B38EM Introduction to Electricity and Magnetism

Lecture 3

Electrostatics (part 1)

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Topics

- Atomic structure
- Charge conservation
- Conductors and insulators
- Coulomb law
- Electric field and field lines
- Electric flux density
- Permittivity of free space

References & Resources

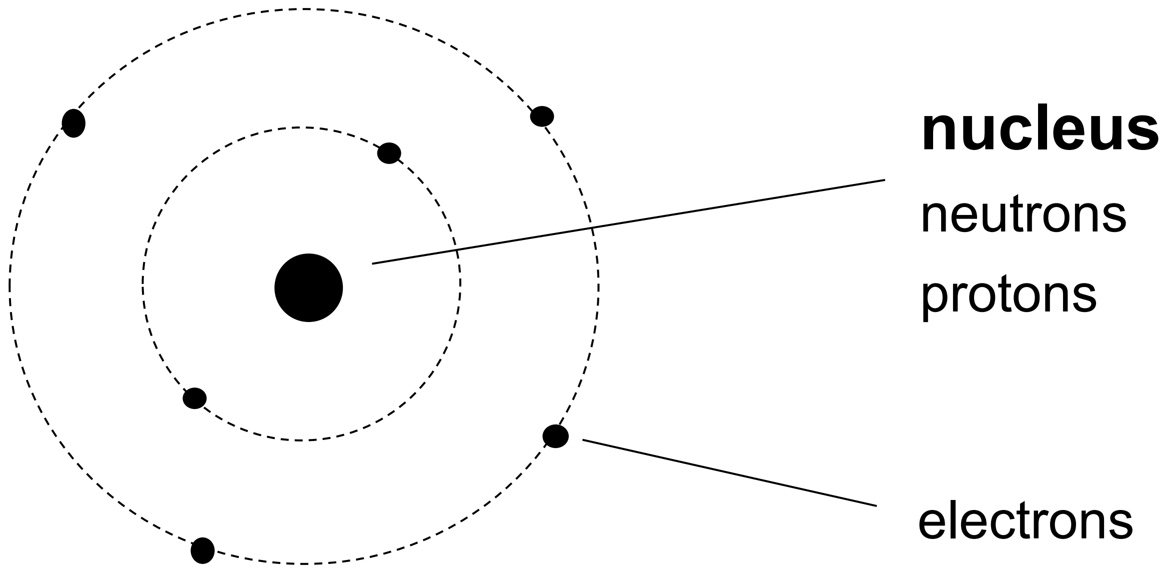
- Elements of Electromagnetics (7th Edition), by Sadiku, Oxford University Press

- Fundamentals of Applied Electromagnetics (7th Edition), by Ulaby and Ravaioli

- Field and Wave Electromagnetics (2nd Edition), by David K. Cheng

- …

Atomic Structure



“Charge” is a property of subatomic particles:

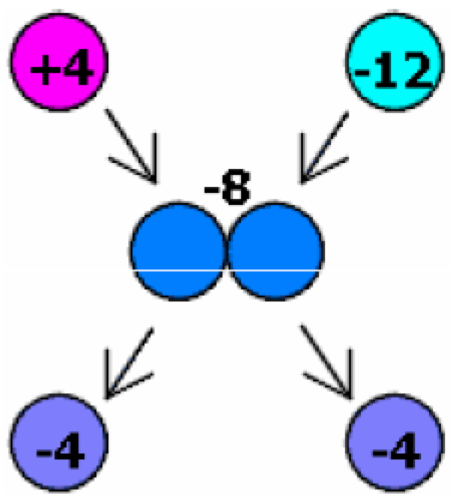
- There are 2 types
  positive (protons) and negative (electrons)

- Like charges repel and opposite charges attract

Atomic structure

Particle	Charge	Mass
Proton	1.6×10^{-19} C	1.67×10^{-27} kg
Electron	1.6×10^{-19} C	9.11×10^{-31} kg
Neutron	0	1.67×10^{-27} kg

Charge Conservation



Charge cannot be created or destroyed.

It can only be transferred from one object to another.

The NET charge stays the same.

