Outline

- Introduction to Machine Learning
- ID3 Decision Tree Learning
- Naïve Bayesian Learning

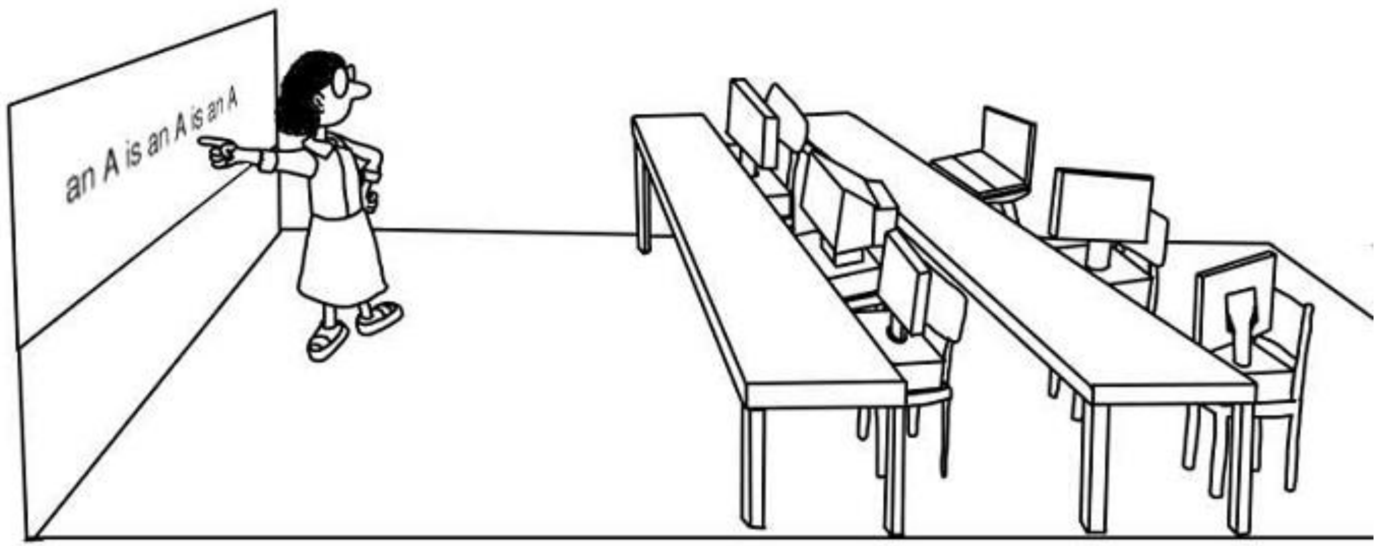
Acknowledgements

- This slide is mainly based on the textbook AIMA (3rd edition)
- Some parts of the slide are adapted from
  - Maria-Florina Balcan, *Introduction to Machine Learning*, 10-401, Spring 2018, Carnegie Mellon University
  - Ryan Urbanowicz, *An Introduction to Machine Learning*, PA CURE Machine Learning Workshop: December 17, School of Medicine, University of Pennsylvania

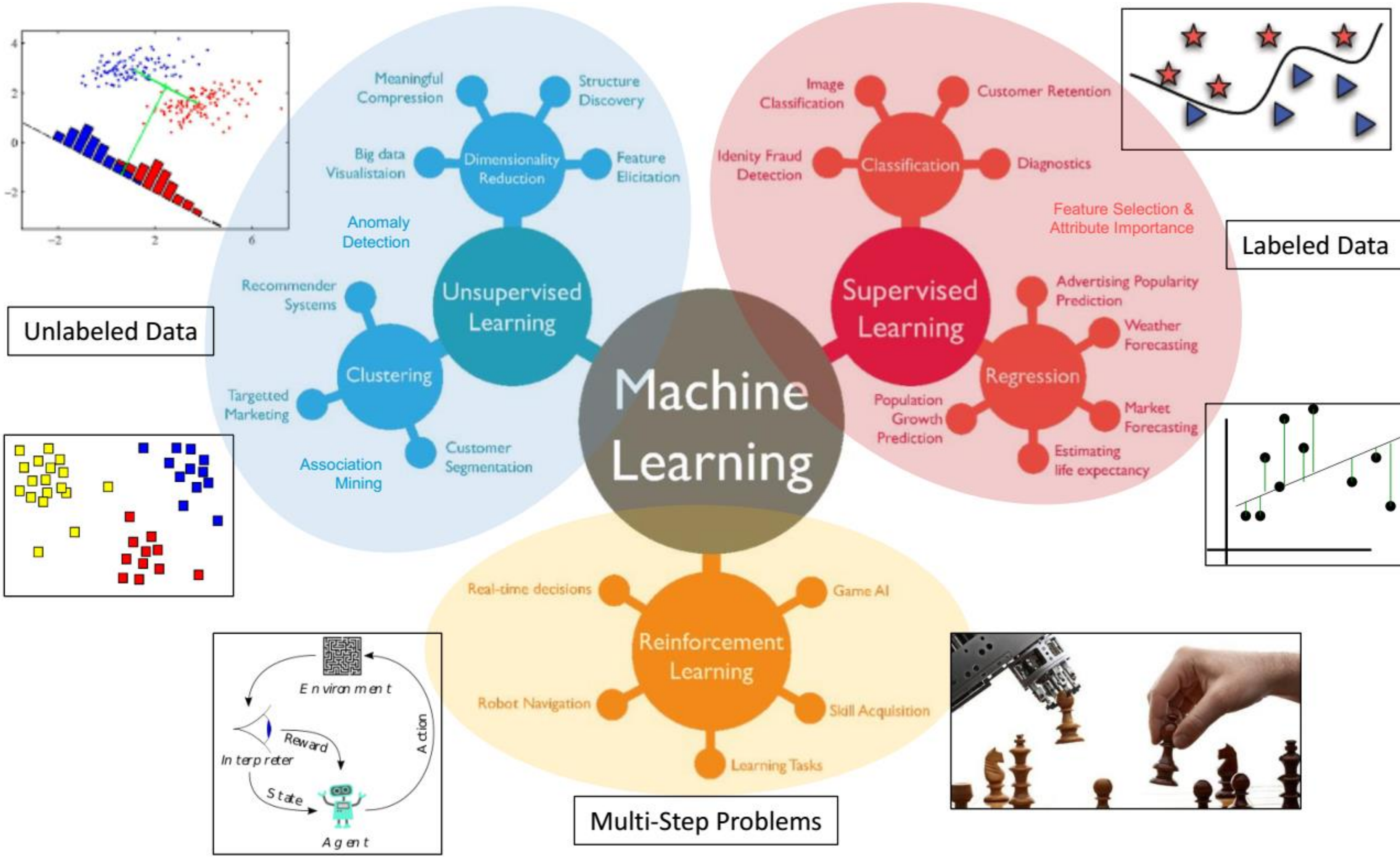
Machine Learning

What is machine learning?

- **Machine learning** involves adaptive mechanisms that enable computers to learn from experience, learn by example and learn by analogy.

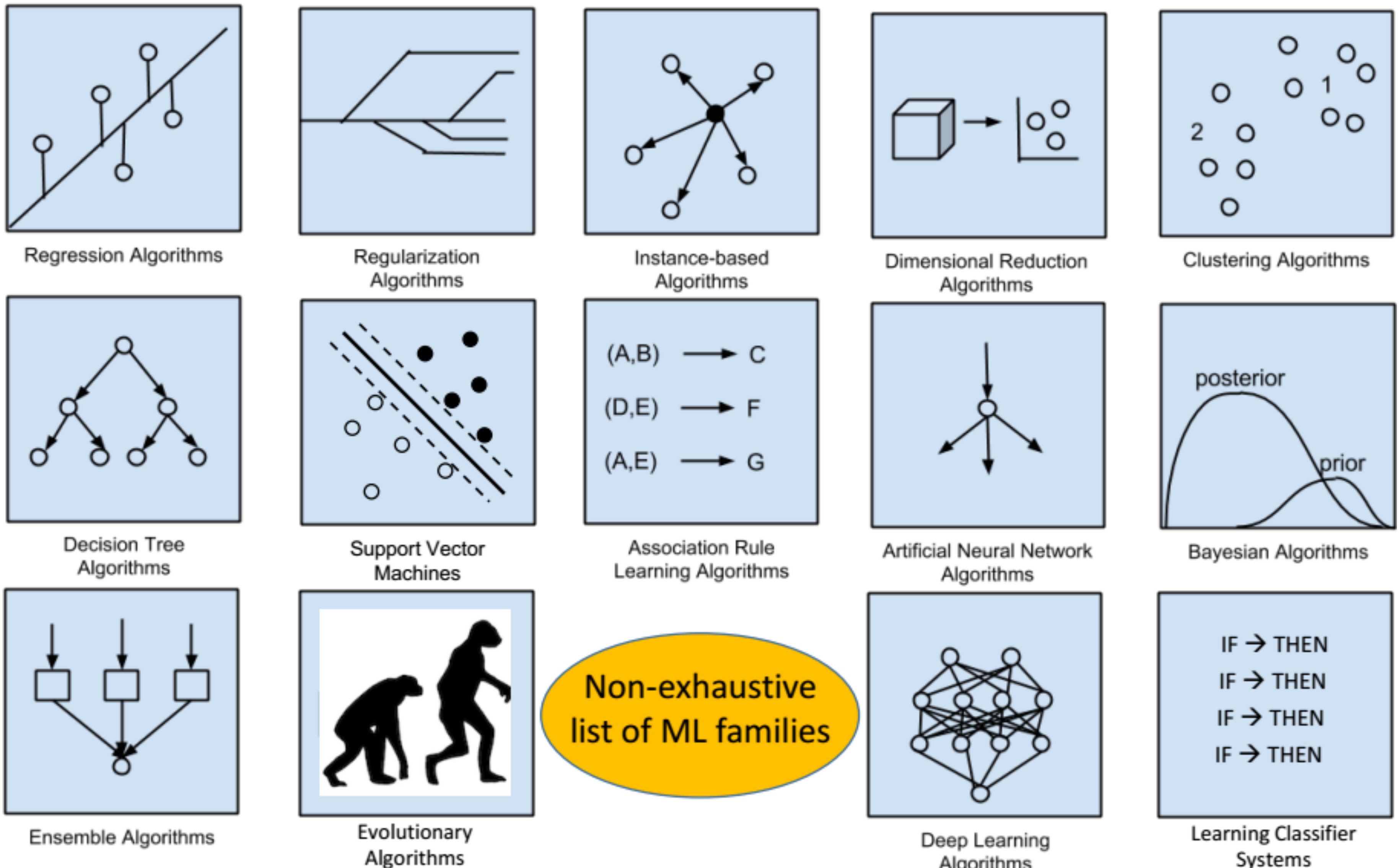


Types of machine learning



Ref: https://ldi.upenn.edu/sites/default/files/Introduction-to-Machine-Learning.pdf

Machine learning algorithms



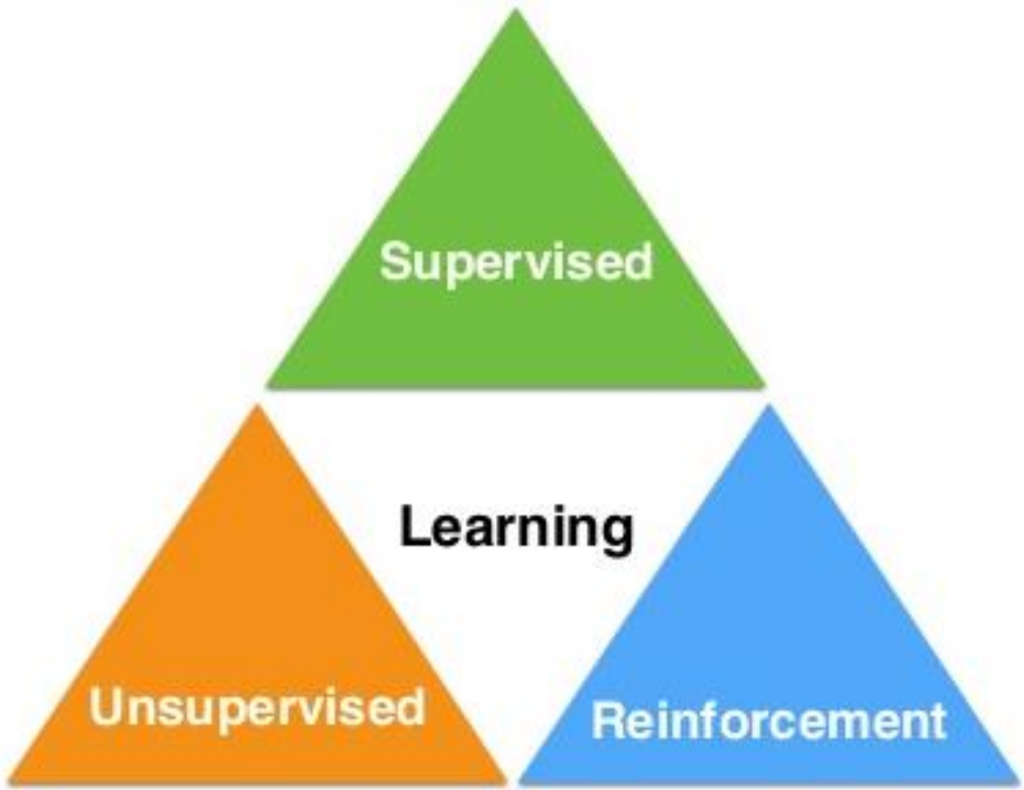
Ref: https://ldi.upenn.edu/sites/default/files/Introduction-to-Machine-Learning.pdf

Types of learning algorithms

SUPERVISED LEARNING	UNSUPERVISED LEARNING	REINFORCEMENT LEARNING
		

Types of learning algorithms

- Labeled data
- Direct feedback
- Predict outcome/future

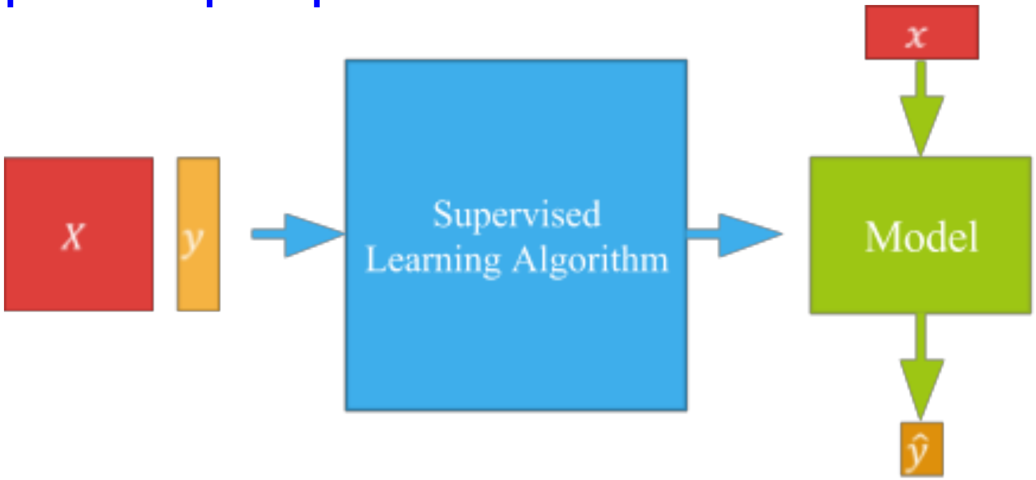


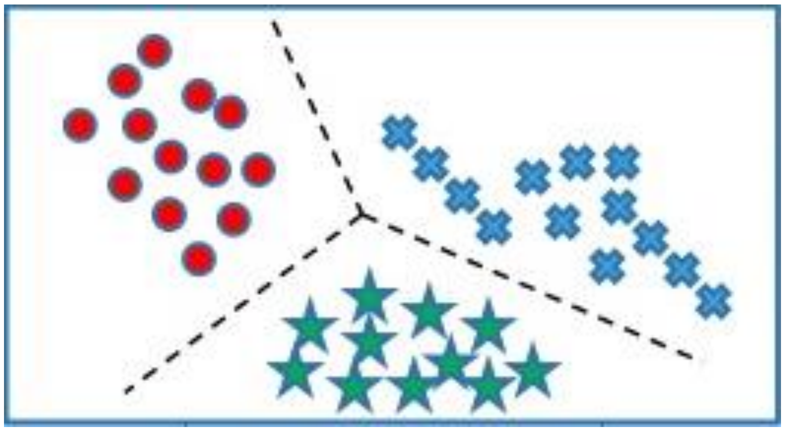
- No labels
- No feedback
- "Find hidden structure"

