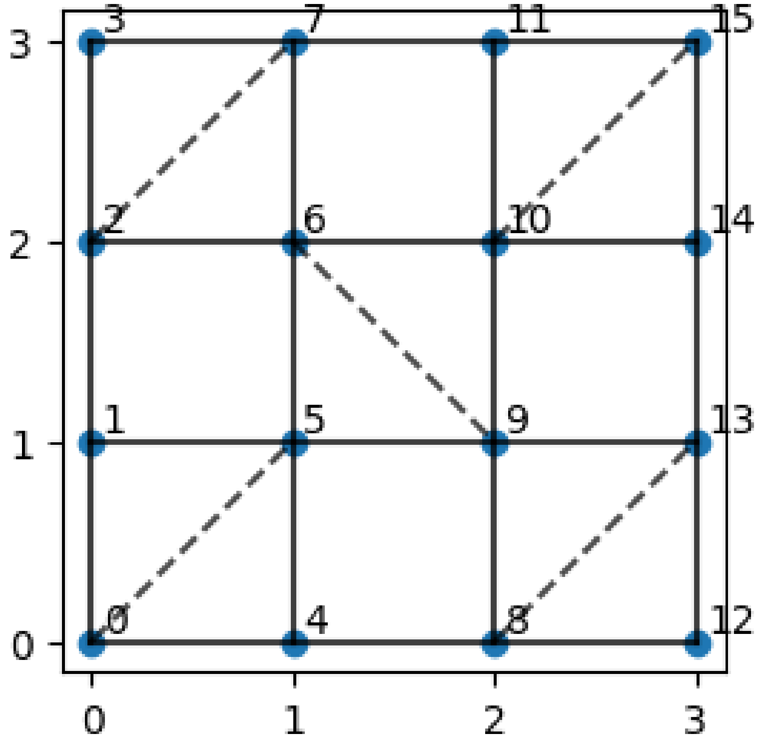
Shastry-Sutherland model

Introduzione ai sistemi quantistici a molti corpi

The Hamiltonian

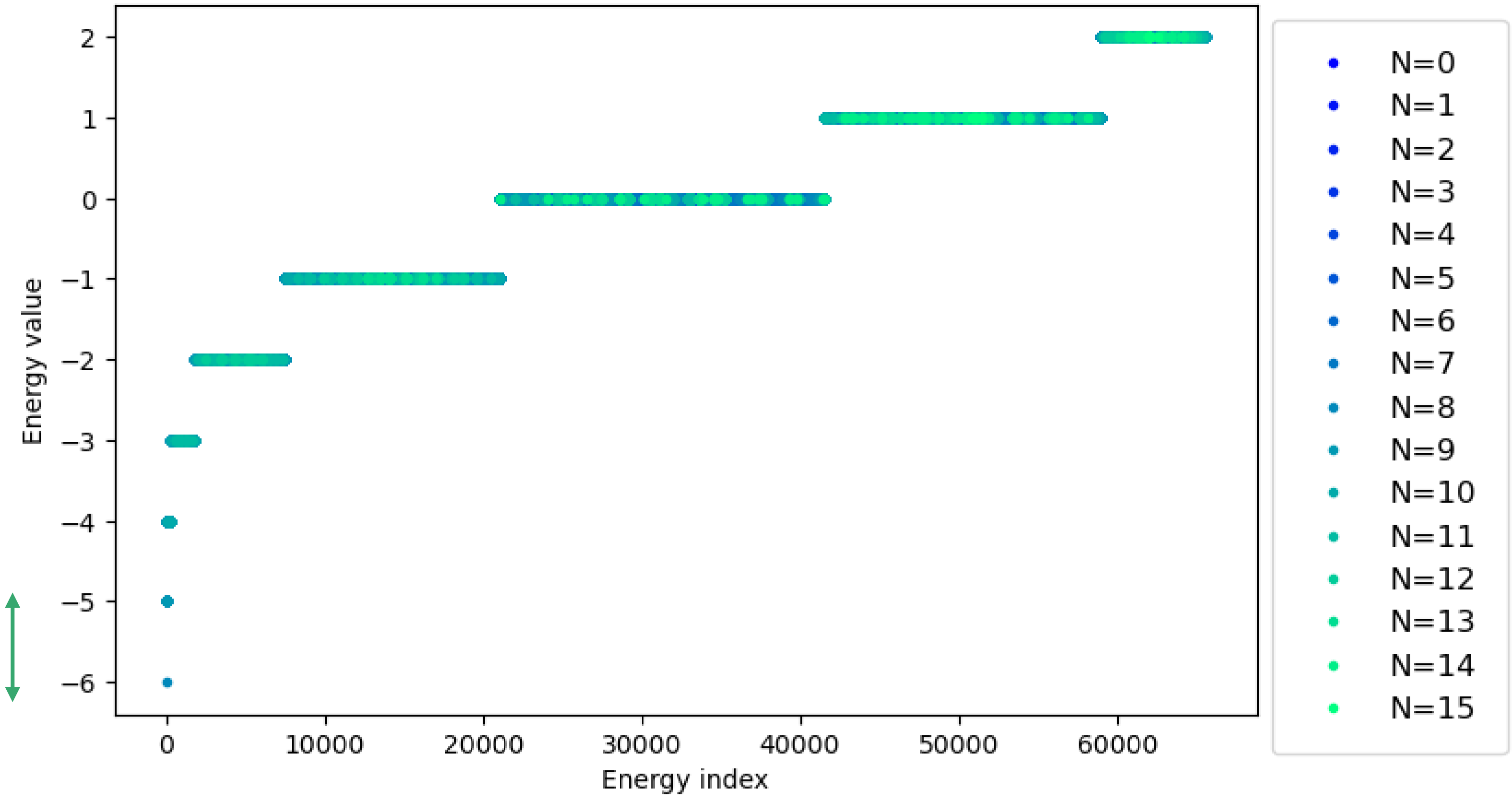
$$\mathcal{H} = J_1 \sum_{\langle i,j \rangle} S_i S_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} S_i S_j$$



