Ch - 9

1.

A. Under what conditions can the conservation of momentum be used? *1pt*

B. Under what conditions can the conservation of energy be used? Write these as short statements **not** as lengthy statements. (0 pts for lengthy verbal statements) *1pt*

C. What are the conditions for a collision to be an elastic collision? *2pts*

D. What is the relation between the center of mass and momentum? Describe it briefly in mathematical terms. *1pt*

2. Consider two billiard balls with masses m_1 and m_2. The first ball is moving with velocity $\vec{v}_{1b}$ toward the second ball, which is at rest. After the collision, the balls move away from each other at an angle θ with velocities $\vec{v}_{1a}$ and $\vec{v}_{2a}$ respectively. *1pt per part*

A. On a coordinate axis, draw a diagram representing the situation

B. Write down the momentum of each ball before the collision

C. Write down the total momentum of the two ball system before the collision

D. Write down the momentum of each ball after the collision

E. Write down the total momentum of the two ball system after the collision

F. Use the law of cosines to write down the total momentum of the two particle system in terms of the given quantities

G. Assume that, after the collision, the velocity of the first ball is at an angle ϕ_1 with respect to its velocity before the collision and the velocity of the second ball is at an angle ϕ_2. Draw a diagram representing this situation.

H. Use the law of sines to write down the total momentum of the two particle system in terms of the given quantities

I. Use these angles and the other given quantities to write down each component (x and y) of the total momentum of the two particle system in terms of the given quantities

J. Write down the KE_{1b}, KE_{2b}, KE_{Tb}

K. Write down the KE_{1a}, KE_{2a}, KE_{Ta}.

L. *Prove that for an elastic collision with $m_1 = m_2$, the deflection angle is $\theta = 90°$.* ***3pts***

M. *Prove that in an elastic collision $m_1 = m_2$ is the* ***necessary*** *condition for the $\theta = 90°$ deflection.* ***3pts***

N. *Obtain θ for an inelastic collision in terms of ΔKE.* ***3pts***

3. In space, a rocket uses exhaust gas to move forward. Assume that initially the system is at rest with the total mass M. The mass of the rocket at any given time is the variable m. The gas of mass dm is exhausted at the constant velocity v with respect to the initial reference frame. The change in the velocity of the rocket as a resulted of this exhausted gas is dv.

A. Obtain a relation connecting these physical quantities. *2pts*

B. How much KE is gained in this process, and where does it come from? *2pts*

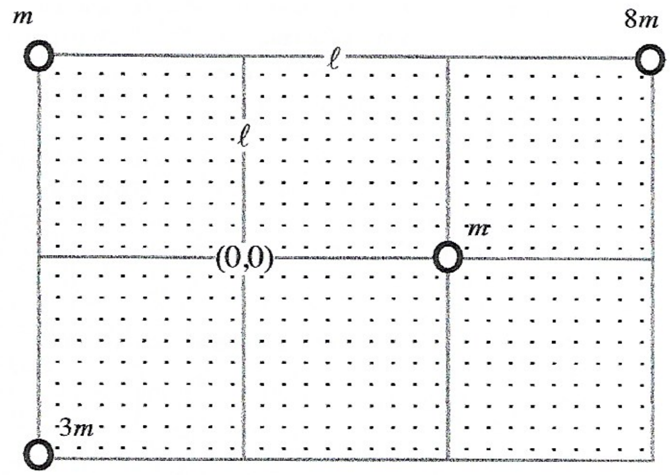
4. Obtain the center of mass of the 4-particle system given in the grid below in terms of ℓ and m. *1pt per part*

