CH, 1, 2

1. a. What is the mass of a mole of $^{197}_{79}Au$?

   b. How many moles and how many atoms are there in a brick of $^{197}_{79}Au$ with a mass of 2.00 kg? $N_A = 6\times10^{23}$. Give the correct number of significant digits in your final answer.
2. Assume that the radius of a typical atom is $1\times10^{-10}\,m$. Calculate how many atoms there are in $1cm^3$.
3. Given the fact that you attend four 90-minute classes a day and that there are 182 school days in a 364-day-year, calculate the additional knowledge a student gains in grades 1-12 in terms of the number of years of school knowledge if s/he studies
   a. 0 hours
   b. 2 hours outside the classroom *everyday of the year on average.*
   c. By the time a student who studies 0 hours a day graduates from high school, what *degree equivalence* will a student who studies 2 hours a day have? Assume 4 years for B.Sc., 2 years for M.Sc., and 4 years for Ph.D.
   d. What is the purpose of asking you this question?
4. Estimate the mass of a. an elephant, b. a squirrel.
   Clearly state your approximations and assumptions and your reasons for your assumptions.
5. (a) How far is the moon? (b) How far is the sun? What is the approximate diameter of (c) the solar system, (d) the milky way galaxy?
6. Consider the vectors **A**=–4**i**+6**j** and **B**=2**i**+2**k**. Obtain
   a. **A+B** b. **A–B** c. **A•B** d. **AxB**
   d. the angle between the vectors **A** and **B**.
7. A commuter airplane starts from an airport and flies to city A located 30 km in a direction 30° north of east. Next, it flies 40 km 60° west of north to city B. Find the location of city B relative to the location of the starting point by using
   (a) graphical method (the law of cosine) (b) component method. Give (c) the components of the final displacement vector,
   (d) the magnitude of the final displacement vector as well as its angle with respect to due north.
8. a. What is the magnitude of the vector **i+j+k?**
   b. What is the magnitude of the vector **i-j+k?**
9. Find the angle between the body diagonal of a cube and one of its edges using vector techniques.
10. Use vectors **A** and **B** together with vector addition and scalar product to obtain the expression you use in geometry in obtaining the third side of a triangle given two sides.
11. Consider **A**=cos α**i**+sin α**j**, **B**=cos β**i**+sin β**j** where α and β are the respective angles that each vector makes with the x-axis.
    a. Show that each vector has a magnitude equal to 1.
    b. Show that the scalar product of these two vectors give the trigonometric identity $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$.
12. Given vectors **A** and **B** how do you obtain a third vector which is perpendicular to both **A** and **B**?
13. Evaluate

    a. $\frac{d}{dx}\left(x^4+x^{-3}+x^{3/5}\right)$ b. $\int\left(x^4+x^{-3}+x^{3/5}\right)dx$
14. Use vectors **A** and **B** together with vector addition and scalar products to obtain the laws of cosines and sines.
15. Obtain the direction of the vector product, **c=*axb***, of the vectors below using the right hand rule.

    A. $\vec{a}\uparrow,\ \vec{b}\rightarrow$ B. $\vec{a}\uparrow,\ \vec{b}\downarrow$ C. $\vec{a}\uparrow,\ \vec{b}\otimes$

    D. $\vec{a}\uparrow,\ \vec{b}\nwarrow$ E. $\vec{a}\downarrow,\ \vec{b}\bullet$
16. Donald Trump, now the President of the United States of America, claimed his worth to be in excess of **$10 billion.** Assuming this is so, do the following calculations
    A. Reportedly he had declared bankruptcy six times and he is about 70 years old, lets assume that he acquired this amount within a 10-year span to account for the bankruptcies. How much money did he gain per (i) second, (ii) minute, (iii) hour, (iv) day to acquire this wealth?
    B. During the last 10 years, the minimum wage has been around $7. To simplify your calculations, assume the minimum wage to be $10 per hour.
    (i) How long does one person have to work at minimum wage *every hour* of his life to make enough income to match that of D. Trump's financial worth?
    (ii) How many people have to work at minimum wage 10 hours per day for 10 years to make enough money to match D. Trump's financial worth?
    (iii) Do the calculations for parts (i) and (ii) if hourly wage is $1 per hour--which is more than what garment workers are paid in several Asian countries. For example, it is $0.25 per hour in Bangladesh.
    (iv) The **average** professor salary is just below $100,000 per year (and just below $50,000 for lecturers). Repeat the calculations in parts (i) and (ii) for a professor and a lecturer.
17. One summer when I was running about 7 miles per day, I started swiming 1 km daily in addition to my run. I noticed that I lost about 3 kg (6.6 lbs) in a span of two weeks. There was no change in my eating habits during this time.
    (i) Calculate how many Calories I burned per mile of run and per km of swim assumming a 7-mile run burns twice the amount of calories as 1-km swim burns.
    (ii) However, if an ordinary 60 kg person burns about 100 Calories per mile of run and about 300 Calories per km of swim, what can you say about my body structure based on this information.
    Relevant info: 1lbs of fat produces about 3,500 Calories and 1 kg about 7,700 Calories. 1 Calorie=1 kcal.

