Logical Agents

LESSON 7

Reading

Chapter 7

Outline

Knowledge-based agents

- Logic: models and entailment (suy diễn, suy luận)
- A simple Logic: propositional (Boolean) logic (logic mệnh đề)
- Inference rules and theorem proving
  - Forward chaining
  - Backward chaining
  - resolution

Knowledge based agents

- Knowledge base (KB) = set of sentences in a formal language
- Declarative (as opposed to procedural) approach to build an agent:
  - Tell it what it needs to know
- Then it can Ask itself what to do - answers should follow from the KB
- Agents can be viewed at the knowledge level
  - i.e., what they know, regardless of how implemented
- Or at the implementation level
  - i.e., data structures in KB and algorithms that manipulate them

A simple knowledge-based agent

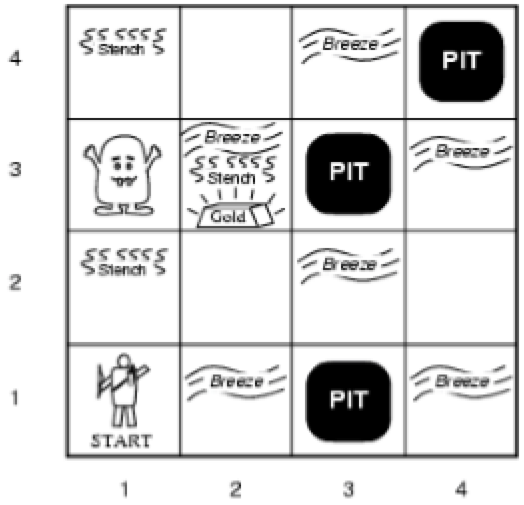
```
function KB-AGENT(percept) returns an action
    static: KB, a knowledge base
            t, a counter, initially 0, indicating time

    TELL(KB, MAKE-PERCEPT-SENTENCE(percept, t))
    action ← ASK(KB, MAKE-ACTION-QUERY(t))
    TELL(KB, MAKE-ACTION-SENTENCE(action, t))
    t ← t + 1
    return action
```

The agent must be able to:

- Represent states, actions, etc. (biểu diễn trạng thái, hành động)
- Incorporate new percepts (tích hợp nhận thức mới)
- Update internal representations of the world (cập nhật biểu diễn bên trong)
- Deduce hidden properties of the world (suy diễn đặc điểm)
- Deduce appropriate actions (suy diễn các hành động tương ứng)

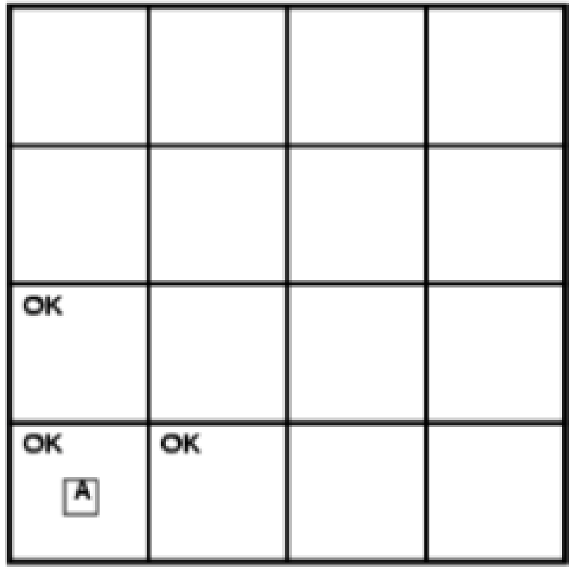
Wumpus World PEAS description

- Performance measure
  - gold +1000, death -1000
  - -1 per step, -10 for using the arrow
- Environment
  - Squares adjacent to wumpus are smelly
  - Squares adjacent to pit are breezy
  - Glitter iff gold is in the same square
  - Shooting kills wumpus if you are facing it
  - Shooting uses up the only arrow
  - Grabbing picks up gold if in same square
  - Releasing drops the gold in same square



- Sensors: Stench, Breeze, Glitter, Bump, Scream
- Actuators: Turn Left, Turn Right, Forward, Grab, Release, Shoot

Exploring a wumpus world



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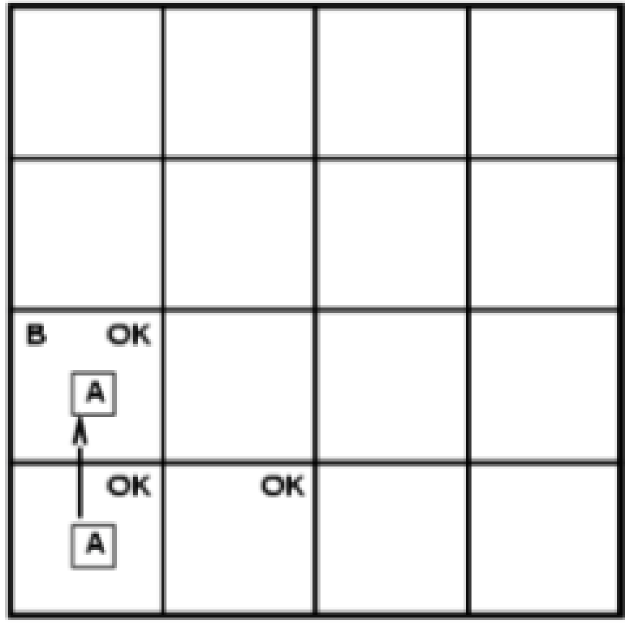

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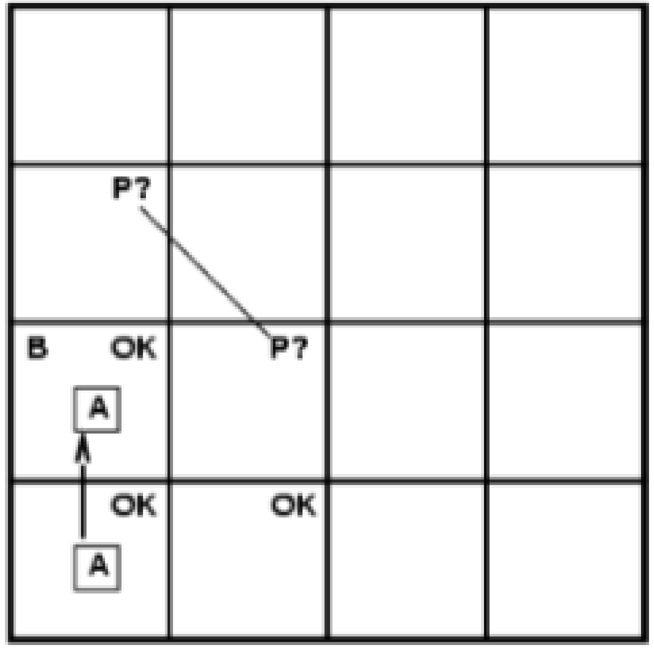

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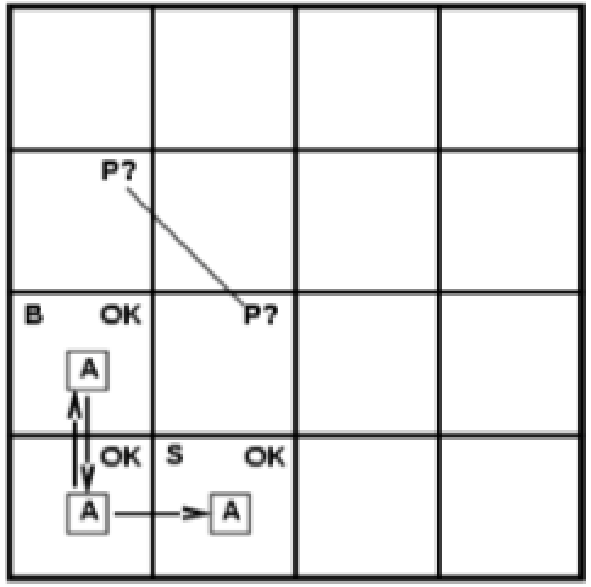

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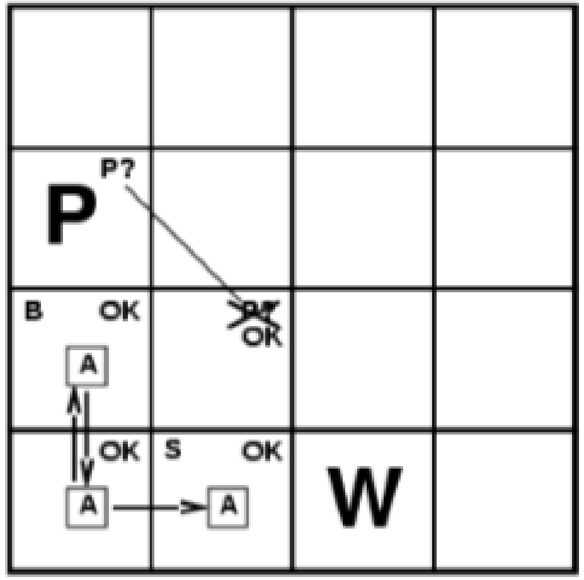


Logics

- Logics are formal languages for representing information such that conclusions can be drawn
- Syntax defines the sentences in the language
- Semantics define the "meaning" of sentences;
  - i.e., define truth of a sentence in a world
- E.g., the language of arithmetic
  - $X+2 \geq y$ is a sentence; $x2+y > \{\}$ is not a sentence
  - $x+2 \geq y$ is true iff the number $x+2$ is no less than the number y
  - $x+2 \geq y$ is true in a world where $x = 7$, $y = 1$
  - $x+2 \geq y$ is false in a world where $x = 0$, $y = 6$