Conductors and Insulators

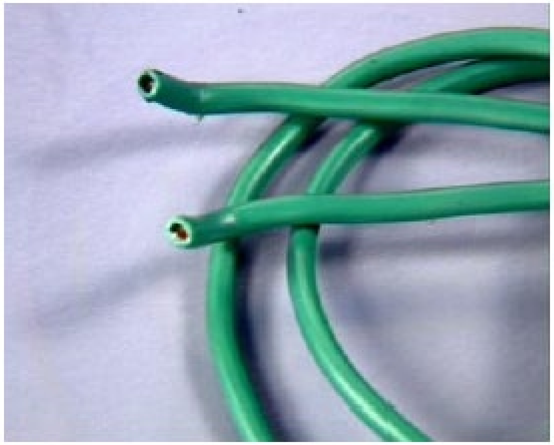
The movement of charge is limited by the material the charge is trying to pass through. There are generally two types of materials:

Conductors:

Allow charge to move (almost) freely though it.

Insulators:

Do not allow (almost) any movement of charge



Conductor = Copper Wire
Insulator = Plastic sheath

Forces Between Charges

- Newton's law works for neutrons, but not for electrons

- The forces between protons and electrons are much greater:
  - electron / electron repulsion
  - proton / proton repulsion
  - electron / proton attraction