A. x-center of mass location

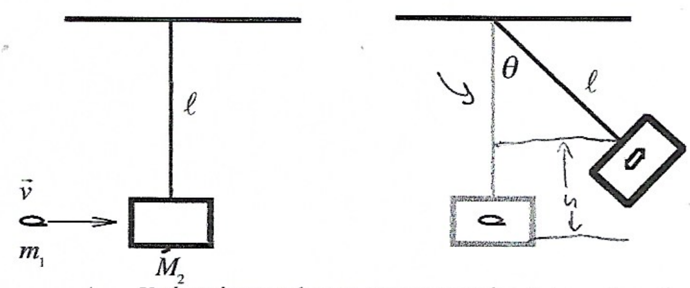
B. y-center of mass location

C. the angle with respect to x axis

D. the center of mass position vector



5. Consider the situation below where the object m_1 moving with velocity $\vec{v}$ strikes the block and is embedded in the block. The block-object system swings up until the string holding the system makes an angle θ with the vertical. *1pt per part*



A. Under what conditions can you use the conservation of momentum for this situation?

B. Write down the total momenta just before and just after the collision to obtain the velocity of the system just after the collision

C. How much KE is lost during this collision/embedding?

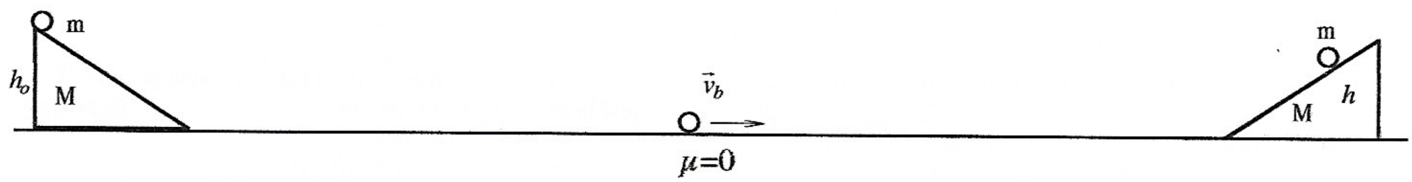
D. Write the height the cm (center of mass) reaches in terms of θ and ℓ.

E. Use the conservation of energy after the collision to obtain the velocity of the system right after the collision in terms of the height cm reaches (in terms of θ and ℓ).

F. Obtain a relation connecting $\vec{v}$ (the velocity of the incoming object right before the collision) and θ in terms of the other given quantities (masses, ℓ, g, etc.)

G. Why can you **not** use the conservation of momentum during the upswing after the collision?

Mesut B. Çakır



6. Consider the set up above where there is no friction between surfaces. The ball of mass m is at a height h_o on the ramp–both at rest and free to slide. The ball moves down on the ramp and reaches $\vec{v}_b$ at the bottom of the ramp. In terms of given quantities and known constants, answer the questions below.
   A. Why and what part of total momentum is conserved during this situation?
   B. Why is total energy conserved during this situation?
   C. Write a conservation of momentum expression between the initial and the final state.
   D. Write a conservation of energy expression between the initial and the final state.
   E. Eliminate the unknowns to obtain an expression between $\vec{v}_b$ and h_o in terms of given quantities and known constants.

7. Consider the set up above where there is no friction between surfaces. The ball of mass m is moving with an initial speed of $\vec{v}_b$ toward the ramp of mass at rest (which is free to slide). The ball moves up on the ramp and reaches a maximum height of h. In terms of given quantities and known constants, answer the questions below.
   A. What will be the relative velocity of the block and the ball when the ball reaches the maximum height up on the ramp?
   B. Why and what part of total momentum is conserved during this collision?
   C. Why is total energy conserved during this collision?
   D. Write a conservation of momentum expression between the initial and the final state.
   E. Write a conservation of energy expression between the initial and the final state.
   F. Eliminate the unknowns to obtain an expression between $\vec{v}_b$ and h in terms of given quantities and known constants.

8. Finally, eliminate $\vec{v}_b$ between problems 6 and 7 to obtain a relation between h_o and h in terms of given quantities and known constants.

9. A rope of legth ℓ held over ground with one end just barely touching the ground. is released at rest . Obtain the normal force exerted by the ground on the rope when the rope has fallen a distance y. If you have difficulty solving the problem, follow the steps below:
   A. How far does the wrope fall during a time internval t?
   B. What is the velocity of each section of the rope at this time?
   C. What is the fraction of the mass that has fallen during this time?
   D. What is the change in the momentum of the rope?
   E. What is net force applied on the rope?
   F. Use your answers above to obtain the normal force.