Limit J1=0: Dimer phase

$$\mathcal{H} = J_2 \sum_{\langle\langle i,j \rangle\rangle} S_i S_j$$

System is gapped



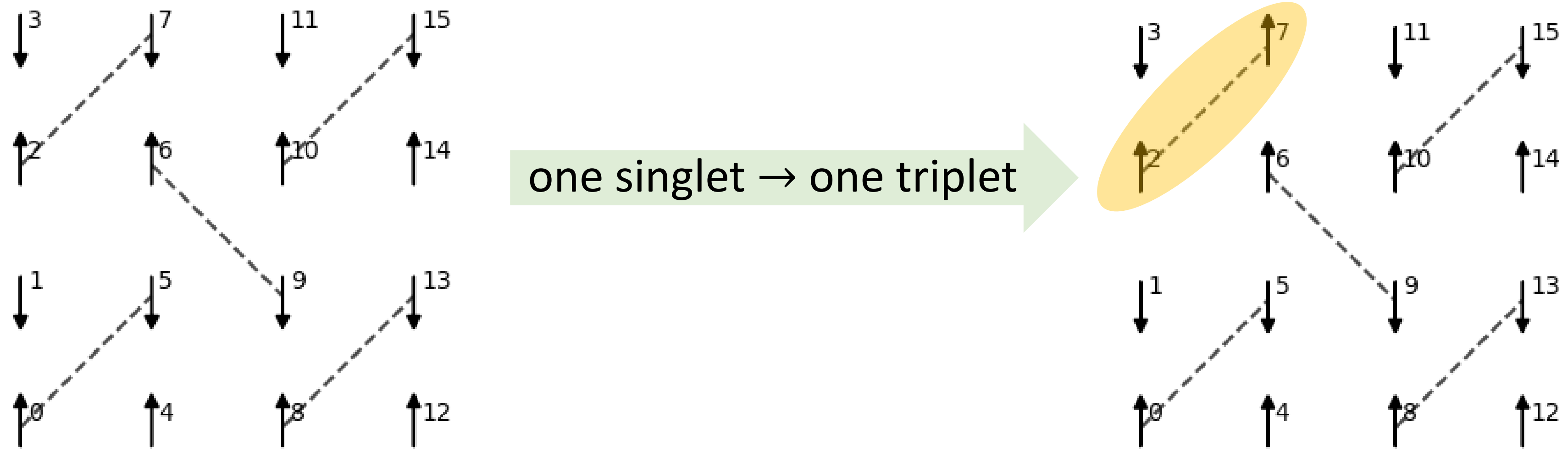
Dimer phase

Ground state: $|GS> = \prod_{\ll i,j \gg} \frac{1}{\sqrt{2}}[|\uparrow_i \downarrow_j> - |\downarrow_i \uparrow_j>]$

Energy:

- $E_{GS} = -\frac{3}{4} J_2 N_{dimer}$
- $E_1 = -\frac{3}{4} J_2 (N_{dimer} - 1) + \frac{1}{4} J_2$

$\Delta = E_1 - E_{GS} = J_2$



Dimer phase: motion of triplets

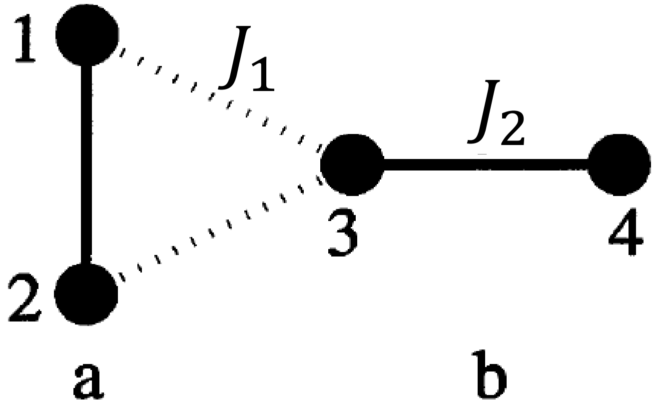
Let consider also a term J_1 that connects dimers between themselves.

