Momentum

1. A 2-kg object has a 6 km/s linear momentum. Obtain its KE.
A. 3 J B. 6 J C. 9 J D. 12 J E. 18 J

2. A 0.5-kg ball, initially at ret, acuires a 4 m/s-speed immediately after being kicked by a 20-N force. For how long did the force act on the ball?
A. 0.01 s B. 0.02 s C. 0.1 s D. 0.2 s E. 1s

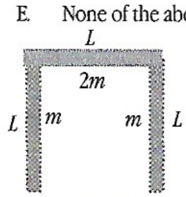
3. A 2-kg box accelerates on a straight line from 4 m/s to 8 m/s due to a force having been applied for 0.5 s. Obtain the average strength of the force.
A. 2 N B. 4 N C. 8 N D. 12 N E. 16 N

4. A ball of mass m trveling horizontally with velocity **v** strikes a massive vertical wall and rebounds back along its original direction with no change in speed. What is the magnitude of the impulse delivered by the wall to the ball?
A. 0 B. $\frac{1}{2}mv$ C. mv D. $2mv$ E. $4mv$

5. A 3-kg mass moving 2 m/s and 5-kg mass moving 2 m/s collide head-on. If the collision is perfectly inelastic, obtain the speed of the masses after the collision.
A. 0.25m/s B. 0.5m/s C. 0.75m/s D. 1m/s E. 2m/s

6. m_1 moves toward m_2 $(=2m_1)$ which is at rest. After the impact, the objects are locked and move together. What fraction is their KE compared to the initial KE of m_1?
A. 1/18 B. 1/9 C. 1/6 D. 1/3 E. None of these

7. Two objects moving toward each other collied and scatter. If no external force acts on the objects but some KE is lost, then the collision is
   A. elastic and the total momentum is conserved
   B. elastic and total linear momentum is not conserved
   C. not elastic and total linear momentum is conserved
   D. not elastic and total linear momentum is not conserved
   E. None of the above.



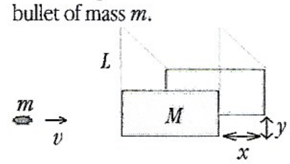
8. Three thin uniform rods each of length L are arranged in an inverted U shape. The two rodds on the arms of the U each has mass m; the third rod has mass $2m$. How far below the midpoint of the horzontal rod is the center of mass if this assembly?
A. L/8 B. L//4 C. 3L/8 D. L/2 E. 3L/4

9. A block of mass M is moving at a speed V. How fast would a bullet of mass m, moving head-on toward the block, need to travel to stop the block as it becomes embedded in the block?
A. $mV/(m+M)$ B. $MV/(m+M)$ C. mV/M D. MV/m E. $(m+M)V/m$

10. Which of the following best describes a perfectly inelastic collision free of external forces?
A.Total $\bar{p}$ is never conserved B.Total $\bar{p}$ is sometimes conserved
C.KE is never conserved D. KE is sometimes conserved
E. KE is always conserved

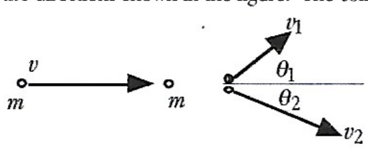
1. A pendulum of mass m, tied by massless cord of length L, is released from a horizontal position (perpendicular to the vertical). At bottom of its path, the pendulum mass strikes a hard plastic block of mass M=4m at rest on a frictionless surface. The collision is elastic.
   A. Find the tension in the cord when the ball is at height $h=L/3$
   B. Find the speed of the block immediately after the collision
   C. To what height h will the ball rebound after the collision?

2. A ballistic pendulum is used to measure the muzzle speed of a bullet of mass m.



   A. In terms of the givens, determine the speed of the bullet.
   B. What fraction of the bullet's original KE is lost in the collision? What happenned to it?
   C. If y is small enough so that y^2 maybe neglected, determine the speed of the bullet in terms of the givens.
   D. Once block begns to swing, is the momentum of the block conserved? Why or why not?

