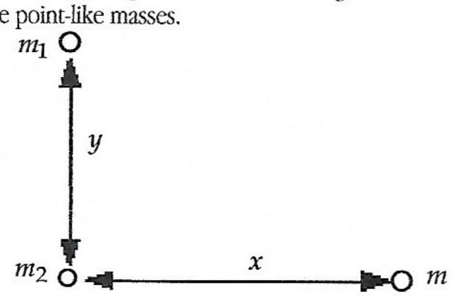
Ch 6,7

FRQ

1. On planet ι.δ., where the locals do nothing but make absolute statements without any rational thought, you measured the gravitational acceleration to be $g_{\iota\delta}$, you are asked to build two different simple harmonic oscillators using a string, a spring, and one or two objects of known masses. Being a great physicist that you are, you know that the oscillations will be sinusoidal, i.e. $y(t) = A\sin(\omega t + \varphi)$.
   A. Draw a picture of each SHO.
   B. Make a list of all relevant physical quantities for each SHO separately.
   C. Use dimensional analysis to obtain an expression for the angular velocity ω of each SHO. Explain your reasoning and show all steps.
   D. Explain mathematicaly how to use each SHO to measure time.
2. You are on planet μεσυτ where the locals eliminated all the irrational absolute statements. You obtained its mass M_{ς} by observing its motion around its star (somehow) and calculated its radius to be R_{ς}.
   A. Obtain the gravitational acceleration g_{ς} on the surface of the planet in terms of the given quantities and known physical constants.
   B. You see a spring on the ground and, being the nerd you are, decide to obtain its spring constant using the mass M and the ruler of length ℓ in your pocket. Draw a free body diagram that shows how you would attempt to measure the spring constant.
   C. Calculate the spring constant k of the spring in terms of known and calculated quantities.
3.
   A. Obtain the gravitational field of earth on its surface in terms of G, M_e, R_e.
   B. Obtain a numerical value for g given that the radius of Earth is $6.4\times10^{6}m$ and its mass is $6.0\times10^{24}kg$, and $G = 6.7\times10^{-11}N\frac{m^2}{kg^2}$.
4. You are given two spings with spring constants k_1 and k_2. Obtain the effective spring constant k_{eff} when the springs are connected
   A. in parallel (side by side)
   B. in series (end-to-end)
   Hint: Assume they are connected to a wall at one end, to a mass at the other end. Use the force applied on the mass and the wall with the action reaction principles.
5. Consider a satellite of mass m revolving on a circular orbit of radius r around a planet of mass M where M>>m. The speed of the satellite in this orbit is v.
   A. Make a list of all the given quantities and the relevant physical constants.
   B. What is the magnitude of the centripetal force needed to keep the satellite in this orbit.
   C. What is the direction of this acceleration?
   D. What is the gravitational force exerted on this satellite by the planet.
   E. What is the direction of the gravitational force exerted on this satellite by the planet?
   F. Obtain an expression for the speed of the satellite in terms of G, M, r.
   G. Write an expression for v in terms of the circumference of the orbit and the period T of the motion.
   H. Use your answers above to obtain a relation between r and T in terms of G and M.
6. Gravitons and gravitational field due to mass M at a distance r. Imagine yourself counting gravitons emitted by a mass M. Assume that g represents the number of gravitons per unit area (graviton flux).
   A. What kind of surface would you use to count all the gravitons emitted by the mass M? Write an expression for this area in terms of the distance from the center of the mass M.
   B. What would be the total number of gravitons flowing through the area you have in part A? Write this in terms of g, r, and any relevant constant.
   C. You are told that the answer you have in part B is proportional to M of the mass and that the proportionality constant is 4πG. Write an expression in terms of g, r, M, 4πG, and any other constants.
   D. Obtain g in terms of r, M, G.
   E. Obtain the gravitational force on a mass m in the gravitational field g you obtained above.
   F. Now you are inside the planet at a distance r<R. What ratio of the mass is going to contribute to the gravitational field ? Call this mass m.
   G. What kind of surface would you use to count all the gravitons emitted by the mass m? Write an expression for this area in terms of the distance from the center of the mass.
   H. What would the total number of gravitons be flowing through the area you have? Write this in terms of g, r, m and any relevant constant.
   I. You are told that the answer you have above is proportional to m of the mass and that the proportionality constant is 4πG. Obtain an expression in terms of g, r, m, 4πG, and any other constants.
   J. Obtain g in terms of r, m, G.
   K. Obtain the gravitational force on a mass m in the gravitational field g you obtained above.
7. On planet ÇακιρμβM of radius R and mass M distributed uniformly, the locals (with egos larger than the planet itself) travel from one pole to another via a frictionless hole through the center of the planet.
   A. Use Gauss' law to show that the force on an inhabitant of mass m inside the planet is given by $\vec{F} = -G\frac{Mm}{R^3}\vec{r}$ [see the preceding problem].
   B. Use the answer in part A to write the Newton's second law for the inhabitant.
   C. Use the fact that $\vec{a} = \frac{d^2\vec{r}}{dt^2}$ and $\vec{r} = \vec{r}_o\cos\omega t$ to show that the inhabitant will experience SHM of angular frequency $\omega = \sqrt{\frac{GM}{R^3}}$
   Use $\frac{d}{dt}\sin\omega t = \omega\cos\omega t,\ \frac{d}{dt}\cos\omega t = -\omega\sin\omega t$

8. What is the magnitude of gravitational force on a satellite in orbit at an altitude of α times the radius of the Earth in terms of G, M_E, R_E, α?
9. How does the mass of a satellite affect the acceleration that it experiences due to the Earth's gravitational force?
10. The magnitude of the gravitational force between the Earth and a satellite depends on what?
11. How does the size of the Moon's gravitational attraction for the Earth compare with the size of the Earth's gravitational attraction for the Moon?
12. Consider a binary star system, a pair of stars bound together by the gravitational attraction between their masses. What does this force depend on?
13. Two members of the opposite sex, of mass m_1 and m_2 respectively, see each other across a room at a party. They are instantly attracted to each other. If the distance between them at this time was r, what was the magnitude of the gravitational force of attraction between them?
14. According to the law of universal gravitation, your mass and the mass of your pen should attract each other. However, if you let go of your pen, it falls down instead of flying toward you. This can be best explained by noting what?
15. What would be the gravitational force upon a test mass placed at the center of the Earth in terms of G, M_E, R_E m [the test mass]?
16. You are an assistant science officer on board the starship Enterprise. Since Spock is incapacitated again, you are called upon to interpret sensor readings that show a planet to have a mass that is α times the Earth mass and a radius that is β times as large as the Earth radius. At its surface, this planet could be expected to have an acceleration due to gravity that is
17. What supplies the centripetal acceleration for a satellite in a circular orbit around the Earth?

Three masses are arranged as shown in the figure below. Assume the masses are point-like masses.