It can be shown that:

$$\mathcal{H}_{ab}^{1}|t_m\rangle_a|s\rangle_b = \frac{J_1}{2}|t_m\rangle_a|t_0\rangle_b - \frac{J_1}{2}|t_0\rangle_a|t_m\rangle_b$$

$$\mathcal{H}_{ab}^{1}|t_0\rangle_a|s\rangle_b = \frac{J_1}{2}|t_1\rangle_a|t_{-1}\rangle_b - \frac{J_1}{2}|t_{-1}\rangle_a|t_1\rangle_b$$

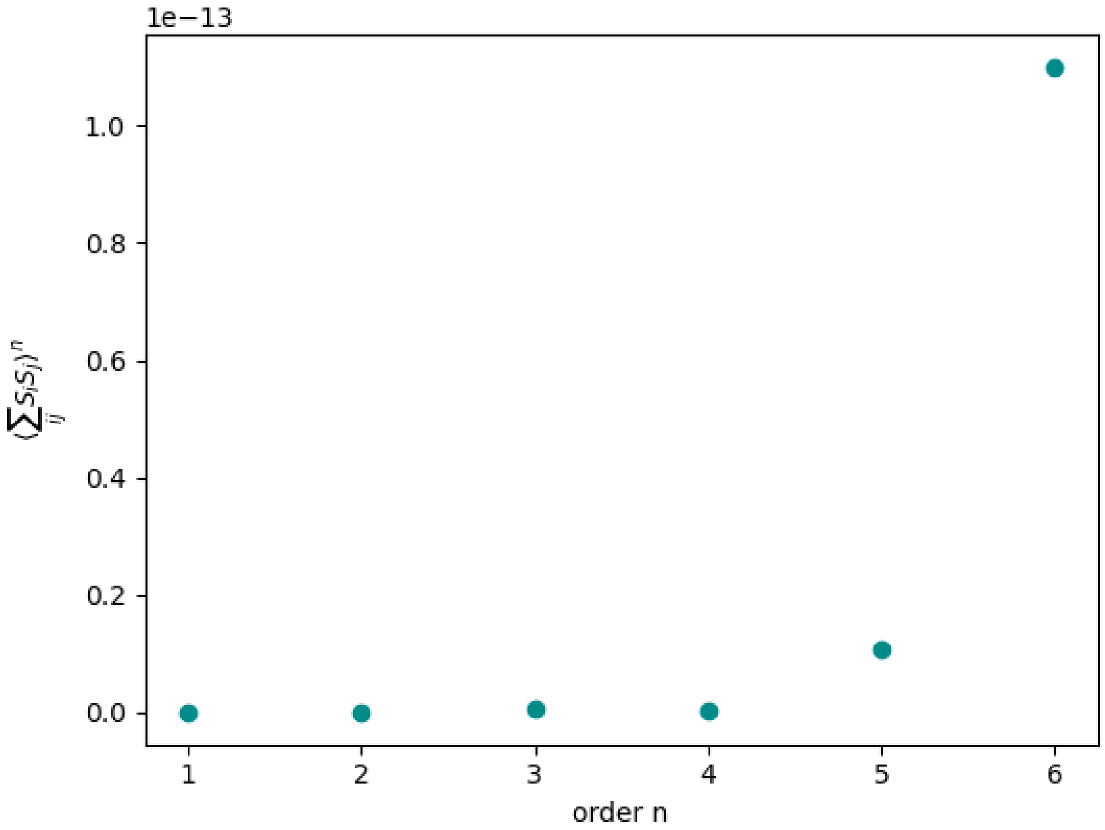
$$\mathcal{H}_{ab}^{1}|s\rangle_a|t_m\rangle_b = 0$$

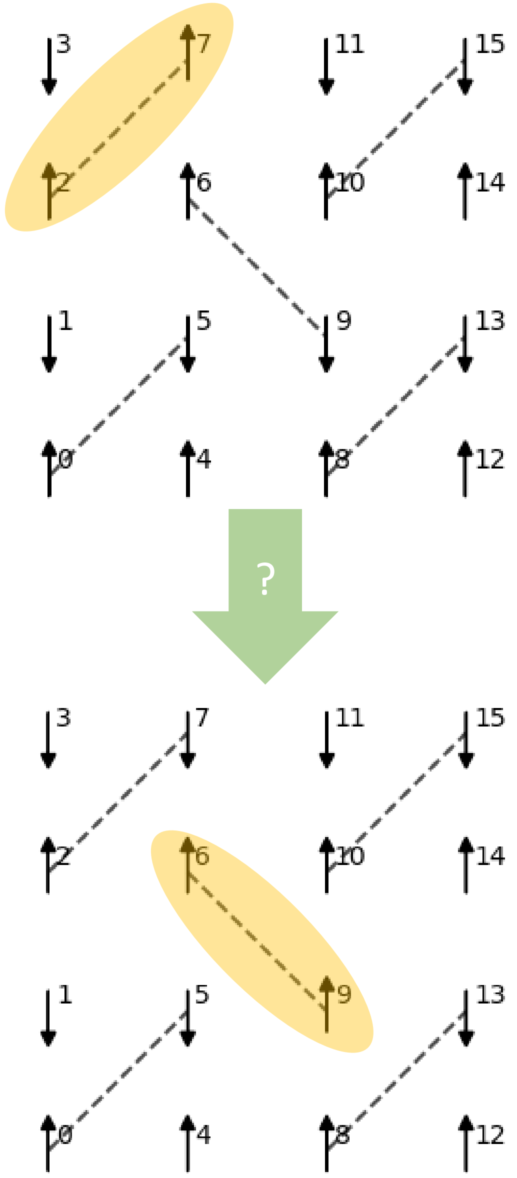


Dimer phase: motion of triplets

First order: $\langle\psi_1{}'|\mathcal{H}_{J_1}|\psi_1\rangle = 0$, where $|\psi_1\rangle$ is a state with one triplet.