- Decision process
- Reward system
- Learn series of actions

Supervised learning

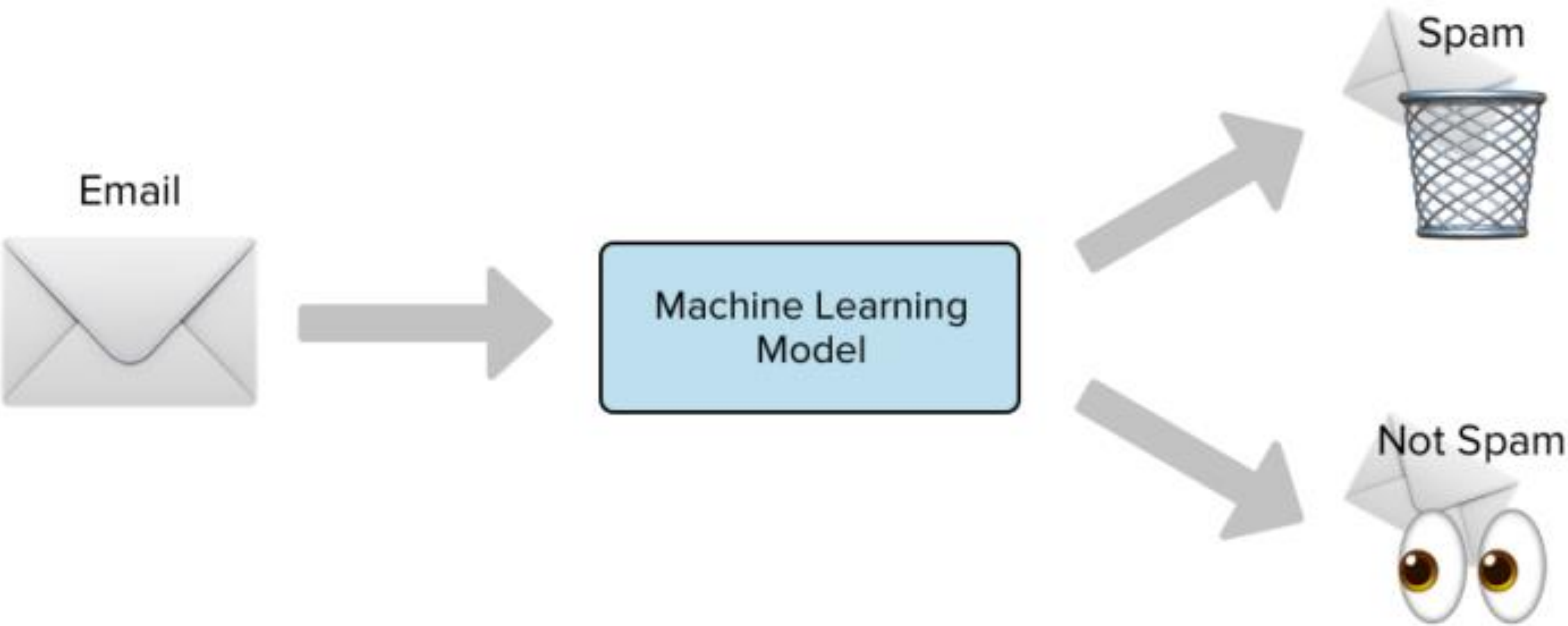
- Learn a function that maps an input to an output based on example input-output pairs





Supervised learning: Examples

- Spam detection: Decide which emails are spam and which are important
  - Use emails seen so far to obtain good prediction rule for future data



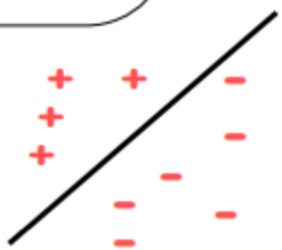
Supervised learning: Examples

- Spam detection: Decide which emails are spam and which are important
  - Represent each message by features. (e.g., keywords, spelling, etc.)

	"money"	"pills"	"Mr."	bad spelling	known-sender	spam?	
	Y	N	Y	Y	N	Y	
	N	N	N	Y	Y	N	
	N	Y	N	N	N	Y	
example	Y	N	N	N	Y	N	label
	N	N	Y	N	Y	N	
	Y	N	N	Y	N	Y	
	N	N	Y	N	N	N	

Reasonable RULES

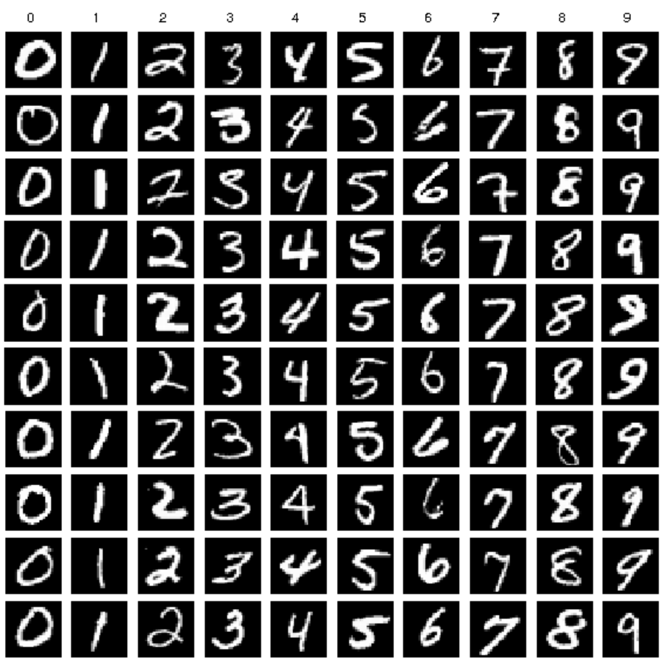
- Predict SPAM if unknown AND (money OR pills)
- Predict SPAM if 2money + 3pills – 5 known > 0



Linearly separable

Supervised learning: Examples

- Object detection and recognition: Localize and identify instances of semantic objects of a certain class (e.g., humans, buildings, or cars) in digital images and videos

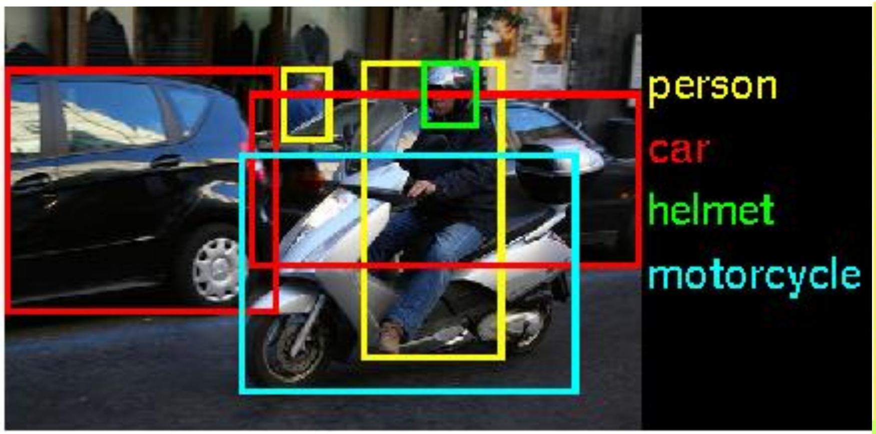


Handwritten digit recognition

Scene text recognition

Supervised learning: Examples

- Object detection and recognition: Localize and identify instances of semantic objects of a certain class (e.g., humans, buildings, or cars) in digital images and videos

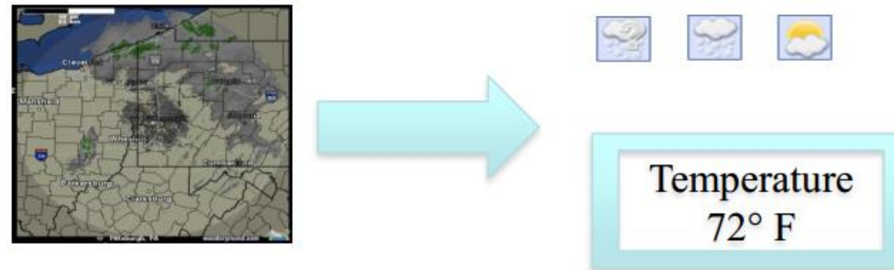


ImageNet object recognition

Indoor scene recognition

Supervised learning: More examples

- Weather prediction: Predict the weather type or the temperature at any given location…



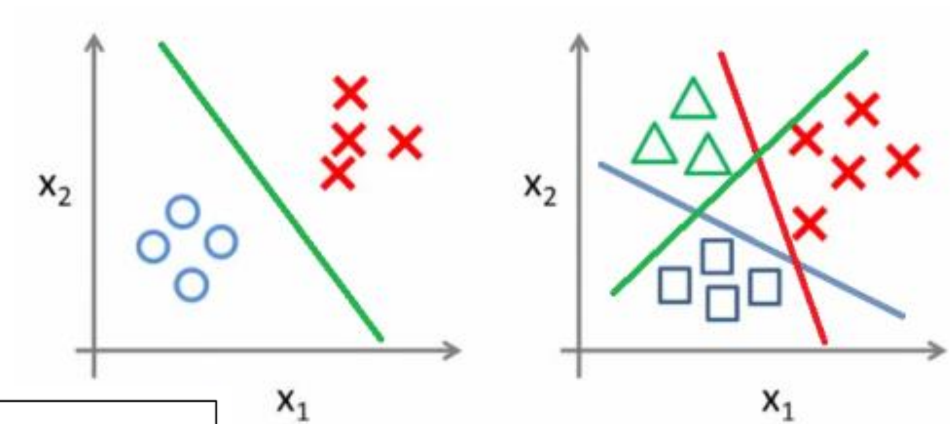
- Medicine: diagnose a disease (or response to chemo drug X, or whether a patient is re-admitted soon?)
  - Input: from symptoms, lab measurements, test results, DNA tests, …
  - Output: one of set of possible diseases, or “none of the above”
  - E.g., audiology, thyroid cancer, diabetes, etc.

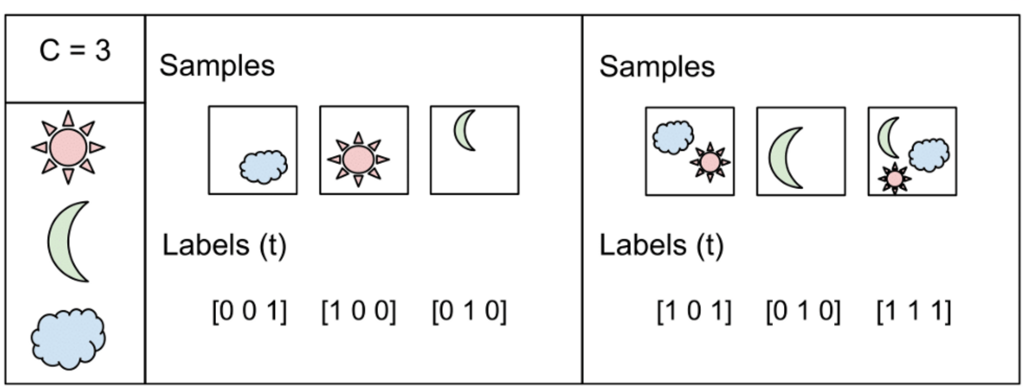
- Computational Economics:
  - Predict if a user will click on an ad so as to decide which ad to show
  - Predict if a stock will rise or fall (with specific amounts)

Classification vs. Regression

- Train a model to predict a categorical dependent variable
- Case studies: predicting disease, classifying images, predicting customer churn, buy or won't buy, etc.

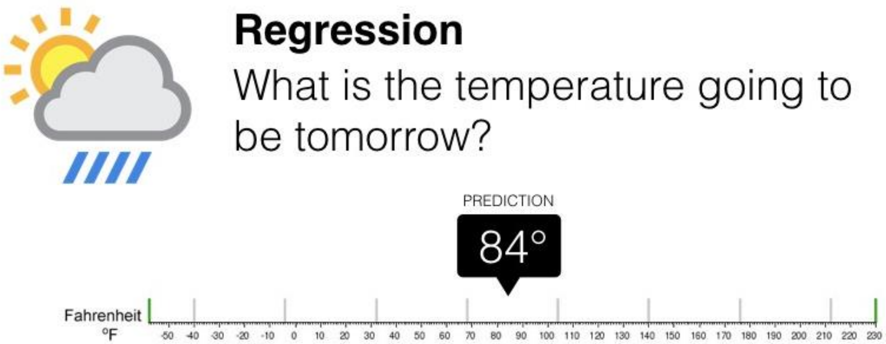
- Binary classification vs.
  Multiclass classification vs.
  Multilabel classification

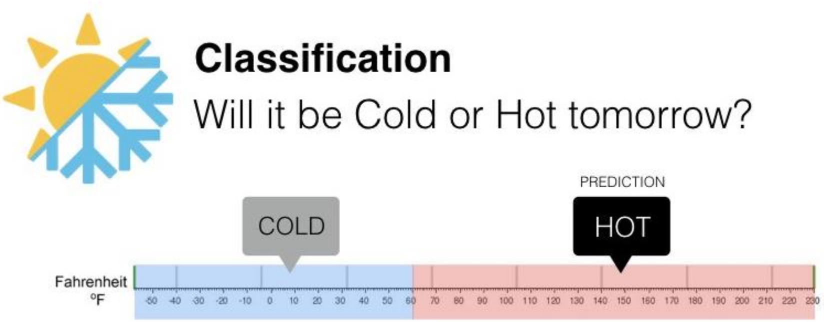




Classification vs. Regression

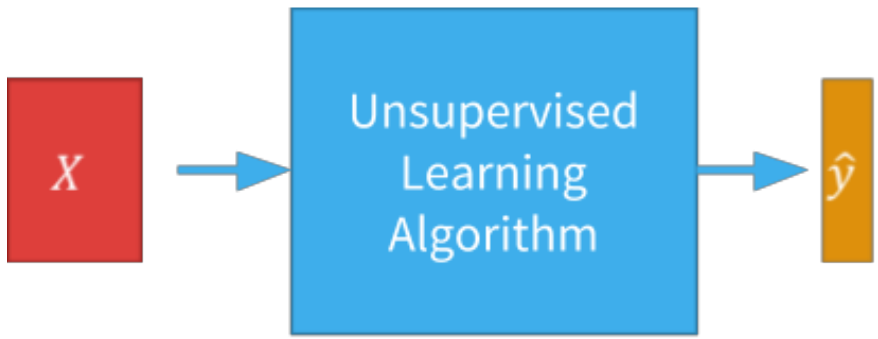
- Train a model to predict a continuous dependent variable
- Case studies: predicting height of children, predicting sales, forecasting stock prices, etc.

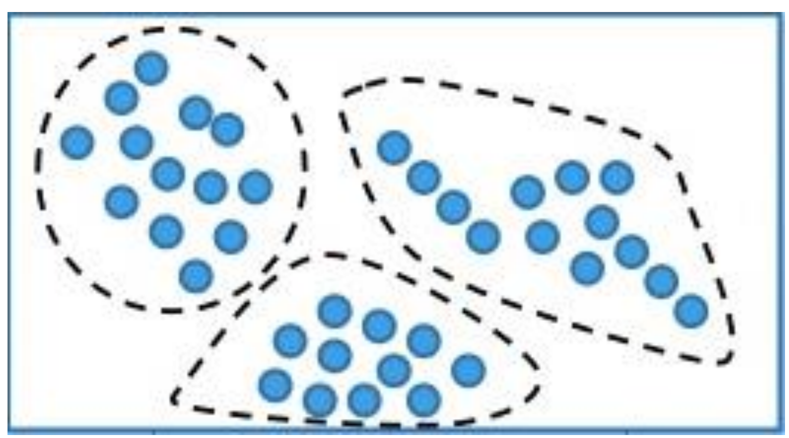




Unsupervised learning

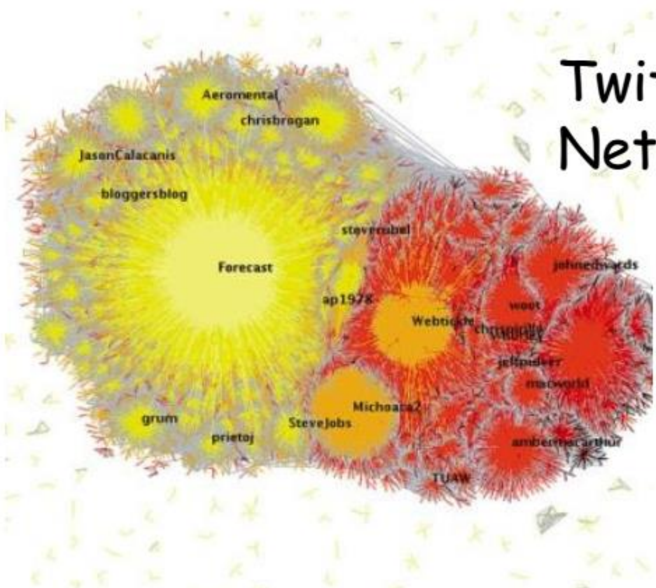
- Infer a function to describe hidden structure from "unlabeled" data
  - A classification (or categorization) is not included in the observations.



