B38EM - Introduction to Electricity and Magnetism

Mock Exam Questions

Q1:

1. Consider the electric field present inside a sphere which carries a volume charge density, $\rho = kr^n$, where k is a constant, r is the distance from the origin, and n is a positive or negative integer.

Calculate the exponent *n* such that the field has a constant magnitude inside the sphere.

Answer: n = -1 and $E(r) = k/2\varepsilon_0$

2. An infinite sheet of charge with surface charge density $\sigma = 10^{-12} \text{ C/m}^2$ passes through the centre of a sphere. If the flux through the surface of the sphere is 3.55 10^{-5} V.m, calculate the radius *R* of the sphere.

Answer: R=1cm

3. Three identical negative charges of -50 nC each are located at A(0,0), B(1,0), C(0,1) in free space (the coordinates are in meters). Find the total electric field at the point M(1/2,1/2).

Answer:
$$\mathbf{E} = \frac{\left(-50 \times 10^{-9}\right)}{4\pi\varepsilon_0\sqrt{2}} (\mathbf{i} + \mathbf{j}) = -317.9(\mathbf{i} + \mathbf{j}) V.m^{-1}$$

4. Using the Gauss's law in its integral form, derive an expression for the electric field strength that exists between two charged perfectly conducting and concentric spheres if the inner sphere has radius a while the outer sphere has radius b (b>a). Assume a charge +Q on the inner sphere and a charge -Q on the outer sphere. The space between the spheres is filled with a liquid of relative permittivity ε_r.

Answer: $E(r) = \frac{Q}{4\pi\varepsilon_0\varepsilon_r r^2}$

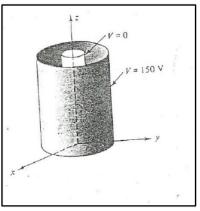
Q2:

1- Find the work done in moving a point charge $Q=-20\mu C$ from the origin to the position M=(4,2,0) expressed in meters in the presence of the electrostatic field intensity:

$$E=(x/2+2y) i +2x j (V/m)$$

Answer: W=400µJ

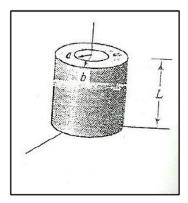
2- Find the potential function and the electric field intensity for the region between two concentric right circular cylinders where V=0 at r=1 mm, and V=150 V at r=20mm as shown in the figure below. The fringing field effects will be neglected.



Answer: $V = 50.1 \ln 2 + 345.9 \text{ Volts};$

 $E = 50.1/r (-\hat{a}_r) (V/m)$

3- Find the capacitance of a coaxial capacitor of length L, where the inner conductor has radius *a* and the outer has radius *b*.



Answer: $C=rac{2\piarepsilon_0arepsilon_r L}{ln(rac{b}{a})}$

Q3:

1) State Ampère's and Faraday's Law (in vacuum) in differential and integral form. Explain all symbols and their meaning. Explain how the integral form can be applied to obtain the magnetic and the electric field from a current distribution and a changing magnetic field respectively.

Answer: Theory, see notes

- 2) Provided is a steady current of I₀=1A which flows down an infinitely long straight cylindrical wire of radius r₀ = 8mm.
 - (i) Explain in which direction the magnetic field is pointing outside the wire.
 - (ii) Provided the current is distributed in the wire such that it is proportional to the distance s from the axis, i.e. $J = k \cdot s$
 - a. Provide an expression for k
 - b. Derive an expression for the magnetic field **B** in dependence on current and distance s from the centre of the wire, both inside and outside the wire. Sketch the magnitude of the magnetic field depending on s in a graph.
 - c. Calculate the numerical value for the magnetic field at a distance $s = r_0 = 8$ mm from the axis of the wire.
 - (iii) Explain what the magnetic field B inside the wire is, if the current is only flowing along the outside surface.

Answers:

i) azimuthial direction; radial component is zero

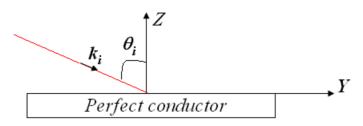
ii) **a.** $k = 3I_0/2\pi r_0^3$ **b.** $B = \mu_0 I_0/2\pi s \, \hat{\phi}$ **c.** $B = 2.5 \cdot 10^{-5} \, \text{T}$ iii) zero

3) Explain what remanence is. Illustrate your answer with a sketch of a corresponding magnetisation curve.

Answers: Theory, see notes.