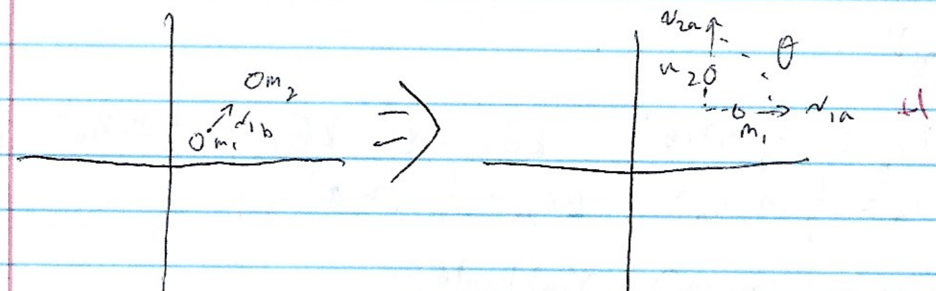
Ch-9

1) a) inelastic, elastic collisions, explosions ~

b) When energy is lost to heat, sound, etc. and you can't calculate where it went ~

c) $KE_0 = KE_f$ +1   d) $\vec{p}_{cm} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \dots$ +1 ~ ~

2) a) ~



b) $m_1: \vec{p} = mv = \boxed{m_1 \vec{v}_{1b}}$ $m_2: \vec{p} = mv = \boxed{0}$ +1

c) $\vec{p}_{total} = \vec{p}_1 + \vec{p}_2 = \boxed{m_1 \vec{v}_{1b}}$ +1

~~d) $p_{after} = p_{before} = m_1 \vec{v}_{1b}$~~

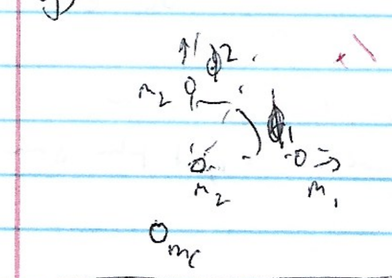
d) $p_{1a} = \boxed{m_1 \vec{v}_{1a}}$ $p_{2a} = \boxed{m_2 \vec{v}_{2a}}$ +1

e) $p_{after} = (m_1 \vec{v}_{1a} + m_2 \vec{v}_{2a}) = p_{before} = \boxed{m_1 \vec{v}_{1b}}$ +1

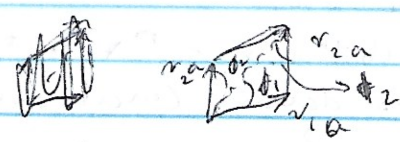
f) $p_{total} = m_{total} v_{total}$

$$= \boxed{(m_1+m_2)\sqrt{v_{1a}^2 + v_{2a}^2 + 2 v_{1a} v_{2a} \cos\theta}}$$ +1 +1

g)



h)



$$\frac{\sin\phi_1}{v_{1a}} = \frac{\sin\phi_2}{v_{2a}} = \frac{\sin\phi_{total}}{v_{total}} \qquad \theta = \phi_1 + \phi_2$$

$p_{total} = m_{total} v_{total}$

$$v_{total} = \frac{v_{2a}\sin(180-\phi_1-\phi_2)}{\sin\phi_2} = \frac{v_{1a}\sin(180-\phi_1-\phi_2)}{\sin\phi_1}$$

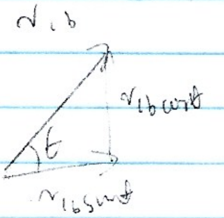
$$= \frac{(m_1+m_2) v_{2a} \sin\theta}{\sin\phi_2}$$

