B38EM Introduction to Electricity and Magnetism

Lecture 4

Electrostatics (part 2)

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Topics

- Conservative Field
- Work of a Force
- Potential & Potential Difference
- Equipotentials
- Relation between Voltage and Electric Field
- Conductors in Electrostatic Fields
- Potential due to Multiple Charges

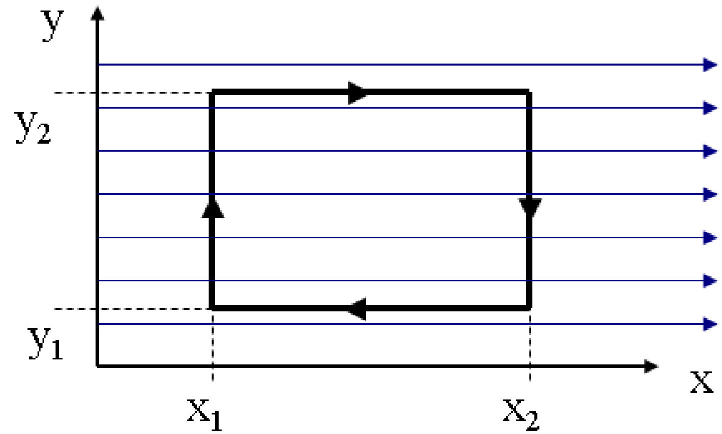
References & Resources

- Elements of Electromagnetics (7th Edition), by Sadiku, Oxford University Press
- Fundamentals of Applied Electromagnetics (7th Edition), by Ulaby and Ravaioli
- Field and Wave Electromagnetics (2nd Edition), by David K. Cheng
- …

Conservative Field

Conservative Vector Field

$$\oint_l \vec{A}.d\vec{l} = 0$$



Vector field

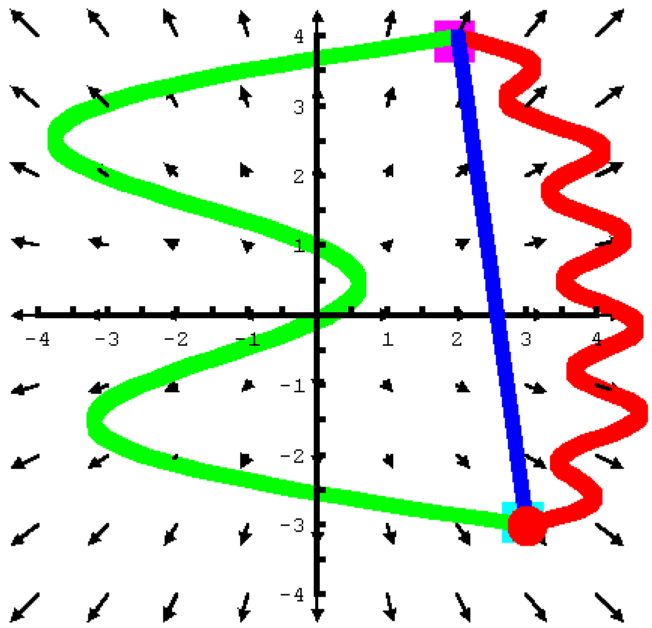
$$\vec{A} = A_1 \vec{a}_x$$

- A vector field with zero circulation is said to be conservative.
- Conservative refers to how energy is conserved around the integral path.
- A zero-curl field can also be described as irrotational.

All electrostatic fields are conservative as with gravitational fields. $\oint_l \vec{E}.d\vec{l} = 0$

Conservative Field

The work required to move a specimen (charge) from one point to another ***is independent of the choice of path connecting the two points***



Electrostatic (like gravitational) field is conservative.

Work of a Force

When a constant force, $\vec{F}$, is applied on an object that moves along distance, $\vec{l}$, then the work, *W*, done by the force is

$$W = \vec{F} \cdot \vec{l} = F \cdot l \cdot \cos\theta$$

When the force, $\vec{F}$, is not constant then the above does not apply.

We can assume an infinitely short distance, $\Delta\vec{l}$, so that the force over this distance is constant.

