Informed Search (Tìm kiếm kinh nghiệm)

LESSON 3-4

Reading

Chapter 4

Material

Sechtions 3.5, 4.1

Excludes memory-bounded heuristic search (3.5)

Outline

- Best-first search
- Greedy best-first search (Tìm kiếm tốt nhất ăn tham)
- A* search
- Heuristics
- Local search algorithms
- Hill Climbing (Leo đồi)
- Beam Search
- Brand-and-Bound Search (Tìm kiếm nhánh cận)

Review: Tree search

- A search strategy is defined by picking the order of node expansion

```
function TREE-SEARCH( problem, fringe) returns a solution, or failure
   fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
   loop do
       if fringe is empty then return failure
       node ← REMOVE-FRONT(fringe)
       if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
       fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

Best-first search

- Idea: use an evaluation function f(n) for each node
  - estimate of "desirability"

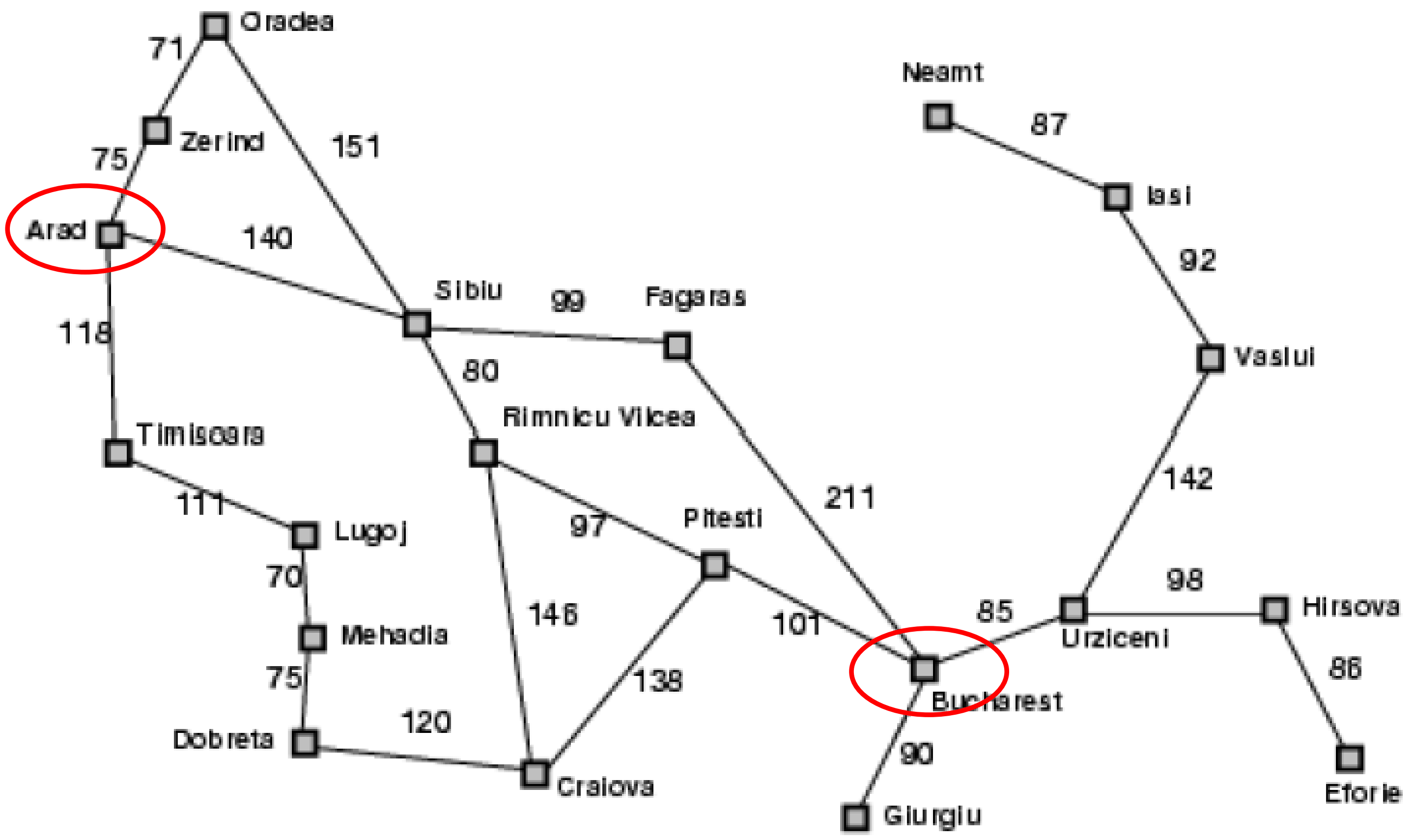
  -> Expand most desirable unexpanded node

- <u>Implementation:</u>

  Order the nodes in fringe in decreasing order of desirability

- Special cases:
  - **Greedy best-first search**
  - **A* search**

Romania with step costs in km



Straight–line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Best First Search: Algorithm

procedure *Best_First_Search*;

begin

1. List L is initialized with start state;

2. **loop do**

2.1 ***if*** *L empty* ***then {****Failure****; stop};***

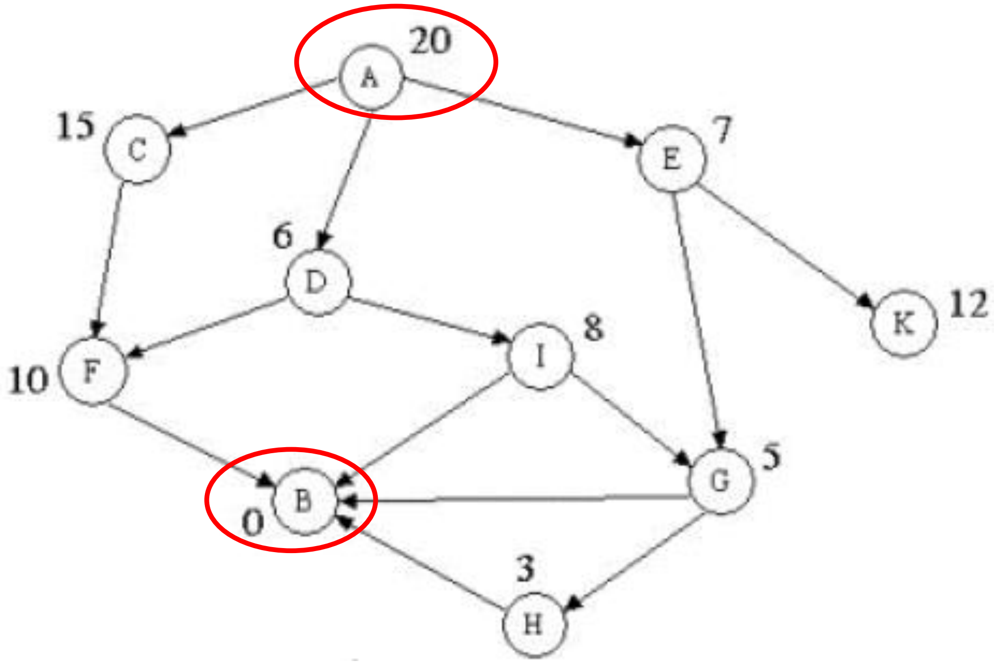
2.2 u <- pop L;

2.3 ***if*** *u final state* ***then*** *{success;* ***stop}***

2.4 ***for*** *each v next u* ***do*** *add v to L and sort L based on Evaluation function;*

end;

BFS: example



Properties of BestFirst Search

Complete? yes,

Time? O(b^m), where m is maximum depth, b is average child size of a node

Space? O(b^m): keeps all nodes in memory

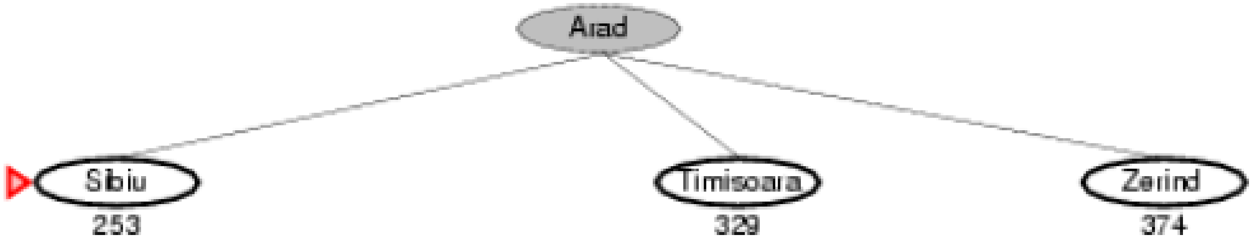
Optimal? Yes

Greedy best-first search

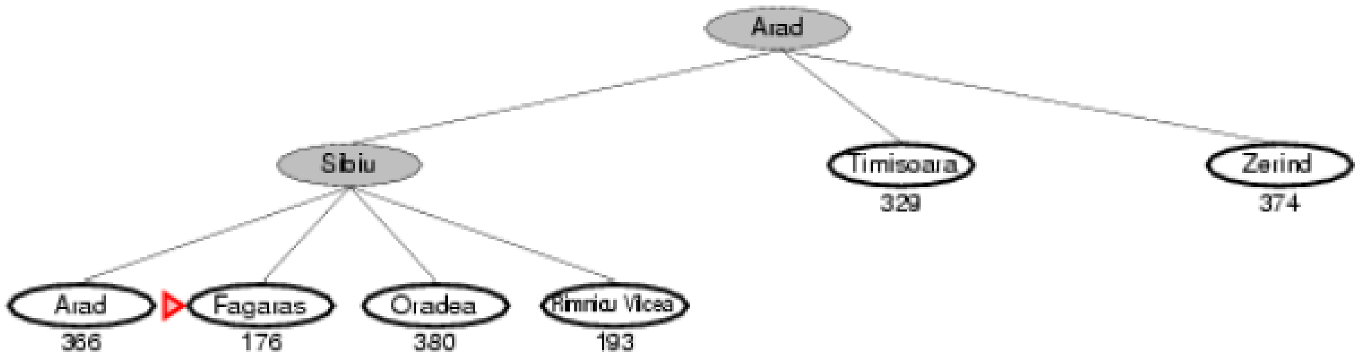
- Greedy best-first search expands the node that appears to be closest to goal

Greedy best-first search example

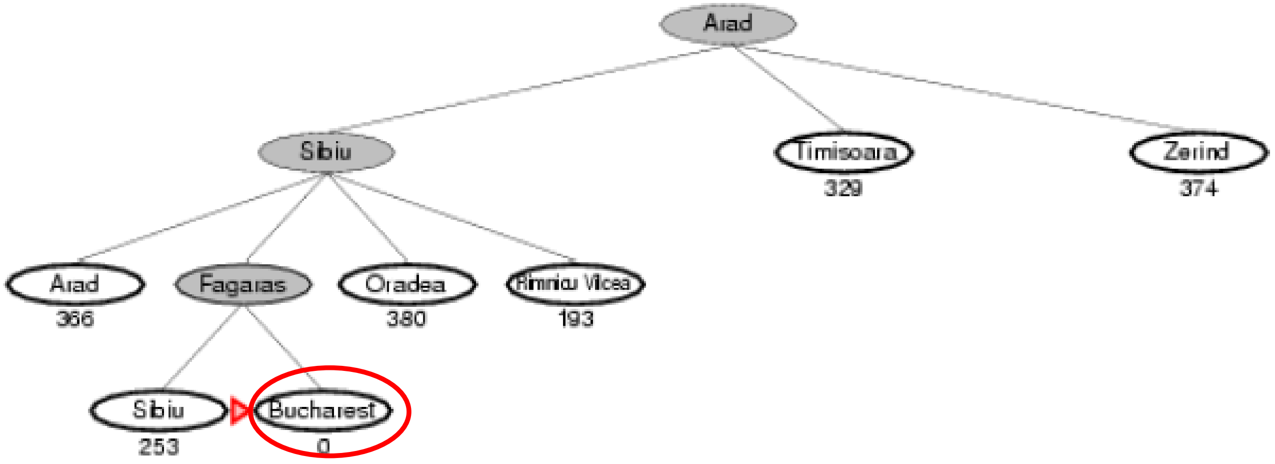
Greedy best-first search example



Greedy best-first search example



Greedy best-first search example



Properties of Greedy best-first search (Measuring problem-solving performance)

- **Complete?** Is the algorithm guaranteed to find a solution when there is one?