Mesut B. Çakır

Black 39+2 (problem ...) Blue 20 Red 9 Total 75 expected

=41

John Yang Ch. 1, 2 take home ~~68~~ 70 accounted

1) $^{197}_{79}Au$ – 197 au/atom

1 mol $^{197}_{97}Au$ = $\boxed{197\ g/mol}$ +1

b) $\frac{200\ kg}{197g/mol} \times \frac{1000g}{1kg} = \boxed{1.02 \times 10^4\ mol}$

$1.02\times10^4\ mol \times 6\times10^{23}\ atoms/mol = \boxed{6.09\times10^{27}\ atoms}$ +1

2) $V_{atom} = \frac{4}{3}\pi r^3 = \frac{4}{3}(\pi)(1\times10^{-10}m) = 4\times10^{-10}\ m^3$

$(1\ cm^3) \times \left(\frac{1m}{100cm}\right)^3 / V_{atom} = (1\ cm^3)\left(\frac{1m}{1000cm}\right)^3 / (4\times10^{-10}\ m^3)$

$= \boxed{2500\ atoms}$

3) a) 0 hours add't'l – $\boxed{0\ years}$ of extra knowledge +1

b) 2 hours/day – 2 hrs/day × 364 days/yr × (4 y)rs = 2912 hrs

(2912 hrs)/(24 hrs/day)/(~~364~~ 182 days/yr) = 0.66 yrs

(doesn't make sense)

b) $\frac{2\ hrs}{1\ day} \times \frac{364\ days}{1\ yr} \times 12\ yrs = 8736\ hrs$ (add't'l) +1

8736 hrs / 24 hrs/day / 182 days/yr = 2 yrs additional

d) to show how much more education you gain from putting extra time into studies +1

c) → half of a B.S or an A.S

4) a) about 1,000 kg, $\boxed{1\times10^6\ g}$ – not really familiar with elephants but 1,000 kg seems reasonable

b) squirrel – seems similar to maybe a bunny, so I'd guess that it's on the order of $\boxed{1\ kg}$.

5) a) no idea but an airplane cruises at about 40,000 ft – on the order of 10^4 m, so I'd guess $\boxed{10^6\ m}$

b) probably 100× the distance from earth to moon, so $\boxed{10^8\ m}$

c) guess – 10,000× distance from earth to sun – $\boxed{10^{12}\ m}$ +1

d) milky way – let's say 1,000,000,000× width of solar system

$= \boxed{10^{21}\ m}$

pretty close

(actually this is correct.) +1

7

6) $\vec{A} = -4\hat{i} + 6\hat{j}$ $\quad \vec{B} = 2\hat{i} + 2\hat{k}$

a) $\vec{A}+\vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$

$= (-4+2)\hat{i} + 6\hat{j} + 2\hat{k} = \boxed{-2\hat{i} + 6\hat{j} + 2\hat{k}}$ +1

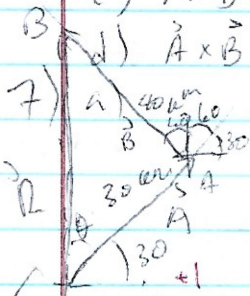
b) $\vec{A} - \vec{B} = \vec{A} + (-\vec{B}) = (-4-2)\hat{i} + 6\hat{j} - 2\hat{k} = \boxed{-6\hat{i} + 6\hat{j} - 2\hat{k}}$ +1

c) $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y\cancel{\hat{j}}^{0} + A_zB_z\cancel{\hat{k}}^{0} = \boxed{-8}$ +1

d) $\vec{A}\times\vec{B} = AB\sin\theta$ ←

e) ?

