 0 W
- How much power is consumed by all three resistors in the circuit?
  - 550 W
  - 250 W
  - 180 W
  - 120 W
  - 0 W
- Calculate the resistance of the circuit below



- Calculate the resistance of the circuit below

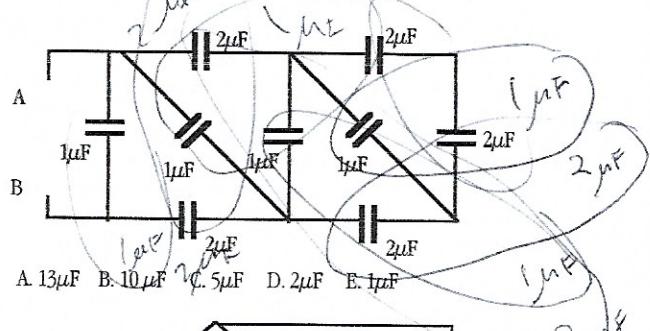
A.  $0.5\Omega$  B.  $1\Omega$  C.  $2\Omega$  D.  $3\Omega$  E.  $6\Omega$

A metal slab of thickness  $a < d$  is placed inside a parallel plate capacitor with area  $A$  and separation  $d$ . Assume that this arrangement acts like two sub-capacitors since the metal slab acts like a wire.

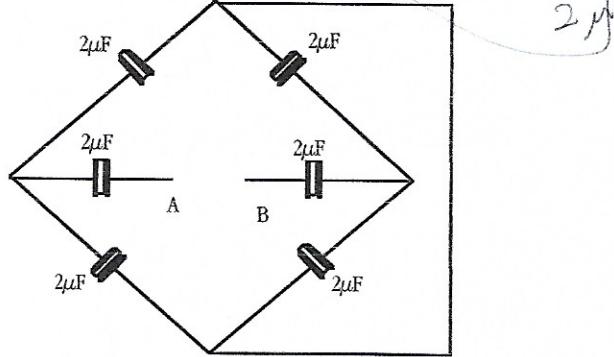


- How are these sub-capacitors connected?
  - Parallel
  - Series
  - Mixed
- What is the equivalent capacitance of this arrangement?
  - $\frac{\epsilon A}{d}$
  - $\frac{\epsilon A}{d-a}$
  - $\frac{2\epsilon A}{d-a}$
  - $\frac{\epsilon A}{2(d-a)}$
  - $\infty$
- What happens to the capacitance when  $a \rightarrow 0$ ?
  - $\frac{\epsilon A}{d}$
  - Becomes 0
  - Becomes  $\infty$
  - Cannot be determined
- What happens to the capacitance when  $a \rightarrow d$ ?
  - $\frac{\epsilon A}{d}$
  - Becomes 0
  - Becomes  $\infty$
  - Cannot be determined

Calculate the equivalent capacitance for each circuit given.



25. A.  $13\mu F$  B.  $10\mu F$  C.  $5\mu F$  D.  $2\mu F$  E.  $1\mu F$

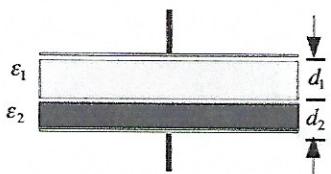


26. A.  $1.5\mu F$  B.  $(2/3)\mu F$  C.  $8\mu F$  D.  $12\mu F$  E.  $6\mu F$

27. Near the surface of the Earth there is an electric field with a magnitude of about  $100 \frac{N}{C}$  directed vertically down. Assume this field is caused by a net free charge on the Earth. Calculate the net charge using the electric field given in the problem. The radius of the Earth is approximately 6400 km.

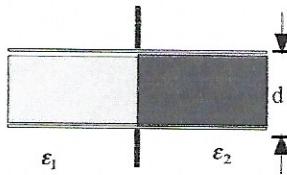
$$A. 100C \quad B. 4.5 \times 10^5 C \quad C. 4\pi\epsilon \times 6400C \quad D. (4\pi\epsilon / 6400)C \quad E.$$

28. Calculate the capacitance for the arrangement (Hint: First identify how many different capacitors you have and how they are connected. Then calculate the equivalent capacitance in terms of the given values).



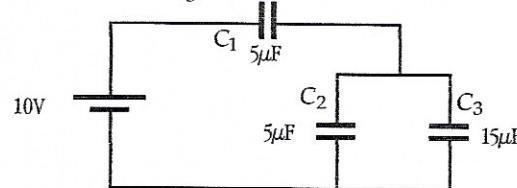
$$A. \frac{(\epsilon_1 + \epsilon_2)A}{d_1 + d_2} \quad B. \frac{\epsilon_1 A}{d_1} + \frac{\epsilon_2 A}{d_2} \quad C. \frac{1}{\frac{d_1}{\epsilon_1 A} + \frac{d_2}{\epsilon_2 A}} \quad D. \frac{1}{\frac{\epsilon_1 A}{d_1} + \frac{\epsilon_2 A}{d_2}}$$

29. Calculate the capacitance for the arrangement (Hint: First identify how many different capacitors you have and how they are connected. Then calculate the equivalent capacitance in terms of the given values).



$$A. \frac{(\epsilon_1 + \epsilon_2)A}{2d} \quad B. \frac{(\epsilon_1 + \epsilon_2)A}{d} \quad C. \frac{2(\epsilon_1 + \epsilon_2)A}{d} \\ D. \frac{\epsilon_1 \epsilon_2 A}{2d(\epsilon_1 + \epsilon_2)} \quad E. \frac{\epsilon_1 \epsilon_2 A}{d(\epsilon_1 + \epsilon_2)} \quad F. \frac{2\epsilon_1 \epsilon_2 A}{d(\epsilon_1 + \epsilon_2)}$$

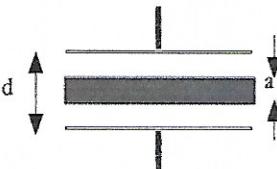
Consider the circuit given.



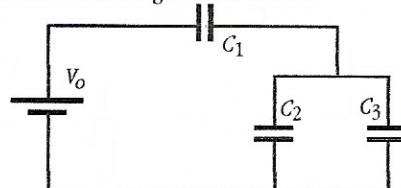
Calculate

30. the equivalent capacitance for the circuit  
A.  $4\mu F$  B.  $(1/4)\mu F$  C.  $5\mu F$  D.  $15\mu F$  E.  $25\mu F$   
 31. the charge stored in the circuit  
A.  $40\mu C$  B.  $30\mu C$  C.  $20\mu C$  D.  $10\mu C$  E.  $5\mu C$   
 32. the energy stored in the circuit  
A.  $200\mu J$  B.  $160\mu J$  C.  $40\mu J$  D.  $30\mu J$  E.  $10\mu J$   
 33. the voltage across the capacitor  $C_1$   
A.  $10 V$  B.  $2 V$  C.  $4 V$  D.  $5 V$  E.  $8 V$  F.  
 34. the voltage across the capacitor  $C_2$   
A.  $10 V$  B.  $2 V$  C.  $4 V$  D.  $5 V$  E.  $8 V$  F.  
 35. the voltage across the capacitor  $C_3$   
A.  $10 V$  B. Same as  $C_1$  C. Same as  $C_2$  D.  $(1/3)C_2$  E.  $3C_2$  F.  
 36. the charge stored on  $C_1$   
A.  $40\mu C$  B.  $30\mu C$  C.  $20\mu C$  D.  $10\mu C$  E.  $5\mu C$   
 37. the charge stored on  $C_2$   
A.  $40\mu C$  B.  $30\mu C$  C.  $20\mu C$  D.  $10\mu C$  E.  $5\mu C$   
 38. the charge stored on  $C_3$   
A.  $10 V$  B. Same as on  $C_1$  C. Same as on  $C_2$   
D.  $(1/3)C_2$  E.  $3$  times of  $C_2$  F.  
 39. the energy stored on  $C_1$   
A.  $200\mu J$  B.  $160\mu J$  C.  $40\mu J$  D.  $30\mu J$  E.  $10\mu J$   
 40. the energy stored on  $C_2$   
A.  $200\mu J$  B.  $160\mu J$  C.  $40\mu J$  D.  $30\mu J$  E.  $10\mu J$   
 41. the energy stored on  $C_3$   
A.  $10 V$  B. Same as on  $C_1$  C. Same as on  $C_2$

