Lab 3
APPLIED LINEAR ALGEBRA FOR IT - 501032

1 Exercises

Exercise 1: Using a command to create the new matrices from vectors **x**=$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \end{pmatrix}$, **b**=$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix}$, **c**=1:1:30 and **d**=1:1:25.

(a) $A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 & 4 \\ 5 & 5 & 5 & 5 & 5 \end{pmatrix}$

(b) $B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix}$

(c) $C = \begin{pmatrix} 1 & 6 & 11 & 16 & 21 & 26 \\ 2 & 7 & 12 & 17 & 22 & 27 \\ 3 & 8 & 13 & 18 & 23 & 28 \\ 4 & 9 & 14 & 19 & 24 & 29 \\ 5 & 10 & 15 & 20 & 25 & 30 \end{pmatrix}$

(d) $D = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \\ 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 25 \end{pmatrix}$

Exercise 2: Write a command that will create a 5×6 matrix with random integer entries with the elements $\in [a, b]$, where $a, b \in Z$

Exercise 3: Write a command to flip the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ horizontally as follows $B = \begin{pmatrix} 3 & 2 & 1 \\ 6 & 5 & 4 \\ 9 & 8 & 7 \end{pmatrix}$

Exercise 4: Write a command to flip the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ vertically as follows $B = \begin{pmatrix} 7 & 8 & 9 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{pmatrix}$

Exercise 5: Enter the matrix $Y = \begin{pmatrix} 1 & 2 & 16 & 31 & 22 \\ 2 & 8 & 12 & 21 & 23 \\ 4 & 9 & 11 & 14 & 25 \\ 3 & 6 & 10 & 16 & 34 \end{pmatrix}$, provide a command to create vectors or matrices as follows:

(a) $x = \begin{pmatrix} 8 & 12 & 21 \end{pmatrix}$

(b) $y = \begin{pmatrix} 16 \\ 12 \\ 11 \\ 10 \end{pmatrix}$

(c) $A = \begin{pmatrix} 8 & 12 & 21 \\ 9 & 11 & 14 \end{pmatrix}$

(d) $B = \begin{pmatrix} 1 & 16 & 22 \\ 2 & 12 & 23 \\ 4 & 11 & 25 \\ 3 & 10 & 34 \end{pmatrix}$

(e) $C = \begin{pmatrix} 2 & 12 & 21 & 23 \\ 4 & 11 & 14 & 25 \\ 3 & 10 & 16 & 34 \end{pmatrix}$

(f) Create a D matrix from Y whose elements is greater than 12

Exercise 6: Given the matrix $A = \begin{pmatrix} 2 & 4 & 1 \\ 6 & 7 & 2 \\ 3 & 5 & 9 \end{pmatrix}$, provide commands to do the following:

(a) Assign the first row of A into a vector called x_1

(b) Assign the last 2 rows of A into the matrix called Y

Exercise 7: Let $A = \begin{pmatrix} 2 & 7 & 9 & 7 \\ 3 & 1 & 5 & 6 \\ 8 & 1 & 2 & 5 \end{pmatrix}$. Write command that will

(a) Assign the even numbered columns of A into a new vector called B

(b) Assign the odd numbered rows into a new vector called C

(c) Convert A to a 4×3 matrix

Exercise 8: A local shop sells three types of ice cream flavours: strawberry, vanilla and chocolate. Strawberry costs 2$, vanilla 1$ and chocolate 3$ each. The sales of each ice cream are as show in the following table.

	Monday	Tuesday	Wednesday	Thursday	Friday
Strawberry (S)	12	15	10	16	12
Vanilla (V)	5	9	14	7	10
Chocolate (C)	8	12	10	9	15

How to evaluate the total sales for each day.