The associated work ΔW then is

$$\Delta W = \vec{F} \cdot \Delta\vec{l} = F \cdot \Delta l \cdot \cos\theta$$

Work of a Force

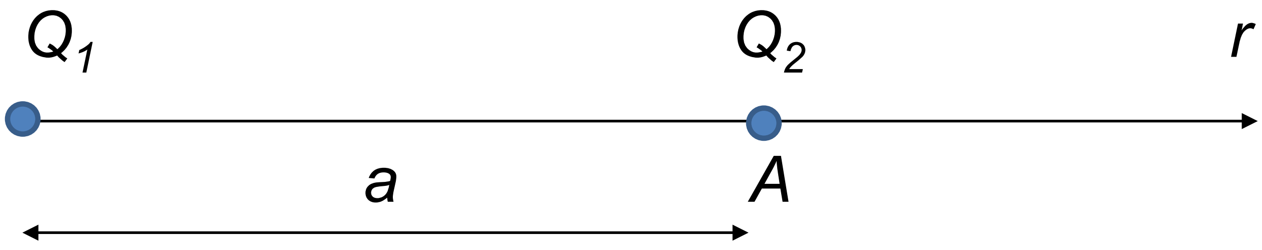
The total work done over the entire distance, $\vec{l}$, is then

$$W = \lim_{\Delta l \to 0} \sum \vec{F} \cdot \Delta \vec{l} = \int \vec{F} \cdot d\vec{l}$$

When charges move in an electric field, work is being spent or released due to the applied electric force.

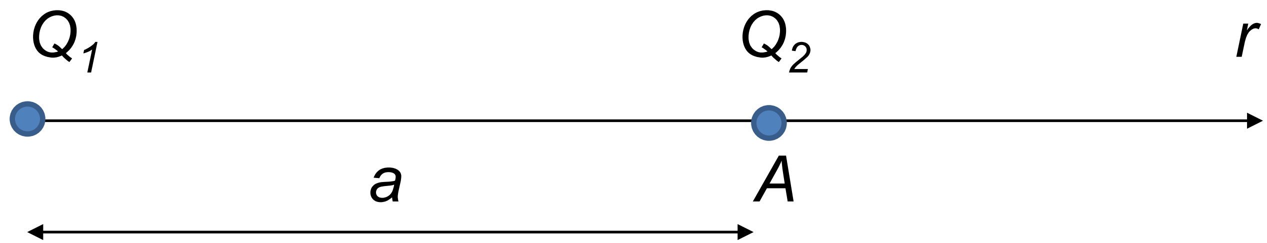
Electric Potential Energy

An assembly of electric charges possesses potential energy. This may be obtained by considering the work (energy) required to assemble the system.



Consider the energy required to bring Q_2 from infinity to A.

Electric Potential Energy



Consider the energy required to bring Q_2 from infinity to A.

$$work\,done = \int_a^\infty \frac{Q_1 \times Q_2}{4\pi\varepsilon_0 r^2} dr$$

$$= \int_a^\infty (force\ on\ Q_2\ at\ r).dr$$

$$= \int_a^\infty E Q_2 dr$$

$$= \frac{Q_1 \times Q_2}{4\pi\varepsilon_0 a}$$

$$work\,done = \frac{Q_1 \times Q_2}{4\pi\varepsilon_0 a}$$

Electric Potential, V

We may also define the potential energy per unit charge, or ***potential***, at a point in the field. Thus

$$work\,done = \int_a^\infty \frac{Q_1 \times 1}{4\pi\varepsilon_0 r^2} dr$$

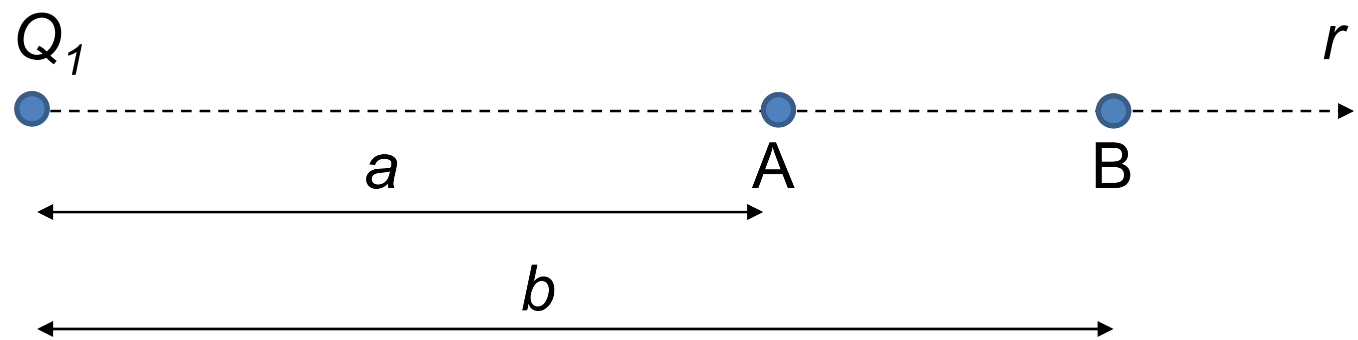
$$= \int_a^\infty (force\ on\ 1\ C\ at\ r).dr$$

$$= \int_a^\infty E.dr$$

$$= \frac{Q_1}{4\pi\varepsilon_0 a}$$

The units of potential are joules per coulomb or *volts* (V).

