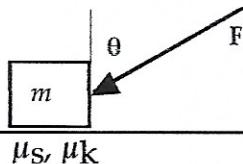


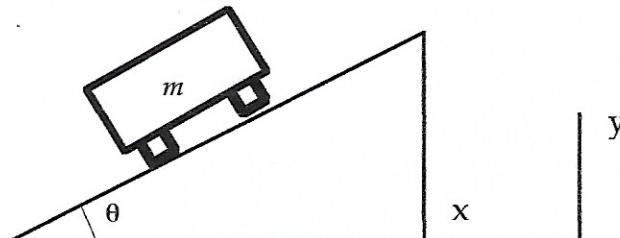
**Question 1** The three masses move smoothly under the effect of an external force parallel to ground. The surface is frictionless and  $F$  is parallel to the ground. Give your answers in terms of the given quantities ( $F$  and masses).

- Draw a free body diagram (FBD) for **each** mass and show **ALL** the forces acting on it.
- Use your FBD to write down Newton's 2nd law for **each** mass.
- Obtain the acceleration of each mass
- Obtain the force acting on each mass by the other



**Questions 2** The object is initially at rest then starts moving.  $F$  is at an angle  $\theta$  with respect to the vertical.

- Draw a FBD and show **ALL** the forces acting on the mass. Do not show any extraneous forces or components.
- Use your FBD to write down Newton's 2nd law for the mass.
- State how your answers will change precisely if the force flips its direction.
- Obtain each force acting on the mass
- Obtain its acceleration prior to motion, the moment it starts, during motion

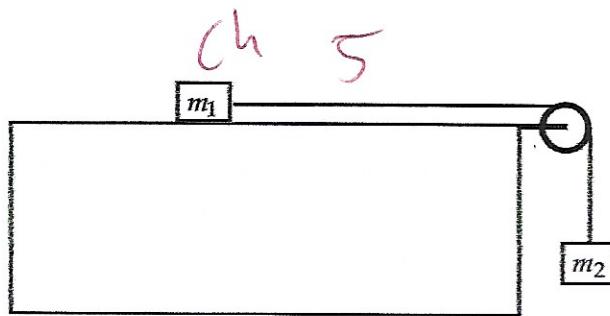


**Question 3** While going at a constant speed, a train makes a turn whose radius is  $r$ . The road is banked at an angle of  $\theta$ . Let  $\mu$  represent the coefficient of friction.

- Draw a FBD and show **ALL** the forces acting on the mass when  $\mu=0$ . Do not show any extraneous forces or components.
- Use your FBD to write down Newton's 2nd law for the mass.
- Obtain all the forces acting on the train
- Obtain the acceleration without friction.
- State the condition for the friction to be (1) up hill, (2) down hill.
- Obtain the acceleration of the train when friction is present.

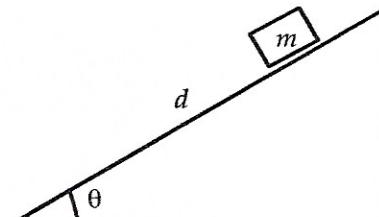
**Question 4** A bucket of water tied to a rope is revolving in a vertical loop of radius  $r$ .

- Draw a FBD and show **ALL** the forces acting on the mass when the bucket is (i) at the top (ii) at the bottom of the loop. Do not show any extraneous forces or components.
- Use your FBD to write down Newton's 2nd law for the bucket.
- When the bucket is at the top of the loop, what has to be its speed  $v_t$  for the rope to remain taut with zero tension?
- If the speed of the bucket at the bottom is  $v_b$ , what is the normal force on the water by the bucket at the bottom of the loop?



**Questions 5** In the system shown above, the block of mass  $m_1$ , is on a rough horizontal table. The string that attaches it to the block of mass  $m_2$  passes over a frictionless pulley of negligible mass. Let  $\mu$  represent either coefficient of friction.

- Draw a FBD and show **ALL** the forces acting on each mass.
- Use your FBD to write down Newton's 2nd law for each mass.
- Obtain all the forces acting on each mass.
- Obtain the acceleration of each mass.
- Obtain the tension in the rope.

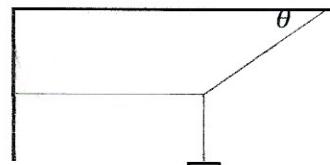


**Question 6** A block of mass  $m$  is placed on a rough inclined table of angle  $\theta$  at a distance  $d$  from the bottom. The coefficients of friction are  $\mu_k$  and  $\mu_s$ . The block starts from rest, then accelerates down hill.

- Draw a FBD and show **ALL** the forces acting on the mass.
- Use your FBD to write down Newton's 2nd law for the mass.
- Obtain all the forces acting on the mass and its acceleration
- In terms of the given quantities in the problem, (i) what will be its speed at the bottom of the hill and (ii) how long will it take to get there?

