1998 Mech MC

total Mech:

$45 \cdot \frac{30}{35} + 42 = \boxed{80.6}$ /90

MC: $\frac{30}{35}$

1) b ✓ 2) e ✓ 3) d ✓

4) $\Sigma F = 0$ when $a = 0$

$\frac{dx}{dt} = 3t^2 - 12t + 9$

$\frac{d^2x}{dt^2} = 6t - 12$

$t = 2$ (b) ✓

5) $2F3R - F3R - F2R - F3R$

$6FR - 6FR - 2FR$ (c) ✓

6) $v = R\omega$ (a) ✓

7) (a) ✓ 8) $F = \frac{GmM}{R^2}$ $F_a = \frac{GmM}{4R^2}$ (d) ✓

9) $\frac{dx}{dt} = -\frac{9}{2}x^2$ $x(t) = -\frac{3}{2}x^3$ (c) d guess

10) $T = 2\pi\sqrt{\frac{g}{L}}$ (e) a

11) e ✓ 12) $F\Delta t = \Delta p = 40$ kg m/s (a) c

13) $mv = (3m) v_f$

$v_f = \frac{1}{3}v$ (a) c

14) $\frac{mv^2}{r} = F_c =$

$= F_s = kx = (100 N/m)(0.03 m)$

$= 3N$ (b) ✓

15) (a) ✓

16) $3U_0 - U_0 = \frac{1}{2}mv^2$

$4U_0 = mv^2$ (c) ✓

$v_f = \sqrt{\frac{4U_0}{m}}$

17) $U(r) = br^{-\frac{3}{2}} + c$

$W = \int F \cdot dr$ $\frac{dW}{dr} = F$

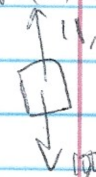
$F = -\frac{3}{2}br^{-\frac{5}{2}}$ (a) ✓

18) (b) ✓

19) 11,000 N $\Sigma F = ma$

$a = 1 m/s^2$

10000 N



$\cancel{v_f} = v_0 + at$ $a = \frac{-v_0}{t}$

$\cancel{v_f^2} = v_0^2 + 2a\Delta x$

$v_0^2 = -2a\Delta x$ $v_0 = \sqrt{2(1)(8)}$

$= \sqrt{16} = 4$ (a) ✓

20) $F = \frac{GMm}{r^2} = \frac{Mv^2}{r}$

$\frac{GM}{D} = \frac{2v^2}{\cancel{D}}$

$v^2 = \frac{GM}{2D}$ (b) ✓

21) $F\cos\theta - f = ma$ (d) ✓

22) $F\sin\theta + N - mg = 0$

$N = mg - F\sin\theta$

$f = \mu N$

$\mu = \frac{f}{N} = \frac{f}{mg - F\sin\theta}$ (e) ✓

23) X N/A

24) (a) ✓ 25) (b) ✓

26) $\bar{v} = \frac{\Delta x}{t}$

$\Delta y = \cancel{v_{0y}t} + \frac{1}{2}gt^2$

$t = \sqrt{\frac{2\Delta y}{g}}$ $v = \frac{3m}{\sqrt{\frac{2(10m)}{10}}} = \frac{3}{\sqrt{2}}$ (c) ✓

27) $W = \int_0^2 F \cdot dx = \int_0^2 40x - 6x^2 dx$ ✓

$= 20x^2 - 2x^3 \Big|_0^2 = 80 - 16 J = 64 J$ (d)

28) (c) ✓ 29) $\frac{dx}{dt} = -A\omega \sin\omega t$

$\frac{dy}{dt} = A\omega\cos\omega t$

$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$

$\frac{dy}{dx} = -\cot\omega t$

$= \frac{\omega}{\sin^2\omega t(-A\omega\sin\omega t)}$

$A\sin^3\omega t$

$\frac{d^2x}{dt^2} = -A\omega^2 \cos\omega t\ \hat{i}$

$\frac{d^2y}{dt^2} = -A\omega^2 \sin\omega t\ \hat{j}$

Assuming $t=0$?