- 1) a) A plane wave is polarized with its electric vector along z. The wave propagates along the y-axis. The electric field is given by $E_z(y,t) = E_0 e^{j(ky-\omega t)}$ Volts/meter. This wave is propagating in vacuum; its amplitude is $E_0 = 7$ V/m and its wavelength is $\lambda = 0.3$ meters.
 - i) What is the frequency of the wave?
 - ii) How large is the magnetic field associated with this wave and in what direction is it oriented?
 - b) Consider an electromagnetic wave whose electric field component can be described by

$$\vec{E} = 10(2j\hat{x} + \hat{y} + \sqrt{3}\hat{z})e^{-j20\pi(\sqrt{3}y - z)}$$



- i) State in which direction this wave is propagating.
- ii) Derive the corresponding magnetic field **B. Hint:** You can use Maxwell's equations.

Answers: 1.i) f = 1 GHz; 1.ii) $B_0 = 23.3$ nT; $H_0 = 18.56 \hat{x}$ mA/m 2.i) Direction: $k=20\pi(\sqrt{3} \hat{y} - \hat{z})$; 2.ii) $\vec{\mathbf{B}} = 10/c_0 \cdot (2\hat{x} - j \hat{y} - j\sqrt{3}\hat{z})e^{-j20\pi(\sqrt{3}y-z)}$

2) A 1.05-GHz generator circuit with series impedance $Z_g = 10 \Omega$ and voltage source given by $v_g(t) = 10 \sin(\omega t + 30^\circ)$ (V) is connected to a load $Z_L = (100 + j50) \Omega$ through a 50- Ω , 67-cm long lossless transmission line. The phase velocity of the line is 0.7c, where c is the velocity of light in a vacuum. Find the wavelength, reflection coefficient at the load, voltage standing wave ratio, input impedance, of the line and draw the schematic diagram of the circuit.

Answers

$$\lambda \text{=-}0.2\text{m}; \;\; \Gamma \text{=-}0.45 \text{e}^{\text{j}26.6^{\circ}}; \; VSWR = 2.63; \;\; Z_{in} \text{=-} 21.9 + \text{j}17.4 \; \Omega;$$

