Hint and Answer set-12

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1. (a) $2\sum_{n=0}^{\infty}\frac{z^{2n+1}}{2n+1}$ converges for $|z|\leq 1$ except $z=1$ and $z=-1$

   (b) everywhere convergent

   (c) $f(z)=\frac{1}{3}+\frac{2i}{9}(z+i)-\frac{7}{27}(z+i)^2+...$

   (d) $2z-1+\frac{1}{2}\left[1-\frac{z-1}{2}+\left(\frac{z-1}{2}\right)^2+...\right]+1-(z-1)+(z-1)^2+...$ and region of converges $|z-1|<1$

2. at $z=0$ the series convergent and at $z=3$ the series absolutely convergent if $lim|\frac{a_{n+1}}{a_n}|<\frac{1}{3}$ and divergent if $lim|\frac{a_{n+1}}{a_n}|>\frac{1}{3}$

3. (a) $\sum_{n=0}^{\infty}(-1)^n\left(2^{-n-1}-(n+4)3^{-n-2}\right)(z-1)^n$ for $|z-1|<2$

   (b) $\sum_{n=0}^{\infty}(-1)^n\frac{2^n}{(z-1)^{n+1}}-\frac{1}{9}\sum_{n=0}^{\infty}(-1)^n(n+4)\left(\frac{z-1}{3}\right)^3$ the sum valid for $2<|z-1|<3$

   (c) $\sum_{n=0}^{\infty}(-1)^n\left[\frac{2^n-3^n}{(z-1)^{n+1}}-\frac{3^n(n+1)}{(z-1)^{n+2}}\right]$

4. (a) 5

   (b) $\frac{1}{6}$

5. (a) $\frac{1}{2z}$

   (b) $\frac{6}{z^3}+\frac{6}{z^2}+\frac{33}{10z}$

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6. (a) $z = 2n\pi i$ is a pole

   (b) $z = 0$ is essential singularity and $z = \dfrac{2}{\pi\,(2n+1)}$ is a pole.

   (c) has a pole of order 2 at $z = \infty$

   (d) essential singularity at $z = 0$

   (e) pole at $z = 0, 2i, -2i$

7. (a) $\text{Res}[f(z), -i] = \dfrac{1}{2}$, $\text{Res}[f(z), i] = \dfrac{1}{2}$ and $\text{Res}[f(z), 0] = -1$

   (b) $\text{Res}[f(z), -i] = \dfrac{i}{4}$ and $\text{Res}[f(z), i] = -\dfrac{i}{4}$

   (c) $\text{Res}[f(z), 0] = 0$

   (d) $\text{Res}[f(z), -i] = \dfrac{i}{4}$ and $\text{Res}[f(z), -i] = -\dfrac{i}{4}$

8. (a) $\dfrac{1}{4!}$

   (b) 2

   (c) $4 - \dfrac{1}{2}log2 + \dfrac{\pi i}{4}$

9. 2

10. (a) $-4\pi i$

    (b) -2

    (c) $\pi i\left(e^2 - 1\right)$

    (d) $(i)\dfrac{2\pi}{1-a^2}(ii)\dfrac{2\pi}{a^2-1}$

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11. 2π

12. (a) $\dfrac{\pi}{2e^a}$

    (b) $\dfrac{1}{2}\sqrt{\dfrac{\pi}{2}}$