Tutorial Sheet-2 **AUTUMN 2018**

MATHEMATICS-I (MA10001) *August 1, 2018*

1. Determine the following limits using L'Hospital rule, if exist:

   a) $\lim_{x\to 0} x\log x$ b) $\lim_{x\to 0} x^x$

   c) $\lim_{x\to 1} x^{\frac{1}{x-1}}$ d) $\lim_{x\to\infty} x^{1/x}$

   e) $\lim_{x\to 0} \frac{\tan^{-1} x}{\sin^{-1} x}$ f) $\lim_{x\to 0}(\frac{1}{e^x-1}-\frac{1}{x})$

   g) $\lim_{x\to\pi} |\sin x|^{\tan x}$ h) $\lim_{x\to 0} |\sin x|^x$

   i) $\lim_{x\to e} \frac{\log(\log x)}{\sin(x-e)}$ j) $\lim_{x\to-\infty} e^{x^2}\sin(e^x)$

   k) $\lim_{x\to 0} \frac{\sin x - x + \frac{x^3}{6}}{x^5}$

2. By Taylor series expansion, using suitable function

   a) find the value of $\sqrt{1.5}$ approximately.

   b) show that $\sin 46^\circ \approx \frac{1}{\sqrt{2}}\left(1+\frac{\pi}{180}\right)$.

3. If $f(x) = e^x$ then using Taylor's theorem, find the smallest interval in which value of $e^{0.1}$ belong. (Take $n = 2$.)

4. Use Taylor's theorem to prove that

   a) $\cos x \geq 1 - \frac{x^2}{2}$ for $-\pi < x < \pi$.

   b) $x - \frac{x^3}{6} < \sin x < x$ for $0 < x < \pi$.

   c) $1 + x + \frac{x^2}{2} + \frac{x^3}{3!} < e^x < 1 + x + \frac{x^2}{2} + \frac{x^3}{3!}e^x$ for all $x > 0$.

5. Prove: If f is continuous at x_0 and there are constatnts a_0 and a_1 such that

$$\lim_{x\to x_0} \frac{f(x) - a_0 - a_1(x - x_0)}{x - x_0} = 0$$

   then $a_0 = f(x_0)$, f' is differentialble at x_0, and $f'(x_0) = a_1$.

6. Using Taylor's series formula, evaluate

a) $\lim_{x \to 0} \frac{\sin x}{\sqrt{1-\cos x}}$ b) $\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{\sin x} \right)$

c) $\lim_{x \to 0} \frac{x - \log(1+x)}{1 - \cos x}$ d) $\lim_{x \to 0} \frac{xe^x - \log(1+x)}{x^2}$.

7. Find the Maclaurin's infinite series expansion for

   a) $f(x) = \cos x$ for all $x \in \mathbb{R}$.

   b) $f(x) = \log(1+x)$ for $(-1, 1]$.

   c) $f(x) = e^x \cos x$ for all $x \in \mathbb{R}$

8. Can the function f(x) defined by $f(x) = e^{1/x}$ for $x \neq 0$ and f(0)=0 be expanded in ascending powers of x by Maclaurin's Theorem?

9. Write the Maclaurin's formula for the function $f(x) = \sqrt[3]{1+x}$ of degree 2. Further estimate the error of the approximate equation $\sqrt[3]{1+x} \approx 1 + \frac{1}{3}x - \frac{1}{9}x^2$ when $x = 0.3$.

10. Using Maclaurin's Theorem expand $\frac{e^x}{1+e^x}$.