3. An object of mass m moves with velocity **v** toward a stationary object of the same mass. After the impact, the objects move off in the directions shown in the figure. The collision is elastic.



   A. If KE_1 is the KE of the first mass before the collision, what is the KE of this mass after the collison in terms of KE_1 & θ_1?
   B. What is the KE of the second mass after the collision in terms of KE_1 & θ_1?
   C. What is the relationship between θ_1 and θ_2?

Black 8   Red 8
Blue 3   Black + Blue 11
Total 19

1) $P = mv$ $v = 3\frac{m}{s}$
$E = \frac{1}{2}mv^2 = 9 \cdot \frac{1}{2} \cdot 2 = 9J$ (C) ✓

2) $J = F\Delta t = \Delta p$
$\frac{2kg \cdot \frac{m}{s}}{20N} = \Delta t$
$\Delta t = 0.1s$ (c) ✓

3) $J = F\Delta t$ $F = \frac{J}{\Delta t} = \frac{4(?)}{0.5} = 16N$ (E) ✓

4) $p_0 = -p_f$ $J = \Delta p = p_f - p_0$
$= 2mv$ (d) ✓

5) $m_1v_1 + m_2v_2 = m_1v_{1f} + m_2v_{2f}$
(e) B ×

6) $m_1v_1 = (m_1 + m_2)v$
$v = \frac{m_1}{3m_1}v_1$ $v = \frac{1}{3}v$
$\frac{1}{9}(mv^2)$ (B) D ×

7) inelastic, momentum conserved (e) ✓

8) 2m ... $L/4$ (B) ✓

9)
$mv + MV = (m+M)0$
$mv = MV$
$v = \frac{MV}{m}$ (d) ✓

10) (E) (C) ×

FRQ

1)
A)

$v_{L/3}$: $mgL = \frac{1}{2}v^2 m + mg\frac{L}{3}$

$v^2 = 2gL - \frac{2gL}{3} = \frac{4}{3}gL$

$v = 2\sqrt{\frac{gL}{3}}$

$F_c = \frac{mv^2}{r} = \frac{m \cdot \frac{4}{3}gL}{L} = \frac{4}{3}mg$

$\theta = \cos^{-1}\left(\frac{2L}{3L}\right) = \cos^{-1}\left(\frac{2}{3}\right)$

$T - mg\cos\theta = \frac{4}{3}mg$

$T = mg\left(\frac{4}{3} + \frac{2}{3}\right) = 2mg$ ~

B) $\frac{1}{2}v_f^2 = mgL$

$v_0 = \sqrt{2gL}$ & $mv_0 = mv_{f1} + 4mv_{f2}$

$v_0 = v_1 + 4v_2$ ~

C) ?

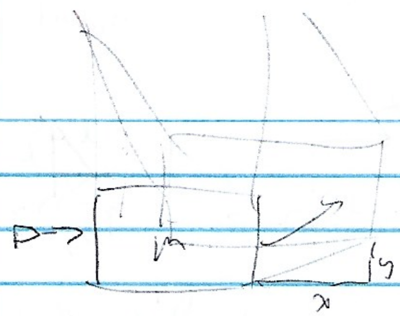
2) A) V B) Same
B) v_h $mv = (m+M)v_f$
$v_f = \frac{mv}{m+M}$
fraction?

C)

7

3R

a)



$(m+M) v_f^2 = (m+M) g y$

$mv = (m+M) v_f$

$v_f^2 = gy \quad v_f = \sqrt{gy}$

$v = \frac{(m+M)\sqrt{gy}}{m}$ ~

b) ? c) $mv = (m+M) v_f$

$v = \frac{v_f (m+M)}{m}$

$\cancel{v_f}^{0} = v_f + at$

$v_f = -at$

$x = v_f t + \frac{1}{2}at^2$

$v_f t = x - \frac{1}{2}at^2$

$v_f = \frac{x}{t} - \frac{1}{2}at$

$-at = \frac{x}{t} - \frac{1}{2}at$

$-\frac{1}{2}at = \frac{x}{t}$

$-at = 2v_f - \frac{2x}{t}$

$v_f = 2v_f - \frac{2x}{t}$

$v_f = \frac{2x}{t}$

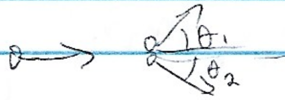
$v = \frac{2x(m+M)}{mt}$?