7) a)



+1

$~~A^2+B^2+2AB\cos\theta~~$

$|R| = \sqrt{A^2+B^2} = \sqrt{(30\,km)^2 + (40\,km)^2} = \boxed{50\ km}$ (a), (d) ↙ No

$\theta_R = \tan^{-1}\left(\frac{B}{A}\right) = \tan^{-1}\left(\frac{40\,km}{30\,km}\right) = 53^0$

$\theta_{geog.} = \theta_R + 30^0 = \boxed{83^0\ N\ of\ E}$ ☆

disagree

b) $A_x = A\cos\theta = (30\,km)\cos(30) = 26\ km$ +1

$A_y = A\sin\theta = (30\,km)\sin(30) = 15\ km$ +1

$B_x = B\cos\theta = (40\,km)\cos(90+60) = -35\ km$ +1

$B_y = B\sin\theta = (40\,km)\sin(90+60) = 20\ km$ +1

$R = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} = \boxed{-9.0\ km\,\hat{i} + 35\ km\,\hat{j}}$ (b), (c) ↘ mag. + dir.

8) a) $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ $\quad |A| = \sqrt{x^2+y^2+z^2} = \sqrt{1^2+1^2+1^2} = \boxed{\sqrt{3}}$ +1

b) $\vec{B} = \hat{i} - \hat{j} + \hat{k}$ $\quad |B| = |A| = \boxed{\sqrt{3}}$ +1

9)  $\vec{B}$ - diagonal $= b(\hat{i} + \hat{j} + \hat{k})$

~~projection onto 2 dimensions:~~  ~~$\theta = 45^0$~~

$|R| = \sqrt{A^2+B^2+2AB\cos\theta} = \sqrt{(30\,km)^2 + (40\,km)^2 + 2(30)(40)\cos(120)}$

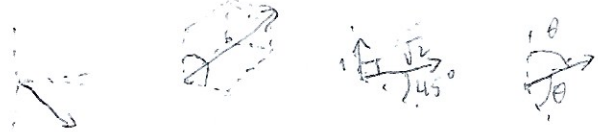
$= \boxed{36.1\ km}$ +1

$R^2 = A^2 + B^2 + 2AB\cos\theta$ $\quad$ ~~$\theta = \cos^{-1}\left(\frac{R^2 - A^2 - B^2}{2AB}\right)$~~

~~$= \cos^{-1}\left(\frac{(36.1\,km)^2 - (30\,km)^2 - (40\,km)^2}{2(30\,km)(40\,km)}\right)$~~

$= 120^0$

11



★ 9) $\vec{D}$ → angle to each plane − 45^0
let $l = 1$. $\vec{D} = \hat{i} + \hat{j} + \hat{k}$
$\theta = 45°$? how?

10)

$\vec{C} = \vec{A} + \vec{B}$ $\quad C = \sqrt{C \cdot C} = \sqrt{C^2} = \sqrt{(\vec{A}+\vec{B})^2}$

$C^2 = (\vec{A}+\vec{B})^2 = A \cdot A + A \cdot B + B \cdot A + B \cdot B$

$C^2 = A^2 + B^2 + 2(A \cdot B)$ +1

$C^2 = A^2 + B^2 + 2AB\cos\theta$ +1

11) $A = \cos\alpha\,\hat{i} + \sin\alpha\,\hat{j}$; $B = \cos\beta\,\hat{i} + \sin\beta\,\hat{j}$

a) $|A| = \sqrt{(A_x)^2 + (A_y)^2}$
$= \sqrt{\cos^2\alpha + \sin^2\alpha}$ (trig identity)
$= \sqrt{1} = 1$ +1

repeat for $\vec{B}$; $|B| = 1$ +1

b) $\vec{A} \cdot \vec{B} = (A_x B_x) + (A_y B_y) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ +1
$\vec{A} \cdot \vec{B} = AB\cos\theta$; where θ is measured between $\vec{A}$ and $\vec{B}$. $|A| = |B| = 1$
$\therefore \vec{A} \cdot \vec{B} = \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ +1

12) cross product (vector product) +1

13) a) $\frac{d}{dx}(x^4 + x^{-3} + x^{3/5}) = 4x^3 - 3x^{-4} + \frac{3}{5}x^{-2/5}$ +1

b) $\int (x^4 + x^{-3} + x^{3/5})\,dx = \frac{1}{5}x^5 - \frac{1}{2}x^{-2} + \frac{5}{8}x^{8/5} + C$ +1

(a) 14) → (10) ~~should be~~

(b) ~~I~~ don't even remember the law of sins

15) a) into the page +1 c) to the left +1 e) to the left +1
b) $\vec{A} \times \vec{B} = \vec{0}$ +1 d) out of the page +1

