Problem Set - 7

AUTUMN 2018

MATHEMATICS - I(MA10001)

August , 2018

1. **Find the order and degree of the following differential equation:**

   (i) $\sqrt{y+\left(\frac{d^2y}{dx^2}\right)^2}=\left(\frac{dy}{dx}\right)^5$

   (ii) $\frac{d^4y}{dx^4}+2k^2\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^4=0$

   (iii) $x^3(y^{'''})^5-\sqrt{xy}=2$

   (iv) $\left(\frac{d^3y}{dx^4}\right)=\frac{7}{3}\left(5+\left(\frac{dy}{dx}\right)^3\right)^{\frac{5}{3}}$

   (v) $[1+\left(\frac{dy}{dx}\right)^3]=c^2\left(\frac{d^2y}{d^2x}\right)^2$

2. **Form the ODE by eliminating the arbitrary constants:**

   (i) $xy=Ae^x+Be^{-x}$

   (ii) $(x-h)^2+(y-k)^2=a^2$, where (h,k) is the centre and a is the radius of the circle.

   (iii) $y=A\cos x+B\cosh x$

   (iv) $y=Ae^x+Be^{2x}$

   (v) Obtain the differential equation of the system of confocal conics $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$, in which λ is the arbitrary parameter and a,b are given constant

   (vi) Obtain the differential equation of all circles each of which touches the axis of confocal conics.

3. **Solve the following Initial Value Problems:**

   (i) $\frac{dy}{dx}+y\tan x=\sin 2x,$ $, y(0)=1$

   (ii) $(x+2y-3)dx=(2x+y-3)dy,$ $, y(1)=2$

4. **Check if the differential equations are homogeneous (reduced it to homogeneous if not), then solve it:**

   (i) $x\sin\frac{y}{x}dy=\left(y\sin\frac{y}{x}-x\right)dx$

   (ii) $y^{'}=\frac{x+y+1}{3x+3y+1}$

   (iii) $(x+2y-3)dy=(2x-y+1)dx$

5. **Check if the differential equations are exact (if not, reduced it to exact using proper Integrating Factor), then solve it:**

   (i) $(2x\cos y+3x^2y)dx+(x^3-x^2\sin y-y)dy=0$

(ii) $(2xy + y - \tan y)dx + (x^2 - x\tan^2 y + \sec^2 y)dy = 0$

(iii) $ydx - xdy + \log x dx = 0$

(iv) $(x^2y - 2xy^2)dx = (x^3 - 3x^2y)dy$

(v) $y(xy + 2x^2y^2)dx + x(xy - x^2y^2) = 0$

(vi) $(x^2 + y^2)dx - 2xydy = 0$

6. **Solve the following ODEs by reducing them to linear differential equations:**

(i) $x\frac{dy}{dx} - (x+1)y = x^2 - x^3$

(ii) $1 + y^2 + (x - e^{-\tan^{-1} y})\frac{dy}{dx}$

(iii) $\frac{dy}{dx} + y\cos x = y^n \sin 2x$

(iv) $\frac{dy}{dx} + x\sin 2y = x^3\cos^2 y$

(v) $\dfrac{dy}{dx} + \dfrac{y}{x} = \dfrac{y^2}{x}(\log x)$

(vi) $\frac{dy}{dx} + x\sin 2y = x^2\cos^2 y$

(vii) $xy' + y = x^3y^6$

(viii) $\frac{dy}{dx} + y = xy^3$