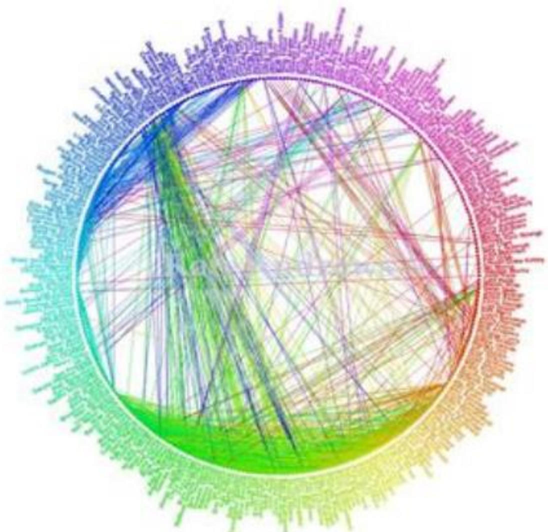


Unsupervised learning: Examples

- Social network analysis: cluster users of social networks by interest (community detection)

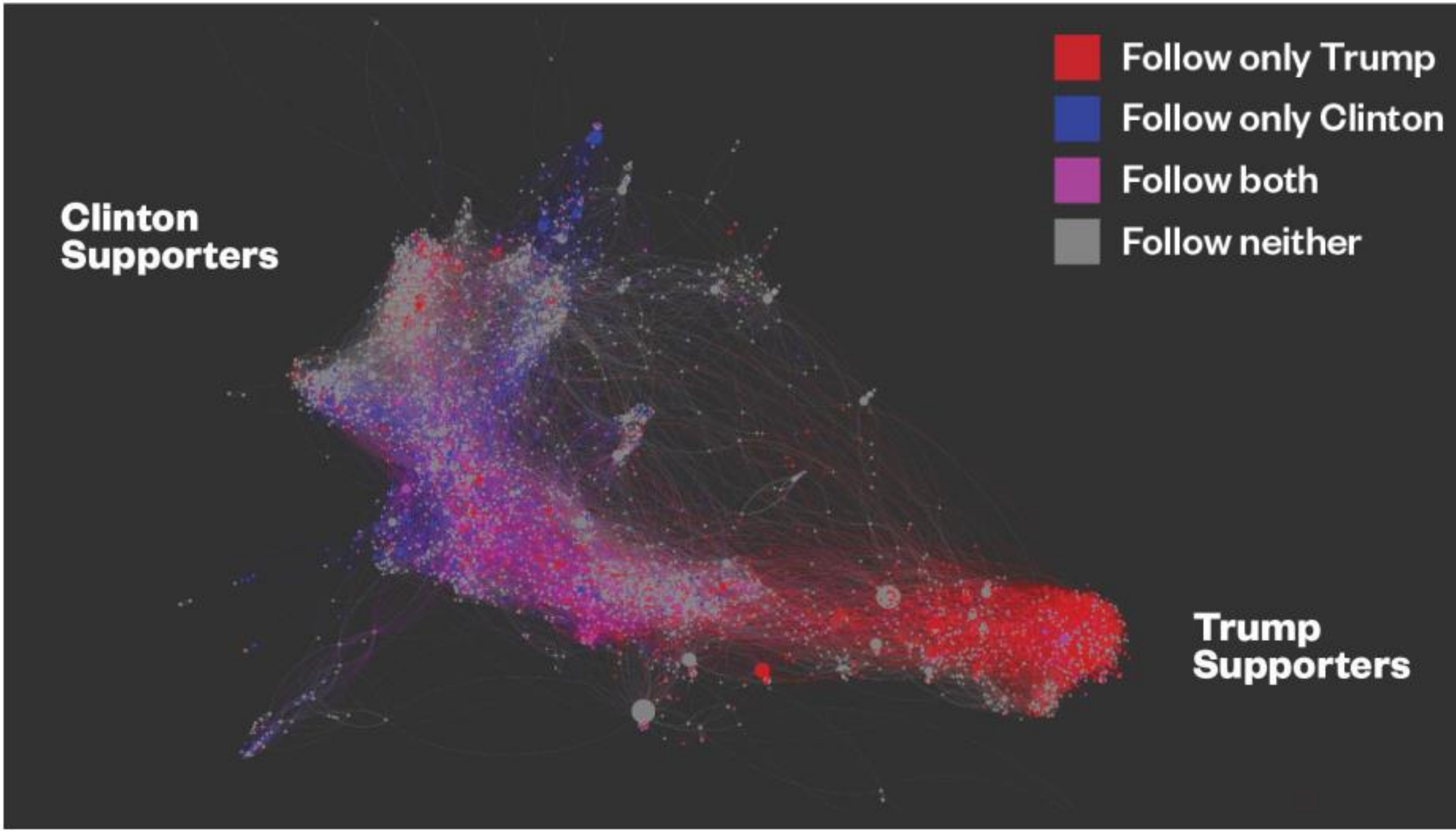


Twitter Network



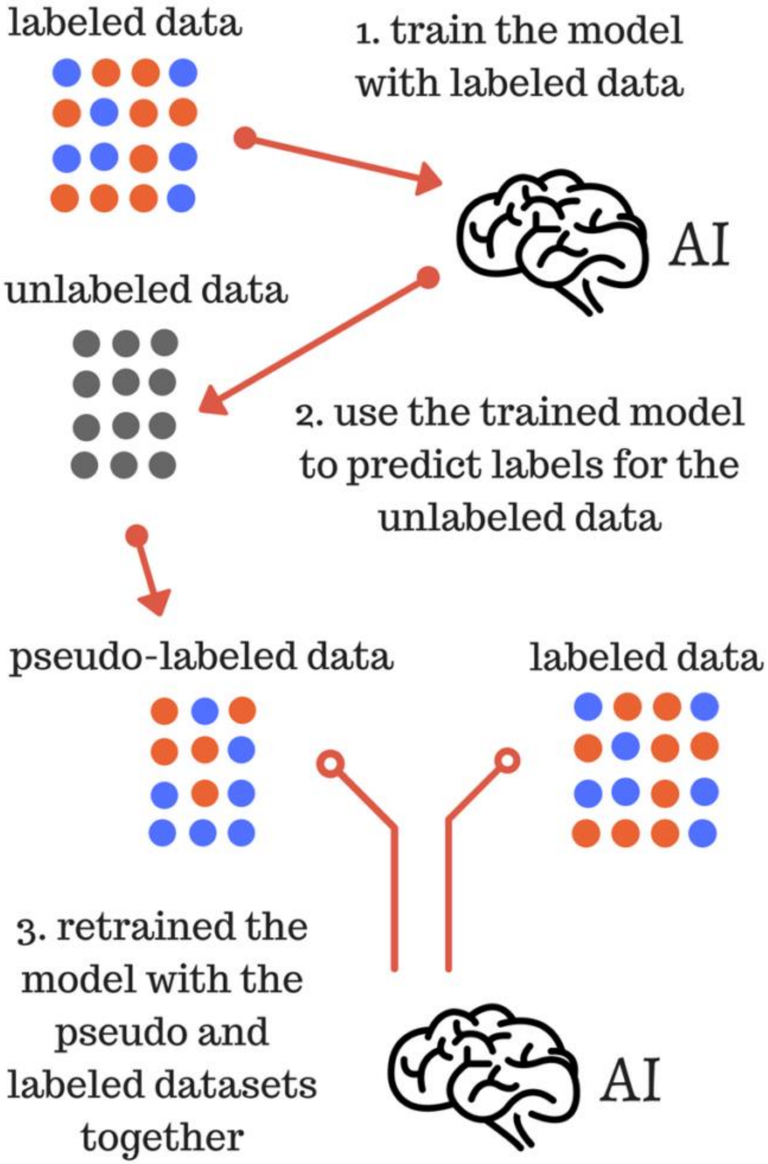
Facebook network

Unsupervised learning: Examples

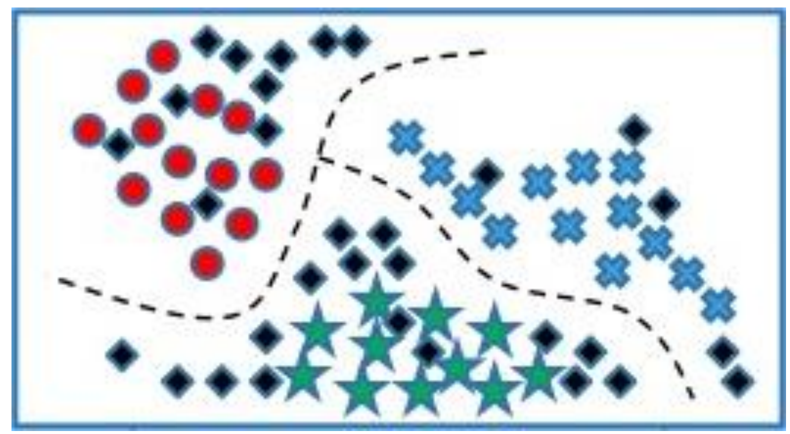


https://news.vice.com/en_us/article/d3xamx/journalists-and-trump-voters-live-in-separate-online-bubbles-mit-analysis-shows

Semi-supervised learning

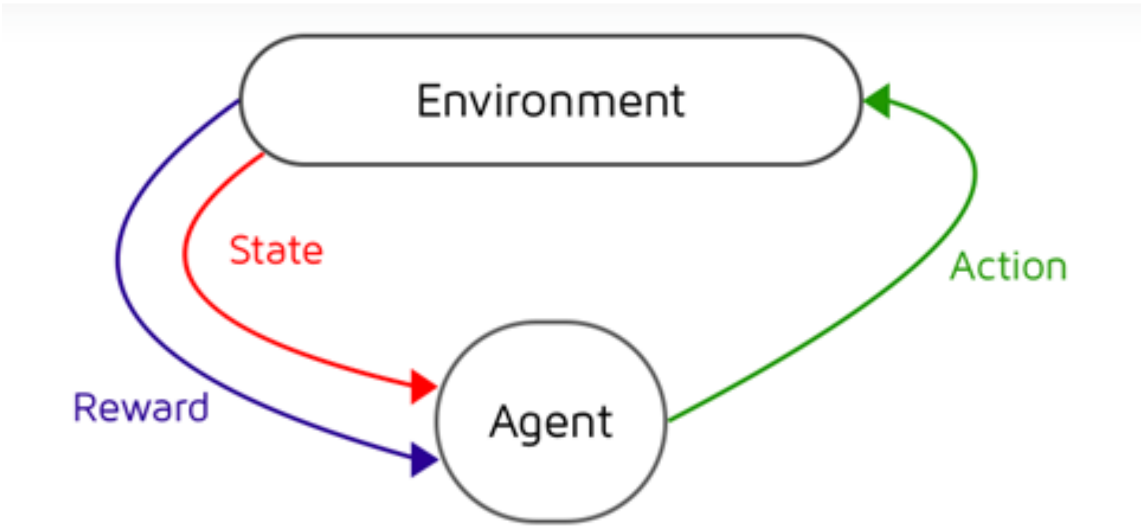


- The model is initially trained with a small amount of labeled data and a large amount of unlabeled data.

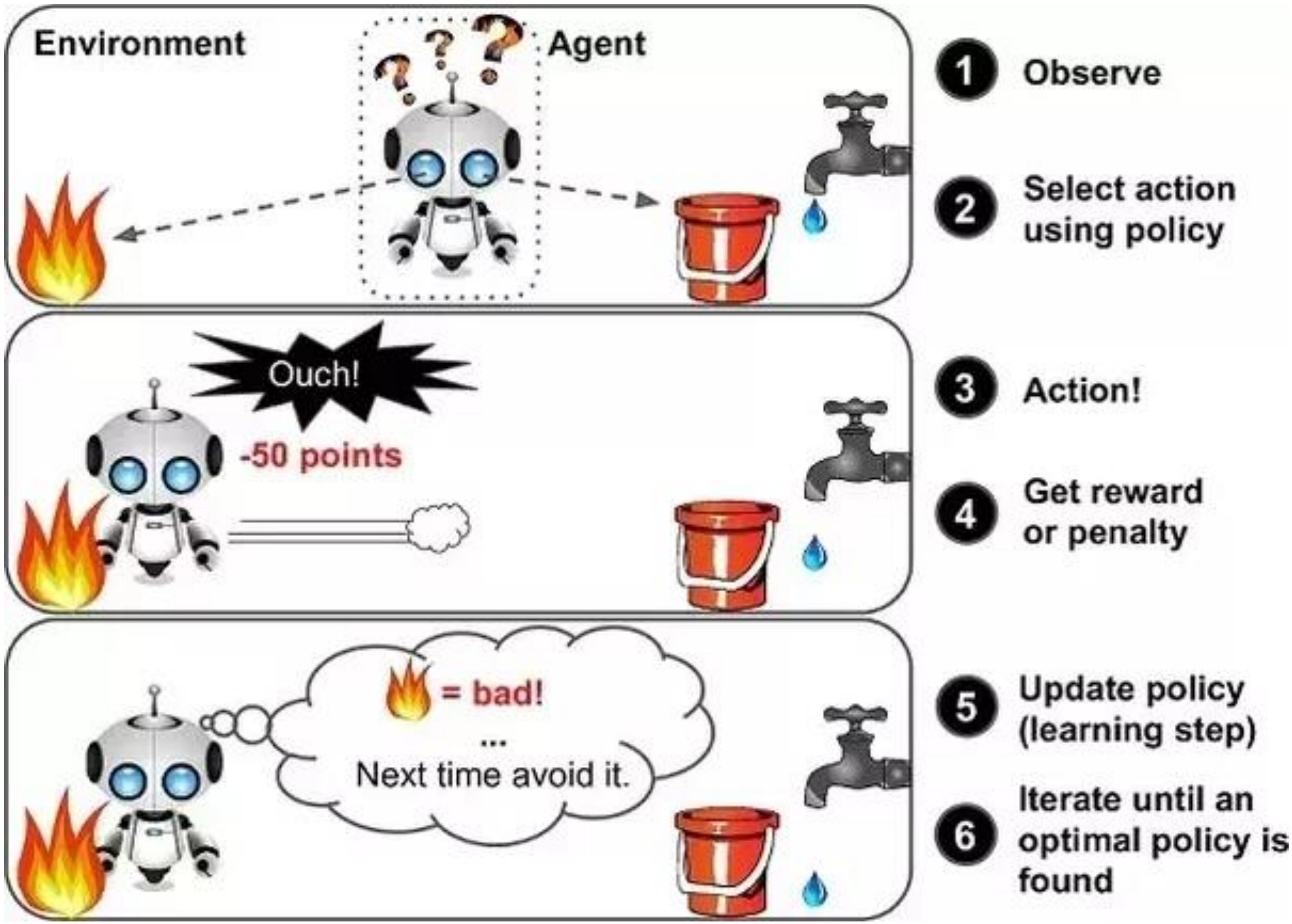


Reinforcement learning

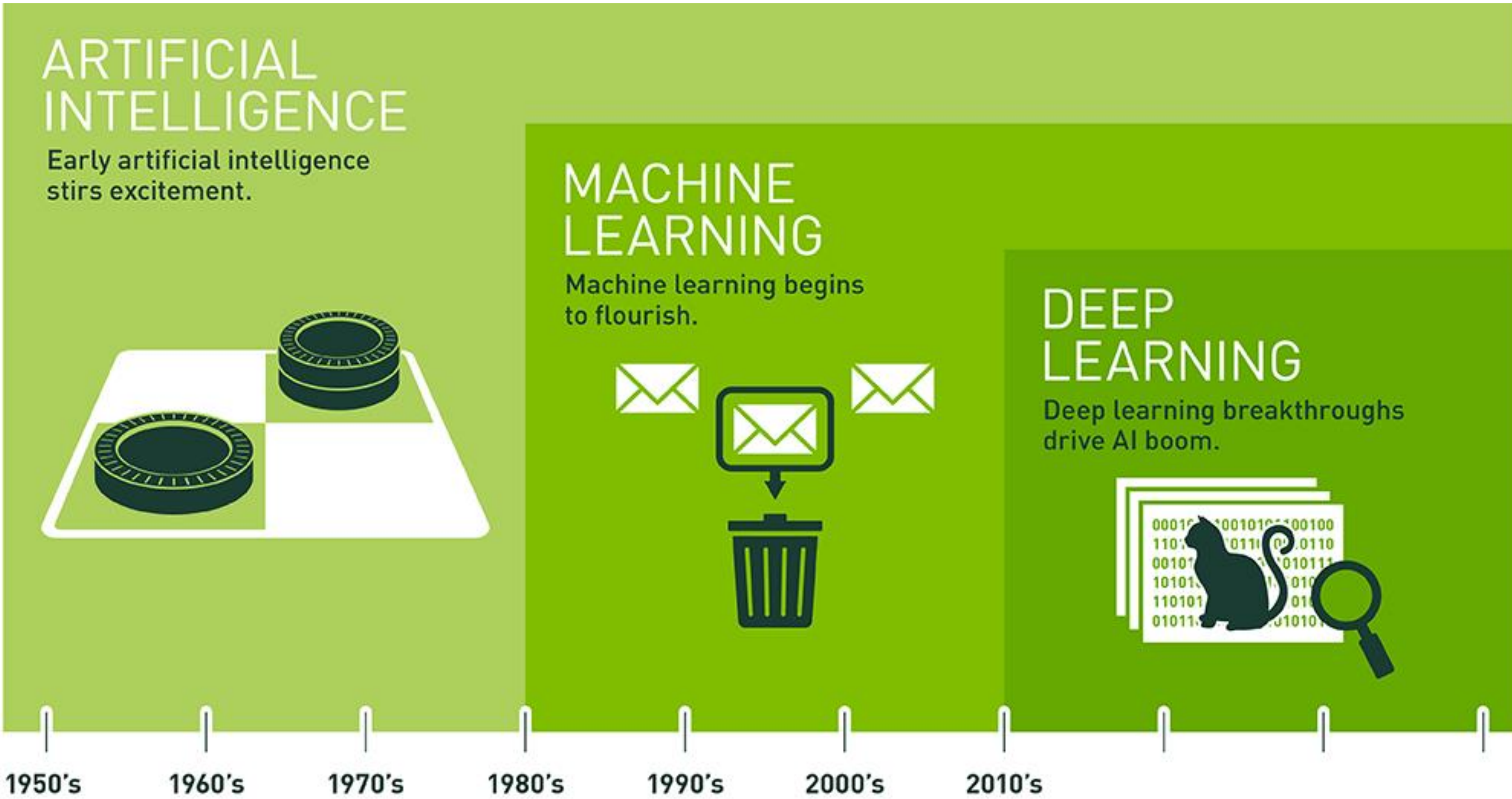
- The agent learns from the environment by interacting with it and receives rewards for performing actions.



