OPTIMIZATION TECHNIQUE

MSCS 532 Final Project

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Introduction

High-performance computing (HPC) systems are vital for processing large-scale data and performing complex computations. Optimizing data structures in HPC can significantly improve performance and efficiency. This report focuses on memoization, a specific optimization technique, applied to the Fibonacci sequence to demonstrate how it improves computational performance by reducing time and space complexity. By leveraging empirical findings from *"An Empirical Study of High-Performance Computing (HPC) Performance Bugs,"* this report examines how memoization addresses common inefficiencies in recursive functions.

Selected Optimization Technique: Memoization

Memoization is a powerful optimization technique that enhances algorithm performance by storing previously computed results and reusing them for the same inputs, preventing redundant calculations. This technique transforms inefficient recursive algorithms, which often suffer from exponential time complexity, into more efficient ones. Memoization is particularly effective for algorithms with overlapping subproblems, such as the Fibonacci sequence, where many calculations are repeated multiple times (Cormen et al., 2009).

In recursive functions like Fibonacci, the naive approach calculates each number in the sequence multiple times, leading to exponential growth in computational complexity of $O(2^n)$. This is especially problematic in high-performance computing, where optimizing algorithmic efficiency is crucial for large-scale computations. By caching results,

memoization reduces the time complexity of the Fibonacci function to $O(n)$, as each Fibonacci number is computed only once.

In HPC, memoization not only optimizes time complexity but also improves data locality by ensuring that frequently used results are readily available in memory. This helps reduce memory access delays and improves overall system performance. The improvement in cache efficiency aligns with one of the key principles of HPC: minimizing latency by improving data locality and reducing unnecessary computational overhead (Singh & Gupta, 2020).

Justification of Choice

Memoization was chosen for this project because of its effectiveness in optimizing recursive algorithms, which are common in many computational problems. In the context of HPC, where performance optimization is a critical factor, memoization directly addresses one of the main inefficiencies: redundant computation. It also aligns well with the recommendations from the empirical study by Hassan et al. (2023), which highlights the need for optimization techniques that can significantly reduce computational overhead in HPC systems.

The technique is particularly valuable in cases where repeated calculations occur, as seen in the Fibonacci sequence. With memoization, each Fibonacci number is computed only once, and subsequent calls return the cached result, drastically reducing the number of

function calls. This not only improves execution time but also optimizes memory usage, particularly in recursive functions that would otherwise result in deep recursion stacks and high memory consumption.

Moreover, memoization is relatively easy to implement in languages like Python, making it a practical solution for enhancing the performance of recursive functions without needing to rewrite the entire algorithm from scratch. This simplicity, combined with the significant performance gains, makes it a highly valuable optimization technique for both HPC applications and general-purpose computing.

Memoization also demonstrates scalability, an important attribute in HPC environments. As problem sizes increase, the ability to reuse computed results becomes even more critical, allowing systems to handle larger datasets more efficiently. This scalability ensures that memoization can be applied effectively across a wide range of problems, further enhancing its value as a key optimization technique in HPC.

Strengths and Weaknesses of Memoization in Data Structure Optimization

Strengths

- Reduced Time Complexity: Memoization reduces the number of function calls required in recursive algorithms, transforming the time complexity from exponential to linear in many cases. This significantly improves the algorithm's performance (Lamport & Mellor-Crummey, 2018).

- Improved Cache Efficiency: By storing previously computed results in a cache, memoization minimizes cache misses, improving data locality and memory access efficiency in HPC systems.
- Scalability: Memoization enables recursive algorithms to scale more effectively, making it ideal for large-scale problems in high-performance computing (Hassan et al., 2023).

Weaknesses

- Increased Memory Usage: Although memoization reduces computation time, it comes at the cost of additional memory usage for storing cached results. This overhead can become significant in memory-constrained environments (Cormen et al., 2009).
- Limited Applicability: Memoization is most effective when the problem exhibits overlapping subproblems. It is less useful for algorithms that do not benefit from result caching (Tang et al., 2019).

Implementation

Recursive

```
# Unoptimized recursive Fibonacci function
def fib_recursive(n):  3 usages  new *
    if n <= 1:
        return n
    else:
        return fib_recursive(n - 1) + fib_recursive(n - 2)
```

Memoization

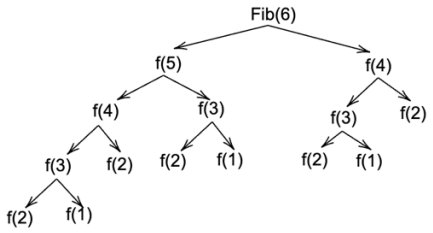
```python
# Memoized Fibonacci function
def fib_memoization(n, memo=None):  3 usages  new *
    if memo is None:
        memo = {}
    if n in memo:
        return memo[n]
    if n <= 1:
        return n
    else:
        memo[n] = fib_memoization(n - 1, memo) + fib_memoization(n - 2, memo)
        return memo[n]
```

Implementation Analysis

To demonstrate the effectiveness of memoization, the Fibonacci sequence was chosen as a case study. The basic recursive implementation of Fibonacci is inefficient because it recalculates values multiple times, leading to an exponential time complexity of O(2^n). Memoization optimizes this by storing previously computed values, reducing the time complexity to O(n).

