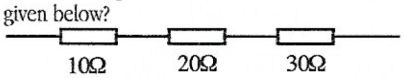
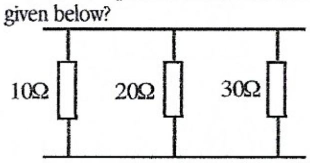
FORM A

1. In terms of conservation laws, what is the physical meaning of Kirchhoff's Junction Rule (KJR)?
   A. Conservation of momentum
   B. Conservation of angular momentum
   C. Conservation of energy
   D. Conservation of charge
   E. None of these
2. In terms of conservation laws, what is the physical meaning of Kirchhoff's Loop Rule (KLR)?
   A. Conservation of momentum
   B. Conservation of angular momentum
   C. Conservation of energy
   D. Conservation of charge
   E. None of these
3. What i sthe equivalent resistance of the combination of the resistors given below?



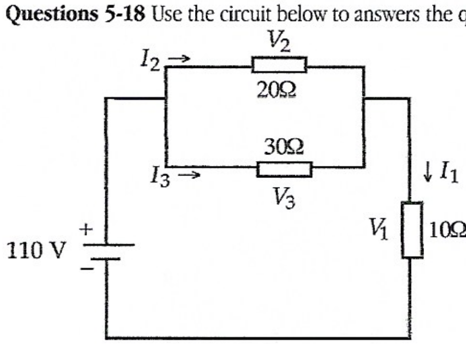
A. 5.45Ω B. 10 Ω C. 15 Ω D. 22 Ω E. 60 Ω

4. What i sthe equivalent resistance of the combination of the resistors given below?

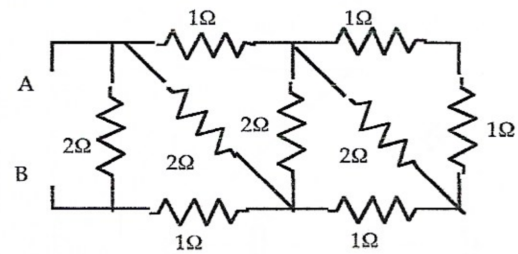


A. 5.45Ω B. 10 Ω C. 15 Ω D. 22 Ω E. 60 Ω

Questions 5-18 Use the circuit below to answers the questions.

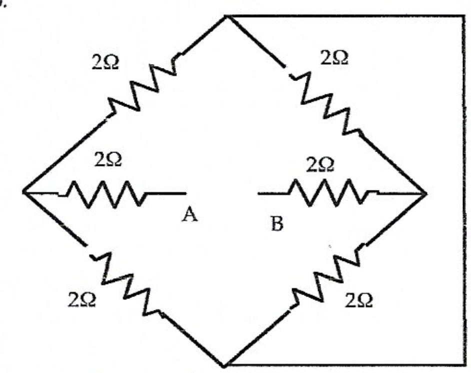


5. How many junctions are there in this circuit?
   A. 1 B. 2 C. 3 D. 4 E. 5
6. How many independent loops are there in this circuit?
   A. 1 B. 2 C. 3 D. 4 E. 5
7. What is the equivalent resistance of this circuit?
   A. 5.45Ω B. 10 Ω C. 15 Ω D. 22 Ω E. 60 Ω
8. What is the magnitude of the current I_1?
   A. 11 A B. 5 A C. 3 A D. 2 A E. 0 A
9. What is the magnitude of the voltage V_1?
   A. 110 V B. 80 V C. 60 V D. 50 V E. 0 V
10. What is the magnitude of the voltage V_2?
    A. 110 V B. 80 V C. 60 V D. 50 V E. 0 V
11. What is the magnitude of the voltage V_3?
    A. Same as V_2 B. Same as V_1
    C. 110 V D. 40 V E. 30 V
12. What is the magnitude of the current I_2?
    A. 11 A B. 5 A C. 3 A D. 2 A E. 0 A
13. What is the magnitude of the current I_3?
    A. 11 A B. 5 A C. 3 A D. 2 A E. 0 A
14. How much power is produced by the battery?
    A. 550 W B. 250 W C. 180 W D. 120 W E. 0 W
15. How much power is consumed by the 10 Ω resistor?
    A. 550 W B. 250 W C. 180 W D. 120 W E. 0 W
16. How much power is consumed by the 20 Ω resistor?
    A. 550 W B. 250 W C. 180 W D. 120 W E. 0 W
17. How much power is consumed by the 30 Ω resistor?
    A. 550 W B. 250 W C. 180 W D. 120 W E. 0 W
18. How much power is consumed by all three resistors in the circuit?
    A. 550 W B. 250 W C. 180 W D. 120 W E. 0 W
19. Calculate the resistance of the circuit below



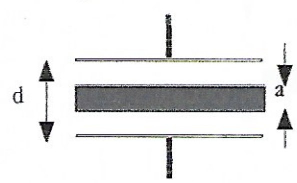
A. 0.5 Ω B. 1Ω C. 2 Ω D. 3Ω E. 6Ω

20.



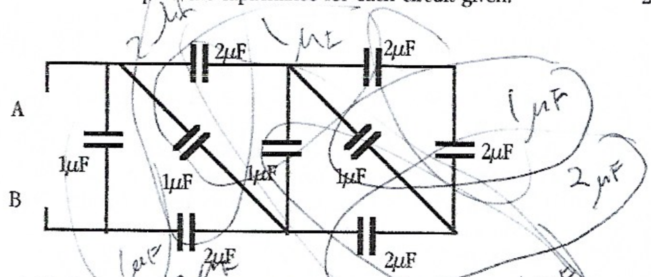
A. 0.5 Ω B. 1Ω C. 2 Ω D. 3Ω E. 6Ω

A metal slab of thickness $a < d$ is placed inside a parallel plate capacitor with area A and separation d. Assume that this arrangement acts like two sub-capacitors since the metal slab acts like a wire.

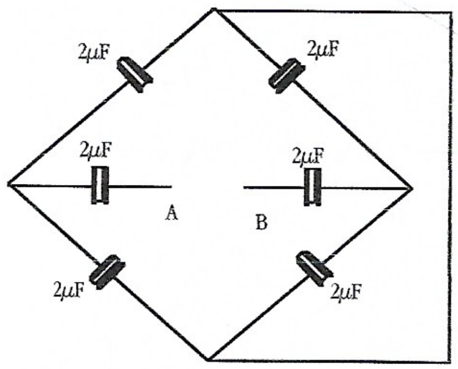


21. How are these sub-capacitors connected?
    A. Parallel B. Series C. Mixed
22. What is the equivalent capacitance of this arrangement?
    A. $\frac{\varepsilon A}{d}$ B. $\frac{\varepsilon A}{d-a}$ C. $\frac{2\varepsilon A}{d-a}$ D. $\frac{\varepsilon A}{2(d-a)}$ E. ∞
23. What happens to the capacitance when $a \to 0$.
    A. $\frac{\varepsilon A}{d}$ B. becomes 0 C. becomes ∞ D.cannot be determined
24. What happens to the capacitance when $a \to d$.
    A. $\frac{\varepsilon A}{d}$ B. becomes 0 C. becomes ∞ D.cannot be determined

Calculate the equivalent capacitance for each circuit given.



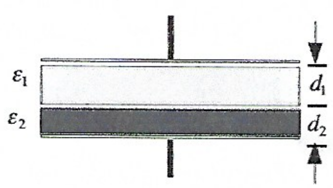
25. A. 13μF B. 10μF C. 5μF D. 2μF E. 1μF



26. A. 1.5μF B. (2/3)μF C. 8μF D. 12μF E. 6μF

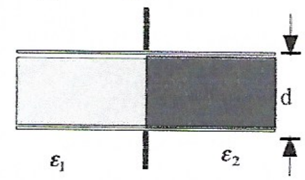
27. Near the surface of the Earth there is an electric field with a magnitude of about 100 $\frac{N}{C}$ directed vertically down. Assume this field is caused by a net free charge on the Earth. Calculate the net charge using the electric field given in the problem. The radius of the Earth is approximately 6400 km.
A. $100C$ B. 4.5×10^5C C. $4\pi\varepsilon\times6400C$ D. $(4\pi\varepsilon/6400)C$ E.

28. Calculate the capacitance for the arrangment (Hint:First identify how many differentcapacitors you have and how they are connected. Then calculate the equivalent capacitance in terms of the given values).



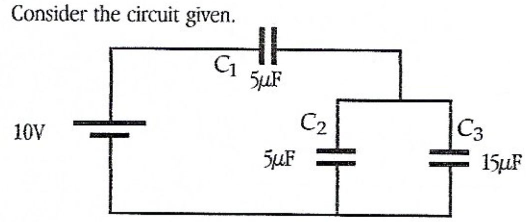
A. $\frac{(\varepsilon_1+\varepsilon_2)A}{d_1+d_2}$ B. $\frac{\varepsilon_1 A}{d_1}+\frac{\varepsilon_2 A}{d_2}$ C. $\frac{1}{\frac{d_1}{\varepsilon_1 A}+\frac{d_2}{\varepsilon_2 A}}$ D. $\frac{1}{\frac{\varepsilon_1 A}{d_1}+\frac{\varepsilon_2 A}{d_2}}$

29. Calculate the capacitance for the arrangment (Hint:First identify how many differentcapacitors you have and how they are connected. Then calculate the equivalent capacitance in terms of the given values).



A. $\frac{(\varepsilon_1+\varepsilon_2)A}{2d}$ B. $\frac{(\varepsilon_1+\varepsilon_2)A}{d}$ C. $\frac{2(\varepsilon_1+\varepsilon_2)A}{d}$
D. $\frac{\varepsilon_1\varepsilon_2 A}{2d(\varepsilon_1+\varepsilon_2)}$ E. $\frac{\varepsilon_1\varepsilon_2 A}{d(\varepsilon_1+\varepsilon_2)}$ F. $\frac{2\varepsilon_1\varepsilon_2 A}{d(\varepsilon_1+\varepsilon_2)}$

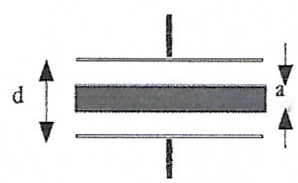
Consider the circuit given.



