2012 MCM

2:08 2:32 2:37 2:48 31/35

1) (e) ✓ 2) (d) ✓ 3) (d) ✓

4) (e) ✓ 5) $x_{cm} = \frac{5(3)+2(5)}{10}$

$= \frac{15+10}{10} = \frac{25}{10} = 2.5$ (d) ✓

6) $kg \cdot \frac{m}{s}$ $N = kg \cdot \frac{m}{s^2}$ (d) ✓

7) $\cancel{v_f^2 = v_0^2 + 2a\Delta x}$

$v_f = \cancel{v_0} + at$ $d = \cancel{v_0 t} + \frac{1}{2}at^2$

$2t \to 4d$

$d_B = 4d_A$ (a) ✓

8) 100N $F = \frac{mv^2}{r}$

$v^2 = \frac{Fr}{m}$

$v = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{100 \cdot 2}{0.5}}$

$= \sqrt{400} = 20$ (d) ✓

9) (a) ✓ 10) $F = ma = m\frac{dv}{dt}$

$m\frac{dv}{dt} = bt$

$m\,dv = bt\,dt$

$mv = \frac{1}{2}bt^2$

$v = \frac{bt^2}{2m}$ (b) ✓

11) $F = m_{total}a$

$= (6\,kg)(2\frac{m}{s^2}) = 12N$ (e) ✓

12) $P = F\bar{v} = \frac{W}{t} = \frac{mgh}{t}$

$t = \frac{mgh}{P} = \frac{(0.1)(10)(2)}{0.5} = 4$ (e) ✓

13) $mgh = \mu mg d$

$d = \frac{h}{\mu} = \frac{5}{0.2} = 25$ (e) ✓

14) (a) ✓ 15) $\frac{dv}{dt} = 6t - 2$

$t=2$ $a = 10$ (d) ✓

16) (b) ✓ 17) $x = A\sin(\omega t)$

$T = 1\,s$

$f = \frac{1}{T} = 1\,Hz$

$\omega = 2\pi f = 2\pi$

$f = 2\pi\sqrt{\frac{m}{k}} = 1$ $\sqrt{\frac{m}{k}} = \frac{1}{2\pi}$

$\frac{k}{m} = 4\pi^2$

$k = 4\pi^2 m = 16\,\frac{N}{m}$

(a) ✓

18) $F = \frac{dp}{dt}$ $\Delta p = F\Delta t$

(c) ✓

19) $\frac{1}{3}ml^2$ $\frac{1}{3}\left(\frac{m}{4}\right)\left(\frac{l}{4}\right)^2 + \frac{1}{3}\left(\frac{3m}{4}\right)\left(\frac{3l}{4}\right)^2$

$= \frac{ml^2}{192} + \frac{27ml^2}{192} = \frac{28ml^2}{192}$

$= \frac{7}{48}ml^2$ (e) ✓

20) $U = -\int F\,dr = +k\int x^3\,dx$

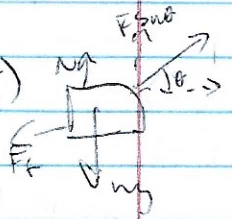
$W = -\Delta PE$

$= \frac{+k}{4}x^4$ (d) ✓

21) $m\frac{dv}{dt} = -kx$ (b) ✓

22) (c) 23) (b) 24) $F = \frac{GMm}{r^2}$ (a)

25)

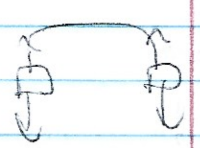


$N + F\sin\theta = mg$

$N = mg - F\sin\theta$

$F_f = \mu N$ (e)

26) $\Sigma F = ma$ $T = mg = 40N$ (d)



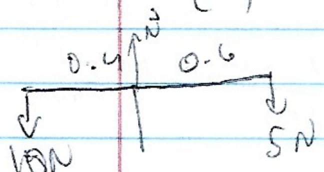
27) $\frac{1}{2}mv^2 = F\Delta x$

$F = \frac{mv^2}{2d} = \frac{(0.2)(30)^2}{2(0.9)} = 100N$ (d)

