

Newton's Laws

1. A person standing on a horizontal floor feels two forces: the downward pull of gravity and the upward normal force of the floor. These two forces
 - A. have equal magnitudes and form an action-reaction pair
 - B. have equal magnitudes but do not form action-reaction pair
 - C. have unequal magnitudes and form an action-reaction pair
 - D. have unequal magnitudes but not an action-reaction pair
 - E. None of the above

2. An 800 N person steps onto a scale on the floor of an elevator. If the elevator accelerates upward at a rate 5 m/s^2 , what will the scale read?

 A. 400 N B. 800 N C. 1000 N D. 1200 N E. 1600 N F. What will be the answer, if the elevator accelerates downward?

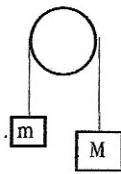
3. A frictionless inclined plane of length 20 m has a maximum vertical height of 5 m. If an object of mass 2 kg is placed on the plane, which of the following best approximates the net force it feels?

 A. 5 N B. 10 N C. 15 N D. 20 N E. 30 N F.

4. A 20 N block is being pushed across a horizontal table by an 18 N force. If the coefficient of kinetic friction between the block and the table is 0.4, find the acceleration of the block.

 A. 0.5 m/s^2 B. 1 m/s^2 C. 5 m/s^2 D. 7.5 m/s^2 E. 9 m/s^2 F.

5. The coefficient of static friction between a box and a ramp is 0.5. The ramp's incline angle is 30° . If the box is placed at rest on the ramp, the box will
 - A. accelerate down the ramp
 - B. accelerate down the ramp but then slow down and stop
 - C. move with constant velocity down the ramp
 - D. not move
 - E. cannot be determined from the information given



6. Assuming massless, frictionless pulley, determine the accelerations of the blocks once they are released from rest. $M > m$.

$$\begin{array}{lll} \text{A. } \frac{mg}{M+m} & \text{B. } \frac{Mg}{M+m} & \text{C. } \frac{Mg}{m} \\ \text{D. } \frac{(M+m)g}{M-m} & \text{E. } \frac{(M-m)g}{M+m} & \end{array}$$

7. If all the forces acting on an object balance so that the net force is zero, then
 - A. the object must be at rest
 - B. the object's speed will decrease
 - C. the object will follow a parabolic trajectory
 - D. the object's direction of motion can change, but not its speed
 - E. None of the above

8. A block of mass m is at rest on a frictionless, horizontal table placed on Earth. An identical block is at rest on a frictionless, horizontal table placed on the surface of the Moon. Let \mathbf{F} be the net force necessary to give the Earth block an acceleration of \mathbf{a} across the table. Given that g_{Moon} is one-sixth of g_{Earth} , the force necessary to give the Moon blocks the same acceleration \mathbf{a} across the table is

 A. $\mathbf{F}/12$ B. $\mathbf{F}/6$ C. $\mathbf{F}/3$ D. \mathbf{F} E. 6

9. A crate of mass 100 kg is at rest on a horizontal floor. The coefficient of static friction between the crate and the floor is 0.4, and the coefficient of kinetic friction is 0.3. A force \mathbf{F} of magnitude 344 N is then applied to the crate, parallel to the floor. Which of the following is true?
 - A. The crate will accelerate across the floor at 0.5 m/s^2
 - B. The static friction force, which is the reaction force to \mathbf{F} as guaranteed by Newton's Third Law, will also have a magnitude of 344
 - C. The crate will slide across the floor at a constant speed of 5 m/s
 - D. The crate will not move
 - E. None of the above

10. Two crates are stacked on top of each other on a horizontal floor, Crate-2 on top of Crate-1. Both crates have the same mass. Compared to the strength of the force \mathbf{F}_1 necessary to push only Crate-1 at a constant speed across the floor, the strength of the force \mathbf{F}_2 necessary to push the stack at the same constant speed across the floor is greater than \mathbf{F}_1 because
 - A. the force of the floor on Crate-1 is greater
 - B. the coefficient of kinetic friction between Crate-1 and the floor is greater
 - C. the force of kinetic friction, but not the normal force, on Crate-1 is greater
 - D. the coefficient of static friction between Crate-1 and the floor is greater
 - E. the weight of Crate-1 is greater

11. An object moves at constant speed in a circular path. Which of the following statements is/are true
 - I. The velocity is constant
 - II. The acceleration is constant
 - III. The net force on the object is zero since its speed is constant
 - A. II only
 - B. I and III only
 - C. II and III only
 - D. I and II only
 - E. None

12. A 60 cm rope is tied to the handle of a bucket which is then whirled in a vertical circle. The mass of the bucket is 3 kg. At the lowest point in its path, the tension in the rope is 50 N. What is the speed of the bucket?

