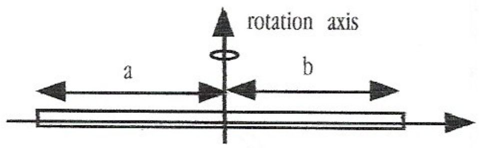
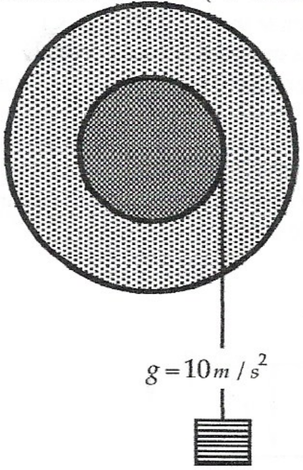
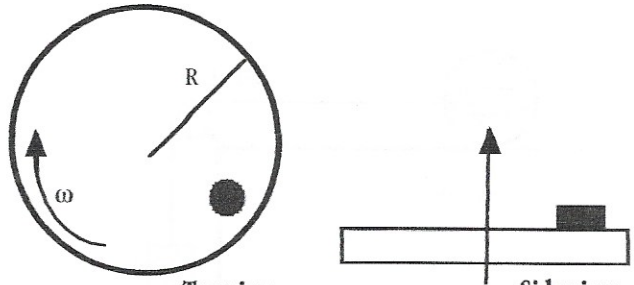


1. In the next few questions, you will be guided through the derivation of the moment of inertia of a uniform stick of mass M, the line density λ and length L about an axis perpendicular to the meter stick at a distance a from one end of the stick and a distance b from the other end as shown in the figure.
   A. What is the value of the line density λ in terms of M and L?
   B. What is the mass of a strip of thickness dx at a distance x from the axis of rotation?
   C. What is the moment of inertia of the strip around the axis?
   D. What is the total moment of inertia of the whole stick around the axis of rotation at a distance a from one end and b from the other?
   E. Integrate your answer above and obtain the result.
2. Under what conditions can the conservation of *angular* momentum be used? ($\vec{T}$ means torque.)



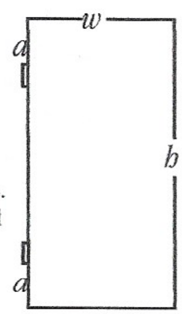
3. Two metal disks, one with radius r_1 cm and mass m_1 and the other with radius r_2 and mass m_2 are welded together and mounted on a frictionless axis through their common center.
   A. What is the moment of inertia of the welded disks?
   B. A light string is wrapped around the edge of the smaller disk in the previous problem, and a mas m_3 is suspended from the free end of the string. Draw a FBD for the welded disks and for the hanging mass.
   C. What is the net torque on the system including the hanging mass?
   D. What is the magnitude of the torque exerted on the welded disks?
   E. The hanging mass is h above ground. Use conservation of energy to calculate its impact speed the moment before it hits the ground.
   F. How do your answers change if the string is wrapped around the bigger disk?
   G. What is the angular acceleration α of the welded disks? The string is wrapped around the smaller disk.
   H. What is the linear acceleration a of the hanging mass? Give your answer in terms of the stated quantities in the previous question.
   I. How will the acceleration of the mass in the previous question change if the string is wrapped around the larger disk?
   J. How does the impact speed (with the ground) of the mass in the previous question change if the string is wrapped around the larger disk? Use kinematics.

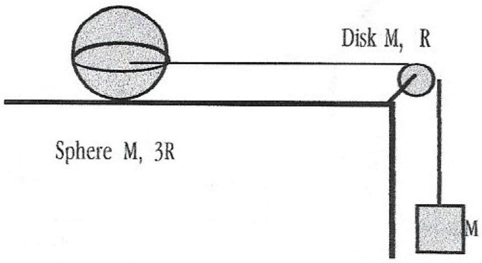


Top view **Side view**

4. A coin of mass m is placed a distance r from the center of a disk rotating at a constant angular speed. The mass and radius of the disk are M and R. The coefficient of static friction is μ.
   A. What is the direction of the force on the coin?
   B. The rotational speed ω is slowly increased to a value ω_{max} at which time the coin just flies off the disk. What is the direction of the net force on the coin while it is in contact with the disk?
   C. Find ω_{max} , the angular velocity of the coin right before it flies off the disk, in terms of m, M, R, g, and μ.
   D. Sketch the path of the coin as it flies off the disk.
   E. Calculate the angular momentum of the disk in the previous question just before the coin leaves the disk.
   F. What is the angular momentum of the coin just before the coin leaves the disk.
   G. Is the total angular momentum of the disk-coin system conserved?
   H. A bycle wheel of radius r completes N revolutions per seconds. Assuming rolling without slipping, how far does the bike travle in one minute?
5. Two children, mass m_1 and m_2, sit on opposite ends of a thin rod with length ℓ and mass m_3. The rod is pivoted at its center and is free to rotate in a horizontal circle without friction.
   A. What is the moment of inertia of the rod and the girls about a vertical axis through the center of the rod?
   B. What is the angular momentum of the system if it is rotating with an angular speed ω_o in a clockwise direction as seen from above?
   C. While the system is rotating the girls pull themselves toward the center of the rod until they are half as far from the center as before. How does the resulting angular momentum compare to the origional one?
   D. How does the resulting angular speed compare to the origional one?
   E. What is the resulting angular speed in terms of ω_o?
   F. What is the change in kinetic energy of the system due to the girls changing their position? Where did the change in KE come from or go to?

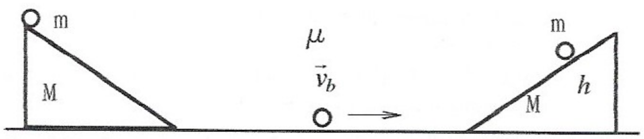
6. Consider a door of mass m, height h, width w being held upright with two hinges at points a away from the bottom and the top of the door as shown in the figure.
   A. Draw a FBD for the door.
   B. Write down the Newton's laws for the forces.
   C. Write down the Newton's laws for the torques.
   D. Solve the relevant equations from parts B and C to obtain all the forces in terms of the given quantities and known physical constants.