- **Time?** How long does it take to find a solution?
- **Space?** How much memory is needed to perform the search?
- **Optimal?** Does the strategy find the optimal solution?

Properties of Greedy best-first search

Complete? No – can get stuck in loops,

Time? O(bm), but a good heuristic can give dramatic improvement

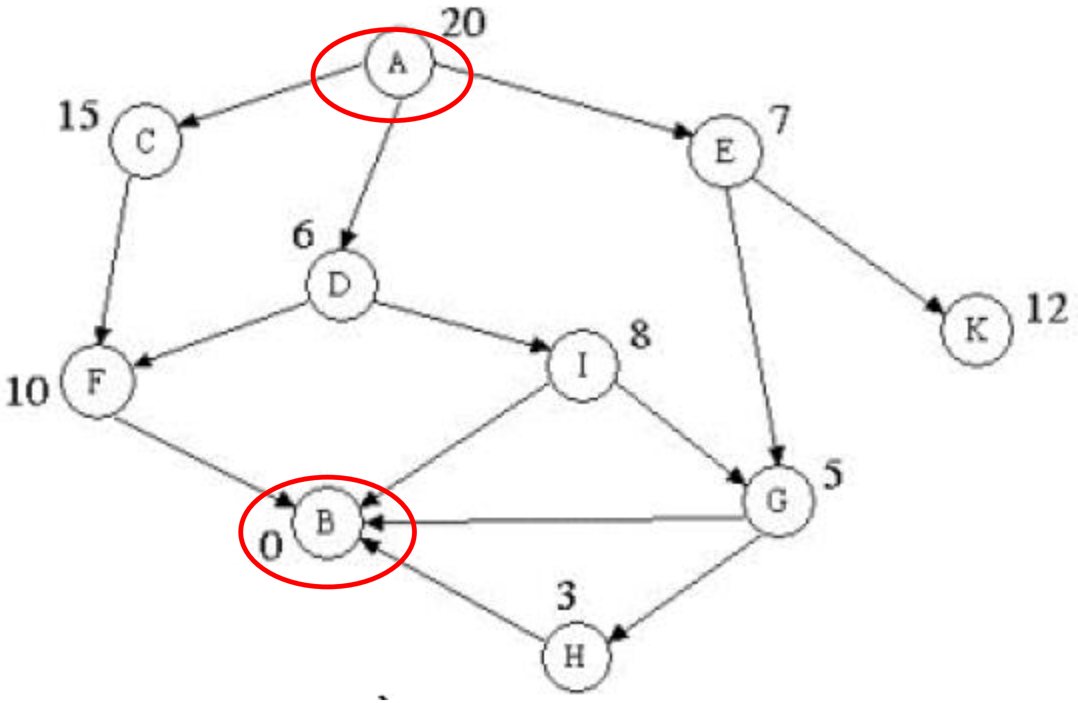
Space? O(bm): keeps all nodes in memory

Optimal? No

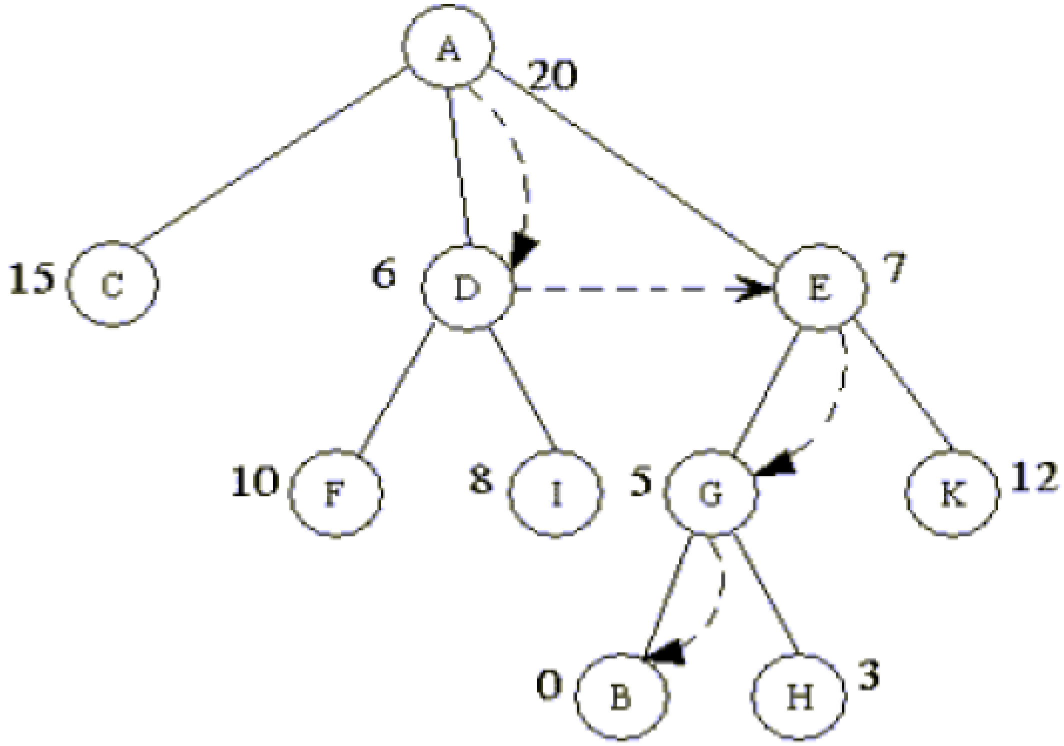
Algorithm of Greedy best-first search

?

Practice?



