Electromagnetism module B38EM
Tutorial 6

Questions and solutions

ε_0 = 8.85x10^{-12} Fm^{-1}, e = 1.6x10^{-19} C, μ_0 = 4π 10^{-7} N/A^2

1.

A horizontal wire with a mass per unit length of 0.2 kg/m carries a current of 4 A in the $+x$-direction. If the wire is placed in a uniform magnetic flux density **B**, what should the direction and minimum magnitude of **B** be in order to magnetically lift the wire vertically upward?
(Hint: The acceleration due to gravity is $\mathbf{g} = -\hat{\mathbf{z}}9.8$ m/s^2.)

Solution: For a length l,

$$\mathbf{F}_g = -\hat{\mathbf{z}}0.2l \times 9.8 = -\hat{\mathbf{z}}1.96l \quad \text{(N)}$$
$$\mathbf{F}_m = \hat{\mathbf{x}}Il \times \mathbf{B}$$

For $\mathbf{F}_m + \mathbf{F}_g = 0$, $\mathbf{F}_m$ has to be along $+\hat{\mathbf{z}}$, which means that $\mathbf{B}$ has to be along $+\hat{\mathbf{y}}$. Hence,

$$1.96l = IlB$$
$$B = \frac{1.96}{I} = 0.49 \text{ (T), and}$$
$$\mathbf{B} = \hat{\mathbf{y}}0.49 \text{ (T)}.$$

2. A rectangular conducting rod of mass m and length L is placed on top of two conducting rails inclined at an angle of α from the horizontal, as in Fig. 2. If the resistance of the conducting rails changes according to $R=R_0 x^2$ and there is a magnetic field $B=B_0x$ directed upwards in the system. The rod slides down the rails due to the force of gravity with an increasing velocity v.

   a) Estimate the retarding force
   b) If the rod was at rest, estimate the time that it will take it to reach a velocity v_0

   Consider: B_0= 1 Tm^{-1}, m=2Kg, L=30cm, α=30°, R_0=3 Ωm^{-2}, v_0=10 ms^{-1}.

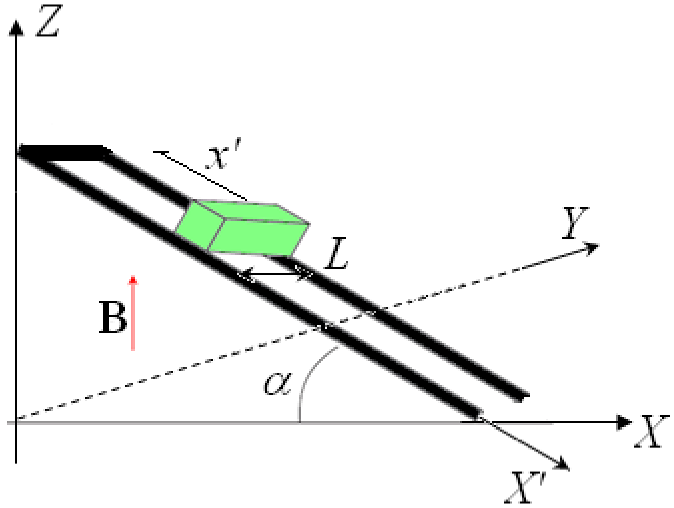
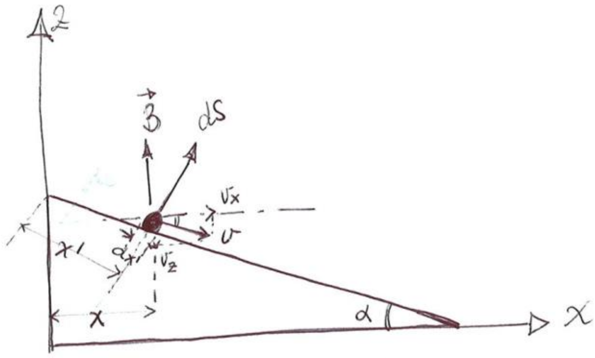


Fig. 2.

