Cumulative 2

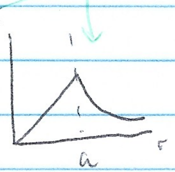
Points 39 TA 2

Total 85

1) $E = \begin{cases} \alpha r \hat{r}, & r \le a \quad (1) \text{ region I} \\ \frac{\beta}{r^2}\hat{r}, & r > a \quad (2) \text{ region II} \end{cases}$

continuity is implied through

a) $\alpha r = \frac{\beta}{r^2}$ when $r = a$ +1



$\alpha a = \frac{\beta}{a^2}$ +1 $\boxed{\alpha = \frac{\beta}{a^3}}$ +1

was in the solution $\alpha = \frac{\beta}{a^3}$ is the same as $\alpha a^3 = \beta$

b) region I: $E = -\frac{dV}{dr}$

$-E\,dV = dr$

$dV = -\frac{dr}{E}$

$V = -\int \frac{1}{E}$

$V = -\int E \cdot dr$

$V = -\alpha \int \hat{r}\, dr$ +1 integrand integrand was correct for region I

$= -\alpha \cdot \frac{1}{2} r^2 + C$

$\Big|_0^r$ $C = 0$ $r = 0$ $E = 0$ $V = 0$

$= \boxed{-\frac{\alpha}{2} r^2}$

~~Region II: $V = -\int E \cdot dr$~~

(−) negative potential means (+) charges are ~~more~~ attracted towards the center and (−) charges are attracted towards $\hat{r}$

region II: $V = -\int_a^r E \cdot dr$ +1 integrand r^{-2}

$= -\beta \int_a^r \frac{dr}{r^2}$ integrand was correct for region II.

$= \frac{\beta}{r}\Big|_a^r$

$= \frac{\beta}{r} - \frac{\beta}{a} = \boxed{\beta\left(\frac{1}{r} - \frac{1}{a}\right)}$

5

c) region I: continuous distribution of charge equally distributed throughout the sphere

uniformly charged sphere

$E = \frac{kQ}{R^3}\vec{r}$ $\quad \alpha = \frac{kQ}{R^3}$ $\quad R = a$

$\sigma = \frac{Q}{V_{olume}}$ $\quad Q = \frac{\alpha a^3}{k}$

$Volume = \frac{4}{3}\pi r^3$

$\sigma = \frac{\alpha a^3}{k} \cdot \frac{3}{4} \cdot \frac{1}{\pi a^3} = \boxed{\frac{3\alpha}{4\pi k}}$

d) $\Sigma F = ma = F_e = (e)E$

$= -e\alpha \vec{r} = ma$

1 pt for the first region

$\boxed{a = \frac{-e\alpha r}{m}\hat{r}}$ +1

towards the center

e) v_{max} is when $r = 0$ +1 this is implied through calcs.

$\frac{1}{2}m v_{max}^2 = qV_0$ +1 → CoE starting

$\frac{1}{2}mv^2 = (-e)\left(-\frac{\alpha a^2}{2}\right)$

$\sqrt{\frac{e\alpha a^2}{m}} = a\sqrt{\frac{e\alpha}{m}}$

$\frac{1}{2}mv^2 = \frac{e\alpha a^2}{2}$

$v^2 = \frac{e\alpha a^2}{m}$ $\quad \boxed{v = \sqrt{\frac{e\alpha a^2}{m}}}$ +1

f) the electron executes SHM +1 from a to $-a$ through the center in the radial direction.

g) yes, it gets all the way to the other side b/c of ~~conserv~~ CoE +1

6

a)

2) $\vec{E}$ is $\perp$ to equipotential lines. E is radially outward or inward from the origin

because voltage increases as r increases, V is $\frac{PE}{q}$, so (+) charges would have more potential energy the farther they get, so they would be attracted towards the origin. therefore, **the $\vec{E}$ points radially towards the origin.**

at (C), $V = -Er$

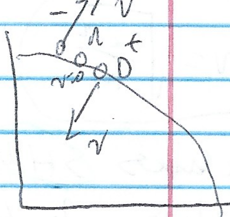
$E = \frac{V}{r} = \frac{20V}{2m} = \boxed{10 \frac{N}{C}}$

at (D), $E = \frac{V}{r} = \frac{30V}{3m} = \boxed{10 \frac{N}{C}}$

both towards the origin... +1 +1

(bi-rection)

