Ch. 10

FR 1) a) $\lambda = \frac{M}{L}$ b) MC 1) units

$M = \lambda L$ $m = \boxed{\lambda\, dx}$ (d)

c) MC 2)

$I = mr^2 = \lambda dx \cdot x^2 = x^2 \lambda dx$ (D)

d) MC 3) $I = \int_{-a}^{b} = \int_{-a}^{b} x^2 \lambda dx$ (d)

e) MC 4) $I = \int_{-a}^{b} x^2 \lambda dx = \lambda \int_{-a}^{b} x^2 dx = \frac{\lambda x^3}{3}\Big|_{-a}^{b}$

MC 5) $\lambda = \frac{m}{L}$ (c)

$= \boxed{\frac{\lambda}{3}(b^3 + a^3)}$ (e)

FR 2) MC 6) (D) when $\Sigma F \Delta t \to 0$ and $\tau_{net} \Delta t \to 0$

FR 3) a)

a) I is additive;

$I = \frac{1}{2} m_1 r_1^2 + \frac{1}{2} m_2 r_2^2$

b)

c) $\boxed{mg}$?

$\Sigma F = ma$

$mg - T = ma$

$T = m(g - a)$

d) $\Sigma \tau = I\alpha$

$r_1 T = I\alpha$

$\Sigma \tau = m r_1 (g - a)$

e) $m_3 g h = \frac{1}{2} m_3 v_f^2 + \frac{1}{2} I \omega_f^2$

$v_f = r_1 \omega$

$= \frac{1}{2}\left(m_3 v_f^2 + \frac{1}{2}(m_1 r_1^2 + m_2 r_2^2)\frac{v_f^2}{r_1^2}\right) = \frac{v_f^2}{2}\left(m_3 + \frac{m_1}{2} + \frac{m_2 r_2^2}{2 r_1^2}\right) = m_3 g h$

$$\boxed{v_f = \sqrt{\frac{2 m_3 g h}{m_3 + \frac{m_1}{2} + \frac{m_2 r_2^2}{r_1^2}}}}$$

f) $\omega_f = \frac{v_f}{r_2}$, so $v_f = \sqrt{\frac{2m_3gh}{m_3 + \frac{m_1 r_1^2}{r_2^2} + \frac{m_2}{2}}}$

g) $\sum \tau = I\alpha$

$m_3(g-a)r_1 = I\alpha$ $\quad \alpha = \frac{2m_3(g-a)}{m_1r_1^2 + m_2r_2^2}$ $\quad a = r_1\alpha$

$= \frac{1}{2}(m_1r_1^2 + m_2r_2^2)\alpha$ $\quad \alpha = \frac{a}{r_1}$

$m_3gr_1 - m_3r_1^2\alpha = I\alpha$

$\alpha(I + m_3r_1^2) = m_3gr_1$

$\sum F = ma$

$m_3g - T = m_3a$

$a = g - \frac{T}{m_3}$

$m_3gr_1 - m_3ar_1 = \frac{1}{2}(m_1r_1^2 + m_2r_2^2)\alpha$

$m_3ar_1 = m_3gr_1 - \frac{\alpha}{2}(m_1r_1^2 + m_2r_2^2)$

$\boxed{a = g - \frac{\alpha(m_1r_1^2 + m_2r_2^2)}{2m_3r_1}}$ $\quad \alpha = \frac{a}{r_1}$

$\alpha = \boxed{\frac{m_3gr_1}{\left(\frac{1}{2}m_1r_1^2 + \frac{1}{2}m_2r_2^2 + m_3r_1^2\right)}}$

h) ~~$\sum F = ma$~~ $\quad a = r_1\alpha = \boxed{\frac{m_3gr_1^2}{\left(\frac{1}{2}m_1r_1^2 + \frac{1}{2}m_2r_2^2 + m_3r_1^2\right)}}$

I) $a = r_2\alpha$; $\alpha = \frac{a}{r_2}$, so the numerator will have m_3gr_2 & $m_3gr_2^2$ & the denominator will have ~~$m_3gr_2^2$~~ $m_3r_1^2$