Potential Difference, ΔV



We may define the potential difference ΔV in moving from B to A:

$$\Delta V_{AB} = \frac{Q_1}{4\pi\varepsilon_0 a} - \frac{Q_1}{4\pi\varepsilon_0 b} = \frac{Q_1}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$$

ΔV_{AB} is the voltage (or potential) of A with respect to B.

Definition of Volt (V)

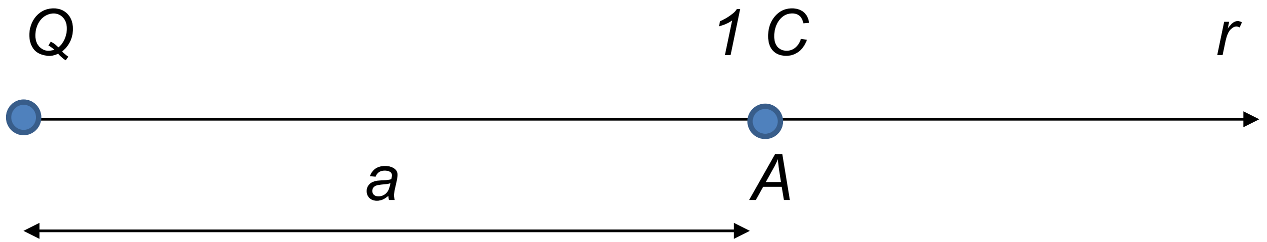
A charge of one coulomb (1 C) receives or delivers an energy of one joule (1 J) when it moves through a voltage of one volt (1 V).

Note: Electric Potential Energy (symbol W, units Joules) is different from Electric Potential (symbol V, units Joules/Coulomb or Volts).

Electric potential is Electric Potential Energy per Coulomb.