Search tree



A* search
(Minimizing the total estimated solution cost)

Idea: avoid expanding paths that are already expensive

Evaluation function $f(n) = g(n) + h(n)$

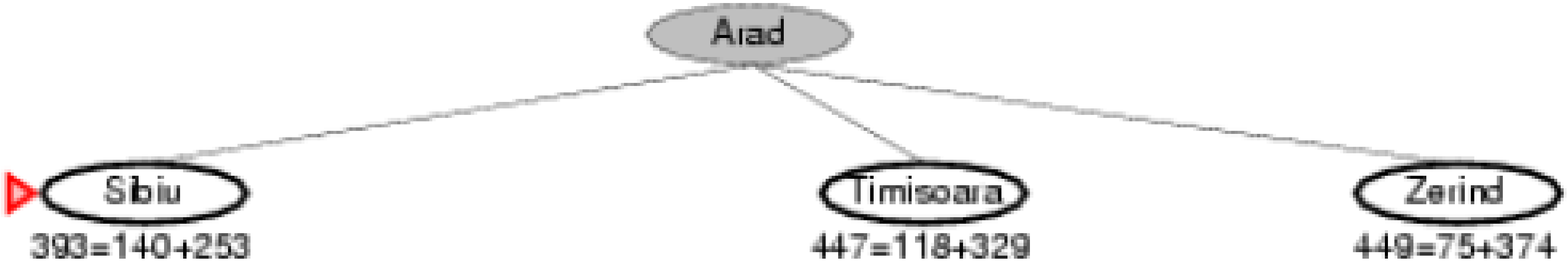
$g(n)$ = cost so far to reach n

$h(n)$ = estimated cost from n to $goal$

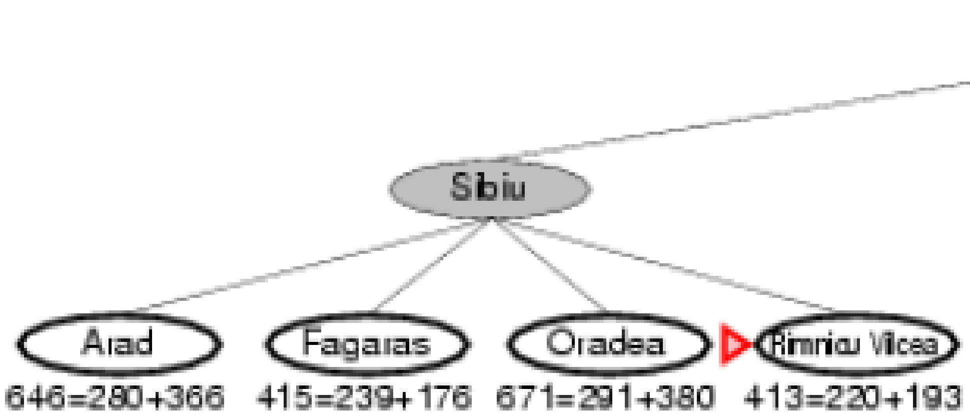
$f(n)$ = estimated total cost of the cheapest solution through n to $goal$

A* search example

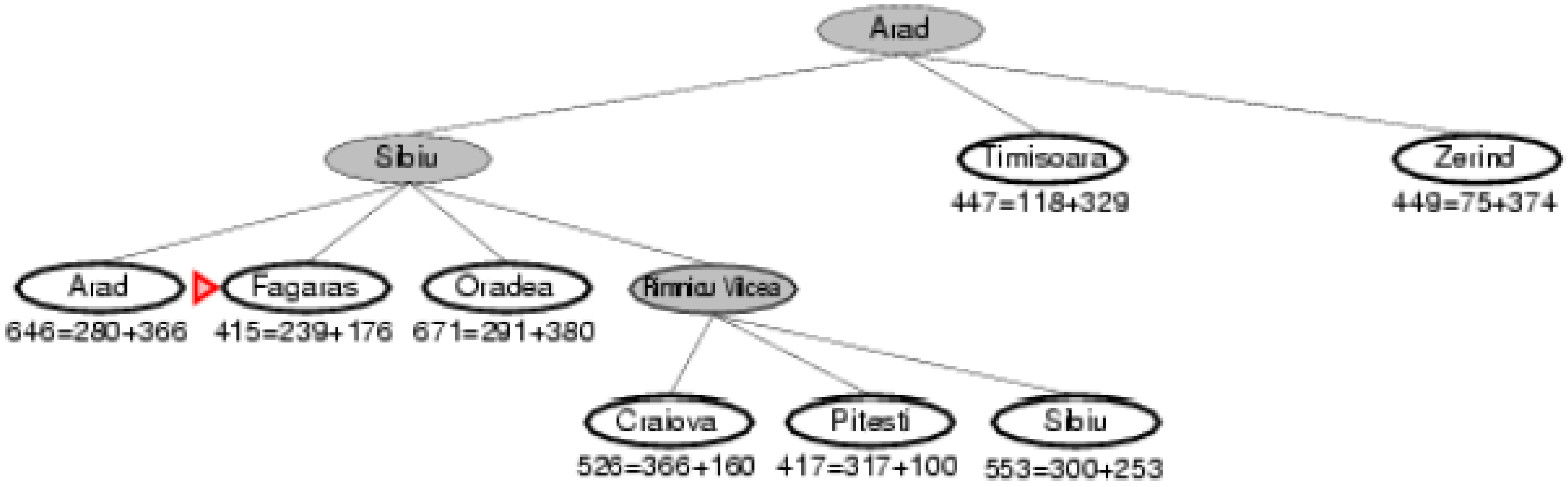
A* search example



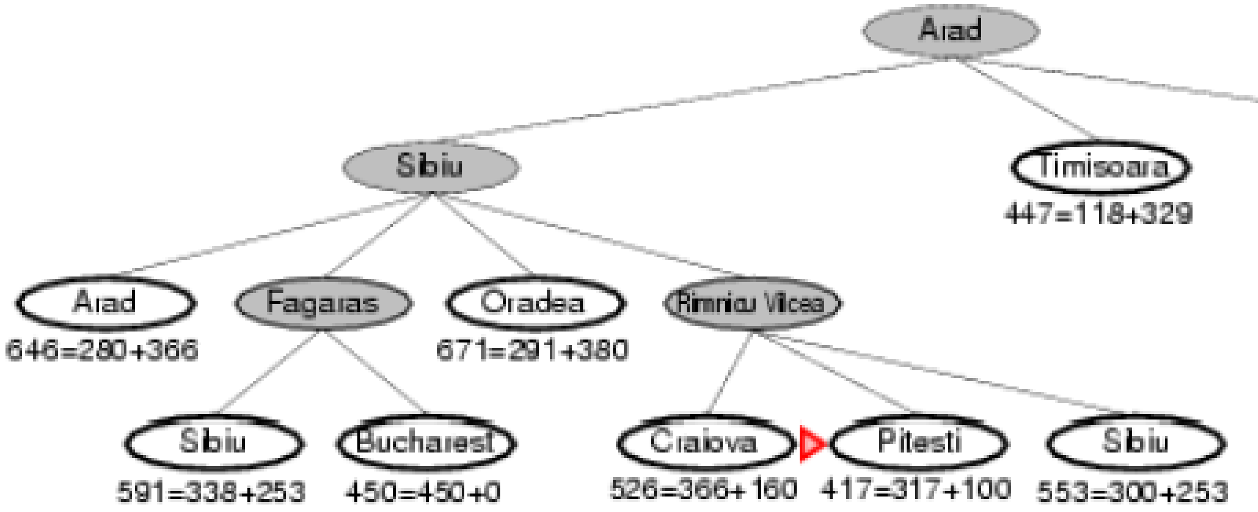
A* search example



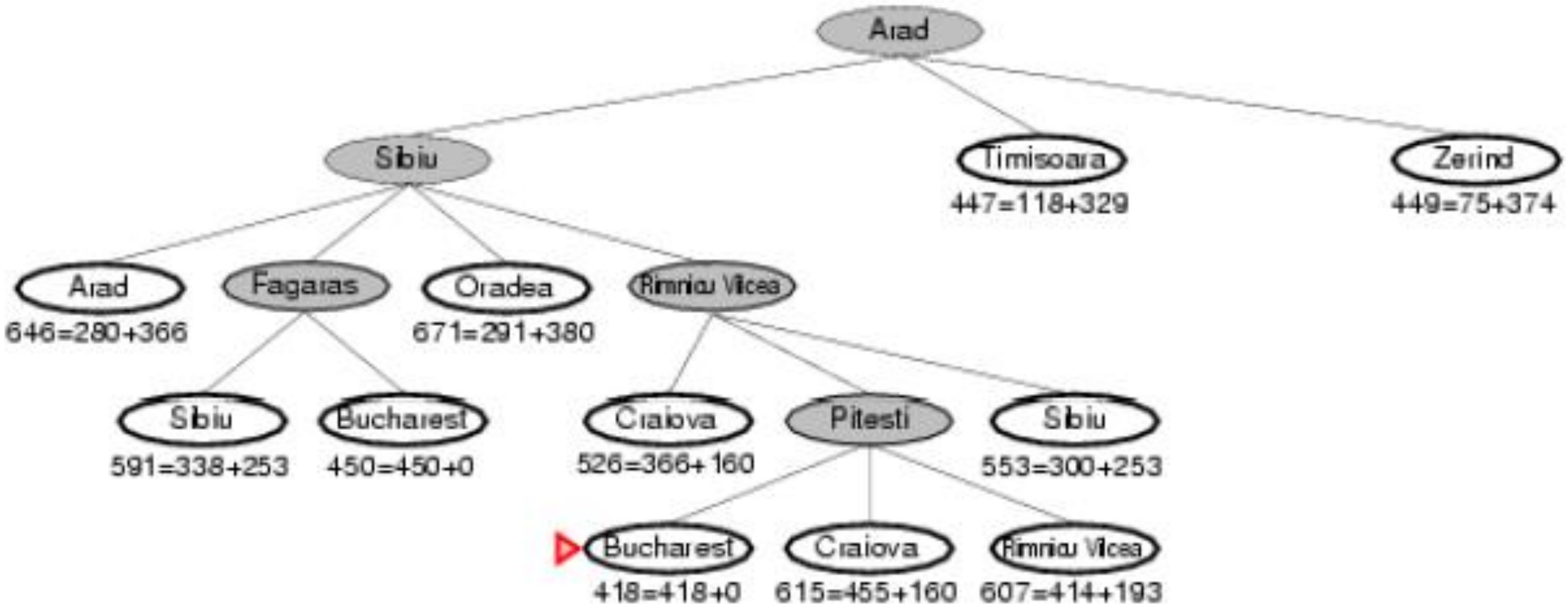
A* search example



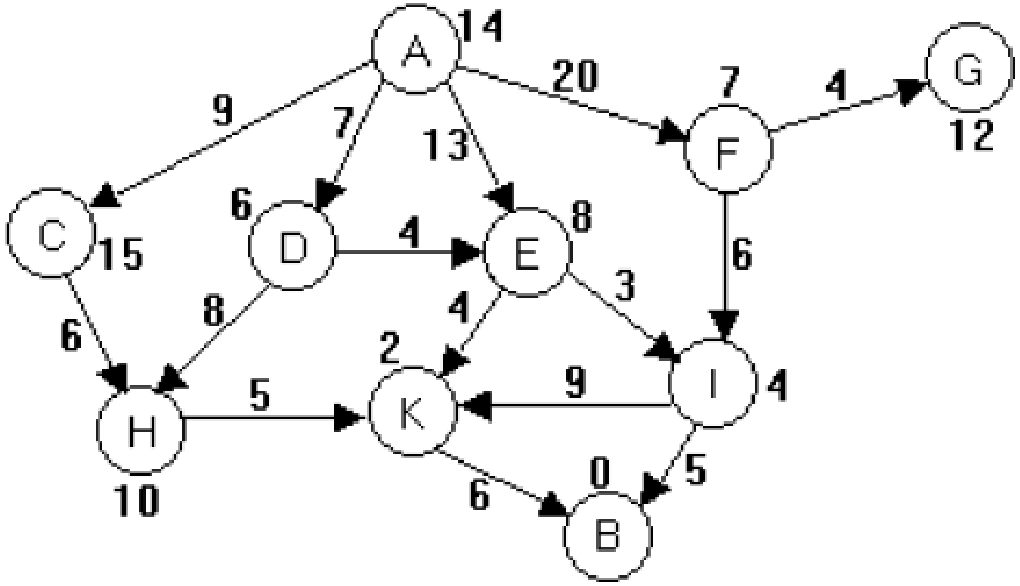
A* search example



A* search example



A* example



Admissible heuristics

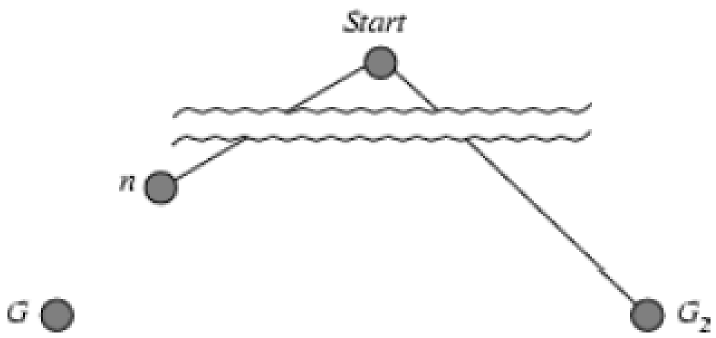
- A heuristic $h(n)$ is admissible if for every node n, $h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from n.

- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic
- Example: hSLD(n) (never overestimates the actual road distance)

- Theorem: If h(n) is admissible, A* using TREE-SEARCH is optimal

Optimality of A* (proof)

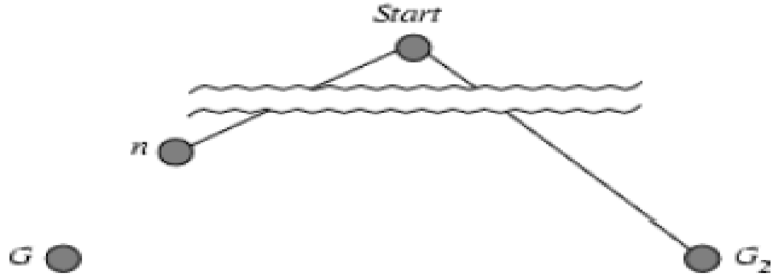
Suppose some suboptimal goal G2 has been generated and is in the fringe. Let n be an unexpanded node in the fringe such that n is on a shortest path to an optimal goal G.



f(G2) = g(G2)	since h(G2) = 0
g(G2) > g(G)	since G2 is suboptimal
f(G) = g(G)	since h(G) = 0
f(G2) > f(G)	from above

Optimality of A* (proof)

- Suppose some suboptimal goal G_2 has been generated and is in the fringe. Let n be an unexpanded node in the fringe such that n is on a shortest path to an optimal goal G.



- $f(G_2) > f(G)$ from previous
- $h(n) \le h^*(n)$ since h is admissible
- $g(n) + h(n) \le g(n) + h^*(n)$
- $f(n) \le f(G)$

Hence $f(G_2) > f(n)$, and A* will never select G_2 for expansion

Consistent heuristics

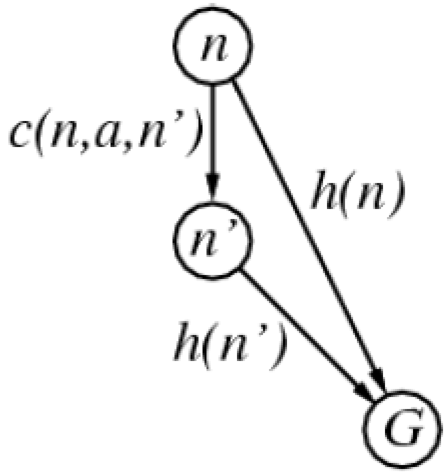
- A heuristic is consistent if for every node n, every successor n' of n generated by any action a,

  $h(n) \le c(n,a,n') + h(n')$

- If h is consistent, we have

$$
\begin{aligned}
f(n') &= g(n') + h(n') \\
&= g(n) + c(n,a,n') + h(n') \\
&\ge g(n) + h(n) = f(n)
\end{aligned}
$$

i.e., f(n) is non-decreasing along any path.



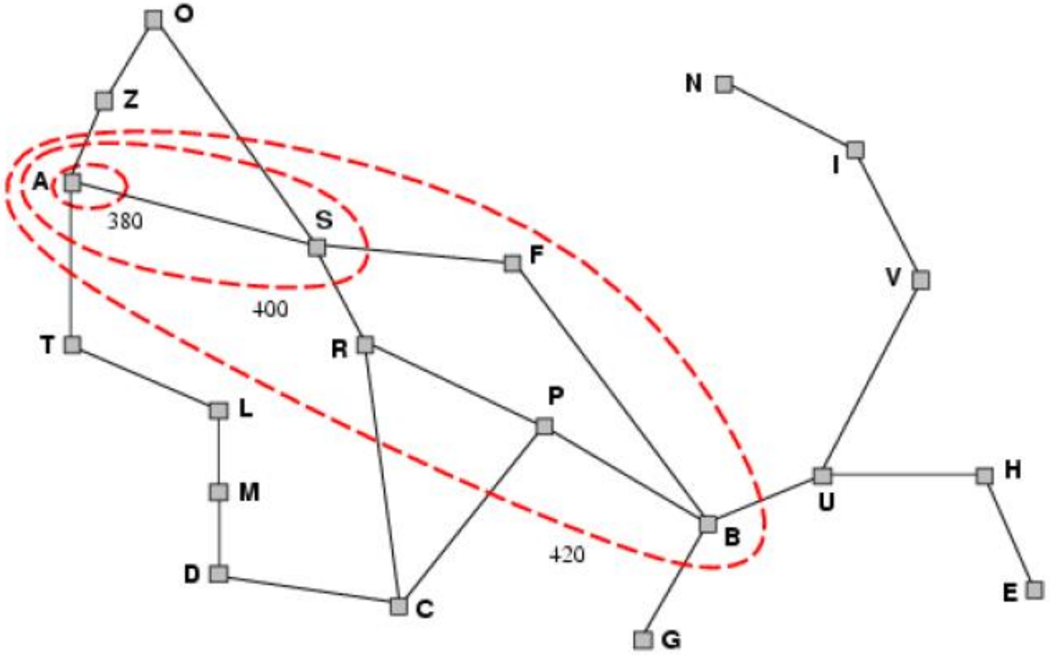
- Theorem: If h(n) is consistent A* using GRAPH SEARCH is optimal

Optimality of A*

A* expands nodes in order of increasing f value

Gradually adds "f-contours" of nodes

Contour i has all nodes with *f=fi*, where *fi < fi+1*



Properties of A*

- Complete? Yes (unless there are infinitely many nodes with $f \leq f(G)$)
- Time? Exponential
- Space? Keeps all nodes in memory
- Optimal? Yes

A* Algorithms

?

A* Algorithms

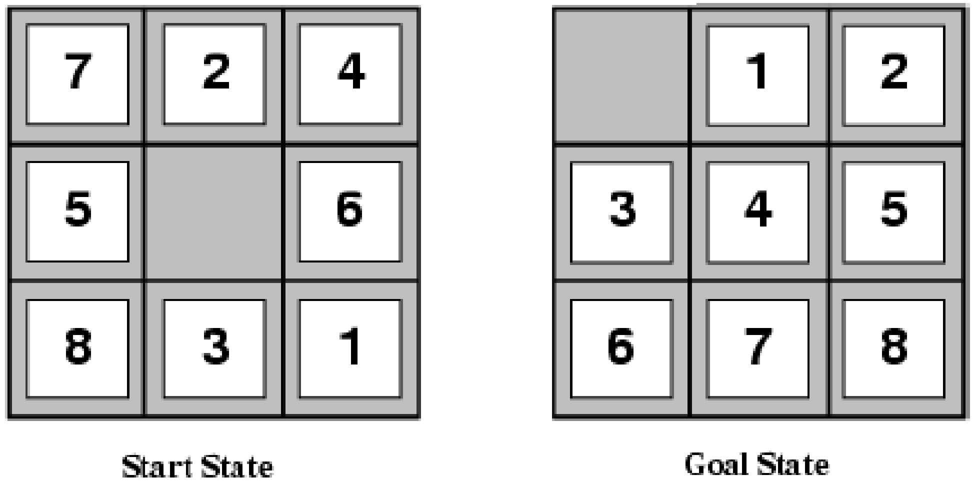
procedure A*;
begin
1. Khởi tạo danh sách L chỉ chứa trạng thái ban đầu;
2. loop do
2.1 if *L rỗng* **then**
    {thông báo thất bại; **stop***};*
2.2 Loại trạng thái u ở đầu danh sách L;
2.3 if *u là trạng thái đích* **then**
    {thông báo thành công; **stop***}*
2.4 **for** *mỗi trạng thái v kề u* **do**
    $\{g(v) \leftarrow g(u) + k(u,v);$
    $f(v) \leftarrow g(v) + h(v);$
    Đặt v vào danh sách L;}
2.5 Sắp xếp L theo thứ tự tăng dần của hàm f sao cho trạng thái có giá trị của hàm f nhỏ nhất ở đầu danh sách;
end*;*

Admissible heuristics

E.g., for the 8-puzzle:

Average solution depth?

Average branching factor?