J) $a = r_2\alpha$

$= \frac{m_3gr_2^2}{\left(\frac{1}{2}m_1r_1^2 + \frac{1}{2}m_2r_2^2 + m_3r_2^2\right)}$ $\quad v_f^2 = \cancel{v_0^2} + 2a\Delta h$

$v_f = \sqrt{\frac{2m_3gr_2^2h}{\frac{1}{2}m_1r_1^2 + \frac{1}{2}m_2r_2^2 + m_3r_2^2}}$

MC 7) $I = \frac{1}{2}(m_1 r_1^2 + m_2 r_2^2) = \frac{1}{2}((0.8\,kg)(0.04\,m)^2 + (1.6\,kg)(0.08\,m)^2)$

$= \boxed{5.76 \times 10^{-3}\ kg \cdot m^2}$ (a)

MC 8)

MC 9) $\sum \tau = m_3 r_1 (g - a) = m_3 r_1 g - \frac{m_3 r_1 (m_3 r_1^2 g)}{(I + m_3 r_1^2)}$

$= (1.5\,kg)(0.04\,m)(9.8\,m/s^2) - \frac{(1.5\,kg)^2 (0.04\,m)^3 (9.8\,m/s^2)}{(5.76 \times 10^{-3}\,kg \cdot m^2 + (1.5\,kg)(0.04\,m)^2)}$

$= 0.42\ N \cdot m$?

MC FR 9B)

$v_f = \sqrt{\frac{2(1.5\,kg)(10\,m/s^2)(2.0\,m)}{(1.5\,kg) + \frac{0.8\,kg}{2} + \frac{(1.6\,kg)(0.08\,m)^2}{(0.04\,m)^2}}}$

$= \boxed{2.7\ m/s}$

MC FR 9C) $v_f = \sqrt{\frac{2(1.5\,kg)(10\,m/s^2)(2.0\,m)}{1.5\,kg + \frac{(0.8\,kg)(0.04\,m)^2}{(0.08\,m)^2} + \frac{1.6\,kg}{2}}} = \boxed{4.9\ m/s}$ g

MC 10) from FR 3)g), $\alpha = \frac{m g r_1}{I + m r_1^2}$ (b)

MC 11) $a = \alpha r_1$ (e)

MC 12) (b) MC 13) (c)

FR 4) / MC 14) a) radially in & tangent (e)

b) MC 15) radially in (c)

c) ω_{max}

MC 16)

$\sum F = m a_c$

$\mu m g = \frac{m v^2}{R}$

$v^2 = \frac{\mu g R}{\cancel{R}}$

$v = \sqrt{\mu g R}$

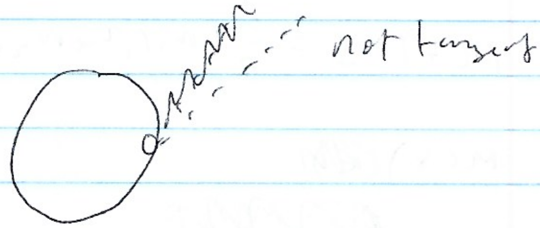
$\frac{m/s^2}{m}$

$v = R\omega$

$\omega = \frac{v}{R}$

$\omega = \frac{v}{r} = \sqrt{\frac{\mu g}{R}}$ (d)

MC FRQ 16 / FR 4) D)



MC 17 / FR 4) E

$L = I\omega_{max}$ $\cancel{= \left(\frac{1}{2}MR^2 + MR^2\right)\omega_{max}}$

$= \frac{1}{2}MR^2\omega_{max}$ (b)

MC 18 / FR 4) F

$L = \vec{r} \times \vec{p} = \cancel{R}\, m v_{max}$ $\qquad \omega = \frac{v}{R} \qquad v = r\omega_{max}$

$= \cancel{\boxed{mR^2\omega_{max}}}$

$\boxed{mr^2\omega_{max}}$ (A)