~

$$= \frac{v_{2a}\sin(180-\theta)}{\sin\phi_2} = \frac{v_{1a}\sin(180-\theta)}{\sin\phi_1}$$

$$= \frac{(m_1+m_2) v_{1a} \sin\theta}{\sin\phi_1}$$

i) from (h): $p_{total} = \frac{(m_1+m_2)\, v_{2a} \sin\theta}{\sin\phi_2} = \frac{(m_1+m_2)\, v_{1a} \sin\theta}{\sin\phi_2}$



$\theta = \theta_0$ of $\vec{v}_{1b}$

$\boxed{2i}$

j) $KE_{1b} = \frac{1}{2} m_1 v_{1b}^2 \qquad KE_{2b} = 0 \qquad KE_{Tb} = \frac{1}{2} m_1 v_{1b}^2$ +1

k) $KE_{1a} = \frac{1}{2} m_1 v_{1a}^2 \qquad KE_{2a} = \frac{1}{2} m_2 v_{2a}^2$ +1

$KE_{Ta} = \frac{1}{2}(m_1+m_2)^2 (v_{1a}^2 + v_{2a}^2)$

l) $m_1 \vec{v}_{1b} + m_2 \vec{v}_{2b} = m_1 \vec{v}_{1a} + m_2 \vec{v}_{2a}$

$\cancel{m}(v_{1b} + v_{2b}) = \cancel{m}(v_{1a} + v_{2a})$

$\underbrace{v_{1b}^2 + v_{2b}^2}_{\text{inner}} = \underbrace{v_{1a}^2 + v_{2a}^2 + 2 v_{1a} v_{2a} \cos\theta}$ +1 +1

in order for [ ] to equal [ ], $\cos\theta$ must $= 0$, +1

so $\theta = 90$ +1

should be COE of v_{1a} and v_{2a}

m) if $m_1 \neq m_2$,

$m_1 v_{1b} + m_2 v_{2b} = m_1 v_{1a} + m_2 v_{2a}$

Mass doesn't cancel out, so we can't get the same result as before?

n) ?

3) a) $P_0 = P_f$

$\cancel{M v_0} = (M - dm)\, dv$

$M\, dv - dm\, dv = 0$

$M\, dv = dm\, dv$

$M \frac{dv}{dm} = dv$

$0 = M\, dv - dm \cdot dv$

$\boxed{m\, dv = dm \cdot dv}$ +1 , but should be in vector form

b) $KE = \frac{1}{2} m (dv)^2 + \frac{1}{2} dm (dv)^2$ +1 ~ ΔKE?

Energy comes from chemical potential energy of the chemical being burned to cause the gas to be released +1

4) a) $x_{cm} = \frac{m(-l) + m(l) + 8m(2l) + 3m(-l)}{(m + m + 8m + 3m)} \hat{i}$

$= \frac{\cancel{m}(-l + l + 16l - 3l)}{\cancel{m}(13)} = \frac{13l}{13} = \boxed{l\, \hat{i}}$ +1

b) $y_{cm} = \frac{m(l) + m(0) + 8m(l) + 3m(-l)}{(m + m + 8m + 3m)} \hat{j}$

$= \frac{\cancel{m}(l + 0 + 8l - 3l)}{13 \cancel{m}} = \boxed{\frac{6l}{13} \hat{j}}$ +1

c) $\theta = \tan^{-1}\left(\frac{y_{cm}}{x_{cm}}\right) = \boxed{\tan^{-1}\left(\frac{6}{13}\right)} = 24.8^\circ$ +1

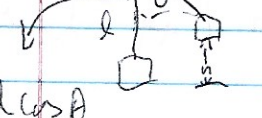
d) $r_{cm} = \sqrt{y_{cm}^2 + x_{cm}^2}$

$= \sqrt{l^2 + \frac{36 l^2}{169}} = l \sqrt{\frac{205}{169}} = \boxed{\frac{l\sqrt{205}}{13} \text{ @ } 24.8^\circ}$ ~ component form