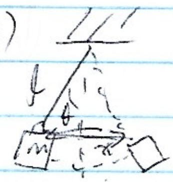
18. What are the x- and y-components of the gravitational *field* due to (a) m_1 and (b) m_2 at the location of m?
19. What are the x- and y-components of the gravitational *force* of (a) m_1 and (b) m_2 on m?
20. What are the x- and y-components and the direction of the *total* gravitational *field* at the location of m
21. What are the x- and y-components and the direction of the *total* gravitational *force* on m?
22. A mass m is distributed uniformly in the shape of a circle of radius R. Find the x, y and z-components of the gravitational field established by the ring along the symmetry axis of the ring at a distance z from the plane of the ring. What is the gravitational field at the center of the ring?
23. A mass m is free to move along a horizontal surface. A spring with a spring constant of k has its left-end fixed and the right-end attached to the mass.
    A. What force must you exert on the mass so that the spring is stretched a vertical distance x? Include magnitude and direction.
    B. When you hold the mass so the spring is stretched x, what force does gravity exert upon the mass?
    C. Explain whether (or not) the forces described in parts A and B represent a force pair as described by Newton's third law.
24. Given the expression $x(t) = A\sin(\omega t + \varphi)$ for x as a function of time, relate each of the following terms to this expression: amplitude, angular frequency, Period T, phase angle, the velocity as a function of time, the acceleration as a function of time
25. A mass m is suspended from a spring with spring constant k. If the mass is pulled down A distace away from its equilibrium point and then released from rest, assuming simple harmonic motion:
    A. How high will it rise from the point of release before stopping?
    B. How long will this trip to the highest point take?
    C. What will be its maximum speed?
    D. How soon after its release will it attain this speed?
26. An object of mass m is on a horizontal surface where the coefficients of friction are μ_s and μ_k. A spring with a spring constant k is connected to it to move it along the horizontal surface. Give your answers to the questions below in terms of the given quantities and the known physical constants.
    A. Minimum how much does the spring have to stretch (x_o) to set the object almost in motion?
    B. What will be the acceleration of the object when the spring is stretched by an amount $x > x_o$?

black ~~44~~ 44
blue ~~28~~ 28
red 6
total 78

Ch. 7

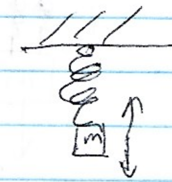
FRQ

1) A) 1)



+1 ✓

2)



+1 ✓

B) 1) g, l, m, θ, x ~ 2) g, k, m, A ~

c) $x(t) = x\sin(\omega t + \phi)$ ~ $y(t) = A\sin(\omega t + \phi)$

$\phi \Rightarrow$ in radians

$\omega t \rightarrow$ must also be in radians

$t \rightarrow s$ ~ ~

$\boxed{\omega = \frac{rad}{s}}$ $\frac{rad}{s} \cdot s = rad$

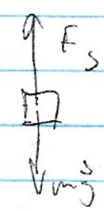
D) $x_f = x_{max}\sin(\omega t + \phi)$ $y(t) = A\sin(\omega t + \phi)$

$\omega t + \phi = \sin^{-1}\left(\frac{x_f}{x_{max}}\right)$ $\omega t + \phi = \sin^{-1}\left(\frac{y}{A}\right)$

$\boxed{t = \frac{\sin^{-1}\left(\frac{y}{A}\right) - \phi}{\omega}}$ ~~~

~~ker~~ $\boxed{t = \frac{\sin^{-1}\left(\frac{x_f}{x_{max}}\right) - \phi}{\omega}}$ ~

2) A) $g = \frac{GM}{R^2}$ $\boxed{g_s = \frac{GM_s}{R_s^2}}$ ✓ +1

B)  $= ma = 0$ +1

C) $F_s\uparrow + mg\downarrow = 0$

$F_s\uparrow = mg\uparrow$

$kx = mg$ ✓

$\boxed{k = \frac{m}{x}g}$ +1

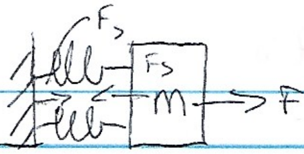
3) A) $\boxed{g_E = \frac{GM_E}{R_E^2}}$ ✓ +1

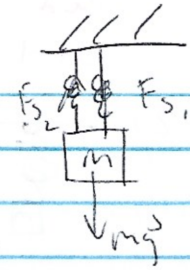
B) $g_E = \frac{(6.7\times10^{-11}\,N\frac{m^2}{kg^2})(6.0\times10^{24}\,kg)}{(6.4\times10^{6}\,m)^2}$

$= \boxed{9.81\,m/s^2}$ ✓ +1

7

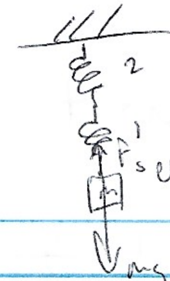
4) A)

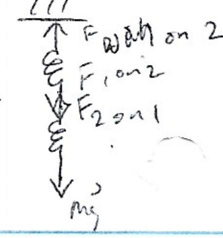




+1

B)





$F_{s_1}\uparrow + F_{s_2}\uparrow + mg\downarrow = ma = 0$

$F_{s\,total}\uparrow + mg\downarrow = 0$

$k_1 x\uparrow + k_2 x\uparrow = mg\uparrow$ +1

$k_{eff}\, x = mg$

$(k_1 + k_2)\, x = mg$

$k_{eff} = \dfrac{mg}{x}$

$(k_1 + k_2) = \dfrac{mg}{x}$

$\boxed{(k_{eff}) = k_1 + k_2}$ parallel ✓ +1

B) $F_{s\,total}\uparrow + mg\downarrow = 0$

$k_{eff}\, x = mg$

$\cancel{k_{eff}} = \dfrac{mg}{x_{total}}$

$F_{wall\ on\ 2}\uparrow + F_{1 on 2}\downarrow + F_{2 on 1}\uparrow + mg\downarrow = 0$

$F_{wall\ on\ 2}\uparrow + k_1 x_1\downarrow + k_2 x_2\uparrow + mg\downarrow = 0$

$k_{eff}\, x_{total}\uparrow + k_1 x_1\downarrow + k_2 x_2\uparrow + mg\downarrow = 0$

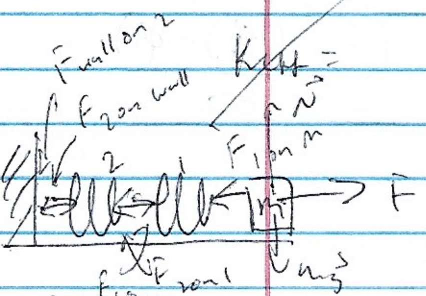
$k_{eff}\, x_{total}\uparrow + k_2 x_2\uparrow = mg\uparrow + k_1 x_1\uparrow$

$k_{eff}\, x_{total}\uparrow = mg\uparrow + k_1 x_1\uparrow - k_2 x_2\uparrow$

$[x_1 + x_2 = x_{total}]$

$k_{eff} = \dfrac{mg}{x_{total}} + \dfrac{k_1 x_1}{x_1 + x_2} - \dfrac{k_2 x_2}{x_1 + x_2}$

$k_{eff} =$



$\Sigma F = ma$

$F\rightarrow + F_{1 on m}\leftarrow + F_{1 on 2}\rightarrow$

$+ F_{2 on 1}\leftarrow + F_{2 on wall}\rightarrow + F_{wall\ on\ 2}\leftarrow = 0$

$F = -F_{wall\ on\ 2}$

$F\rightarrow + k_1 x_1\leftarrow + k_1 x_1\rightarrow + k_2 x_2\leftarrow + k_2 x_2\rightarrow + F_{wall\ on\ 2}\leftarrow = 0$

~

~

~

$\boxed{4B}$

3

5) A) $M, m, r, G,$ ~

B) $F_c = F_g = \boxed{\frac{GMm}{r^2}}$ ~

c) towards the center of planet ✓ +1

d) $\boxed{F_g = \frac{GMm}{r^2}}$ ✓ +1 e) ✓ +1

f) $\frac{mv^2}{r} = \frac{GMm}{r^2}$

$v^2 = \frac{GM}{r}$ $\boxed{v = \sqrt{\frac{GM}{r}}}$ ✓ +1

G) $C = 2\pi r$

$\Delta x = \bar{v}t$

$2\pi r = vT \rightarrow \boxed{v = \frac{2\pi r}{T}}$ ✓ +1

H) $v^2 = \frac{GM}{r}$

$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$ $\boxed{\frac{r^3}{T^2} = \frac{GM}{4\pi^2}}$ ✓ +1