MC 19 / FR 4G

(C) Yes b/c $\sum \tau \Delta t \to 0$

4H) $\omega = N\frac{rev}{s} \times \frac{2\pi\, rad}{1\, rev} = 2\pi N$

$v_{cm} = r\omega = \cancel{2\pi r N}$

$\Delta x = v_{cm} t = \boxed{2\pi r N t}$ $\quad (120\pi r N)$

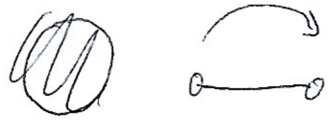
MC 20) $\Delta x = 2\pi(2m)(30)(40s) = 7200\pi m$ (D)

FR 5/~~MC 22~~ =

a) $I = \frac{1}{12}m_3 l^2 + (m_1 + m_2)\left(\frac{l}{2}\right)^2 = \boxed{\frac{1}{12}m_3 l^2 + \frac{1}{4}(m_1 + m_2) l^2}$

MC 21) $I = \frac{1}{12}ml^2 + \frac{1}{4}(2m)l^2 = \frac{1}{2}ml^2 + \frac{1}{12}ml^2$

$= \frac{7}{12}ml^2$ (d)



FR 5b) $L = I\omega_0$ clockwise (into the page)

MC 22) (b)

FR 5c) $I \rightarrow$ becomes smaller ; L is conserved

MC 23) (c)

FR 5d) ω is faster   MC 24) (A)

since I is smaller and L is conserved

FR 5e)

$$L_0 = L_f$$

$$\left(\frac{1}{12} m_3 l^2 + \frac{1}{4}(m_1+m_2)l^2\right)\omega_0 = \left(\frac{1}{12} m_3 l^2 + \frac{1}{16}(m_1+m_2)l^2\right)\omega_f \qquad \left[\frac{l}{4}\right]^2$$

$$\cancel{l^2}\left(\frac{1}{12} m_3 + \frac{3}{12}(m_1+m_2)\right)\omega_0 = \left(\frac{4}{48} m_3 \cancel{l^2} + \frac{3}{48}(m_1+m_2)\cancel{l^2}\right)\omega_f$$

$$\frac{m_3 + 3m_1 + 3m_2}{12}\,\omega_0 = \left(\frac{4m_3 + 3m_1 + 3m_2}{48}\right)\omega_f$$

$$\omega_f = \frac{48(m_3 + 3m_1 + 3m_2)}{12(4m_3 + 3m_1 + 3m_2)}\omega_0 = \frac{4(m_3 + 3m_1 + 3m_2)}{4m_3 + 3m_1 + 3m_2}\omega_0$$

MC 25) $I_0 = \frac{7}{12} ml^2$

$$I_f = \frac{1}{12} ml^2 + 2m\left(\frac{1}{16}\right)l^2 = \frac{1}{12} ml^2 + \frac{1}{8} ml^2$$

$$= \frac{5}{24} ml^2$$

$$I_0\omega_0 = I_f\omega_f$$

$$\omega_f = \frac{I_0}{I_f}\omega_0 = \frac{7\cancel{ml^2}}{\cancel{12}} \times \frac{\cancel{24}^{\,2}}{5\cancel{ml^2}}$$

$$= \frac{14}{5}\omega_0 \quad (b)$$

FR SF / MC FRQ 25

$$\Delta KE = KE_f - KE_0 = \frac{1}{2} I_f \omega_f^2 - \frac{1}{2} I_0 \omega_0^2$$

$$= \frac{1}{2}\left(\frac{\cancel{4m_3+3m_1+3m_2}}{48}\right)\left(\frac{16(m_3+3m_1+3m_2)}{(4m_3+3m_1+3m_2)^{\cancel{2}}}\omega_0^2\right)$$

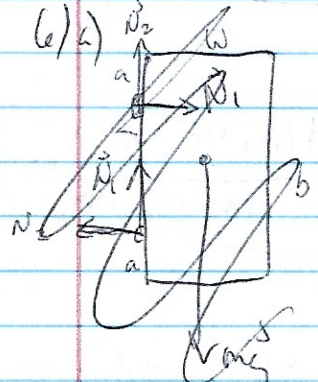
$$-\frac{1}{2}\left(\frac{m_3+3m_1+3m_2}{12}\right)\omega_0^2$$

$$= \frac{\omega_0^2}{6}\frac{(m_3+3m_1+3m_2)}{(4m_3+3m_1+3m_2)} - \frac{\omega_0^2}{24}(m_3+3m_1+3m_2)$$

