Solving problems by searching

LESSON 2

Reading

Chapter 3

Recap from lecture 1

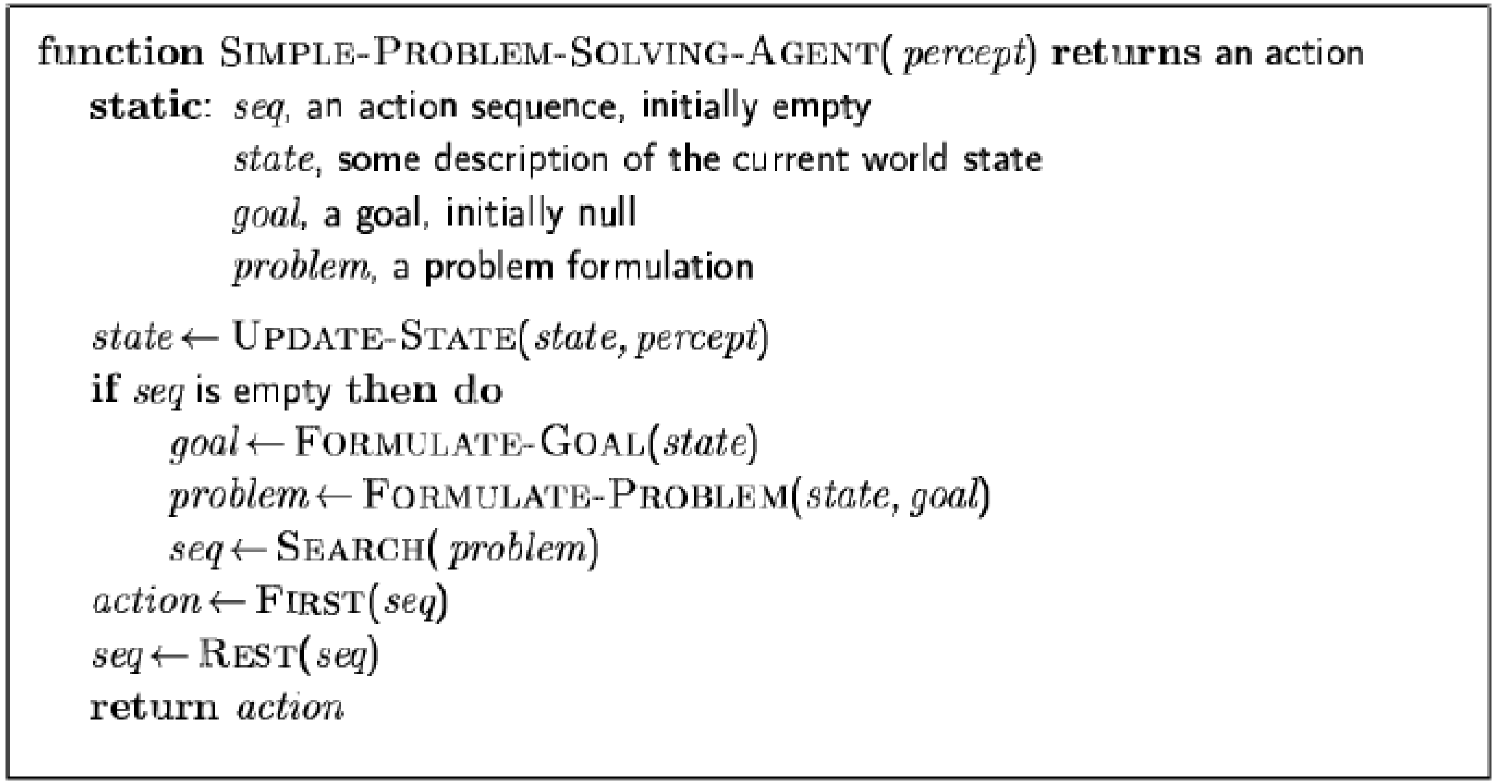
- Introduction
- Agents
- PEAS
- Environment

- Rational Agents – F : mapping P* to A
- Agent architectures: reflex, model, learning

Outline

- Problem solving agents
- Problem types
- Problem formulation
- Example problems
- Basic search algorithms

Problem-solving agents



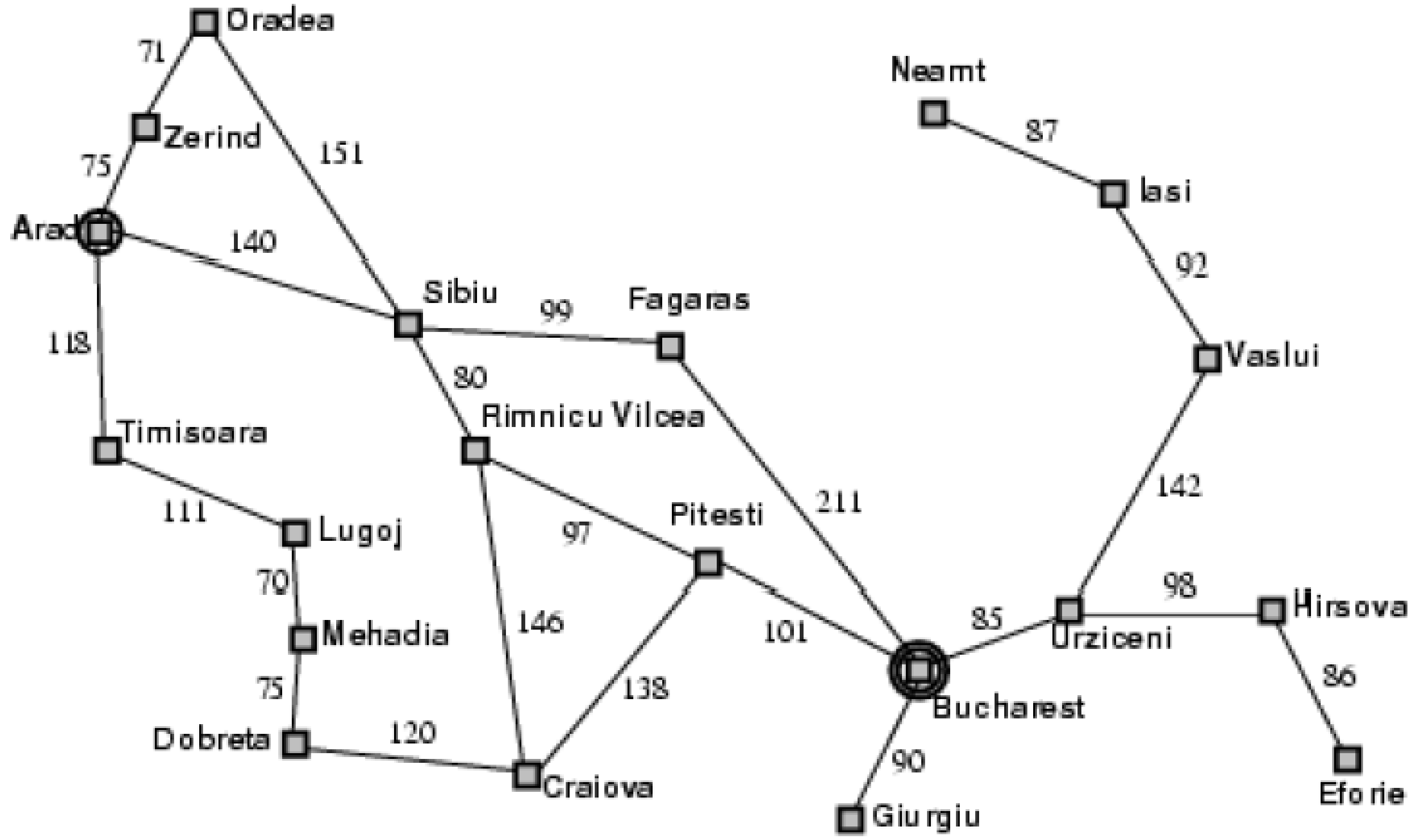
```
function SIMPLE-PROBLEM-SOLVING-AGENT( percept) returns an action
    static: seq, an action sequence, initially empty
            state, some description of the current world state
            goal, a goal, initially null
            problem, a problem formulation

    state ← UPDATE-STATE(state, percept)
    if seq is empty then do
        goal ← FORMULATE-GOAL(state)
        problem ← FORMULATE-PROBLEM(state, goal)
        seq ← SEARCH( problem)
    action ← FIRST(seq)
    seq ← REST(seq)
    return action
```

Example: Romania

- On holiday in Romania; currently in Arad.
- Flight leaves tomorrow from Bucharest
- Formulate goal:
  - be in Bucharest
- Formulate problem:
  - states: various cities
  - actions: drive between cities
- Find solution:
  - Sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest

Example: Romania



Problem types

- Deterministic, fully observable -> single-state problem
  - Agent knows exactly which state it will be in; solution is a sequence
- Non-observable -> sensorless problem (conformant problem)
  - Agent may have no idea where it is; solution is a sequence
- Nondeterministic and/or partially observable -> contingency problem
  - percepts provide new information about current state
  - often interleave search, execution
- Unknown state space -> eploration problem

Example: vacuum world

- Single-state, start in #5.

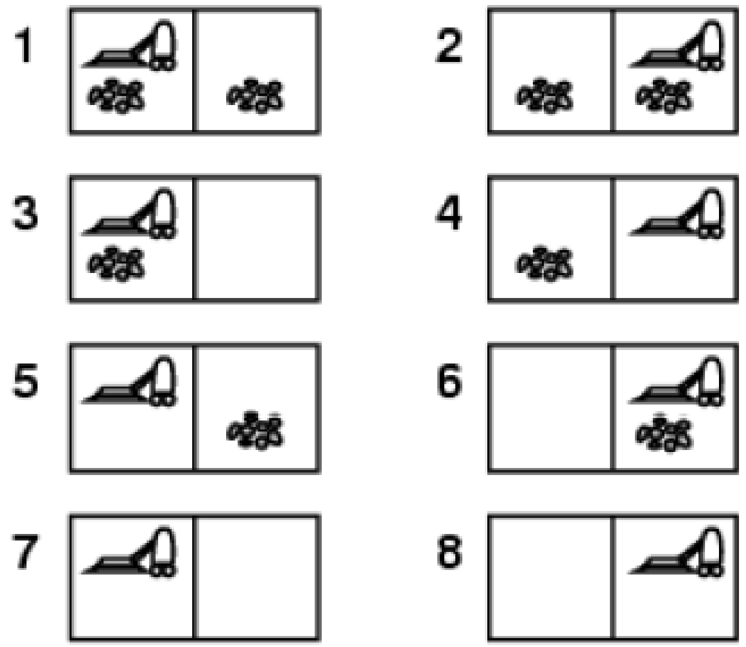
Solution? [*Right, Suck*]

- Sensorless, start in

{*1,2,3,4,5,6,7,8*} e.g.,

Right goes to {*2,4,6,8*}

Solution?



Example: vacuum world

- Sensorless, start in

{*1,2,3,4,5,6,7,8*} e.g.,

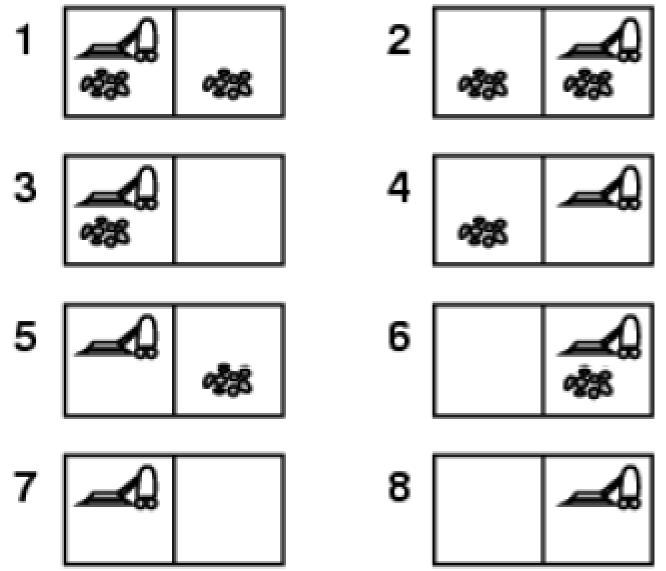
Right goes to {*2,4,6,8*}

Solution?

