Rotational Mechanics

1. A Compact disk has a radius of 6 cm. If the disk rotates about its central axis at an angular speed of 5 rev/s, what is the linear speed of a point on the rim of the disk?
A. 0.3 m/s B. 1.9 m/s C. 7.4 m/s D. 52 m/s E. 83 m/s

2. What is the total distance traveled by a point on the of the disk in 40 min in the previous question?
A. 180 m B. 360 m C. 540 m D. 720 m E. 4.5 km

3. An object of mass 0.5 kg, moving in a circular path of radius 0.25 m, experiences a centripetal acceleration of constant magnitude $9\, m/s^2$. What is the object's angular speed?
A. 2.3 rad/s B. 4.5 rad/s C. 6 rad/s D. 12 rad/s
E. Cannot be determined from the information given

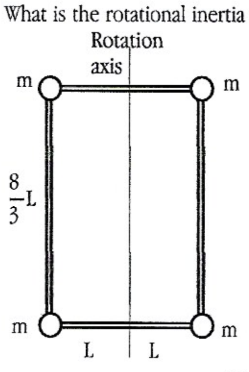
4. An object, originally at rest, begins spinning under uniform angular acceleration. In 10 s, it completes an angular displacement of 60 rad. What is the numerical value of the angular acceleration?
A. 0.3 rad/s^2 B. 0.6 rad/s^2 C. 1.2 rad/s^2
D. 2.4 rad/s^2 E. 3.6 rad/s^2

5. A force **F** is applied to a wrench to tighten a bolt. If the distance from the handle of wrentch to the center of the bolt is 20 cm and F= 20 N, what is the magnitude of the torque produced?
A. 0 N.m B. 1 N.m C. 2 N.m D. 4 N.m E. 10 N.m

6. What is the torque about the a pendulum's suspension point produced by the weight of the bob if the length of the pendulum is 80 cm, the mass of the bob is 0.5 kg and the string of the pendulum makes 60° with the horizontal?
A. 0.49 N.m B. 0.98 N.m C. 1.7 N.m D. 2.0 N.m E. 3.4 N.m

7. A uniform meter stick of mass 1 kg is hanging from a thread attached at the stick's midpoint. One block of mass m=3kg hangs from the 0 cm end of the stick, and another block of unknown mass M hangs at the 80 cm mark. If the stick remains at test in the horizontal position, what is M?
A. 4 kg B. 5 kg C. 6 kg D. 8 kg E. 9 kg

8. What is the rotational inertia of the arrangement below?

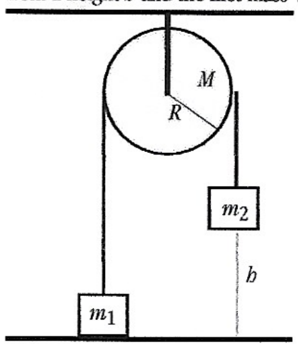


A. $4mL^2$ B. $\frac{27}{3}mL^2$ C. $\frac{64}{9}mL^2$ $\frac{128}{9}$D. mL^2 E. $\frac{256}{9}mL^2$

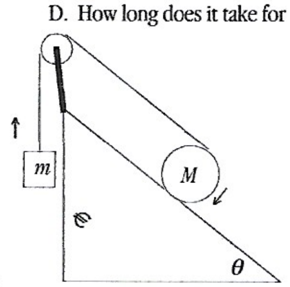
9. The moment of inertia of a solid uniform sphere of mass M and radius R is given by $I = \frac{2}{5}MR^2$. The sphere is releasedfrom rest at top of an plane of heigh h and length L inclined at an angle θ If the sphere rolls without slippping, find its speed at the bootm of the incline.
A. $\sqrt{\frac{10}{7}gh}$ B. $\sqrt{\frac{5}{2}gh}$ C. $\sqrt{\frac{7}{2}gh}$ D. $\sqrt{\frac{2}{7}gL\sin\theta}$ E. $\sqrt{\frac{7}{10}gL\sin\theta}$

10. An object spins with angular velocity ω. If the object's moment of inerti inceases by a factor of 2 without the application of an external torque, what wil be the object's ew angular velocity?
A. $\omega/4$ B. $\omega/2$ C. $\omega/\sqrt{2}$ D. $\sqrt{2}\,\omega$ E. 2ω

1. An Atwood machine consists of a solid disk (pulley, $I = \frac{1}{2}MR^2$) and two masses. The system is set in motion by relassing m_2 from a height b and the first mass m_1 is at rest on the ground.