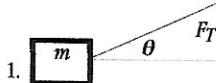
 A. 1 m/s B. 2 m/s C. 3 m/s D. 4 m/s E. 5 m/s

13. What is the critical speed below which the rope would become slack when the bucket reaches the highest point in the circle in the preceding question?

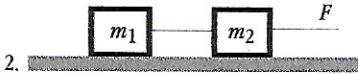
 A. 0.6 m/s B. 1.8 m/s C. 2.4 m/s D. 3.2 m/s E. 4.8 m/s

14. An object moves at a constant speed in a circular path of radius r at a rate of 1 rev/s. What is its acceleration?

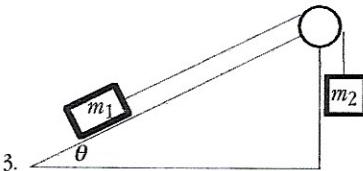
 A. 0 B. $2\pi^2 r/s^2$ C. $2\pi^2 r^2/s^2$ D. $4\pi^2 r/s^2$ E. $4\pi^2 r^2/s^2$



1. A crate of mass m is pulled across a horizontal floor where the coefficient of friction is μ .
- Draw a FBD representing the problem.
 - Calculate the normal force acting on the mass in terms of the givens.
 - Calculate the net force on the crate and its acceleration in terms of the givens.
 - Calculate the angle that leads to the maximum acceleration for the crate.



2. Two blocks, m_1 and m_2 , are connected by a massless string on a frictionless surface. A force F is applied to m_2 .
- Draw a FBD for each mass and clearly label all applied forces.
 - Obtain the acceleration of the blocks.
 - Obtain the tension on the string connecting the masses.
 - Now assume the string connecting the blocks has a mass m .
 - Obtain the acceleration of the masses.
 - Obtain the tension the force the string applies to each mass.



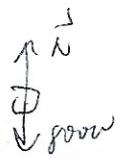
3. The string connecting the two masses and the pulley shown in the figure are frictionless and massless. There is no friction between the mass and the ramp.
- In terms of the given quantities, obtain the value of θ that will cause the masses to (i) accelerate clockwise (ii) move counter-clockwise at a constant speed.
 - Now assume the friction between the mass and the ramp is μ_k . Obtain the value of θ that will cause the masses to (i) accelerate clockwise (ii) move counter-clockwise at a constant speed.

4. A sky diver of mass m is falling with v_0 . She experiences a drag force $F = kv$, where k is a constant.
- Draw a FBD diagram representing the problem. Label clearly all forces acting on the sky diver.
 - Obtain her acceleration in terms of the given quantities.
 - Obtain her terminal velocity.
 - Obtain her velocity as a function of time
 - Plot the velocity and clearly label the axes.
 - Obtain her displacement as a function of time.

5. An amusement park ride consists of a large cylinder that rotates around its central axis as the passengers stand against the inner wall of the cylinder. Once the passengers are moving at a certain speed v , the floor on which they stand is lowered. Each passenger is pinned against the wall of the cylinder as it rotates. The radius of the cylinder is r .
- Draw a FBD and label all forces acting on a passenger of mass m .
 - Describe what conditions must hold to keep the passengers from sliding down the wall of cylinder.
 - Compare the conditions discussed in part (b) for an adult passenger of mass m and a child passenger of mass $m/2$.

6. A curved section of a highway has a radius of curvature of r . The coefficient of friction auto tires and the surface of the highway is μ .
- Draw a FBD and label all the forces acting on a car of mass m traveling along this curved part of the highway.
 - Compute the maximum speed with which a car of mass m could make it around the turn without skidding in terms of the givens.
 - Now, the highway is banked at an angle θ with the horizontal. Draw a FBD and label all the forces acting on a car of mass m traveling along the banked curve.
 - The engineers want to be sure that a car of mass m traveling at a constant speed v could make it safely around the banked turn even on an icy road $\mu=0$ approximately. Calculate this angle in terms of the givens.