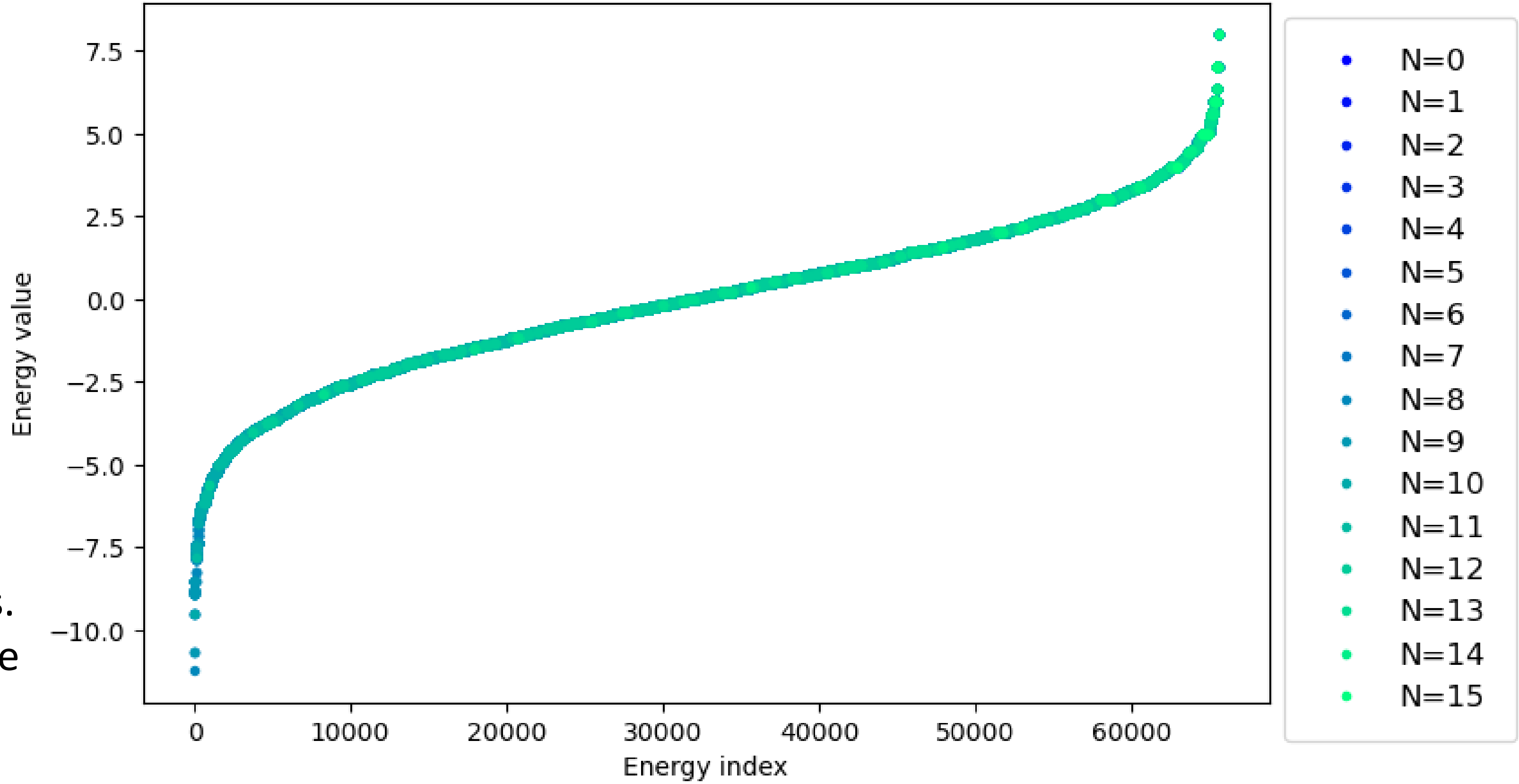
At which order of $\mathcal{H}_{J_1}$ is possible to move one triplet from one bond to a near bond?





Limit J2=0: Heisenberg 2D

$$\mathcal{H} = J_1 \sum_{<i,j>} S_i S_j$$



Gap due to finite-size effects. In the termodinamic limit the system is gapless

Heisenberg 2D

$$\mathcal{H} = J_1 \sum_{<i,j>} S_i S_j = \cdots = -2J_1 S^2 L + 4J_1 S \sum_k \omega_k \alpha_k^+ \alpha_k + 2SJ_1 \sum_k (\omega_k - 1)$$

$$\frac{E_{GS}}{N} = -2J_1 S^2 + 2SJ_1 \underbrace{\frac{1}{N} \sum_k (\omega_k - 1)}_{\simeq -0.158} \simeq -0.658 J_1$$