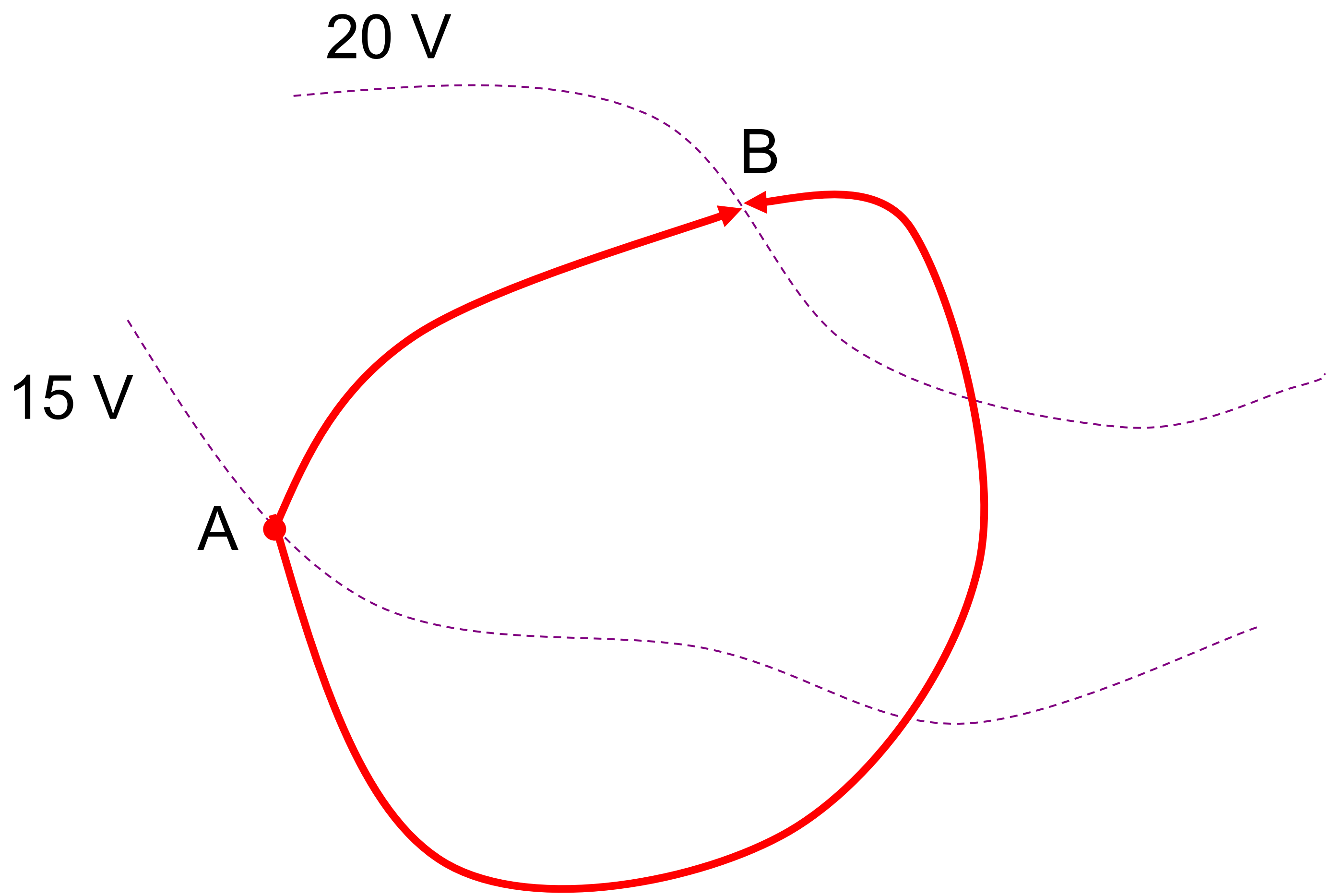
Equipotentials

Electric potential is defined as the energy required to transfer one unit of charge (1 C) from infinity (ground potential) to the point of interest.



For example, the energy to bring 1 C to A is $\dfrac{Q}{4\pi\varepsilon_0 a}$

Equipotentials



Moving a charge of x C from A to B requires 5x Joules *regardless of the path*.

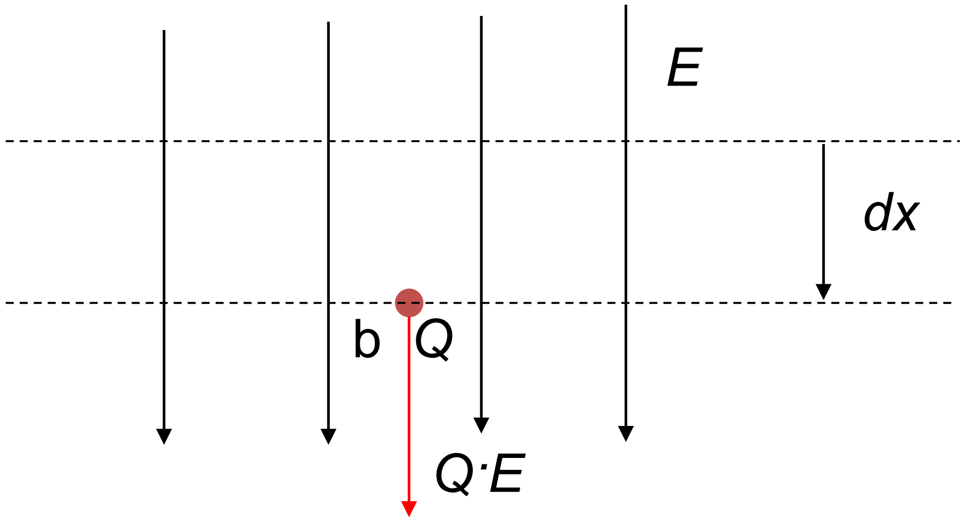
Field Lines & Equipotentials

- In the example of the point charge, the field lines and the equipotentials are normal (at right angles) to each other
- This is *always* the case
- Why?

Movement along an equipotential surface requires no work because such movement is always perpendicular to the electric field.