Newton's law

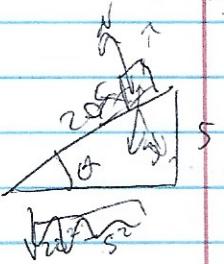


1) A ✓ 2) $\sum F = ma$

B $N - mg = ma$

$$N = 800N + \left(\frac{800}{10}\right)(5\sqrt{3})$$

$$= 800 + 400\sqrt{3} = 1200\sqrt{3} \text{ N}$$



?)

5) $\sum F_y = mg \sin \theta$

$$= (2 \times 10) (10 \times 2) \left(\frac{\sin 30^\circ}{\sin 30^\circ} \right)$$

$$= 5N \quad (\text{A}) \checkmark$$

6) $\sum F = ma$

$$a = \frac{F}{m} - \mu g$$

$$= \frac{(15N)}{2kg} - 0.4 \cdot 10$$

6) $\sum F = ma$



$\sum F = ma$

$$\sum F_{\text{total}} = Mg - mg = (m+m)a$$

$$a = g \left(\frac{M-m}{m+m} \right)$$

(c) ✓

7) ✓ 8) $\sum F = ma$

9) $\sum F = F - \mu_s N$

$\mu_s N = 0.234 \text{ N}$

$$0.4 \cdot 10 \times 9.8 \text{ m/s}^2$$

$$= 39.2 \text{ N}$$

39.2 N < 39.2 N (D) ✓

(D) ✓ (A) ✓

(C) ✓ A

10) $F_c = \frac{mv^2}{r}$

11) $a = g - \frac{v^2}{r} = 5\sqrt{3} \text{ m/s}^2 \quad (\text{C})$

$$v = \sqrt{\frac{Fr}{m}} = \sqrt{50 \cdot 0.4}$$

$$= \sqrt{20 \text{ m/s}} = 4.47 \text{ m/s}$$

(C) ✓ B

12) $\sum F = mg \sin \theta - \mu g \cos \theta$

13) $F_c = mv^2/r$

$g \sin \theta - \mu g \cos \theta = a$ $\theta = 30^\circ$

$$v = \sqrt{rg} = \sqrt{0.6 \cdot 10} = \sqrt{6}$$

$$= 2.4 \sqrt{3} \text{ m/s} \quad (\text{C})$$

$a = g \sin \theta - 0.5 g \cos \theta$

$$= \frac{1}{2}g - \frac{1}{2} - \frac{\sqrt{3}}{2}g$$

$$= g \left(\frac{1}{2} - \frac{\sqrt{3}}{4} \right)$$

14) $T = 1 \text{ s} \quad r = 5 \text{ m}$

$f = \frac{1}{T} = 1 \text{ Hz}$

$\omega = 2\pi f = 2\pi \frac{\text{rad}}{\text{s}}$

$$a_c = \frac{v^2}{r} = \frac{r^2 \omega^2}{r} = r \omega^2$$

$$= r \cdot 4\pi^2$$

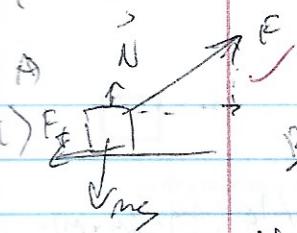
(d) ✓

(A) ✓

DL 4 K 1

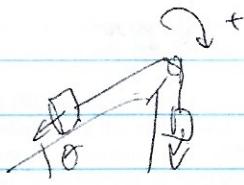
BL 5 R 3

Friction



$$b) F_y = F_{\text{surf}}$$

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~ draw both cases

$$N = mg - F_{\text{surf}} \checkmark$$

$$a) \Sigma F = m_2 g - m_1 g \sin \theta = (m_1 + m_2) a$$

$$c) \Sigma F_x = 0 \quad \Sigma F = F_{\text{cost}} - F_f$$

$$(i) a = \frac{m_2 g - m_1 g \sin \theta}{m_1 + m_2} \checkmark$$

$$= F_{\text{cost}} - \mu(mg - F_{\text{surf}})$$

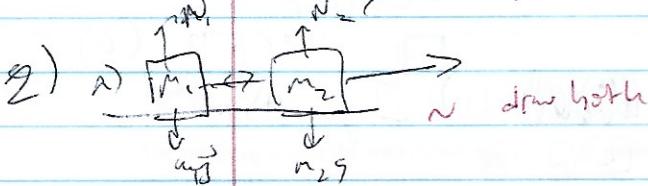
$$= F(\cos \theta + \sin \theta) - \mu mg \quad m_2 g - m_1 g \sin \theta \text{ must be positive}$$