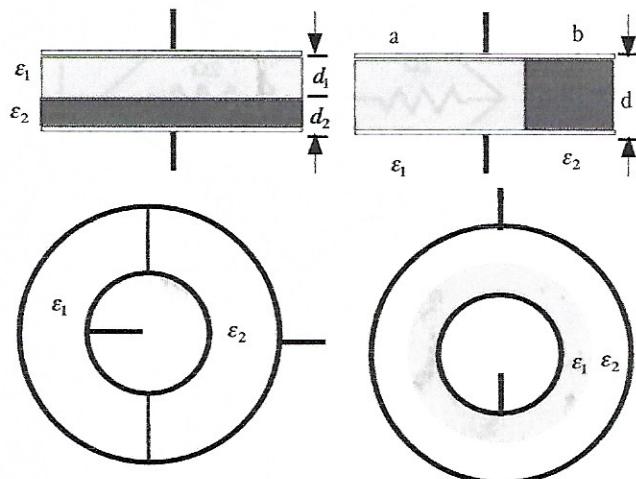
- A conducting sphere of radius  $a$  is surrounded by a concentric, thin conducting spherical shell of radius  $b$ . Each sphere has equal but opposite charges of quantity  $Q$ .
  - Use Gauss' law to show that  $E = \frac{Q}{4\pi\epsilon_0 r^2}$  between the two plates. (*Show every step clearly! Do not fudge!*)
  - Use your answer in part A to calculate the voltage difference between the two spheres?
  - Use your answer in part B to calculate the capacitance of this arrangement.
- A conducting cylinder of radius  $a$  and length  $l$  is surrounded by a concentric, thin conducting cylinder shell of radius  $b$  and length  $l$ . Each cylinder has equal but opposite charges of quantity  $Q$ . (*Show every step clearly! Do not fudge!*)
  - Use Gauss' law to show that  $E = \frac{\lambda}{2\pi\epsilon_0 r}$  between the plates.
  - Use your answer in part A to calculate the voltage difference between the two cylinders?
  - Use your answer in part B to calculate the capacitance of this arrangement.
- Two parallel plates of area  $A$  separated by a distance  $d$ . Each plate has equal but opposite charges of quantity  $Q$ . (*Show every step clearly! Do not fudge!*)
  - Use Gauss' law to show that  $E = \frac{\sigma}{\epsilon_0}$  between the plates.
  - Use your answer in part A to calculate the voltage difference between the two plates?
  - Use your answer in part B to calculate the capacitance of this arrangement.

*Give your final answers in terms of physical constants and given quantities.*
- A metal slab of thickness  $a < d$  is placed inside a parallel plate capacitor with area  $A$  and separation  $d$ . Assume that this arrangement acts like two sub-capacitors since the metal slab acts like a wire.
 
  - Are these sub-capacitors connected in parallel or series?
  - What is the equivalent capacitance of this arrangement?
  - What happens when
    - $a \rightarrow 0$  (i.e. thickness of metal is reduced to zero.)
    - $a \rightarrow d$  (i.e. thickness is increased to cover the space between plates.)
- Use the relation among  $Q$ ,  $I$ ,  $V$ , and  $C$  to derive an expression for the equivalent capacitance of  $n$  capacitors connected in
  - parallel to each other
  - series to each other

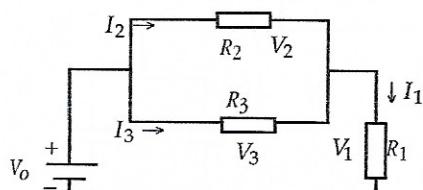
- Consider the circuit given and calculate



- the equivalent capacitance of the circuit
- the charge stored in the circuit
- the energy stored in the circuit
- the voltage across each capacitor
- the charge stored on each capacitor
- the energy stored in each capacitor
- Is the charge stored in the circuit equal to the sum of the charges stored in the elements?
- Is the energy stored in circuit the same as the sum of the energies in the elements?
- Near the surface of the Earth there is an electric field with a magnitude of about  $E_0$  directed vertically down. Assume this field is caused by a net free charge on the Earth.
  - Calculate the net charge using the electric field given in the problem. The radius of the Earth is  $R_E$ .
  - What electrostatic energy density is stored near the surface of the Earth?
- Calculate the capacitance for the following arrangements (Hint: First identify how many different capacitors you have and how they are connected. Then assign each a different value, i.e.,  $C_1$ ,  $C_2$ ,  $C_3$ , ... . Finally, calculate the equivalent capacitance in terms of the given values):
  - Parallel Plate capacitors
  - Spherical capacitors (spheres of radii  $a$ ,  $b$ , and  $r_o$ )
  - Cylindrical capacitors (cylinders of radii  $a$ ,  $b$ , and  $r_o$ )



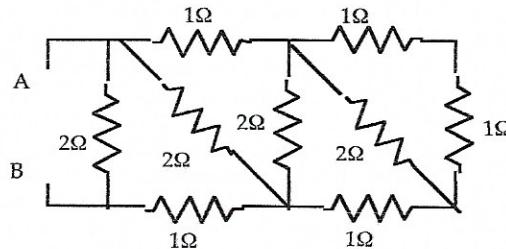
9. Use the circuit below to answer the questions.



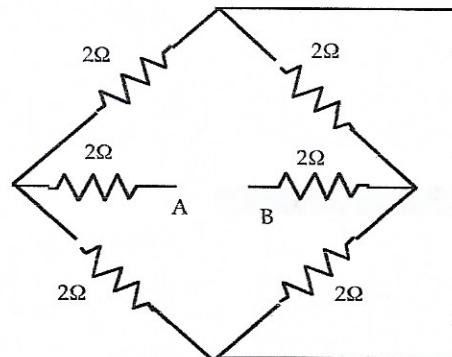
- A. How many junctions and loops are there in this circuit?  
How many are independent?
- B. What is the equivalent resistance of this circuit?
- C. What is the magnitude of the voltage across each element?
- D. What is the magnitude of the current in each branch?
- E. How much power is produced by the battery?
- F. How much power is consumed by each resistor?
- G. How much power is consumed by all three resistors in the circuit?

10. Calculate the resistance of the circuits below

A.

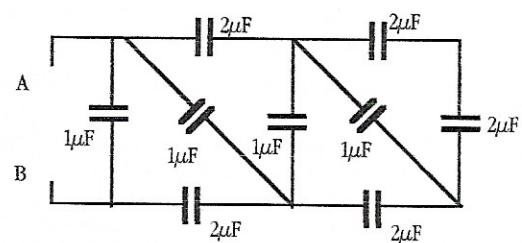


B.

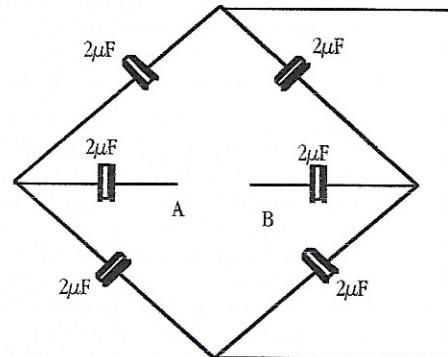


11. Calculate the capacitance of the circuits below

A.

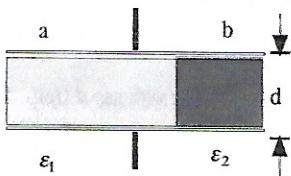


B.

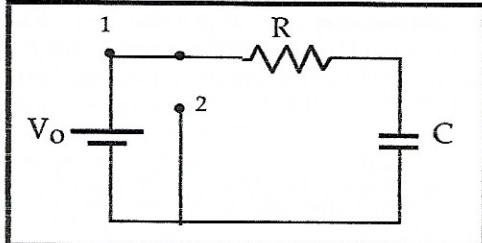


*A word this  
one*

- I. Calculate the capacitance of the arrangement below.



- II. Consider the RC circuit given in the figure. At  $t=0$ , the voltage source is connected to the circuit by closing switch 1 as shown in the figure below.



- A. Use dimensional analysis to obtain the characteristic time frame for this circuit.

*Hint: Use the voltage across R and C in terms of the charge flow in the circuit.*

- B. Write down the Kirchhoff's voltage law for this circuit.

- C. Revise your answer to part B to write a differential equation in terms of the charge or the current only.

- D. Obtain an expression for the charge on the capacitor as a function of time.

- E. Obtain an expression for the current in the loop as a function of time.

F.

1. What is the voltage across the capacitor as a function of time?
2. What is the voltage across the resistor as a function of time?

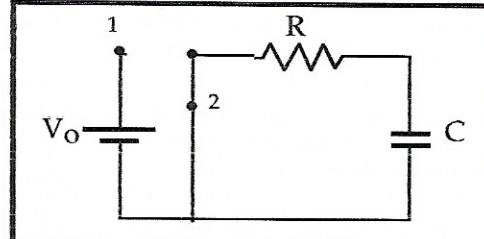
- G. After a long time period with respect to the circuit's characteristic time frame,

1. How much charge will there be on the capacitor?
2. What value will the current through and the voltage across the capacitor reach?
3. What value will the current through and the voltage across the resistor reach?

$$\int_{x_1}^{x_2} \frac{dx}{x-a} = \ln\left(\frac{x_2-a}{x_1-a}\right)$$

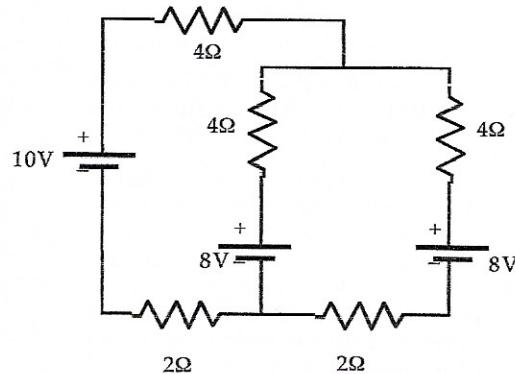
*Even if you cannot evaluate the integrals involved, you will receive full credit for each correctly answered part using dimensional analysis and relevant justification involving the physics of the situation.*

- III. After a long period of time as described in the preceding problem, the voltage source is disconnected by opening switch 1, and switch 2 is closed as shown in the figure below. Take this to be your new  $t=0$ .