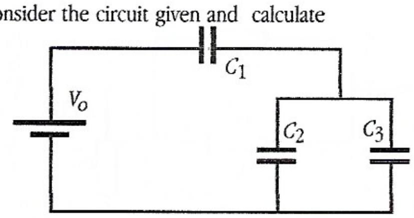
Calculate

30. the equivalent capacitance for the circuit
A. 4μF B. (1/4)μF C. 5μF D. 15μF E. 25μF
31. the charge stored in the circuit
A. 40μC B. 30μC C. 20μC D. 10μC E. 5μC
32. the energy stored in the circuit
A. 200μJ B. 160μJ C. 40μJ D. 30μJ E. 10μJ
33. the voltage across the capacitor C_1
A. 10 V B. 2 V C. 4 V D. 5 V E. 8 V F
34. the voltage across the capacitor C_2
A. 10 V B. 2 V C. 4 V D. 5 V E. 8 V F
35. the voltage across the capacitor C_3
A. 10 V B. Same as C_1 C. Same as C_2 D. (1/3)C_2 E. 3C_2 F.
36. the charge stored on C_1
A. 40μC B. 30μC C. 20μC D. 10μC E. 5μC
37. the charge stored on C_2
A. 40μC B. 30μC C. 20μC D. 10μC E. 5μC
38. the charge stored on C_3
A. 10 V B. Same as on C_1 C. Same as on C_2
D. (1/3) of C_2 E. 3 times of C_2 F.
39. the energy stored on C_1
A. 200μJ B. 160μJ C. 40μJ D. 30μJ E. 10μJ
40. the energy stored on C_2
A. 200μJ B. 160μJ C. 40μJ D. 30μJ E. 10μJ
41. the energy stored on C_3
A. 10 V B. Same as on C_1 C. Same as on C_2

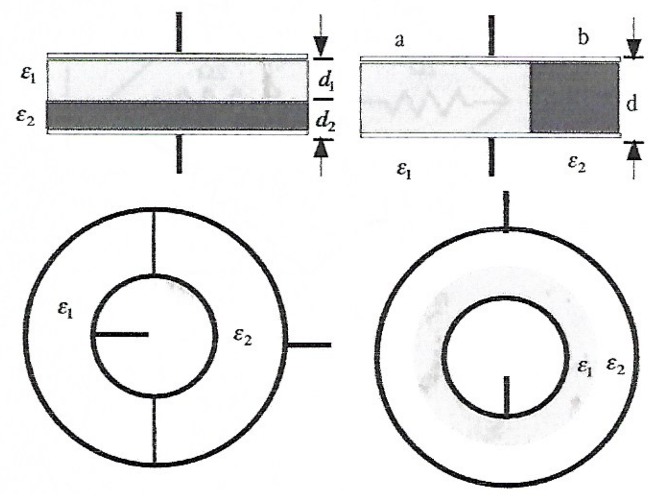
1. A conducting sphere of radius a is surrounded by a concentric, thin conducting spherical shell of radius b. Each sphere has equal but opposite charges of quantity Q.
   A. Use Gauss' law to show that $E = \dfrac{Q}{4\pi\varepsilon_o r^2}$ between the two plates. (*Show every step clearly! Do **not** fudge!*)
   B. Use your answer in part A to calculate the voltage difference between the two spheres?
   C. Use your answer in part B to calculate the capacitance of this arrangement.
2. A conducting cylinder of radius a and length l is surrounded by a concentric, thin conducting cylinder shell of radius b and length l. Each cylinder has equal but opposite charges of quantity Q. (*Show every step clearly! Do **not** fudge!*)
   A. Use Gauss' law to show that $E = \dfrac{\lambda}{2\pi\varepsilon_o r}$ between the plates.
   B. Use your answer in part A to calculate the voltage difference between the two cylinders?
   C. Use your answer in part B to calculate the capacitance of this arrangement.
3. Two parallel plates of area A separated by a distance d. Each plate has equal but opposite charges of quantity Q. (*Show every step clearly! Do **not** fudge!*)
   A. Use Gauss' law to show that $E = \dfrac{\sigma}{\varepsilon_o}$ between the plates.
   B. Use your answer in part A to calculate the voltage difference between the two plates?
   C. Use your answer in part B to calculate the capacitance of this arrangement.
   Give your final answers in terms of physical constants and given quantities.
4. A metal slab of thickness $a < d$ is placed inside a parallel plate capacitor with area A and separation d. Assume that this arrangement acts like two sub-capacitors since the metal slab acts like a wire.



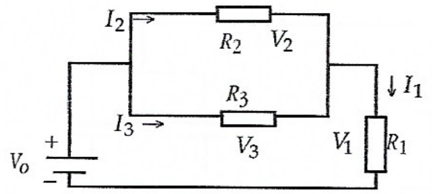
   A. Are these sub-capacitors connected in parallel or series?
   B. What is the equivalent capacitance of this arrangement?
   C. What happens when
      1. $a \to 0$ (i.e. thickness of metal is reduced to zero.)
      2. $a \to d$ (i.e. thickness is increased to cover the space between plates.)
5. Use the relation among Q, I, V, and C to derive an expression for the equivalent capacitance of n capacitors connected in
   A. parallel to each other
   B. series to each other
6. Consider the circuit given and calculate



   A. the equivalent capacitance of the circuit
   B. the charge stored in the circuit
   C. the energy stored in the circuit
   D. the voltage across each capacitor
   E. the charge stored on each capacitor
   F. the energy stored in each capacitor
   G. Is the charge stored in the circuit equal to the sum of the charges stored in the elements
   H. Is the energy stored in circuit the same as the sum of the energies in the elements?
7. Near the surface of the Earth there is an electric field with a magnitude of about E_o directed vertically down. Assume this field is caused by a net free charge on the Earth.
   A. Calculate the net charge using the electric field given in the problem. The radius of the Earth is R_E.
   B. What electrostatic energy density is stored near the surface of the Earth?
8. Calculate the capacitance for the following arrangments (Hint:First identify how many differentcapacitors you have and how they are connected. Then assign each a different value, i.e.,C_1, C_2, C_3, ... Finally, calculate the equivalent capacitance in terms of the given values):
   A. Parallel Plate capacitors
   B. Spherical capacitors (spheres of radii a, b, and r_o)
   C. Cylindrical capacitors (cylinders of radii a, b, and r_o)



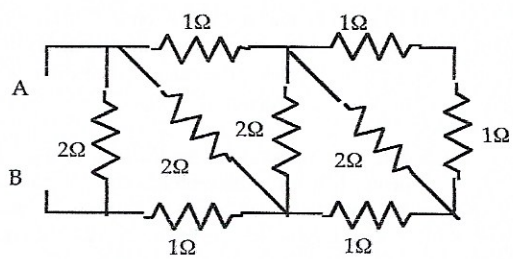
9. Use the circuit below to answers the questions.



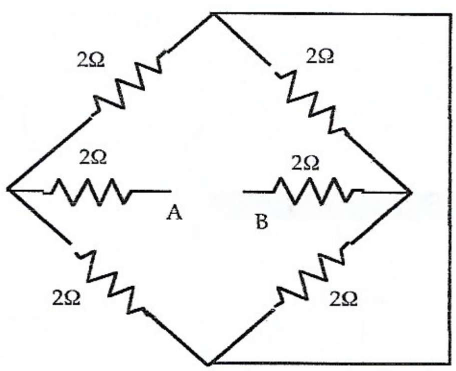
A. How many junctions and loops are there in this circuit? How many are independent?
B. What is the equivalent resistance of this circuit?
C. What is the magnitude of the voltage across each element?
D. What is the magnitude of the current in each branch?
E. How much power is produced by the battery?
F. How much power is consumed by each resistor?
G. How much power is consumed by all three resistors in the circuit?

10. Calculate the resistance of the circuits below

A.

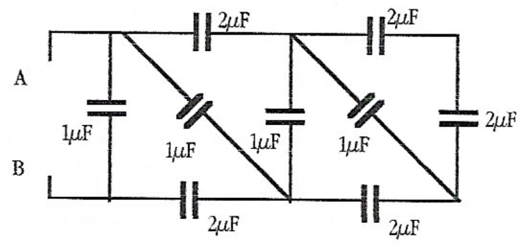


B.

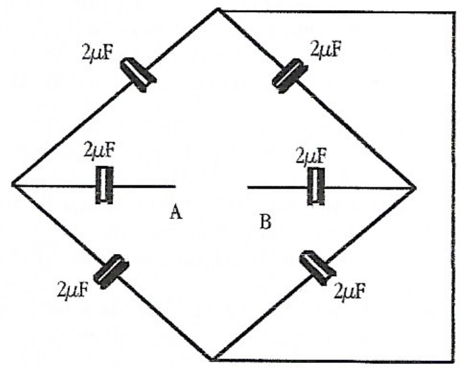


11. Calculate the capacitance of the circuits below

A.

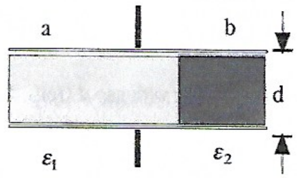


B.

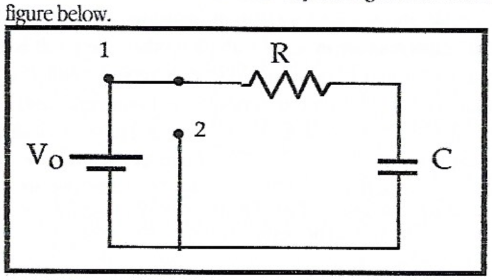


used this one

I. Calculate the capacitance of th arrangement below.



II. Consider the RC circuit given in the figure. At t=0, the voltage source is connected to the circuit by closing switch 1 as shown in the figure below.

