B38EM Introduction to Electricity and Magnetism
Lecture 8

Electromagnetic Waves

Dr. Yuan Ding (Heriot-Watt University)
yuan.ding@hw.ac.uk
yding04.wordpress.com

References & Resources

- Elements of Electromagnetics (7th Edition), by Sadiku, Oxford University Press
- Fundamentals of Applied Electromagnetics (7th Edition), by Ulaby and Ravaioli
- Field and Wave Electromagnetics (2nd Edition), by David Cheng
- ……

Outline & Outcomes

- Recap
- Displacement current and Potential functions
- Nonhomogeneous wave equations
- Homogeneous vector wave equations
- Non-homogenous Helmholtz equations
- Homogenous vector Helmholtz equations

Recap

- Electric charges at rest (electrostatics)
- Steady electric currents – electrons in motion
- Magnetostatic fields
- Electromagnetic induction

Recap

Gauss's Law: Electric Field

In summary:

$$\iint_{S} d\Psi = \iint_{S} D_n dS = \Psi = Q$$

To put Gauss's Theorem in words:

The electric flux emanating from a closed surface is equal to the charge within.

Recap

Gauss's Law: Magnetic Field

In summary:

$$\iint_{s} d\Phi = \iint_{s} B_n dS = \Phi = 0$$

Application of Gauss theorem for magnetic fields. Since there is no free magnetic charge

The magnetic flux emanating from a closed surface is equal to zero.

Recap

Ampere's Law

The *line integral* of the magnetic field strength round a closed path is equal to the current flowing through the closed loop.

$$\oint \vec{H} \cdot d\vec{l} = I = \iint_S \vec{J} \cdot d\vec{s}$$

Recap

Faraday's Law

The induced electromotive force (EMF) in any closed circuit is equal to the time rate of change of the magnetic flux through the circuit.

$$EMF = -\frac{\Delta\Phi}{\Delta t}$$

Recap

$$\iint_S d\Psi = \iint_S D_n dS = \Psi = Q$$

$$\iint_s d\Phi = \iint_s B_n dS = \Phi = 0$$

$$\oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}} = -\frac{\Delta\Phi}{\Delta t}$$

$$\oint \vec{H} \cdot d\vec{l} = I = \iint_S \vec{J} \cdot d\vec{s}$$

Displacement Current

$$\iint_S d\Psi = \iint_S D_n dS = \Psi = Q$$

$$\iint_S d\Phi = \iint_S B_n dS = \Phi = 0$$

$$\oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}} = -\frac{\Delta\Phi}{\Delta t}$$

$$\oint \vec{H} \cdot d\vec{l} = I \quad \Longrightarrow \quad \oint \vec{H} \cdot d\vec{l} = I + \frac{\Delta\Psi}{\Delta t}$$

Displacement Current

Why is it important?

Assume free space (no free charge)

$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \iint_S \vec{B} \cdot d\vec{S}$$

$$\oint_C \vec{H} \cdot d\vec{l} = \frac{\partial}{\partial t} \iint_S \vec{D} \cdot d\vec{S}$$

A variation in the magnetic field, causes an electric field.

A variation in the electric field, causes a magnetic field.

An EM disturbance propagates: EM waves in free space.

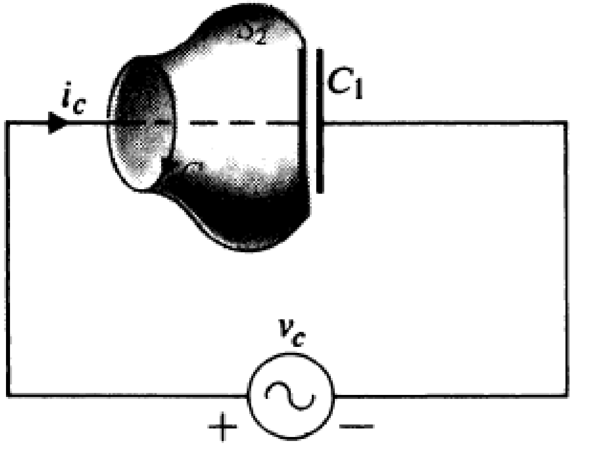
Maxwell's Equations

Example:

An AC voltage source of amplitude V_0 and angular frequency ω, $\upsilon_c = V_0 \sin\omega t$, is connected across a parallel-plate capacitor C_1.

