Notes – D. E. Anagnostou

Appendix & revision

Scalar: magnitude only (time, temperature, volume,…)
Vector: magnitude & direction.

- Magnitude, or "size" of vector, is referred as "displacement", is the scalar portion of the vector and is represented by its length.
- Direction indicates how the vector is oriented relative to some reference axis

APPENDIX

I

$$f(x) = \frac{\sin(x)}{x}$$

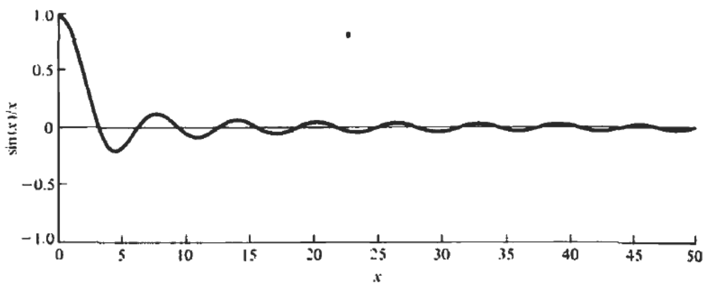


Figure 1.1 Plot of sin $(x)/x$ function.

APPENDIX

II

$$f_N(x) = \left|\frac{\sin(Nx)}{N\sin(x)}\right|$$

$$N = 1, 3, 5, 10, 20$$

APPENDIX

III

COSINE AND SINE INTEGRALS

APPENDIX

VI

IDENTITIES

VI.1 TRIGONOMETRIC

1. Sum or difference:
   - **a.** $\sin(x+y) = \sin x \cos y + \cos x \sin y$
   - **c.** $\cos(x+y) = \cos x \cos y - \sin x \sin y$
   - **e.** $\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$

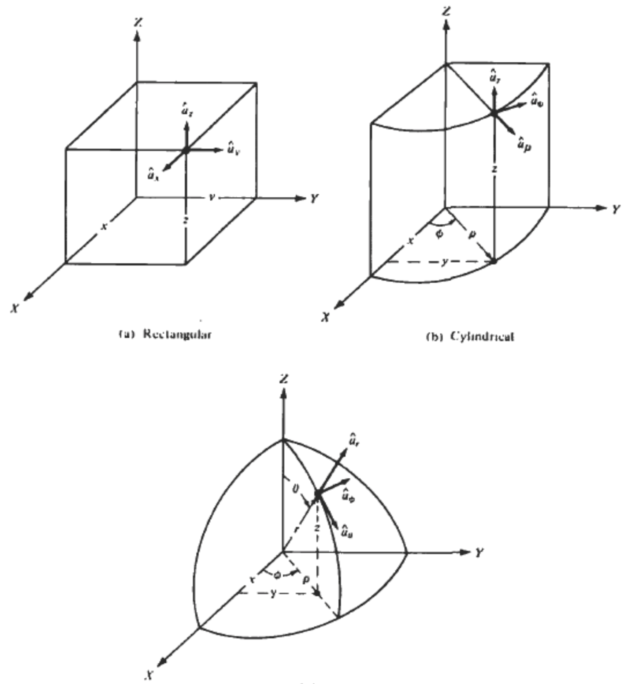
VI.2 HYPERBOLIC

1. Definitions:
   - **a.** Hyperbolic sine: $\sinh x = \frac{1}{2}(e^x - e^{-x})$
   - **b.** Hyperbolic cosine: $\cosh x = \frac{1}{2}(e^x + e^{-x})$
   - **c.** Hyperbolic tangent: $\tanh x = \dfrac{\sinh x}{\cosh x}$
   - **d.** Hyperbolic cotangent: $\coth x = \dfrac{1}{\tanh x} = \dfrac{\cosh x}{\sinh x}$
   - **e.** Hyperbolic secant: $\operatorname{sech} x = \dfrac{1}{\cosh x}$
   - **f.** Hyperbolic cosecant: $\operatorname{csch} x = \dfrac{1}{\sinh x}$

APPENDIX

VII

VECTOR ANALYSIS



(a) Rectangular (b) Cylindrical (c) Spherical

VII.1.3 Rectangular-to-Spherical (and Vice-Versa)

Many times it may be required that a transformation be performed directly from rectangular-to-spherical components. By referring to Figure VII.1, we can write that the rectangular and spherical coordinates are related by

$$\left.\begin{aligned} x &= r \sin\theta \cos\phi \\ y &= r \sin\theta \sin\phi \\ z &= r \cos\theta \end{aligned}\right\} \tag{VII-11}$$

and the rectangular and spherical components by

$$\left.\begin{aligned} A_r &= A_x \sin\theta\cos\phi + A_y \sin\theta\sin\phi + A_z \cos\theta \\ A_\theta &= A_x \cos\theta\cos\phi + A_y \cos\theta\sin\phi - A_z \sin\theta \\ A_\phi &= -A_x \sin\phi + A_y \cos\phi \end{aligned}\right\} \tag{VII-12}$$

which can also be obtained by substituting (VII-6) into (VII-9). In matrix form, (VII-12) can be written as

$$\begin{pmatrix} A_r \\ A_\theta \\ A_\phi \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \tag{VII-12a}$$