- These large forces are due to ***electric charge.***

- We assume $+v_e$ charge for protons and $-v_e$ charge for electrons

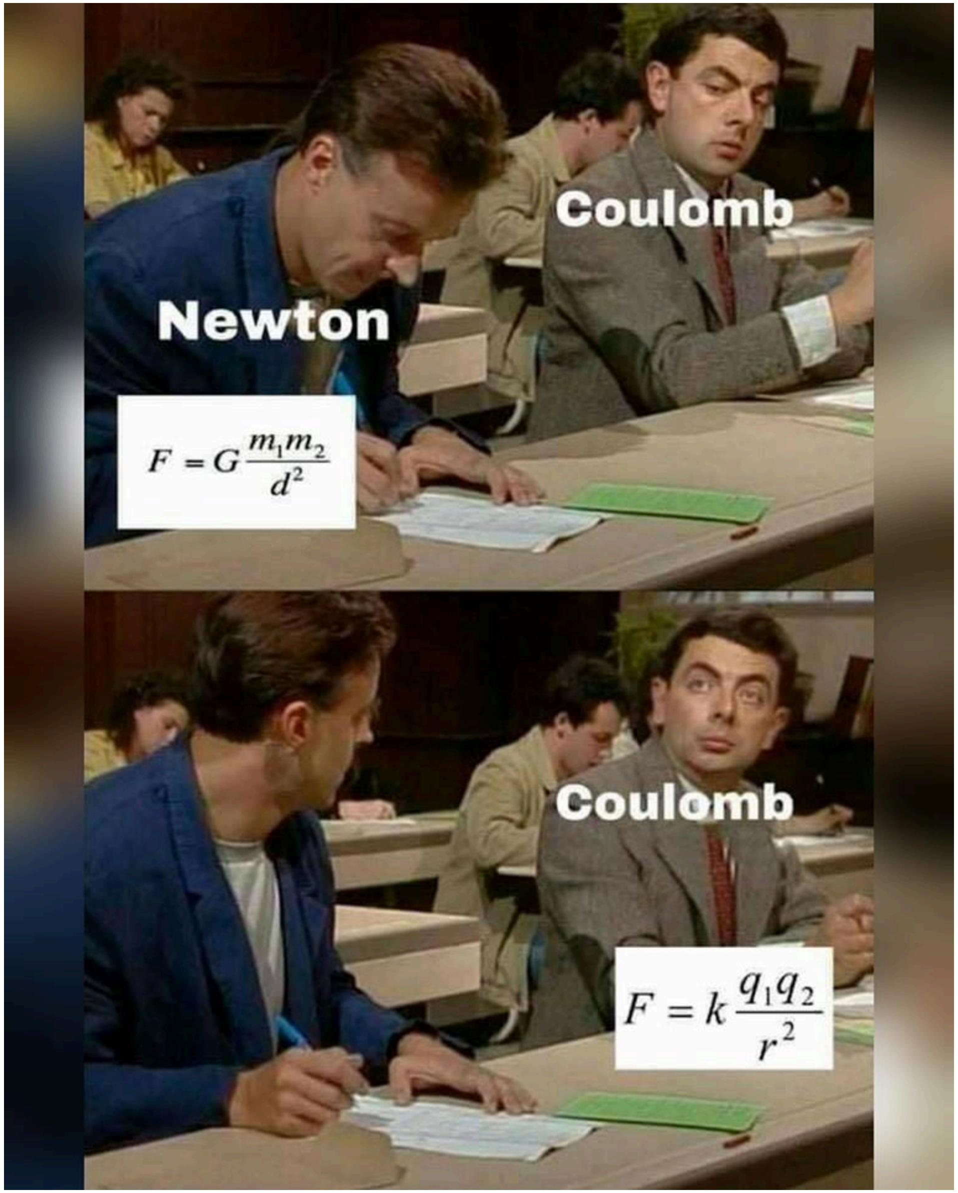
Newton’s Law of Gravitation

Newton’s law of gravitation states that there is a force of attraction between massive particles, inversely proportional to the square of the distance between them:

$$F_g = \lambda_g \frac{m_1 \times m_2}{r^2}$$

λ_g, the constant of gravitation, has the value 6.67×10^{-11}

Forces Between Charges



Forces Between Charges

Experiments show that the force between charges follows an inverse square law:

$$F_e = \lambda_e \frac{Q_1 \times Q_2}{r^2}$$

r: the distance between the charges

The direction of F_e:

like charges repel; unlike charges attract

Q is measured in coulombs (C)

λ_e, the electric constant, is 9×10^9

Coulomb's Law

The inverse square law for the force between charges was written previously as

$$F_e = \lambda_e \frac{Q_1 \times Q_2}{r^2}$$

With $\varepsilon_0 = 1 / 4\pi\lambda_e$ we may write this as

$$F_e = \frac{Q_1 \times Q_2}{4\pi\varepsilon_0 r^2}$$

This is known as ***Coulomb's Law***.

ε_0 is the ***primary electric constant*** or ***permittivity of free space***, given by

$$\varepsilon_0 = \frac{1}{4\pi\lambda_e} = 8.9 \times 10^{-12} \ \ (F/m)$$

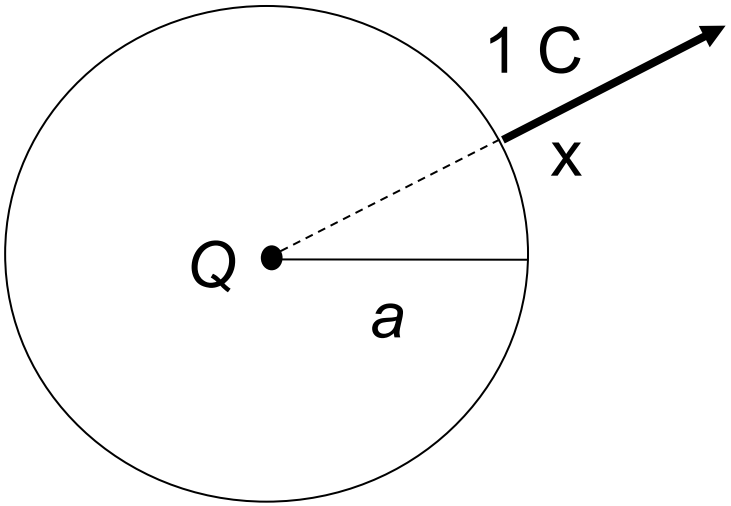
Electric Field

- The force between electric charges is defined in terms of magnitude and direction, i.e. it is a ***vector***.
- The electric field strength is just the force on a unit charge placed at the point of interest.

Electric Field

Electric Field Strength:

Consider a charge Q at the centre of a sphere:



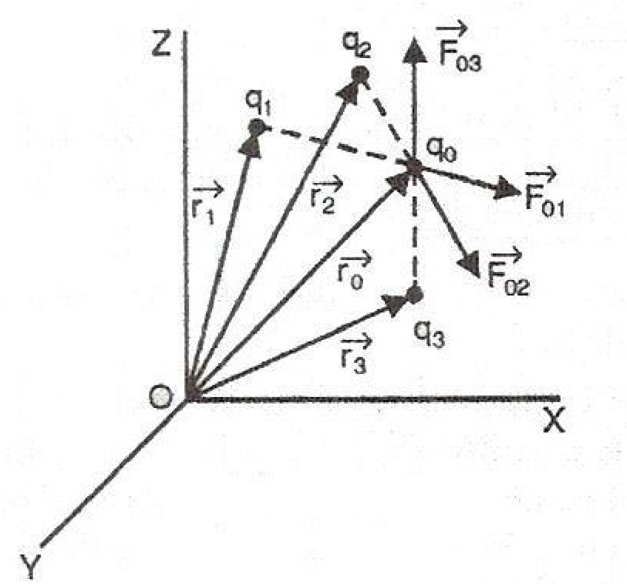
The force on a unit charge at x is $F_e = \lambda_e \dfrac{Q \times 1}{a^2}$

The force on 1 C is the ***electric field strength E***.

