Data Structures and Algorithms

Maze Exploration

Acknowledgement

- The contents of these slides have origin from School of Computing, National University of Singapore.
- We greatly appreciate support from Dr. Steven Halim for kindly sharing these materials.

Policies for students

- These contents are only used for students PERSONALLY.
- Students are NOT allowed to modify or deliver these contents to anywhere or anyone for any purpose.

Recording of modifications

- Currently, there are no modification on these contents.

Outline

Continue Week 05 stuffs (Graph DS Applications)

Two algorithms to traverse a graph

- Depth First Search (DFS) and Breadth First Search (BFS)
- Plus some of their interesting applications

visualgo.net/**dfsbfs.html**

Reference: Mostly from CP3 Section 4.2

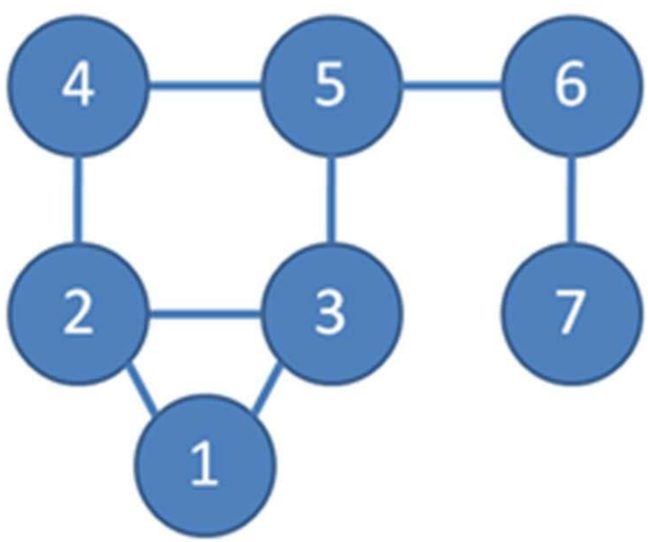
- Not all sections in CP3 chapter 4 are used in CS2010!
  - Some are quite advanced :O

SOME GRAPH DATA STRUCTURE APPLICATIONS

So, what can we do so far? (1)

With just graph DS, not much that we can do…
But here are some:

- Counting **V** (the number of vertices)
  - Very trivial for both AdjMatrix and AdjList: **V = number of rows!**
  - Sometimes this number is stored in separate variable so that we do not have to re-compute every time, that is, O(**1**), especially if the graph never changes



A

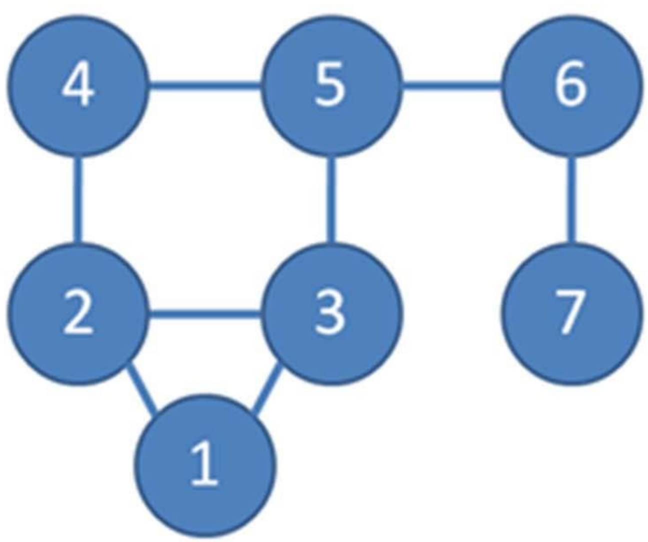
	1	2	3	4	5	6	7
1		1	1				
2	1		1	1			
3	1	1			1		
4		1			1		
5			1	1		1	
6					1		1
7						1	

B

1 → 2 → 3
2 → 1 → 3 → 4
3 → 1 → 2 → 5
4 → 2 → 5
5 → 3 → 4 → 6
6 → 5 → 7
7 → 6

So, what can we do so far? (2)

- Enumerating neighbors of a vertex **v**
  - O(**V**) for AdjMatrix: **scan AdjMatrix[v][j], ∀j ∈ [0..V-1]**
  - O(**k**) for AdjList, **scan AdjList[v]**
    - **k** is the number of neighbors of vertex **v** (output-sensitive algorithm)
  - This is an important difference between AdjMatrix versus AdjList
    - It affects the performance of many graph algorithms. Remember this!



A

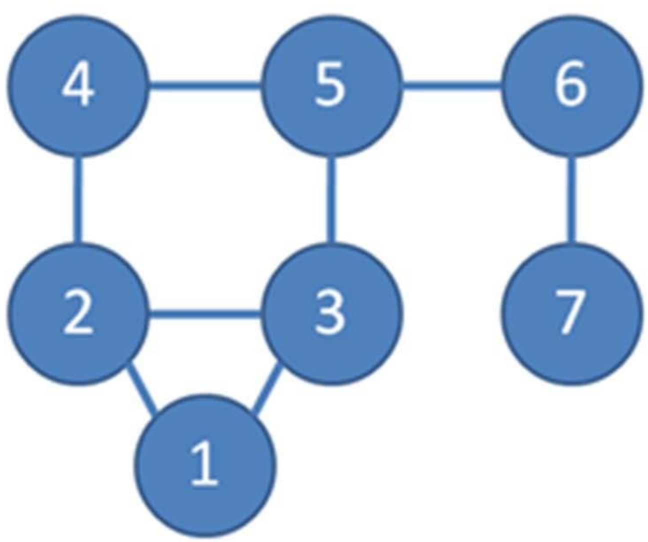
	1	2	3	4	5	6	7
1		1	1				
2	1		1	1			
3	1	1			1		
4		1			1		
5			1	1		1	
6					1		1
7						1	

B

1 → 2 → 3
2 → 1 → 3 → 4
3 → 1 → 2 → 5
4 → 2 → 5
5 → 3 → 4 → 6
6 → 5 → 7
7 → 6

So, what can we do so far? (3)

- Counting **E** (the number of edges)
  - O(**V²**) for AdjMatrix: **count non zero entries in AdjMatrix**
  - O(**V+E**) for AdjList: **sum the length of all V lists**
  - Sometimes this number is stored in separate variable so that we do not have to re-compute every time, i.e. O(**1**), especially if the graph never changes



A

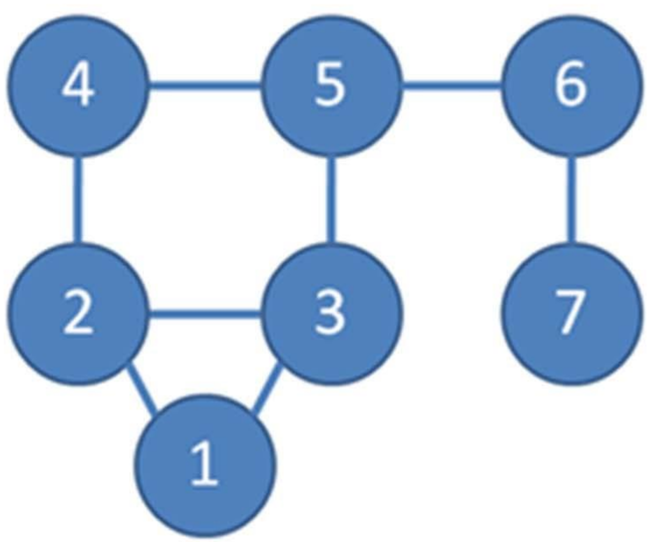
	1	2	3	4	5	6	7
1		1	1				
2	1		1	1			
3	1	1			1		
4		1			1		
5			1	1		1	
6					1		1
7						1	