(a) Verify that the displacement current in the capacitor is the same as the conduction current in the wires.

(b) Determine the magnetic field intensity at a distance *r* from the wire.



Solutions:

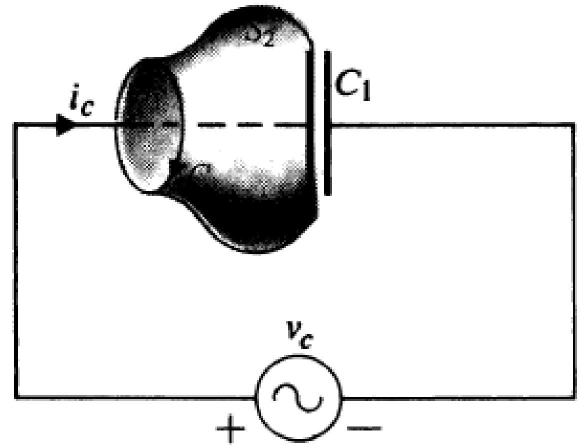
(a) Conduction current:

Displacement current:

(b) Choose different surfaces lead to same result

Maxwell's Equations

Example:



Solution:

(a) Conduction current:

$$i_c = C_1 \frac{dV_c}{dt} = C_1 V_0 w \cos wt \qquad (A)$$

Displacement current:

$$i_D = \frac{d\psi}{dt} = \frac{d(\vec{D}\cdot\vec{A})}{dt} = \frac{d(\varepsilon E A)}{dt} = \varepsilon A \frac{dE}{dt} = \frac{\varepsilon A}{d}\frac{dV_c}{dt}$$

$$= C_1 V_0 w \cos wt \qquad (A).$$

(b) Thus, if we want to calculate H_ϕ, it does not matter what surface we choose, using Ampere's Law, we always get the same result.

Maxwell's Equations

Name	Integral equations	Differential equations
Gauss's law	$\oiint_{\partial\Omega} \mathbf{E} \cdot \mathrm{d}\mathbf{S} = \frac{1}{\varepsilon_0} \iiint_{\Omega} \rho \, \mathrm{d}V$	$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$
Gauss's law for magnetism	$\oiint_{\partial\Omega} \mathbf{B} \cdot \mathrm{d}\mathbf{S} = 0$	$\nabla \cdot \mathbf{B} = 0$
Maxwell–Faraday equation (Faraday's law of induction)	$\oint_{\partial\Sigma} \mathbf{E} \cdot \mathrm{d}\boldsymbol{\ell} = -\frac{\mathrm{d}}{\mathrm{d}t} \iint_{\Sigma} \mathbf{B} \cdot \mathrm{d}\mathbf{S}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
Ampère's circuital law (with Maxwell's addition)	$\oint_{\partial\Sigma} \mathbf{B} \cdot \mathrm{d}\boldsymbol{\ell} = \mu_0 \iint_{\Sigma} \mathbf{J} \cdot \mathrm{d}\mathbf{S} + \mu_0 \varepsilon_0 \frac{\mathrm{d}}{\mathrm{d}t} \iint_{\Sigma} \mathbf{E} \cdot \mathrm{d}\mathbf{S}$	$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$

Charge conservation equation $\nabla \cdot \boldsymbol{J} = -\frac{\partial \rho}{\partial t}$

Lorentz equation $\boldsymbol{F} = q\left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\right)$

Maxwell's Equations

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t},$$

$$\nabla \cdot \mathbf{D} = \rho,$$

$$\nabla \cdot \mathbf{B} = 0.$$

Not all independent

12 unknowns

12 scalar equations

Constituent relations

$$\boldsymbol{D} = \varepsilon \boldsymbol{E}$$

$$\boldsymbol{H} = \boldsymbol{B}/\mu$$

Maxwell's Equations

Integral form

In a physical environment we must deal with finite objects of specified shape and boundaries.

Take the surface integral of both sides of the Curl equations over an open surface *S* with a contour *C* and apply Stokes' theorem.