$a = -A\omega^2 (\cos\omega t\ \hat{i} + \sin\omega t\ \hat{j})$

$= -1.5(4)(1\hat{i})$

$= 6\ \frac{m}{s^2}$ (e) ✓

30) $\Sigma\tau = I\alpha = 0$

$m_1 g a = m_2 g b$ (b) ✓

31) (e) 32) $\Sigma\tau = I\alpha$ $\omega_f = \omega_0 + \alpha t$

$\alpha = \frac{\omega_f}{T}$

$\tau = \frac{I\omega_f}{T}$ (e) ✓

33) $W = \int \tau\, d\theta$ Skipped 32?

$KE_f = \frac{1}{2} I\omega_f^2$

$\frac{W}{t} = \int \tau \cdot \frac{d\theta}{t}$

$\frac{W}{T} = \frac{I\omega_f^2}{2T}$ (b) ✓

34) $\frac{dv}{dt} = g - bv$

$\frac{dv}{g-bv} = dt$

$u = g - bv$ $\frac{du}{dv} = -b$

$t = -\frac{1}{b} \ln|g-bv|$

$dv = \frac{du}{-b}$

$-bt = \ln|g-bv|$

$g - bv = e^{-bt}$

$bv = g - e^{-bt}$

$v = \frac{1}{b}(g - e^{-bt})$ (a) ✓

35) $\frac{1}{2} m v_m^2 = \frac{1}{2} k A^2$

$k = \frac{m v_m^2}{A^2}$ (d) ✓

$\frac{42.5}{45} \rightarrow \frac{42}{45}$ (trunc.)

1998 Mech FR

1) a) (i) ~~$\bar{v} = \frac{x}{t} = \frac{0.5m}{0.5s} = 1 \frac{m}{s}$~~

~~(ii) $\bar{v} = \frac{x}{t} = \frac{0.44m}{0.5s} = 0.88 \frac{m}{s}$~~

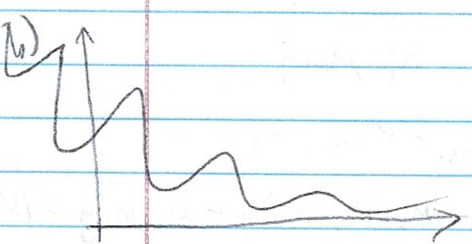
~~(iii) $\bar{v} = \frac{x}{t} =$~~

(i) $\bar{v} = \frac{\Delta x}{t} = \frac{0.2m}{0.2s} = \boxed{1 \frac{m}{s}}$ ✓ +1

(ii) $\bar{v} = \frac{\Delta x}{t} = \frac{0.12m}{0.2s} = \boxed{0.6 \frac{m}{s}}$ ✓ +1

(iii) $\bar{v} = \frac{\Delta x}{t} = \frac{0.04m}{0.2s} = \boxed{0.2 \frac{m}{s}}$ +1 ✓

b)



c) (i) $m_a v_{a_0} + \cancel{m_b v_{b_0}} = \cancel{m_a v_{a_f}} + m_b v_{b_f}$ +1

$(0.40kg)(1 \frac{m}{s}) =$ ~~[illegible]~~

$0.40kg\, v_{af} + 0.60kg(v_{bf})$

+1

$= 0.40kg(0.2 \frac{m}{s}) + 0.60kg(v_{bf})$

$v_b = \frac{0.40kg(1 \frac{m}{s} - 0.2 \frac{m}{s})}{0.60kg} = \boxed{1.2 \frac{m}{s}}$

+1

d) (i) Yes, KE is conserved. +1 +1

(ii) the kinetic energy is stored in spring potential energy.