Reinforcement learning: Example

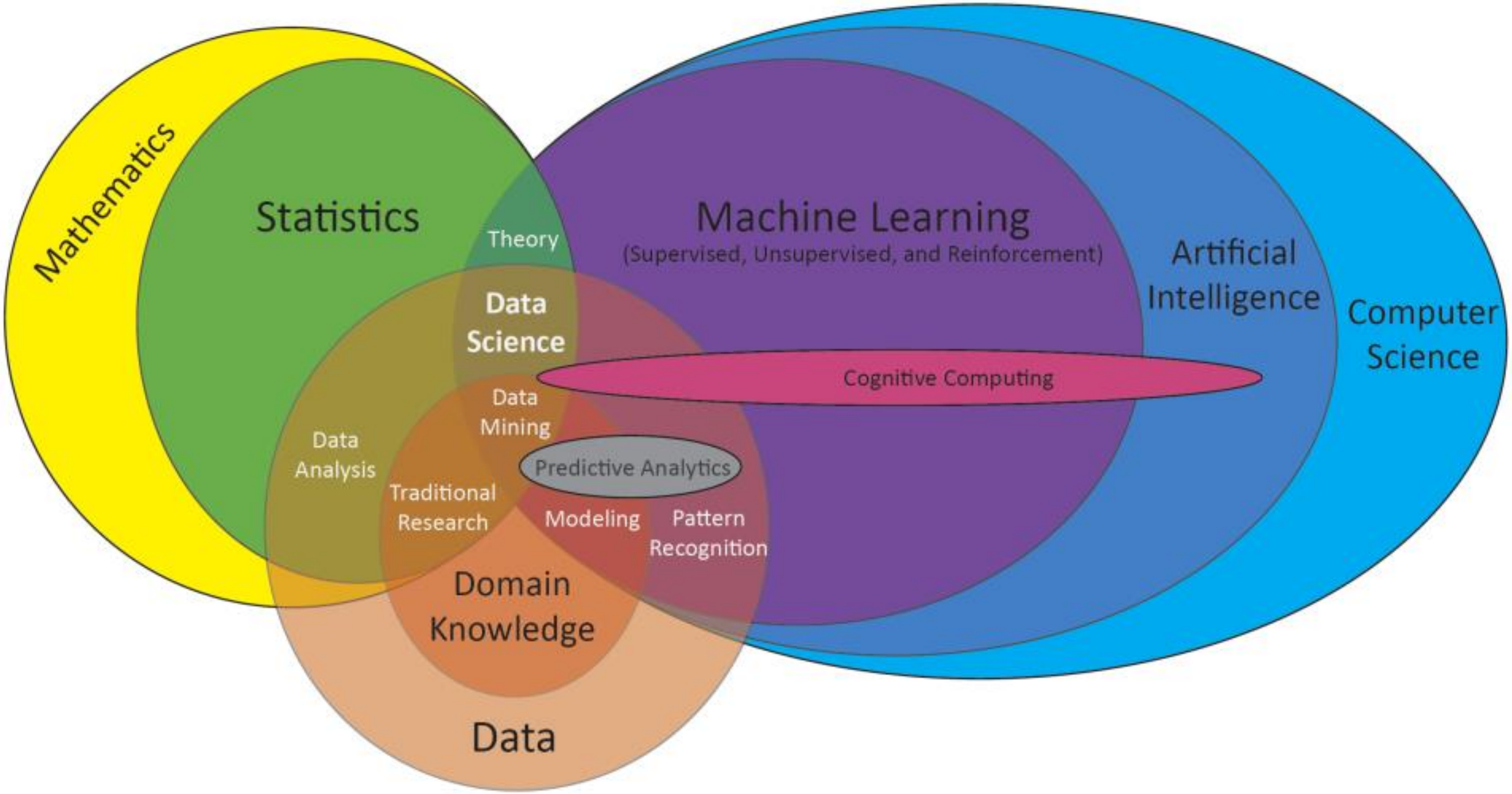


Machine learning and related concepts



Source: https://blogs.nvidia.com/blog/2016/07/29/whats-difference-artificial-intelligence-machine-learning-deep-learning-ai/

Machine learning and related concepts



ID3 Decision Tree Learning

Learning agents – Why learning?

- Unknown environments
  - A robot designed to navigate mazes must learn the layout of each new maze it encounters.
- Environment changes over time
  - An agent designed to predict tomorrow's stock market prices must learn to adapt when conditions change from boom to bust.
- No idea how to program a solution
  - The task to recognizing the faces of family members

Learning element

- Design of a learning element is affected by
  - Which *components* is to be improved
  - What *prior knowledge* the agent already has
  - What *representation* is used for the components
  - What ***feedback*** is available to learn these components

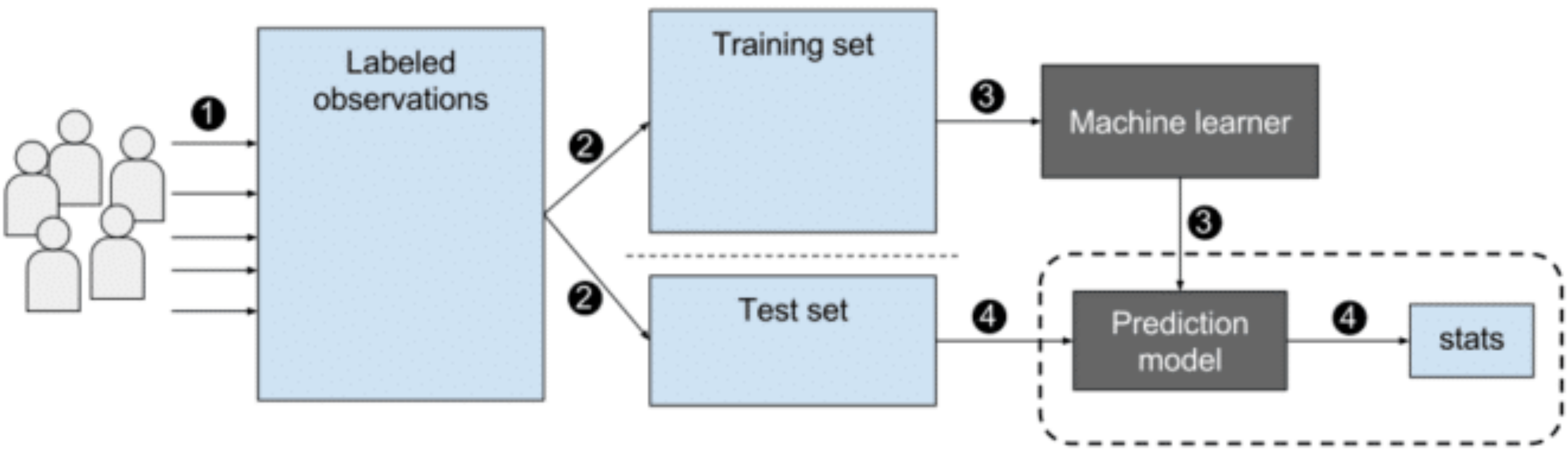
- Type of feedback
  - Supervised learning: correct answers for each example
  - Unsupervised learning: correct answers not given
  - Reinforcement learning: occasional rewards

Supervised learning

- Simplest form: learn a function from examples
- Given a **training set** of N example input-output pairs

$$(x_1, y_1), (x_2, y_2), \dots, (x_N, y_N)$$

  - where each y_j was generated by an unknown function $y = f(x)$
- Find a hypothesis $\boldsymbol{h}$ such that $\boldsymbol{h} \approx \boldsymbol{f}$
- To measure the accuracy of a hypothesis, give it a **test set** of examples that are different with those in the training set.



Supervised learning

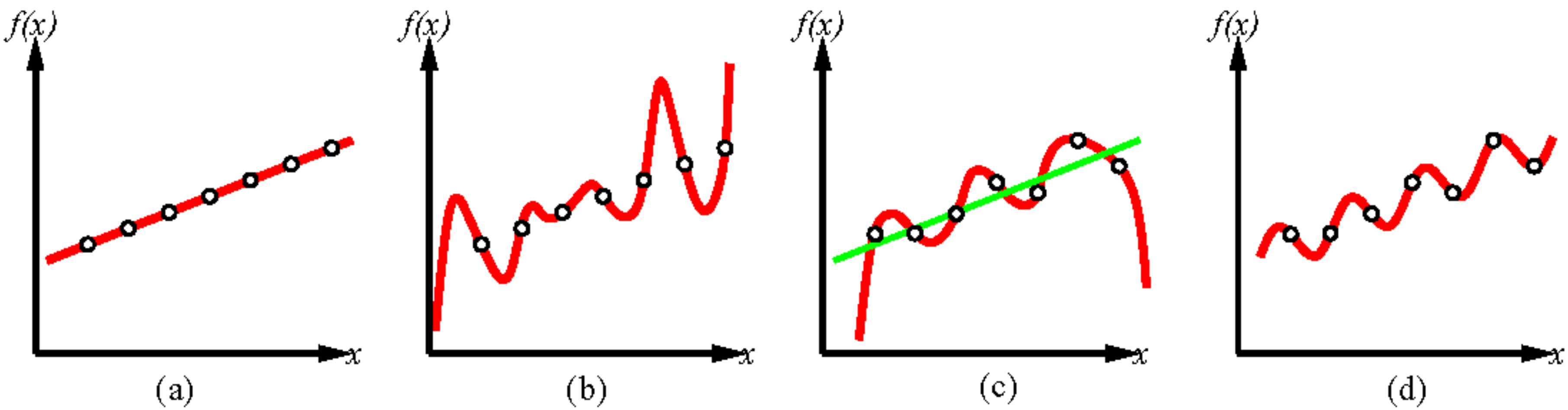
- Construct h so that it agrees with f.
- The hypothesis h is **consistent** if it agrees with f on all observations.
- **Ockham's razor:** Select the simplest consistent hypothesis.

Consistent linear fit

Consistent 7th order polynomial fit

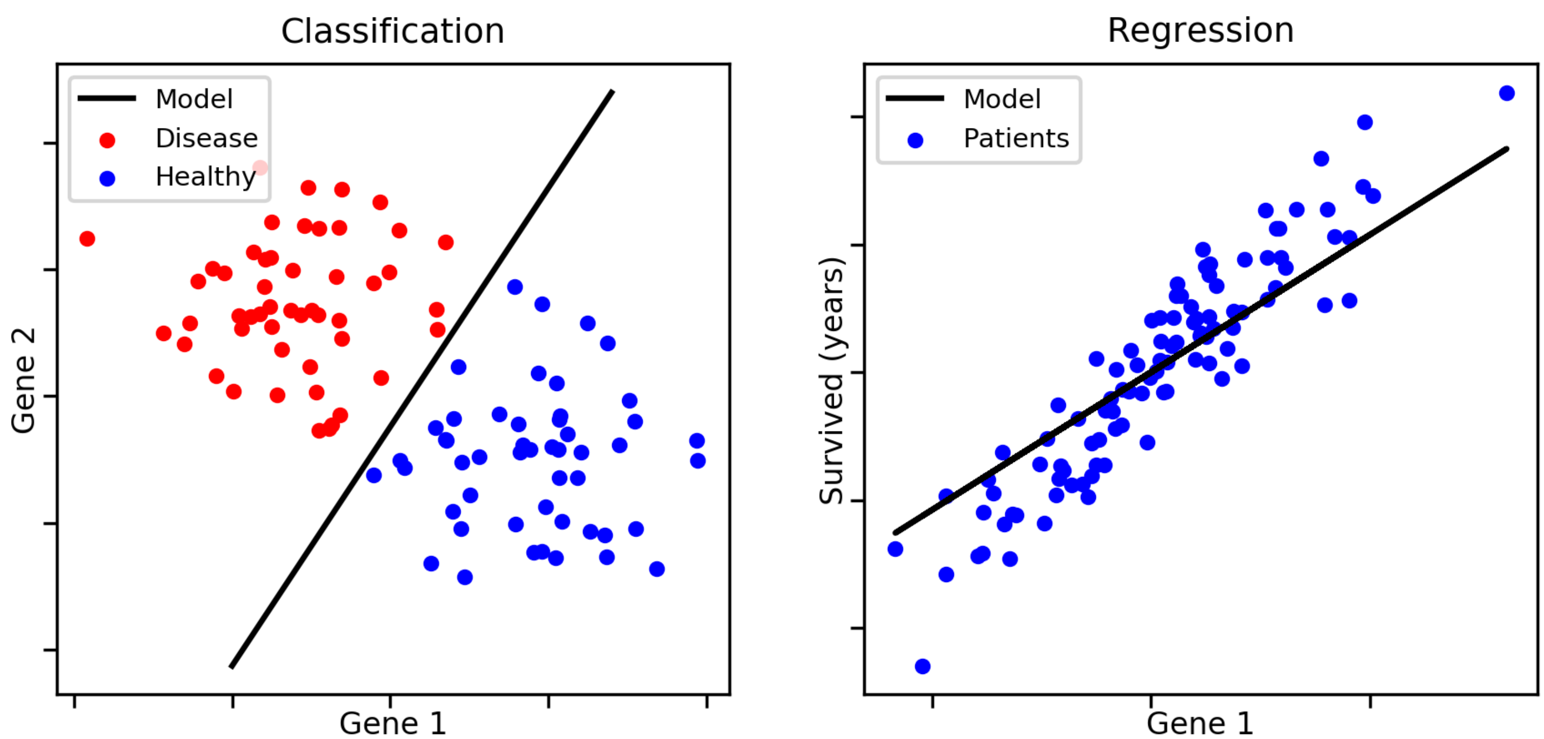
Inconsistent linear fit. Consistent 6th order polynomial fit.

Consistent sinusoidal fit



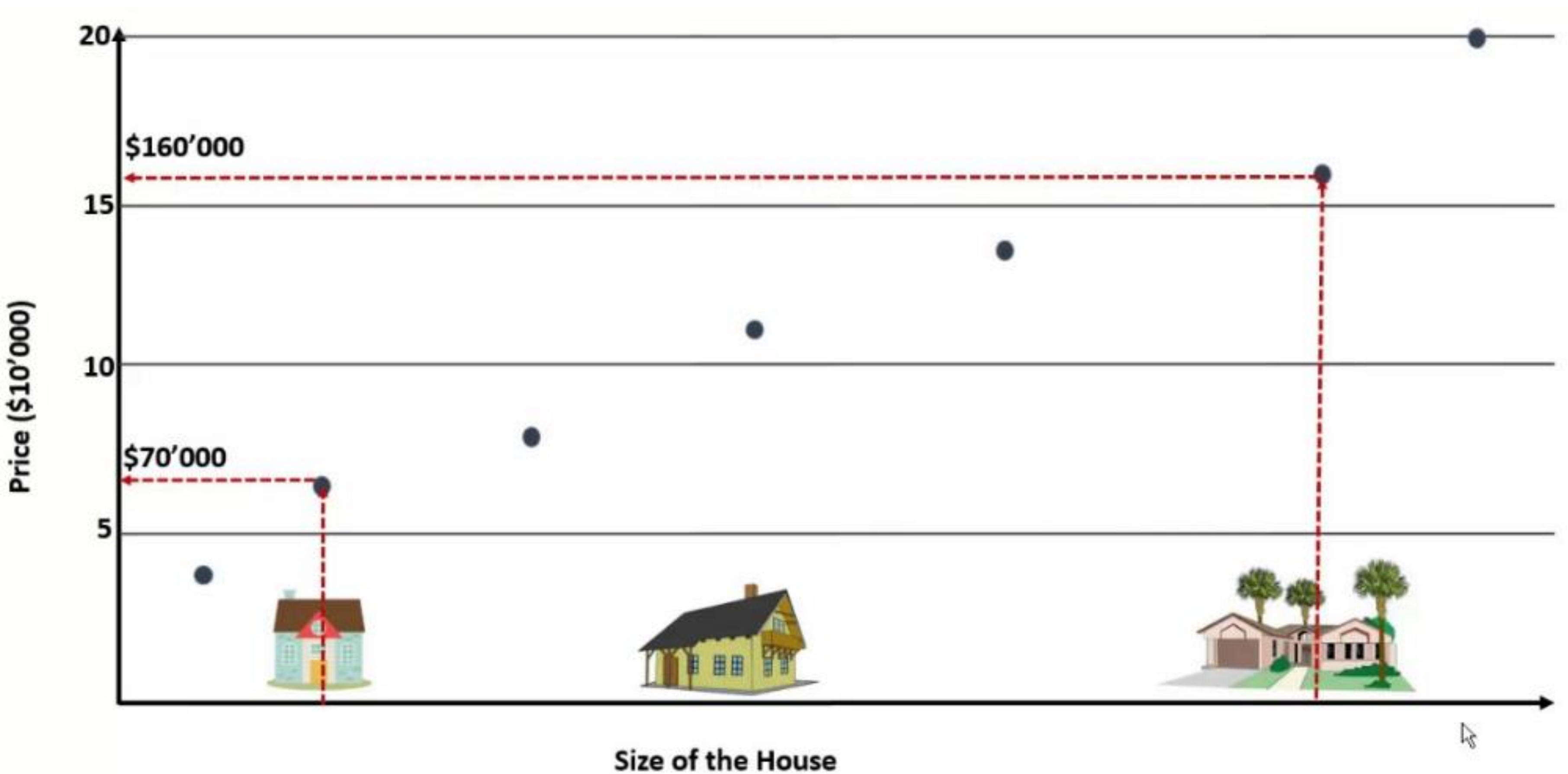
Supervised learning problems

- $\boldsymbol{h(x)} = \textit{the predicted output value for the input } \boldsymbol{x}$
  - Discrete valued function $\rightarrow$ classification
  - Continuous valued function $\rightarrow$ regression