^------- rectangular to spherical transformation matrix.

and the spherical-to-rectangular components related by

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{pmatrix} \begin{pmatrix} A_r \\ A_\theta \\ A_\phi \end{pmatrix} \qquad \text{(VII-13a)}$$

or

$$\left.\begin{aligned} A_x &= A_r \sin\theta\cos\phi + A_\theta \cos\theta\cos\phi - A_\phi \sin\phi \\ A_y &= A_r \sin\theta\sin\phi + A_\theta \cos\theta\sin\phi + A_\phi \cos\phi \\ A_z &= A_r \cos\theta - A_\theta \sin\theta \end{aligned}\right\} \qquad \text{(VII-13b)}$$

VII.2 VECTOR DIFFERENTIAL OPERATORS

The differential operators of gradient of a scalar ($\nabla\psi$), divergence of a vector ($\nabla \cdot \mathbf{A}$), curl of a vector ($\nabla \times \mathbf{A}$), Laplacian of a scalar ($\nabla^2\psi$), and Laplacian of a vector ($\nabla^2\mathbf{A}$) frequently encountered in electromagnetic field analysis will be listed in the rectangular, cylindrical, and spherical coordinate systems.

VII.2.1 Rectangular Coordinates

$$\nabla\psi = \hat{\mathbf{a}}_x \frac{\partial\psi}{\partial x} + \hat{\mathbf{a}}_y \frac{\partial\psi}{\partial y} + \hat{\mathbf{a}}_z \frac{\partial\psi}{\partial z} \qquad \text{(VII-14)}$$

gradient of scalar ψ *(e.g. T°)*

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \qquad \text{(VII-15)}$$

divergence of vector **A**

$$\nabla \times \mathbf{A} = \hat{\mathbf{a}}_x \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{\mathbf{a}}_y \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{\mathbf{a}}_z \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) \qquad \text{(VII-16)}$$

curl of vector **A**

$$\nabla \cdot \nabla\psi = \nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} \qquad \text{(VII-17)}$$

Laplacian of scalar ψ

$$\nabla^2\mathbf{A} = \hat{\mathbf{a}}_x \nabla^2 A_x + \hat{\mathbf{a}}_y \nabla^2 A_y + \hat{\mathbf{a}}_z \nabla^2 A_z \qquad \text{(VII-18)}$$

Laplacian of vector **A**

(**del**) is a spatial derivative and expresses <u>how strongly a quantity varies in space</u>

(**del**) • **E** : If E moves in one direction, (del)·E [=div of E] expresses how much E varies (spreads out or changes) in that direction.

(**del**) × **E** : Measures how much E curls around, or how much it changes in the perpendicular directions

VII.2.3 Spherical Coordinates

$$\nabla\psi = \hat{\mathbf{a}}_r \frac{\partial\psi}{\partial r} + \hat{\mathbf{a}}_\theta \frac{1}{r}\frac{\partial\psi}{\partial\theta} + \hat{\mathbf{a}}_\phi \frac{1}{r\sin\theta}\frac{\partial\psi}{\partial\phi} \tag{VII-24}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi} \tag{VII-25}$$

$$\nabla\times\mathbf{A} = \frac{\hat{\mathbf{a}}_r}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right] + \frac{\hat{\mathbf{a}}_\theta}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi)\right] + \frac{\hat{\mathbf{a}}_\phi}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right] \tag{VII-26}$$

$$\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2} \tag{VII-27}$$

$$\underline{\nabla^2\mathbf{A} = \nabla(\nabla\cdot\mathbf{A}) - \nabla\times\nabla\times\mathbf{A}} \tag{VII-28}$$

note that $\nabla^2\mathbf{A} \neq \hat{\mathbf{a}}_r\nabla^2 A_r + \hat{\mathbf{a}}_\theta\nabla^2 A_\theta + \hat{\mathbf{a}}_\phi\nabla^2 A_\phi$ since the orientation of the unit vectors $\hat{\mathbf{a}}_r$, $\hat{\mathbf{a}}_\theta$, and $\hat{\mathbf{a}}_\phi$ varies with the r, θ, and ϕ coordinates.