APPENDIX: TABLE OF EQUATIONS

Table 1: Vector relations

	Cartesian	Cylindrical	Spherical	
	Coordinates	Coordinates	Coordinates	
Coordinate variables	x,y,z	r, ϕ, z	R, θ, ϕ	
Vector representation A =	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\mathbf{\phi}}A_\phi + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{R}}A_R + \hat{\mathbf{\theta}}A_\theta + \hat{\mathbf{\phi}}A_\phi$	
$\label{eq:magnitude} \textbf{Magnitude of A} \qquad \mathbf{A} =$	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$	
Position vector $\overrightarrow{OP_1} =$	$\hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1,$ for $P(x_1, y_1, z_1)$	$\hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1,$ for $P(r_1, \phi_1, z_1)$	$\hat{\mathbf{R}}R_1$, for $P(R_1, \theta_1, \phi_1)$	
Base vectors properties	$\hat{\mathbf{x}} \cdot \hat{\mathbf{x}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{z}} = 1$	$\hat{\mathbf{r}} \cdot \hat{\mathbf{r}} = \hat{\boldsymbol{\phi}} \cdot \hat{\boldsymbol{\phi}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{z}} = 1$	$\hat{\mathbf{R}} \cdot \hat{\mathbf{R}} = \hat{\mathbf{\theta}} \cdot \hat{\mathbf{\theta}} = \hat{\mathbf{\phi}} \cdot \hat{\mathbf{\phi}} = 1$	
	$\hat{\mathbf{x}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{z}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{x}} = 0$	$\hat{\mathbf{r}} \cdot \hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}} \cdot \hat{\mathbf{z}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{r}} = 0$	$\hat{\mathbf{R}} \cdot \hat{\mathbf{\theta}} = \hat{\mathbf{\theta}} \cdot \hat{\mathbf{\phi}} = \hat{\mathbf{\phi}} \cdot \hat{\mathbf{R}} = 0$	
	$\hat{\mathbf{x}} \times \hat{\mathbf{y}} = \hat{\mathbf{z}}$	$\hat{\mathbf{r}} \times \hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$	$\hat{\mathbf{R}} \times \hat{\mathbf{\theta}} = \hat{\mathbf{\phi}}$	
	$\hat{\mathbf{y}} \times \hat{\mathbf{z}} = \hat{\mathbf{x}}$	$\hat{\mathbf{\phi}} \times \hat{\mathbf{z}} = \hat{\mathbf{r}}$	$\hat{\mathbf{\theta}} \times \hat{\mathbf{\phi}} = \hat{\mathbf{R}}$	
	$\hat{\mathbf{z}} \times \hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{z}} \times \hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\boldsymbol{\phi}} \times \hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$	
Dot product $A \cdot B =$	$A_x B_x + A_y B_y + A_z B_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_R B_R + A_{\theta} B_{\theta} + A_{\phi} B_{\phi}$	
Cross product A × B =	$\left \begin{array}{ccc} \hat{\mathbf{X}} & \hat{\mathbf{y}} & \hat{\mathbf{Z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{array}\right $	$ \begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_{\phi} & A_z \\ B_r & B_{\phi} & B_z \end{vmatrix} $	$\left \begin{array}{ccc} \hat{\mathbf{R}} & \hat{\mathbf{\theta}} & \hat{\mathbf{\phi}} \\ A_R & A_{\theta} & A_{\phi} \\ B_R & B_{\theta} & B_{\phi} \end{array}\right $	
Differential length $dl =$	$\hat{\mathbf{x}} dx + \hat{\mathbf{y}} dy + \hat{\mathbf{z}} dz$	$\hat{\mathbf{r}} dr + \hat{\mathbf{\phi}} r d\phi + \hat{\mathbf{z}} dz$	$\hat{\mathbf{R}} dR + \hat{\mathbf{\theta}} R d\theta + \hat{\mathbf{\phi}} R \sin \theta d\phi$	
Differential surface areas	$d\mathbf{s}_{x} = \hat{\mathbf{x}} dy dz$	$d\mathbf{s}_r = \hat{\mathbf{r}} r d\phi dz$	$d\mathbf{s}_R = \hat{\mathbf{R}}R^2 \sin\theta \ d\theta \ d\phi$	
	$d\mathbf{s}_{y} = \hat{\mathbf{y}} dx dz$	$d\mathbf{s}_{\phi} = \hat{\mathbf{\phi}} dr dz$	$d\mathbf{s}_{\theta} = \hat{\mathbf{\theta}} R \sin \theta \ dR \ d\phi$	
	$d\mathbf{s}_z = \hat{\mathbf{z}} dx dy$	$d\mathbf{s}_z = \hat{\mathbf{z}}r dr d\phi$	$d\mathbf{s}_{\phi} = \hat{\mathbf{\phi}}R dR d\theta$	
Differential volume $dV =$	dx dy dz	r dr dø dz	$R^2 \sin \theta \ dR \ d\theta \ d\phi$	

Table 2: Coordinate transformation relations

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ z = z	$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi$ $\hat{\mathbf{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x \cos \phi + A_y \sin \phi$ $A_\phi = -A_x \sin \phi + A_y \cos \phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}}\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r \cos \phi - A_\phi \sin \phi$ $A_y = A_r \sin \phi + A_\phi \cos \phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2 + y^2 + z^2}$ $\theta = \tan^{-1} \left[\sqrt[+]{x^2 + y^2} / z \right]$ $\phi = \tan^{-1} (y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}} \sin \theta \cos \phi$ $+ \hat{\mathbf{y}} \sin \theta \sin \phi + \hat{\mathbf{z}} \cos \theta$ $\hat{\mathbf{\theta}} = \hat{\mathbf{x}} \cos \theta \cos \phi$ $+ \hat{\mathbf{y}} \cos \theta \sin \phi - \hat{\mathbf{z}} \sin \theta$ $\hat{\mathbf{\phi}} = -\hat{\mathbf{x}} \sin \phi + \hat{\mathbf{y}} \cos \phi$	$A_R = A_x \sin \theta \cos \phi$ $+ A_y \sin \theta \sin \phi + A_z \cos \theta$ $A_\theta = A_x \cos \theta \cos \phi$ $+ A_y \cos \theta \sin \phi - A_z \sin \theta$ $A_\phi = -A_x \sin \phi + A_y \cos \phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}} \sin \theta \cos \phi \\ + \hat{\mathbf{\theta}} \cos \theta \cos \phi - \hat{\mathbf{\phi}} \sin \phi \\ \hat{\mathbf{y}} = \hat{\mathbf{R}} \sin \theta \sin \phi \\ + \hat{\mathbf{\theta}} \cos \theta \sin \phi + \hat{\mathbf{\phi}} \cos \phi \\ \hat{\mathbf{z}} = \hat{\mathbf{R}} \cos \theta - \hat{\mathbf{\theta}} \sin \theta$	$A_x = A_R \sin \theta \cos \phi$ $+ A_{\theta} \cos \theta \cos \phi - A_{\phi} \sin \phi$ $A_y = A_R \sin \theta \sin \phi$ $+ A_{\theta} \cos \theta \sin \phi + A_{\phi} \cos \phi$ $A_z = A_R \cos \theta - A_{\theta} \sin \theta$
Cylindrical to spherical	$R = \sqrt[4]{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}} \sin \theta + \hat{\mathbf{z}} \cos \theta$ $\hat{\mathbf{\theta}} = \hat{\mathbf{r}} \cos \theta - \hat{\mathbf{z}} \sin \theta$ $\hat{\mathbf{\phi}} = \hat{\mathbf{\phi}}$	$A_R = A_r \sin \theta + A_z \cos \theta$ $A_\theta = A_r \cos \theta - A_z \sin \theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}} \sin \theta + \hat{\mathbf{\theta}} \cos \theta$ $\hat{\mathbf{\phi}} = \hat{\mathbf{\phi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}} \cos \theta - \hat{\mathbf{\theta}} \sin \theta$	$A_r = A_R \sin \theta + A_\theta \cos \theta$ $A_\phi = A_\phi$ $A_z = A_R \cos \theta - A_\theta \sin \theta$