**Question 7** A rock of mass  $m$  is inside an elevator. The floor of the elevator is actually a scale.

- Draw a FBD and show **ALL** the forces acting on the rock when the elevator is accelerating (i) up (ii) down.
- Use your FBD to write down Newton's 2nd law for the rock.
- Obtain the scale's reading when the elevator is
  - accelerating up with  $a \uparrow$
  - accelerating down with  $a \downarrow$
  - moving up at a constant velocity of  $v \uparrow$
  - moving down at a constant velocity of  $v \downarrow$
  - at rest

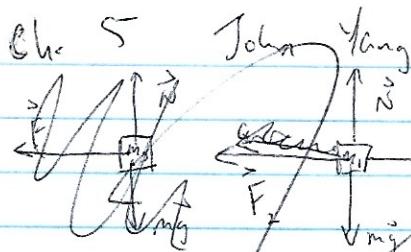


**Questions 8**

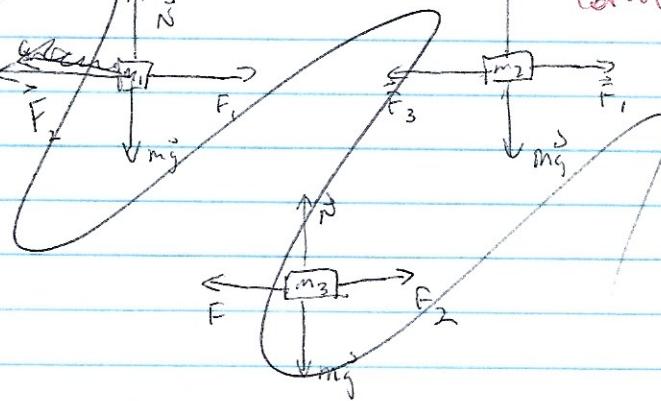
- A mass  $m$  is suspended from a wall and a ceiling as shown in the figure.
- Draw a FBD and show **ALL** the forces acting on the mass.
  - Use your FBD to write down Newton's laws.
  - Obtain the tension in each rope in terms of  $m$ ,  $\theta$ , and  $g$ .
  - Order the three tensions from the largest to the smallest.

black 65  
 blue 9  
 red 14 } 84  
 $\uparrow \text{total} = 90$  expand

1(a)



$\leftarrow$



$$m_1: \sum \vec{F}_1 = m_1 a_{x_1}$$

$$\sum \vec{F}_{y_1} = m_1 a_{y_1} = 0$$

$$N_1 + m_1 g = 0$$

$$N_1 = -m_1 g$$

$$m_2: \sum \vec{F}_2 = m_2 a$$

$$\sum \vec{F}_{y_2} = m_2 a_{y_2} = 0$$

$$N_2 + m_2 g = 0$$

$$N_2 = -m_2 g$$

$$M_3: \sum \vec{F}_3 = m_3 a$$

$$\sum \vec{F}_{y_3} = m_3 a_{y_3} = 0 - \sum \vec{F}_{x_3} = M_3 a_{x_3}$$

$$N_3 + m_3 g = 0$$

$$N_3 = -m_3 g$$

c)

$$m_1: \vec{F}_2 + \vec{F}_1 = m_1 a_{x_1}$$

$$a_{x_1} = \frac{\vec{F}_2 + \vec{F}_1}{m_1}$$

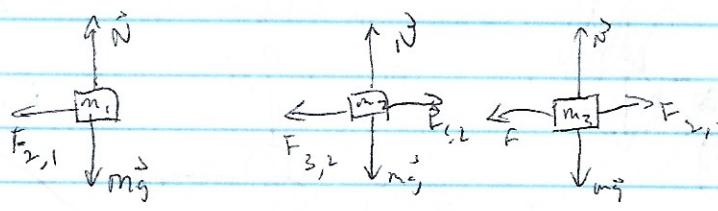
$$m_2: \vec{F} + \vec{F}_2 = m_2 a_{x_2}$$

$$a_{x_2} = \frac{\vec{F} + \vec{F}_2}{m_2}$$

$$d) \vec{F}_3: \vec{F}_1 + \vec{F}_2 = m_1 a_{x_1} \quad \vec{F}_3 + \vec{F}_1 = m_2 a_{x_2} \quad \vec{F} + \vec{F}_2 = m_3 a_{x_3}$$

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d) a)



+11

$$b) \sum F_{1x} = m_1 a \quad \sum F_{1y} = m_1 a_y = 0$$

$$\underline{F_{2,1} = m_1 a_x + 1} \quad \vec{N}_1 + \vec{m}_2 \vec{g} = 0$$

$$\vec{N}_1 = -\vec{m}_2 \vec{g}$$

$$\sum F_{2x} = m_2 a \quad \sum F_{2y} = m_2 a_y = 0$$

$$\cancel{\underline{F_{3,2} + F_{1,2} = m_2 a_x + 1}} \quad \vec{N}_2 = -\vec{m}_2 \vec{g}$$

$$\text{with } \sum F_{3x} = m_3 a \quad \sum F_{3y} = m_3 a_y = 0$$

$$\underline{F + F_{2,3} = m_3 a + 1} \quad \vec{N}_3 = -\vec{m}_3 \vec{g}$$

mark

$$c) \underline{a = \frac{F_{2,1}}{m_1}} \quad a = \frac{F_{3,2} + F_{1,2}}{m_2} \quad a = \frac{F + F_{2,3}}{m_3}$$

$$F_{3,2} = m_2 a - F_{1,2} \quad F_{3,2} = -F_{2,3}$$

$$F_{2,1} = m_1 a \quad F_{1,2} = -F_{2,1}$$

$$-m_1 a = F_{1,2}$$

$$F_{3,2} = m_2 a + m_1 a \rightarrow F_{2,3} = -m_2 a - m_1 a$$

$$a = \frac{F - m_2 a - m_1 a}{m_3}$$

$$m_3 a + m_2 a + m_1 a = F$$

$$a(m_1 + m_2 + m_3) = F + 1$$

d) ~~BB~~

$$F_{3,2} = m_2 a + m_1 a$$

$$= \frac{F m_2}{m_1 + m_2 + m_3} + \frac{F m_1}{m_1 + m_2 + m_3}$$

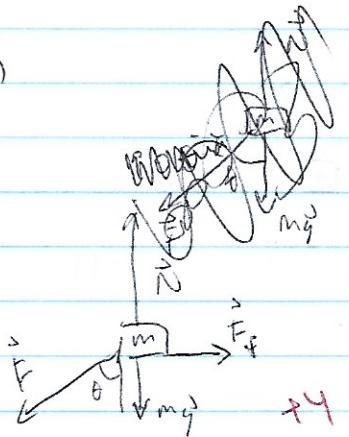
$$\boxed{F_{3,2} = \frac{F(m_1 + m_2)}{m_1 + m_2 + m_3}}$$