From the simulation:

$$\frac{E_{GS}}{N} \simeq -0.702 J_1$$

Heisenberg 2D: introducing a perturbation $J_2 \ll J_1$

$$\mathcal{H} = J_1 \sum_{<i,j>} S_i S_j + J_2 \sum_{\ll i,j \gg} S_i S_j = \mathcal{H}_0 + V$$

$$E_{GS} = E_{GS}^{(0)} + E_{GS}^{(1)} \rightarrow E_{GS}^{(1)} = < \psi_0 | V | \psi_0 > = J_2 \sum_{\ll i,j \gg} < \psi_0 | S_i \, S_j | \psi_0 >$$

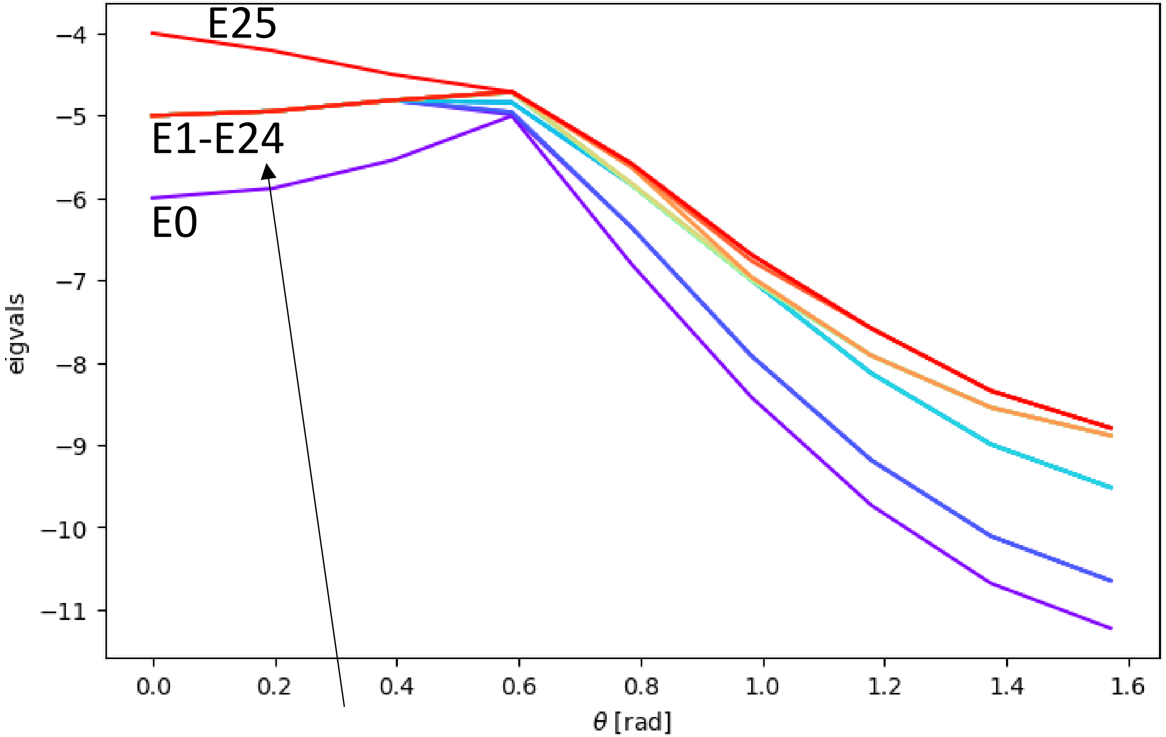
From Miyahara-Ueda: $\frac{E_{AF}}{N} = -0.669 J_1 + 0.102 J_2 \rightarrow \text{Transition at :} \left(\frac{J_1}{J_2}\right)_c = 0.71$

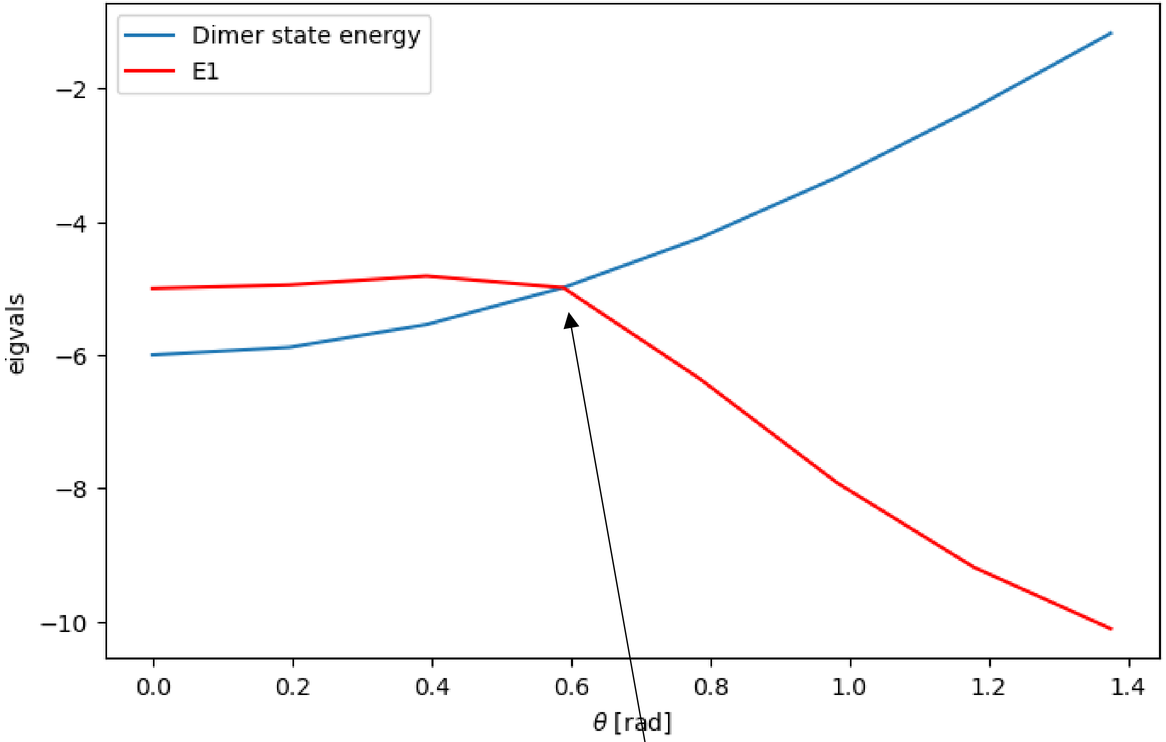
From simulation: $\frac{E_{AF}}{N} = -0.702 J_1 + 0.107 J_2 \rightarrow \text{Transition at:} \left(\frac{J_1}{J_2}\right)_c = 0.687$

