Math Review 1-2

Black 30 Blue 4
Black + Blue 34 Add 4
total 38

Intro:

1. distance → velocity → acceleration
   ↳ $\frac{dx}{dt}$ ↗ ↳ $\frac{d^2x}{dt^2}$ ↗

Vel. → rate of change of distance
Accel. → rate of change of velocity

Momentum → force
↳ $\frac{dp}{dt}$ ↗

Force → rate of change of momentum

2-) acceleration → velocity → distance
integrate

$W = \int \vec{F} \cdot d\vec{r}$

3) $\frac{d}{dx}(ax^m + bx^k) = max^{m-1} + bkx^{k-1}$

a) $\int ax^m + bx^k \, dx = \frac{a}{m}x^{m+1} + \frac{b}{k}x^{k+1} + C$

b) $\frac{d}{dt}(a\cos nt + b\sin kt)$
$= -an\sin nt + bk\cos kt$

$\int a\cos nt + b\sin kt \, dt = \frac{a}{n}\sin nt - \frac{b}{k}\cos kt$

c) $\frac{d}{dx}(ae^{nt} + be^{-kt})$
$= nae^{nt} - bke^{-kt}$

$\int ae^{nt} + be^{-kt} \, dt$
$= \frac{a}{n}e^{nt} - \frac{b}{k}e^{-kt} + C$

Vectors

1) $\vec{A} = (2,-3,0)$ $\vec{B} = (-4,2,0)$

a) $\vec{A} + \vec{B} = -2\hat{i} - \hat{j} = (-2,-1,0)$ +1
$|\vec{A}+\vec{B}| = \sqrt{(-2)^2+(-1)^2}$ @ $\tan^{-1}\left(\frac{-1}{-2}\right)$
$\sqrt{5} = 2.23$ @ 27°

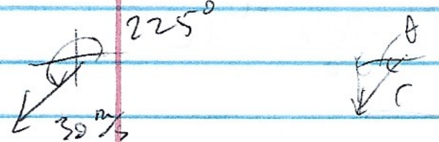
b) $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$
$= (2+4)\hat{i} + (-3-2)\hat{j}$
$= 6\hat{i} - 5\hat{j} = (6,-5,0)$ +1
$|\vec{A}-\vec{B}| = \sqrt{(6^2)+(-5)^2}$ @ $\tan^{-1}\left(\frac{-5}{6}\right)$
$\sqrt{61} = 7.8$ @ -39.8° +1

c) $\vec{A} + 3\vec{B} = (2+(3\cdot-4))\hat{i} + (-3+2(3))\hat{j}$
$= -10\hat{i} + 3\hat{j} = (-10,3,0)$
$|\vec{A}+3\vec{B}| = \sqrt{(10^2)+(3^2)}$ @ $\tan^{-1}\left(\frac{3}{-10}\right)$ +1
$\sqrt{109} = 10.4$ @ -17° +1

d) $A \cdot B = B \cdot A = |A||B|\cos(\theta_A - \theta_B)$
$\left(\sqrt{2^2+3^2}\right)\left(\sqrt{4^2+2^2}\right)\cos\left(\tan^{-1}\left(\frac{-3}{2}\right) - \tan^{-1}\left(\frac{2}{-4}\right)\right)$
$= 14$ -14

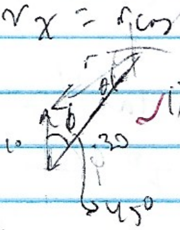
e) $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$ ~