B

1 → 2 → 3
2 → 1 → 3 → 4
3 → 1 → 2 → 5
4 → 2 → 5
5 → 3 → 4 → 6
6 → 5 → 7
7 → 6

So, what can we do so far? (4)

- Checking the existence of edge(**u**, **v**)
  - O(**1**) for AdjMatrix: **see if AdjMatrix[u][v] is non zero**
  - O(**k**) for AdjList: **see if AdjList[u] contains v**
- There are a few others,
  but let's reserve them for PSes or even for test questions ☺



A

	1	2	3	4	5	6	7
1		1	1				
2	1		1	1			
3	1	1			1		
4		1			1		
5			1	1		1	
6					1		1
7						1	

B

1 → 2 → 3
2 → 1 → 3 → 4
3 → 1 → 2 → 5
4 → 2 → 5
5 → 3 → 4 → 6
6 → 5 → 7
7 → 6

Trade-Off

Adjacency Matrix

Pros:

- Existence of edge i-j can be found in O(**1**)
- Good for dense graph/ Floyd Warshall's (Lecture 12)

Cons:

- O(**V**) to enumerate neighbors of a vertex
- O($\mathbf{V}^2$) space

Adjacency List

Pros:

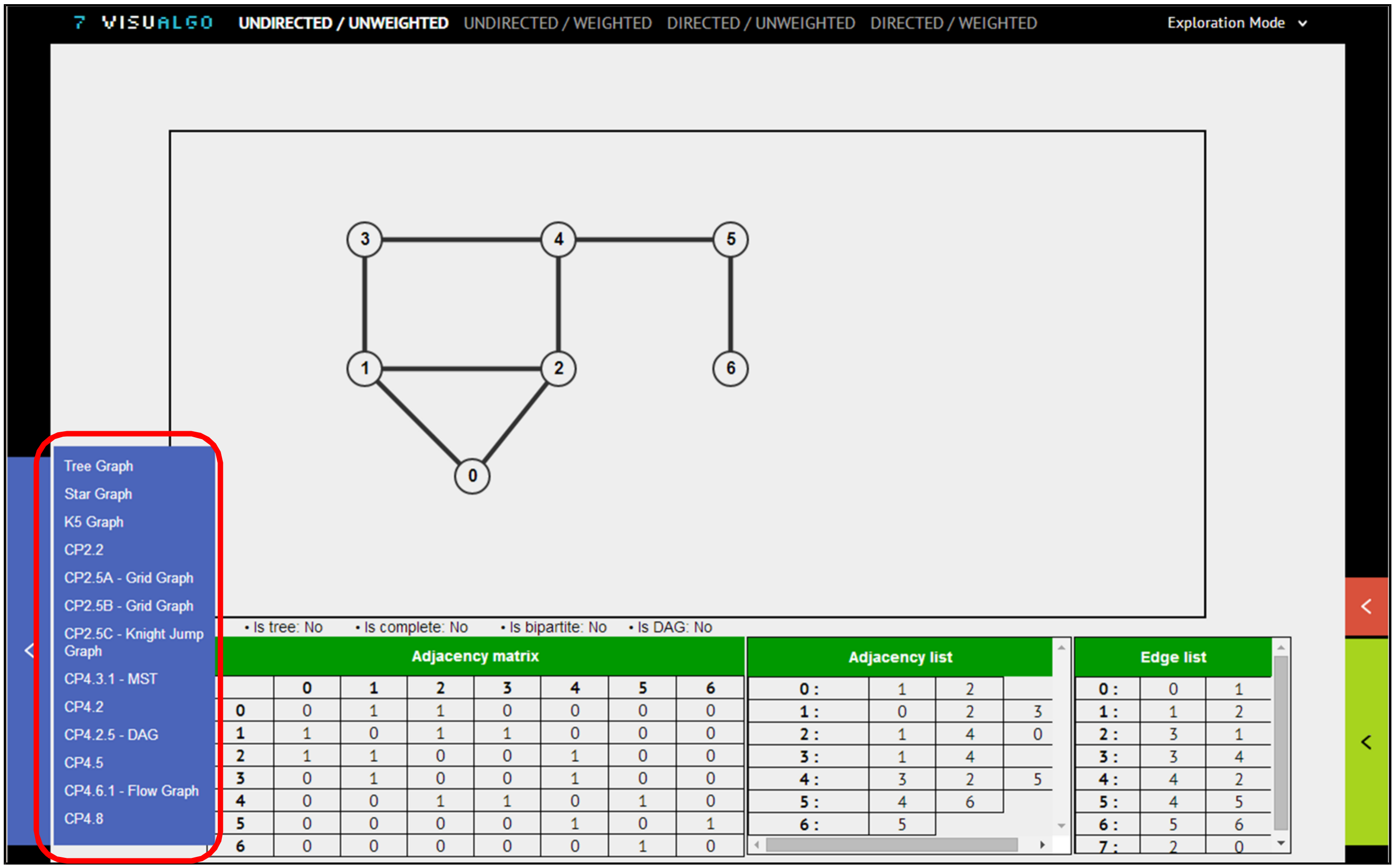
- O(**k**) to enumerate k neighbors of a vertex
- Good for sparse graph/Dijkstra's/ DFS/BFS, O(**V**+**E**) space

Cons:

- O(**k**) to check the existence of edge i-j
- A small overhead in maintaining the list (for sparse graph)

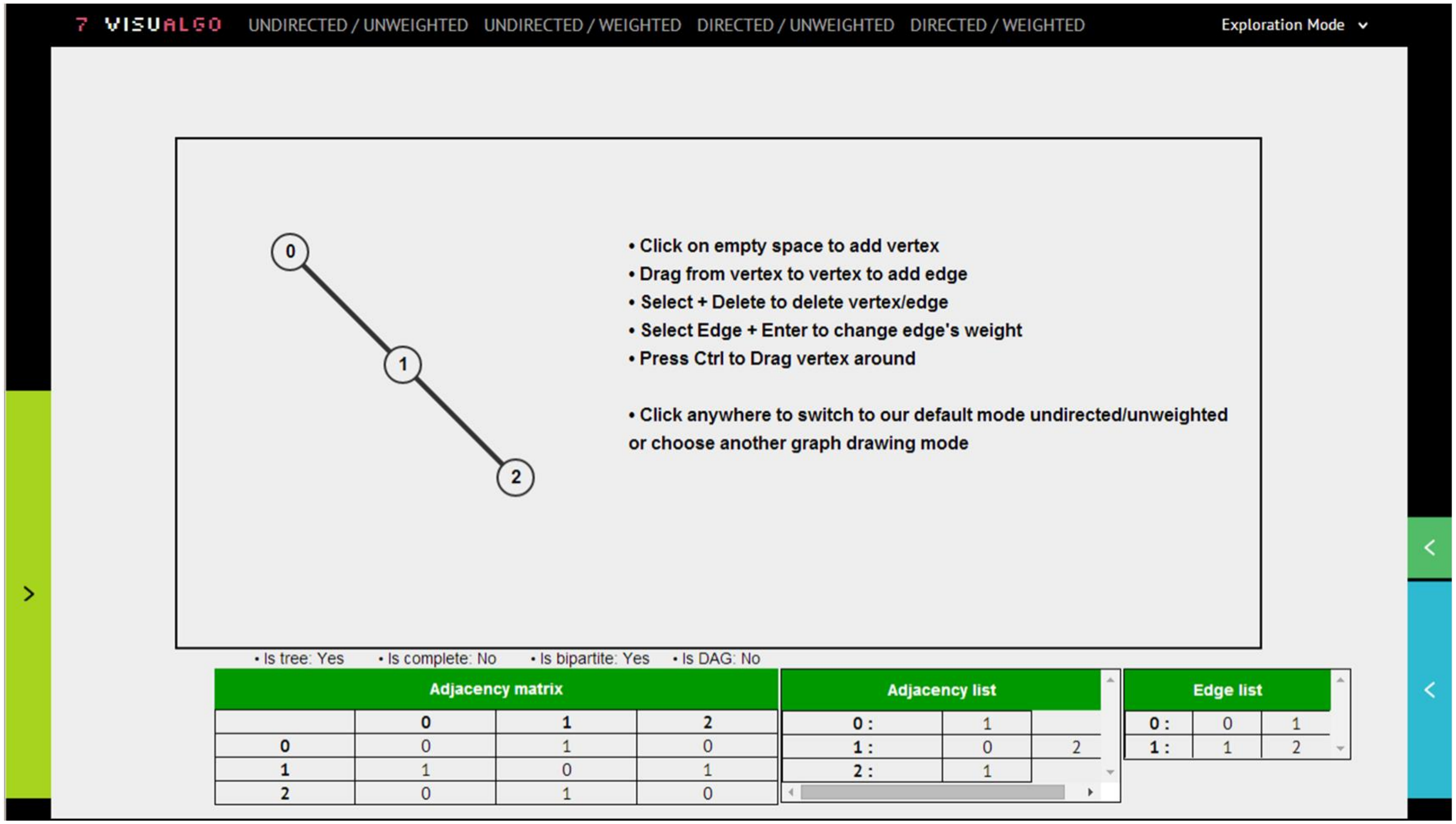
VisuAlgo Graph DS Exploration (1)