5) a) Use cons. of momentum only during the collision w/ the block N

b) $m_1 \vec{v} = (m_1 + M_2) \vec{v}_f$ +1

c) doesn't matter ~ ~

d) 

$l \cos\theta$

$h = l - l\cos\theta$ +1

$= \boxed{l(1 - \cos\theta)}$

e) $KE_0 = \Delta E_{gf}$

$\frac{1}{2} \cancel{m} v_f^2 = \cancel{m} g l (1 - \cos\theta)$ $\sqrt{2gh}$, $h = l(1 - \cos\theta)$

$\boxed{v_f = \sqrt{2gl(1 - \cos\theta)}}$ +1

f) from (b): $m_1 \vec{v} = (m_1 + m_2) v_f$

$m_1 \vec{v} = (m_1 + m_2) \sqrt{2gl(1 - \cos\theta)}$

$\boxed{\vec{v} = \left(\frac{m_1 + m_2}{m_1}\right) \sqrt{2gl(1 - \cos\theta)}}$ +1

g) there's no collision or explosion afterwards ~

6) a) ~~momentum is conserved because the ball didn't interact with anything or lost mass. The ball~~
momentum is conserved because it is an explosion, the momentum of the ball / ramp system is conserved. ~

b) energy is conserved because no nonconservative forces are doing work. +1

c) $0 = Mv_M - mv_b$ +1 ~~d) $mgh_0 = \frac{1}{2}mv_b^2$~~

d) $\cancel{KE_{M_0}} + \cancel{KE_{m_0}} + PE_{g_{m_0}} = KE_{M_f} + KE_{m_f} + \cancel{PE_{g_{m_f}}}$

$$mgh_0 = \frac{1}{2}Mv_f^2 + \frac{1}{2}mv_b^2 \quad +1$$

e) $Mv_f = mv_b$

$v_f = \frac{m}{M}v_b$ $\qquad \cancel{m}gh_0 = \frac{1}{2}\cancel{M}\cdot\frac{m^{\cancel{2}}}{M^{\cancel{2}}}v_b^2 + \frac{1}{2}\cancel{m}v_b^2$

$$gh_0 = \frac{m}{2}\frac{v_b^2}{M} + \frac{1}{2}v_b^2 \quad +1$$

$$\boxed{2gh_0 = \frac{v_b^2(M+m)}{M}} \quad +1$$

7) A) $v_{rel} = 0$ +1

b) Momentum is conserved b/c it is a collision, momentum is conserved for the ball / ramp system ~

c) No work is done by non conservative forces +1

d) $mv_b = Mv_f$ ~

e) $\frac{1}{2}mv_b^2 = \frac{1}{2}Mv_f^2 + mgh$ ~

f) $v_f = \frac{m}{M}v_b$ $\qquad \frac{1}{2}\cancel{m}v_b^2 = \frac{1}{2}\cancel{M}\cdot\frac{m^{\cancel{2}}}{M^{\cancel{2}}}v_b^2 + \cancel{m}gh$ +1

$$\frac{1}{2}v_b^2 = \frac{1}{2}v_b^2\frac{m}{M} + gh$$

$$\boxed{v_b^2\left(1 - \frac{m}{M}\right) = 2gh}$$ +1 double jeopardy

8) from (6)(e) : $2gh_0 = v_b^2 \frac{(M+m)}{M}$ ~

$v_b^2 = \frac{2gh_0 M}{M+m}$

from (7)(c) $v_b^2\left(1-\frac{m}{M}\right) = 2gh$

$v_b^2\left(\frac{M-m}{M}\right) = 2gh$

$v_b^2 = \frac{2ghM}{M-m}$

$\frac{\cancel{2g}h \cancel{M}}{M-m} = \frac{\cancel{2g}h_0\cancel{M}}{M+m}$

$$\boxed{\frac{h}{h_0} = \frac{M-m}{M+m}}$$ ~

9) (9)

~ ~ ~
~ ~ ~
~ ~ ~

i) a) when $J = \Delta p = 0$
b) inelastic collisions, when $\Delta E \neq 0$
d)

1.