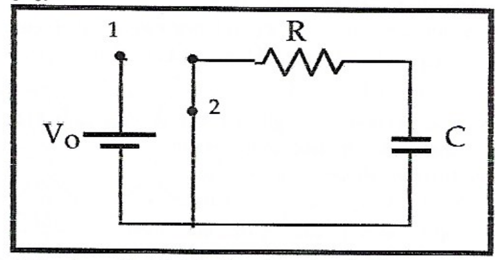


A. Use dimensional analysis to obtain the characteristic time frame for this circuit.
Hint: Use the voltage across R and C in terms of the charge flow in the circuit.
B. Write down the Kirchhoff's voltage law for this circuit.
C. Revise your answer to part B to write a differential equation in terms of the charge or the current only.
D. Obtain an expression for the charge on the capacitor as a function of time.
E. Obtain an expression for the current in the loop as a function of time.
F.
   1. What is the voltage across the capacitor as a function of time?
   2. What is the voltage across the resistor as a function of time?
G. After a long time period with respect to the circuit's characteristic time frame,
   1. How much charge will there be on the capacitor ?
   2. What value will the current through and the voltage across the capacitor reach ?
   3. What value will the current through and the voltage across the resistor reach ?

$$\boxed{\int_{x_1}^{x_2} \frac{dx}{x-a} = \ln\left(\frac{x_2-a}{x_1-a}\right)}$$

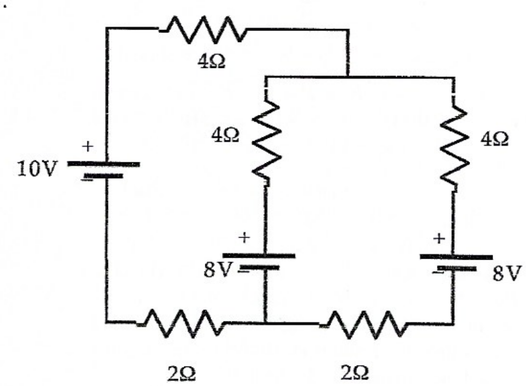
Even if you cannot evaluate the integrals involved, you will receive full credit for each correctly answered part using dimensional analysis and relevant justification involving the physics of the situation.

III. After a long period of time as described in the preceding problem, the voltage source is disconnected by opening switch 1, and switch 2 is closed as shown in the figure below. Take this to be your new t=0.



A. Write down the Kirchhoff's voltage law for this circuit.
B. Revise your answer to part A to write a differential equation in terms of the charge or the current only.
C. Obtain an expression for the charge on the capacitor as a function of time.
D. Obtain an expression for the current in the loop as a function of time.
E.
   1. What is the voltage across the capacitor as a function of time?
   2. What is the voltage across the resistor as a function of time?
F. After a long time period with respect to the circuit's characteristic time frame,
   1. How much charge will there be on the capacitor ?
   2. What value will the current through and the voltage across the capacitor reach ?
   3. What value will the current through and the voltage across the resistor reach ?

IV.



A. Write down the Kirchhoff's voltage rule for each loop.
B. Use your answers to obtain the
   1. current in each resistor
   2. voltage across each resistor
   3. power consumed by each resistor.
C. Is the sum of powers produced by the batteries equal to the sum of the powers consumed by each resistor? Why (not)?

ch 18-19 Test
ML

1) D 2) C 3) $10+20+30=60\Omega$ E

4) $\left(\frac{1}{10}+\frac{1}{20}+\frac{1}{30}\right)^{-1}=5.45\Omega$ A

5) 2 (B) 6) 1 (A)

7) $R_{eq}=\left(\frac{1}{20}+\frac{1}{30}\right)^{-1}+10=22\Omega$ D

8) $I_1=\frac{\Delta V}{R}=\frac{110V}{22\Omega}=5A$ B

9) $V_1=I_1R_1=(5A)(10\Omega)=50V$ D

10) $V_2=110V-50V=60V$ C A

11) $V_3=V_2$

12) $I_2=\frac{\Delta V_2}{R_2}=\frac{60V}{20\Omega}=3A$

13) $I_3=\frac{\Delta V_3}{R_3}=\frac{60V}{30\Omega}=2A$

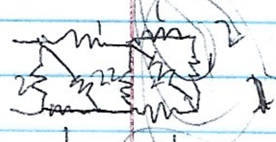
14) $P=I_1\Delta V=(5A)(110V)=550W$ A

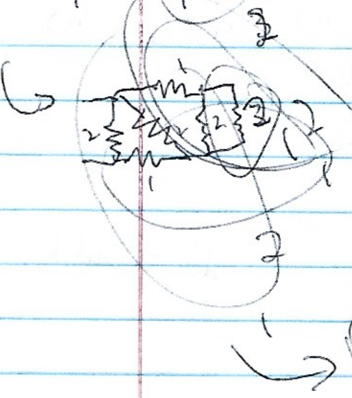
15) $P=I^2R=(5A)^2(10\Omega)=250W$ B

16) $P=I^2R=(3A)^2(20\Omega)=180W$ C

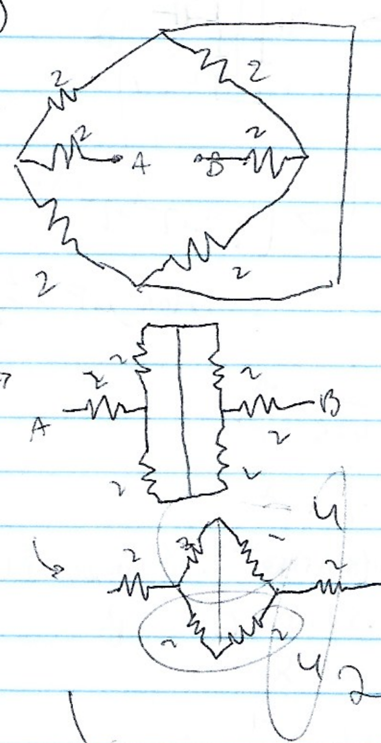
17) $P=I^2R=(2A)^2(30\Omega)=120W$ D

18) $P=P_{bat}=550W$ A

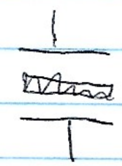
19) 



B

20) 

6Ω E

21) B 

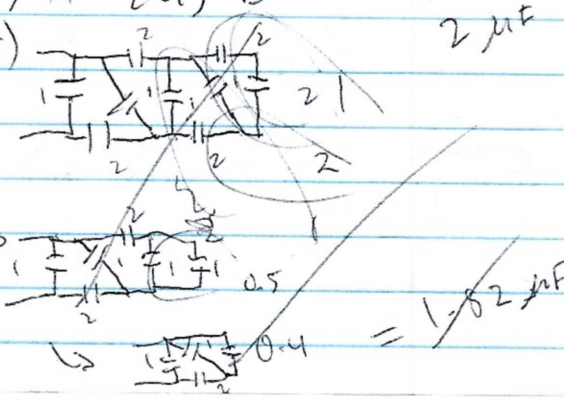
22) $C_{eq}=\left(\frac{1}{C_1}+\frac{1}{C_2}\right)^{-1}$

$C=\frac{\varepsilon_0 A}{d}$ $C_1=\frac{\varepsilon_0 A}{\frac{(d-a)}{2}}=\frac{2\varepsilon_0 A}{d-a}$

$\left(\frac{d-a}{2\varepsilon_0 A}+\frac{d-a}{2\varepsilon_0 A}\right)^{-1}$

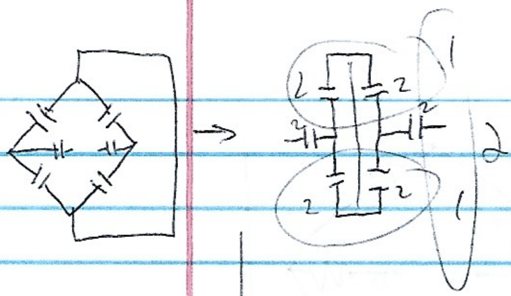
$=\frac{\varepsilon_0 A}{d-a}$ B

23) A 24) B

25) 

$2\,\mu F$ D

~~$=1.82\,\mu F$~~

26 

$\to \left(2^{-1}+2^{-1}+2^{-1}\right)^{-1}$

$= 0.66\,\mu F = \frac{2}{3}\,\mu F$ B

27) $E = \frac{kq}{r^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$

$Q = 4\pi\varepsilon_0 \cdot E r^2$

$= 4\pi(8.85\times10^{-12}\,F/m)(6400{,}000\,m)^2(100\,N/C)$

$= 4.56\times10^5$ C B

28) 2 capacitors in series

$C_{eq} = \left(\frac{1}{C_1}+\frac{1}{C_2}\right)^{-1}$

$C_1 = \frac{K_1\varepsilon_0 A}{d_1} = \frac{\varepsilon_1 A}{d_1}$

$C_2 = \frac{\varepsilon_2 A}{d_2}$

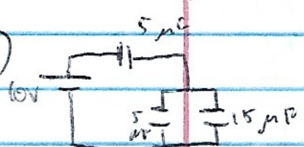
$C_{eq} = \left(\frac{d_1}{\varepsilon_1 A}+\frac{d_2}{\varepsilon_2 A}\right)^{-1}$ C

29) 2 capacitors in parallel

$C_{eq} = C_1 + C_2$

$C_1 = \frac{\varepsilon_1 A}{d}$ $C_2 = \frac{\varepsilon_2 A}{d}$

$C_{eq} = \frac{(\varepsilon_1+\varepsilon_2)A}{d}$ B

30) 