Click each of the sample graphs one by one and verify the content of the corresponding **Adjacency Matrix**, **Adjacency List**, and **Edge List**



VisuAlgo Graph DS Exploration (2)

Now, use your mouse over the currently displayed graph **and start drawing some new vertices and/or edges** and see the updates in AdjMatrix/AdjList/EdgeList structures



	0	1	2
0	0	1	0
1	1	0	1
2	0	1	0

0 :	1	
1 :	0	2
2 :	1	

0 :	0	1
1 :	1	2

GRAPH TRAVERSAL ALGORITHMS

Review – **Binary Tree** Traversal

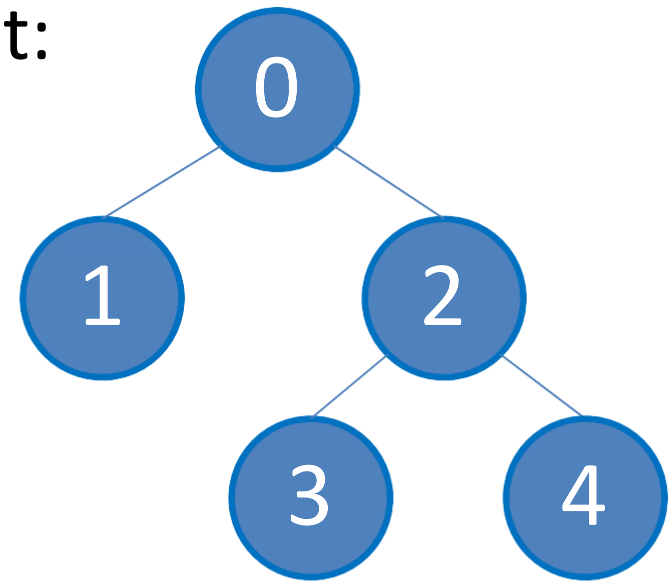
In a binary tree, there are three standard traversal:

- Preorder
- **Inorder**
- Postorder
- (Note: "level order" is just BFS which we will see next)

`pre(u)`
`  visit(u);`
`  pre(u->left);`
`  pre(u->right);`
	`in(u)`
	`  in(u->left);`
	`  visit(u);`
	`  in(u->right);`
		`post(u)`
		`  post(u->left);`
		`  post(u->right);`
		`  visit(u);`

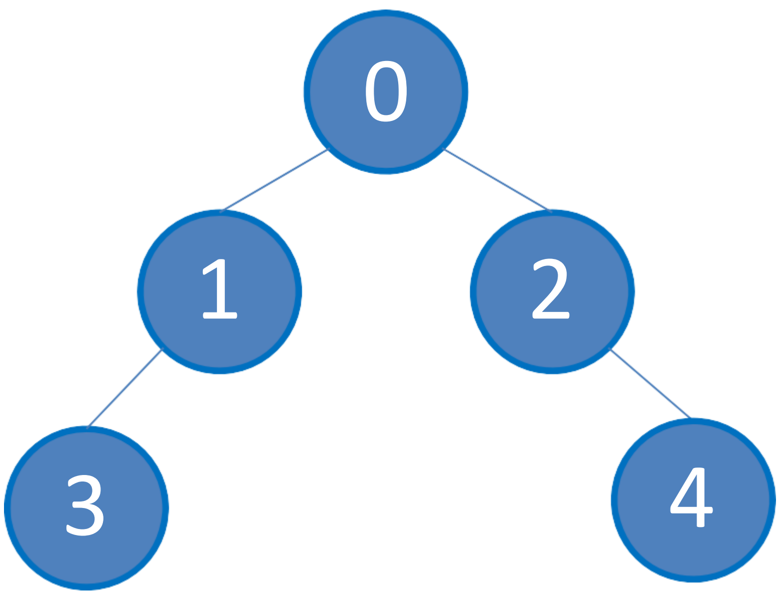
We start binary tree traversal from root:

- pre(root)/in(root)/post(root)
  - pre = 0, 1, 2, 3, 4
  - in = 1, 0, 3, 2, 4
  - post = 1, 3, 4, 2, 0



What is the **Post**Order Traversal of this Binary Tree?

1. 0 1 2 3 4
2. 0 1 3 2 4
3. 3 4 1 2 0
4. 3 1 4 2 0



Traversing a Graph (1)

Two ingredients are needed for a **traversal:**

1. The start
2. The movement

Defining the start ("source")

- In tree, we *normally* start from root
  - Note: Not all tree are rooted though!
    - In that case, we have to select one vertex as the "source", see below
- In general graph, we do not have the notion of root
  - Instead, we start from a distinguished vertex
    - We call this vertex as the **"source" s**

Traversing a Graph (2)

Defining the movement:

- In (binary) tree, we only have (at most) two choices:
  - Go to the **left subtree** or to the **right subtree**
- In general graph, we can have more choices:
  - If **vertex u** and **vertex v** are adjacent/connected with edge (**u**, **v**);
    and we are now in **vertex u**;
    then we can also go to **vertex v** by traversing that edge (**u**, **v**)
- In (binary) tree, there is **no cycle**
- In general graph, we **may have (trivial/non trivial) cycles**
  - We need a way to avoid revisiting **u** → **v** → **u** → **u** → … indefinitely