$$F_{2,1} = m_1 a = \boxed{\frac{F m_1}{m_1 + m_2 + m_3}}$$

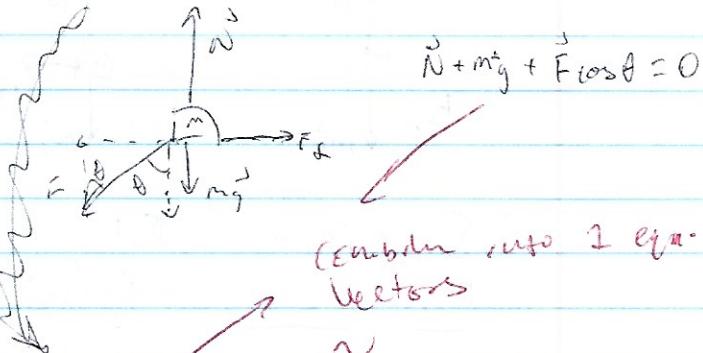
simply

16

2) a)



$$b) \sum F_x = m a_x \quad \sum F_y = m a_y = 0$$



(Combine into 1 eqn. as  
vectors)

$$b) \vec{F}_{\text{sin}\theta} + \vec{F}_f = m a_x = m a$$

c)  $m g$  will be greater than the normal force &  $F_f$  will be less.

$$d) \vec{N} = m \vec{g} \quad ! \quad N = -m g - \vec{F}_{\text{cos}\theta}$$

$$F_f = \mu_k N = \boxed{-\mu_k m g - \mu_k \vec{F}_{\text{cos}\theta}}$$

$$\vec{F}_{\text{sin}\theta} = m a - \vec{F}_f$$

$$\sin\theta \vec{F} = \boxed{m a + \mu_k m g + \mu_k \vec{F}_{\text{cos}\theta}}$$

$$F (\sin\theta - \mu_k \cos\theta) = m (a + \mu_k g)$$

$$\vec{F} = \boxed{\frac{m a - \mu_k m g}{\sin\theta - \mu_k \cos\theta}}$$

e) prior to motion -  $a=0$  since  $\vec{F}_f \leq \mu_k N$

$$\text{start of motion} - \vec{F}_f = \mu_s N$$

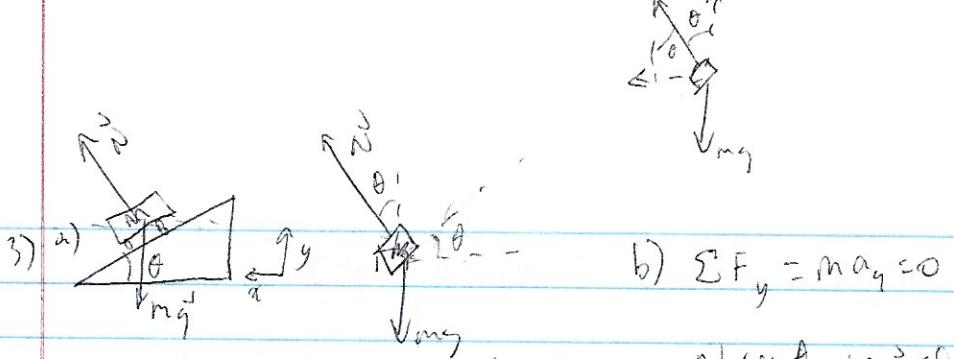
$$a = \frac{\vec{F}_{\text{sin}\theta} + \vec{F}_f}{m} = \frac{\vec{F}_{\text{sin}\theta} + \mu_s N}{m}$$

$$= \frac{F_{\text{sin}\theta} - \mu_s m g - F_{\text{load}}}{m}$$

$$a = \boxed{\frac{F_{\text{sin}\theta} - \mu_s \cos\theta - \mu_s g}{m}} \sim$$

$$\text{during motion} - a = \frac{\vec{F}_{\text{sin}\theta} + \vec{F}_f}{m} = \frac{\vec{F}_{\text{sin}\theta} + \mu_k m g - \vec{F}_{\text{load}}}{m}$$

$$a = \boxed{\frac{F_{\text{sin}\theta} - \mu_k \cos\theta - \mu_k g}{m}} \quad +1$$



b)  $\sum F_x = ma_x = \frac{mv^2}{r}$

$\rightarrow$  combine

$N \sin \theta = \frac{mv^2}{r}$

c)  $(W = mg) + |$

$N \cos \theta + mg = 0$

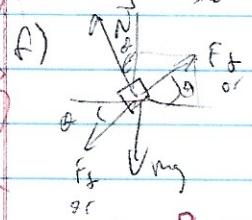
$N \cos \theta = -mg$

$$\left[ N = \frac{mv^2 \sin \theta}{r} = \frac{-mg}{\cos \theta} \right] \sim \sim$$

d)  $a = \frac{\sum F_x}{m} = \frac{N \sin \theta}{m} = \frac{-mg \sin \theta}{m \cos \theta} = (-g \tan \theta) + |$

e) i) if friction is acting uphill, the ~~bank~~ bank on the ~~curve~~ curve  $N \sin \theta$  is too large, which means the curve bank is too steep

iii) if friction is down hill, the curve isn't steep enough so friction needs to provide additional centripetal force.