$C_{eq} = \left(\frac{1}{5}+\frac{1}{20}\right)^{-1}$

$= 4\,\mu F$ A

31) $Q = \frac{C_{eq}}{V} = \cancel{\frac{4\,\mu F}{10\,V} = 4\times10^{-7}}$ C

$Q = CV = 4\,\mu F \cdot 10\,V = \boxed{40\,\mu C}$ A

32) $PE = \frac{1}{2}CV^2 = \cancel{\frac{1}{2}\frac{Q^2}{C}}$

$= \frac{1}{2}(4\,\mu F)(10\,V)^2 = \boxed{2\times10^{-4}\,J}$ A

33) $V_1 = 10\,V$ A 34) $V_2 = 10\,V$ A

35) $V_3 = 10\,V$ A

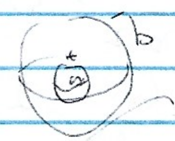
36) $Q = CV = (5\,\mu F)(10\,V) = 50\,\mu C$

37) $C = \frac{Q}{V}$ $Q_2 = C_2V_2 = 50\,\mu C$

38) $Q_3 = CV = 150\,\mu F$

(39 - 41)

FRQ

1) 

$\frac{Q_{enc}}{\varepsilon_0} = \oint E\cdot dS$

$4\pi(b-a)^2\varepsilon_0 E = Q_{enc}$

$E = \frac{kQ_{enc}}{(b-a)^2} = \frac{k(Q)}{(b-a)^2}$

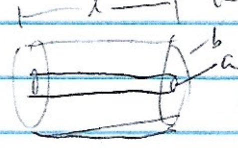
$\oint E$ $E\cdot 4\pi r^2 = \frac{Q_{enc}}{\varepsilon_0}$ $Q_{enc} = Q$

$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q_{enc}}{r^2} = \frac{Q}{4\pi\varepsilon_0 r^2}$

$V = -\int E\cdot dr = -\int\frac{kQ}{r^2}dr$

$= -kQ\int r^{-2}dr = \frac{kQ}{r} = \frac{kQ}{(b-a)}$

c) $C = \frac{Q}{V} = \frac{(b-a)}{k} = 4\pi\varepsilon_0(b-a)$

2) 

$\lambda = \frac{Q}{l}$ $dQ = \lambda\,dl$

2) $\frac{Q}{\varepsilon_0} = \oint E \cdot ds$ $s = 2\pi r l$

a)

$\int E \cdot 2\pi r \, dl = \frac{Q}{\varepsilon_0}$ $Q = \lambda l$

$E l \cdot 2\pi r = \frac{\lambda l}{\varepsilon_0}$

$E = \frac{\lambda}{2\pi r \varepsilon_0}$

b) $V = -\int E \cdot dr = -\int \frac{\lambda}{2\pi r \varepsilon_0} dr$

$= -\frac{\lambda}{2\pi \varepsilon_0} \int_a^b \frac{1}{r} dr$

$= -\frac{\lambda}{2\pi \varepsilon_0} \ln\left|\frac{b}{a}\right|$

c) $C = \frac{Q}{V} = \frac{-\lambda l \cdot 2\pi \varepsilon_0}{\lambda \ln\left|\frac{b}{a}\right|}$

$= \frac{-2\pi \varepsilon_0 l}{\ln\left|\frac{b}{a}\right|}$

3) $\frac{Q_{enc}}{\varepsilon_0} = \oint E \cdot ds$

a)

$EA = \frac{Q}{\varepsilon_0}$ $E = \frac{Q}{A\varepsilon_0}$ $\sigma = \frac{Q}{A}$

$\boxed{E = \frac{\sigma}{\varepsilon_0}}$

b) $V = -\int E \, dr = -\int_d^0 \frac{\sigma}{\varepsilon_0} dr$

$V = \frac{\sigma d}{\varepsilon_0}$

c) $\sigma = \frac{Q}{A}$ $C = \frac{Q}{V} = \frac{Q A \varepsilon_0}{Q \cdot d}$

$= \boxed{\frac{\varepsilon_0 A}{d}}$

4) a) series b) $C_{eq} = \frac{\varepsilon_0 A}{d-a}$ MC 22

c) MC 23, 24

5) $C = \frac{Q}{V}$ $I = \frac{dQ}{dt}$ $\boxed{5}$

6) MC 30-41

7) MC 27

8) a) 2 capacitors in series

MC 28-29

b) ~~$C = \frac{-2\pi \varepsilon_0 l}{\ln\left|\frac{b}{a}\right|}$ parallel~~

~~$C = 2\pi(\varepsilon_1 + \varepsilon_2)$~~

$C = 4\pi \varepsilon_0 (b-a)$ parallel

$C_{eq} = 4\pi(\varepsilon_1 + \varepsilon_2)(b-a)$

c) series $C = \frac{-2\pi \varepsilon_0 l}{\ln\left|\frac{b}{a}\right|}$

$C_{eq} = \frac{1}{\frac{-\ln\left|\frac{b}{a}\right|}{2\pi \varepsilon_1 l} + \frac{-\ln\left|\frac{b}{a}\right|}{2\pi \varepsilon_2 l}}$

9) MC 9-18

10) MC 19-20

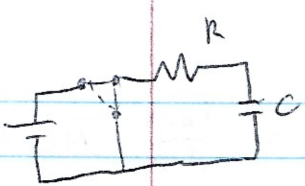
11) MC 25-26

FRQ I) 2 in parallel

$C_1 = \frac{a l \varepsilon_1}{d}$ $C_2 = \frac{b l \varepsilon_2}{d}$

~~$C_{eq} = \frac{(a+b) l (\varepsilon_1 + \varepsilon_2)}{d}$~~

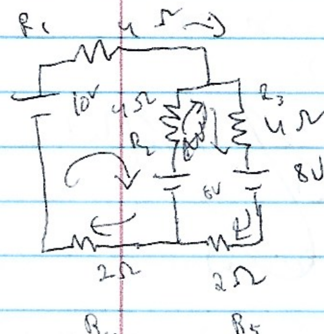
$C_{eq} = \frac{l(a\varepsilon_1 + b\varepsilon_2)}{d}$ ✓

II) 

A) $\tau = RC$ B) $V_c = V_0$

c) $C = \frac{Q}{V}$ $\cancel{Q = CV}$

III) $V_2 = V_1$

IV) 

$\cancel{R_2 + R_3 = R_1 \quad R_4 + R_5}$

$I_2 + I_3 = I_1 \quad I_4 - I_5 - I_2 = 0$

$8V - 8V = \Delta V_2 + \Delta V_3 + \Delta V_5 = 0$

$10V - 8V = \Delta V_1 - \Delta V_2 + \Delta V_4$

$V_2 + V_3 + V_5 = 0$

$V_1 - V_2 + V_4 = 2V$

$V_1 = I_1 R_1$

II) b) voltage in a closed loop is 0

$V_0 = V_0 \quad V_R = IR \quad V_c = \frac{Q}{C}$

$C = \frac{Q}{V}$

$V_0 - IR - \frac{Q}{C} = 0$

c) $\frac{Q}{C} = V_0 - IR$

differential equation?

$I = \frac{dQ}{dt}$

$-V_0 + IR + \frac{Q}{C} = 0$

$\frac{d}{dt}(-V_0) + \frac{d}{dt} IR + \frac{d}{dt}\frac{Q}{C} = 0$

$R\frac{dI}{dt} + \frac{1}{C}\frac{dQ}{dt} = 0$

d) $R\frac{dI}{dt} + \frac{I}{C} = 0$

$\int_{I_0}^{I} \frac{dI}{I} = \int_0^t -\frac{dt}{RC}$

$\ln I - \ln I_0 = -\frac{t}{RC}$

$\ln\left|\frac{I}{I_0}\right| = -\frac{t}{RC}$

$\frac{I}{I_0} = e^{-\frac{t}{RC}}$

$I = I_0 e^{-t/RC}$

$I_0 = \frac{V_0}{R}$

$I = \frac{V_0}{R} e^{-\frac{t}{RC}}$ [e]

$\frac{dQ}{dt} = \frac{V_0}{R} e^{-\frac{t}{RC}}$ $Q_0 = CV_0$

$Q = \int_0^t \frac{V_0}{R} e^{-\frac{t}{RC}} dt$ $u = -\frac{t}{RC}$ $\frac{du}{dt} = -\frac{1}{RC}$ $dt = -RC\,du$