Electric Field

Superposition principle:

A field arising from a number of sources is determined by adding the individual fields from each source.



Electric Field

Example:

Two point charges in free space, Cartesian coordinate

$q_1 = 2 \cdot 10^{-5}$ C located at (1, 3, −1) in metre

$q_2 = -4 \cdot 10^{-5}$ C located at (−3, 1, −2) in metre

a) Find the $\vec{E}$-field at (3, 1, −2)

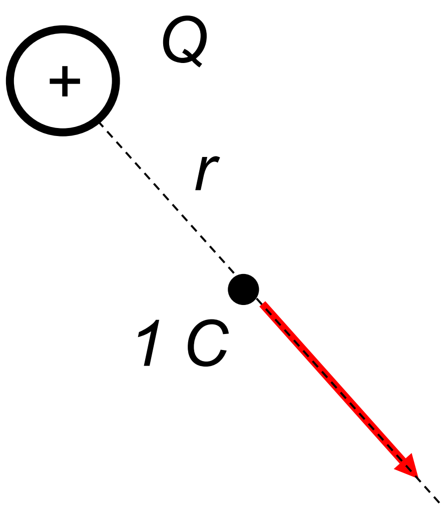
b) Find the force on $q_3 = 8 \cdot 10^{-5}$ C placed at (3, 1, −2)

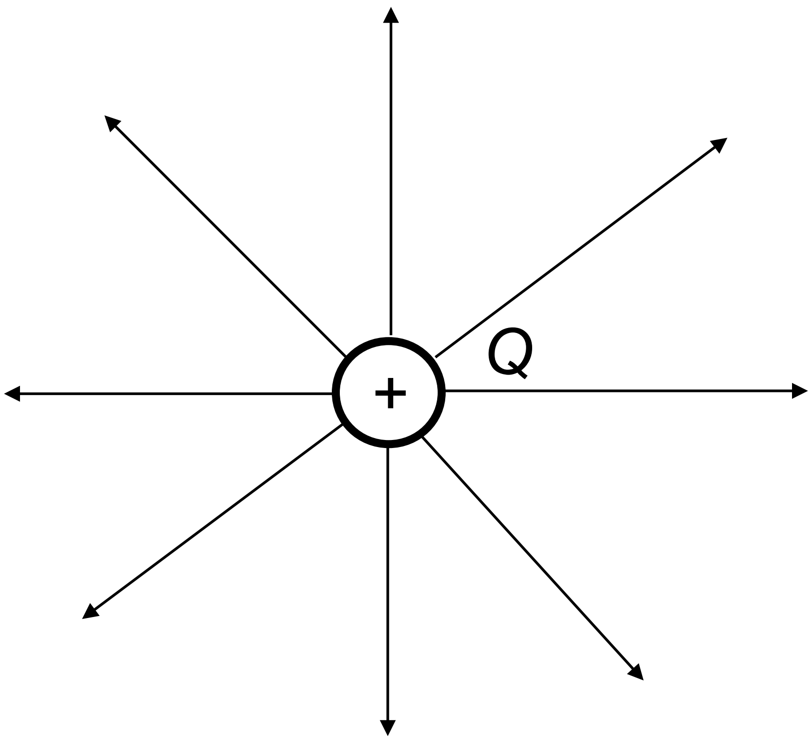
a) $\vec{E} = \frac{\hat{x} - 4\hat{y} - 2\hat{z}}{108\pi\varepsilon_0} \cdot 10^{-5} \quad V/m$

b) $\vec{F} = \frac{2\hat{x} - 8\hat{y} - 4\hat{z}}{27\pi\varepsilon_0} \cdot 10^{-10} \quad N$