[Right, Suck, Left, Suck]

- Contingency
  - Nondeterministic: Suck may dirty a clean carpet
  - Partially observable: location, dirt at current location.
  - Percept: [L, Clean], i.e., start in #5 or #7

Solution?



Example: vacuum world

- Sensorless, start in

{*1,2,3,4,5,6,7,8*} e.g.,

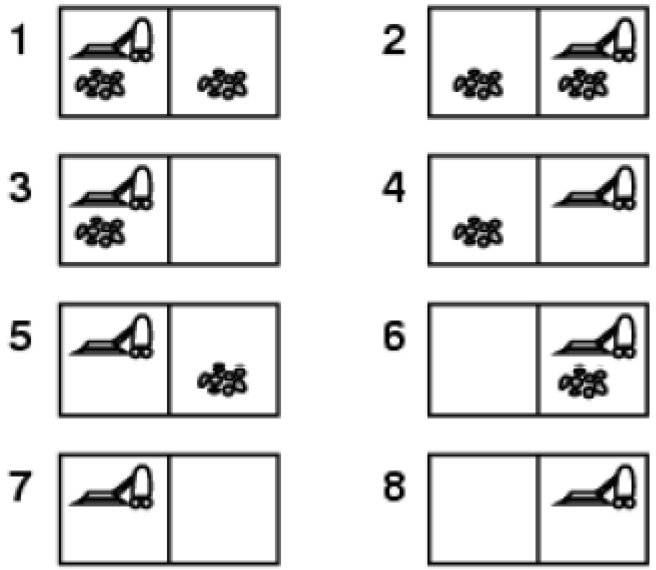
Right goes to {*2,4,6,8*}

Solution?

[Right, Suck, Left, Suck]

- Contingency
  - Nondeterministic: Suck may dirty a clean carpet
  - Partially observable: location, dirt at current location.
  - Percept: [L, Clean], i.e., start in #5 or #7

Solution? [Right, **if** dirt **then** Suck]



Single-state problem formulation

A problem is defined by four items:

1. initial state e.g., "at Arad"

2. actions or successor function S(x) = set of action–state pairs
   - e.g., S(Arad) = {<Arad ⍰ Zerind, Zerind>, … }

3. goal test, can be
   - explicit, e.g., x = "at Bucharest"
   - implicit, e.g., Checkmate(x)

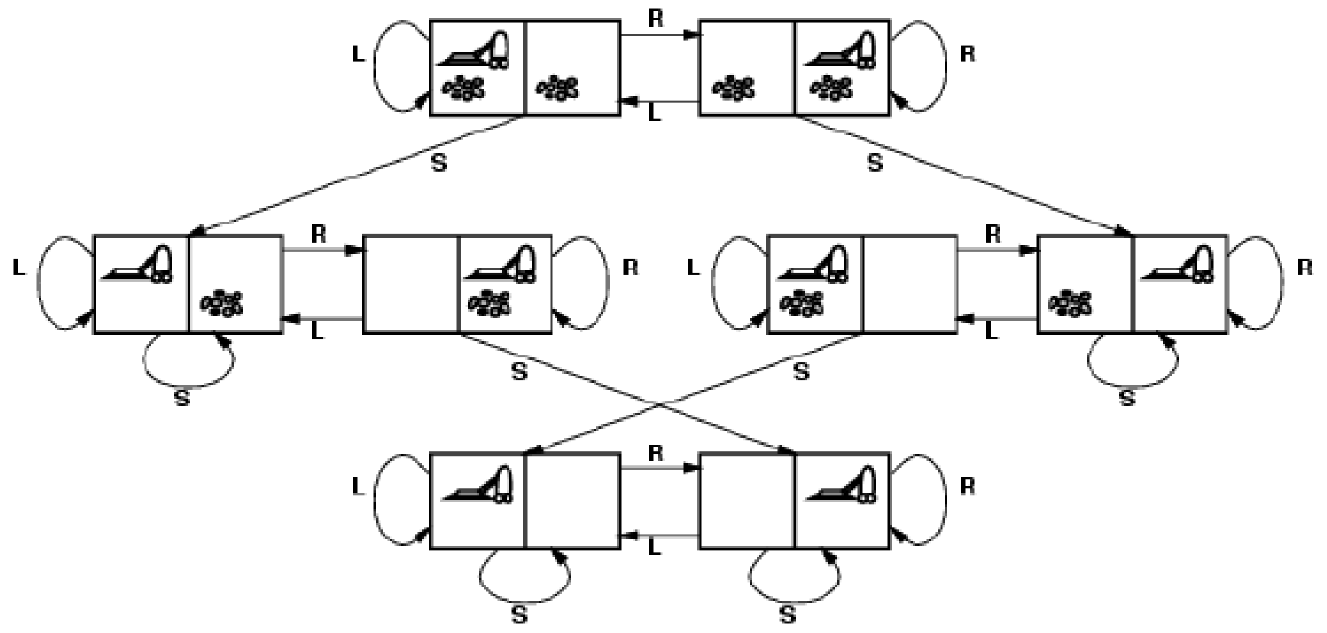
4. path cost (additive)
   - e.g., sum of distances, number of actions executed, etc.
   - c(x,a,y) is the step cost, assumed to be ≥ 0

A solution is a sequence of actions leading from the initial state to a goal state

Selecting a state space

- Real world is absurdly complex
  - state space must be abstracted for problem solving
- (Abstract) state = set of real states
- (Abstract) action = complex combination of real actions
  - e.g., "Arad -> Zerind" represents a complex set of possible routes, detours, rest stops, etc.
- For guaranteed realizability, any real state "in Arad“ must get to some real state "in Zerind"
- (Abstract) solution =
  - set of real paths that are solutions in the real world
- Each abstract action should be "easier" than the original

Vacuum world state space graph



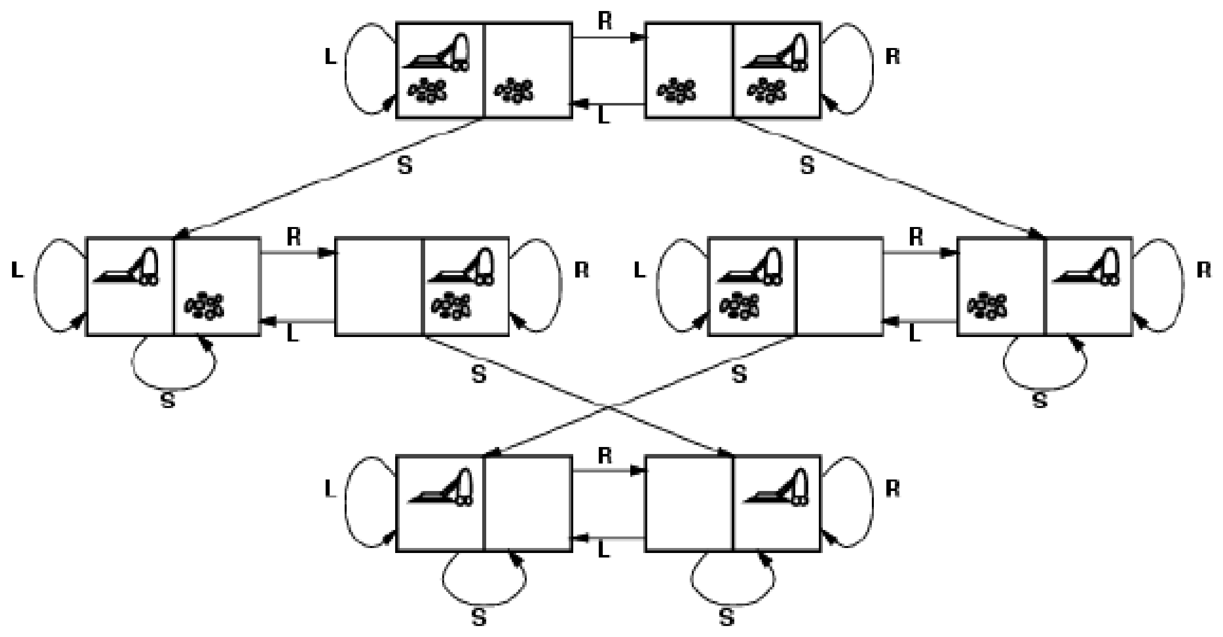
States?

Actions?

Goal test?

Path cost?

Vacuum world state space graph



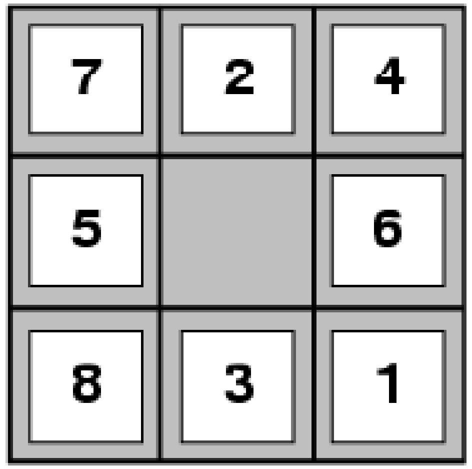
States? Integer dirt and robot location

Actions? Left, Right, Suck

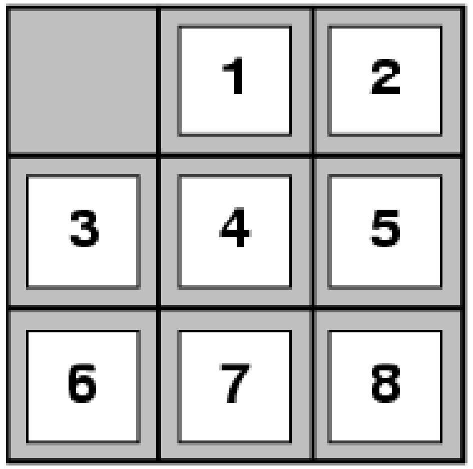
Goal test? No dirt at all locations

Path cost? 1 per action

Example: The 8-puzzle



Start State



Goal State

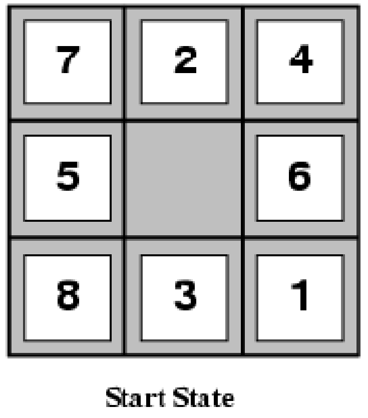
States?

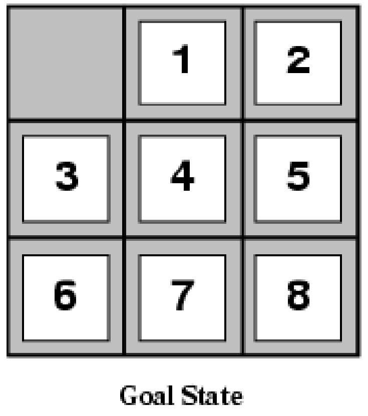
Actions?

Goal test?

Path cost?

