Answer and hints of Problem Set - 6

Autumn 2018

MATHEMATICS-I(MA10001) 18/07/2018

1. **Ans:** $f(x,y) = 1 + 2x + 2x^2 + xy + y^2$.

2. **Ans:** $f(x,y) = 1 - \frac{\pi^2}{8}(x-1)^2 - \frac{\pi}{2}(x-1)(y-\frac{\pi}{2}) - \frac{1}{2}(y-\frac{\pi}{2})^2$+Remainder term.

3. **Hint:** Use Taylor's series expansion and then substitute $x = \frac{51}{100}\pi$ and $y = 0.99$.
   Ans: $f(x,y) = e[1+(y-1)-\frac{1}{2}(x-\frac{\pi}{2})^2+\frac{1}{2}(y-1)^2]$ and $f(\frac{51}{100}\pi, 0.99) = 2.68989$.

4. **Hint:** Use Taylor's series expansion for three variables.
   Ans: $f(x,y,z) = x + y + xz + yz$.

5. **Ans:** $f(x,y) = (\pi+e)+(x-1)(2\pi+e)+\frac{(x-1)^2}{2}(2\pi+e)+2(x-1)(y-\pi)+\frac{(x-1)^3}{6}e^{\xi}+(x-1)^2(y-\pi)-(y-\pi)^3\cos\eta$, where $\xi = 1+(x-1)\theta;\ \eta = \pi + (y-\pi)\theta;\ 0 < \theta < 1$

6. **Hint:** Expand by Taylor's series up to second order and then write the remainder term.

7. **Hint**: First of all find the stationary points and then apply second order derivative test for two variables.

   (a) **Ans:** (0,0),(-3,0),(0,3/2) are saddle points and (-1,1/2) is local minimum.

   (b) **Ans:** (0,0) is neither maximum nor minimum,($\sqrt{2}$,-$\sqrt{2}$) and (-$\sqrt{2}$,$\sqrt{2}$) are local maximum

   (c) **Ans:** (2,1), (-2,-3) are local maximum and (-2,1), (2,-3) are saddle points.

   (d) **Ans:** (4,0) local maxima, (6,0) local minima and (5,1),(5,-1) are saddle points.

   (e) **Ans:** (-4,-8), (-4,4),(8,4) are saddle points and (0,0) is a point of maxima.

8. **Hint**: Use second order derivative test for two variables.

9. **Hint:** First check the extremum value at stationary point and then on the boundary.
   Ans: Absolute minimum value is -4 which occurs at (1,2/3) and absolute maximum value is 49 which occurs at (2,3) and (0,3).

10. **Hint:** To check on the boundary use polar co-ordinates.
    Ans: $f(x,y)$ attains absolute maximum value 3/2 at (1/$\sqrt{2}$, 1/$\sqrt{2}$), (-1/$\sqrt{2}$, -1/$\sqrt{2}$) and absolute minimum value 0 at (0,0).

11. **Ans:** $f(x,y)$ has absolute maximum value $\frac{3+\sqrt{2}}{2}$ at (-1/$\sqrt{2}$,0) and absolute minimum value $\frac{-1}{12}$ at ($\frac{1}{6}$,0).

12. **Hint:** (1) First take two points on ellipse and line then find the distance between them and then (2) use Lagrange's multiplier method to extremize it.
    Ans: Shortest distance is $\sqrt{5}$.

13. **Hint:** Use Lagrange's method of multipliers.

14. **Hint:** Use Lagrange's method of multipliers.
    Ans: $f(x,y)$ has absolute maximum value $(1+5\sqrt{2})$ at $(\frac{\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}, \frac{2-\sqrt{2}}{2})$ and absolute minimum value $(1-5\sqrt{2})$ at $(-\frac{\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2}, \frac{2+\sqrt{2}}{2})$.

15. **Hint:** Use Lagrange's method of multipliers.
    Ans: $(a+b+c)^3$

16. **Ans:** Equilateral triangle.

17. **Hint:** Use Lagrange's method of multipliers.
    Ans: Smallest distance is $\frac{a}{\sqrt{3}}\sqrt{(7-4\sqrt{3})}$ and largest distance is $\frac{a}{\sqrt{3}}\sqrt{(7+4\sqrt{3})}$.

18. **Hint:** Find the surface area of the box and then minimize it taking volume as a constraint.
    Ans: Dimensions are 4cm, 4cm, 2cm.

19. **Hint:** Use Lagrange's method of multipliers.
    Ans: Largest and smallest distances are $3\sqrt{6}, \sqrt{6}$ respectively.

20. **Hint:** Use Lagrange's method of multipliers.

21. **Hint:** Use Lagrange's method of multipliers.

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