Electric Field

Point Charge

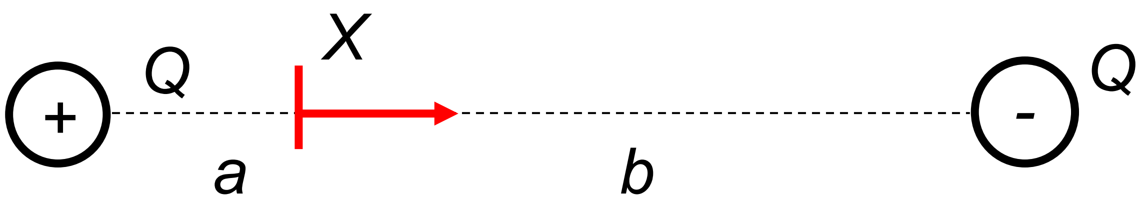




$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Electric Field

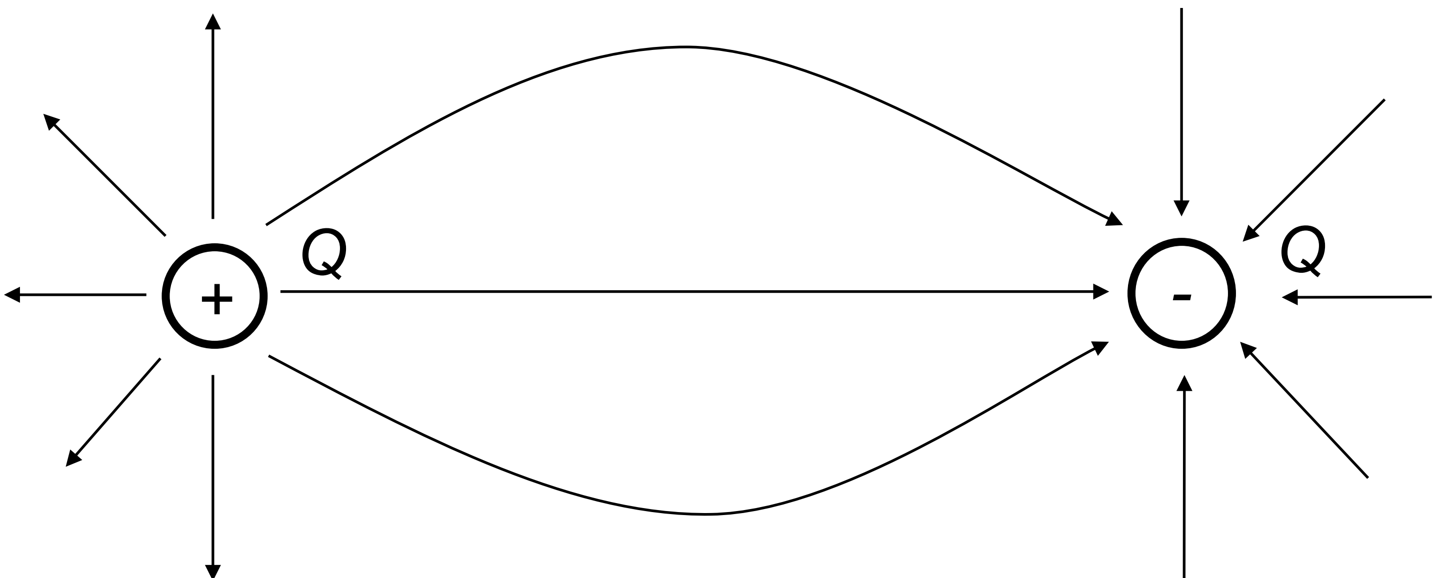
Two equal and opposite charges



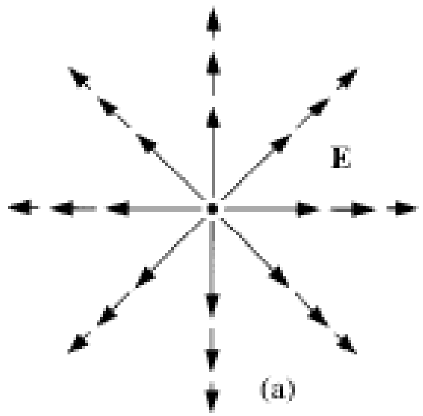
$$E_x = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a^2} + \frac{1}{b^2}\right)$$

Electric Field

Two equal and opposite charges

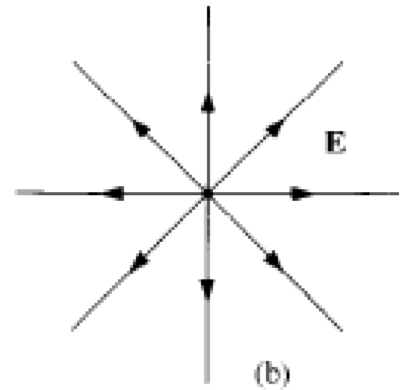


Field Lines



We visualize the *electric vector field* **E(r)** by sketching a few representative vectors.

