

B38EM – Tutorial 1 (Week 2)

- 1) A triangle has vertices located at points $A(3, 1, -10)$, $B(1, 7, -10)$, and $C(10, 6, -10)$ [units of meters]. Find the angle associated with vertex C , the area of the triangle, and the perimeter of the triangle.

- 2) **Surface Area in Spherical Coordinates:** The spherical strip shown in the Figure is a section of a sphere of radius 3 cm. Find the area of the strip.

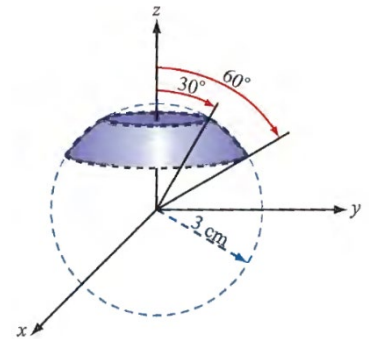


Table 3-2: Coordinate transformation relations.

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{r} = \hat{x} \cos \phi + \hat{y} \sin \phi$ $\hat{\phi} = -\hat{x} \sin \phi + \hat{y} \cos \phi$ $\hat{z} = \hat{z}$	$A_r = A_x \cos \phi + A_y \sin \phi$ $A_\phi = -A_x \sin \phi + A_y \cos \phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r \cos \phi$ $y = r \sin \phi$ $z = z$	$\hat{x} = \hat{r} \cos \phi - \hat{\phi} \sin \phi$ $\hat{y} = \hat{r} \sin \phi + \hat{\phi} \cos \phi$ $\hat{z} = \hat{z}$	$A_x = A_r \cos \phi - A_\phi \sin \phi$ $A_y = A_r \sin \phi + A_\phi \cos \phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}[\sqrt{x^2 + y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{R} = \hat{x} \sin \theta \cos \phi + \hat{y} \sin \theta \sin \phi + \hat{z} \cos \theta$ $\hat{\theta} = \hat{x} \cos \theta \cos \phi + \hat{y} \cos \theta \sin \phi - \hat{z} \sin \theta$ $\hat{\phi} = -\hat{x} \sin \phi + \hat{y} \cos \phi$	$A_R = A_x \sin \theta \cos \phi + A_y \sin \theta \sin \phi + A_z \cos \theta$ $A_\theta = A_x \cos \theta \cos \phi + A_y \cos \theta \sin \phi - A_z \sin \theta$ $A_\phi = -A_x \sin \phi + A_y \cos \phi$
Spherical to Cartesian	$x = R \sin \theta \cos \phi$ $y = R \sin \theta \sin \phi$ $z = R \cos \theta$	$\hat{x} = \hat{R} \sin \theta \cos \phi + \hat{\theta} \cos \theta \cos \phi - \hat{\phi} \sin \phi$ $\hat{y} = \hat{R} \sin \theta \sin \phi + \hat{\theta} \cos \theta \sin \phi + \hat{\phi} \cos \phi$ $\hat{z} = \hat{R} \cos \theta - \hat{\theta} \sin \theta$	$A_x = A_R \sin \theta \cos \phi + A_\theta \cos \theta \cos \phi - A_\phi \sin \phi$ $A_y = A_R \sin \theta \sin \phi + A_\theta \cos \theta \sin \phi + A_\phi \cos \phi$ $A_z = A_R \cos \theta - A_\theta \sin \theta$
Cylindrical to spherical	$R = \sqrt{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{R} = \hat{r} \sin \theta + \hat{z} \cos \theta$ $\hat{\theta} = \hat{r} \cos \theta - \hat{z} \sin \theta$ $\hat{\phi} = \hat{\phi}$	$A_R = A_r \sin \theta + A_z \cos \theta$ $A_\theta = A_r \cos \theta - A_z \sin \theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R \sin \theta$ $\phi = \phi$ $z = R \cos \theta$	$\hat{r} = \hat{R} \sin \theta + \hat{\theta} \cos \theta$ $\hat{\phi} = \hat{\phi}$ $\hat{z} = \hat{R} \cos \theta - \hat{\theta} \sin \theta$	$A_r = A_R \sin \theta + A_\theta \cos \theta$ $A_\phi = A_\phi$ $A_z = A_R \cos \theta - A_\theta \sin \theta$

- 3) **Cartesian to Cylindrical Transformations:** Given point $P_1 = (3, -4, 3)$ and vector $\mathbf{A} = \hat{x}2 - \hat{y}3 + \hat{z}4$, defined in Cartesian coordinates, express P_1 and \mathbf{A} in cylindrical coordinates and evaluate \mathbf{A} at P_1 .

- 4) **Gradient of scalar function:** Find the gradient of the following scalar function and then evaluate it at the given point.

$$V_1 = 24x^2 - 3y + z \quad \text{at } (1, 2, 3) \text{ in Cartesian coordinates.}$$

- 5) **Calculating the Divergence:** Determine the divergence of the vector field and then evaluate it at the indicated point:

$$\mathbf{P} = x^2yz \mathbf{a}_x + xz \mathbf{a}_z \quad \text{at point: } (3, 2, 4)$$

- 6) **Calculating the Curl:** Determine the curl of the vector field and then evaluate it at the indicated point:

$$\mathbf{P} = x^2yz \mathbf{a}_x + xz \mathbf{a}_z \quad \text{at } (5, 5, 1)$$