$\sum F_y = 0 \Rightarrow 0$

$N \cos \theta + F_f \sin \theta - mg = 0$

(a) +3

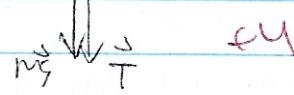
$\sum F_x = ma \Rightarrow N \sin \theta \pm F_f \cos \theta = ma$

$a = \frac{\sum F_x}{m} = \frac{N \sin \theta \pm F_f \cos \theta}{m} \sim$

$= -g \tan \theta \pm \frac{\mu mg (\cos \theta)}{m} \sim$

$= [-g \tan \theta \pm \mu g] \sim$

ii) or iii)



cu

9

$$b) \sum F_y = m a_x$$

$$c) T + mg = m a = \frac{m v^2}{r}$$

$$\cancel{T} + mg = m a \quad +1$$

$$T=0 \quad mg = \cancel{m v^2} \quad +1$$

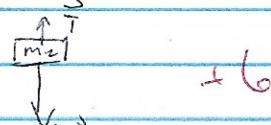
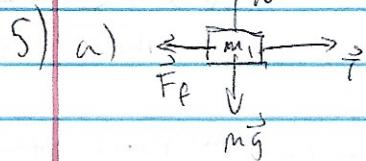
$$v^2 = r \quad \boxed{v = \sqrt{rg}} \quad +1$$

$$d) \cancel{N} + mg = m v^2$$

$$\sum F_y = m a$$

$$N - mg = m a = \cancel{m v^2}$$

$$\boxed{N = m \left( g + \frac{v^2}{r} \right) \uparrow} \quad +1$$



$$b) \sum F_{xy} = m a_y = 0$$

$$\sum F_{1x_2} = m_1 a_1$$

$$\cancel{T_1 + F_f} = m_1 a_1$$

$$N + mg = 0$$

$$N = -mg$$

$$\sum F_{2y} = m_2 a_2$$

$$mg - \cancel{T_2} = m_2 a_2$$

$$c) F_f = \mu N = \boxed{-\mu mg} \quad +1 \quad \text{but wrong}$$

$$F_{g2} = \boxed{m_2 g} \quad +1$$

$$T_1 = m_1 a_1 - F_f = \boxed{m_1 a_1 + \mu m_1 g} \quad +1$$

$$T_2 = m_2 g - m_2 a_2$$

$$d) a_1 = a_2 = a \quad T_1 = T_2 = T$$

$$m_2 g - m_2 a = m_1 a + \mu m_1 g$$

$$a(m_1 + m_2) = m_2 g - \mu m_1 g$$

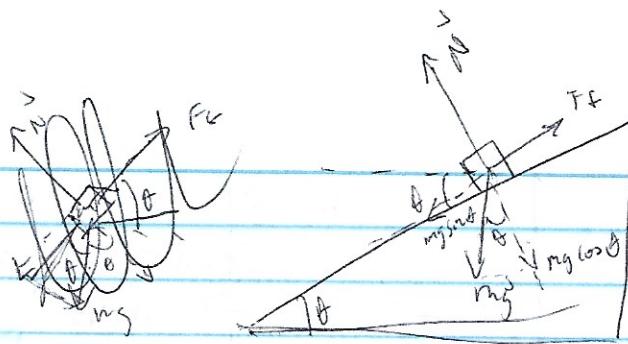
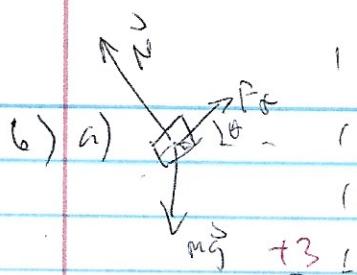
$$a = \boxed{\frac{m_2 g - \mu m_1 g}{m_1 + m_2}} \quad +1$$

$$e) T = m_2 g - m_2 a = m_2 g - \boxed{m_2 (m_2 g - \mu m_1 g)}$$

$$= \boxed{\frac{m_1 m_2 g + m_2^2 g - m_2^2 g + \mu m_1 m_2 g}{m_1 + m_2}} \quad (m_1 + m_2)$$

$$= \boxed{\frac{m_1 m_2 g (\mu + 1)}{m_1 + m_2}} \quad +1$$

14



$$b) \sum F_x = ma \quad \sum F_y = ma = 0$$

$$mg \sin \theta + F_f = ma \quad N + mg \cos \theta = 0 \quad +1$$

$$c) w = mg \downarrow; +1 \quad N = mg \cos \theta; +1$$

$$F_f = ma - mg \sin \theta = \mu_k N = \mu_k mg \cos \theta \quad +1$$

$$a = \frac{\sum F_x}{m} = \frac{mg \sin \theta + \mu_k mg \cos \theta}{m}$$

$$= [g \sin \theta - \mu_k g \cos \theta] = g (\sin \theta - \mu_k \cos \theta) \quad +1$$