Example: The 8-puzzle





States? Locations of tiles

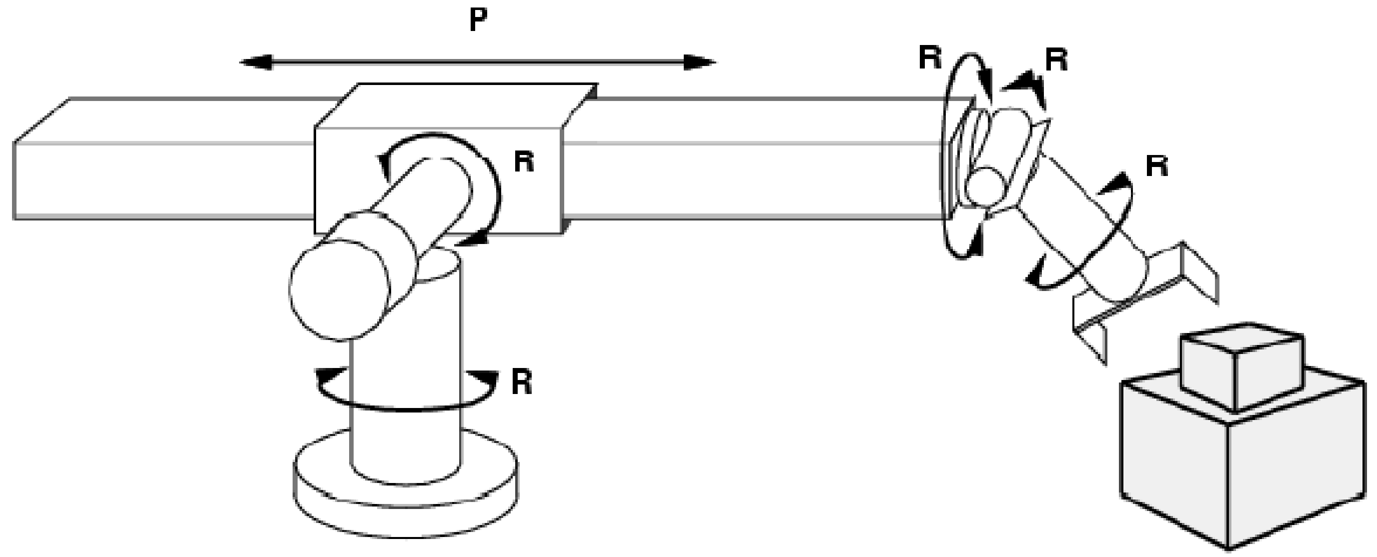
Actions? Move bank left, right, up, down

Goal test? = goal state (given)

Path cost? 1 per move

Note that: optimal solution of n-Puzzle family is NP-hard

Example: robotic assembly



States?

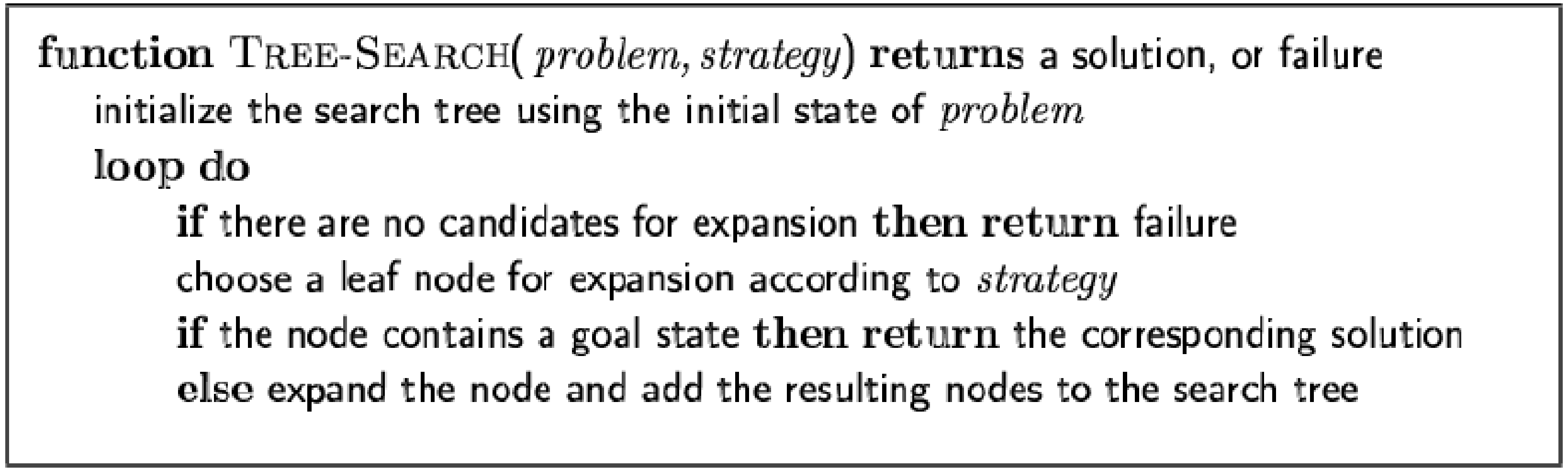
Actions?

Goal test?

Path cost?

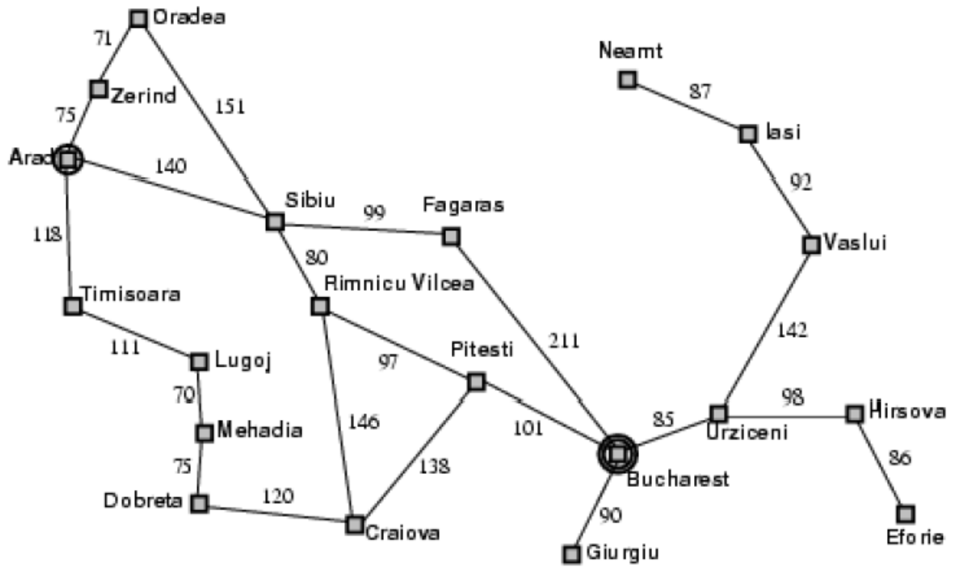
Tree search algorithms

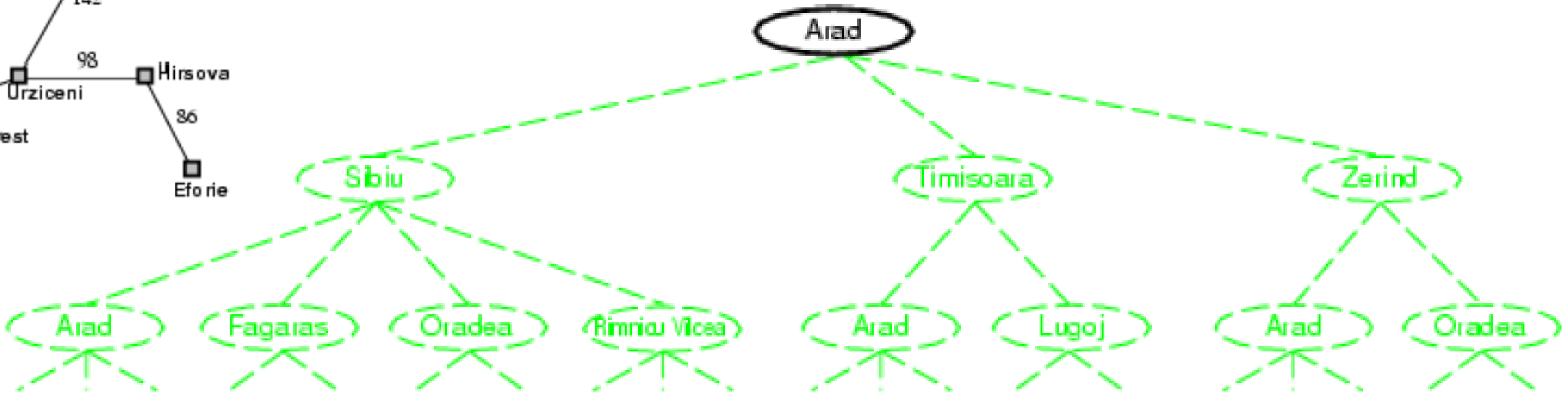
- Basic idea:
  - offline, simulated exploration of state space by generating successors of already-explored states (a.k.a.~expanding states)



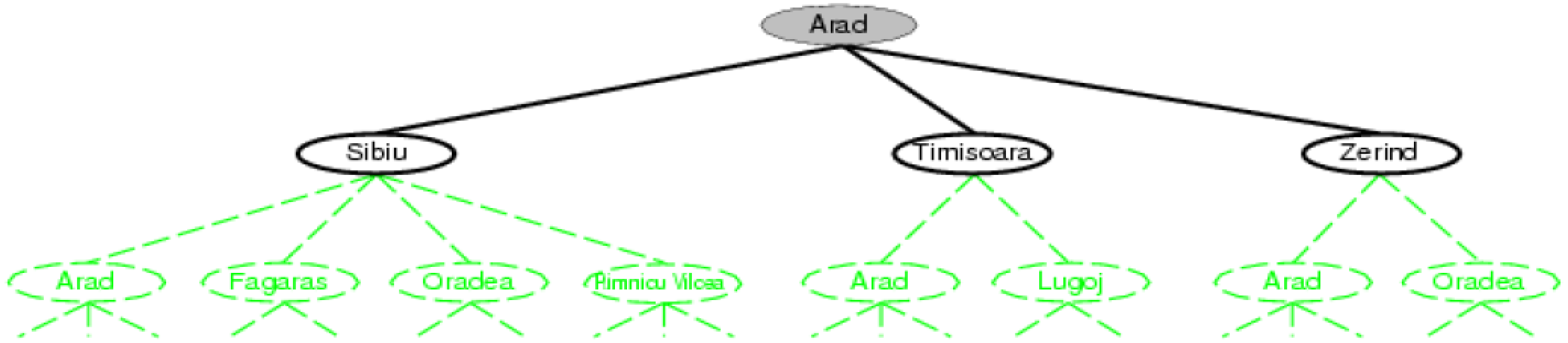
```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
```

