Universal Gravity

1. If a distance be between two point particles is doubled, then the gravitational force between them.
   A. decreases by a factor of 4
   B. decreases by a factor of 2
   C. increases by a factor of 2
   D. increases by a factor of 4
   E. Cannot be determined without knowing the masses

2. On the surface of the earth, an object of mass m has weight w. If this object is transported to an altitude that's twice the radius of the earth, at the new location, its mass and weight are
   A. m/2, w/2 B. m, w/2 C. m/2, w/4 D. m, w/4 E. m, w/9 F.

3. A moon of mass M orbits a planet of mas 100 M. If the strength of the gravitational force exerted by the planet on the moon is F_P and the strength of the gravitational force exerted by the moon on the planet is F_m, which statement is true?
   A. $F_P = 100\,F_m$ B. $F_P = 10\,F_m$ C. $F_P = F_m$
   D. $F_P = F_m/10$ E. $F_P = F_m/100$ F.

4. Pluto has 1/500 the mass and 1/15 the radius of Earth. What is the value of g on its surface?
   A. $0.3\,m/s^2$ B. $1.6\,m/s^2$ C. $2.4\,m/s^2$ D. $4.5\,m/s^2$ E. $7.1\,m/s^2$ F

5. A satellite in a near circular orbit of radius R around Earth has kinetic energy K. When the satellite moves to a new orbit of radius 2R, what will be its new kinetic energy?
   A. K/4 B. K/2 C/ K D. 2K E. 4K F.

6. A moon of Jupiter has a nearly circular orbit of radius R and a period of T. What is the mass of Jupiter?
   A. $2\pi R/T$ B. $4\pi^2 R/T^2$ C. $2\pi R^3/(GT^2)$
   D. $4\pi R^2/(GT^2)$ E. $2\pi R^3/(GT^2)$

7. Two large bodies, A of mass *m* and B of mass *4m*, are seperated by a distance R. At what distance from A, along the line joining the bodies, would teh gravitational force on an object be equal to zero?
   A. R/16 B. R/8 C. R/5 D. R/4 E. R/3 F.

8. The mean distance from Saturn to the Sun is 9 times greater tan the mean distance from Earth to the Sun. How long is a Saturn year in Earth years?
   A. 18 B. 27 C. 81 D. 243 E. 729 F.

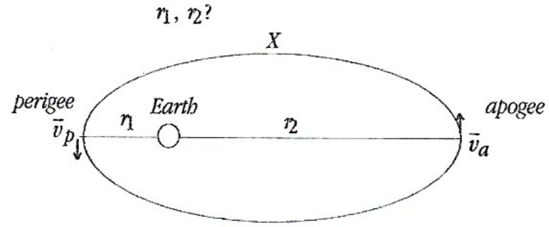
9. The Moon has mass M and radius R. A small object is dropped from a distance 3R from the Moon's center. The object's impact speed when it strikes the surface of the Moon is equal to
   A. $\sqrt{\frac{GM}{3R}}$ B. $\sqrt{\frac{2GM}{3R}}$ C. $\sqrt{\frac{3GM}{4R}}$ D. $\sqrt{\frac{4GM}{3R}}$ E. $\sqrt{\frac{3GM}{2R}}$ F.

10. A planet orbits the Sun in an elliptical orbit of eccentricity e. What is the ratio of the planet's speed at perihelionto its speed at aphelion?
    A. 1/(1–*e*) B. *e*/(1–*e*) C. 1/(1+*e*) D. *e*/(1+*e*) E. (1+*e*)/(1-*e*) F

11. Two satellits A and B orbit a planet in circular orbits with radii R_A and $R_B = 3R_A$. What is the relation between the velocities?
    A. $v_B = v_A$ B. $v_B = 3v_A$ C. $v_B = 9v_A$ D. $v_B = \sqrt{3}v_A$ E. $v_B = \frac{v_A}{\sqrt{3}}$

1. Donsider two uniform spherical bodies in deep space with masses m_1 and m_2. Starting from rest from a distance *R* apart, they are gravitationally attracted to each other.
   A. Calculate the acceleration of m_1 when the spheres are a distance R/2 apart.
   B. Calculate the acceleration of m_2 when the spheres are a distance R/2 apart.
   C. Calculate the speed of m_1 when the spheres are a distance R/2 apart.
   D. Calculate the speed of m_2 when the spheres are a distance R/2 apart.
   E. Now assume that the spheres orbit their center of mass with the same orbital period T. Determine the radii of their orbits.