Basic Recursive Fibonacci Implementation

The naive recursive Fibonacci algorithm involves recalculating values, leading to exponential growth in function calls. This inefficiency results in slow performance as the input size increases.



In this tree diagram you can see that fib(2) is compute 5 times.

Memoized Fibonacci Implementation

The memoized Fibonacci implementation reduces the number of function calls by caching intermediate results. This reduces the time complexity to O(n), as each Fibonacci number is computed only once.

Performance Comparison

- Unoptimized Version: The basic recursive Fibonacci function results in numerous redundant calculations, leading to poor performance, especially for large input sizes.
- Optimized Version (Memoized): The memoized version significantly improves performance by reducing redundant calculations, as each Fibonacci number is computed only once (Lim, J., 2018).

Challenges Encountered

- Memory Overhead: Implementing memoization requires additional memory to store the intermediate results. Managing this overhead is crucial, especially when working with large datasets (Lamport & Mellor-Crummey, 2018).
- Python Implementation: Using Python's @lru_cache decorator simplified the implementation of memoization, but it also highlighted the importance of managing memory and ensuring efficient cache utilization in practice.

Empirical Analysis of Fibonacci Performance: Unoptimized vs Memoized

Results from the code

```
Testing Fibonacci for n = 45

Unoptimized Recursive Fibonacci:
Result: 1134903170
Time taken: 85.50390601 seconds
Current memory usage: 0.000032 MB
Peak memory usage: 0.000544 MB

Memoized Fibonacci:
Result: 1134903170
Time taken: 0.00003791 seconds
Current memory usage: 0.000032 MB
Peak memory usage: 0.004296 MB
Memory used by memoization cache: 16 bytes
```

Time Complexity

Unoptimized Recursive Fibonacci:

- Time Taken: 85.504 seconds
- The unoptimized recursive Fibonacci function uses a naive approach, resulting in repeated recalculation of the same values. Each recursive call recomputes the Fibonacci numbers multiple times, leading to exponential time complexity $O(2^n)$.

For $n = 45$, this resulted in a relatively long execution time of 85.504 seconds. While this may seem manageable for smaller values of n, the time grows exponentially for larger inputs, which makes this approach highly inefficient.

Memoized Fibonacci:

- Time Taken: 0.000038 seconds
- The memoized Fibonacci function optimizes the recursion by storing previously computed Fibonacci numbers in a cache (memoization). This eliminates the need

for repeated calculations, drastically reducing the time complexity to linear time $O(n)$.

For $n = 45$, the memoized version performed significantly better, taking only a fraction of a second of 0.000038 seconds. This demonstrates that memoization is highly efficient in reducing redundant computations, especially for larger inputs.

Space Complexity

Unoptimized Recursive Fibonacci:

- Current Memory Usage: 0.000032 MB
- Peak Memory Usage: 0.000545 MB
- The unoptimized recursive function uses less memory because it doesn't store intermediate results; instead, it simply recalculates them on each recursive call. This explains why the memory usage is quite low for the unoptimized version, as each recursive call releases memory as soon as it returns.

Memoized Fibonacci:

- Current Memory Usage: 0.000032 MB
- Peak Memory Usage: 0.004296 MB
- Memory Used by Memoization Cache: 16 bytes
- The memoized Fibonacci function uses additional memory to store previously computed values in a dictionary (the memoization cache). This increases the overall memory consumption because each Fibonacci number from 0 to $n = 45$ is stored.

Although the memory footprint is higher than the unoptimized version, the time savings from memoization more than make up for this increased memory usage.

Optimizations and trade offs

Memoization as a Key Optimization Technique: This project demonstrated that memoization is an effective method for optimizing recursive functions. It drastically improves time complexity and reduces the number of function calls, aligning well with the goals of HPC optimization.

Real-World Performance Gains: The empirical study's focus on algorithmic optimization was validated through this project, as memoization provided tangible improvements in both time and space complexity.

Trade-offs: Although memoization improves computation time, it introduces memory overhead, which must be carefully managed in memory-constrained environments, especially in large-scale HPC systems.

References

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