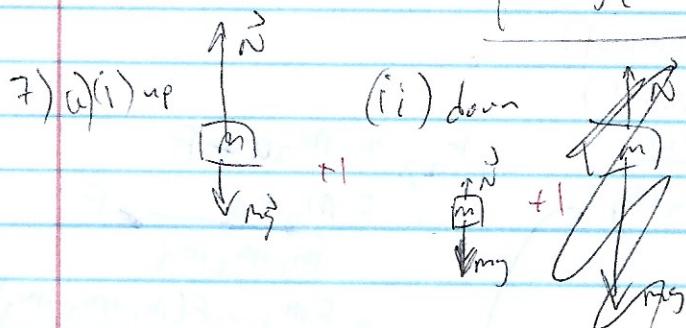
$$d) v_f^2 = v_0^2 + 2ad$$

$$(i) v_f = \sqrt{v_0^2 + 2ad} = \sqrt{2g(\sin \theta - \mu_k \cos \theta)}$$

$$= \sqrt{2gd(\sin \theta - \mu_k \cos \theta)} \quad +1$$

$$(ii) v_f = v_0 + at$$

$$t = \frac{v_f}{a} = \frac{\sqrt{2g(\sin \theta - \mu_k \cos \theta)}}{g(\sin \theta - \mu_k \cos \theta)}$$



$$b) \sum F = ma$$

$$\vec{N} - \vec{mg} = \vec{ma}$$

$\sim$  vector

$$c) N = mg \downarrow + ma \uparrow$$

$$= [mg \downarrow - ma \uparrow]$$

switch

$$2) N = [mg \uparrow + ma \uparrow]$$

$\sim$

$$3) \text{ case } 3, 4) a = 0; \vec{N} - \vec{mg} = \vec{ma} = 0$$

+1

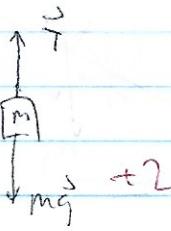
$$N = \boxed{mg} +1$$

$$5) a = 0; N = \boxed{mg} +1$$

14

8)

a)

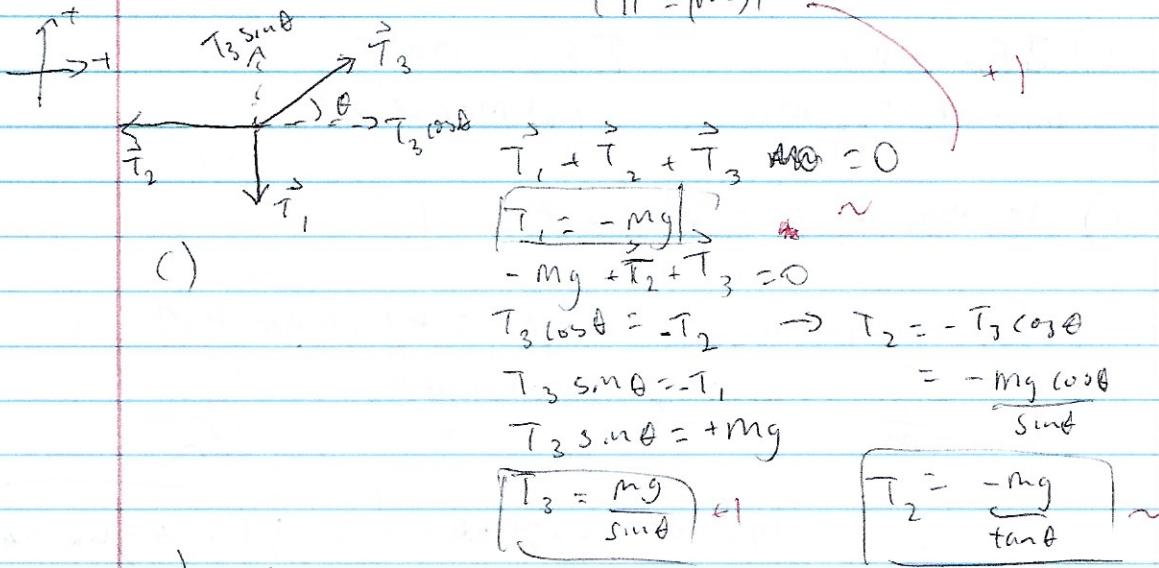


$$\sum F = ma \approx 0$$

$$T + mg = ma \approx 0$$

$$T = -mg$$

$$|T| = (mg)$$



$$d) |T_1| = mg$$

$$|T_2| = \frac{mg}{\tan \theta}$$

$$|T_3| = \frac{mg}{\sin \theta}$$

$$\theta < 90^\circ$$

$$\text{let } \theta = 30^\circ \quad \sin \theta = 0.5 \quad \tan \theta = 0.6$$

$$\frac{mg}{\sin \theta} = 2mg \quad \frac{mg}{\tan \theta} = mg(0.6^{-1}) = 1.7mg$$

$\left\{ \begin{array}{l} \text{smallest } T_1 \text{ (vertical)} \\ T_2 \text{ (horizontal)} \end{array} \right.$

A

$\left\{ \begin{array}{l} \text{largest } T_3 \text{ (diagonal)} \end{array} \right.$