②



$$(\vec{v}\times\vec{B}) = \begin{cases} v_x . B . \sin 90^\circ = v_x . B \\ v_z . B \sin(180^\circ) = 0 \end{cases}$$

$$\varphi = \int \vec{B}\cdot d\vec{S} = \begin{cases} \vec{B} = B_0 \cdot x\,\hat{z} \\ d\vec{S} = ds\cos\alpha\,\hat{z} + ds\sin\alpha\,\hat{x} \\ \quad \hookrightarrow ds = L \cdot d \end{cases}$$

$$\varphi = \int B_0 \cdot \underset{\substack{\downarrow \\ x = x'\cdot\cos\alpha}}{x} \cdot ds \cdot \cos\alpha = \int B_0 \cdot x' \cdot \cos^2\alpha \cdot L\,dx' = B_0 \cdot L \cdot \frac{x'^2}{2}\Big|_0^{x'} \cos^2\alpha$$

$$\varphi = B_0 L \cdot \cos^2\alpha \cdot x'/2$$

$$F_m = i \cdot \int d\vec{l} \times \vec{B}$$

In order to calculate i we need the voltage induced:

$$\mathcal{E} = \left|\frac{d\varphi}{dt}\right| = \frac{d\left[B_0 \cdot L \cdot \cos^2\alpha \cdot x'^2/2\right]}{dt} = B_0 \cdot L \cdot \cos^2\alpha \cdot \frac{\cancel{1}}{\cancel{2}} \cdot \cancel{2}\,x' \overset{v}{\frac{dx'}{dt}}$$

Thus,

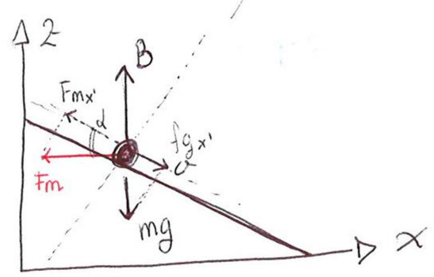
$$i = \frac{\mathcal{E}}{R} = \frac{B_0 \cdot L \cdot \cancel{x'} \cos^2\alpha}{R_0 \cdot x'^{\cancel{2}}} = \frac{B_0 \cdot L \cdot \cos^2\alpha \cdot v}{R_0\, x'}$$

In order to determine the direction of flow of the induced current consider that as x' ↑ the flux ↑ therefore should oppose this increase

$$F_m = i \int B \cdot dl \cdot \sin \overset{90^\circ}{\theta} = i \int_0^L B_0 \cdot \underset{x = x'\cos\alpha}{x} \cdot dl =$$

$$= \frac{B_0 \cdot L \cdot \cos^2\alpha \cdot v}{R_0 \cdot \cancel{x'}} \cdot B_0 \cdot \cancel{x'} \cos\alpha \cdot \int_0^L dl = \frac{(B_0 \cdot L)^2 \cdot \cos^3\alpha \, v}{R_0}$$

b)



$$\sum \vec{F} = m\vec{a}$$

$$\underbrace{mg\sin\alpha}_{f_{g_{x'}}} - \overbrace{F_m \cdot \cos\alpha}^{F_{mx'}} = m \cdot \frac{dv}{dt}$$

$$mg\sin\alpha - \frac{(B_0 \cdot L)^2}{R_0} \cos^4\alpha \cdot v = m \frac{dv}{dt}$$

$$\int_0^t dt = \int_0^{v_0} \frac{m R_0}{R_0 m g \sin\alpha - (B_0 L)^2 \cos^4\alpha \cdot v} \cdot dv$$

We Know that

$$\int \frac{c}{ax+b} = \frac{c}{a} \ln[ax+b] + C$$

Therefore

$$\int_0^{v_0} \frac{mR_0}{R_0 mg\sin\alpha - (B_0 L)^2 \cos^4\alpha\, v}\, dv = \frac{-mR_0}{(B_0 L)^2\cos^4\alpha} \cdot \ln\left\{R_0 mg\sin\alpha - (B_0 L)^2 \cos^4\alpha\, v\right\}\Big|_0^{v_0}$$

$$= \frac{-mR_0}{(B_0 L)^2\cos^4\alpha} \cdot \left[\ln\left\{R_0 mg\sin\alpha - (B_0 L)^2\cos^4\alpha\, v_0\right\} - \ln\left\{R_0 mg\sin\alpha\right\}\right]$$