$t = \frac{1}{4}T = \frac{1}{4}\sqrt{\frac{g}{L}}$

$v = \frac{2x(m+M)4}{m\sqrt{\frac{g}{L}}}$

$= \frac{8x(m+M)}{m\sqrt{\frac{g}{L}}}$ ~

d) no, b/c the strings are external forces. ✓

3) a) 

$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$

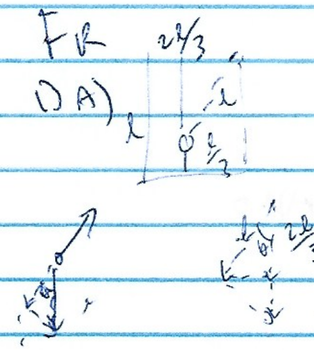
$v^2 = v_1^2 + v_2^2$ ~

b) ? c) $\theta_1 = \theta_2$?

$\theta_1 + \theta_2 = 90$ ✓, but no reasons

1

FR

1) A) 

$mgl = mg\frac{l}{3} + \frac{1}{2}mv^2$

$\frac{2}{3}gl = \frac{1}{2}v^2$

$v = \sqrt{\frac{4}{3}gl}$

$T - mg\cos\theta = \frac{mv^2}{l} = \frac{m}{\cancel{l}} \cdot \frac{4}{3}g\cancel{l}$

$T - mg\left(\frac{2l/3}{l}\right) = \frac{4}{3}mg$

$T = mg\left(\frac{2}{3} + \frac{4}{3}\right) = 2mg$ ~

1) b)

$mgh = \frac{1}{2}mv_f^2 \qquad h = L$

$v_f = \sqrt{2gL}$

$\frac{1}{2}mv_{b_0}^2 + \cancel{\frac{1}{2}Mv_{L_0}^2} = \frac{1}{2}mv_{b_f}^2 + \frac{1}{2}Mv_{L_f}^2$

$mv_{b_0} + \cancel{Mv_{L_0}} = mv_{b_f} + Mv_{L_f}$

$\cancel{v_{Lf} = \frac{m(v_{b_0} - v_{b_f})}{M}}$

$v_{b_f} = \frac{mv_{b_0} - Mv_{Lf}}{m}$

$= v_{b_0} - \frac{M}{m}v_{Lf}$

$\frac{1}{2}Mv_{Lf}^2 = \frac{1}{2}mv_{b_0}^2 - \frac{1}{2}mv_{bf}^2$

$v_{Lf}^2 = \frac{m}{M}(v_{b_0}^2 - v_{bf}^2)$

$= \frac{m}{M}\left(2gL - \left(\sqrt{2gL} - \frac{M}{m}v_{Lf}\right)^2\right)$

$\cancel{2\frac{m}{M}}$

$= \frac{m}{M}\left(2gL - \left(2gL - 2\sqrt{2gL}\frac{M}{m}v_{Lf} + \frac{M^2}{m^2}v_{Lf}^2\right)\right)$

$= \frac{m}{M}\left(2\sqrt{2gL}\frac{M}{m}v_{Lf} - \frac{M^2}{m^2}v_{Lf}^2\right)$

$\frac{M}{m}v_{Lf}^{\cancel{2}} = 2\sqrt{2gL}\frac{M}{m}\cancel{v_{Lf}} - \left(\frac{M}{m}\right)^2 v_{Lf}^{\cancel{2}}$