- A. Write down the Kirchhoff's voltage law for this circuit.
- B. Revise your answer to part A to write a differential equation in terms of the charge or the current only.
- C. Obtain an expression for the charge on the capacitor as a function of time.
- D. Obtain an expression for the current in the loop as a function of time.
- E.
  1. What is the voltage across the capacitor as a function of time?
  2. What is the voltage across the resistor as a function of time?
- F. After a long time period with respect to the circuit's characteristic time frame,
  1. How much charge will there be on the capacitor?
  2. What value will the current through and the voltage across the capacitor reach?
  3. What value will the current through and the voltage across the resistor reach?

IV.



- A. Write down the Kirchhoff's voltage rule for each loop.
- B. Use your answers to obtain the
  1. current in each resistor
  2. voltage across each resistor
  3. power consumed by each resistor.
- C. Is the sum of powers produced by the batteries equal to the sum of the powers consumed by each resistor? Why (not)?

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ML

1) D 2) C 3)  $10 + 20 + 30 = 60 \Omega E$  20)

4)  $(\frac{1}{10} + \frac{1}{20} + \frac{1}{30})^{-1} = 5.45 \Omega A$

5) 2 (B) 6) 1 (A)

7)  $R_{eq} = (\frac{1}{20} + \frac{1}{30})^{-1} + 10 = 22 \Omega$

8)  $I_1 = \frac{\Delta U}{R} = \frac{110V}{22\Omega} = 5A$  B

9)  $V_1 = I_1 R_1 = (5A)(10\Omega) = 50V$  D

10)  $V_2 = 110V - 50V = 60V$  C

11)  $V_3 = V_2$

12)  $I_2 = \frac{\Delta V_2}{R_2} = \frac{60V}{20\Omega} = 3A$

13)  $I_3 = \frac{\Delta V_3}{R_3} = \frac{60V}{30\Omega} = 2A$

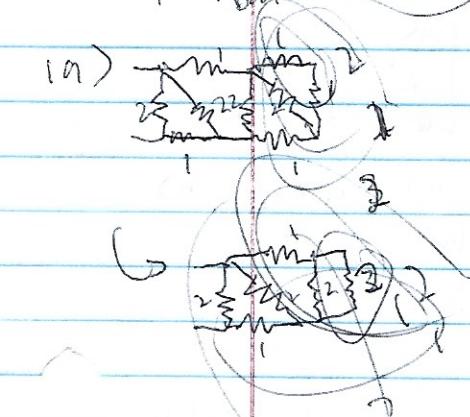
14)  $P = I_1 \Delta U = (5A)(10V) = 50W$  A

15)  $P = I^2 R = (5A)^2 (10\Omega) = 250W$  B

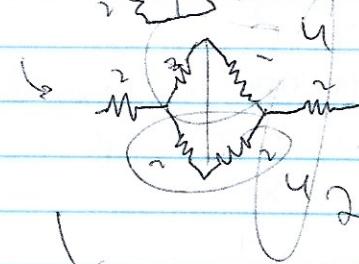
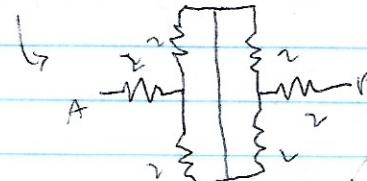
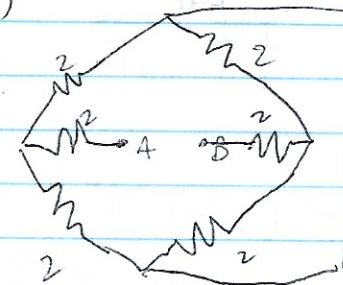
16)  $P = I^2 R = (3A)^2 (20\Omega) = 180W$  C

17)  $P = I^2 R = (2A)^2 (30\Omega) = 120W$  D

18)  $P = P_{out} = 550W$  A



$\rightarrow 3.5\Omega$  B



$\rightarrow 6 \Omega E$

21) B  $\frac{1}{\text{_____}}$

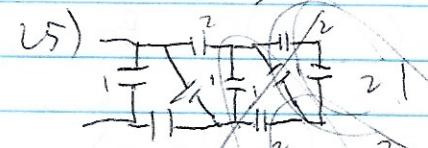
22)  $C_{eq} = \left( \frac{1}{C_1} + \frac{1}{C_2} \right)^{-1}$

$C = \frac{C_1 C_2}{d-a} \quad C_1 = \frac{C_0 A}{(\frac{d-a}{2})^2} = \frac{2C_0 A}{d-a}$

$\left( \frac{d-a}{2C_0 A} + \frac{d-a}{2C_0 A} \right)^{-1}$

$= \frac{C_0 A}{d-a} B$

23) A 24) B



$2 \mu F$  D

$\rightarrow 1.62 \mu F$

26) 
$$C_{eq} = \left( \frac{1}{5\mu F} + \frac{1}{15\mu F} \right)^{-1} = 3\mu F$$

27)  $E = \frac{kQ}{r^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r^2}$   
 $Q = \epsilon_0 \epsilon_r \cdot E r^2$   
 $= 4\pi(8.85 \times 10^{-12} \text{ F/V}) (6400,000 \text{ m})^2 (100 \text{ V})$   
 $= 1.56 \times 10^5 \text{ C}$

28) 2 capacitors in series  
 $C_{eq} = \left( \frac{1}{C_1} + \frac{1}{C_2} \right)^{-1}$   
 $C_1 = \frac{k_1 \epsilon_0 A}{d_1} = \frac{\epsilon_1 A}{d_1}$   
 $C_2 = \frac{\epsilon_2 A}{d_2}$   
 $C_{eq} = \left( \frac{d_2}{\epsilon_1 A} + \frac{d_1}{\epsilon_2 A} \right)^{-1}$

29) 2 capacitors in parallel  
 $C_{eq} = C_1 + C_2$   
 $C_1 = \frac{\epsilon_1 A}{d} \quad C_2 = \frac{\epsilon_2 A}{d}$   
 $C_{eq} = \frac{(\epsilon_1 + \epsilon_2) A}{d}$

30) 
 $C_{eq} = \left( \frac{1}{5\mu F} + \frac{1}{15\mu F} \right)^{-1} = 4\mu F \text{ A}$

31)  $Q = \frac{C \Delta V}{V} = \frac{4\mu F \cdot 10V}{10V} = 4 \times 10^{-6} \text{ C}$   
 $Q = CV = 4\mu F \cdot 10V = 40 \mu C$

32)  $P_E = \frac{1}{2} CV^2 = \frac{1}{2} \cdot 4\mu F \cdot (10V)^2 = 2 \times 10^{-6} \text{ J}$

33)  $V_1 = 10V \text{ A}$   
 $V_2 = 10V \text{ A}$   
 $V_3 = 10V \text{ A}$

34)  $\Delta = CV = (4\mu F)(10V) = 40 \mu C$   
 $C_2 = \frac{Q}{V_2} = \frac{40 \mu C}{10V} = 4 \mu F$   
 $C_3 = \frac{Q}{V_3} = \frac{40 \mu C}{10V} = 4 \mu F$

35)  $F = k \frac{Q_1 Q_2}{r^2}$   
 36)  $\frac{Q}{r^2} = \frac{Q_1}{r_1^2} + \frac{Q_2}{r_2^2}$

37)  $E = \frac{kQ}{r^2} = \frac{kQ}{(b-a)^2}$   
 $E = \frac{kQ}{r^2} = \frac{kQ}{(b-a)^2}$

38)  $E = \frac{kQ}{r^2} = \frac{kQ}{(b-a)^2}$   
 $E = \frac{kQ}{r^2} = \frac{kQ}{(b-a)^2}$

39)  $E = \frac{kQ}{r^2} = \frac{kQ}{(b-a)^2}$   
 $E = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q_{ext}}{r^2} = \frac{Q}{4\pi\epsilon_0 r^2}$

40)  $V = - \int E \cdot dr = - \int \frac{kQ}{r^2} dr = -kQ \int r^{-2} dr = \frac{kQ}{r} = \frac{kQ}{b-a}$   
 $C = \frac{Q}{V} = \frac{(b-a)}{4\pi\epsilon_0 (b-a)}$