12

16) a) $\bar{v} = \frac{\Delta x}{t} = \$10,000,000,000 / 10\ yrs = \$1B/yr$

(iv) $\frac{\$1B}{1yr} \times \frac{1yr}{365\ days} = 2.7\ M/day$ ~ +1, order of magnitude is close enough

(iii) $\frac{\$1B}{1yr} \times \frac{1yr}{365\ days} \times \frac{1\ day}{24\ hrs} = \$110,000/hr$ +1

(ii) $\frac{\$1B}{1yr} \times \frac{1yr}{365\ days} \times \frac{1\ day}{24\ hrs} \times \frac{1\ hr}{60\ mins} = \$1,900/min$ +1

(i) " " " " $\times \frac{1\ min}{60\ s} = \$32/s$ +1

b) $t = \frac{\Delta x}{\bar{v}} = \$10B / \$10/hr = 1B\ hrs$

(i) ~~1B hrs x 24 hrs~~ $1B\ hrs \times \frac{1\ day}{24\ hrs} \times \frac{1\ yr}{365\ days} = \boxed{110,000\ years}$

$1B\ hrs / 10\ hrs/day = 100\ M\ days$

(ii) $100\ M\ days \times \frac{1\ yr}{365\ days} \times \frac{1\ person}{10\ yrs} = \boxed{27,000\ people}$

(iii) $t = \frac{\Delta x}{\bar{v}} = \$10B\ hrs = \boxed{1.1\ M\ yrs}$ (from (i))

$\$10B\ hrs \times \frac{1\ day}{10\ hrs} \times \frac{1\ yr}{365\ days} \times \frac{1\ person}{10\ yrs} = \boxed{270,000\ people}$

(iv) $t = \frac{\Delta x}{\bar{v}} = \$10B / \$100,000/yr = 100,000\ yrs$ +1 professors

" $50,000/yr = 200,000\ yrs$ lecturers

10,000 people professors

20,000 people lecturers +1

working 10 years.

17) burned 3 kg of fat $= 3(7,700\ cal)$

$= 231,000\ cal$

~~$2r + 1S = 231,000$~~

~~$r + S = 8$~~

~~$S = 2(7)(14)r + (1)(14)S$~~

~~$= 231,000$~~

6

17) (i) (calories 7 mile run) = 2(calories (1 km swim))

3.5 mile run = 1km swim

$3.5r = s$

~~[scribble]~~ 231,000 cal / 14 days = 16500 cal/day

+1

$7r + s = 16500$

$3.5r = s$

$7r + 3.5r = 16500$

$\boxed{r = 1571 \text{ cal/mile} \quad s = 5500 \text{ cal/km}}$ +1 +1

(ii) your body burns more energy than the average person, meaning you ~~either have super~~ probably have around 16 x the amount of mitochondria in your muscles as the average person. lol

3

4) a) textbook - mass of a person is 10^2 kg so maybe 5×10^3 kg is reasonable

b) squirrel - 1 L of water weighs 10^0 kg, so squirrels probably weigh about 4 kg

5) a) dia. of earth is ~~about~~ 10^7 m, so distance to earth is probably on the order of 10^{10} m close

b) distance to sun is much greater than distance to moon, so 10^{14} m too high

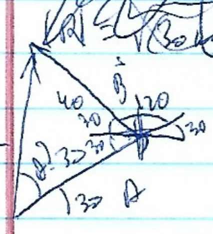
c) 10^{13} m (as stated in textbook) +1

d) 10^{21} m +1

6) d) $\vec{A}\times\vec{B} = (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$

7) (a) $= 12\hat{i} + 8\hat{j} - 12\hat{k}$ +1

~~$R^2 = A^2 + B^2 + 2AB\cos\theta$~~



$a = 30$ km $r = 36.1$ km

$b = 40$ km

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin R}{r}$

$\angle B = \sin^{-1}\left(\frac{b \sin R}{r}\right) = \sin^{-1}\left(\frac{40 \text{ km} \sin(60^\circ)}{36.1}\right)$

$\theta = 73.7^\circ + 30^\circ$ +1

$= 103^\circ$

$= \boxed{13^\circ \text{ W of N}}$

73.7°

9)  $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Cube - plane is rotated 45^0

