Ch 6, 16

USE $G=10^{-10}\frac{m^2N}{kg^2}$, $k=10^{+10}\frac{m^2N}{C^2}$ **unless otherwise stated**

1. Consider a spherical object of radius r mass M and charge Q uniformly distributed through out its volume.
Imagine yourself counting gravitons and photons emitted by it. Assume that g represents the number of gravitons per unit area (graviton flux) and E the photons.
   A. What kind of surface would you use to count all the gravitons and photons emitted by it? Write an expression for this area in terms of the distance from the center of the object.
   B. What would be the total number of (i) gravitons, (ii) photons flowing through the area you have in part A? Write these in terms of g, r, E, and any relevant constant.
   C. You are told that the answer you have in part B is proportional to (i) M of the mass and that the proportionality constant is $4\pi G$. Write an expression in terms of g, r, M, $4\pi G$, and any other constants. (ii) Q of the charge and that the proportionality constant is $4\pi k$. Write an expression in terms of E, Q, $4\pi k$.
   D. Obtain (i) g in terms of r, M, G, (ii) E interms of r, Q, k.
   E. Obtain (i) the gravitational force on a mass m in the gravitational field g you obtained above, (ii)the electric force on a charge q in the electric field E you obtained above.
   F. Now you are inside the planet at a distance $r<R$. (i) What ratio of the mass is going to contribute to the gravitational field ? Call this mass m. (ii) What ratio of the charge is going to contribute to the electric field ? Call this charge q.
   G. What kind of surface would you use to count all the gravitons or the photons emitted by the mass m and charge q? Write an expression for this area in terms of the distance from the center of the mass and the charge.
   H. What would the total number of (i) gravitons, (ii) photons be flowing through the area you have? Write these in terms of g, E, r, m and any relevant constant.
   I. You are told that the answer you have above is proportional to (i) m of the mass and that the proportionality constant is $4\pi G$. Obtain an expression in terms of g, r, m , $4\pi G$, and any other constants (ii) q of the charge and that the proportionality constant is $4\pi k$. Obtain an expression in terms of E, r, q , $4\pi G$, and any other constants.
   J. Obtain (i) g in terms of r, m , G, (ii) E in terms of r, q, k.
   K. Obtain (i) the gravitational force on a mass m in the gravitational field g you obtained above (ii) the electric force on a charge q in the electric field E you obtained above

2. You are on planet μεουτ where the locals eliminated all the irrational absolute statements. You obtained its mass $M_ç$ by observing its motion around its star (somehow) and calculated its radius to be $R_ç$.
   A. Obtain the gravitational acceleration $g_ç$ on the surface of the planet in terms of the given quantities and known physical constants.
   B. You see a spring on the ground and, being the nerd you are, decide to obtain its spring constant using the mass M and the ruler of length ℓ in your pocket. Draw a free body diagram that shows how you would attempt to measure the spring constant.
   C. Calculate the spring constant k of the spring in terms of known and calculated quantities.

3.
   A. Obtain the gravitational field of earth on its surface in terms of G, M_e, R_e.
   B. Obtain a numerical value for g given that the radius of Earth is $6.4\times10^6 m$ and its mass is $6.0\times10^{24} kg$, and $G=6.7\times10^{-11} N\frac{m^2}{kg^2}$.

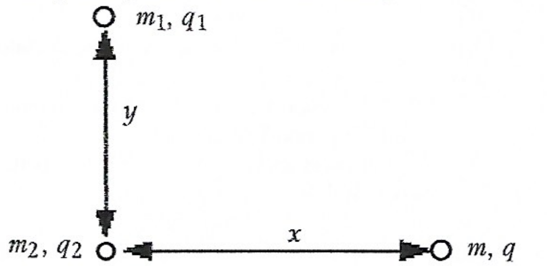
4. Consider a satellite of mass m revolving on a circular orbit of radius r around a planet of mass M where M>>m. The speed of the satellite in this orbit is v.
   A. Make a list of all the given quantities and the relevant physical constants.
   B. What is the magnitude of the centripetal force needed to keep the satellite in this orbit.
   C. What is the direction of this acceleration?
   D. What is the gravitational force exerted on this satellite by the planet.
   E. What is the direction of the gravitational force exerted on this satellite by the planet?
   F. Obtain an expression for the speed of the satellite in terms of G, M, r.
   G. Write an expression for v in terms of the circumference of the orbit and the period T of the motion.
   H. Use your answers above to obtain a relation between r and T in terms of G and M.

5. On planet ÇακιρμβΜ of radius R and mass M distributed uniformly, the locals (with egos larger than the planet itself) travel from one pole to another via a frictionless hole through the center of the planet.
   A. Use Gauss' law to show that the force on an inhabitant of mass m inside the planet is given by $\vec{F}=-G\frac{Mm}{R^3}\vec{r}$ [see the preceding problem] .
   B. Use the answer in part A to write the Newton's second law for the inhabitant.
   C. Use the fact that $\vec{a}=\frac{d^2\vec{r}}{dt^2}$ and $\vec{r}=\vec{r}_o\cos\omega t$ to show that the inhabitant will experience SHM of angular frequency $\omega=\sqrt{\frac{GM}{R^3}}$
   Use $\frac{d}{dt}\sin\omega t=\omega\cos\omega t,\quad \frac{d}{dt}\cos\omega t=-\omega\sin\omega t$

6. What is the magnitude of gravitational force on a satellite in orbit at an altitude of α times the radius of the Earth in terms of G, M_E, R_E, α?
7. How does the mass of a satellite affect the acceleration that it experiences due to the Earth's gravitational force?
8. The magnitude of the gravitational force between the Earth and a satellite depends on what?
9. How does the size of the Moon's gravitational attraction for the Earth compare with the size of the Earth's gravitational attraction for the Moon?
10. Consider a binary star system, a pair of stars bound together by the gravitational attraction between their masses. What does this force depend on?

