Notes

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Dec 12

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Measurements and Uncertainties

1.1 Measurements in Physics

1.1.1 Physical Quantities and Units

Physical Quantities

All measurements are made of the seven basic quantities. The table shows their quantities and their units.

Quantity	SI Unit	Symbol
Time	second	s
Distance	meter	m
Mass	kilogram	kg
Current	ampere	A
Temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

Derived Quantities

These quantities are derived from and Physical Quantities. The table below show some common derived quantities.

Quantity	Symbol	Unit	SI Unit	Usual Symbol
Velocity	v	$m \cdot s^{-1}$		
Acceleration	a	$m \cdot s^{-2}$		
Force	F	$kg \cdot m \cdot s^{-2}$	Newton (N)	
Energy, Work	E, W	$kg \cdot m^2 \cdot s^{-2}$	Joule (J)	
Power	P	$kg \cdot m^2 \cdot s^{-3}$	Watt (W)	$J \cdot s^{-1}$
Momentum	p	$kg \cdot m^{-1} \cdot s^{-2}$		
Pressure	p	$kg \cdot m^{-1} \cdot s^{-2}$	Pascal (Pa)	$N \cdot m^{-2}$
Gravitation Field Strength	g	$m \cdot s^{-2}$		$N \cdot kg^{-1}$
Electric Resistance	R	$kg\cdot m^2\cdot s^{-3}\cdot A^{-2}$	Ohm (Ω)	$V \cdot A^{-1}$
Frequency	f	s^{-1}	herts (Hz)	

1.1.2 Physical Quantities and Calculations

When plugging in quantities into an equation, always be aware of unit conversion. A method to avoid calculation error with unit conversion is to treat the units as mathematical entities. This is called quantity calculus.

1.1.3 Scientific Notation

Scientific Notation leaves only one digit, and concatenate the rest as powers of ten. For example: $4.82 \cdot 10^4$. The decimal places indicates how many digits of significant figures is the number accurate to.

SI Prefix

A common list of SI Prefixes can be found below:

Factor	Name	Symbol	Factor	Name	Symbol
10^{1}	deca	da	10^{-1}	deci	\overline{d}
10^{2}	hecto	h	10^{-2}	centi	c
10^{3}	kilo	k	10^{-3}	milli	m
10^{6}	mega	M	10^{-6}	micro	μ
10^{9}	$_{ m giga}$	G	10^{-9}	nano	n
$10^{1}2$	tera	T	10^{-12}	pico	p
$10^{1}5$	$_{ m peta}$	P	10^{-15}	femto	f
$10^{1}8$	exa	E	10^{-18}	atto	a
$10^{2}1$	zetta	Z	10^{-21}	zepto	z
$10^{2}4$	yotta	Y	10^{-24}	yocto	y

Significant Figures

Significant Figures determines the amount of digits that is accurate in a certain calculation. To find the significant figure of a number, we use the following rules:

- Left most non-zero digit is the most significant digit
- If there is no decimal points, the rightmost digit non zero digit is the least significant digit
- If there is a decimal point, the rightmost digit is the least significant digit
- All digits between the most significant digit and the least significant digit is significant.

When doing a calculation, always round to the smallest significant figure. When rounding, round up with numbers above and including 5, and down with numbers below 5.

1.1.4 Uncertainties and Error

In physics, there will always be issues with uncertainty and errors. It is important to take these into account when doing calculations with the data.

1.1.5 Quantifying Uncertainties

1.1.6 Combining Uncertainties

1.1.7 Displaying Uncertainties

1.2 Vectors and Scalers

Physical quantities can be grouped into two different categories, scalers and vectors.

1.2.1 Define Scalers and Vectors

Scalers are unit of measurement that only describe the magnitude of an object. For example, distance is a scaler: it only describe how far you went.

Vectors are a unit of measurement that describe both the magnitude and a direction. For example, displacement is a vector. It describe the direction of movement and the distance traveled.

Below is a list of common scalers and vectors:

Scaler	Vector
speed	velocity
distance	displacement
energy, work, power	acceleration
temperature	force
pressure	momentum
mass	impulse
volume	electric field strength

1.2.2 Working with Vectors

We tend to break vectors into the x direction and the y direction. This way, we can represent the vectors with positive and negative values. For example, a force applied on a object upwards can be indicated with a positive sign, and a downwards force represented by a negative sign.