41) 
 $\lambda = \frac{Q}{l}$   
 $dQ = \lambda dl$

$$2) \text{a)} \oint E \cdot d\sigma \quad S = 2\pi r l \quad 4) \text{a)} \text{series} \quad b) C_{\text{el}} = \frac{\epsilon_0 A}{d-a} \quad \text{MC 22}$$

$$\int E \cdot 2\pi r dl = \frac{Q}{\epsilon_0} \quad A = \pi r l$$

$$E \cdot l \cdot 2\pi r = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{2\pi r \epsilon_0}$$

c) MC 23, 24

$$5) C = \frac{Q}{V} \quad I = \frac{dQ}{dt} \quad \boxed{5}$$

6) MC 30-41

7) MC 27

8) a) 2 capacitors in series

MC 28-29

$$b) \text{if } -2\pi \epsilon_0 l \neq \text{parallel}$$

$$C = \frac{1}{2\pi \epsilon_0 (a + \epsilon_1 l)}$$

(=  $4\pi \epsilon_0 (b-a)$  parallel)

$$C_{\text{eq}} = 4\pi \epsilon_0 (a + \epsilon_1 l)(b-a)$$

$$\text{series} \quad C = \frac{1}{\frac{1}{4\pi \epsilon_0 a} + \frac{1}{4\pi \epsilon_0 b}}$$

$$\left\{ C_{\text{eq}} = \frac{1}{\frac{1}{4\pi \epsilon_1 a} + \frac{1}{4\pi \epsilon_2 b}} \right.$$

$$3) \frac{Q_{\text{ext}}}{\epsilon_0} = \oint E \cdot d\sigma$$

$$\text{a)} \quad EA = \frac{Q}{\epsilon_0} \quad E = \frac{Q}{A \epsilon_0} \quad \sigma = \frac{Q}{A}$$

$$\boxed{E = \frac{\sigma}{\epsilon_0}}$$

a) MC 3-4

b) MC 4a-20

c) MC 25-26

FHQ I) 2 in parallel

$$C_1 = \frac{ad\epsilon_1}{d} \quad C_2 = \frac{bd\epsilon_2}{d}$$

$$\cancel{C_{\text{eq}} = \frac{1}{\frac{1}{ad\epsilon_1} + \frac{1}{bd\epsilon_2}}}$$

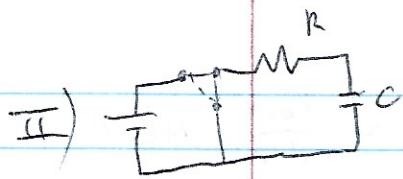
$$\cancel{C_{\text{eq}} = \frac{1}{d} (a\epsilon_1 + b\epsilon_2)} \quad \checkmark$$

$$b) V = - \int E \cdot dr = - \int_a^b \frac{E}{\epsilon_0} dr$$

$$V = \frac{\sigma d}{\epsilon_0}$$

$$C = \frac{Q}{V} = \frac{A \epsilon_0}{d \cdot d}$$

$$= \frac{\epsilon_0 A}{d}$$



$$c) \frac{Q}{C} = V_0 - IR$$

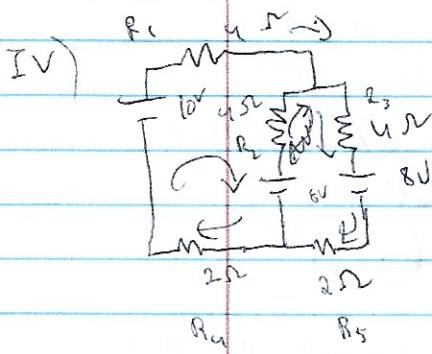
- A)  $Z = R + jC$   
B)  $V_C = V_0$

$$c) \frac{CQ}{V}$$

and different approach?

$$\text{defn } I = \frac{dQ}{dt}$$

$$\text{II}) V_2 = V_C$$



$$-V_0 + IR + \frac{Q}{C} = 0$$

$$\frac{d}{dt}(-V_0) + \frac{1}{R} IR + \frac{1}{C} \frac{dQ}{dt} = 0$$

$$R \frac{dI}{dt} + \frac{1}{C} \frac{dQ}{dt} = 0$$

$$d) R \frac{dI}{dt} + \frac{I}{C} = 0$$

$$\frac{dI}{I} = -\frac{dt}{RC}$$

$$\ln I - \ln I_0 = -\frac{t}{RC}$$

$$\ln \left| \frac{I}{I_0} \right| = -\frac{t}{RC}$$

$$\frac{I}{I_0} = e^{-\frac{t}{RC}}$$

$$I = I_0 e^{-\frac{t}{RC}}$$

$$I_0 = \frac{V_0}{R}$$

$$I = \frac{V_0}{R} e^{-\frac{t}{RC}} \quad [e]$$

$$\frac{dQ}{dt} = \frac{V_0}{R} e^{-\frac{t}{RC}} \quad Q = CV_0$$

$$Q = \int_0^t \frac{V_0}{R} e^{-\frac{t}{RC}} dt \quad u = -\frac{t}{RC} \quad \frac{du}{dt} = -\frac{1}{RC}$$

$$Q(t) = \frac{V_0}{R} \cdot -RC \cdot e^{-\frac{t}{RC}} \Big|_0^t \quad \rightarrow$$

$$Q_0 = CV_0$$

$$Q(t) = -V_0 C \cdot e^{-\frac{t}{RC}} \Big|_{t=0} = -Q_0 e^{-\frac{t}{RC}} + Q_0$$

$$Q(t) = Q_0 (1 - e^{-\frac{t}{RC}}) \quad [d]$$

Q)

$$a) C = \frac{Q}{V} \quad V = \frac{Q}{C}$$

$$\frac{Q(t)}{C} = \frac{Q_0}{C} (1 - e^{-\frac{t}{RC}})$$

$$i) V(t) = V_0 (1 - e^{-\frac{t}{RC}})$$

$$ii) \Delta V = IR$$

$$V_0 - V_c = IR \approx V_R$$

$$V_R = V_0 e^{-\frac{t}{RC}} \cdot R$$

$$V_R = V_0 e^{-\frac{t}{RC}}$$

$$g) i) Q = CV_0 e^{\frac{t}{RC}}$$

ii) current doesn't actually pass through the capacitor

$$V_c = V_0$$

$$3) I_R = 0, V_R = 0$$

$$III) \quad C = \frac{Q}{V} \quad V = \frac{Q}{C}$$

$$a) \frac{Q}{C} = IR$$

$$b) \frac{d}{dt} \frac{Q}{C} = \frac{1}{RC} \cancel{IR}$$

$$\frac{dQ}{dt} \cdot \frac{1}{C} = \frac{dI}{dt} R$$

$$c) \frac{I}{C} = \frac{dI}{dt} \cdot R$$

$$d) \frac{dI}{I} = \frac{1}{RC} dt$$

$$\int \frac{dI}{I} = \int_0^t \frac{1}{RC} dt$$

$$\ln \left| \frac{I}{I_0} \right| = \frac{t}{RC}$$

$$\frac{I}{I_0} = e^{\frac{t}{RC}} \quad \rightarrow = \frac{V_0}{R} e^{\frac{t}{RC}}$$

$$I = I_0 e^{\frac{t}{RC}} \quad R I_0 = V_0$$

$$\frac{dQ}{dt} = I_0 e^{\frac{t}{RC}} \quad I_0 = \frac{V_0}{R}$$

$$dQ = \frac{V_0}{R} e^{\frac{t}{RC}} dt \quad u = \frac{t}{RC}$$

$$du = \frac{1}{RC} dt \quad dt = RC du$$

$$Q(t) = \frac{V_0}{R} \int_0^t e^{u \cdot RC} du$$

$$Q(t) = \frac{V_0}{R} \cdot RC \cdot e^{\frac{t}{RC}}$$

$$= V_0 C e^{\frac{t}{RC}} = Q_0 e^{\frac{t}{RC}}$$

$$Q = CV \quad V = \frac{Q}{C}$$

$$V(t) = V_0 e^{\frac{t}{RC}}$$

$$V_R = V_0 - V(t) = V_0 (1 - e^{\frac{t}{RC}})$$

$$f) Q = CV = 0$$

$$I_R = 0 \quad I_C = 0$$

$$\Delta V = 0 \quad \Delta V = 0$$

- KJR involves flow of charges into and from a junction. Therefore, it is nothing but the law of conservation of electric charges. *1pt*
- KLR involves adding all the voltages changes (which are the electric potential energies per unit charge) in a loop. Therefore, it is nothing but the conservation of energy. *1pt*
- These resistors are in series. We simply add them: *1pt*  

$$R_{\text{eq}} = 10 \Omega + 20 \Omega + 30 \Omega = 60 \Omega$$
- These resistors are in parallel. We add their inverses, then invert the result:  $1/(R_{\text{eq}}) = 1/(10 \Omega) + 1/(20 \Omega) + 1/(30 \Omega) = 11/(60 \Omega)$  *1pt*  

$$R_{\text{eq}} = (60/11) \Omega = 5.45 \Omega$$
 *1pt*
- There are two junctions although they both give the same KJR equation; that is they are **not** independent. *1pt*
- There are **two independent** loops: *1pt*
  - Loop 1: Follow  $I_1$  and  $I_2$  (the outer loop)
  - Loop 2: Follow  $I_1$  and  $I_3$  (the inner loop)
  - Loop 3: Follow  $I_2$ , then  $-I_3$  (the closed box-loop)