11. Two members of the opposite sex, of mass m_1 and m_2 respectively, see each other across a room at a party. They are instantly attracted to each other. If the distance between them at this time was r, what was the magnitude of the gravitational force of attraction between them?
12. According to the law of universal gravitation, your mass and the mass of your pen should attract each other. However, if you let go of your pen, it falls down instead of flying toward you. This can be best explained by noting what?
13. What would be the gravitational force upon a test mass placed at the center of the Earth in terms of G, M_E, R_E m [the test mass]?
14. You are an assistant science officer on board the starship Enterprise. Since Spock is incapacitated again, you are called upon to interpret sensor readings that show a planet to have a mass that is α times the Earth mass and a radius that is β times as large as the Earth radius. At its surface, this planet could be expected to have an acceleration due to gravity that is
15. What supplies the centripetal acceleration for a satellite in a circular orbit around the Earth?

Three objects are arranged as shown in the figure below. Assume the masses and charges are point-like masses and charges.



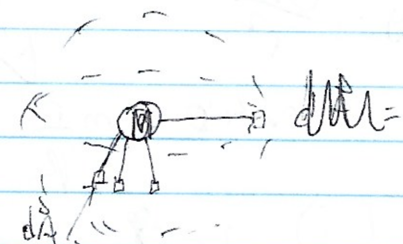
16. (i)What are the x- and y-components of the gravitational *field* due to (a) m_1 and (b) m_2 at the location of m?
(ii)What are the x- and y-components of the electric *field* due to (a) q_1 and (b) q_2 at the location of q?
17. What are the x- and y-components of the gravitational *force* of (a) m_1 and (b) m_2 on m?
(ii)What are the x- and y-components of the electric *force* due to (a) q_1 and (b) q_2 on q?
18. (i) What are the x- and y-components and the direction of the *total* gravitational *field* at the location of m
(ii) What are the x- and y-components and the direction of the *total* electric *field* at the location of q
19. (i) What are the x- and y-components and the direction of the *total* gravitational *force* on m?
(ii) What are the x- and y-components and the direction of the *total* electric *force* on q?
20. (i) A mass m is distributed uniformly in the shape of a circle of radius R. Find the x, y and z-components of the gravitational field established by the ring along the symmetry axis of the ring at a distance z from the plane of the ring. What is the gravitational field at the center of the ring?
(ii) A charge q is distributed uniformly in the shape of a circle of radius R. Find the x, y and z-components of the electric field established by the ring along the symmetry axis of the ring at a distance z from the plane of the ring. What is the electric field at the center of the ring?

Ch. 6, 16 John Yang

black 41
blue 17
red 13
71

Total
exp
71

1) A)



spherical gaussian surface

$\vec{A} = 4\pi r^2$ +1

B) (i) # gravitons $= g \cdot A = 4\pi g r^2$ +1

(ii) # photons $= E \cdot A = 4\pi E r^2$ +1

C) (i) $4\pi G M = 4\pi g r^2 \hat{r}$ +1

(ii) $4\pi k Q = 4\pi E r^2 \hat{r}$ +1

D) (i) $\cancel{4\pi} G M = \cancel{4\pi} g r^2 \hat{r}$ (ii) $\cancel{4\pi} k Q = \cancel{4\pi} E r^2 \hat{r}$

$g = \frac{GM}{r^2}\hat{r}$ ~ signage $\qquad E = \frac{kQ}{r^2}\hat{r}$ ~ signage

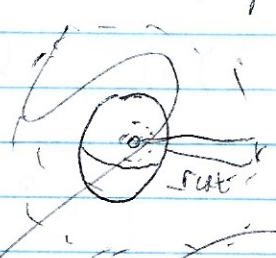
E) (i) $\Sigma \vec{F} = m\vec{a} = \vec{F}_g = m\vec{g}$ (ii) $\Sigma F = F_E = ma$

$F_e = qE$

$F_g = m\left(\frac{GM}{r^2}\right) = \boxed{\frac{GMm}{r^2}}$ +1 signage $\qquad = q\left(\frac{kQ}{r^2}\right)$

$\boxed{F_e = \frac{kQq}{r^2}}$ +1 signage

F)



$g_m = \frac{Gm}{((R-r)+r_{ext})^2} \qquad g_M = \frac{GM}{(R+r_{ext})^2}$

$\frac{g_m}{g_M} = \frac{Gm\,(R+r_{ext})^2}{((R-r)+r_{ext})^2\, GM} = \frac{m}{M}\left(\frac{R^2+2Rr_{ext}+r_{ext}^2}{(R-r)^2+2(R-r)(r_{ext})+(r_{ext})^2}\right)$

$= \frac{m}{M}\left(\frac{R^2+2Rr_{ext}+r_{ext}^2}{R^2-2Rr+r^2+2Rr_{ext}-2rr_{ext}+r_{ext}^2}\right)$

$\frac{m}{M}\left(\frac{R^2+2Rr_{ext}+r_{ext}^2}{(R^2+2Rr_{ext}+r_{ext}^2)+(-2Rr+r^2-2rr_{ext})}\right)$