28) $(1500\,kg)(4\,m/s) - (3000\,kg)(3\,m/s) = (4500\,kg)(v_f)$

$\xrightarrow{+}$ $\frac{2}{3}$ (a)

29)



$\Sigma\tau = I\alpha = 10\cdot 0.4 - 0.6\cdot 5$

$= 4 - 3.0 = 1$ (a)

30) $F = kx = ma$

$a = \left(\frac{k}{m}\right)x$ (b) a

31) $\Delta x = \cancel{v_0 t} + \frac{1}{2}at^2$ $\uparrow +$

$d_1 = -\frac{1}{2}gt^2$ $d_1 = h - d_2$

$d_2 = v_0 t + \frac{1}{2}gt^2$ $h - v_0 t - \cancel{\frac{1}{2}gt^2} = -\cancel{\frac{1}{2}gt^2}$

$h = v_0 t$ $t = \frac{h}{v_0}$ (c)

32) $\frac{1}{2}kx^2 - ax = 0$

$x\left(\frac{1}{2}kx - a\right) = 0$

$x = 0, \frac{+2a}{k}$ (e) b

33) $J = \Delta p = m\Delta v = m(v_f - v_0)$ (b)

34) (c) 35) $\frac{dy}{dx} = \frac{15\cos(3t)}{-15\sin(3t)}$

$= -\cot(3t)$ (c) b

-3

2012 FRM

7:49
7:24

38/45

1) ~~$v(t) = A\omega\cos(\omega t)$~~
a) ~~$x(t) = A\sin(\omega t)$~~

$x(t) = A\cos(\omega t)$

$v(t) = -A\omega\sin(\omega t)$ +1

$A\omega$ = amplitude = 1.6 m/s

$T = 0.7$ s +1 $f = \frac{1}{T}$ $\omega = 2\pi f$ +1

$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.7} = 9.0\ \frac{1}{s}$

$\boxed{v(t) = -1.6\sin(9.0t)}$ -1

-0.16

b) $x(t) = A\cos(\omega t)$ +1

$A\omega$ = amplitude = 1.6 m/s

$A = \frac{amp.}{\omega} = \frac{1.6\ m/s}{9.0\ 1/s} = 0.18$ m

+1 $x(t) = \boxed{0.18\cos(9.0t)}$

consistently

0.018

c) $\frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$

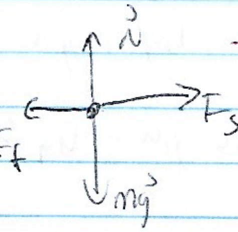
$k = \frac{\frac{1}{2}mv_{max}^2}{A^2}$ +1 COE

$= \frac{\frac{1}{2}(0.38\text{ kg})(1.6\ m/s)^2}{(0.18\text{ m})^2}$

$= \boxed{12\ N/m}$ -1 wrong

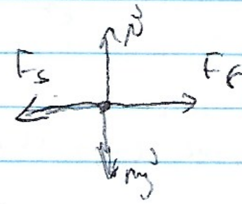
-2

(d) (i) toward equil.



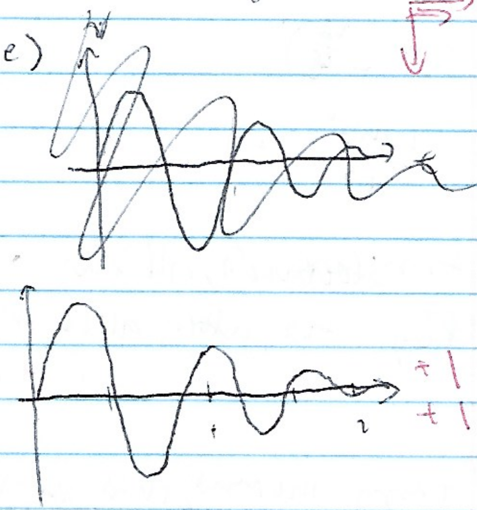
+1
+1
+1

(ii) away from equil.