$\cancel{v_{Lf}\left(1 - \frac{M}{m}\right) = 2\sqrt{2gL}}$

$\cancel{v_{Lf}\left(\frac{m-M}{m}\right) = 2\sqrt{2gL}}$

$\cancel{v_{Lf} = \frac{2M}{m-M}\sqrt{2gL}}$

$\cancel{M = 4m}$

$v_{Lf}\left(1 + \frac{M}{m}\right) = 2\sqrt{2gL}$

$v_{Lf}\left(\frac{m+M}{m}\right) = 2\sqrt{2gL}$

$v_{Lf} = 2\sqrt{2gL}\cdot\frac{m}{m+M} = 2\sqrt{2gL}\cdot\frac{m}{m+4m}$

$M = 4m$

$= \frac{2}{5}\sqrt{2gL}$ ✓

c) $v_{bf} = v_{b_0} - \frac{M}{m}v_{Lf}$

$= \sqrt{2gL} - \frac{M}{m}\cdot\frac{2}{5}\sqrt{2gL}$

$= \sqrt{2gL} - \frac{8}{5}\sqrt{2gL}$

$= -\frac{3}{5}\sqrt{2gL}$

$\cancel{m}gh_f = \frac{1}{2}\cancel{m}v_f^2$

$h = \frac{1}{2}\left(\frac{9}{25}\cdot 2gL\right)$

$h = \frac{9}{25}L$ ✓

2

2) a–c)

a) $mv = (m+M)v_f$

$\frac{1}{2}\cancel{(m+M)}v_f^2 = \cancel{(m+M)}gy$

$v = \frac{m+M}{m}v_f$

$v_f = \sqrt{2gy}$

$$\boxed{v = \frac{m+M}{m}\sqrt{2gy}}$$

✓

b) $\frac{1}{2}mv^2 \rightarrow KE_0 = \frac{1}{2}m\left(\frac{M+M}{m}\right)^2(2gy)$

$KE_{f\,\text{total}} = \frac{1}{2}(m+M)(2gy)$

$= (m+M)gy$

$KE_{f\,\text{bullet}} = m+M\,gy \cdot \frac{m}{M}$

$= \frac{m(m+M)}{M}gy$

$KE_0 = m\left(\frac{m+M}{m}\right)^2 gy$

$= \frac{(m+M)^2}{m}gy$

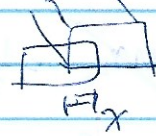
$\frac{KE_0}{KE_f} = \frac{(m+m)^2\cancel{gy}}{m} \times \frac{M}{(m)\cancel{(m+M)gy}}$

$= \frac{M(M+m)}{m^2}$ ~

Lost in the collison – sound, heat, vibration, etc.

c) $v = \frac{m+M}{m}v_f$

pendulum w/ small ∠ approx



$T = \sqrt{\frac{g}{L}}$

$x = v_0t + \frac{1}{2}at^2$

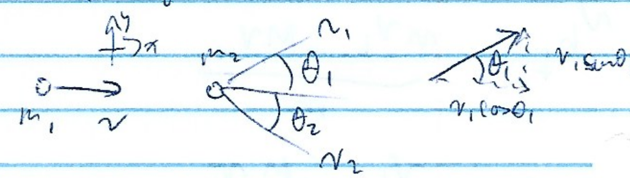
$t = \frac{1}{4}T$

~~vsx~~ $\cancel{v_f} = v_0 + at$

~~$v_0 = -at$~~

$v_f^2 - v_0^2 = 2a\Delta x$??

3)



$m_1v_{\cancel{x}} = m_1v_x + m_2v_{2x}$

$m_1v_{1y} = m_2v_{2y}$

$m_1v = m_1v_1\cos\theta_1 + m_2v_2\cos\theta_2$

$m_1v_1\sin\theta_1 = m_2v_2\sin\theta_2$

a) elastic – KE conserved

$KE_1 = \frac{1}{2}mv^2$

~~KE~~

$KE_1 = KE_f = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

???

b) c) /

2) b) $KE_f = \frac{1}{2}(m+M)v_f^2 = \frac{1}{2}\left(\frac{m^2}{m+M}\right)v^2$