$$1) d) F_{3,2} = \frac{F(m_1 + m_2)}{m_1 + m_2 + m_3}$$

$$F_{2,3} = m_3 a + F$$

$$= \frac{m_3 F}{m_1 + m_2 + m_3} + F$$

$$= F m_3 - \underbrace{F(m_1 + m_2 + m_3)}_{m_1 + m_2 + m_3}$$

$$= - \frac{F(m_1 + m_2)}{m_1 + m_2 + m_3}$$

$$\boxed{F_{3,2} = -F_{2,3}}$$

+1

$$F_{2,1} = \frac{F_m}{m_1 + m_2 + m_3}$$

$$F_{1,2} = m_2 a - F_{3,2}$$

$$= \frac{F(m_1 + m_2)}{m_1 + m_2 + m_3} + m_2 a$$

$$= \frac{m_2(m_1 + m_2 + m_3)}{m_1 + m_2 + m_3}$$

$$= \frac{F m_2 - F(m_1 + m_2)}{m_1 + m_2 + m_3}$$

$$F_{2,1} = -F_{1,2}$$

$$+ | = \frac{-F m_1}{m_1 + m_2 + m_3}$$

2) b)  $\sum \vec{F} = m \vec{a}$

$$(N + m\vec{g} + \vec{F}_f + \vec{F} = m\vec{a}) + |$$

c)  $p \cos \theta - \sum F_x = m a_x = m a$

$$F_f = \mu_s N$$

$$F \sin \theta - F_f = m a$$

$$a = \frac{F \sin \theta - F_f}{m}$$

$$= \frac{F \sin \theta - \mu_s m g - \mu_s F \cos \theta}{m}$$

$$= \frac{F}{m} (\sin \theta - \mu_s \cos \theta) - \mu_s g + | + |$$

3) b)  $\sum \vec{F} = m \vec{a}$

$$\vec{N} + \vec{F}_f + \vec{m\vec{g}} = m\vec{a} + |$$

c) (Signum)  $(N = \frac{m\vec{g}}{\cos \theta}) \sim$

e) (i) ~~Newton's law~~ -  ~~$F_f = \mu_s N$~~  ~~Newton's law~~

$$\text{OR } \sum F = m a = \frac{m v^2}{r}$$

$$N \sin \theta \pm F_f \cos \theta = \frac{m v^2}{r}$$

~~if  $F_f < 0$ ,  $\theta > 90^\circ$~~

if  $F_f < 0$ , going too slowly

if  $F_f > 0$ , going too fast (+ feels good for fast on curves with friction)

if  $v < \sqrt{\mu_s g r}$  not enough

$$3) \text{a) } \sum F = ma$$

$$\bullet N \sin \theta \pm F_f = ma \quad N \cos \theta = mg \uparrow$$

$$a = \frac{N \sin \theta \pm F_f}{m} \quad N = \frac{mg}{\cos \theta}$$

$$= \frac{mg}{\cos \theta} \left( \frac{\sin \theta}{m} \right) \pm \frac{mN}{m}$$

$$= g \tan \theta \pm \frac{m \cdot mg}{\cos \theta} \sim$$

$$a = \underbrace{g \tan \theta \pm \frac{mg}{\cos \theta}}_{\sim}$$

a) b)  $\sum F_y = ma$  bottom  $\uparrow$   
 $\frac{T}{T} + mg = ma$  top  $\uparrow$   
 $T - mg = ma \uparrow$   $T + mg = ma \downarrow + \uparrow$

5) b)  $\sum F_1 = m_1 a$   $\sum F_2 = m_2 a$   
 $\vec{N}_1 + \vec{mg} + \vec{F}_f = m_1 a$  ~~all~~  $\vec{T} + \vec{mg} = m_2 a \sim$   
 $\Rightarrow \boxed{N_1 = m_1 g} \quad \boxed{N_2 = -m_2 g} + \uparrow$

c)  $\sum F \uparrow = ma$   $N \uparrow + mg \downarrow = m a \uparrow$   
 $N \uparrow = m a \uparrow + mg \uparrow + \uparrow$   
 $= m(a+g)$

b)  $\sum F \uparrow = ma$   $N \uparrow + mg \downarrow = ma$   
 $\boxed{N \uparrow + mg \downarrow = m a \uparrow} \quad 2) \text{ a) } N \downarrow + mg \uparrow = ma \downarrow$   
 $+ \uparrow \quad N \downarrow = ma \downarrow - mg \uparrow$   
 $N \downarrow = mg \uparrow - ma \uparrow + \uparrow$   
 $= m(g-a)$

$$8) c) \sum \vec{F} = m\vec{a} = 0$$

$$\vec{T} + mg\hat{j} = m\vec{a} = 0$$

$$T\hat{i} = mg\hat{j} \quad +1$$

$$T_2 = T_3 \cos\theta = \boxed{\frac{mg \cos\theta}{\sin\theta}} \quad +1$$

$$2$$

$$3) c) N \cos\theta - \mu N \sin\theta = mg \quad +1$$

$$\mu = \frac{mg}{\cos\theta + \mu N \sin\theta} \quad +1$$

$$F_f \leq \frac{\mu mg}{\cos\theta + \mu \sin\theta} \quad +1$$

- e) if friction uphill, too slow  
if friction downhill, too fast

$$a) N \sin\theta + F_f \cos\theta = ma$$

$$\frac{mg \sin\theta}{\cos\theta + \mu \sin\theta} + \frac{\mu mg \cos\theta}{\cos\theta + \mu \sin\theta} = ma \quad +1$$

$$a = g \frac{\sin\theta + \mu \cos\theta}{\cos\theta + \mu \sin\theta} \quad +1$$

$$b) T - \mu mg = m, a \quad +1$$

$$c) d = \bar{v}t = \frac{vt}{2} \quad ; \quad v_f = \int a dt = g(\sin\theta - \mu \cos\theta)t = g(\sin\theta - \mu \cos\theta) \frac{2d}{v_f}$$

$$v_f = \sqrt{2dg(\sin\theta - \mu \cos\theta)}$$

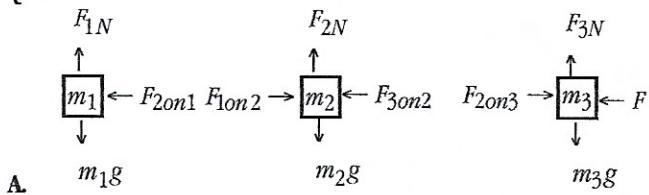
$$a = g(\sin\theta - \mu \cos\theta) \quad v = gt(\sin\theta - \mu \cos\theta)$$