2. A satellite of mass *m* is in an elliptical orbit.
   A. Determine the speed v_p of the satellite at perigee (the point on the ellips closest to the Earth) in terms of r_1, r_2, *M* and *G*.
   B. Determine the speed v_a of the satellite at apogee (the point on the ellips fartest from the Earth) in terms of r_1, r_2, *M* and *G*.
   C. Express the ratio v_p/v_a in the simplest form.
   D. What is the satellite's angular momentum when it is at apogee
   E. Determine the speed of the satellite when it is at the midpoint between perigee and apogee marked as *X* in in terms of r_1, r_2, *M* and *G*.
   F. Determine the period of the satellite's orbit.
   G. What is the eccentricity of the satellite's orbit, in terms of r_1, r_2?



Black 4 Red 16

Blue 2 Total 22

Black+Blue 6

PR 7 Grav

MC) 1) $F_g = \frac{GMm}{r^2}$ (A) ✓

2) $F_g = \frac{GMm}{r^2} = \frac{GM}{4r^2}$ (d) × c

3) (c) ✓

4) $g = \frac{GM}{r^2} = \frac{G(\frac{1}{500}M)}{(\frac{1}{15}r)^2}$

$= \frac{1}{500} \cdot \frac{1}{15} \cdot \frac{1}{15} (9.8 m/s)$

(d) $= 0.000087 m/s$

(F) ×

5) (A)? (B)

6) $F_g = F_c = \frac{GMm}{R^2}$

$f = \frac{1}{T}$ $\omega = 2\pi f$

$\omega = \frac{2\pi}{T}$

$F_g = \frac{GMm}{R^{2}} = F_c$

$= \frac{mv^2}{R}$

$\frac{GM}{R} = v^2$ $v = \frac{2\pi R}{T}$

$\frac{GM}{R} = \frac{4\pi^2 R^2}{T^2}$

$M = \frac{4\pi^2 R^3}{GT^2}$ (d) or (e) ✓e

which one is printed wrong

R3 3

7) $F_1 \; F_g$

0m —— 4m

$F = \frac{GMm}{r^2}$ $\frac{Gm\cancel{m_t}}{r_1^2} = \frac{G4m\cancel{m_t}}{r_2^2}$

$r_1 + r_2 = R$

$r_1 = R - r_2$

$\frac{G}{r_1^2} = \frac{4G}{r_2^2}$

$\frac{(R-r_2)^2}{\cancel{G}} = \frac{r_2^2}{4\cancel{G}}$

$R^2 - 2Rr_2 + r_2^2 - \frac{1}{4}r_2^2 = 0$

$\frac{3}{4}r_2^2 - 2Rr_2 + R^2 = 0$

$r_2 = \frac{2R \pm \sqrt{4R^2 - 3R^2}}{\frac{3}{2}}$

$= (2R + R) \cdot \frac{2}{3}, \; R \cdot \frac{2}{3}$

$= \cancel{2R}, \; \frac{2}{3}R$

$r_2 = \frac{2}{3}R$ $r_1 = \frac{1}{3}R$ (e) ✓

8) $\frac{\cancel{2\pi} r_1}{T_1} = \frac{9 \cancel{2\pi} r_1}{T_2}$

$\frac{T_2}{9\cancel{r_1}} = \frac{T_1}{\cancel{r_1}}$

$T_2 = 9T_1$?? (A)? (B) ×

R1 1

9) $U = \frac{GMm}{R}$ $\quad KE_f = U_0 = \frac{1}{2}mv_f^2$

$\frac{1}{2}\cancel{m}v_f^2 = \frac{GM\cancel{m}}{(3R)}$

$v_f = \sqrt{\frac{2GM}{3R}}$ (b) (b)

10) ?! × (E)

11) (e)?