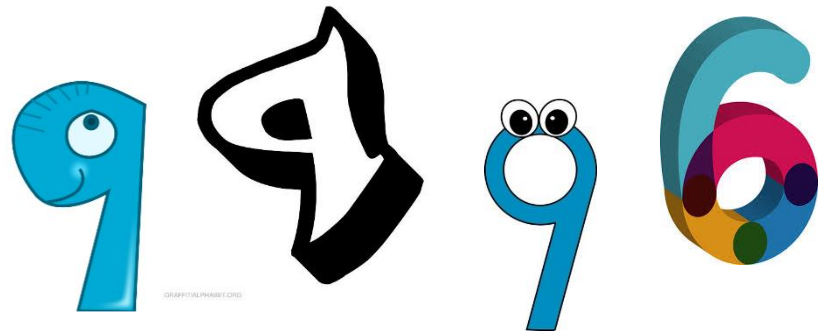
Regression vs. Classification

- Estimating the price of a house



Regression vs. Classification

- Is this number 9?
  - 2 classes: Yes/No



- Will you pass or fail the exam?
  - 2 classes: Fail/Pass

- Is this an apple, an orange or a tomato?
  - 3 classes: Apple/Orange/Tomato

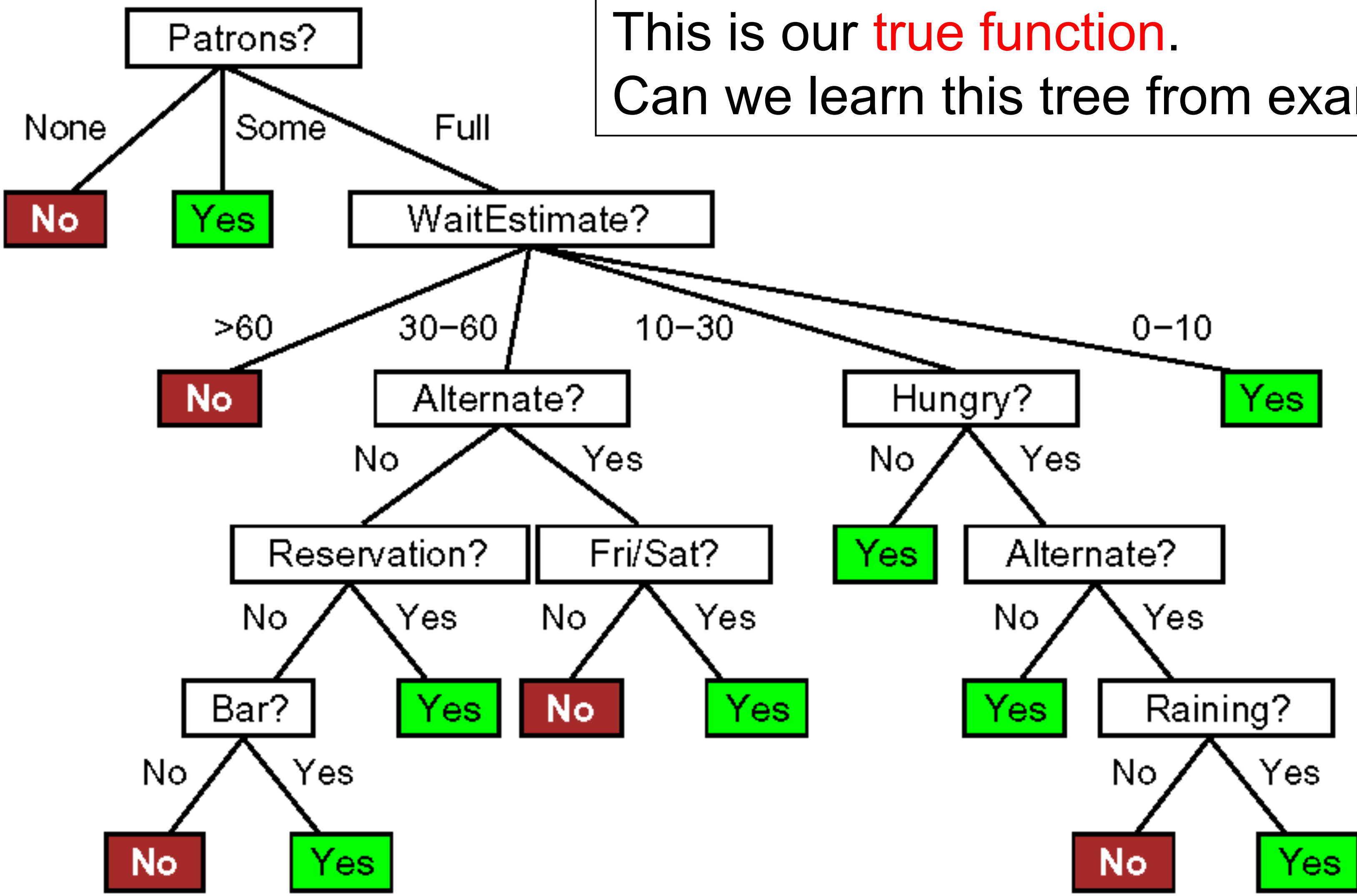
A classification problem example

Predicting whether a certain person will wait to have a seat in a restaurant.

A classification problem example

- The decision is based on the following attributes
  1. **Alternate:** is there an alternative restaurant nearby?
  2. **Bar:** is there a comfortable bar area to wait in?
  3. **Fri/Sat:** is today Friday or Saturday?
  4. **Hungry:** are we hungry?
  5. **Patrons:** number of people in the restaurant (None, Some, Full)
  6. **Price:** price range ($, $$, $$$)
  7. **Raining:** is it raining outside?
  8. **Reservation:** have we made a reservation?
  9. **Type:** kind of restaurant (French, Italian, Thai, Burger)
  10. **WaitEstimate:** estimated waiting time (0-10, 10-30, 30-60, >60)

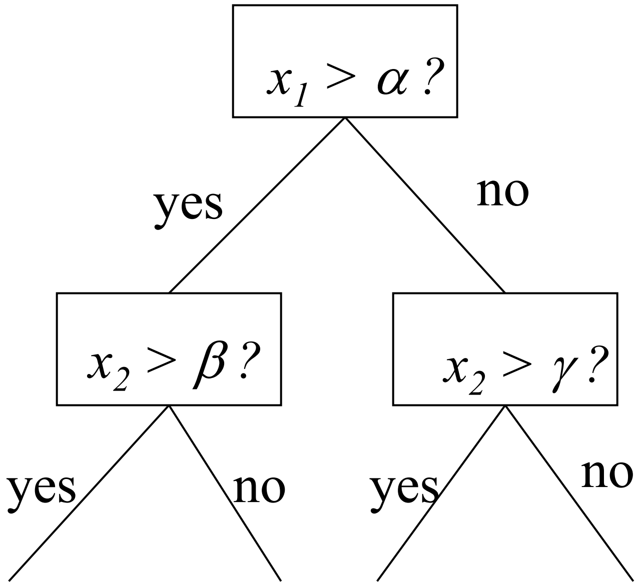
The wait@restaurant decision tree



This is our true function.
Can we learn this tree from examples?

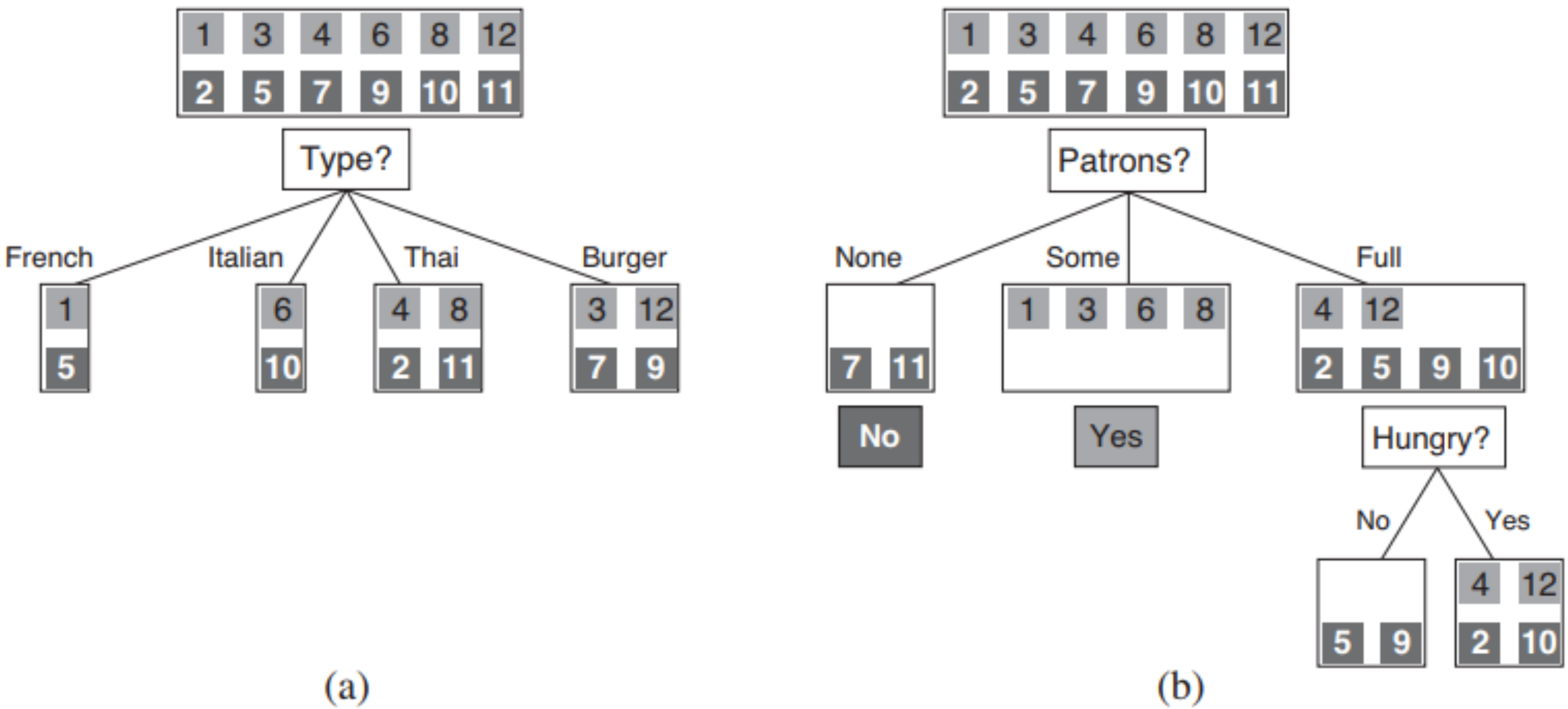
Learning decision trees

- **Divide and conquer:** Split data into smaller and smaller subsets
- Splits usually on a single variable



- After splitting up, each outcome is a new decision tree learning problem with fewer examples and one less attribute.

Learning decision trees



Splitting the examples by testing on attributes

Learning decision trees

1. The remaining examples are **all** positive (or **all** negative), $\rightarrow$ DONE, it is possible to answer Yes or No.
   - E.g., in Figure (b), None and Some branches
2. There are **some** positive and **some** negative examples $\rightarrow$ choose the **best** attribute to split them
   - E.g., in Figure (b), Hungry is used to split the remaining examples

Learning decision trees

3. **No** examples left at a branch $\rightarrow$ return a default value.
   - No example has been observed for a combination of attribute values
   - The default value is calculated from the plurality classification of all the examples that were used in constructing the node's parent.
   - These are passed along in the variable parent examples
4. **No** attributes left but both positive and negative examples $\rightarrow$ return the plurality classification of remaining ones.
   - Examples of the same description, but different classifications
   - Usually an error or noise in the data, nondeterministic domain, or no observation of an attribute that would distinguish the examples.

Decision-tree learning algorithm

function DECISION-TREE-LEARNING(*examples*, *attributes*, *parent examples*)
returns a tree
  if *examples* is empty
      then return PLURALITY-VALUE(*parent examples*)
  else if all *examples* have the same classification
      then return the classification
  else if *attributes* is empty
    then return PLURALITY-VALUE(*examples*)
  else
    ...

No examples left

remaining examples are all pos/all neg

No attributes left but examples are still pos & neg

Decision-tree learning algorithm

function DECISION-TREE-LEARNING$(examples, attributes, parent\ examples)$
returns a tree

...

else

$A \leftarrow argmax_{a \in attributes}$IMPORTANCE$(a, examples)$

$tree$ ← a new decision tree with root test A

for each value v_k of A **do**

$exs \leftarrow \{e : e \in examples$ **and** $e.A = vk\}$

$subtree \leftarrow$ DECISION-TREE-LEARNING$(exs, attributes - A, examples)$

add a branch to $tree$ with label $(A = v_k)$ and subtree $subtree$

return $tree$

Inductive learning of decision tree

- **Simplest:** Construct a decision tree with one leaf for every example = memory based learning.

  → *Not very good generalization.*

- **Advanced:** Split on each variable so that the purity of each split increases (i.e. either only yes or only no)
  - E.g., using Entropy to measure the purity of data

A purity measure with entropy

- **Entropy** is a measure of the uncertainty of a random variable V with values v_k.

An indicator of how messy your data is

$$H(V) = \sum_k P(v_k) \log_2 \frac{1}{P(v_k)} = -\sum_k P(v_k) \log_2 P(v_k)$$

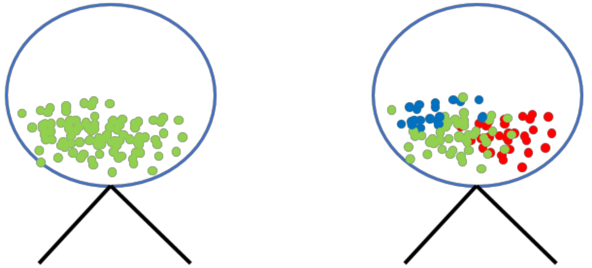
  - v_k is a class in V (e.g., yes/no in binary classification)
  - $P(v_k)$ is the proportion of the number of elements in class v_k to the number of elements in V

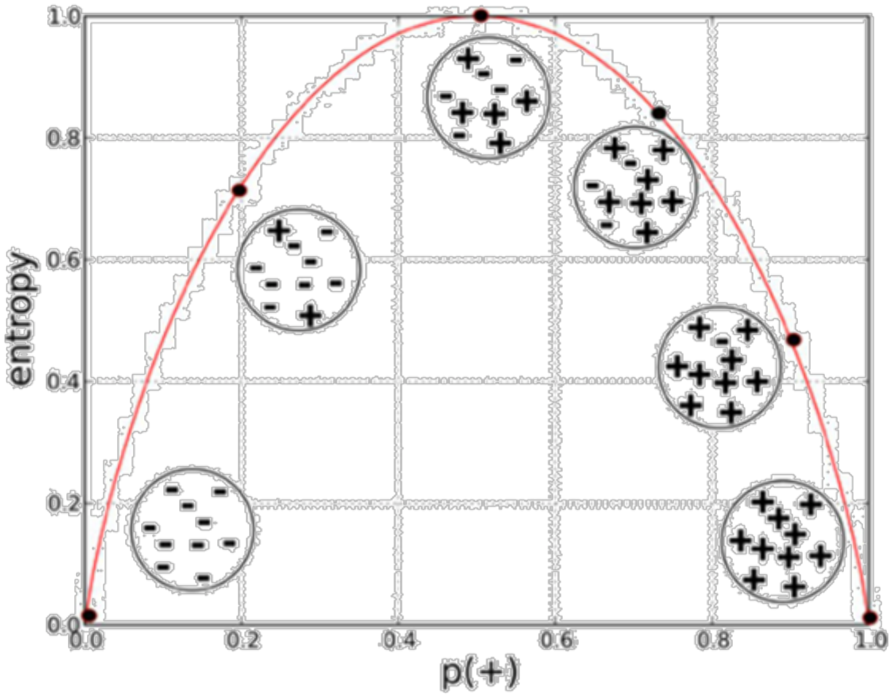
A purity measure with entropy

- Entropy is maximal when all possibilities are equally likely.
- Entropy is zero in a pure ”yes” (or pure ”no”) node.

Totally pure

More impure





Provost, Foster; Fawcett, Tom. Data Science for Business: What You Need to Know about Data Mining and Data-Analytic Thinking

- Decision tree aims to **decrease** the entropy in each node.