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \Rightarrow \quad \oint_C \mathbf{E} \cdot d\boldsymbol{\ell} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad \Rightarrow \quad \oint_C \mathbf{H} \cdot d\boldsymbol{\ell} = \int_S \left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{s}.$$

Maxwell's Equations

Integral form

Taking the volume integral of both sides of the divergence equations over a volume V with a closed surface *S* and using divergence theorem

$$\nabla \cdot \mathbf{D} = \rho, \quad \Rightarrow \quad \oint_S \mathbf{D} \cdot d\mathbf{s} = \int_V \rho \, dv$$

$$\nabla \cdot \mathbf{B} = 0. \quad \Rightarrow \quad \oint_S \mathbf{B} \cdot d\mathbf{s} = 0$$

Maxwell's Equations

Potential functions

$$\nabla \times \boldsymbol{F} = \begin{vmatrix} \hat{\boldsymbol{x}} & \hat{\boldsymbol{y}} & \hat{\boldsymbol{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

$$\nabla \cdot \boldsymbol{F} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \cdot \left(F_x, F_y, F_z \right)$$

You can prove: $\nabla \cdot \left(\nabla \times \boldsymbol{F} \right) = 0$

$$\nabla \cdot \boldsymbol{B} = 0 \qquad \Rightarrow \qquad \boldsymbol{B} = \nabla \times \boldsymbol{A}$$

A: vector magnetic potential

Substitute into Faradays' Law $\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}(\boldsymbol{B})$

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}(\nabla \times \boldsymbol{A})$$

$$\nabla \times \left(\boldsymbol{E} + \frac{\partial \boldsymbol{A}}{\partial t} \right) = 0 \qquad \Rightarrow \qquad \boldsymbol{E} + \frac{\partial \boldsymbol{A}}{\partial t} = -\nabla V$$

Maxwell's Equations

Potential functions

$$\boldsymbol{E} = -\nabla V - \frac{\partial \boldsymbol{A}}{\partial t} \qquad (V/m)$$

In the static case, $\frac{\partial}{\partial t} = 0$ and above equation reduces to $\boldsymbol{E} = -\nabla V$

For time-varying fields, ***E*** depends on both *V* and ***A***.

E and ***B*** are coupled.

Maxwell's Equations

Nonhomogeneous wave equations for vector potential ***A***

$$\nabla^2 \boldsymbol{A} - \mu\varepsilon \frac{\partial^2 \boldsymbol{A}}{\partial t^2} = -\mu \boldsymbol{J}$$

Nonhomogeneous wave equations for scalar potential *V*

$$\nabla^2 V - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\varepsilon}$$

Maxwell's Equations

Electromagnetic Boundary Conditions

It is necessary to know the boundary conditions for ***E***, ***D***, ***H***, and ***B*** in order to solve the electromagnetic problems in contiguous regions.

For **curl equations**, apply the integral form to a flat closed path at a boundary with top and bottom sides in the two touching media yielding the boundary conditions for ***the tangential components***

For **divergence equations**, apply the integral form to a shallow pillbox at an interface with top and bottom surface yielding the boundary conditions for ***the normal components***

Maxwell's Equations

Electromagnetic Boundary Conditions

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \quad \Leftrightarrow \quad \oint_C \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{d\Phi}{dt} \quad \Leftrightarrow \quad E_{1t} = E_{2t}$$

The tangential component of an E field is continuous across an interface.

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} - \frac{\partial \boldsymbol{D}}{\partial t} \quad \Leftrightarrow \quad \oint_C \boldsymbol{H} \cdot d\boldsymbol{l} = I + \int_s \frac{\partial \boldsymbol{D}}{\partial t} \cdot d\boldsymbol{s}$$

$$\Leftrightarrow \quad \widehat{\boldsymbol{a}} \times \left(\boldsymbol{H}_1 - \boldsymbol{H}_2 \right) = \boldsymbol{J}_s$$

The tangential component of an H field is discontinuous across an interface if a surface current exists.

Maxwell's Equations

Electromagnetic Boundary Conditions

$$\nabla \cdot \boldsymbol{D} = \rho \quad \Leftrightarrow \quad \oint_s \boldsymbol{D} \cdot d\boldsymbol{s} = Q \quad \Leftrightarrow \quad \widehat{\boldsymbol{a}} \cdot \left(\boldsymbol{D}_1 - \boldsymbol{D}_2 \right) = \rho_s$$

The normal component of a D field is discontinuous across an interface if a surface charge exists.

$$\nabla \cdot \boldsymbol{B} = 0 \quad \Leftrightarrow \quad \oint_s \boldsymbol{B} \cdot d\boldsymbol{s} = 0 \quad \Leftrightarrow \quad \boldsymbol{B}_{1n} = \boldsymbol{B}_{2n}$$

The normal component of a B field is continuous across an interface.

