Introduction to Electric and Magnetic Fields B38EM
Tutorial Week #3 – Solutions

$\varepsilon_0 = 8.85\text{x}10^{-12}\ \text{Fm}^{-1}$, $q_{e^-} = 1.6\text{x}10^{-19}$ C

1. A charge of Q_1=3 10^{-4} C is located at M(1,2,3) and a charge of Q_2 = -10^{-4} C is located at N(2,0,5) in a vacuum. Calculate the force exerted on Q_2 by Q_1?

Solution:

Force exerted on charge Q_2 by charge Q_1: $F_{21} = \dfrac{Q_2 Q_1}{4\pi\varepsilon_0}\dfrac{R_2 - R_1}{|R_2 - R_1|^3}$

Find the position vectors:

$$R_1 = 2\hat{x} + 5\hat{z}$$
$$R_2 = \hat{x} + 2\hat{y} + 3\hat{z}$$

The vector drawn between Q1 and Q2:

$$R_2 - R_1 = \hat{x} + 2\hat{y} + 3\hat{z} - (2\hat{x} + 5\hat{z}) = -\hat{x} + 2\hat{y} - 2\hat{z}$$

For a three-dimensional vector a(a1,a2,a3), the formula for its magnitude is:

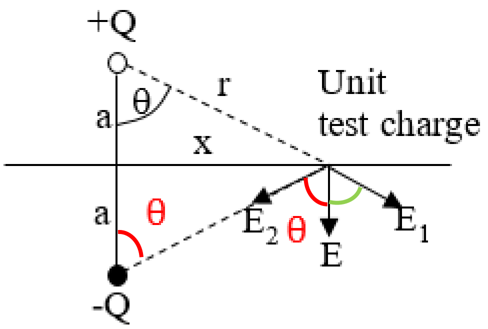
$$\|a\| = \sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}$$

$$F_{21} = \frac{-10^{-4} * 3 * 10^{-4}}{4 * \pi * 8.85 * 10^{-12}} \frac{-\hat{x} + 2\hat{y} - 2\hat{z}}{\left(\sqrt{(-1)^2 + 2^2 + (-2)^2}\right)^3} = -\frac{269.89}{27}(-\hat{x} + 2\hat{y} - 2\hat{z})$$
$$= (\hat{x} - 2\hat{y} + 2\hat{z})\ 9.99N$$

2. Two equal but opposite charges Q are separated by a distance 2a. Use Coulomb's law and the principle of superposition to find expressions for the electric field **E** and electric potential V along the line through the midpoint between the charges and normal to the axis of the charges.

Solution:

The horizontal contributions of the electric field produced by the two charges will cancel each other as the charges are of opposite polarity and of same magnitude. It means that the field has only a vertical component.



Triangle: $\cos\theta = \frac{a}{r}, r^2 = a^2 + x^2$

$$|E_1| = |E_2| = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$\cos\theta = \frac{|E|}{|E_2|} \Rightarrow |E| = |E_2|\cos\theta$$

$$\cos\theta = \frac{|E|}{|E_1|} \Rightarrow |E| = |E_1|\cos\theta$$

$$|E| = E_{1y} + E_{2y} = |E_1|\cos\theta + |E_2|\cos\theta$$

$$|E| = \frac{2Q}{4\pi\varepsilon_0(a^2 + x^2)}\frac{a}{\sqrt{a^2 + x^2}}$$

$$E(x) = \frac{Qa}{2\pi\varepsilon_0(\alpha^2 + x^2)^{\frac{3}{2}}}$$

In the case of same polarity of charges and equal magnitude, the situation will be the opposite ie no vertical contribution of the total electrical field but a horizontal component.
Because the *E*-field along the line passing through the midpoint between the charges and normal to the axis of the charges has only horizontal component, therefore the electric potential V along that line is always zero.

3. A charge of 3 *nC* is located at the origin, and another charge of +5 *nC* is located at 0.3 m along the positive *x*-axis. Determine the position where the electric field is zero. (remark: *nC* means nanoCoulomb, which is 10^{-9} C.)

Solution:

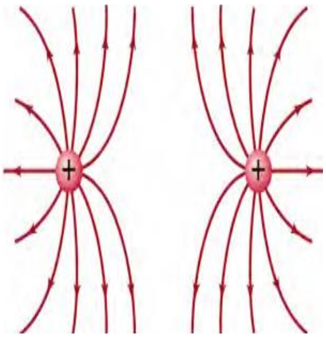


Figure 1 Electrostatic distributions of double charge

It follows from symmetry considerations that **E** is zero at some point situated between the charges on the x-axis.