$\Delta KE = KE_f - KE_0 = \frac{1}{2}\frac{m^2 v^2}{m+M} - \frac{1}{2}mv^2$

$= -\frac{M}{m+M} KE_0$ ✓

$v^2 = v_{1a}^2 + 2v_{1a}v_{2a}\cos(\theta_1+\theta_2) + v_{2a}^2$

$v^2 = v_{1a}^2 + v_{2a}^2$; $\theta_1 + \theta_2 = 90°$ ✓

1) a) $T = \frac{3}{2}mg$ ✓

5

3) a) $mv = mv_{1a}\cos\theta_1 + mv_{2a}\cos\theta_2$

$mv_{1a}\sin\theta_1 = mv_{2a}\sin\theta_2$

$\frac{1}{2}mv^2 = \frac{1}{2}mv_{1a}^2 + \frac{1}{2}mv_{2a}^2$

$v_{2a}\cos\theta_2 = v - v_{1a}\cos\theta_1$

$v_{2a}\sin\theta_2 = v_{1a}\sin\theta_1$

$v_{2a}^2 = (v - v_{1a}\cos\theta_1)^2 + v_{1a}^2\sin^2\theta_{1a}$

$v_{2a}^2 = v^2 - 2vv_{1a}\cos\theta_1 + v_{1a}^2$

$v^2 - v_{1a}^2 = v_{2a}^2 = v^2 - 2vv_{1a}\cos\theta_1 + v_{1a}^2$ ✓

$2v_{1a}^2 = 2vv_{1a}\cos\theta_{1a}$

$v_{1a} = v\cos\theta_{1a}$

$KE_{1a} = \frac{1}{2}mv_{1a}^2 = \cos^2\theta_{1a}\frac{1}{2}mv^2 = \cos^2\theta_{1a}KE_1$ ✓

b) $KE_{2a} = \frac{1}{2}mv_{2a}^2 = \frac{1}{2}mv^2 - \frac{1}{2}mv_{1a}^2$

$= KE_1 - \cos^2\theta_{1a}KE_1 = KE_1(1-\cos^2\theta_{1a})$

$= \sin^2\theta_{1a}KE$ ✓

c) $v^2 = v_{1a}^2\cos^2\theta_{1a} + 2v_{1a}v_{2a}\cos\theta_{1a}\cos\theta_{2a}$

$+ v_{2a}^2\cos^2\theta_2$

$0 = v_{1a}^2\sin^2\theta_{1a} - 2v_{1a}v_{2a}\sin\theta_{1a}\sin\theta_{2a}$

$+ v_{2a}^2\sin^2\theta_{2a}$

CHAPTER 5 MC

1. C $p=mv \Rightarrow v=3\frac{m}{s} \Rightarrow KE=9J$

2. C $J=F\Delta t=mv \Rightarrow \Delta t=0.1s$

3. E $\vec{J}=\vec{F}_{av}\Delta t=\Delta\vec{p} \Rightarrow F_{av}=16N$

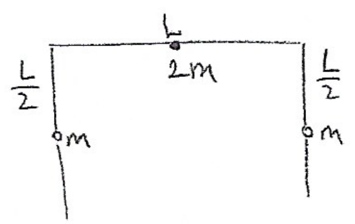
4. D $\Delta\vec{p}=m\vec{v}_2-m\vec{v}_1=m(-\vec{v})-m\vec{v}$
$=2m\vec{v}$

5. B Inelastic $m_1\vec{v}_1+m_2\vec{v}_2=(m_1+m_2)\vec{v}$
$\vec{v}=\frac{m_1\vec{v}_1+m_2\vec{v}_2}{m_1+m_2}=-0.5\frac{m}{s}$

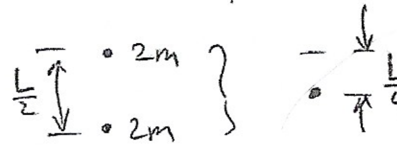
6. D C.o.$\vec{P}$: $m_1\vec{v}_1+m_2\vec{v}_2=(m_1+m_2)\vec{v}_a$
$\Rightarrow v_a=\frac{1}{3}v_1$, $v_2=0$
$\frac{KE_a}{KE_b}=\frac{\frac{1}{2}m_T v_a^2}{\frac{1}{2}m_1v_1^2}=\frac{\frac{1}{2}3m_1(\frac{1}{3}v_1)^2}{\frac{1}{2}m_1v_1^2}=\frac{1}{3}$

