Hints and solutions - 4 Autumn 2018

MATHEMATICS-I(MA10001) 25/07/2018

1. Put $x = r\cos\theta$ and $y = r\sin\theta$.
2. Hint:
   (a) Take path $x^2 = my$.
   (b) Take path $y = x - mx^3$.
3. Value of $f_x(x, y)$ and $f_y(x, y)$:
   (a) $f_x(x, y) = 2x$ and $f_y(x, y) = 2y$
   (b) $f_x(x, y) = 3\cos(3x + 4y)$ and $f_y(x, y) = 4\cos(3x + 4y)$
   (c) $f_x(x, y) = -ye^{-x} + y$ and $f_y(x, y) = e^{-x} + x$
   (d) $f_x(x, y) = 2x + y$ and $f_y(x, y) = x + 3y^2$
   (e) $f_x(x, y) = \sin y + 2x$ and $f_y(x, y) = x\cos y$
   (f) $f_x(x, y) = ye^{xy} + \frac{1}{y}$ and $f_y(x, y) = xe^{xy} - \frac{x}{y^2}$
4. Value of $f_x(0, 0)$, $f_y(0, 0)$, $f_x(0, y)$ and $f_y(x, 0)$:
   (a) $f_x(0, 0) = 0$, $f_y(0, 0) = 0$, $f_x(0, y) = 1$ and $f_y(x, 0) = 1$
   (b) $f_x(0, 0) = 0$, $f_y(0, 0) = 0$, $f_x(0, y) = y$ and $f_y(x, 0) = x$
   (c) $f_x(0, 0) = 0$, $f_y(0, 0) = 0$, $f_x(0, y)$ does not exist and $f_y(x, 0)$ does not exist.
   (d) $f_x(0, 0) = 2$, $f_y(0, 0) = -2$, $f_x(0, y) = e^y + e^{-y}$ and $f_y(x, 0) = -e^x - e^{-x}$

(e) $f_x(0,0) = 0$, $f_y(0,0) = 0$, $f_x(0,y) = y$ and $f_y(x,0) = -x$

5. Ans:
   (a) Differentiable
   (b) Not differentiable
6. Take $x = r\cos\theta, y = r\sin\theta$
7. Use definition.
8. Differentiability of the function at $(0,0)$:
   (a) Differentiable
   (b) Not differentiable
9. $f_{xx}(0,0) = 0$, $f_{xy}(0,0)$ does not exist, $f_{yx}(0,0) = 0$ and $f_{yy}(0,0) = 0$. Not differentiable.
10. Differentiable
11. Value of $f_{yxx}(x,y)$ and $f_{xyx}(x,y)$:
    (a) $f_{yxx}(x,y) = 36x^2\cos 3y$ and $f_{xyx}(x,y) = 36x^2\cos 3y$
    (b) $f_{yxx}(x,y) = 60x^3y^2$ and $f_{xyx}(x,y) = 60x^3y^2$
    (c) $f_{yxx}(x,y) = 2xe^{xy}\sec^2 x\tan x + 2e^{xy}\sec^2 x + 2xye^{xy}\sec^2 x + 2ye^{xy}\tan x + xy^2e^{xy}\tan x + 12xy$
    and $f_{xyx}(x,y) = 2xe^{xy}sec^2xtanx + 2e^{xy}sec^2x + 2xye^{xy}sec^2x + 2ye^{xy}tanx + xy^2e^{xy}\tan x + 12xy$
    (d) $f_{yxx}(x,y) = 6x\cos y - 3y^2\cos x$ and $f_{xyx}(x,y) = 6x\cos y - 3y^2\cos x$
    (e) $f_{yxx}(x,y) = e^x\dfrac{1}{y} + \dfrac{1}{x^2}\sin y$ and $f_{xyx}(x,y) = e^x\dfrac{1}{y} + \dfrac{1}{x^2}\sin y$

(f) $f_{yxx}(x,y) = 12xy - 4y^3\cos(xy^2) + 2xy^5\sin(xy^2)$ and $f_{xyx}(x,y) = 12xy - 4y^3\cos(xy^2) + 2xy^5\sin(xy^2)$

12. Total differential

(a) $dw = (2x + y^2 + y^2z^3)dx + (2xy + 2xyz^3)dy + 3xy^2z^2dz$

(b) $dz = \dfrac{y}{x^2+y^2}dx - \dfrac{x}{x^2+y^2}dy$

(c) $du = 2e^{(x^2+y^2+z^2)}(xdx + ydy + zdz)$

(d) $dw = 3\cos(3x+4y)dx + 4\cos(3x+4y)dy + 5e^z dz$

(e) $dw = yzdx + (\dfrac{z}{y} + \ln z + xz)dy + (\ln y + \dfrac{y}{z} + xy)dz$

(f) $du = \dfrac{1}{\sqrt{x^2+y^2+z^2}}(xdx + ydy + zdz)$,

(g) $dw = [e^x\sin(y+2z) - 2xy^2]dx + [e^x\cos(y+2z) - 2x^2y]dy + 2e^x\cos(y+2z)dz$

(h) $dw = \dfrac{1}{y}e^{\frac{x}{y}}dx - \dfrac{1}{y^2}(xe^{\frac{x}{y}} + ze^{\frac{z}{y}})dy + \dfrac{1}{y}e^{\frac{z}{y}}dz.$