6) A) sphere - $A = 4\pi R^2$ ✓ +1 e) $F_g = mg = \boxed{\frac{GMm}{R^2}}$ +1

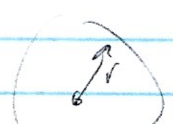
B) $g \cdot A = 4\pi g R^2$ ✓ +1

c) $4\pi g R^2 = 4\pi GM$ ✓ +1 f) $V_M = \frac{4}{3}\pi R^3$ & $V_m = \frac{4}{3}\pi r^3$ & $\frac{V_m}{V_M} = \frac{r^3}{R^3} \rightarrow \frac{mr^3}{R^3}$ ratio of mass +1 ✓

d) $\boxed{g = \frac{GM}{R^2}}$ ~ vector

G) sphere - $A = 4\pi r^2$ ✓ +1 H) $gA = 4\pi g r^2$ ✓ +1 I) $4\pi g r^2 = 4\pi \frac{Gmr^3}{R^3}$ +1

J) $g = \frac{Gmr^3}{R^3 r^2} = \frac{Gmr}{R^3}$ ✓ +1 sign K) $F_g = mg = \boxed{\frac{GMm_2 r}{R^3}}$ ✓ sign +1



7) A) $g = \frac{GM}{r^2} \rightarrow m = \frac{mr^3}{R^3}$ B) $\sum F = ma$

$g = \frac{Gm\vec{r}^3}{R^3 r^2} = \frac{Gm\vec{r}}{R^3}$ $-\frac{GMm\vec{r}}{R^3} = ma$ ✓ +1

$F_g = Mg = \frac{GMmr}{R^3} \cdot -\hat{r} = \boxed{-\frac{GMm\vec{r}}{R^3}}$ ✓ +1

18

c) $\vec{r}_R = \vec{r}_0 \cos\omega t$

$v = -r\omega\sin\omega t$

$a = -r\omega^2\cos\omega t$ $\quad a = g$

$g = -r\omega^2 \cancel{\cos\omega t}$

$-\frac{GM\vec{r}}{R^3} = -r\omega^2 \cancel{\cos\omega t}$ ~

$\omega^2 = \frac{GM}{R^3}$ $\quad \boxed{\omega = \sqrt{\frac{GM}{R^3}}}$ ~ ~

8) $F_g = \frac{GM_E m}{r^2}$

$r = (\alpha+1)R_E$

$F_g = \frac{GM_E m}{((\alpha+1)R_E)^2}$ ✓ +1

9) It doesn't since $g = \frac{GM}{r^2}$ ← source ✓ +1

10) M, m, r ✓ +1

11) equal + opposite ✓ +1

12) m_1, m_2, r ✓ +1

13) $\boxed{F_g = \frac{Gm_1m_2}{r^2}}$ ✓ +1

14) Mass of the earth is much larger than both you or [illegible] ✓ +1

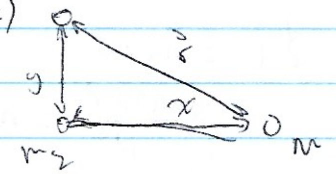
15) 0 since all the mass is evenly around it ✓ +1

16) $g_E = \frac{GM_E}{R_E^2}$ $\quad g_2 = \frac{G(\alpha M_E)}{(\beta R_E)^2} = \boxed{\frac{\alpha}{\beta^2} g_E}$ ✓ +1

17) Force of gravity ✓ +1

|18-22|

18) a)



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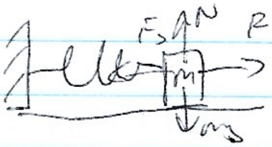
23) 

A) $\Sigma F = ma = 0$

$\vec{F} \rightarrow + F_s \leftarrow = 0$

$F \rightarrow = F_s \rightarrow$

$\boxed{F = kx \rightarrow}$ ✓ +1

B) $\boxed{F_g = mg \downarrow}$, independent of F_s ? ~

C) They don't since the forces ~~are~~ ~~[illegible]~~ are orthogonal so they are independent of each other.) ~

24) $x(t) = A\sin(\omega t + \phi)$ ✓ +1

$A = \dfrac{x(t)}{\sin(\omega t + \phi)}$

~~[illegible]~~

$\omega t + \phi = \sin^{-1}\left(\dfrac{x(t)}{A}\right)$ $\phi = \sin^{-1}\left(\dfrac{x(t)}{A}\right) - \omega t$

$\omega = \dfrac{\sin^{-1}\left(\frac{x(t)}{A}\right) - \phi}{t}$

$T = \frac{1}{f}$

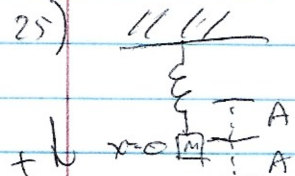
$\omega = 2\pi f$ $f = \dfrac{\omega}{2\pi}$

$T = \dfrac{2\pi}{\omega}$? $\omega = \dfrac{2\pi}{T}$ $\dfrac{2\pi}{T} = \dfrac{\sin^{-1}\left(\frac{x(t)}{A}\right) - \phi}{t}$

$T = \dfrac{2\pi t}{\sin^{-1}\left(\frac{x(t)}{A}\right) - \phi}$

$v(t) = x'(t) = A\omega\cos(\omega t + \phi)$ ✓ +1 ~

$a(t) = v'(t) = -A\omega^2\sin(\omega t + \phi)$ ✓ +1

25) 

A) $y(t) = A\sin(\omega t + \phi)$

at max length, $\sin(\omega t + \phi) = +1$

at max compression, $\sin(\omega t + \phi) = -1$

distance $= A$ ✗ ~

b) $-A = A\sin(\omega t + \phi)$, $\phi = 0$

$-1 = \sin(\omega t)$

$\omega t = \sin^{-1}(-1)$

$\omega t = -\frac{\pi}{2}$ ~

$\boxed{t = -\dfrac{\pi}{2\omega}}$ ~

c) $v(t) = A\omega\cos(\omega t + \phi)$, $\phi = 0$

v_{max} occurs when $\cos(\omega t + \phi) = 1$

$\cos(\omega t) = 1$

~~[illegible]~~

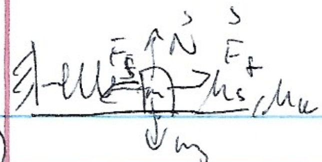
$v(t) = A\omega(\cos(\omega t) = 1)$

$\boxed{v(t) = A\omega}$ close ~

d) $\cos(\omega t) = 1$

$\omega t = \cancel{0}, \pi$ $\boxed{t = \dfrac{\pi}{\omega}}$ ~

4



2(a)

A) $F_s \leftarrow + N\uparrow + mg\downarrow + F_f \rightarrow = 0$ $\quad$ $N\uparrow + mg\downarrow = 0$

$kx_0 \leftarrow + \mu_s mg \rightarrow = 0$ $\quad$ $N = mg$

$kx_0 = \mu_s mg$

$$\boxed{x_0 = \frac{\mu_s mg}{k}}$$ ✓ +1

B) $\Sigma F = ma$

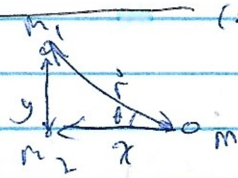
$F_s \leftarrow + F_f \rightarrow = ma$

$kx \leftarrow + \mu_k mg \vec{\rightarrow} = ma \leftarrow$ +1

$m\overleftarrow{a} = kx \leftarrow - \mu_k mg \leftarrow$

$$\boxed{a = \frac{kx}{m} - \mu_k g}$$ ~ 2

$\cos\theta = \frac{x}{r} = \frac{-\hat{i}}{\hat{r}}$



18) a) $g_1 = \frac{Gm_1}{r^2}$ $\quad$ $r = \sqrt{x^2+y^2}$

$= \frac{Gm_1}{x^2+y^2}\left((-\hat{i}\cos\theta) + \hat{j}\sin\theta\right)$ +1

$= \frac{Gm_1}{x^2+y^2} - \hat{i}\frac{x}{\sqrt{x^2+y^2}} + \frac{Gm_1}{x^2+y^2}\frac{\hat{j}y}{\sqrt{x^2+y^2}}$