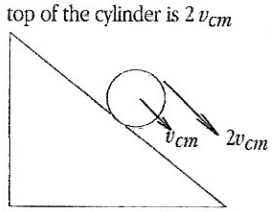


A. What is the speed of m_2 just before it strikes the ground?
B. What is the angular speed of the pulley at this moment?
C. What's the angular displacement of the pulley?
D. How long does it take for m_2 to fall to the floor?

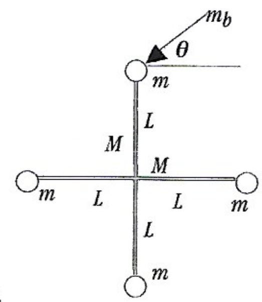


2. Answer the questions below for the set up above.

A. Show that "rolling without slipping" means that the speed of the cylinder's center of mass, v_{cm}, is equal to $R\omega$, where ω is its angular speed.
B. Show that relative to the contact point, the speed of the top of the cylinder is $2\,v_{cm}$



C. What is the relationshiop between the magnitude of the acceleraton of the block and the linear acceleration of the cylinder?
D. What is the acceleration of the cylinder?
E. What is the acceleration of the block?



3. Two uniform bars of mass M and length 2L meet at right angles at their midpoints to form a rigid assemly that can freely rotate about the intersection point. A solid ball of mass m is attached to the each end of the bars. A bullet of mass m_b is shot with velocity **v** and becomes embeddd in the targeted ball.

A. Show that the moment of inertia o feach rod about the rotation axis is $ML^2/3$.
B. Determine the angular velocity of the assembly after the bullet has becom lodged in the targeted clay ball.
C. What is the resulting linear speed of each clay ball?
D. Determine the ratio of the final KE of the assembly to the KE of the bullet before impact.

PR 6 RM

Black 12 Red 7
Blue 4
Black + Blue 16 Total 23

1) $v = r\omega = 0.06\,m \times \frac{5\,rev}{s} \cdot \frac{2\pi\,rad}{rev}$

$= 1.9\ m/s$ (b) ✓

2) $\Delta\theta = \bar{\omega} t$

$s = r\theta = r\omega t$

$= (1.9\,m/s)(40\,m)(60\,s)$

$= 4560\,m$ (e) ✓

3) $a_c = \frac{v^2}{r}$ $\omega = \frac{v}{r}$

$v = r\omega$

$a_c = \frac{r^2\omega^2}{r}$

$= r\omega^2$

$\omega = \sqrt{\frac{a_c}{r}} = \sqrt{\frac{9\,m/s^2}{0.25\,m}} = 6\ rad/s$ (c) ✓

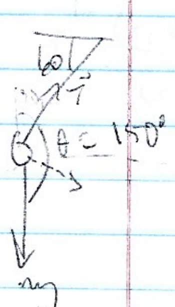
4) $\Delta\theta = \cancel{\omega_0 t}^{\,0} + \frac{1}{2}\alpha t^2$

$\omega_f = \omega_0 + \alpha t$

$\alpha = \frac{2\Delta\theta}{t^2} = \frac{2(60\,rad)}{(10\,s)^2}$

$= 1.2\ rad/s^2$ (c) ✓

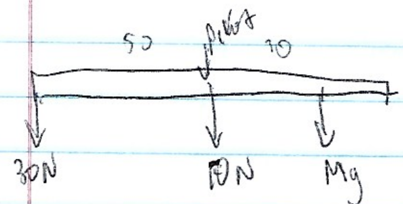
5) $\tau = r \times F = 0.2\,m \cdot 20\,N = 4\,N{\cdot}m$ (d) ✓

6) 