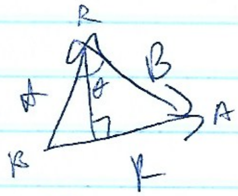
$\vec{R} = \vec{A} + \vec{B}$

$R^2 = A^2 + B^2 + 2AB\cos\theta$

$(\sqrt{3})^2 = (1)^2 + (\sqrt{2})^2 + 2(1)(\sqrt{2})\cos\theta$

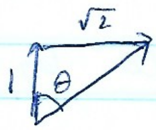
$\theta = \cos^{-1}\left(\frac{3-1-2}{2\sqrt{2}}\right) = 0$

(14)(b) $\frac{\sin A}{|A|} = \frac{\sin B}{|B|} = \frac{\sin R}{|R|}$



$\vec{R} = \vec{A} + \vec{B}$ $\sin A = \cos(90 - A)$

9)



$\theta = \tan^{-1}\left(\frac{\sqrt{2}}{1}\right) = \boxed{54.7^\circ}$ +1

1) b) $\frac{200g}{197 g/mol} = \boxed{1.02 \text{ mol}}$

$1.02 \text{ mol} \times 6\times10^{23} \frac{\text{atoms}}{\text{mol}} = \boxed{6.02\times10^{23} \text{ atoms}}$ +1

2) approx. atom as a cube

$V_{atom} = (2r)^3 = (2 \cdot 1\times10^{-10} m)^3 = 8\times10^{-30} m^3$ +1

$1 cm^3 \times \left(\frac{1m}{100cm}\right)^3 = 1\times10^{-6} m^3 / 8\times10^{-30} m^3/\text{atom} = \boxed{1.3\times10^{23} \text{ atoms}}$ +1

3) b) ~~8736 hrs / (4490 mins)~~

$8736 \text{ hrs} \times \frac{1 \text{ day}}{4490 \text{ mins}} \times \frac{60 \text{ mins}}{1 \text{ hr}} \times \frac{1 \text{ yr}}{182 \text{ days}} = \boxed{8 \text{ years}}$ +1

c) 8 yrs - B.S + M.S + 1/2 of a ph.D +1

4) a) approx. as a rect. prism w/ a similar density to water

elephants are probably 2 m tall and 3 m long, and 1 m wide

$V = lwh = 6 m^3 \quad \rho = \frac{m}{V} = 1 \frac{g}{cm^3}$ +1

$= 6\times10^6 cm^3 \quad m = \rho V = (1 \frac{g}{cm^3})(6\times10^6 cm^3) = \boxed{6,000 \text{ kg}}$ +1

b) again imagine a cube

10 cm wide, 10 cm tall, 20 cm tall +1

$V = lwh = 2,000 cm^3$

$m = \rho V = (1 \frac{g}{cm^3})(2000 cm^3) = \boxed{2 \text{ kg}}$ +1

5) a), b) don't know how to approach - -

6) e) $\theta_{AB} = \cos^{-1}\left(\frac{A_xB_x + \cancel{A_yB_y} + \cancel{A_zB_z}}{|A||B|}\right)$

$= \cos^{-1}\left(\frac{(-4\hat{i})(2\hat{i})}{(\sqrt{4^2+6^2})(\sqrt{2^2+2^2})}\right) = \boxed{113°}$ +1

7) $\theta_{R,A} = \cos^{-1}\left(\frac{R \cdot A}{|R||A|}\right) = \cos^{-1}\left(\frac{R_xA_x + R_yA_y}{RA}\right)$

$= \cos^{-1}\left(\frac{(-9.0 km\hat{i})(30km)\cos 30\hat{i} + (35 km\hat{j})(30km)\sin 30\hat{j}}{(36.1 km)(30 km)}\right)$

$= 74.4°$

$\theta_{geo.} = 74.4° + 30° - 90° = \boxed{14.4° \text{ W of N}}$ +1

7) b) $\vec{R} = -9.0\,km\,\hat{i} + 35\,km\,\hat{j}$

$|R| = \sqrt{9^2+35^2} = \boxed{36.1\,km}$

$\theta_R = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{35}{-9}\right) = -75.6^\circ$ 2nd quad.