$\sin\theta = \frac{y}{r} = \frac{\hat{j}}{\hat{r}}$

$$= \boxed{\frac{Gm_1(-\vec{x}+\vec{y})}{(x^2+y^2)^{3/2}}} = \frac{Gm_1 x}{(x^2+y^2)^{3/2}} - \hat{i} + \frac{Gm_1 y}{(x^2+y^2)^{3/2}}\hat{j}$$ ✓ +1

b) $g_2 = \frac{Gm_2}{r^2} = \boxed{\frac{Gm_2}{x^2} - \hat{i}}$ ✓ +1

19) a) $F_g = mg$ $\quad$ $F_{g_1} = \frac{Gm_1 m(-\vec{x}+\vec{y})}{(x^2+y^2)^{3/2}}$ ✓ +1

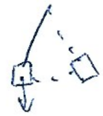
b) $F_g = mg = F_{g_2} = \boxed{\frac{Gm_2 m}{x^2} - \hat{i}}$ ✓ +1

20) $g_{total} = g_1 + g_2 = \left(\frac{Gm_1 x}{(x^2+y^2)^{3/2}} + \frac{Gm_2}{x^2}\right) - \hat{i} + \frac{Gm_1 y}{(x^2+y^2)^{3/2}}\hat{j}$ ✓ +1

21) $F_{total} = F_1 + F_2$

$= mg_{total} = \left(\frac{Gm_1 m x}{(x^2+y^2)^{3/2}} + \frac{Gm_2 m}{x^2}\right) - \hat{i} + \frac{Gm_1 m y}{(x^2+y^2)^{3/2}}\hat{j}$ ✓ +1

7



1) B) 1) g, l, m +1 2) g, k, m

$F_s = -kx$

c) $g\left[\frac{m}{s^2}\right]$ $l[m]$

+1 $\left[kg\frac{m}{s^2}\right] = [k][m]$

$g\left[\frac{m}{s^2}\right]$ $\hookrightarrow k\left[\frac{kg}{s^2}\right]$

$m[kg]$ $\omega\left[\frac{rad}{s}\right] \Rightarrow \left[\frac{1}{s}\right]$ +1

$m[kg]$ $\omega\left[\frac{1}{s}\right]$

$\omega\left[\frac{1}{s}\right]$ $\left[\frac{m}{s^2}\right]/[m]$

$\hookrightarrow \left[\frac{1}{s^2}\right] \Rightarrow \sqrt{\left[\frac{1}{s^2}\right]} = \left[\frac{1}{s}\right]$ +1

$\left[\frac{1}{s}\right] \rightarrow \left[\frac{kg}{s^2}\right]/[kg]$

$\hookrightarrow \left[\frac{1}{s^2}\right] \rightarrow \sqrt{\left[\frac{1}{s^2}\right]}$

$\downarrow$

$\left[\frac{1}{s}\right]$

$\omega = \sqrt{\frac{g}{l}}$ kg can't cancel out with anything so it's not included +1

$\omega = \sqrt{\frac{k}{m}}$

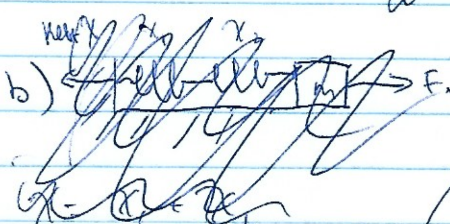
D)

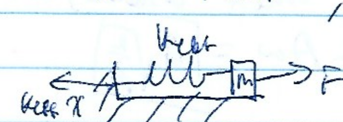
$T = \frac{1}{f}$ $\omega = 2\pi f$

$f = \frac{\omega}{2\pi}$

$T = \frac{2\pi}{\omega}$ → each back & forth, constant oscillators +1

4) b)

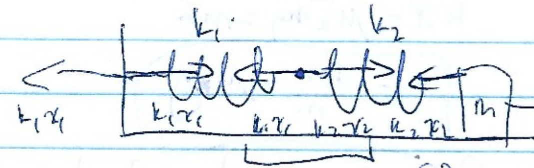




$\hookrightarrow x = x_1 + x_2$

+1

$F - k_{eff}x = 0$



$k_1x_1 = k_2x_2$

$k_{eff}x = k_1x_1$ $x_2 = \frac{k_1x_1}{k_2}$ +1

$k_{eff}(x_1 + x_2) = k_1x_1$ +1

$k_{eff}\left(x_1 + \frac{k_1x_1}{k_2}\right) = k_1x_1$ +1

$k_{eff}x_1\left(1 + \frac{k_1}{k_2}\right) = k_1x_1$

$k_{eff}\left(1 + \frac{k_1}{k_2}\right) = k_1$

$k_{eff}\left(\frac{k_2 + k_1}{k_2}\right) = k_1$ +1

$\boxed{k_{eff} = \frac{k_1k_2}{k_1 + k_2}}$

11

5) a) M, m, r, G, v +1

b) $F_c = m a_c = \boxed{\frac{mv^2}{r}}$ +1

6) d) ~~$g = \frac{GM}{R^2}$~~ $\boxed{g = -\frac{GM}{r^2}\hat{r}}$ +1

24) $x(t) = A\sin(\omega t + \phi)$

amplitude $= A$, angular frequency $= \omega$, $T = \frac{1}{f}$ $\omega = 2\pi f$

phase angle $= \phi$ $f = \frac{\omega}{2\pi}$

+1 $\boxed{T = \frac{2\pi}{\omega}}$

25) $\omega = \sqrt{\frac{k}{m}}$

A) distance = $2A$ +1 → equi pt

B) $t = \frac{T}{2}$ $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

$\boxed{t = \pi\sqrt{\frac{m}{k}}}$ +1

c) $v(t) = A\omega\cos(\omega t)$

v_{max} when $\cos(\omega t) = 1$

$v = A\omega = \boxed{A\sqrt{\frac{k}{m}}}$ +1

d) $t = \frac{1}{4}T = \boxed{\frac{\pi}{2}\sqrt{\frac{m}{k}}}$ +1

26) B) $\Sigma F = ma$

$kx - \mu_k mg = ma$

$\boxed{a = \frac{kx}{m} - \mu_k g}$ ~

8

~~4) b) $k_{eff} = \frac{k_1 x_1}{x_1 + x_2}$ +1~~

7) c) $\frac{d^2 r}{dt^2} = -\frac{GM_m}{R^3}\vec{r}$ $-\omega^2 r = -\frac{GM_m}{R^3}\vec{r}$

$\omega^2 = \frac{GM_m}{R^3}$

$\omega = \sqrt{\frac{GM_m}{R^3}}$ +1 +1

$\frac{4\pi^2}{T^2} = \frac{GM_m}{R^3}$ +1

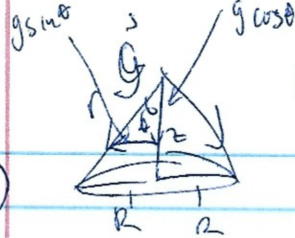
23) b) $mg = -kx$ +1

c) not action/reaction since they act on the same mass & obj. +1

26) b) $\Sigma F = kx_0 - \mu_k mg$

$= (\mu_s - \mu_k)mg$ +1

6

22) 

$\cos\theta = \frac{A}{H}$

$r^2 = z^2 + R^2$

$\vec{g} \neq \theta$

$g\sin\theta$'s cancel due to symmetry +1

$$g_z = g\cos\theta = \frac{Gm}{r^2} \cdot \frac{z}{r} = \frac{Gmz}{(z^2+R^2)^{3/2}} \quad +1$$

2

83, 81 (20, 2

1.