$\tau = rF\sin 150$

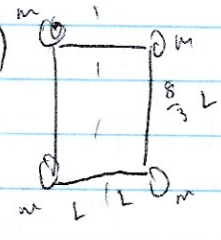
$= 0.8\,m \cdot 0.5\,kg \cdot 9.8\,m/s^2 \cdot \sin 150$ ✓

$= 1.96\,N{\cdot}m$ (d)

7) 

(7) $30N \cdot 0.5\,m = 10M \cdot 0.3\,m$

$M = \frac{30N \cdot 0.5\,m}{0.3\,m \cdot 10} = 5\,kg$ (b) ✓

(8) 

~~$I_m = 4mL^2$~~

~~$I_{//} = \frac{4}{3}mL^2$~~

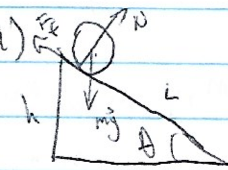
~~$I_\perp = 2mL^2$~~

~~$I_{total} = \frac{22}{3}mL^2$ (b)?~~

rods are massless;

$I = 4mL^2$

(a) ✓

9) 

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v^2}{R^2}\right)$

$mgh = \frac{1}{2}mv^2 + \frac{1}{5}mv^2$

$mgh = \frac{7}{10}mv^2$

$v = \sqrt{\frac{10}{7}gh}$ (A) ✓

10) $\frac{1}{2}\cancel{I}\omega^2 = \cancel{\tfrac{1}{2}}\,2\cancel{I}\,\omega_f^2$

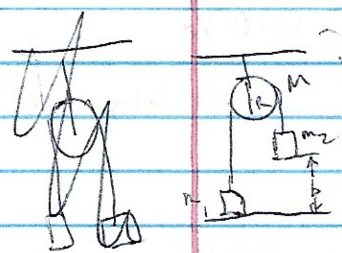
$\omega_f^2 = \frac{1}{2}\omega^2$

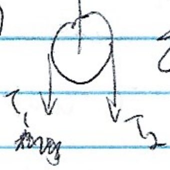
$\omega_f = \frac{1}{\sqrt{2}}\omega$ (c) ~ B

6

3 1R

FR

1)  α, a, v

A) 

$\sum \tau = T_2R - T_1R = I\alpha$

$= \frac{1}{2}MR^2 \cdot \frac{a}{R}$

$= \frac{1}{2}MRa$

$\sum F_2 = m_2g - T_2$ $\quad \sum F_1 = T_1 - m_1g = m_1a$

$= m_2a$

$T_2 = m_2g - m_2a = m_2(g-a)$

$T_1 = m_1a - m_1g$

$= m_1(a-g)$

$m_2R(g-a) - m_1R(a-g) = \frac{1}{2}MRa$

$m_2R(g-a) + m_1R(g-a) = \frac{1}{2}MRa$

$(g-a)(m_2R + m_1R) = \frac{1}{2}MRa$

$\frac{1}{2}Ma = (g-a)(m_2+m_1)$

$\frac{M}{2(m_1+m_2)}a = g - a$

$a\left(\frac{M}{2(m_1+m_2)} + 1\right) = g$

$a\left(\frac{M+2(m_1+m_2)}{2(m_1+m_2)}\right) = g$

$a = \frac{2g(m_1+m_2)}{M+2(m_1+m_2)}$

$\Delta x = \cancel{v_0t} + \frac{1}{2}at^2$

$v_f^2 = \cancel{v_0^2} + 2a\Delta x$

$v_f = \sqrt{2ab}$

$= \sqrt{\frac{2 \cdot 2bg(m_1+m_2)}{M+2(m_1+m_2)}}$

$= 2\sqrt{\frac{bg(m_1+m_2)}{M+2(m_1+m_2)}}$ ~

b) $\alpha = \frac{a}{r} = \frac{2g(m_1+m_2)}{r(M+2(m_1+m_2))}$ ~ $\quad a = r\alpha$

c) $S = r\theta = b$

$\theta = \frac{b}{r}$ ✓

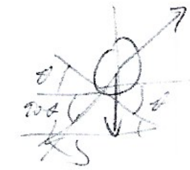
d) $b = \cancel{v_0t} + \frac{1}{2}at^2$