given for the correct direction

b) 

for the electron, it will move radially away from the center +1 1)b)a

the neutron will stay in place. +1 1)b)b

the proton will move towards the center. +1 1)b)c

(see solutions) 5 7

c) $\Delta V = 10V$

(1) $KE = -\Delta PE = q\Delta V = 10V(10^{-19}C)$
$= \boxed{10^{-18} J}$

for both electron & proton +1 +1 1)b

$KE_f = 0$ +1

(2) electron: $KE = \frac{1}{2}mv^2$ +1

$\frac{1}{2}mv^2 = 10eV = 10^{-18}J$ is simplified since I just solved for KE above.

$v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2(10^{-18}J)}{10^{-30}kg}}$ +1

$= \boxed{1.414 \times 10^{6} \frac{m}{s}}$

proton: $v = \sqrt{\frac{2KE}{m}}$

$= \sqrt{\frac{2(10^{-18}J)}{10^{-27}kg}} = \boxed{4.47 \times 10^{4} \frac{m}{s}}$ +1

$v_n = 0$ +1

3) $B = \begin{cases} \beta r \hat{\theta} & r \le r_0 \\ \frac{\lambda}{r} \hat{\theta}, & r > r_0 \end{cases}$

+1 → cont. is implied

a) when $r = r_0$, $\beta r = \frac{\lambda}{r}$ +1

$\beta r = \frac{\lambda}{r_0}$ is implied since IS that $r = r_0$

$$\boxed{\beta = \frac{\lambda}{r^2}}$$ +1

equiv to $\lambda = \beta r_0^2$

b) at $r = r_0$, $F = qvB \sin\theta$, $\sin\theta = 1$

TA 1 using $\sin\theta$ instead of $\times$ product

$= q v_0 \beta r_0$

v_0 is towards the center

$= -e v_0 \beta r_0 = ma$

$$\boxed{a = \frac{-e v_0 \beta r_0}{m}}$$

TA 1 sign

perpendicular to the velocity

c) $v_{max} = v_0$? +1 speed remains v_0

d) the electron takes an elliptical orbit within the inner region

e) $I = \frac{dQ}{dt}$ $\quad \frac{dr}{dt} = v$

4 TA 1

4) a) grav. field points towards the origin.

at (C): $V = \frac{PE}{m}$ $\quad g = \frac{V}{r}$

$= \frac{20\ J/kg}{2m}$

$= \boxed{10\ m/s^2}$

at P: $g = \frac{V}{r} = \frac{30\ J/kg}{3m} = \boxed{10\ m/s^2}$

both towards center. +1 +1 direction

b) they will all move towards the origin. +1 +1 +1

higher to lower potential is towards the origin for each one

c) $\Delta KE = -\Delta PE = m\Delta V = 10\ J/kg \cdot m$ +1

(1) electron: $\Delta KE = (10\ J/kg)(10^{-30}\ kg)$

$= \boxed{10^{-29}\ J}$ +1 c

proton and neutron: $KE = (10\ J/kg)(10^{-27}\ kg)$

$= \boxed{10^{-26}\ J}$ +1 +1 a, b

(2) electron: $v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2(10^{-29}\ J)}{10^{-30}\ kg}}$

$= \boxed{4.47\ m/s}$

proton & neutron: $v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2(10^{-26}\ J)}{10^{-27}\ kg}}$ see side

$= \boxed{4.47\ m/s}$

+1 +1 +1

1) c) 2

Since the masses are all (+), then they would obviously go to the origin.

12

1.

A. Continuity of the electric field at $r = a$ *[1 pt]*

requires that $\alpha a = \frac{\beta}{a^2}$ *.[1 pt]*

Therefore, we need $\beta = \alpha a^3$ *.[1 pt]*

B. $V(r) = \int_{\infty}^{r} \vec{E}.d\vec{r} = \int_{\infty}^{r} \frac{\beta}{r^2} dr$ *[1 pt] Correct limits*

[1 pt] Correct integrand.

$= \frac{\beta}{r}, \ r > a$ *[1 pt]*

$V(r) = \int_{\infty}^{r} \vec{E}.d\vec{r} = -\int_{\infty}^{a} \frac{\beta}{r^2} dr - \int_{a}^{r} \alpha r dr, \ r < a$

[1+1 pt] Correct limits for each region
[1+1 pt] Correct integrand for each region.

