Problem Set - 12

MATHEMATICS-I(MA10001) Autumn 2018

1. Expand the following functions in Taylor's series and determine the region of convergence

   (a) $log\left(\dfrac{1+z}{1-z}\right)$ about $z = 0$

   (b) $\sin z$ about $\frac{\pi}{4}$

   (c) $\dfrac{1}{z^2+4}$ about $z = -i$

   (d) $\dfrac{2z^3+1}{z^2+z}$ about 1.

2. Can the Series $\sum_{n=1}^{\infty} a_n z^n$converges at $z = 0$ and diverges $z = 3$?

3. Find all possible Laurent series expansion of the function $f(z) = \dfrac{1}{(z+1)(z+2)^2}$ in the region

   (a) $|z-1| < 2$,

   (b) $2 < |z-1| < 3$,

   (c) $|z-1| > 3$

4. (a) $\sum_{n=-\infty}^{\infty} a_n z^n$ Laurent series expansion of $f(z) = \dfrac{1}{2z^2-13z+15}$ in the annulus $\frac{3}{2} < |z| < 5$ then $\frac{a_1}{a_2}$ =?

   (b) The coefficient of $(z-\pi)^2$ in Laurent series expansion of $f(z) = \dfrac{sinz}{z-\pi}$ around π.

5. Write down the principal part of the Laurent Series:

   (a) $\dfrac{e^z}{z+sinz}$

(b) $\dfrac{e^z}{z - sinz}$

6. Find the singularity and classify them:

   (a) $\dfrac{1}{e^z - 1}$

   (b) $\tan \dfrac{1}{z}$

   (c) $z^2 + 1$

   (d) e^z

   (e) $\dfrac{1}{z(z^2+4)}$

7. Find the residue at all singular point:

   (a) $\dfrac{1}{z^3 + z^5}$

   (b) $\dfrac{z^2}{(z^2+1)^2}$

   (c) $zsin(\dfrac{1}{z})$

   (d) $f(z) = \dfrac{z^2}{\left(z^2+1\right)^2}$

8. Using Cauchy Residue Formula find the value of

   (a) $\dfrac{1}{2\pi i}\displaystyle\int_{|z|=2} z^7 cos(\dfrac{1}{z^2})dz = ?$

   (b) $\dfrac{i}{4-\pi}\displaystyle\int_{|z|=4} \dfrac{dz}{zcosz} = ?$

   (c) $\Omega = \{z \in \mathbb{C} | Imz > 0\}$ C be the curve lying in ω with initial and final point $-1+2i$ and $1+2i$ then $\displaystyle\int \dfrac{1+2z}{1+z}dz = ?$

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9. Find the value of $\dfrac{(1-|a|^2)}{\pi}\displaystyle\int_{|z|=1}\dfrac{|dz|}{|z+a|^2}$ where $a \in \mathbb{C}, |a| < 1$

10. Evaluate

    (a) $I = \displaystyle\int_C \dfrac{f(z)}{(z-1)(z-2)}$ where, $f(z) = sin\dfrac{\pi z}{2} + cos\dfrac{\pi z}{2}, C : |z| = 3$

    (b) $C = \{z \in \mathbb{C} | |z-i| = 2\}$,then $\dfrac{1}{2\pi}\displaystyle\int_C \dfrac{z^2-4}{z^2+4} = ?$

    (c) Γ be the given circle, $z = 4e^{i\theta}$, $\theta : 0$ to 2π then $\displaystyle\int_\Gamma \dfrac{e^z}{z^2-2z}dz = ?$

    (d) $I = \displaystyle\int_0^{2\pi}\dfrac{d\theta}{1-2acos\theta+a^2}$, and $(i)|a| < 1, (ii)|a| > 1$

11. Evaluate the integral $I = \displaystyle\int_0^{2\pi} e^{\cos\theta}\cos(\sin\theta)d\theta$

12. use Cauchy integral formula find the value of:

    (a) $\displaystyle\int_0^\infty \dfrac{\cos ax}{x^2+1}dx$

    (b) $\displaystyle\int_0^\infty cosx^2dx$ [Assume the value of he Gaussian integral $\displaystyle\int_{-\infty}^\infty e^{-x^2}dx = \sqrt{\pi}$]