Entailment

- Entailment means that one thing follows from another:

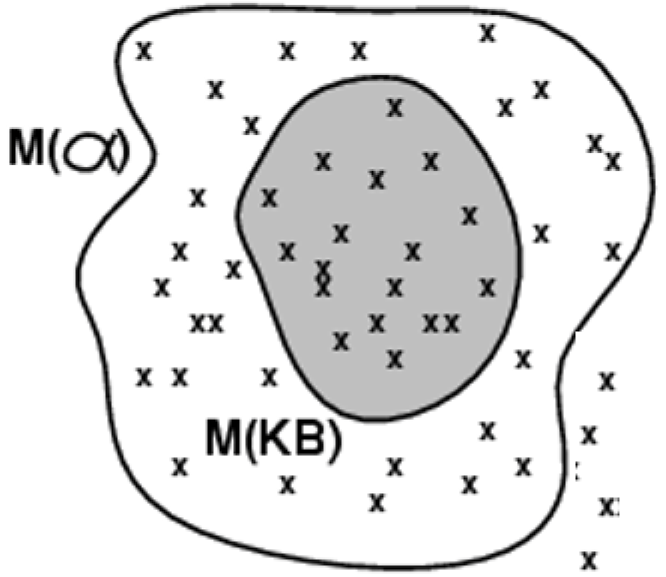
$$KB \models \alpha$$

- Knowledge base *KB* entails sentence α if and only if α is true in all worlds where *KB* is true
  - E.g., a KB containing "Today is sunny" and "Yesterday was rainy" entails "Either today is sunny or yesterday was rainy"
  - E.g., x+y = 4 entails 4 = x+y
  - Entailment is a relationship between sentences (i.e., syntax) that is based on semantics

Models

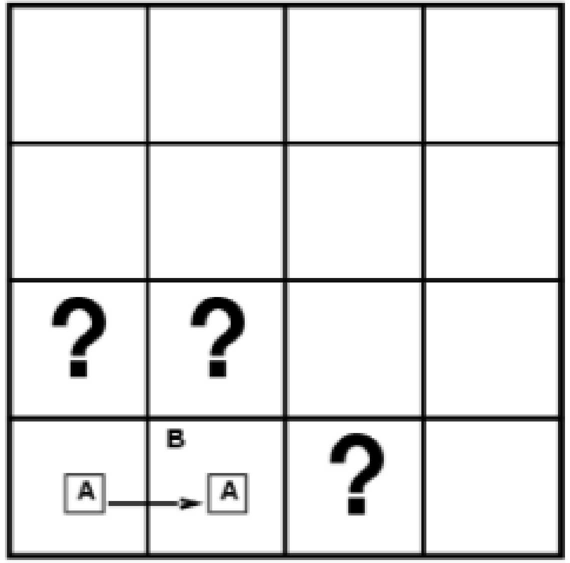
- Logicians often think in terms of models ("possible worlds"), which are formally structured worlds with respect to which truth can be evaluated
- We say m is a model of a sentence α if α is true in m
- $M(\alpha)$ is the set of all models of α
- Then KB $\models \alpha$ iff $M(KB) \subseteq M(\alpha)$
  - E.g. KB = Today is sunny and yesterday was rainy

    α = Today is sunny

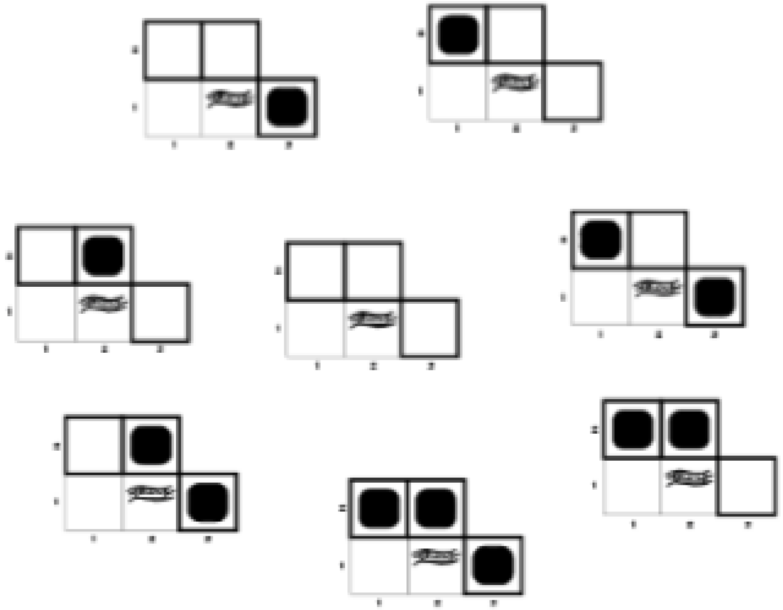


Entailment in the wumpus world

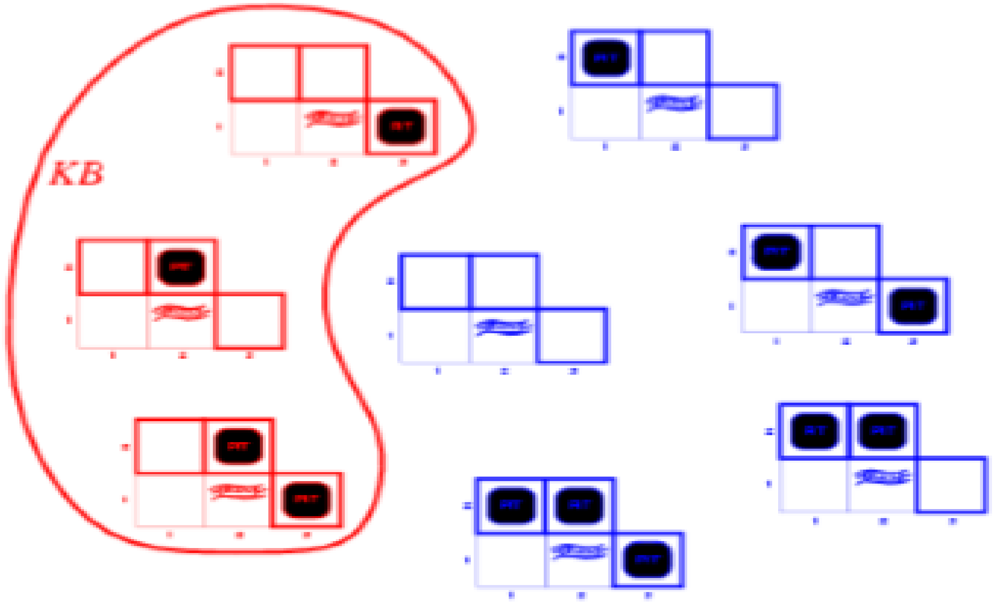
- Situation after detecting nothing in [1,1], moving right, breeze in [2,1]
- Consider possible models for KB assuming only pits
- 3 Boolean choices ⇒ 8 possible models



Wumpus world models

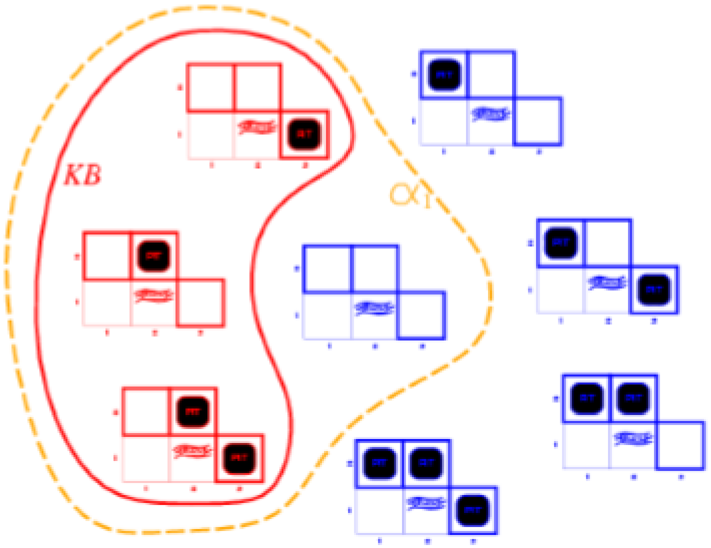


Wumpus models



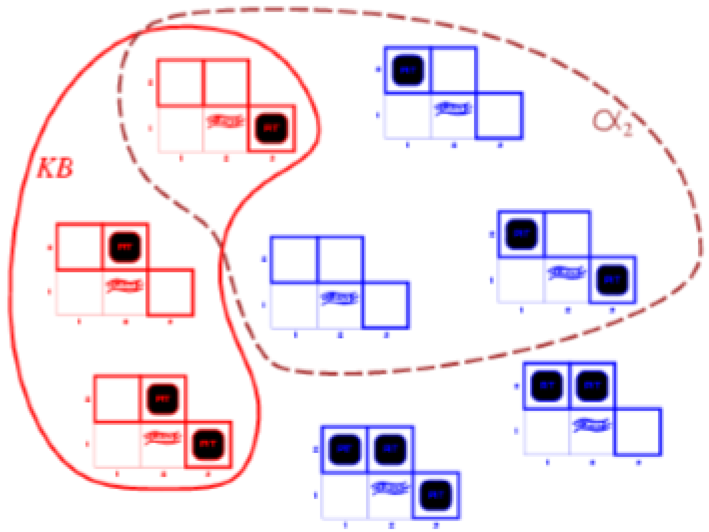
KB = wumpus-world rules + observations

Wumpus models



- *KB* = wumpus-world rules + observations
- α_1 = "[1,2] is safe", *KB* $\models \alpha_1$, proved by model checking

Wumpus models



- KB = wumpus-world rules + observations
- α_2 = "[2,2] is safe", $KB \not\models \alpha_2$

Inference

- Define: $KB \vdash_i \alpha$ = sentence α can be derived from KB by procedure i
  - Soundness: i is sound if whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$
  - Completeness: i is complete if whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$
- An inference procedure will answer any question whose answer follows from what is known by the KB.