VII.3 VECTOR IDENTITIES

VII.3.2 Differentiation

Divergence of a curl of a vector = always zero

$$\underline{\nabla\cdot(\nabla\times\mathbf{A}) = 0} \tag{VII-40}$$

$$\nabla\times\nabla\psi = 0 \tag{VII-41}$$

$$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi \tag{VII-42}$$

$$\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi \tag{VII-43}$$

$$\underline{\nabla\cdot(\mathbf{A} + \mathbf{B}) = \nabla\cdot\mathbf{A} + \nabla\cdot\mathbf{B}} \tag{VII-44}$$

$$\underline{\nabla\times(\mathbf{A} + \mathbf{B}) = \nabla\times\mathbf{A} + \nabla\times\mathbf{B}} \tag{VII-45}$$

$$\nabla\cdot(\psi\mathbf{A}) = \mathbf{A}\cdot\nabla\psi + \psi\nabla\cdot\mathbf{A} \tag{VII-46}$$

$$\nabla\times(\psi\mathbf{A}) = \nabla\psi\times\mathbf{A} + \psi\nabla\times\mathbf{A} \tag{VII-47}$$

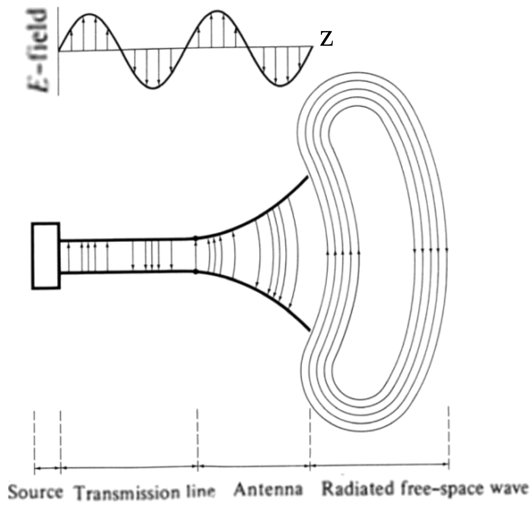
$$\nabla(\mathbf{A}\cdot\mathbf{B}) = (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A} + \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) \tag{VII-48}$$

$$\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot\nabla\times\mathbf{A} - \mathbf{A}\cdot\nabla\times\mathbf{B} \tag{VII-49}$$

$$\nabla\times(\mathbf{A}\times\mathbf{B}) = \mathbf{A}\,\nabla\cdot\mathbf{B} - \mathbf{B}\nabla\cdot\mathbf{A} + (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B} \tag{VII-50}$$

$$\underline{\nabla\times\nabla\times\mathbf{A} = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}} \tag{VII-51}$$

- IEEE Std 145-1983: An <u>antenna</u> is the device (of a transmitting or receiving system) that provides a means for radiating or receiving electromagnetic waves (radio waves).
- It provides a <u>transition</u> from a <u>guided wave</u> on a transmission line to a <u>“free-space” wave</u> (and vice versa when receiving).



1873
<u>Maxwell’s Equations</u>

Gauss' Laws

$$\nabla \cdot D = \rho$$

$$\nabla \cdot B = \rho_m$$

scalar

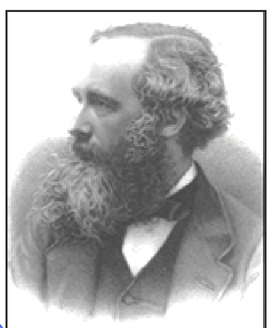
Faraday's Law:

$$\nabla \times E = -\frac{\partial B}{\partial t} - \boldsymbol{M}$$

Ampere's Law:

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

Vector eqs



Maxwell

Under *static* conditions, none of the quantities appearing in Maxwell’s equations are functions of time (i.e., $\partial/\partial t = 0$). ***This happens when all charges are permanently fixed in space, or, if they move, they do so at a steady rate so that ρ_v and J are constant in time.*** Under these circumstances, the time derivatives of **B** and **D** in Eqs. (4.1b) and (4.1d) vanish, and Maxwell’s equations reduce to

Electrostatics

$$\nabla \cdot \mathbf{D} = \rho_v, \quad (4.2a)$$

$$\nabla \times \mathbf{E} = 0. \quad (4.2b)$$

Magnetostatics

$$\nabla \cdot \mathbf{B} = 0, \quad (4.3a)$$

$$\nabla \times \mathbf{H} = \mathbf{J}. \quad (4.3b)$$

D=Electric flux density	[Cb/m^2]
B=Magnetic flux density	[Wb/m^2]
E=Electric field intensity	[V/m]
H=Magnetic field intensity	[A/m]
ρ=Charge density (ρ_m for magnetic)	[Cb/m^3 and Wb/m^3]
J=Electric current density	[A/m^2]
M=Magnetic current density	[V/m^2]

Maxwell’s Equations <u>are always true</u>.