Information about the **strength** of the field is contained in the **length of the arrows.**

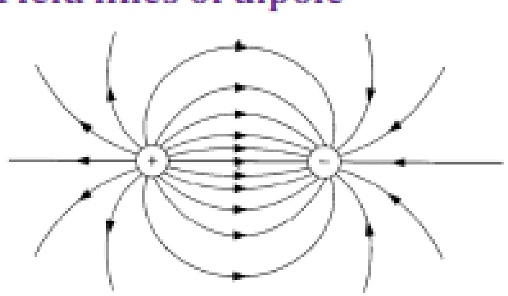


A better way to represent the *electric vector field* consists of connecting of arrows and drawing the so-called **field lines**.

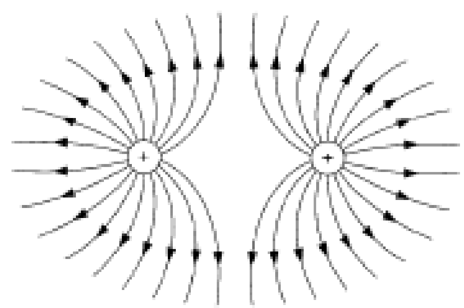
The **magnitude** of **E(r)** is indicated by the ***density*** of the field lines.

The closer the lines, the stronger the field

Field lines of dipole

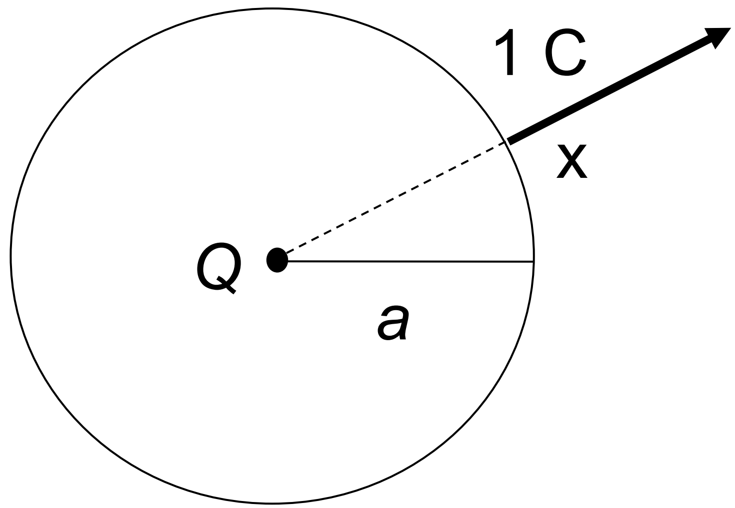


Equal but opposite charges



Equal charges

Electric Flux Density (D)

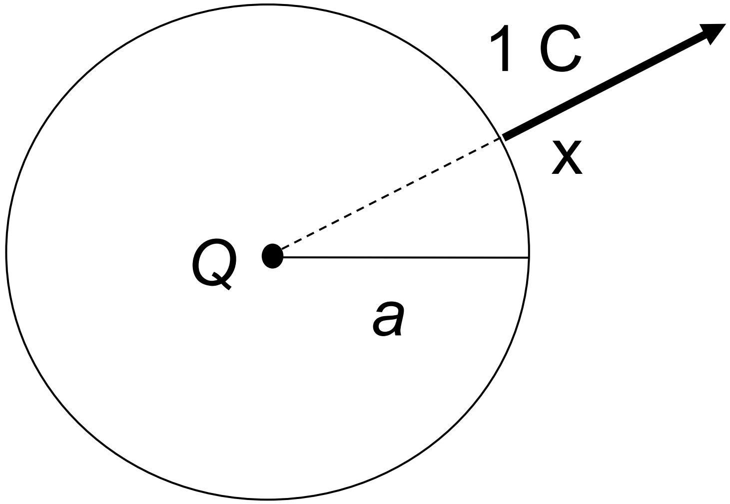


$$E = \frac{Q}{4\pi\varepsilon_0 a^2}$$

The surface area of the sphere S is $4\pi a^2$, and we define

$$D = \frac{Q}{S} \qquad \Rightarrow D = \varepsilon_0 E$$

Electric Flux (Ψ)