Table 3: Constitutive parameters of materials

Parameter	Units	Free-space Value
Electrical permittivity $arepsilon$	F/m	$\varepsilon_0 = 8.854 \times 10^{-12} \text{ (F/m)}$ $\simeq \frac{1}{36\pi} \times 10^{-9} \text{ (F/m)}$
Magnetic permeability μ	H/m	$\mu_0 = 4\pi \times 10^{-7} \text{ (H/m)}$
Conductivity σ	S/m	0

Table 4: Attributes of electrostatics and magnetostatics

Attribute	Electrostatics	Magnetostatics
Sources	Stationary charges ρ_{v}	Steady currents J
Fields and Fluxes	E and D	H and B
Constitutive parameter(s)	$arepsilon$ and σ	μ
Governing equations	$\nabla \cdot \mathbf{D} = \rho_{\mathbf{v}}$ $\nabla \times \mathbf{E} = 0$ $\oint_{S} \mathbf{D} \cdot d\mathbf{s} = Q$ $\oint_{C} \mathbf{E} \cdot d\mathbf{l} = 0$	$\nabla \cdot \mathbf{B} = 0$ $\nabla \times \mathbf{H} = \mathbf{J}$ $\oint_{S} \mathbf{B} \cdot d\mathbf{s} = 0$ $\oint_{C} \mathbf{H} \cdot d\mathbf{l} = \mathbf{I}$
Potential	Scalar V , with $\mathbf{E} = -\nabla V$	Vector A, with $B = \nabla \times A$
Energy density	$w_{\rm e} = \frac{1}{2} \varepsilon E^2$	$w_{\rm m} = \frac{1}{2}\mu H^2$
Force on charge q	$\mathbf{F}_{\mathbf{e}} = q\mathbf{E}$	$\mathbf{F}_{\mathbf{m}} = q\mathbf{u} \times \mathbf{B}$
Circuit element(s)	C and R	L

Table 5: Time-invariant fields

Charge Distribution	Electric Field Intensity $\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}$
Point Charge	$\bar{E} = \frac{Q}{4\pi\varepsilon_0 R^2} \hat{a}_R$
Infinite Line of Charge	$ar{E} = rac{ ho_\ell}{2\piarepsilon_0 R} \hat{a}_R$
Infinite Sheet of Charge	$\vec{E} = \frac{\rho_s}{2\varepsilon_0} \hat{a}_R$
Current Distribution	Magnetic Field Intensity $ec{B}=\mu_0^{}\mu_r^{}ec{H}$
Infinite Line Current	$ar{H} = rac{I}{2\pi ho}\hat{a}_{\phi}$
Infinite Sheet of Current	$\vec{H} = \frac{1}{2}\vec{K} \times \hat{a}_n$

Table 6: Transmission-line parameters R', L', G', and C' for three types of lines.