Solution: BFS and DFS ☺

Breadth First Search (BFS) – Ideas

- Start from **s**
- If a vertex **v** is reachable from **s**, then all neighbors of **v** will also be reachable from **s** (recursive definition)
- BFS visits vertices of G in *breadth-first* manner (when viewed from source vertex s)
  - Q: How to maintain such order?
    - A: Use queue **Q**, initially, it contains only **s**
  - Q: How to differentiate visited vs unvisited vertices (to avoid cycle)?
    - A: 1D array/Vector **visited** of size V,
visited[v] = 0 initially, and **visited[v] = 1** when **v** is visited
  - Q: How to memorize the path?
    - A: 1D array/Vector **p** of size V,
p[v] denotes the **p**redecessor (or **p**arent) of **v**

BFS Pseudo Code

```
for all v in V
  visited[v] ← 0
  p[v]  ←  -1
Q  ←{s} // start from s
visited[s]  ←     1

while Q is not empty
  u  ←Q.dequeue()
  for all v adjacent to u // order of neighbor
    if visited[v] = 0 // influences BFS
      visited[v] ← true // visitation sequence
      p[v]  ←  u
      Q.enqueue(v)

// after BFS stops, we can use info stored in visited/p
```

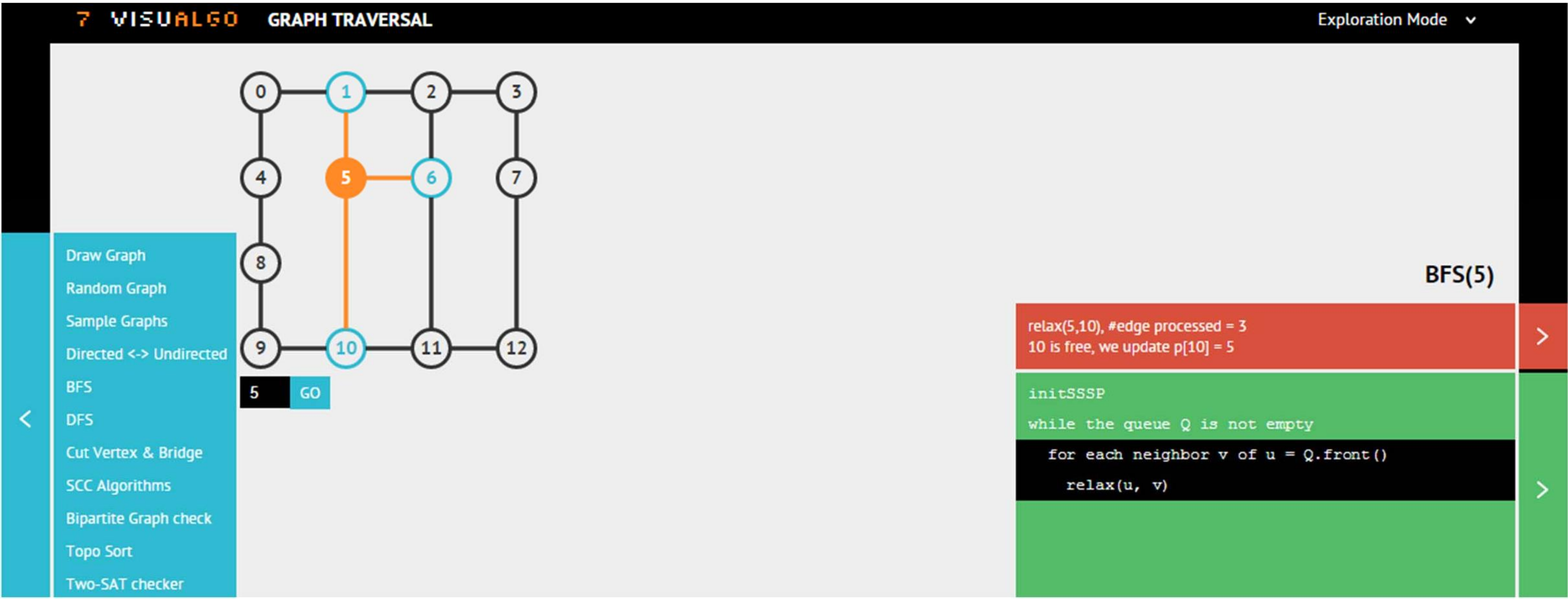
Initialization phase

Main loop

Graph Traversal: BFS(s)

Ask VisuAlgo to perform various Breadth-First Search operations on the sample Graph (CP3 4.3, Undirected)

In the screen shot below, we show the start of **BFS(5)**



BFS Analysis

```
for all v in V
  visited[v] ← 0
  p[v]  ←  -1
Q  ←{s} // start from s
visited[s]  ←
    1
while Q is not empty
  u  ←Q.dequeue()
  for all v adjacent to u // order of neighbor
    if visited[v] =  0 // influences BFS
      visited[v]  ←     true // visitation
      sequence  p[v]  ← u
      Q.enqueue(v)

// we can then use information stored in visited/p
```

Time Complexity: O(**V**+**E**)

- Each vertex is only in the queue once ~ O(**V**)
- Every time a vertex is dequeued, all its **k** neighbors are scanned; After all vertices are dequeued, all **E** edges are examined ~ O(**E**) → assuming that we use **Adjacency List**!
- Overall: O(**V**+**E**)

Depth First Search (DFS) – Ideas

- Start from **s**
- If a vertex **v** is reachable from **s**, then all neighbors of **v** will also be reachable from **s** (recursive definition)
- DFS visits vertices of G in *depth-first* manner (when viewed from source vertex s)
  - Q: How to maintain such order?
    - A: Stack **S**, but we will simply use recursion (an implicit stack)
  - Q: How to differentiate visited vs unvisited vertices (to avoid cycle)?
    - A: 1D array/Vector **visited** of size V,
      visited[v] = 0 initially, and **visited[v] = 1** when **v** is visited
  - Q: How to memorize the path?
    - A: 1D array/Vector **p** of size V,
      p[v] denotes the **p**redecessor (or **p**arent) of **v**

DFS Pseudo Code

```
DFSrec(u)
  visited[u] ← 1 // to avoid cycle
  for all v adjacent to u // order of neighbor
    if visited[v] = 0 //  influences DFS
      p[v] ← u // visitation sequence
      DFSrec(v) // recursive (implicit stack)
```