The wait@restaurant training data

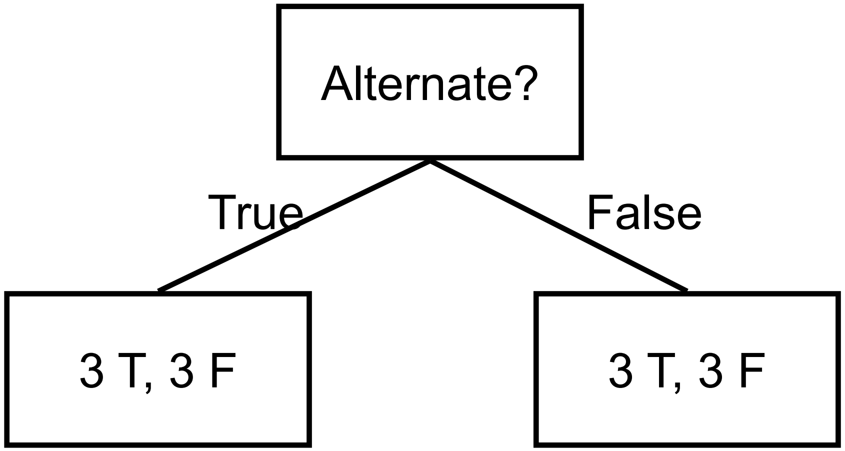
T = True, F = False

Example	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
X_1	T	F	F	T	Some	$$$	F	T	French	0–10	T
X_2	T	F	F	T	Full	$	F	F	Thai	30–60	F
X_3	F	T	F	F	Some	$	F	F	Burger	0–10	T
X_4	T	F	T	T	Full	$	F	F	Thai	10–30	T
X_5	T	F	T	F	Full	$$$	F	T	French	>60	F
X_6	F	T	F	T	Some	$$	T	T	Italian	0–10	T
X_7	F	T	F	F	None	$	T	F	Burger	0–10	F
X_8	F	F	F	T	Some	$$	T	T	Thai	0–10	T
X_9	F	T	T	F	Full	$	T	F	Burger	>60	F
X_{10}	T	T	T	T	Full	$$$	F	T	Italian	10–30	F
X_{11}	F	F	F	F	None	$	F	F	Thai	0–10	F
X_{12}	T	T	T	T	Full	$	F	F	Burger	30–60	T

6 True,
6 False

$$H(S) = -\left({}^{6}/_{12}\right)\log_2\left({}^{6}/_{12}\right) - \left({}^{6}/_{12}\right)\log_2\left({}^{6}/_{12}\right)$$
$$= 1$$

Decision tree learning example



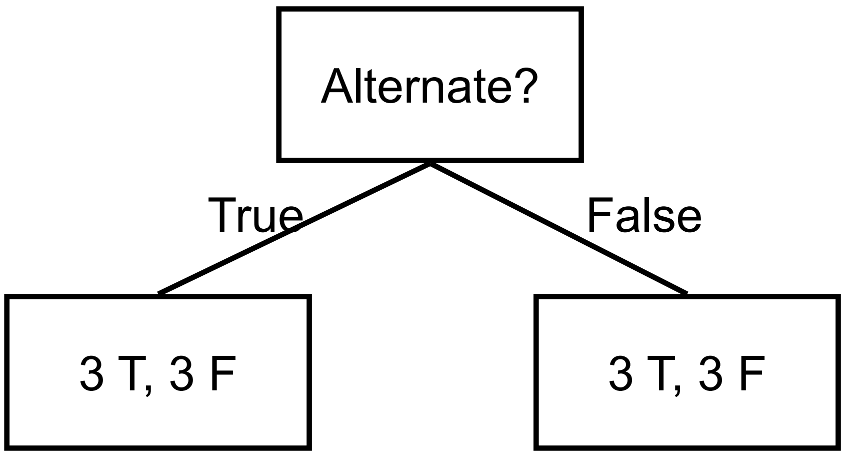
Example	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
X_1	T	F	F	T	Some	$$$	F	T	French	0–10	T
X_2	T	F	F	T	Full	$	F	F	Thai	30–60	F
X_3	F	T	F	F	Some	$	F	F	Burger	0–10	T
X_4	T	F	T	T	Full	$	F	F	Thai	10–30	T
X_5	T	F	T	F	Full	$$$	F	T	French	>60	F
X_6	F	T	F	T	Some	$$	T	T	Italian	0–10	T
X_7	F	T	F	F	None	$	T	F	Burger	0–10	F
X_8	F	F	F	T	Some	$$	T	T	Thai	0–10	T
X_9	F	T	T	F	Full	$	T	F	Burger	>60	F
X_{10}	T	T	T	T	Full	$$$	F	T	Italian	10–30	F
X_{11}	F	F	F	F	None	$	F	F	Thai	0–10	F
X_{12}	T	T	T	T	Full	$	F	F	Burger	30–60	T

- Calculate Average Entropy of attribute Alternate

$$AE_{Alternate} = P(Alt = T) \times H(Alt = T) + P(Alt = F) \times H(Alt = F)$$

$$AE_{Alternate} = \frac{6}{12}\left[-\left(\frac{3}{6}\log_2\frac{3}{6}\right)-\left(\frac{3}{6}\log_2\frac{3}{6}\right)\right] + \frac{6}{12}\left[-\left(\frac{3}{6}\log_2\frac{3}{6}\right)-\left(\frac{3}{6}\log_2\frac{3}{6}\right)\right] = 1$$

Decision tree learning example

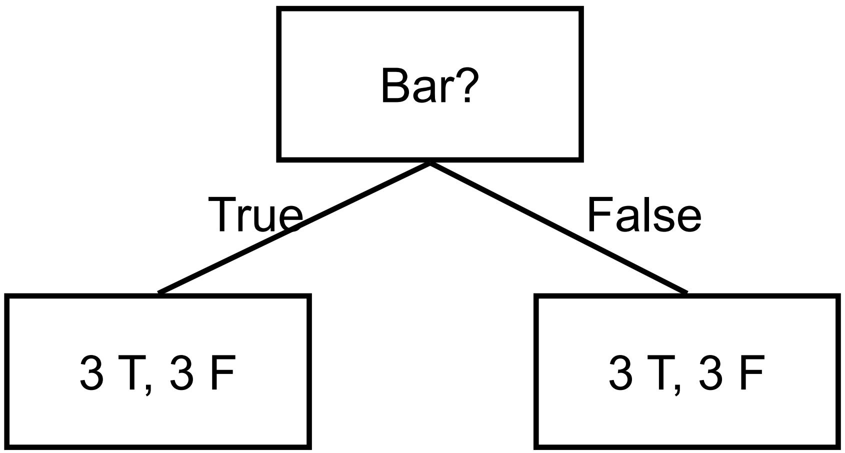


Example	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
X_1	T	F	F	T	Some	$$$	F	T	French	0–10	T
X_2	T	F	F	T	Full	$	F	F	Thai	30–60	F
X_3	F	T	F	F	Some	$	F	F	Burger	0–10	T
X_4	T	F	T	T	Full	$	F	F	Thai	10–30	T
X_5	T	F	T	F	Full	$$$	F	T	French	>60	F
X_6	F	T	F	T	Some	$$	T	T	Italian	0–10	T
X_7	F	T	F	F	None	$	T	F	Burger	0–10	F
X_8	F	F	F	T	Some	$$	T	T	Thai	0–10	T
X_9	F	T	T	F	Full	$	T	F	Burger	>60	F
X_{10}	T	T	T	T	Full	$$$	F	T	Italian	10–30	F
X_{11}	F	F	F	F	None	$	F	F	Thai	0–10	F
X_{12}	T	T	T	T	Full	$	F	F	Burger	30–60	T

- **Information Gain** is the difference in entropy from before to after the set S is split on the selected attribute.

$$IG(Alternate, S) = H(S) - AE_{Alternate} = 1 - 1 = 0$$

Decision tree learning example

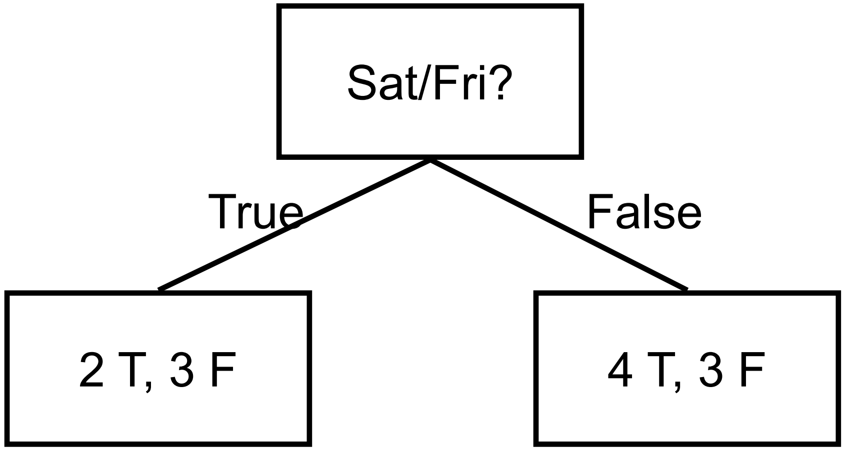


Example	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
X_1	T	F	F	T	Some	$$$	F	T	French	0–10	T
X_2	T	F	F	T	Full	$	F	F	Thai	30–60	F
X_3	F	T	F	F	Some	$	F	F	Burger	0–10	T
X_4	T	F	T	T	Full	$	F	F	Thai	10–30	T
X_5	T	F	T	F	Full	$$$	F	T	French	>60	F
X_6	F	T	F	T	Some	$$	T	T	Italian	0–10	T
X_7	F	T	F	F	None	$	T	F	Burger	0–10	F
X_8	F	F	F	T	Some	$$	T	T	Thai	0–10	T
X_9	F	T	T	F	Full	$	T	F	Burger	>60	F
X_{10}	T	T	T	T	Full	$$$	F	T	Italian	10–30	F
X_{11}	F	F	F	F	None	$	F	F	Thai	0–10	F
X_{12}	T	T	T	T	Full	$	F	F	Burger	30–60	T

$$AE_{Bar} = \frac{6}{12}\left[-\left(\frac{3}{6}\log_2\frac{3}{6}\right)-\left(\frac{3}{6}\log_2\frac{3}{6}\right)\right]+\frac{6}{12}\left[-\left(\frac{3}{6}\log_2\frac{3}{6}\right)-\left(\frac{3}{6}\log_2\frac{3}{6}\right)\right]=1$$

$$IG(Bar,S) = H(S) - AE_{Bar} = 1 - 1 = {\color{red}0}$$

Decision tree learning example

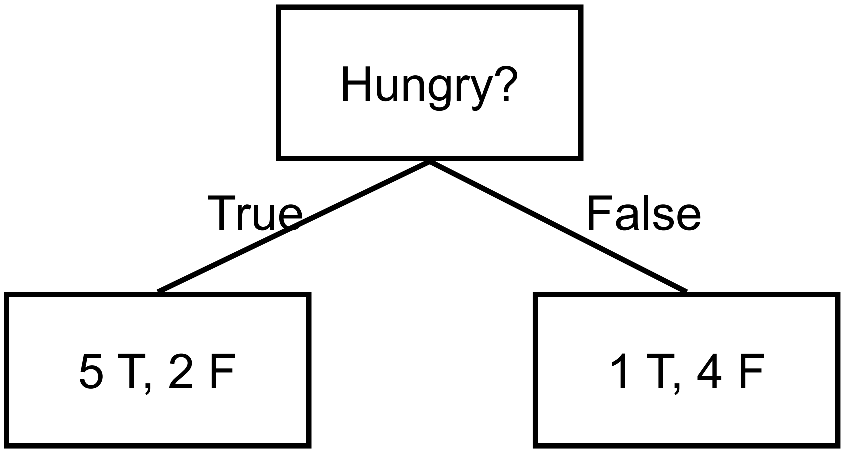


Example	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
X_1	T	F	F	T	Some	$$$	F	T	French	0–10	T
X_2	T	F	F	T	Full	$	F	F	Thai	30–60	F
X_3	F	T	F	F	Some	$	F	F	Burger	0–10	T
X_4	T	F	T	T	Full	$	F	F	Thai	10–30	T
X_5	T	F	T	F	Full	$$$	F	T	French	>60	F
X_6	F	T	F	T	Some	$$	T	T	Italian	0–10	T
X_7	F	T	F	F	None	$	T	F	Burger	0–10	F
X_8	F	F	F	T	Some	$$	T	T	Thai	0–10	T
X_9	F	T	T	F	Full	$	T	F	Burger	>60	F
X_{10}	T	T	T	T	Full	$$$	F	T	Italian	10–30	F
X_{11}	F	F	F	F	None	$	F	F	Thai	0–10	F
X_{12}	T	T	T	T	Full	$	F	F	Burger	30–60	T

$$AE_{Sat/Fri?} = \frac{5}{12}\left[-\left(\frac{2}{5}\log_2\frac{2}{5}\right)-\left(\frac{3}{5}\log_2\frac{3}{5}\right)\right]+\frac{7}{12}\left[-\left(\frac{4}{7}\log_2\frac{4}{7}\right)-\left(\frac{3}{7}\log_2\frac{3}{7}\right)\right]$$
$$= 0.979$$

$$IG(Sat/Fri?, S) = H(S) - AE_{Sat/Fri?} = 1 - 0.979 = 0.021$$

Decision tree learning example

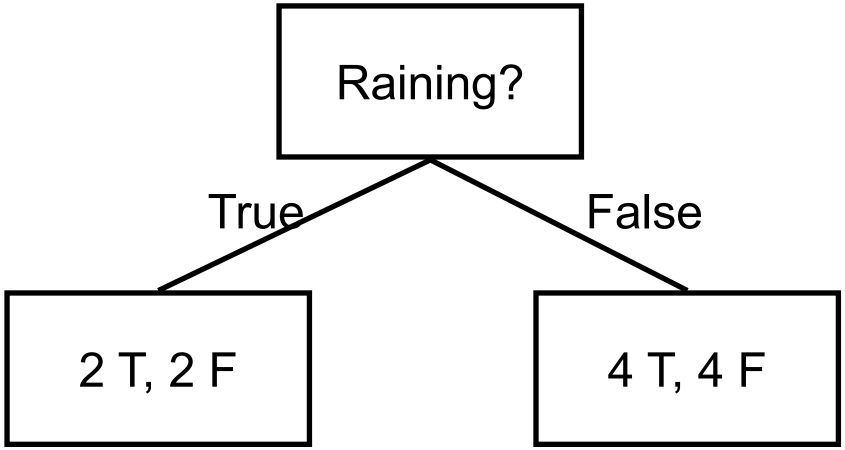


Example	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
X_1	*T*	*F*	*F*	*T*	*Some*	*$$$*	*F*	*T*	*French*	*0–10*	*T*
X_2	*T*	*F*	*F*	*T*	*Full*	*$*	*F*	*F*	*Thai*	*30–60*	*F*
X_3	*F*	*T*	*F*	*F*	*Some*	*$*	*F*	*F*	*Burger*	*0–10*	*T*
X_4	*T*	*F*	*T*	*T*	*Full*	*$*	*F*	*F*	*Thai*	*10–30*	*T*
X_5	*T*	*F*	*T*	*F*	*Full*	*$$$*	*F*	*T*	*French*	*>60*	*F*
X_6	*F*	*T*	*F*	*T*	*Some*	*$$*	*T*	*T*	*Italian*	*0–10*	*T*
X_7	*F*	*T*	*F*	*F*	*None*	*$*	*T*	*F*	*Burger*	*0–10*	*F*
X_8	*F*	*F*	*F*	*T*	*Some*	*$$*	*T*	*T*	*Thai*	*0–10*	*T*
X_9	*F*	*T*	*T*	*F*	*Full*	*$*	*T*	*F*	*Burger*	*>60*	*F*
X_{10}	*T*	*T*	*T*	*T*	*Full*	*$$$*	*F*	*T*	*Italian*	*10–30*	*F*
X_{11}	*F*	*F*	*F*	*F*	*None*	*$*	*F*	*F*	*Thai*	*0–10*	*F*
X_{12}	*T*	*T*	*T*	*T*	*Full*	*$*	*F*	*F*	*Burger*	*30–60*	*T*

$$AE_{Hungry} = \frac{7}{12}\left[-\left(\frac{5}{7}\log_2\frac{5}{7}\right)-\left(\frac{2}{7}\log_2\frac{2}{7}\right)\right]+\frac{5}{12}\left[-\left(\frac{1}{5}\log_2\frac{1}{5}\right)-\left(\frac{4}{5}\log_2\frac{4}{5}\right)\right]$$
$$= 0.804$$

$$IG(Hungry, S) = H(S) - AE_{Hungry} = 1 - 0.804 = 0.196$$

Decision tree learning example

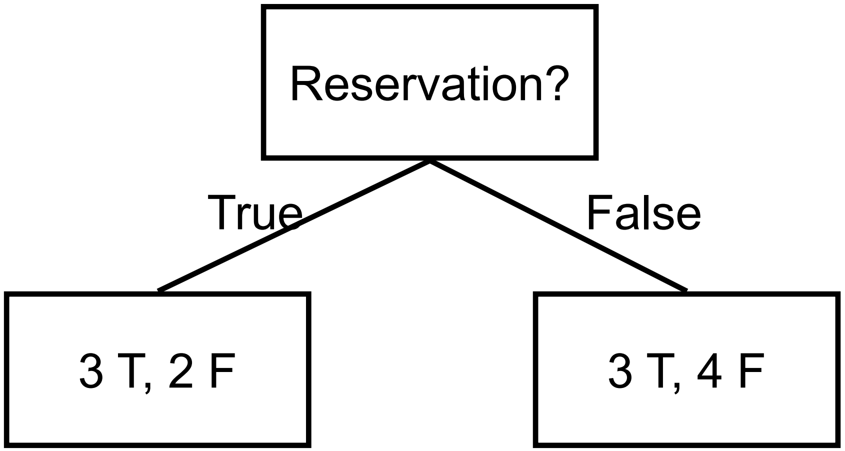


Example	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
X_1	T	F	F	T	Some	\$\$\$	F	T	French	0–10	T
X_2	T	F	F	T	Full	\$	F	F	Thai	30–60	F
X_3	F	T	F	F	Some	\$	F	F	Burger	0–10	T
X_4	T	F	T	T	Full	\$	F	F	Thai	10–30	T
X_5	T	F	T	F	Full	\$\$\$	F	T	French	>60	F
X_6	F	T	F	T	Some	\$\$	T	T	Italian	0–10	T
X_7	F	T	F	F	None	\$	T	F	Burger	0–10	F
X_8	F	F	F	T	Some	\$\$	T	T	Thai	0–10	T
X_9	F	T	T	F	Full	\$	T	F	Burger	>60	F
X_{10}	T	T	T	T	Full	\$\$\$	F	T	Italian	10–30	F
X_{11}	F	F	F	F	None	\$	F	F	Thai	0–10	F
X_{12}	T	T	T	T	Full	\$	F	F	Burger	30–60	T

$$AE_{Raining} = \frac{4}{12}\left[-\left(\frac{2}{4}\log_2\frac{2}{4}\right)-\left(\frac{2}{4}\log_2\frac{2}{4}\right)\right]+\frac{8}{12}\left[-\left(\frac{4}{8}\log_2\frac{4}{8}\right)-\left(\frac{4}{8}\log_2\frac{4}{8}\right)\right]=1$$