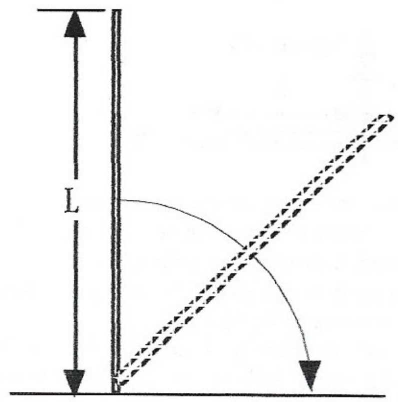
7. A uniform solid sphere of mass M and radius 3R rests on a horizontal table. A string is attached to a frictionless axle that goes through the center of the sphere. The sphere is free to rotate about the axle. The string runs over a pulley, in the shape of a uniform disk, that has mass M and radius R. The pulley is free to rotate on a frictionless axle through its center. A mass M is attached to the string and the whole system moves without slipping. The system is released from rest.

A. Draw a free body diagram for each mass and label all forces
B. Write down Newton's second law for the rotational dynamics for each object
C. What is the magnitude of the linear acceleration?
D. What is the magnitude of the angular acceleration?
E. What is the tension in each part of the string?



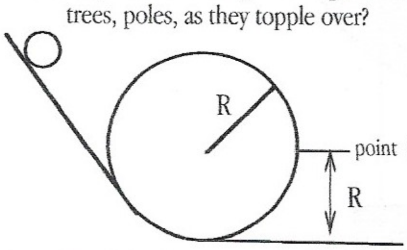
8. Consider the set up above where a spherical ball ralls without slipping but there is no friction between the triangular blocks and the surface they stand on. The spherical ball of radius r and mass m is at a height h_o on the ramp--both at rest and the block is free to slide. The ball moves down on the ramp and reaches $\vec{v}_b$ at the bottom of the ramp. In terms of given quantities and known constants, answer the questions below.

A. Why and what part of total linear momentum is conserved during this situation?
B. Why is total energy conserved during this situation?
C. Write a conservation of linear momentum expression between the initial and the final state.
D. Write a conservation of energy expression between the initial and the final state (include both linear and rotational).
E. Eliminate the unknowns to obtain an expression between $\vec{v}_b$ and h_o in terms of given quantities and known constants.
F. What will be the relative velocity of the block and the ball when the ball reaches the maximum height up on the right-ramp?
G. Write a conservation of linear momentum expression between the initial and the final state.
H. Write a conservation of energy expression between the initial and the final state (include both linear and rotational).
I. Obtain an expression between $\vec{v}_b$ & h_o
J. Finally, eliminate $\vec{v}_b$ betweento obtain a relation between h_o and h in terms of the given quantities and known constants.



9. A long, uniform rod of length L is balanced vertically on one end, which rests on a rough horizontal surface. After a moment the rod begins to fall, rotating around its bottom end that remains where it was without slipping. The rod makes an angle θ with the vertical as it falls.

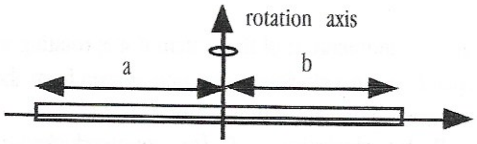
A. When the center of mass has fallen a vertical distance h, write down PE, KE, E_T, obtain ω in terms of g, L and θ.
B. Express the radial acceleration at the top of the rod as a function of θ, where θ is measured from the vertical.
C. Express the tangential acceleration at the top of the rod as a function of θ.
D. Can the resultant linear acceleration exceed g? For what angles θ does this occur?
E. Does your answer in part D explain what happens to buildings, trees, poles, as they topple over?



10. A small marble of mass m and radius r rolls without slipping along the track shown. The marble is released from rest somewhere along the straight section.

A. Draw a free body diagram for the marble at the top of the loop.
B. Calculate the energy of the marble initially before it is released and at the top of the loop. (Consider both translational & rotational energy).
C. From what minimum height above the bottom of the track must the marble be released in order that it just stays on the track at the top of the loop? (That is, the marble stays on the circular path without any aid from the track.) Assume that R>>r.
D. If the marble is released from a height of 6R above the bottom of the track, what is the horizontal component of the force acting on it at point Q?

11. Which will roll down an incline faster, a can of regular fruit juice or a can of frozen fruit juice? ***Explain your answer!*** Consider rotatioal kinematics, dynamics, and the moment of inertia for the liquid and the solid states.



1.
In the next few questions, you will be guided through the derivation of the moment of inertia of a uniform stick of mass M, the line density λ and length L about an axis perpendicular to the meter stick at a distance a from one end of the stick and a distance b from the other end as shown in the figure.
What is the mass of a strip of thickness dx at a distance x from the axis of rotation?
A. $x^3\lambda dx$ B. $x^2\lambda dx$ C. $x\lambda dx$ D. λdx E. λ

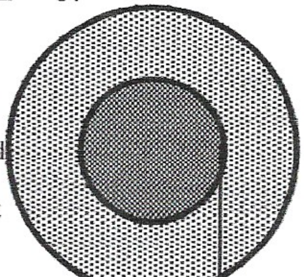
2. What is the moment of inertia of the strip around the axis?
A. λ B. λdx C. $x\lambda dx$ D. $x^2\lambda dx$ E.

3. What is the total moment of inertia of the whole stick around the axis of rotation at a distance a from one end and b from the other?
A. $\int_a^b \lambda$ B. $\int_{-a}^b \lambda dx$ C. $\int_a^b x\lambda dx$ D. $\int_{-a}^b x^2\lambda dx$ E.

4. Integrate your answer above and obtain the result.
A. $\lambda(b+a)$ B. $\frac{1}{2}\lambda(b^2-a^2)$ C. $\frac{1}{2}\lambda(b^2+a^2)$
D. $\frac{1}{3}\lambda(b^3-a^3)$ E. $\frac{1}{3}\lambda(b^3+a^3)$ F.