To separate the X and Y direction, we usually use trig function (sin cos tan).

Mechanics

2.1 Motion

2.1.1 Basic Concept of Motion

Distance is a measurement of how far an object have traveled. Distance is a scaler. The vector counterpart of distance, displacement (s) measures the distance from the start to the end of a motion. It also includes the measurement of direction. The SI Unit of displacement is meters. If a object travels in a circle, the distance is the circumference of the circle, but the displacement would he zero

Speed denotes the rate of change of distance. Speed is a vector. Velocity measures both rate of change of displacement and change in direction. A change in *velocity* might not be a change in speed.

Acceleration is the change in velocity. Acceleration is a vector. There is no scaler equivalent of acceleration. Acceleration also measures the change it direction. Thus, in circular motion, an object is *always* accelerating.

Average Versus Instantaneous

We commonly measure rate of change with respect to time. For example, to find the velocity of a bike, we measure the displacement over a period of 5 seconds. This is the average velocity.

Average velocity accounts for all the changes in velocity during the time period of measurement. Instantaneous velocity is the velocity of the bike at the exact time. This instantaneous velocity may not be exactly the same as the average velocity.

2.2 Graphing Motion

Often times, motion of an object can be graphed. These graphs gives us a lot of information.

2.2.1 Position Time Graph

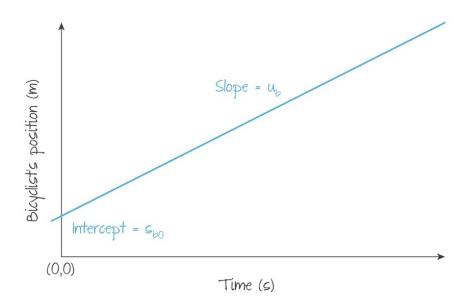


Figure 2.1: A position time graph of a bike over time

The change in Y values is the displacement. The change in X value is the change in time. The gradient of the line is the velocity.

Note, if the curve is non-linear, the average velocity would be the average slope. The instantaneous velocity is the slope of the tangent line.

2.2.2 Velocity Time graph

The Y intercept of the graph is the initial velocity. The change in Y is the change in velocity. The change in X is the change in time. The gradient of the graph is the acceleration (use tangent line if we need the instantaneous acceleration). The area under the curve is the displacement.

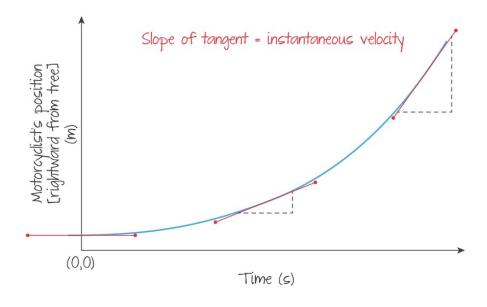


Figure 2.2: A curved position time graph

2.2.3 Acceleration Time graph

The area under the curve is the change in velocity.

2.2.4 Free fall air resistance

2.3 SUVAT

The SUVAT equations describe the relationship acceleration, displacement, velocity, and time.

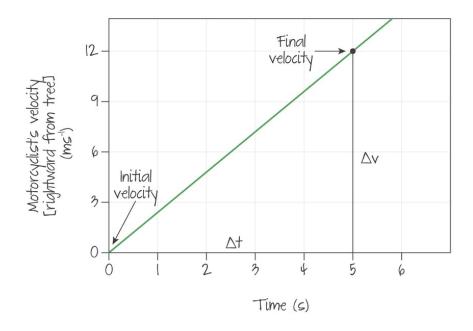
$$v = u + at (2.1)$$

$$s = ut + \frac{1}{2}at^2 \tag{2.2}$$

$$v^2 = u^2 + 2as (2.3)$$

$$s = \left(\frac{v+u}{2}\right)t\tag{2.4}$$

v final velocity, u is initial velocity. t is time. a is acceleration.



2.4 Projectile Motion

In a projectile motion, a object is accelerated in a certain direction. During flight, no additional force acts on said object. Projectile motion ends when the object impact the ground.

