Problem Set - 9

AUTUMN 2018

MATHEMATICS-I (MA10001)

1. Solve the following differential equations:
   (a) $(x^2D^2 - 3xD + 4)y = 2x^2$,
   (b) $(x^2D^2 + 7xD + 13)y = \log x$
   (c) $(x^2D^2 - 4xD + 6)y = x^4$
   (d) $(x^2D^2 + xD - 1)y = x^m$, $\quad [m \neq \pm 1]$
   (e) $(x^2D^2 - 3xD + 5)y = \sin(\log x)$
   (f) $(x^2D^2 - xD + 1)y = 2\log x$
   (g) $(x^2D^2 - (2m-1)xD + (m^2+n^2))y = n^2x^m \log x$
   (h) $(x^2D^2 - xD + 2)y = x\log x$
   (i) $(x^2D^2 - 3xD + 5)y = x^2\sin(\log x)$
   (j) $(x^4D^4 + 6x^3D^3 + 9x^2D^2 + 3xD + 1)y = (1 + \log x)^2$

2. Solve the following differential equations:

   (a) $(1+x)^2y'' - 4(1+x)y' + 6y = 6(1+x)$
   (b) $(x+1)^2y'' + (x+1)y' = (2x+3)(2x+4)$,
   (c) $(1+2x)^2y'' - 6(1+2x)y' + 16y = 8(1+2x)^2$

3. Apply the method of variation of parameters to solve the following differential equations:

   (a) $y'' - 2y' = e^x \sin x$
   (b) $y'' - 6y' + 9y = \dfrac{e^{3x}}{x^2}$
   (c) $y'' - 2y' + y = e^x \log x$
   (d) $y''' + y' = \tan x$
   (e) $y''' - 2y'' - 21y' - 18y = 3 + 4e^{-t}$
   (f) $y'' - 2y' + 2y = e^x \tan x$

4. Using the method of variation of parameters, solve

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x \sin x$$

   with $y(0) = 0$ and $(\frac{dy}{dx})_{x=0} = 0$

5. Solve the following system of differential equations :

(a) $\dfrac{dx}{dt} - y = t, \qquad \dfrac{dy}{dt} + x = 1$

(b) $\dfrac{dx}{dt} + 2y + x = e^t, \qquad \dfrac{dy}{dt} + 2x + y = 3e^t$

(c) $\dfrac{dx}{dt} + 2x - 3y = t, \qquad \dfrac{dy}{dt} - 3x + 2y = e^{2t}$

(d) $\dfrac{dx}{dt} + \dfrac{dy}{dt} + 2x + y = e^t, \qquad \dfrac{dy}{dt} + 5x + 3y = t$