$$IG(Raining, S) = H(S) - AE_{Hungry} = 1 - 1 = 0$$

Decision tree learning example



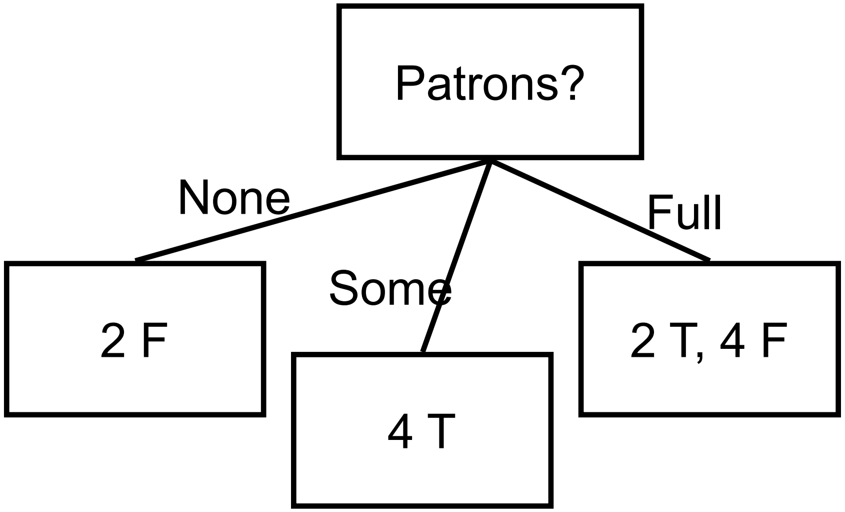
Example	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
X_1	T	F	F	T	Some	$$$	F	T	French	0–10	T
X_2	T	F	F	T	Full	$	F	F	Thai	30–60	F
X_3	F	T	F	F	Some	$	F	F	Burger	0–10	T
X_4	T	F	T	T	Full	$	F	F	Thai	10–30	T
X_5	T	F	T	F	Full	$$$	F	T	French	>60	F
X_6	F	T	F	T	Some	$$	T	T	Italian	0–10	T
X_7	F	T	F	F	None	$	T	F	Burger	0–10	F
X_8	F	F	F	T	Some	$$	T	T	Thai	0–10	T
X_9	F	T	T	F	Full	$	T	F	Burger	>60	F
X_{10}	T	T	T	T	Full	$$$	F	T	Italian	10–30	F
X_{11}	F	F	F	F	None	$	F	F	Thai	0–10	F
X_{12}	T	T	T	T	Full	$	F	F	Burger	30–60	T

$$AE_{Reservation} = \frac{5}{12}\left[-\left(\frac{3}{5}\log_2\frac{3}{5}\right)-\left(\frac{2}{5}\log_2\frac{2}{5}\right)\right]+\frac{7}{12}\left[-\left(\frac{3}{7}\log_2\frac{3}{7}\right)-\left(\frac{4}{7}\log_2\frac{4}{7}\right)\right]$$

$$= 0.979$$

$$IG(Reservation, S) = H(S) - AE_{Reservation} = 1 - 0.979 = 0.021$$

Decision tree learning example



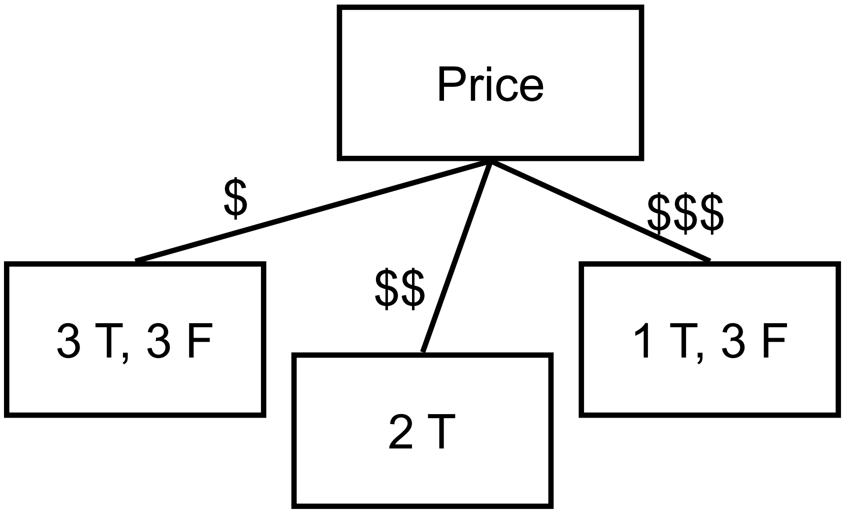
Example	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
X_1	T	F	F	T	Some	$$$	F	T	French	0–10	T
X_2	T	F	F	T	Full	$	F	F	Thai	30–60	F
X_3	F	T	F	F	Some	$	F	F	Burger	0–10	T
X_4	T	F	T	T	Full	$	F	F	Thai	10–30	T
X_5	T	F	T	F	Full	$$$	F	T	French	>60	F
X_6	F	T	F	T	Some	$$	T	T	Italian	0–10	T
X_7	F	T	F	F	None	$	T	F	Burger	0–10	F
X_8	F	F	F	T	Some	$$	T	T	Thai	0–10	T
X_9	F	T	T	F	Full	$	T	F	Burger	>60	F
X_{10}	T	T	T	T	Full	$$$	F	T	Italian	10–30	F
X_{11}	F	F	F	F	None	$	F	F	Thai	0–10	F
X_{12}	T	T	T	T	Full	$	F	F	Burger	30–60	T

$$AE_{Patron} = \frac{2}{12}\left[-\left(\frac{0}{2}\log_2\frac{0}{2}\right)-\left(\frac{2}{2}\log_2\frac{2}{2}\right)\right]+\frac{4}{12}\left[-\left(\frac{4}{4}\log_2\frac{4}{4}\right)-\left(\frac{0}{4}\log_2\frac{0}{4}\right)\right]$$

$$+\frac{6}{12}\left[-\left(\frac{2}{6}\log_2\frac{2}{6}\right)-\left(\frac{4}{6}\log_2\frac{4}{6}\right)\right] = 0.541$$

$$IG(Patron, S) = H(S) - AE_{Patron} = 1 - 0.541 = 0.459$$

Decision tree learning example

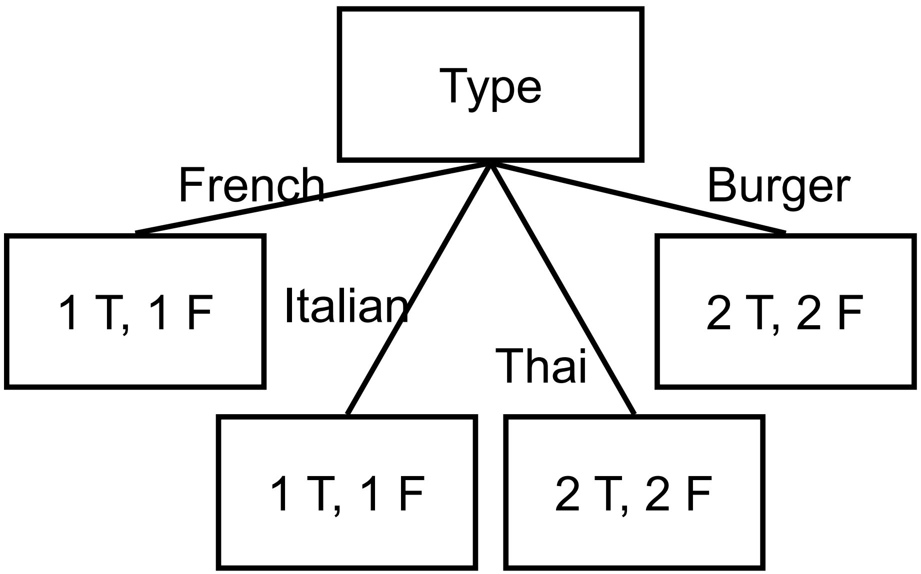


Example	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
X_1	T	F	F	T	Some	\$\$\$	F	T	French	0–10	T
X_2	T	F	F	T	Full	\$	F	F	Thai	30–60	F
X_3	F	T	F	F	Some	\$	F	F	Burger	0–10	T
X_4	T	F	T	T	Full	\$	F	F	Thai	10–30	T
X_5	T	F	T	F	Full	\$\$\$	F	T	French	>60	F
X_6	F	T	F	T	Some	\$\$	T	T	Italian	0–10	T
X_7	F	T	F	F	None	\$	T	F	Burger	0–10	F
X_8	F	F	F	T	Some	\$\$	T	T	Thai	0–10	T
X_9	F	T	T	F	Full	\$	T	F	Burger	>60	F
X_{10}	T	T	T	T	Full	\$\$\$	F	T	Italian	10–30	F
X_{11}	F	F	F	F	None	\$	F	F	Thai	0–10	F
X_{12}	T	T	T	T	Full	\$	F	F	Burger	30–60	T

$$
\begin{aligned}
&AE_{Price} \\
&= \frac{6}{12}\left[-\left(\frac{3}{6}\log_2\frac{3}{6}\right)-\left(\frac{3}{6}\log_2\frac{3}{6}\right)\right]+\frac{2}{12}\left[-\left(\frac{2}{2}\log_2\frac{2}{2}\right)-\left(\frac{0}{2}\log_2\frac{0}{2}\right)\right] \\
&+\frac{4}{12}\left[-\left(\frac{1}{4}\log_2\frac{1}{4}\right)-\left(\frac{3}{4}\log_2\frac{3}{4}\right)\right]=0.770
\end{aligned}
$$

$$IG(Price, S) = H(S) - AE_{Price} = 1 - 0.770 = 0.23$$

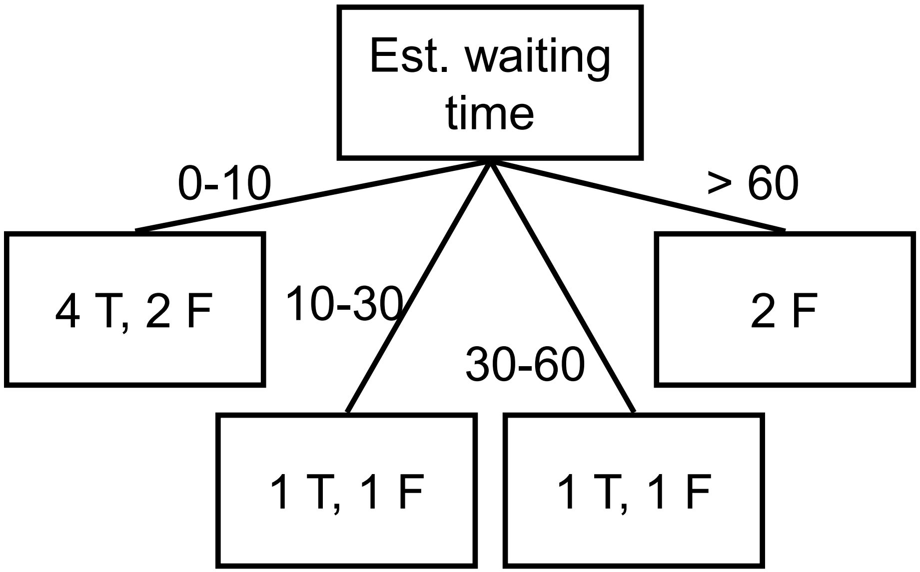
Decision tree learning example



Example	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
X_1	T	F	F	T	Some	$$$	F	T	French	0–10	T
X_2	T	F	F	T	Full	$	F	F	Thai	30–60	F
X_3	F	T	F	F	Some	$	F	F	Burger	0–10	T
X_4	T	F	T	T	Full	$	F	F	Thai	10–30	T
X_5	T	F	T	F	Full	$$$	F	T	French	>60	F
X_6	F	T	F	T	Some	$$	T	T	Italian	0–10	T
X_7	F	T	F	F	None	$	T	F	Burger	0–10	F
X_8	F	F	F	T	Some	$$	T	T	Thai	0–10	T
X_9	F	T	T	F	Full	$	T	F	Burger	>60	F
X_{10}	T	T	T	T	Full	$$$	F	T	Italian	10–30	F
X_{11}	F	F	F	F	None	$	F	F	Thai	0–10	F
X_{12}	T	T	T	T	Full	$	F	F	Burger	30–60	T

$$AE_{Type}$$
$$= \frac{2}{12}\left[-\left(\frac{1}{2}\log_2\frac{1}{2}\right)-\left(\frac{1}{2}\log_2\frac{1}{2}\right)\right]+\frac{2}{12}\left[-\left(\frac{1}{2}\log_2\frac{1}{2}\right)-\left(\frac{1}{2}\log_2\frac{1}{2}\right)\right]$$
$$+\frac{4}{12}\left[-\left(\frac{2}{4}\log_2\frac{2}{4}\right)-\left(\frac{2}{4}\log_2\frac{2}{4}\right)\right]+\frac{4}{12}\left[-\left(\frac{2}{4}\log_2\frac{2}{4}\right)-\left(\frac{2}{4}\log_2\frac{2}{4}\right)\right]=1$$
$$IG(Type,S)=H(S)-AE_{Type}=1-1=0$$

Decision tree learning example



Example	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
X_1	T	F	F	T	Some	$$$	F	T	French	0–10	T
X_2	T	F	F	T	Full	$	F	F	Thai	30–60	F
X_3	F	T	F	F	Some	$	F	F	Burger	0–10	T
X_4	T	F	T	T	Full	$	F	F	Thai	10–30	T
X_5	T	F	T	F	Full	$$$	F	T	French	>60	F
X_6	F	T	F	T	Some	$$	T	T	Italian	0–10	T
X_7	F	T	F	F	None	$	T	F	Burger	0–10	F
X_8	F	F	F	T	Some	$$	T	T	Thai	0–10	T
X_9	F	T	T	F	Full	$	T	F	Burger	>60	F
X_{10}	T	T	T	T	Full	$$$	F	T	Italian	10–30	F
X_{11}	F	F	F	F	None	$	F	F	Thai	0–10	F
X_{12}	T	T	T	T	Full	$	F	F	Burger	30–60	T

$$AE_{Est.waiting\ time}$$

$$= \frac{6}{12}\left[-\left(\frac{4}{6}\log_2\frac{4}{6}\right)-\left(\frac{2}{6}\log_2\frac{2}{6}\right)\right]+\frac{2}{12}\left[-\left(\frac{1}{2}\log_2\frac{1}{2}\right)-\left(\frac{1}{2}\log_2\frac{1}{2}\right)\right]$$

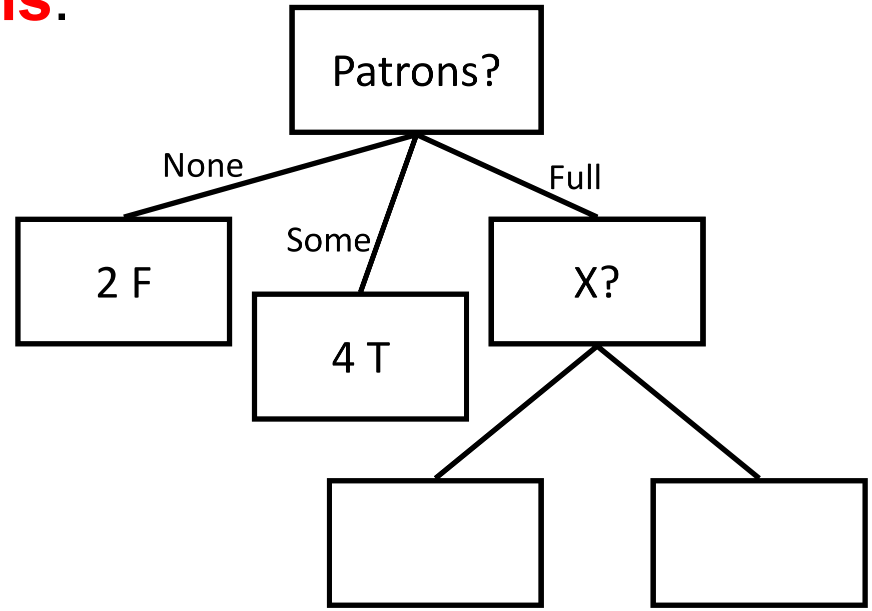
$$+\frac{2}{12}\left[-\left(\frac{1}{2}\log_2\frac{1}{2}\right)-\left(\frac{1}{2}\log_2\frac{1}{2}\right)\right]+\frac{2}{12}\left[-\left(\frac{0}{2}\log_2\frac{0}{2}\right)-\left(\frac{2}{2}\log_2\frac{2}{2}\right)\right]=0.792$$

$$IG(Est.waiting\ time, S) = H(S) - AE_{Est.waiting\ time} = 1 - 0.792$$

$$= \color{red}{0.208}$$

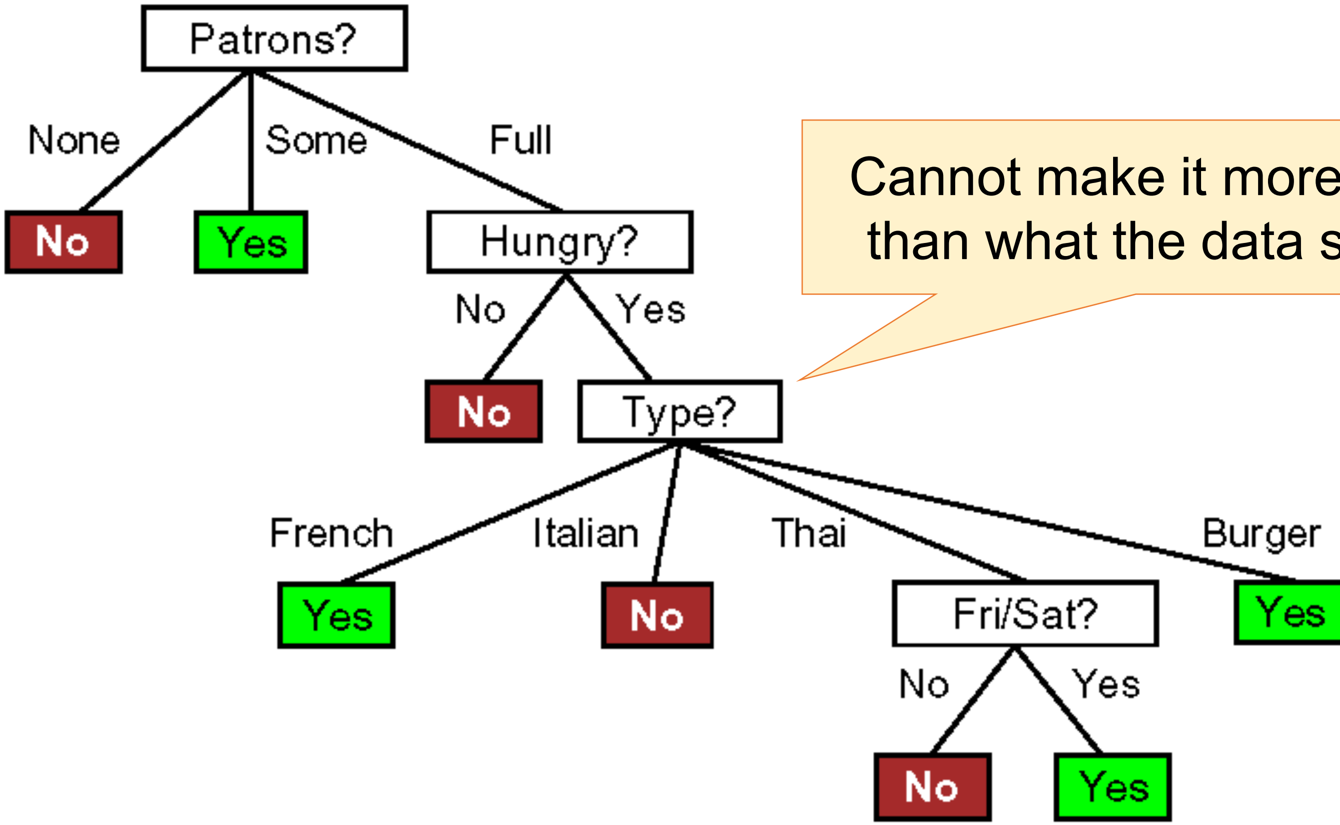
Decision tree learning example

- Largest Information Gain (0.459) / Smallest Entropy (0.541) achieved by splitting on **Patrons**.