A.  [1pt]  [1pt]

B. Pendulum: m, g, ℓ [1pt] Spring: m, g, k [1pt]

C. Lets do a dimensional analysis of the relevant quantities:

$$\omega\left[\frac{1}{s}\right],\ m\,[kg],\ g\left[\frac{m}{s^2}\right],\ \ell\,[m],\ k\left[\frac{N}{m}\right]=\left[\frac{kg}{s^2}\right] \quad [1pt]$$

Pendulum: m, g, k. g has m and s in its units, and l has m.

$$\frac{g}{\ell}\left[\frac{\frac{m}{s^2}}{m}\right]=[s^{-2}]\text{. Square root gives } [s^{-1}]:\ \omega=\sqrt{\frac{g}{\ell}} \quad [1pt]$$

Spring: m, g, ℓ. k has kg and s, g has m and s, m has kg.

$$\frac{k}{m}\left[\frac{\frac{kg}{s^2}}{kg}\right]=[s^{-2}]\text{. Square root gives } [s^{-1}]:\ \omega=\sqrt{\frac{k}{m}} \quad [1pt]$$

D. Use the fact that $T=\frac{2\pi}{\omega}$. Each full oscillation measures one period. By counting the number of oscillations, you can measure time. This is how grandfather clocks work. For example, you can choose a combination of g & ℓ and m & k that gives $\omega = 2\pi$, thus $T = 1s$. [1pt]

2. You are on planet μεουτ where the locals eliminated all the irrational absolute statements. You obtained its mass $M_ç$ by observing its motion around its star (somehow) and calculated its radius to be $R_ç$.

A. $g_ç = G\frac{M_ç}{R_ç^2}$ [1pt]

B. $\uparrow kx$, $\downarrow mg$ [1pt]

C. $m\vec{g} - k\vec{x} = 0$, $k = \frac{m}{x}g$ [1pt]

3. 11

A. $g_e = G\frac{M_e}{R_e^2}$ [1pt] See Gauss' Law question for derivation.

B. $g_e = 6.7\times10^{-11} N\frac{m^2}{kg^2}\frac{6.0\times10^{24}kg}{(6.4\times10^6 m)^2}$, $g_e = 9.81\frac{m}{s^2}$

[1pt]

4. You are given two spings with spring constants k_1 and k_2. Obtain the effective spring constant k_{eff} when the springs are connected

A. Parallel (side by side): $\begin{matrix}-k_1\vec{x}\leftarrow \\ -k_2\vec{x}\leftarrow\end{matrix} \rightarrow \vec{F}$. [1pt]

$-k_1\vec{x} - k_2\vec{x} + \vec{F} = 0$ [1pt]

$F = |k_{eff}\vec{x}| = |-k_1\vec{x} - k_2\vec{x}| = |k_1 + k_2|\vec{x}$ [1pt]

$k_{eff} = k_1 + k_2$ [1pt]

B. Series (end to end):

x: total stretch, x_1 stretch of 1, x_2 stretch of 2.

$x = x_1 + x_2$ [1pt] Say #1 is connected to the wall:

Net force: $k_{eff}x = k_{eff}(x_1 + x_2) = k_2x_2$ [1pt]

$k_{eff} = \frac{k_1x_1}{x_1 + x_2}$ [1pt]

9

Use action-reaction at the connection point: $k_1x_1 = k_2x_2$ [1pt]

$x_2 = \frac{k_1x_1}{k_2}$. Substitute this in the eqn above

$$k_{eff} = \frac{k_1x_1}{x_1 + \frac{k_1x_1}{k_2}} = \frac{k_1k_2}{k_1+k_2}\ [1pt] \Rightarrow k_{eff} = \frac{k_1k_2}{k_1+k_2}$$

5.

A. M, m, r, g, v [1pt]

B. $F_C = ma_c = m\frac{v^2}{r}$ [1pt]

C. Toward M (radially in) [1pt]

D. $F_G = G\frac{Mm}{r^2}$ [1pt]

E. Toward M (radially in) [1pt]

F. $F_C = F_G$, $m\frac{v^2}{r} = G\frac{Mm}{r^2}$, $v^2 = \frac{GM}{r}$ [1pt]

G. $v = \frac{2\pi r}{T}$, $v^2 = \frac{4\pi^2r^2}{T^2}$ [1pt]

H. $\frac{4\pi^2r^2}{T^2} = \frac{GM}{r}$, $\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$ [1pt]

10

6.

A. Spherical symmetry--a sphere-like shape: $A = 4\pi r^2$ [1pt]

B. $C_GM = gA = g4\pi r^2$ [1pt]

C. $4\pi GM = gA = g4\pi r^2$ [1pt]

D. $g = \frac{4\pi GM}{4\pi r^2} = \frac{GM}{r^2}$, $\vec{g} = -\frac{GM}{r^3}\vec{r}$ [1pt]

E. $\vec{F}_G = m\vec{g} = -\frac{GMm}{r^3}\vec{r}$ [1pt]

F. $M_r = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3}M = \frac{r^3}{R^3}M$, $r < R$ [1pt]

G. $A_r = 4\pi r^2$ [1pt]

H. $C_GM_r = gA = g4\pi r^2$ [1pt]

I. $4\pi GM_r = 4\pi G\frac{r^3}{R^3}M = gA = g4\pi r^2$ [1pt]

J. $g = \frac{4\pi Gr^3}{4\pi r^2R^3}M = \frac{GM}{R^3}r$, $\vec{g} = -\frac{GM}{R^3}\vec{r} = -\frac{GM}{R^2}\hat{r}$, [1pt] 10

$\vec{g} = 0$ at $\vec{r} = 0$ [1pt]

K. $\vec{F}_G = m\vec{g} = -\frac{GMm}{R^3}\vec{r}$. Notice that $\vec{F}_G = 0$ at $\vec{r} = 0$ [1pt]

7.

A. See 6.K above. [1pt]

B. $\vec{F}_G = m\vec{a}$, $-\frac{GMm}{R^3}\vec{r} = m\vec{a}$, R is constant (r of planet) [1pt]

C. $\frac{d^2\vec{r}}{dt^2} = -\frac{GMm}{R^3}\vec{r}$, $-\omega^2\vec{r} = -\frac{GMm}{R^3}\vec{r}$, $\omega^2 = \frac{GMm}{R^3}$ [1pt]

$\omega = \sqrt{\frac{GMm}{R^3}}$. Remember $\omega = \frac{2\pi}{T}$ which gives

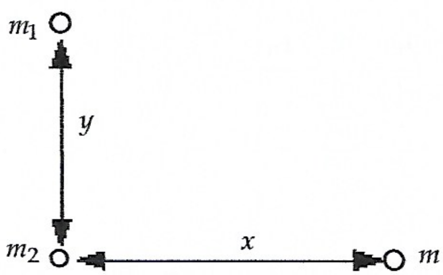
[1pt]

6

$$\frac{4\pi^2}{T^2}=\frac{GMm}{R^3}$$ which is Keppler's 3rd law. *[1pt]*

8. $F=G\frac{M_E m}{r^2}=G\frac{M_E m}{(1+\alpha)^2 R_E^2}=\frac{1}{(1+\alpha)^2}\frac{GM_E m}{R_E^2}$ *[1pt]*
9. Since $a=\frac{F}{m}=\frac{GM}{r^2}$. M satellite does not affect its a. *[1pt]*
10. M, m, r *[1pt]*
11. The same magnitude, opposite direction. Action-Reaction. *[1pt]*
12. Masses and r. *[1pt]*
13. $F=G\frac{m_1 m_2}{r^2}$ *[1pt]*
14. The mass of the Earth infinitely larger than your mass. *[1pt]*
15. 0 *[1pt]*
16. $g=\frac{G(\alpha M_E)}{(\beta R_E)^2}=\frac{\alpha}{\beta^2}\frac{GM_E}{R_E}=\frac{\alpha}{\beta^2}g_E$ *[1pt]*
17. Gravitational attraction between the satellite and the Earth. *[1pt]*

11



18. (a) $\vec{g}_1=\frac{Gm_1}{x^2+y^2}(-\vec{i}\cos\theta,\vec{j}\sin\theta)$ *[1pt]*

$$\vec{g}_1=\frac{Gm_1}{\left(x^2+y^2\right)^{3/2}}\left[-\vec{i}x+\vec{j}y\right]$$ *[1pt]*