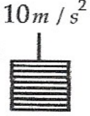
5. What is the value of the line density λ in terms of M and L?
A. ML B. $\frac{L}{M}$ C. $\frac{M}{L}$ D. $\frac{M}{L^2}$ E.

6. Under what conditions can the conservation of *angular* momentum be used? ($\vec{T}$ means torque.)
A. Only when $\vec{F}_{net}$ and $\vec{T}_{net}$ are zero.
B. Only when the impact of rotation time Δt is zero.
C. Only when $\vec{F}_{net}$, $\vec{T}_{net}$, and Δt are zero.
D. Only when $\vec{F}_{net}\Delta t \to 0$ and $\vec{T}_{net}\Delta t \to 0$.
E. When $\vec{T}_{net}\Delta t \to 0$
F.



7. Two metal disks, one with radius 4.0 cm and mass 0.8 kg and the other with radius 8.0 cm and mass 1.6 kg are welded together and mounted on a frictionless axis through their common center. What is the moment of inertia of the welded disks?
A. $5.76\times10^{-3}\,kg.m^2$ B. $11.52\times10^{-3}\,kg.m^2$
C. $6.4\times10^{-4}\,kg.m^2$ D. $5.12\times10^{-3}\,kg.m^2$
E.

$g = 10\,m/s^2$



8. A light string is wrapped around the edge of the smaller disk in the previous problem, and a 1.5 kg block is suspended from the free end of the string. What is the net torque on the system including the hanging mass?
A. $0.6Nm$ B. $0.9Nm$ C. $1.2Nm$ D. $1.8Nm$ E.

9. What is the magnitude of the torque exerted on the welded disks?
A. $0.6Nm$ B. $0.9Nm$ C. $1.2Nm$ D. $1.8Nm$ E.

FRQ 9A Draw a FBD for the welded disks and for the hanging mass.
FRQ 9B The hanging mass is 2.0 m above ground. Use conservation of energy to calculate its impact speed the moment before it hits the ground.
FRQ 9C How does your answer change if the string is wrapped around the bigger disk?

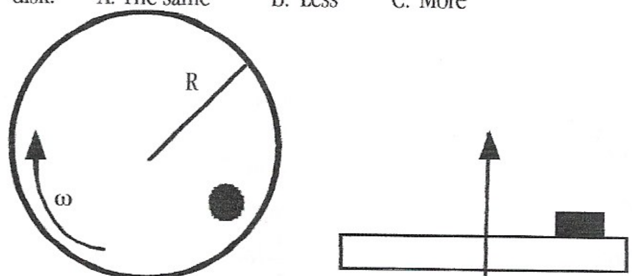
FORM A

10. Assume that the hanging mass is m, the moment of inertia of the welded disks is I_D about the rotation axis, the radius of the smaller disk is r_1 and the larger r_2. What is the angular acceleration α of the welded disks? The string is wrapped around the smaller disk.
A. $\frac{mgr_2}{I_D+mr_2^2}$ B. $\frac{mgr_1}{I_D+mr_1^2}$ C. $\frac{mgr_1}{I_D+mr_2^2}$
D. $\frac{mgr_2}{I_D+mr_1^2}$ E. $\frac{mgr_1}{I_D}$ F.

11. What is the linear acceleration a of the hanging mass? Give your answer in terms of the stated quantities in the previous question.
A. g B. $\alpha\, r_2$ C. $\alpha\, r_1\, r_2$ D. $\alpha(r_1+r_2)/2$ E. $\alpha\, r_1$

12. How will the acceleration of the mass in the previous question change if the string is wrapped around the larger disk?
A. The same B. Less C. More

13. How does the impact speed (with the ground) of the mass in the previous question change if the string is wrapped around the larger disk? A. The same B. Less C. More



14. **Top view** **Side view**
A coin of mass m is placed a distance r from the center of a disk rotating at a constant angular speed. The mass and radius of the disk are M and R. The coefficient of static friction is μ. What is the direction of the force on the coin?
A. Radially outward B. Radially in C. Tangent to the circle
D. It is a combination of radially out and tangent to the circle
E. It is a combination of radially in and tangent to the circle. F.

15. The rotational speed ω is slowly increased to a value ω_{max} at which time the coin just flies off the disk. What is the direction of the net force on the coin while it is in contact with the disk?
A. Radially outward B. Tangent to the circle C. Radially in
D. It is a combination of radially out and tangent to the circle
E. It is a combination of radially in and tangent to the circle. F.

16. Find ω_{max}, the angular velocity of the coin right before it flies off the disk, in terms of m, M, R, g, and μ.
A. $\sqrt{\frac{g}{r}}$ B. $\sqrt{\frac{g}{R}}$ C. $\sqrt{\frac{\mu g}{r}}$ D. $\sqrt{\frac{\mu g}{R}}$ E. $\sqrt{\frac{\mu g}{R+r}}$ F.

FRQ 16 Sketch the path of the coin as it flies off the disk.

17. Calculate the angular momentum of the disk in the previous question just before the coin leaves the disk.
A. $MR^2\omega_{max}$ B. $\frac{1}{2}MR^2\omega_{max}$ C. $\frac{2}{5}MR^2\omega_{max}$
D. $\mu MR^2\omega_{max}$ E. $\frac{1}{2}\mu MR^2\omega_{max}$ F.

18. What is the angular momentum of the coin just before the coin leaves the disk.
A. $mr^2\omega_{max}$ B. $\frac{1}{2}mR^2\omega_{max}$ C. $\frac{2}{5}mR^2\omega_{max}$
D. $\mu mr^2\omega_{max}$ E. $\frac{1}{2}\mu mR^2\omega_{max}$ F.

19. Is the total angular momentum of the disk-coin system conserved?
A. Yes, it is conserved because the elapsed impact time is zero.
B. Yes, it is conserved because the torque applied is zero.
C. Yes, it is conserved becasue the net torque on the system times the elapsed time is negligible.
D. No, it is not conserved because gravity applies torque
E No, it is not conserved because the net force on the system is not zero.