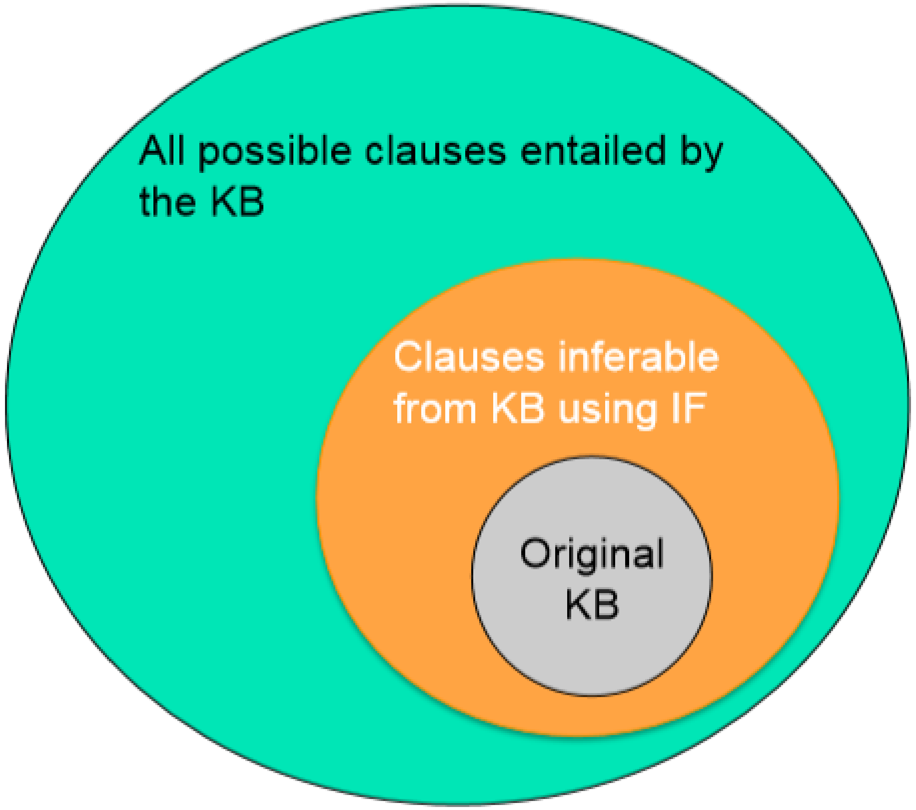
"Entailment is like the needle (α) being in the haystack (KB) and inference is like finding it"

We want to know: Is a set of inference operators **complete** and **sound**?

Completeness

Completeness: *i* is complete if whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$

- An incomplete inference algorithm cannot reach all possible conclusions
  - Equivalent to completeness in search (chapter 3)



Propositional logic: Syntax

- Propositional logic is the simplest logic – illustrates basic ideas

- The proposition symbols P_1, P_2 etc are sentences
  - If S is a sentence, $\neg S$ is a sentence (negation)
  - If S_1 and S_2 are sentences, $S_1 \wedge S_2$ is a sentence (conjunction)
  - If S_1 and S_2 are sentences, $S_1 \vee S_2$ is a sentence (disjunction)
  - If S_1 and S_2 are sentences, $S_1 \Rightarrow S_2$ is a sentence (implication)
  - If S_1 and S_2 are sentences, $S_1 \Leftrightarrow S_2$ is a sentence (biconditional)

Propositional logic: Semantics

Each model specifies true/false for each proposition symbol

E.g.	$P_{1,2}$	$P_{2,2}$	$P_{3,1}$
	false	true	false

With these symbols, 8 possible models can be enumerated automatically.
Rules for evaluating truth with respect to a model *m*:

$\neg S$	is true iff	S is false	
$S_1 \wedge S_2$	is true iff	S_1 is true and	S_2 is true
$S_1 \vee S_2$	is true iff	S_1 is true or	S_2 is true
$S_1 \Rightarrow S_2$	is true iff	S_1 is false or	S_2 is true
i.e.,	is false iff	S_1 is true and	S_2 is false
$S_1 \Leftrightarrow S_2$	is true iff	$S_1 \Rightarrow S_2$ is true and	$S_2 \Rightarrow S_1$ is true

Simple recursive process evaluates an arbitrary sentence, e.g.,

$$\neg P_{1,2} \wedge (P_{2,2} \vee P_{3,1}) = \mathit{true} \wedge (\mathit{true} \vee \mathit{false}) = \mathit{true} \wedge \mathit{true} = \mathit{true}$$

Truth tables for connectives

p	q	$\sim p$	$\sim q$	$p \Rightarrow q$	$\sim p \Rightarrow \sim q$	$q \Rightarrow p$	$\sim q \Rightarrow \sim p$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

Wumpus world sentences

Let $P_{i,j}$ be true if there is a pit in [i, j].

Let $B_{i,j}$ be true if there is a breeze in [i, j].

- $\neg P_{1,1}$
- $\neg B_{1,1}$
- $B_{2,1}$

How do we translate "Pits cause breezes in adjacent squares”?

Truth tables for inference

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	KB	α_1
false	*false*	*false*	*false*	*false*	*false*	*false*	*false*	*true*
false	*false*	*false*	*false*	*false*	*false*	*true*	*false*	*true*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*false*	*true*
false	*true*	*false*	*false*	*false*	*false*	*true*	<u>*true*</u>	<u>*true*</u>
false	*true*	*false*	*false*	*false*	*true*	*false*	<u>*true*</u>	<u>*true*</u>
false	*true*	*false*	*false*	*false*	*true*	*true*	<u>*true*</u>	<u>*true*</u>
false	*true*	*false*	*false*	*true*	*false*	*false*	*false*	*true*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*false*

$R_1 = \neg P_{1,1}$

$R_4 = \neg B_{1,1}$

$R_5 = B_{2,1}$

$\alpha_1 = \neg P_{1,2}$? (Is 1,2 safe from pits)?

Inference by enumeration

Depth-first enumeration of all models is sound and complete

```
function TT-ENTAILS?(KB, α) returns true or false
   symbols ← a list of the proposition symbols in KB and α
   return TT-CHECK-ALL(KB, α, symbols, [ ])

function TT-CHECK-ALL(KB, α, symbols, model) returns true or false
   if EMPTY?(symbols) then
       if PL-TRUE?(KB, model) then return PL-TRUE?(α, model)
       else return true
   else do
       P ← FIRST(symbols); rest ← REST(symbols)
       return TT-CHECK-ALL(KB, α, rest, EXTEND(P, true, model) and
                  TT-CHECK-ALL(KB, α, rest, EXTEND(P, false, model)
```

For n symbols, time complexity is $O(2^n)$, space complexity is $O(n)$

Logical equivalence

- Two sentences are logically equivalent iff true in same models: $\alpha \equiv \beta$ iff $\alpha \models \beta$ and $\beta \models \alpha$

$$
\begin{aligned}
(\alpha \land \beta) &\equiv (\beta \land \alpha) \quad \text{commutativity of } \land \\
(\alpha \lor \beta) &\equiv (\beta \lor \alpha) \quad \text{commutativity of } \lor \\
((\alpha \land \beta) \land \gamma) &\equiv (\alpha \land (\beta \land \gamma)) \quad \text{associativity of } \land \\
((\alpha \lor \beta) \lor \gamma) &\equiv (\alpha \lor (\beta \lor \gamma)) \quad \text{associativity of } \lor \\
\neg(\neg\alpha) &\equiv \alpha \quad \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) \quad \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \lor \beta) \quad \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha)) \quad \text{biconditional elimination} \\
\neg(\alpha \land \beta) &\equiv (\neg\alpha \lor \neg\beta) \quad \text{de Morgan} \\
\neg(\alpha \lor \beta) &\equiv (\neg\alpha \land \neg\beta) \quad \text{de Morgan} \\
(\alpha \land (\beta \lor \gamma)) &\equiv ((\alpha \land \beta) \lor (\alpha \land \gamma)) \quad \text{distributivity of } \land \text{ over } \lor \\
(\alpha \lor (\beta \land \gamma)) &\equiv ((\alpha \lor \beta) \land (\alpha \lor \gamma)) \quad \text{distributivity of } \lor \text{ over } \land
\end{aligned}
$$

Validity and satisfiability Validity and satisfiability

A sentence is valid if it is true in all models,

e.g., *True*, $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the Deduction Theorem:

$KB \models \alpha$ if and only if $(KB \Rightarrow \alpha)$ is valid

A sentence is satisfiable if it is true in some model

e.g., $A \vee B$, C

A sentence is unsatisfiable if it is true in no models

e.g., $A \wedge \neg A$

Satisfiability is connected to inference via the following:

$KB \models \alpha$ if and only if $(KB \wedge \neg \alpha)$ is unsatisfiable

Proof methods

- Proof methods divide into (roughly) two kinds:

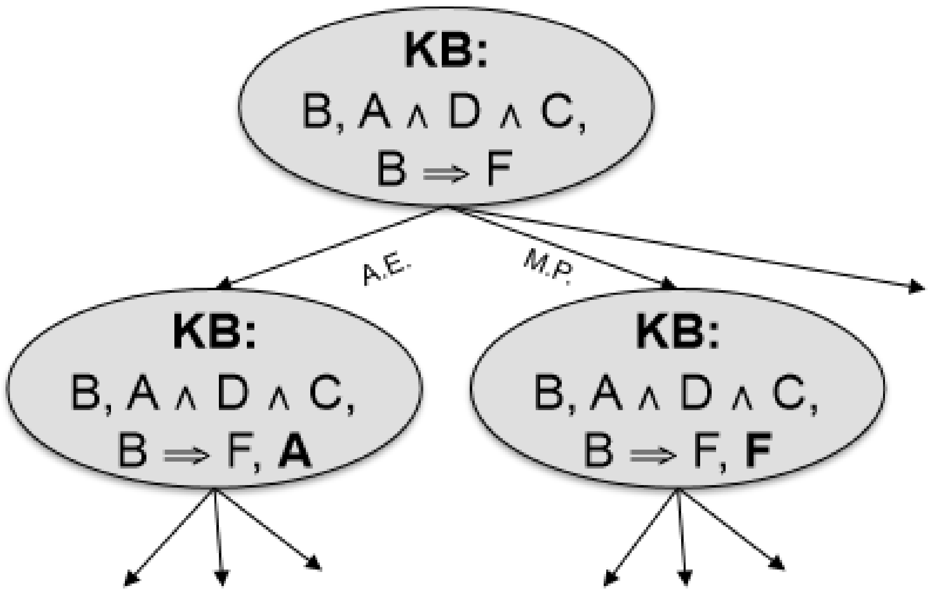
- Application of inference rules
  - Legitimate (sound) generation of new sentences from old
  - Proof = a sequence of inference rule applications Can use inference rules as operators in a standard search algorithm
  - Typically require transformation of sentences into a normal form

- Model checking
  - truth table enumeration (always exponential in n)
  - improved backtracking, e.g., Davis-Putnam-Logemann-Loveland (DPLL)
  - heuristic search in model space (sound but incomplete)
  - e.g., min-conflicts like hill-climbing algorithms

Applying inference rules

Equivalent to a search problem

- KB state = node
- Inference rule application = edge



Resolution

Conjunctive Normal Form (CNF)

conjunction of "disjunctions of literals" (clauses)

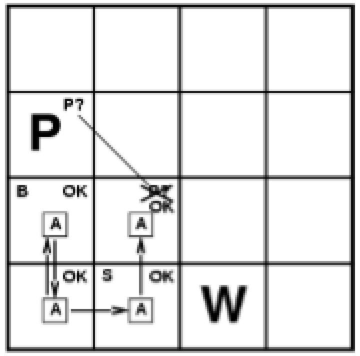
E.g., $(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$

- Resolution inference rule (for CNF):

$$\frac{\ell_i \vee \ldots \vee \ell_k, \qquad m_1 \vee \ldots \vee m_n}{\ell_i \vee \ldots \vee \ell_{i-1} \vee \ell_{i+1} \vee \ldots \vee \ell_k \vee m_1 \vee \ldots \vee m_{j-1} \vee m_{j+1} \vee \ldots \vee m_n}$$

where ℓ_i and m_j are complementary literals.

E.g., $\dfrac{P_{1,3} \vee P_{2,2}, \qquad \neg P_{2,2}}{P_{1,3}}$



- Resolution is sound and complete for propositional logic

Soundness of Resolution

Same truth value

$$\neg(\ell_i \lor \ldots \lor \ell_{i-1} \lor \ell_{i+1} \lor \ldots \lor \ell_k) \Rightarrow \ell_i$$

$$\frac{\neg m_j \Rightarrow (m_1 \lor \ldots \lor m_{j-1} \lor m_{j+1} \lor \ldots \lor m_n)}{\neg(\ell_i \lor \ldots \lor \ell_{i-1} \lor \ell_{i+1} \lor \ldots \lor \ell_k) \Rightarrow (m_1 \lor \ldots \lor m_{j-1} \lor m_{j+1} \lor \ldots \lor m_n)}$$

where ℓ_i and m_j are complementary literals.

If ℓ_i true, then m_j is false, hence $(m_1 \lor \ldots \lor m_{j-1} \lor m_{j+1} \lor \ldots \lor m_n)$ must be true.

If m_j true, then ℓ_i is false, hence $(\ell_i \lor \ldots \lor \ell_{i-1} \lor \ell_{i+1} \lor \ldots \lor \ell_k)$

Conversion to CNF

$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

1. Eliminate $\Leftrightarrow$, replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$.
   $(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$

2. Eliminate $\Rightarrow$, replacing $\alpha \Rightarrow \beta$ with $\neg \alpha \vee \beta$.
   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$

3. Move $\neg$ inwards using de Morgan's rules and double-negation:
   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$

4. Apply distributivity law ($\wedge$ over $\vee$) and flatten:
   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$

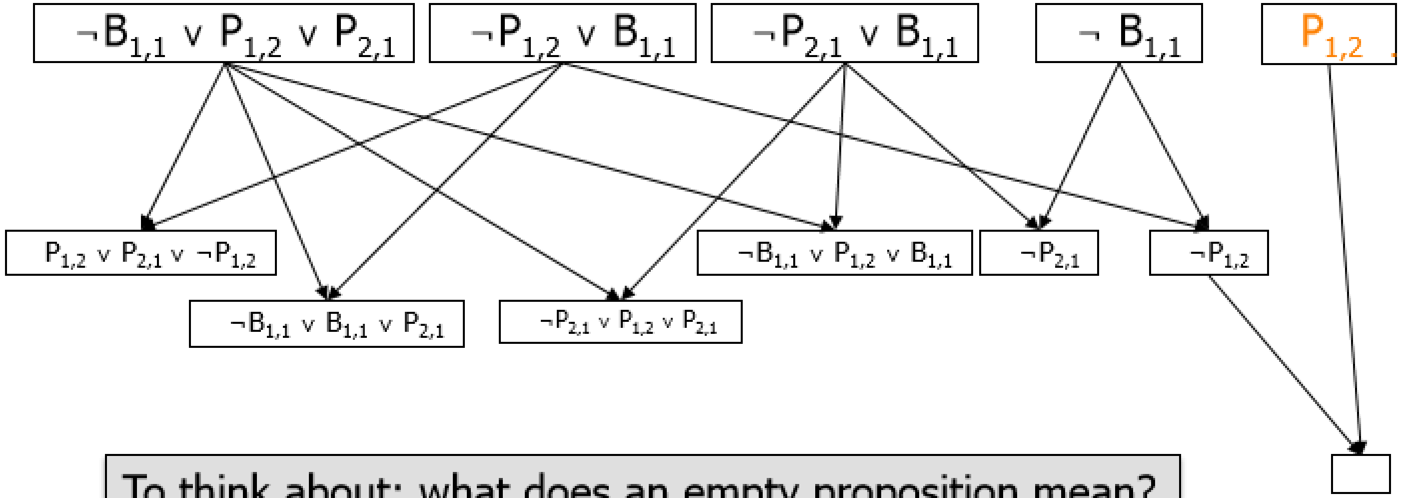
Resolution algorithm

- Proof by contradiction, i.e., show $KB \wedge \neg \alpha$ unsatisfiable

```
function PL-RESOLUTION(KB, α) returns true or false
    clauses ← the set of clauses in the CNF representation of KB ∧ ¬α
    new ← { }
    loop do
        for each Ci, Cj in clauses do
            resolvents ← PL-RESOLVE(Ci, Cj)
            if resolvents contains the empty clause then return true
            new ← new ∪ resolvents
        if new ⊆ clauses then return false
        clauses ← clauses ∪ new
```

Resolution example

- $KB = (B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1}$
- $\alpha = \neg P_{1,2}$ (negate the premise for proof by refutation)



To think about: what does an empty proposition mean?

The power of false

- Given: $(P) \land (\neg P)$
- Prove: Z

$\neg$ P	Given
P	Given
$\neg$ Z	Given
	Unsatisfiable

- Can we prove $\neg$Z, using the givens above?

Forward and backward chaining

- Horn Form (restricted)
  KB = conjunction of Horn clauses
  - Horn clause =
    - proposition symbol; or
    - (conjunction of symbols) $\Rightarrow$ symbol
  - E.g., $C \wedge (B \Rightarrow A) \wedge (C \wedge D \Rightarrow B)$
- Modus Ponens (for Horn Form): complete for Horn KBs

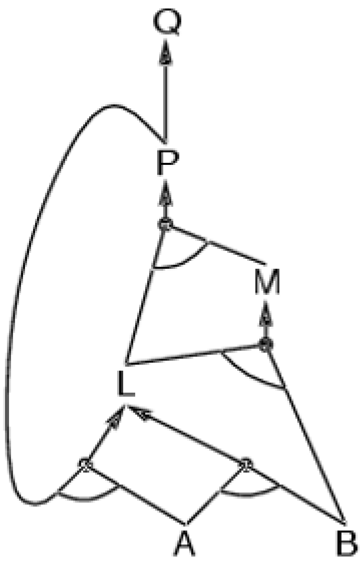
$$\frac{\alpha_1, \ldots, \alpha_n, \qquad \alpha_1 \wedge \ldots \wedge \alpha_n \Rightarrow \beta}{\beta}$$

- Can be used with forward chaining or backward chaining.
- These algorithms are very natural and run in linear time

Forward chaining

- Idea: fire any rule whose premises are satisfied in the *KB*,
  - add its conclusion to the *KB*, until query is found