A. When the net impulse on the system approaches zero. That is $\vec{F}_{net}\Delta t \to 0$ *1 pt*

B. When the net work done on or by the system approaches zero. That is $\vec{F}_{net} \bullet \Delta\vec{r} \to 0$ *1 pt*

C. (1) Total momentum of the system is conserved. *1 pt*

(1) Total *kinetic* energy of the system is conserved. *1 pt*

D. $M_T\vec{R}_{cm} = \sum m_i\vec{r}_i$. The derivative of this gives *1 pt*

$\frac{d}{dt}(M_T\vec{R}_{cm}) = \frac{d}{dt}(\sum m_i\vec{r}_i)$ *1 pt*

$M_T\vec{v}_{cm} = \sum m_i\vec{v}_i$ that is $\vec{P}_{cm} = \sum \vec{p}_i$ *1 pt*

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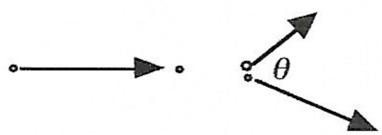
2. Consider two billiard balls with masses m_1 and m_2. The first ball is moving with velocity $\vec{v}_{1b}$ toward the second ball, which is at rest. After the collision, the balls move away from each other at an angle θ with velocities $\vec{v}_{1a}$ and $\vec{v}_{2a}$ respectively.

A.



1 pt

B. $m_1\vec{v}_{1b}, 0$ *1 pt*

C. $\vec{P}_{Tb} = m_1\vec{v}_{1b} + m_2\vec{v}_{2b} = m_1\vec{v}_{1b}$ *1 pt*

D. $m_1\vec{v}_{1a}, m_2\vec{v}_{2a}$ *1 pt*

E. $\vec{P}_{Ta} = m_1\vec{v}_{1a} + m_2\vec{v}_{2a}$ *1 pt*

F. $m_1\vec{v}_{1b} = m_1\vec{v}_{1a} + m_2\vec{v}_{2a}$ *1+1 pt*

$m_1^2v_{1b}^2 = m_1^2v_{1a}^2 + m_2^2v_{2a}^2 + 2m_1m_2v_{1a}v_{2a}\cos\theta$

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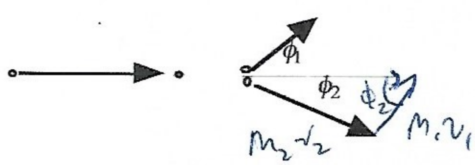
G.



1 pt

H. $\frac{m_1v_{1b}}{\sin(\phi_1+\phi_2)} = \frac{m_1\vec{v}_{1a}}{\sin\phi_2} = \frac{m_2\vec{v}_{2a}}{\sin\phi_1}$ *1 pt*

I.

$m_1v_{1bx} = m_1v_{1ax} + m_2v_{2ax}$

$0 = m_1v_{1ay} - m_2v_{2ay}$ *1 pt*

J. $KE_{1b} = \frac{1}{2}m_1v_{1b}^2$, $KE_{2b} = 0$, $KE_{Tb} = \frac{1}{2}m_1v_{1b}^2$ *1 pt*

K. $KE_{1a} = \frac{1}{2}m_1v_{1a}^2$, $KE_{2a} = \frac{1}{2}m_2v_{2a}^2$, *1 pt*

$KE_{Ta} = \frac{1}{2}m_1v_{1a}^2 + \frac{1}{2}m_2v_{2a}^2$ *1 pt*

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L. From part F with $m_1 = m_2$, we get

$v_{1b}^2 = v_{1a}^2v_{2a}^2 + 2v_{1a}v_{2a}\cos\theta$ *1 pt*

From part J and K, using conservation of energy with $m_1 = m_2$ gives $v_{1b}^2 = v_{1a}^2 + v_{2a}^2$ *1 pt*