Maxwell's Equations

Electromagnetic Boundary Conditions

When $\rho_s = 0 \qquad \boldsymbol{J}_s = 0$

$$E_{1t} = E_{2t} \rightarrow \frac{D_{1t}}{D_{2t}} = \frac{\epsilon_1}{\epsilon_2}$$

$$H_{1t} = H_{2t} \rightarrow \frac{B_{1t}}{B_{2t}} = \frac{\mu_1}{\mu_2}$$

$$D_{1n} = D_{2n} \rightarrow \epsilon_1 E_{1n} = \epsilon_2 E_{2n}$$

$$B_{1n} = B_{2n} \rightarrow \mu_1 H_{1n} = \mu_2 H_{2n}$$

Maxwell's Equations

Wave equations

For given charge ρ and current distribution $\boldsymbol{J}$, first find $\boldsymbol{A}$ and V, then calculate $\boldsymbol{E}$ and $\boldsymbol{B}$.

$$\nabla^2 \boldsymbol{A} - \mu\varepsilon \frac{\partial^2 \boldsymbol{A}}{\partial t^2} = -\mu \boldsymbol{J} \qquad \boldsymbol{E} = -\nabla V - \frac{\partial \boldsymbol{A}}{\partial t}$$

$$\Rightarrow$$

$$\nabla^2 V - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\varepsilon} \qquad \boldsymbol{B} = \nabla \times \boldsymbol{A}$$

Source of $\boldsymbol{E}$**:** (1) charges (static or varying); (2) varying magnetic field.
Source of $\boldsymbol{B}$**:** electric current (free electron or varying polarised charges).

Maxwell’s Equations

Wave equations

$$\nabla^2 V - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\varepsilon}$$

$$\Delta f = \nabla^2 f = \nabla \cdot \nabla f$$

$$\begin{aligned}\Delta f &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 f}{\partial \varphi^2} \\ &= \frac{1}{r}\frac{\partial^2}{\partial r^2}(rf) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 f}{\partial \varphi^2}\end{aligned}$$

Assume a point charge at time *t*, $\rho(t)\Delta v'$, at the origin of a spherical coordinate, and at a location rather than the origin we have

$$\frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2 \frac{\partial V}{\partial R}\right) - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = 0$$

Introduce: $U(R,t) = R \cdot V(R,t)$

$$\frac{\partial^2 U}{\partial R^2} - \mu\varepsilon \frac{\partial^2 U}{\partial t^2} = 0$$

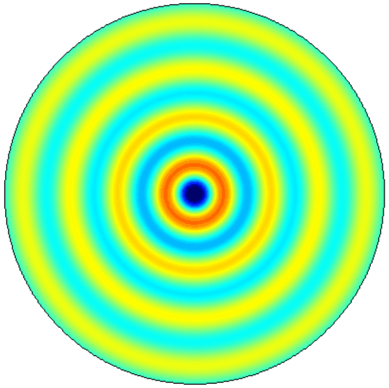
One-dimensional homogeneous wave equation

Maxwell's Equations

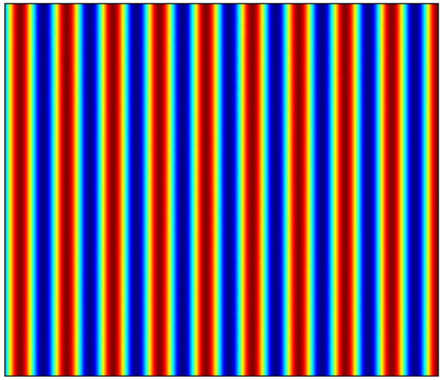
Wave equations

Variable transformation

$V\left(R,t\right)$ $U\left(R,t\right)$



$$V \cdot R = U$$



Maxwell's Equations

Wave equations

$$\frac{\partial^2 U}{\partial R^2} - \mu\varepsilon \frac{\partial^2 U}{\partial t^2} = 0$$

Any twice differentiable function of ($t - R\sqrt{\mu\varepsilon}$) or ($t + R\sqrt{\mu\varepsilon}$) is a solution of the wave equation.
We only select a function of ($t - R\sqrt{\mu\varepsilon}$).

$$U\left(R,t\right) = f(t - R\sqrt{\mu\varepsilon})$$

$$U\left(R+\Delta R, t+\Delta t\right) = f(t+\Delta t - \left(R+\Delta R\right)\sqrt{\mu\varepsilon})$$

If $\Delta t = \Delta R\sqrt{\mu\varepsilon}$ the function remains the same.