2) 225°



$v_y = r\sin\theta$ $= 30\sin(225) = -21\ m/s\ \hat{j}$ +1

$v_x = r\cos\theta = 30\cos(225) = -21\ m/s\ \hat{i}$ +1



B) ~1) $r = \sqrt{A^2+B^2-2AB\cos\phi}$

$= \sqrt{10^2+30^2-2(10)(30)\cos(45)}$

$= 24\ m/s$ ✓ +1

~2) $r = A\frac{\sin c}{\sin a} = B\frac{\sin c}{\sin B}$

3) $r = -11\hat{j} - 21\hat{i}$

$|r| = \sqrt{11^2+21^2} = 24\ m/s$

$\theta = \tan^{-1}\left(\frac{-11}{-21}\right) = 207^\circ$ ✓ +1

4) $\vec{r} = 24\ m/s$ @ 28° S of E — S of W, or 218 w/E

5) $\vec{r} = -21\hat{i} - 11\hat{j} + 0\hat{k}$ ✓ +1

kinematics

(2) A) It starts slowing down ✓ +1

b) (0, 1): $\bar{v} = 10\ m/s$ ✓ +1

(1, 7): $\bar{v} = 5\ m/s$ ~

[1, 5] $\bar{v} = 10\ m/s$

7

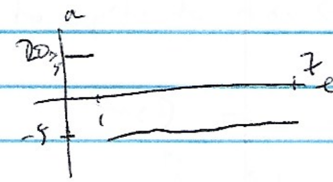
c) $\Delta x = \int v\,dt$

$= (1)(20)(\tfrac{1}{2}) + (4)(20)(\tfrac{1}{2})$

$-(2)(10)(\tfrac{1}{2})$

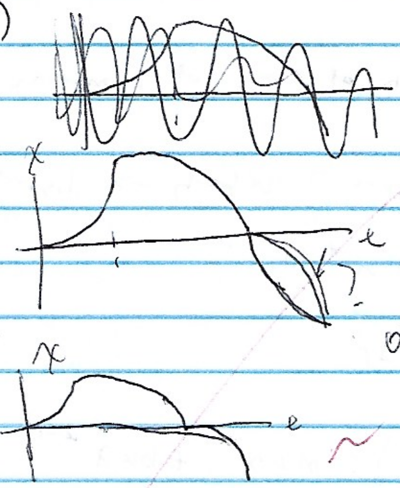
$= 10 + 40 - 10 = 40\ m$ ✓ +1

d) $a = \frac{dv}{dt}$



✓ +1

e)



(curving + starts at x=0) ~

~

$v = \frac{dx}{dt}$

$dx = v\,dt$

$x = vt$

$v = at = \frac{dx}{dt}$

(3)

$v_{0y} = v_0\sin\theta_0$ $v_{0x} = v_0\cos\theta_x$

$v_y^2 = v_{0y}^2 + 2a_y h_{max}$

2

$-v_{0y}^2 = 2a_y h_{max} = 2g h_{max}$

$h_{max} = \frac{-v_0^2 \sin^2\theta_0}{2g}$ ✓ Sign convention

+1

b) $R = v_{0x} t$ $\quad v_{fy} = v_{0y} + g t_{up}$

$t_{up} = \frac{-v_{0y}}{g}$

$t = \frac{-2v_{0y}}{g} = \frac{-2v_0 \sin\theta}{g}$

$R = v_0 \cos\theta_0 \cdot \frac{-2v_0 \sin\theta}{g}$

$= \frac{-2v_0^2 \sin\theta\cos\theta}{g}$] can simplify more alg.

c) $R = \frac{-2v_0^2 \sin\theta\cos\theta}{g}$

maximize $\sin\theta\cos\theta$ $\quad 0 < \theta < 90$

~~$\frac{d}{d\theta}\sin\theta\cos\theta = \ldots$~~ $-\sin^2\theta + \cos^2\theta$

~~this~~ $\sin\theta\cos\theta$ is largest

when $\theta = 45$ ✓ +1

D) $h = v_{0y} t + \frac{1}{2} a_y t^2$

$v_0 \sin\theta\, t + \frac{1}{2} g t^2 - h = 0$

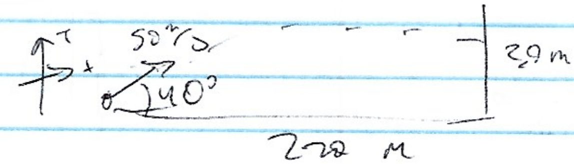
~~$t = \frac{-v_0\sin\theta \pm \sqrt{v_0^2 \sin^2\theta + 2h(\ldots)}}{g}$~~

$\frac{1}{2} g t^2 + v_0 \sin\theta\, t - h = 0$

2

$t = \frac{-v_0 \sin\theta \pm \sqrt{v_0^2 \sin^2\theta + 2gh}}{g}$] simplify

(4)

A) 

from (13), ~~$R = \frac{-2v_0^2 \sin\theta\cos\theta}{g}$~~

~~$= \frac{-2(50\,m/s)^2 \sin 40 \cos 40}{-9.8\,m/s^2}$~~

$h = v_{0y} t + \frac{1}{2} a_y t^2$

$v_f^2 = v_0^2 + 2a\Delta x$

$v_{fy} = \sqrt{v_{0y}^2 + 2g\Delta y}$

b) $v_x t = \Delta x = R$

$t = \frac{R}{v_{0x}} = \frac{R}{v_0 \cos\theta} = \frac{220\,m}{50\,m/s \cos 40}$