Substituting this in the first equation gives

$v_{1a}^2 + v_{2a}^2 = v_{1a}^2v_{2a}^2 + 2v_{1a}v_{2a}\cos\theta$ *1 pt*

As a result, we must have $2v_{1a}v_{2a}\cos\theta = 0$ *1 pt*

Since v_{1a} and v_{2a} are not zero *1 pt*

we must have have $\cos\theta = 0$, i.e. $\theta = 90°$ *1 pt* /6

M. From part F, we have *1 pt*

$m_1^2v_{1b}^2 = m_1^2v_{1a}^2 + m_2^2v_{2a}^2 + 2m_1m_2v_{1a}v_{2a}\cos\theta$

$m_1^2v_{1b}^2 = m_1^2v_{1a}^2 + m_2^2v_{2a}^2$ with $\theta = 90°$ *1 pt*

or $v_{1b}^2 = v_{1a}^2 + \frac{m_2^2}{m_1^2}v_{2a}^2$

From part J and K, using CoE, we have

$m_1v_{1b}^2 = m_1v_{1a}^2 + m_2v_{2a}^2$ *1 pt*

or $v_{1b}^2 = v_{1a}^2 + \frac{m_2}{m_1}v_{2a}^2$

Substituting this result in the momentum equation gives

$v_{1a}^2 + \frac{m_2}{m_1}v_{2a}^2 = v_{1a}^2 + \frac{m_2^2}{m_1^2}v_{2a}^2$ *1 pt*

$\Rightarrow \frac{m_2}{m_1} = \frac{m_2^2}{m_1^2}$, $\frac{m_2}{m_1} = 1$ & $m_2 = m_1$ *1 pt* /5

N. From CoP, we have *1 pt*

$m_1^2v_{1b}^2 = m_1^2v_{1a}^2 + m_2^2v_{2a}^2 + 2m_1m_2v_{1a}v_{2a}\cos\theta$

$\cos\theta = \frac{p_{1b}^2 - p_{1a}^2 - p_{2a}^2}{2p_{1a}p_{2a}}$ *1 pt*

/2

3.

A. $p = m\vec{v}$, $dp = (dm)\vec{v} + m(d\vec{v}) = 0$ *1 pt*

B. $KE_b = 0$, $KE_a = \frac{1}{2}(dm)v^2 + \frac{1}{2}m(dv)^2$ *1 pt*

$\Delta KE_a = \frac{1}{2}(dm)v^2 + \frac{1}{2}m(dv)^2 - 0$ *1 pt*

This comes from burning fuel. Burning the fuel releases the electromagnetic potential energy stored within the molecular bonds. *1 pt*

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4. Obtain the center of mass of the 4-particle system given in the grid below in terms of ℓ and m.

A. $X_{cm} = \frac{-m\ell - 3m\ell + m\ell + 2\times 8m\ell}{m + 3m + m + 8m} = \ell$ *1 pt*

B. $Y_{cm} = \frac{m\ell - 3m\ell + m0 + 8m\ell}{m + 3m + m + 8m} = \frac{6}{13}\ell$ *1 pt*

C. $\tan\theta = \frac{Y_{cm}}{X_{cm}} = \frac{6}{13}$, ~~$\theta = 86°$~~ *1 pt*

$\theta = 25°$

D. $\vec{R}_{cm} = (\ell, \frac{6}{13}\ell)$ *1 pt*

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Mesut B. Çakır

5.

A. When the interaction time is short enough that the force from the string (tension) has negligible effect on the system, i.e. $\vec{F}_{net}\Delta\bar{t}_{impact} \to 0$ 1 pt

or $\vec{F}_{net}\Delta\bar{t}_{impact} << p_b - p_a$

B. $m_1 v = (m_1 + m_2)v_a,\ v_a = \dfrac{m_1 v}{m_1 + m_2}$ 1 pt

C. $KE_a - KE_b = \dfrac{1}{2}(m_1 + m_2)v_a^2 - \dfrac{1}{2}m_1 v^2$ 1 pt

$KE_a - KE_b = \dfrac{1}{2}(m_1 + m_2)\dfrac{m_1^2 v^2}{(m_1 + m_2)^2} - \dfrac{1}{2}m_1 v^2$ 1 pt