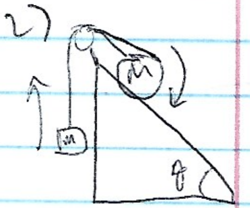
$t = \sqrt{\frac{2b}{a}}$

$= \sqrt{\frac{2b(M+2(m_1+m_2))}{2g(m_1+m_2)}}$

$= \sqrt{\frac{b(M+2(m_1+m_2))}{g(m_1+m_2)}}$ ~

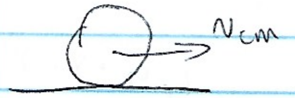
almost!!



2) 

$v = R\omega$

$s = r\theta$



$s = r\theta$ $\quad \theta = 2\pi$

$C = 2\pi r$

$\Delta x = \bar{v} t$

$2\pi r = v_{cm} t$

$v_{cm} = \frac{2\pi r}{t}$

$f = \frac{1}{t}$; $\omega = 2\pi f$,

$\frac{2\pi}{t} = \omega$

$v_{cm} = R\omega$ ✓

b) 

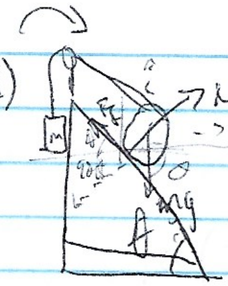
$v = r\omega$

has rotational ω_r as well as orbital ω_o ; $\omega_o = \omega_r$

$v_{top} = r\omega = 2R\omega$

$= 2 v_{cm}$ ✓

c) $a_m = a_{cm}$ (system) ~

d) 

$\Sigma \tau = I\alpha = \mu N - T$

$= \mu Mg \cos(90 - \theta)$

$= \mu Mg \cos\theta - T$

$= \frac{1}{2} MR^2 \cdot \frac{a}{R}$

$= \frac{1}{2} MRa$

$\Sigma F = m_{total} a = Mg\sin\theta - mg - F_f$

$= (m+M)a$

$\cancel{a = \frac{Mg\sin\theta - mg}{m+M}}$

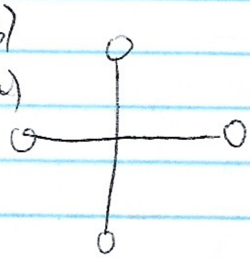
$\cancel{= g\,\frac{M\sin\theta - m}{m+M}}$

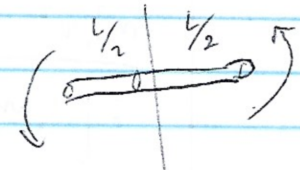
$Mg\sin\theta - mg - \mu Mg\cos\theta = (m+M)a$

$a = g \cdot \frac{M\sin\theta - \mu M\cos\theta - m}{m+M}$ ~

e) $a_{block} = a =$

3)

a) 



$\lambda = \frac{m}{L}$

$dm = \lambda dL$

$dr = dL$ $\quad I = mr^2$

$= 2\int_0^{L/2} \lambda L^2 dL$?

$I_{total} = \frac{4}{3} ML^2 + 4mL^2$ ~

$L = r \times p = rp\sin\theta$

2

$)\ L_0 = L_f$

$I\omega_0 = I\omega_f$

$I\cancel{\omega_0} + L m_b v = \left(I + m_b L^2\right)\omega_f$

$$\omega_f = \frac{m_b v L}{\frac{4}{3}ML^2 + 4mL^2 + m_b L^2} = \frac{m_b v}{\frac{4}{3}ML + 4mL + m_b L}$$

c) $v = r\omega = \dfrac{m_b v L^2}{\frac{4}{3}ML^2 + 4mL^2 + m_b L^2}$

$$= \frac{m_b v}{\frac{4}{3}M + 4m + m_b}$$

d) $KE_f = \frac{1}{2} I \omega_f^2$

$$= \frac{1}{2} I \cdot \frac{(m_b v L)^2}{I^2}$$

$$= \frac{(m_b v L)^2}{2\left(\frac{4}{3}ML^2 + 4mL^2 + m_b L^2\right)}$$

$$= \frac{m_b^2 v^2}{2\left(\frac{4}{3}M + 4m + m_b\right)}$$

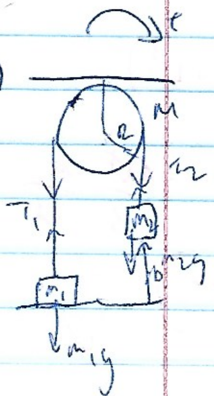
$$= \frac{m_b^2 v^2}{\frac{8}{3}M + 4m + m_b}$$