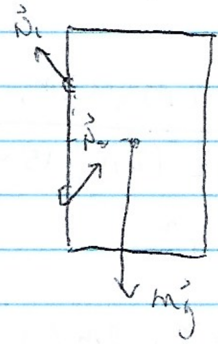
$$= \boxed{\frac{\omega_0^2(m_3+3m_1+3m_2)}{6}\left(\frac{1}{4m_3+3m_1+3m_2} - \frac{1}{4}\right)}$$

KE comes from change in moment of inertia?

6) a)



~~b)~~ a)



b) $\Sigma F = ma = 0$

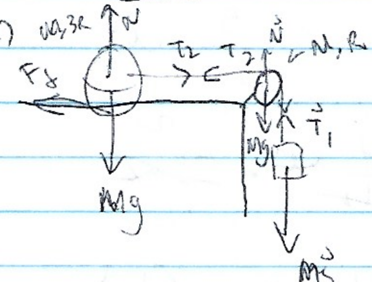
$\vec{N}_1 + \vec{N}_2 + m\vec{g} = 0$

c) $\Sigma \tau = I\alpha = 0$

$\frac{w}{2} m\vec{g} + \left(\frac{h}{2} - a\right)\vec{N}_1 + \left(\frac{h}{2} - a\right)\vec{N}_2 = 0$

d) $\boxed{6D}$

7) a)



b) sphere $\rightarrow$ $\Sigma \tau = I\alpha$

$\mu Mg = I\alpha = \frac{2}{5} M (3R)^2 \alpha$

$\mu \cancel{M} g = \frac{18}{5} \cancel{M} R^2 \alpha$

$\mu g = \frac{18}{5} R^2 \alpha$

$\Sigma F = Ma$

$T_2 - F_f = Ma$

$T_2 - \mu Mg = Ma$

$T_2 = \mu Mg + Ma$

$+\circlearrowright$

7b) cont'd

pulley - $\Sigma\tau = I\alpha$

$T_1 R - T_2 R = I\alpha$

$= \frac{1}{2} MR^2 \alpha$

block - $\Sigma F = ma$

$Mg - T_1 = Ma$

c) $a = g - \frac{T_1}{M}$

$T_1 R - T_2 R = \frac{1}{2} MR^2 \alpha = \frac{1}{2} MRa$

$T_1 \cancel{R} = T_2 \cancel{R} + \frac{1}{2} M\cancel{R}a$

$T_1 = T_2 + \frac{1}{2} Ma$

$T_2 = \mu Mg + Ma$

$T_1 = \mu Mg + \frac{3}{2} Ma$

$a = g - \mu g - \frac{3}{2} a$

$\frac{5}{2} a = g - \mu g = g(1-\mu)$

$\boxed{a = \frac{2}{5} g(1-\mu)}$

d) sphere $\Rightarrow$ $\mu g = \frac{18}{5} R^2 \alpha$

$\alpha = \frac{5\mu g}{18R^2}$

~~$\alpha = \frac{a}{R} = \frac{2g(1-\mu)}{5R}$~~

e) $T_1 = \mu Mg + \frac{3}{2} Ma$

$= \mu Mg + \frac{3}{\cancel{2}} M \frac{\cancel{2}}{5} g(1-\mu)$

$= \mu Mg + \frac{3}{5} Mg(1-\mu) = Mg\left(\mu + \frac{3}{5} - \frac{3}{5}\mu\right)$