$$d = \int v dt = \frac{1}{2} gt^2 (\sin\theta - \mu \cos\theta)$$

$$t = \sqrt{\frac{2d}{\sin\theta - \mu \cos\theta}} \quad +5$$

8) d)  $F_3 > F_1, F_2$ ;  $F_1, F_2$ : depends on  $\theta$  + 2

**Question 1**



- A.  $m_1g$       m<sub>2</sub>g      m<sub>3</sub>g  
1 pt for each correctly drawn and labeled force vector. [11 pts]  
B. All masses have the same acceleration since they move together. In addition, for each mass,  $\vec{F}_N + m\vec{g} = 0$ ; therefore, we have

$$\begin{aligned}\vec{F}_{2on1} &= m_1\vec{a} \\ \vec{F}_{1on2} + \vec{F}_{3on2} &= m_2\vec{a} \\ \vec{F}_{2on3} + \vec{F} &= m_3\vec{a} \\ \vec{F} &= (m_1 + m_2 + m_3)\vec{a}\end{aligned}$$

- C. They all have the same acceleration  $\vec{a} = \frac{\vec{F}}{m_1 + m_2 + m_3}$   
D.  $\vec{F}_{2on1} = m_1\vec{a} = \frac{m_1\vec{F}}{m_1 + m_2 + m_3} = -\vec{F}_{1on2}$   
 $\vec{F}_{3on2} = m_2\vec{a} - \vec{F}_{1on2} = \frac{(m_1 + m_2)\vec{F}}{m_1 + m_2 + m_3} = -\vec{F}_{2on3}$

**Questions 2**

- 
- A. 1 pt for each correctly drawn and labeled force vector [4 points]  
Take off 1 pt for any component or extraneous force vector.  
B.  $F + F_\mu + F_N + mg = m\vec{a}$   
or  $F \sin\theta - F_f = ma$ ,  $F_N - F \cos\theta - mg = 0$ ,  $F_f \leq \mu F_N$   
C.  $\vec{F} + \vec{F}_\mu + \vec{F}_N + \vec{mg} = m\vec{a}$   
or  $F \sin\theta - F_f = ma$ ,  $F_N + F \cos\theta - mg = 0$ ,  $F_f \leq \mu F_N$   
D.  $\vec{F}_g = mg \downarrow$   
 $\vec{F}_N = (mg + F \cos\theta) \uparrow$   
 $\vec{F}_f \leq \mu(mg + F \cos\theta) \rightarrow$  against the motion.  
E. (i)  $a = \frac{F}{m} \sin\theta - \mu_s(g + \frac{F}{m} \cos\theta)$   
(ii) & (iii)  $a = \frac{F}{m} \sin\theta - \mu_k(g + \frac{F}{m} \cos\theta)$

**Question 3**

- 
- A. What you need to keep in mind in this problem is that the acceleration is toward the center and parallel to the ground  
NOT the surface of the incline!  
1 pt for each correctly drawn and labeled force vector. Either direction (uphill or downhill) for friction is acceptable but you must specify that it is against the direction of motion. [3 pts]  
Take off 1 pt for any component or extraneous force vector.

B.  $\vec{F}_\mu + \vec{F}_N + m\vec{g} = m\vec{a}$

or  $F_N \sin\theta \pm F_f \cos\theta = ma$ ,  $a = \frac{v^2}{r}$   
 $F_N \cos\theta \pm F_f \sin\theta - mg = 0$ ,  $F_f \leq \mu F_N$

C. From the second equation above  
 $F_N \cos\theta \pm \mu F_N \sin\theta = mg$

$$F_N = \frac{mg}{\cos\theta \pm \mu \sin\theta} \quad 1 pt, \quad F_f \leq \frac{\mu mg}{\cos\theta \pm \mu \sin\theta} \quad 1 pt$$

D. If  $\mu = 0$ , the x-direction 2nd law equation gives

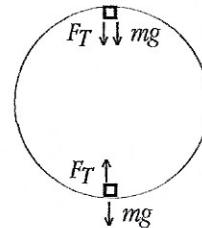
$$\begin{aligned}F_N \sin\theta \pm F_f \cos\theta &= ma, \quad F_N \sin\theta \pm 0 = ma \\ a &= g \frac{\sin\theta}{\cos\theta} = g \tan\theta\end{aligned}$$

E. If the speed is low, i.e.  $a < gr \tan\theta$  &  $v < \sqrt{gr \tan\theta}$ , the friction will be up hill; otherwise,  $a > gr \tan\theta$  &  $v > \sqrt{gr \tan\theta}$  and down hill.

F.  $F_N \sin\theta \mp F_f \cos\theta = ma$

$$\begin{aligned}\frac{mgs \in \theta}{\cos\theta \pm \mu \sin\theta} \mp \frac{\mu mg \cos\theta}{\cos\theta \pm \mu \sin\theta} &= ma \\ a &= g \frac{\sin\theta \mp \mu \cos\theta}{\cos\theta \pm \mu \sin\theta}\end{aligned}$$

**Question 4**



A.

1 pt for each correctly drawn and labeled force vector

Take off 1 pt for any component or extraneous force vector. [4 pts]

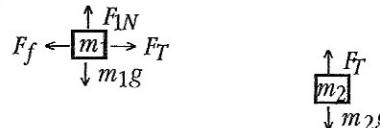
B. (Top)  $\vec{F}_T + \vec{mg} = \vec{ma}$ ,  $F_T + mg = ma$   
(Bottom)  $\vec{F}_T + \vec{mg} = \vec{ma}$ ,  $F_T - mg = ma$

C. If the tension is zero at the top, and the basket completes the loop while still hanging on the loop, we have to have

$$0 + mg = ma = m \frac{v^2}{r} \quad 1 pt, \quad v = \sqrt{gr} \quad 1 pt$$

D.  $F_T - mg = ma \quad 1 pt, \quad F_N = F_T = m(a + g) = m \left( g + \frac{v^2}{r} \right) \quad 1 pt$

**Questions 5**



A.