However, Loop 3 = (Loop 1 – Loop 2), i.e. not an independent one. Thus, there are only 2 *independent* loops which give two distinct KLR equations (see question 20 below).
- $20 \Omega$  and  $30 \Omega$  are parallel to each other. Therefore, we reduce them first:  $1/R_{\text{parallel}} = 1/(20 \Omega) + 1/(30 \Omega) = 5/(60 \Omega)$ ,  $R_{\text{parallel}} = 12 \Omega$  *1pt*  
That combination is in series with  $10 \Omega$ ; therefore,  

$$R_{\text{eq}} = 10 \Omega + R_{\text{parallel}} = 22 \Omega$$
 *1pt*
- $I_1$  is the current that is coming out of and going into the battery which is the same current that goes through the equivalent R: *1pt*  

$$I_1 = 110V / R_{\text{eq}} = 5A$$
 *1pt*
- $V_1 = I_1 R_{\text{parallel}} = I_1 R_{\text{parallel}} = 60V$  ( $I_1$  goes through  $R_{\text{parallel}}$ ) *1+1 pts*
- and 11: Since  $20 \Omega$  and  $30 \Omega$  are parallel, they have the same voltage:  $V_1 = V_2 = V_3$ . We can find them in multiple ways:  

$$V_1 = 110V - V_1 = 60V$$
  

$$V_1 = I_1 R_{\text{parallel}} = I_1 R_{\text{parallel}} = 60V$$
 ( $I_1$  goes through  $R_{\text{parallel}}$ ) *1+1 pts*
- and 13. Use your answers from 10 and 11 to solve  
 $V_1 = V_2 = V_3 \Rightarrow 60V = I_1 R_{\text{parallel}} = I_2 20 \Omega = I_3 30 \Omega$ . Thus  
 $I_2 = 3A$  and  $I_3 = 2A$ . Notice  $I_2 + I_3 = 5A = I_1$  as should be. *1+1 pts*
- 14–18:  $P = VI$  or  $P = I^2 R$  or  $P = V^2 / R$  for each element *1pt each*  
YOU HAVE TO SEPARATE THE ANSWERS and SHOW WORK IN YOUR CORRECTIONS!  

$$P_{\text{battery}} = 110V \times 5A = 550W$$
  
The sum of these give the total: the same as above.
- Ask me to show you in class. In a complicated circuit, you first need to reduce the elements that are clearly either (i) in series, or (ii) in parallel. In this case the upper right most  $1\Omega$  resistors are in series with each other, the result is in parallel to the diagonal  $2\Omega$  resistor, when combined reduce to  $1\Omega$  again the result which is in series to the lower right hand  $1\Omega$ , so on so forth. After all the work is done, we get  $1\Omega$ . *1pt for showing the first step, 1pt for the second, 1pt for the correct final answer.* *3pts*
- Ask me to show you in class. You first need to recognize the fact that the wire connecting the top and the bottom between the  $2\Omega$  resistors causes the top and the bottom points to shrink to a single point (*1pt*). As a result, we have a pair of  $2\Omega$  resistors connected in parallel which reduces  $1\Omega$  (*1pt*), and this result is connected to the second pair in series. As a result, we have two  $1\Omega$ 's connected in series to the remaining two  $2\Omega$  resistors—they are all in series, thus the equivalent resistance is  $6\Omega$  (*1pt*). *3pts*
- In series since everything goes through each element one after the other. Therefore, the two capacitors are in series. *1pt*
- The space between each capacitor is  $\frac{d-a}{2}$  (*1pt*). As a result, each

capacitor's capacitance  $C = \frac{\epsilon A}{d-a} = \frac{2\epsilon A}{d-a}$  (*1pt*). However, now

there are two of them in series. Therefore,

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C}, C_{\text{eq}} = \frac{C}{2} = \frac{\epsilon A}{d-a}$$
 (*1pt*) *3pts*

- When  $a \rightarrow 0$ , we have only one capacitor with gap  $d$  (*1pt*). As a

$$\text{result, } C_{\text{eq}} = \frac{\epsilon A}{d-a}$$
 (*1pt*) *2pts*

- When  $a \rightarrow d$ , we have  $C_{\text{eq}} = \frac{\epsilon A}{0} \rightarrow \infty$ . A best capacitor is the one made purely of an excellent conductor. *1pt*

- This is just like the one we have with the resistors. The upper right most  $2\mu F$  capacitors are in series with each other, the result is in parallel to the diagonal  $1\mu F$  resistor, when combined reduce to  $2\mu F$  again the result which is in series to the lower right hand  $2\mu F$ , so on so forth. After all the work is done, we get  $2\mu F$ . *1pt for showing the first step, 1pt for the second, 1pt for the correct final answer.* *3pts*.
- Just like the resistor one, except you need to remember we are dealing with capacitors. The result is  $\frac{2}{3}\mu F$ . *3pts*

- Gauss' law gives

$$E = \frac{kQ}{R^2} \Rightarrow Q = \frac{ER^2}{k} = \frac{100N(6.4 \times 10^6 m)^2}{9 \times 10^9 \frac{Nm^2}{C^2}} = 4.55 \times 10^6 C$$
 *2pts*

$$28. C = \left( \frac{1}{C_1} + \frac{1}{C_2} \right)^{-1} = \frac{1}{\frac{d_1}{\epsilon_1 A_1} + \frac{d_2}{\epsilon_2 A_2}}$$
 *2pts*

$$29. C = C_1 + C_2 = \frac{\epsilon_1 \frac{A}{2}}{d} + \frac{\epsilon_2 \frac{A}{2}}{d} = (\epsilon_1 + \epsilon_2) \frac{A}{2d}$$
 *2pts*

$$30. \frac{1}{C} = \frac{1}{5\mu F} + \frac{1}{5\mu F + 15\mu F} \Rightarrow C = 4\mu F$$
 *2pts*

$$31. Q = VC = 40 \mu C$$
 *1pt*

$$32. U = \frac{1}{2} QV = 200 \mu J$$
 *1pt*

- When capacitors are in series, they share the same charge just like

$$\text{the resistors in series sharing the same current. } V_1 = \frac{Q}{C_1} = 8V$$

$$34. V_2 = \frac{Q}{C_2} = 2V \text{ Notice that } V = V_1 + V_2 (= V_3) = 8V + 2V = 10V$$
 *1pt*

$$35. \text{Since they are parallel, same as } C_2.$$
 *1pt*

$$36. \text{The same as the charge stored in the circuit } 40 \mu C \text{ or } Q_1 = V_1 C_1$$
 *1pt*

$$37. Q_2 = V_2 C_2 = 10 \mu C$$
 *1pt*

$$38. \text{Since the voltage the same and the capacitance is 3 times } C_2, \text{ so is the charge.}$$
 *1pt*

$$39. U = \frac{1}{2} Q_1 V_1 = 160 \mu J$$
 *1pt*

$$40. U = \frac{1}{2} Q_2 V_2 = 10 \mu J$$
 *1pt*

$$41. U = \frac{1}{2} Q_3 V_3 = 30 \mu J$$
 *1pt*

FRQ

1. The capacitor is made of two parallel capacitors, side by side—not one after the other.

$$C = \frac{\epsilon_1 a c}{d} + \frac{\epsilon_2 b c}{d} = \frac{(a\epsilon_1 + b\epsilon_2)c}{d}$$

1pt

2.

A.  $IR = \frac{Q}{C}, \frac{Q}{t}R = \frac{Q}{C}, \tau = RC$

2pts

B.  $-V_o + IR + \frac{Q}{C} = 0$

C.  $R \frac{dI}{dt} + \frac{I}{C} = 0$  or  $-V_o + R \frac{dQ}{dt} + \frac{Q}{C} = 0$

D.  $\frac{dQ}{dt} = -\frac{1}{R} \left( \frac{Q}{C} - V_o \right), u = Q - V_o C, \frac{du}{dt} = -\frac{dt}{RC}$

$$\ln u = -\frac{t}{RC} + K, u = e^K e^{-\frac{t}{RC}}, (Q - V_o C) = e^K e^{-\frac{t}{RC}}$$

$$Q = V_o C \left( 1 - e^{-\frac{t}{RC}} \right), t=0, Q=0$$

E.  $\frac{dI}{I} = -\frac{1}{RC} dt, I = I_o e^{\frac{-t}{RC}} = \frac{V_o}{R} e^{-\frac{t}{RC}}, t=0, I = \frac{V_o}{R}$

F. 1.  $V_C = \frac{Q}{C} = V_o \left( 1 - e^{-\frac{t}{RC}} \right)$ , 2.  $V_R = IR = V_o e^{-\frac{t}{RC}}$

G. 1.  $Q = V_o C$ , 2.  $I_R = 0, V_R = 0$ , 3.  $I_C = 0, V_C = 0$

3.

A.  $IR + \frac{Q}{C} = 0$

B.  $\frac{dQ}{dt} = -\frac{dt}{RC}$

C.  $\ln Q = -\frac{t}{RC} + K, Q = Q_o e^{-\frac{t}{RC}} = V_o C e^{-\frac{t}{RC}}$

D.  $I = \frac{dQ}{dt} = -\frac{V_o}{R} e^{-\frac{t}{RC}}$

E. 1.  $V_C = \frac{Q}{C} = V_o e^{-\frac{t}{RC}}$ , 2.  $V_R = IR = -V_o e^{-\frac{t}{RC}}$

F. 1.  $Q_C = 0$ , 2.  $V_C = 0, I_C = 0$ , 3.  $V_R = 0, I_R = 0$

4.