Tree search example

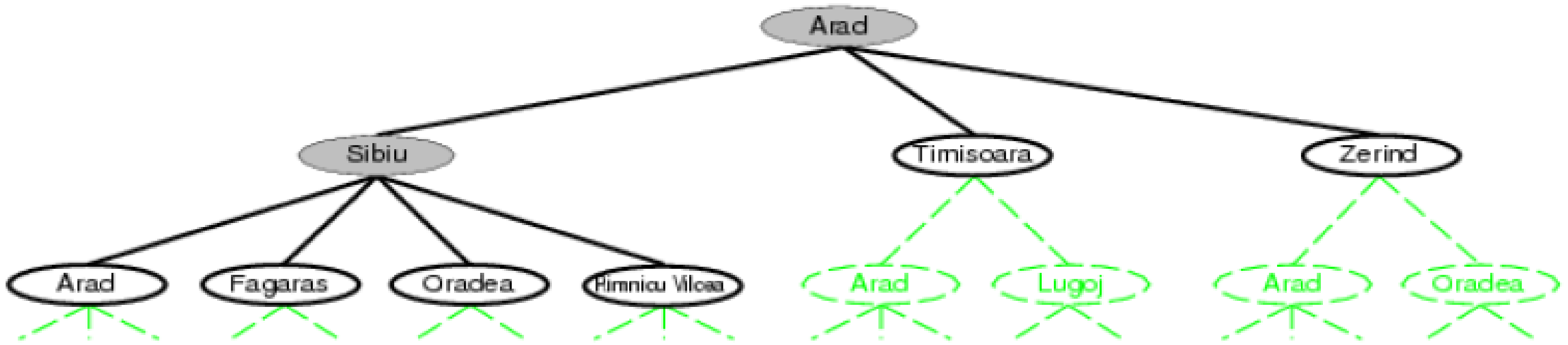




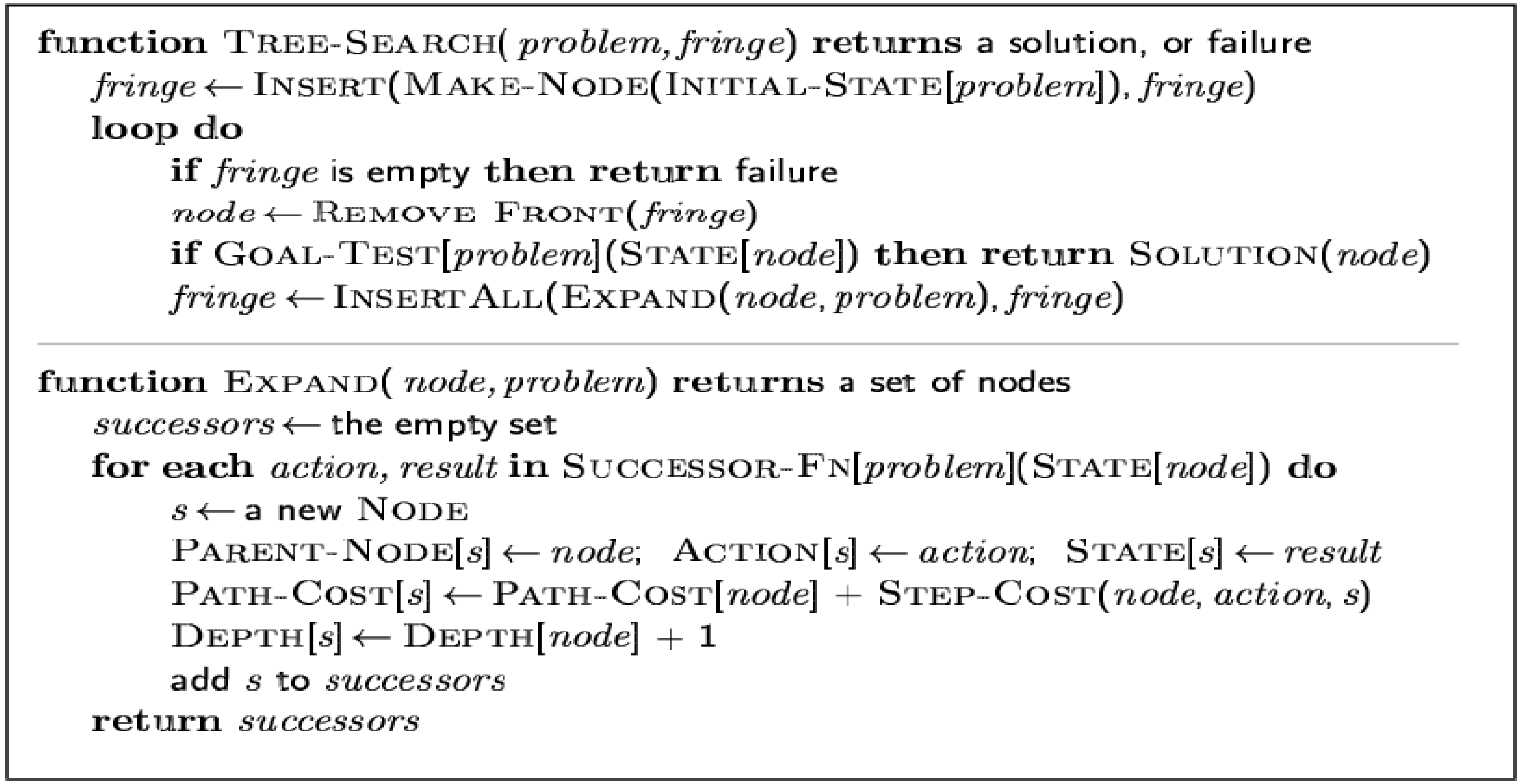
Tree search example



Tree search example



Implementation: general tree search

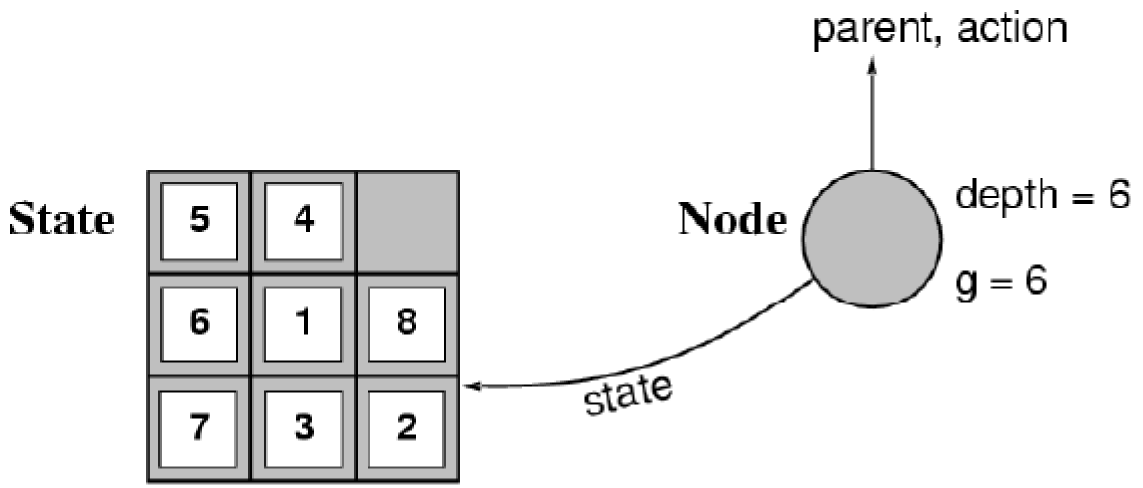


```
function TREE-SEARCH( problem, fringe) returns a solution, or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE FRONT(fringe)
        if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
        fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