FR

1) m_1 — — — — m_2

a) $F_G = \frac{GM_1M_2}{r^2} = m_1a_1$

$a_1 = \frac{GM_2}{r^2} = \frac{1}{4}\cdot\frac{GM_2}{R^2}$

~~$a_2 = ... a_1$~~ b) $a_2m_2 = \frac{GM_1M_2}{r^2}$

$a_2 = \frac{1}{4}\frac{GM_1}{R^2}$ ~

b) ~~$U_0 = \frac{GMm}{R}$ $\quad E_0 = E_f$~~

~~$\frac{GM_1M_2}{R} = \frac{GM_1M_2}{\frac{1}{2}R} + \frac{1}{2}m_1v^2$~~

~~$\frac{1}{2}m_1v^2 = \frac{GM_1M_2}{R} - \frac{2GM_1M_2}{R}$~~

~~$v^2 = \frac{2GM_2}{R} - \frac{4GM_2}{R}$~~ ^

~~$= \sqrt{\frac{-2GM_2}{R}}$~~

R2

c) $v_f = v_0 + at$

$v_f^2 = \cancel{v_0^2} + 2a\Delta x$

~~$v_f = \sqrt{2a\Delta x}$~~

~~$= \sqrt{\frac{2}{4}\frac{GM_2}{R^2}\cdot\frac{R}{2}}$~~

2) ? ^

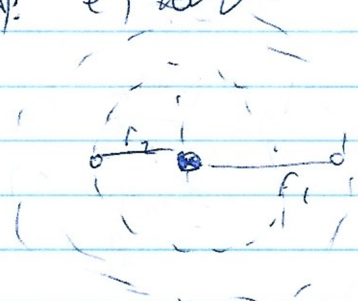
1) a) m_1 ——— m_2

$F = \frac{GM_1M_2}{(\frac{R}{2})^2} = \frac{4GM_1M_2}{R^2} = M_1a$

$a = \frac{4GM_2}{R^2}$ ✓

b) $a_2 = \frac{4GM_1}{R^2}$ ✓

c) ~~d)~~ e) ~~...~~ $\quad m_2 > m_1$



$F_{c_1} = \frac{\cancel{m}v^2}{r_1} = \frac{G\cancel{M_1}M_2}{R^2}$

$\frac{r_1}{v^2} = \frac{R^2}{GM_2}$

$r_1 = \frac{R^2v^2}{GM_2}$ $\quad v = \frac{2\pi r_1}{T}$

$r_1 = \frac{4\pi^2r_1^2R^2}{GM_2T^2}$

2

$$1 = \frac{4\pi^2 \cdot r_1 R^2}{G m_2 T^2}$$

$$\boxed{r_1 = \frac{G m_2 T^2}{4\pi^2 R^2}}$$

~

$$F_{c2} = \frac{\cancel{m_2} v^2}{r_2} = \frac{G m_1 \cancel{m_2}}{R^2}$$

$$\frac{r_2}{v^2} = \frac{R^2}{G m_1}$$

$$r_2 = \frac{v^2 R^2}{G m_1}$$

$$v = \frac{2\pi r_2}{T}$$

$$r_2 = \frac{4\pi^2 r_2^2 R^2}{G m_1 T^2}$$

$$1 = \frac{4\pi^2 r_2 R^2}{G m_1 T^2}$$

$$\boxed{r_2 = \frac{G m_1 T^2}{4\pi^2 R^2}}$$

~

MC) 2) m=m

$$F_g = \frac{GmM}{R^2} - \frac{GmM}{(2R)^2} = \frac{1}{4}\frac{GmM}{R^2}$$

i) c-e) 2)

c) $m_1\vec{v}_1 + m_2\vec{v}_2 = 0 \quad m_1 v_1 - m_2 v_2 = 0$

$$v_2 = \frac{m_1}{m_2} v_1$$

$$0 - G\frac{m_1 m_2}{R} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 - G\frac{m_1 m_2}{\left(\frac{R}{2}\right)}$$

$$G\frac{m_2}{R} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2\left(\frac{m_1}{m_2}v_1\right)^2$$

$$G\frac{m_2}{R} = \frac{1}{2}\left(m_1 + m_2\left(\frac{m_1}{m_2}\right)^2\right)v_1^2$$

$$v_1 = m_2\sqrt{\frac{2G}{(m_1+m_2)m_1 R}}$$

✓

d) $v_2 = \frac{m_1}{m_2}v_1 = \frac{m_1}{m_2}m_2\sqrt{\frac{2G}{(m_1+m_2)m_1 R}}$