$$D = \varepsilon_0 E$$

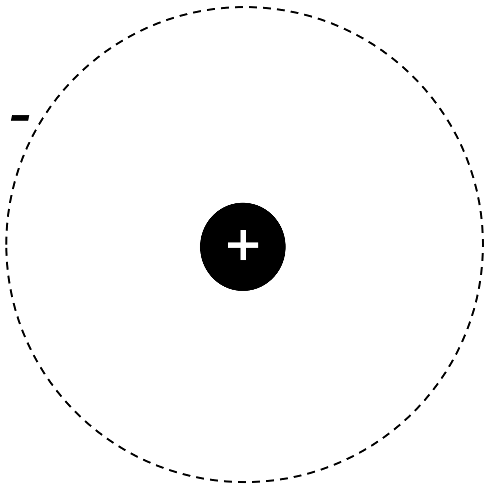
$$D = \frac{Q}{S}$$

The total flux is density times area:

$$\Psi = DS = Q$$

The total flux is independent of the surface area and equal to the net charge enclosed (Gauss Law)

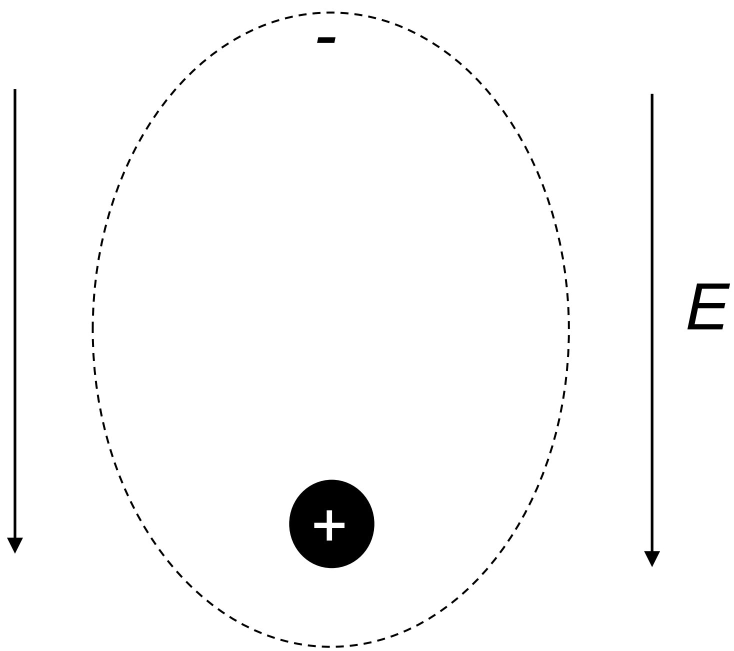
Dielectric (Insulator)



When an electric field is applied the atom is *polarised*.

The separation of positive and negative electrical charges is measured by the *dipole moment*

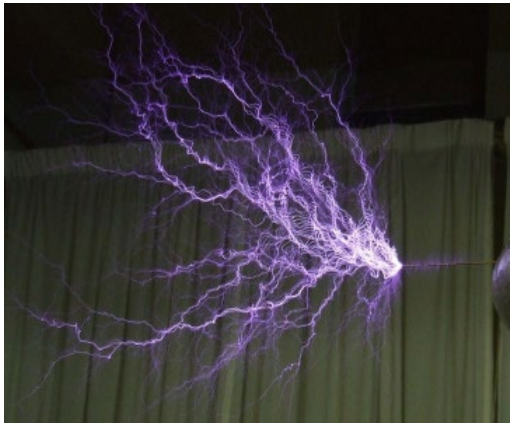
The polarisability of a dielectric is described by its relative dielectric permittivity, ε_r



$$D = \varepsilon_r \varepsilon_o E$$

Dielectric (Insulator)

Dielectric field strength (breakdown voltage):
the maximum electric field strength that it can withstand intrinsically without breaking down, i.e., without experiencing failure of its insulating properties.



Air	***3 MV/m***
Mica	***118 MV/m***
PTFE	***150 MV/m***