$KE_0 = \frac{1}{2} m_b v_b^2$

$$\frac{KE_f}{KE_0} = \frac{m_b^{\cancel{2}} \cancel{v_b^2}}{\frac{8}{3}M + 4m + m_b} \cdot \frac{2}{\cancel{m_b} \cancel{v_b^2}}$$

$$= \frac{2 m_b}{\frac{8}{3}M + 4m + m_b}$$

1) a)



assume 0 normal force

$$\Sigma \tau = I\alpha = \frac{1}{2}MR^2\frac{a}{R} = T_2R - T_1R$$

$$\frac{1}{2}mRa = T_2R - T_1R$$

$$\Sigma F_2 = m_2a = m_2g - T_2 = m_2a$$

$$\Sigma F_1 = m_1a = T_1 - m_1g = m_1a$$

$$T_1 = m_1a + m_1g$$

$$T_2 = m_2g - m_2a$$

$$(m_2g - m_2a)R - (m_1a + m_1g)R = \frac{1}{2}MRa$$

$$\frac{1}{2}Ma = m_2g - m_2a - m_1a - m_1g$$

$$a\left(\frac{1}{2}M + m_2 + m_1\right) = m_2g - m_1g$$

$$a = g\left(\frac{m_2 - m_1}{\frac{1}{2}M + m_2 + m_1}\right)$$

$$\cancel{= 2g\left(\frac{m_2 - m_1}{M + 2m_2 + 2m_1}\right)}$$

A) $v_f^2 = \cancel{v_0^2}^{\,0} + 2ab$

$$v_f = \sqrt{2ab}$$

$$= \sqrt{2gb\left(\frac{m_2 - m_1}{\frac{1}{2}M + m_2 + m_1}\right)}$$

✓

1

B) $v = r\omega$

$$\omega = \frac{v}{R} = \frac{1}{R}\sqrt{2gb\left(\frac{m_2 - m_1}{\frac{1}{2}M + m_1 + m_2}\right)}$$ ✓

~~c) $\Delta\theta$~~

d) $v_f = \cancel{v_0} + at$ $\quad \Delta x = \cancel{v_0 t} + \frac{1}{2}at^2$

$\cancel{t = \frac{\Delta v}{a}}$

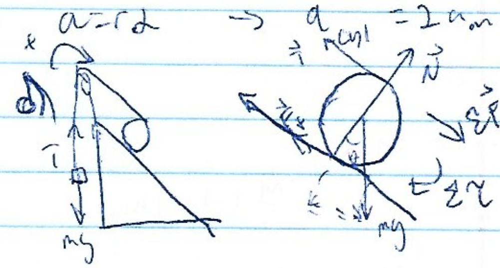
$$2b = at^2 \qquad t^2 = \frac{2b}{a}$$

$$t = \sqrt{\frac{2b}{a}} = \sqrt{\frac{2b\left(\frac{1}{2}M + m_1 + m_2\right)}{g(m_2 - m_1)}}$$ ✓

2) c), d), e)

c) b/c the pivot point is on the perimeter, ✓

$a = r\alpha \quad \rightarrow \quad a_{m(y)} = 2a_{cm}$? why?