U

7

F)  little m has its own spherical surface of radius r

(i) $g = \frac{Gm}{r^2}$

ratio: $m\frac{r}{R}$ ~

(ii) $q\frac{r}{R}$ ~

G) a spherical surface where $A = 4\pi r^2$ +1

H) (i) $g \cdot A = 4\pi g r^2$ +1 (ii) $E \cdot A = 4\pi E r^2$ +1

~~I) (i) $4\pi G m = 4\pi g r^2$ (ii) $4\pi k q = 4\pi E r^2$~~

~~J) (i) $g = \frac{GM}{r^2}$ (ii) $E = \frac{kq}{r^2}$~~

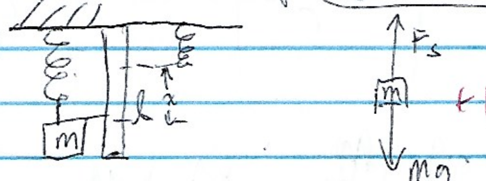
~~K)~~

I) (i) $4\pi G m \frac{r}{R} = 4\pi g r^2$ +1 (ii) $4\pi k q \frac{r}{R} = 4\pi E r^2$ +1

J) (i) $g = \frac{Gmr}{R \cdot r^2} = \frac{Gm}{Rr}$ ~ (ii) $E = \frac{kqr}{R \cdot r^2} = \frac{kq}{Rr}$ ~

K) (i) $F_g = mg = \frac{Gm^2}{Rr}$ ~ (ii) $F_E = qE = \frac{kq^2}{Rr}$ ~

2) A) $g = \frac{GM}{r^2}$ $\boxed{g_c = \frac{GM_c}{R_c^2}}$ +1

B)  +1

$\sum \vec{F} = M\vec{a} = 0$

$\vec{F}_s + M\vec{g}_c = 0$

$F_s \uparrow = M g_c \uparrow$

$k\Delta l = \frac{GM_c M}{R_c^2}$ ~

$\boxed{k = \frac{GM_c M \Delta l}{R_c^2}}$

3) A) $g = \frac{GM}{r^2}$

$\boxed{g_E = \frac{GM_E}{R_E^2}}$ +1

B) $g_E = \frac{(6.67 \times 10^{-11} N\frac{m^2}{kg^2})(6.0 \times 10^{24} kg)}{(6.4 \times 10^{6} m)^2} = \boxed{9.8 \frac{m}{s^2}}$ +1

9

4) A) Mass of planet = M
Mass of satellite = m
Speed of satellite = v
gravitational constant = G
radius of orbit = r +1

B) $F_c = \Sigma F = F_g = ma = mg = \boxed{\frac{mv^2}{r}}$ +1

c) $\hat{r}$ - towards the center of the planet +1

d) $\boxed{F_g = \frac{GMm}{r^2}}$ +1 e) towards the center of the planet +1

f) $\frac{mv^2}{r} = F_g = \frac{GMm}{r^2}$ +1

$v^2 = \frac{GM}{r}$ +1 $\boxed{v = \sqrt{\frac{GM}{r}}}$

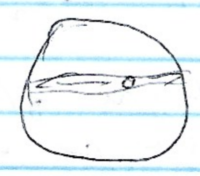
g) $\Delta x = \bar{v} t$
$2\pi r = vT$
$\boxed{v = \frac{2\pi r}{T}}$ +1

h) $\sqrt{\frac{GM}{r}} = \frac{2\pi r}{T}$

$\boxed{\frac{r}{T} = \frac{1}{2\pi}\sqrt{\frac{GM}{r}}}$ ~

5) A) $F_g = \frac{GMm}{r^2}$. from problem 1) F) (i) $\rightarrow m = \frac{mr}{R}$

$\vec{F}_g = \frac{GM\frac{mr}{R}}{R^2} = \frac{GMmr}{R^3}$



if $\hat{r}$ points away from the center,

$\boxed{F_g = -\frac{GMm\hat{r}}{R^3}}$ (no point available)

B) $\Sigma F = ma$

$F_g = ma$ $\quad \boxed{-\frac{GMm\hat{r}}{R^3} = ma}$ +1 $\boxed{a = -\frac{GM\hat{r}}{R^3}}$

9

c) $\vec{a} = \frac{d^2\vec{r}}{dt^2}$ $\vec{r} = \vec{r}_0 \cos\omega t$

~~a=~~ $v = r_0 \omega \sin\omega t$

$a = \cancel{-} r_0 \omega^2 \cos\omega t = \frac{\cancel{r} GM \cancel{r}}{R^3}$

$$\underbrace{\omega^2 \cos\omega t}_{SHM} = \frac{GM}{R^3}$$

$$\boxed{\omega = \sqrt{\frac{GM}{R^3}}}$$

6) $F_g = \frac{GMm}{r^2}$ +1 $\boxed{F_g = \frac{GM_E m}{(\alpha R_E + 1)^2}}$

7) it doesn't +1 since $g = \frac{GM}{R^2}$ +1

8) Mass of Earth, mass of satellite, distance from center of earth

9) It is equal and opposite (newton's 3rd law) +1 +1

10) Mass of each star and their distance +1

11) $F_g = \frac{GMm}{r^2}$ $\boxed{F_g = \frac{Gm_1 m_2}{r^2}}$

12) We are on earth and the magnitude of earth's gravitational force on the pen is much greater than your gravitational attraction to the pen. +1