```
function EXPAND( node, problem) returns a set of nodes
    successors ← the empty set
    for each action, result in SUCCESSOR-FN[problem](STATE[node]) do
        s ← a new NODE
        PARENT-NODE[s] ← node;   ACTION[s] ← action;   STATE[s] ← result
        PATH-COST[s] ← PATH-COST[node] + STEP-COST(node, action, s)
        DEPTH[s] ← DEPTH[node] + 1
        add s to successors
    return successors
```

Implementation: state vs. nodes

- A state is a (representation of) a physical configuration
- A node is a data structure constituting part of a search tree includes state, parent node, action, path cost g(x), depth



- The Expand function creates new nodes, filling in the various fields and using the SuccessorFn of the problem to create the corresponding states.

Search strategies

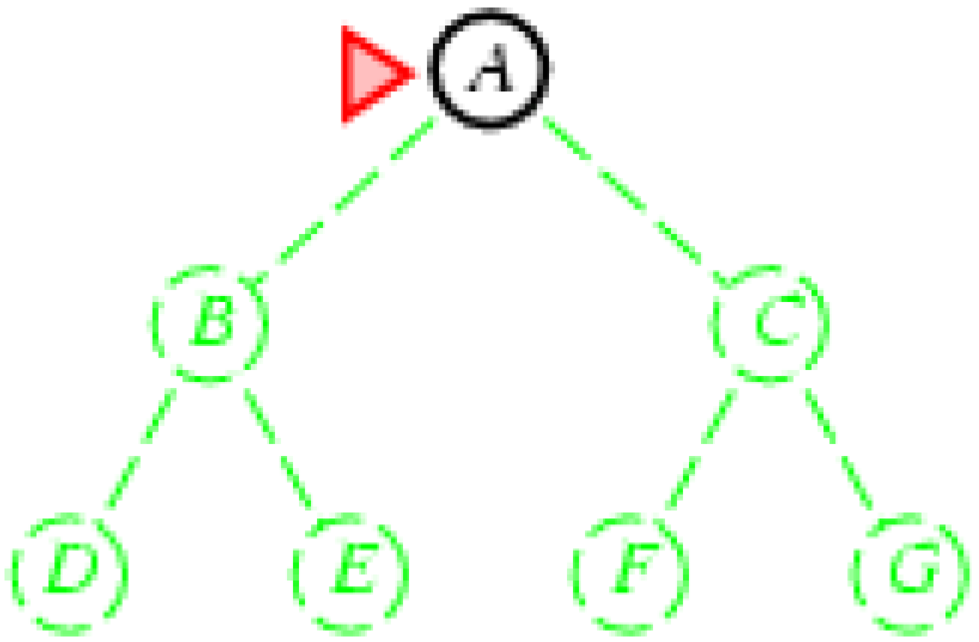
- A search strategy is defined by picking the order of node expansion
- Strategies are evaluated along the following dimensions:
  - completeness: does it always find a solution if one exists?
  - time complexity: number of nodes generated
  - space complexity: maximum number of nodes in memory
  - optimality: does it always find a least-cost solution?
- Time and space complexity are measured in terms of
  - b: maximum branching factor of the search tree
  - d: depth of the least-cost solution
  - m: maximum depth of the state space (may be ∞)

Uninformed search strategies

- Uninformed search strategies use only the information available in the problem definition
- Breadth-first search
- Uniform-cost search
- Depth-first search
- Depth-limited search
- Iterative deepening search

Breadth-first search

- Expand shallowest unexpanded node
- Implementation:
  - fringe is a FIFO queue, i.e., new successors go at end