$KE_a - KE_b = \dfrac{-m_2}{m_1 + m_2}\dfrac{1}{2}m_1 v^2 = \dfrac{-m_2}{m_1 + m_2}KE_b$ 1 pt

D. $h = \ell - \ell\cos\theta$ 1 pt

E. $\dfrac{1}{2}(m_1 + m_2)v_a^2 = (m_1 + m_2)gh,\ v_a = \sqrt{2gh}$ 1 pt

F. $\dfrac{m_1 v}{m_1 + m_2} = \sqrt{2gh},\ v = \dfrac{m_1 + m_2}{m_1}\sqrt{2g\ell(1 - \cos\theta)}$ 1 pt

G. Because there is net force acting on the system as given by $\vec{F}_{net} = m\vec{g} + \vec{F}_{Tension}$ 1 pt. This net force is what causes the system to come to a full stop at the highest point.

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6.

A. The x-component of the total momentum is conserved since there is no net force in the x-direction acting on the system (no friction) 1 pt. However, there is a net force acting on the system in the y-direction (gravity); as a result, the cm of the system moves down as the ball rolls down the incline. 1 pt Therefore, the y-component of the momentum is not conserved.

B. No net work is done on the system by external forces except gravity. The work done by gravity changes PE into KE, hence the total energy of the system remains constant. 1 pt

C. $0 = mv_b - Mv_A,\ v_A = \dfrac{m}{M}v_b$ 1 pt

D. $mgh_o = \dfrac{1}{2}mv_b^2 + \dfrac{1}{2}Mv_A^2$ 1 pt

E. $mgh_o = \dfrac{1}{2}mv_b^2 + \dfrac{1}{2}Mv_A^2 = \dfrac{1}{2}(mv_b^2 + \dfrac{m^2}{M}v_b^2)$ 1 pt

$2gh_o = (1 + \dfrac{m}{M})v_b^2,\ v_b^2 = \dfrac{2Mgh_o}{m + M}$ 1 pt

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7.

A. 0. They will be moving together with the same velocity at that instant. 1 pt

B. Same answer as in 6.A. 1 pt

C. Same answr as in 6.B. 1 pt

D. $mv_b = (m + M)v,\ v = \dfrac{m}{m + M}v_b$ 1 pt

E. $\dfrac{1}{2}mv_b^2 = mgh + \dfrac{1}{2}(m + M)v^2$ 1 pt

F. $\dfrac{1}{2}mv_b^2 = mgh + \dfrac{1}{2}(m + M)\dfrac{m^2 v_b^2}{(m + M)^2}$ 1 pt

$2gh = \dfrac{Mv_b^2}{m + M}$ 1 pt

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8. $2gh = \dfrac{Mv_b^2}{m + M} = \dfrac{M}{m + M}\dfrac{2Mgh_o}{m + M}$ 1 pt

$h = \dfrac{M^2 h_o}{(m + M)^2}$ 1 pt

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9. We will choose downward direction to be +.

A. $y = \dfrac{1}{2}gt^2$ 1 pt

B. $v = gt$ 1 pt

C. $\dfrac{y}{\ell}$ 1 pt

D. Momentum of the remaining part is

$p = (m - \dfrac{y}{\ell}m)gt = m(1 - \dfrac{1}{2\ell}gt^2)gt = mg(t - \dfrac{gt^3}{2\ell})$ 1 pt

$\dfrac{dp}{dt} = mg(1 - \dfrac{3gt^2}{2\ell}) = mg(1 - \dfrac{3y}{\ell})$ 1 pt

E. $F_{net} = mg - F_N$ 1 pt

F. $F_{net} = \dfrac{dp}{dt}$ 1 pt

$mg - F_N = mg - mg\dfrac{3y}{\ell}$ 1 pt

$\vec{F}_N = 3mg\dfrac{y}{\ell}\uparrow$ 1 pt

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