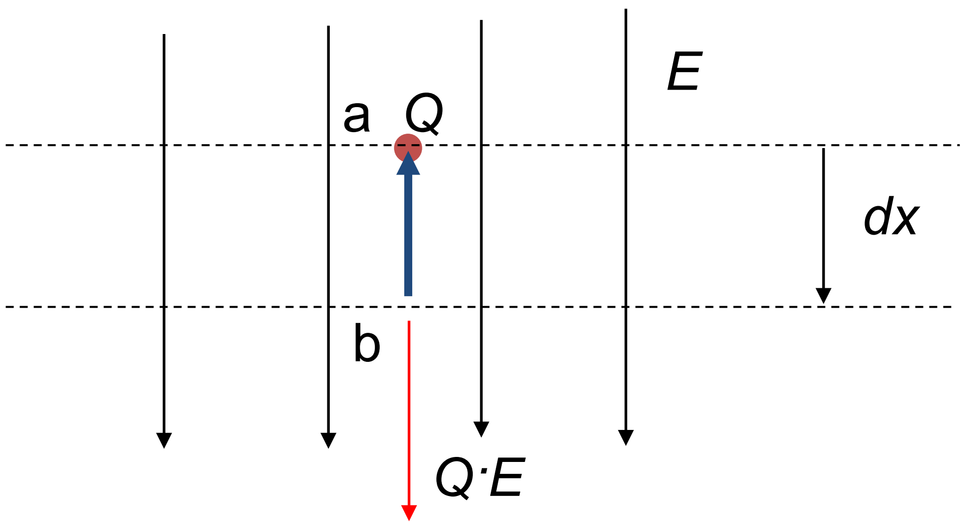
Electric Field & Voltage

Electric field strength *E* = force on unit charge



Electric Field & Voltage

Electric field strength E = force on unit charge



work done = gain in potential energy = $Q \cdot E \cdot dx$

gain in potential = $E \cdot dx = E\,(x_b - x_a)$

In general, $V_{AB} = \int_A^B \vec{E} \cdot d\vec{l}$ volts

Electric Field & Voltage

Since the voltage can be obtained integrating the electric field…

… the electric field can be obtained by differentiating the voltage:

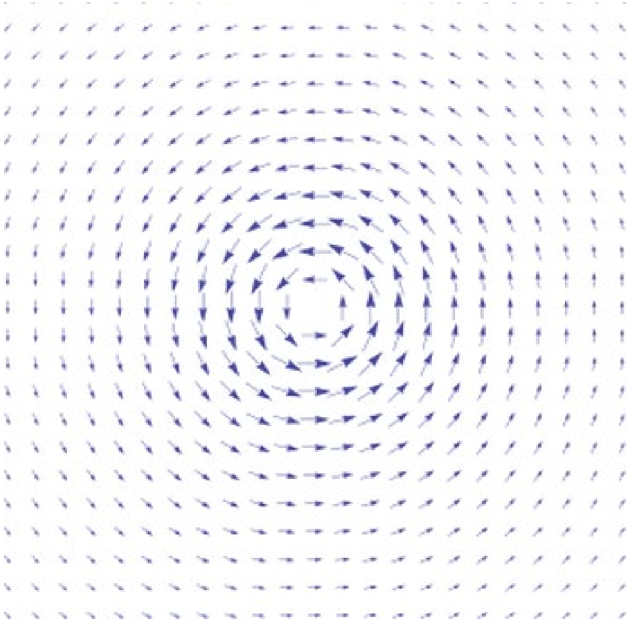
$$\vec{E} = \hat{x}\frac{dV}{dx} + \hat{y}\frac{dV}{dy} + \hat{z}\frac{dV}{dz} = \nabla V$$

Non-conservative Field

For conservative fields, the circulation is zero.

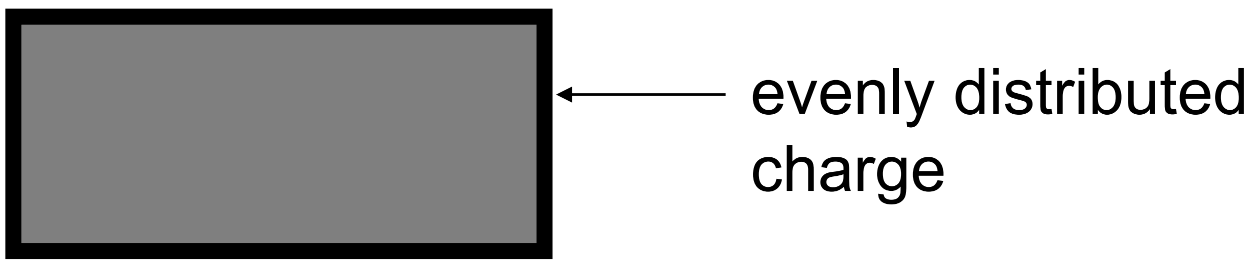
Fields for which the circulation is non-zero are non conservative.

Potential then cannot be defined at least as scalar as we need to indicate the direction of the path.



Conductors in Electrostatic Fields

Electric charge can move easily through a conductor. The charge will distribute itself on the conductor surface, because like charges repel.