When analyzing projectile motion, split the variables into horizontal and vertical components. After the initial force, the horizontal acceleration is zero (horizontal velocity maintains constant), while the vertical acceleration remains constant (g, acceleration due to gravity). To find the net force/acceleration/velocity, we can use Pythagorean Theorem to combine the x and y components.

$$V_{net} = \sqrt{{V_{vertical}}^2 + {V_{horizontal}}^2}$$

2.5 Forces

2.5.1 Newton's Laws of Motion

- 1. An object continues to remain stationary or to move at a constant velocity unless an external force acts on it.
- 2. $F = ma, f = \frac{\Delta mv}{t}$, impulse is equal to change in momentum.

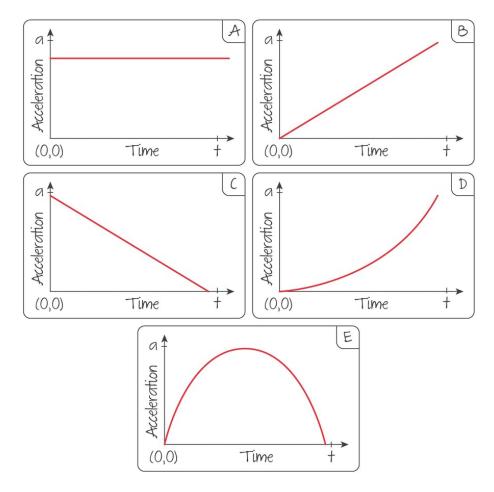


Figure 2.3: Acceleration Time graph

3. Every action has an equal and opposite reaction.

2.5.2 Free-Body Diagram

Free body diagrams are often used to show the forces acting on an object. Forces are drawn as arrows, with the direction of the arrow representing the direction of the force. And the length of the arrow representing magnitude. All forces need to have clear labels.

When drawing a free-body diagram, it is easier to represent the mass as a dot. All arrows start from the dot.

Note, if an object falls a long period of time, it may hit terminal veloc-

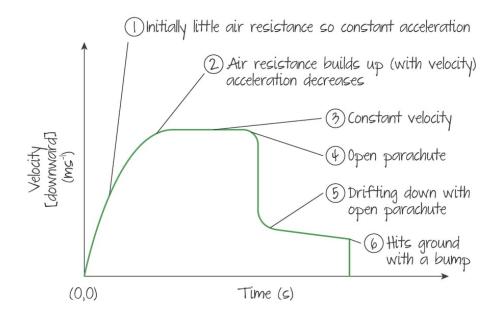


Figure 2.4: Velocity time graph of an object in free fall

ity. This is when air resistance increases a velocity increases, and eventually counteracts the force due to gravity.

2.5.3 Translational Equilibrium

Net force is the combination of all the forces acting on a body. Adding forces as vectors requires taking in account of direction. An object will accelerate according to the net force. If the net force of an object is zero, then the object is in translational equilibrium. This means the object is moving at constant velocity, or stationary.

2.6 Friction

Solid friction is a type of friction that occurs where two surfaces are in contact. There is two types of solid friction, static friction and dynamic friction.

2.6.1 Static Friction

Static friction occurs when there is no movement between two objects. Static friction increases with the force acting on an object until hitting a certain point. After reaching the point, the object begins to accelerate, and friction transfers to dynamic friction.

Static friction can be represented by:

$$F_t \le \mu_s R \tag{2.5}$$

2.6.2 Dynamic Friction

Dynamic friction happens when an object is moving. Dynamic friction is modeled by this equation:

$$F_t = \mu_d R \tag{2.6}$$

Where F_t is the force due to friction. μ is the coefficient of friction. R is the normal force acting on the object.

2.7 Work, Energy, and Power

Energy is stored in many forms. Below is a list of some common ways of storing energy.

- Kinetic
- Gravitational Potential
- Electric/Magnetic
- Chemical
- Nuclear
- Elastic
- Thermal
- Mass
- Vibration
- Light

Energy is often transformed into many different forms. One way to describe energy is through work.

2.7.1 Work

Work (W), has the SI Unit of Joules (J). The definition of work is the amount of energy required to move a mass of 1kg over a distance of 1m. The equation for work is:

$$W = F \cdot s \tag{2.7}$$

Where s is displacement, and F is force.