+1

2) a) (i) $mv_0 = 3m(v_f)$ +1

$v_f = \frac{v_0}{3}$ +1

$KE = \frac{1}{2}(3m)v_f^2$

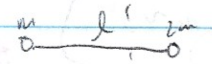
$= \frac{1}{2}(3m)\left(\frac{v_0}{3}\right)^2$

$= \boxed{\frac{1}{6}mv_0^2}$ +1

(ii) $KE_0 = \frac{1}{2}mv_0^2$

$\Delta KE = KE_f - KE_0 = \boxed{-\frac{1}{3}mv_0^2}$ +1 +1

b)



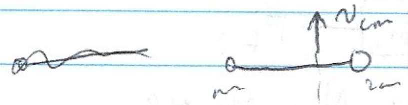
(i) $x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_{total}}$ +1

$= \frac{(m)(0m) + 2m(l)}{3m}$

+1

$= \boxed{\frac{2}{3}l}$ from the left end

(ii)



+1

(iii) $mv_0 = 3m(v_f)$

$v_f = \boxed{\frac{v_0}{3}}$ +1

~~$v_{cm_0} = v_{cm_f}$~~

(iv) $L_0 = L_f$

$I = m\left(\frac{2l}{3}\right)^2 + 2m\left(\frac{l}{3}\right)^2$

$= \frac{4}{9}ml^2 + \frac{2}{9}ml^2$

$= \frac{2}{3}ml^2$ +1

+1

$r_p = I\omega_f$

$\omega_f = \frac{\frac{1}{3}l \cdot mv_0}{\frac{2}{3}ml^2} = \boxed{\frac{v_0}{2l}}$ +1

(v) $KE_0 = \frac{1}{2} m v_0^2$

$KE_f = \underbrace{\frac{1}{6} m v_0^2}_{\text{linear (parallel)}} + \underbrace{\frac{1}{2}\left(\frac{2}{3} m\ell^2\right)\left(\frac{v_0}{2\ell}\right)^2}_{\text{rotational}}$ +1

$= \frac{1}{6} m v_0^2 + \frac{1}{3} m\ell^2 \cdot \frac{v_0^2}{4\ell^2}$

$= \frac{3}{12} m v_0^2 = \boxed{\frac{1}{4} m v_0^2}$ +1

$\Delta KE = KE_f - KE_0 = \boxed{-\frac{1}{2} m v_0^2}$ brah!!