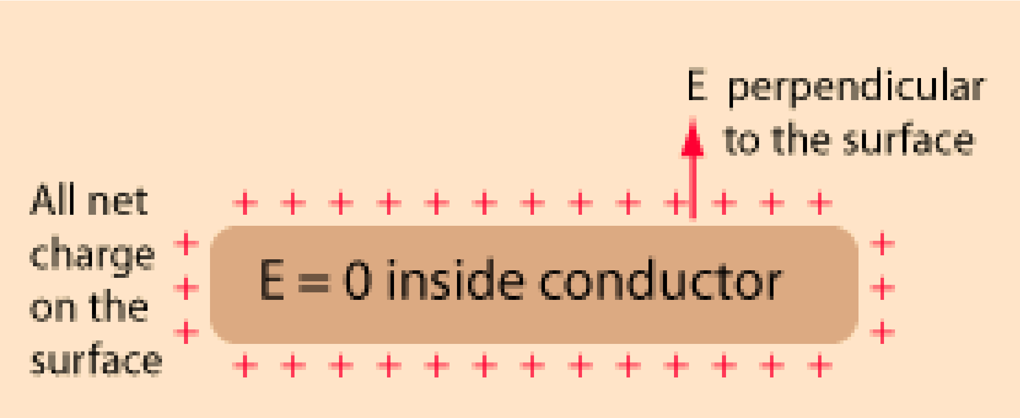


A field E in the conductor would cause a flow of charge. Because there is no flow in the *static* case, $E = 0$, and **the conductor is at a uniform potential**.

Conductors at Equilibrium

For a conductor at equilibrium:

1. The net electric charge of a conductor resides entirely on its surface.
2. The electric field inside the conductor is zero.
3. The external electric field at the surface of the conductor is perpendicular to that surface.
4. The potential on the conductor surface is constant



Conductors at Equilibrium

1. *Charges only at surface:* The mutual repulsion of like charges from Coulomb's Law demands that the charges be as far apart as possible, hence on the surface of the conductor.

2. *Electric field is zero inside:* Any net electric field in the conductor would cause charge to move since it is abundant and mobile (Also through Gauss law)

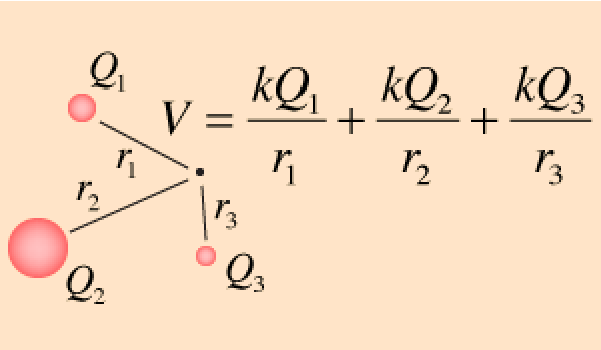
3. *Electric fields always normal to conductor surfaces*: otherwise it would exert a force parallel to the surface and produce charge motion.

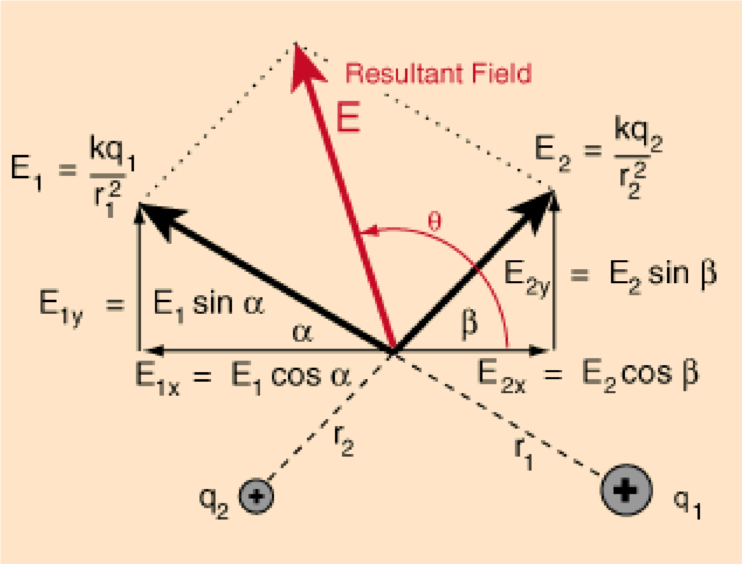
4. The potential is constant along the conductor surface (as a result of 3)