$$a = \frac{F}{m} (\sin \theta + \cos \theta) - \mu g \checkmark$$

$$m_2 g - m_1 g \sin \theta$$

$$d) \max \text{surf cost} \geq \theta = 45^\circ \sim$$

$$\sin \theta = \frac{m_2}{m_1}$$



$$\theta \geq \sin^{-1}\left(\frac{m_2}{m_1}\right) \checkmark$$

$$(ii) \Sigma F = m_2 g - m_1 g \sin \theta \approx 0$$

$$e) \Sigma F = m_1 a \quad a = F$$

$$\sin \theta = \frac{m_1}{m_2} \sim$$

$$F = (m_1 + m_2) a$$

$$a = \frac{F}{m_1 + m_2} \checkmark$$

$$b) m_2 g - m_1 g \sin \theta - \mu m_1 g \cos \theta = (m_1 + m_2) a$$

$$f) a_1 = a_2$$

$$i) a = \frac{m_2 g - m_1 g \sin \theta - \mu m_1 g \cos \theta}{m_1 + m_2} \checkmark$$

$$\Sigma F_1 = T_2 - m_1 a$$

$$m_2 g = m_1 g \sin \theta + \mu m_1 g \cos \theta$$

$$\therefore (\sin \theta + \mu \cos \theta) = \frac{m_2}{m_1} \checkmark$$

$$T = m_1 a = \frac{m_1 F}{m_1 + m_2} \checkmark$$

$$ii) m_2 g - m_1 g \sin \theta - \mu m_1 g \cos \theta = 0$$

$$m_1 (\sin \theta + \mu \cos \theta) = m_2 g$$

$$\sin \theta + \mu \cos \theta = \frac{m_2}{m_1}$$

$$j) a = \frac{F}{m_1 + m_2} \checkmark \quad F = ? \star$$

6

4

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4) $F = kx$

47

$$B) \Sigma F = ma = mg - kv$$

$$a = g - \frac{\kappa v}{m}$$

$$c) mg = kx$$

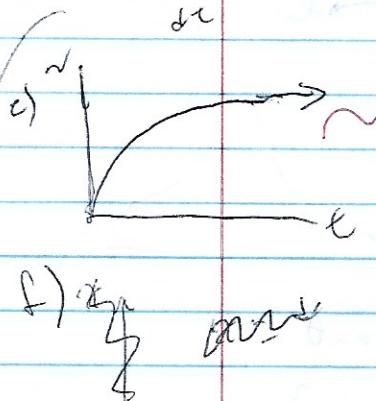
$$v = \cancel{mg} \quad \checkmark$$

d) Many -kr

$$m \cdot \frac{dv}{dt} = mg - kv$$

$$\frac{dx}{dt} = g - \frac{kx}{m}$$

$$m \cdot \frac{dv}{dt} + kv = mg$$



$$(E) m dv = (mg - \frac{mv}{\alpha}) dt$$

$$mv = mgt - kv$$

$$v(m+kt) = mg t$$

V - my t
met

$$f) \frac{mgt}{mkt} = \frac{dx}{dt}$$

$$\frac{m g t}{m + k t} dt = d\pi$$

$$x = \int_{\text{mette}}^{\text{mige}} dt$$

u = mette
 $\frac{du}{dt} = k$
 $dt = \frac{du}{k}$

mg

~~my~~ ~~my~~ ~~my~~

$$\frac{m}{mg_t} = -\frac{1}{gt}$$

$$V = \int \frac{k}{mg} - \frac{1}{g} t dt$$

$$= \frac{kt}{mg} - \frac{1}{g} \int \frac{1}{c} dt$$

$$f(x) = \frac{e^{tx}}{x^g} - \frac{1}{g} \ln|x| \sim$$

$$f'(x) = \frac{dy}{dt} = v$$

51

b/c the return from B
non inertial, Centrifugal
force is included.