The wave propagates along positive *R* direction at a speed of $u = \frac{1}{\sqrt{\mu\varepsilon}}$

$$V = \frac{1}{R} f\left(t - R/u\right)$$

Maxwell's Equations

Wave equations

$$\frac{\partial^2 U}{\partial R^2} - \mu\varepsilon \frac{\partial^2 U}{\partial t^2} = 0 \qquad U(R,t) = f(t - R\sqrt{\mu\varepsilon}) \qquad V = \frac{1}{R} f\left(t - R/u\right)$$

Electric potential due to a charge distribution over a volume V'

$$V(R,t) = \frac{1}{4\pi\varepsilon}\int_{v'} \frac{\rho\left(t - R/u\right)}{R}\,dv' \qquad \text{(V)}$$

Retarded scalar potential.

Thus, we cannot take $f(t + R\sqrt{\mu\varepsilon})$ solutions as they are physically impossible.

Maxwell’s Equations

Wave equations

Similarly,

Magnetic vector potential due to a current distribution over a volume V’

$$\boldsymbol{A}(R,t) = \frac{\mu}{4\pi}\int_{v'} \frac{\boldsymbol{J}\left(t - R/u\right)}{R} dv' \qquad \text{(Wb/m)}$$

Retarded vector potential.

It takes time for electromagnetic waves to travel and to be felt at a distance.

Maxwell's Equations

Source-free wave equations

$\rho = 0$, and $\boldsymbol{J} = 0$

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t}, \qquad \nabla \times \nabla \times \mathbf{E} = -\mu \frac{\partial}{\partial t}(\nabla \times \mathbf{H}) = -\mu\epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\nabla \times \mathbf{H} = \epsilon \frac{\partial \mathbf{E}}{\partial t}, \qquad \nabla \times \nabla \times \mathbf{E} = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\nabla^2 \mathbf{E}$$

$$\nabla \cdot \mathbf{E} = 0, \qquad \Downarrow$$

$$\nabla \cdot \mathbf{H} = 0.$$

$$\nabla^2 \mathbf{E} - \mu\epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0;$$

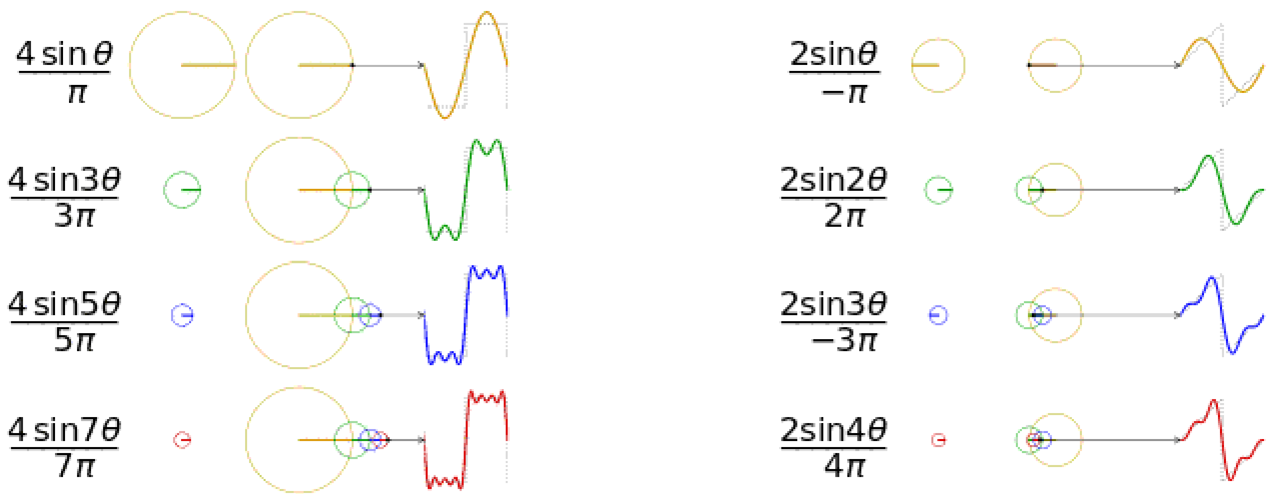
Homogeneous vector wave equations

$$\nabla^2 \mathbf{E} - \frac{1}{u^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0. \qquad \nabla^2 \mathbf{H} - \frac{1}{u^2} \frac{\partial^2 \mathbf{H}}{\partial t^2} = 0.$$