$= \frac{\beta}{a} + \frac{1}{2}\alpha a^2 - \frac{1}{2}\alpha r^2$ *[1 pt]*

or $V(r) = \frac{3}{2}\alpha a^2 - \frac{1}{2}\alpha r^2$

C. $C_F Q_{enc} = E.A$ *[1 pt] For any correct Gauss' Law*

$4\pi k Q_{enc} = E 4\pi r^2$ *[1 pt]*

$= \frac{\beta}{r^2} 4\pi r^2 = 4\pi\beta, \ r > a$ *[1 pt] Correct substitution*

$Q = \frac{\beta}{k} = 4\pi\varepsilon_o\beta, \ r > a$ *[1 pt]*

$4\pi k Q_{enc} = E 4\pi r^2 = 4\pi\alpha r^3, \ r \le a$
[1 pt] Correct substitution

$Q_{enc} = \frac{4\pi\alpha}{k} r^3 = \varepsilon_o \alpha r^3, \ r \le a$ *[1 pt] either constant.*

D. $m\vec{a} = -e\vec{E}$ *[1 pt] Newton's Law*

$$\vec{a} = \begin{cases} \frac{-e}{m}\alpha r\hat{r}, & r \le a \\ \frac{-e\beta}{mr^2}\hat{r}, & r > a \end{cases}$$ *[1 pt] for each region, total [2 pts]*

E. The electron reaches its maximum velocity at $r = 0$ where its *potential energy* is **minimum** (where the electric potential is maximum).*[1 pt] for the statement or using this information in the calculations.*
Correct CoE statement *[1 pt]*

$\frac{1}{2}mv_o^2 - \frac{1}{2}mv_a^2 = e\left(V(0) - V(a)\right) = \frac{1}{2}e\alpha a^2$ *[1 pt]*

$v_o = \sqrt{\frac{e}{m}\alpha a^2 + v_a^2} = a\sqrt{\frac{e\alpha}{m}}$ *[1 pt]*

F. This is a simple harmonic motion*[1 pt]* given by $r = a\sin(\omega t + \phi)$ *[1 pt]* (or $r = a\cos(\omega t + \varphi)$) where ϕ is determined by the initial conditions. In this case it is 90° since r=a at t=0, i.e. $r = a\cos(\omega t)$
The electron exits the region at the velocity that it enters after reaching its maximum speed at the center of the region. The angular frequency of the motion is

$\omega = \sqrt{\frac{e\alpha}{m}}$ *[1 pt]* and the period of the motion is

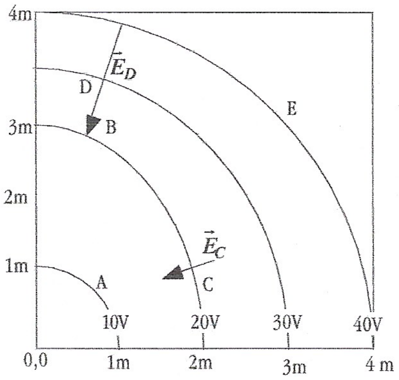
$T = 2\pi\sqrt{\frac{m}{e\alpha}}$ *.[1 pt]*

G. Yes. It gets there*[1 pt]* in half the period, that is

$t = \pi\sqrt{\frac{m}{e\alpha}}$ *[1 pt]*

2.

A.



$E = -\frac{dV}{dr} \approx -\frac{\Delta V}{\Delta r}$ *[1 pt]*

$E_C = -\frac{30V - 10V}{3m - 1m} = -10\frac{V}{m}$ *[1 pt]*

Correct direction on the figure[1 pt]

$E_D = -\frac{40V - 20V}{4m - 3m} = -20\frac{V}{m}$ *[1 pt]*

Correct direction on the figure[1 pt]

B. They will all move from higher to lower potential for each one.*[1 pt]* That is
The electron will move toward 40V equipotential *[1 pt]* following E_D inthe opposite direction*[1 pt]*. a
The proton will move toward 20V equipotential*[1 pt]* following E_D *[1 pt]*. b
The neutron will stay at D*[1 pt]* since it has no electric charge*[1 pt]*. c

C.