-1

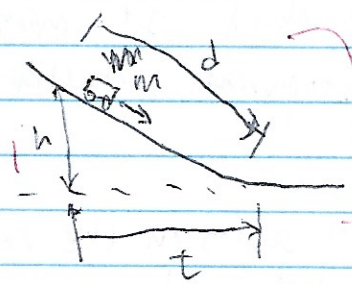
e)



+1
+1 +1

2) track, cart, meterstick, stopwatch,
a) ~~balance~~, balance. +1

b)



+1

+1

place the cart at varying heights on the track, h, and time how long it takes to get to the bottom. +1 +1 +1
measure the distance along the track.
measure the ~~mass of the cart w/ the balance~~

consid. the initial potential

(c) energy is given by mgh. +1

the final energy is given by $\frac{1}{2}mv^2$, kinetic energy.

$v_f = v_0 + at$ $\quad d = v_0 t + \frac{1}{2}at^2$

$a = \frac{2d}{t^2}$

$v_f = \frac{2d}{t}$ +2

thus, the final kinetic energy is given by

$KE = \frac{1}{2}m\left(\frac{2d}{t}\right)^2$

$= \frac{2md^2}{t^2}$ +1

before the transformation, all the energy is PE_g, and after, all of it is KE. +1 +1

(d) If the energy increased, then maybe either the distance down the ramp was measured too large or the time was measured too short. It's more likely that the stopwatch was clicked too early. +2

e) There is friction acting on the axles of the cart's wheels, which dissipates some of the energy b/c it is a nonconserving force. There is also rotational KE of the wheels that we aren't taking into account. +2

#0

3) a) $\sum F = ma = -F_f = M\frac{dv}{dt}$ +1

$-\mu Mg = M\frac{dv}{dt}$ $\quad$ a(i) $\boxed{-\mu g = \frac{dv}{dt}}$ +1

$dv = -\mu g\, dt$ $\quad v = -\mu g t + C$

b(i) $\boxed{v = v_0 - \mu g t}$ +1 $\quad C = v_0$

-1

a(ii) $\sum \tau = I\alpha = I\frac{d\omega}{dt} = F_f R$ +1

integration implied

$\mu M g R = MR^2 \frac{d\omega}{dt}$ +1

$\boxed{\frac{\mu g}{R} = \frac{d\omega}{dt}}$ +1

-1

b(ii) $\boxed{\omega = \frac{\mu g t}{R}}$ $\quad \omega = \frac{\mu g t}{R} + C$ $\quad C = 0$ $\quad \omega_0 = 0$

+1

c) $v = v_0 - \mu g t$

at $d = L$,

$v = R\omega$

$R\omega = v_0 - \mu g t$

$\mu g t = v_0 - R\omega$

$t = \frac{v_0 - R\omega}{\mu g}$

$\omega = \frac{\mu g t}{R} = \frac{\mu g}{R}\left(\frac{v_0 - R\omega}{\mu g}\right)$

$= \frac{v_0}{R} - \omega = 0$ +1

$\omega = \frac{v_0}{2R}$

$t = \frac{v_0 - \frac{v_0}{2R}\cdot R}{\mu g}$

$= \boxed{\frac{v_0}{2\mu g}}$ +1

-2

d) $v = R\omega = \cancel{R} \cdot \frac{\mu g t}{\cancel{R}} = \mu g t$

$= \mu g \left(\frac{v_0}{2\mu g}\right) = \boxed{\frac{1}{2} v_0}$ +1

(e) $\frac{1}{2} m v_0^2 = \mu M g L + \frac{1}{2} m v_f^2 + \frac{1}{2} I \omega^2$

$\mu \cancel{M} g L = \frac{1}{2} \cancel{M} v_0^2 - \frac{1}{2} \cancel{M} \left(\frac{v_0}{2}\right)^2 - \frac{1}{2} (\cancel{M} R^2) \left(\frac{v_0}{2R}\right)^2$

$\mu g L = \frac{1}{2} v_0^2 - \frac{1}{8} v_0^2 - \frac{1}{2} \left(\cancel{R^2} \cdot \frac{v_0^2}{4\cancel{R^2}}\right)$