(b) $\vec{g}_2=G\frac{m_2}{x^2}(-\vec{i})$ *[1pt]*

19. (a) $\vec{F}_1=\frac{Gm_1 m}{\left(x^2+y^2\right)^{3/2}}\left[-\vec{i}x+\vec{j}y\right]$ *[1pt]*

(b) $\vec{F}_2=G\frac{m_2 m}{x^2}(-\vec{i})$ *[1pt]*

20. $\vec{g}=\vec{g}_1+\vec{g}_2=-\vec{i}\left[\frac{Gm_1 x}{\left(x^2+y^2\right)^{3/2}}+G\frac{m_2}{x^2}\right]+\vec{j}\left[\frac{Gm_1 y}{\left(x^2+y^2\right)^{3/2}}\right]$

[1pt]

$$\tan\theta=\frac{g_y}{g_x}=\frac{-m_1 yx^2}{m_1x^3+m_2\left(x^2+y^2\right)^{3/2}}$$ ~~*[1pt]*~~

21. $\vec{F}_1+\vec{F}_2=-\vec{i}\left[\frac{Gm_1 mx}{\left(x^2+y^2\right)^{3/2}}+G\frac{m_2 m}{x^2}\right]+\vec{j}\left[\frac{Gm_1 my}{\left(x^2+y^2\right)^{3/2}}\right]$

[1pt]

$$\tan\theta=\frac{-m_1 yx^2}{m_1x^3+m_2\left(x^2+y^2\right)^{3/2}}$$ ~~*[1pt]*~~

X 7

22. All the mass is at a distance $\sqrt{R^2+z^2}$ along the z-axis. Only the z-component survives due to symmetry; everything in the x-y plane cancels in pairs, i.e. *[1pt]*

$$g\cos\theta=\frac{Gm}{r^2}\frac{z}{r}=\frac{Gmz}{\left(R^2+z^2\right)^{3/2}}$$ along -z. *[1pt]*

At the center, $\theta=\frac{\pi}{2}$ and g=0.

23.
   A. $-k\vec{x}$ *[1pt]*
   B. $m\vec{g}=-k\vec{x}$ *[1pt]*
   C. They are not action-reaction forces since they both act on the the same mass and the same object. *[1pt]*

24. $x(t)=A\sin(\omega t+\varphi)$ displacement at time t *[1pt]*

$v(t)=\omega A\cos(t+\varphi)$ velocity at time t *[1pt]*

$a(t)=-\omega^2 A\sin(t+\varphi)$ acceleration at time t *[1pt]*

A amplitude, ω angular frequency, $T=\frac{1}{\omega}$, φ phase angle *[1pt]*

9

25. A mass m is suspended from a spring with spring constant k. If the mass is pulled down A distace away from its equilibrium point and then released from rest, assuming simple harmonic motion:
   A. $2A$ *[1pt]*
   B. $\frac{T}{2}=\pi\sqrt{\frac{m}{k}}$ *[1pt]*
   C. $A\sqrt{\frac{k}{m}}$ *[1pt]*
   D. $\frac{T}{4}=\frac{\pi}{2}\sqrt{\frac{m}{k}}$ *[1pt]*

4

26. An object of mass m is on a horizontal surface where the coefficients of friction are μ_s and μ_k. A spring with a spring constant k is connected to it to move it along the horizontal surface. Give your answers to the questions below in terms of the given quantities and the known physical constants.
   A. $F_{spring}=F_{friction}$, $kx_o=\mu_s mg$ *[1pt]* $x_o=\frac{\mu_s mg}{k}$ *[1pt]*
   B. $F_{net}=F_{spring}-F_{friction}=kx-\mu_k mg$. *[1pt]*

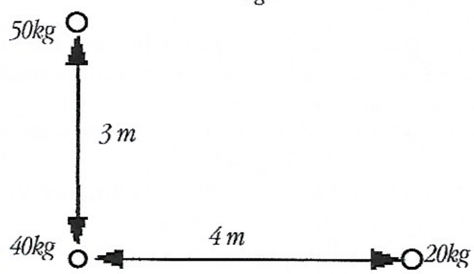
   If it barely starts moving

   $F_{net}=kx_o-\mu_k mg=(\mu_s-\mu_k)mg$ *[1pt]*

4

USE $G=10^{-10}\ \frac{m^2N}{kg^2}$ **FORM B**

Three masses are arranged as shown in the figure below. Assume these are point-like masses. **Use** $G=10^{-10}\ \frac{m^2N}{kg^2}$



1. What are the x- and y-components of the gravitational *field* (g_x, g_y) due to the 40 kg mass at the *location* of the 20 kg mass?
   A. $(0, 2.5\times10^{-10}\ \frac{m}{s^2})$ B. $(5\times10^{-10}\ \frac{m}{s^2}, 0)$
   C. $(0, 10\ \frac{m}{s^2})$ D. $(0, 40\times10^{-10}\ \frac{m}{s^2})$
   E. $(2.5\times10^{-10}\ \frac{m}{s^2}, 0)$
2. What are the x- and y-components of the gravitational *force* of the 40 kg mass on the 20 kg mass?
   A. $(0, 2.5\times10^{-10}N)$ B. $(4\times10^{-8}N, 0)$ C. $(10^{-9}N, 0)$
   D. $(50\times10^{-10}N, 0)$ E. $(200N, 200N)$
3. What are the x- and y-components of the gravitational *field* due to the 50 kg mass at the *location* of the 20 kg mass?
   A. $(0,\ 2\times10^{-10}\ \frac{m}{s^2})$ B. $(1.6\times10^{-10}\ \frac{m}{s^2}, 0)$
   C. $(1.6\times10^{-10}\ \frac{m}{s^2},\ 2.4\times10^{-9}\ \frac{m}{s^2})$
   D. $(1.6\times10^{-10}\ \frac{m}{s^2},\ 1.2\times10^{-10}\ \frac{m}{s^2})$ E. $(10\frac{m}{s^2}, 10\frac{m}{s^2})$
4. What are the x- and y-components of the gravitational *force* of the 50 kg mass on the 20 kg mass?
   A. $(0,\ 40\times10^{-10}N)$ B. $(32\times10^{-10}N, 0)$
   C. $(32\times10^{-10}N,\ 48\times10^{-10}N)$
   D. $(32\times10^{-10}N,\ 24\times10^{-10}N)$ E. $(200N, 200N)$
5. What are the x- and y-components of the *total* gravitational *field* at the location of the 20 kg mass?
   A. $(0,\ 1.2\times10^{-10}\ \frac{m}{s^2})$ B. $(1.6\times10^{-10}\ \frac{m}{s^2}, 0)$
   C. $(4.1\times10^{-10}\ \frac{m}{s^2}, 1.2\times10^{-10}\ \frac{m}{s^2})$
   D. $(1.6\times10^{-10}\ \frac{m}{s^2},\ 4.1\times10^{-10}\ \frac{m}{s^2})$ E. (0,0)
6. What is the direction of the *total* gravitational *field* at the location of the 20 kg mass with respect to the horizontal. (2 significant digits)
   A. 0° B. 16° C. 30° D. 45° E. 57°
7. What are the x- and y-components of the *total* gravitational *force* on the 20 kg mass?
   A. $(24\times10^{-10}N, 24\times10^{-10}N)$
   B. $(82\times10^{-10}N, 82\times10^{-10}N)$
   C. $(82\times10^{-10}N, 0)$
   D. $(0, 24\times10^{-10}N)$
   E. $(82\times10^{-10}N, 24\times10^{-10}N)$
8. What is the direction of the total gravitational *force* on the 20 kg mass with respect to the horizontal? (2 significant digits)
   A. 0° B. 16° C. 30° D. 45° E. 57°

A mass m is distributed uniformly in the shape of a circle of radius R.