13) $F_g = \frac{GM_E m}{r^2}$ where $\vec{r} = |R_E|$ away from center

$F_g = \frac{GM_E m}{(R_E - r)^2} = 0$ +1

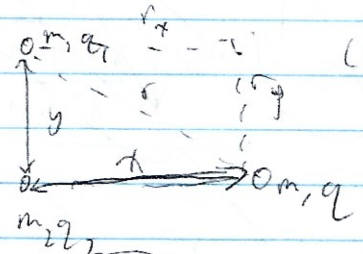
8

14) $g = \frac{GM}{r^2}$ $\quad g_E = \frac{GM_E}{(R_E)^2}$ $\quad g_p = \frac{G(\alpha M_E)}{(\beta R_E)^2}$ +1

$\boxed{g_p = \frac{\alpha}{\beta^2} g_E}$ +1

15) force of gravity +1

16)



(i) a) $\cancel{r_1 = x\hat{i} - y\hat{j}}$

$\cancel{g_1 = \frac{Gm_1}{r^2} = \frac{Gm_1}{x^2}\hat{i} - \frac{Gm_1}{y^2}\hat{j}}$

$\cancel{= \frac{Gm_1}{(x\hat{i} - y\hat{j})^2} = \frac{Gm_1}{x^2\hat{i}\cdot\hat{i} - 2x\hat{i}y\hat{j} + y^2\hat{j}\cdot\hat{j}}}$

$\cancel{g_1 = \frac{Gm_1}{x^2+y^2}}$

b) $\boxed{g_2 = \frac{Gm_2}{x^2}\hat{i}}$ +1

(ii) a) $\cancel{E_1 = \frac{kq_1}{r_1^2}\hat{r} = \frac{kq}{x^2+y^2}}$

b) $\boxed{E_2 = \frac{kq_2}{x^2}}$ +1

(i) a) $r_{1x} = x$, $r_{1y} = -y$

$\boxed{g_{1x} = \frac{Gm_1}{x^2}\hat{i} \quad g_{1y} = -\frac{Gm_1}{y^2}\hat{j}}$ ~

17) (i) a) $F_{g_{1x}} = \frac{Gm_1m}{x^2}\hat{i}$ b) $F_{g_2} = \frac{Gm_2m}{x^2}\hat{i}$ +1

$F_{g_{1y}} = \frac{Gm_1m}{y^2}\hat{j}$ ~

(ii) a) $\boxed{E_{1x} = \frac{kq_1}{x^2}\hat{i} \quad E_{1y} = \frac{kq_1}{y^2}\hat{j}}$ ~

(ii) a) $F_{E_{1x}} = \frac{kq_1q}{x^2}\hat{i}$ b) $F_{E_2} = \frac{kq_1q}{x^2}\hat{i}$ +1

$F_{E_{1y}} = \frac{kq_1q}{y^2}\hat{j}$ ~

7

$\cancel{r_1 = x\hat{i},\ y\hat{j} \quad |r^2| = x^2+y^2}$

18) $\cancel{(i)\quad g_m = g_1 + g_2 = \frac{2Gm_1}{x^2}\hat{i} + \frac{Gm_1}{y^2}\hat{j}}$

$\cancel{|g_m| = \sqrt{\left(\frac{2Gm_1}{x^2}\right)^2 + \left(\frac{Gm_1}{y^2}\right)^2} = \sqrt{\frac{4G^2m_1^2}{x^4} + \frac{G^2m_1^2}{y^4}}}$

$\cancel{= \sqrt{\frac{4y^4G^2m_1^2 + x^4G^2m_1^2}{x^4+y^4}}}$

$\cancel{= \sqrt{\frac{G^2m_1^2(4y^4+x^4)}{x^4+y^4}} = \boxed{Gm_1\sqrt{\frac{x^4+4y^4}{x^4+y^4}}}}$

$\cancel{= \sqrt{G^2m_1^2\left(\frac{4}{x^4}+\frac{1}{y^4}\right)} = \boxed{Gm_1\sqrt{\frac{4}{x^4}+\frac{1}{y^4}}}}$

$\cancel{(ii)\quad E_q = E_1 + E_2 =}$

(1) a) $g_m = g_1 + g_2 = \left(\frac{G(m_1+m_2)}{x^2}\hat{i}\right) - \frac{Gm_1}{y^2}\hat{j}$

$|g_m| = \sqrt{\frac{G^2(m_1+m_2)^2}{x^4} + \frac{G^2m_1^2}{y^4}} = \boxed{G\sqrt{\frac{(m_1+m_2)^2}{x^4} + \frac{m_1^2}{y^4}}}$ ~

b) $E_q = E_1 + E_2 = \sqrt{\frac{k^2(q_1+q_2)^2}{x^4} + \frac{k^2q_1^2}{y^4}}$

$= \boxed{k\sqrt{\frac{(q_1+q_2)^2}{x^4} + \frac{q_1^2}{y^4}}}$ ~

$\theta = \tan^{-1}\left(\frac{E_y}{E_x}\right) = \tan^{-1}\left(\frac{-kq_1}{y^2}\cdot\frac{x^2}{k(q_1+q_2)}\right)$ ~

$= \boxed{\tan^{-1}\left(\frac{-x^2q_1}{y^2(q_1+q_2)}\right)}$

(a)(2)

$\theta = \tan^{-1}\left(\frac{g_y}{g_x}\right) = \tan^{-1}\left(\frac{-Gm_1}{y^2}\cdot\frac{x^2}{G(m_1+m_2)}\right)$

$= \boxed{\tan^{-1}\left(\frac{-x^2m_1}{y^2(m_1+m_2)}\right)}$ ~

1a) (i) $F_g = \sqrt{\left(\frac{Gm(m_1+m_2)}{x^2}\right)^2 + \left(\frac{Gm_1m}{y^2}\right)^2}$