$= 5.7\,s$ ✓ +1

a) $h_y = v_{0y} t + \frac{1}{2} a_y t^2$

$= (50\,m/s \sin 40)(5.7\,s) + \frac{1}{2}(-9.8\,m/s^2)(5.7\,s)^2$

$= 24.0\,m$ ✓ +1

↳ $h_{wall} = 30\,m$

↳ shorter

c) ↗

$\frac{1}{2} a_y t^2 + v_{0y} t - 30\,m = 0$

d) $h_y(t) = v_{0y} t + \frac{1}{2} a_y t^2 = 30\,m$

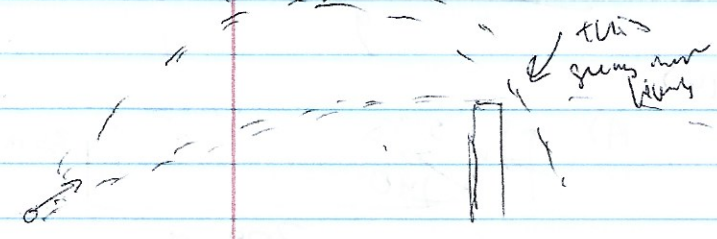
$t = \frac{-v_{0y} \pm \sqrt{v_{0y}^2 + 4 \cdot 30\,m \cdot \frac{1}{2} a_y}}{a_y}$

$= \frac{-(50 \sin 40) \pm \sqrt{(50 \sin 40)^2 + 4(30)(\frac{1}{2})(-9.8)}}{-9.8\,m/s^2}$

$= -34.3 \ldots$ ~~-2000~~

2

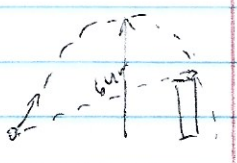
d) if it barely clears the wall,



If $\theta = 45°$, $h_{max} = \frac{-v_0^2(\sin\theta)^2}{2g}$

$= \frac{-(50\ m/s)^2(\sin^2(45°))}{2(-9.8\ m/s^2)}$

$= 64\ m$



if $h_{max} = 30_y$ $\theta = 29°$

if $\theta = 29°$, $R = \frac{-2v_0^2 \sin\theta\cos\theta}{g}$

$= 216\ m$ → hits the ground early

If $\theta = 45$, $R = 255$

$\frac{1}{2}a_y t^2 + v_{0y} t - 30m = 0$

$-4.9t^2 + 50\sin\theta\, t - 30m = 0$

$v_{f_y}^2 = v_{0_y}^2 + 2a\Delta y$

$\Delta x = v_0 t = v_0 \cos\theta\, t$

$= 50\cos\theta\, t = 220\ m$ +1

$t = \frac{220\ m}{50\cos\theta}$

$-4.9\left(\frac{220\ m}{50\cos\theta}\right)^2 + 50\cdot\frac{220\ m}{50\cos\theta}\cdot\sin\theta = 30$

$\sec^2\theta(-94.9) + 220\tan\theta = 30$

$\theta = 44.8,\ 53.0$ ✓ +1

2

1) uniform circular motion

II, I (B) × (A)

2) 

(E) × (F)

3) I - False; change direction

II - speed or velocity? true

III - False; constant acc

(c) ✓ +1

4) (c) - always g ✓ +1

5) $\Delta x = v_0 t + \frac{1}{2}at^2$

$t = \sqrt{\frac{2\Delta x}{a}} = \sqrt{\frac{2(20\ m)}{5\ m/s^2}}$

$= 2.9\ s$ (A) ✓ +1

6) $v_f = v_0 + at$ $v_f^2 = v_0^2 + 2a\Delta x$

$\Delta x = \frac{2v_f^2}{a} = \frac{2(30\ m/s)^2}{(4.5\ m/s^2)}$

(D) $= 180\ m$ ~

7) $v_{f_y} = v_{0_y} + at$ $v_f^2 = v_0^2 + 2a\Delta y$

$t = \frac{v_{f_y}}{a} = \frac{\sqrt{2a_y}}{a} = \frac{\sqrt{2(9.8\ m/s^2)(80\ m)}}{9.8\ m/s^2}$

$= 4\ s$ (C) ✓ +1

8) $v_{f_y} = v_{0_y} + at$

$t_{up} = \frac{-v_{0_y}}{a} = \frac{-v_0\sin\theta}{g}$

$t_{total} = \frac{-2v_0\sin\theta}{g} = \frac{-2(10\ m/s)\sin 30}{-9.8\ m/s^2}$

$= 1.0\ s$ (b) ✓ +1

5

9) $v_{kx} = v_{0x} = 30\ m/s$

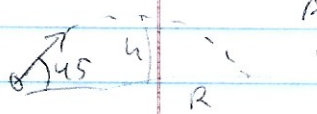
$t = 4s$

$v_{ky} = v_{0y} + a_y t$

$v_{ky} = (9.6\ m/s^2)(4s) = 39.2\ m/s$

$v_{tot} = \sqrt{39.2^2 + 30^2} = 50\ m/s$ (C) ✓ +1

10)



