RPCAKit: MATLAB RPCA Library

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1 RPCA

Robust Principal Component Analysis (RPCA) [?] refers to the following problem

$$\begin{aligned} &\min_{\mathbf{A},\mathbf{N}} \|\mathbf{A}\| + \lambda \|\mathbf{N}\|_q \\ &\text{s.t.} \quad \mathbf{X} = \mathbf{A} + \mathbf{N} \end{aligned} \tag{1}$$

where $\mathbf{X}$ is observed data, $\mathbf{A}$ is the original low-rank data $\mathbf{N}$ is some noise and q is a placeholder for the noise type. For example if our data is corrupted by sparse noise we set $q = 1$. In other words the goal is to recover $\mathbf{A}$ only knowing $\mathbf{X}$ and the type of noise the data has been affected by. It has been shown that the solution to this objective can exactly recover $\mathbf{A}$ under reasonable conditions [?]. RPCAKit provides MATLAB functions to solve this problem for Gaussian, sparse and sample specific noise.

2 Function Listing

Objective	Function
$\min_{\mathbf{A},\mathbf{N}} \|\mathbf{A}\| + \frac{\lambda}{2}\|\mathbf{N}\|_F^2$ s.t. $\mathbf{X} = \mathbf{A} + \mathbf{N}$	rpca_fro
$\min_{\mathbf{A},\mathbf{N}} \|\mathbf{A}\| + \lambda\|\mathbf{N}\|_1$ s.t. $\mathbf{X} = \mathbf{A} + \mathbf{N}$	rpca_l1
$\min_{\mathbf{A},\mathbf{N}} \|\mathbf{A}\| + \lambda\|\mathbf{N}\|_{1,2}$ s.t. $\mathbf{X} = \mathbf{A} + \mathbf{N}$	rpca_l1l2

3 Implementation

We use Linearised ADMM with Adaptive Penalty [?] to solve the RPCA problem. First form the Augmented Lagrangian

$$\min_{\mathbf{A},\mathbf{N}} \|\mathbf{A}\| + \lambda\|\mathbf{N}\|_q + \langle \mathbf{Y}, \mathbf{X} - \mathbf{A} - \mathbf{N} \rangle + \frac{\mu}{2}\|\mathbf{X} - \mathbf{A} - \mathbf{N}\|_F^2$$

1. Fix others and solve for $\mathbf{A}$

$$\min_{\mathbf{A}} \|\mathbf{A}\| + \langle \mathbf{Y}, \mathbf{X} - \mathbf{A} - \mathbf{N} \rangle + \frac{\mu}{2} \|\mathbf{X} - \mathbf{A} - \mathbf{N}\|_F^2$$

$$\min_{\mathbf{A}} \|\mathbf{A}\| + \frac{\mu}{2} \|\mathbf{X} - \mathbf{A} - \mathbf{N} + \frac{1}{\mu}\mathbf{Y}\|_F^2$$

   which has a closed form solution defined by the singular value shrinking operator [**?**, **?**].

2. Fix others and solve for $\mathbf{N}$

$$\min_{\mathbf{N}} \lambda \|\mathbf{N}\|_q + \langle \mathbf{Y}, \mathbf{X} - \mathbf{A} - \mathbf{N} \rangle + \frac{\mu}{2} \|\mathbf{X} - \mathbf{A} - \mathbf{N}\|_F^2$$

$$\min_{\mathbf{N}} \lambda \|\mathbf{N}\|_q + \frac{\mu}{2} \|\mathbf{X} - \mathbf{A} - \mathbf{N} + \frac{1}{\mu}\mathbf{Y}\|_F^2$$

   where the solutions have various closed form solutions, see [**?**, **?**].

3. Update $\mathbf{Y}$

$$\mathbf{Y} = \mathbf{Y} + \mu(\mathbf{X} - \mathbf{A} - \mathbf{N})$$

4. Update μ

$$\mu = \min(\mu_{\max}, \gamma\mu)$$

   where ρ is defined as

$$\gamma = \begin{cases} \gamma_0 & \text{if } \ \mu_k \frac{\max(\|\mathbf{A}_{k+1} - \mathbf{A}_k\|_F, \|\mathbf{N}_{k+1} - \mathbf{N}_k\|_F)}{\|\mathbf{X}\|_F} < \epsilon_2 \\ 1 & \text{otherwise,} \end{cases}$$

   $\mu_{\max} >> \mu_0$ and $\epsilon > 0$.

5. Check stopping criteria

$$\|\mathbf{X} - \mathbf{A}^{k+1} - \mathbf{N}^{k+1}\|_F < \epsilon_1, \ \mu_k \frac{\max(\|\mathbf{A}_{k+1} - \mathbf{A}_k\|_F, \|\mathbf{N}_{k+1} - \mathbf{N}_k\|_F)}{\|\mathbf{X}\|_F} < \epsilon_2$$