$= \boxed{Gm\sqrt{\frac{(m_1+m_2)^2}{x^4} + \frac{m_1^2}{y^4}}}$ ~

$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{\cancel{Gm}m_1}{y^2} \cdot \frac{-x^2}{\cancel{Gm}(m_1+m_2)}\right)$

$= \boxed{\tan^{-1}\left(\frac{-x^2 m_1}{y^2(m_1+m_2)}\right)}$ ~

(ii) $F_E = \sqrt{\left(\frac{kq(q_1+q_2)}{x^2}\right)^2 + \left(\frac{kq_1q}{y^2}\right)^2}$

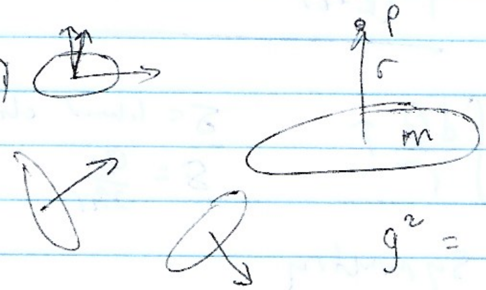
$= \boxed{kq\sqrt{\frac{(q_1+q_2)^2}{x^4} + \frac{q_1^2}{y^4}}}$ ~

$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$

$= \tan^{-1}\left(\frac{kq_1q}{y^2} \cdot \frac{-x^2}{kq(q_1+q_2)}\right)$

$= \boxed{\tan^{-1}\left(\frac{-x^2 q_1}{y^2(q_1+q_2)}\right)}$ ~

2a) (i)



$\boxed{g = \frac{Gm}{r^2}\hat{r}}$ ~~z direction~~

$g^2 = g_x^2 + g_y^2 + g_z^2$

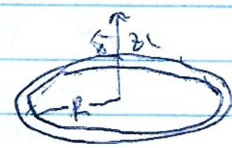
g_{center} $r = 0$ $\therefore g = 0$

(ii) $\boxed{g = \frac{kq}{r^2}\hat{r}}$ $E_{center} = 0$ +1

1

2b) (i) [Ex. 6-12]

$\vec{g} = -G\int \frac{dM}{r^2}\hat{r}$



+1

linear mass density $\lambda = \frac{m}{2\pi R}$

$m = \lambda 2\pi R$

$dm = \lambda ds$

$dg\cos\theta$

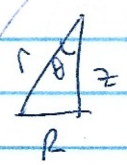
$dg = G\frac{dM}{r^2}\cos\theta(-\hat{z})$

$g = G\int \frac{dM}{r^2}\cos\theta\,-\hat{z}$

1

$= G\int \frac{\lambda ds}{r^2}\cos\theta\,-\hat{z} = \frac{G\lambda\cos\theta}{r^2}\int ds\,-\hat{z}$

$$\vec{g} = -\frac{G\lambda\cos\theta}{r^2} 2\pi R\,\hat{z} \qquad \lambda = \frac{M}{2\pi R}$$



$r^2 = R^2 + z^2$ +1

$$\cos\theta = \frac{z}{r} = \frac{z}{\sqrt{R^2+z^2}}$$

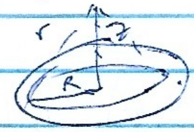
$$\vec{g} = -\frac{GM\cos\theta}{r^2\,\cancel{2\pi R}}\cancel{2\pi R}\,\hat{z}$$

$$= -\frac{GM\cos\theta}{r^2} = \frac{-GMz}{(\sqrt{R^2+z^2})(R^2+z^2)}\hat{z}$$

$$= \boxed{-\frac{GMz}{(R^2+z^2)^{3/2}}\hat{z}}$$ +1

(ii)? ~ ~ ~

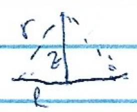
(ii)



$$E = -k\int\frac{dQ}{r^2}\hat{r}$$

δ = linear charge density

$$\delta = \frac{Q}{2\pi r}$$



– symmetry

$$dE = \frac{-k\,d\delta}{r^2}\cos\theta\ \hat{z}$$

$r = \sqrt{R^2+z^2}$ +1

$$E = \int\frac{-k\,d\delta}{r^2}\cos\theta\ \hat{z} \qquad \cos\theta = \frac{z}{r} = \frac{z}{\sqrt{R^2+z^2}}$$

$$= \frac{-k\cos\theta}{2\pi r\cdot r^2}\int d\delta\ \hat{z}$$

$$= \frac{-kQ\cos\theta}{r^2} = \boxed{\frac{-kQz}{(R^2+z^2)^{3/2}}\hat{z}}$$ +1

if $z=0$, $E=0$ +1

1) b) (i) $4\pi GM = 4\pi g r^2$

$$\boxed{g = -\frac{GM\hat{r}}{r^3}}$$ +1

(ii) $4\pi kQ = 4\pi E r^2$

$$\boxed{E = \frac{kQ\vec{r}}{r^3}}$$ +1

7

1) F) $\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \frac{r^3}{R^3} \rightarrow$ divide volumes

(i) $\left(m\frac{r^3}{R^3}\right)$ +1 (ii) $\left(q\frac{r^3}{R^3}\right)$ +1

I)(i) $\cancel{4\pi}\, g r^2 = \cancel{4\pi}\, G \frac{M r^3}{R^3}$

J(i) $g = \frac{GMr^3}{R^3 r^2} = \boxed{\frac{GM\vec{r}}{R^3}}$ +1

I)(ii) $4\pi E r^2 = 4\pi k \frac{q r^3}{R^3}$

J(ii) $\boxed{E = \frac{kq\vec{r}}{R^3}}$ +1

K)(i) $F_g = mg = \boxed{\frac{GMm\vec{r}}{R^3}}$ +1

(ii) $F_e = QE = \boxed{\frac{kQq\vec{r}}{R^3}}$ +1

2) C) $\vec{F}_g + \vec{F}_s = 0$

$mg\downarrow + kx\uparrow = 0$

$kx\uparrow = mg\uparrow$

$k = \boxed{\frac{m}{x}g}$ +1

4) h) from (f)