$$= \frac{mR_0}{(B_0 L)^2\cos^4\alpha} \cdot \ln\left\{\frac{R_0 mg\sin\alpha}{R_0 mg\sin\alpha - (B_0 L)^2\cos^4\alpha\, v_0}\right\}$$

for $m = 2\,Kg$ $\quad L = 30\,cm$

$R_0 = 3\,\Omega\, m^{-2}$ $\quad v_0 = 10\, m/s$

$B_0 = 1\, T\, m^{-1}$

$\alpha = 30^\circ$

$$t = \frac{2\,(3)}{(1 \cdot 0{,}3)^2 \cos^4(30^\circ)} \cdot \ln\left\{\frac{3\cdot(2)\cdot 9{,}8 \cdot \sin(30)}{3\cdot(2)\cdot 9{,}8\cdot \sin(30) - (1\cdot 0{,}3)^2 \cos^4(30^\circ)\cdot 10}\right\}$$

$= 2{,}0586\ s$

3. Consider an infinitely large sheet of thickness b lying in the xy plane with a uniform current density $\mathbf{J}=J_0\hat{\mathbf{x}}$. Find the magnetic field everywhere.

$\vec{J} = J_0 \hat{y}$

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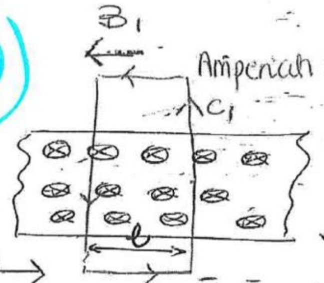
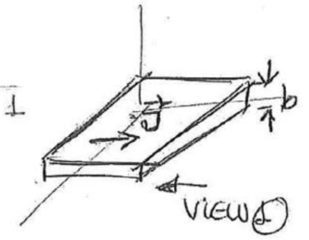
Applying Ampere's law

$\oint \bar{B} d\bar{l} = \mu_0 I_{enc}$



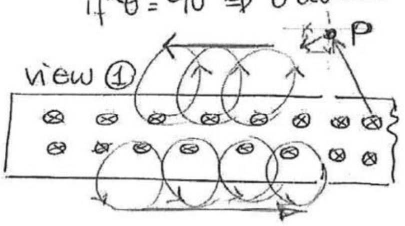


Only contribution from ① & ② because $\bar{B} d\bar{l} = |B||dl| \cdot \cos\theta$

if $\theta = 90° \Rightarrow \bar{B}\, d\bar{l} = 0$.

$I_{enc} = \iint \bar{J} \cdot d\bar{a} =$

$= J_0 \cdot b \times \ell$ 5



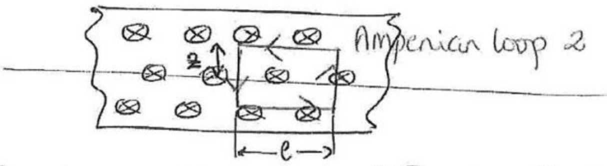
Therefore : Amperian loop 1

$\oint \bar{B}\, d\bar{l} = (J_0\, b \cdot \ell)\, \mu_0$

$B_1 \times \cancel{\ell} + B_2 \times \cancel{\ell} = (J_0 b \cancel{\ell})\, \mu_0$ 5

But $B_1 = B_2 \Rightarrow \boxed{B = \frac{J_0 b \mu_0}{2}}$

$$\vec{B} = \begin{cases} -\frac{\mu_0 J_0 b}{2}\hat{y} & z > \frac{b}{2} \\ -\mu_0 J z\, \hat{y} & -\frac{b}{2} < z < \frac{b}{2} \\ \frac{\mu_0 J_0 b}{2}\hat{y} & z < -\frac{b}{2} \end{cases}$$



Amperian loop 2 $\quad I_{enc} = \iint \bar{J}\, d\bar{a} = J_0 \cdot (2z \cdot \ell)$ 5

$\oint B dl = J_0 (2z\ell) \cdot \mu_0$

$B(2\ell) = J_0 \cancel{2} z \cancel{\ell} \mu_0 \Rightarrow \boxed{B = J_0 \mu_0 |z|}$ 5