Maxwell's Equations

Time-Harmonic Fields

- Arbitrary periodic time functions can be expanded into Fourier series of harmonic sinusoidal components
- Sinusoidal time variations of source functions will produce sinusoidal variations of ***E*** and ***H*** with the same frequency



Maxwell’s Equations

Phasors

$$i(t) = I\cos(\omega t + \phi)$$

amplitude, frequency, and phase

Not convenient for differentiation or integration

Example:

A series RLC circuit with an applied voltage $v(t) = V\cos(\omega t)$

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i dt = v(t)$$

$$I\left[-\omega L\sin(\omega t + \phi) + R\cos(\omega t + \phi) + \frac{1}{\omega C}\sin(\omega t + \phi)\right] = V\cos(\omega t)$$

Maxwell's Equations

Phasors

Now

$$\frac{di}{dt} = \mathrm{Re}\left(j\omega I_s e^{j(\omega t+\phi)} \right) \qquad \int i\,dt = \mathrm{Re}\left(\frac{I_s}{j\omega} e^{j(\omega t+\phi)} \right)$$

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt = v(t)$$

$$\Downarrow$$

$$\left[R + j\left(\omega L - \frac{1}{\omega C} \right) \right] I_s \cdot e^{j\phi} = V_s$$

Phasor: $I_s \cdot e^{j\phi}$

Maxwell's Equations

Time-harmonic electromagnetics

$$\boldsymbol{E}\left(x,y,z,t\right)=\mathrm{Re}\left[\boldsymbol{E}\left(x,y,z\right)e^{j\omega t}\right]$$

$$\boldsymbol{H}\left(x,y,z,t\right)=\mathrm{Re}\left[\boldsymbol{H}\left(x,y,z\right)e^{j\omega t}\right]$$

Time-harmonic Maxwell's equations in simple medium

$$\begin{aligned}\nabla\times\mathbf{E}&=-\frac{\partial\mathbf{B}}{\partial t},\\ \nabla\times\mathbf{H}&=\mathbf{J}+\frac{\partial\mathbf{D}}{\partial t},\\ \nabla\cdot\mathbf{D}&=\rho,\\ \nabla\cdot\mathbf{B}&=0.\end{aligned}\quad\Rightarrow\quad\begin{aligned}\nabla\times\mathbf{E}&=-j\omega\mu\mathbf{H},\\ \nabla\times\mathbf{H}&=\mathbf{J}+j\omega\epsilon\mathbf{E},\\ \nabla\cdot\mathbf{E}&=\rho/\epsilon,\\ \nabla\cdot\mathbf{H}&=0.\end{aligned}$$

Maxwell's Equations

Time-harmonic electromagnetics

Non-homogenous wave equations

$$\nabla^2 V - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\varepsilon} \qquad \nabla^2 \boldsymbol{A} - \mu\varepsilon \frac{\partial^2 \boldsymbol{A}}{\partial t^2} = -\mu \boldsymbol{J}$$

$$\Rightarrow$$

$$\nabla^2 V + k^2 V = -\frac{\rho}{\varepsilon}$$

Non-homogenous Helmholtz equations

$$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J}$$

Wavenumber: $k = \omega\sqrt{\mu\varepsilon} = \dfrac{\omega}{u}$

Maxwell's Equations

Time-harmonic electromagnetics

Non-homogenous Helmholtz equations

$$\nabla^2 V + k^2 V = -\frac{\rho}{\varepsilon} \qquad \nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J}$$

$$\Downarrow$$

$$V(R) = \frac{1}{4\pi\varepsilon}\int_{v'} \frac{\rho e^{-jkR}}{R}\, dv' \qquad \boldsymbol{A}(R) = \frac{\mu}{4\pi}\int_{v'} \frac{\boldsymbol{J} e^{-jkR}}{R}\, dv'$$

Formal procedure to determine ***E*** and ***H*** due harmonic charges and currents

1. Find phasors $V(R)$ and $\mathbf{A}(R)$
2. Find phasors $\mathbf{E}(R) = -\nabla V - j\omega\mathbf{A}$ and $\mathbf{B}(R) = \nabla \times \mathbf{A}$.
3. Find instantaneous $\mathbf{E}(R, t) = \mathcal{Re}[\mathbf{E}(R)e^{j\omega t}]$ and $\mathbf{B}(R, t) = \mathcal{Re}[\mathbf{B}(R)e^{j\omega t}]$ for a cosine reference.