Recursive phase

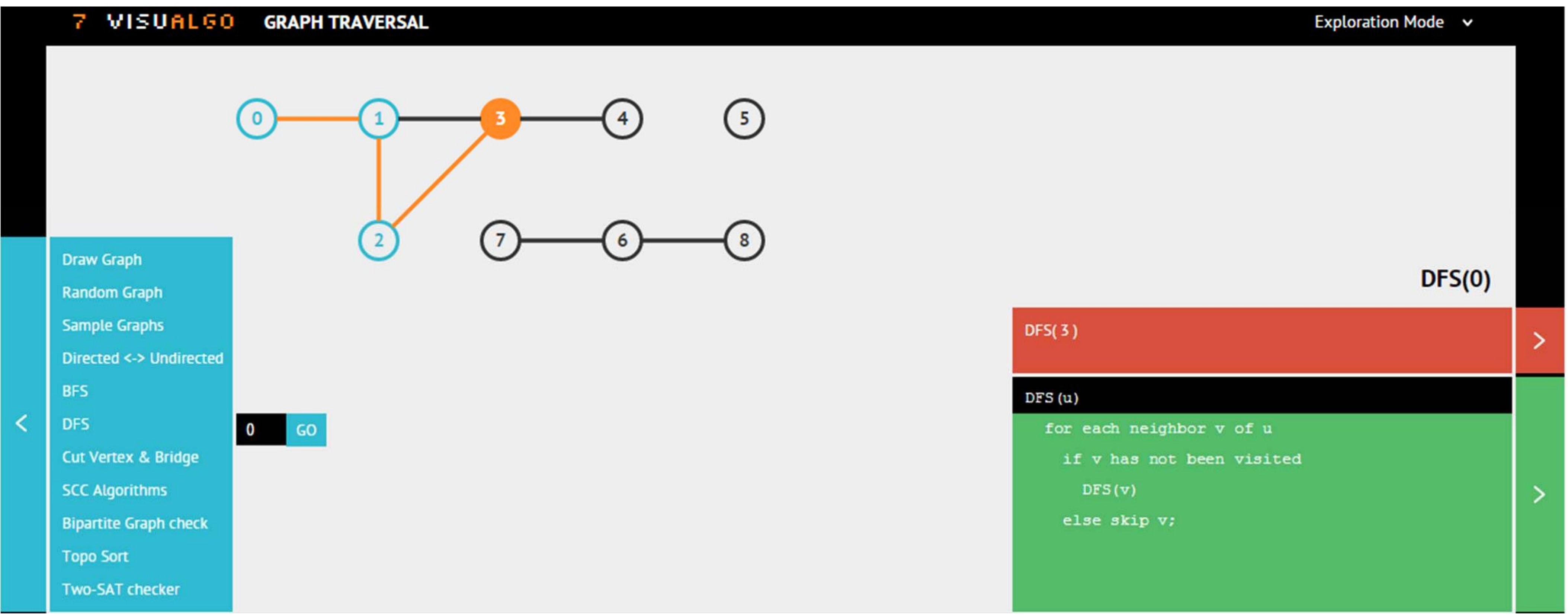
```
// in the main method
for all v in V
  visited[v] ← 0
  p[v] ← -1
DFSrec(s) // start the
recursive call from s
```

Initialization phase, same as with BFS

Graph Traversal: DFS(s)

Ask VisuAlgo to perform various Breadth-First Search operations on the sample Graph (CP3 4.1, Undirected)

In the screen shot below, we show the start of **DFS(0)**



DFS Analysis

```
DFSrec(u)
  visited[u] ← 1 // to avoid cycle
  for all v adjacent to u // order of neighbor
    if visited[v] = 0 //  influences DFS
      p[v] ← u // visitation sequence
      DFSrec(v) // recursive (implicit stack)

// in the main method
for all v in V
  visited[v] ← 0
  p[v] ← -1
DFSrec(s) // start the
recursive call from s
```

Time Complexity: O(**V**+**E**)

- Each vertex is only visited once O(**V**), then it is flagged to avoid cycle
- Every time a vertex is visited, all its **k** neighbors are scanned; Thus after all vertices are visited, we have examined all **E** edges ~ O(**E**) → assuming that we use **Adjacency List**!
- Overall: O(**V**+**E**)

Path Reconstruction Algorithm (1)

```
// iterative version (will produce reversed output)
Output "(Reversed) Path:"
i  ←t // start from end of path: suppose vertex t
while i != s
  Output i
  i  ←p[i] // go back to predecessor of i
Output s



// try it on this array p, t = 4
// p = {-1, 0, 1, 2, 3, -1, -1, -1}
```

Path Reconstruction Algorithm (2)

```
void backtrack(u)
  if (u == -1) // recall: predecessor of s is -1
    stop
  backtrack(p[u]) // go back to predecessor of u
  Output u // recursion like this reverses the order

// in main method
// recursive version (normal path)
Output "Path:"
backtrack(t); // start from end of path (vertex t)
// try it on this array p, t = 4
// p = {-1, 0, 1, 2, 3, -1, -1, -1}
```

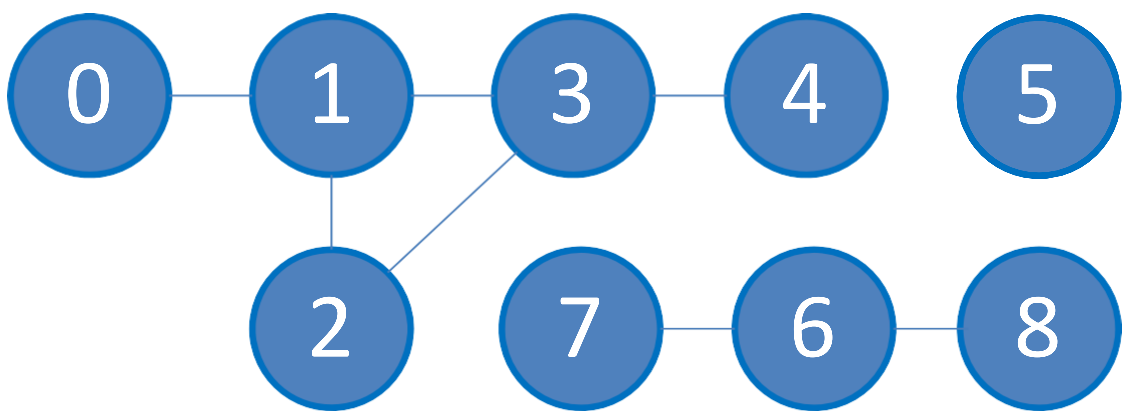
SOME GRAPH TRAVERSAL APPLICATIONS

What can we do with BFS/DFS? (1)

Several stuffs, let's see ***some of them***:

- Reachability test
  - Test whether vertex **v** is reachable from vertex **u**?
  - Start BFS/DFS from **s = u**
  - If **visited[v] = 1** after BFS/DFS terminates, then **v** is *reachable* from **u**; otherwise, **v** is *not reachable* from **u**

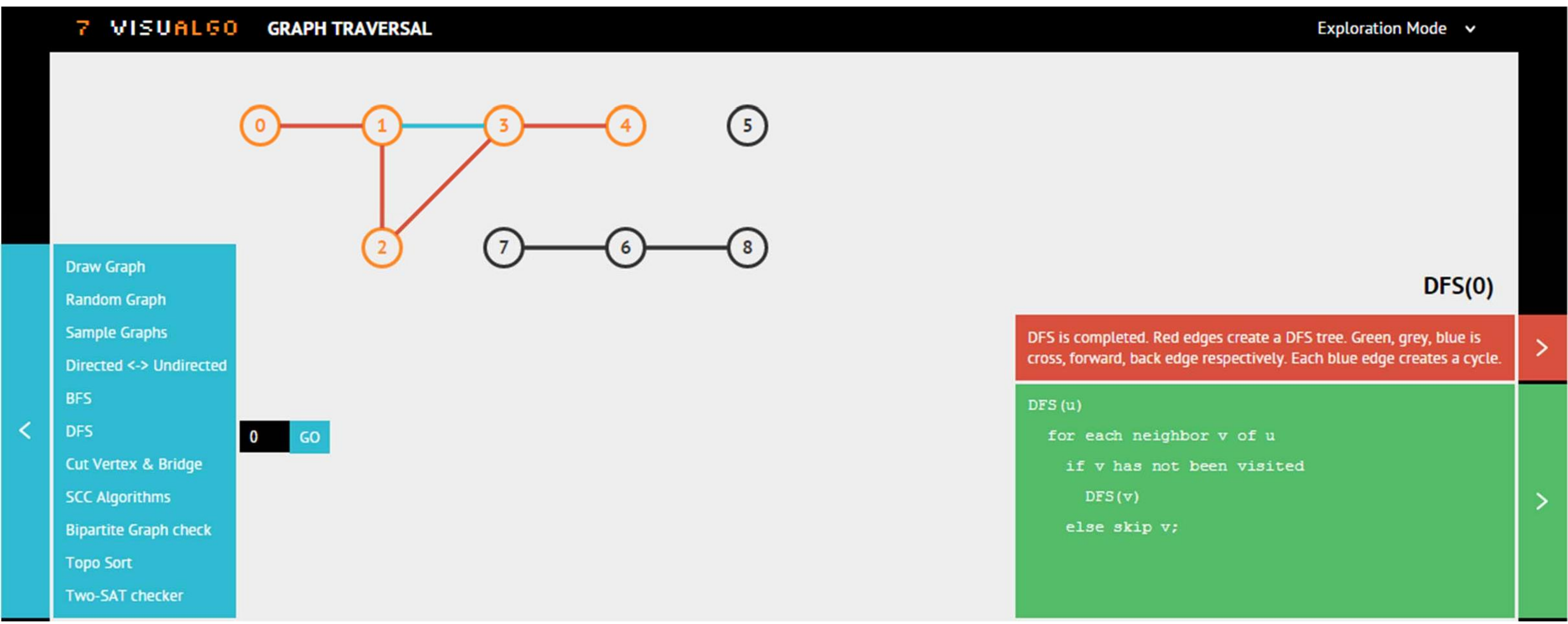
```
BFS(u) // DFSrec(u)
if visited[v] == 1
  Output "Yes"
else
  Output "No"
```