$$a_b = 2a_{cm}$$

$$\Sigma F = ma_b$$

$$-mg + T = ma_b$$

$$T - mg = m(2a_{cm})$$

$$\Sigma \tau = I\alpha = \frac{1}{2}MR^2\frac{a_{cm}}{R}$$

$$\cancel{R}\mu Mg\cos\theta - T\cancel{R} = \frac{1}{2}MRa_{cm}$$

$$\mu Mg\cos\theta - T = \frac{1}{2}Ma_{cm}$$

$$T = \mu Mg\cos\theta - \frac{1}{2}Ma_{cm}$$

$$T = 2ma_{cm} + mg = \mu Mg\cos\theta - \frac{1}{2}Ma_{cm}$$

$$2ma_{cm} + \frac{1}{2}Ma_{cm} = \mu Mg\cos\theta - mg$$

$$a_{cm}\left(2m + \frac{1}{2}M\right) = \mu Mg\cos\theta - mg$$

3

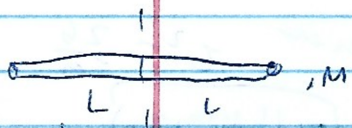
$a_{cm}(2m+\frac{1}{2}M) = \mu M g\cos\theta - mg$

$a_{cm} = g\left(\frac{\mu M\cos\theta - m}{2m+\frac{1}{2}M}\right)$ ~

c) $a_b = 2a_{cm}$

$= 2g\left(\frac{\mu M\cos\theta - m}{2m+\frac{1}{2}M}\right)$ ~

3) a)



??? ~

b) d)

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2) d) $T - mg = ma_b = 2ma_{cm}$

$Mg\sin\theta - F_f - T = Ma_{cm}$

$RF_f - RT = I\alpha$

$RF_f - RF_T = \frac{1}{2}MR^2\frac{a_{cm}}{R}$

$F_f - T = \frac{1}{2}Ma_{cm}$

$Mg\sin\theta - 2T = \frac{3}{2}Ma_{cm}$

$2T - 2mg = 4ma_{cm}$

$Mg\sin\theta - 2mg = \left(\frac{3M}{2}+4m\right)a_{cm}$

$a_{cm} = \left(\frac{M\sin\theta - 2m}{\frac{3}{2}M+4m}\right)g$ ✓

e) $a_b = 2a_{cm} = 2\left(\frac{M\sin\theta - 2m}{\frac{3}{2}M+4m}\right)g$ ✓

3) a)

$2L, M$

linear density $\frac{M}{2L} = \lambda$ $\quad dm = \lambda\,dx$

$I = \int r^2\,dm = \int_{-L}^{L} x^2\lambda\,dx = \lambda\int_{-L}^{L} x^2\,dx$

$= \lambda\left[\frac{1}{3}x^3\right]_{-L}^{L} = \frac{\lambda}{3}(L^3+L^3)$

$= \frac{2L^3}{3}\lambda = \frac{2L^3}{3}\cdot\frac{M}{2L}$

$= \boxed{\frac{1}{3}ML^2}$ ✓

b) $\vec{l} = \vec{r}\times\vec{p}$ $\quad l = I\omega$

$l = rp_\perp = rm_bv_\perp = rm_bv\cos\theta$

$= Lm_bv\cos\theta$

$Lm_bv\cos\theta = \left(4mL^2 + 2\left(\frac{1}{3}ML^2\right) + m_bL^2\right)\omega_f$

$\omega_f = \frac{m_bv\cos\theta}{L(4m+\frac{2}{3}M+m_b)}$ ✓

c) $v = r\omega$ $\quad v_f = L\omega_f$

$v_f = \left(\frac{m_bv\cos\theta}{4m+\frac{2}{3}M+m_b}\right)$ ✓

d) $\frac{KE_f}{KE_0} = \frac{\frac{1}{2}I\omega_f^2}{\frac{1}{2}m_bv^2}$

$= \frac{(4mL^2+\frac{2}{3}ML^2+m_bL^2)\left(\frac{m_bv\cos\theta}{L(4m+\frac{2}{3}M+m_b)}\right)^2}{m_bv^2}$

$= \boxed{\frac{m_b\cos^2\theta}{4m+\frac{2}{3}M+m_b}}$ ✓

6

CHAPTER 6 MC

1. B $v=\omega r=\left(5\frac{rev}{s}\cdot\frac{2\pi}{rev}\right)\cdot 0.06m = 1.9\frac{m}{s}$

2. E $d = vt = \omega r t = 4500m$

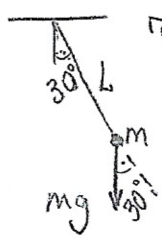
3. C $a_c = \frac{v^2}{r} = \omega^2 r \Rightarrow \omega = 6\,rad/s$