20. A bycle wheel of radius 2 m completes 30 revolutions per seconds. Assuming rolling without slipping, how far does the bike travle in one minute?
A. $60\,m$ B. $1800\pi\,m$ C. $3600\pi\,m$ D. $7200\pi\,m$ E. $7200m$ F.

21. Two girls, each of mass m, sit on opposite ends of a thin rod with length ℓ and mass m (the same as each girl's mass). The rod is pivoted at its center and is free to rotate in a horizontal circle without friction. What is the moment of inertia of the rod and the girls about a vertical axis through the center of the rod?
A. $m\ell^2$ B. $3m\ell^2$ C. $\frac{1}{3}m\ell^2$ D. $\frac{7}{12}m\ell^2$ E. $\frac{25}{12}m\ell^2$ F.

FORM A

22. What is the angular momentum of the system if it is rotating with an angular speed ω_o in a clockwise direction as seen from above? Assume the moment of inertia of the system is I.
A. 0 B. $I\omega_o$ clockwise C. $I\omega_o$ counterclockwise
D. $3\,I\omega_o$ clockwise E. $3\,I\omega_o$ counterclockwise

23. While the system is rotating the girls pull themselves toward the center of the rod until they are half as far from the center as before. How does the resulting angular momentum compare to the origional one? A. Faster B. Slower C. The same

24. While the system is rotating the girls pull themselves toward the center of the rod until they are half as far from the center as before. How does the resulting angular speed compare to the origional one? A. Faster B. Slower C. The same

25. What is the resulting angular speed in terms of ω_o?
A. $\frac{5}{14}\omega_o$ B. $\frac{14}{5}\omega_o$ C. $4\omega_o$ D. $\frac{1}{4}\omega_o$ E. $2\omega_o$ F.

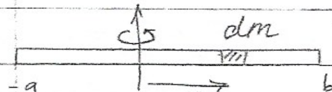
FRQ 25 What is the change in kinetic energy of the system due to the girls changing their position? Where did the change in KE come from or go to?

mr^2
$\frac{1}{2}mr^2$
$\frac{1}{3}mr^2$
$\frac{2}{5}mr^2$
$\frac{1}{12}mr^2$
$I_{cm} + md^2$

$m\vec{a}$
$m\vec{v}$
$\frac{1}{2}mv^2$
mgh
$\frac{1}{2}I\omega^2$
$I\vec{\omega}$
$I\vec{\alpha}$

$G\frac{m_1m_2}{r^2}$
$G\frac{m}{r}$
$m\frac{v^2}{r}$

$\vec{r} \times \vec{p}$
$\vec{r} \times F$
μF
$\vec{r} \times \vec{p}$
$\vec{v}_o + \vec{a}t$
$\vec{s}_o + \vec{v}_o t + \frac{1}{2}\vec{a}t^2$

1\. 

7

A. $\lambda = \frac{M}{L}$ 1/2

B. $dm = \lambda dx$ 1/2

or $dm = \frac{M}{L} dx$

C. $dI = dm\, x^2$ 1/

or $dI = \lambda x^2 dx$ or $\frac{M}{L} x^2 dx$

D. $I = \int_{-a}^{b} dm\, x^2 = \frac{M}{L}\int_{-a}^{b} x^2 dx = \frac{M}{L}\frac{x^3}{3}\Big|_{-a}^{b}$

1/2 for limits   1 for integral

$= \frac{M}{3L}(b^3 - (-a)^3)$ 1 substitution

$I = \frac{M}{3L}(b^3 + a^3)$ 1 obtaining the answer

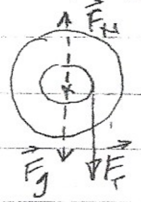
2\. $\vec{\tau} = \frac{d\vec{L}}{dt}$ $\Delta\vec{L} = \vec{\tau}\Delta t$

1 Conservation of $\vec{L}$ can be used when $\vec{\tau}\Delta t \to 0$

$\vec{\tau} \to 0$ 1/2 point, $\Delta t \to 0$ 1/2 point

3\. A $I = I_1 + I_2 = \frac{1}{2}(m_1 r_1^2 + m_2 r_2^2)$ 1

17

B 

$\vec{F}_N$ & $\vec{F}_g$ 1

$\vec{F}_T$ 1

C $\vec{\tau}_{net} = \vec{r}_1 \times m_3\vec{g}$ or $\tau_{net} = m_3 g r_1$ 1/

D $\vec{\tau}_1 = \vec{r}_1 \times \vec{F}_T$ or $\tau_1 = r_1 F_T$ 1/

E $m_3 gh = \frac{1}{2}m_3 v^2 + \frac{1}{2}I_T\omega^2$

1 for CoE 1 for correct CE

$m_3 gh = \frac{1}{2}m_3 v^2 + \frac{1}{2}I_T\frac{v^2}{r_1^2}$

1 for using $v = \omega r_1$

$v = \sqrt{\frac{2m_3 g h r_1^2}{m_3 r_1^2 + I_T}}$ or any equivalent 1

F Wherever we see r_1, we substitute r_2 & the resultant v becomes larger 1/

G We can solve this problem in two ways:

(i) Use Newton's 2nd law individually

$m_3 a = m_3 g - F_T$ 1

$I_T\alpha = r_1 F_T$ 1 & eliminate F_T

or (ii) $(m_3 r_1^2 + I_T)\alpha = m_3 g r_1$ 2

Either method gives $\alpha = \frac{m_3 g r_1}{m_3 r_1^2 + I_T}$ 1/

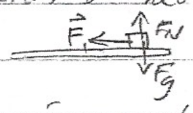
H Use the method in G to solve for a or $a = \alpha r_1$ & $a = \left(\frac{1}{1 + I_T/m_3 r_1^2}\right) g$ 1/

I The larger, the larger α & a. 1/
Just substitute r_2 in H & G.