$= \frac{1}{4} v_0^2 = \mu g L$

$\boxed{L = \frac{v_0^2}{4\mu g}}$ -2

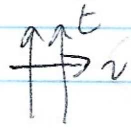
-2

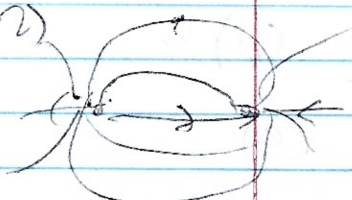
2012 E MC

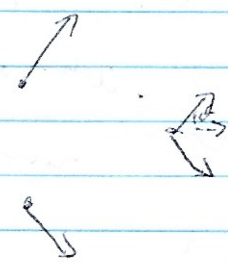
9:19 9:50

10:04

25/35

1)  (d) ✓

2)  (C) ✓

3) $F_1 = \frac{kQq}{a^2}$ 

$F_2 = \frac{kQ_c}{a^2}$

$2F\cos\theta$ (e) ✓

4) $\frac{kQq}{a} + \frac{kQ_a}{a}$ (d) a

5) (d) ✓ 6) $\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$

$I = \frac{V}{R}$ (d) ✓

7) (b) ✓ 8) (b) ✓ 9) (a) ✓

10) $C = QV$ $V = \frac{C}{Q}$ $C = \frac{k\varepsilon_0 A}{d}$

III, I, IV (C)

10) $V = \frac{Q}{C}$ $\frac{Qd}{\varepsilon_0 A}$ II, I, III

(b) ✓

11) $V = Ex = \frac{Q}{C}$

$E = \frac{Q}{Cd} = \frac{Q}{\varepsilon_0 A}$ (e) d

-2

12) $0 = ax - bx^2$

$x(a - bx)$

$x = \frac{a}{b}$ (C) b

13) $BA\cos\theta$

$= 1.2 \cdot 0.2 \cdot 0.1 \cos 30$

$=$ (C) b

14) (a) ✓ 15) (a) e

16) ~~I~~ II ~~III~~ (b) ✓

$P = I^2 R$

$P = \frac{V^2}{R}$

17) (a) ✓ 18)

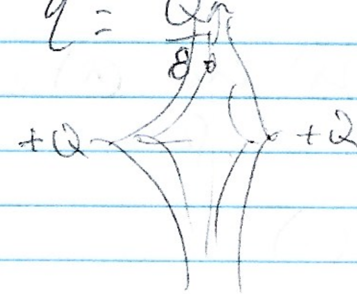
$\frac{F}{2} = \frac{kQ^2}{l^2} - \frac{kQq}{(\frac{l}{2})^2}$ $F = \frac{kQ^2}{l^2}$

$\frac{kQ^2}{2l^2} = \frac{kQ^2}{l^2} - \frac{4kQq}{l^2}$

$\frac{1}{2}Q^2 = Q^2 - 4Qq$

$4Qq = \frac{Q^2}{2}$

$q = \frac{Q}{8}$ (d) ✓

19)  (C) b

-4

20) $V = Er$

$E = \frac{V\uparrow}{r\downarrow}$ $\quad \uparrow V = \frac{Q}{C\downarrow}$ $\quad C = \frac{\kappa\epsilon_0 A}{d}$

(d) ✓

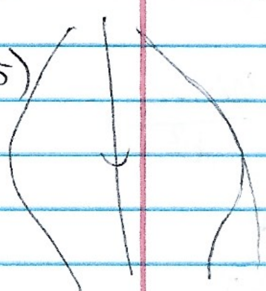
21) $P = I^2R = \frac{V^2}{R} = \left(\frac{V}{2}\right)^2\frac{1}{R}$