50

$$B) \frac{F}{m} = \frac{m \cdot g^2}{r} \quad \left\{ \begin{array}{l} F = mg = 0 \\ MN - mg = 0 \end{array} \right.$$

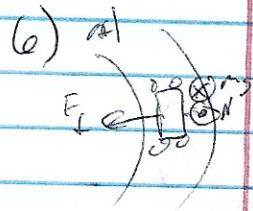
$$\underline{\text{Naph}} = \text{phen}$$

$$V \cong \left[\frac{r_2}{m} \right]$$

\sim $M_{\text{out}} \cdot t$
solar fuel M

c) mass doesn't matter here.

✓



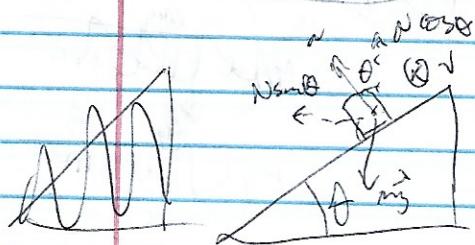
$$b) \frac{mv^2}{r} = \mu mg$$

$$v = \sqrt{r\mu g}$$



$$v = \sqrt{r\mu g}$$

c)



$$d) \frac{mv^2}{r} = N \sin \theta \quad \frac{mv^2}{r} = N \sin \theta$$

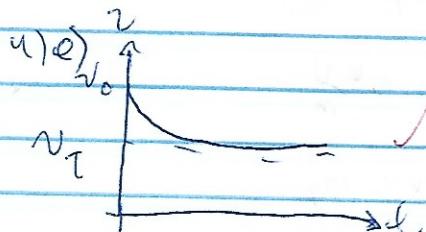
$$N \cos \theta = mg \quad \frac{v^2}{r} = \mu g \sin \theta$$

$$N = \frac{mg}{\cos \theta}$$

$$\sqrt{r} = \sqrt{g \tan \theta}$$

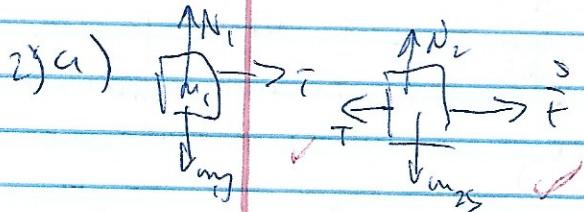
$\tan \theta \leq \mu$

3



$$e) b) \frac{v^2}{r} = g$$

$$\mu = \frac{gr}{v^2}$$



$$e) d) \frac{v^2}{r} = g + \mu g$$

$$\tan \theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1} \left(\frac{v}{rg} \right)$$

f) d) maximize $\sin \theta \cos \theta$

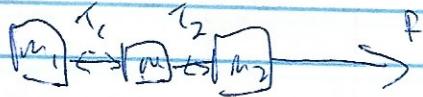
$$\frac{d}{d\theta} \sin \theta \cos \theta = 0$$

✓

$$\sin^2 \theta - \cos^2 \theta = 0$$

$$\cos^2 \theta = \sin^2 \theta \quad \theta = 45^\circ$$

✓



Newton's Law

$$gF = ma$$

$$T_1 + T_2 - F = ma$$

$$e) A) (i) 2F = m_2 g - m_1 g \sin \theta = 0$$

$$m_1 g \sin \theta = m_2 g$$

$$\sin \theta = \frac{m_2}{m_1}$$

$$\theta \leq \sin^{-1} \left(\frac{m_2}{m_1} \right)$$

✓

1d) maximum ~~sinθ + μcosθ~~

$$\frac{d}{d\theta} (\cos\theta + \mu \sin\theta) = 0$$

$$-\sin\theta + \mu \cos\theta = 0$$

$$\mu = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

q.u, h

2) f) $a = \frac{F}{m_1 + m_2}$

$$F + F_1 - F_2 = m_1 + m_2 a$$

$$\sim F_2 - F_1 = F - (m_1 + m_2)a \\ = F \left(\frac{m_2}{m_1 + m_2 + m} \right)$$