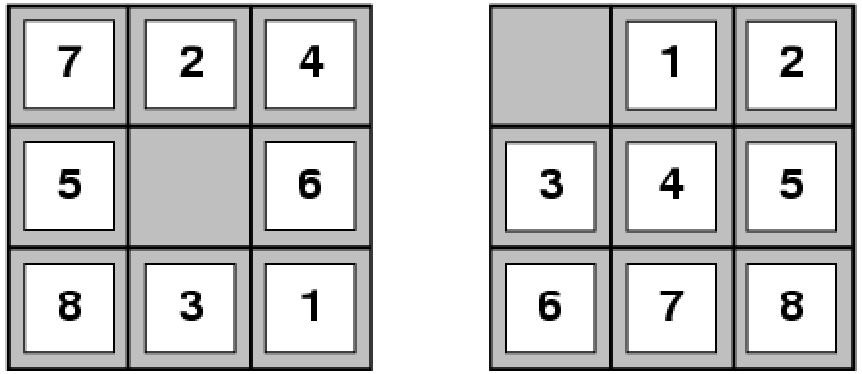
Admissible heuristics

E.g., for the 8-puzzle:

h1(n) = number of misplaced tiles

h2(n) = total Manhattan distance

(i.e., no. of squares from desired location of each tile)



Start State Goal State

h1(S) = ?

h2(S) = ?

Admissible heuristics

E.g., for the 8-puzzle:

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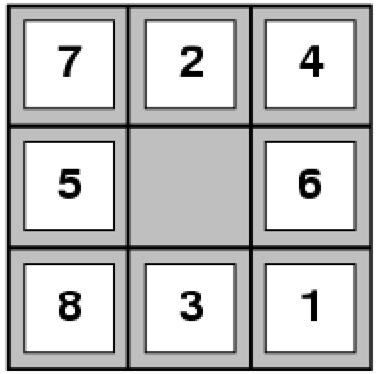
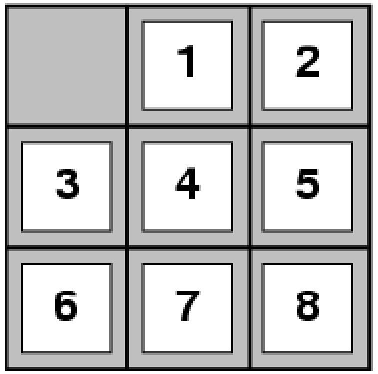
h2(n) = total Manhattan distance

(i.e., no. of squares from desired location of each tile)



Start State



Goal State

h1(S) = ? 8

h2(S) = ? 3+1+2+2+2+3+3+2 = 18

Dominance

- If h2(n) ≥ h1(n) for all n (both admissible)

  then h2 dominates h1

  h2 is better for search

Typical search costs (average number of nodes expanded):

- d=12 IDS = 3,644,035 nodes

  A*(h1) = 227 nodes

  A*(h2) = 73 nodes

- d=24 IDS = too many nodes

  A*(h1) = 39,135 nodes

  A*(h2) = 1,641 nodes

Relaxed problems

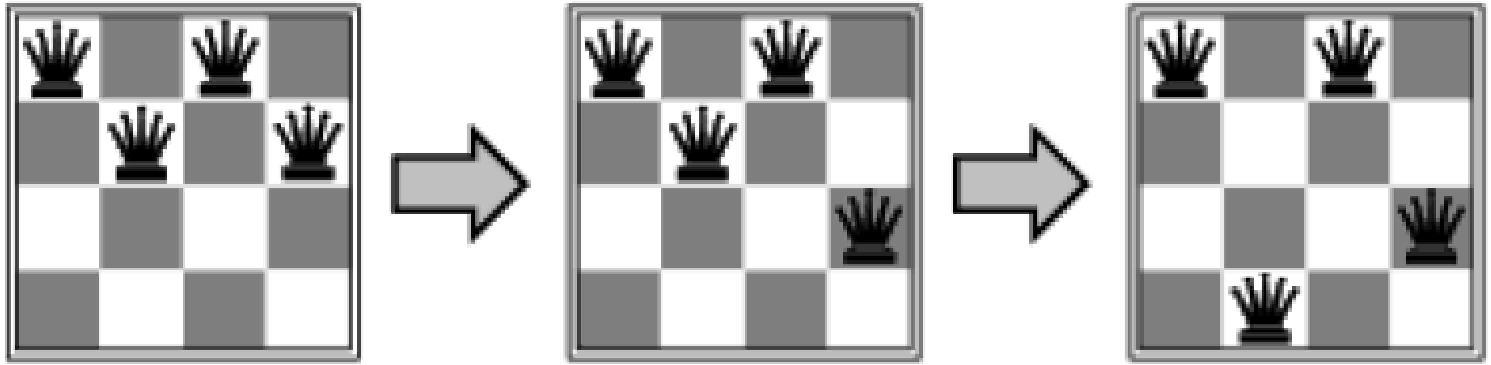
- A problem with fewer restrictions on the actions is called a relaxed problem
- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem
- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then h1(n) gives the shortest solution
- If the rules are relaxed so that a tile can move to any adjacent square, then h2(n) gives the shortest solution

Local search algorithms

- In many optimization problems, the path to the goal is irrelevant; the goal state itself is the solution
- State space = set of "complete" configurations
- Find configuration satisfying constraints, e.g., nqueens
- In such cases, we can use local search algorithms keep a single "current" state, try to improve it

Example: n-queens