$= \frac{1}{4}\frac{V^2}{R}$

$P = I^2R = \frac{V^2}{R} = \frac{V^2}{R}$ (e) ✓

22) (e) ✓ $F \otimes I$

23) (b) ✓ 24) (e) ✓

25)  (d)? ✓

26) $\mathcal{E} = \frac{dB}{dt}A$ $\quad \frac{dB}{dt} \alpha t$ (b) ✓

27) (b) ✓ 28) $\frac{kQ}{R}$ (a) ✓

29) $\frac{1}{2}LI^2$ (c) ✓

30) (c) ✓

−1

31) $\tau = RL = \frac{L}{R}$? $\frac{R}{L}$? (c) e

32) $Q =$ $\quad V = \frac{Q}{C}$ (d) b

33) 2 in series (d) c

34) (b) ✓ 35) $\oint B \cdot dl = \mu_0 I_{enc}$

$\mu_0 I \cdot \frac{\text{area of wedge}}{\text{area of circle}}$

$A = \pi r^2$ $\quad A_p = \pi r^2 \cdot \frac{\theta}{2\pi}$

$\frac{\theta \pi r^2}{2 \cdot \pi r^2}$ $\quad = \frac{\theta r^2}{2}$

(a) ✓

−3

2012 E FR

10:02 10:20 10:25

36/45

1) a) $\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$ +1

$Q_{enc} = Q_i$

$E(r) \cdot 4\pi r^2 = \frac{Q_i}{\varepsilon_0}$ +1

$$\boxed{E(r) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_i}{r^2} \quad 0.10m < r < 0.20m}$$ +1

b) $\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$

$E(r) 4\pi r^2 = \frac{Q_0 - Q_i}{\varepsilon_0}$

$$\boxed{E(r) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_0 - Q_i}{r^2} \quad r > 0.20m}$$ -2

c) $V(r) - V(0) = -\int E \cdot dr$ +1

$V(r) = V(0) - \int \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_0 - Q_i}{r^2} dr$

$= V_0 + \frac{1}{4\pi\varepsilon_0} \frac{Q_0 - Q_i}{r}$

$V_0 = 0$?

$$V(r) = \frac{1}{4\pi\varepsilon_0} \frac{Q_0 - Q_i}{r}$$ -1

d) $\Delta V = 200V$

$\Delta V = -\int_{0.10m}^{0.20m} E \cdot dr$

$= -\int_{0.10m}^{0.20m} \frac{kQ_i}{r^2} dr = \frac{kQ_i}{r} \Big|_{0.10m}^{0.20m}$

$= \frac{kQ_i}{0.20m} - \frac{kQ_i}{0.10m} = 200V$

$Q_i \left(\frac{k}{0.20} - \frac{k}{0.10} \right) = 200V$

$Q_i = \frac{200V}{(9.0\times10^9 \frac{Nm^2}{C^2}) \cdot \left(\frac{1}{0.20m} - \frac{1}{0.10m}\right)}$

$= -4.4\times10^{-9}\ C$

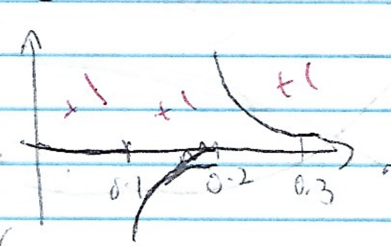
Or $E(0.20m) = \frac{kQ_i}{r^2} = \frac{k(Q_0 - Q_i)}{r^2}$

$Q_i = Q_0 - Q_i$

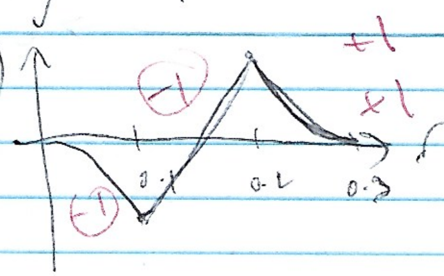
$Q_0 = 2Q_i = 8.8\times10^{-9}\ C$

$Q_T = 4.4\times10^{-9}\ C$ -1

(e)



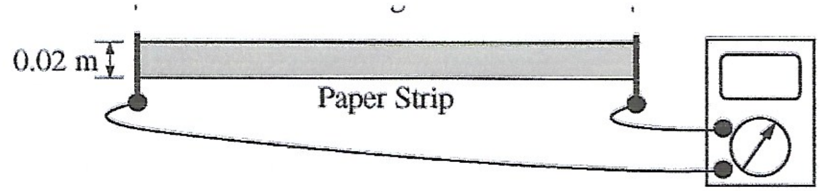
(f)



