Intro to Deep Learning HW4 Yifan Li yl506

Q1. Variational Lower Bound

$$\log p(x) = D_{KL}(q(z|x)||p(z|x)) + \mathcal{L}(q;x)$$

$$D_{KL}(q(z|x)||p(z|x)) = \int q(z|x) \log \frac{q(z|x)}{p(z|x)} dz$$

a) show that $D_{KL}(q(z|x)||p(z|x)) = 0$ if and only if $q(z|x) = p(z|x)$

- $\Leftarrow$ direction: if $p(z|x) = q(z|x)$, $D_{KL} = \int q(z|x) \log \frac{q(z|x)}{p(z|x)} dz = \int q(z|x) \log(1) dz$

$$= \int q(z|x) \cdot 0 \, dz = 0$$

- $\Rightarrow$ direction: if $D_{KL}(q(z|x)||p(z|x)) = 0$, we have $D_{KL} = \int q(z|x) \log \frac{q(z|x)}{p(z|x)} dz$

$$= E_{z\sim q}\left[\log \frac{q(z|x)}{p(z|x)}\right] = 0 \Rightarrow \log \frac{q(z|x)}{p(z|x)} = 0 \Rightarrow p(z|x) = q(z|x)$$

b) show that the equality above holds for $\mathcal{L}(q;x) = \int q(z|x) \log \frac{p(x,z)}{q(z|x)} dz$

WTS: $\log p(x) = D_{KL}(q(z|x)||p(z|x)) + \mathcal{L}(q;x) = \int q(z|x) \log \frac{q(z|x)}{p(z|x)} dz + \int q(z|x) \log \frac{p(x,z)}{q(z|x)} dz$

$$\text{R.H.S.} = \int q(z|x) \left(\log \frac{q(z|x)}{p(z|x)} + \log \frac{p(x,z)}{q(z|x)}\right) dz = \int q(z|x) [\log q(z|x) - \log p(z|x)$$

$$+ \log p(x,z) - \log q(z|x)] dz = \int q(z|x) [\log p(x,z) - \log p(z|x)] dz$$

$$= \int q(z|x) [\log(p(z|x)p(x)) - \log p(z|x)] dz$$

$$= \int q(z|x) [\log p(z|x) + \log p(x) - \log p(z|x)] dz = \int q(z|x) \log p(x) dz$$

$$= \log p(x) \int q(z|x) dz = \log p(x) \cdot 1 = \log p(x) = \text{L.H.S.}$$

Q5. Bookkeeping

a) About 15 hours.

b) I adhere to the Duke Community Standard in the completion of this assignment.