$v^2 = \frac{GM}{r}$

from (g) $v = \frac{2\pi r}{T}$

$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$

$\boxed{\frac{r^3}{T^2} = \frac{GM}{4\pi^2}}$ +1

8

5) c) $a = \frac{d^2\vec{r}}{dt^2}$ $\vec{r} = \vec{r}_0 \cos\omega t$

$g = \frac{d^2 r}{dt^2} \rightarrow \frac{d^2 r}{dt^2} = \frac{GM_{B}}{R_{B}^2}$

$\frac{d}{dt}(-r\omega \sin\omega t) = \frac{GM}{R^2}$

$-r\omega^2 \sin\omega t = \frac{GM}{R^2}$

$-r\omega^2 = \frac{GM}{R^2}$

$\omega^2 = \frac{GM}{R^3}$

$\boxed{\omega = \sqrt{\frac{-GM}{R^3}}}$ ~

(c) $F_g = \frac{GMm}{r^2}$ B $r = (d+1)r_E$

$\boxed{F_g = \frac{GM_E m}{((d+1)r_E)^2}}$ +1

(6) (i) a) $r = \sqrt{x^2+y^2}$ $r^2 = x^2+y^2$

$g_1 = \frac{GM}{r^2} = \boxed{\frac{GM_1}{x^2+y^2}\hat{r}}$ ~

(ii) a) $E_1 = \frac{kq}{r^2} = \boxed{\frac{kq_1}{x^2+y^2}\hat{r}}$ ~

(7) (i) a) $F_g = mg = \boxed{\frac{GM_1 M}{x^2+y^2}\hat{r}}$ ~

(ii) a) $F_E = qE$

$= \boxed{\frac{kq_1 q}{x^2+y^2}\hat{r}}$ ~

~~18) (i) $g_{total} = g_1 + g_2 =$~~

5) c) $\frac{d^2 r}{dt^2} = \frac{-GM}{R^3}\hat{r}$ $\quad -\omega^2 r = \frac{-GM}{R^3}\hat{r}$

$$\omega^2 = \frac{GM}{R^3}$$

$$\boxed{\omega = \sqrt{\frac{GM}{R^3}}} \quad +1$$

16) (i) a) $g_1 = \frac{Gm_1}{x^2+y^2}(-\hat{i}\cos\theta, \hat{j}\sin\theta) = \frac{Gm_1}{(x^2+y^2)^{3/2}}(-\hat{i}x+\hat{j}y)$ $\quad +1$

(ii) a) $E_1 = \frac{kq_1}{x^2+y^2}(\hat{i}\cos\theta, -\hat{j}\sin\theta) = \frac{kq_1}{(x^2+y^2)^{3/2}}(\hat{i}x-\hat{j}y)$ $\quad +1$

17) (i) a $\rightarrow$ 16)(i)(a) $\cdot m \rightarrow F_g = mg = \frac{Gm_1 m}{(x^2+y^2)^{3/2}}(-\hat{i}x+\hat{j}y)$ $\quad +1$

(ii) a 16)(ii)(a) $\cdot m \rightarrow F_E = qE = \frac{kq_1 q}{(x^2+y^2)^{3/2}}(\hat{i}x-\hat{j}y)$ $\quad +1$

18) $g = g_1 + g_2 = -\hat{i}\left(\frac{Gm_1 x}{(x^2+y^2)^{3/2}} + \frac{Gm_2}{x^2}\right) + \hat{j}\left(\frac{Gm_1 y}{(x^2+y^2)^{3/2}}\right)$ $\quad +1$

$\tan\theta = \frac{g_y}{g_x} = \frac{m_1 y x^2}{-m_1 x^3 - m_2(x^2+y^2)^{3/2}}$ $\quad +1$

$E = E_1 + E_2 = \hat{i}\left(\frac{kq_1 x}{(x^2+y^2)^{3/2}} + \frac{kq_2}{x^2}\right) - \hat{j}\left(\frac{kq_1 y}{(x^2+y^2)^{3/2}}\right)$ $\quad +1$

$\tan\theta = \frac{E_y}{E_x} = \frac{-q_1 y x^2}{q_1 x^3 + q_2(x^2+y^2)^{3/2}}$ $\quad +1$

19) $F_1 + F_2 = -\hat{i}\left(\frac{Gm_1 m x}{(x^2+y^2)^{3/2}} + \frac{Gm_2 m}{x^2}\right) + \hat{j}\left(\frac{Gm_1 m y}{(x^2+y^2)^{3/2}}\right)$ $+1$ $\quad \tan\theta = \frac{m_1 y x^2}{-m_1 x^3 - m_2(x^2+y^2)^{3/2}}$ $+1$

$F_1 + F_2 = \hat{i}\left(\frac{kq_1 q x}{(x^2+y^2)^{3/2}} + \frac{kq_2 q}{x^2}\right) - \hat{j}\left(\frac{kq_1 q y}{(x^2+y^2)^{3/2}}\right)$ $\quad \tan\theta = \frac{-m_1 y x^2}{m_1 x^3 + m_2(x^2+y^2)^{3/2}}$ $+1$

+1

72 exp.