3) a) (i) $\vec{N}_1$ ↑ [m_1] +1

$N_1 = m_1 g$

(ii) [m_1] → F_f

$F_f = \mu_{s1} N = \mu_{s1} m g$

(iii) [m_2] → T +1

$\vec{T}$ ↑ [M] ↓ mg $\quad \sum F = 0 \quad T = Mg$

(iv) N_2 ↑ [m_2] $\quad N_2 = (m_1 + m_2) g$ +1

(v) F_f ← [m_2] $\quad F_f = \mu_{s2}(m_1 + m_2) g$ +1/2

b) $Mg - \mu_{s2}(m_1 + m_2) g = 0$ +1 +1

$M = \mu_{s2}(m_1 + m_2)$ +1

c) $\sum F_{\text{system total}} = ma$ +1

$Mg - \mu_{k2}(m_1 + m_2) g = ~~ma~~ (M + m_1 + m_2) a$ +1

$\boxed{a = \frac{Mg - \mu_{k2}(m_1 + m_2) g}{M + m_1 + m_2}}$ +1

d) $\sum F_1 = m_1 a_1$

(i) $F_f = m_1 a_1$

$\mu_{k1} \cancel{m_1} g = \cancel{m_1} a_1$ +1

$a_1 = \mu_{k1} g$ +1

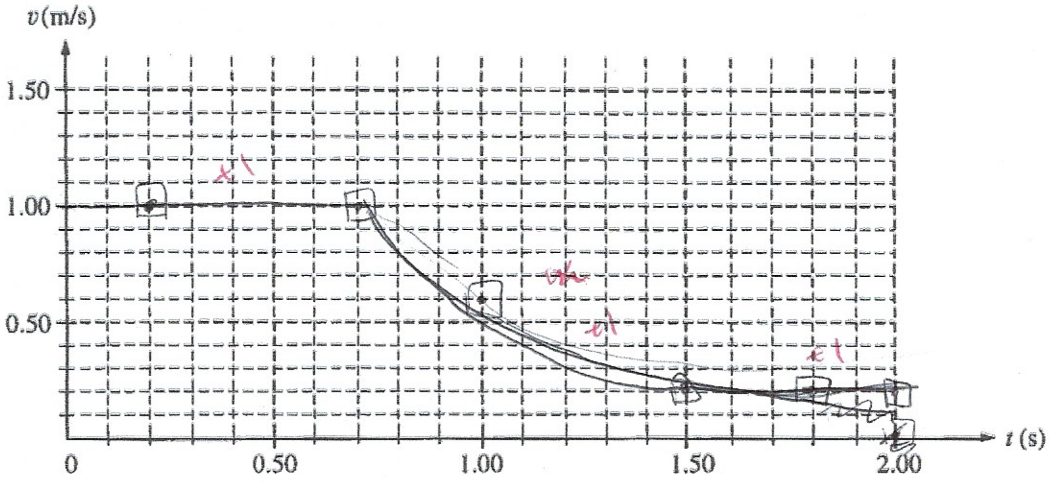
(ii) $\sum F_{sys.} = m_{total} a_2$

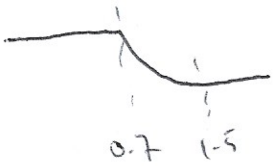
$Mg - \mu_{k2}(m_1 + m_2) g - \mu_{k1} m_1 g = (M + m_2) a_2$ +1

$\boxed{a_2 = \frac{Mg - \mu_{k2}(m_1 + m_2) g - \mu_{k1} m_1 g}{M + m_2}}$ +1

-2.5

1) (b) On the axes below, sketch a graph, consistent with the data above, of the speed of glider A as a function of time t for the 2.00 s interval.

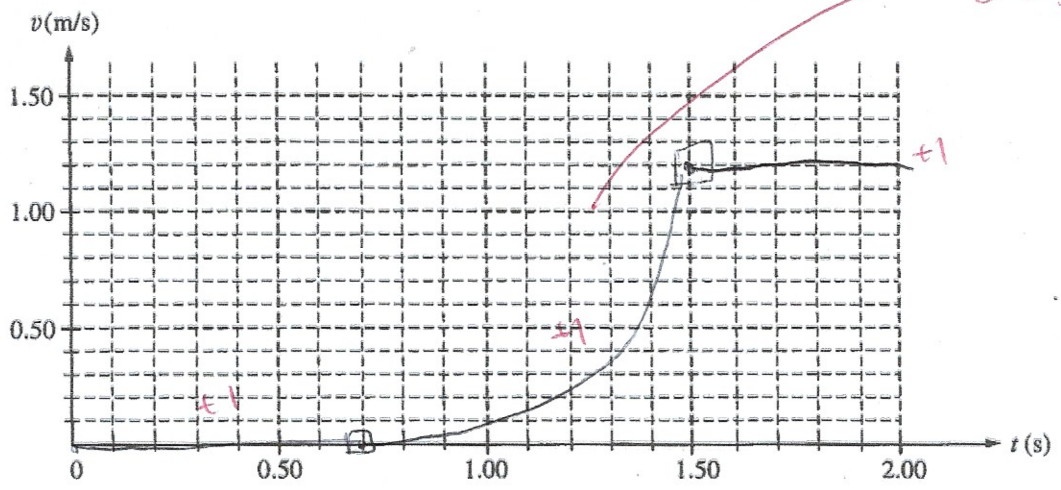




(c) i. Use the data to calculate the speed of glider B immediately after it separates from the spring.

ii. On the axes below, sketch a graph of the speed of glider B as a function of time t.

Monotonic, +S slope



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