9. Obtain the gravitational field established by the ring along the symmetry axis of the ring at a vertical distance z from the plane of the ring.
   A. $G\frac{m}{z^2}$ B. $\frac{Gm}{R^2}$ C. $\frac{Gm}{(z^2+R^2)}$
   D. $\frac{Gmz}{(z^2+R^2)^{3/2}}$ E. 0
10. What is the gravitational field at the center of the ring?
    A. $G\frac{m}{z^2}$ B. $\frac{Gm}{R^2}$ C. $\frac{Gm}{(z^2+R^2)}$
    D. $\frac{Gmz}{(z^2+R^2)^{3/2}}$ E. 0
11. A 2.0 kg mass is free to move along a horizontal surface. A spring with a spring constant of $k=30\frac{N}{m}$ has its left-end fixed and the right-end attached to the mass. What force must you exert on the mass so that the spring is stretched 0.40 m?
    A. 30 N B. 20 N C. 12N D. 4N E. 2N
12. When you hold the mass so the spring is stretched 0.40 m, what force do you exert upon the mass?
    A. 30 N B. 20 N C. 12N D. 4N E. 2N
13. Are the forces described above represent a force pair as described by Newton's third law.
    A. Yes, b/c they are equal and in opposite direction
    B. Yes, b/c they are acting on the same system
    C. No, b/c they are two different forces
    D. No, b/c they are in the same direction
    E. No, b/c they are acting on the same system

Given the expression $x(t)=A\sin(\omega t+\varphi)$ for x as a function of time, relate each of the following terms to this expression:

14. amplitude
    A. A B. ω C. φ D. 1/ω E. 2π/ω
15. angular frequency
    A. A B. ω C. φ D. 1/ω E. 2π/ω
16. Period T
    A. A B. ω C. φ D. 1/ω E. 2π/ω
17. phase angle
    A. A B. ω C. φ D. 1/ω E. 2π/ω
18. the velocity as a function of time
    A. $A\sin(\omega t+\varphi)$ B. $\omega A\sin(\omega t+\varphi)$
    C. $A\cos(\omega t+\varphi)$ D. $\omega A\cos(\omega t+\varphi)$
    E. $-\omega^2 A\sin(\omega t+\varphi)$
19. the acceleration as a function of time
    A. $A\sin(\omega t+\varphi)$ B. $\omega A\sin(\omega t+\varphi)$
    C. $A\cos(\omega t+\varphi)$ D. $\omega A\cos(\omega t+\varphi)$
    E. $-\omega^2 A\sin(\omega t+\varphi)$

USE $G=10^{-10}\ \frac{m^2N}{kg^2}$

FORM B

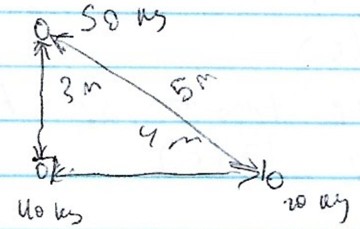
20. A mass m is suspended from a spring with spring constant k. If the mass is pulled down a distance A away from its equilibrium point and then released from rest, assuming simple harmonic motion: How high will it rise from the point of release before stopping?
    A. 0 B. $A/2$ C. A D. $2A$ E. $3A$
21. How long will this trip to the highest point take?
    A. $2\pi\sqrt{\frac{m}{k}}$ B. $\pi\sqrt{\frac{m}{k}}$ C. $2\pi\sqrt{\frac{k}{m}}$ D. $\pi\sqrt{\frac{k}{m}}$ E. $\sqrt{\frac{m}{k}}$
22. What will be its maximum speed?
    A. $A\sqrt{\frac{m}{k}}$ B. $A\sqrt{\frac{m}{k}}$ C. $A\sqrt{\frac{k}{m}}$ D. $A\sqrt{\frac{k}{m}}$ E. $\sqrt{\frac{m}{k}}$
23. How soon after its release will it attain this speed?
    A. $\pi\sqrt{\frac{m}{k}}$ B. $\frac{\pi}{2}\sqrt{\frac{m}{k}}$ C. $\pi\sqrt{\frac{k}{m}}$ D. $\frac{\pi}{2}\sqrt{\frac{k}{m}}$ E. $\sqrt{\frac{m}{k}}$

An object of mass m is on a horizontal surface where the coefficients of friction are μ_s and μ_k. A spring with a spring constant k is connected to it to move it along the horizontal surface. Give your answers to the questions below in terms of the given quantities and the known physical constants.

24. Minimum how much does the spring have to stretch (x_o) to set the object almost in motion?
    A. $\mu_s\frac{mg}{k}$ B. $\mu_k\frac{mg}{k}$ C. $\frac{mg}{k}$ D. $\mu_k kmg$ E. $\mu_s kmg$
25. What will be the acceleration of the object when the spring is stretched by an amount $x > x_o$?
    A. $\frac{kx}{m}$ B. $\frac{k}{m}(x-x_o)$ C. $\frac{k}{m}x_o$
    D. $\frac{k}{m}x-\mu_s g$ E. $\frac{k}{m}x-\mu_k g$
26. A satellite in orbit at an altitude of 0.25 times the radius of the Earth will experience a gravitational force whose magnitude is::
    A. zero
    B. 0.062 times what it would be here on the Earth' surface .
    C. 0.64 times what it would be here on the Earth' surface ..
    D. 1.6 times what it would be here on the Earth' surface .
    E. 64
27. How does the mass of a satellite affect the acceleration that it experiences due to the Earth's gravitational force?
    A. It doesn't.
    B. Acceleration is directly proportional to satellite mass.
    C. Acceleration is inversely proportional to satellite mass.
    D. Acceleration is inversely proportional to the cube of the satellite mass.
    E. All of these.
28. The magnitude of the gravitational force between the Earth and a satellite is:
    A. Not affected by the size of the satellite mass.
    B. Directly proportional to the satellite mass.
    C. Inversely proportional to the satellite mass.
    D. Inversely proportional to the cube of the satellite mass.
    E. None of these.
29. How does the size of the Moon's gravitational attraction for the Earth compare with the size of the Earth's gravitational attraction for the Moon?
    A. They are the same.
    B. There is no gravitational force between the Earth and the Moon.
    C. The moon's gravitational attraction for the Earth is smaller because the Moon's mass is smaller.
    D. The Moon's gravitational attraction for the Earth is greater because the Earth's mass is greater
    E. The Moon does not have any gravitational attraction for the Earth; it is the Earth that attracts the Moon.
30. Consider a binary star system, a pair of stars bound together by the gravitational attraction between their masses. This force between them is:
    A. Independent of their masses.
    B. Directly proportional to the larger star's mass.
    C. Directly proportional to the sum of the stars' masses.
    D. Directly proportional to the product of the stars' masses.
    E. All of these.
31. Two members of the opposite sex, of mass 100 kg and 50 kg respectively, see each other across a room at a party. They are instantly attracted to each other. If the distance between them at this time was 10 m, what was the magnitude of the gravitational force of attraction between them.
    A. $5\times10^{-9}N$ B. $4\times10^{-9}N$ C. $8N$ D. 10 N E. $1\times10^{3}N$
32. According to the law of universal gravitation, your mass and the mass of your pen should attract each other. However, if you let go of your pen, it falls down instead of flying toward you. This can be best explained by noting that:
    A. The Sun's mass is greater than yours is.
    B. The Earth's mass is greater than yours is.
    C. You and your pen, and especially you, are not point masses.
    D. The Earth's gravitational force of attraction for the pen is greater than your gravitational attraction for it.
    E. be attracted toward you, especially if you wear a white shirt or blouse.
33. What would be the gravitational force upon a test mass placed at the center of the Earth? [m is the test mass]
    A. 0 B. mg C. $\frac{mg}{R_E^2}$ D. ∞ E. $\frac{M_E g}{R_E^2}$
34. You are an assistant science officer on board the starship Enterprise. Since Spock is incapacitated again, you are called upon to interpret sensor readings that show a planet to have a mass that is 4.0 times the Earth mass and a radius that is 3.0 times as large as the Earth radius. At its surface, this planet could be expected to have an acceleration due to gravity that is:
    A. 0.44 times that here on the surface of Earth.
    B. 0.75 times that here on the surface of Earth.
    C. 1.3 times that here on the surface of Earth.
    D. 2.3 times that here on the surface of Earth.
    E. 9 times that here on the surface of Earth.
35. What supplies the centripetal acceleration for a satellite in a circular orbit around the Earth?
    A. In a perfectly circular orbit, the satellite has constant velocity, zero acceleration, so the total force upon it is zero.
    B. Earth's gravitational force upon the satellite.
    C. The velocity with which the satellite was launched.
    D. No force is required since the satellite is weightless when in orbit.
    E. Atmospheric friction.