1. Both the electron and the proton will gain $10eV$ kinetic energy (or $10^{-18}J$)*[1+1 pts]*. Neutron stays where it is.*[1 pt]*

2. $\frac{1}{2}mv^2 = 10eV$ *[1 pt]*, $v = \sqrt{20\frac{eV}{m}}$ *[1 pt]*

$v_e = \sqrt{20\frac{10^{-19}}{10^{-30}}}\frac{m}{s} = 1.4\times10^6\frac{m}{s}$ *[1 pt]*

$v_p = \sqrt{20\frac{10^{-19}}{10^{-27}}}\frac{m}{s} = 4.5\times10^4\frac{m}{s}$ *[1 pt]*

$v_n = 0$ or it will not get there on its own. *[1 pt]*

3.

A. Continuity of the magnetic field*[1 pt]* requires $\beta r_o = \frac{\lambda}{r_o}$ *[1 pt]*, $\lambda = \beta r_o^2$ *[1 pt]*

B. $\vec{a} = \frac{\vec{F}}{m} = -\frac{e}{m}\vec{v}_o \times \vec{B}$ *[1 pt]*

$= -\frac{e}{m}\beta r v_o(-\hat{r} \times \hat{\theta}) = \frac{e}{m}\beta r v_o \hat{k}$ *[1 pt]*

The direction of the accelerations continually changes (rotates).*[1 pt]*

C. The speed of the electron remains v_o *[1 pt]* since the magnetic force is always perpedicular to the velocity; however, the direction of velocity changes as described in B. *[1 pt]*

D. $a = \frac{v_o^2}{R} = \frac{e}{m}\beta r v_o$, $R = \frac{m v_o}{e\beta r}$ *[1 pt]*

The radius of the motion gets larger as the electron gets closer to the center.*[1 pt]*

There are three distinct possibilities

1. The electron completes a semi-ellipse and goes back into the region it came from*[1 pt]*
2. The electron completes a quarter-ellipse moves tangent to the z-axis. *[1 pt]*
3. The electron completes a quarter-ellipse goes through the center and moves to the other side and completes a reverse quarter-ellipse.*[1 pt]*

E. The current is in the z-direction.*[1 pt]*

$B 2\pi r = \mu_o I_{enc}$ *[1 pt]*

$\frac{\lambda}{r} 2\pi r = \mu_o I_{enc}$ *[1 pt] correct substitution*

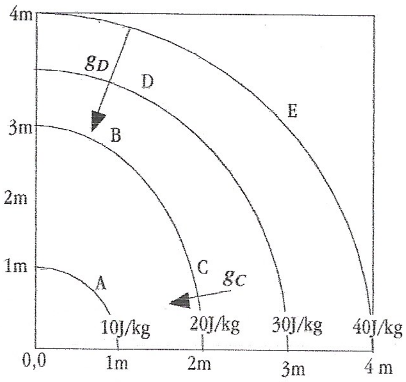
$I = \frac{2\pi}{\mu_o}\lambda$, $r > r_o$ *[1 pt]*

$\beta r 2\pi r = \mu_o I_{enc}$ *[1 pt] correct substitution*

$I = \frac{2\pi}{\mu_o}\beta r^2$, $r \le r_o$ *[1 pt]*

4.

A.



$g = -\frac{dV_g}{dr} \approx -\frac{\Delta V_g}{\Delta r}$ *[1 pt]*

$g_C = -\frac{30-10}{3-1}\frac{m}{s^2} = -10\frac{m}{s^2}$ *[1 pt]*

Correct direction on the figure[1 pt]

$g_D = -\frac{40-20}{4-3}\frac{m}{s^2} = -20\frac{m}{s^2}$ *[1 pt]*

Correct direction on the figure[1 pt]

B. They will all move from higher to lower potential for each one.*[1+1+1 pt]* That is they will all follow the direction of the gravitational field and will move toward $20\frac{J}{kg}$ equipotential line.*[1 pt]*

C.

1. They will all gain $10M\frac{J}{kg}$ *[1 pt]* d

   $KE_p = 10^{-26}\,J$ *[1 pt]* a

   $KE_n = 10^{-26}\,J$ *[1 pt]* b

   $KE_e = 10^{-29}\,J$ *[1 pt]* c

2. $\frac{1}{2}Mv^2 = 10M\frac{J}{kg}$, $v = 4.5\frac{m}{s}$ *[1+1+1 pts]*