$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$

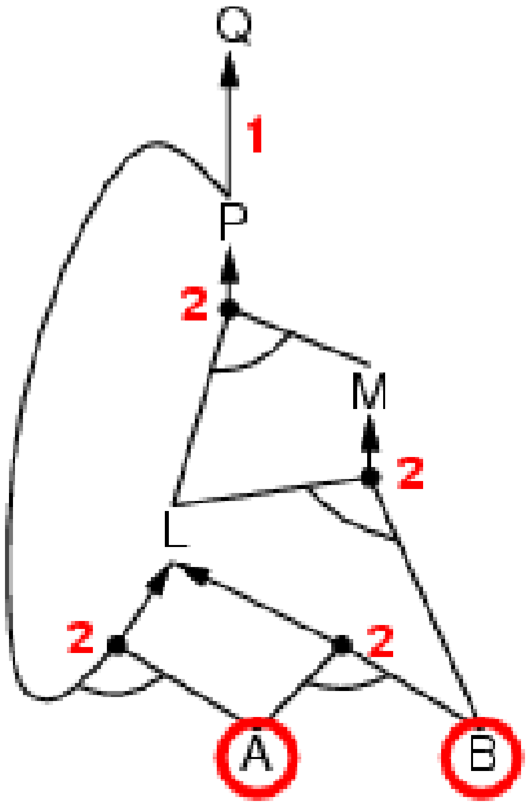


Forward chaining algorithm

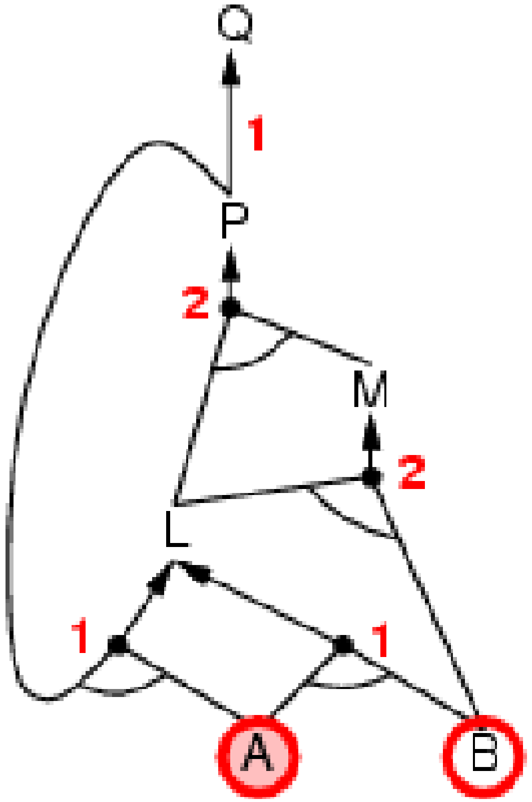
```
function PL-FC-ENTAILS?(KB, q) returns true or false
  local variables: count, a table, indexed by clause, initially the number of premises
                   inferred, a table, indexed by symbol, each entry initially false
                   agenda, a list of symbols, initially the symbols known to be true

  while agenda is not empty do
      p ← POP(agenda)
      unless inferred[p] do
          inferred[p] ← true
          for each Horn clause c in whose premise p appears do
              decrement count[c]
              if count[c] = 0 then do
                  if HEAD[c] = q then return true
                  PUSH(HEAD[c], agenda)
  return false
```

- Forward chaining is sound and complete for Horn KB

Forward chaining example



Forward chaining example



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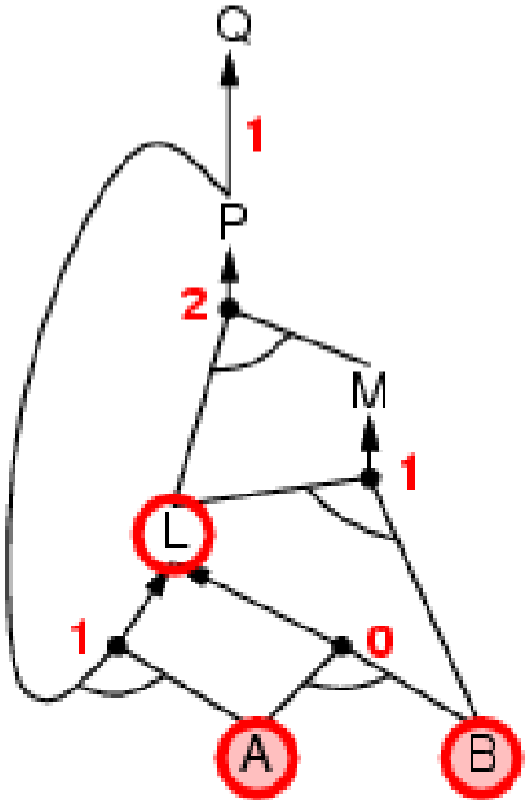

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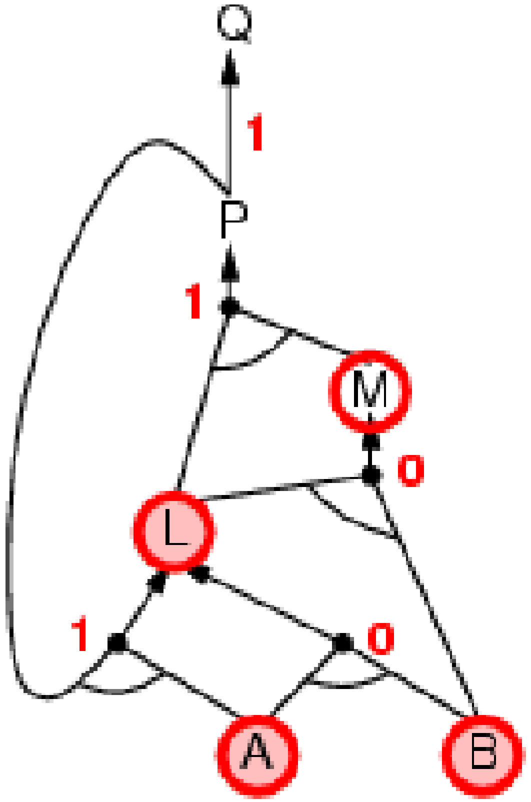
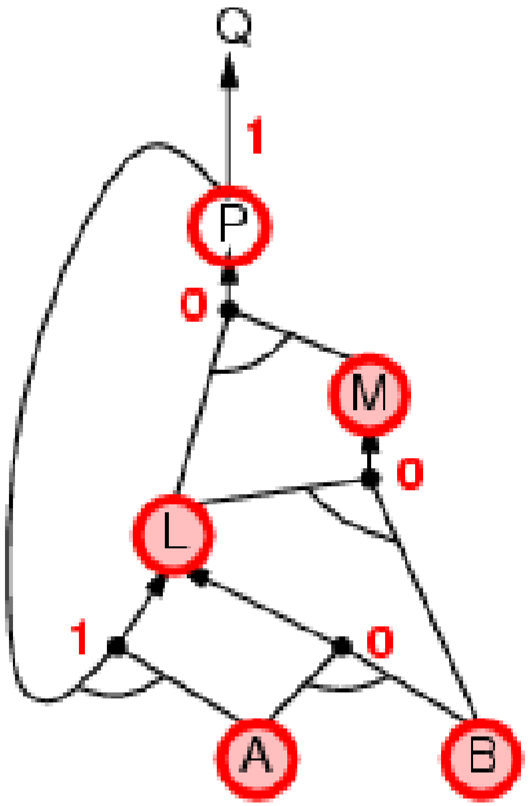
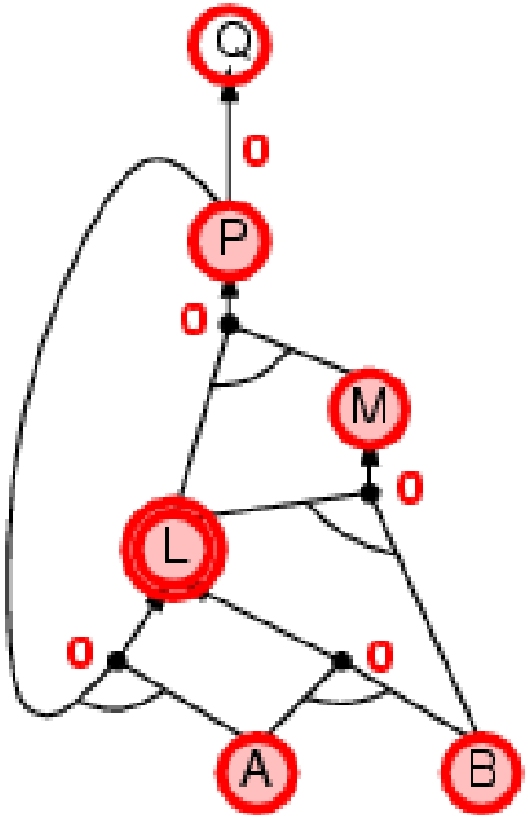