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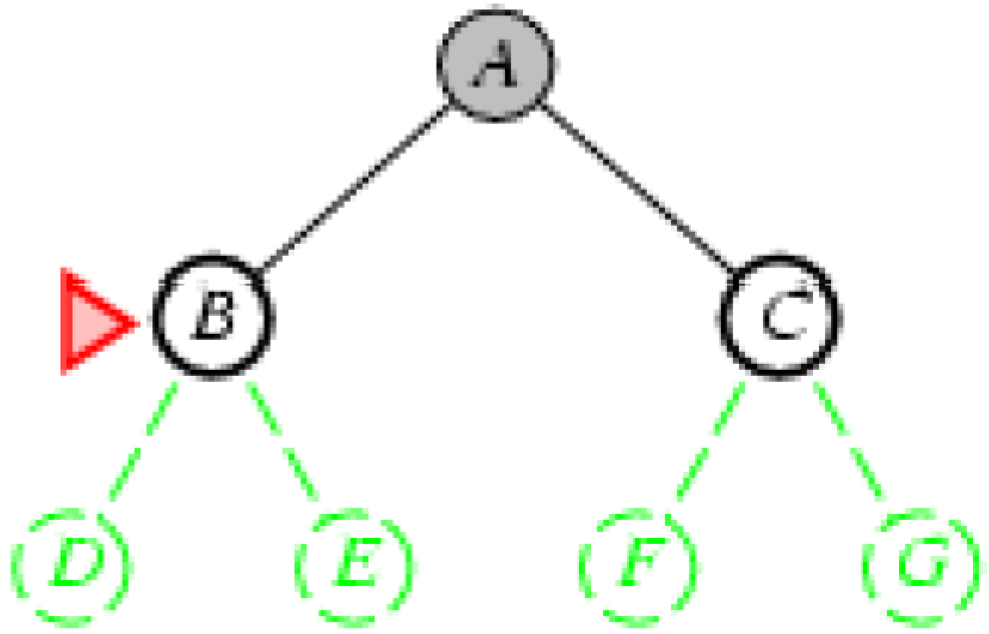

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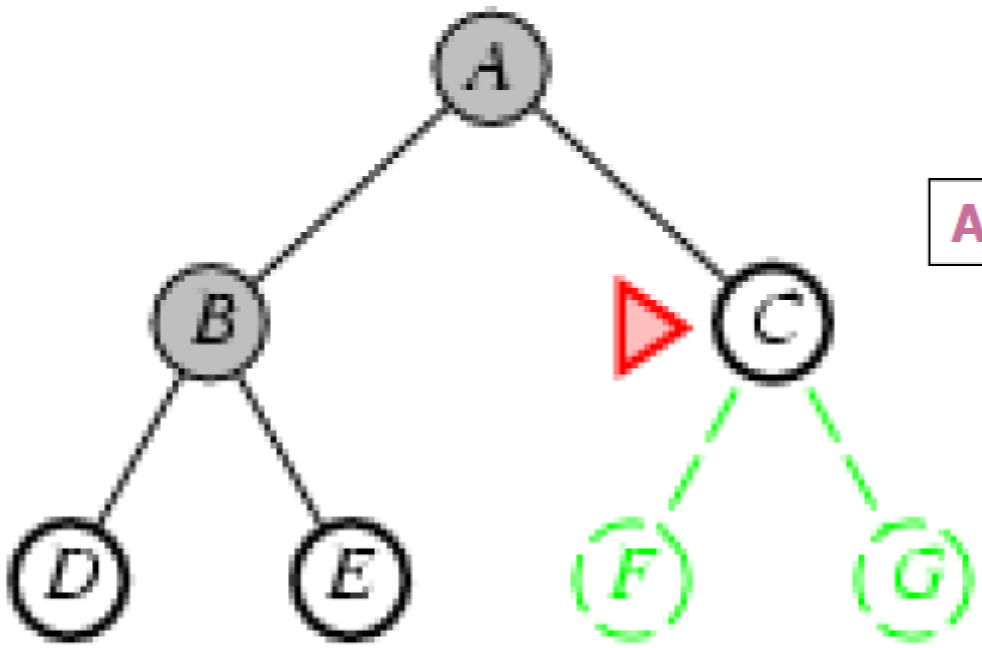

Animation time!

Breadth-first search

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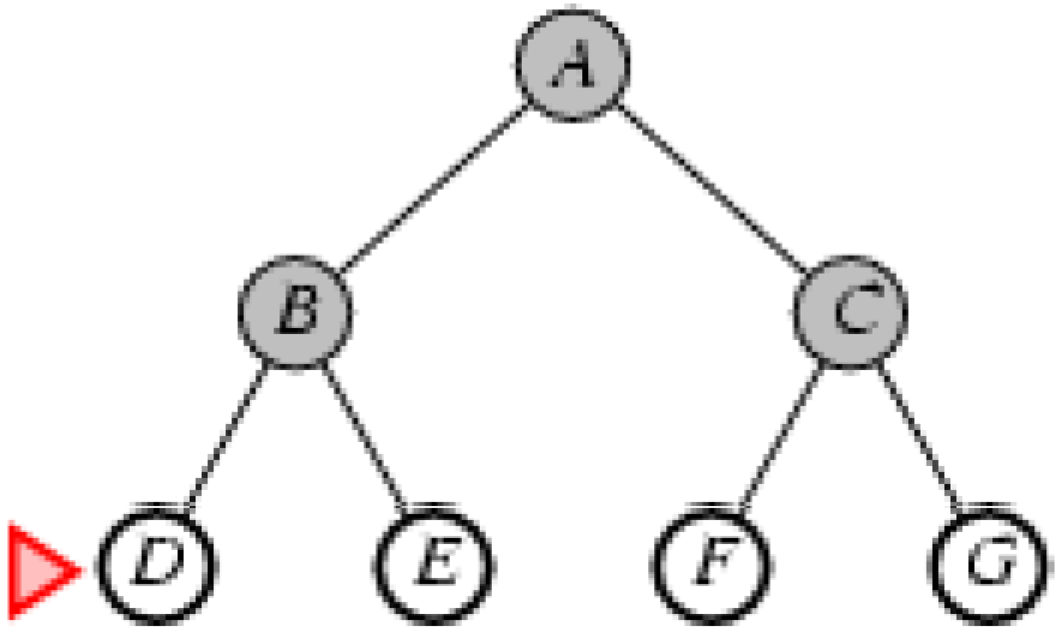


Properties of breadth-first search

- Complete?
- Time?
- Space?
- Optimal?

Uniform-cost search

- Expand least-cost unexpanded node
- Implementation:
  - Fringe = queue ordered by path cost
- Equivalent to breadth-first if step costs all equal

Complete?

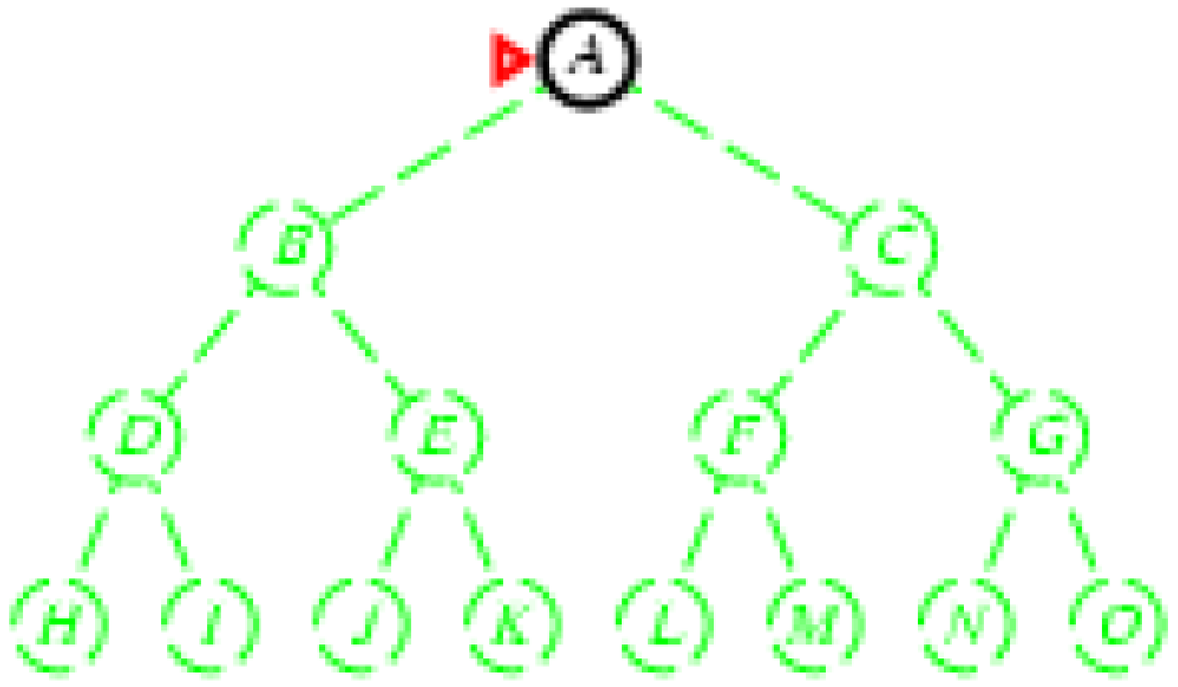
Time?

Space?

Optimal

Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - fringe = LIFO queue, i.e., put successors at front



Depth-first search

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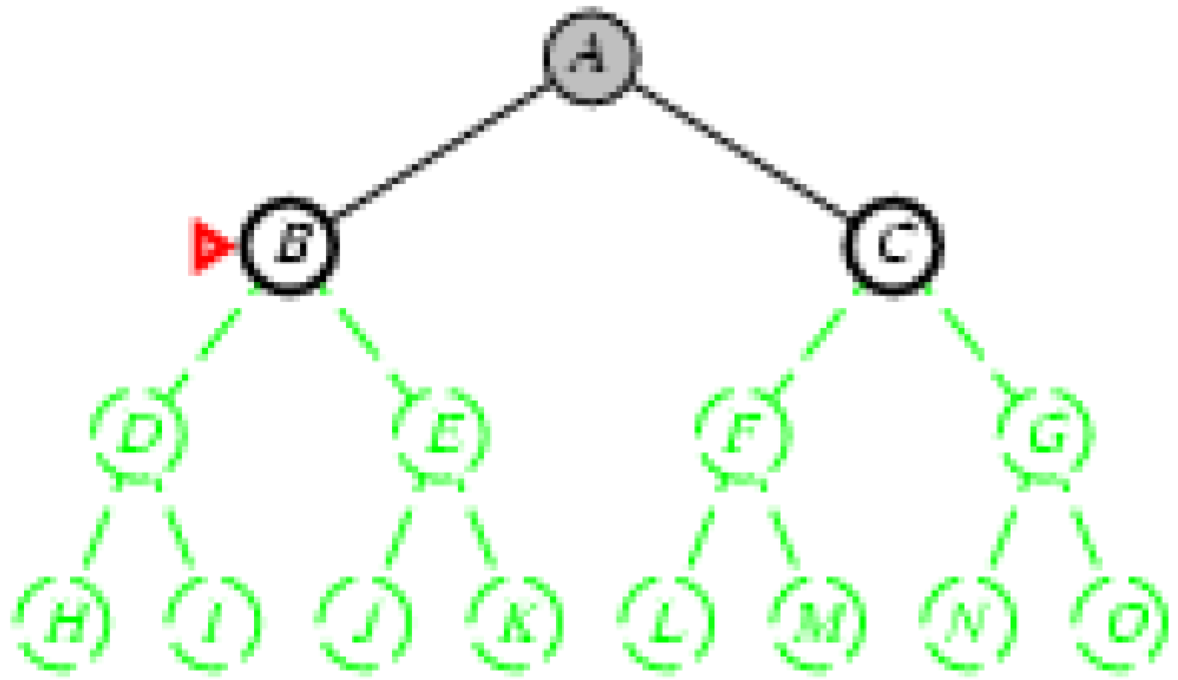

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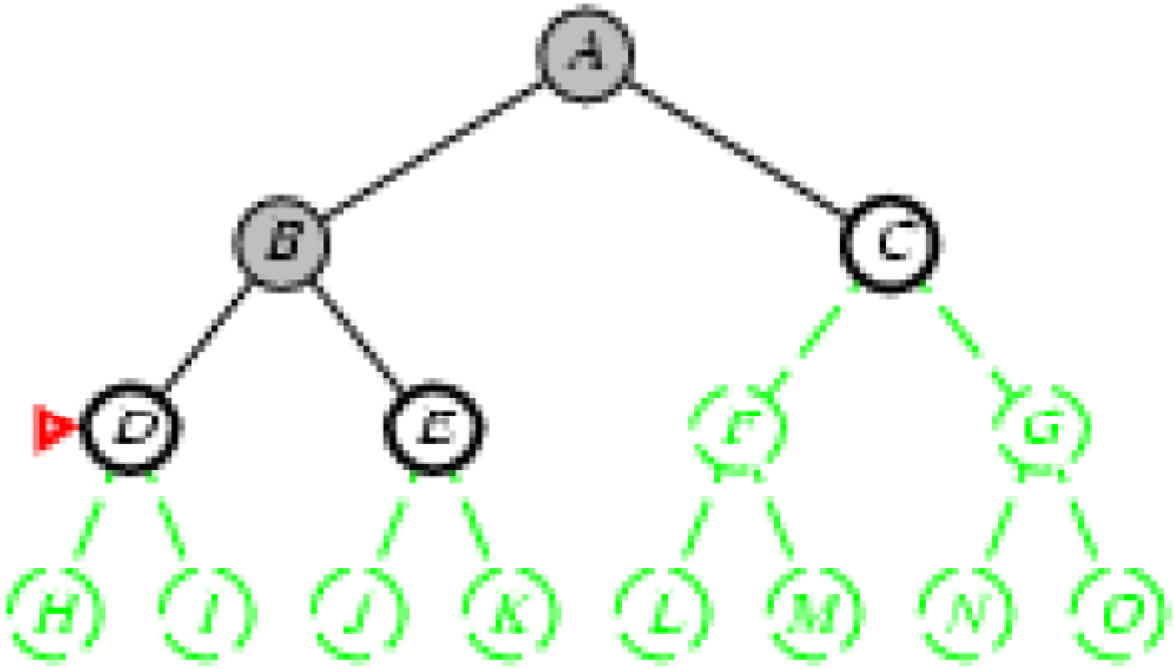

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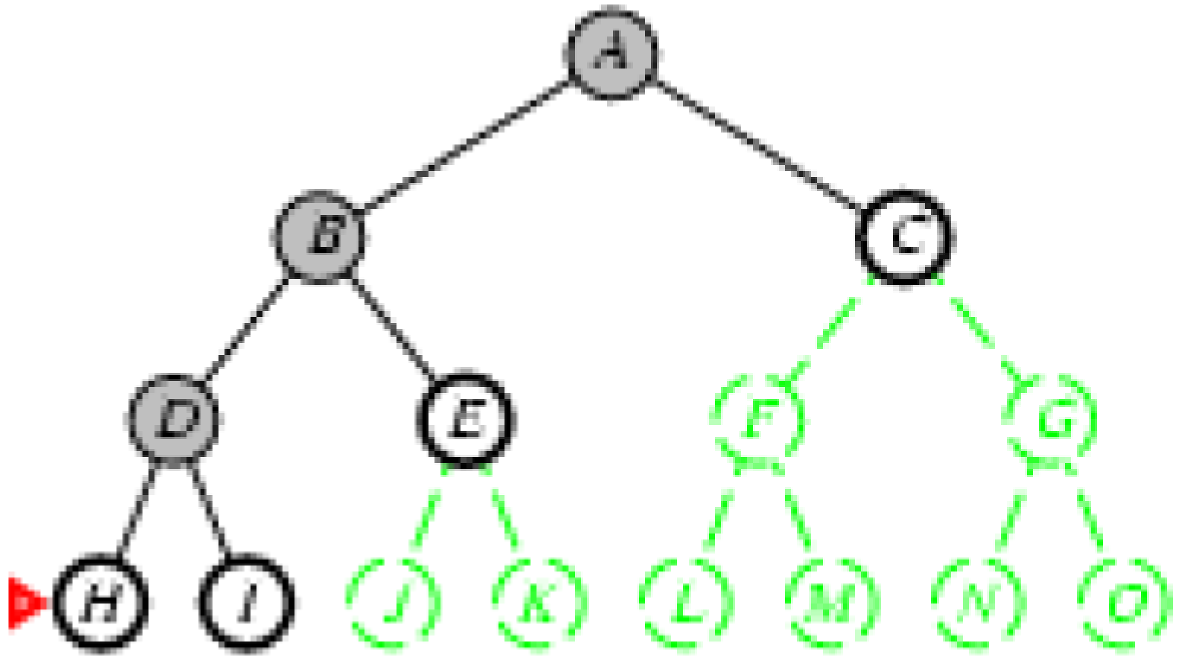

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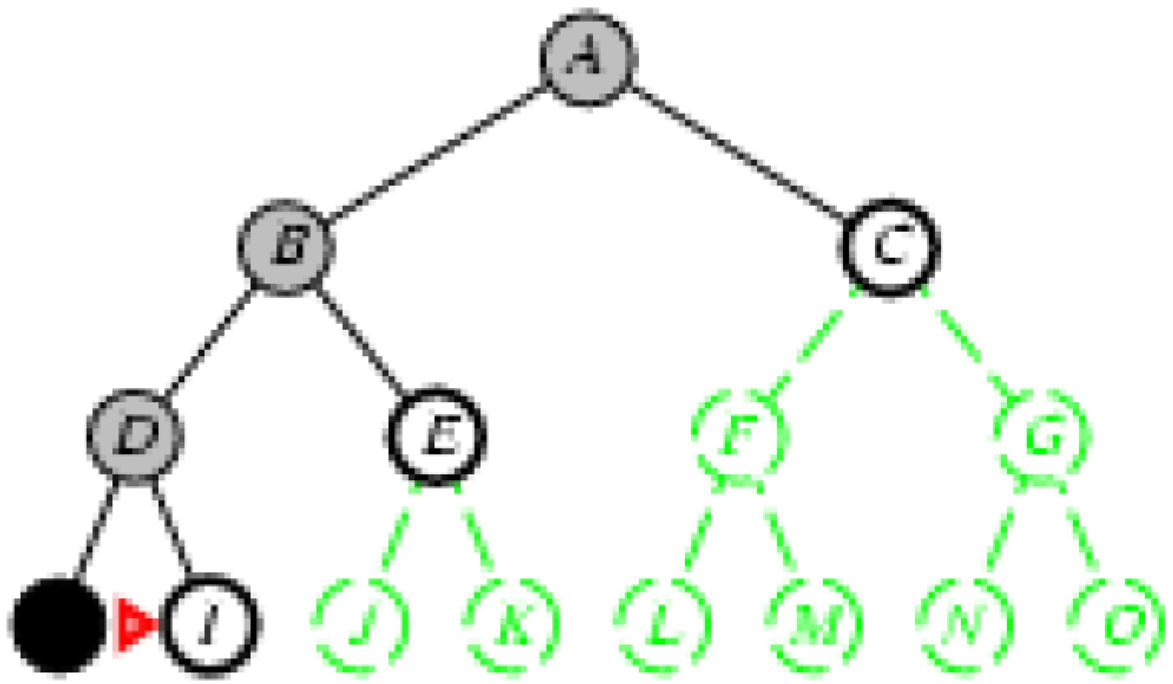

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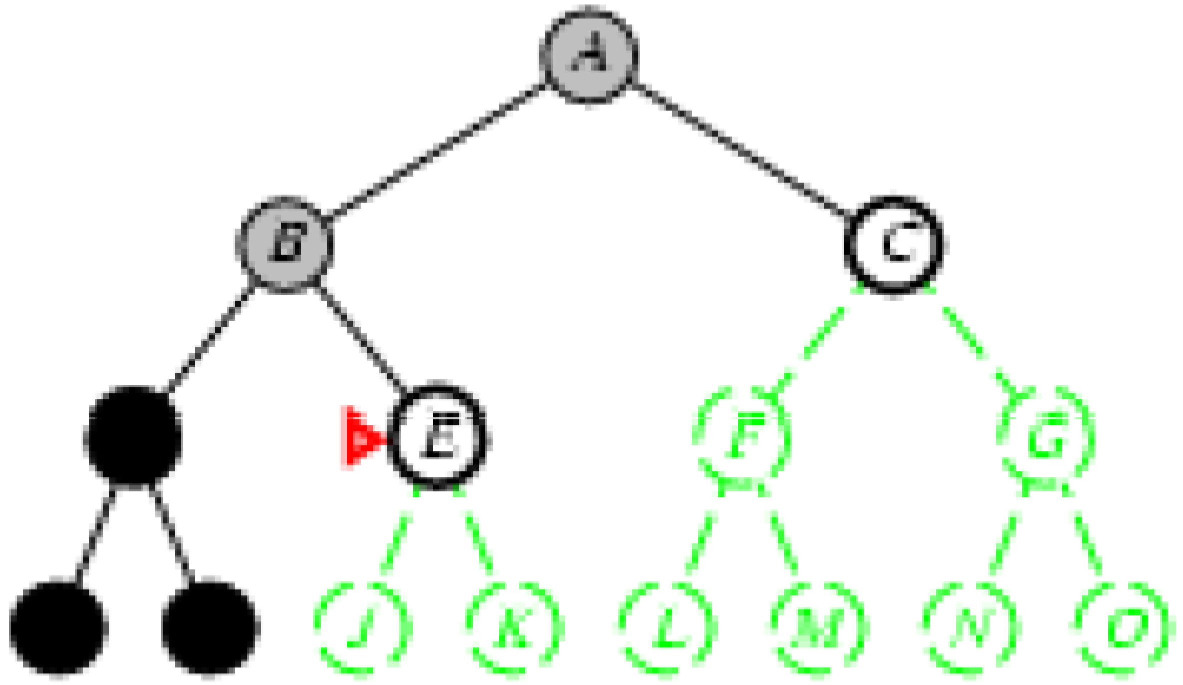

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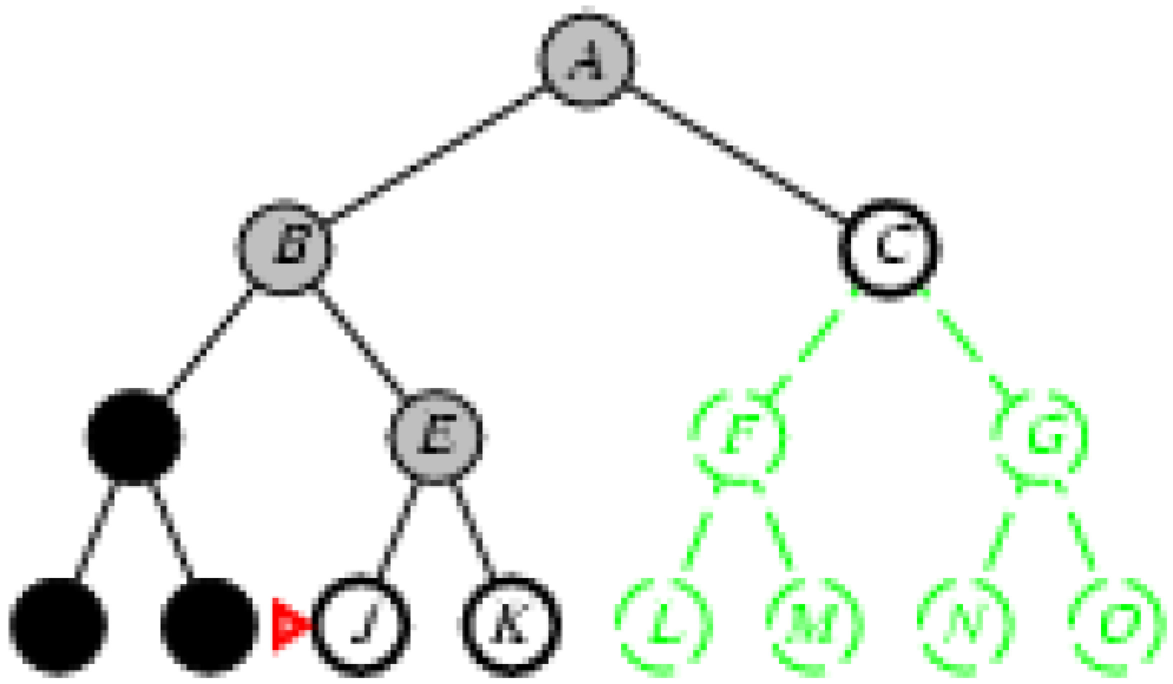

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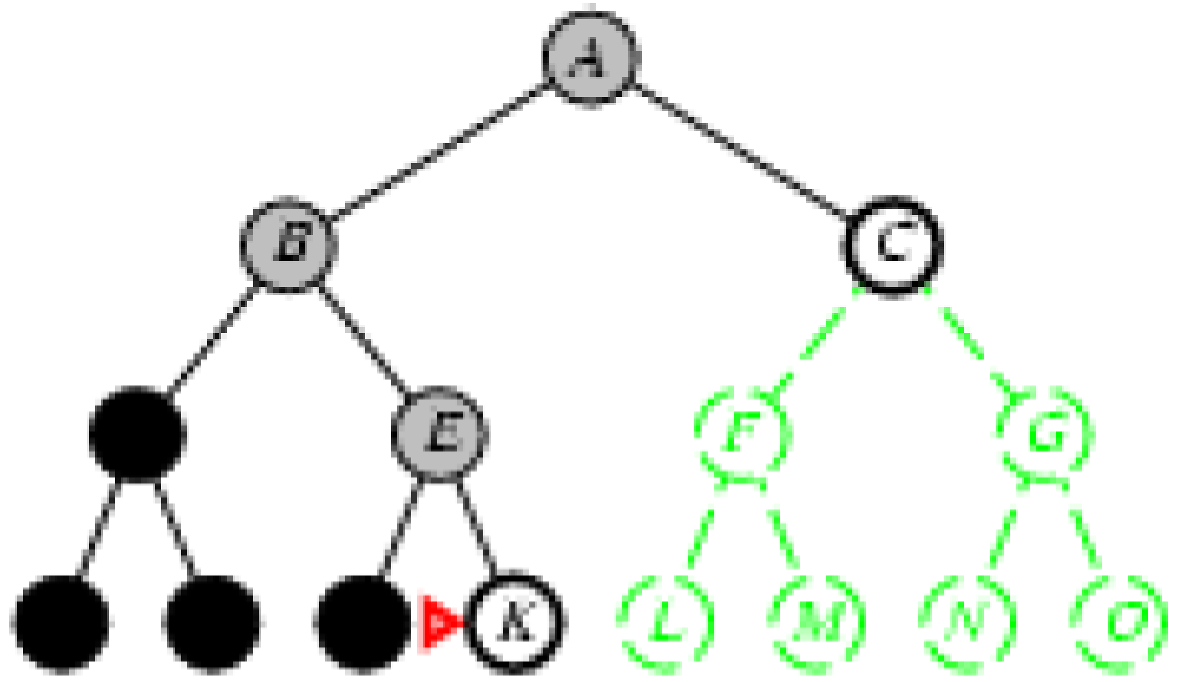

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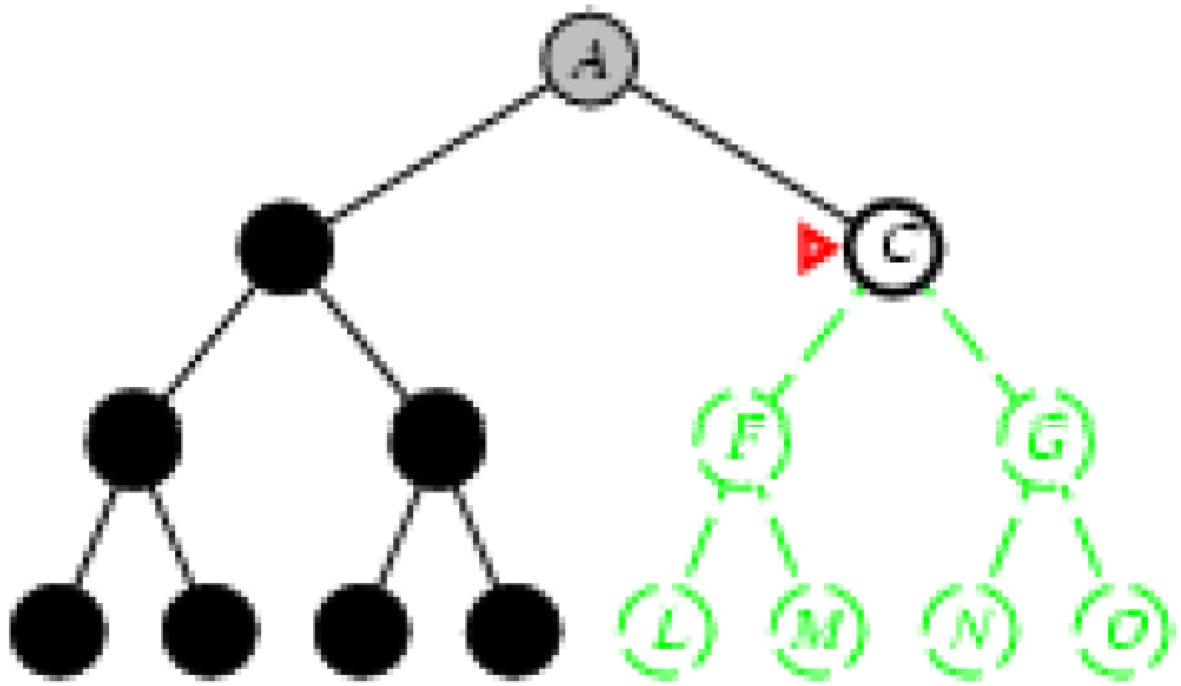


Properties of depth-first search

Complete?

Time?

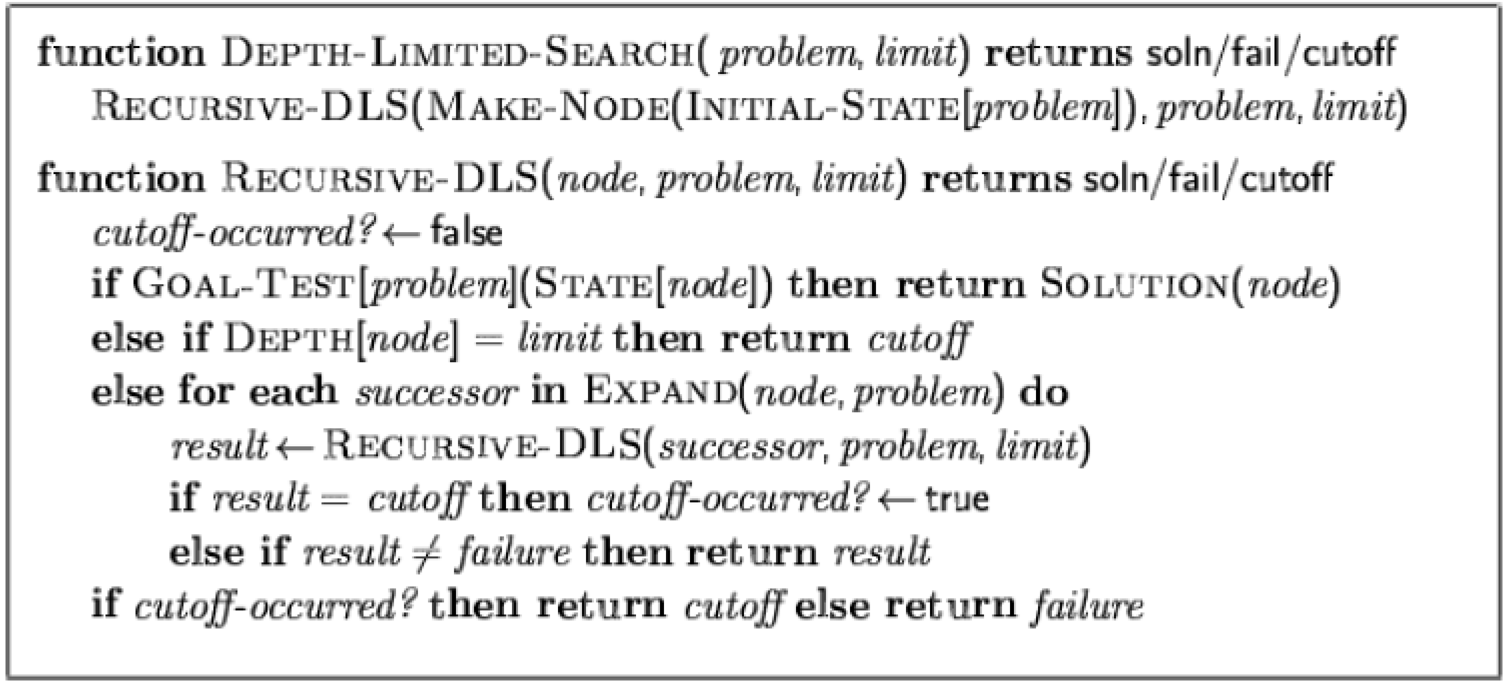
Space?

Optimal?

Depth-limited search

= depth-first search with depth limit l,

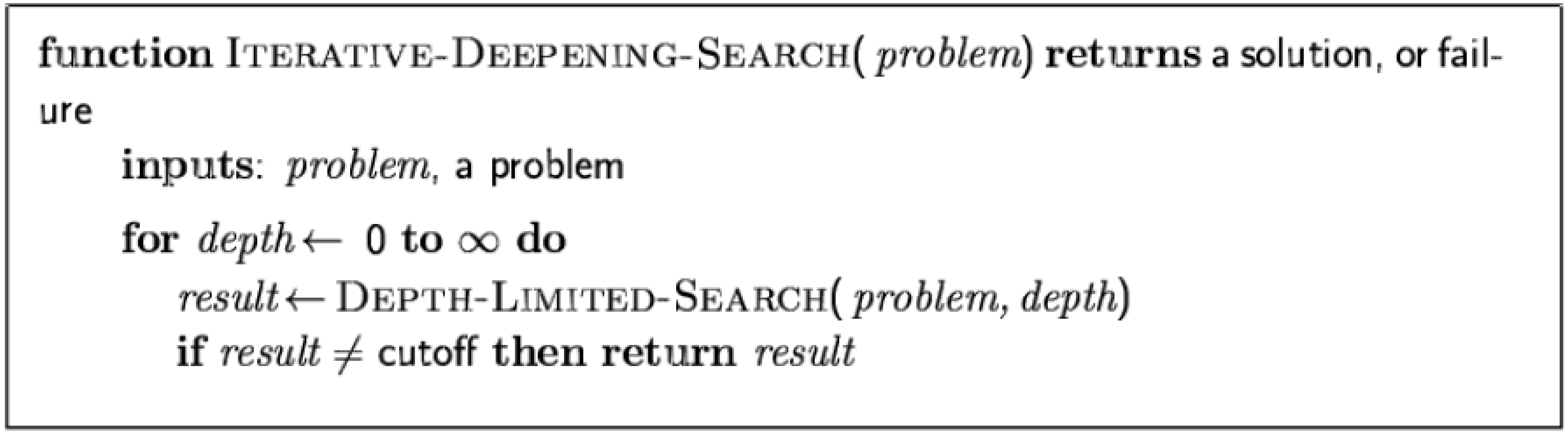
i.e., nodes at depth l have no successors