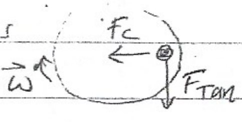
J Substitute r_2 in E 1/

$r_2 > r_1 \Rightarrow v$ is larger

4\. A Since ω is constant, this is a simple circular motion & the net force is toward the center 1/

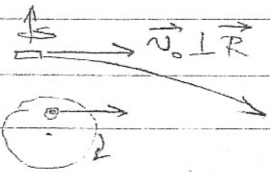


B. the net force is a combination of the centripetal & the tangential forces 1/



C $F_f = F_c = m\frac{v^2}{R}$ any centripetal motion force eqn 1/

$\mu_s mg = m\omega^2 R$, $\omega_{max} = \sqrt{\mu_s g/R}$ 1/

D 

$v_0 = \omega_{max} R$ 1/

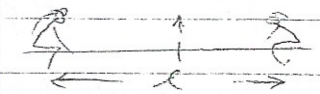
E $\vec{L}_D = I_D \vec{\omega}_{max} = \frac{1}{2} m_D R^2 \sqrt{\mu_s g/R}\,\hat{k}$ ½

F $\vec{L}_C = I_C \vec{\omega}_{max} = m_C R^2 \sqrt{\mu_s g/R}\,\hat{k}$ ½

G Not when the disk is speeding ½
Yes at the moment the disk is leaving for $\Delta t \to 0$

H. $d = vt = \omega r t = N\frac{2\pi}{s} r\, 60s$
$= 360 N\pi r$ ½

5 A



$I_T = I_\ell + I_G$ ½

$= (m_1+m_2+\frac{m_3}{12})\ell^2$ ½

B $\vec{L} = I_T\vec{\omega}_0 = (m_1+m_2+\frac{m_3}{12})\ell^2\omega_0(-\hat{k})$ ½

C $\vec{L}_b = \vec{L}_a$ L is conserved ½
b/c motion is along re
$\vec{\tau}_{net} = 0.$

D $\omega_f > \omega_0$ since $L_f = L_0$ ½

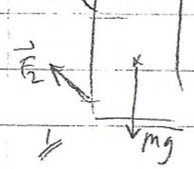
E $I_f\omega_f = I_0\omega_0$ $\quad \omega_f = \frac{I_0}{I_f}\omega_0$ ½

F $\Delta KE = KE_f - KE_i = \frac{1}{2} I_f\omega_f^2 - \frac{1}{2} I_0\omega_0^2$ ½

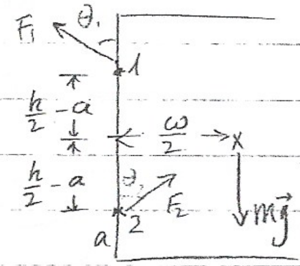
$= \frac{1}{2} I_0\omega_0^2\left(\frac{I_0}{I_f} - 1\right)$

$= \frac{1}{2}\left(\frac{12m_1+12m_2+m_3}{3m_1+3m_2+m_3}\right)9(m_1+m_2)\omega_0^2$ ½

6. A



½



B $\vec{F}_1 + \vec{F}_2 + m\vec{g} = 0$

$\vec{F}_{1y} + \vec{F}_{2y} = m\vec{g}$ (1) & $\vec{F}_{1x} + \vec{F}_{2x} = 0$ (2)

C Choose rotation axis to eliminate some of the force

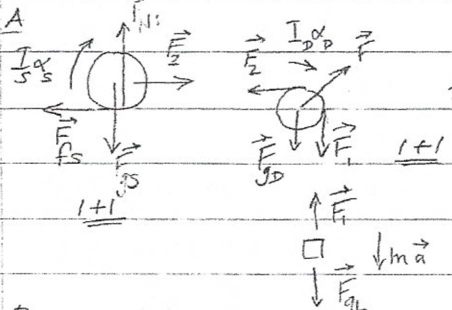
$0 = \sum_{(2)} \vec{\tau} = mg\frac{\omega}{2} - F_{1x}(h-2a)\,\hat{k}$ (3) ½

$0 = \sum_{(1)} \vec{\tau} = mg\frac{\omega}{2} - F_{2x}(h-2a)\,\hat{k}$ (4) ½

$0 = \sum_{(cm)} \vec{\tau} = F_{1y}\frac{\omega}{2} + F_{2y}\frac{\omega}{2} - F_{1x}\left(\frac{h}{2}-a\right) - F_{2x}\left(\frac{h}{2}-a\right)$ ½

D From (3) & (4)
$F_{1x} = F_{2x} = \frac{w}{2h-4a}\,mg$ same mag. opp. dir. ½

7 A



1 pt per force pair

1/1

B

$I_{S.cp.}\alpha_S = 3RF_2$ or $I_S\alpha_S = 3R\,F_{fs}$ 1/1

$I_D\alpha_D = R(F_1 - F_2)$ 1/1

$ma = mg - F_1$ 1/1

C

$a = 3R\alpha_S - R\alpha_D$ 1/1

$I_{S.cp} = m(3R)^2 + I_S$ 1/1 $I_{scp} = \frac{7}{5}m(3R)^2$

$I_D = \frac{1}{2}mR^2$

$\left(\frac{I_{scp}}{(3R)^2} + \frac{I_D}{R^2} + m\right)a = mg$ 1/1

$\left(\frac{7}{5} + \frac{1}{2} + 1\right)ma = mg$, $a = \frac{10}{29}g$ 1/1

D $\alpha_S = a/3R = \frac{10}{87}g$, $\alpha_D = \frac{10}{29R}g$
1/1 1/1

E $ma = mg - F_1$, $F_1 = \frac{19}{29}mg$ 1/1

$\frac{1}{2}mR^2\alpha_D = R(F_1 - F_2)$, $F_2 = F_1 - \frac{1}{2}ma$

$F_2 = \frac{14}{29}mg$ 1/1

$\frac{2}{5}m(3R)^2\alpha_S = 3R\,F_{fs}$ Not asked

$F_{fs} = \frac{2}{5}ma = \frac{4}{29}mg$ in the problem

8 A the horizontal part of the linear momentum
1/1 is conserved since there are no forces in that direction

B the only external net force is gravity
1/1 which is a conservative force. therefore the total energy is conserved.