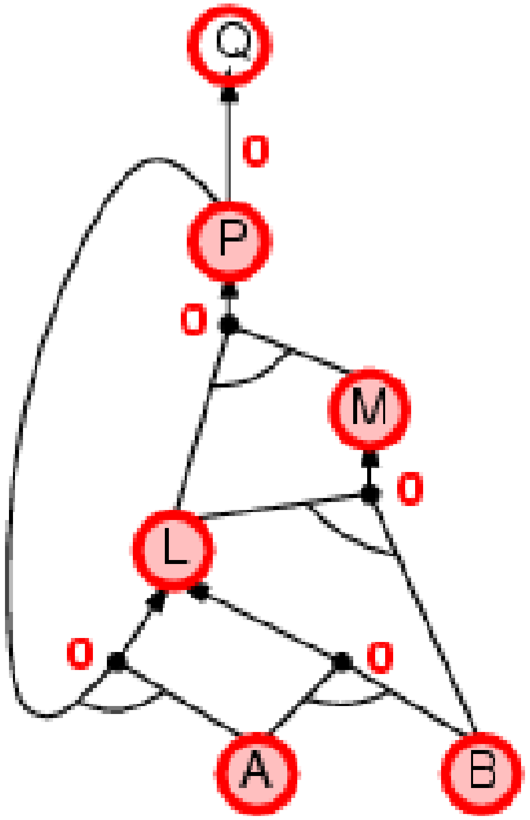
Forward chaining example



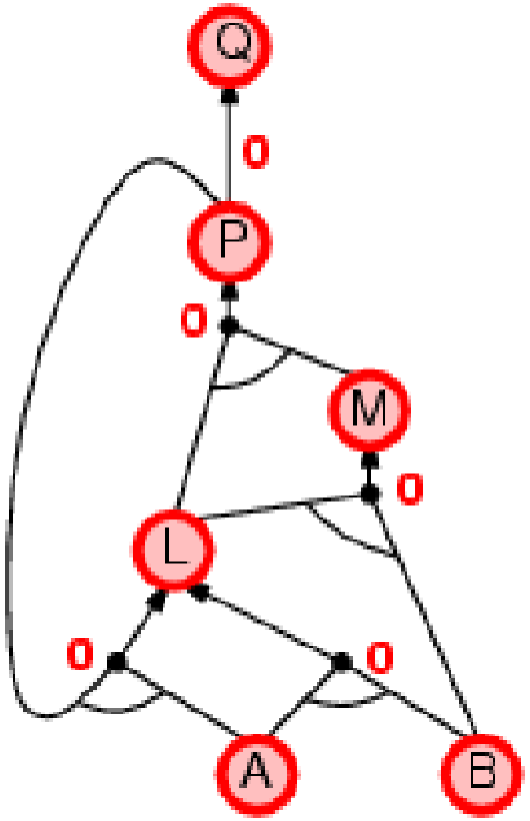
Forward chaining example



Forward chaining example



Forward chaining example



Proof of completeness

FC derives every atomic sentence that is entailed by *KB*

1. FC reaches a fixed point where no new atomic sentences are derived
2. Consider the final state as a model *m*, assigning true/false to symbols
3. Every clause in the original *KB* is true in *m*

   $a_1 \wedge \ldots \wedge a_k \Rightarrow b$
4. Hence *m* is a model of *KB*
5. If $KB \models q$, *q* is true in every model of *KB*, including *m*

Backward chaining

Idea: work backwards from the query q :

- to prove q by BC,

  check if q is known already, or

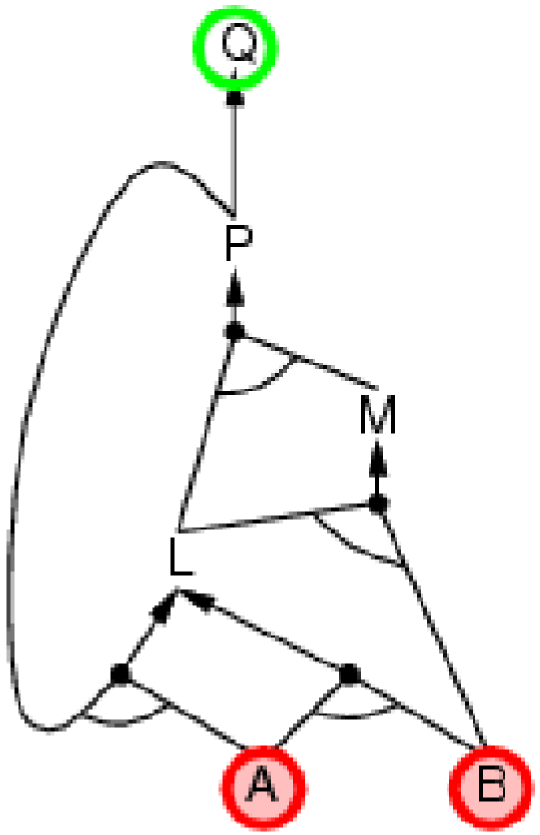
  prove by BC all premises of some rule concluding q

Avoid loops: check if new subgoal is already on the goal stack

Avoid repeated work: check if new subgoal

- 1. has already been proved true, or
- 2. has already failed

Backward chaining example



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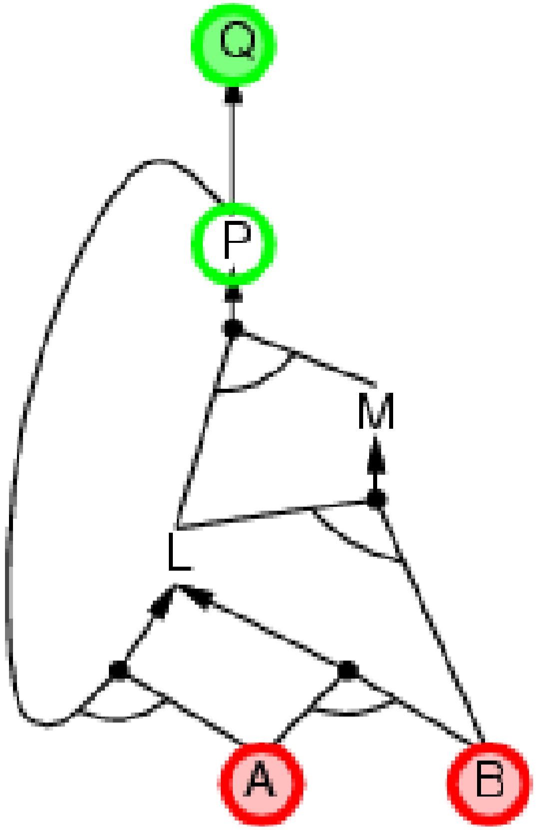

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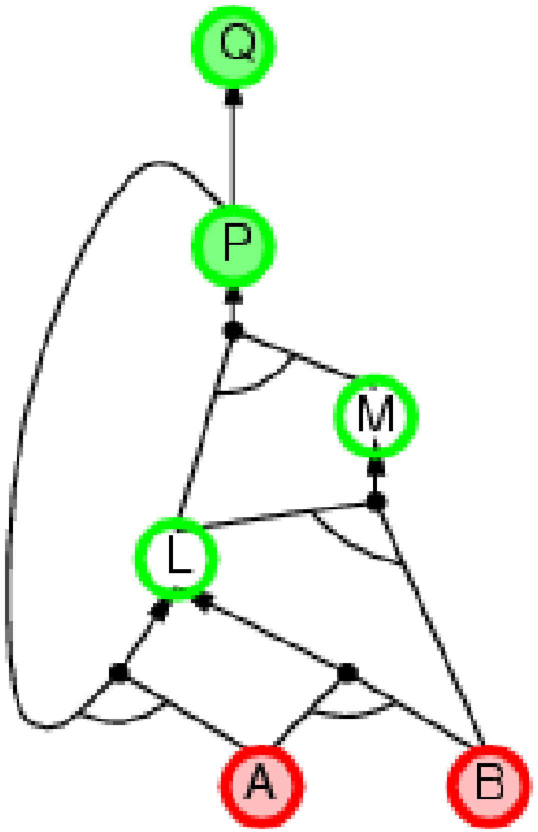

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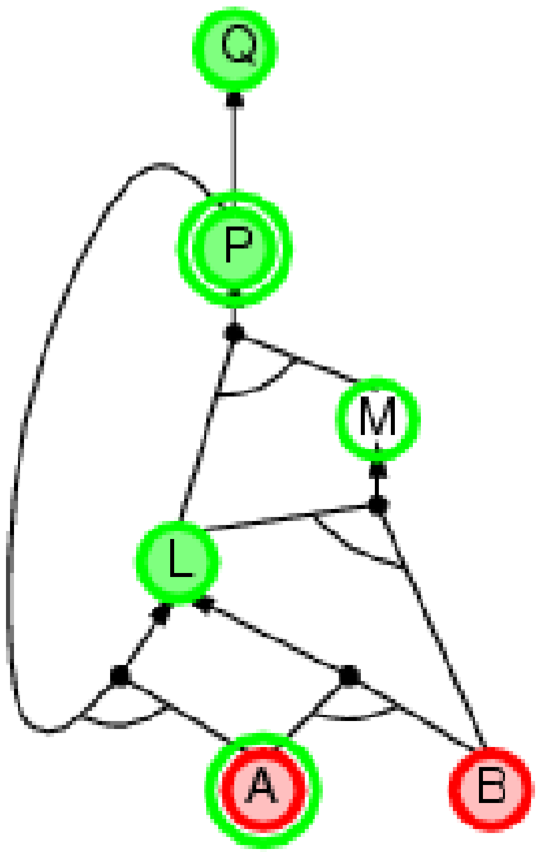