```
function DEPTH-LIMITED-SEARCH( problem, limit) returns soln/fail/cutoff
    RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[problem]), problem, limit)

function RECURSIVE-DLS(node, problem, limit) returns soln/fail/cutoff
    cutoff-occurred? ← false
    if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
    else if DEPTH[node] = limit then return cutoff
    else for each successor in EXPAND(node, problem) do
        result ← RECURSIVE-DLS(successor, problem, limit)
        if result = cutoff then cutoff-occurred? ← true
        else if result ≠ failure then return result
    if cutoff-occurred? then return cutoff else return failure
```

Iterative deepening search

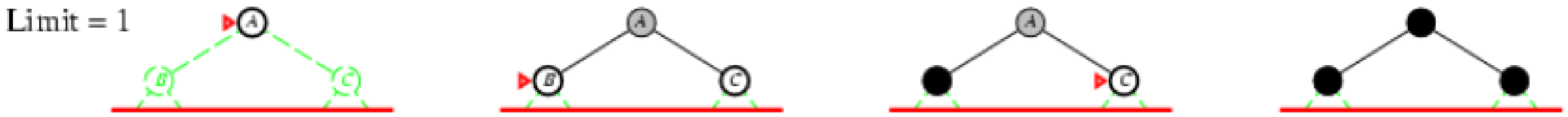


