Problem Set - 11

MATHEMATICS-I(MA10001) Autumn 2018

1. (a) Evaluate $\int_{1-i}^{2+i} (2x + iy + 1)dz$, along the paths
   - (i) $x = t + 1, y = 2t^2 - 1$
   - (ii) the straight line joining $1 - i$ and $2 + i$

   (b) Evaluate $\int_{0}^{1+i} (x - y + ix^2)dz$ along
   - (i) the straight line from $z = 0$ and $z = 1 + i$
   - (ii) real axis from $z = 0$ and $z = 1$ and then a line parallel to imaginary axis from $z = 1$ to $z = 1 + i$

   (c) Find the value of the integral $\int_C (z + 1)^2 dz$ where C is the boundary of the rectangle with vertices at the points $1+i$, $-1+i$,$-1-i$ and $1 - i$

   (d) Compute $\int_\Gamma |z|dz$ where Γ is the left half of the unit circle $|z| = 1$ from $z = -i$ to $z = i$

   (e) Find the value of $\int_C (z^2 - iz)dz$ along the curve C:$y = x^3 - 3x^2 + 4x - 1$ joining points $(1, 1)$ and $(2, 3)$

2. (a) Verify that the value of the integral $\int_C z^2 dz$ is same in all case:
   - (i) C is the straight line joining the point A(0,0) and B(1,2)
   - (ii) C is the straight line path from A(0,0) to P(1,0) followed by the straight line path from P(1,0) to B(1,2)
   - (iii) C be the parabolic path $y = 2x^2$ joining the point A(0,0) and B(1,2)

   (b) Integrate xz along the straight line from $A(1, 1)$ to $B(2, 4)$in the complex plane.Is the value same if the path of integration from A to B is along the curve $x = t$, $y = t^2$?

   (c) Evaluate the function f defined by the integral $f(z) = \oint_{|w|=1} \frac{e^{w^2}-1}{w-z} dw$

Problem Set - 11

MATHEMATICS-I(MA10001) Autumn 2018

3. $F(a) = \oint_C \frac{(4z^2+z+5)}{(z-a)} dz$, where C: $(\frac{x}{2})^2 + (\frac{y}{3})^2 = 1$ taken in counter clockwise sense. Find $F(3.5)$, $F(i)$, $F'(-1)$ and $F''(-i)$

4. (a) Evaluate $\oint_C \frac{3z^2+z+1}{(z^2-1)(z+3)} dz$, where C is the circle $|z| = 2$

   (b) Compute $\oint_C \frac{e^{z^2}}{(z-2)} dz$ over the contour C, C: $|z-(2+i)| = 3$

   (c) Compute $\oint_C \frac{1}{(z^2+4)^2} dz$ over the contour C : $|z-i| = \frac{3}{2}$

   (d) Evaluate $\oint_C \frac{1}{(z-a)^n} dz$, where n is any integer and C is any closed curve containing 'a'.

5. (a) Show that $|\oint_C \frac{e^z}{z^2+1} dz| \leq \frac{4}{3}\pi e^2$ where C: $|z| = 2$

   (b) Estimate an upper bound for $|\oint_{|z|=3} \frac{Log(z)}{z-4i} dz|$

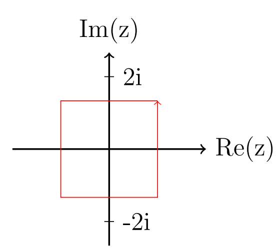
6. (a) Find the value $\oint_{|z|=1} \frac{4z^2-4z+1}{(z-2)(z^2+4)} dz$

   (b) Compute $\oint_{|z+1-i|=2} \frac{z+4}{z^2+2z+5} dz$

   (c) Compute $\oint_{|z|=6} (\frac{e^{2iz}}{z^4} - \frac{z^4}{(z-i)^3}) dz$

   (d) Evaluate the integral $\oint_{|z|=1} \frac{dz}{2-\bar{z}}$

7. Evaluate $\oint_C \frac{cosz}{z(z^2+8)} dz$ over the contour shown



Problem Set - 11

MATHEMATICS-I(MA10001) Autumn 2018

8. Evaluate $\oint_C \frac{z^2+2}{z^2-2}$ where C is the circle

   (a) $|z| = \frac{3}{2}$

   (b) $|z-1| = 1$

   (c) $|z| = \frac{1}{2}$

9. verify Cauchy's theorem for the function $z^3 - iz^2 - 5z + 2i$ if C is

   (a) the circle $|z| = 1$

   (b) the circle $|z-1| = 2$

   (c) the ellipse $|z-3i| + |z+3i| = 20$

10. If $0 < r < R$, evaluate the integral $I = \oint_C \frac{R+z}{z(R-z)} dz$,where C:$|z| = r$. Further using this result deduce that

    (a) $\int_0^{2\pi} \frac{d\theta}{R^2 - 2rR\cos\theta + r^2} = \frac{2\pi}{R^2 - r^2}$

    (b) $\int_0^{2\pi} \frac{\sin\theta d\theta}{R^2 - 2rR\cos\theta + r^2} = 0$

11. By integrating $f(z) = \frac{1}{(R-z)}$ over C:$|z| = r$,$0 < r < R$ and using the result of exercise 10, show that $\int_0^{2\pi} \frac{R\cos\theta}{R^2 - 2rR\cos\theta + R^2} d\theta = \frac{2\pi r}{R^2 - r^2}$