Potential due to Multiple Charges

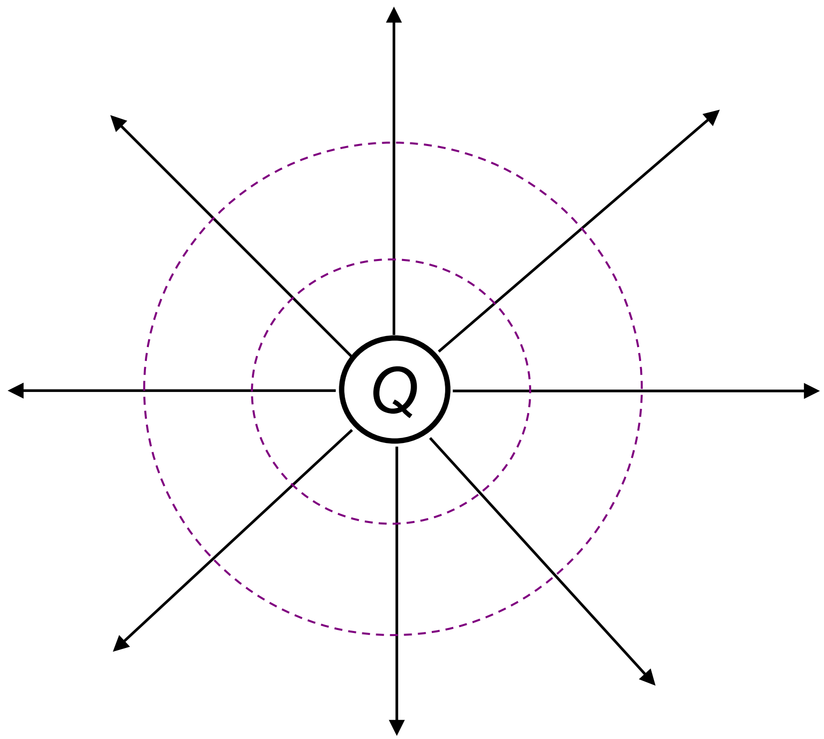
The electric potential produced by any number of point charges can be calculated from the point charge expression by simple addition since voltage is a scalar quantity.

Simpler than the vector sum required to calculate the electric field.