Reachability Test

Ask VisuAlgo to perform various DFS (or BFS) operations on the sample Graph (CP3 4.1, Undirected)

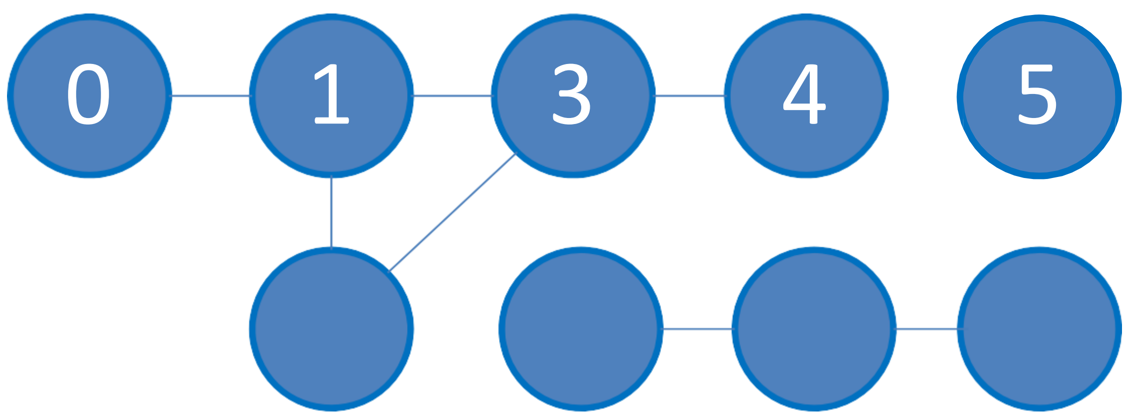
Below, we show vertices that are reachable from vertex 0



What can we do with BFS/DFS? (2)

- Identifying component(s)
  - Component is sub graph in which any 2 vertices are connected to each other by at least one path, and is connected to no additional vertices
  - With BFS/DFS, we can identify/label/count components in graph G
  - Solution:

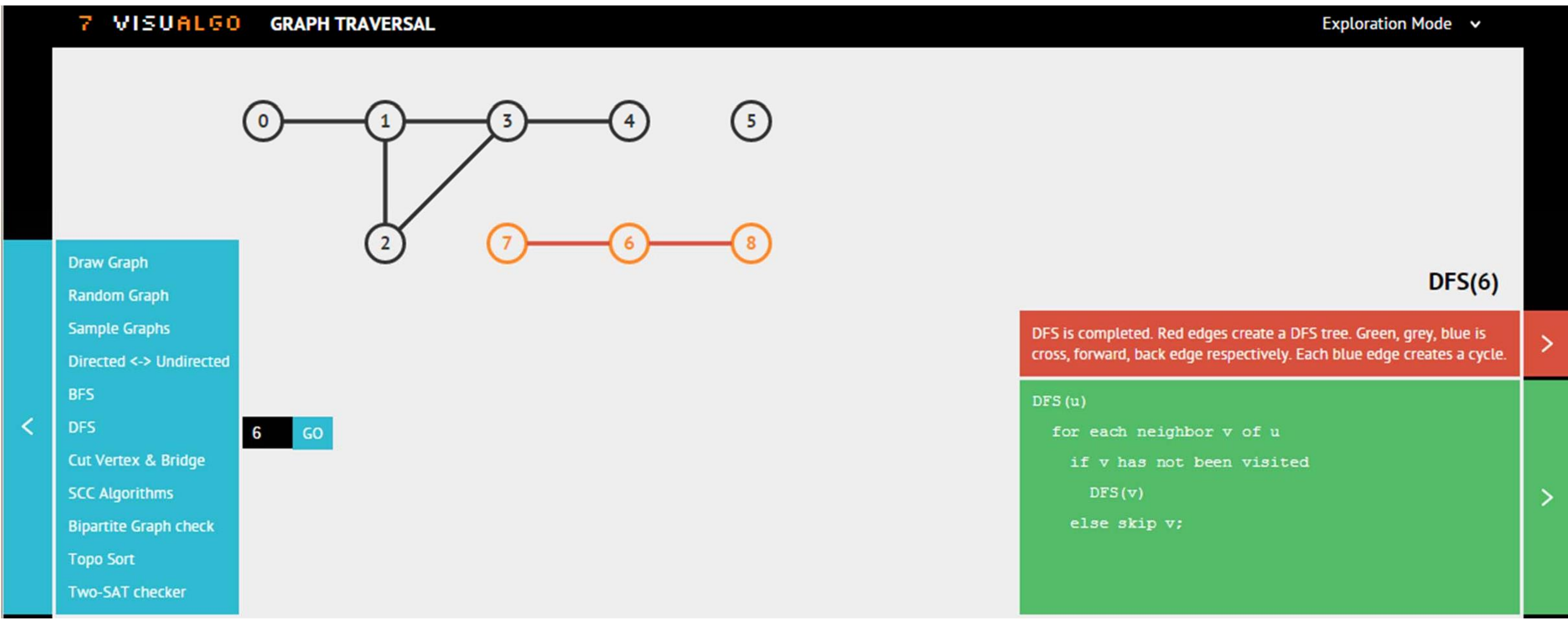
```
CC ← 0
for all v in V
  visited[v] ← 0
for all v in V // O(V)?
  if visited[v] == 0
    CC ← CC + 1
    DFSrec(v)//O(V+E)?
    // BFS from v
    // is also OK
```



Reachability Test

Ask VisuAlgo to perform various DFS (or BFS) operations on the sample Graph (CP3 4.1, Undirected)

Call **DFS(0)/BFS(0)**, **DFS(5)/BFS(5)**, then **DFS(6)/BFS(6)**

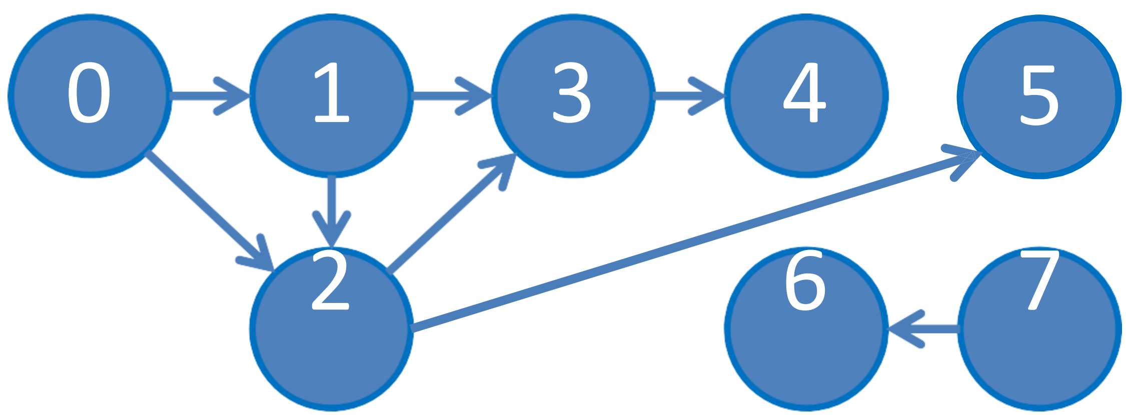


What is the time complexity for "counting connected component"?