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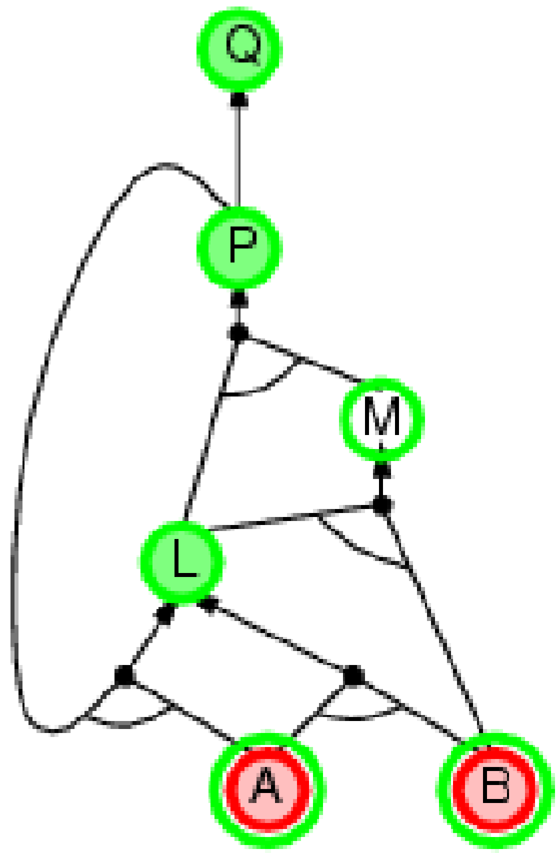

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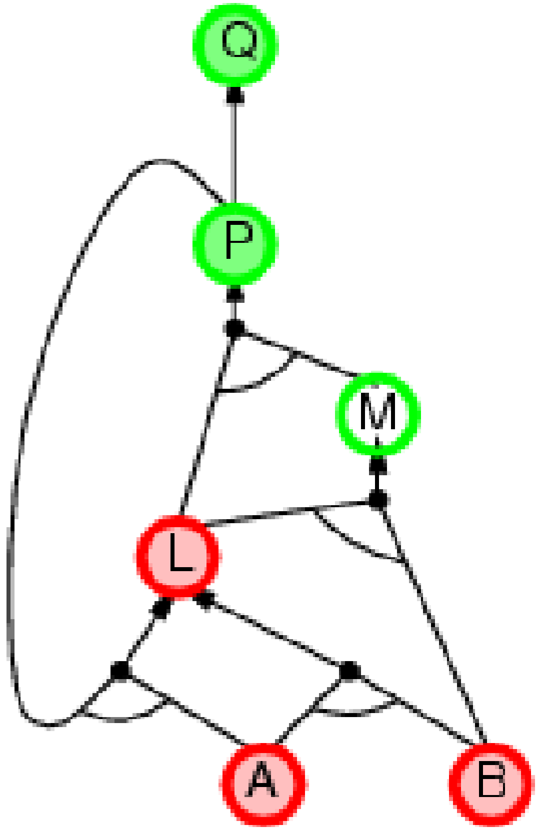

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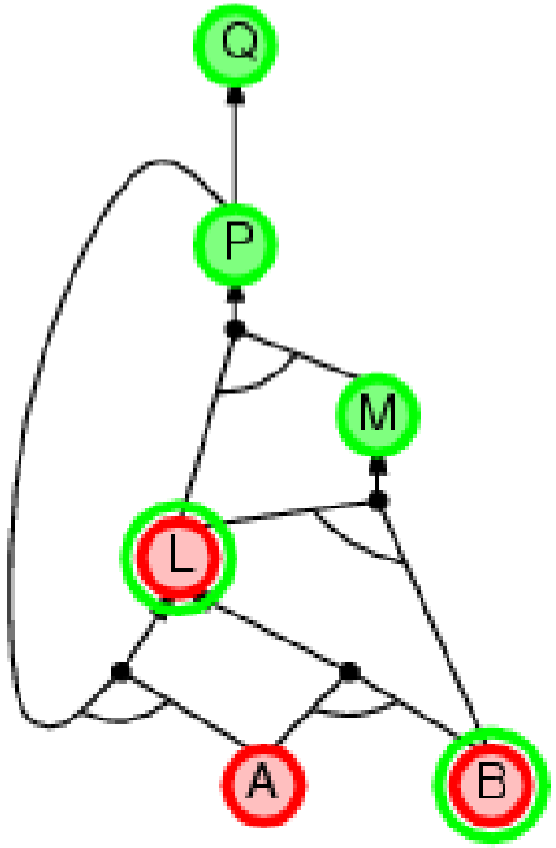

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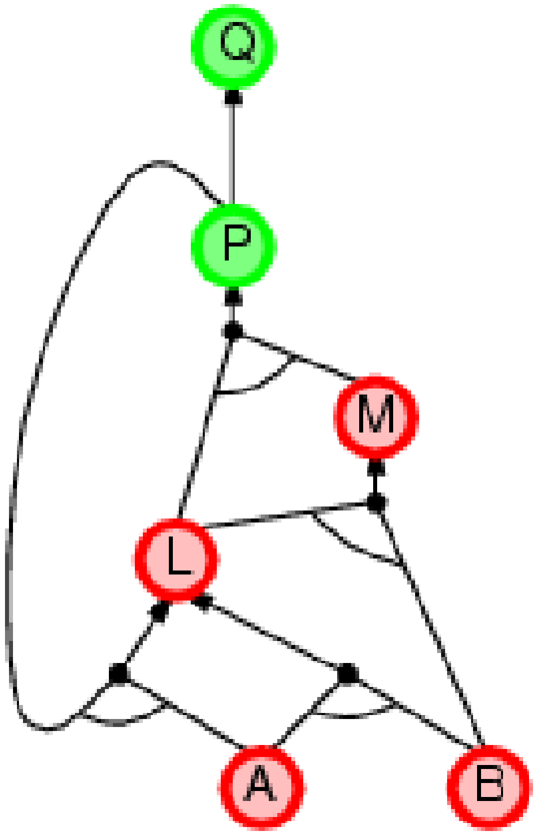

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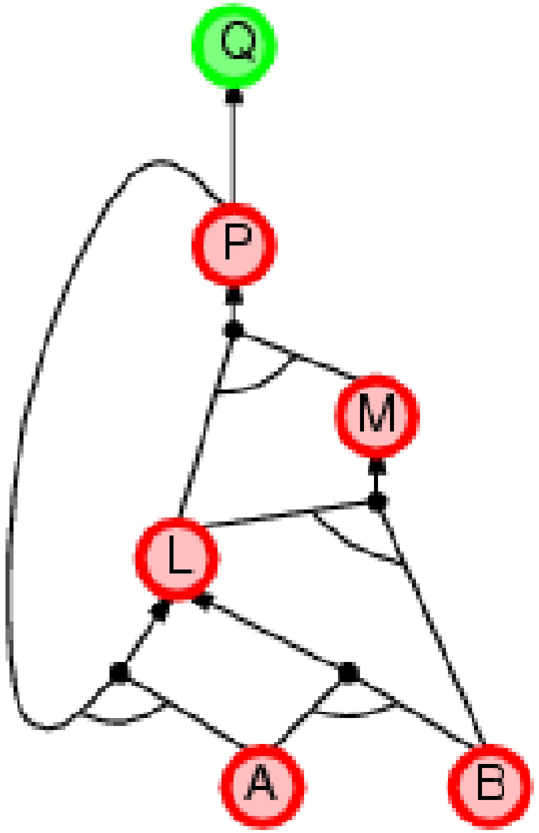

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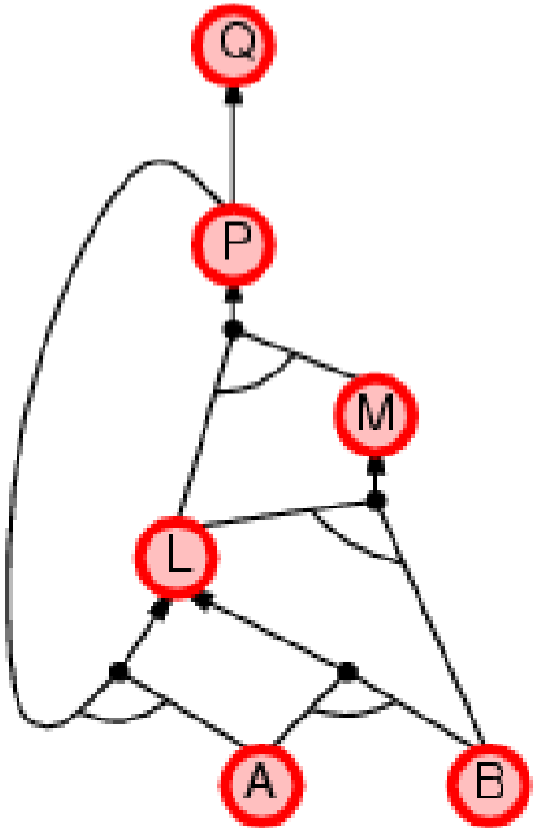


Forward vs. backward chaining

- FC is data-driven, automatic, unconscious processing,
  - e.g., object recognition, routine decisions
- May do lots of work that is irrelevant to the goal
- BC is goal-driven, appropriate for problem-solving,
  - e.g., Where are my keys? How do I get into a PhD program?
- Complexity of BC can be much less than linear in size of KB

Efficient propositional inference

- Two families of efficient algorithms for propositional inference:

- Complete backtracking search algorithms
  - DPLL algorithm (Davis, Putnam, Logemann, Loveland)

- Incomplete local search algorithms WalkSAT algorithm

The DPLL algorithm

Determine if an input propositional logic sentence (in CNF) is satisfiable.

Improvements over truth table enumeration:

1. Early termination

   A clause is true if any literal is true.

   A sentence is false if any clause is false.

2. Pure symbol heuristic

   Pure symbol: always appears with the same "sign" in all clauses.

   e.g., In the three clauses (A ∨ ¬B), (¬B ∨ ¬C), (C ∨ A), A and B are pure, C is impure.