h) i) $a = \frac{dv}{dt} \quad \frac{dv}{dt} = \frac{k}{m} (v - g \frac{m}{k})$

$$\frac{dv}{v - g \frac{m}{k}} = -\frac{k}{m} dt$$

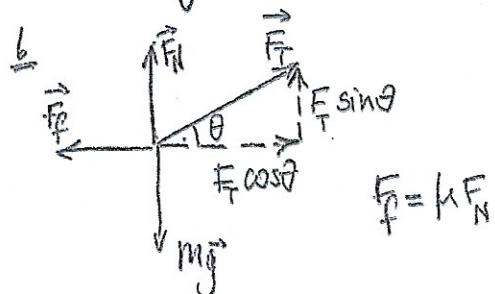
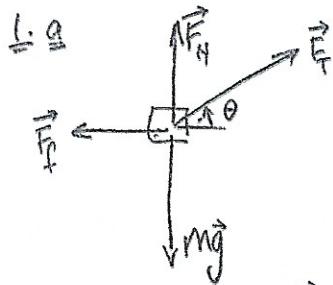
$$\ln(v - g \frac{m}{k}) = -\frac{k}{m} t + C$$

$$v - g \frac{m}{k} = e^C e^{-\frac{k}{m} t}$$

at $t=0, v=v_0 ; e^C = v_0 - g \frac{m}{k}$

$$v(t) = \frac{m}{k} g (v_0 - g \frac{m}{k}) e^{-\frac{k}{m} t}$$

CHAPTER 3 OG



$$F_N + F \sin \theta - mg = 0 \Rightarrow F_N = mg - F \sin \theta$$

$$\therefore F \cos \theta - F_f = ma$$

$$a = \frac{1}{m} (F \cos \theta - (\mu(mg - F \sin \theta)))$$

$$\therefore a = \frac{1}{m} (F \cos \theta + \mu \sin \theta - \mu mg)$$

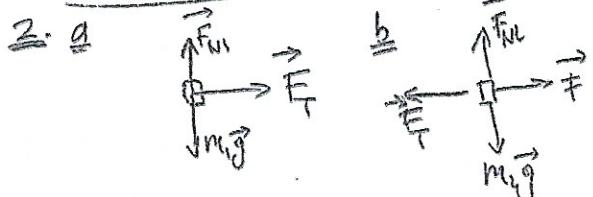
Since m, μ, g , & F are fixed, we can only change $\cos \theta + \mu \sin \theta$

Optimum θ : $\frac{d}{d\theta} (\cos \theta + \mu \sin \theta) = 0$
i.e. the slope of $\cos \theta + \mu \sin \theta$.

$$\frac{d}{d\theta} (\cos \theta + \mu \sin \theta) = -\sin \theta + \mu \cos \theta = 0$$

$$\mu = \tan \theta \Rightarrow \theta = \tan^{-1} \mu$$

Substitute this in $\cos \theta + \mu \sin \theta$ to see (by graphing e.g.) that this indeed maximizes a .



$$\therefore F + (F \rightarrow) + (F \leftarrow) = (m_1 + m_2) \vec{a}$$

$$\vec{a} = \frac{\vec{F}}{(m_1 + m_2)}$$

$$\therefore \vec{F} = m_1 \vec{a} = \left(\frac{m_1}{m_1 + m_2} \right) \vec{F}$$

(i) In part (c) include m_2 string

$$\vec{a} = \frac{\vec{F}}{(m_1 + m_2 + m_3)}$$

(ii) In part (c) F_{T1} & F_{T2} are no longer equal. Therefore

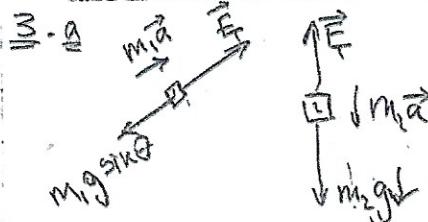
$$\vec{F} + (F_{T1} \rightarrow) + (F_{T2} \leftarrow) = (m_1 + m_2) \vec{a}$$

$$F_{T2} - F_{T1} = \vec{F} (m_1 + m_2) a$$

$$|F_{T2} - F_{T1}| = F \left(\frac{m_3}{m_1 + m_2 + m_3} \right)$$

$$\text{Alternatively } F_{T2} - F_{T1} = m_3 a$$

$$F_{T2} - F_{T1} = \left(\frac{m_3}{m_1 + m_2 + m_3} \right) F$$



Either

$$\begin{cases} F - m_1 g \sin \theta = m_1 a \\ m_2 g - F = m_2 a \end{cases}$$

$$\begin{aligned} (m_1 + m_2) a &= m_2 g - m_1 g \sin \theta \\ a &= \left(\frac{m_2 - m_1 \sin \theta}{m_1 + m_2} \right) g \end{aligned}$$