- Put n queens on an n × n board with no two queens on the same row, column, or diagonal



Hill-climbing search

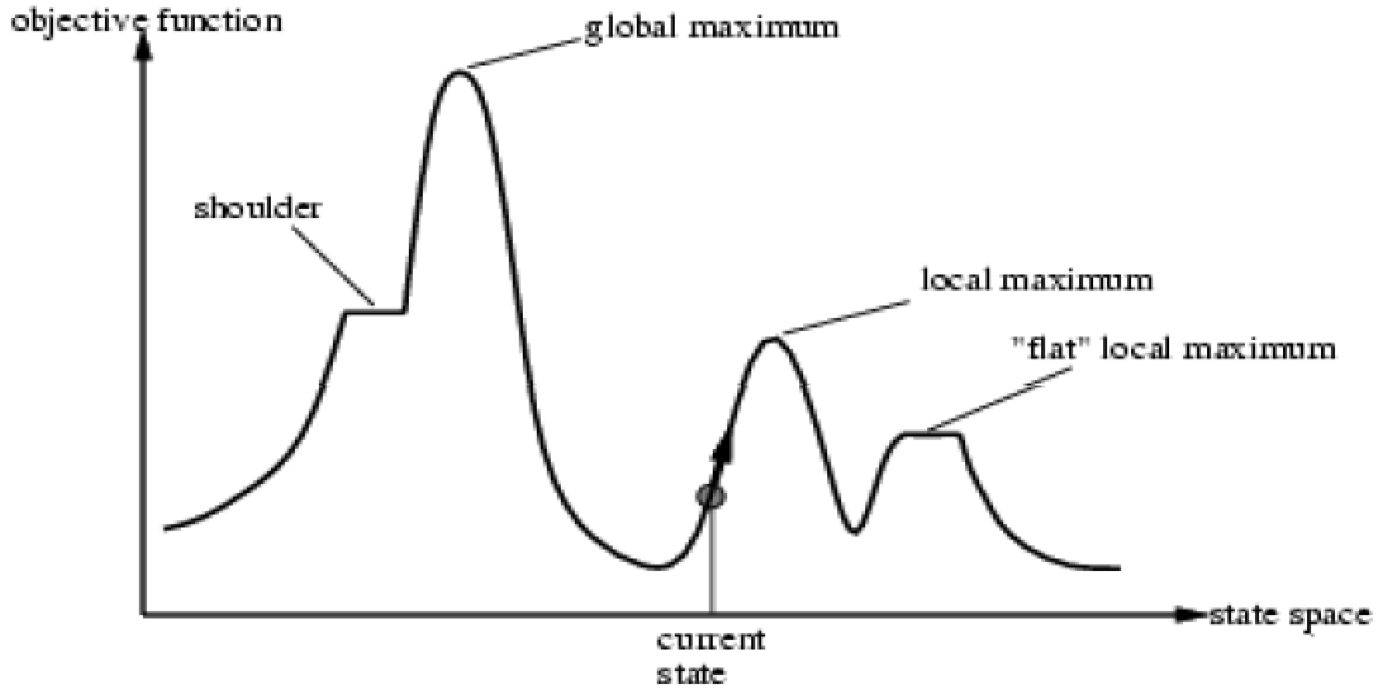
- "Like climbing Everest in thick fog with amnesia"

```
function HILL-CLIMBING(problem) returns a state that is a local maximum
    inputs: problem, a problem
    local variables: current, a node
                     neighbor, a node

    current ← MAKE-NODE(INITIAL-STATE[problem])
    loop do
        neighbor ← a highest-valued successor of current
        if VALUE[neighbor] ≤ VALUE[current] then return STATE[current]
        current ← neighbor
```

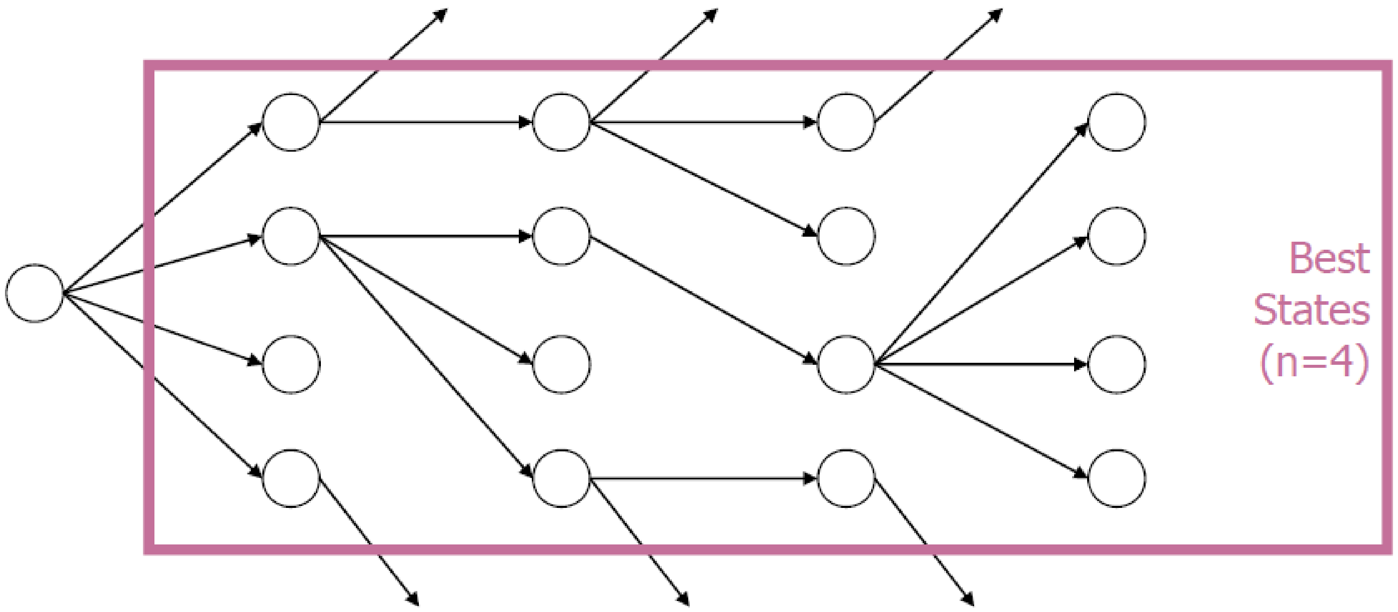
Hi-climbing search

Problem: depending on initial state, can get stuck in local maxima



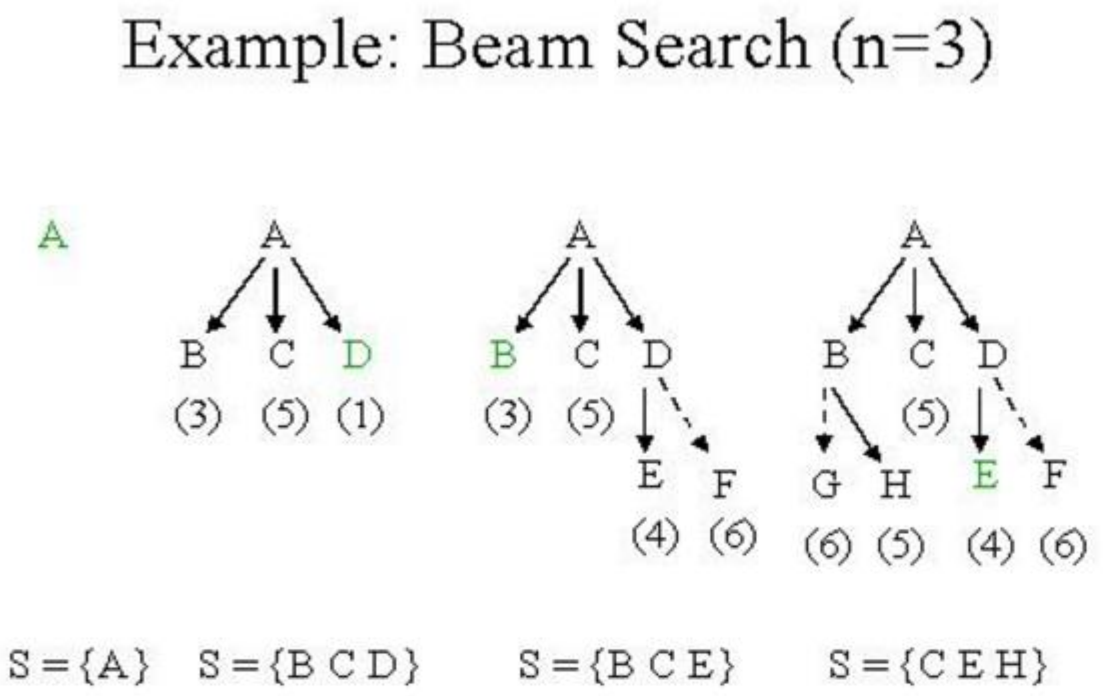
Local Beam search

- Why keep just one best state?



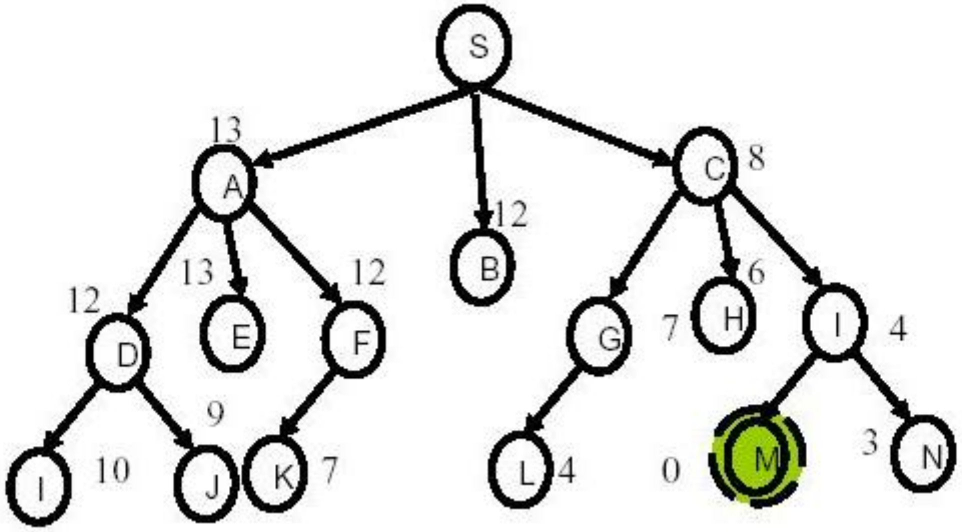
- Can be used with randomization

Local Beam Search: example



Exercise (1)

1) Hill climbing
2) Best First Search
3) Beam search (k=2)



Exercise (2)

E.g., for the 8-puzzle:

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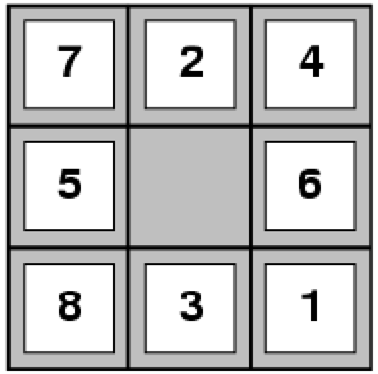
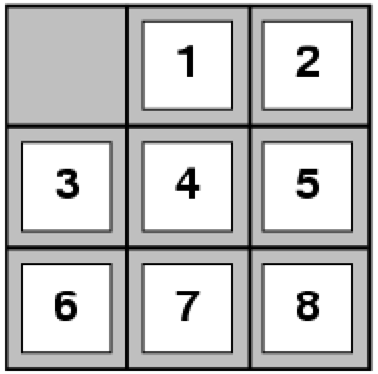
(i.e., no. of squares from desired location of each tile)



Start State

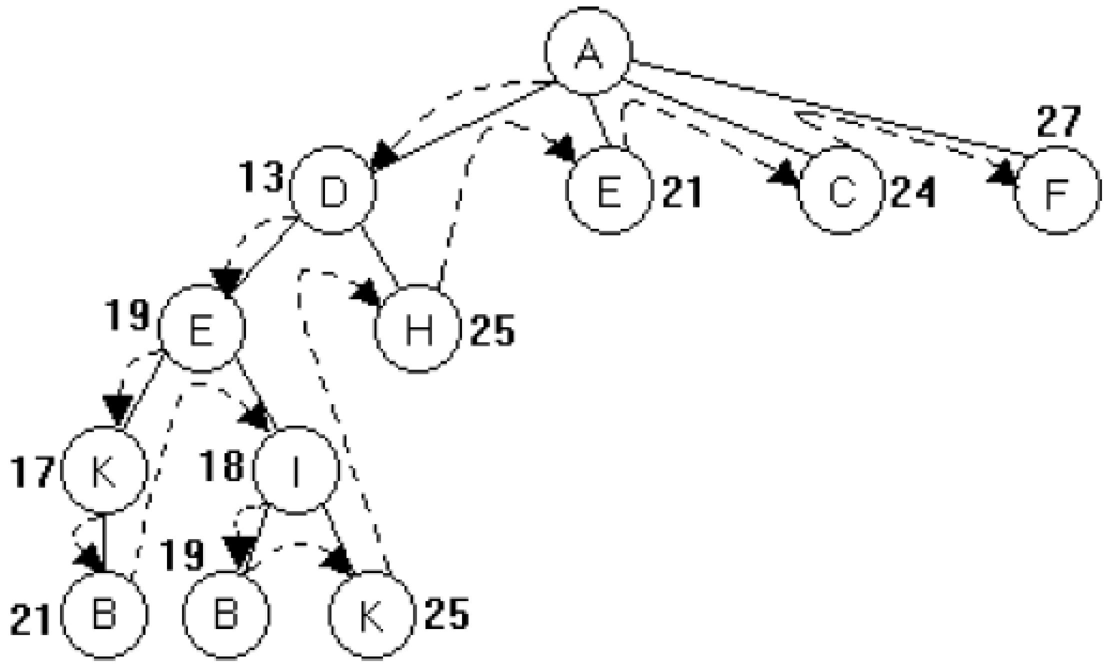


Goal State

h1(S) = ? 8

h2(S) = ? 3+1+2+2+2+3+3+2 = 18

Brand-and-Bound Search (Tìm kiếm nhánh cận)



Simulated annealing search

- Idea: escape local maxima by allowing some "bad“ moves but gradually decrease their frequency

```
function SIMULATED-ANNEALING( problem, schedule) returns a solution state
   inputs: problem, a problem
           schedule, a mapping from time to "temperature"
   local variables: current, a node