-3 -3

$R = \frac{\rho L}{A}$

plot R vs. $\frac{L}{A}$



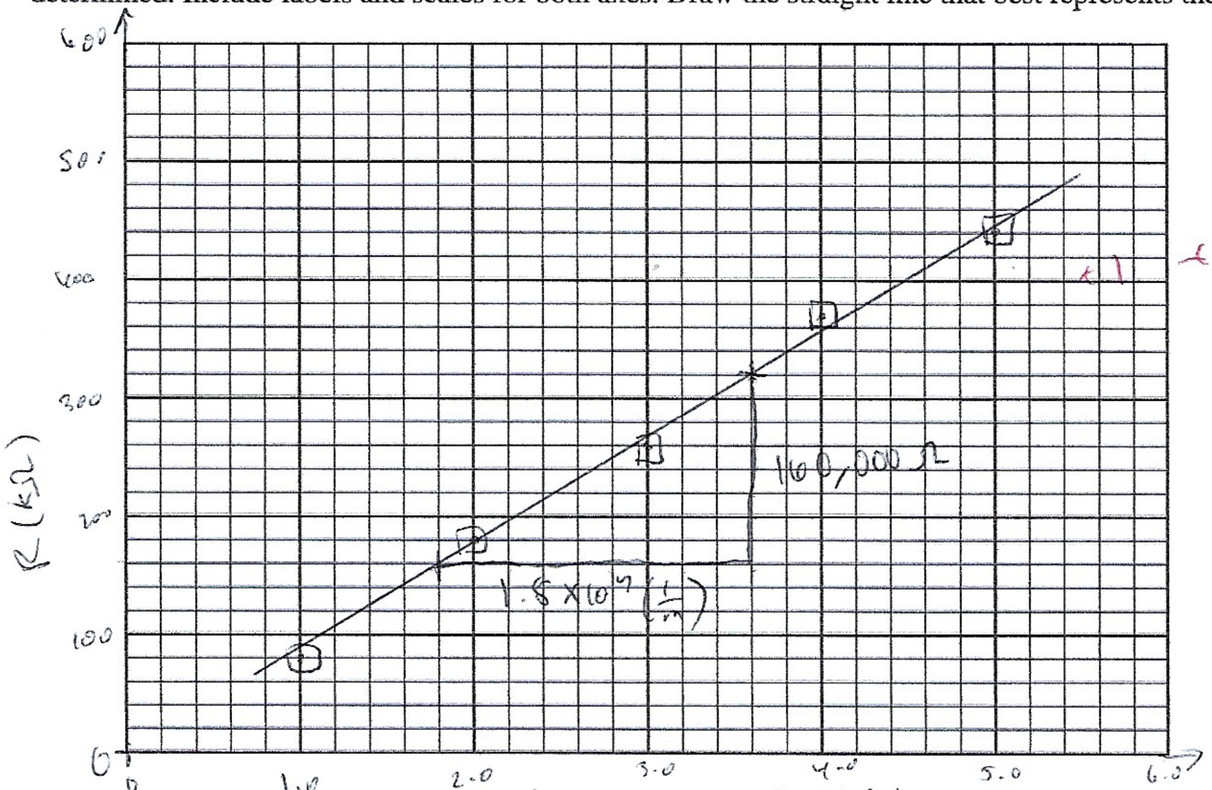
E&M. 2.

A physics student wishes to measure the resistivity of slightly conductive paper that has a thickness of 1.0×10^{-4} m. The student cuts a sheet of the conductive paper into strips of width 0.02 m and varying lengths, making five resistors labeled R1 to R5. Using an ohmmeter, the student measures the resistance of each strip, as shown above. The data are recorded below.

$A = 0.02 \cdot 1\times10^{-4}$ m $= 2\times10^{-6}$ m^2

Resistor	R1	R2	R3	R4	R5
Length (m)	0.020	0.040	0.060	0.080	0.100
Resistance (Ω)	80,000	180,000	260,000	370,000	440,000
L/A	1×10^4	2×10^4	3×10^4	4×10^4	5×10^4

(a) Use the grid below to plot a linear graph of the data points from which the resistivity of the paper can be determined. Include labels and scales for both axes. Draw the straight line that best represents the data.