$Q(t) = \frac{V_0}{R} \cdot -RC \cdot e^{-\frac{t}{RC}} \Big|_0^t$

$Q_0 = CV_0$

$$Q(t) = -V_0 C \cdot e^{-t/RC}\Big|_0^t \qquad e^0 = 1$$

$$= -Q_0 e^{-t/RC} + Q_0$$

$$Q(t) = Q_0(1 - e^{-t/RC}) \qquad [d]$$

f) $C = \frac{Q}{V}$ $\quad V = \frac{Q}{C}$

$$\frac{Q(t)}{C} = \frac{Q_0}{C}(1 - e^{-t/RC})$$

1) $V(t) = V_0(1 - e^{-t/RC})$

2) $\Delta V = IR$

$$V_0 - V_c = IR = V_R$$

$$V_R = \frac{V_0}{R} e^{-t/RC} \cdot R$$

$$V_R = V_0 e^{-t/RC}$$

g) 1) $Q = CV_0$

2) current doesn't actually "pass" through the capacitor

$V_c = V_0$

3) $I_R = 0$, $V_R = 0$

III) $C = \frac{Q}{V}$ $\quad V = \frac{Q}{C}$

a) $\frac{Q}{C} = IR$

b) $\frac{d}{dt}\frac{Q}{C} = \frac{d}{dt} IR$

$$\frac{dQ}{dt} \cdot \frac{1}{C} = \frac{dI}{dt} R$$

c) $\frac{I}{C} = \frac{dI}{dt} \cdot R$

e) $\frac{dI}{I} = \frac{1}{RC} dt$

$$\int_{I_0}^{I} \frac{dI}{I} = \int_0^t \frac{1}{RC} dt$$

$$\ln\left|\frac{I}{I_0}\right| = \frac{t}{RC}$$

$$\frac{I}{I_0} = e^{t/RC}$$

$$I = I_0 e^{t/RC} \rightarrow = \frac{V_0}{R} e^{t/RC}$$

$RI_0 = V_0$

$I_0 = \frac{V_0}{R}$

$$\frac{dQ}{dt} = I_0 e^{t/RC}$$

$$dQ = \frac{V_0}{R} e^{t/RC} dt$$

$u = \frac{t}{RC}$

$du = \frac{1}{RC} dt$

$dt = RC\,du$

$$Q(t) = \frac{V_0}{R}\int_0^t e^u \cdot RC\,du$$

$$Q(t) = \frac{V_0}{R} \cdot RC\, e^{t/RC}$$

$$= V_0 C e^{t/RC} = Q_0 e^{t/RC}$$

$Q = CV \qquad V = \frac{Q}{C}$

$$V(t) = V_0 e^{t/RC}$$

$$V_R = V_0 - V(t) = V_0(1 - e^{t/RC})$$

f) $Q = CV = 0$

$I_R = 0 \quad I_c = 0$

$\Delta V = 0 \quad \Delta V = 0$

IV) $-10V + 4I_1 + 4I_2 + 8V + 2I_1 = 0$

$-8V - 4I_2 + 4I_3 + 8V + 2I_3 = 0$

$-10V + 4I_1 + 4I_3 + 8V + 2I_3 + 2I_1 = 0$

$I_1 = I_2 + I_3$

$I_1 = \frac{5}{21} A$, $I_2 = \frac{3}{21} A$ $I_3 = \frac{2}{21} A$

$V_1 = \frac{20}{21} V$ $V_2 = \frac{12}{21} V$ $V_3 = \frac{8}{21} V$

$V_4 = \frac{4}{21} V$ $V_5 = \frac{10}{21} V$

1. KJR involves flow of charges into and from a junction. Therefore, it is nothing but the law of conservation of electric charges. *1pt*
2. KLR involves adding all the voltages changes (which are the electric potential energies per unit charge) in a loop. Therefore, it is nothing but the conservation of energy. *1pt*
3. These resistors are in series. We simply add them:
   $R_{eq}=10\ \Omega+20\ \Omega+30\Omega=60\ \Omega$ *1pt*
4. These resistors are in parallel. We add their inverses, then invert the result: $1/(R_{eq})=1/(10\Omega)+1/(20\Omega)+1/(30\Omega)=11/(60\Omega)$ *1pt*
   $R_{eq}=(60/11)\Omega=5.45\Omega$ *1pt*
5. There are two junctions although they both give the same KJR equation; that is they are **not** independent. *1pt*
6. There are **two** *independent* loops: *1pt*
   Loop 1: Follow I_1 and I_2 (the outer loop)
   Loop 2: Follow I_1 and I_3 (the inner loop)
   Loop 3: Follow I_2, then $-I_3$ (the closed box-loop)
   However, Loop 3 =(Loop 1 – Loop 2), i. e. not an independent one. Thus, there are only 2 *independent* loops which give two distinct KLR equations (see question 20 below).
7. 20 Ω and 30Ω are parallel to each other. Therefore, we reduce them first: $1/R_{[]}=1/(20\Omega)+1/(30\Omega)=5/(60\Omega)$, $R_{[]}=12\Omega$ *1pt*
   That combination is in series with 10Ω; therefore,
   $R_{eq}=10\Omega+R_{[]}=22\Omega$ *1pt*
8. I_1 is the current that is coming out of and going into the battery which is the same current that goes through the equivalent R:
   $I_1=110V/R_{eq}=5A$ *1pt*
9. $V_1=I_1 10\Omega=50V$ *1pt*
10. and 11: Since 20 Ω and 30Ω are parallel, they have the same voltage: $V_{[]}=V_2=V_3$. We can find them in multiple ways:
   $V_{[]}=110V-V_1=60V$
   $V_{[]}=I_{[]}R_{[]}=I_1R_{[]}=60V$ (I_1 goes through $R_{[]}$) *1+1 pts*

12. and 13. Use your answers from 10 and 11 to solve
$V_{[]}=V_2=V_3 \Rightarrow 60V=I_1R_{[]}=I_2 20\Omega=I_3 30\Omega$. Thus
$I_2=3A$ and $I_3=2A$. Notice $I_2+I_3=5A=I_1$ as should be. *1+1 pts*

14–18: $P=VI$ or $P=I^2R$ or $P=V^2/R$ for each element *1pt each*
YOU HAVE TO SEPARATE THE ANSWERS and **SHOW WORK** IN YOUR CORRECTIONS!

$$P_{battery}=110V\times 5A=550W$$
$$=P_{total-consumed-in-circuit}$$
$$P_{10\Omega}=250W,\ P_{20\Omega}=180W,\ P_{30\Omega}=120W$$

The sum of these give the total: the same as above.

19. Ask me to show you in class. In a complicated circuit, you first need to reduce the elements that are clearly either (i) in series, or (ii) in parallel. In this case the uper right most 1Ω resistros are in series with each other, the result is in parallel to the diagonal 2Ω resistor, when combined reduce to 1Ω again the result which is in series to the lower right hand 1Ω, so on so forth. After all the work is done, we get 1Ω. *1pt for showing th e first step, 1pt for the second, 1 pt for the correct final answer.* *3pts.*
20. Ask me to show you in class. You first need to recognize the fact that the wire connecting the top and the bottom between the 2Ω resistors causes the top and the bottom points to shrik to a single point (*1pt*). As a result, we have a pair of 2Ω resistors connected in parallel which reduces 1Ω (*1pt*), and this result is connected to the second pair in series. As a result, we have two 1Ω's connnected in series to the remaining two 2Ω resistors--they are all in series, thus the equivalent resistance is 6Ω (*1pt*). *3pts.*
21. In series since everything goes through each element one aftet the other. Therefore, the two capacitors are in series. *1pt*
22. The space between each capacitor is $\frac{d-a}{2}$(*1pt*). As a result, each capacitor's capacitance $C=\frac{\varepsilon A}{\frac{d-a}{2}}=\frac{2\varepsilon A}{d-a}$(*1pt*). However, now there are two of them in series. Therefore,
   $\frac{1}{C_{eq}}=\frac{1}{C}+\frac{1}{C}=\frac{2}{C}$, $C_{eq}=\frac{C}{2}=\frac{\varepsilon A}{d-a}$(*1pt*) *3pts*
23. When $a\to 0$, we have only one capacitor with gap d (*1pt*). As a result, $C_{eq}=\frac{\varepsilon A}{d-a}$(*1pt*) *2pts*
24. When $a\to d$, we have $C_{eq}=\frac{\varepsilon A}{0}\to\infty$. A best capacitor is the one made purely of an excellent conductor. *1pt*
25. This is just like the one we have with the resistors. The uper right most $2\mu F$ capacitors are in series with each other, the result is in parallel to the diagonal $1\mu F$ resistor, when combined reduce to $2\mu F$ again the result which is in series to the lower right hand $2\mu F$, so on so forth. After all the work is done, we get $2\mu F$. *1pt for showing th e first step, 1pt for the second, 1 pt for the correct final answer.* *3pts.*
26. Just like the resistor one, except you need to remember we are dealing with capacitors. The result is $\frac{2}{3}\mu F$. *3pts*
27. Gauss' law gives
$$E=\frac{kQ}{R^2}\Rightarrow Q=\frac{ER^2}{k}=\frac{100\frac{N}{C}\left(6.4\times10^6 m\right)^2}{9\times10^9\frac{Nm^2}{C^2}}=4.55\times10^6 C$$ *2pts*
28. $C=\left(\frac{1}{C_1}+\frac{1}{C_2}\right)^{-1}=\frac{1}{\frac{d_1}{\varepsilon_1 A_1}+\frac{d_2}{\varepsilon_2 A_2}}$ *2pts*
29. $C=C_1+C_2=\frac{\varepsilon_1\frac{A}{2}}{d}+\frac{\varepsilon_2\frac{A}{2}}{d}=(\varepsilon_1+\varepsilon_2)\frac{A}{2d}$ *2pts*
30. $\frac{1}{C}=\frac{1}{5\mu F}+\frac{1}{5\mu F+15\mu F}\Rightarrow C=4\mu F$ *2pts*
31. $Q=VC=40\mu C$ *1pt*
32. $U=\frac{1}{2}QV=200\mu J$ *1pt*
33. Whencapacitors are in series, they share the same charge just like the resistors in series sharing the same current. $V_1=\frac{Q}{C_1}=8V$
34. $V_2=\frac{Q}{C_2}=2V$ Notice that $V=V_1+V_2(=V_3)=8V+2V=10V$ *1pt*
35. Since they are parallel, same as C_2. *1pt*
36. The same as the charge stored in the circuit 40 μC or $Q_1=V_1C_1$ *1pt*
37. $Q_2=V_2C_2=10\mu C$ *1pt*
38. Since the voltage the same and the capacitance is 3 times C_2, so is the charge. *1pt*
39. $U=\frac{1}{2}Q_1V_1=160\mu J$ *1pt*
40. $U=\frac{1}{2}Q_2V_2=10\mu J$ *1pt*
41. $U=\frac{1}{2}Q_3V_3=30\mu J$ *1pt*

FRQ

1. The capacitor is made of two parallel capacitors, side by side--not one afte the other. *1pt*

$$C=\frac{\varepsilon_1 ac}{d}+\frac{\varepsilon_2 bc}{d}=\frac{(a\varepsilon_1+b\varepsilon_2)c}{d}$$ *1pt*

2.