1 pt for each correctly drawn and labeled force vector

Take off 1 pt for any component or extraneous force vector. [6 pts]

B.  $\vec{F}_T + \vec{F}_f + \vec{F}_N + m_1\vec{g} = m_1\vec{a}$ ,  $F_T - \mu m_1 g = m_1 a$   
 $\vec{F}_T + m_2\vec{g} = m_2\vec{a}$ ,  $m_2 g - F_T = m_2 a$

C.  $\vec{F}_1g = m_1g \downarrow \quad 1 pt, \quad \vec{F}_N = m_1g \uparrow \quad 1 pt, \quad \vec{F}_f = \mu m_1 g \leftarrow \quad 1 pt$   
 $\vec{F}_2g = m_2g \downarrow \quad 1 pt$

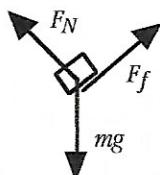
D. Solving  $F_T - \mu m_1 g = m_1 a$  and  $m_2 g - F_T = m_2 a$

simultaneously gives  $(m_2 - \mu m_1)g = (m_1 + m_2)a \quad 1 pt$   
and  $a = \frac{m_2 - \mu m_1}{m_1 + m_2} g \quad 1 pt$

E.  $F_T - \mu m_1 g = m_1 a$  or  $m_2 g - F_T = m_2 a$  gives

$$F_T = (1 + \mu) \frac{m_1 m_2}{m_1 + m_2} g$$

Question 6



A. The acceleration is along the incline.

1 pt for each correctly drawn and labeled force vector. Either direction (uphill or downhill) for friction is acceptable but you must specify that it is against the direction of motion. [3 pts]

Take off 1 pt for any component or extraneous force vector.

B.  $\vec{F}_\mu + \vec{F}_N + \vec{mg} = \vec{ma}$  or

1pt

$$m g \sin \theta - \mu m g \cos \theta = m a$$

C.  $F_g = m g$  1pt,  $F_N = m g \cos \theta$  1pt,  $F_f \leq \mu m g \cos \theta$  1pt

D. We can solve the problem two different ways, find  $v_f$  first or  $t$  first. Since initial velocity is 0 and  $a$  is constant, we have

$$d = v_{av} t = \frac{v_f}{2} t \quad \text{1pt and}$$

$$v_f = \int a dt = g(\sin \theta - \mu \cos \theta) t = g(\sin \theta - \mu \cos \theta) \frac{2d}{v_f} \quad \text{1pt}$$

$$v_f = \sqrt{2dg(\sin \theta - \mu \cos \theta)} \quad \text{1pt}$$

$$a = g(\sin \theta - \mu \cos \theta)$$

$$v = g(\sin \theta - \mu \cos \theta) t$$

$$d = \int v dt = \frac{1}{2} g(\sin \theta - \mu \cos \theta) t^2 \quad \text{1pt}$$

$$t = \sqrt{\frac{2d}{\sin \theta - \mu \cos \theta}} \quad \text{1pt}$$

Question 7

A. (i)  $\uparrow F_N = \uparrow ma$

1pt

60

(ii)  $\downarrow F_N = \downarrow ma$

1pt

B. In both cases,  $\vec{F}_N + \vec{mg} = \vec{ma}$ . Remember, this is the vector notation! **Not** the scalar form!

1pt

C. 1.  $F_N \uparrow + mg \downarrow = ma \uparrow$ ,  $F_N \uparrow = ma \uparrow - mg \downarrow = m(g + a) \uparrow$

1pt

This is so because  $F_N$  has to account for both  $g$  and  $a$ .

2.  $F_N \uparrow + mg \downarrow = ma \downarrow$ ,  $F_N \uparrow = ma \downarrow - mg \downarrow = m(g - a) \uparrow$

1pt

3.  $\vec{a} = 0$ ,  $F_N \uparrow = mg \uparrow$

1pt

4.  $\vec{a} = 0$ ,  $F_N \uparrow = mg \uparrow$

1pt

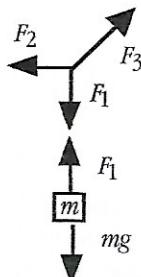
5.  $\vec{a} = 0$ ,  $F_N \uparrow = mg \uparrow$

1pt

In other words, if you want to loose weight, weight yourself in an elevator accelerating down, better yet on free fall.

Questions 8

1pt



A.

Since the question asks forces acting on the mass, you need to show only the two forces acting on it.

1 pt for each correctly drawn and labeled force vector acting on the mass. Take off 1 pt for any component or extraneous force vector acting on the mass. [2pts]

B.  $\vec{F}_1 + \vec{mg} = 0$ ,  $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$

1pt

C.  $F_1 = mg$

1pt

$$F_1 - F_3 \sin \theta = 0, F_3 = \frac{mg}{\sin \theta}$$

1pt

$$F_2 - F_3 \cos \theta = 0, F_2 = \frac{mg \cos \theta}{\sin \theta}$$

1pt

D.  $F_3$  is larger than both  $F_1$  and  $F_2$  [1pt], but the relative sizes of  $F_1$  and  $F_2$  depend on the angle  $\theta$  since smaller the  $\theta$ , the larger  $F_2$  [1pt]

85 90