+1 +1

+1

+1

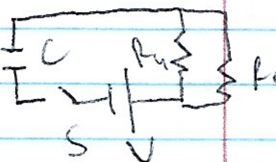
(b) Using the graph, calculate the resistivity of the paper.

$\rho = \text{slope} = \frac{\Delta y}{\Delta x} = \frac{160{,}000\ \Omega}{(1.8\times10^4\ \frac{1}{m})} = \boxed{8.89\ \Omega\cdot m}$ +3

Unauthorized copying or reuse of

2) a)-b) see front

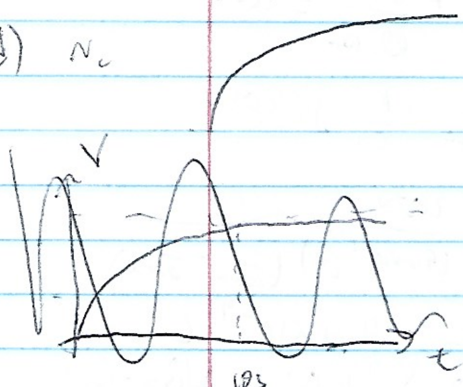
c) $\tau = RC = \left(\frac{1}{370k\Omega}+\frac{1}{440k\Omega}\right)^{-1}(10\mu F)$



$= \boxed{2.0\ s}$ +3

+1 units

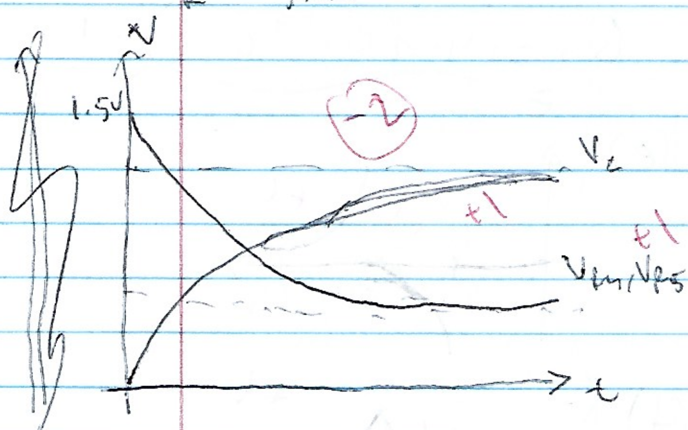
d) V_c



at $t=0$, $V_c = 0$

$V = IR$

$\cancel{I = \frac{V}{R} = \frac{1.5V}{370{,}000\Omega}}$



$V = \frac{Q}{C}$

-2

3) a) $\Phi = BA = \boxed{B_0 L h_0}$ +1

b) because area is decreasing, $\Delta\Phi$ is $\odot$

Lenz's law; ind. current opposes change in flux. B_{ind} is $\otimes$, so current is cw +1

+1

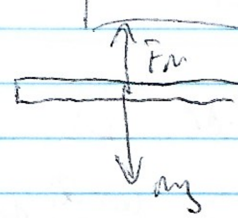
c) $\mathcal{E} = \frac{d\Phi}{dt} = IR$ +1

+1

$= B\frac{dA}{dt} = BL\frac{dh}{dt} = BLv$

$\boxed{I = \frac{B_0 L v}{R}}$ +1

d)



+1

$\Sigma F = ma = m\frac{dv}{dt} = mg - F_m = mg - ILB_0$

+1

$m\frac{dv}{dt} = mg - ILB_0$ +1

$\frac{dv}{dt} = g - \frac{ILB_0}{m} = g - \frac{B_0 L v \cdot LB_0}{mR}$

$\frac{dv}{dt} = g - \frac{B_0^2 L^2 v}{mR}$ +1

c) $\Sigma F = ma = 0 = mg - F_m$

+1

$F_m = mg$

$ILB_0 = mg = \frac{B_0 L v}{R} \cdot LB_0$

$mg = \frac{B_0^2 L^2 v_T}{R}$

$$\boxed{v_T = \frac{mgR}{B_0^2 L^2}}$$ +1

d) increases
+1
If the resistor increases, then the magnetic force on the bar will decrease, which means the +1 terminal speed will be higher.

0 on mag

-1