A. $IR=\frac{Q}{C},\ \frac{Q}{\tau}R=\frac{Q}{C},\ \tau=RC$ *2pts*

B. $-V_0+IR+\frac{Q}{C}=0$

C. $R\frac{dI}{dt}+\frac{I}{C}=0$ or $-V_0+R\frac{dQ}{dt}+\frac{Q}{C}=0$

D. $\frac{dQ}{dt}=-\frac{1}{R}\left(\frac{Q}{C}-V_0\right),\ u=Q-V_cC,\ \frac{du}{dt}=-\frac{dt}{RC}$

$\ln u=-\frac{t}{RC}+K,\ u=e^{K}e^{-\frac{t}{RC}},\ (Q-V_cC)=e^{K}e^{-\frac{t}{RC}}$

$Q=V_cC\left(1-e^{-\frac{t}{RC}}\right),\ t=0,\ Q=0$

E. $\frac{dI}{I}=-\frac{1}{RC}dt,\ I=I_0e^{\frac{-t}{RC}}=\frac{V_0}{R}e^{-\frac{t}{RC}},\ t=0,\ I=\frac{V_0}{R}$

F. 1. $V_C=\frac{Q}{C}=V_0\left(1-e^{-\frac{t}{RC}}\right)$, 2. $V_R=IR=V_0e^{-\frac{t}{RC}}$

G. 1. $Q=V_0C$, 2. $I_R=0,\ V_R=0$, 3. $I_C=0,\ V_C=0$

3.

A. $IR+\frac{Q}{C}=0$

B. $\frac{dQ}{Q}=-\frac{dt}{RC}$

C. $\ln Q=-\frac{t}{RC}+K,\ Q=Q_0e^{-\frac{t}{RC}}=V_0Ce^{-\frac{t}{RC}}$

D. $I=\frac{dQ}{dt}=-\frac{V_0}{R}e^{-\frac{t}{RC}}$

E. 1. $V_C=\frac{Q}{C}=V_0e^{-\frac{t}{RC}}$, 2. $V_R=IR=-V_0e^{-\frac{t}{RC}}$

F. 1. $Q_C=0$, 2. $V_C=0,\ I_C=0$, 3. $V_R=0,\ I_R=0$

4.

A. $-10V+4I_1+4I_2+8V+2I_1=0$ (1)

$-8V-4I_2+4I_3+8V+2I_3=0$ (2)

$-10V+4I_1+4I_3+8V+2I_3+2I_1=0$ (3)

$I_1=I_2+I_3$ (4)

B. $-2+6I_1+4I_2=0,\ 3I_1+2I_2=1$

$-4I_2+6I_3=0,\ I_3=\frac{2}{3}I_2$

$I_1-I_2=\frac{2}{3}I_2,\ I_1=\frac{5}{3}I_2$

$3\times\frac{5}{3}I_2+2I_2=1,\ I_2=\frac{1}{7}A$

$I_1=\frac{5}{21}A,\ I_2=\frac{3}{21}A,\ I_3=\frac{2}{21}A,\ I_1=I_2+I_3$

$V_1=\frac{20}{21}V,\ V_2=\frac{12}{21}V,\ V_3=\frac{8}{21}V,\ V_4=\frac{4}{21}V,\ V_5=\frac{10}{21}V$

$P_i=I_iV_i$

C. Yes. $P_{Battery}=\sum I_iV_i$

5.

A. $R_{eq}=\frac{1}{\frac{1}{2\Omega}+\frac{1}{3\Omega}}+5\Omega=\frac{6}{5}\Omega+\frac{25}{5}\Omega=\frac{31}{5}\Omega$

B. $I_1=\frac{62V}{(31/5)\Omega}=10A,\ V_1=I_1 5\Omega=50V$

$V_2=V_3=62V-50V=12V$

We could also use $I_1=I_2+I_3,\ V_2=V_3,\ I_2 2\Omega=I_3 3\Omega$

C. $2I_2=3I_3,\ I_1=I_2+I_3=I_2+\frac{2}{3}I_2=\frac{5}{3}I_2=10A$

$I_2=6A,\ I_3=4A$

D. $P_1=V_1I_1=500W,\ P_2=V_2I_2=72W,\ P_3=V_3I_3=48W$

$P=VI_1=620W$

E. $P=VI_1=620W=P_1+P_2+P_3$. Yes, it should be since the power consumed cannot exceed power produced, and power produced needs to be consumed.

19. 1st Jct: $I_1-I_2-I_3=0$; 2nd Jct: $I_2+I_3-I_1=0$. *1pt*

20. Loop1 (upper): $-110V+V_2+V_1=0$ *1pt*

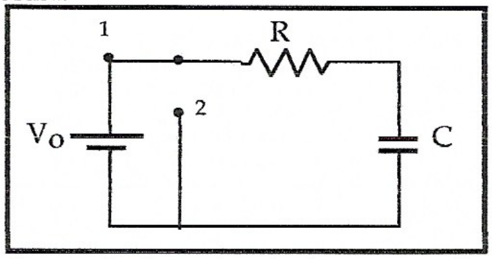
Loop 2 (lower): $-110V+V_3+V_1=0$ *1pt*

Loop 3 (closed box): $V_2-V_3=0$

Notice that Loop 3 =(Loop 1 – Loop 2), that is **not** independent.

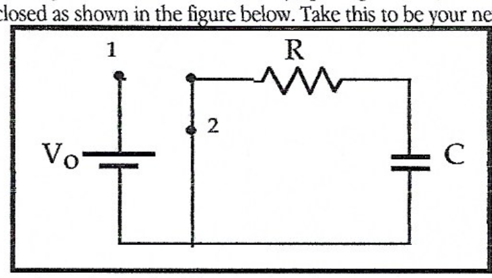
21. I will go over this in class more explicitly. For your corrections, use your text: Figures 18.16 and 18. 17 give the graphic representation of the charge on the capacitor. The current through the resistor starts from a maximum value of V/R and exponentially decreases to 0 in both cases (looks like 18.b in both cases--parts A and B of 21).

I. Consider the RC circuit given in the figure. At t=0, the voltage source is connected to the circuit by closing switch 1 as shown in the figure below.



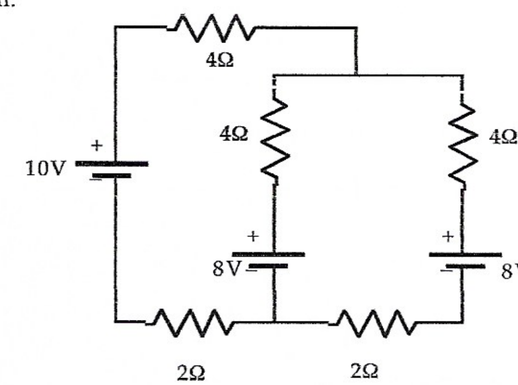
A. Use dimensional analysis to obtain the characteristic time frame for this circuit. *Hint: Use the voltage across R and C in terms of the charge flow in the circuit.*
B. Write down the Kirchhoff's voltage law for this circuit.
C. Revise your answer to part B to write a differential equation in terms of the charge or the current only.
D. Obtain an expression for the charge on the capacitor as a function of time.
E. Obtain an expression for the current in the loop as a function of time.
F. 1. What is the voltage across the capacitor as a function of time?
   2. What is the voltage across the resistor as a function of time?
G. After a long time period with respect to the circuit's characteristic time frame,
   1. How much charge will there be on the capacitor ?
   2. What value will the current through and the voltage across the capacitor reach ?
   3. What value will the current through and the voltage across the resistor reach ?

II. After a long period of time as described in the preceding problem, the voltage source is disconnected by opening switch1, and switch2 is closed as shown in the figure below. Take this to be your new t=0.



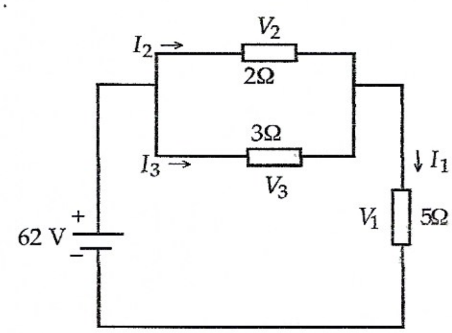
A. Write down the Kirchhoff's voltage law for this circuit.
B. Revise your answer to part A to write a differential equation in terms of the charge or the current only.
C. Obtain an expression for the charge on the capacitor as a function of time.
D. Obtain an expression for the current in the loop as a function of time.
E. 1. What is the voltage across the capacitor as a function of time?
   2. What is the voltage across the resistor as a function of time?
F. After a long time period with respect to the circuit's characteristic time frame,
   1. How much charge will there be on the capacitor ?
   2. What value will the current through and the voltage across the capacitor reach ?
   3. What value will the current through and the voltage across the resistor reach ?

III.



A. Write down the Kirchhoff's voltage rule for each loop.
B. Use your answers to obtain the
   1. current in each resistor
   2. voltage across each resistor
   3. power consumed by each resistor.
C. Is the sum of powers produced by the batteries equal to the sum of the powers consumed by each resistor? Why (not)?

IV.



Calculate
   A. the equivalent resistance of the circuit
   B. the voltage
   C. the current
   D. the power consumed
      for each element.
   E. Do individual powers consumed by each resistor add up to the power produced by the battery? Why, why not?

$$\boxed{\int_{x_1}^{x_2} \frac{dx}{x-a} = \ln\left(\frac{x_2-a}{x_1-a}\right)}$$

Even if you cannot evaluate the integrals involved, you will receive full credit for each correctly answered part using dimensional analysis and relevant justification involving the physics of the situation.