1. Hm… you can call O(**V**+**E**) DFS/BFS up to **V** times…
   I think it is O(**V***(**V**+**E**)) = O(**V^2** + **VE**)
2. It is O(**V**+**E**)...
3. Maybe some other time complexity, it is O(_______)

What can we do with BFS/DFS? (3)

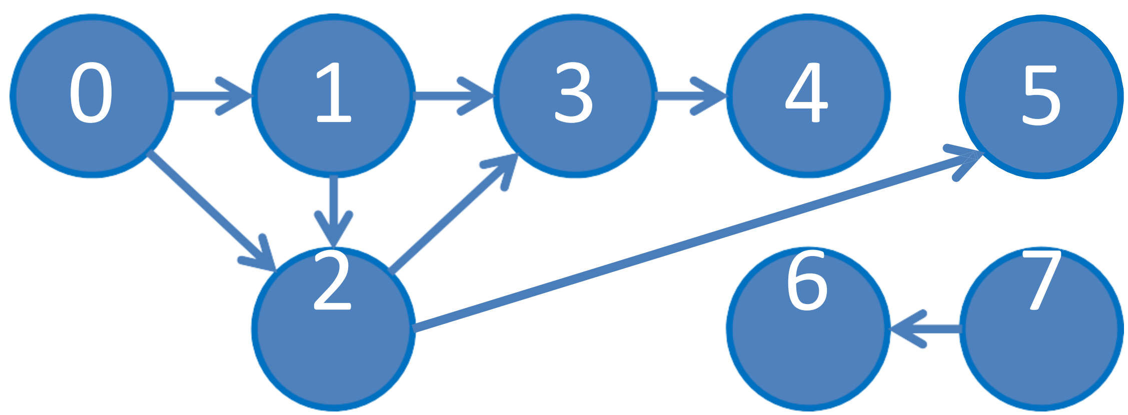
- Topological Sort
  - Topological sort of a DAG is a linear ordering of its vertices in which each vertex comes before all vertices to which it has outbound edges
  - Every DAG has one *or more* topological sorts
  - One of the main purpose of finding topological sort: for Dynamic Programming (DP) on DAG (will be discussed a few weeks later…)



Reminder to myself:
slow down here

What can we do with BFS/DFS? (4)

- Topological Sort
  - If the graph is a DAG, then simply running **DFS** on it (and at the same time record the vertices in "post-order" manner) will give us one valid topological order
    - "Post-order" = process vertex **u** after all children of **u** have been visited
  - See pseudo code in the next slide



DFS for TopoSort – Pseudo Code

Simply look at the codes in red/underlined

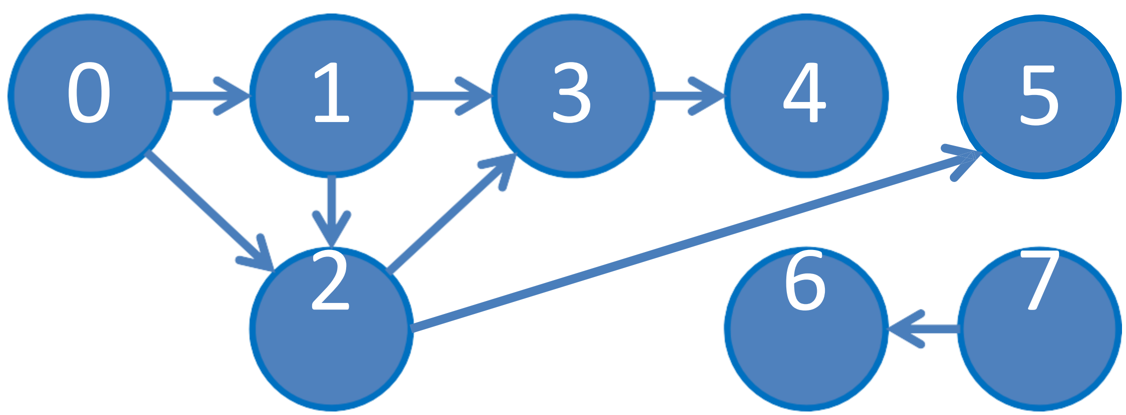
```
DFSrec(u)
  visited[u]  ←1 // to avoid cycle
  for all v adjacent to u // order of neighbor
    if visited[v] =  0 // influences DFS
      p[v]  ←u // visitation sequence
      DFSrec(v) // recursive (implicit stack)
  append u to the back of toposort // "post-order"

// in the main method
for all v in V
  visited[v]  ←0
  p[v]  ←-1
clear toposort
for all v in V
  if visited[v] == 0
    DFSrec(s) // start the recursive call from s
reverse toposort and output it
```

toposort is a kind of List (Vector)

What can we do with BFS/DFS? (5)

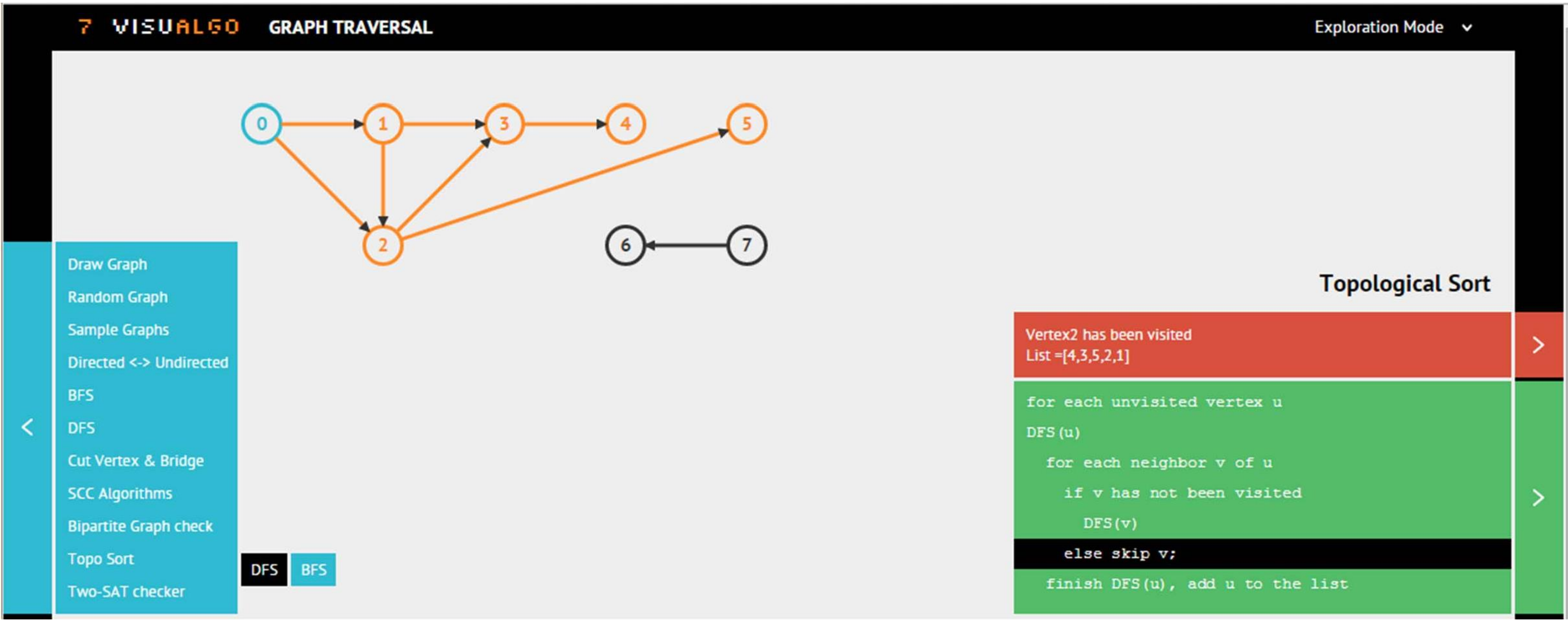
- Topological Sort
  - Suppose we have visited all neighbors of 0 recursively with DFS
  - toposort list = [list of vertices reachable from 0] - vertex 0
    - Suppose we have visited all neighbors of 1 recursively with DFS
    - toposort list = [[list of vertices reachable from 1] - vertex 1] - vertex 0
    - and so on…
  - We will eventually have = [<u>4, 3, 5, 2, 1, 0</u>, 6, 7]
  - Reversing it, we will have = [7, 6, 0, 1, 2, 5, 3, 4]