Note, if the force is exerted at an angle to the direction of displacement, we have to use trig ratios to find the component of the force that is parallel to the direction of displacement. Often times, work is multiplied by $\cos\theta$ (where θ is the angle between the force and the direction of motion) in order to account the difference in angles.

Sometimes, there is a resistive force done against the direction of travel. In this case, there would be work done by the force and work done by the resistive force.

2.7.2 Force Distance Graph

In a force distance graph, area under the curve is the work done.

2.7.3 Power

Power (P) is defined as the rate of doing work. The SI Unit of power is watts.

$$P = \frac{W}{t} \tag{2.8}$$

In which t represents time.

2.7.4 Kinetic Energy

Kinetic Energy (KE) is defined as the energy objects have due to its motion. The SI Unit of kinetic energy is Joules.

$$KE = \frac{1}{2}mv^2 \tag{2.9}$$

2.7.5 Gravitational Potential Energy

Gravitational Energy (GPE) is defined as the energy an object gains due to its height.

$$GPE = mgh (2.10)$$

In which, m is mass, g is the acceleration due to gravity, and h is the height from a reference point.

g is a constant. IB defines the acceleration due to gravitational on Earth's surface as $9.8m \cdot s^{-2}$.

2.7.6 Conservation of Energy

In a close system, energy is always conserved. For example, when dropping an object from a height, gravitational energy converts of kinetic energy. You can write this in an equation:

$$\frac{1}{2}mv^2 = mgh \tag{2.11}$$

Sometimes, there is some energy lost in the transition. For example, kinetic energy may be lost to heat (thermal energy).

2.7.7 Elastic Potential Energy

The force due to elasticity can be modeled by Hooke's law.

$$F = kx (2.12)$$

Where x is the displacement from the spring's central position and k is the spring constant. The central point is also where the spring is at equilibrium, where the net force is zero.

2.7.8 Efficiency

The efficiency of a system is how much energy was useful.

$$efficiency = \frac{\text{useful work out}}{\text{total energy in}}$$
 (2.13)

2.8 Momentum

Momentum (p) is the product of the mass of an object and its velocity.

$$p = mv (2.14)$$

Where p is momentum, m is mass, and v is velocity. Momentum is a vector.

2.8.1 Impulse

The change in momentum is equals to impulse. Impulse is defined by force multiplied by time.

$$pv = ft (2.15)$$

2.8.2 Conservation of Momentum

In a close system, momentum is always conserved. This means the change in momentum of one object may contribute to the change in momentum of another object.

2.8.3 Force-Time Graphs

The area under a force-time graph is the impulse, or the change in momentum.

2.8.4 Collisions

There is three types of collisions: elastic, inelastic, and completely inelastic collision.

In an elastic collision, kinetic energy of the system is conserved. In an inelastic collision, some kinetic energy is waste, converted into heat, sound, etc. In a completely inelastic collision, the two objects stick together, kinetic energy is not conserved.

Note, in all collisions, momentum is conserved.

In an explosion, momentum is not conserved, as there is an external force acting on the object.

2.8.5 Energy and Momentum

Since the equation for kinetic energy and momentum are both based on mass and velocity, we can find a relationship between the kinetic energy and the momentum of an object. This is a useful shortcut.

$$KE = \frac{p^2}{2m} \tag{2.16}$$

Thermal Physics

3.1 Temperature and Energy Transfer

Temperature is the average internal kinetic energy of an object. The SI Unit of temperature in Kelvins (K). However, Celsius is sometimes used.

$$K = C + 273 (3.1)$$

In general, temperature will aways attempt to reach thermal equilibrium. Meaning that objects will aways try to reach the same temperature over time.

A temperature of 0K is called absolute temperature. This is when the internal energy of a mas is zero.

3.1.1 Internal Energy

Substances consist of particles in constant random motion. The internal energy of a molecule is the total potential and random kinetic energy of all the particles in a substance.

3.1.2 Specific Heat Capacity

The Specific Heat Capacity is defined as the amount of energy needed to raise a unit mass of an object by one Kelvin. This equation models the relationship between change in temperature and energy.

$$Q = mc\Delta T \tag{3.2}$$

Where Q is the energy required, m is the mass, c is the specific heat capacity, and ΔT is the change in temperature.