   Make a pure symbol literal true.

3. Unit clause heuristic

   Unit clause: only one literal in the clause

   The only literal in a unit clause must be true.

Least constraining value

Most constrained value

What are correspondences between DPLL and in general CSPs?

The DPLL algorithm

```
function DPLL-SATISFIABLE?(s) returns true or false
   inputs: s, a sentence in propositional logic

   clauses ← the set of clauses in the CNF representation of s
   symbols ← a list of the proposition symbols in s
   return DPLL(clauses, symbols, [ ])
```

```
function DPLL(clauses, symbols, model) returns true or false

   if every clause in clauses is true in model then return true
   if some clause in clauses is false in model then return false
   P, value ← FIND-PURE-SYMBOL(symbols, clauses, model)
   if P is non-null then return DPLL(clauses, symbols–P, [P = value|model])
   P, value ← FIND-UNIT-CLAUSE(clauses, model)
   if P is non-null then return DPLL(clauses, symbols–P, [P = value|model])
   P ← FIRST(symbols); rest ← REST(symbols)
   return DPLL(clauses, rest, [P = true|model]) or
          DPLL(clauses, rest, [P = false|model])
```

The WalkSAT algorithm

- Incomplete, local search algorithm
- Evaluation function: The min-conflict heuristic of minimizing the number of unsatisfied clauses
- Balance between greediness and randomness

The WalkSAT algorithm

```
function WALKSAT(clauses, p, max-flips) returns a satisfying model or failure
   inputs: clauses, a set of clauses in propositional logic
           p, the probability of choosing to do a "random walk" move
           max-flips, number of flips allowed before giving up

   model ← a random assignment of true/false to the symbols in clauses
   for i = 1 to max-flips do
      if model satisfies clauses then return model
      clause ← a randomly selected clause from clauses that is false in model
      with probability p flip the value in model of a randomly selected symbol
            from clause
     else flip whichever symbol in clause maximizes the number of satisfied clauses
   return failure
```

Hard satisfiability problems

- Consider random 3-CNF sentences. e.g.,

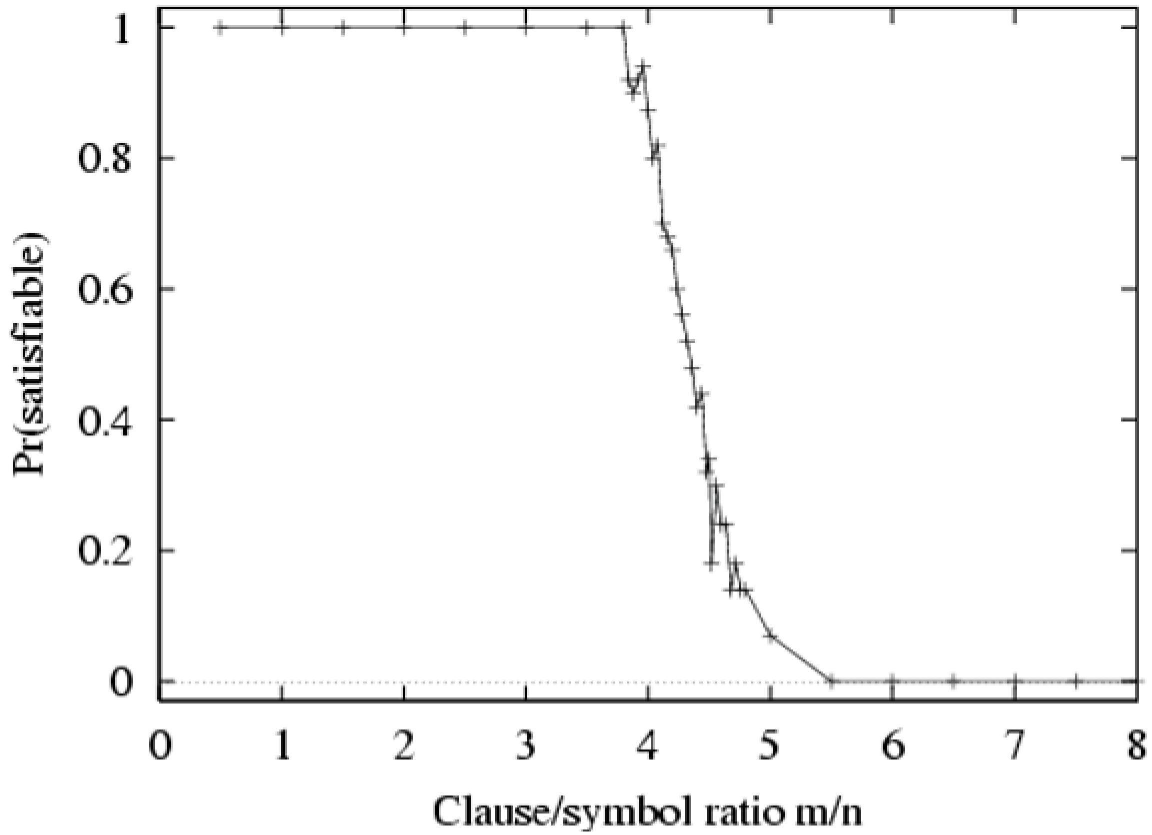
$(\neg D \lor \neg B \lor C) \land (B \lor \neg A \lor \neg C) \land (\neg C \lor \neg B \lor E) \land (E \lor \neg D \lor B) \land (B \lor E \lor \neg C)$

m = number of clauses

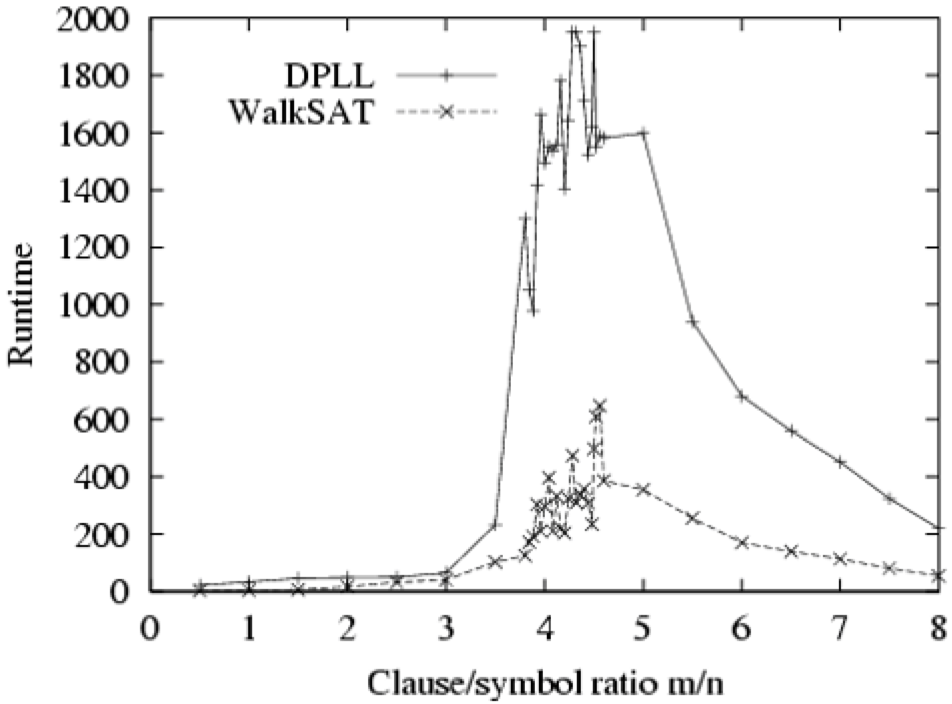
n = number of symbols

- Hard problems seem to cluster near $m/n = 4.3$ (critical point)

Hard satisfiability problems



Hard satisfiability problems



- Median runtime for 100 satisfiable random 3-CNF sentences, n = 50

Inference-based agents in the wumpus world

A wumpus-world agent using propositional logic:

$\neg P_{1,1}$
$\neg W_{1,1}$
$B_{x,y} \Leftrightarrow (P_{x,y+1} \vee P_{x,y-1} \vee P_{x+1,y} \vee P_{x-1,y})$
$S_{x,y} \Leftrightarrow (W_{x,y+1} \vee W_{x,y-1} \vee W_{x+1,y} \vee W_{x-1,y})$
$W_{1,1} \vee W_{1,2} \vee \ldots \vee W_{4,4}$
$\neg W_{1,1} \vee \neg W_{1,2}$
$\neg W_{1,1} \vee \neg W_{1,3}$
…

Have to propositionalize each of these *x,y* rules

Expressing that there is exactly one wumpus

$\Rightarrow$ 64 distinct proposition symbols, 155 sentences

Expressiveness limitation of propositional logic

- KB contains "physics" sentences for every single square

- For every time *t* and every location [*x*,*y*],

$$L^{t}_{x,y} \land FacingRight^{t} \land Forward^{t} \Rightarrow L^{t}_{x+1,y}$$

- Rapid proliferation of clauses

Summary

- Logical agents apply inference to a knowledge base to derive new information and make decisions
- Basic concepts of logic:
  - syntax: formal structure of sentences
  - semantics: truth of sentences w.r.t. models
  - entailment: necessary truth of one sentence given another
  - inference: deriving sentences from other sentences
  - soundness: derivations produce only entailed sentences
  - completeness: derivations can produce all entailed sentences
- The wumpus world requires the ability to represent partial and negated information, reason by cases, etc.
- Resolution is complete for propositional logic Forward, backward chaining are linear-time, complete for Horn clauses
- Propositional logic lacks expressive power