1.

A. Spherical symmetry--a sphere-like shape: $A=4\pi r^2$ *1pt*

B. $C_G M = gA = g4\pi r^2$ *1pt* $C_E Q = EA = E4\pi r^2$ *1pt*

C. $4\pi GM = gA = g4\pi r^2$ *1pt* $C_E Q = EA = E4\pi r^2$ *1pt*

D. $g=\dfrac{4\pi GM}{4\pi r^2}=\dfrac{GM}{r^2},\ \vec{g}=-\dfrac{GM}{r^3}\vec{r}$ *1pt* $E=\dfrac{4\pi kQ}{4\pi r^2}=\dfrac{kQ}{r^2},\ \vec{g}=-\dfrac{GM}{r^3}\vec{r}$ *1pt*

E. $\vec{F}_G = m\vec{g} = -\dfrac{GMm}{r^3}\vec{r}$ *1pt* $\vec{F}_E = q\vec{E} = \dfrac{kQq}{r^3}\vec{r}$ *1pt*

F. $M_r = \dfrac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3}M = \dfrac{r^3}{R^3}M,\ r<R$ *1pt* $Q_r = \dfrac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3}Q = \dfrac{r^3}{R^3}Q,\ r<R$ *1pt*

G. $A_r = 4\pi r^2$ *1pt*

H. $C_G M_r = gA = g4\pi r^2$ *1pt* $C_G Q_r = EA = E4\pi r^2$ *1pt*

I. $4\pi GM_r = 4\pi G\dfrac{r^3}{R^3}M = gA = g4\pi r^2$ *1pt* $4\pi kQ_r = 4\pi k\dfrac{r^3}{R^3}E = EA = E\,4\pi r^2$ *1pt*

J. $g=\dfrac{4\pi Gr^3}{4\pi r^2 R^3}M=\dfrac{GM}{R^3}r,\ \vec{g}=-\dfrac{GM}{R^3}\vec{r},\ \vec{g}=0$ at $\vec{r}=0$ *1pt* $E=\dfrac{4\pi kr^3}{4\pi r^2 R^3}Q=\dfrac{kQ}{R^3}r,\ \vec{E}=\dfrac{kQ}{R^3}\vec{r},\ \vec{E}=0$ at $\vec{r}=0$ *1pt*

K. $\vec{F}_G = m\vec{g} = -\dfrac{GMm}{R^3}\vec{r}$. Notice that $\vec{F}_G=0$ at $\vec{r}=0$ *1pt* $\vec{F}_E = q\vec{E} = \dfrac{kQq}{R^3}\vec{r}$. Notice that $\vec{F}_E=0$ at $\vec{r}=0$ *1pt*

−11 −9

2. A. $g_ç = G\dfrac{M_ç}{R_ç^2}$ *1pt* B. $\begin{matrix}\uparrow -k\vec{x}\\ \downarrow m\vec{g}\end{matrix}$ *1pt* C. $m\vec{g}-k\vec{x}=0,\ k=\dfrac{m}{x}g$ *1pt*

/3

3. A. $g_e = G\dfrac{M_e}{R_e^2}$ *1pt* See Gauss' Law question for the derivation.

A. $g_e = 6.7\times10^{-11}N\dfrac{m^2}{kg^2}\dfrac{6.0\times10^{24}kg}{(6.4\times10^6 m)^2},\ g_e = 9.81\,\dfrac{m}{s^2}$ *1pt*

/2

4. A. M, m, r, g, v *1pt* B. $F_c = ma_c = m\dfrac{v^2}{r}$ *1pt* C. Toward M (radially in) *1pt* D. $F_G = G\dfrac{Mm}{r^2}$ *1pt* E. Toward M (radially in) *1pt* /5

F. $F_c = F_G$ or $m\dfrac{v^2}{r}=G\dfrac{Mm}{r^2}$ *1pt*, $v^2=\dfrac{GM}{r}$ *1pt* G. $v=\dfrac{2\pi r}{T}$ or $v^2=\dfrac{4\pi^2 r^2}{T^2}$ *1pt* H. $\dfrac{4\pi^2 r^2}{T^2}=\dfrac{GM}{r},\ \dfrac{r^3}{T^2}=\dfrac{GM}{4\pi^2}$ *1pt* /4

5. A. See above. B. $\vec{F}_G = m\vec{a},\ -\dfrac{GMm}{R^3}\vec{r} = m\vec{a}$, R is constant (radius of planet) *1pt* C. $\dfrac{d^2\vec{r}}{dt^2}=-\dfrac{GM}{R^3}\vec{r}$ or $-\omega^2\vec{r}=-\dfrac{GM}{R^3}\vec{r}$ *1pt* $\omega^2=\dfrac{GM}{R^3}$ or

$\omega=\sqrt{\dfrac{GM}{R^3}}$ *1pt* Remember $\omega=\dfrac{2\pi}{T}$ which gives $\dfrac{4\pi^2}{T^2}=\dfrac{GM}{R^3}$ which is Keppler's 3rd law. /3

6. $F=G\dfrac{M_E m}{r^2}=G\dfrac{M_E m}{(1+\alpha)^2 R_E^2}=\dfrac{1}{(1+\alpha)^2}\dfrac{GM_E m}{R_E^2}$ *1pt+1pt*

7. Since $a=\dfrac{F}{m}=\dfrac{GM}{r^2}$ *1pt*. The mass of the satellite does not affect its a. *1pt*

8. M, m, r *1pt*

9. The same magnitude, opposite direction. Action-Reaction. *1pt*

10. M and r. *1pt*

11. $F=G\dfrac{m_1 m_2}{r^2}$ *1pt*

12. The mass of the Earth infinitely larger than your mass. *1pt*

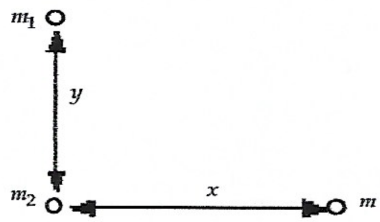
13. 0 *1pt* /10

14. $g=\dfrac{G(\alpha M_E)}{(\beta R_E)^2}=\dfrac{\alpha}{\beta^2}\dfrac{GM_E}{R_E}=\dfrac{\alpha}{\beta^2}g_E$ *1pt+1pt*

15. Gravitational attraction between the satellite and the Earth. *1pt*

/3

16.