                    next, a node
                    T, a "temperature" controlling prob. of downward steps

   current ← MAKE-NODE(INITIAL-STATE[problem])
   for t ← 1 to ∞ do
       T ← schedule[t]
       if T = 0 then return current
       next ← a randomly selected successor of current
       ΔE ← VALUE[next] – VALUE[current]
       if ΔE > 0 then current ← next
       else current ← next only with probability e^(ΔE/T)
```

Properties of simulated annealing search

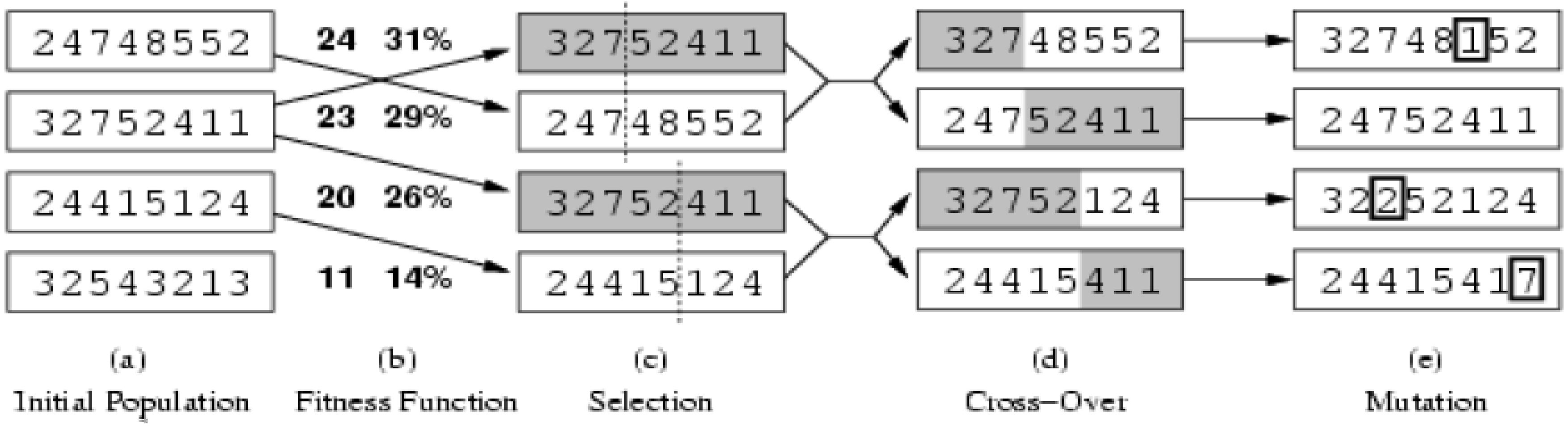
- One can prove: If T decreases slowly enough, then simulated annealing search will find a global optimum with probability approaching 1

- Widely used in VLSI layout, airline scheduling, etc

Genetic algorithms

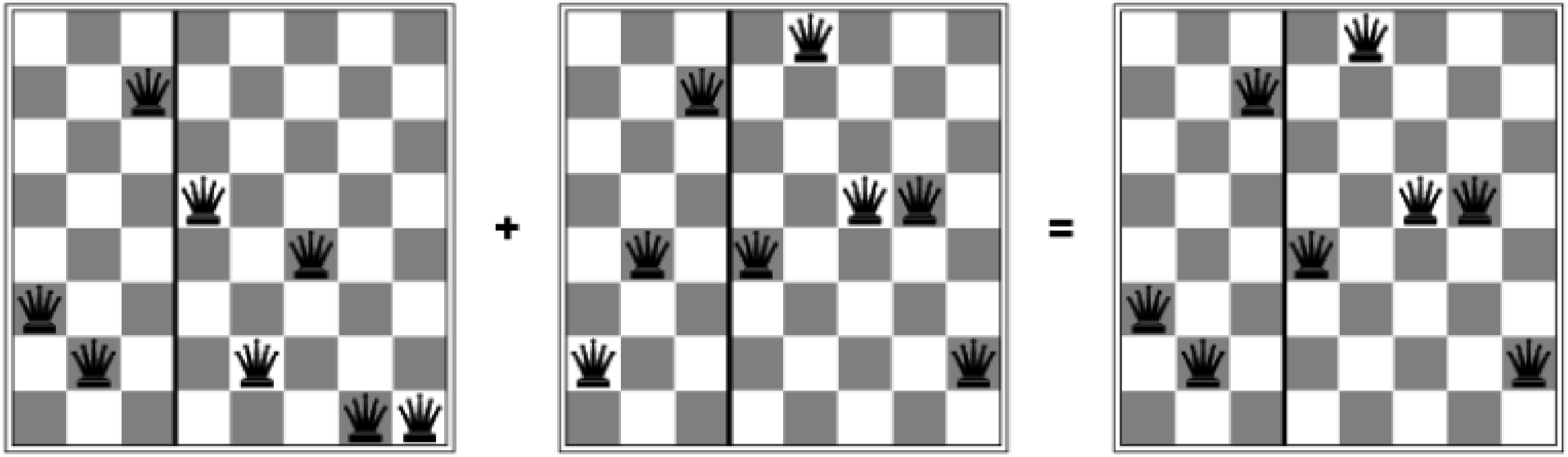
- A successor state is generated by combining two parent states
- Start with k randomly generated states (population)
- A state is represented as a string over a finite alphabet (often a string of 0s and 1s)
- Evaluation function (fitness function). Higher values for better states
- Produce the next generation of states by selection, and mutation

Genetic algorithms



- Fitness function: number of non-attacking pairs of queens (min = 0, max = 8 × 7/2 = 28)
- 24/(24+23+20+11) = 31%
- 23/(24+23+20+11) = 29% etc.

Genetic algorithms



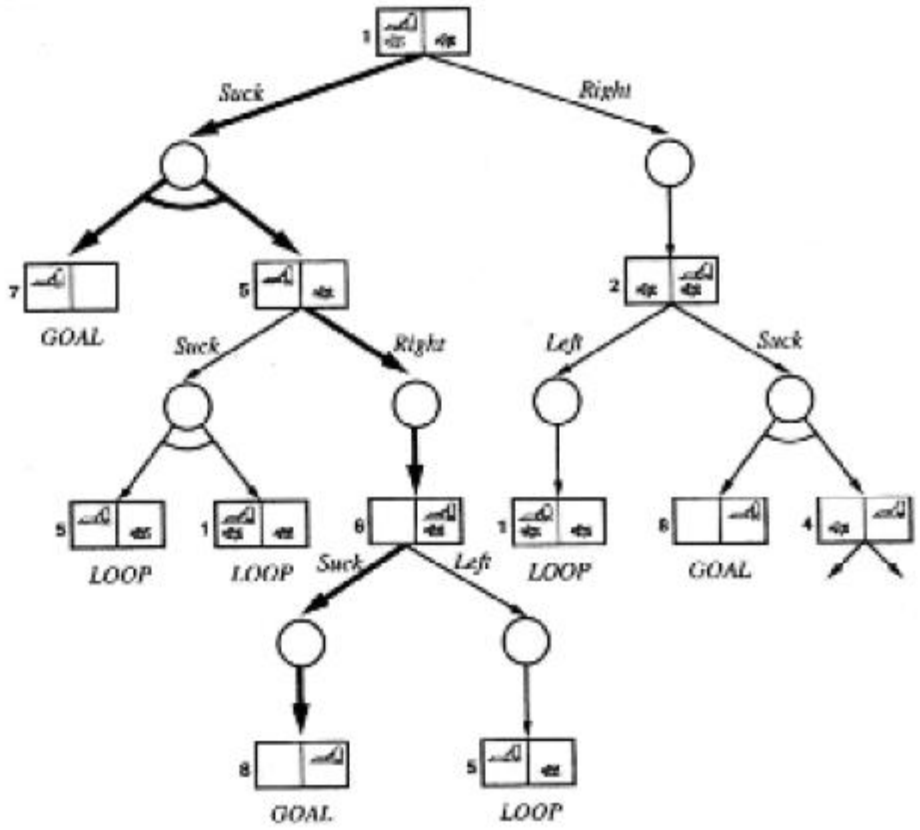
Search w/ non-determinism

- Fully observable, deterministic environments
  - Sensors, precepts no use

- Consider erratic actuators
  - Action leads to a **set** of possible states
  - Plan will not be a set sequence, may have loops contingencies (if-then-else)

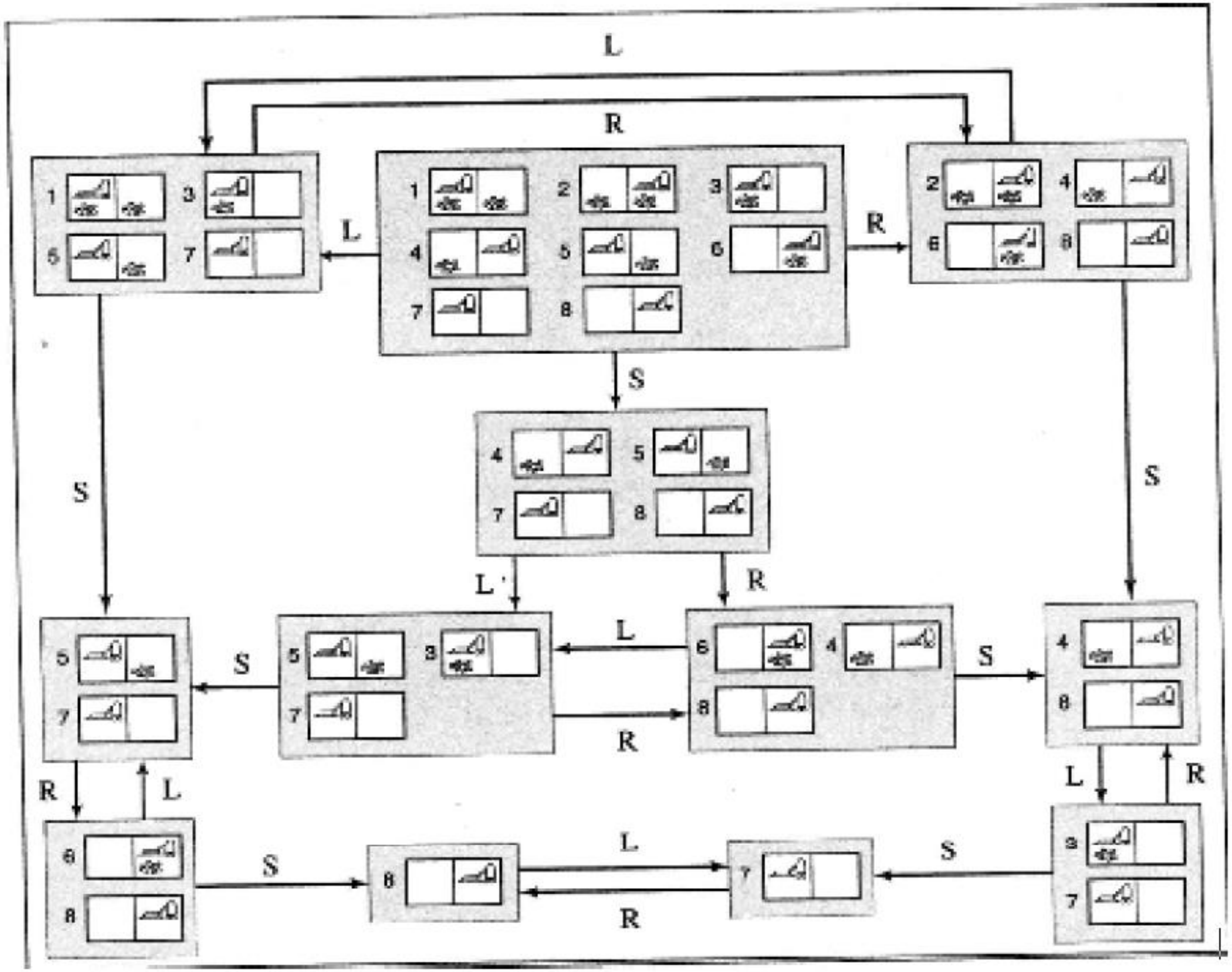
And-Or Search Tree

What does the “LOOP” label mean here?