A.  $-10V + 4I_1 + 4I_2 + 8V + 2I_1 = 0$

$-8V - 4I_2 + 4I_3 + 8V + 2I_3 = 0$

$-10V + 4I_1 + 4I_3 + 8V + 2I_3 + 2I_1 = 0$

$I_1 = I_2 + I_3$

B.  $-2 + 6I_1 + 4I_2 = 0, 3I_1 + 2I_2 = 1$

$-4I_2 + 6I_3 = 0, I_3 = \frac{2}{3}I_2$

$I_1 - I_2 = \frac{2}{3}I_2, I_1 = \frac{5}{3}I_2$

$$3 \times \frac{5}{3}I_2 + 2I_2 = 1, I_2 = \frac{1}{7}A$$

$$I_1 = \frac{5}{21}A, I_2 = \frac{3}{21}A, I_3 = \frac{2}{21}A, I_1 = I_2 + I_3$$

$$V_1 = \frac{20}{21}V, V_2 = \frac{12}{21}V, V_3 = \frac{8}{21}V, V_4 = \frac{4}{21}V, V_5 = \frac{10}{21}V$$

$$P_i = I_i V_i$$

C. Yes.  $P_{Battery} = \sum I_i V_i$

5.

A.  $R_{eq} = \frac{1}{\frac{1}{2\Omega} + \frac{1}{3\Omega}} + 5\Omega = \frac{6}{5}\Omega + \frac{25}{5}\Omega = \frac{31}{5}\Omega$

B.  $I_1 = \frac{62V}{(31/5)\Omega} = 10A, V_1 = I_1 5\Omega = 50V$

$V_2 = V_3 = 62V - 50V = 12V$

We could also use  $I_1 = I_2 + I_3, V_2 = V_3, I_2 2\Omega = I_3 3\Omega$

C.  $2I_2 = 3I_3, I_1 = I_2 + I_3 = I_2 + \frac{2}{3}I_2 = \frac{5}{3}I_2 = 10A$

$I_2 = 6A, I_3 = 4A$

D.  $P_1 = V_1 I_1 = 500W, P_2 = V_2 I_2 = 72W, P_3 = V_3 I_3 = 48W$   
 $P = VI_1 = 620W$

E.  $P = VI_1 = 620W = P_1 + P_2 + P_3$ . Yes, it should be since the power consumed cannot exceed power produced, and power produced needs to be consumed.

19.

1st Jct:  $I_1 - I_2 - I_3 = 0$ ; 2nd Jct:  $I_2 + I_3 - I_1 = 0$ .

20.

Loop 1 (upper):  $-110V + V_2 + V_1 = 0$

Loop 2 (lower):  $-110V + V_3 + V_1 = 0$

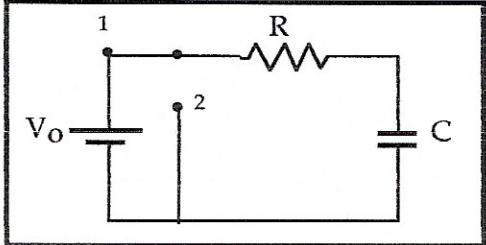
Loop 3 (closed box):  $V_2 - V_3 = 0$

Notice that Loop 3 = (Loop 1 – Loop 2), that is

**not** independent.

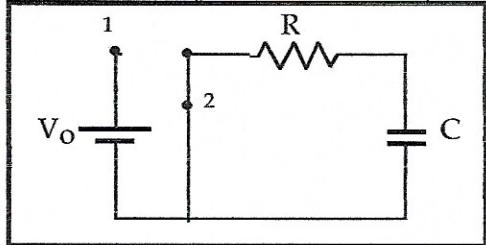
21. I will go over this in class more explicitly. For your corrections, use your text: Figures 18.16 and 18.17 give the graphic representation of the charge on the capacitor. The current through the resistor starts from a maximum value of  $V/R$  and exponentially decreases to 0 in both cases (looks like 18.b in both cases-parts A and B of 21).

- I. Consider the RC circuit given in the figure. At  $t=0$ , the voltage source is connected to the circuit by closing switch 1 as shown in the figure below.



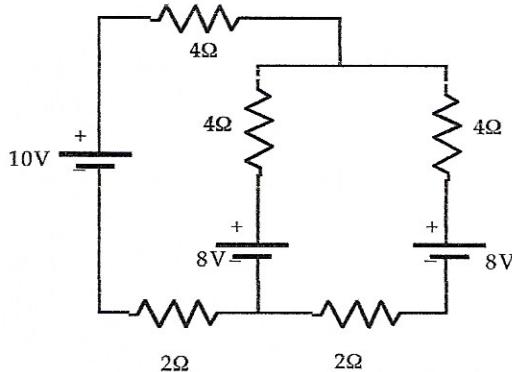
- A. Use dimensional analysis to obtain the characteristic time frame for this circuit. Hint: Use the voltage across  $R$  and  $C$  in terms of the charge flow in the circuit.
- B. Write down the Kirchhoff's voltage law for this circuit.
- C. Revise your answer to part B to write a differential equation in terms of the charge or the current only.
- D. Obtain an expression for the charge on the capacitor as a function of time.
- E. Obtain an expression for the current in the loop as a function of time.
- F. 1. What is the voltage across the capacitor as a function of time?  
2. What is the voltage across the resistor as a function of time?
- G. After a long time period with respect to the circuit's characteristic time frame,
  - 1. How much charge will there be on the capacitor ?
  - 2. What value will the current through and the voltage across the capacitor reach ?
  - 3. What value will the current through and the voltage across the resistor reach ?

- II. After a long period of time as described in the preceding problem, the voltage source is disconnected by opening switch 1, and switch 2 is closed as shown in the figure below. Take this to be your new  $t=0$ .



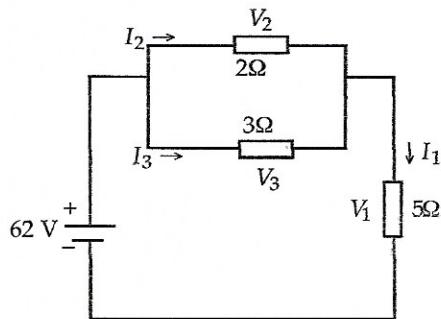
- A. Write down the Kirchhoff's voltage law for this circuit.
- B. Revise your answer to part A to write a differential equation in terms of the charge or the current only.
- C. Obtain an expression for the charge on the capacitor as a function of time.
- D. Obtain an expression for the current in the loop as a function of time.
- E. 1. What is the voltage across the capacitor as a function of time?  
2. What is the voltage across the resistor as a function of time?
- F. After a long time period with respect to the circuit's characteristic time frame,
  - 1. How much charge will there be on the capacitor ?
  - 2. What value will the current through and the voltage across the capacitor reach ?
  - 3. What value will the current through and the voltage across the resistor reach ?

III.



- A. Write down the Kirchhoff's voltage rule for each loop.
- B. Use your answers to obtain the
  1. current in each resistor
  2. voltage across each resistor
  3. power consumed by each resistor.
- C. Is the sum of powers produced by the batteries equal to the sum of the powers consumed by each resistor? Why (not)?

IV.



Calculate

- A. the equivalent resistance of the circuit
- B. the voltage
- C. the current
- D. the power consumed for each element.
- E. Do individual powers consumed by each resistor add up to the power produced by the battery?  
Why, why not?

$$\int_{x_1}^{x_2} \frac{dx}{x-a} = \ln\left(\frac{x_2-a}{x_1-a}\right)$$

*Even if you cannot evaluate the integrals involved, you will receive full credit for each correctly answered part using dimensional analysis and relevant justification involving the physics of the situation.*