$\theta = 180 - \theta_R = \boxed{104.4^\circ}$ +1

14) a) $\vec{C} = \vec{A} - \vec{B}$

$\vec{C} \cdot \vec{C} = (A-B)\cdot(A-B)$

$C^2 = A^2 - A\cdot B - B\cdot A + B^2$

$C^2 = A^2 + B^2 - 2A.B = A^2 + B^2 - 2AB\cos\theta$ +1

b) $\frac{\sin A}{A} = \frac{\sin B}{B} = \frac{\sin C}{C}$

area $= |A\times B| = |B\times C| = |C\times A|$ +1

$= \frac{AB\sin C = BC\sin A = AC\sin B}{ABC}$

$\frac{\sin C}{C} = \frac{\sin A}{A} = \frac{\sin B}{B}$ +1

16) b) $10B / $10/hr = 1B hours $\times \frac{1\text{ day}}{24\text{ hrs}} = 4.2\times10^7$ days

$= 42{,}000{,}000$ days

4.2×10^7 days $\times \frac{1\text{ yr}}{365\text{ days}} = \boxed{1.1\times10^5\text{ yrs}}$

(ii) 1B hrs $\times \frac{1\text{ day}}{10\text{ hrs}} = 100M$ days

~~100M days $\times \frac{1\text{ yr}}{365\text{ days}} \times \frac{1\text{ person}}{10\text{ yrs}} = 3\times10^4$ people~~

$\frac{10\text{ hrs}}{1\text{ day}} \times \frac{\$10}{hr} = \$100/\text{day}$ $\quad \frac{\$100}{1\text{ day}} \times \frac{365\text{ days}}{1\text{ yr}} \times 10\text{ yrs} = \$3.65\times10^5$

$\frac{\$10B}{\$3.65\times10^5} = \boxed{27{,}000\text{ people}}$

(iii) → part (ii) answers ×10 $= \boxed{1.1\times10^6\text{ yrs}}$

$\boxed{270{,}000\text{ people}}$

20 blue

1) b) 10.2 mol , 6.02×10^{24} atoms 1

5) a) 1 light second = 3×10^{8} m 1

b) 8 light minutes = 1.4×10^{11} m 1

(6) b) (i) O(1M yrs)
(ii) 1M people
(iii) 10M yrs, 10M people] 5

(7) (ii) means his body is heavier (or other inconsistencies in data recording 1

1. A. ${}^{197}_{79}Au$: $M = 197\frac{g}{mole} = 0.197\frac{kg}{mole}$ *1 pt*

B. $\frac{2.0kg}{0.197\frac{kg}{mole}} = 10.2\ moles$ *1 pt*

$10.2 moles \times 6\times10^{23}\frac{atoms}{mole} = 6\times10^{24} atoms$ *1 pt*

Assumptions:

i. Each atom takes a cubic space (not so in general)

ii. We can pack them tightly (this is not so in general)

2.

$V = (2r)^3 = 2\bullet10^{-8}cm)^3 = 8x10^{-24}cm^3$

$N = \frac{1cm^3}{8x10^{-24}cm^3} = 1.25x10^{-23}atoms$ *1 pt each line*

3. $1\ sch\bullet yr = 182\ dy\frac{4}{dy}\frac{1.5hr}{bl} = 6\bullet182\ hrs$ *1 pt*

A. 0hrs--> 0yrs *1 pt*

B. $12yr\bullet364\frac{dy}{yr}2\frac{hrs}{dy} = 12\bullet364\bullet2hrs$

$\frac{12\bullet364\bullet2hrs}{6\bullet182\ hrs} = 8yrs$ *1 pt*

C. Traditionally: 4yrs=BS, MS=2yrs. *1 pt*

The student who studies extra two hours a day for the year will had BS+MS+2 yrs.

D. For you to appreciate the value of studying daily and regularly. *1 pt*

4. Assume an elephant and a squirrel are shaped like cylinders on their sides. They both have a density equal to that of waters--they both can stay a float.

A. $r = \frac{1}{2}m$, $\rho = 10^3\frac{kg}{m^3}$, $l = 3$ m *1 pt*

$\pi r^2 l$ $V = \cancel{2\pi rl} = 9.4m^3$, $M = \rho V = 9.4x10^3kg$ *1 pt*

B. r=3cm, $\rho = 1\frac{g}{cm^3}$, l=20 cm w/o the tail

$V = 2\pi rl = 565cm^3$ *1 pt*

$M = \rho V = 565\ g = 1.25\ lbs$ *1 pt*

5. (a) 1 light.second (seconds) *1 pt*

(b) 8 light.minutes (minutes) *1 pt*

(c) 3 light.hours (hours) *1 pt*

(d) 100,000 light.years (100 thousand years) *1 pt*

6. **A**=–4**i**+6**j** , **B**=2**i**+2**k**.