Search w/ partial observations

- Conformant problem – no observations
  - Useful! Solutions are independent of initial state
  - **Coerce** the state space into a subset of possible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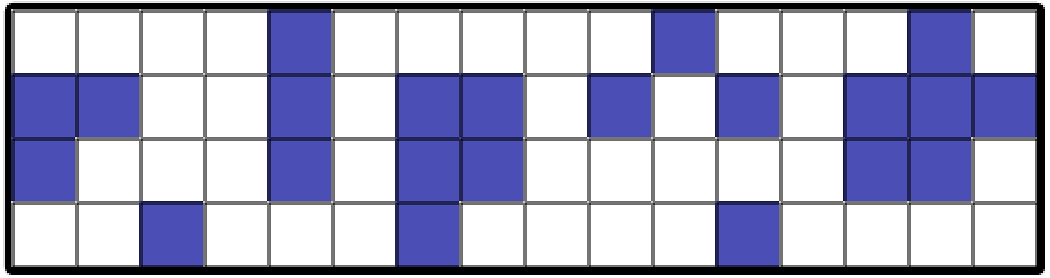
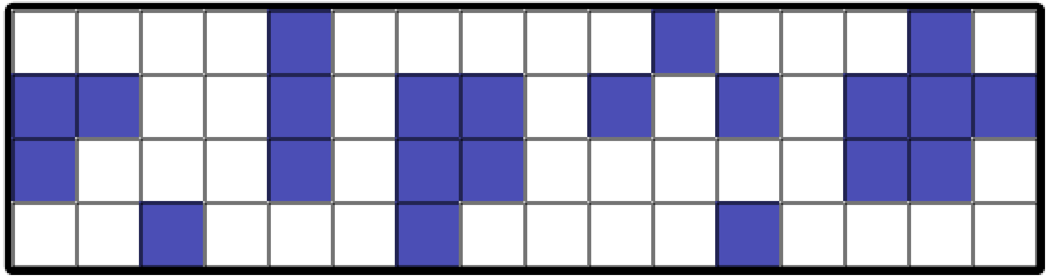
Localization

What about a really big set of initial?

Initial State:



After observing NSW:



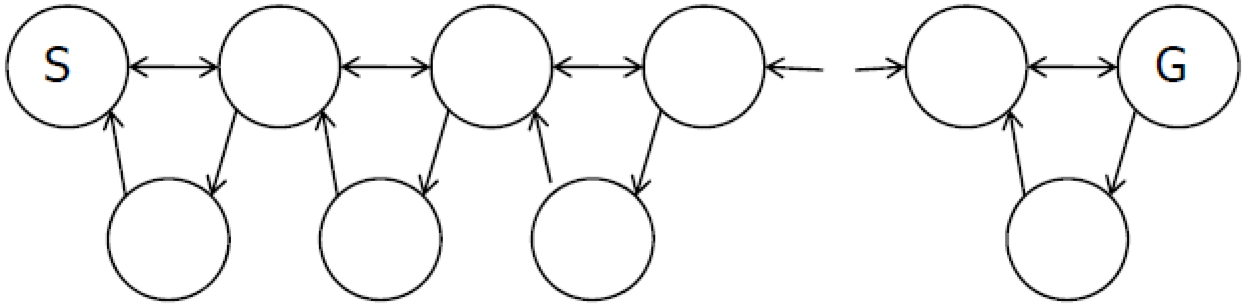
Online search and exploration

- Many problems are offline
  - Do search for action and then perform action

- Online search interleave search & execution
  - Necessary for exploration problems
  - New observations only possible after acting

Exploratory Search

- In an unknown state space, how to pick an action?
  - Any random action will do … but



- Favor those that allow more exploration of the search space
  - Graph-search to track of states previously seen

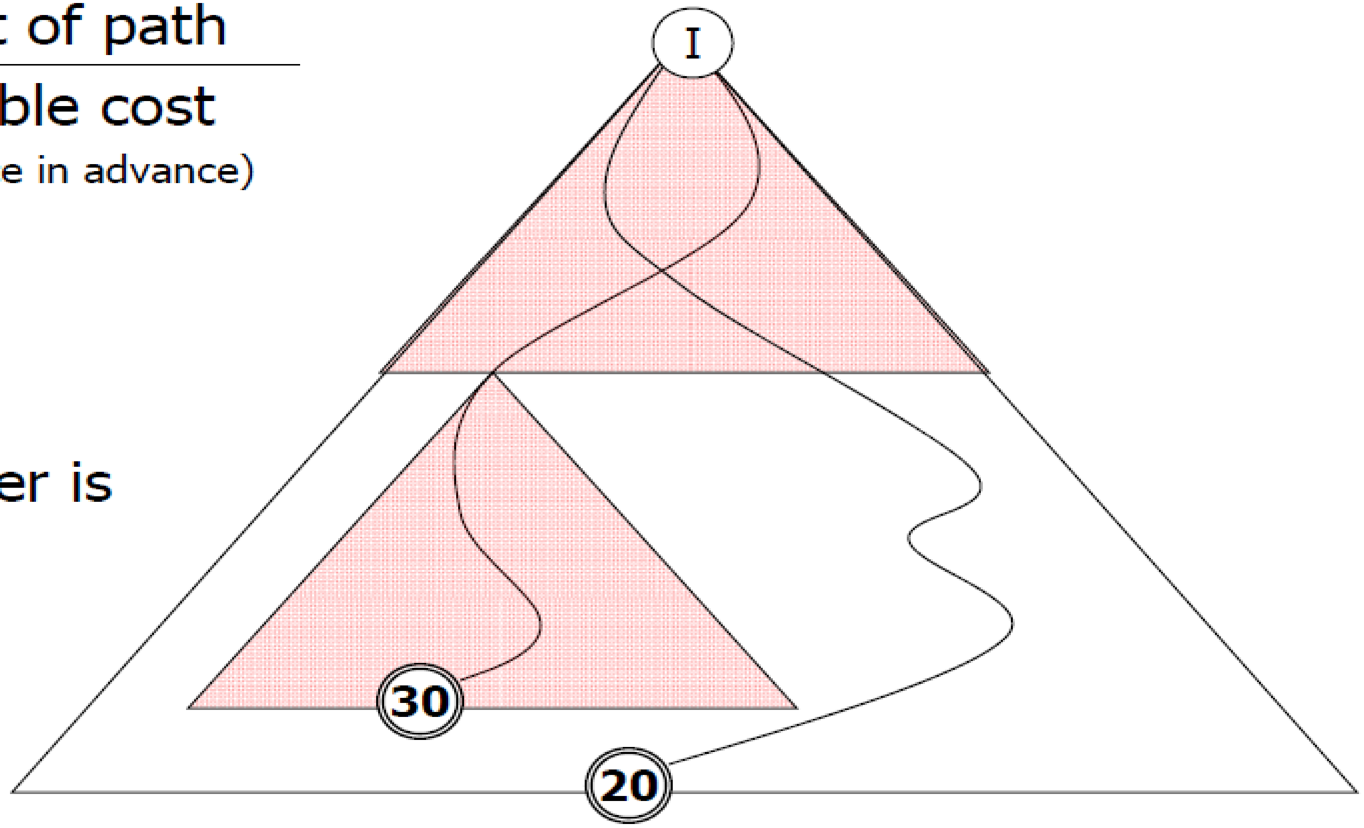
Assessing Online Agents: Competitive Ratio

$$\frac{\text{Actual cost of path}}{\text{Best possible cost}}$$

(if agent knew space in advance)

30/20 = 1.5

For cost, lower is better



Exploration problems

- Exploration problems: agent physically in some part of the state space.
  - e.g. solving a maze using an agent with local wall sensors
  - Sensible to expand states easily accessible to agent (i.e. local states)
    - Local search algorithms apply (e.g., hill-climbing)