1. **Field • Area**$_{\text{enclosing source}}$ $= C_F \times$ Source$_{\text{enclosed}}$

A. Between the two spheres, the only charge enclosed by the Gaussian surface is the charge on the conducting sphere which is Q. The external sphere does not create any net electric field inside.

$$\vec{E}\cdot 4\pi r^2\hat{r} = 4\pi kQ = \frac{Q}{\varepsilon_o},\ \vec{E} = \frac{Q}{4\pi\varepsilon_o r^2}\hat{r}$$

B. The voltage difference between the two spheres is

$$V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{r} = -\int_a^b \frac{Q}{4\pi\varepsilon_o r^2}dr = \left.\frac{Q}{4\pi\varepsilon_o r}\right]_a^b$$

$$V_b - V_a = \frac{Q}{4\pi\varepsilon_o}\left(\frac{1}{b} - \frac{1}{a}\right)$$

C. $Q = VC,\ C = \dfrac{Q}{|V_b - V_a|} = \dfrac{4\pi\varepsilon_o ab}{b-a}$

2. **Field • Area**$_{\text{enclosing source}}$ $= C_F \times$ Source$_{\text{enclosed}}$

A. Between the two cylinders, the only charge enclosed by the Gaussian surface is the charge on the conducting cylinder which is Q. The external cylinder does not create any net electric field inside.

$$\vec{E}\cdot 2\pi r\ell\hat{r} = 4\pi kQ = \frac{Q}{\varepsilon_o},\ \vec{E} = \frac{Q}{2\pi\varepsilon_o r\ell}\hat{r} = \frac{\lambda}{2\pi\varepsilon_o r}\hat{r}$$

B. The voltage difference between the two spheres is

$$V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{r} = -\int_a^b \frac{\lambda}{2\pi\varepsilon_o r}dr = \left.\frac{\lambda}{2\pi\varepsilon_o}\ln r\right]_a^b$$

$$V_b - V_a = \frac{\lambda}{2\pi\varepsilon_o}(\ln b - \ln a) = \frac{\lambda}{2\pi\varepsilon_o}\ln\left(\frac{b}{a}\right)$$

C. $Q = VC,\ C = \dfrac{Q}{|V_b - V_a|} = \dfrac{Q}{\dfrac{\lambda}{2\pi\varepsilon_o}\ln\left(\dfrac{b}{a}\right)} = \dfrac{2\pi\varepsilon_o\ell}{\ln\left(\dfrac{b}{a}\right)}$

$$\frac{C}{\ell} = \frac{2\pi\varepsilon_o}{\ln\left(\dfrac{b}{a}\right)}$$

3. **Field • Area**$_{\text{enclosing source}}$ $= C_F \times$ Source$_{\text{enclosed}}$ a

A. Consider the + plate. A rectangular Gaussian surface encloses the plate. We will assume that the sides are very small and the field through the sides is negligible compared to the rest of the field flux. On both the upper and the lower parts of the areas point outwards and the fields going through aso point outwards. As a result, we have

$$\vec{E}_+\cdot\vec{A} = E_{up}\cdot A_{up} + E_{down}\cdot A_{down} = 2EA = \frac{Q}{\varepsilon_o}$$

$$\vec{E}_+ = \frac{Q}{2A\varepsilon_o}\uparrow = \frac{\sigma}{2\varepsilon_o}\uparrow$$

Between the plates, this field points toward the – plate. For the – plate, we instead have

$$\vec{E}_-\cdot\vec{A} = E_{up}\cdot A_{up} + E_{down}\cdot A_{down} = 2EA = \frac{Q}{\varepsilon_o}$$

$$\vec{E}_- = \frac{Q}{2A\varepsilon_o}\uparrow = \frac{\sigma}{2\varepsilon_o}\uparrow$$

Between the plates, this field points toward the – plate. As a result, the net electric field between the plates is

$$\vec{E} = \vec{E}_+ + \vec{E}_- = \frac{\sigma}{\varepsilon_o}\uparrow$$

B. The voltage difference between the two spheres is

$$V_+ - V_- = -\int_a^b \vec{E}\cdot d\vec{r} = \frac{\sigma d}{\varepsilon_o}$$

C. $Q = VC,\ C = \dfrac{Q}{|V_+ - V_-|} = \dfrac{Q}{\dfrac{\sigma d}{\varepsilon_o}} = \dfrac{\varepsilon_o A}{d}$

4.

A. Effectively this arrangement forms two capacitors in series, since the same current would flow through both. The effective gap between plates for each capacitor *(d-a)/2*.

B. In series, the inverse of capacitors add

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{(d-a)/2}{\varepsilon_o A} + \frac{(d-a)/2}{\varepsilon_o A} = \frac{d-a}{\varepsilon_o A}$$

$$C = \frac{\varepsilon_o A}{d-a}$$

C.

1. $\lim_{a\to 0} C = \dfrac{\varepsilon_o A}{d}$ the same as when there is nothing between the plates.
2. $\lim_{d\to a} C = \dfrac{\varepsilon_o A}{0} = \infty$ this is the same as a conductor.

5.

A. When capacitors are in series, the total voltage is the sum of the voltages:

$V = V_1 + V_2 + \ldots$

Since they are in series, the same charge goes through each; as a result,

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \ldots = Q\sum\frac{1}{C_i},\ \frac{1}{C_{eq}} = \sum\frac{1}{C_i}$$

B. In parallel, the charge coming into the circuit is divided among the parallel arms. The total charge is the sum of the charges in each arm

$Q = Q_1 + Q_2 + \ldots$

SInce they are parallel, each arm has the same voltage; as a result,

$$VC_{eq} = VC_1 + VC_2 + \ldots = V\sum C_i,\ C_{eq} = \sum C_i$$

6.

A. The parallel ones reduce to $C_{[]} = C_2 + C_3$. The combination is in series to the first one

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2 + C_3} = \frac{C_1 + C_2 + C_3}{C_1(C_2 + C_3)}$$

$$C_{eq} = \frac{C_1(C_2 + C_3)}{C_1 + C_2 + C_3}$$

B. $Q = V_o C_{eq} = \dfrac{V_o C_1(C_2 + C_3)}{C_1 + C_2 + C_3}$

C. $E = \dfrac{1}{2}QV_o = \dfrac{V_o^2 C_1(C_2 + C_3)}{2(C_1 + C_2 + C_3)}$

D. C_1 and $C_{[]}$ have the same charge across them which is the charge coming into the circuit, in part B.

$$V_1 = \frac{Q}{C_1} = \frac{V_o(C_2 + C_3)}{C_1 + C_2 + C_3},\ V_{[]} = \frac{Q}{C_{[]}} = \frac{V_o C_1}{C_1 + C_2 + C_3}$$

C_2 and C_3 have the same voltge as [] since they are parallel.

E. The charge on C_1 and $C_{[]}$ is the same as in part B since this is the charge coming into the circuit from the source. However, it splits between C_2 and C_3 proportional to each capacitor but the fraction of the total.

$Q_2 = \frac{C_2}{C_{[]}}Q,\ Q_3 = \frac{C_3}{C_{[]}}Q$

Notice that the sum is the same as that of $C_{[]}$.

F. $E_1 = \frac{1}{2}QV_1,\ E_2 = \frac{1}{2}Q_2V_2,\ E_3 = \frac{1}{2}Q_3V_3$

G. Yes, as it should be. You must prove this.

H. Yes, as it should be. You must prove this.

7.

A. On the surface of the earth, the electric potential is

$E_o = \frac{Q}{4\pi\varepsilon_o R_E^2},\ Q = E_o 4\pi\varepsilon_o R_E^2$

B. $\frac{1}{2}\varepsilon_o E_o^2$

1.

A. $IR = \frac{Q}{C},\ \frac{Q}{\tau}R = \frac{Q}{C},\ \tau = RC$ *2pts*

B. $-V_o + IR + \frac{Q}{C} = 0$

C. $R\frac{dI}{dt} + \frac{I}{C} = 0$ or $-V_o + R\frac{dQ}{dt} + \frac{Q}{C} = 0$

D. $\frac{dQ}{dt} = -\frac{1}{R}\left(\frac{Q}{C} - V_o\right),\ u = Q - V_cC,\ \frac{du}{dt} = -\frac{dt}{RC}$

$\ln u = -\frac{t}{RC} + K,\ u = e^K e^{-\frac{t}{RC}},\ (Q - V_cC) = e^K e^{-\frac{t}{RC}}$

$Q = V_cC\left(1 - e^{-\frac{t}{RC}}\right),\ t = 0,\ Q = 0$

E. $\frac{dI}{I} = -\frac{1}{RC}dt,\ I = I_o e^{\frac{-t}{RC}} = \frac{V_o}{R}e^{-\frac{t}{RC}},\ t = 0,\ I = \frac{V_o}{R}$

F. 1. $V_C = \frac{Q}{C} = V_o\left(1 - e^{-\frac{t}{RC}}\right)$, 2. $V_R = IR = V_o e^{-\frac{t}{RC}}$

G. 1. $Q = V_oC$, 2. $I_R = 0,\ V_R = 0$, 3. $I_C = 0,\ V_C = 0$

2.

A. $IR + \frac{Q}{C} = 0$

B. $\frac{dQ}{Q} = -\frac{dt}{RC}$

C. $\ln Q = -\frac{t}{RC} + K,\ Q = Q_o e^{-\frac{t}{RC}} = V_oCe^{-\frac{t}{RC}}$

D. $I = \frac{dQ}{dt} = -\frac{V_o}{R}e^{-\frac{t}{RC}}$

E. 1. $V_C = \frac{Q}{C} = V_o e^{-\frac{t}{RC}}$, 2. $V_R = IR = -V_o e^{-\frac{t}{RC}}$

F. 1. $Q_C = 0$, 2. $V_C = 0,\ I_C = 0$, 3. $V_R = 0,\ I_R = 0$

3.

A. $-10V + 4I_1 + 4I_2 + 8V + 2I_1 = 0$ (1)

$-8V - 4I_2 + 4I_3 + 8V + 2I_3 = 0$ (2)

$-10V + 4I_1 + 4I_3 + 8V + 2I_3 + 2I_1 = 0$

(3)

$I_1 = I_2 + I_3$ (4)

B. $-2 + 6I_1 + 4I_2 = 0,\ 3I_1 + 2I_2 = 1$

$-4I_2 + 6I_3 = 0,\ I_3 = \frac{2}{3}I_2$

$I_1 - I_2 = \frac{2}{3}I_2,\ I_1 = \frac{5}{3}I_2$

$3 \times \frac{5}{3}I_2 + 2I_2 = 1,\ I_2 = \frac{1}{7}A$

$I_1 = \frac{5}{21}A,\ I_2 = \frac{3}{21}A,\ I_3 = \frac{2}{21}A,\ I_1 = I_2 + I_3$

$V_1 = \frac{20}{21}V,\ V_2 = \frac{12}{21}V,\ V_3 = \frac{8}{21}V,\ V_4 = \frac{4}{21}V,\ V_5 = \frac{10}{21}V$

$P_i = I_iV_i$

C. Yes. $P_{Battery} = \sum I_iV_i$

4.