Maxwell's Equations

Time-harmonic electromagnetics (source-free)

$\rho = 0$, and $\boldsymbol{J} = 0$

$$
\begin{aligned}
\nabla \times \mathbf{E} &= -j\omega\mu\mathbf{H}, \\
\nabla \times \mathbf{H} &= \mathbf{J} + j\omega\epsilon\mathbf{E}, \\
\nabla \cdot \mathbf{E} &= \rho/\epsilon, \\
\nabla \cdot \mathbf{H} &= 0.
\end{aligned}
\quad \Rightarrow \quad
\begin{aligned}
\nabla \times \mathbf{E} &= -j\omega\mu\mathbf{H}, \\
\nabla \times \mathbf{H} &= j\omega\epsilon\mathbf{E}, \\
\nabla \cdot \mathbf{E} &= 0, \\
\nabla \cdot \mathbf{H} &= 0.
\end{aligned}
$$

Homogenous vector wave equations

$$\nabla^2\mathbf{E} - \frac{1}{u^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = 0.$$

$$\nabla^2\mathbf{H} - \frac{1}{u^2}\frac{\partial^2\mathbf{H}}{\partial t^2} = 0.$$

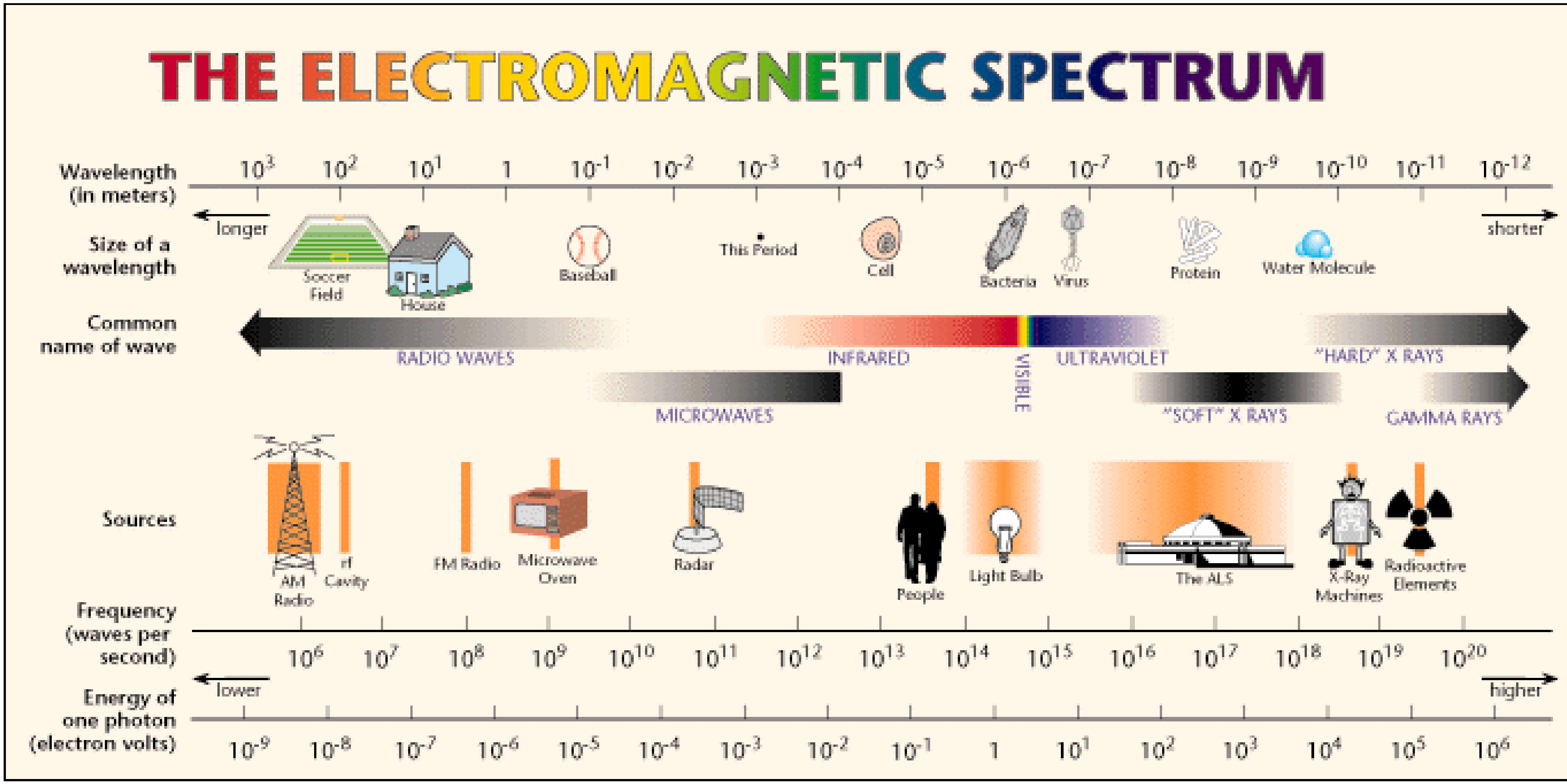
Homogenous vector Helmholtz equations

$$\nabla^2\mathbf{E} + k^2\mathbf{E} = 0$$

$$\nabla^2\mathbf{H} + k^2\mathbf{H} = 0,$$

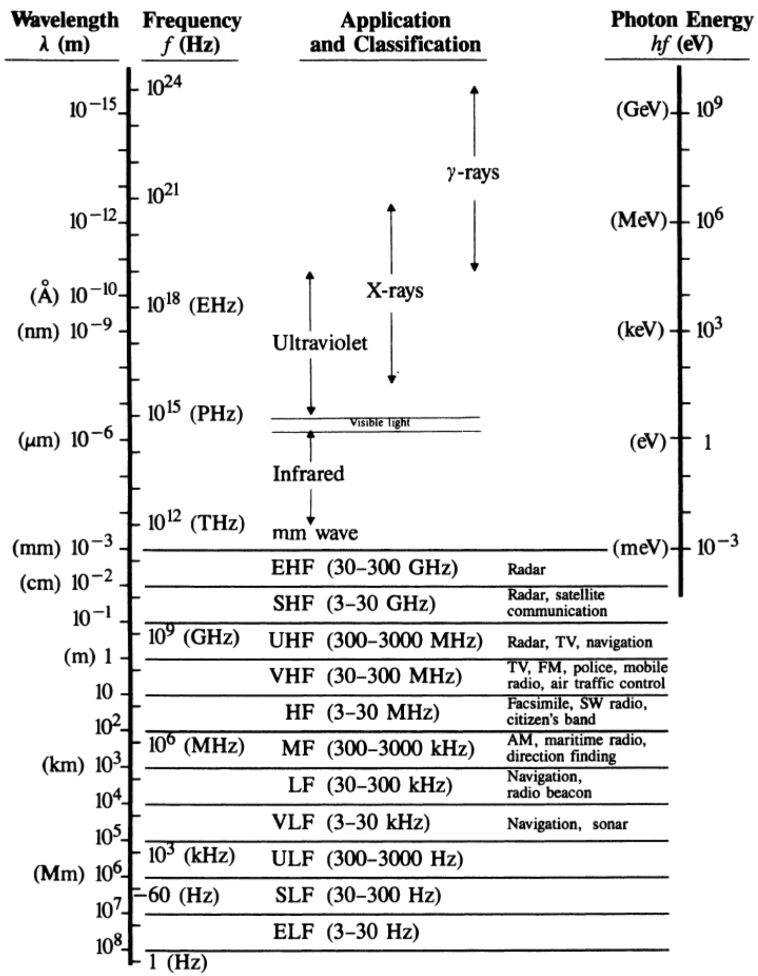
Maxwell's Equations

The electromagnetics spectrum



Maxwell's Equations

The electromagnetics spectrum



Band Designations for Microwave Frequency Ranges

Old†	New	Frequency Ranges (GHz)
Ka	K	26.5–40
K	K	20–26.5
K	J	18–20
Ku	J	12.4–18
X	J	10–12.4
X	I	8–10
C	H	6–8
C	G	4–6
S	F	3–4
S	E	2–3
L	D	1–2
UHF	C	0.5–1