Topological Sort

Ask VisuAlgo to perform Topo Sort (DFS) operation on the sample Graph (CP3 4.4, Directed)

Below, we show partial execution of the DFS variant



Trade-Off

O(V+E) DFS

- Pros:
  - Slightly easier? to code (this one depends)
  - Use less memory
  - Has some extra features (not in CS2010 syllabus but useful for your PS3)
- Cons:
  - Cannot solve SSSP on unweighted graphs

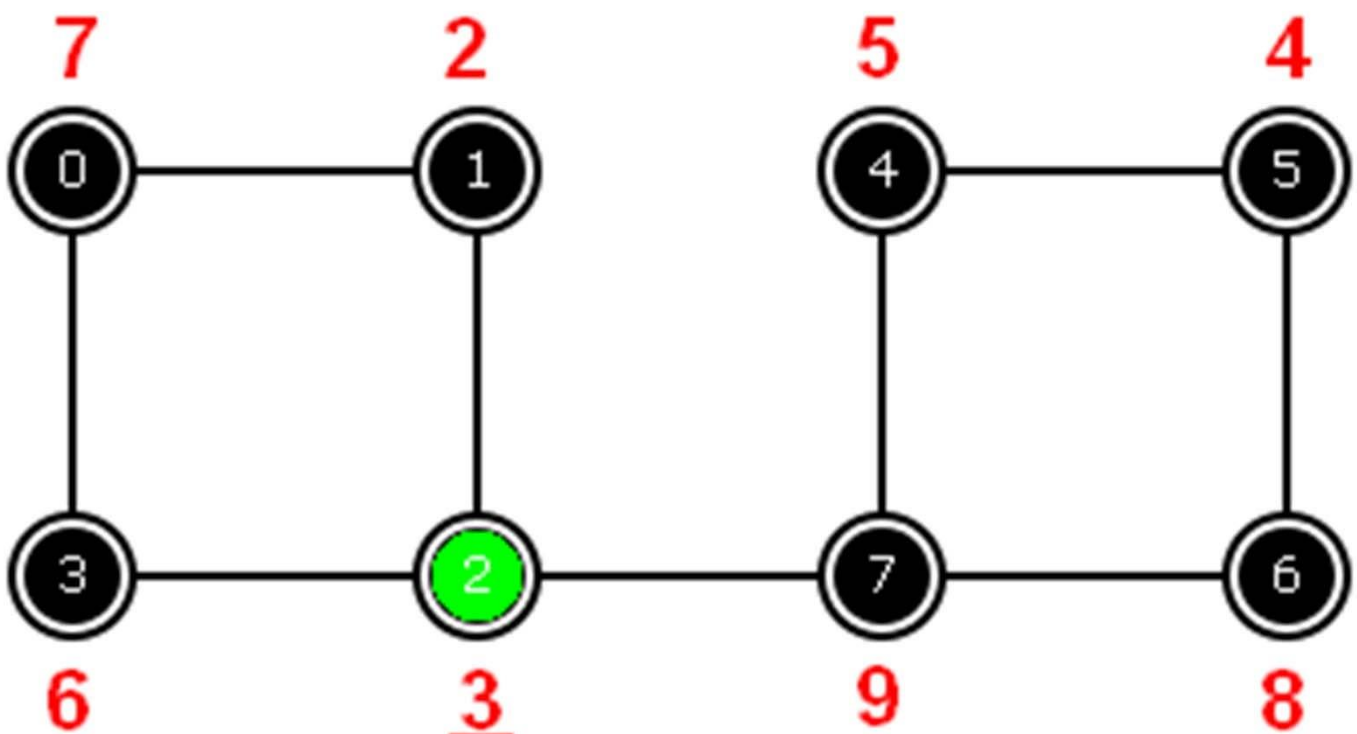
O(V+E) BFS

- Pros:
  - Can solve SSSP on unweighted graphs (revisited in latter lectures)
- Cons:
  - Slightly longer? to code (this one depends)
  - Use more memory (especially for the queue)

Hospital Tour Problem (PS3)

Given a layout of a hospital…

- Determine which room(s) is/are the 'important room(s)'
- Among those room(s), pick one with the lowest rating score



Online Quiz 1 (Tomorrow)

(Thu, 17 Sep 2015, during your lab session)

Try OQ1 Preview (test ID: 31) if you have not done so

http://visualgo.net**/test.html**

You can always challenge yourself more with this:

http://visualgo.net/training.html?diff=Hard&n=20&tl=40&
module=heap,bst,avl,ufds,bitmask,graphds

Written Quiz 1 (This Saturday)

(Sat, 19 Sep 2015, LT19, SR@LT19, TR9)

3 Sections only, 90 minutes:

- Most basic questions about Binary Heap/BST/AVL/UFDS/ Bitmask/Graph Data Structures have been <u>automated in the Online Quiz 1</u>
- So this one is definitely (much) harder than Online Quiz 1…
  - Disclaimer: Doing well in OQ1 may not correlate with doing well in WQ1

Material:

- Lecture 1-2-3-4-5, Tutorial 1-2-3-4, Lab Demos 1-2-3-4, PS1-2
  - IMPORTANT: UFDS, bitmask, and Graph DSes are included
  - Lecture 06 (DFS/BFS) is excluded
- CP3: page 36-54 ☺

Summary

In this lecture, we have looked at:

- Some applications of Graph Data Structures
  - Continuation from Lecture 05
- Graph Traversal Algorithms: Start + Movement
  - Breadth-First Search: uses queue, breadth-first
  - Depth-First Search: uses stack/recursion, depth-first
  - Both BFS/DFS uses “flag” technique to avoid cycling
  - Both BFS/DFS generates BFS/DFS “Spanning Tree”
  - Some applications: Reachability, CC, Toposort