Changing J_1 and J_2

We can define the coupling constants through trigonometric functions:

$$J_1 = \sin\theta, J_2 = \cos\theta$$





deg=8 for Sz=+1 | ↑↑⟩
deg=8 for Sz=-1 | ↑↓⟩ + |↓↑⟩
deg=8 for Sz=0 | ↓↓⟩

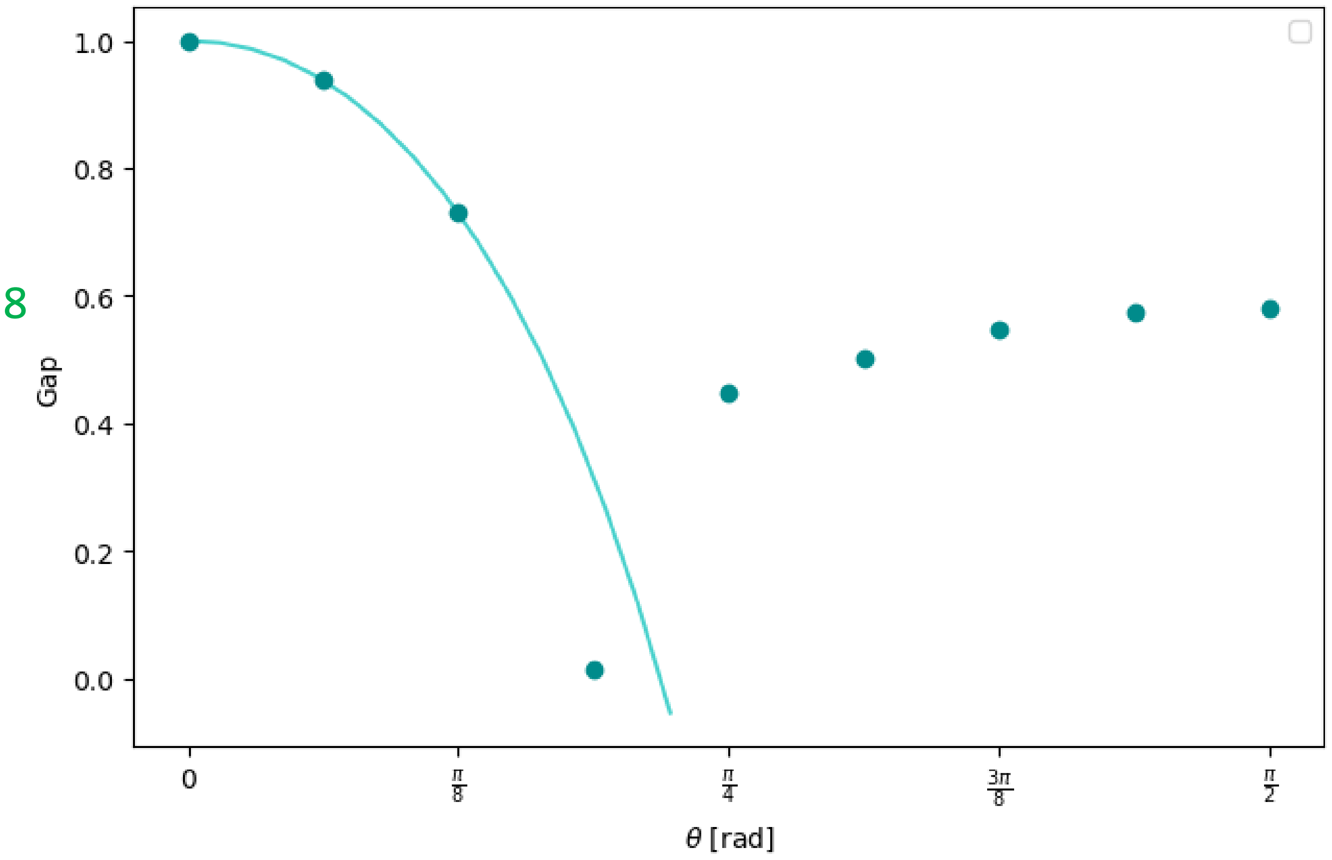
J1/J2=0.66818

Changing J_1 and J_2: gap

We can define the coupling costants through trigonometric funtions: $J_1/J_2 = \tan\theta$