- Continue making new splits, always purifying nodes

Decision tree learning example

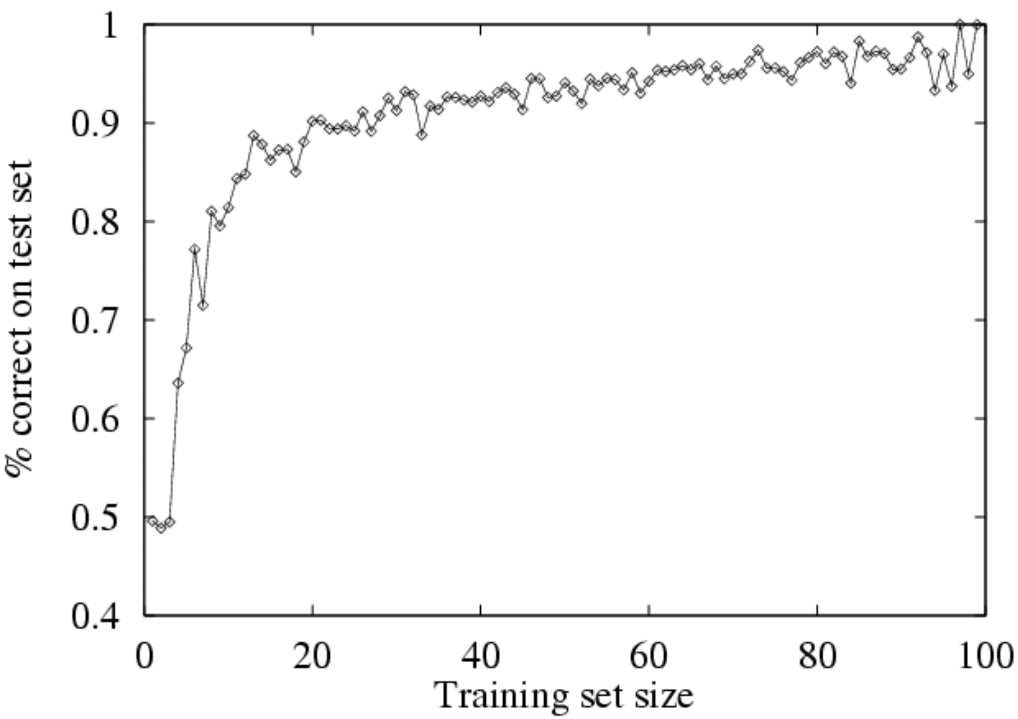


Cannot make it more complex than what the data supports.

Induced tree (from examples)

Performance measurement

- How do we know that $h \approx f$?
  1. Use theorems of computational or statistical learning theory
  2. Try h on a new test set of examples
     - Use the same distribution over example space as training set



Learning curve = % correct on test set as a function of training set size

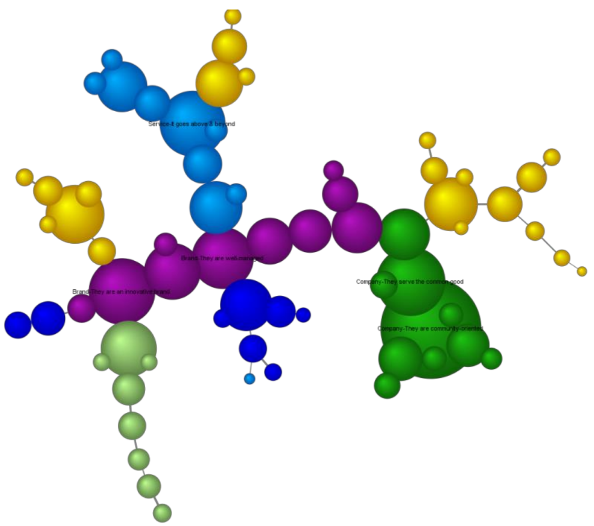
Quiz 01: ID3 decision tree

- The data represent files on a computer system. Possible values of the class variable are “infected”, which implies the file has a virus infection, or “clean” if it doesn't.
- Derive decision tree for virus identification.

No.	Writable	Updated	Size	Class
1	Yes	No	Small	Infected
2	Yes	Yes	Large	Infected
3	No	Yes	Med	Infected
4	No	No	Med	Clean
5	Yes	No	Large	Clean
6	No	No	Large	Clean

Bayesian Approaches

- *naïve Bayesian Classification*
- *Bayesian Belief Networks*



Bayesian classification

- A statistical classifier performs probabilistic prediction, i.e., predicts class membership probabilities
- Foundation: Based on **Bayes' Theorem**

Likelihood

Class Prior Probability

$$P(c \mid x) = \frac{P(x \mid c)\,P(c)}{P(x)}$$

Posterior Probability

Predictor Prior Probability

$$P(c \mid X) = P(x_1 \mid c) \times P(x_2 \mid c) \times \cdots \times P(x_n \mid c) \times P(c)$$

Bayesian classification

- Performance
  - A simple Bayesian classifier (e.g., naïve Bayesian), has comparable performance with decision tree and selected neural networks.
- Incremental
  - Each training example can incrementally increase/decrease the probability that a hypothesis is correct
  - That is, prior knowledge can be combined with observed data.
- Standard
  - Even when Bayesian methods are computationally intractable, they can provide a standard of optimal decision making against which other methods can be measured

The Bayes' Theorem

- Total Probability Theorem: $P(B) = \sum_{i=1}^{M} P(B|A_i)P(A_i)$
- Let $\mathbf{X}$ be a data sample ("evidence") with unknown class label and H be a hypothesis that $\mathbf{X}$ belongs to class C
- **Bayes' Theorem:** $\boldsymbol{P(H \mid \mathbf{X}) = \frac{P(\mathbf{X} \mid H)P(H)}{P(\mathbf{X})}}$
- Classification is to determine $P(H \mid \mathbf{X})$, i.e. the probability that the hypothesis H holds given the observed data sample $\mathbf{X}$.

The buying computer dataset

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31…40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31…40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31…40	medium	no	excellent	yes
31…40	high	yes	fair	yes
>40	medium	no	excellent	no

The Bayes' Theorem

- $P(H)$ (prior probability): the initial probability
  - E.g., $\mathbf{X}$ will buy computer, regardless of age, income, …
- $P(\mathbf{X})$: the probability that sample data is observed
  - E.g., $\mathbf{X}$ is 31..40 and has a medium income, regardless of the buying
- $P(\mathbf{X} \mid H)$ (likelihood): the probability of observing the sample $\mathbf{X}$, given that the hypothesis holds
  - E.g., Given that $\mathbf{X}$ will buy computer, the probability that $\mathbf{X}$ is 31..40 and has a medium income
- $P(H \mid \mathbf{X}) = \frac{P(\mathbf{X} \mid H)P(H)}{P(\mathbf{X})}$(posterior probability)
  - E.g., given that $\mathbf{X}$ is 31..40 and has a medium income, the probability that $\mathbf{X}$ will buy computer

The Bayes' Theorem

- Informally, $P(H \mid \mathbf{X}) = \frac{P(\mathbf{X} \mid H)P(H)}{P(\mathbf{X})}$ can be viewed as

$$posteriori \ = \ likelihood * prior \ / \ evidence$$

- $\mathbf{X}$ belongs to C_i iff the probability $P(C_i | \mathbf{X})$ is the highest among all the $P(C_k | \mathbf{X})$ for all the k classes

- **Practical difficulty**
  - Require initial knowledge of many probabilities
  - Significant computational cost involved

Classification with Bayes' Theorem

- Let D be a training set of tuples and associated class labels
- Each tuple is represented by a n-attribute $\mathbf{X} = (x_1, x_2, \ldots, x_n)$
- Suppose there are m classes $C_1, C_2, \ldots, C_m$
- Classification is to derive the maximum posteriori $P(C_i | \mathbf{X})$ from **Bayes' theorem**

$$P(C_i \mid \mathbf{X}) = \frac{P(\mathbf{X} \mid C_i) P(C_i)}{P(\mathbf{X})}$$

- $P(X)$ is constant for all classes, only $P(\mathbf{X} \mid C_i) P(C_i)$ needs to be maximized.

naïve Bayesian classifier

- Class-conditional independence: There are no dependence relationships **among the attributes**
- The **naïve Bayesian classification** formula is written as

$$P(\mathbf{X} \mid C_i) = \prod_{k=1}^{n} P(x_k \mid C_i) = P(x_1 \mid C_i) \times P(x_2 \mid C_i) \times \cdots \times P(x_n \mid C_i)$$

  - A_k is categorical: $P(x_k \mid C_i)$ is the number of tuples in C_i having value x_k for A_k divided by $|C_{i,D}|$ (# of tuples of C_i in D)
  - A_k is continuous: $P(x_k \mid C_i) = g\left(x_k, \mu_{C_i}, \sigma_{C_i}\right)$ with the Gaussian distribution $g(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
- Count class distributions only $\rightarrow$ computation cost reduced

naive Bayesian for the example dataset

P(buys_computer = “yes”)	9/14
P(buys_computer = “no”)	5/14

	buys_computer = “yes”	buys_computer = “no”
age = “<=30”	2/9	3/5
age = “31…40”	4/9	0/5
age = “>40”	3/9	2/5
income = “low”	3/9	1/5
income = “medium”	4/9	2/5
income = “high”	2/9	2/5
student = “yes”	6/9	1/5
student = “no”	3/9	4/5
credit_rating = “fair”	6/9	2/5
credit_rating = “excellent”	3/9	3/5

naive Bayesian for the example dataset

age	income	student	credit_rating	buys_computer
<=30	medium	yes	fair	?

- $P(\mathbf{X}|\ C_i)$
  - $P(\mathbf{X}\,|\,buys\_computer\ =\ \text{“}yes\text{”})\ =\ 2/9\ *\ 4/9\ *\ 6/9\ *\ 6/9\ =\ 0.044$
  - $P(\mathbf{X}\,|\,buys\_computer\ =\ \text{“}no\text{”})\ =\ 3/5\ *\ 2/5\ *\ 1/5\ *\ 2/5\ =\ 0.019$
- $P(\mathbf{X}|\ C_i) * P(C_i)$
  - $P(\mathbf{X}\,|\,buys\_computer\ =\ \text{“}yes\text{”}) * P(buys\_computer\ = \text{“}yes\text{”}) = 0.028$
  - $P(\mathbf{X}|\,buys\_computer\ =\ \text{“}no\text{”}) * P(buys\_computer\ =\ \text{“}no\text{”}) = 0.007$
- $P(C_i\,|\,\mathbf{X})$
  - $P(buys\_computer\ =\ \text{“}yes\text{”}\,|\,\mathbf{X}) = 0.8$
  - $P(buys\_computer\ =\ \text{“}no\text{”}\,|\,\mathbf{X}) = 0.2$

Therefore, $\mathbf{X}$ belongs to class (“buys_computer = yes”)

Avoiding the zero-probability problem

- The naïve Bayesian prediction requires each conditional probability be **non-zero**.

$$P(\mathbf{X} \mid C_i) = \prod_{k=1}^{n} P(x_k \mid C_i)$$

- Otherwise, the predicted probability will be zero
- For example,

age	income	student	credit_rating	buys_computer
31…40	medium	yes	fair	?

  - $P(\mathbf{X} \mid buys\_computer = \text{“}no\text{”}) = 0 * 2/5 * 1/5 * 2/5 = 0$
  - Therefore, the conclusion is always **yes** regardless the value of $P(\mathbf{X} \mid buys\_computer = \text{“}yes\text{”})$

Avoiding the zero-probability problem

- **Laplacian correction** (or Laplacian estimator)

$$P(C_i) = \frac{|C_i| + 1}{|D| + m} \qquad P(x_k \mid C_i) = \frac{|x_k \cup C_i| + 1}{|C_i| + r}$$

  - where m is the number of classes, $|x_k \cup C_i|$ denotes the number of tuples contains both $A_k = x_k$ and C_i, and r is the number of values of attribute A_k
- The "corrected" probability estimates are close to their "uncorrected" counterparts

naive Bayesian for the example dataset

P(buys_computer = “yes”)	10/16
P(buys_computer = “no”)	6/16

	buys_computer = “yes”	buys_computer = “no”
age = “<=30”	3/12	4/8
age = “31…40”	5/12	1/8
age = “>40”	4/12	3/8
income = “low”	4/12	2/8
income = “medium”	5/12	3/8
income = “high”	3/12	3/8
student = “yes”	7/11	2/7
student = “no”	4/11	5/7
credit_rating = “fair”	7/11	3/7
credit_rating = “excellent”	4/11	4/7

naive Bayesian for the example dataset

age	income	student	credit_rating	buys_computer
31..40	medium	yes	fair	?

- $P(\mathbf{X}|\, C_i)$
  - $P(\mathbf{X}\,|\, buys\_computer\ =\ \text{“}yes\text{”})\ = 5/12 * 5/12 * 7/11 * 7/11\ =\ 0.070$
  - $P(\mathbf{X}\,|\, buys\_computer\ =\ \text{“}no\text{”})\ = 1/8\ * 3/8\ * 2/7 * 3/7 =\ 0.006$
- $P(\mathbf{X}|\, C_i) * P(C_i)$
  - $P(\mathbf{X}\,|\, buys\_computer\ =\ \text{“}yes\text{”}) * P(buys\_computer\ = \text{“}yes\text{”}) = 0.044$
  - $P(\mathbf{X}|\, buys\_computer\ =\ \text{“}no\text{”}) * P(buys\_computer\ =\ \text{“}no\text{”}) = 0.002$
- $P(C_i\,|\,\mathbf{X})$
  - $P(buys\_computer\ =\ \text{“}yes\text{”}\,|\,\mathbf{X}) = 0.953$
  - $P(buys\_computer\ =\ \text{“}no\text{”}\,|\,\mathbf{X}) = 0.047$

Therefore, $\mathbf{X}$ belongs to class (“buys_computer = yes”)

Handling missing values

- If the values of some attributes are missing, these attributes are omitted from the product of probabilities
- As a result, the estimation is less accurate
- For example,

age	income	student	credit_rating	buys_computer
?	medium	yes	fair	?

Algorithm efficiency

- Advantages
  - Easy to implement
  - Good results obtained in most of the cases
- Disadvantages
  - Class conditional independence $\rightarrow$ loss of accuracy
  - Practically, dependencies exist among variables, which cannot be modeled by Naïve Bayes
    - E.g., in medical records, patients' profile (age, family history, etc.), symptoms (fever, cough etc.), disease (lung cancer, diabetes, etc.)
- *How to deal with these dependencies?*
  - Bayesian Belief Networks

Quiz 02: naïve Bayesian classification

- The data represent files on a computer system. Possible values of the class variable are “infected”, which implies the file has a virus infection, or “clean” if it doesn't.
- Derive naïve Bayesian probabilities for virus identification in either cases, with or without Laplacian correction.

No.	Writable	Updated	Size	Class
1	Yes	No	Small	Infected
2	Yes	Yes	Large	Infected
3	No	Yes	Med	Infected
4	No	No	Med	Clean
5	Yes	No	Large	Clean
6	No	No	Large	Clean

THE END