C $0 = M\vec{v}_B + m\vec{v}_b$, $\vec{v}_B = \frac{-m}{M}\vec{v}_b$ 1/1

D $mgh_0 = \frac{1}{2}Mv_B^2 + \frac{1}{2}mv_b^2 + \frac{1}{2}I_b\omega_b^2$ 1/1

E $mgh_0 = \frac{1}{2}M\frac{m^2v_b^2}{M^2} + \frac{1}{2}mv_b^2 + \frac{1}{2}\frac{2}{5}mv_b^2$ 1/1

$2gh_0 = \left(\frac{m}{M} + \frac{7}{5}\right)v_b^2$

F they will be moving together, therefore, 0 1/1

G $m\vec{v}_b = (m+M)\vec{v}_f$, $v_f = \frac{m}{m+M}v_b$ 1/1

H $\frac{1}{2}mv_b^2 = \frac{1}{2}(m+M)v_f^2 + \frac{1}{2}I_S\omega_S^2 + mgh$ 1/1

I $v_b^2 = \left(1 + \frac{M}{m}\right)\left(\frac{m}{m+M}\right)^2 v_b^2 + \frac{2}{5}\left(\frac{m}{m+M}\right)^2 v_b^2 + 2gh$

$\left[1 - \left(\frac{m}{m+M}\right) - \frac{2}{5}\left(\frac{m}{m+M}\right)^2\right]v_b^2 = 2gh$

$\left[\frac{M}{M+m} - \frac{2}{5}\left(\frac{m}{M+m}\right)^2\right]v_b^2 = 2gh$

J $\left[\frac{M}{M+m} - \frac{2}{5}\left(\frac{m}{M+m}\right)^2\right]\frac{2gh_0}{\left(\frac{m}{M} + \frac{7}{5}\right)} = 2gh$ 1/1

$$h = \left(\frac{\frac{M}{M+m} - \frac{2}{5}\left(\frac{m}{M+m}\right)^2}{\frac{m}{M} + \frac{7}{5}}\right)h_0$$

1 6pts

A $\lambda = \frac{M}{L}$ 1pt B $dm = \lambda dx$ 1pt

C $dI = x^2 dm \overset{or}{=} x^2 \lambda dx$ 1pt

D, E $I = \int_{-a}^{b} x^2 \lambda dx = \lambda \int_{-a}^{b} x^2 dx = \lambda \frac{x^3}{3}\Big|_{-a}^{b}$ 1pt 1pt

$= \frac{\lambda}{3}(b^3 + a^3) = \frac{M}{3L}(b^3 + a^3)$. 1pt

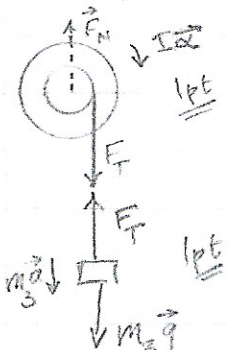
2 1pt

$\vec{\tau}_{net} \to 0,\ \Delta t \to 0 \} \Rightarrow \vec{\tau}_{net}\Delta t \to 0$ 1pt

3 A $I_1 = \frac{1}{2}m_1 r_1^2$, $I_2 = \frac{1}{2}m_2 r_2^2$

$I_T = \frac{1}{2}(m_1 r_1^2 + m_2 r_2^2)$ 1pt

B



C $\vec{\tau}_{net} = \vec{r}_1 \times m_3\vec{g}$ 1pt

or $\tau_{net} = m_3 g r_1$

D $\vec{\tau}_D = \vec{r}_1 \times \vec{F}_T$ or

$\tau_D = r_1 F_T$ 1pt

E $PE_0 = KE_f$ 1pt

$m_3 gh = \frac{1}{2}I\omega^2 + \frac{1}{2}m_3 v^2$ 1pt

$v = \omega r_1$ 1pt

$m_3 gh = \frac{1}{2}\left[(m_1 r_1^2 + m_2 r_2^2)\frac{v^2}{r_1^2} + 2m_3 v^2\right]$

$v = \sqrt{\frac{2m_3 gh}{(2m_3 + m_1 + m_2 \frac{r_2^2}{r_1^2})}}$ 1pt

F $v = \omega r_1$ or $v = \sqrt{\frac{2m_3 gh}{(2m_3 + m_1 \frac{r_1^2}{r_2^2} + m_2)}}$ 1pt

It will fall faster.

G Method 1 $I_{system} = \frac{1}{2}(m_1 r_1^2 + m_2 r_2^2) + m_3 r_1^2$ 1pt

$\vec{\tau}_{net\text{-}system} = m_3 g r_1$ 1pt

$m_3 g r_1 = I_{system}\alpha$ 1pt

$\alpha = 2m_3 g r_1 / (m_1 r_1^2 + m_2 r_2^2 + 2m_3 r_1^2)$ 1pt

Method 2 $F_T r_1 = \frac{1}{2}(m_1 r_1^2 + m_2 r_2^2)\alpha$ 1pt

$m_3 g - F_T = m_3 a$ 1pt $a = \alpha r_1$ 1pt

$F_T = m(g - \alpha r_1)$

$m_3(g - \alpha r_1)r_1 = \frac{1}{2}(m_1 r_1^2 + m_2 r_2^2)\alpha$

$m_3 g r_1 = \left[\frac{1}{2}(m_1 r_1^2 + m_2 r_2^2 + 2m_3 r_1^2\right]\alpha$

$\alpha = 2m_3 g r_1 / (m_1 r_1^2 + m_2 r_2^2 + 2m_3 r_1^2)$ 1pt

H $a = \alpha r_1 = 2m_3 g r_1^2 / (m_1 r_1^2 + m_2 r_2^2 + 2m_3 r_1^2)$ 1pt

$a = \left[2m_3 / (m_1 + m_2 \frac{r_2^2}{r_1^2} + 2m_3)\right] g$

I $I_{system} = \frac{1}{2}(m_1 r_1^2 + m_2 r_2^2) + m_3 r_2^2$ &

$\vec{\tau}_{net\text{-}system}\ m_3 g r_2$

Instead of $F_T r_1$ we have $F_T r_2$ & $a = \alpha r_2$

the final answers are

$\alpha = 2m_3 g r_2 / (m_1 r_1^2 + m_2 r_2^2 + m_3 r_2^2)$

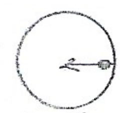
$a = \left[2m_3 / (m_1 \frac{r_1^2}{r_2^2} + m_2 + 2m_3)\right] g$ 1pt

In this case both a & α are greater than they were in part G & H.