$$= m_1\sqrt{\frac{2G}{(m_1+m_2)m_1 R}}$$

✓

e) $\frac{m_1 v_1^2}{r_1} = G\frac{m_1 m_2}{(r_1+r_2)^2} = \frac{m_2 v_2^2}{r_2}$

$$\frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2} \qquad v = \frac{2\pi r}{T}$$

$$\frac{m_1}{r_1}\left(\frac{2\pi r_1}{T}\right)^2 = \frac{m_2}{r_2}\left(\frac{2\pi r_2}{T}\right)^2$$

$$m_1 r_1 = m_2 r_2$$

$$\frac{T^2}{(r_1+r_2)^3} = \frac{4\pi^2}{G(m_1+m_2)} \Rightarrow r_1 + r_2 = \sqrt[3]{\frac{G(m_1+m_2)T^2}{4\pi^2}}$$

$$r_1 + r_2 = \sqrt[3]{\frac{G(m_1+m_2)T^2}{4\pi^2}} = r_1 + \frac{m_1}{m_2}r_1$$

$$r_1 = \left(\frac{m_2}{m_1+m_2}\right)\left(\frac{G(m_1+m_2)T^2}{4\pi^2}\right)^{\frac{1}{3}}$$

$$r_2 = \left(\frac{m_1}{m_1+m_2}\right)\left(\frac{G(m_1+m_2)T^2}{4\pi^2}\right)^{1/3}$$

✓

2) a) $E = -G\frac{mM}{2a}$ $\quad 2a = r_1 + r_2$

$K_1 = E - U_1 = -G\frac{mM}{r_1+r_2} + G\frac{mM}{r_1}$

$= \frac{GmM}{(r_1+r_2)} \cdot \frac{r_2}{r_1}$

$\frac{1}{2}mv_1^2 = \frac{GmM}{r_1+r_2} \cdot \frac{r_2}{r_1}$

$v_1 = \sqrt{\frac{2GM}{(r_1+r_2)}\frac{r_2}{r_1}}$ ✓

b) $K_2 = E - U_2 = -G\frac{mM}{r_1+r_2} + G\frac{mM}{r_2}$

$= \frac{GmM}{(r_1+r_2)} \cdot \frac{r_1}{r_2}$ $\quad v_2 = \sqrt{\frac{2GMr_1}{(r_1+r_2)r_2}}$ ✓

c) $\frac{v_1}{v_2} = \frac{\sqrt{\frac{2GM}{(r_1+r_2)} \cdot \frac{r_2}{r_1}}}{\sqrt{\frac{2GM}{(r_1+r_2)} \cdot \frac{r_1}{r_2}}} = \frac{r_2}{r_1}$ ✓

d) $L_2 = r_2 m v_2 = r_2 m\sqrt{\frac{2GMr_1}{r_2(r_1+r_2)}}$ ✓

$= m\sqrt{\frac{2GMr_1r_2}{r_1+r_2}}$

e) $a = \frac{1}{2}(r_1+r_2)$ $\quad 2a = r_1 + r_2$

$K_x = E - U_x = -\frac{GmM}{r_1+r_2} + \frac{GmM}{\frac{1}{2}(r_1+r_2)}$

$= G\frac{mM}{r_1+r_2}$; $K_x = \frac{1}{2}mv_x^2$ ✓

$v_x = \sqrt{\frac{2GM}{r_1+r_2}}$

f) $\frac{T^2}{a^3} = \frac{4\pi^2}{GM} \rightarrow T = \sqrt{\frac{4\pi^2 a^3}{GM}}$

$= \sqrt{\frac{4\pi^2\left(\frac{1}{2}(r_1+r_2)\right)^3}{GM}} = \pi\sqrt{\frac{(r_1+r_2)^3}{2GM}}$ ✓

g) $e = \frac{c}{a} = \frac{a-r_1}{a} = \frac{\frac{1}{2}(r_1+r_2)-r_1}{\frac{1}{2}(r_1+r_2)}$