$\vec{E}$

(a) $\vec{g}_1 = \dfrac{Gm_1}{x^2+y^2}(-\vec{i}\cos\theta, \vec{j}\sin\theta) = \dfrac{Gm_1}{\left(x^2+y^2\right)^{3/2}}\left[-\vec{i}x+\vec{j}y\right]$ *1pt* $\qquad \vec{E}_1 = \dfrac{kq_1}{x^2+y^2}(\vec{i}\cos\theta, -\vec{j}\sin\theta) = \dfrac{kq_1}{\left(x^2+y^2\right)^{3/2}}\left[\vec{i}x-\vec{j}y\right]$ *1pt*

(b) $\vec{g}_2 = G\dfrac{m_2}{x^2}(-\vec{i})$ *1pt* $\qquad \vec{E}_2 = k\dfrac{q_2}{x^2}(\vec{i})$ *1pt*

17. (a) $\vec{F}_1 = \dfrac{Gm_1 m}{\left(x^2+y^2\right)^{3/2}}\left[-\vec{i}x+\vec{j}y\right]$ *1pt* $\qquad \vec{F}_1 = \dfrac{kq_1 q}{\left(x^2+y^2\right)^{3/2}}\left[\vec{i}x-\vec{j}y\right]$ *1pt*

(b) $\vec{F}_2 = G\dfrac{m_2 m}{x^2}(-\vec{i})$ *1pt* $\qquad \vec{F}_2 = k\dfrac{q_2 q}{x^2}(-\vec{i})$ *1pt*

18. $\vec{g} = \vec{g}_1 + \vec{g}_2 = -\vec{i}\left[\dfrac{Gm_1 x}{\left(x^2+y^2\right)^{3/2}} + G\dfrac{m_2}{x^2}\right] + \vec{j}\left[\dfrac{Gm_1 y}{\left(x^2+y^2\right)^{3/2}}\right]$ *1pt* $\qquad \vec{E} = \vec{E}_1 + \vec{E}_2 = \vec{i}\left[\dfrac{kq_1 x}{\left(x^2+y^2\right)^{3/2}} + \dfrac{kq_2}{x^2}\right] - \vec{j}\left[\dfrac{kq_1 y}{\left(x^2+y^2\right)^{3/2}}\right]$ *1pt*

$\tan\theta = \dfrac{g_y}{g_x} = \dfrac{m_1 y x^2}{-m_1 x^3 - m_2\left(x^2+y^2\right)^{3/2}}$ *1pt* $\qquad \tan\theta = \dfrac{E_y}{E_x} = \dfrac{-q_1 y x^2}{q_1 x^3 + q_2\left(x^2+y^2\right)^{3/2}}$ *1pt*

19. $\vec{F}_1 + \vec{F}_2 = -\vec{i}\left[\dfrac{Gm_1 m x}{\left(x^2+y^2\right)^{3/2}} + G\dfrac{m_2 m}{x^2}\right] + \vec{j}\left[\dfrac{Gm_1 m y}{\left(x^2+y^2\right)^{3/2}}\right]$ *1pt* $\qquad \vec{F}_1 + \vec{F}_2 = \vec{i}\left[\dfrac{kq_1 q x}{\left(x^2+y^2\right)^{3/2}} + \dfrac{kq_2 q}{x^2}\right] - \vec{j}\left[\dfrac{kq_1 q y}{\left(x^2+y^2\right)^{3/2}}\right]$ *1pt*

$\tan\theta = \dfrac{m_1 y x^2}{-m_1 x^3 - m_2\left(x^2+y^2\right)^{3/2}}$ *1pt* /8 $\qquad \tan\theta = \dfrac{-m_1 y x^2}{m_1 x^3 + m_2\left(x^2+y^2\right)^{3/2}}$ *1pt* /8

20. (i) All the mass is at a distance $\sqrt{R^2+z^2}$ along the z-axis. *1pt* Only the z-component survives due to symmetry; everything in the x-y plane cancels in pairs, i.e. $g\cos\theta = -\dfrac{Gm}{r^2}\dfrac{z}{r} = \dfrac{-Gmz}{\left(R^2+z^2\right)^{3/2}}$ along -z *1 pt*

At the center, $\theta = \dfrac{\pi}{2}$ and g=0 *1 pt*

(ii) All the charge is at a distance $\sqrt{R^2+z^2}$ along the z-axis *1 pt.* Only the z-component survives due to symmetry; everything in the x-y plane cancels in pairs, i.e. $E\cos\theta = \dfrac{kq}{r^2}\dfrac{z}{r} = \dfrac{kqz}{\left(R^2+z^2\right)^{3/2}}$ along -z *1 pt*

At the center, $\theta = \dfrac{\pi}{2}$ and E=0 *1 pt*

/6