1. **Field • Area**  $\text{enclosing source} = C_F \times \text{Source}_{\text{enclosed}}$
- A. Between the two spheres, the only charge enclosed by the Gaussian surface is the charge on the conducting sphere which is  $Q$ . The external sphere does not create any net electric field inside.
- $$\vec{E} \cdot 4\pi r^2 \hat{r} = 4\pi k Q = \frac{Q}{\epsilon_0}, \quad \vec{E} = \frac{Q}{4\pi \epsilon_0 r^2} \hat{r}$$
- B. The voltage difference between the two spheres is
- $$V_b - V_a = - \int_a^b \vec{E} \cdot d\vec{r} = - \int_a^b \frac{Q}{4\pi \epsilon_0 r^2} dr = \frac{Q}{4\pi \epsilon_0} \left[ \frac{1}{b} - \frac{1}{a} \right]$$
- $$V_b - V_a = \frac{Q}{4\pi \epsilon_0} \left( \frac{1}{b} - \frac{1}{a} \right)$$
- C.  $Q = VC, \quad C = \frac{Q}{|V_b - V_a|} = \frac{4\pi \epsilon_0 ab}{b-a}$
2. **Field • Area**  $\text{enclosing source} = C_F \times \text{Source}_{\text{enclosed}}$
- A. Between the two cylinders, the only charge enclosed by the Gaussian surface is the charge on the conducting cylinder which is  $Q$ . The external cylinder does not create any net electric field inside.
- $$\vec{E} \cdot 2\pi r \ell \hat{r} = 4\pi k Q = \frac{Q}{\epsilon_0}, \quad \vec{E} = \frac{Q}{2\pi \epsilon_0 r \ell} \hat{r} = \frac{\lambda}{2\pi \epsilon_0 r} \hat{r}$$
- B. The voltage difference between the two spheres is
- $$V_b - V_a = - \int_a^b \vec{E} \cdot d\vec{r} = - \int_a^b \frac{\lambda}{2\pi \epsilon_0 r} dr = \frac{\lambda}{2\pi \epsilon_0} \ln r \Big|_a^b$$
- $$V_b - V_a = \frac{\lambda}{2\pi \epsilon_0} (\ln b - \ln a) = \frac{\lambda}{2\pi \epsilon_0} \ln \left( \frac{b}{a} \right)$$
- C.  $Q = VC, \quad C = \frac{Q}{|V_b - V_a|} = \frac{Q}{\frac{\lambda}{2\pi \epsilon_0} \ln \left( \frac{b}{a} \right)} = \frac{2\pi \epsilon_0 \ell}{\ln \left( \frac{b}{a} \right)}$
- $$\frac{C}{\ell} = \frac{2\pi \epsilon_0}{\ln \left( \frac{b}{a} \right)}$$
3. **Field • Area**  $\text{enclosing source} = C_F \times \text{Source}_{\text{enclosed}}$
- A. Consider the + plate. A rectangular Gaussian surface encloses the plate. We will assume that the sides are very small and the field through the sides is negligible compared to the rest of the field flux. On both the upper and the lower parts of the areas point outwards and the fields going through also point outwards. As a result, we have
- $$\vec{E}_+ \cdot \vec{A} = E_{up} \cdot A_{up} + E_{down} \cdot A_{down} = 2EA = \frac{Q}{\epsilon_0}$$
- $$\vec{E}_+ = \frac{Q}{2A\epsilon_0} \uparrow = \frac{\sigma}{2\epsilon_0} \uparrow$$
- Between the plates, this field points toward the - plate. For the - plate, we instead have
- $$\vec{E}_- \cdot \vec{A} = E_{up} \cdot A_{up} + E_{down} \cdot A_{down} = 2EA = \frac{Q}{\epsilon_0}$$
- $$\vec{E}_- = \frac{Q}{2A\epsilon_0} \uparrow = \frac{\sigma}{2\epsilon_0} \uparrow$$
- Between the plates, this field points toward the - plate. As a result, the net electric field between the plates is
- $$\vec{E} = \vec{E}_+ + \vec{E}_- = \frac{\sigma}{\epsilon_0} \uparrow$$
- B. The voltage difference between the two spheres is
4.  $V_+ - V_- = - \int_a^b \vec{E} \cdot d\vec{r} = \frac{\sigma d}{\epsilon_0}$
- C.  $Q = VC, \quad C = \frac{Q}{|V_+ - V_-|} = \frac{Q}{\frac{\sigma d}{\epsilon_0}} = \frac{\epsilon_0 A}{d}$
5. A. Effectively this arrangement forms two capacitors in series, since the same current would flow through both. The effective gap between plates for each capacitor  $(d-a)/2$ .
- B. In series, the inverse of capacitors add
- $$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{(d-a)/2}{\epsilon_0 A} + \frac{(d-a)/2}{\epsilon_0 A} = \frac{d-a}{\epsilon_0 A}$$
- $$C = \frac{\epsilon_0 A}{d-a}$$
- C.
1.  $\lim_{a \rightarrow 0} C = \frac{\epsilon_0 A}{d}$  the same as when there is nothing between the plates.
  2.  $\lim_{d \rightarrow a} C = \frac{\epsilon_0 A}{0} = \infty$  this is the same as a conductor.
- A. When capacitors are in series, the total voltage is the sum of the voltages:
- $$V = V_1 + V_2 + \dots$$
- Since they are in series, the same charge goes through each; as a result,
- $$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \dots = Q \sum \frac{1}{C_i}, \quad \frac{1}{C_{eq}} = \sum \frac{1}{C_i}$$
- B. In parallel, the charge coming into the circuit is divided among the parallel arms. The total charge is the sum of the charges in each arm
- $$Q = Q_1 + Q_2 + \dots$$
- Since they are parallel, each arm has the same voltage; as a result,
- $$VC_{eq} = VC_1 + VC_2 + \dots = V \sum C_i, \quad C_{eq} = \sum C_i$$
6. A. The parallel ones reduce to  $C_{||} = C_2 + C_3$ . The combination is in series to the first one
- $$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2 + C_3} = \frac{C_1 + C_2 + C_3}{C_1(C_2 + C_3)}$$
- $$C_{eq} = \frac{C_1(C_2 + C_3)}{C_1 + C_2 + C_3}$$
- B.  $Q = V_0 C_{eq} = \frac{V_0 C_1 (C_2 + C_3)}{C_1 + C_2 + C_3}$
- C.  $E = \frac{1}{2} Q V_0 = \frac{V_0^2 C_1 (C_2 + C_3)}{2(C_1 + C_2 + C_3)}$
- D.  $C_1$  and  $C_{||}$  have the same charge across them which is the charge coming into the circuit, in part B.
- $$V_1 = \frac{Q}{C_1} = \frac{V_0 (C_2 + C_3)}{C_1 + C_2 + C_3}, \quad V_{||} = \frac{Q}{C_{||}} = \frac{V_0 C_1}{C_1 + C_2 + C_3}$$
- $C_2$  and  $C_3$  have the same voltage as  $||$  since they are parallel.
- E. The charge on  $C_1$  and  $C_{||}$  is the same as in part B since this is the charge coming into the circuit from the source. However, it splits between  $C_2$  and  $C_3$  proportional to

each capacitor but the fraction of the total.

$$Q_2 = \frac{C_2}{C_{\text{JJ}}} Q, \quad Q_3 = \frac{C_3}{C_{\text{JJ}}} Q$$

Notice that the sum is the same as that of  $C_{\text{JJ}}$ .

- F.  $E_1 = \frac{1}{2} Q V_1, \quad E_2 = \frac{1}{2} Q_2 V_2, \quad E_3 = \frac{1}{2} Q_3 V_3$   
 G. Yes, as it should be. You must prove this.  
 H. Yes, as it should be. You must prove this.
7. A. On the surface of the earth, the electric potential is  
 $E_o = \frac{Q}{4\pi\epsilon_o R_E^2}, \quad Q = E_o 4\pi\epsilon_o R_E^2$

B.  $\frac{1}{2}\epsilon_o E_o^2$

1. A.  $IR = \frac{Q}{C}, \quad \frac{Q}{\tau}R = \frac{Q}{C}, \quad \tau = RC$  2pts

B.  $-V_o + IR + \frac{Q}{C} = 0$

C.  $R \frac{dI}{dt} + \frac{I}{C} = 0 \quad \text{or} \quad -V_o + R \frac{dQ}{dt} + \frac{Q}{C} = 0$

D.  $\frac{dQ}{dt} = -\frac{1}{R} \left( \frac{Q}{C} - V_o \right), \quad u = Q - V_c C, \quad \frac{du}{dt} = -\frac{dt}{RC}$

$$\ln u = -\frac{t}{RC} + K, \quad u = e^K e^{-\frac{t}{RC}}, \quad (Q - V_c C) = e^K e^{-\frac{t}{RC}}$$

$$Q = V_c C \left( 1 - e^{-\frac{t}{RC}} \right), \quad t = 0, \quad Q = 0$$

E.  $\frac{dl}{I} = -\frac{1}{RC} dt, \quad I = I_o e^{-\frac{t}{RC}} = \frac{V_o}{R} e^{-\frac{t}{RC}}, \quad t = 0, \quad I = \frac{V_o}{R}$

F. 1.  $V_C = \frac{Q}{C} = V_o \left( 1 - e^{-\frac{t}{RC}} \right)$ , 2.  $V_R = IR = V_o e^{-\frac{t}{RC}}$

G. 1.  $Q = V_o C$ , 2.  $I_R = 0$ , 3.  $I_C = 0$ ,  $V_C = 0$

2.

A.  $IR + \frac{Q}{C} = 0$

B.  $\frac{dQ}{Q} = -\frac{dt}{RC}$

C.  $\ln Q = -\frac{t}{RC} + K, \quad Q = Q_o e^{-\frac{t}{RC}} = V_o C e^{-\frac{t}{RC}}$

D.  $I = \frac{dQ}{dt} = -\frac{V_o}{R} e^{-\frac{t}{RC}}$

E. 1.  $V_C = \frac{Q}{C} = V_o e^{-\frac{t}{RC}}$ , 2.  $V_R = IR = -V_o e^{-\frac{t}{RC}}$

F. 1.  $Q_C = 0$ , 2.  $V_C = 0$ , 3.  $I_C = 0$ , 3.  $V_R = 0$ ,  $I_R = 0$

3.