$= \frac{r_2-r_1}{r_2+r_1}$ ✓

2

CHAPTER 7 MC

1. A $F_G \propto \frac{1}{r^2} \Rightarrow r \to 2r,\ F_G \to \frac{1}{4} F_G$

2. E $R \to 3R \quad F \to \frac{1}{9} F$

3. C Action-Reaction $\vec{F}_{M\ on\ E} = -\vec{F}_{E\ on\ M}$

4. D $g = G\frac{M}{R^2} \Rightarrow g_{Pluto} = G\frac{M_{pl}}{R_{pl}^2}$

$$g_{pl} = G\frac{\frac{1}{500}M_E}{(\frac{1}{15}R_E)^2} = \frac{225}{500}\, G\frac{M_E}{R_E} \approx 4.5\frac{m}{s^2}$$

5. B $F_G = F_c \Rightarrow G\frac{Mm}{R^2} = \frac{mv^2}{R}$

$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}\, G\frac{Mm}{R}.$

$\Rightarrow R \to 2R \Rightarrow KE \to \frac{1}{2}KE$

6. E $F_G = F_c \Rightarrow G\frac{Mm}{R^2} = m\frac{v^2}{R} = \frac{m}{R}\left(\frac{2\pi R}{T}\right)^2$

$\Rightarrow M = \frac{4\pi^2 R^3}{GT^2}$

7. E

$$G\frac{m_A m}{x^2} = G\frac{m_B m}{(R-x)^2}$$

$\frac{m}{x^2} = \frac{4m}{(R-x)^2} \Rightarrow (R-x)^2 = 4x^2$

$R - x = \pm 2x$

$R = (1 \pm 2)x$

$x = -\frac{R}{2},\ \frac{R}{3}$

If the m is between A & B $x = \frac{R}{3}$.

8. B $T^2 \propto R^3$ (see 6 above)

9. D C.o.E: $KE_i + U_i = KE_f + U_f$

$0 - G\frac{Mm}{3R} = \frac{1}{2}mv_f^2 - G\frac{Mm}{R}$

$\Rightarrow v_f = \sqrt{\frac{4GM}{3R}}$

10. E C.o.L: $mv_2r_2 = mv_1r_1 \Rightarrow \frac{v_2}{v_1} = \frac{r_1}{r_2}$

OE

1 a $F_G = F_c \Rightarrow m_1 a_1 = G\frac{m_1 m_2}{(\frac{R}{2})^2}$

$a_1 = 4G\frac{m_2}{R^2}$ toward 2

b $1 \leftrightarrow 2$ in a above

$a_2 = 4G\frac{m_1}{R^2}$ toward 1

c No external forces on the ~~two~~-sphere system $\Rightarrow \vec{P}_T$ is conserved.

$m_1\vec{v}_1 + m_2\vec{v}_2 = 0$

$m_1 v_1 - m_2 v_2 = 0 \Rightarrow v_2 = \frac{m_1}{m_2}v_1$

C.o.E: $K_i + U_i = K_f + U_f$

$0 - G\frac{m_1 m_2}{R} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 - G\frac{m_1 m_2}{(R/2)}$

$G\frac{m_2}{R} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2\left(\frac{m_1}{m_2}v_1\right)^2$

$\Rightarrow G\frac{m_2}{R} = \frac{1}{2}\left(m_1 + m_2\left(\frac{m_1}{m_2}\right)^2\right)v_1^2$

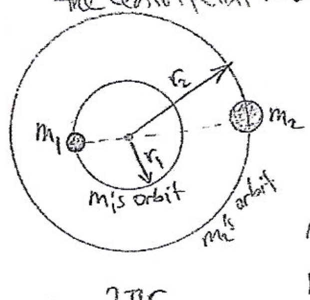
$v_1 = m_2\sqrt{\frac{2G}{(m_1+m_2)m_1 R}}$

(d) from (c)

$v_2 = \frac{m_1}{m_2}v_1 = \frac{m_1}{m_2}m_2\sqrt{\frac{2G}{(m_1+m_2)Rm_1}}$

$v_2 = m_1\sqrt{\frac{2G}{(m_1+m_2)Rm_1}}$

(e) Gravitational is the source of the centripetal motion.