Ch. 7 John Yang

MC Form B

1) 

$$g = \frac{GM}{r^2} = \frac{\left(10^{-10} \frac{m^2N}{kg^2}\right)(40\,kg)}{(4m)^2} + \hat{i}$$

$$= 2.5 \times 10^{-9} \frac{m}{s^2}\ \hat{i}$$

(E)

2) $$F_g = G\frac{Mm}{r^2} = \frac{\left(10^{-10} \frac{Nm^2}{kg^2}\right)(40\,kg)(20\,kg)}{(4m)^2} - \hat{i}$$

$$= -5 \times 10^{-8}\ N\ \hat{i} \quad \text{(B)}$$

3) $$g_x = \frac{GM}{r_x^2} = \frac{\left(10^{-10} \frac{Nm^2}{kg^2}\right)(50\,kg)}{(4m)^2} = 3.13 \times 10^{-9} \frac{m}{s^2}$$

$$g_y = \frac{GM}{r_y^2} = \frac{\left(10^{-10} \frac{Nm^2}{kg^2}\right)(50\,kg)}{(3m)^2} = 5.56 \times 10^{-9} \frac{m}{s^2}$$

(C)

4) $$F_{g_x} = \frac{GMm}{r_x^2} = \frac{\left(10^{-10} \frac{Nm^2}{kg^2}\right)(50\,kg)(20\,kg)}{(4m)^2} = 6.3 \times 10^{-8} N$$

$$F_{g_y} = \frac{GMm}{r_y^2} = \frac{\left(10^{-10} \frac{Nm^2}{kg^2}\right)(50\,kg)(20\,kg)}{(3m)^2} = 1.1 \times 10^{-7} N \quad \text{(D)}$$

5) $$\cancel{g_{total} = \sqrt{(g_{x_1} + g_{x_2})^2 + g_y^2} = \sqrt{2.5 \times 10^{-9} \frac{m}{s^2} + 3}}$$

Other $$g_x = g_{x_1} + g_{x_2} = \left(2.5 \times 10^{-9} \frac{m}{s^2}\right) + \left(3.13 \times 10^{-9} \frac{m}{s^2}\right)$$

$$= 5.6 \times 10^{-9} \frac{m}{s^2}$$

$$g_y = g_{y_2} = 5.6 \times 10^{-9} \frac{m}{s^2}$$??

$\boxed{1-8}$

9) (D), but only b/c I remember from the other test

10) (E) → 0, all the mass is distributed about the ring

11) $F_s = kx = (30 \, N/m)(0.40 \, m) = 12 \, N$ (C)

12) $\Sigma F = ma = 0 \quad F = F_s = 12 \, N$ (C)

13) (C) → the external force is indep. from the spring force

14) (A) 15 (B) 16 (E) - from Erq 17 (C)

18 (D) 19 (E) from Erq

20) (C) 21 & $t = -\frac{\pi}{2\omega}$ $t = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}$

$\omega = \frac{2\pi}{T}$ $\omega = 2\pi T$

$\omega = 4\pi^2\sqrt{\frac{m}{k}}$

$\omega = \frac{2\pi}{2\pi\sqrt{\frac{m}{k}}} = \sqrt{\frac{k}{m}}$

24) (A)

25) (E)

26) $F_g = \frac{GMm}{(1.25R_E)^2} = 0.64 \, F_{g\,surface}$ (C)

27) (A)

28) (B) 30) (D) 31) $F_g = \frac{GMm}{r^2} = \frac{(10^{-10} \frac{Nm^2}{kg^2})(100 \, kg)(50 \, kg)}{100 \, m}$

$= 5 \times 10^{-8} \, N$ (A)

29) (A)

32) (A)

33) (A)

34) $F_g = \frac{\alpha}{\beta^2} g_E$

$= \frac{4}{9} = 0.44$ (A)

35) (B)

From FRQ

MC 1) $g_2 = \frac{Gm_2}{x^2}\,-\hat{i} = \frac{(10^{-10}\frac{Nm^2}{kg^2})(40kg)}{(4m)^2} = 2.5\times10^{-10}\,m/s^2\,-\hat{i}$ (E)

2) $F_{g_2} = \frac{Gm_2 m}{x^2}\,-\hat{i} = \frac{(10^{-10}\frac{Nm^2}{kg^2})(40kg)(20kg)}{(4m)^2} = 5.0\times10^{-9} = 50\times10^{-10}$ (D)

3) $g_1 = \frac{Gm_1 x}{(x^2+y^2)^{3/2}}\,-\hat{i} + \frac{Gm_1 y}{(x^2+y^2)^{3/2}}\,\hat{j}$

$= \frac{(10^{-10}\frac{Nm^2}{kg^2})(50kg)(4m)}{((4m)^2+(3m)^2)^{3/2}}\,-\hat{i} + \frac{(10^{-10}\frac{Nm^2}{kg^2})(50kg)(3m)}{((4m)^2+(3m)^2)^{3/2}}\,\hat{j}$

$= 1.6\times10^{-10}\,m/s^2\,\hat{i} + 1.2\times10^{-10}\,m/s^2\,\hat{j}$ (D)

4) $F_g = mg_1 = (80kg)(1.6\times10^{-10}\,m/s^2\,\hat{i} + 1.2\times10^{-10}\,m/s^2\,\hat{j})$
$= 3.2\times10^{-9}N\,\hat{i} + 2.4\times10^{-9}N\,\hat{j}$ (D)

5) $g_{total} = \left(\frac{Gm_1 x}{(x^2+y^2)^{3/2}} + \frac{Gm_2}{x^2}\right)-\hat{i} + \frac{Gm_1 y}{(x^2+y^2)^{3/2}}\,\hat{j}$

$= \left(\frac{(10^{-10}\frac{Nm^2}{kg^2})(50kg)(4m)}{((4m)^2+(3m)^2)^{3/2}} + \frac{(10^{-10}\frac{Nm^2}{kg^2})(40kg)\cancel{(4m)}}{(4m)^2\cancel{[(4m)^2+(3m)^2]^{3/2}}}\right)\hat{i} + \frac{(10^{-10}\frac{Nm^2}{kg^2})(50kg)(3m)}{((4m)^2+(3m)^2)}\,\hat{j}$

$= \cancel{2.56\times10^{-10}\,m/s^2}\,\hat{i} + 1.2\times10^{-10}\,m/s^2\,\hat{j}$ (C)

$4.1\times10^{-10}\,m/s^2\,\hat{i}$

6) $\theta = \tan^{-1}\left(\frac{g_y}{g_x}\right) = \cancel{\tan^{-1}\left(\frac{1.2\times10^{-10}\,m/s^2}{4.1\times10^{-10}\,m/s^2}\right)} = \cancel{2\times10}$

$= \cancel{\tan^{-1}\left(\frac{4.1}{1.2}\right)} \tan^{-1}\left(\frac{1.2}{4.1}\right) = 16^\circ$ (B)

7) $F_g = mg = (20kg)(4.1\times10^{-10}\,m/s^2\,\hat{i} + 1.2\times10^{-10}\,m/s^2\,\hat{j})$
$= 8.2\times10^{-9}\,N\,\hat{i} + 2.4\times10^{-9}\,N\,\hat{j}$ (E)

8) $\theta = \tan^{-1}\left(\frac{2.4}{8.2}\right) = 16^\circ$ (B)