3.1.3 Specific Latent Heat

Latent heat is the amount of energy required to shift an object from one state to another. This equation models the relationship between latent heat and energy.

$$Q = mL (3.3)$$

Where Q is the energy required, m is the mass, and L is the latent heat.

Latent heat of fusion is the energy require to change the phase from liquid to solid of 1kg of substance. Latent heat of vaporization is the energy required to change the phase of 1kg of liquid into a gas.

3.1.4 Phase Change Diagram

This diagram shows the internal energy of a substance as it changes state.

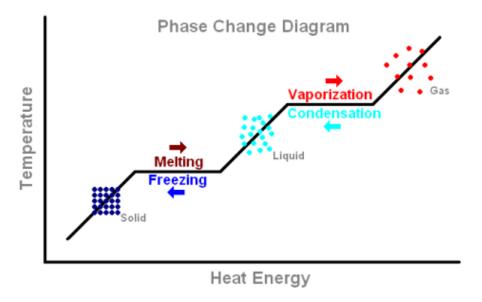


Figure 3.1: A Phase Change Diagram

3.2 Modeling a gas

3.2.1 Mole and the Avogadro Constant

The mole is used for measuring the amount of substance. A mol of gas contains $6.02 \cdot 10^23$ atoms. This is also the Avogadro constant.

3.2.2 Molar Mass

The molar mass of an substance is the mass of one mol of said substance.

Mass of substance = Substance Molar Mass \times Mols of Substance (3.4)

3.2.3 Ideal Gas Law

The Ideal Gas Law sum up the relationship between pressure, volume, temperature, and moles of molecules of a gas.

$$pV = nRT (3.5)$$

Where p is pressure (SI Unit is Pascals or Pa), V is volume, n moles of molecule (SI Unit in moles or Mol), R is the ideal gas constant $(8.3145 \cdot 10^3)$, and T is the temperature (in Kelvins).

3.2.4 Assumptions of an Ideal Gas

- A gas consist of many tiny molecules in random motion
- Each molecule has negligible volume when compared with the volume of the gas
- Molecules undergo perfectly elastic collisions
- There is no intermolecular force, the energy of the molecules is only kinetic
- The duration of a collision is negligible compared with the time between the collisions
- Forces of individual molecules with average out to an uniformed pressure

3.2.5 Total Internal Energy of an Ideal Gas

The total internal energy of an ideal gas is the sum of the average random kinetic energy of each particle inside the gas.

$$\sum KE = \frac{3}{2}Nk_BT \tag{3.6}$$

Where k_B is the Boltzmann Constant, N is the amount of particles, and T is the temperature.

Note, the internal energy of each individual particle is the equation above without N.

$$KE = \frac{3}{2}k_BT\tag{3.7}$$

3.2.6 Real Gases

Ideal gas is ideas. Ideal gases cannot be liquefied. Thus, as a gas near high pressure or low temperatures, it may behave like a real gas instead of an ideal gas. A real gas is a non idea gas.

Circular Motion and Gravitation

4.1 Circular Motion

When an object is moving in a circle, it is undergoing circular motion. A uniformed circular motion is when the linear velocity stays constant. We will only be looking at uniformed circular motion in IB.

In circular motion, the linear velocity is constant, but velocity is always changing. There is a constant acceleration towards the center of the circle. The velocity of the object is always a tangent to the circle.

In order for a circular motion to exist, an object must be moving tangentially to another object, while a centripetal force is exerted on the object.

4.1.1 Angular Displacement

Angular displacement (θ) is the angle an object travels in circular motion. This is measured in Radians (rad). To convert between radians and degrees:

$$\pi \ rad = 180^{\circ} \tag{4.1}$$

4.1.2 Angular Velocity

Angular velocity (ω) measures the change in angle overtime.

$$\omega = \frac{\theta}{t} \tag{4.2}$$

Where t is time.

4.1.3 Period and Frequency

Period (T) is defined as the time taken to complete one revolution around the circle. The SI Unit of period is seconds (s).

$$T = \frac{2\pi}{\omega} \tag{4.3}$$

 2π is the total angle of the circle, and ω is the angular velocity.