$\Delta = \cos(\theta)\,[1 - \tan^2(\theta) - 1/2\tan^3(\theta) - 1/8\tan^4(\theta)]$

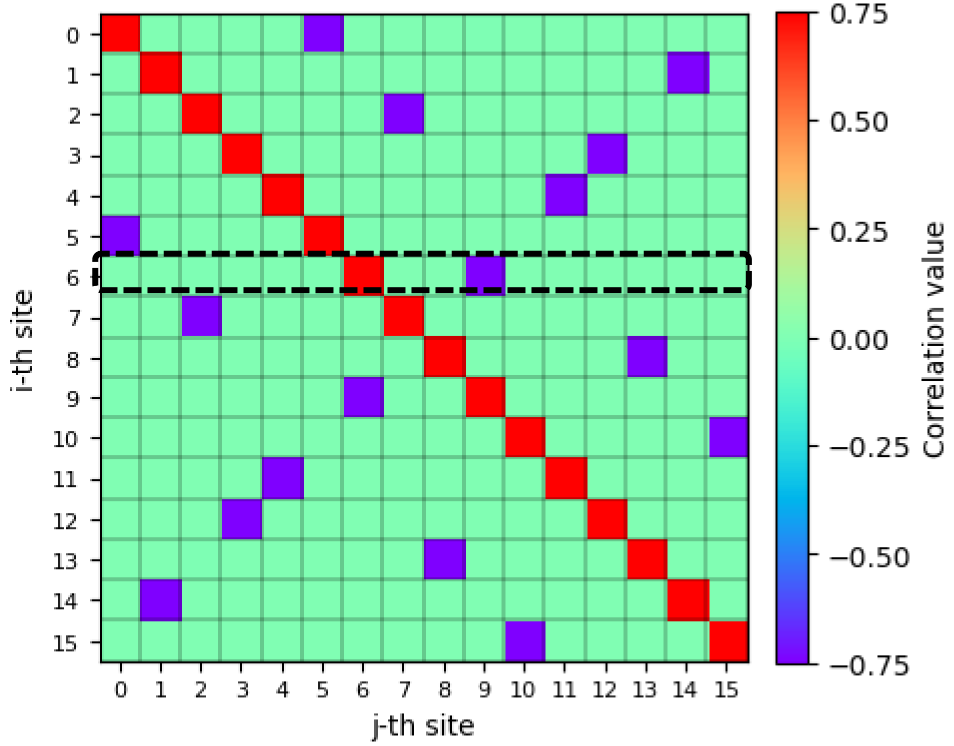
Transition between the gapless and the gaped phase: J1/J2=0.66818