$$(i) m_2 - m_1 \sin \theta > 0 : \frac{m_2}{m_1} > \sin \theta \text{ clockwise}$$

$$m_2 - m_1 \sin \theta < 0 : \frac{m_2}{m_1} < \sin \theta \text{ counter-clockwise}$$

$$(ii) m_2 = m_1 \sin \theta \quad \theta = \sin^{-1} \left(\frac{m_2}{m_1} \right)$$

$$(b) \text{ Friction } F_f = \mu_k F_N = \mu_k m_1 g \cos \theta$$

include this in a :

$$\begin{aligned} m_2 g \cos \theta - F_f &= m_2 a \\ (m_1 + m_2) a &= m_2 g - m_1 g \sin \theta - \mu_k m_1 g \cos \theta \\ a &= \left[\frac{m_2 - m_1 (\sin \theta + \mu_k \cos \theta)}{m_1 + m_2} \right] g \end{aligned}$$

constant velocity $\vec{a} = 0$.

$$m_2 - m_1 (\sin \theta + \mu_k \cos \theta) = 0 \Rightarrow \sin \theta + \mu_k \cos \theta = \frac{m_2}{m_1}$$

4(a)

(b) $\vec{m\ddot{a}} = \vec{F}_r + \vec{mg}$
 $\vec{a} = \left(\frac{mg - k\vec{v}}{m}\right) \uparrow$

(c) Terminal velocity: $\vec{a} = 0$
 $mg = k\vec{v}_T \Rightarrow \vec{v}_T = \frac{mg}{k}$

(d) $t=0 \Rightarrow v=v_0$
 $t \rightarrow \infty \Rightarrow v=v_T = \frac{mg}{k}$ smoothly

(e) $a = \frac{dv}{dt} \Rightarrow \frac{dv}{dt} = \frac{k}{m}(v - g \frac{m}{k})$

$$\frac{dv}{v - g \frac{m}{k}} = -\frac{k}{m} dt$$

Integrate: $\ln\left|v - g \frac{m}{k}\right| = -\frac{k}{m}t + C$ (d)

Solution $v - g \frac{m}{k} = e^C e^{-\frac{k}{m}t}$

@ $t=0, v=v_0 \Rightarrow e^C = v_0 - g \frac{m}{k}$

$v(t) = \frac{m}{k}g + (v_0 - \frac{m}{k}g)e^{-\frac{k}{m}t}$

5(a)

(b) You need $F_f \geq mg$
 $\mu_s F_N \geq mg$
 F_N accounts for F_c
 $\therefore F_f = m \frac{v^2}{r}$

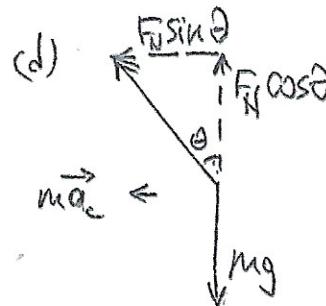
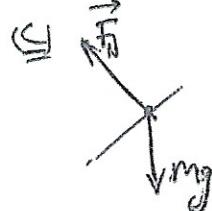
$\Rightarrow \mu_s m \frac{v^2}{r} \geq mg \Rightarrow \mu_s \geq \frac{r}{v^2} g$

(c) $\mu_s \geq \frac{r}{v^2} g$ is independent of m .
 m does not matter.

6(a)

(b) F_f accounts for $F_c \therefore$
 $F_f = m \frac{v^2}{r}$
 $\mu_s F_N = m \frac{v^2}{r}$
 $F_N = mg$

$$v_{max} = \sqrt{\mu_s g r}$$



$$F_N \cos \theta - mg = 0$$

$$F_N = \frac{1}{\cos \theta} mg \quad (1)$$

$$m\vec{a}_c = F_N \sin \theta$$

$$F_N = \frac{1}{\sin \theta} m \frac{v^2}{r} \quad (2)$$

$$(1) = (2)$$

$$\Rightarrow \frac{1}{\cos \theta} mg = \frac{1}{\sin \theta} m \frac{v^2}{r}$$

$$\tan \theta = \frac{v^2}{gr}$$

$$\theta = \tan^{-1} \left(\frac{v^2}{gr} \right)$$