A. $R_{eq} = \frac{1}{\frac{1}{2\Omega} + \frac{1}{3\Omega}} + 5\Omega = \frac{6}{5}\Omega + \frac{25}{5}\Omega = \frac{31}{5}\Omega$

B. $I_1 = \frac{62V}{(31/5)\Omega} = 10A,\ V_1 = I_1 5\Omega = 50V$

$V_2 = V_3 = 62V - 50V = 12V$.

We could also use $I_1 = I_2 + I_3,\ V_2 = V_3,\ I_2 2\Omega = I_3 3\Omega$

C. $2I_2 = 3I_3,\ I_1 = I_2 + I_3 = I_2 + \frac{2}{3}I_2 = \frac{5}{3}I_2 = 10A$

$I_2 = 6A,\ I_3 = 4A$

D. $P_1 = V_1I_1 = 500W,\ P_2 = V_2I_2 = 72W,\ P_3 = V_3I_3 = 48W$

$P = VI_1 = 620W$

E. $P = VI_1 = 620W = P_1 + P_2 + P_3$. Yes, it should be since the power consumed cannot exceed power produced, and power produced needs to be consumed.

1. KJR involves flow of charges into and from a junction. Therefore, it is nothing but the law of conservation of electric charges. *1pt*
2. KLR involves adding all the voltages changes (which are the electric potential energies per unit charge) in a loop. Therefore, it is nothing but the conservation of energy. *1pt*
3. These resistors are in series. We simply add them:
   $R_{eq}=10\ \Omega+20\ \Omega+30\Omega=60\ \Omega$ *1pt*
4. These resistors are in parallel. We add their inverses, then invert the result:
   $1/(R_{eq})=1/(10\Omega)+1/(20\Omega)+1/(30\Omega)=11/(60\Omega)$ *1pt*
   $R_{eq}=(60/11)\Omega=5.45\Omega$ *1pt*
5. There are two junctions although they both give the same KJR equation; that is they are **not** independent. *1pt*
6. There are **two** *independent* loops: *1pt*
   Loop 1: Follow I_1 and I_2 (the outer loop)
   Loop 2: Follow I_1 and I_3 (the inner loop)
   Loop 3: Follow I_2, then $-I_3$ (the closed box-loop)
   However, Loop 3 =(Loop 1 – Loop 2), i. e. not an independent one.
   Thus, there are only 2 *independent* loops which give two distinct KLR equations (see question 20 below).
7. 20 Ω and 30Ω are parallel to each other. Therefore, we reduce them first: $1/R_{[]}=1/(20\Omega)+1/(30\Omega)=5/(60\Omega)$, $R_{[]}=12\Omega$ *1pt*
   That combination is in series with 10Ω; therefore,
   $R_{eq}=10\Omega+R_{[]}=22\Omega$ *1pt*
8. I_1 is the current that is coming out of and going into the battery which is the same current that goes through the equivalent R:
   $I_1=110V/R_{eq}=5A$ *1pt*
9. $V_1=I_1 10\Omega=50V$ *1pt*
10. and 11:
   1pt
11. and 10. Since 20 Ω and 30Ω are parallel, they have the same voltage: $V_{[]}=V_2=V_3$. We can find them in multiple ways:
   $V_{[]}=110V-V_1=60V$
   $V_{[]}=I_{[]}R_{[]}=I_1R_{[]}=60V$ (I_1 goes through $R_{[]}$) *1pt*
12. and 13. Use your answers from 10 and 11 to solve
   1pt
13. and 12. $V_{[]}=V_2=V_3 \Rightarrow 60V=I_1R_{[]}=I_2 20\Omega=I_3 30\Omega$. Thus $I_2=3A$ and $I_3=2A$. Notice $I_2+I_3=5A=I_1$ as should be. *1pt*

14–18: $P=VI$ or $P=I^2R$ or $P=V^2/R$ for each element *1pt each*
YOU HAVE TO SEPARATE THE ANSWERS and **SHOW WORK** IN YOUR CORRECTIONS!

$$P_{battery}=110V\times 5A=550W$$
$$=P_{total-consumed-in-circuit}$$
$$P_{10\Omega}=250W,\ P_{20\Omega}=180W,\ P_{30\Omega}=120W$$

The sum of these give the total: the same as above.

19. Ask me to show you in class. In a complicated circuit, you first need to reduce the elements that are clearly either (i) in series, or (ii) in parallel. In this case the uper right most 1Ω resistros are in series with each other, the result is in parallel to the diagonal 2Ω resistor, when combined reduce to 1Ω again the result which is in series to the lower right hand 1Ω, so on so forth. After all the work is done, we get 1Ω. *1pt for showing th e first step, 1pt for the second, 1 pt for the correct final answer.* *3pts.*
20. Ask me to show you in class. You first need to recognize the fact that the wire connecting the top and the bottom between the 2Ω resistors causes the top and the bottom points to shrik to a single point (*1pt*). As a result, we have a pair of 2Ω resistors connected in parallel which reduces 1Ω (*1pt*), and this result is connected to the second pair in series. As a result, we have two 1Ω's connnected in series to the remaining two 2Ω resistors--they are all in series, thus the equivalent resistance is 6Ω (*1pt*). *3pts.*
21. In series since everything goes through each element one aftet the other. Therefore, the two capacitors are in series. *1pt*
22. The space between each capacitor is $\frac{d-a}{2}$(*1pt*). As a result, each capacitor's capacitance $C=\frac{\varepsilon A}{\frac{d-a}{2}}=\frac{2\varepsilon A}{d-a}$(*1pt*). However, now there are two of them in series. Therefore,
   $\frac{1}{C_{eq}}=\frac{1}{C}+\frac{1}{C}=\frac{2}{C}$, $C_{eq}=\frac{C}{2}=\frac{\varepsilon A}{d-a}$(*1pt*) *3pts*
23. When $a\to 0$, we have only one capacitor with gap d (*1pt*). As a result, $C_{eq}=\frac{\varepsilon A}{d-a}$(*1pt*) *2pts*
24. When $a\to d$, we have $C_{eq}=\frac{\varepsilon A}{0}\to\infty$. A best capacitor is the one made purely of an excellent conductor. *1pt*
25. This is just like the one we have with the resistors. The uper right most 2μF capacitors are in series with each other, the result is in parallel to the diagonal 1μF resistor, when combined reduce to 2μF again the result which is in series to the lower right hand 2μF, so on so forth. After all the work is done, we get 2μF. *1pt for showing th e first step, 1pt for the second, 1 pt for the correct final answer.*
   3pts.
26. Just like the resistor one, except you need to remember we are dealing with capacitors. The result is $\frac{2}{3}\mu F$. *3pts*
27. Gauss' law gives
   $$E=\frac{kQ}{R^2}\Rightarrow Q=\frac{ER^2}{k}=\frac{100\frac{N}{C}\left(6.4\times10^6 m\right)^2}{9\times10^9\frac{Nm^2}{C^2}}=4.55\times10^6 C$$ *2pts*
28. $C=\left(\frac{1}{C_1}+\frac{1}{C_2}\right)^{-1}=\frac{1}{\frac{d_1}{\varepsilon_1 A_1}+\frac{d_2}{\varepsilon_2 A_2}}$ *2pts*
29. $C=C_1+C_2=\frac{\varepsilon_1\frac{A}{2}}{d}+\frac{\varepsilon_2\frac{A}{2}}{d}=(\varepsilon_1+\varepsilon_2)\frac{A}{2d}$ *2pts*
30. $\frac{1}{C}=\frac{1}{5\mu F}+\frac{1}{5\mu F+15\mu F}\Rightarrow C=4\mu F$ *2pts*
31. $Q=VC=40\mu C$ *1pt*
32. $U=\frac{1}{2}QV=200\mu J$ *1pt*
33. Whencapacitors are in series, they share the same charge just like the resistors in series sharing the same current.
   $V_1=\frac{Q}{C_1}=8V$
34. $V_2=\frac{Q}{C_2}=2V$ Notice that $V=V_1+V_2(=V_3)=8V+2V=10V$ *1pt*
35. Since they are parallel, same as C_2. *1pt*
36. The same as the charge stored in the circuit 40 μC or $Q_1=V_1C_1$ *1pt*
37. $Q_2=V_2C_2=10\mu C$ *1pt*

38. Since the voltage the same and the capacitance is 3 times C_2, so is the charge. *1pt*

39. $U = \frac{1}{2}Q_1V_1 = 160\mu J$ *1pt*

40. $U = \frac{1}{2}Q_2V_2 = 10\mu J$ *1pt*

41. $U = \frac{1}{2}Q_3V_3 = 30\mu J$ *1pt*

FRQ The capacitor is made of two parallel capacitors, side by side--not one afte the other. *1pt*

$$C = \frac{\varepsilon_1 ac}{d} + \frac{\varepsilon_2 bc}{d} = \frac{(a\varepsilon_1 + b\varepsilon_2)c}{d}$$ *1pt*

19. 1st Jct: $I_1 - I_2 - I_3 = 0$; 2nd Jct: $I_2 + I_3 - I_1 = 0$. *1pt*

20. Loop1 (upper): $-110V + V_2 + V_1 = 0$ *1pt*

Loop 2 (lower): $-110V + V_3 + V_1 = 0$ *1pt*

Loop 3 (closed box): $V_2 - V_3 = 0$

Notice that Loop 3 =(Loop 1 – Loop 2), that is **not** independent.

21. I will go over this in class more explicitly. For your corrections, use your text: Figures 18.16 and 18. 17 give the graphic representation of the charge on the capacitor. The current through the resistor starts from a maximum value of V/R and exponentially decreases to 0 in both cases (looks like 18.b in both cases--parts A and B of 21).