Frequency (f) is defined as the revolutions that can be completed in one second. The SI Unit of frequency is Hertz (Hz). One hertz is one revolution per second.

$$f = \frac{1}{T} \tag{4.4}$$

4.1.4 Linear and Angular Velocity

Linear velocity measures the speed of rotation.

$$v = \frac{2\pi r}{T} \tag{4.5}$$

Where r as the radius of the circle.

The relationship between linear velocity and angular velocity is:

$$v = \omega r \tag{4.6}$$

4.1.5 Centripetal Acceleration

Centripetal acceleration is the acceleration of the object as it undergoes circular motion.

$$a = \frac{v^2}{r} = \omega^2 r = v\omega \tag{4.7}$$

4.1.6 Centripetal Force

Centripetal force is the resultant force required to keep an object in circular motion. Similar to the acceleration, centripetal force points to the center of the circle.

$$F = ma = m\frac{v^2}{r} = m\omega^2 r = mv\omega \tag{4.8}$$

The centripetal force itself is not a force. It is a combination of other forces. For example, when a car undergoes into circular motion, friction between the car and ground is the centripetal force. For a satellite that is undergoing circular motion, its centripetal force is gravity.

4.2 Newton's Law of Gravitation

Gravity is a force one mass exerts on another.

4.2.1 Gravitational Field Strength

Gravitational field strength is the force per unit mass experienced by a point mass placed at a specific point.

$$g = \frac{F}{m} \tag{4.9}$$

Where g is the acceleration due to gravity, or gravitational field strength. F is the force due to gravity, and m is the mass of the object.

4.2.2 Newton's Law of Gravitation

The force due to gravity can be calculated using the equation below.

$$F = \frac{GMm}{r^2} \tag{4.10}$$

Where F is the force, G is Universal Gravitational Constant $(6.67 \cdot 10^{-11})$, m and M are the mass of the objects, and r is the distance from the center of mass.

If we plug this equation into the equation for gravitational field strength, we find another equation for gravitational field strength:

$$g = \frac{GM}{r^2} \tag{4.11}$$

4.2.3 Orbit

For an object to remain in orbit, it means it is circular motion with a planet. Likely, the only centripetal force acting on the object is gravity due to the planet. Thus, we can model this relationship in an equation.

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \tag{4.12}$$

The orbit velocity of a satellite can be calculated using the equation above. By simplifying, we get the resultant equation:

$$V_{orbit} = \sqrt{\frac{GM}{r}} \tag{4.13}$$

Further expansion of this equation can give us Kepler's third law.

$$T^2 \propto r^3 \tag{4.14}$$

Fields

5.0.1 Fields

A field is said to exist when one object can exert a force on another object at a distance.

A gravitational field is associate with a mass. Every mass has its own gravitational field. A gravitational field only attracts, there is no repulsive gravitational field.

The gravitational field strength of an mass is g, or acceleration due to gravity.

5.0.2 Field Lines

Field lines help visualize and fields. Field lines show the direction of the force due to a field.

Equipotential lines are drawn to represent where the potential of a field is always the same. In a gravitational field line diagram, equipotential lines are farther apart from each other the farther the lines are from the center.

5.1 Fields at Work

5.1.1 Gravitational Potential

Gravitational potential difference is the work done in moving a unit mass between two points. Gravitational potential is the work done in moving a unit mass infinitely far away to a certain point. The equation for gravitational potential is:

$$V_g = -\frac{GM}{r} \tag{5.1}$$

Gravitational potential is always negative because the closer an object is to another object, the more work has been done to move the object from infinite distance, and thus the more work we will have to do to put it back.

5.1.2 Gravitational Potential Energy

Similar to gravitational potential, gravitational potential energy is the energy required to move a mass from infinite far away to a certain point.

$$GPE = \frac{GMm}{r} \tag{5.2}$$

5.1.3 Escaping the Earth

The total energy of a satellite is made up of its kinetic and gravitational potential energy. In order for a satellite to escape a planet's gravitational field, the object will have to convert all of its potential energy into kinetic energy.

$$\Delta - \frac{GMm}{r} = \Delta \frac{1}{2}mv^2 \tag{5.3}$$

Simplify this equation to find the escape velocity, which is $\sqrt{2gr}$.