7. C $\vec{F}_{net}=0 \Rightarrow \vec{P}_T$=conserved
If $\Delta KE\neq 0 \Rightarrow$ inelastic

8. B



$\Rightarrow$



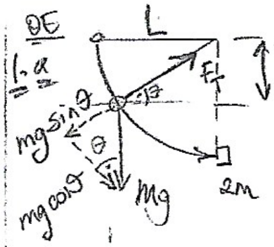
$y_{cm}=\frac{L}{4}$ or $y_{cm}=\frac{0\cdot 2m+\frac{L}{2}m+\frac{L}{2}m}{4m}$

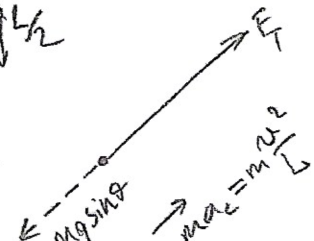
9. D $mv=MV \Rightarrow v=\frac{M}{m}V$

10. C Inelastic Collision $\Delta KE\neq 0$.

OE

1. a





$\frac{mv^2}{L}=F_T-mg\sin\theta$

$KE_1+PE_1=KE_2+PE_2$
$0+mgL=\frac{1}{2}mv^2+mg\frac{L}{2}$
$mv^2=mgL \Rightarrow \frac{mv^2}{L}=mg$

$mg=F_T-mg\sin\theta$
$F_T=mg(1+\sin\theta)$
$\sin\theta=\frac{L/2}{L}=\frac{1}{2}$
$F_T=\frac{3}{2}mg$

b $KE_1+PE_1=KE_2+PE_2$ C.o.E.
$0+mgL=\frac{1}{2}mv^2+0$
$v=2gL$

Elastic
KE is conserved, $\vec{P}$ is conserved
KE: $\frac{1}{2}m_1v_{1b}^2+\frac{1}{2}m_2\underset{0}{v_{2b}^2}=\frac{1}{2}m_1v_{1a}^2+\frac{1}{2}m_2v_{2a}^2$ (1)
$\vec{P}$: $m_1\vec{v}_{1b}+m_2\underset{0}{\vec{v}_{2b}}=m_1\vec{v}_{1a}+m_2\vec{v}_{2a}$ (2)
Obtain v_{1a} from (2), substitute in (1)
$\Rightarrow v_{2a}=\frac{2m_1v_1}{m_1+m_2}=\frac{2m}{m+4m}\sqrt{2gL}$
$v_{2a}=\frac{2}{5}\sqrt{2gL}$

c) Substitute v_{2a} in (2)
$v_{1a}=\frac{m_1-m_2}{m_1+m_2}v_1=\frac{m-4m}{m+4m}\sqrt{2gL}$
$v_{1a}=-\frac{3}{5}\sqrt{2gL}$
C.o.E: $KE_i+PE_i=KE_f+PE_f$
$\frac{1}{2}mv_{1a}^2+0=0+mgh$
$h=\frac{9}{25}L$

2-a C.o.$\vec{P}$: $mv = (m+M)v_a$

$$v_a = \left(\frac{m}{m+M}\right)v$$

C.o.E. $KE_1 + PE_1 = KE_2 + PE_2$

$$\frac{1}{2}(m+M)v_a^2 + 0 = 0 + (m+M)\,g\,y$$

$$\frac{1}{2}\left(\frac{mv}{m+M}\right)^2 = gy$$

$$v = \left(\frac{m+M}{m}\right)\sqrt{2gy} \;*$$

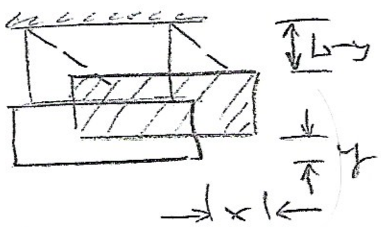
b $KE_a = \frac{1}{2}(m+M)v_a^2 = \frac{1}{2}\left(\frac{m^2}{m+M}\right)v^2$

$$\Delta KE = KE_a - KE_1 = \frac{1}{2}\frac{m^2v^2}{m+M} - \frac{1}{2}mv^2$$

$$= -\left(\frac{M}{m+M}\right)KE_1$$

← this fraction is lost

c



$$(L-y)^2 + x^2 = L^2$$

$$L^2 - 2yL + \underbrace{y^2}_{\approx 0} + x^2 = L^2 \Rightarrow y = \frac{x^2}{2L}$$