A. **A**+**B** = (-4+2)**i**+ 6**j** +2**k**=-2**i**+ 6**j** +2**k** *1 pt*

B. **A**–**B** = (-4-2)**i** + 6**j** - 2**k**=-6**i** 6 6**j** -2**k** *1 pt*

C. **A**•**B** = (-4•2)(**i**•**i**)+0+0=-8 *1 pt*

D. **A**x**B** = (-4•2)**i**x**k**+(6•2)**j**x**i**+(6•2)**j**x**k** *1 pt*

=8**j**-12**k**+12**i**

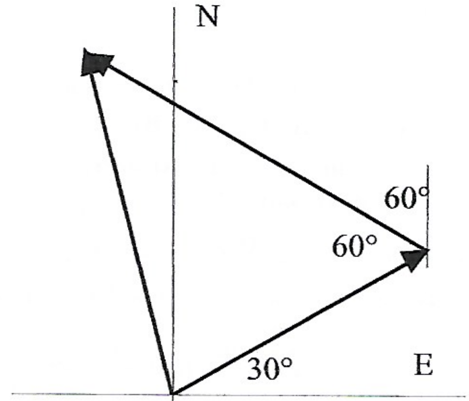
=12**i**+8**j**-12**k**

E. Use**A**•**B** = ABcos θ or |**A**x**B**|= ABsin θ *1 pt*

$\sqrt{(-4)^2+6^2}\bullet\sqrt{2^2+2^2}\cos\theta = -8$ *1 pt*

$\theta = 113°$

7.



1 pt

A. $R = \sqrt{30^2+40^2-2\bullet30\bullet40\bullet\cos60°} = 36\ km$ *1 pt*

$B^2 = R^2 + A^2 - 2\bullet R\bullet A\bullet\cos\theta$ *1 pt*

$\cos\theta = \frac{40^2-36^2-30^2}{-2\bullet36\bullet30} = 0.276$ *1 pt*

$\theta = 103°, \theta_N = 13.98°$ *1 pt*

B. $A_E = 30\bullet\cos30°km = 26km$ *1 pt*

$B_E = 40°\cos150°km = -40\bullet\sin60°km = -34.6km$ *1 pt*

$A_N = 30\bullet\sin30°km = 15km$ *1 pt*

$B_N = 40°\sin150°km = 20km$ *1 pt*

$R_E = A_E + B_E = -8.6km$, $R_N = A_N + B_N = 35km$

$R = \sqrt{R_E^2 + R_N^2} = 36\ km$ *1 pt*

$\tan\theta_N = \left|\frac{R_E}{R_N}\right| = 0.246$, $\theta_N = 13.8°$ *1 pt*

8. (A) $\sqrt{1^2+1^2+1^2} = \sqrt{3}$ *1 pt*

(B) $\sqrt{1^2+1^2+1^2} = \sqrt{3}$ *1 pt*

9. $1\bullet\sqrt{1^2+1^2+1^2}\cos\theta = \hat{i}\bullet\hat{i} + 0 + 0$ *1 pt*

$\cos\theta = \frac{1}{\sqrt{3}}$, $\theta = 55°$ *1 pt*

10. $\vec{C} = \vec{A} + \vec{B}$

$C^2 = (\vec{A}+\vec{B})\bullet(\vec{A}+\vec{B}) = A^2 + B^2 + 2\vec{A}\bullet\vec{B}$ *1 pt*

$C^2 = A^2 + B^2 + 2AB\cos\theta$ *1 pt*

11. (A) **A.A**=**i.i**$\cos\alpha\cos\alpha$ +**j.j**$\sin\alpha\sin\alpha$

+**i.j**$\cos\alpha\sin\alpha$ +**j.i**$\sin\alpha\cos\alpha$ *1 pt*

$A^2 = \cos^2\alpha + \sin^2\alpha = 1$ *1 pt*

Similarly $B^2 = \cos^2\beta + \sin^2\beta = 1$ *1 pt*

(B)**A.B**=**i.i**$\cos\alpha\cos\beta$ +**j.j**$\sin\alpha\sin\beta$

+**i.j**$\cos\alpha\sin\beta$ +**j.i**$\sin\alpha\cos\beta$ *1 pt*

$AB\cos(\alpha-\beta) = \cos\alpha\bullet\cos\beta + \sin\alpha\bullet\sin\beta$ *1 pt*

A=B=1; $\cos(\alpha-\beta) = \cos\alpha\bullet\cos\beta + \sin\alpha\bullet\sin\beta$ *1pt*