Exercise 9: Let T be a (transition) matrix of a Markov chain and p be a probability vector. Then the probability that the chain is in a particular state after k steps is given by the vector p_k:

$$p_k = T^k p$$

Determine p_k by any appropriate program for

$$T = \begin{pmatrix} 0.6 & 0.7 \\ 0.4 & 0.3 \end{pmatrix}, \qquad p = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}, \qquad \text{and } k = 1, 2, 10, 100, 100000.$$

Exercise 10: Let $A = \begin{pmatrix} -1 & 4 & 8 \\ -9 & 1 & 2 \end{pmatrix}$ $B = \begin{pmatrix} 5 & 8 \\ 0 & -6 \\ 5 & 6 \end{pmatrix}$ $C = \begin{pmatrix} -4 & 1 \\ 6 & 5 \end{pmatrix}$ $D = \begin{pmatrix} -6 & 3 & 1 \\ 8 & 9 & -2 \\ 6 & -1 & 5 \end{pmatrix}$.

Computing the following, if possible

(a) (AB^T)

(b) (BC^T)

(c) $(C - C^T)$

(d) $(D - D^T)$

(e) $((D^T)^T)$

(f) $(2C^T)$

(g) $(A^T + B)$

(h) $((A^T + B)^T)$

(i) $((2A^T - 5B)^T)$

(j) $((-D)^T)$

(k) $(-(D)^T)$

(l) $((C^2)^T)$

(m) $((C^T)^2)$

Exercise 11: Let $A = \begin{pmatrix} 2 & 4 & 1 \\ 6 & 7 & 2 \\ 3 & 5 & 9 \end{pmatrix}$

(a) Is A matrix square or not?

(b) Is A matrix symmetric or not?

(c) Is A matrix skew-symmetric or not?

(d) Find Upper triangular matrix of A.

(e) Find Lower triangular matrix of A.

Exercise 12: Write a command to compute the determinant of the matrices below:

$$A = \begin{pmatrix} 6 & 0 & 0 & 5 \\ 1 & 7 & 2 & -5 \\ 2 & 0 & 0 & 0 \\ 8 & 3 & 1 & 8 \end{pmatrix}, B = \begin{pmatrix} 1 & -2 & 5 & 2 \\ 0 & 0 & 3 & 0 \\ 2 & -6 & -7 & 5 \\ 5 & 0 & 4 & 4 \end{pmatrix}, C = \begin{pmatrix} 3 & 5 & -8 & 4 \\ 0 & -2 & 3 & -7 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 2 \end{pmatrix},$$

$$D = \begin{pmatrix} 4 & 0 & 0 & 0 \\ 7 & -1 & 0 & 0 \\ 2 & 6 & 3 & 0 \\ 5 & -8 & 3 & 0 \\ 5 & -8 & 4 & -3 \end{pmatrix}, E = \begin{pmatrix} 4 & 0 & -7 & 3 & -5 \\ 0 & 0 & 2 & 0 & 0 \\ 7 & 3 & -6 & 4 & -8 \\ 5 & 0 & 5 & 2 & -3 \\ 0 & 0 & 9 & -1 & 2 \end{pmatrix}, F = \begin{pmatrix} 6 & 3 & 2 & 4 & 0 \\ 9 & 0 & -4 & 1 & 0 \\ 8 & -5 & 6 & 7 & 1 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 2 & 3 & 2 & 0 \end{pmatrix}$$

Exercise 13: Is it true that $\det(A+B)=\det A+\det B$? To find out, generate random 5×5 matrices A and B, and compute $\det(A+B)-\det A-\det B$. Repeat the calculations for three other pairs of $n \times n$ matrices, for various values of n.

Exercise 14: Is it true that $\det AB=(\det A)(\det B)$?. Repeat the calculation for four pairs of random matrices.

Exercise 15: Let A and B be the following 3×3 matrices $\begin{pmatrix} 2 & 4 & \frac{5}{2} \\ \frac{-3}{4} & 2 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & \frac{-1}{2} & \frac{3}{4} \\ \frac{3}{2} & \frac{1}{2} & -2 \\ \frac{1}{4} & 1 & \frac{1}{2} \end{pmatrix}$

(a) Calculate $A^{-1}B^{-1}$, $(AB)^{-1}$, and $(BA)^{-1}$

(b) Find $(A^{-1})^T$ and $(A^T)^{-1}$