$\frac{m_1 v_1^2}{r_1} = G\frac{m_1 m_2}{(r_1+r_2)^2}$

$= m_2\frac{v_2^2}{r_2}$

Action-reaction

$m_1\frac{v_1^2}{r_1} = m_2\frac{v_2^2}{r_2}$

$v = \frac{2\pi r}{T}$

$\Rightarrow \frac{m_1}{r_1}\left(\frac{2\pi r_1}{T}\right)^2 = \frac{m_2}{r_2}\left(\frac{2\pi r_2}{T}\right)^2,$

$\Rightarrow m_1 r_1 = m_2 r_2 \quad (1)$

See derivation of Kepler's 3rd law:

$$\frac{T^2}{(r_1+r_2)^3} = \frac{4\pi^2}{G(m_1+m_2)} \Rightarrow r_1+r_2 = \sqrt[3]{\frac{G(m_1+m_2)T^2}{4\pi^2}}$$

Substitute from above (1) $r_2 = \frac{m_1}{m_2} r_1$

$$r_1+r_2 = \sqrt[3]{\frac{G(m_1+m_2)T^2}{4\pi^2}}$$

$$r_1 + \frac{m_1}{m_2} r_1 = \sqrt[3]{\quad " \quad}$$

$$\Rightarrow r_1 = \left(\frac{m_2}{m_1+m_2}\right)\left(\frac{G(m_1+m_2)T^2}{4\pi^2}\right)^{1/3}$$

$$r_2 = \frac{m_1}{m_2} r_1 = \left(\frac{m_1}{m_1+m_2}\right)\left(\frac{G(m_1+m_2)T^2}{4\pi^2}\right)^{1/3}$$

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2. a $E = -G\frac{mM}{2a}$, $2a = r_1+r_2$

$$K_1 = E - U_1 = -G\frac{mM}{r_1+r_2} - \left(-G\frac{mM}{r_1}\right)$$

$$K_1 = \frac{GmM}{(r_1+r_2)}\frac{r_2}{r_1}$$

$$\frac{1}{2}mv_1^2 = G\frac{mM}{(r_1+r_2)}\frac{r_2}{r_1} \Rightarrow v_1 = \sqrt{\frac{2GM}{(r_1+r_2)}\frac{r_2}{r_1}}$$

b $K_2 = E - U_2 = -G\frac{mM}{r_1+r_2} - \left(-G\frac{mM}{r_2}\right)$

$$K_2 = G\frac{mM}{(r_1+r_2)}\frac{r_1}{r_2}$$

$$\Rightarrow v_2 = \sqrt{\frac{2GM}{(r_1+r_2)}\frac{r_1}{r_2}}$$

c From a & b

$$\frac{v_1}{v_2} = \frac{\sqrt{\frac{2GM}{(r_1+r_2)}\frac{r_2}{r_1}}}{\sqrt{\frac{2GM}{(r_1+r_2)}\frac{r_1}{r_2}}} = \frac{r_2}{r_1}$$

Or use conservation of angular mom.

$$mv_1r_1 = mv_2r_2 \Rightarrow \frac{v_1}{v_2} = \frac{r_2}{r_1}$$

d $L_2 = r_2 m v_2 = r_2 m\sqrt{\frac{2GMr_1}{r_2(r_1+r_2)}}$

$$L_2 = m\sqrt{\frac{2GMr_1r_2}{r_1+r_2}}$$

e X to c. of Earth $a = \frac{1}{2}(r_1+r_2)$

$$2a = r_1+r_2$$

$$K_x = E - U_x = -G\frac{mM}{r_1+r_2} - \left(-\frac{GmM}{\frac{1}{2}(r_1+r_2)}\right)$$

$$K_x = G\frac{mM}{r_1+r_2}, \quad K_x = \frac{1}{2}mv_x^2$$

$$\Rightarrow v_x = \sqrt{\frac{2GM}{r_1+r_2}}$$

f See the derivation of Kepler's 3rd law

$$\frac{T^2}{a^3} = \frac{4\pi^2}{GM} \Rightarrow T = \sqrt{\frac{4\pi^2a^3}{GM}} = \sqrt{\frac{4\pi^2\left(\frac{1}{2}(r_1+r_2)\right)^3}{GM}}$$

$$T = \pi\sqrt{\frac{(r_1+r_2)^3}{2GM}}$$

g $e = \frac{c}{a}$

$$e = \frac{c}{a} = \frac{a-r_1}{a} = \frac{\frac{1}{2}(r_1+r_2)-r_1}{\frac{1}{2}(r_1+r_2)} = \frac{r_2-r_1}{r_2+r_1}$$