Substitute in part a *

$$v = \left(\frac{m+M}{m}\right)\sqrt{2gy} = \left(\frac{m+M}{m}\right)x\sqrt{\frac{g}{L}}$$

d $\vec{P}$ is conserved during impact <u>not</u> on the way up!

3. a C.o.$\vec{P}$

x: $mv = mv_{1a}\cos\theta_{1a} + mv_{2a}\cos\theta_{2a}$ (1)

y: $0 = mv_{1a}\sin\theta_{1a} - mv_{2a}\sin\theta_{2a}$ (2)

C.o.KE (elastic)

$$\frac{1}{2}mv^2 + 0 = \frac{1}{2}mv_{1a}^2 + \frac{1}{2}mv_{1a}^2 \quad (3)$$

From (1) $v_{2a}\cos\theta_{2a} = v - v_{1a}\cos\theta_{1a}$ (4)

(2) $v_{2a}\sin\theta_{2a} = v_{1a}\sin\theta_{1a}$ (5)

$(4)^2 + (5)^2$ side by side & use $\sin^2\theta + \cos^2\theta = 1$

$$v_{2a}^2 = (v - v_{1a}\cos\theta_{1a})^2 + v_{1a}^2\sin^2\theta_{1a}$$

$$v_{2a}^2 = v^2 - 2vv_{1a}\cos\theta_{1a} + v_{1a}^2 \quad (6)$$

Use eq'n (3) in (6) (after cancelling m's)

$$v^2 - v_{1a}^2 = v_{2a}^2 = v^2 - 2vv_{1a}\cos\theta_{1a} + v_{1a}^2$$

$$2v_{1a}^2 = 2vv_{1a}\cos\theta_{1a}$$

$$v_{1a} = v\cos\theta_{1a}$$

$$KE_{1a} = \frac{1}{2}mv_{1a}^2 = \cos^2\theta_{1a}\frac{1}{2}mv^2 = \cos^2\theta_{1a}KE_1$$

(b) From (3) $v_{2a}^2 = v^2 - v_{1a}^2$ & $KE_{1a} = \cos^2\theta_{1a}KE_1$

$$KE_{2a} = \frac{1}{2}mv_{2a}^2 = \frac{1}{2}mv^2 - \frac{1}{2}mv_{1a}^2$$

$$= KE_1 - \cos^2\theta_{1a}KE_1 = KE_1(1-\cos^2\theta_{1a})$$

$$= \sin^2\theta_{1a}KE$$

(c) $(1)^2 + (2)^2$ in (a) & eliminate m's

$$v^2 = v_{1a}^2\cos^2\theta_{1a} + 2v_{1a}v_{2a}\cos\theta_{1a}\cos\theta_{2a} + v_{2a}^2\cos^2\theta_{2a}$$

$$0 = v_{1a}^2\sin^2\theta_{1a} - 2v_{1a}v_{2a}\sin\theta_{1a}\sin\theta_{2a} + v_{2a}^2\sin^2\theta_{2a}$$

+

$$v^2 = v_{1a}^2 + 2v_{1a}v_{2a}\cos(\theta_{1a}+\theta_{2a}) + v_{2a}^2$$

$$v^2 = v_{1a}^2 + v_{2a}^2 \text{ when } \theta_{1a}+\theta_{2a} = 90^\circ$$

(We used $\cos\theta_{1a}\cos\theta_{2a} - \sin\theta_{1a}\sin\theta_{2a} = \cos(\theta_{1a}+\theta_{2a})$)