Parameter	Coaxial	Two-Wire	Parallel-Plate	Unit
R'	$\frac{R_{\rm S}}{2\pi} \left(\frac{1}{a} + \frac{1}{b} \right)$	$\frac{2R_{\rm s}}{\pi d}$	$\frac{2R_{\rm S}}{w}$	Ω/m
L'	$\frac{\mu}{2\pi} \ln(b/a)$	$\frac{\mu}{\pi} \ln \left[(D/d) + \sqrt{(D/d)^2 - 1} \right]$	$\frac{\mu h}{w}$	H/m
G'	$\frac{2\pi\sigma}{\ln(b/a)}$	$\frac{\pi\sigma}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\frac{\sigma w}{h}$	S/m
C'	$\frac{2\pi\varepsilon}{\ln(b/a)}$	$\frac{\pi\varepsilon}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\frac{\varepsilon w}{h}$	F/m

Notes: (1) Refer to Fig. ?? for definitions of dimensions. (2) μ , ε , and σ pertain to the insulating material between the conductors. (3) $R_{\rm s} = \sqrt{\pi f \mu_{\rm c}/\sigma_{\rm c}}$. (4) $\mu_{\rm c}$ and $\sigma_{\rm c}$ pertain to the conductors. (5) If $(D/d)^2 \gg 1$, then $\ln\left[(D/d) + \sqrt{(D/d)^2 - 1}\right] \simeq \ln(2D/d)$.

Table 7: Transmission lines and waves parameters

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Voltage Reflection Coefficient	$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$	Phase constant or wave number, β	$\beta = \omega \sqrt{\mu_0 \mu_r}$
Voltage Standing Wave Ratio	$SWR = \frac{1 + \Gamma }{1 - \Gamma }$	Wavelength, λ	$\lambda = \frac{2\pi}{\beta}$
Input Impedance	$Z_{in} = Z_0 \left(\frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \right)$	Phase velocity, u_p	$u_p = \frac{\omega}{\beta}$
Electric Field	$\vec{E} = -\eta \hat{k} \times \vec{H}$	Intrinsic impedance,	$\eta = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}}$
Magnetic Field	$\vec{H} = \frac{1}{\eta}\hat{k} \times \vec{E}$	Propagation constant, γ	$\gamma = \alpha + j\beta$

Table 9: Characteristic parameters of transmission lines.

	Propagation Constant $\gamma = \alpha + j\beta$	Phase Velocity u _p	Characteristic Impedance Z_0
General case	$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$	$u_{\rm p} = \omega/\beta$	$Z_0 = \sqrt{\frac{(R' + j\omega L')}{(G' + j\omega C')}}$
Lossless $(R' = G' = 0)$	$\alpha = 0, \; \beta = \omega \sqrt{\varepsilon_{\rm f}}/c$	$u_{\rm p} = c/\sqrt{\varepsilon_{\rm r}}$	$Z_0 = \sqrt{L'/C'}$
Lossless coaxial	$\alpha = 0, \; \beta = \omega \sqrt{\varepsilon_{\scriptscriptstyle m I}}/c$	$u_{\rm p} = c/\sqrt{\varepsilon_{\rm r}}$	$Z_0 = (60/\sqrt{\epsilon_{\rm f}}) \ln(b/a)$
Lossless two-wire	$\alpha = 0, \ \beta = \omega \sqrt{\varepsilon_{\rm r}}/c$	$u_{\rm p} = c/\sqrt{\varepsilon_{\rm r}}$	$Z_0 = (120/\sqrt{\varepsilon_{\overline{\mathbf{r}}}})$ $\cdot \ln[(D/d) + \sqrt{(D/d)^2 - 1}]$
			$Z_0 \simeq (120/\sqrt{\epsilon_{\rm r}}) \ln(2D/d),$ if $D \gg d$
Lossless parallel-plate	$\alpha = 0, \; \beta = \omega \sqrt{\varepsilon_{\rm r}}/c$	$u_{\rm p} = c/\sqrt{\varepsilon_{\rm r}}$	$Z_0 = (120\pi/\sqrt{\varepsilon_{\rm f}})(h/w)$

Notes: (1) $\mu = \mu_0$, $\varepsilon = \varepsilon_r \varepsilon_0$, $c = 1/\sqrt{\mu_0 \varepsilon_0}$, and $\sqrt{\mu_0 / \varepsilon_0} \simeq (120\pi) \Omega$, where ε_r is the relative permittivity of insulating material. (2) For coaxial line, a and b are radii of inner and outer conductors. (3) For two-wire line, d = wire diameter and D = separation between wire centers. (4) For parallel-plate line, w = width of plate and h = separation between the plates.