J $v = at$ & $h = \frac{1}{2}at^2$ $t = \sqrt{2h/a}$ 1pt

$v = \sqrt{2ha} = \sqrt{\frac{2m_3 gh}{m_1 \frac{r_1^2}{r_2^2} + m_2 + 2m_3}}$ 1pt

4 A Towards the center



centripetal 1pt

B

this one is the tangential force since it is speeding up

1pt

C $\vec{F}_{net} = m\vec{a}_c$ 1pt $F_f \le \mu F_N = \mu mg$

$\mu mg = mv^2/R = m\omega^2 R$ 1pt

$\omega_{max} = \sqrt{\mu g / R}$ 1pt

D ↑ &

1pt

E $L_D = I_D \omega = \frac{1}{2} M R^2 \sqrt{Mg/R}$ 1pt

F $L_c = I_c \omega = \frac{1}{2} m R^2 \sqrt{Mg/R}$ 1pt

G Until the moment ω_{max} is reached, the angular momenta are increasing. At the moment that the coin flies off, the angular momenta are conserved since $\vec{\tau}_{net} \Delta t \to 0$ 1pt

H $d = 60 \cdot N \, 2\pi r = 120 N \pi r$ 1pt

5 A $I = (m_1 + m_2)\left(\frac{\ell}{2}\right)^2 + \frac{1}{12} m_3 \ell^2$ 1pt

$= \frac{1}{12}(3m_1 + 3m_2 + m_3)\ell^2$

B $L = I \omega_0 = \frac{1}{12}(3m_1 + 3m_2 + m_3)\ell^2 \omega_0$ 1pt

C $L_{new} = L_0$ since there is no net external torque 1pt

$\vec{F}_{girls} \times \vec{r} = F_{girls}\, r \sin 180° = 0$

D $L_{new} = L_0$, $I_{new} \omega_{new} = I_0 \omega_0$

E $\omega_{new} = \frac{I_0}{I_{new}} \omega_0$, $I_{new} = (m_1 + m_2)\left(\frac{\ell}{4}\right)^2 + \frac{1}{12} m_3 \ell^2$ 1pt

$\omega_{new} = \dfrac{\frac{1}{12}(3m_1 + 3m_2 + m_3)}{\frac{1}{16}(m_1 + m_2 + \frac{4}{3} m_3)} \omega_0$ 1pt

$\omega_{new} = \frac{4}{3}\left(\dfrac{3m_1 + 3m_2 + m_3}{m_1 + m_2 + \frac{4}{3} m_3}\right)\omega_0$

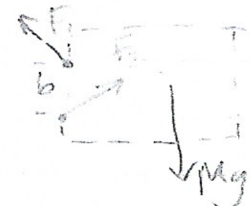
the resulting angular speed is larger b/c $I_{new} < I_0$ 1pt

F the increase in KE comes from the girls' muscles & food. 1pt

$\Delta KE = \frac{1}{2} I_{new} \omega_{new}^2 - \frac{1}{2} I_0 \omega_0^2$ 1pt

Do the substitutions

6 A
9pts 3pts



1pt for each correct force vector.

B $\vec{F}_1 + \vec{F}_2 + m\vec{g} = 0$ 1pt

C $\vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_g = 0$ about any axis 1pt

D $F_{1x} = F_{2x}$ & $F_{1y} + F_{2y} = mg$ 1pt

I set $b = h - 2a$ 1pt

(2) $\tau_{net} = -F_{1x} b + mg \frac{w}{2} = 0 \Rightarrow F_{1x} = \frac{w}{2b} mg$ 1pt

(1) $\tau_{net} = -F_{2x} b + mg \frac{w}{2} = 0 \Rightarrow F_{2x} = \frac{w}{2b} mg$

(cm) $\tau_{net} = F_{1y} \frac{w}{2} + F_{2y} \frac{w}{2} - F_{1x} \frac{b}{2} - F_{2x} \frac{b}{2} = 0$

Due to symmetry we will get $F_{1y} = F_{2y}$

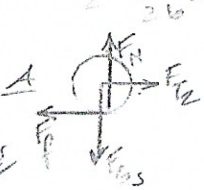
From the first equation above

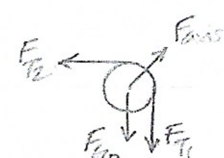
$F_{1y} + F_{2y} = mg$ $F_{1y} = \frac{1}{2} mg$

$\vec{F}_1 = -\frac{w}{2b} mg \hat{i} + \frac{1}{2} mg \hat{j}$ 1pt

$\vec{F}_2 = \frac{w}{2b} mg \hat{i} + \frac{1}{2} mg \hat{j}$ 1pt

7 A
11pts





1pt per force pair

1pt

B

Sphere (cp) $\tau_{net} = 3R F_{T2} = (I_S + M(3R)^2)\alpha_S$ 1pt

Disk (Disk axis) $\tau_{net} = R(F_{T1} - F_{T2}) = I_D \alpha_D$ 1pt

Mass (Disk axis) $\tau_{net} = R(Mg - F_{T1}) = MR^2 \alpha_D$ 1pt

C $a_S = a_D = a_M$, $a_S = \alpha_S 3R$, $a_D = \alpha_D R$ 1pt

Obtain F_T from each equation:

$F_{T2} = (\frac{I_S}{9R^2} + M)a = \frac{7}{5}Ma$

$F_{T1} - F_{T2} = \frac{I_D}{R^2}a = \frac{1}{2}Ma$ 1pt

$Mg - F_{T1} = Ma$

$Mg = (\frac{7}{5} + \frac{1}{2} + 1)Ma = 2.9Ma$ 1pt

$a = \frac{1}{2.9}g$ 1pt

D $\alpha_S = a/3R = \frac{1}{8.7}\frac{g}{R}$ 1pt

$\alpha_D = \alpha_M = \frac{a}{R} = \frac{1}{2.9}\frac{g}{R}$ 1pt

E Use the equations in C to obtain each tension

$F_{T1} = M(g-a) = \frac{1.9}{2.9}Mg$ 1pt

$F_{T2} = \frac{7}{5}Ma = \frac{7}{14.4}Mg$ 1pt

8 A p_y is not conserved while the ball
13pts 1pt is rolling down because of gravity

1pt p_x is conserved since there is no net force in the x-direction

B the only work is done by gravity which is a conservative force.
1pt Otherwise, there is no net external force

C $0 = M\vec{v}_\Delta + m\vec{v}_b$ 1pt $v_\Delta = \frac{m}{M}v_b$

D $mgh = \frac{1}{2}Mv_\Delta^2 + \frac{1}{2}mv_b^2 + \frac{1}{2}I\omega_b^2$ 1pt

E $mgh_0 = \frac{1}{2}\frac{m^2}{M}v_b^2 + \frac{1}{2}mv_b^2 + \frac{1}{2}\frac{2}{5}mr^2\omega_b^2$

rolling w/o slipping $\omega_b = v_b/r$ 1pt

$mgh_0 = \frac{1}{2}mv_b^2(\frac{m}{M} + 1 + \frac{2}{5})$

$2gh_0 = (\frac{7}{5} + \frac{m}{M})v_b^2$ 1pt

F $v_{rel} = 0$ 1pt

G $m\vec{v}_b = (m+M)\vec{v}$ 1pt $v = \frac{m}{m+M}v_b$

H $\frac{1}{2}mv_b^2 + \frac{1}{2}I\omega_b^2 = \frac{1}{2}(m+M)v^2 + mgh$ 1pt

I $\frac{1}{2}mv_b^2 + \frac{1}{2}\frac{2}{5}mr^2\omega_b^2 = \frac{1}{2}\frac{m^2}{(m+M)}v_b^2 + mgh$ 1pt

$(\frac{7}{5} - \frac{m}{m+M})v_b^2 = 2gh$ 1pt

J From E $v_b^2 = \frac{2gh_0}{(\frac{7}{5} + \frac{m}{M})}$. Substitute

this in I $\frac{\frac{7}{5} - \frac{m}{m+M}}{\frac{7}{5} + \frac{m}{M}} 2gh_0 = 2gh$

$h = \left(\frac{\frac{7}{5} - \frac{m}{m+M}}{\frac{7}{5} + \frac{m}{M}}\right) h_0$ 1pt

9 A
13pts

$KE = \frac{1}{2}\frac{M\ell^2}{3}\omega^2$ 1pt

$PE = Mg\frac{\ell}{2}\cos\theta$ 1pt

$E = \frac{1}{6}M\ell^2\omega^2 + \frac{1}{2}Mg\ell\cos\theta$ 1pt

$E_0 = Mg\frac{\ell}{2} = E = \frac{1}{6}M\ell^2\omega^2 + \frac{1}{2}Mg\ell\cos\theta$ 1pt

$\frac{1}{3}\ell\omega^2 = g(1-\cos\theta)$

$\omega = \sqrt{3\frac{g}{\ell}(1-\cos\theta)}$ 1pt

B $a_r = \omega^2\ell = 3g(1-\cos\theta)$ 1pt

C $\tau_{net} = I\alpha$ 1pt

$Mg\frac{\ell}{2}\sin\theta = \frac{M\ell^2}{3}\alpha$, $a_t = \ell\alpha$ 1pt

$a_t = \frac{3}{2}g\sin\theta$ 1pt

D $a = \sqrt{a_r^2 + a_t^2} = 3g\sqrt{\frac{\sin^2\theta}{4} + (1-\cos\theta)^2}$ 1pt

Yes, $a > g$ is possible 1pt

$a = 3\sqrt{\frac{3}{4}\cos^2\theta - 2\cos\theta + \frac{5}{4}}\,g$

$a > g \Rightarrow 3\sqrt{\frac{3}{4}u^2 - 2u + \frac{5}{4}} > 1$ ½

where $u = \cos\theta$

$\frac{3}{4}u^2 - 2u + \frac{41}{36} > 0$

$u = \frac{4}{3}\left(1 \pm \sqrt{\frac{7}{12}}\right) = 0.31,\ 2.35$

$\cos\theta = 0.31$

For $\cos\theta < 0.31 \Rightarrow \theta > 71.6°$ ½

10. 9pts

A 

1pt

B $E_0 = PE = mgh$ 1pt

$E_f = mg2R + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ 1pt

$= mg2R + \frac{7}{10}mv^2$, $v = \omega r$ 1pt
rolling w/o slipping

C We need all the ma_c to be accounted by gravity

$mg = ma_c = m\frac{v^2}{R} \Rightarrow v^2 = gR$ 1pt

Substitute this in the energy equation

$mgh = mg2R + \frac{7}{10}mgR$ 1pt

$h = 2.7R$ 1pt

D $mg6R = mgR + \frac{7}{10}mv^2$ 1pt

$F_x = m\frac{v^2}{R} = \frac{50}{7}mg$ 1pt

11 3pts

$mgh = \frac{1}{2}mv_\ell^2$ for the fluid since it flows rather than rolls 1pt

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{4}mv^2$
$= \frac{3}{2}mv_f^2$ since the frozen one rolls 1pt

$v_\ell = \sqrt{2gh} > v_f = \sqrt{\frac{2}{3}gh}$ 1pt

Liquid one gets there first.

MC

1 D
2 D
3 D
4 E
5 C
6 E
7 A
8 A
9 E
10 B
11 E
12 C
13 A
14 B
15 E
16 C
17 B
18 A
19 C
20 D
21 D
22 B
23 C
24 A
25 B