4. C $\Delta\theta = \frac{1}{2}\alpha t^2+\omega t \Rightarrow \alpha = \frac{2\Delta\theta}{t^2}$

$\alpha = 1.2\ rad/s^2$

5. D $\vec{\tau} = \vec{r}\times\vec{F} = \vec{4}\ Nm$

6. D

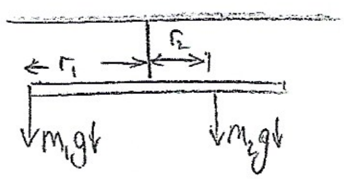


$\tau = rF\sin\theta$

$= Lmg\sin\theta$

$\tau = 2.0\ Nm$

7. B



$m_1 g r_1 = m_2 g r_2$

$m_2 = \frac{r_1}{r_2} m_1 = 5kg$

8. A $I = \Sigma m_i r_i^2 = 4mL^2$

9. A C.o.E. $KE_i+PE_i=KE_f+PE_f$

$0+Mgh = \left(\frac{1}{2}I\omega^2+\frac{1}{2}Mv_{cm}^2\right)+0$

$I_s=\frac{2}{5}MR^2$, $\omega=\frac{v}{R}$ $\quad = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v_{cm}^2}{R^2}\right)+\frac{1}{2}Mv_{cm}^2$

$\Rightarrow v_{cm} = \sqrt{\frac{10}{7}gh}$

10. B C.o.$\vec{L}$: $\vec{L}_i = \vec{L}_f$

$I_1\omega_1 = I_2\omega_2$

$\omega_2 = \frac{I_1}{I_2}\omega_1 = \frac{\omega}{2}$

OE

1.a Use C.o.E. $KE_i+PE_i=KE_f+PE_f$

$0+m_2gh = \frac{1}{2}m_1v^2+\frac{1}{2}m_2v^2+\frac{1}{2}I\omega^2+m_1gh$

Substitute $I=\frac{1}{2}MR^2$ & $\omega = v/R$ to get

$(m_2-m_1)gh = \frac{1}{2}\left(m_1+m_2+\frac{1}{2}M\right)v^2$

$v = \sqrt{\frac{2(m_2-m_1)}{(m_1+m_2+\frac{1}{2}M)}gh}$

b $\omega = \frac{v}{R}$ $\quad \omega = \frac{1}{R}\sqrt{\frac{2(m_2-m_1)}{(m_1+m_2+\frac{1}{2}M)}gh}$

c The angular displacement must result in an arc length disp of h.

I.E. $h = (\Delta\theta)\cdot R$

$\Delta\theta = \frac{h}{R}$

d $v_{av} = \frac{\Delta s}{\Delta t} \Rightarrow \Delta t = \frac{\Delta s}{v_{av}} = \frac{\Delta s}{v/2} = \frac{2h}{v}$

$\Delta t = 2h\sqrt{\frac{m_1+m_2+\frac{1}{2}M}{2(m_2-m_1)gh}} = \sqrt{h\left[\frac{2(m_1+m_2)+M}{(m_2-m_1)g}\right]}$

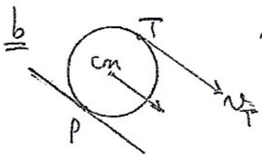
---

2 a The angular displacement $\Delta\theta$ corresponds to arc length displacement $\Delta s = R(\Delta\theta)$ which must be the same as linear displacement

$\Rightarrow \frac{\Delta s}{\Delta t} = R\frac{\Delta\theta}{\Delta t}$

$v_{cm} = R\omega$

b



$v_{cm} = R\omega$

$v_T = 2R\omega = 2v_{cm}$

or $\vec{v}_p = \omega R \leftarrow$

$\vec{v}_T = \omega R \rightarrow$

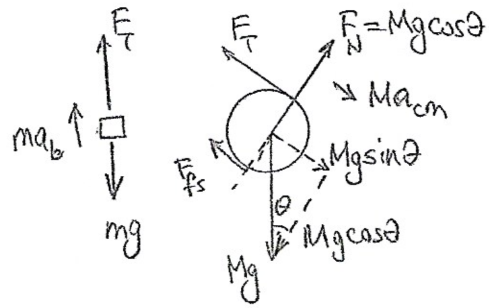
$\vec{v}_T - \vec{v}_p = (\omega R\rightarrow)-(\omega R\leftarrow)$