Let it be the point with coordinates (x,0,0). Then:

$$|E_1| = |E_2| \Rightarrow \frac{q_1}{4\pi\varepsilon_0 x^2} = \frac{q_2}{4\pi\varepsilon_0(0.3 - x)^2} \Rightarrow \frac{q1}{x^2} = \frac{q_2}{(0.3 - x)^2}$$

$\boldsymbol{x = 0.131}$ $and\ x = -1.03$

(Note that $\boldsymbol{E_1}$ and $\boldsymbol{E_2}$ have opposite direction, thus *x* should be 0.131m.)

4. Four positive identical charges of 50nC each are located at $A(1,0,0)$, $B(-1,0,0)$, $C(0,1,0)$, and $D(0,-1,0)$ in free space (the coordinates are in meters). Find the total force on the charge at A.

Find position vectors: $R_A = \hat{x}$
$R_B = -\hat{x}$
$R_C = \hat{y}$
$R_D = -\hat{y}$

Find forces: $F_{AA} = 0$

$$F_{AB} = \frac{Q_AQ_B}{4\pi\varepsilon_0}\frac{R_{A-}R_B}{|R_A - R_B|^3} = \frac{Q_AQ_B}{4\pi\varepsilon_0}\frac{2\hat{x}}{\left(\sqrt{(2)^2}\right)^3} = \frac{Q_AQ_B}{4\pi\varepsilon_0}\frac{1}{8}2\hat{x}$$

$$= \boldsymbol{\frac{Q_AQ_B}{4\pi\varepsilon_0}\frac{1}{4}\hat{x}}$$

$$F_{AC} = \frac{Q_AQ_C}{4\pi\varepsilon_0}\frac{R_{A-}R_C}{|R_A - R_C|^3} = \frac{Q_AQ_C}{4\pi\varepsilon_0}\frac{\hat{x} - \hat{y}}{\left(\sqrt{(-1)^2 + 1^2}\right)^3}$$

$$= \frac{Q_AQ_C}{4\pi\varepsilon_0}\frac{1}{\sqrt{2}^3}(\hat{x} - \hat{y})$$

$$= \frac{Q_AQ_C}{4\pi\varepsilon_0}\frac{1}{\left(\sqrt{2}\right)^2\left(\sqrt{2}\right)^1}(\hat{x} - \hat{y})$$

$$= \frac{Q_AQ_C}{4\pi\varepsilon_0}\frac{1}{2\sqrt{2}}(\hat{x} - \hat{y})$$

$$= \boldsymbol{\frac{Q_AQ_C}{4\pi\varepsilon_0}\frac{1}{2}\frac{\sqrt{2}}{2}(\hat{x} - \hat{y})}$$

$$F_{AD} = \frac{Q_AQ_D}{4\pi\varepsilon_0}\frac{R_{A-}R_D}{|R_A - R_D|^3} = \frac{Q_AQ_D}{4\pi\varepsilon_0}\frac{\hat{x} + \hat{y}}{\left(\sqrt{1^2 + 1^2}\right)^3}$$

$$= \frac{Q_AQ_D}{4\pi\varepsilon_0}\frac{1}{\sqrt{2}^3}(\hat{x} + \hat{y})$$

$$= \boldsymbol{\frac{Q_AQ_D}{4\pi\varepsilon_0}\frac{1}{2}\frac{\sqrt{2}}{2}(\hat{x} + \hat{y})}$$

Total Force:
$$F = \frac{(50 \times 10^{-9})^2}{4\pi\varepsilon_0}\left(\frac{1}{4} + \frac{1}{2}\frac{\sqrt{2}}{2} + \frac{1}{2}\frac{\sqrt{2}}{2}\right)\hat{x} = 21.5\,\hat{x}\,\mu N$$

5. A charged circular annulus is defined by two circles of centre O, and radii *a* and R (*a*<R), of surface charge density σ.

a. Calculate the E-field generated by these charges at a point M, of height z, situated directly above the point O.

b. What becomes the expression of the field when *a* tends to zero. Draw the curve E(z) as a function of z.

c. When *a* is different from zero, what becomes the expression of the field when R increases to infinity. Draw the curve E(z) as a function of z.

d. What is the expression of the field when a tends to zero and R goes to infinity. Draw the curve E(z) as a function of z.

Solution:

a. For symmetry, the field at M is parallel to axis *Oz*. Consider a small annulus of thickness *dr* of radius *r* and centre *O*. A point *P* belongs to this annulus. The field created by this circular elemental annulus, along the *Oz* axis, is given by:

$$dE = \frac{1}{4\pi\varepsilon_0}.\frac{\sigma 2\pi r dr}{MP^2}.\cos\alpha$$

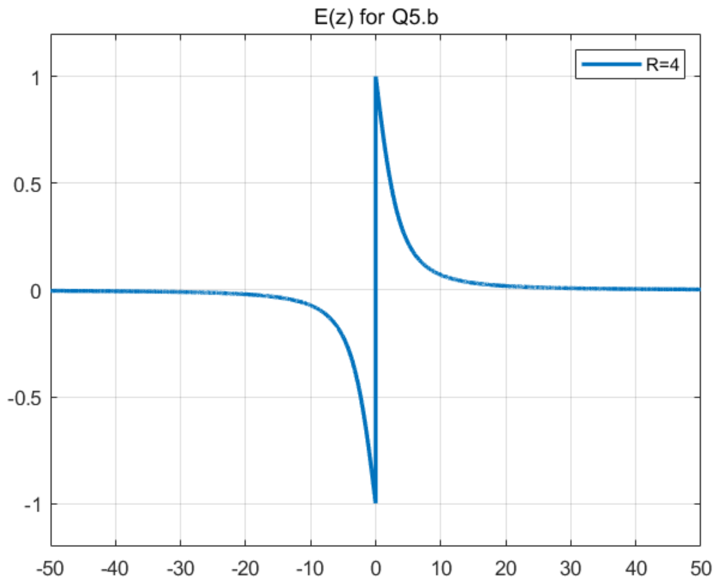
But $\cos\alpha = \dfrac{z}{\sqrt{z^2 + r^2}}, \quad MP = \sqrt{z^2 + r^2}$

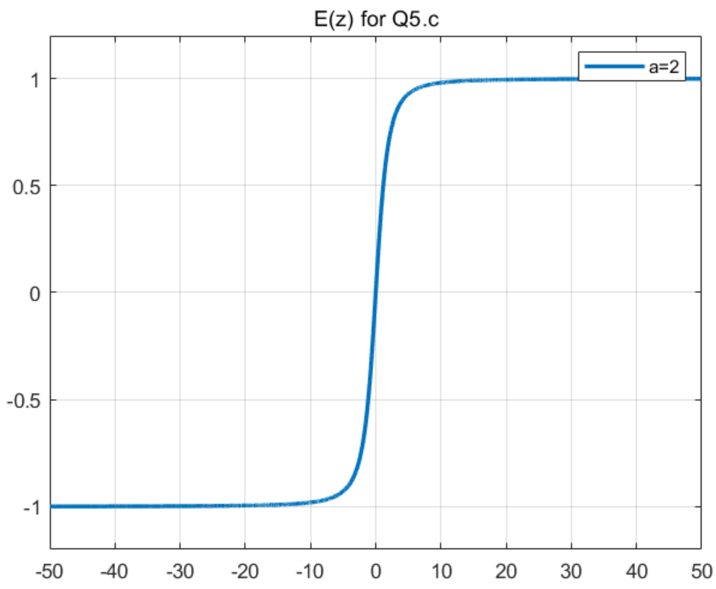
Therefore $E = \int_R^a dE = \dfrac{\sigma}{2\varepsilon_0}\left[\dfrac{z}{\sqrt{z^2 + a^2}} - \dfrac{z}{\sqrt{z^2 + R^2}}\right]$

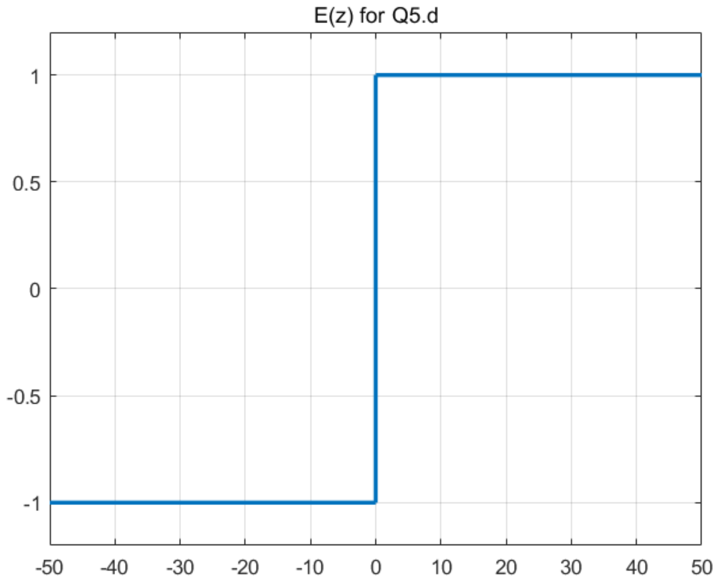
b. First case. If a tends to zero, then: $E = \dfrac{\sigma}{2\varepsilon_0}\left[\dfrac{z}{|z|} - \dfrac{z}{\sqrt{z^2 + R^2}}\right]$

c. If *a* is different from zero and R tends to infinity: $E = \dfrac{\sigma}{2\varepsilon_0}\left[\dfrac{z}{\sqrt{z^2 + a^2}}\right]$

d. If *a* is zero and R tends to infinity $E = \dfrac{\sigma}{2\varepsilon_0}\left[\dfrac{z}{|z|}\right]$






E(z) for Q5.d
1
0.5
0
-0.5
-1
-50
-40
-30
-20
-10
0
10
20
30
40
50