```
function Iterative-Deepening-Search( problem) returns a solution, or fail-
ure
    inputs: problem, a problem
    for depth ← 0 to $\infty$ do
        result ← Depth-Limited-Search( problem, depth)
        if result $\neq$ cutoff then return result
```

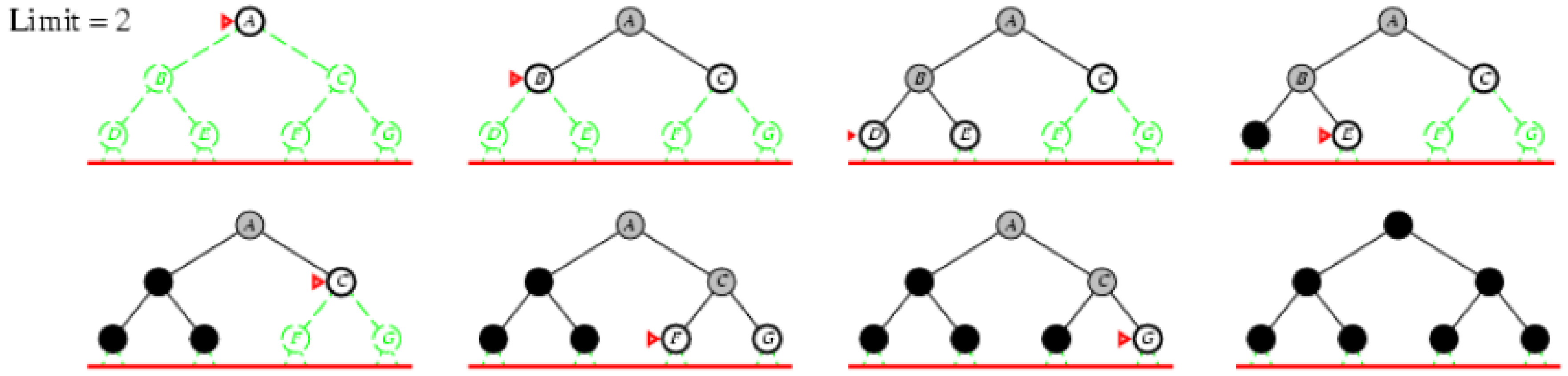
Iterative deepening search

Limit = 0

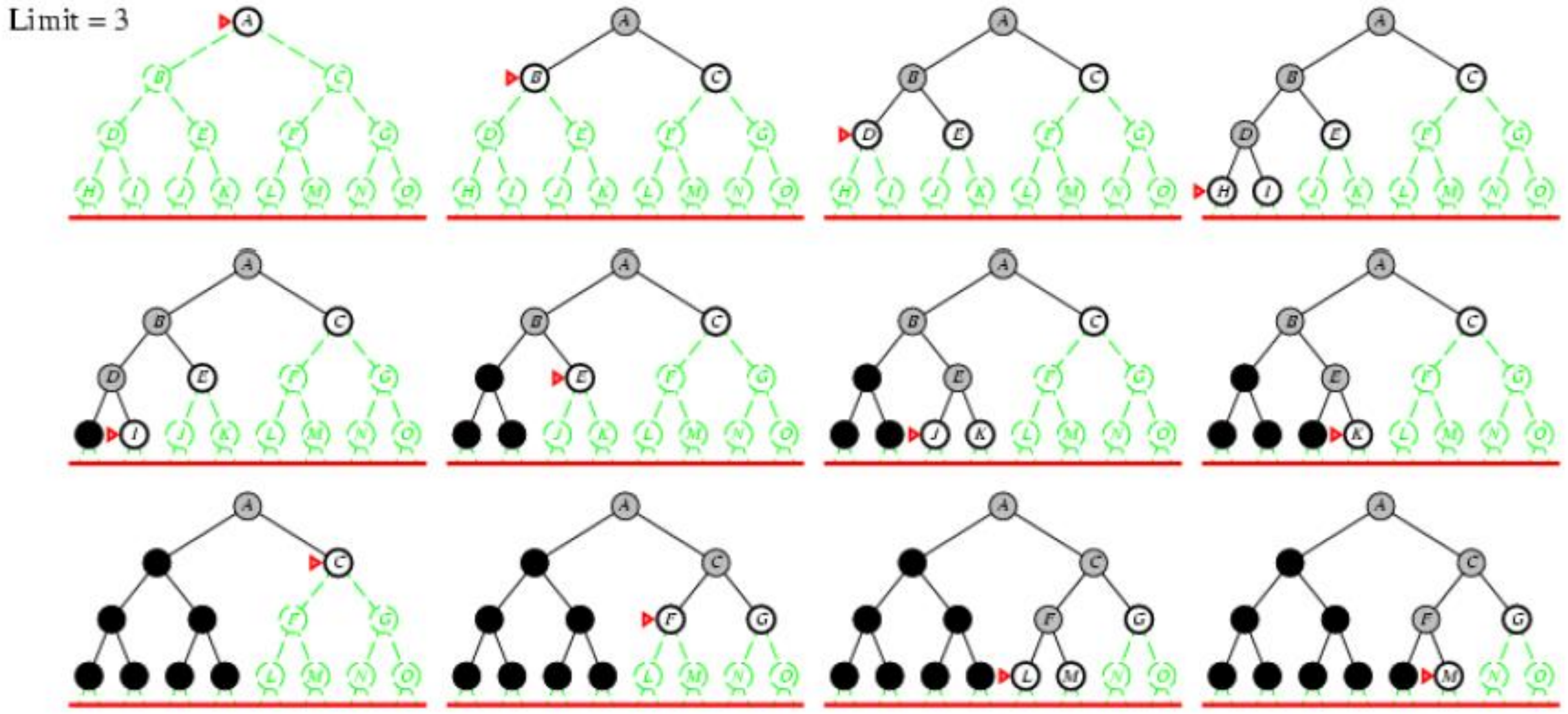
Iterative deepening search



Iterative deepening search



Iterative deepening search



Iterative deepening search

Number of nodes generated in a depth-limited search to depth *d* with branching factor *b*:

$$N_{DLS} = b^0 + b^1 + b^2 + \ldots + b^{d-2} + b^{d-1} + b^d$$

Number of nodes generated in an iterative deepening search to depth *d* with branching factor *b*:

$$N_{IDS} = (d+1)b^0 + d\, b^1 + (d-1)b^2 + \ldots + 3b^{d-2} + 2b^{d-1} + 1b^d$$

For $b = 10$, $d = 5$,

- $N_{DLS} = 1 + 10 + 100 + 1{,}000 + 10{,}000 + 100{,}000 = 111{,}111$
- $N_{IDS} = 6 + 50 + 400 + 3{,}000 + 20{,}000 + 100{,}000 = 123{,}456$

Overhead = (123,456 - 111,111)/111,111 = 11%

Iterative deepening search

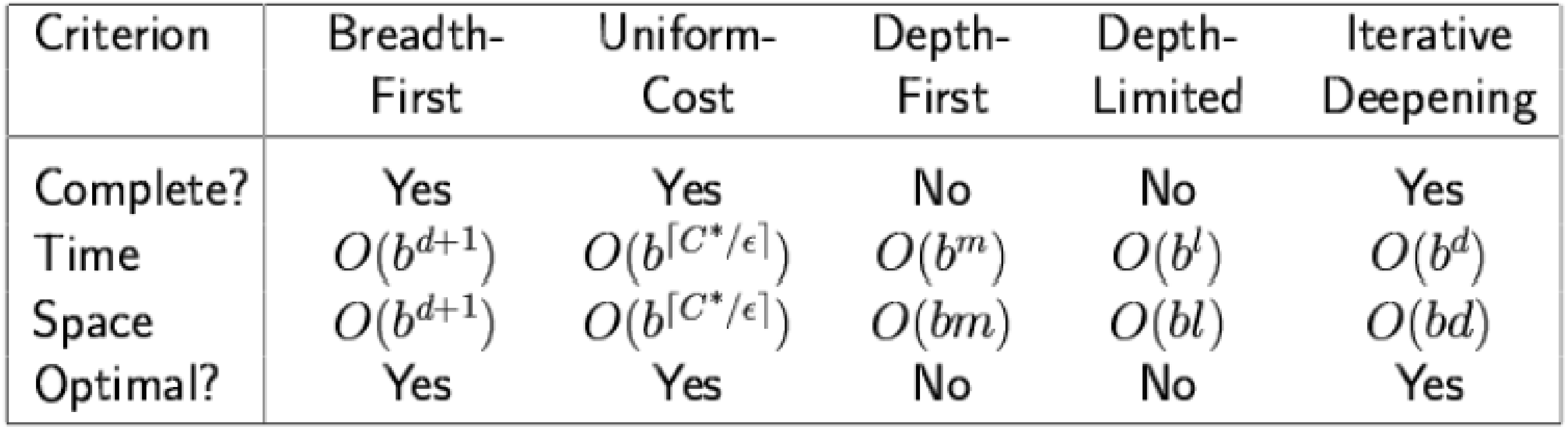
Complete?

Time?

Space?

Optimal?

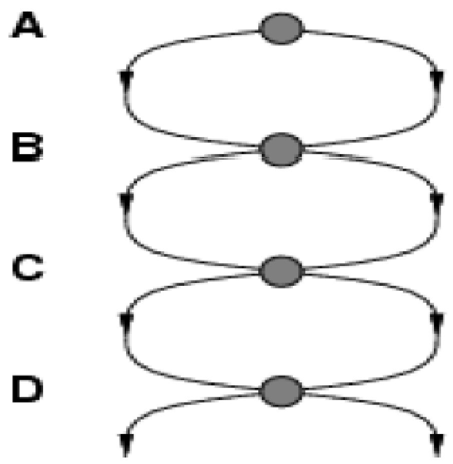
Summary of algorithms

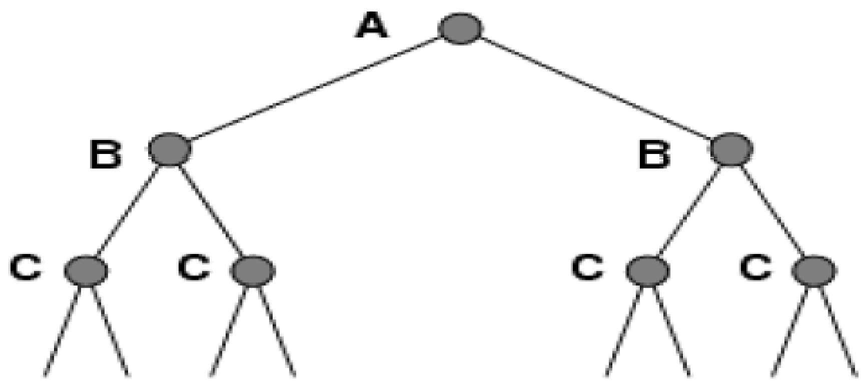


Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes	Yes	No	No	Yes
Time	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(b^m)$	$O(b^l)$	$O(b^d)$
Space	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(bm)$	$O(bl)$	$O(bd)$
Optimal?	Yes	Yes	No	No	Yes

Repeated states

- Failure to detect repeated states can turn a linear problem into an exponential one!





Graph search

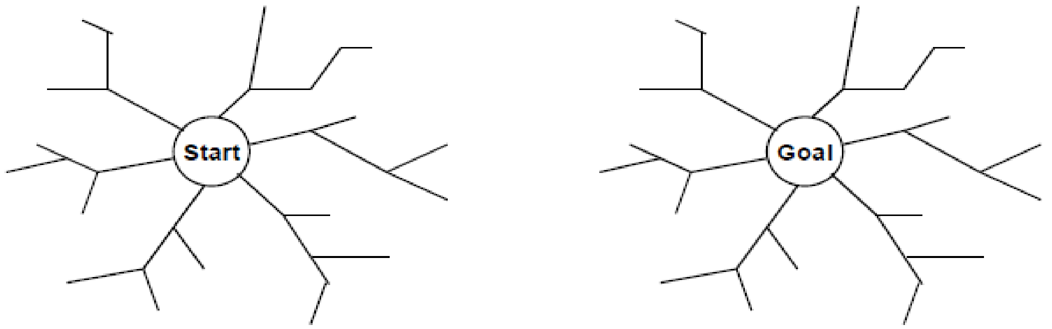
```
function GRAPH-SEARCH( problem, fringe) returns a solution, or failure
    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
        if STATE[node] is not in closed then
            add STATE[node] to closed
            fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

Bidirectional Search

Simultaneously search both forward (from the initial state) and backward (from the goal state)

Stop when the two searches meet.

Intuition = 2 * O($b^{d/2}$) is smaller than O(b^d)



Summary

- Problem formulation usually requires abstracting away realworld details to define a state space that can feasibly be explored
- Variety of uninformed search strategies
- Iterative deepening search uses only linear space and not much more time than other uninformed algorithms