$= 2\omega R\rightarrow = 2v_{cm}$

c The speed of the block will be the same as v_T since the rope is in contact w/ point T & wrapping around the cylinder. Therefore,

$v_b = v_T = 2v_{cm} \Rightarrow a_b = 2a_{cm}$

d



(1) $F_T - mg = ma_b = 2ma_{cm}$ block

(2) $Mg\sin\theta - F_f - F_T = Ma_{cm}$ cylinder

$RF_f - RF_T = I\alpha$

$RF_f - RF_T = \frac{1}{2}MR^2 \frac{a_{cm}}{R}$

(3) $F_f - F_T = \frac{1}{2}Ma_{cm}$

(2)+(3) $\Rightarrow$ $Mg\sin\theta - 2F_T = \frac{3}{2}Ma_{cm}$ (4)

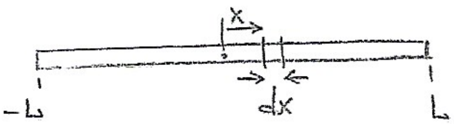
From (1) $\Rightarrow$ $2F_T - 2mg = 4ma_{cm}$ (5)

(4)+(5) $\Rightarrow$ $Mg\sin\theta - 2mg = \left(\frac{3M}{2} + 4m\right)a_{cm}$

$$a_{cm} = \left(\frac{M\sin\theta - 2m}{\frac{3}{2}M + 4m}\right)g$$

(d) $a_b = 2a_{cm} \Rightarrow a_b = 2\left(\frac{M\sin\theta - 2m}{\frac{3}{2}M + 4m}\right)g$

3 a See class notes.



Length: 2L

Density: $\frac{M}{2L} = \lambda \Rightarrow dm = \lambda dx$

$$I = \int r^2 dm = \int_{-L}^{L} x^2 \lambda \, dx = \lambda \int_{-L}^{L} x^2 dx$$

r=x

$$= \lambda\left[\frac{x^3}{3}\right]_{-L}^{L} = \frac{\lambda}{3}\left(L^3 - (-L)^3\right)$$

$$= \frac{\lambda}{3} 2L^3 = \frac{2}{3}\lambda L^3 = \frac{2}{3}\frac{M}{2L}L^3$$

$$I = \frac{1}{3}ML^2$$

b $\vec{\ell} = \vec{r} \times \vec{p}$ & $\vec{\ell} = I\vec{\omega}$ $\vec{\ell}$: angular mom.

$\ell = rp_\perp = rm_b v_\perp = rm_b v\cos\theta$ in this case

$= Lm_b v\cos\theta$

$$Lm_b v\cos\theta = \left(4mL^2 + 2\left(\frac{1}{3}mL^2\right) + m_b L^2\right)\omega_f$$

$$\omega_f = \frac{m_b v\cos\theta}{L(4m + \frac{2}{3}M + m_b)}$$

c $v = \omega R \Rightarrow v_f = L\omega_f$

$$v_f = \left(\frac{m_b v\cos\theta}{4m + \frac{2}{3}M + m_b}\right)$$

d $\frac{KE_a}{KE_b} = \frac{\frac{1}{2}I\omega_f^2}{\frac{1}{2}m_b v^2}$ ← rotational KE ← only translational KE

$$\frac{KE_a}{KE_b} = \frac{\left(4mL^2 + \frac{2}{3}ML^2 + m_b L^2\right)\left[\frac{m_b v\cos\theta}{L(4m + \frac{2}{3}M + m_b)}\right]^2}{m_b v^2}$$

$$\frac{KE_a}{KE_b} = \left(\frac{m_b \cos^2\theta}{4m + \frac{2}{3}M + m_b}\right)$$