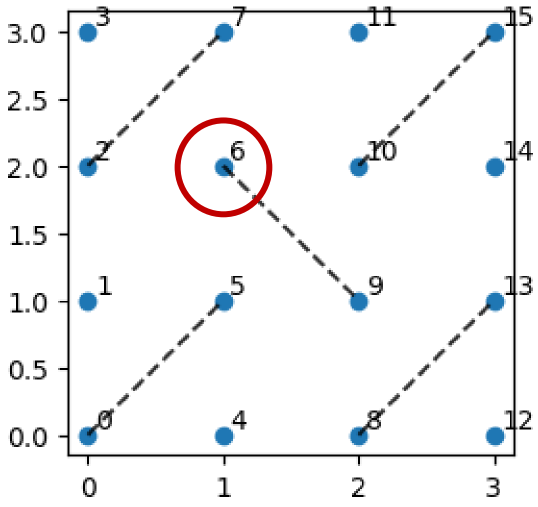


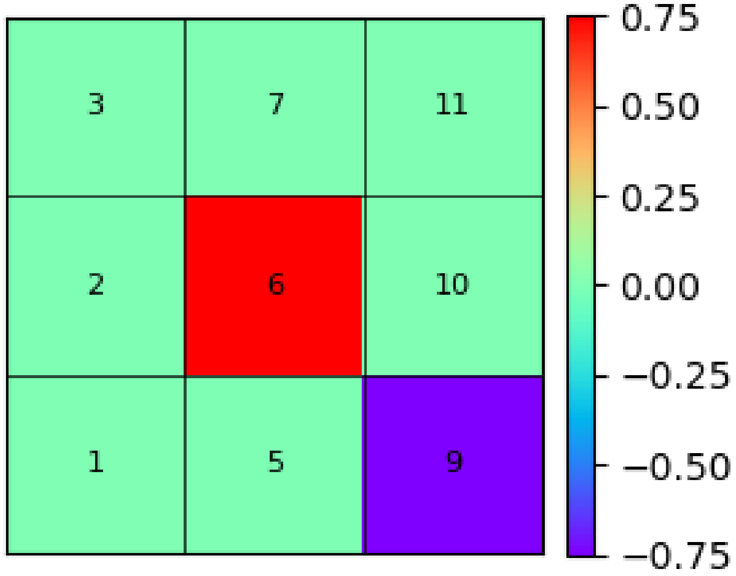
In the termodinamic limit, gap →0

Correlations: dimer phase

$$\langle GS|S_iS_j|GS\rangle$$

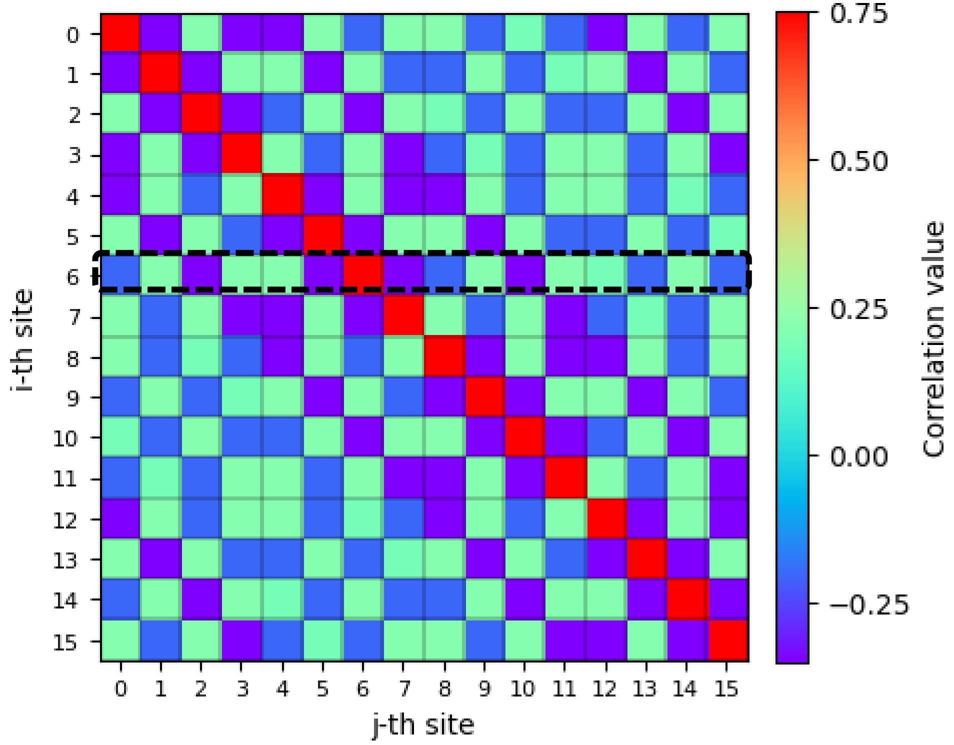


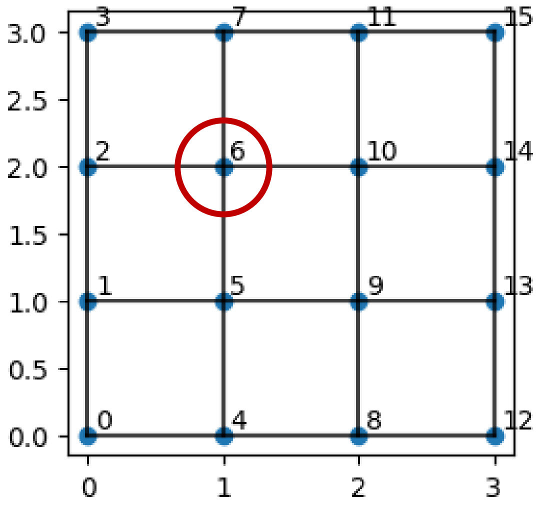


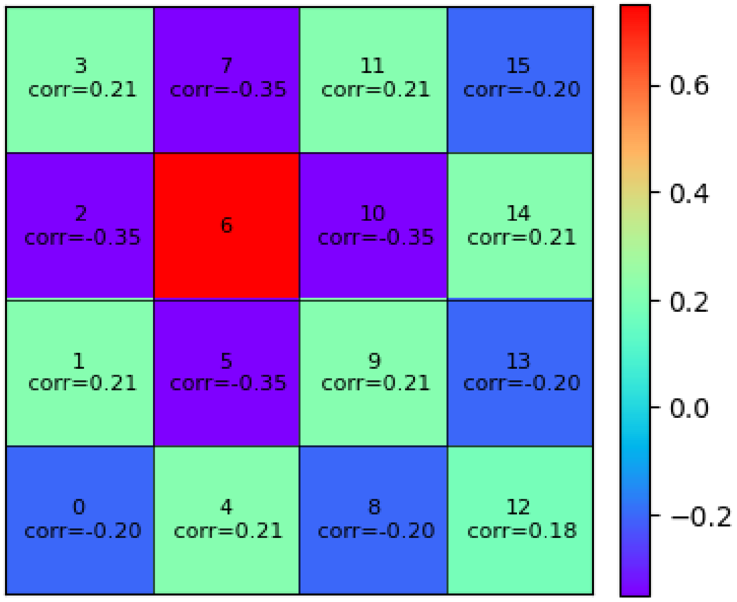


Correlations with nearest neighbors are 0, the only correlation survived is the one between the dimer sites

Correlations: Heisenberg 2D







NN correlation = -0.35 → antiferromagnetic order is still present