A.  $-10V + 4I_1 + 4I_2 + 8V + 2I_1 = 0$  (1)

B.  $-8V - 4I_2 + 4I_3 + 8V + 2I_3 = 0$  (2)

C.  $-10V + 4I_1 + 4I_3 + 8V + 2I_3 + 2I_1 = 0$  (3)

D.  $I_1 = I_2 + I_3$  (4)

B.  $-2 + 6I_1 + 4I_2 = 0, \quad 3I_1 + 2I_2 = 1$

$-4I_2 + 6I_3 = 0, \quad I_3 = \frac{2}{3}I_2$

$I_1 - I_2 = \frac{2}{3}I_2, \quad I_1 = \frac{5}{3}I_2$

$3 \times \frac{5}{3}I_2 + 2I_2 = 1, \quad I_2 = \frac{1}{7}A$

$I_1 = \frac{5}{21}A, \quad I_2 = \frac{3}{21}A, \quad I_3 = \frac{2}{21}A, \quad I_1 = I_2 + I_3$

$V_1 = \frac{20}{21}V, \quad V_2 = \frac{12}{21}V, \quad V_3 = \frac{8}{21}V, \quad V_4 = \frac{4}{21}V, \quad V_5 = \frac{10}{21}V$

$P_i = I_i V_i$

C. Yes.  $P_{\text{Battery}} = \sum I_i V_i$

A.  $R_{\text{eq}} = \frac{\frac{1}{2\Omega} + \frac{1}{3\Omega}}{\frac{1}{2\Omega} + \frac{1}{3\Omega}} + 5\Omega = \frac{6}{5}\Omega + \frac{25}{5}\Omega = \frac{31}{5}\Omega$

B.  $I_1 = \frac{62V}{(31/5)\Omega} = 10A, \quad V_1 = I_1 5\Omega = 50V$

$V_2 = V_3 = 62V - 50V = 12V$

We could also use  $I_1 = I_2 + I_3, \quad V_2 = V_3, \quad I_2 2\Omega = I_3 3\Omega$

C.  $2I_2 = 3I_3, \quad I_1 = I_2 + I_3 = I_2 + \frac{2}{3}I_2 = \frac{5}{3}I_2 = 10A$

$I_2 = 6A, \quad I_3 = 4A$

D.  $P_1 = V_1 I_1 = 500W, \quad P_2 = V_2 I_2 = 72W, \quad P_3 = V_3 I_3 = 48W$   
 $P = VI_1 = 620W$

E.  $P = VI_1 = 620W = P_1 + P_2 + P_3$ . Yes, it should be since the power consumed cannot exceed power produced, and power produced needs to be consumed.

1. KJR involves flow of charges into and from a junction. Therefore, it is nothing but the law of conservation of electric charges. *1pt*
2. KLR involves adding all the voltages changes (which are the electric potential energies per unit charge) in a loop. Therefore, it is nothing but the conservation of energy. *1pt*
3. These resistors are in series. We simply add them: *1pt*  

$$R_{eq} = 10\Omega + 20\Omega + 30\Omega = 60\Omega$$
4. These resistors are in parallel. We add their inverses, then invert the result: *1pt*  

$$\frac{1}{R_{eq}} = \frac{1}{(10\Omega)} + \frac{1}{(20\Omega)} + \frac{1}{(30\Omega)} = \frac{1}{11/(60\Omega)}$$
  

$$R_{eq} = (60/11)\Omega = 5.45\Omega$$
5. There are two junctions although they both give the same KJR equation; that is they are **not** independent. *1pt*
6. There are **two independent** loops: *1pt*  
 Loop 1: Follow  $I_1$  and  $I_2$  (the outer loop)  
 Loop 2: Follow  $I_1$  and  $I_3$  (the inner loop)  
 Loop 3: Follow  $I_2$ , then  $-I_3$  (the closed box-loop)  
 However, Loop 3 = (Loop 1 – Loop 2), i.e. not an independent one.  
 Thus, there are only 2 *independent* loops which give two distinct KLR equations (see question 20 below).
7.  $20\Omega$  and  $30\Omega$  are parallel to each other. Therefore, we reduce them first:  $1/R_{||} = 1/(20\Omega) + 1/(30\Omega) = 5/(60\Omega)$ ,  $R_{||} = 12\Omega$  *1pt*  
 That combination is in series with  $10\Omega$ ; therefore, *1pt*  

$$R_{eq} = 10\Omega + R_{||} = 22\Omega$$
8.  $I_1$  is the current that is coming out of and going into the battery which is the same current that goes through the equivalent R: *1pt*  

$$I_1 = 110V/R_{eq} = 5A$$
9.  $V_1 = I_1 10\Omega = 50V$  *1pt*
10. and 11: *1pt*
11. and 10. Since  $20\Omega$  and  $30\Omega$  are parallel, they have the same voltage:  $V_{||} = V_2 = V_3$ . We can find them in multiple ways:  
 $V_{||} = 110V - V_1 = 60V$   
 $V_{||} = I_1 R_{||} = I_1 R_{||} = 60V$  ( $I_1$  goes through  $R_{||}$ ) *1pt*
12. and 13. Use your answers from 10 and 11 to solve *1pt*
13. and 12.  $V_{||} = V_2 = V_3 \Rightarrow 60V = I_1 R_{||} = I_2 20\Omega = I_3 30\Omega$ . Thus  $I_2 = 3A$  and  $I_3 = 2A$ . Notice  $I_2 + I_3 = 5A = I_1$  as should be. *1pt*
- 14–18:  $P = VI$  or  $P = I^2 R$  or  $P = V^2/R$  for each element *1pt each*  
 YOU HAVE TO SEPARATE THE ANSWERS and SHOW WORK IN YOUR CORRECTIONS!
- $P_{battery} = 110V \times 5A = 550W$   
 $= P_{total-consumed-in-circuit}$   
 $P_{10\Omega} = 250W$ ,  $P_{20\Omega} = 180W$ ,  $P_{30\Omega} = 120W$
- The sum of these give the total: the same as above.
19. Ask me to show you in class. In a complicated circuit, you first need to reduce the elements that are clearly either (i) in series, or (ii) in parallel. In this case the upper right most  $1\Omega$  resistors are in series with each other, the result is in parallel to the diagonal  $2\Omega$  resistor, when combined reduce to  $1\Omega$  again the result which is in series to the lower right hand  $1\Omega$ , so on so forth. After all the work is done, we get  $1\Omega$ . *1pt for showing the first step, 1pt for the second, 1pt for the correct final answer.*
20. Ask me to show you in class. You first need to recognize the fact that the wire connecting the top and the bottom between the  $2\Omega$  resistors causes the top and the bottom points to shrink to a single point (*1pt*). As a result, we have a pair of  $2\Omega$  resistors connected in parallel which reduces  $1\Omega$  (*1pt*), and this result is connected to the second pair in series. As a result, we have two  $1\Omega$ 's connected in series to the remaining two  $2\Omega$  resistors—they are all in series, thus the equivalent resistance is  $6\Omega$  (*1pt*). *3pts*
21. In series since everything goes through each element one after the other. Therefore, the two capacitors are in series. *1pt*
22. The space between each capacitor is  $\frac{d-a}{2}$  (*1pt*). As a result, each capacitor's capacitance  $C = \frac{\epsilon A}{d-a} = \frac{2\epsilon A}{d-a}$  (*1pt*). However, now there are two of them in series. Therefore,  

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C}$$
,  $C_{eq} = \frac{C}{2} = \frac{\epsilon A}{d-a}$  (*1pt*) *3pts*
23. When  $a \rightarrow 0$ , we have only one capacitor with gap  $d$  (*1pt*). As a result,  $C_{eq} = \frac{\epsilon A}{d-a}$  (*1pt*) *2pts*
24. When  $a \rightarrow d$ , we have  $C_{eq} = \frac{\epsilon A}{0} \rightarrow \infty$ . A best capacitor is the one made purely of an excellent conductor. *1pt*
25. This is just like the one we have with the resistors. The upper right most  $2\mu F$  capacitors are in series with each other, the result is in parallel to the diagonal  $1\mu F$  resistor, when combined reduce to  $2\mu F$  again the result which is in series to the lower right hand  $2\mu F$ , so on so forth. After all the work is done, we get  $2\mu F$ . *1pt for showing the first step, 1pt for the second, 1pt for the correct final answer.*
26. Just like the resistor one, except you need to remember we are dealing with capacitors. The result is  $\frac{2}{3}\mu F$ . *3pts*
27. Gauss' law gives
- $$E = \frac{kQ}{R^2} \Rightarrow Q = \frac{ER^2}{k} = \frac{100 \frac{N}{C} (6.4 \times 10^6 m)^2}{9 \times 10^9 \frac{Nm^2}{C^2}} = 4.55 \times 10^6 C$$
- 2pts*

38. Since the voltage the same and the capacitance is 3 times  $C_2$ , so is the charge. *1pt*
39.  $U = \frac{1}{2}Q_1V_1 = 160\mu J$  *1pt*
40.  $U = \frac{1}{2}Q_2V_2 = 10\mu J$  *1pt*
41.  $U = \frac{1}{2}Q_3V_3 = 30\mu J$  *1pt*

FRQ The capacitor is made of two parallel capacitors, side by side--not one after the other. *1pt*

$$C = \frac{\epsilon_1 ac}{d} + \frac{\epsilon_2 bc}{d} = \frac{(a\epsilon_1 + b\epsilon_2)c}{d}$$
 *1pt*

19. 1st Jct:  $I_1 - I_2 - I_3 = 0$ ; 2nd Jct:  $I_2 + I_3 - I_1 = 0$ . *1pt*
20. Loop 1 (upper):  $-110V + V_2 + V_1 = 0$  *1pt*  
 Loop 2 (lower):  $-110V + V_3 + V_1 = 0$  *1pt*  
 Loop 3 (closed box):  $V_2 - V_3 = 0$
- Notice that Loop 3 = (Loop 1 - Loop 2), that is **not** independent.

21. I will go over this in class more explicitly. For your corrections, use your text: Figures 18.16 and 18.17 give the graphic representation of the charge on the capacitor. The current through the resistor starts from a maximum value of  $V/R$  and exponentially decreases to 0 in both cases (looks like 18.b in both cases--parts A and B of 21).