12. Use their vector product: **A** x **B** *1 pt*

If they are parallel, **C** that gives **C**•**A**=**C**•**B**=0. *1 pt*

Mesut B. Çakır

13. a. $3x^2 - 2x^{-3} + \frac{2}{5}x^{-3/5}$ 1 pt

b. $\frac{1}{4}x^4 - x^{-1} + \frac{5}{7}x^{7/5} + C$ 1 pt

14. **C=A-B**, $C^2 = A^2 + B^2 - 2AB\cos\alpha$ 1 pt

Area=$|\mathbf{A}\times\mathbf{B}| = |\mathbf{A}\times\mathbf{C}| = |\mathbf{C}\times\mathbf{B}|$ 1 pt

$= AB\sin\gamma = AC\sin\beta = AB\sin\alpha$ divide by ABC

$\sin\gamma/C = \sin\beta/B = \sin\alpha/A$ 1 pt

15. in $\otimes$, 0, left $\leftarrow$, out $\odot$, left $\leftarrow$ *1 pt each*

16. Lets start by calculating the number of seconds in a year. $365 days \times 24 \frac{hours}{days} \times 60 \frac{minutes}{hour} \times 60 \frac{s}{hours} = 3\times10^7 s$

We will approximate this to $1\times10^7 s$ to simplify our calculations.

A.

(i) $\$10 billion/10 years = \$1 billion/ year = \$100/s$ 1 pt

(ii) \$6,000/min. Order of mag=\$ 1 thousand 1 pt

(iii) \$360,000/hour. Order of mag=\$ 100 thousand 1 pt

(iv) \$8.64 million/day. Order of mag=\$ 10 million 1 pt

B.

(i) $\$10 / hour = \$3\times10^{-3}/s = \$30,000/ year$ 1 pt

In **one thousand years**, s/he will make \$30 million, and in **one million years** s/he will make \$30 billion. S/he has to work in the order of one million years every single hour of his/her day. 1 pt

(ii) One person makes $\$10,000/ year$ working 10 hours per day. Therefore, we need one million people to make \$10 billion. 1 pt

(iii) This is one thenth of the minimum wage. Therefore, we need 10 times more. The answers are
Ten million years and 1 pt

Ten million people (40 million in Bangladesh)1pt

In other words, whole nations of people have worked to acquire this wealth for the individual in question.

(iv) \$100,000 is 10 times more than the minimum wage yearly total; therefore, we need one tenth of (i) and (ii) The answers are

***Only* 100 thousand years for a professor or 200 thousand years for a lecturer.** 1 pt

***Only* 100 thousand professors or 200 thousand lecturers need to work for one year or**
***Only* 10 thousand professors or 20 thousand lectureres need to work for ten years** 1 pt

In other words, we cannot pay people "decent" living wages if we want to acquire "obscene" amount of wealth.

Remember, these are only order of magnitude calculations..

17. $3kg \times 7700 \frac{Cal}{kg} = 23{,}100 Cal$. 1 pt

Divide this by 14 days, to get the daily consumption 1,650 *Cal* 1 pt

(i) 1,100 Cal for run and 550 Cal for swim. 1 pt

Divide this by 7 to get the Calories per mile 1 pt

157 Cal per mile of run and 550 Cal per km of swim.

(ii) These are 30-50% more than what would be for a 60-kg person. Assuming a direct corrolation between mass and energy consumed, we would conlclude I am about 50% more than 60 kg. My mass is 78 kg. The over estimation can be due to several reasons, such as that I lost less than 3 kg (e.g. my measurements had some inconsistencies such as when I measured my mass or weather there was too much or too little liquid in my body, etc.), that my eating habits were different during the time frame (I may have consumed less calories than usual), that calories per mile and per km are different from stated (see below), my body density (that I am denser as a result display much more watter while swimming or experience more drag force hence loose more energy), etc.

With better data and information, we can do more precise calculations than this. 1 pt

According to some website (I am not sure how reliable it is, it seems to make some sense)

	Running	Swimming Fast	Slow
130lb	103 Cal/mile	590 Cal/hour	413
155lb	123 Cal/mile	704 Cal/hour	493
180lb	143 Cal/mile	817 Cal/hour	572
205lb	163 Cal/mile	931 Cal/hour	651

13) a) $4x^3 + (-3x^{-4}) + \frac{3}{5}x^{-\frac{2}{5}}$

$\frac{1}{5}x^5 + \frac{x^{-2}}{-2} + \frac{x^{8/5}}{\frac{8}{5}} + C$

$\frac{1}{5}x^5 - \frac{1}{2}x^{-2} + \frac{5}{8}x^{8/5} + C$

Mesut B. Çakır