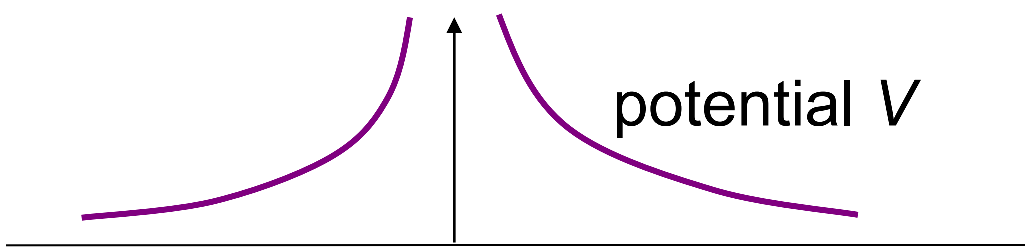




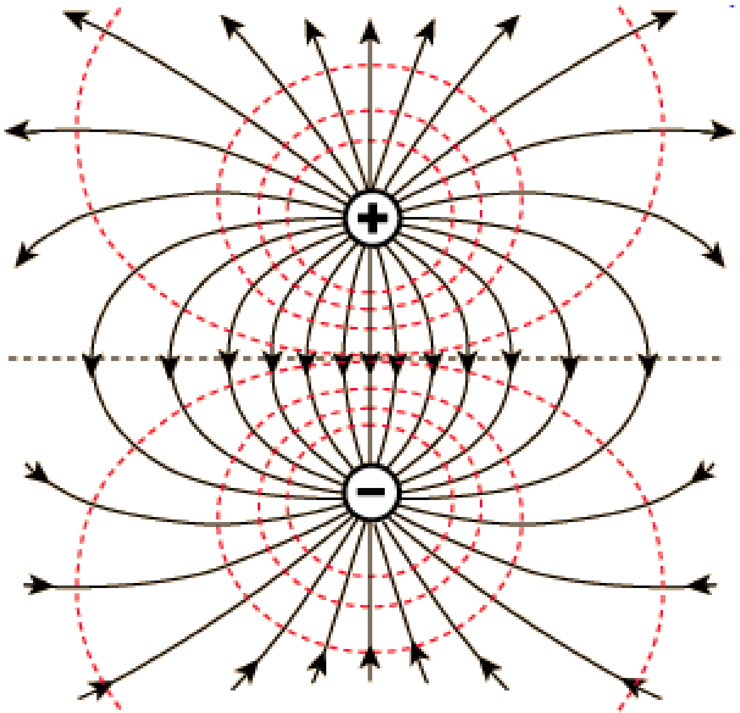
Point Charge

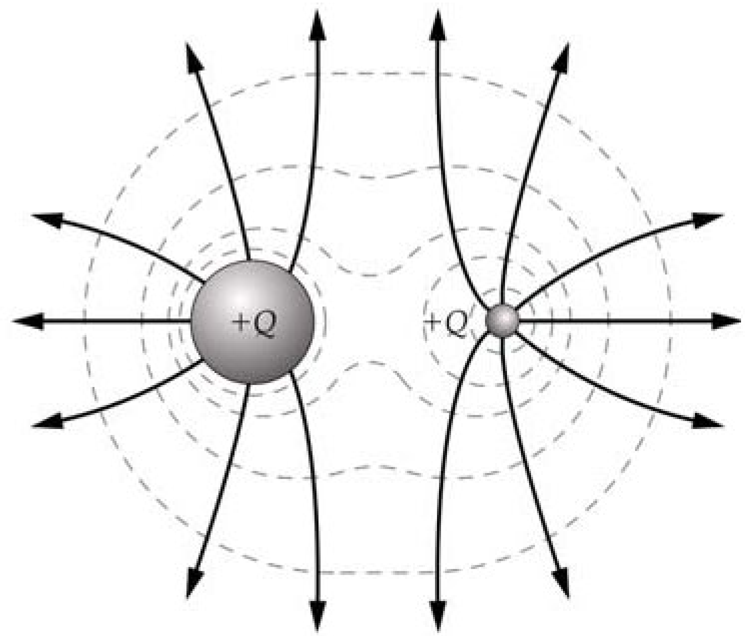


$$V = \frac{Q}{4\pi\varepsilon_0 a}$$



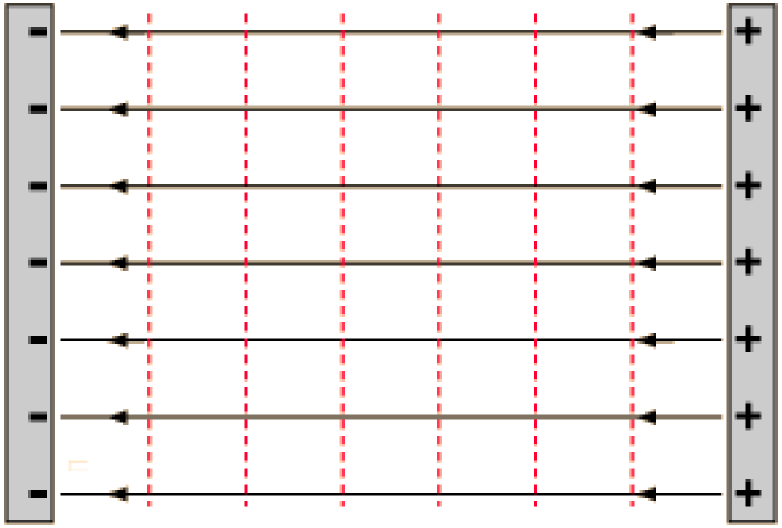
Two Point Charges





Parallel Plates

Uniform E-field



Electric field between two oppositely charged plates (assuming large surface and short separation) is uniform; therefore potential varies linearly.

Why Potential … ?

For conservative fields, the potential is an equivalent description to the electric field:

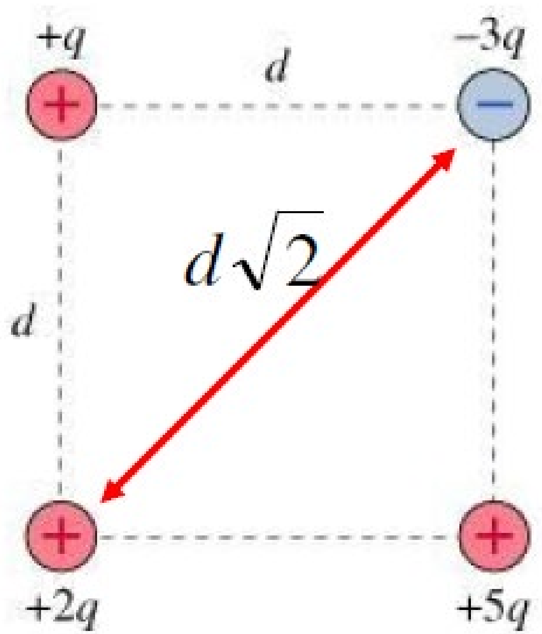
If we know $\vec{E}(x, y, z)$ we can derive $V(x, y, z)$ and vice versa.

BUT

It is scalar (rather than vector) and hence often easier to use.

Example

Assume 4 charges, $+q$, $+2q$, $+5q$, $-3q$ each at a corner of a square with side d, as shown. Find the potential at the centre of the square.



Example

Solution:

The distance from each corner to the centre is

$$r = \frac{d\sqrt{2}}{2}$$

Using the superposition principle we can write

$$V = \frac{+q}{4\pi\varepsilon_o r} + \frac{+2q}{4\pi\varepsilon_o r} + \frac{+5q}{4\pi\varepsilon_o r} + \frac{-3q}{4\pi\varepsilon_o r}$$

So that

$$V = \frac{+5q}{4\pi\varepsilon_o r} = \frac{+5q * 2}{4\pi\varepsilon_o d\sqrt{2}} = \frac{5\sqrt{2}q}{4\pi\varepsilon_o d}$$

Home exercise: Find the electric field at the centre of the square.