$= Mg\left(\frac{2}{5}\mu + \frac{3}{5}\right)$

$\boxed{= \frac{1}{5} Mg(2\mu + 3)}$

$T_2 = \mu Mg + Ma = \mu Mg + \frac{2}{5} Mg(1-\mu)$

$= Mg\left(\mu + \frac{2}{5} - \frac{2}{5}\mu\right)$

$\boxed{= \frac{1}{5} Mg(3\mu + 2)}$

8) a) Linear momentum is const b/c $\Sigma F \Delta t \to 0$

b) No external forces doing work

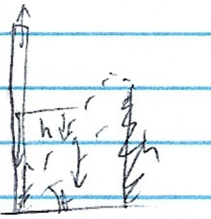
c) $m v_0 = M v_f + m v_b$

d) $mgh_0 = \frac{1}{2} m v_b^2 + \frac{1}{2} I \omega^2$

$= \frac{1}{2} m v_b^2 + \frac{1}{2} \cdot \frac{2}{5} m r^2 \cdot \frac{v_b^2}{r^2}$

$\boxed{mgh_0 = \frac{7}{10} m v_b^2}$

e) $\boxed{8e}$

9) 

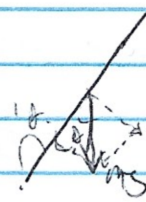
A) $PE_g = mg(L/2 - h)$

$KE_r = \frac{1}{2} I \omega^2 = \frac{1}{6} mL^2 \omega^2$

$E_{total} = mg(\frac{L}{2} - h) + \frac{1}{6} mL^2 \omega^2$

+

↓+



$\Sigma \tau = I\alpha$

$mg \sin\theta \frac{L}{2} = I\alpha$

$= \frac{1}{3} mL^2 \alpha$

$\alpha = \frac{3g\sin\theta}{2L}$

$\omega_f^2 = \cancel{\omega_0^2} + 2\alpha\theta$

$\boxed{\omega_f = \sqrt{\frac{3g\sin\theta \cdot \theta}{L}}}$

9b) ~~$a = r\alpha = \frac{3}{2} g \sin\theta$~~

$$\boxed{\alpha = \frac{3 g \sin\theta}{2L}}$$ (from a)

c) $a = r\alpha = \boxed{\frac{3}{2} g \sin\theta}$

d) $a > g$ when $\sin\theta > \frac{2}{3}$

$$\theta > 42^\circ$$

e) what happens to buildings, trees and things when they topple over? The tops accelerate faster than g?! Sure

10) a)

b) $E_0 = PE_g = mgh_0$

$E_f = mg2R + \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I \omega^2$

$= mg2R + \frac{1}{2} m v_{cm}^2 + \frac{1}{2} \cdot \frac{2}{5} m r^2 \cdot \frac{v_{cm}^2}{r^2}$

$= mg2R + \frac{7}{10} m v_{cm}^2$

c) ~~$mgh_0 = mg2R$~~

$\sum F_{up} = ma = \frac{m v_{cm}^2}{R}$

$mg = \frac{m v_{cm}^2}{R}$

$v_{cm}^2 = gR$

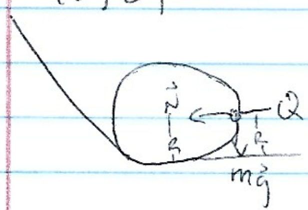
$mgh_0 = mg2R + \frac{7}{10} m v_{cm}^2$

$mgh_0 = mg2R + \frac{7}{10} m (gR)$

$h_0 = 2R + \frac{7}{10} R$

$$\boxed{h_0 = \frac{27}{10} R}$$

10) d)



$$\cancel{m}g6R = \cancel{m}gR + \frac{7}{10}\cancel{m}v_{cm}^2$$

$$5gR = \frac{7}{10}v_{cm}^2$$

$$v_{cm}^2 = \frac{50}{7}gR$$

$$F_c = \frac{mv_{cm}^2}{R} = \frac{50}{7}gR\cdot\frac{m}{R} = \boxed{\frac{50}{7}mg}$$

11) frozen juice rolls faster.

Consider that → both m is the same, r is the same, geometry of the can is the same

Liquid - stuff on the inside sloshes around & dissipates KE

also, liquid juice has some air inside; mass continuously moving to the bottom of the can as it rolls; increases its moment of inertia

6d, 8e-j