A) False
B) false
C) false
D) false
E) true ✓ +1

(1) $x(t) = 7 + 10t - 6t^2$

$v(t) = x'(t) = 10 - 12t$

$v(4) = 10 - 12(4)$

$= -38\ m/s$ (E)? +1

(5) $a(t) = 6t\ m/s^3$

$v_0 = 2\ m/s \quad x_0 = 4m$

A) $v(t) = \int a(t)\,dt$

$= \int 6t\,dt = 3t^2 + C$

$C = v(0) = 2\ m/s$

$v(t) = 3t^2 + 2\ m/s$

$3t^2 + 2 = 14\ m/s$

$t^2 = 4 \quad \boxed{t = 2s}$ ✓ +1

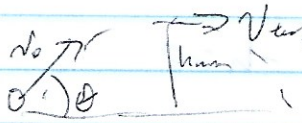
B) $x(t) = \int v(t)\,dt = \int 3t^2 + 2\,dt$

$= \frac{3}{3}t^3 + 2t + C \quad C = x(0) = 4$

$x(t) = t^3 + 2t + 4$ 4

$x(3) = 27 + 6 + 4 = \boxed{37m}$ ✓ +1

16)



A) (1)
B) (1)
C) (4), (5)
D) (1) 1

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1) d) $A \cdot B = B \cdot A$

$= AB\cos\theta$

$\cos\left(\tan^{-1}\left(\frac{-3}{2}\right) - \tan^{-1}\left(\frac{2}{-4}\right)\right)$ (unsimplified)

$= 0.868$

$\cos\left(\tan^{-1}\left(\frac{2}{-4}\right) - \tan^{-1}\left(\frac{-3}{2}\right)\right)$

$= 0.868$

when does the (-) come from?

c) $A \times B = (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$

$= (A_2 B_3 - A_3 B_2)\hat{i}$

$= (2.2 - (-3)(-4))\hat{i}$

$= \boxed{-8\hat{i}}$ ✓ +1

$\vec{B} \times \vec{A} = -\vec{A} \times \vec{B} = \boxed{8\hat{i}}$ ✓ +1

2

(13) b) $\Delta y = v_{0y} t + \frac{1}{2} g t^2 = h$

$t^2\left(\frac{1}{2}g\right) + v_{0y} t - h = 0$

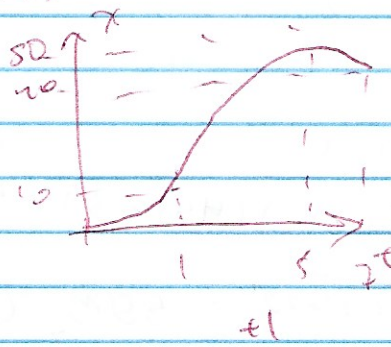
$t = \frac{-v_{0y} \pm \sqrt{v_{0y}^2 + 2gh}}{g}$ $\quad b^2 - 4ac$ / $2a$

$\Delta t = \frac{-v_{0y} + \sqrt{v_{0y}^2 + 2gh}}{g} + \frac{+v_{0y} + \sqrt{v_{0y}^2 + 2gh}}{g}$

$= \frac{2\sqrt{v_{0y}^2 + 2gh}}{g}$ (sign convention) ✓ +1

1

(12) e)



+1

Vectors 2B) 1, 2

1) $\frac{v_e}{\sin\beta} = \frac{v_p}{\sin\alpha} = \frac{v_r}{\sin 45}$

$v_R^2 = v_p^2 + v_w^2 - 2 v_p v_w \cos 45$

$v_R = \sqrt{900 + 100 - 2 \cdot 30 \cdot 10} = 24\ m/s$

$\sin\beta = \frac{v_w}{v_R} \sin 45 = 0.236$

$\beta = 14^\circ$ w/ E ✓ +1

$\theta_E = 211^\circ$, $\theta = 31^\circ$, $\theta_E = 210^\circ$; std +1

Vec. 1) $A \cdot B = B \cdot A = -14$ ✓ +1 3

(13) b) $-2\frac{v_0^2 \sin\theta \cos\theta}{g}$

$= -\frac{v_0^2 \sin 2\theta}{g}$ ✓1

1